Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.4/55-1.1.3.4-b

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**299**]. This is test number [55].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (299)	0.00 (0)
Mathematica	100.00 (299)	0.00 (0)
Maple	71.91 (215)	28.09 (84)
Fricas	63.21 (189)	36.79 (110)
Sympy	62.54 (187)	37.46 (112)
Mupad	38.80 (116)	61.20 (183)
Maxima	38.13 (114)	61.87 (185)
Giac	34.45 (103)	65.55 (196)
Reduce	27.42 (82)	72.58 (217)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

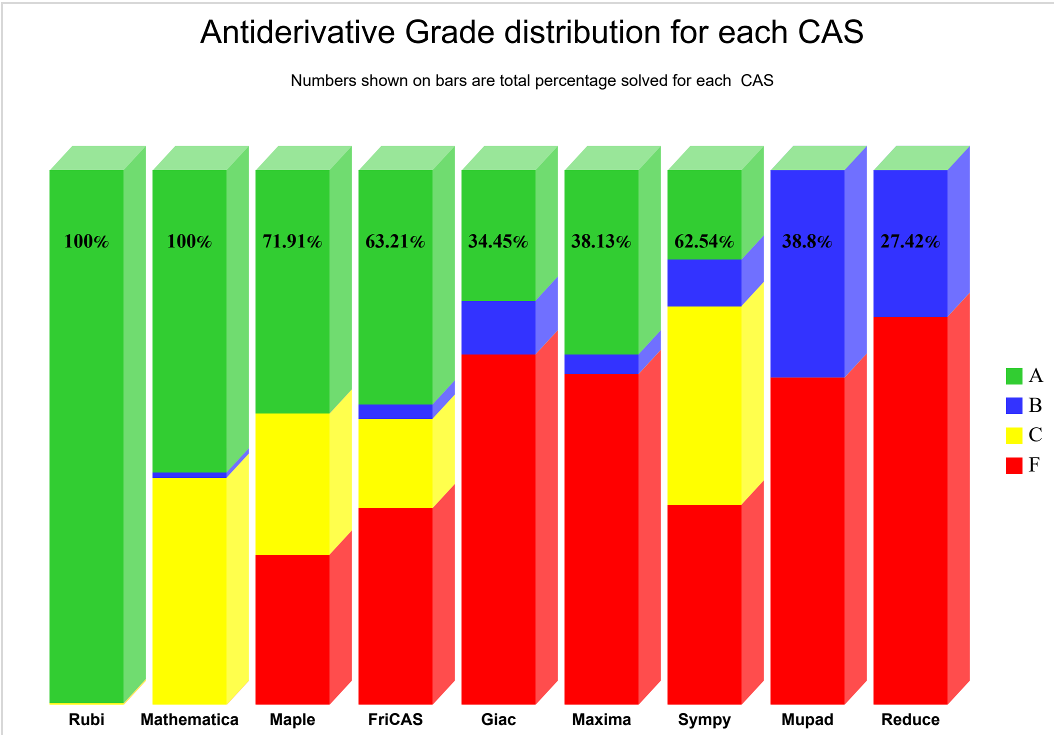
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

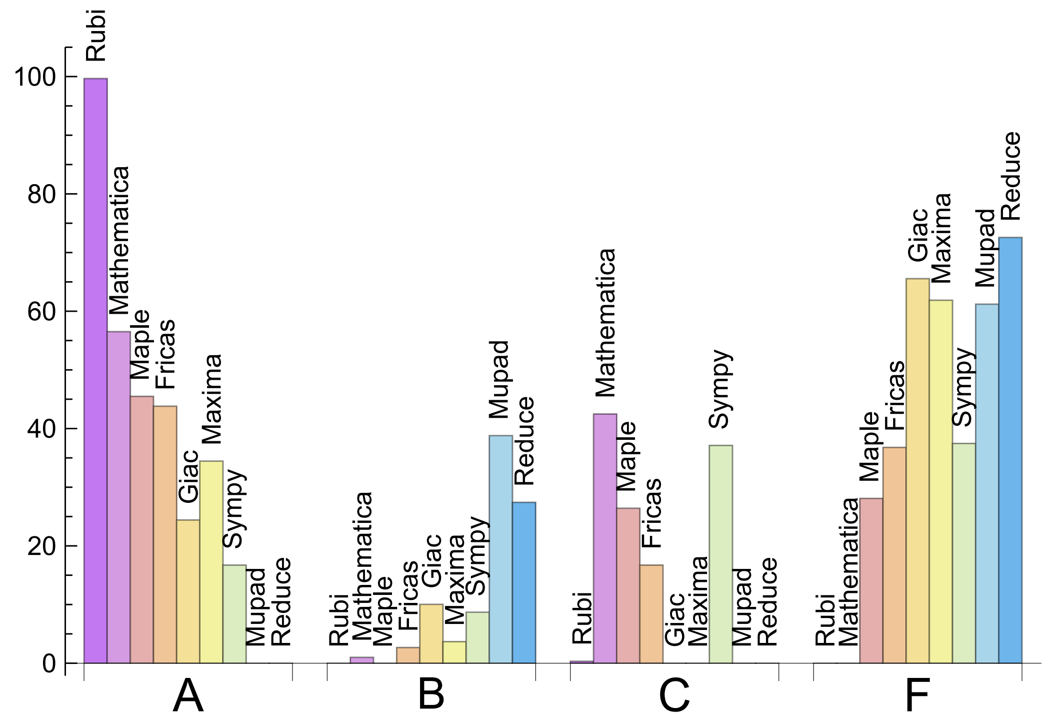
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.666	0.000	0.334	0.000
Mathematica	56.522	1.003	42.475	0.000
Maple	45.485	0.000	26.421	28.094
Fricas	43.813	2.676	16.722	36.789
Maxima	34.448	3.679	0.000	61.873
Giac	24.415	10.033	0.000	65.552
Sympy	16.722	8.696	37.124	37.458
Mupad	0.000	38.796	0.000	61.204
Reduce	0.000	27.425	0.000	72.575

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	84	100.00	0.00	0.00
Fricas	110	78.18	21.82	0.00
Sympy	112	49.11	50.89	0.00
Mupad	183	0.00	100.00	0.00
Maxima	185	94.59	0.00	5.41
Giac	196	97.45	0.51	2.04
Reduce	217	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Giac	0.14
Fricas	0.21
Reduce	0.29
Rubi	0.62
Maple	1.36
Mupad	3.42
Mathematica	5.14
Sympy	27.19

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	119.17	0.81	90.00	0.79
Maple	136.68	0.76	112.00	0.75
Sympy	147.53	1.38	80.00	0.71
Maxima	149.04	1.16	118.00	1.13
Giac	191.82	1.40	121.00	1.29
Rubi	219.08	1.08	127.00	1.00
Fricas	393.62	2.57	245.00	1.14
Mupad	645.71	3.20	90.50	0.95
Reduce	1075.61	7.54	240.50	2.08

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

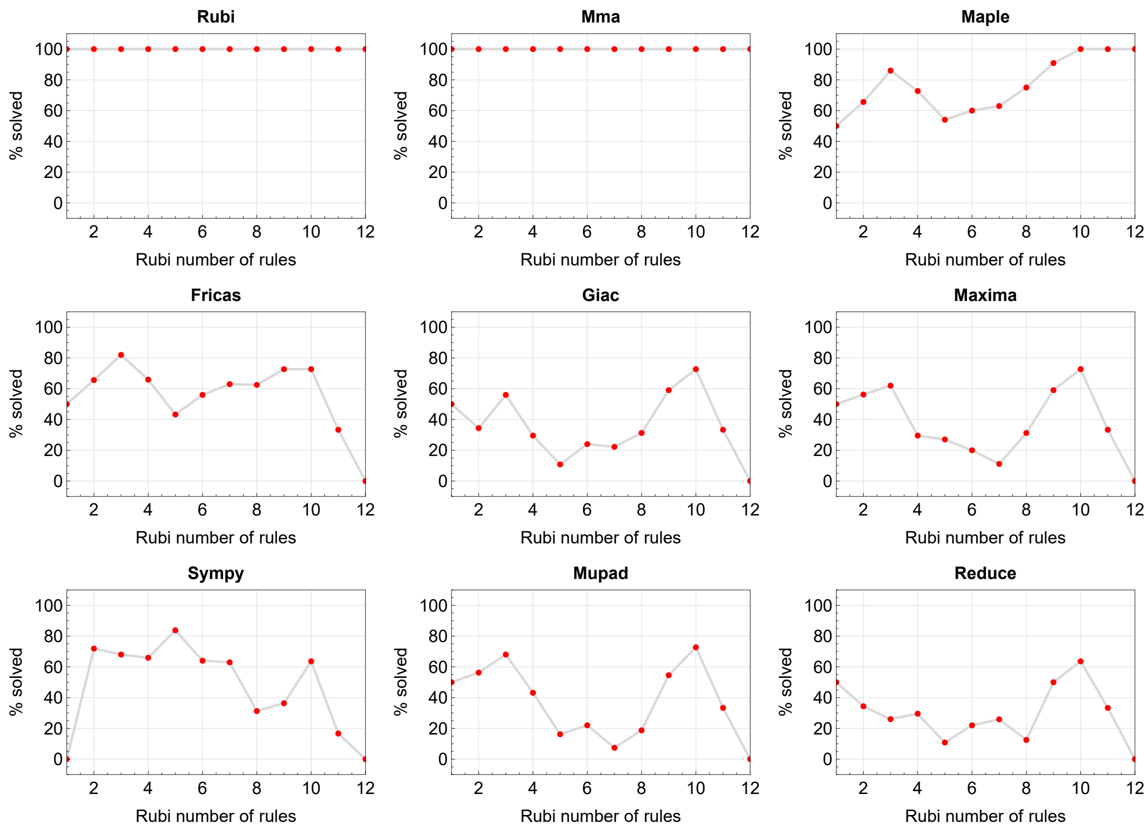


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

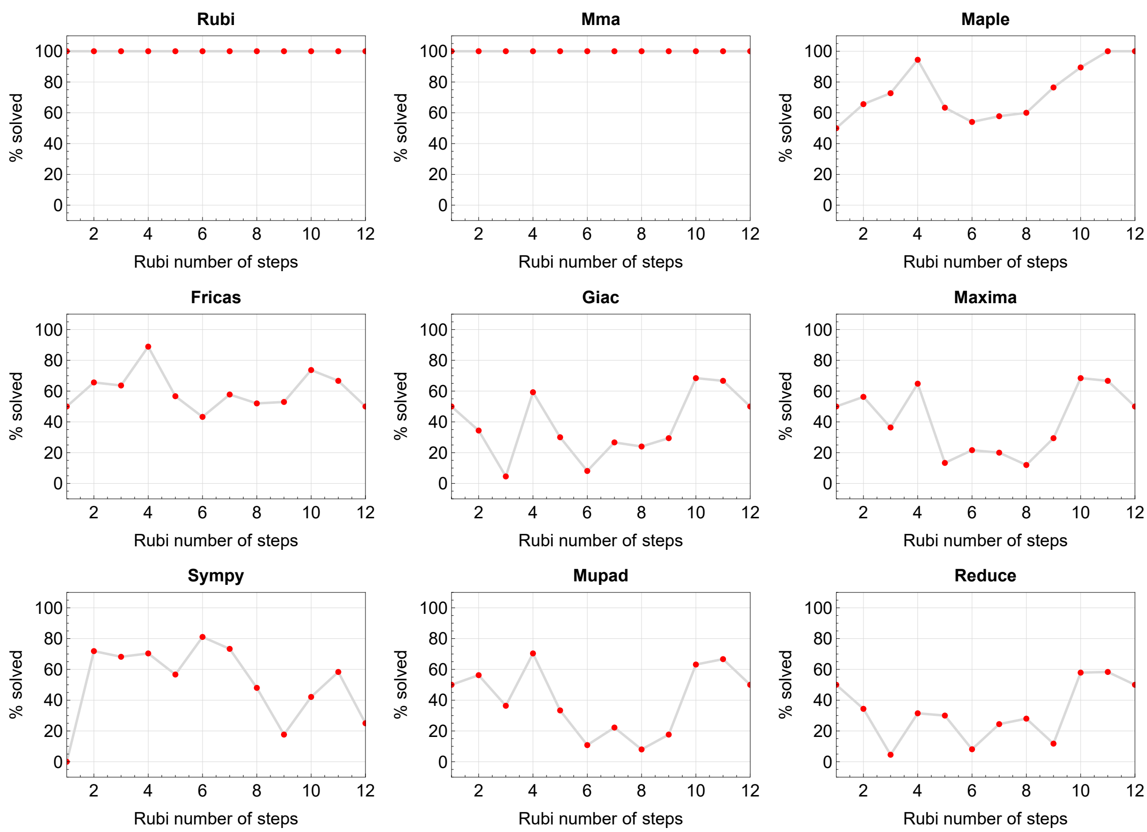


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

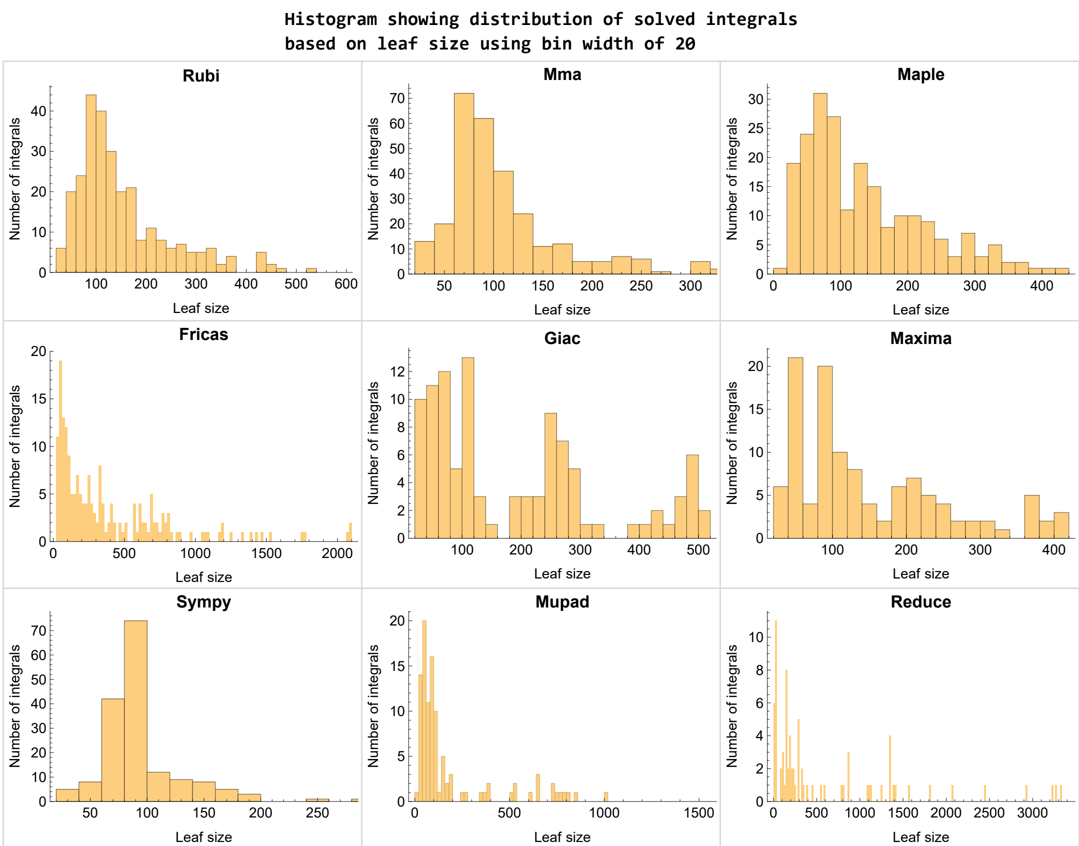


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

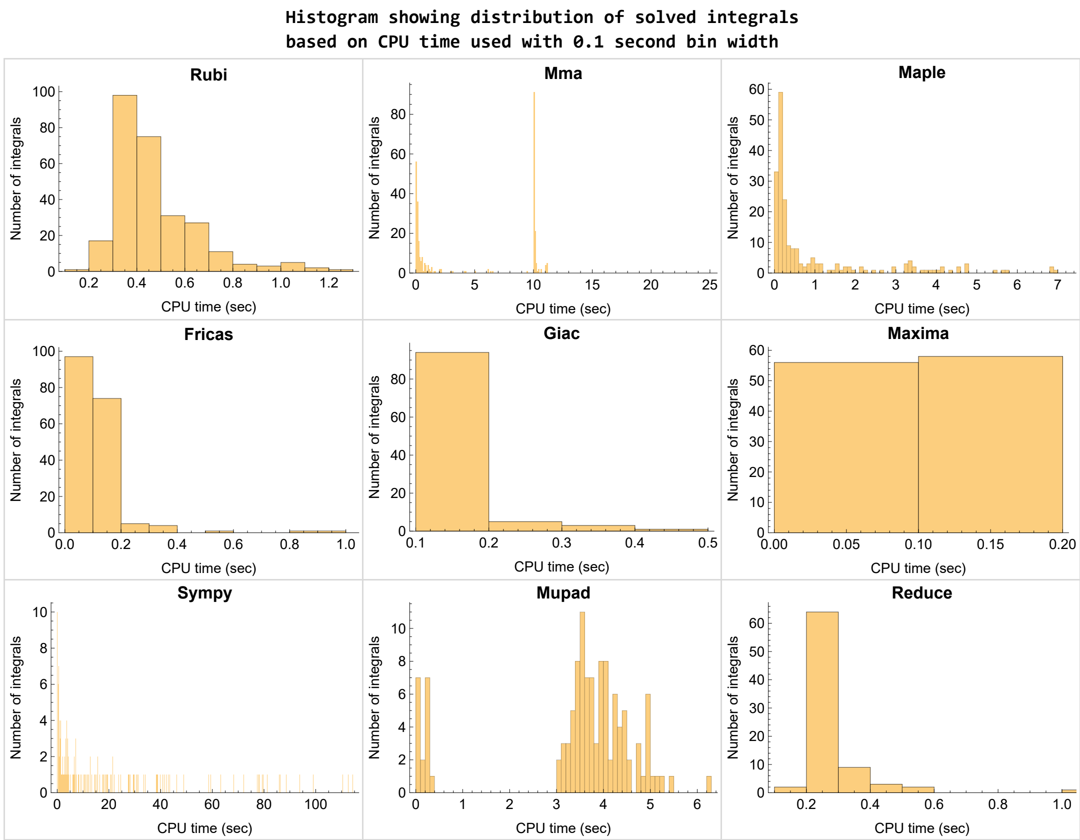


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

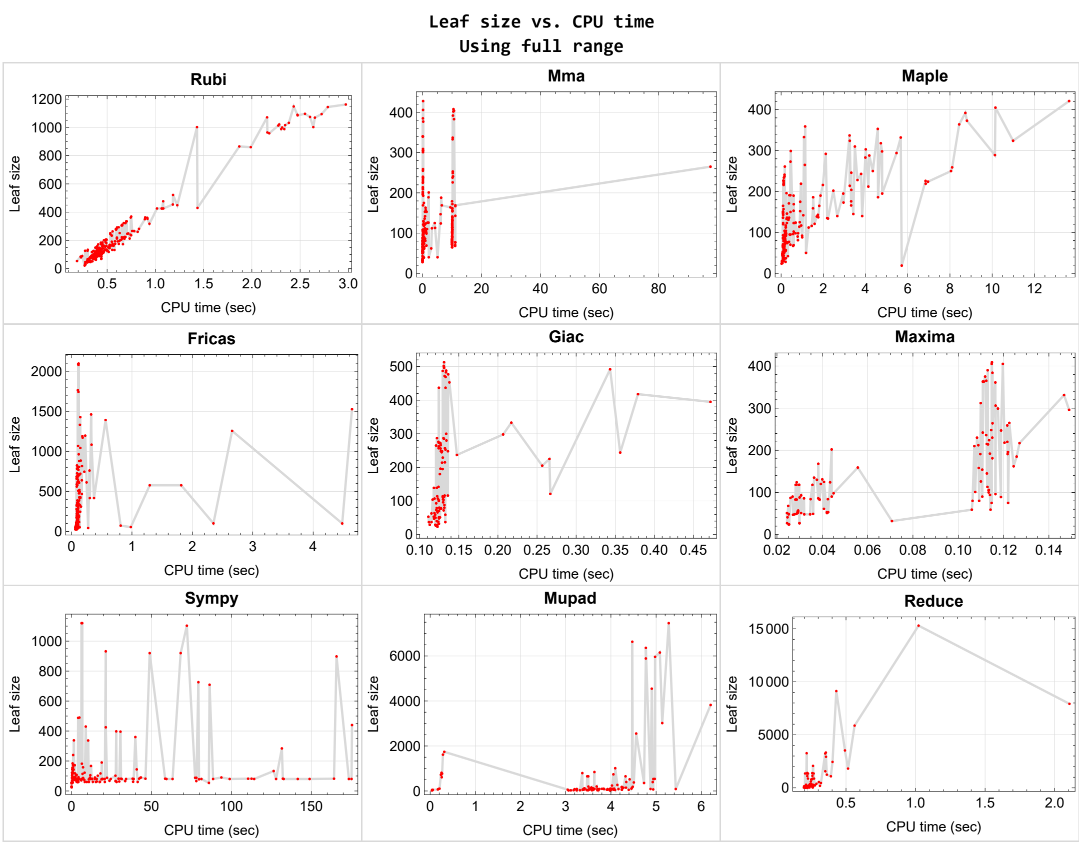


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {140, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270}

Mathematica {229, 230, 231, 233, 245, 247, 248, 249, 264, 265, 266, 267, 268, 269, 270, 272, 273, 282, 283, 288, 292}

Maple $\{229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

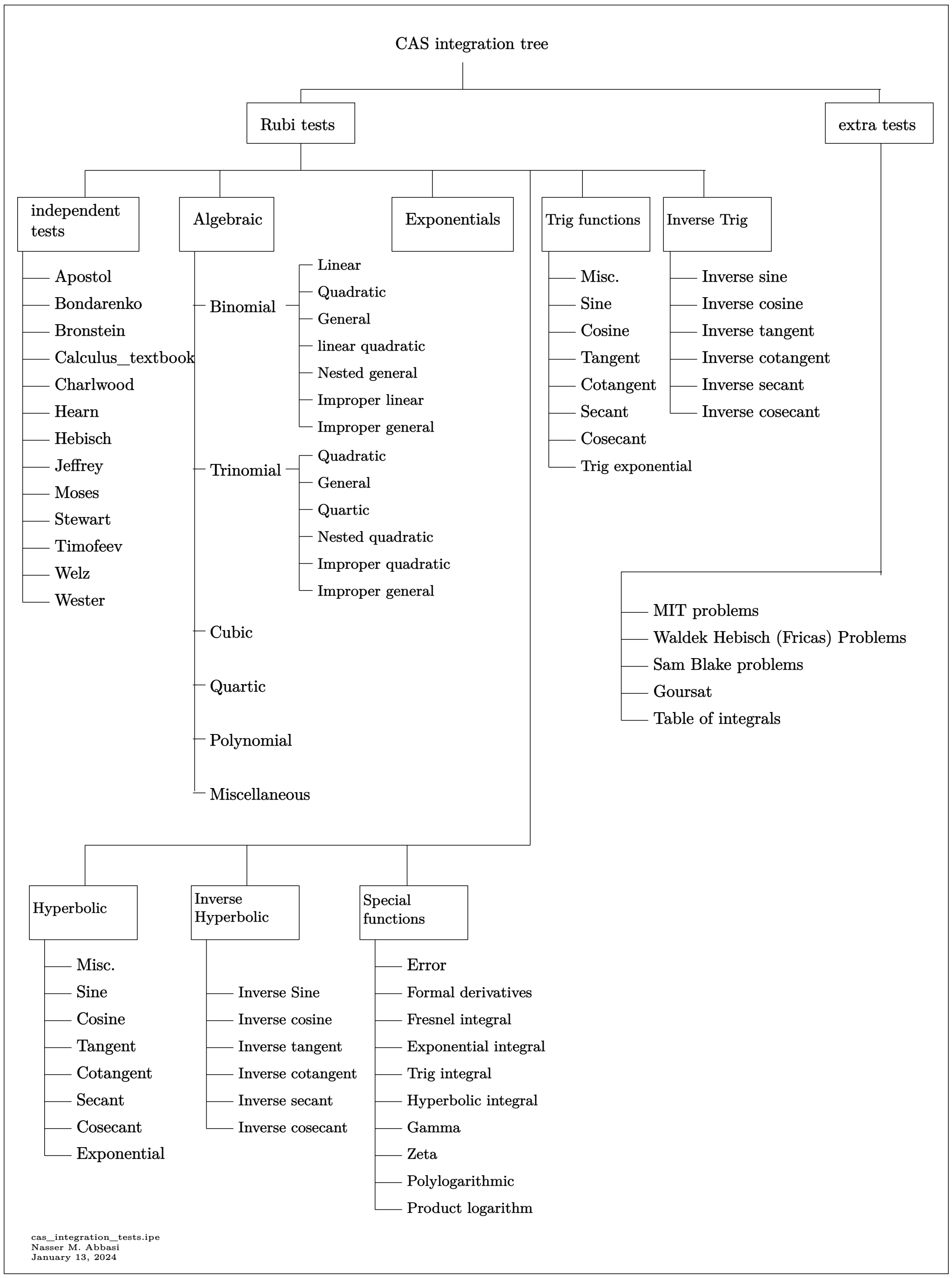
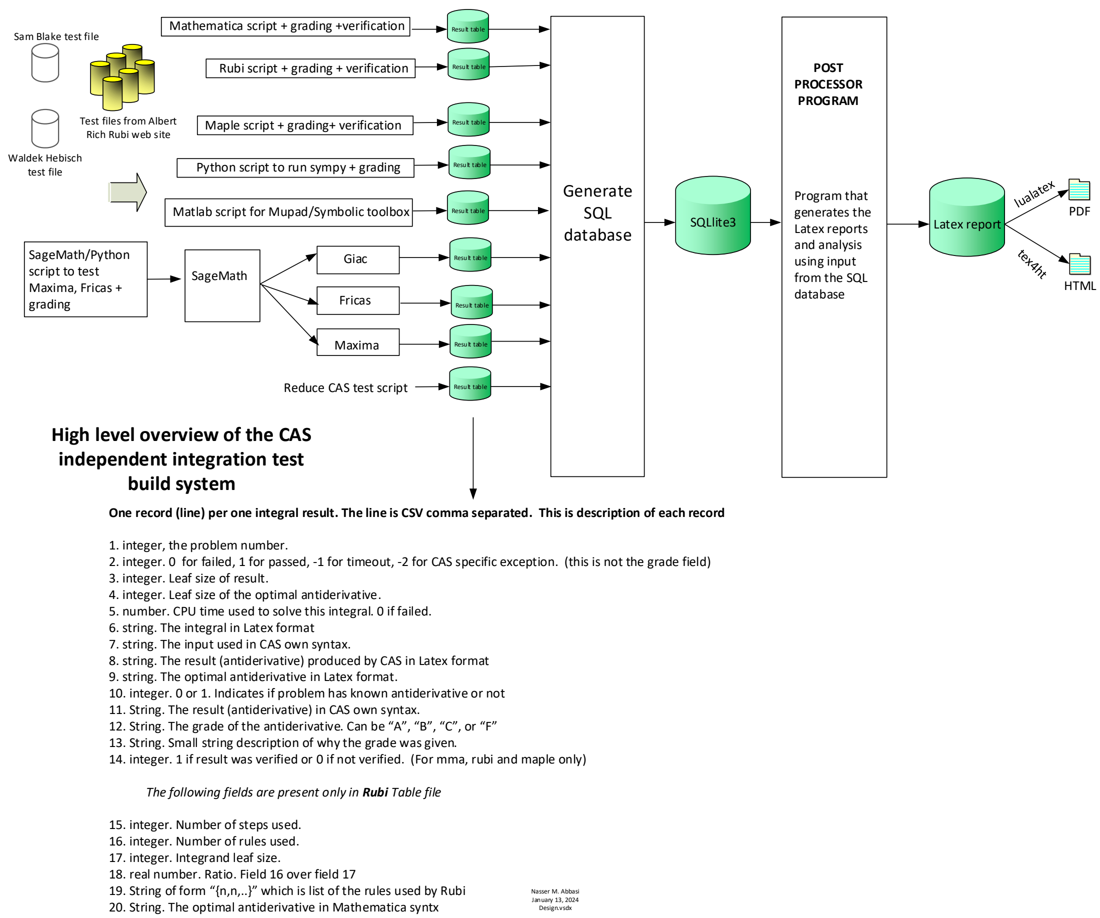


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { }

C grade { 274 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 165, 166, 167, 168, 169, 170, 177, 178, 179, 180, 181, 182, 183, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 275, 276, 277, 279, 280, 281, 282, 284, 285, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

B grade { 278, 283, 288 }

C grade { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 82, 83, 84, 85, 86, 87, 88, 99, 100, 101, 102, 103, 104, 105, 106, 117, 118, 119, 120, 121, 122, 123, 124, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 188, 189, 190, 193, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 22, 23, 24, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 165, 166, 167, 168, 169, 170, 177, 178, 179, 180, 181, 182, 183, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { }

C grade { 11, 12, 13, 17, 18, 19, 20, 21, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 214, 215, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274 }

F normal fail { 81, 82, 83, 84, 85, 86, 87, 88, 99, 100, 101, 102, 103, 104, 105, 106, 117, 118, 119, 120, 121, 122, 123, 124, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 188, 189, 190, 191, 192, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 79, 80, 89, 90, 91, 96, 97, 98, 107, 108, 109, 114, 115, 116, 125, 126, 127, 132, 133, 134, 145, 146, 147, 152, 153, 154, 167, 168, 169, 170, 179, 180, 181, 182, 183, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 236, 237, 238, 239, 240, 241, 244, 254, 256, 257, 258, 259, 263, 274 }

B grade { 242, 243, 252, 253, 255, 260, 261, 262 }

C grade { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 73, 74, 75, 76, 92, 93, 94, 110, 111, 112, 113, 128, 129, 130, 131, 148, 149, 150, 151, 165, 166, 177, 178, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218 }

F normal fail { 33, 81, 82, 83, 84, 85, 86, 87, 88, 99, 100, 101, 102, 103, 104, 105, 106, 117, 118, 119, 120, 121, 122, 123, 124, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 188, 189, 190, 191, 192, 229, 246, 249, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { 77, 95, 230, 231, 232, 233, 234, 245, 247, 248, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 276, 277 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 70, 71, 72, 73, 77, 78, 79, 80, 89, 90, 91, 92, 95, 96, 97, 98, 107, 108, 109, 110, 111, 113, 114, 115, 116, 125, 126, 127, 128, 131, 132, 133, 134, 145, 146, 147, 148, 149, 151, 152, 153, 154, 165, 166, 167, 168, 169, 170, 177, 178, 179, 180, 181, 182, 183, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 218 }

B grade { 74, 75, 76, 93, 94, 112, 129, 130, 150, 215, 217 }

C grade { }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 81, 82, 83, 84, 85, 86, 87, 88, 99, 100, 101, 102, 103, 104, 105, 106, 117, 118, 119, 120, 121, 122, 123, 124, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 174, 175, 176, 184, 185, 186, 187, 188, 189, 190, 191, 192, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { 219, 220, 221, 235, 236, 237, 252, 253, 254, 255 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 16, 17, 18, 19, 20, 21, 22, 23, 24, 70, 71, 72, 89, 90, 91, 107, 108, 109, 125, 126, 127, 145, 146, 147, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 219, 220, 221, 222, 223, 235, 236, 237, 238, 239, 242, 243, 252, 253, 254, 255, 256, 257 }

B grade { 12, 13, 14, 15, 73, 74, 92, 93, 110, 111, 128, 129, 148, 149, 209, 214, 215, 216, 217, 218, 226, 227, 228, 244, 258, 259, 260, 261, 262, 263 }

C grade { }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298 }

F(-1) timedout fail { 299 }

F(-2) exception fail { 224, 225, 240, 241 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 70, 71, 72, 73, 74, 77, 78, 79, 80, 89, 90, 91, 92, 93, 96, 97, 98, 107, 108, 109, 110, 111, 114, 115, 116, 125, 126, 127, 128, 129, 132, 133, 134, 145, 146, 147, 148, 149, 152, 153, 154, 167, 168, 169, 170, 179, 180, 181, 182, 183, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 235, 236, 237, 238, 239, 252, 253, 254, 255, 256, 257 }

C grade { }

F normal fail { }

F(-1) timedout fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 184, 185, 186, 187, 188, 189, 190, 191, 192, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 70, 71, 72, 73, 89, 90, 91, 92, 107, 108, 109, 110, 125, 126, 127, 128, 148, 214, 215, 216, 218, 219, 220, 221, 237, 238 }

B grade { 78, 79, 80, 96, 97, 98, 114, 115, 116, 132, 133, 134, 145, 146, 147, 152, 153, 154, 167, 168, 169, 196, 197, 201, 204, 222 }

C grade { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 65, 66, 67, 68, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 87, 88, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 111, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 172, 173, 174, 175, 279, 280, 281, 286, 287, 294 }

F normal fail { 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 282, 283, 285, 288 }

F(-1) timedout fail { 56, 62, 63, 64, 69, 165, 170, 171, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 217, 252, 253, 254, 284, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

C grade { }

F normal fail { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	26	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.79	0.85
time (sec)	N/A	0.272	0.005	0.211	0.030	0.063	0.022	0.123	0.205	0.046

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	26	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.79	0.85
time (sec)	N/A	0.276	0.005	0.055	0.030	0.068	0.021	0.113	0.222	3.244

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	24	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.86	0.89
time (sec)	N/A	0.273	0.005	0.062	0.025	0.069	0.021	0.121	0.261	0.038

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	26	28
N.S.	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.84	0.90
time (sec)	N/A	0.275	0.009	0.039	0.024	0.065	0.054	0.120	0.238	3.113

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	29	24	23	26	24
N.S.	1	1.00	1.00	0.86	0.86	1.04	0.86	0.82	0.93	0.86
time (sec)	N/A	0.272	0.009	0.021	0.025	0.065	0.072	0.123	0.197	3.063

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	37	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.67	0.93
time (sec)	N/A	0.330	0.007	0.089	0.029	0.070	0.027	0.119	0.209	0.059

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	37	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.67	0.93
time (sec)	N/A	0.326	0.006	0.086	0.031	0.080	0.026	0.118	0.209	0.053

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	35	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.70	0.96
time (sec)	N/A	0.308	0.006	0.085	0.032	0.063	0.029	0.120	0.206	0.047

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	51	53	53	53	37	51
N.S.	1	1.00	1.00	1.00	0.96	1.00	1.00	1.00	0.70	0.96
time (sec)	N/A	0.310	0.012	0.077	0.024	0.063	0.065	0.111	0.230	0.052

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	51	48	37	48
N.S.	1	1.00	1.00	0.98	0.96	1.06	1.02	0.96	0.74	0.96
time (sec)	N/A	0.316	0.012	0.069	0.025	0.090	0.076	0.125	0.219	0.054

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	236	225	60	230	587	102	256	5	770
N.S.	1	1.30	1.24	0.33	1.26	3.23	0.56	1.41	0.03	4.23
time (sec)	N/A	0.749	0.110	0.112	0.108	0.086	0.352	0.127	0.206	0.264

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	224	199	45	191	683	122	248	5	68
N.S.	1	1.33	1.18	0.27	1.13	4.04	0.72	1.47	0.03	0.40
time (sec)	N/A	0.690	0.085	0.056	0.122	0.083	0.355	0.136	0.222	0.170

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	219	196	42	212	560	87	245	1	720
N.S.	1	1.34	1.20	0.26	1.29	3.41	0.53	1.49	0.01	4.39
time (sec)	N/A	0.660	0.072	0.076	0.109	0.086	0.308	0.128	0.203	0.238

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	223	202	123	191	694	122	248	5	68
N.S.	1	1.34	1.21	0.74	1.14	4.16	0.73	1.49	0.03	0.41
time (sec)	N/A	0.675	0.091	0.096	0.110	0.105	0.359	0.129	0.211	3.379

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	225	209	120	215	579	88	248	5	795
N.S.	1	1.33	1.24	0.71	1.27	3.43	0.52	1.47	0.03	4.70
time (sec)	N/A	0.683	0.129	0.096	0.113	0.097	0.393	0.127	0.211	3.367

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	240	227	136	210	725	143	265	5	87
N.S.	1	1.30	1.23	0.74	1.14	3.94	0.78	1.44	0.03	0.47
time (sec)	N/A	0.743	0.146	0.088	0.107	0.092	0.480	0.121	0.210	0.183

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	252	255	87	264	686	141	287	154	810
N.S.	1	1.19	1.21	0.41	1.25	3.25	0.67	1.36	0.73	3.84
time (sec)	N/A	0.630	0.142	0.090	0.115	0.102	0.788	0.122	0.227	0.251

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	267	240	71	220	782	156	280	147	102
N.S.	1	1.34	1.21	0.36	1.11	3.93	0.78	1.41	0.74	0.51
time (sec)	N/A	0.779	0.144	0.089	0.122	0.091	0.908	0.134	0.199	0.207

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	261	232	65	242	659	117	273	144	731
N.S.	1	1.38	1.23	0.34	1.28	3.49	0.62	1.44	0.76	3.87
time (sec)	N/A	0.817	0.143	0.085	0.111	0.084	0.659	0.126	0.229	0.243

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	249	214	67	214	768	148	270	112	87
N.S.	1	1.32	1.14	0.36	1.14	4.09	0.79	1.44	0.60	0.46
time (sec)	N/A	0.720	0.107	0.082	0.113	0.114	0.644	0.127	0.213	3.520

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	247	212	65	236	648	112	266	112	740
N.S.	1	1.33	1.14	0.35	1.27	3.48	0.60	1.43	0.60	3.98
time (sec)	N/A	0.709	0.110	0.082	0.112	0.104	0.436	0.127	0.234	4.055

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	265	237	149	217	778	155	273	150	97
N.S.	1	1.35	1.20	0.76	1.10	3.95	0.79	1.39	0.76	0.49
time (sec)	N/A	0.780	0.179	0.094	0.127	0.104	0.552	0.124	0.229	3.590

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	267	238	147	248	678	124	276	159	843
N.S.	1	1.36	1.21	0.75	1.26	3.44	0.63	1.40	0.81	4.28
time (sec)	N/A	0.759	0.158	0.090	0.115	0.094	0.507	0.124	0.255	3.635

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	282	259	165	247	824	184	300	168	117
N.S.	1	1.31	1.20	0.76	1.14	3.81	0.85	1.39	0.78	0.54
time (sec)	N/A	0.831	0.175	0.092	0.119	0.090	0.654	0.134	0.207	3.546

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	177	114	140	0	98	80	0	138	0
N.S.	1	0.99	0.64	0.78	0.00	0.55	0.45	0.00	0.77	0.00
time (sec)	N/A	0.436	10.107	3.841	0.000	0.088	1.642	0.000	0.242	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	147	89	116	0	72	80	0	98	0
N.S.	1	0.99	0.60	0.78	0.00	0.49	0.54	0.00	0.66	0.00
time (sec)	N/A	0.389	10.122	1.450	0.000	0.087	1.454	0.000	0.247	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	73	96	0	58	78	0	65	0
N.S.	1	1.00	0.62	0.81	0.00	0.49	0.66	0.00	0.55	0.00
time (sec)	N/A	0.342	0.023	0.567	0.000	0.086	1.345	0.000	0.254	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	78	98	0	61	82	0	80	0
N.S.	1	1.00	0.66	0.82	0.00	0.51	0.69	0.00	0.67	0.00
time (sec)	N/A	0.346	10.030	0.684	0.000	0.082	1.118	0.000	0.276	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	149	78	121	0	75	90	0	82	0
N.S.	1	0.99	0.52	0.81	0.00	0.50	0.60	0.00	0.55	0.00
time (sec)	N/A	0.378	10.043	1.586	0.000	0.084	1.428	0.000	0.298	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	179	78	145	0	102	94	0	82	0
N.S.	1	0.99	0.43	0.80	0.00	0.56	0.52	0.00	0.45	0.00
time (sec)	N/A	0.445	10.049	3.422	0.000	0.099	1.797	0.000	0.336	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	282	91	140	0	139	80	0	108	0
N.S.	1	0.96	0.31	0.47	0.00	0.47	0.27	0.00	0.37	0.00
time (sec)	N/A	0.598	10.086	2.662	0.000	0.109	1.528	0.000	0.228	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	252	75	125	0	109	80	0	73	0
N.S.	1	0.95	0.28	0.47	0.00	0.41	0.30	0.00	0.28	0.00
time (sec)	N/A	0.533	10.055	0.882	0.000	0.078	1.454	0.000	0.243	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	245	76	123	0	0	82	0	74	0
N.S.	1	0.96	0.30	0.48	0.00	0.00	0.32	0.00	0.29	0.00
time (sec)	N/A	0.533	10.025	0.628	0.000	0.000	1.235	0.000	0.235	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	277	78	142	0	112	88	0	82	0
N.S.	1	0.93	0.26	0.48	0.00	0.38	0.30	0.00	0.28	0.00
time (sec)	N/A	0.594	10.029	1.051	0.000	0.079	1.252	0.000	0.269	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	102	195	0	127	80	0	253	0
N.S.	1	1.00	0.58	1.11	0.00	0.72	0.45	0.00	1.44	0.00
time (sec)	N/A	0.451	10.077	4.790	0.000	0.098	11.363	0.000	0.310	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	147	78	135	0	110	80	0	212	0
N.S.	1	1.01	0.54	0.93	0.00	0.76	0.55	0.00	1.46	0.00
time (sec)	N/A	0.396	10.079	2.183	0.000	0.090	5.064	0.000	0.253	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	71	123	0	93	78	0	174	0
N.S.	1	1.00	0.56	0.98	0.00	0.74	0.62	0.00	1.38	0.00
time (sec)	N/A	0.358	0.026	0.594	0.000	0.088	3.198	0.000	0.285	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	147	86	137	0	115	82	0	203	0
N.S.	1	0.99	0.58	0.93	0.00	0.78	0.55	0.00	1.37	0.00
time (sec)	N/A	0.384	10.069	1.762	0.000	0.087	7.113	0.000	0.299	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	178	72	173	0	132	90	0	203	0
N.S.	1	0.99	0.40	0.96	0.00	0.73	0.50	0.00	1.13	0.00
time (sec)	N/A	0.450	10.035	2.956	0.000	0.106	14.626	0.000	0.376	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	311	104	226	0	220	80	0	269	0
N.S.	1	0.96	0.32	0.70	0.00	0.68	0.25	0.00	0.83	0.00
time (sec)	N/A	0.657	10.072	6.844	0.000	0.091	19.072	0.000	0.362	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	281	79	166	0	185	80	0	228	0
N.S.	1	0.96	0.27	0.57	0.00	0.63	0.27	0.00	0.78	0.00
time (sec)	N/A	0.591	10.068	3.319	0.000	0.097	7.445	0.000	0.282	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	264	71	152	0	166	80	0	189	0
N.S.	1	0.96	0.26	0.55	0.00	0.61	0.29	0.00	0.69	0.00
time (sec)	N/A	0.557	10.048	0.902	0.000	0.094	4.159	0.000	0.270	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	277	71	165	0	167	82	0	199	0
N.S.	1	0.94	0.24	0.56	0.00	0.57	0.28	0.00	0.67	0.00
time (sec)	N/A	0.592	10.025	1.808	0.000	0.095	6.647	0.000	0.281	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	306	72	202	0	190	88	0	203	0
N.S.	1	0.94	0.22	0.62	0.00	0.58	0.27	0.00	0.62	0.00
time (sec)	N/A	0.657	10.039	2.475	0.000	0.124	11.494	0.000	0.330	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	205	132	250	0	186	80	0	456	0
N.S.	1	0.99	0.64	1.21	0.00	0.90	0.39	0.00	2.20	0.00
time (sec)	N/A	0.495	10.105	8.050	0.000	0.097	98.956	0.000	0.408	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	172	106	186	0	165	80	0	415	0
N.S.	1	0.99	0.61	1.08	0.00	0.95	0.46	0.00	2.40	0.00
time (sec)	N/A	0.439	10.097	4.592	0.000	0.095	38.871	0.000	0.379	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	100	159	0	152	80	0	374	0
N.S.	1	1.03	0.65	1.03	0.00	0.99	0.52	0.00	2.43	0.00
time (sec)	N/A	0.399	10.076	1.510	0.000	0.095	24.503	0.000	0.329	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	101	153	0	154	78	0	340	0
N.S.	1	1.00	0.64	0.97	0.00	0.98	0.50	0.00	2.17	0.00
time (sec)	N/A	0.394	10.039	0.592	0.000	0.098	18.734	0.000	0.309	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	172	114	176	0	176	82	0	374	0
N.S.	1	0.98	0.65	1.00	0.00	1.00	0.47	0.00	2.12	0.00
time (sec)	N/A	0.407	10.059	3.319	0.000	0.092	46.260	0.000	0.405	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	207	81	212	0	191	90	0	374	0
N.S.	1	0.99	0.39	1.01	0.00	0.91	0.43	0.00	1.78	0.00
time (sec)	N/A	0.491	10.047	4.134	0.000	0.090	93.775	0.000	0.556	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	310	109	219	0	268	80	0	438	0
N.S.	1	0.96	0.34	0.68	0.00	0.83	0.25	0.00	1.36	0.00
time (sec)	N/A	0.656	10.154	6.849	0.000	0.107	59.404	0.000	0.360	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	293	92	190	0	247	80	0	398	0
N.S.	1	0.97	0.30	0.63	0.00	0.82	0.26	0.00	1.31	0.00
time (sec)	N/A	0.620	10.075	1.867	0.000	0.116	31.384	0.000	0.331	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	292	79	175	0	224	80	0	362	0
N.S.	1	0.97	0.26	0.58	0.00	0.74	0.26	0.00	1.20	0.00
time (sec)	N/A	0.603	10.066	0.905	0.000	0.089	19.377	0.000	0.367	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	306	80	204	0	252	82	0	370	0
N.S.	1	0.94	0.25	0.63	0.00	0.78	0.25	0.00	1.14	0.00
time (sec)	N/A	0.649	10.041	3.327	0.000	0.091	30.985	0.000	0.369	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	335	81	243	0	273	88	0	374	0
N.S.	1	0.94	0.23	0.68	0.00	0.76	0.25	0.00	1.05	0.00
time (sec)	N/A	0.699	10.042	3.790	0.000	0.086	78.137	0.000	0.430	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	234	156	289	0	245	0	0	703	0
N.S.	1	0.98	0.66	1.21	0.00	1.03	0.00	0.00	2.95	0.00
time (sec)	N/A	0.548	10.141	10.131	0.000	0.101	0.000	0.000	0.516	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	131	224	0	229	80	0	663	0
N.S.	1	1.00	0.64	1.09	0.00	1.12	0.39	0.00	3.23	0.00
time (sec)	N/A	0.484	10.121	6.960	0.000	0.094	175.224	0.000	0.524	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	188	128	195	0	211	80	0	623	0
N.S.	1	1.03	0.70	1.07	0.00	1.15	0.44	0.00	3.40	0.00
time (sec)	N/A	0.484	10.099	2.944	0.000	0.111	127.996	0.000	0.466	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	185	125	186	0	207	80	0	583	0
N.S.	1	1.02	0.69	1.02	0.00	1.14	0.44	0.00	3.20	0.00
time (sec)	N/A	0.451	10.103	1.518	0.000	0.109	141.575	0.000	0.458	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	184	110	190	0	214	78	0	547	0
N.S.	1	0.98	0.59	1.02	0.00	1.14	0.42	0.00	2.93	0.00
time (sec)	N/A	0.435	10.080	0.590	0.000	0.091	79.773	0.000	0.453	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	201	113	214	0	235	82	0	589	0
N.S.	1	0.99	0.55	1.05	0.00	1.15	0.40	0.00	2.89	0.00
time (sec)	N/A	0.467	10.066	3.252	0.000	0.107	164.254	0.000	0.534	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	236	83	250	0	250	0	0	589	0
N.S.	1	0.98	0.35	1.04	0.00	1.04	0.00	0.00	2.45	0.00
time (sec)	N/A	0.545	10.079	4.345	0.000	0.097	0.000	0.000	0.767	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	369	155	324	0	386	0	0	731	0
N.S.	1	0.96	0.40	0.84	0.00	1.01	0.00	0.00	1.90	0.00
time (sec)	N/A	0.750	10.150	10.991	0.000	0.096	0.000	0.000	0.579	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	335	131	259	0	340	0	0	691	0
N.S.	1	0.96	0.37	0.74	0.00	0.97	0.00	0.00	1.97	0.00
time (sec)	N/A	0.703	10.122	8.105	0.000	0.101	0.000	0.000	0.543	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	322	114	228	0	327	80	0	651	0
N.S.	1	0.96	0.34	0.68	0.00	0.98	0.24	0.00	1.95	0.00
time (sec)	N/A	0.690	10.140	3.618	0.000	0.096	173.565	0.000	0.475	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	323	98	216	0	322	80	0	611	0
N.S.	1	0.95	0.29	0.64	0.00	0.95	0.24	0.00	1.80	0.00
time (sec)	N/A	0.673	10.106	1.980	0.000	0.084	114.290	0.000	0.464	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	323	123	219	0	322	80	0	575	0
N.S.	1	0.94	0.36	0.64	0.00	0.94	0.23	0.00	1.68	0.00
time (sec)	N/A	0.675	10.067	0.941	0.000	0.093	81.278	0.000	0.430	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	335	84	246	0	335	82	0	585	0
N.S.	1	0.94	0.24	0.69	0.00	0.94	0.23	0.00	1.65	0.00
time (sec)	N/A	0.699	10.050	3.326	0.000	0.089	112.655	0.000	0.476	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	362	83	283	0	356	0	0	589	0
N.S.	1	0.94	0.21	0.73	0.00	0.92	0.00	0.00	1.52	0.00
time (sec)	N/A	0.745	10.054	3.992	0.000	0.094	0.000	0.000	0.687	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	118	76	172	107	35	79
N.S.	1	1.04	0.78	0.66	1.15	0.74	1.67	1.04	0.34	0.77
time (sec)	N/A	0.413	0.068	0.079	0.028	0.074	0.968	0.119	0.285	3.732

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	84	52	121	79	35	57
N.S.	1	1.05	0.77	0.67	1.15	0.71	1.66	1.08	0.48	0.78
time (sec)	N/A	0.368	0.052	0.060	0.039	0.076	0.545	0.122	0.248	3.517

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	49	30	71	47	35	35
N.S.	1	1.09	0.74	0.67	1.07	0.65	1.54	1.02	0.76	0.76
time (sec)	N/A	0.324	0.036	0.057	0.036	0.076	0.378	0.118	0.237	3.497

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	82	76	79	75	173	63	196	35	56
N.S.	1	1.08	1.00	1.04	0.99	2.28	0.83	2.58	0.46	0.74
time (sec)	N/A	0.350	0.092	0.141	0.122	0.083	15.791	0.126	0.264	4.060

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	92	94	85	154	707	80	253	35	101
N.S.	1	0.99	1.01	0.91	1.66	7.60	0.86	2.72	0.38	1.09
time (sec)	N/A	0.374	0.192	0.093	0.111	0.130	19.522	0.133	0.278	4.454

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	124	105	113	265	809	80	0	35	0
N.S.	1	0.98	0.83	0.89	2.09	6.37	0.63	0.00	0.28	0.00
time (sec)	N/A	0.404	0.639	0.224	0.123	0.101	7.336	0.000	0.267	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	94	88	143	181	716	78	0	31	0
N.S.	1	0.99	0.93	1.51	1.91	7.54	0.82	0.00	0.33	0.00
time (sec)	N/A	0.343	0.016	0.098	0.115	0.103	3.751	0.000	0.232	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	81	93	89	0	71	0	31	57
N.S.	1	1.01	1.00	1.15	1.10	0.00	0.88	0.00	0.38	0.70
time (sec)	N/A	0.324	0.293	0.089	0.119	0.000	3.051	0.000	0.239	3.911

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	35	107	0	35	36
N.S.	1	1.00	0.75	0.68	1.04	0.66	2.02	0.00	0.66	0.68
time (sec)	N/A	0.308	0.246	0.093	0.029	0.095	3.201	0.000	0.201	3.725

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	58	490	0	35	58
N.S.	1	0.99	0.74	0.65	1.07	0.69	5.83	0.00	0.42	0.69
time (sec)	N/A	0.368	0.334	0.128	0.044	0.086	4.930	0.000	0.215	4.009

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	82	1120	0	35	105
N.S.	1	0.97	0.74	0.63	1.06	0.70	9.57	0.00	0.30	0.90
time (sec)	N/A	0.392	0.429	0.176	0.043	0.096	6.684	0.000	0.251	4.070

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	149	93	0	0	0	80	0	35	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.52	0.00	0.23	0.00
time (sec)	N/A	0.522	10.066	0.000	0.000	0.000	3.781	0.000	0.225	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	119	74	0	0	0	80	0	35	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.65	0.00	0.28	0.00
time (sec)	N/A	0.453	10.075	0.000	0.000	0.000	3.438	0.000	0.256	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	113	77	0	0	0	82	0	35	0
N.S.	1	0.95	0.65	0.00	0.00	0.00	0.69	0.00	0.29	0.00
time (sec)	N/A	0.461	10.035	0.000	0.000	0.000	2.807	0.000	0.225	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	114	78	0	0	0	71	0	35	0
N.S.	1	0.93	0.63	0.00	0.00	0.00	0.58	0.00	0.28	0.00
time (sec)	N/A	0.458	10.034	0.000	0.000	0.000	2.615	0.000	0.231	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	147	92	0	0	0	60	0	35	0
N.S.	1	0.96	0.60	0.00	0.00	0.00	0.39	0.00	0.23	0.00
time (sec)	N/A	0.420	10.072	0.000	0.000	0.000	4.299	0.000	0.234	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	75	0	0	0	60	0	33	0
N.S.	1	0.96	0.61	0.00	0.00	0.00	0.49	0.00	0.27	0.00
time (sec)	N/A	0.382	10.049	0.000	0.000	0.000	3.675	0.000	0.246	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	116	77	0	0	0	61	0	33	0
N.S.	1	0.97	0.64	0.00	0.00	0.00	0.51	0.00	0.28	0.00
time (sec)	N/A	0.381	10.029	0.000	0.000	0.000	3.264	0.000	0.294	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	144	78	0	0	0	66	0	35	0
N.S.	1	0.96	0.52	0.00	0.00	0.00	0.44	0.00	0.23	0.00
time (sec)	N/A	0.412	10.039	0.000	0.000	0.000	4.187	0.000	0.296	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	105	80	68	118	76	172	100	35	79
N.S.	1	1.05	0.80	0.68	1.18	0.76	1.72	1.00	0.35	0.79
time (sec)	N/A	0.412	0.065	0.121	0.036	0.075	1.964	0.124	0.230	3.195

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	56	49	83	51	121	70	35	57
N.S.	1	1.06	0.79	0.69	1.17	0.72	1.70	0.99	0.49	0.80
time (sec)	N/A	0.364	0.051	0.105	0.029	0.094	0.947	0.125	0.215	3.045

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	33	31	47	29	70	37	34	35
N.S.	1	1.12	0.77	0.72	1.09	0.67	1.63	0.86	0.79	0.81
time (sec)	N/A	0.329	0.034	0.088	0.027	0.091	0.633	0.112	0.213	3.176

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	85	73	79	75	164	63	195	34	55
N.S.	1	1.16	1.00	1.08	1.03	2.25	0.86	2.67	0.47	0.75
time (sec)	N/A	0.341	0.095	0.112	0.115	0.093	29.880	0.129	0.232	3.460

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	99	95	104	158	607	82	257	35	101
N.S.	1	1.04	1.00	1.09	1.66	6.39	0.86	2.71	0.37	1.06
time (sec)	N/A	0.364	0.173	0.227	0.109	0.125	38.212	0.132	0.267	3.839

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	96	90	98	185	614	80	0	35	0
N.S.	1	0.99	0.93	1.01	1.91	6.33	0.82	0.00	0.36	0.00
time (sec)	N/A	0.369	0.522	0.237	0.126	0.097	4.580	0.000	0.248	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	80	79	89	90	0	73	0	35	0
N.S.	1	1.01	1.00	1.13	1.14	0.00	0.92	0.00	0.44	0.00
time (sec)	N/A	0.343	0.376	0.184	0.111	0.000	2.199	0.000	0.213	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	55	35	105	0	35	35
N.S.	1	1.00	0.75	0.68	1.04	0.66	1.98	0.00	0.66	0.66
time (sec)	N/A	0.304	0.357	0.155	0.029	0.100	2.951	0.000	0.206	3.241

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	90	58	488	0	35	58
N.S.	1	0.99	0.74	0.65	1.07	0.69	5.81	0.00	0.42	0.69
time (sec)	N/A	0.367	0.503	0.210	0.027	0.081	4.141	0.000	0.253	3.400

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	124	82	1120	0	35	105
N.S.	1	0.97	0.74	0.63	1.06	0.70	9.57	0.00	0.30	0.90
time (sec)	N/A	0.408	0.587	0.286	0.029	0.092	6.262	0.000	0.221	3.587

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	91	0	0	0	60	0	35	0
N.S.	1	0.99	0.75	0.00	0.00	0.00	0.49	0.00	0.29	0.00
time (sec)	N/A	0.383	10.074	0.000	0.000	0.000	3.875	0.000	0.217	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	75	0	0	0	60	0	33	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.65	0.00	0.36	0.00
time (sec)	N/A	0.340	10.046	0.000	0.000	0.000	3.792	0.000	0.211	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	0	0	0	61	0	33	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.67	0.00	0.36	0.00
time (sec)	N/A	0.345	10.030	0.000	0.000	0.000	3.254	0.000	0.269	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	78	0	0	0	66	0	35	0
N.S.	1	0.96	0.64	0.00	0.00	0.00	0.54	0.00	0.29	0.00
time (sec)	N/A	0.392	10.030	0.000	0.000	0.000	3.800	0.000	0.222	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	122	90	0	0	0	80	0	35	0
N.S.	1	0.99	0.73	0.00	0.00	0.00	0.65	0.00	0.28	0.00
time (sec)	N/A	0.468	10.066	0.000	0.000	0.000	4.294	0.000	0.214	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	0	0	0	78	0	31	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.84	0.00	0.33	0.00
time (sec)	N/A	0.397	0.023	0.000	0.000	0.000	2.817	0.000	0.224	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	78	0	0	0	82	0	31	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.86	0.00	0.33	0.00
time (sec)	N/A	0.401	10.031	0.000	0.000	0.000	2.187	0.000	0.220	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	124	78	0	0	0	90	0	35	0
N.S.	1	0.99	0.62	0.00	0.00	0.00	0.72	0.00	0.28	0.00
time (sec)	N/A	0.463	10.033	0.000	0.000	0.000	2.409	0.000	0.218	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	77	68	118	88	172	114	73	88
N.S.	1	1.04	0.76	0.67	1.17	0.87	1.70	1.13	0.72	0.87
time (sec)	N/A	0.417	0.066	0.139	0.030	0.085	1.199	0.129	0.229	3.702

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	56	49	83	63	121	77	73	60
N.S.	1	1.07	0.80	0.70	1.19	0.90	1.73	1.10	1.04	0.86
time (sec)	N/A	0.366	0.053	0.122	0.028	0.082	0.695	0.127	0.307	3.601

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	33	30	48	41	70	38	73	32
N.S.	1	1.09	0.75	0.68	1.09	0.93	1.59	0.86	1.66	0.73
time (sec)	N/A	0.321	0.036	0.110	0.035	0.077	0.470	0.124	0.245	3.580

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	80	88	94	256	66	214	70	70
N.S.	1	1.12	0.96	1.06	1.13	3.08	0.80	2.58	0.84	0.84
time (sec)	N/A	0.361	0.129	0.135	0.114	0.105	33.579	0.129	0.254	3.927

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	118	105	112	192	797	82	285	69	145
N.S.	1	0.99	0.88	0.94	1.61	6.70	0.69	2.39	0.58	1.22
time (sec)	N/A	0.395	0.292	0.217	0.114	0.101	58.568	0.131	0.271	4.276

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	118	102	196	224	805	80	0	73	0
N.S.	1	0.94	0.82	1.57	1.79	6.44	0.64	0.00	0.58	0.00
time (sec)	N/A	0.403	1.057	0.255	0.114	0.124	11.843	0.000	0.221	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	82	94	106	276	71	0	69	0
N.S.	1	1.05	0.95	1.09	1.23	3.21	0.83	0.00	0.80	0.00
time (sec)	N/A	0.340	0.045	0.158	0.113	0.102	6.062	0.000	0.213	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	36	52	46	102	0	68	39
N.S.	1	1.00	0.78	0.71	1.02	0.90	2.00	0.00	1.33	0.76
time (sec)	N/A	0.296	0.348	0.161	0.029	0.109	16.421	0.000	0.281	3.466

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	62	55	93	69	396	0	71	96
N.S.	1	0.98	0.75	0.66	1.12	0.83	4.77	0.00	0.86	1.16
time (sec)	N/A	0.333	0.582	0.211	0.031	0.085	30.625	0.000	0.221	3.803

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	111	86	74	130	94	920	0	71	103
N.S.	1	0.97	0.75	0.65	1.14	0.82	8.07	0.00	0.62	0.90
time (sec)	N/A	0.386	0.878	0.285	0.038	0.094	48.904	0.000	0.225	3.996

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	122	80	0	0	0	80	0	73	0
N.S.	1	0.99	0.65	0.00	0.00	0.00	0.65	0.00	0.59	0.00
time (sec)	N/A	0.449	10.064	0.000	0.000	0.000	15.747	0.000	0.222	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	72	0	0	0	80	0	73	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.86	0.00	0.78	0.00
time (sec)	N/A	0.404	10.059	0.000	0.000	0.000	7.215	0.000	0.237	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	0	0	0	82	0	72	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.93	0.00	0.82	0.00
time (sec)	N/A	0.416	10.026	0.000	0.000	0.000	12.752	0.000	0.211	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	118	72	0	0	0	88	0	71	0
N.S.	1	0.96	0.59	0.00	0.00	0.00	0.72	0.00	0.58	0.00
time (sec)	N/A	0.461	10.029	0.000	0.000	0.000	22.188	0.000	0.213	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	79	0	0	0	60	0	73	0
N.S.	1	0.96	0.65	0.00	0.00	0.00	0.49	0.00	0.60	0.00
time (sec)	N/A	0.392	10.057	0.000	0.000	0.000	15.274	0.000	0.292	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	60	0	71	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.71	0.00	0.84	0.00
time (sec)	N/A	0.343	10.051	0.000	0.000	0.000	10.874	0.000	0.251	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	0	61	0	70	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.66	0.00	0.76	0.00
time (sec)	N/A	0.349	10.034	0.000	0.000	0.000	17.812	0.000	0.253	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	117	72	0	0	0	66	0	71	0
N.S.	1	0.96	0.59	0.00	0.00	0.00	0.54	0.00	0.58	0.00
time (sec)	N/A	0.380	10.031	0.000	0.000	0.000	27.521	0.000	0.251	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	74	68	118	88	172	114	73	91
N.S.	1	1.04	0.73	0.67	1.17	0.87	1.70	1.13	0.72	0.90
time (sec)	N/A	0.413	0.071	0.133	0.030	0.081	1.816	0.122	0.236	3.578

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	56	49	83	64	119	77	73	60
N.S.	1	1.07	0.80	0.70	1.19	0.91	1.70	1.10	1.04	0.86
time (sec)	N/A	0.364	0.049	0.115	0.039	0.084	1.274	0.126	0.229	3.570

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	34	31	48	42	70	37	73	32
N.S.	1	1.12	0.79	0.72	1.12	0.98	1.63	0.86	1.70	0.74
time (sec)	N/A	0.328	0.037	0.103	0.028	0.081	0.673	0.115	0.254	3.491

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	100	81	88	96	249	68	215	70	71
N.S.	1	1.16	0.94	1.02	1.12	2.90	0.79	2.50	0.81	0.83
time (sec)	N/A	0.368	0.136	0.139	0.116	0.095	23.743	0.132	0.257	3.716

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	108	136	196	689	82	262	69	147
N.S.	1	1.03	0.89	1.12	1.62	5.69	0.68	2.17	0.57	1.21
time (sec)	N/A	0.401	0.262	0.279	0.122	0.110	42.272	0.124	0.206	4.333

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	139	107	134	230	697	80	0	73	0
N.S.	1	1.10	0.85	1.06	1.83	5.53	0.63	0.00	0.58	0.00
time (sec)	N/A	0.444	1.055	0.315	0.115	0.099	14.539	0.000	0.204	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	86	97	113	273	73	0	73	0
N.S.	1	1.04	0.95	1.07	1.24	3.00	0.80	0.00	0.80	0.00
time (sec)	N/A	0.369	0.746	0.183	0.113	0.095	8.044	0.000	0.198	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	36	57	45	105	0	72	47
N.S.	1	1.00	0.76	0.71	1.12	0.88	2.06	0.00	1.41	0.92
time (sec)	N/A	0.303	0.469	0.164	0.029	0.084	11.749	0.000	0.276	4.093

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	62	57	98	70	398	0	71	58
N.S.	1	0.98	0.75	0.69	1.18	0.84	4.80	0.00	0.86	0.70
time (sec)	N/A	0.352	0.549	0.200	0.045	0.083	27.981	0.000	0.256	4.244

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	111	81	76	135	94	920	0	71	82
N.S.	1	0.97	0.71	0.67	1.18	0.82	8.07	0.00	0.62	0.72
time (sec)	N/A	0.401	0.958	0.270	0.036	0.079	68.282	0.000	0.207	3.929

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	104	0	0	0	60	0	73	0
N.S.	1	1.08	0.68	0.00	0.00	0.00	0.39	0.00	0.48	0.00
time (sec)	N/A	0.458	10.076	0.000	0.000	0.000	17.590	0.000	0.257	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	133	78	0	0	0	60	0	73	0
N.S.	1	1.11	0.65	0.00	0.00	0.00	0.50	0.00	0.61	0.00
time (sec)	N/A	0.408	10.059	0.000	0.000	0.000	9.566	0.000	0.225	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	75	0	0	0	60	0	71	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.59	0.00	0.70	0.00
time (sec)	N/A	0.366	10.070	0.000	0.000	0.000	8.284	0.000	0.223	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	86	0	0	0	61	0	70	0
N.S.	1	0.99	0.70	0.00	0.00	0.00	0.50	0.00	0.57	0.00
time (sec)	N/A	0.377	10.040	0.000	0.000	0.000	13.345	0.000	0.246	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	146	73	0	0	0	66	0	71	0
N.S.	1	0.96	0.48	0.00	0.00	0.00	0.43	0.00	0.47	0.00
time (sec)	N/A	0.423	10.044	0.000	0.000	0.000	27.642	0.000	0.266	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	163	101	0	0	0	80	0	73	0
N.S.	1	1.08	0.67	0.00	0.00	0.00	0.53	0.00	0.48	0.00
time (sec)	N/A	0.568	10.075	0.000	0.000	0.000	20.758	0.000	0.243	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	135	78	0	0	0	80	0	73	0
N.S.	1	1.12	0.65	0.00	0.00	0.00	0.67	0.00	0.61	0.00
time (sec)	N/A	0.476	10.059	0.000	0.000	0.000	12.747	0.000	0.208	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	72	0	0	0	78	0	69	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.76	0.00	0.68	0.00
time (sec)	N/A	0.422	0.031	0.000	0.000	0.000	10.798	0.000	0.224	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	120	85	0	0	0	82	0	68	0
N.S.	1	0.98	0.69	0.00	0.00	0.00	0.67	0.00	0.55	0.00
time (sec)	N/A	0.457	10.029	0.000	0.000	0.000	29.402	0.000	0.203	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	151	72	0	0	0	90	0	71	0
N.S.	1	0.97	0.46	0.00	0.00	0.00	0.58	0.00	0.46	0.00
time (sec)	N/A	0.512	10.031	0.000	0.000	0.000	43.412	0.000	0.245	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	105	80	68	118	99	338	105	113	90
N.S.	1	1.05	0.80	0.68	1.18	0.99	3.38	1.05	1.13	0.90
time (sec)	N/A	0.422	0.069	0.155	0.029	0.085	1.480	0.130	0.297	3.637

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	56	49	84	75	240	63	113	60
N.S.	1	1.06	0.79	0.69	1.18	1.06	3.38	0.89	1.59	0.85
time (sec)	N/A	0.376	0.053	0.135	0.035	0.094	1.008	0.115	0.307	3.703

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	48	34	30	49	52	143	32	113	32
N.S.	1	1.09	0.77	0.68	1.11	1.18	3.25	0.73	2.57	0.73
time (sec)	N/A	0.328	0.038	0.100	0.029	0.079	1.309	0.125	0.254	3.701

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	116	94	104	105	335	88	229	110	85
N.S.	1	1.14	0.92	1.02	1.03	3.28	0.86	2.25	1.08	0.83
time (sec)	N/A	0.400	0.162	0.161	0.113	0.129	43.028	0.129	0.280	3.743

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	141	129	132	218	879	82	287	109	177
N.S.	1	0.97	0.88	0.90	1.49	6.02	0.56	1.97	0.75	1.21
time (sec)	N/A	0.434	0.329	0.309	0.120	0.127	132.243	0.130	0.283	4.239

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	162	126	131	259	884	80	0	113	0
N.S.	1	1.07	0.83	0.87	1.72	5.85	0.53	0.00	0.75	0.00
time (sec)	N/A	0.493	2.037	0.351	0.122	0.152	38.697	0.000	0.224	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	99	117	126	358	102	0	113	0
N.S.	1	1.07	0.91	1.07	1.16	3.28	0.94	0.00	1.04	0.00
time (sec)	N/A	0.413	1.345	0.217	0.110	0.109	27.469	0.000	0.249	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	47	37	34	51	54	190	0	109	33
N.S.	1	0.77	0.61	0.56	0.84	0.89	3.11	0.00	1.79	0.54
time (sec)	N/A	0.274	0.007	0.133	0.042	0.107	18.883	0.000	0.264	3.314

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	62	57	88	81	360	0	108	70
N.S.	1	0.99	0.78	0.72	1.11	1.03	4.56	0.00	1.37	0.89
time (sec)	N/A	0.331	0.551	0.224	0.030	0.093	39.944	0.000	0.213	3.523

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	108	86	76	131	105	726	0	111	147
N.S.	1	0.96	0.76	0.67	1.16	0.93	6.42	0.00	0.98	1.30
time (sec)	N/A	0.372	0.796	0.285	0.040	0.088	79.394	0.000	0.216	4.259

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	165	109	0	0	0	80	0	113	0
N.S.	1	1.09	0.72	0.00	0.00	0.00	0.53	0.00	0.74	0.00
time (sec)	N/A	0.529	10.088	0.000	0.000	0.000	63.217	0.000	0.251	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	135	93	0	0	0	80	0	113	0
N.S.	1	1.11	0.76	0.00	0.00	0.00	0.66	0.00	0.93	0.00
time (sec)	N/A	0.495	10.067	0.000	0.000	0.000	34.112	0.000	0.229	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	0	0	0	80	0	113	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.78	0.00	1.10	0.00
time (sec)	N/A	0.439	10.052	0.000	0.000	0.000	19.409	0.000	0.237	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	118	80	0	0	0	82	0	112	0
N.S.	1	0.98	0.66	0.00	0.00	0.00	0.68	0.00	0.93	0.00
time (sec)	N/A	0.464	10.028	0.000	0.000	0.000	29.570	0.000	0.239	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	145	81	0	0	0	88	0	111	0
N.S.	1	0.95	0.53	0.00	0.00	0.00	0.58	0.00	0.73	0.00
time (sec)	N/A	0.526	10.050	0.000	0.000	0.000	77.343	0.000	0.230	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	159	107	0	0	0	60	0	113	0
N.S.	1	1.05	0.71	0.00	0.00	0.00	0.40	0.00	0.75	0.00
time (sec)	N/A	0.449	10.094	0.000	0.000	0.000	41.381	0.000	0.266	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	129	101	0	0	0	60	0	113	0
N.S.	1	1.10	0.86	0.00	0.00	0.00	0.51	0.00	0.97	0.00
time (sec)	N/A	0.393	10.080	0.000	0.000	0.000	28.204	0.000	0.274	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	0	0	0	60	0	111	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.59	0.00	1.09	0.00
time (sec)	N/A	0.358	10.078	0.000	0.000	0.000	20.830	0.000	0.301	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	121	118	0	0	0	61	0	110	0
N.S.	1	0.99	0.97	0.00	0.00	0.00	0.50	0.00	0.90	0.00
time (sec)	N/A	0.375	10.049	0.000	0.000	0.000	38.532	0.000	0.273	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	146	81	0	0	0	66	0	111	0
N.S.	1	0.96	0.53	0.00	0.00	0.00	0.43	0.00	0.73	0.00
time (sec)	N/A	0.420	10.037	0.000	0.000	0.000	78.090	0.000	0.319	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	191	147	150	296	963	0	0	153	0
N.S.	1	1.05	0.81	0.82	1.63	5.29	0.00	0.00	0.84	0.00
time (sec)	N/A	0.549	6.113	0.445	0.149	0.119	0.000	0.000	0.246	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	143	112	123	145	426	133	0	153	0
N.S.	1	1.09	0.85	0.94	1.11	3.25	1.02	0.00	1.17	0.00
time (sec)	N/A	0.450	3.191	0.273	0.113	0.129	126.474	0.000	0.232	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	37	53	68	284	0	153	56
N.S.	1	1.00	0.61	0.56	0.80	1.03	4.30	0.00	2.32	0.85
time (sec)	N/A	0.312	2.130	0.175	0.042	0.106	131.628	0.000	0.222	3.610

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	88	60	52	86	87	709	0	149	87
N.S.	1	0.97	0.66	0.57	0.95	0.96	7.79	0.00	1.64	0.96
time (sec)	N/A	0.333	0.010	0.158	0.032	0.097	86.381	0.000	0.264	3.629

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	103	81	76	125	116	898	0	148	82
N.S.	1	0.96	0.76	0.71	1.17	1.08	8.39	0.00	1.38	0.77
time (sec)	N/A	0.372	0.791	0.324	0.041	0.125	165.828	0.000	0.218	3.848

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	133	105	95	168	140	0	0	151	173
N.S.	1	0.93	0.73	0.66	1.17	0.98	0.00	0.00	1.06	1.21
time (sec)	N/A	0.420	1.137	0.404	0.038	0.097	0.000	0.000	0.237	4.241

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	194	133	0	0	0	0	0	153	0
N.S.	1	1.07	0.73	0.00	0.00	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.581	10.137	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	117	0	0	0	80	0	153	0
N.S.	1	1.08	0.77	0.00	0.00	0.00	0.53	0.00	1.01	0.00
time (sec)	N/A	0.541	10.106	0.000	0.000	0.000	148.982	0.000	0.234	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	135	98	0	0	0	80	0	153	0
N.S.	1	1.02	0.74	0.00	0.00	0.00	0.61	0.00	1.16	0.00
time (sec)	N/A	0.476	10.080	0.000	0.000	0.000	132.766	0.000	0.228	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	122	0	0	0	80	0	153	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.60	0.00	1.14	0.00
time (sec)	N/A	0.487	10.061	0.000	0.000	0.000	88.586	0.000	0.209	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	147	84	0	0	0	82	0	152	0
N.S.	1	0.97	0.56	0.00	0.00	0.00	0.54	0.00	1.01	0.00
time (sec)	N/A	0.523	10.034	0.000	0.000	0.000	110.497	0.000	0.246	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	183	176	83	0	0	0	0	0	151	0
N.S.	1	0.96	0.45	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.584	10.035	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	220	169	169	331	1042	0	0	193	0
N.S.	1	1.03	0.79	0.79	1.55	4.89	0.00	0.00	0.91	0.00
time (sec)	N/A	0.602	6.534	0.599	0.147	0.145	0.000	0.000	0.298	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	169	124	136	162	494	0	0	193	0
N.S.	1	1.10	0.81	0.89	1.06	3.23	0.00	0.00	1.26	0.00
time (sec)	N/A	0.528	6.198	0.378	0.124	0.135	0.000	0.000	0.253	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	37	53	79	0	0	193	156
N.S.	1	1.00	0.61	0.56	0.80	1.20	0.00	0.00	2.92	2.36
time (sec)	N/A	0.315	5.099	0.226	0.043	0.111	0.000	0.000	0.224	3.568

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	62	55	86	102	0	0	193	116
N.S.	1	0.97	0.63	0.56	0.87	1.03	0.00	0.00	1.95	1.17
time (sec)	N/A	0.373	2.961	0.182	0.038	0.105	0.000	0.000	0.224	3.417

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	115	80	71	120	121	0	0	189	105
N.S.	1	0.95	0.66	0.59	0.99	1.00	0.00	0.00	1.56	0.87
time (sec)	N/A	0.378	0.016	0.177	0.039	0.114	0.000	0.000	0.246	3.402

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	132	102	95	159	151	0	0	188	120
N.S.	1	0.98	0.76	0.70	1.18	1.12	0.00	0.00	1.39	0.89
time (sec)	N/A	0.417	1.282	0.449	0.056	0.110	0.000	0.000	0.231	3.602

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	162	126	114	202	175	0	0	191	241
N.S.	1	0.94	0.73	0.66	1.17	1.01	0.00	0.00	1.10	1.39
time (sec)	N/A	0.468	1.698	0.861	0.044	0.120	0.000	0.000	0.257	4.023

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	223	154	0	0	0	0	0	193	0
N.S.	1	1.05	0.73	0.00	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	0.653	10.132	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	193	139	0	0	0	0	0	193	0
N.S.	1	1.06	0.76	0.00	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.610	10.123	0.000	0.000	0.000	0.000	0.000	0.222	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	164	120	0	0	0	0	0	193	0
N.S.	1	1.01	0.74	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.528	10.122	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	164	134	0	0	0	0	0	193	0
N.S.	1	0.99	0.81	0.00	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	0.532	10.074	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	164	123	0	0	0	0	0	193	0
N.S.	1	0.97	0.73	0.00	0.00	0.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.555	10.066	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	176	84	0	0	0	0	0	192	0
N.S.	1	0.97	0.46	0.00	0.00	0.00	0.00	0.00	1.06	0.00
time (sec)	N/A	0.557	10.039	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	213	205	83	0	0	0	0	0	191	0
N.S.	1	0.96	0.39	0.00	0.00	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.647	10.044	0.000	0.000	0.000	0.000	0.000	0.255	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	108	0	0	0	0	0	0	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.441	0.098	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	97	81	0	0	0	0	0	119	0
N.S.	1	1.09	0.91	0.00	0.00	0.00	0.00	0.00	1.34	0.00
time (sec)	N/A	0.408	0.114	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	106	19	32	24	0	37	24	18
N.S.	1	1.00	4.82	0.86	1.45	1.09	0.00	1.68	1.09	0.82
time (sec)	N/A	0.267	0.118	5.714	0.071	0.085	0.000	0.133	0.209	3.983

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	87	82	78	84	100	0	88	180	88
N.S.	1	0.97	0.91	0.87	0.93	1.11	0.00	0.98	2.00	0.98
time (sec)	N/A	0.441	0.041	0.219	0.042	2.349	0.000	0.132	0.252	4.877

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	66	66	65	68	72	0	70	150	68
N.S.	1	0.94	0.94	0.93	0.97	1.03	0.00	1.00	2.14	0.97
time (sec)	N/A	0.375	0.025	0.138	0.025	0.813	0.000	0.123	0.263	4.936

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	43	43	49	42	144	51	115	51
N.S.	1	0.98	0.81	0.81	0.92	0.79	2.72	0.96	2.17	0.96
time (sec)	N/A	0.348	0.020	0.124	0.025	0.275	40.790	0.124	0.255	4.394

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	44	31	32	41	31	138	51	99	1012
N.S.	1	0.98	0.69	0.71	0.91	0.69	3.07	1.13	2.20	22.49
time (sec)	N/A	0.296	0.020	0.098	0.025	0.087	0.962	0.126	0.224	4.096

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	61	54	0	73	127	58
N.S.	1	1.02	0.87	0.89	0.98	0.87	0.00	1.18	2.05	0.94
time (sec)	N/A	0.384	0.025	0.144	0.041	0.981	0.000	0.121	0.261	4.242

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	88	82	87	99	0	112	185	87
N.S.	1	0.99	1.01	0.94	1.00	1.14	0.00	1.29	2.13	1.00
time (sec)	N/A	0.438	0.033	0.175	0.026	4.481	0.000	0.131	0.313	5.435

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	132	104	98	100	576	0	112	222	532
N.S.	1	1.18	0.93	0.88	0.89	5.14	0.00	1.00	1.98	4.75
time (sec)	N/A	0.576	0.110	0.197	0.113	1.294	0.000	0.129	0.232	4.939

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	99	82	81	80	416	932	80	184	518
N.S.	1	1.08	0.89	0.88	0.87	4.52	10.13	0.87	2.00	5.63
time (sec)	N/A	0.415	0.104	0.158	0.106	0.303	21.416	0.131	0.250	4.414

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	66	60	59	325	0	59	153	379
N.S.	1	0.99	0.84	0.76	0.75	4.11	0.00	0.75	1.94	4.80
time (sec)	N/A	0.346	0.028	0.147	0.106	0.142	0.000	0.120	0.254	4.507

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	66	60	59	309	0	59	153	399
N.S.	1	0.99	0.84	0.76	0.75	3.91	0.00	0.75	1.94	5.05
time (sec)	N/A	0.324	0.035	0.140	0.114	0.156	0.000	0.133	0.218	4.340

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	100	169	81	80	416	1103	80	193	354
N.S.	1	1.09	1.84	0.88	0.87	4.52	11.99	0.87	2.10	3.85
time (sec)	N/A	0.413	0.151	0.158	0.109	0.372	72.161	0.125	0.245	4.733

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	133	193	101	101	576	0	103	231	535
N.S.	1	1.19	1.72	0.90	0.90	5.14	0.00	0.92	2.06	4.78
time (sec)	N/A	0.553	0.247	0.186	0.107	1.816	0.000	0.130	0.277	4.970

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	449	377	234	375	1196	0	469	327	6361
N.S.	1	1.34	1.13	0.70	1.12	3.57	0.00	1.40	0.98	18.99
time (sec)	N/A	1.227	0.253	0.132	0.112	0.227	0.000	0.133	0.251	4.773

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	426	340	226	363	1427	0	453	281	2553
N.S.	1	1.30	1.04	0.69	1.11	4.36	0.00	1.39	0.86	7.81
time (sec)	N/A	1.083	0.123	0.124	0.111	0.145	0.000	0.138	0.250	4.560

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	426	340	218	361	1067	0	437	281	5889
N.S.	1	1.30	1.04	0.67	1.10	3.26	0.00	1.34	0.86	18.01
time (sec)	N/A	1.065	0.076	0.119	0.116	0.108	0.000	0.125	0.267	4.773

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	426	340	218	363	1331	0	477	281	6633
N.S.	1	1.30	1.04	0.67	1.11	4.07	0.00	1.46	0.86	20.28
time (sec)	N/A	1.070	0.081	0.115	0.111	0.139	0.000	0.136	0.244	4.473

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	426	340	226	365	1171	0	437	281	6153
N.S.	1	1.30	1.04	0.69	1.12	3.58	0.00	1.34	0.86	18.82
time (sec)	N/A	1.017	0.084	0.111	0.112	0.184	0.000	0.133	0.237	5.083

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	477	385	237	384	1461	0	488	336	5962
N.S.	1	1.41	1.14	0.70	1.14	4.32	0.00	1.44	0.99	17.64
time (sec)	N/A	1.081	0.142	0.161	0.115	0.324	0.000	0.134	0.253	4.977

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	454	406	241	390	1255	0	472	354	7459
N.S.	1	1.34	1.19	0.71	1.15	3.69	0.00	1.39	1.04	21.94
time (sec)	N/A	1.181	0.191	0.161	0.113	2.658	0.000	0.132	0.241	5.279

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	522	428	261	405	1526	0	483	382	4547
N.S.	1	1.46	1.20	0.73	1.13	4.27	0.00	1.35	1.07	12.74
time (sec)	N/A	1.183	0.209	0.176	0.115	4.643	0.000	0.134	0.287	4.904

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	363	305	157	306	2079	430	513	1355	610
N.S.	1	1.42	1.20	0.62	1.20	8.15	1.69	2.01	5.31	2.39
time (sec)	N/A	0.895	0.175	0.115	0.116	0.115	8.898	0.131	0.268	0.256

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	318	301	151	405	1742	337	496	1347	1615
N.S.	1	1.28	1.21	0.61	1.63	7.00	1.35	1.99	5.41	6.49
time (sec)	N/A	0.939	0.168	0.089	0.120	0.114	10.400	0.131	0.227	0.283

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	353	309	221	299	2090	425	499	1356	651
N.S.	1	1.43	1.25	0.89	1.21	8.46	1.72	2.02	5.49	2.64
time (sec)	N/A	0.919	0.176	0.123	0.118	0.110	21.434	0.131	0.206	3.499

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	350	305	216	409	1764	0	487	1388	1740
N.S.	1	1.45	1.26	0.89	1.69	7.29	0.00	2.01	5.74	7.19
time (sec)	N/A	0.891	0.175	0.119	0.115	0.105	0.000	0.129	0.257	0.313

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	360	305	227	312	2096	440	502	1403	652
N.S.	1	1.43	1.21	0.90	1.24	8.32	1.75	1.99	5.57	2.59
time (sec)	N/A	0.917	0.186	0.112	0.110	0.117	175.519	0.131	0.221	3.476

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	124	0	280	167	139	1108	196
N.S.	1	1.00	0.97	0.99	0.00	2.24	1.34	1.11	8.86	1.57
time (sec)	N/A	0.502	0.224	0.691	0.000	0.084	12.168	0.118	0.221	3.502

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	98	88	84	0	195	122	96	583	87
N.S.	1	1.05	0.95	0.90	0.00	2.10	1.31	1.03	6.27	0.94
time (sec)	N/A	0.391	0.118	0.193	0.000	0.089	8.177	0.130	0.247	3.246

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	62	0	156	92	66	218	54
N.S.	1	1.00	0.99	0.89	0.00	2.23	1.31	0.94	3.11	0.77
time (sec)	N/A	0.355	0.078	0.147	0.000	0.098	4.354	0.123	0.238	3.372

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	71	0	377	158	79	802	199
N.S.	1	1.00	0.95	0.84	0.00	4.44	1.86	0.93	9.44	2.34
time (sec)	N/A	0.395	0.113	0.166	0.000	0.104	7.201	0.127	0.255	3.685

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	107	96	0	507	0	107	2930	269
N.S.	1	1.05	0.93	0.83	0.00	4.41	0.00	0.93	25.48	2.34
time (sec)	N/A	0.457	0.288	0.235	0.000	0.111	0.000	0.123	0.356	4.143

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	128	142	102	0	720	0	0	870	0
N.S.	1	1.07	1.18	0.85	0.00	6.00	0.00	0.00	7.25	0.00
time (sec)	N/A	0.506	0.864	0.595	0.000	0.145	0.000	0.000	0.262	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	107	77	0	617	0	0	218	0
N.S.	1	0.98	1.18	0.85	0.00	6.78	0.00	0.00	2.40	0.00
time (sec)	N/A	0.390	0.277	0.373	0.000	0.109	0.000	0.000	0.239	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	96	69	0	281	0	121	455	0
N.S.	1	1.00	1.26	0.91	0.00	3.70	0.00	1.59	5.99	0.00
time (sec)	N/A	0.385	0.332	0.593	0.000	0.098	0.000	0.267	0.320	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	118	92	0	329	0	225	1091	0
N.S.	1	1.07	1.07	0.84	0.00	2.99	0.00	2.05	9.92	0.00
time (sec)	N/A	0.517	0.470	0.994	0.000	0.116	0.000	0.266	0.391	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	173	153	139	0	413	0	492	1815	0
N.S.	1	1.09	0.96	0.87	0.00	2.60	0.00	3.09	11.42	0.00
time (sec)	N/A	0.664	0.832	1.692	0.000	0.135	0.000	0.343	0.515	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	671	1002	241	303	0	0	0	0	139	0
N.S.	1	1.49	0.36	0.45	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	2.637	10.330	4.010	0.000	0.000	0.000	0.000	0.455	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	749	964	161	273	0	0	0	0	20	0
N.S.	1	1.29	0.21	0.36	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.161	10.128	0.444	0.000	0.000	0.000	0.000	0.237	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	674	988	333	294	0	0	0	0	24	0
N.S.	1	1.47	0.49	0.44	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.336	10.196	5.468	0.000	0.000	0.000	0.000	0.798	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	857	1069	141	332	0	0	0	0	144	0
N.S.	1	1.25	0.16	0.39	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	2.653	10.100	5.668	0.000	0.000	0.000	0.000	0.497	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	786	1096	65	299	0	0	0	0	23	0
N.S.	1	1.39	0.08	0.38	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.550	10.031	0.457	0.000	0.000	0.000	0.000	0.281	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	809	1021	138	318	0	0	0	0	24	0
N.S.	1	1.26	0.17	0.39	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.283	10.143	4.724	0.000	0.000	0.000	0.000	0.628	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	93	0	289	0	102	794	102
N.S.	1	1.00	0.88	0.89	0.00	2.78	0.00	0.98	7.63	0.98
time (sec)	N/A	0.447	0.243	0.207	0.000	0.110	0.000	0.124	0.250	3.984

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	59	0	205	0	64	295	58
N.S.	1	1.00	0.99	0.80	0.00	2.77	0.00	0.86	3.99	0.78
time (sec)	N/A	0.344	0.107	0.154	0.000	0.100	0.000	0.119	0.213	3.920

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	92	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	1.80	0.78
time (sec)	N/A	0.307	0.048	0.094	0.000	0.089	7.227	0.122	0.214	3.970

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	65	0	385	114	71	874	652
N.S.	1	1.00	0.94	0.76	0.00	4.53	1.34	0.84	10.28	7.67
time (sec)	N/A	0.225	0.155	0.158	0.000	0.125	6.868	0.122	0.270	4.323

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	518	0	104	3329	396
N.S.	1	1.09	0.93	0.79	0.00	4.43	0.00	0.89	28.45	3.38
time (sec)	N/A	0.272	0.296	0.230	0.000	0.130	0.000	0.116	0.355	4.925

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	136	98	0	745	0	0	1128	0
N.S.	1	1.09	1.11	0.80	0.00	6.06	0.00	0.00	9.17	0.00
time (sec)	N/A	0.300	0.725	0.805	0.000	0.212	0.000	0.000	0.259	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	108	70	0	638	0	0	250	0
N.S.	1	0.99	1.19	0.77	0.00	7.01	0.00	0.00	2.75	0.00
time (sec)	N/A	0.242	0.383	0.441	0.000	0.171	0.000	0.000	0.243	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	165	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	3.06	0.00
time (sec)	N/A	0.188	0.545	0.322	0.000	0.143	0.000	0.131	0.238	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	67	0	332	0	116	547	0
N.S.	1	1.00	1.25	0.84	0.00	4.15	0.00	1.45	6.84	0.00
time (sec)	N/A	0.230	0.402	0.569	0.000	0.128	0.000	0.136	0.305	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	121	88	0	418	0	205	1240	0
N.S.	1	1.10	1.05	0.77	0.00	3.63	0.00	1.78	10.78	0.00
time (sec)	N/A	0.297	0.755	1.000	0.000	0.146	0.000	0.257	0.362	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	660	1002	249	298	0	0	0	0	142	0
N.S.	1	1.52	0.38	0.45	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	1.432	10.247	4.783	0.000	0.000	0.000	0.000	0.494	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	625	958	65	265	0	0	0	0	38	0
N.S.	1	1.53	0.10	0.42	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.180	10.049	1.125	0.000	0.000	0.000	0.000	0.233	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	625	860	161	191	0	0	0	0	35	0
N.S.	1	1.38	0.26	0.31	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.989	10.049	0.436	0.000	0.000	0.000	0.000	0.245	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	662	988	337	288	0	0	0	0	38	0
N.S.	1	1.49	0.51	0.44	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.304	10.223	4.178	0.000	0.000	0.000	0.000	0.271	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	804	1089	65	292	0	0	0	0	38	0
N.S.	1	1.35	0.08	0.36	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.472	10.076	2.116	0.000	0.000	0.000	0.000	0.229	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	656	865	65	191	0	0	0	0	38	0
N.S.	1	1.32	0.10	0.29	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.870	10.037	0.436	0.000	0.000	0.000	0.000	0.264	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	833	999	141	310	0	0	0	0	38	0
N.S.	1	1.20	0.17	0.37	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.314	10.093	3.495	0.000	0.000	0.000	0.000	0.240	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	182	175	171	0	622	0	180	3265	186
N.S.	1	1.18	1.14	1.11	0.00	4.04	0.00	1.17	21.20	1.21
time (sec)	N/A	0.492	0.473	0.369	0.000	0.117	0.000	0.126	0.217	4.483

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	142	2072	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.15	16.85	1.17
time (sec)	N/A	0.459	0.317	0.257	0.000	0.123	0.000	0.124	0.263	4.466

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	1344	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	13.58	0.96
time (sec)	N/A	0.377	0.214	0.191	0.000	0.107	0.000	0.132	0.219	4.027

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	86	71	0	302	0	93	872	84
N.S.	1	0.99	0.99	0.82	0.00	3.47	0.00	1.07	10.02	0.97
time (sec)	N/A	0.362	0.210	0.148	0.000	0.110	0.000	0.121	0.227	4.167

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	816	0	139	9123	3017
N.S.	1	1.17	0.94	1.11	0.00	6.18	0.00	1.05	69.11	22.86
time (sec)	N/A	0.480	0.400	0.276	0.000	0.173	0.000	0.122	0.430	5.136

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1189	0	257	15292	3822
N.S.	1	1.15	0.88	1.03	0.00	6.43	0.00	1.39	82.66	20.66
time (sec)	N/A	0.602	1.043	0.391	0.000	0.180	0.000	0.124	1.022	6.207

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	213	189	143	0	1391	0	333	5868	0
N.S.	1	1.12	0.99	0.75	0.00	7.28	0.00	1.74	30.72	0.00
time (sec)	N/A	0.734	2.027	1.743	0.000	0.564	0.000	0.217	0.562	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1083	0	298	3238	0
N.S.	1	1.18	1.09	0.83	0.00	7.68	0.00	2.11	22.96	0.00
time (sec)	N/A	0.523	1.389	0.954	0.000	0.331	0.000	0.207	0.350	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	244	1561	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	2.62	16.78	0.00
time (sec)	N/A	0.395	0.838	0.797	0.000	0.146	0.000	0.356	0.351	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	2444	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	23.50	0.00
time (sec)	N/A	0.383	1.130	0.783	0.000	0.170	0.000	0.148	0.403	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	418	3526	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	2.81	23.66	0.00
time (sec)	N/A	0.527	1.080	1.303	0.000	0.249	0.000	0.379	0.492	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	395	7920	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	1.90	38.08	0.00
time (sec)	N/A	0.680	2.115	2.221	0.000	0.297	0.000	0.472	2.106	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	852	1162	404	421	0	0	0	0	0	0
N.S.	1	1.36	0.47	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.973	10.678	13.645	0.000	0.000	0.000	0.000	5.819	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	776	1094	402	373	0	0	0	0	1632	0
N.S.	1	1.41	0.52	0.48	0.00	0.00	0.00	0.00	2.10	0.00
time (sec)	N/A	2.723	10.449	8.805	0.000	0.000	0.000	0.000	3.837	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	1032	253	337	0	0	0	0	62	0
N.S.	1	1.43	0.35	0.47	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.383	10.256	3.243	0.000	0.000	0.000	0.000	1.063	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	1013	238	324	0	0	0	0	62	0
N.S.	1	1.64	0.39	0.53	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.273	10.177	3.253	0.000	0.000	0.000	0.000	0.979	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	1016	392	333	0	0	0	0	59	0
N.S.	1	1.41	0.54	0.46	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.345	10.299	1.085	0.000	0.000	0.000	0.000	0.255	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	778	1074	408	364	0	0	0	0	442	0
N.S.	1	1.38	0.52	0.47	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	2.603	10.485	8.438	0.000	0.000	0.000	0.000	2.083	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	854	1144	383	405	0	0	0	0	778	0
N.S.	1	1.34	0.45	0.47	0.00	0.00	0.00	0.00	0.91	0.00
time (sec)	N/A	2.787	10.674	10.155	0.000	0.000	0.000	0.000	4.921	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1159	1085	162	353	0	0	0	0	62	0
N.S.	1	0.94	0.14	0.30	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.474	10.090	4.580	0.000	0.000	0.000	0.000	0.981	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1169	1071	172	359	0	0	0	0	62	0
N.S.	1	0.92	0.15	0.31	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.158	10.128	1.139	0.000	0.000	0.000	0.000	0.253	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1248	1148	226	392	0	0	0	0	445	0
N.S.	1	0.92	0.18	0.31	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	2.434	10.214	8.730	0.000	0.000	0.000	0.000	2.041	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	175	53	50	0	51	0	0	19	0
N.S.	1	3.57	1.08	1.02	0.00	1.04	0.00	0.00	0.39	0.00
time (sec)	N/A	0.621	0.205	1.180	0.000	0.135	0.000	0.000	0.193	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	27	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.440	11.075	0.000	0.000	0.000	0.000	0.000	9.510	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	25	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.442	11.054	0.000	0.000	0.000	0.000	0.000	9.467	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	31	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.382	11.057	0.000	0.000	0.000	0.000	0.000	88.315	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0	32	0
N.S.	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.439	11.122	0.000	0.000	0.000	0.000	0.000	101.012	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	195	164	0	0	0	182	0	1018	0
N.S.	1	1.04	0.87	0.00	0.00	0.00	0.97	0.00	5.41	0.00
time (sec)	N/A	0.624	9.442	0.000	0.000	0.000	6.472	0.000	0.275	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	117	110	0	0	0	117	0	268	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	1.02	0.00	2.33	0.00
time (sec)	N/A	0.405	1.532	0.000	0.000	0.000	2.227	0.000	0.230	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	27	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.40	0.00
time (sec)	N/A	0.300	0.589	0.000	0.000	0.000	0.604	0.000	0.233	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0	42	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.368	4.153	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	66	0
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.368	10.007	0.000	0.000	0.000	0.000	0.000	0.313	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	11.100	0.000	0.000	0.000	0.000	0.000	0.401	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	205	200	167	0	0	0	0	0	0	0
N.S.	1	0.98	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.633	11.126	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	132	113	0	0	0	117	0	701	0
N.S.	1	1.12	0.96	0.00	0.00	0.00	0.99	0.00	5.94	0.00
time (sec)	N/A	0.425	4.259	0.000	0.000	0.000	18.109	0.000	0.297	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	38	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.54	0.00
time (sec)	N/A	0.311	1.343	0.000	0.000	0.000	0.741	0.000	0.225	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0	66	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	0.364	11.129	0.000	0.000	0.000	0.000	0.000	0.322	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	11.114	0.000	0.000	0.000	0.000	0.000	0.464	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	11.118	0.000	0.000	0.000	0.000	0.000	0.615	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	369	430	188	0	0	0	0	0	0	0
N.S.	1	1.17	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.437	6.323	0.000	0.000	0.000	0.000	0.000	0.516	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	214	265	0	0	0	0	0	0	0
N.S.	1	1.03	1.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.660	97.510	0.000	0.000	0.000	0.000	0.000	0.324	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	108	0	0	0	0	0	0	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.024	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	174	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	2.64	0.00
time (sec)	N/A	0.297	0.030	0.000	0.000	0.000	85.952	0.000	0.220	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	0	0	0	0	0	28	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.361	0.169	0.000	0.000	0.000	0.000	0.000	0.302	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	0	0	0	0	0	39	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.366	0.218	0.000	0.000	0.000	0.000	0.000	1.498	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	1026	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	10.16	0.00
time (sec)	N/A	0.438	0.179	0.000	0.000	0.000	0.000	0.000	0.707	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	0	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	0.089	0.000	0.000	0.000	0.000	0.000	0.598	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	0.159	0.000	0.000	0.000	0.000	0.000	1.072	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [24] had the largest ratio of [.550000000000000044]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	2	2	1.00	18	0.111
3	A	2	2	1.00	15	0.133
4	A	2	2	1.00	18	0.111
5	A	2	2	1.00	18	0.111
6	A	2	2	1.00	20	0.100
7	A	2	2	1.00	20	0.100
8	A	2	2	1.00	17	0.118
9	A	2	2	1.00	20	0.100
10	A	2	2	1.00	20	0.100
11	A	11	10	1.30	20	0.500
12	A	10	9	1.33	20	0.450
13	A	10	9	1.34	17	0.529
14	A	10	9	1.34	20	0.450
15	A	10	9	1.33	20	0.450
16	A	11	10	1.30	20	0.500
17	A	3	3	1.19	20	0.150
18	A	11	10	1.34	20	0.500
19	A	11	10	1.38	20	0.500
20	A	10	9	1.32	20	0.450
21	A	10	9	1.33	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	11	10	1.35	20	0.500
23	A	11	10	1.36	20	0.500
24	A	12	11	1.31	20	0.550
25	A	4	4	0.99	22	0.182
26	A	3	3	0.99	22	0.136
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	22	0.091
29	A	3	3	0.99	22	0.136
30	A	4	4	0.99	22	0.182
31	A	6	6	0.96	22	0.273
32	A	5	5	0.95	22	0.227
33	A	5	5	0.96	22	0.227
34	A	6	6	0.93	22	0.273
35	A	4	4	1.00	22	0.182
36	A	3	3	1.01	22	0.136
37	A	2	2	1.00	19	0.105
38	A	3	3	0.99	22	0.136
39	A	4	4	0.99	22	0.182
40	A	7	7	0.96	22	0.318
41	A	6	6	0.96	22	0.273
42	A	5	5	0.96	22	0.227
43	A	6	6	0.94	22	0.273
44	A	7	7	0.94	22	0.318
45	A	5	5	0.99	22	0.227
46	A	4	4	0.99	22	0.182
47	A	3	3	1.03	22	0.136
48	A	3	3	1.00	19	0.158
49	A	4	4	0.98	22	0.182
50	A	5	5	0.99	22	0.227
51	A	7	7	0.96	22	0.318
52	A	6	6	0.97	22	0.273
53	A	6	6	0.97	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	7	0.94	22	0.318
55	A	8	8	0.94	22	0.364
56	A	6	6	0.98	22	0.273
57	A	5	5	1.00	22	0.227
58	A	4	4	1.03	22	0.182
59	A	4	4	1.02	22	0.182
60	A	4	4	0.98	19	0.211
61	A	5	5	0.99	22	0.227
62	A	6	6	0.98	22	0.273
63	A	9	9	0.96	22	0.409
64	A	8	8	0.96	22	0.364
65	A	7	7	0.96	22	0.318
66	A	7	7	0.95	22	0.318
67	A	7	7	0.94	22	0.318
68	A	8	8	0.94	22	0.364
69	A	9	9	0.94	22	0.409
70	A	4	3	1.04	22	0.136
71	A	4	3	1.05	22	0.136
72	A	4	3	1.09	22	0.136
73	A	9	8	1.08	22	0.364
74	A	9	8	0.99	22	0.364
75	A	7	6	0.98	22	0.273
76	A	6	5	0.99	19	0.263
77	A	6	5	1.01	22	0.227
78	A	2	2	1.00	22	0.091
79	A	3	3	0.99	22	0.136
80	A	4	4	0.97	22	0.182
81	A	8	7	0.97	22	0.318
82	A	7	6	0.97	22	0.273
83	A	7	6	0.95	22	0.273
84	A	7	6	0.93	22	0.273
85	A	7	6	0.96	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	5	0.96	20	0.250
87	A	6	5	0.97	22	0.227
88	A	7	6	0.96	22	0.273
89	A	4	3	1.05	22	0.136
90	A	4	3	1.06	22	0.136
91	A	4	3	1.12	22	0.136
92	A	7	6	1.16	22	0.273
93	A	7	6	1.04	22	0.273
94	A	6	5	0.99	22	0.227
95	A	6	5	1.01	22	0.227
96	A	2	2	1.00	22	0.091
97	A	3	3	0.99	22	0.136
98	A	4	4	0.97	22	0.182
99	A	6	5	0.99	22	0.227
100	A	5	4	1.00	20	0.200
101	A	5	4	1.00	22	0.182
102	A	6	5	0.96	22	0.227
103	A	7	6	0.99	22	0.273
104	A	6	5	1.00	19	0.263
105	A	6	5	1.00	22	0.227
106	A	7	6	0.99	22	0.273
107	A	4	3	1.04	22	0.136
108	A	4	3	1.07	22	0.136
109	A	4	3	1.09	22	0.136
110	A	9	8	1.12	22	0.364
111	A	10	9	0.99	22	0.409
112	A	7	6	0.94	22	0.273
113	A	6	5	1.05	19	0.263
114	A	2	2	1.00	22	0.091
115	A	3	3	0.98	22	0.136
116	A	4	4	0.97	22	0.182
117	A	7	6	0.99	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	1.00	22	0.227
119	A	6	5	1.00	22	0.227
120	A	7	6	0.96	22	0.273
121	A	6	5	0.96	22	0.227
122	A	5	4	1.00	20	0.200
123	A	5	4	1.00	22	0.182
124	A	6	5	0.96	22	0.227
125	A	4	3	1.04	22	0.136
126	A	4	3	1.07	22	0.136
127	A	4	3	1.12	22	0.136
128	A	7	6	1.16	22	0.273
129	A	8	7	1.03	22	0.318
130	A	7	6	1.10	22	0.273
131	A	6	5	1.04	22	0.227
132	A	2	2	1.00	22	0.091
133	A	3	3	0.98	22	0.136
134	A	4	4	0.97	22	0.182
135	A	7	6	1.08	22	0.273
136	A	6	5	1.11	22	0.227
137	A	5	4	1.00	20	0.200
138	A	6	5	0.99	22	0.227
139	A	7	6	0.96	22	0.273
140	A	8	7	1.08	22	0.318
141	A	7	6	1.12	22	0.273
142	A	6	5	1.00	19	0.263
143	A	7	6	0.98	22	0.273
144	A	8	7	0.97	22	0.318
145	A	4	3	1.05	22	0.136
146	A	4	3	1.06	22	0.136
147	A	4	3	1.09	22	0.136
148	A	10	9	1.14	22	0.409
149	A	11	10	0.97	22	0.455

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	8	7	1.07	22	0.318
151	A	7	6	1.07	22	0.273
152	A	2	2	0.77	19	0.105
153	A	3	3	0.99	22	0.136
154	A	4	4	0.96	22	0.182
155	A	8	7	1.09	22	0.318
156	A	7	6	1.11	22	0.273
157	A	6	5	1.00	22	0.227
158	A	7	6	0.98	22	0.273
159	A	8	7	0.95	22	0.318
160	A	7	6	1.05	22	0.273
161	A	6	5	1.10	22	0.227
162	A	5	4	1.00	20	0.200
163	A	6	5	0.99	22	0.227
164	A	7	6	0.96	22	0.273
165	A	9	8	1.05	22	0.364
166	A	8	7	1.09	22	0.318
167	A	2	2	1.00	22	0.091
168	A	3	3	0.97	19	0.158
169	A	4	4	0.96	22	0.182
170	A	5	5	0.93	22	0.227
171	A	9	8	1.07	22	0.364
172	A	8	7	1.08	22	0.318
173	A	7	6	1.02	22	0.273
174	A	7	6	1.00	22	0.273
175	A	8	7	0.97	22	0.318
176	A	9	8	0.96	22	0.364
177	A	10	9	1.03	22	0.409
178	A	9	8	1.10	22	0.364
179	A	2	2	1.00	22	0.091
180	A	3	3	0.97	22	0.136
181	A	4	4	0.95	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	5	5	0.98	22	0.227
183	A	6	6	0.94	22	0.273
184	A	10	9	1.05	22	0.409
185	A	9	8	1.06	22	0.364
186	A	8	7	1.01	22	0.318
187	A	8	7	0.99	22	0.318
188	A	8	7	0.97	22	0.318
189	A	9	8	0.97	22	0.364
190	A	10	9	0.96	22	0.409
191	A	3	3	1.02	22	0.136
192	A	4	3	1.09	26	0.115
193	A	1	1	1.00	34	0.029
194	A	4	3	0.97	22	0.136
195	A	4	3	0.94	22	0.136
196	A	4	3	0.98	22	0.136
197	A	4	3	0.98	22	0.136
198	A	4	3	1.02	22	0.136
199	A	4	3	0.99	22	0.136
200	A	7	6	1.18	22	0.273
201	A	5	4	1.08	22	0.182
202	A	4	3	0.99	22	0.136
203	A	4	3	0.99	20	0.150
204	A	6	5	1.09	22	0.227
205	A	7	6	1.19	22	0.273
206	A	11	10	1.34	22	0.455
207	A	10	9	1.30	22	0.409
208	A	10	9	1.30	22	0.409
209	A	10	9	1.30	22	0.409
210	A	10	9	1.30	19	0.474
211	A	4	4	1.41	22	0.182
212	A	12	11	1.34	22	0.500
213	A	5	5	1.46	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	4	4	1.42	22	0.182
215	A	2	2	1.28	19	0.105
216	A	4	4	1.43	22	0.182
217	A	4	4	1.45	22	0.182
218	A	4	4	1.43	22	0.182
219	A	4	3	1.00	24	0.125
220	A	6	5	1.05	24	0.208
221	A	5	4	1.00	24	0.167
222	A	5	4	1.00	24	0.167
223	A	7	6	1.05	24	0.250
224	A	8	7	1.07	24	0.292
225	A	7	6	0.98	22	0.273
226	A	7	6	1.00	24	0.250
227	A	8	7	1.07	24	0.292
228	A	9	8	1.09	24	0.333
229	A	9	9	1.49	24	0.375
230	A	8	8	1.29	21	0.381
231	A	10	10	1.47	24	0.417
232	A	3	3	1.25	24	0.125
233	A	11	11	1.39	24	0.458
234	A	4	4	1.26	24	0.167
235	A	4	3	1.00	24	0.125
236	A	5	4	1.00	24	0.167
237	A	4	3	1.00	24	0.125
238	A	5	4	1.00	24	0.167
239	A	7	6	1.09	24	0.250
240	A	8	7	1.09	24	0.292
241	A	7	6	0.99	24	0.250
242	A	4	3	1.00	22	0.136
243	A	7	6	1.00	24	0.250
244	A	8	7	1.10	24	0.292
245	A	9	9	1.52	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	8	8	1.53	24	0.333
247	A	6	6	1.38	21	0.286
248	A	10	10	1.49	24	0.417
249	A	11	11	1.35	24	0.458
250	A	6	6	1.32	24	0.250
251	A	4	4	1.20	24	0.167
252	A	7	6	1.18	24	0.250
253	A	7	6	1.17	24	0.250
254	A	5	4	0.99	24	0.167
255	A	5	4	0.99	24	0.167
256	A	7	6	1.17	24	0.250
257	A	8	7	1.15	24	0.292
258	A	10	9	1.12	24	0.375
259	A	8	7	1.18	24	0.292
260	A	6	5	1.04	24	0.208
261	A	5	4	1.04	22	0.182
262	A	8	7	1.05	24	0.292
263	A	9	8	1.02	24	0.333
264	A	12	12	1.36	24	0.500
265	A	11	11	1.41	24	0.458
266	A	9	9	1.43	24	0.375
267	A	9	9	1.64	24	0.375
268	A	10	10	1.41	21	0.476
269	A	11	11	1.38	24	0.458
270	A	12	12	1.34	24	0.500
271	A	3	3	0.94	24	0.125
272	A	4	4	0.92	24	0.167
273	A	5	5	0.92	24	0.208
274	C	7	6	3.57	20	0.300
275	A	5	4	1.00	28	0.143
276	A	5	4	1.00	28	0.143
277	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	5	4	1.00	28	0.143
279	A	4	4	1.04	26	0.154
280	A	3	3	1.02	24	0.125
281	A	2	2	1.00	17	0.118
282	A	2	2	1.00	26	0.077
283	A	2	2	1.00	26	0.077
284	A	2	2	1.00	26	0.077
285	A	5	5	0.98	26	0.192
286	A	3	3	1.12	24	0.125
287	A	2	2	1.00	17	0.118
288	A	2	2	1.00	26	0.077
289	A	2	2	1.00	26	0.077
290	A	2	2	1.00	26	0.077
291	A	6	6	1.17	24	0.250
292	A	4	4	1.03	24	0.167
293	A	3	3	1.02	22	0.136
294	A	2	2	1.00	15	0.133
295	A	2	2	1.00	24	0.083
296	A	2	2	1.00	24	0.083
297	A	3	3	1.00	24	0.125
298	A	1	1	1.00	22	0.045
299	A	3	3	1.00	24	0.125

CHAPTER 3

Listing of integrals

3.1 $\int x^4(a+bx^4)(A+Bx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^4\left(a+bx^4\right)\left(A+Bx^4\right)\,dx=\frac{1}{5}aAx^5+\frac{1}{9}(Ab+aB)x^9+\frac{1}{13}bBx^{13}$$

output
```
1/5*a*A*x^5+1/9*(A*b+B*a)*x^9+1/13*b*B*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^4\right)\left(A+Bx^4\right)\,dx=\frac{1}{5}aAx^5+\frac{1}{9}(Ab+aB)x^9+\frac{1}{13}bBx^{13}$$

input
```
Integrate[x^4*(a + b*x^4)*(A + B*x^4),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^9)/9 + (b*B*x^13)/13
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^4)\left(A+Bx^4\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^8(aB+Ab)+aAx^4+bBx^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}x^9(aB+Ab)+\frac{1}{5}aAx^5+\frac{1}{13}bBx^{13}$$

input
```
Int[x^4*(a + b*x^4)*(A + B*x^4),x]
```

output
```
(a*A*x^5)/5 + ((A*b + a*B)*x^9)/9 + (b*B*x^13)/13
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{5}}{5}+\frac{(Ab+Ba)x^{9}}{9}+\frac{bB\,x^{13}}{13}$	28
norman	$\frac{bB\,x^{13}}{13}+\left(\frac{Ab}{9}+\frac{Ba}{9}\right)x^{9}+\frac{aA\,x^{5}}{5}$	29
gosper	$\frac{1}{13}bB\,x^{13}+\frac{1}{9}x^{9}Ab+\frac{1}{9}x^{9}Ba+\frac{1}{5}aA\,x^{5}$	30
risch	$\frac{1}{13}bB\,x^{13}+\frac{1}{9}x^{9}Ab+\frac{1}{9}x^{9}Ba+\frac{1}{5}aA\,x^{5}$	30
parallelrisch	$\frac{1}{13}bB\,x^{13}+\frac{1}{9}x^{9}Ab+\frac{1}{9}x^{9}Ba+\frac{1}{5}aA\,x^{5}$	30
orering	$\frac{x^{5}(45Bb\,x^{8}+65Ab\,x^{4}+65Ba\,x^{4}+117Aa)}{585}$	32

input
```
int(x^4*(b*x^4+a)*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*A*x^5+1/9*(A*b+B*a)*x^9+1/13*b*B*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{1}{13}\,Bbx^{13}+\frac{1}{9}\,(Ba+Ab)x^9+\frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x^4+a)*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/13*B*b*x^13 + 1/9*(B*a + A*b)*x^9 + 1/5*A*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{Aax^5}{5}+\frac{Bbx^{13}}{13}+x^9\left(\frac{Ab}{9}+\frac{Ba}{9}\right)$$

input
```
integrate(x**4*(b*x**4+a)*(B*x**4+A),x)
```

output
```
A*a*x**5/5 + B*b*x**13/13 + x**9*(A*b/9 + B*a/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{1}{13}\,Bbx^{13}+\frac{1}{9}\,(Ba+Ab)x^9+\frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x^4+a)*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/13*B*b*x^13 + 1/9*(B*a + A*b)*x^9 + 1/5*A*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{1}{13}\,Bbx^{13}+\frac{1}{9}\,Bax^9+\frac{1}{9}\,Abx^9+\frac{1}{5}\,Aax^5$$

input
```
integrate(x^4*(b*x^4+a)*(B*x^4+A),x, algorithm="giac")
```

output
```
1/13*B*b*x^13 + 1/9*B*a*x^9 + 1/9*A*b*x^9 + 1/5*A*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{B\,b\,x^{13}}{13}+\left(\frac{A\,b}{9}+\frac{B\,a}{9}\right)\,x^9+\frac{A\,a\,x^5}{5}$$

input
```
int(x^4*(A + B*x^4)*(a + b*x^4),x)
```

output
```
x^9*((A*b)/9 + (B*a)/9) + (A*a*x^5)/5 + (B*b*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x^4(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{x^5(45b^2x^8+130ab\,x^4+117a^2)}{585}$$

input
```
int(x^4*(b*x^4+a)*(B*x^4+A),x)
```

output
```
(x**5*(117*a**2 + 130*a*b*x**4 + 45*b**2*x**8))/585
```

3.2 $\int x^2(a+bx^4)(A+Bx^4)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2\left(a+bx^4\right)\left(A+Bx^4\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{7}(Ab+aB)x^7+\frac{1}{11}bBx^{11}$$

output
```
1/3*a*A*x^3+1/7*(A*b+B*a)*x^7+1/11*b*B*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^4\right)\left(A+Bx^4\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{7}(Ab+aB)x^7+\frac{1}{11}bBx^{11}$$

input
```
Integrate[x^2*(a + b*x^4)*(A + B*x^4),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^7)/7 + (b*B*x^11)/11
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right) \left(A+Bx^4\right) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^6(aB+Ab)+aAx^2+bBx^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{11}bBx^{11}$$

input
```
Int[x^2*(a + b*x^4)*(A + B*x^4),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^7)/7 + (b*B*x^11)/11
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{7}}{7}+\frac{bB\,x^{11}}{11}$	28
norman	$\frac{bB\,x^{11}}{11}+\left(\frac{Ab}{7}+\frac{Ba}{7}\right)x^{7}+\frac{aA\,x^{3}}{3}$	29
gosper	$\frac{1}{11}bB\,x^{11}+\frac{1}{7}x^{7}Ab+\frac{1}{7}x^{7}Ba+\frac{1}{3}aA\,x^{3}$	30
risch	$\frac{1}{11}bB\,x^{11}+\frac{1}{7}x^{7}Ab+\frac{1}{7}x^{7}Ba+\frac{1}{3}aA\,x^{3}$	30
parallelrisch	$\frac{1}{11}bB\,x^{11}+\frac{1}{7}x^{7}Ab+\frac{1}{7}x^{7}Ba+\frac{1}{3}aA\,x^{3}$	30
orering	$\frac{x^{3}(21Bb\,x^{8}+33Ab\,x^{4}+33Ba\,x^{4}+77Aa)}{231}$	32

input
```
int(x^2*(b*x^4+a)*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/7*(A*b+B*a)*x^7+1/11*b*B*x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{1}{11}\,Bbx^{11}+\frac{1}{7}\,(Ba+Ab)x^7+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^4+a)*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/11*B*b*x^11 + 1/7*(B*a + A*b)*x^7 + 1/3*A*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^4)(A+Bx^4)\,dx = \frac{Aax^3}{3} + \frac{Bbx^{11}}{11} + x^7\left(\frac{Ab}{7} + \frac{Ba}{7}\right)$$

input
```
integrate(x**2*(b*x**4+a)*(B*x**4+A),x)
```

output
```
A*a*x**3/3 + B*b*x**11/11 + x**7*(A*b/7 + B*a/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^4)(A+Bx^4)\,dx = \frac{1}{11}\,Bbx^{11} + \frac{1}{7}\,(Ba+Ab)x^7 + \frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^4+a)*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/11*B*b*x^11 + 1/7*(B*a + A*b)*x^7 + 1/3*A*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^4)(A+Bx^4)\,dx = \frac{1}{11}\,Bbx^{11} + \frac{1}{7}\,Bax^7 + \frac{1}{7}\,Abx^7 + \frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^4+a)*(B*x^4+A),x, algorithm="giac")
```

output
```
1/11*B*b*x^11 + 1/7*B*a*x^7 + 1/7*A*b*x^7 + 1/3*A*a*x^3
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{B\,b\,x^{11}}{11}+\left(\frac{A\,b}{7}+\frac{B\,a}{7}\right)\,x^7+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x^4)*(a + b*x^4),x)
```

output
```
x^7*((A*b)/7 + (B*a)/7) + (A*a*x^3)/3 + (B*b*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x^2(a+bx^4)\left(A+Bx^4\right)\,dx=\frac{x^3(21b^2x^8+66ab\,x^4+77a^2)}{231}$$

input
```
int(x^2*(b*x^4+a)*(B*x^4+A),x)
```

output
```
(x**3*(77*a**2 + 66*a*b*x**4 + 21*b**2*x**8))/231
```

3.3 $\int (a + bx^4) (A + Bx^4) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a + bx^4) (A + Bx^4) \, dx = aAx + \frac{1}{5}(Ab + aB)x^5 + \frac{1}{9}bBx^9$$

output
```
a*A*x+1/5*(A*b+B*a)*x^5+1/9*b*B*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a + bx^4) (A + Bx^4) \, dx = aAx + \frac{1}{5}(Ab + aB)x^5 + \frac{1}{9}bBx^9$$

input
```
Integrate[(a + b*x^4)*(A + B*x^4),x]
```

output
```
a*A*x + ((A*b + a*B)*x^5)/5 + (b*B*x^9)/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)\left(A+Bx^4\right)\,dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(x^4(aB+Ab)+aA+bBx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aB+Ab)+aAx+\frac{1}{9}bBx^9$$

input
```
Int[(a + b*x^4)*(A + B*x^4),x]
```

output
```
a*A*x + ((A*b + a*B)*x^5)/5 + (b*B*x^9)/9
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{9}}{9}$	25
norman	$\frac{bB\,x^{9}}{9}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+aAx$	26
gosper	$\frac{1}{9}bB\,x^{9}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+aAx$	27
risch	$\frac{1}{9}bB\,x^{9}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+aAx$	27
parallelrisch	$\frac{1}{9}bB\,x^{9}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+aAx$	27
orering	$\frac{x\left(5Bb\,x^{8}+9Ab\,x^{4}+9Ba\,x^{4}+45Aa\right)}{45}$	30

input
```
int((b*x^4+a)*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/5*(A*b+B*a)*x^5+1/9*b*B*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^4\right)\left(A+Bx^4\right)\,dx=\frac{1}{9}\,Bbx^9+\frac{1}{5}\,(Ba+Ab)x^5+Aax$$

input
```
integrate((b*x^4+a)*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/9*B*b*x^9 + 1/5*(B*a + A*b)*x^5 + A*a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)\left(A+Bx^4\right)\,dx = Aax+\frac{Bbx^9}{9}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate((b*x**4+a)*(B*x**4+A),x)
```

output
```
A*a*x + B*b*x**9/9 + x**5*(A*b/5 + B*a/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^4\right)\left(A+Bx^4\right)\,dx = \frac{1}{9}\,Bbx^9+\frac{1}{5}\,(Ba+Ab)x^5+Aax$$

input
```
integrate((b*x^4+a)*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/9*B*b*x^9 + 1/5*(B*a + A*b)*x^5 + A*a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^4\right)\left(A+Bx^4\right)\,dx = \frac{1}{9}\,Bbx^9+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+Aax$$

input
```
integrate((b*x^4+a)*(B*x^4+A),x, algorithm="giac")
```

output
```
1/9*B*b*x^9 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + A*a*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^4)\,(A+Bx^4)\ dx=\frac{B\,b\,x^9}{9}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+A\,a\,x$$

input
```
int((A + B*x^4)*(a + b*x^4),x)
```

output
```
x^5*((A*b)/5 + (B*a)/5) + A*a*x + (B*b*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^4)\,(A+Bx^4)\ dx=\frac{x(5b^2x^8+18ab\,x^4+45a^2)}{45}$$

input
```
int((b*x^4+a)*(B*x^4+A),x)
```

output
```
(x*(45*a**2 + 18*a*b*x**4 + 5*b**2*x**8))/45
```

3.4 $\int \frac{(a+bx^4)(A+Bx^4)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2} dx = -\frac{aA}{x} + \frac{1}{3}(Ab+aB)x^3 + \frac{1}{7}bBx^7$$

output
```
-a*A/x+1/3*(A*b+B*a)*x^3+1/7*b*B*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2} dx = -\frac{aA}{x} + \frac{1}{3}(Ab+aB)x^3 + \frac{1}{7}bBx^7$$

input
```
Integrate[((a + b*x^4)*(A + B*x^4))/x^2,x]
```

output
```
-((a*A)/x) + ((A*b + a*B)*x^3)/3 + (b*B*x^7)/7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(x^2(aB+Ab)+\frac{aA}{x^2}+bBx^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3(aB+Ab)-\frac{aA}{x}+\frac{1}{7}bBx^7$$

input
```
Int[((a + b*x^4)*(A + B*x^4))/x^2,x]
```

output
```
-((a*A)/x) + ((A*b + a*B)*x^3)/3 + (b*B*x^7)/7
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{bB\,x^7}{7}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}-\frac{aA}{x}$	30
norman	$\frac{\frac{Bb\,x^8}{7}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^4-Aa}{x}$	30
risch	$\frac{bB\,x^7}{7}+\frac{Ab\,x^3}{3}+\frac{Ba\,x^3}{3}-\frac{aA}{x}$	30
gosper	$-\frac{-3Bb\,x^8-7Ab\,x^4-7Ba\,x^4+21Aa}{21x}$	32
parallelrisch	$\frac{3Bb\,x^8+7Ab\,x^4+7Ba\,x^4-21Aa}{21x}$	32
orering	$-\frac{-3Bb\,x^8-7Ab\,x^4-7Ba\,x^4+21Aa}{21x}$	32

input
```
int((b*x^4+a)*(B*x^4+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b*B*x^7+1/3*A*b*x^3+1/3*B*a*x^3-a*A/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^4)\,(A+Bx^4)}{x^2}\,dx = \frac{3\,Bbx^8+7\,(Ba+Ab)x^4-21\,Aa}{21\,x}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^2,x, algorithm="fricas")
```

output
```
1/21*(3*B*b*x^8 + 7*(B*a + A*b)*x^4 - 21*A*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}\,dx = -\frac{Aa}{x} + \frac{Bbx^7}{7} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

input
```
integrate((b*x**4+a)*(B*x**4+A)/x**2,x)
```

output
```
-A*a/x + B*b*x**7/7 + x**3*(A*b/3 + B*a/3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{3}\,(Ba+Ab)x^3 - \frac{Aa}{x}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^2,x, algorithm="maxima")
```

output
```
1/7*B*b*x^7 + 1/3*(B*a + A*b)*x^3 - A*a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}\,dx = \frac{1}{7}\,Bbx^7 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 - \frac{Aa}{x}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^2,x, algorithm="giac")
```

output
```
1/7*B*b*x^7 + 1/3*B*a*x^3 + 1/3*A*b*x^3 - A*a/x
```

Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}\,dx = x^3\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^7}{7}$$

input
```
int(((A + B*x^4)*(a + b*x^4))/x^2,x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) - (A*a)/x + (B*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^2}\,dx = \frac{3b^2x^8+14ab\,x^4-21a^2}{21x}$$

input
```
int((b*x^4+a)*(B*x^4+A)/x^2,x)
```

output
```
( - 21*a**2 + 14*a*b*x**4 + 3*b**2*x**8)/(21*x)
```

3.5 $\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx = -\frac{aA}{3x^3} + (Ab+aB)x + \frac{1}{5}bBx^5$$

output
```
-1/3*a*A/x^3+(A*b+B*a)*x+1/5*b*B*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx = -\frac{aA}{3x^3} + (Ab+aB)x + \frac{1}{5}bBx^5$$

input
```
Integrate[((a + b*x^4)*(A + B*x^4))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (A*b + a*B)*x + (b*B*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(Ab\left(\frac{aB}{Ab}+1\right)+\frac{aA}{x^4}+bBx^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{5}bBx^5$$

input
```
Int[((a + b*x^4)*(A + B*x^4))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (A*b + a*B)*x + (b*B*x^5)/5
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{bB\,x^{5}}{5}+Abx+Bax-\frac{aA}{3x^{3}}$	24
risch	$\frac{bB\,x^{5}}{5}+Abx+Bax-\frac{aA}{3x^{3}}$	24
norman	$\frac{\frac{Bb\,x^{8}}{5}+(Ab+Ba)x^{4}-\frac{Aa}{3}}{x^{3}}$	28
gosper	$-\frac{-3Bb\,x^{8}-15Ab\,x^{4}-15Ba\,x^{4}+5Aa}{15x^{3}}$	32
parallelrisch	$\frac{3Bb\,x^{8}+15Ab\,x^{4}+15Ba\,x^{4}-5Aa}{15x^{3}}$	32
orering	$-\frac{-3Bb\,x^{8}-15Ab\,x^{4}-15Ba\,x^{4}+5Aa}{15x^{3}}$	32

input
```
int((b*x^4+a)*(B*x^4+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*b*B*x^5+A*b*x+B*a*x-1/3*a*A/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^4)\,(A+Bx^4)}{x^4}\,dx=\frac{3\,Bbx^8+15\,(Ba+Ab)x^4-5\,Aa}{15\,x^3}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*B*b*x^8 + 15*(B*a + A*b)*x^4 - 5*A*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx = -\frac{Aa}{3x^3} + \frac{Bbx^5}{5} + x(Ab+Ba)$$

input
```
integrate((b*x**4+a)*(B*x**4+A)/x**4,x)
```

output
```
-A*a/(3*x**3) + B*b*x**5/5 + x*(A*b + B*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx = \frac{1}{5}Bbx^5 + (Ba+Ab)x - \frac{Aa}{3\,x^3}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^4,x, algorithm="maxima")
```

output
```
1/5*B*b*x^5 + (B*a + A*b)*x - 1/3*A*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4}\,dx = \frac{1}{5}Bbx^5 + Bax + Abx - \frac{Aa}{3\,x^3}$$

input
```
integrate((b*x^4+a)*(B*x^4+A)/x^4,x, algorithm="giac")
```

output
```
1/5*B*b*x^5 + B*a*x + A*b*x - 1/3*A*a/x^3
```

Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4} \, dx = x(Ab+Ba) - \frac{Aa}{3x^3} + \frac{Bbx^5}{5}$$

input
```
int(((A + B*x^4)*(a + b*x^4))/x^4,x)
```

output
```
x*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)(A+Bx^4)}{x^4} \, dx = \frac{3b^2x^8+30ab\,x^4-5a^2}{15x^3}$$

input
```
int((b*x^4+a)*(B*x^4+A)/x^4,x)
```

output
```
( - 5*a**2 + 30*a*b*x**4 + 3*b**2*x**8)/(15*x**3)
```

3.6 $\int x^4(a+bx^4)^2(A+Bx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^4(a+bx^4)^2(A+Bx^4)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{9}a(2Ab+aB)x^9+\frac{1}{13}b(Ab+2aB)x^{13}+\frac{1}{17}b^2Bx^{17}$$

output
```
1/5*a^2*A*x^5+1/9*a*(2*A*b+B*a)*x^9+1/13*b*(A*b+2*B*a)*x^13+1/17*b^2*B*x^1
7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^4)^2(A+Bx^4)\,dx=\frac{1}{5}a^2Ax^5+\frac{1}{9}a(2Ab+aB)x^9+\frac{1}{13}b(Ab+2aB)x^{13}+\frac{1}{17}b^2Bx^{17}$$

input
```
Integrate[x^4*(a + b*x^4)^2*(A + B*x^4),x]
```

output
```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^9)/9 + (b*(A*b + 2*a*B)*x^13)/13 + (b^2
*B*x^17)/17
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^4)^2 (A+Bx^4)\, dx$$

$$\Big\downarrow \text{950}$$

$$\int (a^2Ax^4 + bx^{12}(2aB+Ab) + ax^8(aB+2Ab) + b^2Bx^{16})\, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2Ax^5 + \frac{1}{13}bx^{13}(2aB+Ab) + \frac{1}{9}ax^9(aB+2Ab) + \frac{1}{17}b^2Bx^{17}$$

input
```
Int[x^4*(a + b*x^4)^2*(A + B*x^4),x]
```

output

```
(a^2*A*x^5)/5 + (a*(2*A*b + a*B)*x^9)/9 + (b*(A*b + 2*a*B)*x^13)/13 + (b^2
*B*x^17)/17
```


Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^{2} B \,x^{17}}{17}+\frac{\left(b^{2} A +2 a b B \right) x^{13}}{13}+\frac{\left(2 a b A +a^{2} B \right) x^{9}}{9}+\frac{a^{2} A \,x^{5}}{5}$	52
norman	$\frac{a^{2} A \,x^{5}}{5}+\left(\frac{2}{9} a b A +\frac{1}{9} a^{2} B \right) x^{9}+\left(\frac{1}{13} b^{2} A +\frac{2}{13} a b B \right) x^{13}+\frac{b^{2} B \,x^{17}}{17}$	52
gosper	$\frac{1}{5} a^{2} A \,x^{5}+\frac{2}{9} x^{9} a b A +\frac{1}{9} x^{9} a^{2} B +\frac{1}{13} x^{13} b^{2} A +\frac{2}{13} x^{13} a b B +\frac{1}{17} b^{2} B \,x^{17}$	54
risch	$\frac{1}{5} a^{2} A \,x^{5}+\frac{2}{9} x^{9} a b A +\frac{1}{9} x^{9} a^{2} B +\frac{1}{13} x^{13} b^{2} A +\frac{2}{13} x^{13} a b B +\frac{1}{17} b^{2} B \,x^{17}$	54
parallelrisch	$\frac{1}{5} a^{2} A \,x^{5}+\frac{2}{9} x^{9} a b A +\frac{1}{9} x^{9} a^{2} B +\frac{1}{13} x^{13} b^{2} A +\frac{2}{13} x^{13} a b B +\frac{1}{17} b^{2} B \,x^{17}$	54
orering	$\frac{x^{5} \left(585 B \,b^{2} x^{12}+765 A \,b^{2} x^{8}+1530 B a b \,x^{8}+2210 A a b \,x^{4}+1105 B \,a^{2} x^{4}+1989 a^{2} A \right)}{9945}$	56

input
```
int(x^4*(b*x^4+a)^2*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/17*b^2*B*x^17+1/13*(A*b^2+2*B*a*b)*x^13+1/9*(2*A*a*b+B*a^2)*x^9+1/5*a^2*
A*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4\left(a+bx^4\right)^2\left(A+Bx^4\right)\,dx=\frac{1}{17}\,Bb^2x^{17}+\frac{1}{13}\,(2\,Bab+Ab^2)x^{13}+\frac{1}{9}\,(Ba^2+2\,Aab)x^9+\frac{1}{5}\,Aa^2x^5$$

input
```
integrate(x^4*(b*x^4+a)^2*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/17*B*b^2*x^17 + 1/13*(2*B*a*b + A*b^2)*x^13 + 1/9*(B*a^2 + 2*A*a*b)*x^9
+ 1/5*A*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^4 (a + bx^4)^2 (A + Bx^4) \, dx = \frac{Aa^2x^5}{5} + \frac{Bb^2x^{17}}{17} + x^{13}\left(\frac{Ab^2}{13} + \frac{2Bab}{13}\right) + x^9 \cdot \left(\frac{2Aab}{9} + \frac{Ba^2}{9}\right)$$

input
```
integrate(x**4*(b*x**4+a)**2*(B*x**4+A),x)
```

output
```
A*a**2*x**5/5 + B*b**2*x**17/17 + x**13*(A*b**2/13 + 2*B*a*b/13) + x**9*(2
*A*a*b/9 + B*a**2/9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4 (a + bx^4)^2 (A + Bx^4) \, dx = \frac{1}{17} Bb^2x^{17} + \frac{1}{13} (2 Bab + Ab^2)x^{13} + \frac{1}{9} (Ba^2 + 2 Aab)x^9 + \frac{1}{5} Aa^2x^5$$

input
```
integrate(x^4*(b*x^4+a)^2*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/17*B*b^2*x^17 + 1/13*(2*B*a*b + A*b^2)*x^13 + 1/9*(B*a^2 + 2*A*a*b)*x^9
+ 1/5*A*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^4(a+bx^4)^2(A+Bx^4)\,dx = \frac{1}{17}Bb^2x^{17}+\frac{2}{13}Babx^{13}+\frac{1}{13}Ab^2x^{13}+\frac{1}{9}Ba^2x^9+\frac{2}{9}Aabx^9+\frac{1}{5}Aa^2x^5$$

input
```
integrate(x^4*(b*x^4+a)^2*(B*x^4+A),x, algorithm="giac")
```

output
```
1/17*B*b^2*x^17 + 2/13*B*a*b*x^13 + 1/13*A*b^2*x^13 + 1/9*B*a^2*x^9 + 2/9*
A*a*b*x^9 + 1/5*A*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^4)^2(A+Bx^4)\,dx = x^9\left(\frac{B\,a^2}{9}+\frac{2\,A\,b\,a}{9}\right)+x^{13}\left(\frac{A\,b^2}{13}+\frac{2\,B\,a\,b}{13}\right)+\frac{A\,a^2\,x^5}{5}+\frac{B\,b^2\,x^{17}}{17}$$

input
```
int(x^4*(A + B*x^4)*(a + b*x^4)^2,x)
```

output
```
x^9*((B*a^2)/9 + (2*A*a*b)/9) + x^13*((A*b^2)/13 + (2*B*a*b)/13) + (A*a^2*
x^5)/5 + (B*b^2*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int x^4(a+bx^4)^2(A+Bx^4)\,dx = \frac{x^5(195b^3x^{12}+765a\,b^2x^8+1105a^2b\,x^4+663a^3)}{3315}$$

input
```
int(x^4*(b*x^4+a)^2*(B*x^4+A),x)
```

output
```
(x**5*(663*a**3 + 1105*a**2*b*x**4 + 765*a*b**2*x**8 + 195*b**3*x**12))/33
15
```

3.7 $\int x^2(a+bx^4)^2(A+Bx^4)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^2(a+bx^4)^2(A+Bx^4)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{7}a(2Ab+aB)x^7+\frac{1}{11}b(Ab+2aB)x^{11}+\frac{1}{15}b^2Bx^{15}$$

output
```
1/3*a^2*A*x^3+1/7*a*(2*A*b+B*a)*x^7+1/11*b*(A*b+2*B*a)*x^11+1/15*b^2*B*x^1
5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^4)^2(A+Bx^4)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{7}a(2Ab+aB)x^7+\frac{1}{11}b(Ab+2aB)x^{11}+\frac{1}{15}b^2Bx^{15}$$

input
```
Integrate[x^2*(a + b*x^4)^2*(A + B*x^4),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^7)/7 + (b*(A*b + 2*a*B)*x^11)/11 + (b^2
*B*x^15)/15
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right)^2 \left(A+Bx^4\right) \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(a^2Ax^2+bx^{10}(2aB+Ab)+ax^6(aB+2Ab)+b^2Bx^{14}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{11}bx^{11}(2aB+Ab)+\frac{1}{7}ax^7(aB+2Ab)+\frac{1}{15}b^2Bx^{15}$$

input
```
Int[x^2*(a + b*x^4)^2*(A + B*x^4),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^7)/7 + (b*(A*b + 2*a*B)*x^11)/11 + (b^2
*B*x^15)/15
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^{2} B \,x^{15}}{15}+\frac{\left(b^{2} A +2 a b B \right) x^{11}}{11}+\frac{\left(2 a b A +a^{2} B \right) x^{7}}{7}+\frac{a^{2} A \,x^{3}}{3}$	52
norman	$\frac{a^{2} A \,x^{3}}{3}+\left(\frac{2}{7} a b A +\frac{1}{7} a^{2} B \right) x^{7}+\left(\frac{1}{11} b^{2} A +\frac{2}{11} a b B \right) x^{11}+\frac{b^{2} B \,x^{15}}{15}$	52
gosper	$\frac{1}{3} a^{2} A \,x^{3}+\frac{2}{7} x^{7} a b A +\frac{1}{7} x^{7} a^{2} B +\frac{1}{11} x^{11} b^{2} A +\frac{2}{11} x^{11} a b B +\frac{1}{15} b^{2} B \,x^{15}$	54
risch	$\frac{1}{3} a^{2} A \,x^{3}+\frac{2}{7} x^{7} a b A +\frac{1}{7} x^{7} a^{2} B +\frac{1}{11} x^{11} b^{2} A +\frac{2}{11} x^{11} a b B +\frac{1}{15} b^{2} B \,x^{15}$	54
parallelrisch	$\frac{1}{3} a^{2} A \,x^{3}+\frac{2}{7} x^{7} a b A +\frac{1}{7} x^{7} a^{2} B +\frac{1}{11} x^{11} b^{2} A +\frac{2}{11} x^{11} a b B +\frac{1}{15} b^{2} B \,x^{15}$	54
orering	$\frac{x^{3} \left(77 B \,b^{2} x^{12}+105 A \,b^{2} x^{8}+210 B a b \,x^{8}+330 A a b \,x^{4}+165 B \,a^{2} x^{4}+385 a^{2} A \right)}{1155}$	56

input
```
int(x^2*(b*x^4+a)^2*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/15*b^2*B*x^15+1/11*(A*b^2+2*B*a*b)*x^11+1/7*(2*A*a*b+B*a^2)*x^7+1/3*a^2*
A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2\left(a+bx^4\right)^2\left(A+Bx^4\right)\,dx=\frac{1}{15}\,Bb^2x^{15}+\frac{1}{11}\,(2\,Bab+Ab^2)x^{11}+\frac{1}{7}\,(Ba^2+2\,Aab)x^7+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^4+a)^2*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/15*B*b^2*x^15 + 1/11*(2*B*a*b + A*b^2)*x^11 + 1/7*(B*a^2 + 2*A*a*b)*x^7
+ 1/3*A*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^4)^2(A+Bx^4)\,dx = \frac{Aa^2x^3}{3}+\frac{Bb^2x^{15}}{15}+x^{11}\left(\frac{Ab^2}{11}+\frac{2Bab}{11}\right)+x^7\cdot\left(\frac{2Aab}{7}+\frac{Ba^2}{7}\right)$$

input
```
integrate(x**2*(b*x**4+a)**2*(B*x**4+A),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**15/15 + x**11*(A*b**2/11 + 2*B*a*b/11) + x**7*(2
*A*a*b/7 + B*a**2/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^4)^2(A+Bx^4)\,dx = \frac{1}{15}Bb^2x^{15}+\frac{1}{11}(2Bab+Ab^2)x^{11}+\frac{1}{7}(Ba^2+2Aab)x^7+\frac{1}{3}Aa^2x^3$$

input
```
integrate(x^2*(b*x^4+a)^2*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/15*B*b^2*x^15 + 1/11*(2*B*a*b + A*b^2)*x^11 + 1/7*(B*a^2 + 2*A*a*b)*x^7
+ 1/3*A*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2 (a + bx^4)^2 (A + Bx^4) \, dx = \frac{1}{15} Bb^2x^{15} + \frac{2}{11} Babx^{11} + \frac{1}{11} Ab^2x^{11} + \frac{1}{7} Ba^2x^7 + \frac{2}{7} Aabx^7 + \frac{1}{3} Aa^2x^3$$

input
```
integrate(x^2*(b*x^4+a)^2*(B*x^4+A),x, algorithm="giac")
```

output
```
1/15*B*b^2*x^15 + 2/11*B*a*b*x^11 + 1/11*A*b^2*x^11 + 1/7*B*a^2*x^7 + 2/7*
A*a*b*x^7 + 1/3*A*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2 (a + bx^4)^2 (A + Bx^4) \, dx = x^7 \left(\frac{B\,a^2}{7} + \frac{2\,A\,b\,a}{7} \right) + x^{11} \left(\frac{A\,b^2}{11} + \frac{2\,B\,a\,b}{11} \right) + \frac{A\,a^2\,x^3}{3} + \frac{B\,b^2\,x^{15}}{15}$$

input
```
int(x^2*(A + B*x^4)*(a + b*x^4)^2,x)
```

output
```
x^7*((B*a^2)/7 + (2*A*a*b)/7) + x^11*((A*b^2)/11 + (2*B*a*b)/11) + (A*a^2*
x^3)/3 + (B*b^2*x^15)/15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int x^2 (a + bx^4)^2 (A + Bx^4) \, dx = \frac{x^3(77b^3x^{12} + 315a\,b^2x^8 + 495a^2b\,x^4 + 385a^3)}{1155}$$

input
```
int(x^2*(b*x^4+a)^2*(B*x^4+A),x)
```

output
```
(x**3*(385*a**3 + 495*a**2*b*x**4 + 315*a*b**2*x**8 + 77*b**3*x**12))/1155
```

3.8 $\int (a+bx^4)^2 (A+Bx^4) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^4)^2 (A+Bx^4) \, dx = a^2Ax + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{9}b(Ab+2aB)x^9 + \frac{1}{13}b^2Bx^{13}$$

output
```
a^2*A*x+1/5*a*(2*A*b+B*a)*x^5+1/9*b*(A*b+2*B*a)*x^9+1/13*b^2*B*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 (A+Bx^4) \, dx = a^2Ax + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{9}b(Ab+2aB)x^9 + \frac{1}{13}b^2Bx^{13}$$

input
```
Integrate[(a + b*x^4)^2*(A + B*x^4),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^9)/9 + (b^2*B*x^13)
/13
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^2 (A+Bx^4)\, dx$$

$$\Big\downarrow \text{ 897}$$

$$\int (a^2A+bx^8(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^{12})\, dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2Ax+\frac{1}{9}bx^9(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{13}b^2Bx^{13}$$

input
```
Int[(a + b*x^4)^2*(A + B*x^4),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^9)/9 + (b^2*B*x^13)
/13
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B x^{13}}{13}+\frac{(b^2 A+2abB)x^9}{9}+\frac{(2abA+a^2 B)x^5}{5}+a^2 Ax$	49
norman	$\frac{b^2 B x^{13}}{13}+\left(\frac{1}{9}b^2 A+\frac{2}{9}abB\right)x^9+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+a^2 Ax$	49
gosper	$\frac{1}{13}b^2 B x^{13}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+a^2 Ax$	51
risch	$\frac{1}{13}b^2 B x^{13}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+a^2 Ax$	51
parallelrisch	$\frac{1}{13}b^2 B x^{13}+\frac{1}{9}x^9 b^2 A+\frac{2}{9}x^9 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+a^2 Ax$	51
orering	$\frac{x(45B\,b^2 x^{12}+65A\,b^2 x^8+130Bab\,x^8+234Aab\,x^4+117B\,a^2 x^4+585a^2 A)}{585}$	54

input
```
int((b*x^4+a)^2*(B*x^4+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*B*x^13+1/9*(A*b^2+2*B*a*b)*x^9+1/5*(2*A*a*b+B*a^2)*x^5+a^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^4)^2 (A+Bx^4)\,dx=\frac{1}{13}\,Bb^2x^{13}+\frac{1}{9}\left(2\,Bab+Ab^2\right)x^9+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5+Aa^2x$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A),x, algorithm="fricas")
```

output
```
1/13*B*b^2*x^13 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
A*a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^4\right)^2 \left(A+Bx^4\right)\,dx = Aa^2x+\frac{Bb^2x^{13}}{13}+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate((b*x**4+a)**2*(B*x**4+A),x)
```

output
```
A*a**2*x + B*b**2*x**13/13 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**5*(2*A*a*b/5
 + B*a**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(A+Bx^4\right)\,dx = \frac{1}{13}\,Bb^2x^{13}+\frac{1}{9}\left(2\,Bab+Ab^2\right)x^9+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5+Aa^2x$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A),x, algorithm="maxima")
```

output
```
1/13*B*b^2*x^13 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/5*(B*a^2 + 2*A*a*b)*x^5 +
A*a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^2 \left(A+Bx^4\right)\,dx = \frac{1}{13}\,Bb^2x^{13}+\frac{2}{9}\,Babx^9+\frac{1}{9}\,Ab^2x^9+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+Aa^2x$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A),x, algorithm="giac")
```

output
```
1/13*B*b^2*x^13 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/5*B*a^2*x^5 + 2/5*A*a*
b*x^5 + A*a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2 \left(A+Bx^4\right)\,dx = x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right) + x^9\left(\frac{A\,b^2}{9}+\frac{2\,B\,a\,b}{9}\right)+\frac{B\,b^2\,x^{13}}{13}+A\,a^2\,x$$

input
```
int((A + B*x^4)*(a + b*x^4)^2,x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (B*b^2*x^1
3)/13 + A*a^2*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \left(a+bx^4\right)^2 \left(A+Bx^4\right)\,dx = \frac{x(15b^3x^{12}+65a\,b^2x^8+117a^2b\,x^4+195a^3)}{195}$$

input
```
int((b*x^4+a)^2*(B*x^4+A),x)
```

output
```
(x*(195*a**3 + 117*a**2*b*x**4 + 65*a*b**2*x**8 + 15*b**3*x**12))/195
```

3.9 $\int \frac{(a+bx^4)^2(A+Bx^4)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^2}\,dx = -\frac{a^2A}{x}+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{11}b^2Bx^{11}$$

output
```
-a^2*A/x+1/3*a*(2*A*b+B*a)*x^3+1/7*b*(A*b+2*B*a)*x^7+1/11*b^2*B*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^2}\,dx = -\frac{a^2A}{x}+\frac{1}{3}a(2Ab+aB)x^3+\frac{1}{7}b(Ab+2aB)x^7+\frac{1}{11}b^2Bx^{11}$$

input
```
Integrate[((a + b*x^4)^2*(A + B*x^4))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^11)/11
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^2}+bx^6(2aB+Ab)+ax^2(aB+2Ab)+b^2Bx^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{x}+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

input
```
Int[((a + b*x^4)^2*(A + B*x^4))/x^2,x]
```

output
```
-((a^2*A)/x) + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*
x^11)/11
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{\frac{B b^2 x^{12}}{11}+\left(\frac{1}{7} b^2 A+\frac{2}{7} a b B\right) x^8+\left(\frac{2}{3} a b A+\frac{1}{3} a^2 B\right) x^4-a^2 A}{x}$	53
default	$\frac{b^2 B\, x^{11}}{11}+\frac{A\, b^2 x^7}{7}+\frac{2 B a b\, x^7}{7}+\frac{2 A a b\, x^3}{3}+\frac{B\, a^2 x^3}{3}-\frac{a^2 A}{x}$	54
risch	$\frac{b^2 B\, x^{11}}{11}+\frac{A\, b^2 x^7}{7}+\frac{2 B a b\, x^7}{7}+\frac{2 A a b\, x^3}{3}+\frac{B\, a^2 x^3}{3}-\frac{a^2 A}{x}$	54
gosper	$-\frac{-21 B\, b^2 x^{12}-33 A\, b^2 x^8-66 B a b\, x^8-154 A a b\, x^4-77 B\, a^2 x^4+231 a^2 A}{231 x}$	56
parallelrisch	$\frac{21 B\, b^2 x^{12}+33 A\, b^2 x^8+66 B a b\, x^8+154 A a b\, x^4+77 B\, a^2 x^4-231 a^2 A}{231 x}$	56
orering	$-\frac{-21 B\, b^2 x^{12}-33 A\, b^2 x^8-66 B a b\, x^8-154 A a b\, x^4-77 B\, a^2 x^4+231 a^2 A}{231 x}$	56

input
```
int((b*x^4+a)^2*(B*x^4+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/11*B*b^2*x^12+(1/7*b^2*A+2/7*a*b*B)*x^8+(2/3*a*b*A+1/3*a^2*B)*x^4-a
^2*A)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2}\,dx = \frac{21\,Bb^2x^{12}+33\,(2\,Bab+Ab^2)x^8+77\,(Ba^2+2\,Aab)x^4-231\,Aa^2}{231\,x}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^2,x, algorithm="fricas")
```

output
```
1/231*(21*B*b^2*x^12 + 33*(2*B*a*b + A*b^2)*x^8 + 77*(B*a^2 + 2*A*a*b)*x^4
 - 231*A*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} dx = -\frac{Aa^2}{x} + \frac{Bb^2x^{11}}{11} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^3 \cdot \left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**4+a)**2*(B*x**4+A)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**11/11 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**3*(2*A*a*b/
3 + B*a**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} dx = \frac{1}{11} Bb^2x^{11} + \frac{1}{7} (2 Bab + Ab^2)x^7 + \frac{1}{3} (Ba^2 + 2 Aab)x^3 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^2,x, algorithm="maxima")
```

output
```
1/11*B*b^2*x^11 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*(B*a^2 + 2*A*a*b)*x^3 -
A*a^2/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} dx = \frac{1}{11} Bb^2x^{11} + \frac{2}{7} Babx^7 + \frac{1}{7} Ab^2x^7 + \frac{1}{3} Ba^2x^3 + \frac{2}{3} Aabx^3 - \frac{Aa^2}{x}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^2,x, algorithm="giac")
```

output
```
1/11*B*b^2*x^11 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/3*B*a^2*x^3 + 2/3*A*a*
b*x^3 - A*a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} \, dx = x^3 \left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^7 \left(\frac{A\,b^2}{7} + \frac{2\,B\,a\,b}{7}\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^{11}}{11}$$

input
```
int(((A + B*x^4)*(a + b*x^4)^2)/x^2,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^7*((A*b^2)/7 + (2*B*a*b)/7) - (A*a^2)/x
+ (B*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^2} \, dx = \frac{7b^3x^{12} + 33a\,b^2x^8 + 77a^2b\,x^4 - 77a^3}{77x}$$

input
```
int((b*x^4+a)^2*(B*x^4+A)/x^2,x)
```

output
```
( - 77*a**3 + 77*a**2*b*x**4 + 33*a*b**2*x**8 + 7*b**3*x**12)/(77*x)
```

3.10 $\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4} dx = -\frac{a^2A}{3x^3} + a(2Ab+aB)x + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{9}b^2Bx^9$$

output
```
-1/3*a^2*A/x^3+a*(2*A*b+B*a)*x+1/5*b*(A*b+2*B*a)*x^5+1/9*b^2*B*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4} dx = -\frac{a^2A}{3x^3} + a(2Ab+aB)x + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{9}b^2Bx^9$$

input
```
Integrate[((a + b*x^4)^2*(A + B*x^4))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^
9)/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^4} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^4}+bx^4(2aB+Ab)+a(aB+2Ab)+b^2Bx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}+\frac{1}{5}bx^5(2aB+Ab)+ax(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

input
```
Int[((a + b*x^4)^2*(A + B*x^4))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^
9)/9
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B x^9}{9}+\frac{A b^2 x^5}{5}+\frac{2 B a b x^5}{5}+2 a b A x+a^2 B x-\frac{a^2 A}{3 x^3}$	49
risch	$\frac{b^2 B x^9}{9}+\frac{A b^2 x^5}{5}+\frac{2 B a b x^5}{5}+2 a b A x+a^2 B x-\frac{a^2 A}{3 x^3}$	49
norman	$\frac{\frac{B b^2 x^{12}}{9}+\left(\frac{1}{5} b^2 A+\frac{2}{5} a b B\right) x^8+\left(2 a b A+a^2 B\right) x^4-\frac{a^2 A}{3}}{x^3}$	52
gosper	$-\frac{-5 B b^2 x^{12}-9 A b^2 x^8-18 B a b x^8-90 A a b x^4-45 B a^2 x^4+15 a^2 A}{45 x^3}$	56
parallelrisch	$\frac{5 B b^2 x^{12}+9 A b^2 x^8+18 B a b x^8+90 A a b x^4+45 B a^2 x^4-15 a^2 A}{45 x^3}$	56
orering	$-\frac{-5 B b^2 x^{12}-9 A b^2 x^8-18 B a b x^8-90 A a b x^4-45 B a^2 x^4+15 a^2 A}{45 x^3}$	56

input
```
int((b*x^4+a)^2*(B*x^4+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*B*x^9+1/5*A*b^2*x^5+2/5*B*a*b*x^5+2*a*b*A*x+a^2*B*x-1/3*a^2*A/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4}\,dx = \frac{5\,Bb^2x^{12}+9\,(2\,Bab+Ab^2)x^8+45\,(Ba^2+2\,Aab)x^4-15\,Aa^2}{45\,x^3}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^4,x, algorithm="fricas")
```

output
```
1/45*(5*B*b^2*x^12 + 9*(2*B*a*b + A*b^2)*x^8 + 45*(B*a^2 + 2*A*a*b)*x^4 -
15*A*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4}\,dx = -\frac{Aa^2}{3x^3}+\frac{Bb^2x^9}{9}+x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+x\left(2Aab+Ba^2\right)$$

input
```
integrate((b*x**4+a)**2*(B*x**4+A)/x**4,x)
```

output
```
-A*a**2/(3*x**3) + B*b**2*x**9/9 + x**5*(A*b**2/5 + 2*B*a*b/5) + x*(2*A*a*
b + B*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4}\,dx = \frac{1}{9}\,Bb^2x^9+\frac{1}{5}\,(2\,Bab+Ab^2)x^5+\left(Ba^2+2\,Aab\right)x-\frac{Aa^2}{3\,x^3}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^4,x, algorithm="maxima")
```

output
```
1/9*B*b^2*x^9 + 1/5*(2*B*a*b + A*b^2)*x^5 + (B*a^2 + 2*A*a*b)*x - 1/3*A*a^
2/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2(A+Bx^4)}{x^4}\,dx = \frac{1}{9}\,Bb^2x^9+\frac{2}{5}\,Babx^5+\frac{1}{5}\,Ab^2x^5+Ba^2x+2\,Aabx-\frac{Aa^2}{3\,x^3}$$

input
```
integrate((b*x^4+a)^2*(B*x^4+A)/x^4,x, algorithm="giac")
```

output
```
1/9*B*b^2*x^9 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + B*a^2*x + 2*A*a*b*x - 1/3*
A*a^2/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^4}\,dx = x^5 \left(\frac{A\,b^2}{5}+\frac{2\,B\,a\,b}{5}\right)+x\,\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{3\,x^3}+\frac{B\,b^2\,x^9}{9}$$

input
```
int(((A + B*x^4)*(a + b*x^4)^2)/x^4,x)
```

output
```
x^5*((A*b^2)/5 + (2*B*a*b)/5) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3) + (B
*b^2*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^4)^2 (A+Bx^4)}{x^4}\,dx = \frac{5b^3x^{12}+27a\,b^2x^8+135a^2b\,x^4-15a^3}{45x^3}$$

input
```
int((b*x^4+a)^2*(B*x^4+A)/x^4,x)
```

output
```
( - 15*a**3 + 135*a**2*b*x**4 + 27*a*b**2*x**8 + 5*b**3*x**12)/(45*x**3)
```

3.11 $\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx = \frac{(Ab-aB)x}{b^2}+\frac{Bx^5}{5b}+\frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{9/4}} - \frac{\sqrt[4]{a}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{9/4}}$$

output
```
(A*b-B*a)*x/b^2+1/5*B*x^5/b-1/4*a^(1/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4
)*x/a^(1/4))*2^(1/2)/b^(9/4)-1/4*a^(1/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4
)*x/a^(1/4))*2^(1/2)/b^(9/4)-1/4*a^(1/4)*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx$$
$$= \frac{40\sqrt[4]{b}(Ab-aB)x+8b^{5/4}Bx^5-10\sqrt{2}\sqrt[4]{a}(-Ab+aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}\sqrt[4]{a}(-Ab+aB)\text{arc}}{}$$

input
```
Integrate[(x^4*(A + B*x^4))/(a + b*x^4),x]
```

output
```
(40*b^(1/4)*(A*b - a*B)*x + 8*b^(5/4)*B*x^5 - 10*Sqrt[2]*a^(1/4)*(-(A*b) +
 a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*a^(1/4)*(-(A*b)
 + a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*a^(1/4)*(-(A*b
) + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2
]*a^(1/4)*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(40*b^(9/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {959, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(Ab-aB)\int \frac{x^4}{bx^4+a}dx}{b}+\frac{Bx^5}{5b}$$
$$\Big\downarrow \text{843}$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^4+a}dx}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\Big\downarrow$ 755

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\Big\downarrow$ 1476

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\Big\downarrow$ 1082

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\downarrow$ 217

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\downarrow$ 1479

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\frac{Bx^5}{5b}$$

$\Big\downarrow$ 25

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+$$

$$\frac{Bx^5}{5b}$$

$\Big\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

$\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{Bx^5}{5b}$$

input

```
Int[(x^4*(A + B*x^4))/(a + b*x^4),x]
```

output
```
(B*x^5)/(5*b) + ((A*b - a*B)*(x/b - (a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*S
qrt[a])))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.33

method	result	size
risch	$\frac{B x^5}{5b}+\frac{Ax}{b}-\frac{Bax}{b^2}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(-Ab+Ba)\ln\left(x-\_R\right)}{\_R^3}\right)}{4b^3}$	60
default	$\frac{\frac{1}{5}bB\,x^5+Abx-Bax}{b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b^2}$	132

input
```
int(x^4*(B*x^4+A)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b+1/b*A*x-1/b^2*B*a*x+1/4/b^3*a*sum((-A*b+B*a)/_R^3*ln(x-_R),_R=
RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 587, normalized size of antiderivative = 3.23

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx$$

$$=\frac{4\,Bbx^5-5\,b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4}{b^9}\right.\right.}{}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/20*(4*B*b*x^5 - 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4
*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b
+ 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b
)*x) - 5*I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^
2*b^3 + A^4*a*b^4)/b^9)^(1/4)*log(I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2
*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*x) +
5*I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 +
 A^4*a*b^4)/b^9)^(1/4)*log(-I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a
^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*x) + 5*b^2*
(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b
^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4
*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*x) - 20*(B*a - A*b)*x
)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx = \frac{Bx^5}{5b}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right) + \mathrm{RootSum}\left(256t^4b^9+A^4ab^4-4A^3Ba^2b^3+6A^2B^2a^3b^2-4AB^3a^4b+B^4a^5,\left(t\mapsto t\log\left(\frac{4tb^2}{-Ab+Ba}+\right.\right.\right.$$

input
```
integrate(x**4*(B*x**4+A)/(b*x**4+a),x)
```

output
```
B*x**5/(5*b) + x*(A/b - B*a/b**2) + RootSum(256*_t**4*b**9 + A**4*a*b**4 -
 4*A**3*B*a**2*b**3 + 6*A**2*B**2*a**3*b**2 - 4*A*B**3*a**4*b + B**4*a**5,
 Lambda(_t, _t*log(4*_t*b**2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx$$

$$= \frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{8\,b^2}$$

$$+ \frac{Bbx^5-5\,(Ba-Ab)x}{5\,b^2}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqr
t(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a -
A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/
4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(1/4)))*a/b^2 + 1/5*(B*b*x^5 - 5*(B*a - A*b)*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.41

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8b^3} + \frac{Bb^4x^5-5\,Bab^3x+5\,Ab^4x}{5\,b^5}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a
 - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/b^3 + 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2
+ sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^3 - 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a
- (a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^3 + 1/
5*(B*b^4*x^5 - 5*B*a*b^3*x + 5*A*b^4*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.23

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^4))/(a + b*x^4),x)
```

output
```
x*(A/b - (B*a)/b^2) + (B*x^5)/(5*b) - ((-a)^(1/4)*atan((((-a)^(1/4)*(A*b -
 B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(16*A*a
^2*b^2 - 16*B*a^3*b)*(A*b - B*a))/(4*b^(9/4)))*1i)/(4*b^(9/4)) + ((-a)^(1/
4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4
)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a))/(4*b^(9/4)))*1i)/(4*b^(9/4)))/(
((-a)^(1/4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - (
(-a)^(1/4)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a))/(4*b^(9/4))))/(4*b^(9/
4)) - ((-a)^(1/4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))
/b + ((-a)^(1/4)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a))/(4*b^(9/4))))/(4
*b^(9/4))))*(A*b - B*a)*1i)/(2*b^(9/4)) - ((-a)^(1/4)*atan((((-a)^(1/4)*(A
*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(16
*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a)*1i)/(4*b^(9/4))))/(4*b^(9/4)) + ((-a)
^(1/4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^
(1/4)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a)*1i)/(4*b^(9/4))))/(4*b^(9/4)
))/(((-a)^(1/4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b
 - ((-a)^(1/4)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a)*1i)/(4*b^(9/4)))*1i
)/(4*b^(9/4)) - ((-a)^(1/4)*(A*b - B*a)*((4*x*(B^2*a^4 + A^2*a^2*b^2 - 2*A
*B*a^3*b))/b + ((-a)^(1/4)*(16*A*a^2*b^2 - 16*B*a^3*b)*(A*b - B*a)*1i)/(4*
b^(9/4)))*1i)/(4*b^(9/4))))*(A*b - B*a))/(2*b^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^4(A+Bx^4)}{a+bx^4}\,dx = \frac{x^5}{5}$$

input
```
int(x^4*(B*x^4+A)/(b*x^4+a),x)
```

output
```
x**5/5
```

3.12 $\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{Bx^3}{3b} - \frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

output
```
1/3*B*x^3/b+1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
1/4)/b^(7/4)+1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
1/4)/b^(7/4)-1/4*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1
/2)*x^2))*2^(1/2)/a^(1/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx$$
$$= \frac{8\sqrt[4]{a}b^{3/4}Bx^3 - 6\sqrt{2}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 6\sqrt{2}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 3\sqrt{2}(Ab}{24\sqrt[4]{a}b^{7/4}}$$

input
```
Integrate[(x^2*(A + B*x^4))/(a + b*x^4),x]
```

output
```
(8*a^(1/4)*b^(3/4)*B*x^3 - 6*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*x)/a^(1/4)] + 6*Sqrt[2]*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)] + 3*Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - 3*Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2])/(24*a^(1/4)*b^(7/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {959, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(Ab-aB)\int \frac{x^2}{bx^4+a}dx}{b} + \frac{Bx^3}{3b}$$
$$\Big\downarrow \text{826}$$
$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}} - \frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b} + \frac{Bx^3}{3b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \text{217}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$\downarrow$ 25

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

$\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}+\frac{Bx^3}{3b}$$

input
```
Int[(x^2*(A + B*x^4))/(a + b*x^4),x]
```

output
```
(B*x^3)/(3*b) + ((A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(
Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[
2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/
b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.27

method	result	size
risch	$\frac{B x^{3}}{3 b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b \_Z^{4}+a \right)}\frac{\left(A b -B a \right) \ln \left(x -\_R \right)}{\_R}}{4 b^{2}}$	45
default	$\frac{B x^{3}}{3 b}+\frac{\left(A b -B a \right) \sqrt{2}\, \left(\ln \left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8 b^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}$	120

input
```
int(x^2*(B*x^4+A)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/b+1/4/b^2*sum((A*b-B*a)/_R*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 683, normalized size of antiderivative = 4.04

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{4\,Bx^3+3\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)\right.}{}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/12*(4*B*x^3 + 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3
*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b +
6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3
*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x) - 3*I*b*(-(B^4*a^4 - 4*A*B^3*a^
3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(I*a*
b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x
) + 3*I*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a*b^7))^(1/4)*log(-I*a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B
^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*
a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x) - 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A
^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(-a*b^5*(-(B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7
))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x))/b
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.72

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{Bx^3}{3b}$$
$$+\text{RootSum}\left(256t^4ab^7 + A^4b^4 - 4A^3Bab^3 + 6A^2B^2a^2b^2 - 4AB^3a^3b + B^4a^4, \left(t \mapsto t\log\left(-\frac{}{-A^3b^3+3A}\right.\right.\right.$$

input
```
integrate(x**2*(B*x**4+A)/(b*x**4+a),x)
```

output
```
B*x**3/(3*b) + RootSum(256*_t**4*a*b**7 + A**4*b**4 - 4*A**3*B*a*b**3 + 6*
A**2*B**2*a**2*b**2 - 4*A*B**3*a**3*b + B**4*a**4, Lambda(_t, _t*log(-64*_
t**3*a*b**5/(-A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2*a**2*b + B**3*a**3) +
 x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{Bx^3}{3\,b}$$
$$-\frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \right.}{8\,b}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/3*B*x^3/b - 1/8*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x +
 sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sq
rt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(s
qrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt
(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(120) = 240$.

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.47

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{Bx^3}{3\,b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba - \left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^4}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/3*B*x^3/b - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1
/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) - 1/4*sqrt(2)*
((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/8*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^
3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*s
qrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx = \frac{B\,x^3}{3\,b} + \frac{\mathrm{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,\left(A\,b-B\,a\right)}{2\,{(-a)}^{1/4}\,b^{7/4}} - \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,\left(A\,b-B\,a\right)}{2\,{(-a)}^{1/4}\,b^{7/4}}$$

input
```
int((x^2*(A + B*x^4))/(a + b*x^4),x)
```

output
```
(B*x^3)/(3*b) + (atan((b^(1/4)*x)/(-a)^(1/4))*(A*b - B*a))/(2*(-a)^(1/4)*b
^(7/4)) - (atanh((b^(1/4)*x)/(-a)^(1/4))*(A*b - B*a))/(2*(-a)^(1/4)*b^(7/4
))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{x^2(A+Bx^4)}{a+bx^4}\,dx=\frac{x^3}{3}$$

input
```
int(x^2*(B*x^4+A)/(b*x^4+a),x)
```

output
```
x**3/3
```

3.13 $\int \frac{A+Bx^4}{a+bx^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int \frac{A+Bx^4}{a+bx^4}\,dx=\frac{Bx}{b}-\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
B*x/b+1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b
^(5/4)+1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b
^(5/4)+1/4*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^
2))*2^(1/2)/a^(3/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^4}{a+bx^4}\,dx$$
$$=\frac{8a^{3/4}\sqrt[4]{b}Bx-2\sqrt{2}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt{2}(Ab-}{8a^{3/4}b^{5/4}}$$

input
```
Integrate[(A + B*x^4)/(a + b*x^4),x]
```

output
```
(8*a^(3/4)*b^(1/4)*B*x - 2*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)] + 2*Sqrt[2]*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] - Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2] + Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(8*a^(3/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{a+bx^4}\,dx$$
$$\Big\downarrow \text{913}$$
$$\frac{(Ab-aB)\int \frac{1}{bx^4+a}dx}{b}+\frac{Bx}{b}$$
$$\Big\downarrow \text{755}$$
$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$\Big\downarrow$ 25

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$\Big\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

$\Big\downarrow$ 1103

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{Bx}{b}$$

input

```
Int[(A + B*x^4)/(a + b*x^4),x]
```

output
```
(B*x)/b + ((A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2
]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{Bx}{b}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)}\frac{(Ab-Ba)\ln\left(x-\_R\right)}{\_R^{3}}}{4b^{2}}$	42
default	$\frac{Bx}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ba}$	120

input
```
int((B*x^4+A)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/b+1/4/b^2*sum((A*b-B*a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.41

$$\int \frac{A + Bx^4}{a + bx^4} \, dx$$
$$= \frac{b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}} \log\left(ab\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}} - (Ba -$$

input
```
integrate((B*x^4+A)/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^
4*b^4)/(a^3*b^5))^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2
*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*x) + I*b*(-
(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a
^3*b^5))^(1/4)*log(I*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*x) - I*b*(-(B^4*a^
4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5)
)^(1/4)*log(-I*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*
B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*x) - b*(-(B^4*a^4 - 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4)*
log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*x) + 4*B*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{A + Bx^4}{a + bx^4} \, dx = \frac{Bx}{b}$$
$$+ \operatorname{RootSum}\left(256t^4a^3b^5 + A^4b^4 - 4A^3Bab^3 + 6A^2B^2a^2b^2 - 4AB^3a^3b + B^4a^4, \left(t \mapsto t\log\left(-\frac{4tab}{-Ab + Ba}\right.\right.\right.$$

input
```
integrate((B*x**4+A)/(b*x**4+a),x)
```

output
```
B*x/b + RootSum(256*_t**4*a**3*b**5 + A**4*b**4 - 4*A**3*B*a*b**3 + 6*A**2
*B**2*a**2*b**2 - 4*A*B**3*a**3*b + B**4*a**4, Lambda(_t, _t*log(-4*_t*a*b
/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^4}{a+bx^4}\,dx = \frac{Bx}{b} - \frac{\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8b}$$

input
```
integrate((B*x^4+A)/(b*x^4+a),x, algorithm="maxima")
```

output
```
B*x/b - 1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + 2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*(B*a - A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4
)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(117) = 234$.

Time = 0.13 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^4}{a+bx^4}\,dx = \frac{Bx}{b} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba - (ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba - (ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba - (ab^3)^{\frac{1}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba - (ab^3)^{\frac{1}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^2}$$

input
```
integrate((B*x^4+A)/(b*x^4+a),x, algorithm="giac")
```

output
```
B*x/b - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*((a*b^
3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a*b^2) - 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/
4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) + 1/8*sqrt(2)
*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.39

$$\int \frac{A+Bx^4}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(a + b*x^4),x)
```

output
```
(B*x)/b - (atan((((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2) -
 ((16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a))/(4*(-a)^(3/4)*b^(5/4)))*1i)/(4*
(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b
^2) + ((16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a))/(4*(-a)^(3/4)*b^(5/4)))*1i
)/(4*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*
B*a*b^2) - ((16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a))/(4*(-a)^(3/4)*b^(5/4)
)))/(4*(-a)^(3/4)*b^(5/4)) - ((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*
A*B*a*b^2) + ((16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a))/(4*(-a)^(3/4)*b^(5/
4))))/(4*(-a)^(3/4)*b^(5/4))))*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (a
tan((((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2) - ((16*B*a^2*
b^2 - 16*A*a*b^3)*(A*b - B*a)*1i)/(4*(-a)^(3/4)*b^(5/4))))/(4*(-a)^(3/4)*b
^(5/4)) + ((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2) + ((16*B
*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a)*1i)/(4*(-a)^(3/4)*b^(5/4))))/(4*(-a)^(3
/4)*b^(5/4)))/(((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2) - (
(16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a)*1i)/(4*(-a)^(3/4)*b^(5/4)))*1i)/(4
*(-a)^(3/4)*b^(5/4)) - ((A*b - B*a)*(x*(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*
b^2) + ((16*B*a^2*b^2 - 16*A*a*b^3)*(A*b - B*a)*1i)/(4*(-a)^(3/4)*b^(5/4))
)*1i)/(4*(-a)^(3/4)*b^(5/4))))*(A*b - B*a))/(2*(-a)^(3/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \frac{A+Bx^4}{a+bx^4}\,dx=x$$

input
```
int((B*x^4+A)/(b*x^4+a),x)
```

output
```
x
```

3.14 $\int \frac{A+Bx^4}{x^2(a+bx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 167

$$\int \frac{A+Bx^4}{x^2(a+bx^4)}\,dx = -\frac{A}{ax}+\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}b^{3/4}} - \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}b^{3/4}} + \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{5/4}b^{3/4}}$$

output
```
-A/a/x-1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/
b^(3/4)-1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/
b^(3/4)+1/4*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x
^2))*2^(1/2)/a^(5/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)}\,dx$$
$$= \frac{-8\sqrt[4]{a}Ab^{3/4}+2\sqrt{2}(Ab-aB)x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt{2}(Ab-aB)x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt{2}(A}{8a^{5/4}b^{3/4}x}$$

input
```
Integrate[(A + B*x^4)/(x^2*(a + b*x^4)),x]
```

output
```
(-8*a^(1/4)*A*b^(3/4) + 2*Sqrt[2]*(A*b - a*B)*x*ArcTan[1 - (Sqrt[2]*b^(1/4
)*x)/a^(1/4)] - 2*Sqrt[2]*(A*b - a*B)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)] - Sqrt[2]*(A*b - a*B)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + Sqrt[2]*(A*b - a*B)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/(8*a^(5/4)*b^(3/4)*x)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {955, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)}\,dx$$
$$\Big\downarrow \text{955}$$
$$-\frac{(Ab-aB)\int \frac{x^2}{bx^4+a}dx}{a}-\frac{A}{ax}$$
$$\Big\downarrow \text{826}$$
$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{217}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 25$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 27$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

$$\Big\downarrow \; 1103$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{A}{ax}$$

input
```
Int[(A + B*x^4)/(x^2*(a + b*x^4)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*
a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.74

method	result
default	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{A}{ax}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^5b^3\_Z^4+A^4b^4-4A^3Ba\,b^3+6A^2B^2a^2b^2-4A\,B^3a^3b+B^4a^4\right)}\_R\ln\left(\left(5\_R^4a^5b^3+4A^4b^4-16A^3Ba\,b^3+24A\right.\right.\right.}{4}$

input
```
int((B*x^4+A)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(A*b-B*a)/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-A/a/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 694, normalized size of antiderivative = 4.16

$$\int \frac{A+Bx^4}{x^2\,(a+bx^4)}\,dx =$$

$$-\frac{ax\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^5b^3}\right)^{\frac{3}{4}}-\left(\right.\right.}{}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*
B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B
^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x) - I*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b
+ 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(I*a^4*
b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)
*x) + I*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3
 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-I*a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 -
3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*x) - a*x*(-(B^4*a^4 - 4*A*B^3*a^3
*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-a^
4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4
*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^
3)*x) + 4*A)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^4}{x^2(a+bx^4)}\,dx = -\frac{A}{ax} + \text{RootSum}\left(256t^4a^5b^3 + A^4b^4 - 4A^3Bab^3 + 6A^2B^2a^2b^2 - 4AB^3a^3b + B^4a^4, \left(t \mapsto t\log\left(\frac{}{-A^3b^3+3A}\right.\right.\right.$$

input
```
integrate((B*x**4+A)/x**2/(b*x**4+a),x)
```

output
```
-A/(a*x) + RootSum(256*_t**4*a**5*b**3 + A**4*b**4 - 4*A**3*B*a*b**3 + 6*A
**2*B**2*a**2*b**2 - 4*A*B**3*a**3*b + B**4*a**4, Lambda(_t, _t*log(64*_t*
*3*a**4*b**2/(-A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2*a**2*b + B**3*a**3)
+ x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^4}{x^2(a+bx^4)}\,dx = \frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\,\mathrm{l}}{}\right.}{8a} - \frac{A}{ax}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqr
t(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b
)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a - A/(a*x
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(120) = 240$.

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^4}{x^2(a+bx^4)}\,dx = -\frac{A}{ax} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba - (ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^2b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba - (ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^2b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba - (ab^3)^{\frac{3}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^2b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba - (ab^3)^{\frac{3}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^2b^3}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a),x, algorithm="giac")
```

output
```
-A/(a*x) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/4*sqrt(2)*(
(a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) - 1/8*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b
^3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/
8*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b
)^(1/4) + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^4}{x^2\,(a+bx^4)}\,dx=\frac{\mathrm{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{2\,(-a)^{5/4}\,b^{3/4}}-\frac{A}{a\,x}-\frac{\mathrm{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{2\,(-a)^{5/4}\,b^{3/4}}$$

input
```
int((A + B*x^4)/(x^2*(a + b*x^4)),x)
```

output
```
(atan((b^(1/4)*x)/(-a)^(1/4))*(A*b - B*a))/(2*(-a)^(5/4)*b^(3/4)) - A/(a*x
) - (atanh((b^(1/4)*x)/(-a)^(1/4))*(A*b - B*a))/(2*(-a)^(5/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^4}{x^2\,(a+bx^4)}\,dx=-\frac{1}{x}$$

input
```
int((B*x^4+A)/x^2/(b*x^4+a),x)
```

output
```
( - 1)/x
```

3.15 $\int \frac{A+Bx^4}{x^4(a+bx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx=-\frac{A}{3ax^3}+\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

output
```
-1/3*A/a/x^3-1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^
(7/4)/b^(1/4)-1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^
(7/4)/b^(1/4)-1/4*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(
1/2)*x^2))*2^(1/2)/a^(7/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx$$
$$= \frac{-\frac{8a^{3/4}A}{x^3}+\frac{6\sqrt{2}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right.}{\sqrt[4]{b}}}{24a^{7/4}}$$

input
```
Integrate[(A + B*x^4)/(x^4*(a + b*x^4)),x]
```

output
```
((-8*a^(3/4)*A)/x^3 + (6*Sqrt[2]*(A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/b^(1/4) - (6*Sqrt[2]*(A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/b^(1/4) + (3*Sqrt[2]*(A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4) + (3*Sqrt[2]*(-(A*b) + a*B)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(24*a^(7/4))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {955, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx$$
$$\Big\downarrow \text{955}$$
$$-\frac{(Ab-aB)\int \frac{1}{bx^4+a}dx}{a}-\frac{A}{3ax^3}$$
$$\Big\downarrow \text{755}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$\big\downarrow$ 25

$$-\frac{(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$\big\downarrow$ 27

$$-\frac{(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

$\big\downarrow$ 1103

$$-\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{A}{3ax^3}$$

input
```
Int[(A + B*x^4)/(x^4*(a + b*x^4)),x]
```

output
```
-1/3*A/(a*x^3) - ((A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/
(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))
/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

method	result
default	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2}-\frac{A}{3a\,x^3}$
risch	$-\frac{A}{3a\,x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(b\,a^7\_Z^4+A^4b^4-4A^3Ba\,b^3+6A^2B^2a^2b^2-4A\,B^3a^3b+B^4a^4\right)}\_R\ln\left(\left(-5\_R^4a^7b-4A^4b^4+16A^3Ba\,b^3-2\cdots\right.\right.\right.}{4}$

input
```
int((B*x^4+A)/x^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-A*b+B*a)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/3*A/a/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.43

$$\int\frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx=$$

$$-\frac{3\,ax^3\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}-\cdots\right.}{\cdots}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a),x, algorithm="fricas")
```

output
```
-1/12*(3*a*x^3*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*
b^3 + A^4*b^4)/(a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B
^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*x) + 3*
I*a*x^3*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4)/(a^7*b))^(1/4)*log(I*a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^
2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*x) - 3*I*a*x
^3*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^
4)/(a^7*b))^(1/4)*log(-I*a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*x) - 3*a*x^3*(-(
B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^
7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^
3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*x) + 4*A)/(a*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx = -\frac{A}{3ax^3}$$
$$+\mathrm{RootSum}\left(256t^4a^7b+A^4b^4-4A^3Bab^3+6A^2B^2a^2b^2-4AB^3a^3b+B^4a^4,\left(t\mapsto t\log\left(\frac{4ta^2}{-Ab+Ba}+\right.\right.\right.$$

input
```
integrate((B*x**4+A)/x**4/(b*x**4+a),x)
```

output
```
-A/(3*a*x**3) + RootSum(256*_t**4*a**7*b + A**4*b**4 - 4*A**3*B*a*b**3 + 6
*A**2*B**2*a**2*b**2 - 4*A*B**3*a**3*b + B**4*a**4, Lambda(_t, _t*log(4*_t
*a**2/(-A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx$$
$$=\frac{\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8a}$$
$$-\frac{A}{3\,ax^3}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqr
t(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a -
A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/
4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(1/4)))/a - 1/3*A/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(120) = 240$.

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx = \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^2b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^2b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^2b} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^2b} - \frac{A}{3ax^3}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)
*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^2*b) + 1/8*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*
log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b) - 1/8*sqrt(2)*((a*b^3
)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^2*b) - 1/3*A/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.70

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^4*(a + b*x^4)),x)
```

output
```
- A/(3*a*x^3) - (atan((((x*(4*A^2*a^3*b^5 + 4*B^2*a^5*b^3 - 8*A*B*a^4*b^4)
 - ((A*b - B*a)*(16*A*a^5*b^4 - 16*B*a^6*b^3))/(4*(-a)^(7/4)*b^(1/4)))*(A*
b - B*a)*1i)/(4*(-a)^(7/4)*b^(1/4)) + ((x*(4*A^2*a^3*b^5 + 4*B^2*a^5*b^3 -
 8*A*B*a^4*b^4) + ((A*b - B*a)*(16*A*a^5*b^4 - 16*B*a^6*b^3))/(4*(-a)^(7/4
)*b^(1/4)))*(A*b - B*a)*1i)/(4*(-a)^(7/4)*b^(1/4)))/(((x*(4*A^2*a^3*b^5 +
4*B^2*a^5*b^3 - 8*A*B*a^4*b^4) - ((A*b - B*a)*(16*A*a^5*b^4 - 16*B*a^6*b^3
))/(4*(-a)^(7/4)*b^(1/4)))*(A*b - B*a))/(4*(-a)^(7/4)*b^(1/4)) - ((x*(4*A^
2*a^3*b^5 + 4*B^2*a^5*b^3 - 8*A*B*a^4*b^4) + ((A*b - B*a)*(16*A*a^5*b^4 -
16*B*a^6*b^3))/(4*(-a)^(7/4)*b^(1/4)))*(A*b - B*a))/(4*(-a)^(7/4)*b^(1/4))
))*(A*b - B*a)*1i)/(2*(-a)^(7/4)*b^(1/4)) - (atan((((x*(4*A^2*a^3*b^5 + 4*
B^2*a^5*b^3 - 8*A*B*a^4*b^4) - ((A*b - B*a)*(16*A*a^5*b^4 - 16*B*a^6*b^3)*
1i)/(4*(-a)^(7/4)*b^(1/4)))*(A*b - B*a))/(4*(-a)^(7/4)*b^(1/4)) + ((x*(4*A
^2*a^3*b^5 + 4*B^2*a^5*b^3 - 8*A*B*a^4*b^4) + ((A*b - B*a)*(16*A*a^5*b^4 -
 16*B*a^6*b^3)*1i)/(4*(-a)^(7/4)*b^(1/4)))*(A*b - B*a))/(4*(-a)^(7/4)*b^(1
/4)))/(((x*(4*A^2*a^3*b^5 + 4*B^2*a^5*b^3 - 8*A*B*a^4*b^4) - ((A*b - B*a)*
(16*A*a^5*b^4 - 16*B*a^6*b^3)*1i)/(4*(-a)^(7/4)*b^(1/4)))*(A*b - B*a)*1i)/
(4*(-a)^(7/4)*b^(1/4)) - ((x*(4*A^2*a^3*b^5 + 4*B^2*a^5*b^3 - 8*A*B*a^4*b^
4) + ((A*b - B*a)*(16*A*a^5*b^4 - 16*B*a^6*b^3)*1i)/(4*(-a)^(7/4)*b^(1/4))
)*(A*b - B*a)*1i)/(4*(-a)^(7/4)*b^(1/4))))*(A*b - B*a))/(2*(-a)^(7/4)*b^(1
/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)}\,dx=-\frac{1}{3x^3}$$

input
```
int((B*x^4+A)/x^4/(b*x^4+a),x)
```

output
```
( - 1)/(3*x**3)
```

3.16 $\int \frac{A+Bx^4}{x^6(a+bx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 184

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx=-\frac{A}{5ax^5}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}}+\frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}}-\frac{\sqrt[4]{b}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{9/4}}$$

output
```
-1/5*A/a/x^5+(A*b-B*a)/a^2/x+1/4*b^(1/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/
4)*x/a^(1/4))*2^(1/2)/a^(9/4)+1/4*b^(1/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/
4)*x/a^(1/4))*2^(1/2)/a^(9/4)-1/4*b^(1/4)*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4
)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^4}{x^6(a+bx^4)}\,dx$$
$$= \frac{-\frac{8a^{5/4}A}{x^5}+\frac{40\sqrt[4]{a}(Ab-aB)}{x}-10\sqrt{2}\sqrt[4]{b}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}\sqrt[4]{b}(Ab-aB)\arctan\left(1+\right.}{}$$

input
```
Integrate[(A + B*x^4)/(x^6*(a + b*x^4)),x]
```

output
```
((-8*a^(5/4)*A)/x^5 + (40*a^(1/4)*(A*b - a*B))/x - 10*Sqrt[2]*b^(1/4)*(A*b
 - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*b^(1/4)*(A*b
- a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 5*Sqrt[2]*b^(1/4)*(A*b -
a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*b^
(1/4)*(-(A*b) + a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2
])/(40*a^(9/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {955, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^6(a+bx^4)}\,dx$$
$$\Big\downarrow \text{955}$$
$$-\frac{(Ab-aB)\int \frac{1}{x^2(bx^4+a)}dx}{a}-\frac{A}{5ax^5}$$
$$\Big\downarrow \text{847}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int\frac{x^2}{bx^4+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 826

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 1476

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 1082

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 217$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \; 1479$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a} -\right.}{a}$$

$$\frac{A}{5ax^5}$$

$\downarrow$ 25

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right.}{a}$$

$$\frac{A}{5ax^5}$$

$\downarrow$ 27

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

$\downarrow$ 1103

$$-\frac{(Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{5ax^5}$$

input

```
Int[(A + B*x^4)/(x^6*(a + b*x^4)),x]
```

output
```
-1/5*A/(a*x^5) - ((A*b - a*B)*(-(1/(a*x)) - (b*((-(ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*
x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4
)))/(2*Sqrt[b])))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

method	result
default	$\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{A}{5a\,x^5}-\frac{-Ab+Ba}{a^2x}$
risch	$\frac{\frac{(Ab-Ba)x^4}{a^2}-\frac{A}{5a}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^4+A^4b^5-4A^3Ba\,b^4+6A^2B^2a^2b^3-4A\,B^3a^3b^2+B^4a^4b\right)}\_R\ln\left(\left(5\_R^4a^9+4A^4b^5-16A^3Ba\right.\right.\right.}{4}$

input
```
int((B*x^4+A)/x^6/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*(A*b-B*a)/a^2/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/5*A/a/x^5-1/a^2*(-A*b+B*a)/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.94

$$\int\frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx$$
$$=\frac{5\,a^2x^5\left(-\frac{B^4a^4b-4\,AB^3a^3b^2+6\,A^2B^2a^2b^3-4\,A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{B^4a^4b-4\,AB^3a^3b^2+6\,A^2B^2a^2b^3-4\,A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{3}{4}}-\right.}{}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a),x, algorithm="fricas")
```

output
```
1/20*(5*a^2*x^5*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3
*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*
A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B
^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*x) - 5*I*a^2*x^5*(-(B^4*a^4*b - 4*A*
B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(
I*a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 +
 A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*
b^4)*x) + 5*I*a^2*x^5*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 -
 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-I*a^7*(-(B^4*a^4*b - 4*A*B^3*a^3
*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*
b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*x) - 5*a^2*x^5*(-(B^4*a^4*b
 - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/
4)*log(-a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a
*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3
- A^3*b^4)*x) - 20*(B*a - A*b)*x^4 - 4*A*a)/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^4}{x^6\,(a+bx^4)}\,dx$$
$$=\text{RootSum}\left(256t^4a^9+A^4b^5-4A^3Bab^4+6A^2B^2a^2b^3-4AB^3a^3b^2+B^4a^4b,\left(t\mapsto t\log\left(-\frac{}{-A^3b^4+3A^2}\right.\right.\right.$$
$$+\frac{-Aa+x^4\cdot(5Ab-5Ba)}{5a^2x^5}$$

input
```
integrate((B*x**4+A)/x**6/(b*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**9 + A**4*b**5 - 4*A**3*B*a*b**4 + 6*A**2*B**2*a**2*b*
*3 - 4*A*B**3*a**3*b**2 + B**4*a**4*b, Lambda(_t, _t*log(-64*_t**3*a**7/(-
A**3*b**4 + 3*A**2*B*a*b**3 - 3*A*B**2*a**2*b**2 + B**3*a**3*b) + x))) + (
-A*a + x**4*(5*A*b - 5*B*a))/(5*a**2*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx = -\frac{\left(Bab-Ab^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\right.}{8\,a^2} - \frac{5\,(Ba-Ab)x^4+Aa}{5\,a^2x^5}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*(B*a*b - A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*
a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2 -
 1/5*(5*(B*a - A*b)*x^4 + A*a)/(a^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx = -\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^3b^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4a^3b^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^3b^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8a^3b^2} - \frac{5Bax^4-5Abx^4+Aa}{5a^2x^5}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^2) - 1/4*sqrt(2)*((a*b^3)^(3
/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4)
)/(a/b)^(1/4))/(a^3*b^2) + 1/8*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*
A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^2) - 1/8*sqrt(2)*
((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(a^3*b^2) - 1/5*(5*B*a*x^4 - 5*A*b*x^4 + A*a)/(a^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx = \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,x}{a^{1/4}}\right)\,(A\,b-B\,a)}{2\,a^{9/4}} - \frac{\frac{A}{5\,a}-\frac{x^4\,(A\,b-B\,a)}{a^2}}{x^5} - \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,x}{a^{1/4}}\right)\,(A\,b-B\,a)}{2\,a^{9/4}}$$

input
```
int((A + B*x^4)/(x^6*(a + b*x^4)),x)
```

output
```
((-b)^(1/4)*atan(((-b)^(1/4)*x)/a^(1/4))*(A*b - B*a))/(2*a^(9/4)) - (A/(5*
a) - (x^4*(A*b - B*a))/a^2)/x^5 - ((-b)^(1/4)*atanh(((-b)^(1/4)*x)/a^(1/4)
)*(A*b - B*a))/(2*a^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.03

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)}\,dx=-\frac{1}{5x^5}$$

input
```
int((B*x^4+A)/x^6/(b*x^4+a),x)
```

output
```
( - 1)/(5*x**5)
```

3.17 $\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{(Ab-2aB)x}{b^3}+\frac{Bx^5}{5b^2}+\frac{a(Ab-aB)x}{4b^3(a+bx^4)} + \frac{\sqrt[4]{a}(5Ab-9aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{a}(5Ab-9aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}b^{13/4}} - \frac{\sqrt[4]{a}(5Ab-9aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}b^{13/4}}$$

output
```
(A*b-2*B*a)*x/b^3+1/5*B*x^5/b^2+1/4*a*(A*b-B*a)*x/b^3/(b*x^4+a)-1/16*a^(1/
4)*(5*A*b-9*B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(13/4)-1/1
6*a^(1/4)*(5*A*b-9*B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(13/
4)-1/16*a^(1/4)*(5*A*b-9*B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b
^(1/2)*x^2))*2^(1/2)/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.21

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$= \frac{160\sqrt[4]{b}(Ab-2aB)x+32b^{5/4}Bx^5+\frac{40a\sqrt[4]{b}(Ab-aB)x}{a+bx^4}-10\sqrt{2}\sqrt[4]{a}(-5Ab+9aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10}{}$$

input
```
Integrate[(x^8*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
(160*b^(1/4)*(A*b - 2*a*B)*x + 32*b^(5/4)*B*x^5 + (40*a*b^(1/4)*(A*b - a*B
)*x)/(a + b*x^4) - 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[1 + (Sqr
t[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*a^(1/4)*(-5*A*b + 9
*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(160*b^(13/4
))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{x^9(Ab-aB)}{4ab(a+bx^4)}-\frac{(5Ab-9aB)\int \frac{x^8}{bx^4+a}dx}{4ab}$$
$$\Big\downarrow \text{831}$$

$$\frac{x^9(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(5Ab-9aB)\int\left(\frac{x^4}{b}+\frac{a^2}{b^2(bx^4+a)}-\frac{a}{b^2}\right)dx}{4ab}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^9(Ab-aB)}{4ab\left(a+bx^4\right)}-$$

$$\frac{(5Ab-9aB)\left(-\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{9/4}}+\frac{a^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{9/4}}-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{9/4}}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{\right.}{4\right.}}{4ab}$$

input
```
Int[(x^8*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
((A*b - a*B)*x^9)/(4*a*b*(a + b*x^4)) - ((5*A*b - 9*a*B)*(-((a*x)/b^2) + x
^5/(5*b) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^
(9/4)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(9
/4)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4
*Sqrt[2]*b^(9/4)) + (a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2])/(4*Sqrt[2]*b^(9/4))))/(4*a*b)
```

Defintions of rubi rules used

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.41

method	result
risch	$\frac{B\,x^{5}}{5b^{2}}+\frac{Ax}{b^{2}}-\frac{2Bax}{b^{3}}+\frac{\left(\frac{1}{4}abA-\frac{1}{4}a^{2}B\right)x}{b^{3}\left(b\,x^{4}+a\right)}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{4}+a\right)}\frac{\left(-5Ab+9Ba\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16b^{4}}$
default	$\frac{\frac{1}{5}bB\,x^{5}+Abx-2Bax}{b^{3}}-\frac{a\left(\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x}{b\,x^{4}+a}+\frac{\left(5Ab-9Ba\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.\right.}{32a}\right.}{b^{3}}$

input
```
int(x^8*(B*x^4+A)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*x^5/b^2+1/b^2*A*x-2/b^3*B*a*x+(1/4*a*b*A-1/4*a^2*B)*x/b^3/(b*x^4+a)+
1/16/b^4*a*sum((-5*A*b+9*B*a)/_R^3*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 686, normalized size of antiderivative = 3.25

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/80*(16*B*b^2*x^9 - 16*(9*B*a*b - 5*A*b^2)*x^5 - 5*(b^4*x^4 + a*b^3)*(-(6
561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b
^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*
b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4
) - (9*B*a - 5*A*b)*x) - 5*(I*b^4*x^4 + I*a*b^3)*(-(6561*B^4*a^5 - 14580*A
*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b
^13)^(1/4)*log(I*b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a
^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a - 5*A*b)
*x) - 5*(-I*b^4*x^4 - I*a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150
*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(-I*
b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3
*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a - 5*A*b)*x) + 5*(b^4*x^4
+ a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 450
0*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14
580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b
^4)/b^13)^(1/4) - (9*B*a - 5*A*b)*x) - 20*(9*B*a^2 - 5*A*a*b)*x)/(b^4*x^4
+ a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.67

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx^5}{5b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab-Ba^2)}{4ab^3+4b^4x^4}$$
$$+\text{RootSum}\Bigg(65536t^4b^{13} + 625A^4ab^4 - 4500A^3Ba^2b^3 + 12150A^2B^2a^3b^2 - 14580AB^3a^4b + 6561B^4a^5, \Bigg($$

input
```
integrate(x**8*(B*x**4+A)/(b*x**4+a)**2,x)
```

output
```
B*x**5/(5*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(4*a*b**3 +
 4*b**4*x**4) + RootSum(65536*_t**4*b**13 + 625*A**4*a*b**4 - 4500*A**3*B*
a**2*b**3 + 12150*A**2*B**2*a**3*b**2 - 14580*A*B**3*a**4*b + 6561*B**4*a*
*5, Lambda(_t, _t*log(16*_t*b**3/(-5*A*b + 9*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.25

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = -\frac{(Ba^2-Aab)x}{4\,(b^4x^4+ab^3)} + \frac{\left(\frac{2\sqrt{2}(9\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(9\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(9\,Ba-5\,Ab)\log\left(\sqrt{b}x^2+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,b^3} + \frac{Bbx^5-5\,(2\,Ba-Ab)x}{5\,b^3}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(B*a^2 - A*a*b)*x/(b^4*x^4 + a*b^3) + 1/32*(2*sqrt(2)*(9*B*a - 5*A*b)
*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - 5*A*b)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - 5*A*b)*log(sqrt(b)*x
^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(9*B
*a - 5*A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4
)*b^(1/4)))*a/b^3 + 1/5*(B*b*x^5 - 5*(2*B*a - A*b)*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.36

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,b^4} + \frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,b^4} + \frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,b^4} - \frac{\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-5\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,b^4} - \frac{Ba^2x-Aabx}{4\,(bx^4+a)b^3} + \frac{Bb^8x^5-10\,Bab^7x+5\,Ab^8x}{5\,b^{10}}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/b^4 + 1/16*sqrt(2)*(9*(a*b^3)^(
1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/b^4 + 1/32*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4
)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/b^4 - 1/32*sqrt(2)*(9*
(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/b^4 - 1/4*(B*a^2*x - A*a*b*x)/((b*x^4 + a)*b^3) + 1/5*(B*b^8*x
^5 - 10*B*a*b^7*x + 5*A*b^8*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.84

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^8*(A + B*x^4))/(a + b*x^4)^2,x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/4 - (A*a*b)/4))/(a*b^3 + b^4*x^4) +
(B*x^5)/(5*b^2) + ((-a)^(1/4)*atan((((-a)^(1/4)*(5*A*b - 9*B*a)*((x*(81*B^
2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3) - ((-a)^(1/4)*(5*A*b - 9*B
*a)*(36*B*a^3 - 20*A*a^2*b))/(16*b^(13/4)))*1i)/(16*b^(13/4)) + ((-a)^(1/4
)*(5*A*b - 9*B*a)*((x*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3
) + ((-a)^(1/4)*(5*A*b - 9*B*a)*(36*B*a^3 - 20*A*a^2*b))/(16*b^(13/4)))*1i
)/(16*b^(13/4)))/(((-a)^(1/4)*(5*A*b - 9*B*a)*((x*(81*B^2*a^4 + 25*A^2*a^2
*b^2 - 90*A*B*a^3*b))/(4*b^3) - ((-a)^(1/4)*(5*A*b - 9*B*a)*(36*B*a^3 - 20
*A*a^2*b))/(16*b^(13/4))))/(16*b^(13/4)) - ((-a)^(1/4)*(5*A*b - 9*B*a)*((x
*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3) + ((-a)^(1/4)*(5*A*
b - 9*B*a)*(36*B*a^3 - 20*A*a^2*b))/(16*b^(13/4))))/(16*b^(13/4))))*(5*A*b
 - 9*B*a)*1i)/(8*b^(13/4)) + ((-a)^(1/4)*atan((((-a)^(1/4)*(5*A*b - 9*B*a)
*((x*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3) - ((-a)^(1/4)*(
5*A*b - 9*B*a)*(36*B*a^3 - 20*A*a^2*b)*1i)/(16*b^(13/4))))/(16*b^(13/4)) +
 ((-a)^(1/4)*(5*A*b - 9*B*a)*((x*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3
*b))/(4*b^3) + ((-a)^(1/4)*(5*A*b - 9*B*a)*(36*B*a^3 - 20*A*a^2*b)*1i)/(16
*b^(13/4))))/(16*b^(13/4)))/(((-a)^(1/4)*(5*A*b - 9*B*a)*((x*(81*B^2*a^4 +
 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3) - ((-a)^(1/4)*(5*A*b - 9*B*a)*(36
*B*a^3 - 20*A*a^2*b)*1i)/(16*b^(13/4)))*1i)/(16*b^(13/4)) - ((-a)^(1/4)*(5
*A*b - 9*B*a)*((x*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/(4*b^3)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{-10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-5b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\right.}{40b^3}$$

input
```
int(x^8*(B*x^4+A)/(b*x^4+a)^2,x)
```

output
```
( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a -
5*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(b)*x**2)*a + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(
2)*x + sqrt(a) + sqrt(b)*x**2)*a - 40*a*b*x + 8*b**2*x**5)/(40*b**3)
```

3.18 $\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 199

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx^3}{3b^2} - \frac{(Ab-aB)x^3}{4b^2(a+bx^4)} - \frac{(3Ab-7aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} + \frac{(3Ab-7aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} - \frac{(3Ab-7aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}$$

output
```
1/3*B*x^3/b^2-1/4*(A*b-B*a)*x^3/b^2/(b*x^4+a)+1/16*(3*A*b-7*B*a)*arctan(-1
+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/b^(11/4)+1/16*(3*A*b-7*B*a)*ar
ctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/b^(11/4)-1/16*(3*A*b-7*B
*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(1/
4)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.21

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$= \frac{32b^{3/4}Bx^3 - \frac{24b^{3/4}(Ab-aB)x^3}{a+bx^4} + \frac{6\sqrt{2}(-3Ab+7aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{6\sqrt{2}(3Ab-7aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{3\sqrt{2}(3Ab-7}{}}{96b^{11/4}}$$

input
```
Integrate[(x^6*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
(32*b^(3/4)*B*x^3 - (24*b^(3/4)*(A*b - a*B)*x^3)/(a + b*x^4) + (6*Sqrt[2]*
(-3*A*b + 7*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(1/4) + (6*Sqr
t[2]*(3*A*b - 7*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(1/4) + (3
*Sqrt[2]*(3*A*b - 7*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(1/4) + (3*Sqrt[2]*(-3*A*b + 7*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/a^(1/4))/(96*b^(11/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{x^7(Ab-aB)}{4ab(a+bx^4)} - \frac{(3Ab-7aB)\int \frac{x^6}{bx^4+a}dx}{4ab}$$
$$\Big\downarrow \text{843}$$

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\int\frac{x^2}{bx^4+a}dx}{b}\right)}{4ab}$$

$\Big\downarrow$ 826

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\Big\downarrow$ 1476

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\Big\downarrow$ 1082

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 217$$

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 1479$$

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 25

$$\frac{x^7(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 27

$$\frac{x^7(Ab-aB)}{4ab\,(a+bx^4)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\big\downarrow$ 1103

$$\frac{x^7(Ab-aB)}{4ab\,(a+bx^4)}-\frac{(3Ab-7aB)\left(\frac{x^3}{3b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

input
```
Int[(x^6*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
((A*b - a*B)*x^7)/(4*a*b*(a + b*x^4)) - ((3*A*b - 7*a*B)*(x^3/(3*b) - (a*(
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b
]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]
*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{B\,x^{3}}{3b^{2}}+\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{3}}{b^{2}\left(b\,x^{4}+a\right)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{4}+a\right)}\frac{(3Ab-7Ba)\ln\left(x-\_R\right)}{\_R}}{16b^{3}}$	71
default	$\frac{B\,x^{3}}{3b^{2}}+\frac{\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{3}}{b\,x^{4}+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^{2}}$	148

input
```
int(x^6*(B*x^4+A)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/b^2+(-1/4*A*b+1/4*B*a)*x^3/b^2/(b*x^4+a)+1/16/b^3*sum((3*A*b-7*B
*a)/_R*ln(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 782, normalized size of antiderivative = 3.93

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/48*(16*B*b*x^7 + 4*(7*B*a - 3*A*b)*x^3 + 3*(b^3*x^4 + a*b^2)*(-(2401*B^4
*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*
b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A
^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*
a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*x) - 3*(I*b^3*x^4 +
I*a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A
^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(I*a*b^8*(-(2401*B^4*a^4 - 411
6*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^
11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3
)*x) - 3*(-I*b^3*x^4 - I*a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*
A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-I*a*b
^8*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a
*b^3 + 81*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*
A^2*B*a*b^2 - 27*A^3*b^3)*x) - 3*(b^3*x^4 + a*b^2)*(-(2401*B^4*a^4 - 4116*
A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11
))^(1/4)*log(-a*b^8*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*
b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A
*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*x))/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx^3}{3b^2} + \frac{x^3(-Ab+Ba)}{4ab^2+4b^3x^4}$$
$$+\operatorname{RootSum}\left(65536t^4ab^{11}+81A^4b^4-756A^3Bab^3+2646A^2B^2a^2b^2-4116AB^3a^3b+2401B^4a^4, \left(t\mapsto t\right.\right.$$

input
```
integrate(x**6*(B*x**4+A)/(b*x**4+a)**2,x)
```

output
```
B*x**3/(3*b**2) + x**3*(-A*b + B*a)/(4*a*b**2 + 4*b**3*x**4) + RootSum(655
36*_t**4*a*b**11 + 81*A**4*b**4 - 756*A**3*B*a*b**3 + 2646*A**2*B**2*a**2*
b**2 - 4116*A*B**3*a**3*b + 2401*B**4*a**4, Lambda(_t, _t*log(-4096*_t**3*
a*b**8/(-27*A**3*b**3 + 189*A**2*B*a*b**2 - 441*A*B**2*a**2*b + 343*B**3*a
**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{(Ba-Ab)x^3}{4\,(b^3x^4+ab^2)} + \frac{Bx^3}{3\,b^2} - \frac{(7\,Ba-3\,Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{32\,b^2}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(B*a - A*b)*x^3/(b^3*x^4 + a*b^2) + 1/3*B*x^3/b^2 - 1/32*(7*B*a - 3*A*
b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx^3}{3\,b^2} + \frac{Bax^3-Abx^3}{4\,(bx^4+a)b^2} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^5} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^5} + \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^5} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^5}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/3*B*x^3/b^2 + 1/4*(B*a*x^3 - A*b*x^3)/((b*x^4 + a)*b^2) - 1/16*sqrt(2)*(
7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) - 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a -
 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)
^(1/4))/(a*b^5) + 1/32*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)
*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/32*sqrt(2)*(7*(a
*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.51

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{B\,x^3}{3\,b^2} - \frac{x^3\,\left(\frac{A\,b}{4}-\frac{B\,a}{4}\right)}{b^3\,x^4+a\,b^2} + \frac{\mathrm{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(3\,A\,b-7\,B\,a)}{8\,(-a)^{1/4}\,b^{11/4}} + \frac{\mathrm{atan}\left(\frac{b^{1/4}\,x\,1\mathrm{i}}{(-a)^{1/4}}\right)\,(3\,A\,b-7\,B\,a)\,1\mathrm{i}}{8\,(-a)^{1/4}\,b^{11/4}}$$

input

```
int((x^6*(A + B*x^4))/(a + b*x^4)^2,x)
```

output

```
(B*x^3)/(3*b^2) - (x^3*((A*b)/4 - (B*a)/4))/(a*b^2 + b^3*x^4) + (atan((b^(
1/4)*x)/(-a)^(1/4))*(3*A*b - 7*B*a))/(8*(-a)^(1/4)*b^(11/4)) + (atan((b^(1
/4)*x*1i)/(-a)^(1/4))*(3*A*b - 7*B*a)*1i)/(8*(-a)^(1/4)*b^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 3b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\right.}{24b^2}$$

input

```
int(x^6*(B*x^4+A)/(b*x^4+a)^2,x)
```

output

```
(6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2))) - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) - 3*b**(1/4
)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*
x**2) + 3*b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(b)*x**2) + 8*b*x**3)/(24*b**2)
```

3.19 $\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx}{b^2} - \frac{(Ab-aB)x}{4b^2(a+bx^4)} - \frac{(Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{9/4}} + \frac{(Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}b^{9/4}} + \frac{(Ab-5aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{3/4}b^{9/4}}$$

output
```
B*x/b^2-1/4*(A*b-B*a)*x/b^2/(b*x^4+a)+1/16*(A*b-5*B*a)*arctan(-1+2^(1/2)*b
^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+1/16*(A*b-5*B*a)*arctan(1+2^(1/2
)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+1/16*(A*b-5*B*a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.23

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$= \frac{32\sqrt[4]{b}Bx - \frac{8\sqrt[4]{b}(Ab-aB)x}{a+bx^4} + \frac{2\sqrt{2}(-Ab+5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}(Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{\sqrt{2}(-Ab+5aB)\log}{}}{32b^{9/4}}$$

input
```
Integrate[(x^4*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
(32*b^(1/4)*B*x - (8*b^(1/4)*(A*b - a*B)*x)/(a + b*x^4) + (2*Sqrt[2]*(-(A*
b) + 5*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*
(A*b - 5*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*
(-(A*b) + 5*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4) + (Sqrt[2]*(A*b - 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/a^(3/4))/(32*b^(9/4))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {957, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{x^5(Ab-aB)}{4ab(a+bx^4)} - \frac{(Ab-5aB)\int \frac{x^4}{bx^4+a}dx}{4ab}$$
$$\Big\downarrow \text{843}$$

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^4+a}dx}{b}\right)}{4ab}$$

$\downarrow$ 755

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1476

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1082

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 217

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1479

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 25$$

$$\frac{x^5(Ab-aB)}{4ab\left(a+bx^4\right)}-\frac{(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^5(Ab-aB)}{4ab(a+bx^4)}-(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{x^5(Ab-aB)}{4ab(a+bx^4)}-(Ab-5aB)\left(\frac{x}{b}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

input
```
Int[(x^4*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
((A*b - a*B)*x^5)/(4*a*b*(a + b*x^4)) - ((A*b - 5*a*B)*(x/b - (a*((-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 +
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1
/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)
*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x}{b^2(b\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(Ab-5Ba)\ln\left(x-\_R\right)}{\_R^3}}{16b^3}$	65
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x}{b\,x^4+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}}{b^2}$	142

input
```
int(x^4*(B*x^4+A)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+(-1/4*A*b+1/4*B*a)*x/b^2/(b*x^4+a)+1/16/b^3*sum((A*b-5*B*a)/_R^3*l
n(x-_R),_R=RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.49

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$=\frac{16\,Bbx^5+(b^3x^4+ab^2)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+\cdots}{}\right.\right.}{}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*B*b*x^5 + (b^3*x^4 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 15
0*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(a*b^2*(
-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A
^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*x) - (-I*b^3*x^4 - I*a*b^2)*(-(62
5*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^9))^(1/4)*log(I*a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^
2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)
*x) - (I*b^3*x^4 + I*a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2
*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(-I*a*b^2*(-(625*
B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4
)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*x) - (b^3*x^4 + a*b^2)*(-(625*B^4*a^4 -
 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^
9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*x) + 4*(5*B
*a - A*b)*x)/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx}{b^2} + \frac{x(-Ab+Ba)}{4ab^2+4b^3x^4}$$
$$+\mathrm{RootSum}\left(65536t^4a^3b^9 + A^4b^4 - 20A^3Bab^3 + 150A^2B^2a^2b^2 - 500AB^3a^3b + 625B^4a^4, \left(t \mapsto t\log\left(-\right.\right.\right.$$

input
```
integrate(x**4*(B*x**4+A)/(b*x**4+a)**2,x)
```

output
```
B*x/b**2 + x*(-A*b + B*a)/(4*a*b**2 + 4*b**3*x**4) + RootSum(65536*_t**4*a
**3*b**9 + A**4*b**4 - 20*A**3*B*a*b**3 + 150*A**2*B**2*a**2*b**2 - 500*A*
B**3*a**3*b + 625*B**4*a**4, Lambda(_t, _t*log(-16*_t*a*b**2/(-A*b + 5*B*a
) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.28

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{(Ba-Ab)x}{4\,(b^3x^4+ab^2)} + \frac{Bx}{b^2} - \frac{\frac{2\sqrt{2}(5\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(5\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(5\,Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,b^2}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*(B*a - A*b)*x/(b^3*x^4 + a*b^2) + B*x/b^2 - 1/32*(2*sqrt(2)*(5*B*a - A
*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*B*a - A*b)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a - A*b)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*B*
a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b
^(1/4)))/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.44

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{Bx}{b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,ab^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^3} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,ab^3} + \frac{Bax-Abx}{4\,(bx^4+a)b^2}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
B*x/b^2 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/
2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/16*sqrt(2)*
(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) - 1/32*sqrt(2)*(5*(a*b^3)^(1/4)*B*a -
(a*b^3)^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) +
1/32*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/4*(B*a*x - A*b*x)/((b*x^4 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.87

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^4))/(a + b*x^4)^2,x)
```

output
```
(B*x)/b^2 - (x*((A*b)/4 - (B*a)/4))/(a*b^2 + b^3*x^4) + (atan(((((x*(A^2*b
^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) - ((A*b - 5*B*a)*(4*A*a*b^2 - 20*B*a^
2*b))/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B*a)*1i)/(16*(-a)^(3/4)*b^(9/4)) +
 (((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) + ((A*b - 5*B*a)*(4*A*a*b
^2 - 20*B*a^2*b))/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B*a)*1i)/(16*(-a)^(3/4
)*b^(9/4)))/((((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) - ((A*b - 5*B
*a)*(4*A*a*b^2 - 20*B*a^2*b))/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B*a))/(16*
(-a)^(3/4)*b^(9/4)) - (((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) + ((
A*b - 5*B*a)*(4*A*a*b^2 - 20*B*a^2*b))/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B
*a))/(16*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a)*1i)/(8*(-a)^(3/4)*b^(9/4)) +
(atan(((((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) - ((A*b - 5*B*a)*(4
*A*a*b^2 - 20*B*a^2*b)*1i)/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B*a))/(16*(-a
)^(3/4)*b^(9/4)) + (((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(4*b) + ((A*b
 - 5*B*a)*(4*A*a*b^2 - 20*B*a^2*b)*1i)/(16*(-a)^(3/4)*b^(9/4)))*(A*b - 5*B
*a))/(16*(-a)^(3/4)*b^(9/4)))/((((x*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/(
4*b) - ((A*b - 5*B*a)*(4*A*a*b^2 - 20*B*a^2*b)*1i)/(16*(-a)^(3/4)*b^(9/4))
)*(A*b - 5*B*a)*1i)/(16*(-a)^(3/4)*b^(9/4)) - (((x*(A^2*b^2 + 25*B^2*a^2 -
 10*A*B*a*b))/(4*b) + ((A*b - 5*B*a)*(4*A*a*b^2 - 20*B*a^2*b)*1i)/(16*(-a)
^(3/4)*b^(9/4)))*(A*b - 5*B*a)*1i)/(16*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a)
)/(8*(-a)^(3/4)*b^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.76

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$= \frac{2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x}{8b^2}$$

input
```
int(x^4*(B*x^4+A)/(b*x^4+a)^2,x)
```

output
```
(2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + b**(3/4)*
a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x*
*2) - b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2) + 8*b*x)/(8*b**2)
```

3.20 $\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 188

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{(Ab-aB)x^3}{4ab\,(a+bx^4)} - \frac{(Ab+3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}b^{7/4}} - \frac{(Ab+3aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$

output
```
1/4*(A*b-B*a)*x^3/a/b/(b*x^4+a)+1/16*(A*b+3*B*a)*arctan(-1+2^(1/2)*b^(1/4)
*x/a^(1/4))*2^(1/2)/a^(5/4)/b^(7/4)+1/16*(A*b+3*B*a)*arctan(1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/b^(7/4)-1/16*(A*b+3*B*a)*arctanh(2^(1/2)*a^
(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(5/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8\sqrt[4]{a}b^{3/4}(-Ab+aB)x^3}{a+bx^4} - 2\sqrt{2}(Ab+3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(Ab+3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) -}{32a^{5/4}b}$$

input
```
Integrate[(x^2*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
((-8*a^(1/4)*b^(3/4)*(-(A*b) + a*B)*x^3)/(a + b*x^4) - 2*Sqrt[2]*(A*b + 3*
a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(A*b + 3*a*B)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + Sqrt[2]*(A*b + 3*a*B)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Sqrt[2]*(A*b + 3*a*B)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(5/4)*b^(7/4))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {957, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx$$
$$\Big\downarrow \; 957$$
$$\frac{(3aB+Ab)\int \frac{x^2}{bx^4+a}dx}{4ab} + \frac{x^3(Ab-aB)}{4ab\,(a+bx^4)}$$
$$\Big\downarrow \; 826$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4ab}+\frac{x^3(Ab-aB)}{4ab\,(a+bx^4)}$$

input
```
Int[(x^2*(A + B*x^4))/(a + b*x^4)^2,x]
```

output
```
((A*b - a*B)*x^3)/(4*a*b*(a + b*x^4)) + ((A*b + 3*a*B)*((-(ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4))
+ Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4
)*b^(1/4)))/(2*Sqrt[b])))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{(Ab-Ba)x^3}{4ab(b\,x^4+a)}+\frac{\sum\limits_{\_R=\mathrm{RootOf}\left(b\_Z^4+a\right)}\frac{(Ab+3Ba)\ln\left(x-\_R\right)}{\_R}}{16a\,b^2}$	67
default	$\frac{(Ab-Ba)x^3}{4ab(b\,x^4+a)}+\frac{(Ab+3Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a\,b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	142

input
```
int(x^2*(B*x^4+A)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(A*b-B*a)*x^3/a/b/(b*x^4+a)+1/16/a/b^2*sum((A*b+3*B*a)/_R*ln(x-_R),_R=
RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 768, normalized size of antiderivative = 4.09

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*(B*a - A*b)*x^3 - (a*b^2*x^4 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a
^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log
(a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a
*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*
a*b^2 + A^3*b^3)*x) + (I*a*b^2*x^4 + I*a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^
3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(
I*a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*
a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B
*a*b^2 + A^3*b^3)*x) + (-I*a*b^2*x^4 - I*a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*
a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*lo
g(-I*a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^
2*B*a*b^2 + A^3*b^3)*x) + (a*b^2*x^4 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^3*a^
3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*log(
-a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a
*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2*B*
a*b^2 + A^3*b^3)*x))/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.79

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{x^3(Ab-Ba)}{4a^2b+4ab^2x^4} + \mathrm{RootSum}\left(65536t^4a^5b^7+A^4b^4+12A^3Bab^3+54A^2B^2a^2b^2+108AB^3a^3b+81B^4a^4, \left(t\mapsto t\log\left(\frac{}{A^3b}\right.\right.\right.$$

input
```
integrate(x**2*(B*x**4+A)/(b*x**4+a)**2,x)
```

output
```
x**3*(A*b - B*a)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**5*b**
7 + A**4*b**4 + 12*A**3*B*a*b**3 + 54*A**2*B**2*a**2*b**2 + 108*A*B**3*a**
3*b + 81*B**4*a**4, Lambda(_t, _t*log(4096*_t**3*a**4*b**5/(A**3*b**3 + 9*
A**2*B*a*b**2 + 27*A*B**2*a**2*b + 27*B**3*a**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = -\frac{(Ba-Ab)x^3}{4\,(ab^2x^4+a^2b)} + \frac{(3\,Ba+Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}\,b^{\frac{3}{4}}} + \right.}{32\,ab}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(B*a - A*b)*x^3/(a*b^2*x^4 + a^2*b) + 1/32*(3*B*a + A*b)*(2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*s
qrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1
/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.44

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = -\frac{Bax^3-Abx^3}{4\,(bx^4+a)ab} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^4} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^4} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^2b^4}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(B*a*x^3 - A*b*x^3)/((b*x^4 + a)*a*b) + 1/16*sqrt(2)*(3*(a*b^3)^(3/4)
*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(
a/b)^(1/4))/(a^2*b^4) + 1/16*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*
A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4)
 - 1/32*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(x^2 + sqrt(2
)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/32*sqrt(2)*(3*(a*b^3)^(3/4)*B*a
 + (a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^
4)
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(A\,b+3\,B\,a)}{8\,(-a)^{5/4}\,b^{7/4}} - \frac{\operatorname{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(A\,b+3\,B\,a)}{8\,(-a)^{5/4}\,b^{7/4}} + \frac{x^3\,(A\,b-B\,a)}{4\,a\,b\,(b\,x^4+a)}$$

input
```
int((x^2*(A + B*x^4))/(a + b*x^4)^2,x)
```

output
```
(atanh((b^(1/4)*x)/(-a)^(1/4))*(A*b + 3*B*a))/(8*(-a)^(5/4)*b^(7/4)) - (at
an((b^(1/4)*x)/(-a)^(1/4))*(A*b + 3*B*a))/(8*(-a)^(5/4)*b^(7/4)) + (x^3*(A
*b - B*a))/(4*a*b*(a + b*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\right.}{8b^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int(x^2*(B*x^4+A)/(b*x^4+a)^2,x)
```

output
```
(b**(1/4)*a**(3/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + log( - b**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(b)*x**2) - log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2)))/(8*a*b)
```

3.21 $\int \frac{A+Bx^4}{(a+bx^4)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{A+Bx^4}{(a+bx^4)^2} \, dx = \frac{(Ab-aB)x}{4ab(a+bx^4)} - \frac{(3Ab+aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
1/4*(A*b-B*a)*x/a/b/(b*x^4+a)+1/16*(3*A*b+B*a)*arctan(-1+2^(1/2)*b^(1/4)*x
/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/16*(3*A*b+B*a)*arctan(1+2^(1/2)*b^(1/4
)*x/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+1/16*(3*A*b+B*a)*arctanh(2^(1/2)*a^(1
/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(7/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^4}{(a+bx^4)^2}\,dx$$
$$= \frac{-\frac{8a^{3/4}\sqrt[4]{b}(-Ab+aB)x}{a+bx^4} - 2\sqrt{2}(3Ab+aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) + 2\sqrt{2}(3Ab+aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right) -}{32a^{7/4}b^5}$$

input
```
Integrate[(A + B*x^4)/(a + b*x^4)^2,x]
```

output
```
((-8*a^(3/4)*b^(1/4)*(-(A*b) + a*B)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*A*b + a*
B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*A*b + a*B)*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*A*b + a*B)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*A*b + a*B)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {910, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(aB+3Ab)\int \frac{1}{bx^4+a}dx}{4ab}+\frac{x(Ab-aB)}{4ab\,(a+bx^4)}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{217}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^4)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{(aB+3Ab)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{4ab}+\frac{x(Ab-aB)}{4ab\left(a+bx^4\right)}$$

input
```
Int[(A + B*x^4)/(a + b*x^4)^2,x]
```

output
```
((A*b - a*B)*x)/(4*a*b*(a + b*x^4)) + ((3*A*b + a*B)*((-(ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a])))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{(Ab-Ba)x}{4ab(b\,x^4+a)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^4+a\right)}\frac{(3Ab+Ba)\ln\left(x-\_R\right)}{\_R^3}}{16a\,b^2}$	65
default	$\frac{(Ab-Ba)x}{4ab(b\,x^4+a)}+\frac{(3Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2b}$	140

input
```
int((B*x^4+A)/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(A*b-B*a)*x/a/b/(b*x^4+a)+1/16/a/b^2*sum((3*A*b+B*a)/_R^3*ln(x-_R),_R=
RootOf(_Z^4*b+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 648, normalized size of antiderivative = 3.48

$$\int\frac{A+Bx^4}{(a+bx^4)^2}\,dx$$

$$=\frac{(ab^2x^4+a^2b)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3\cdots}{a^7b^5}\right.\right.}{}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*((a*b^2*x^4 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 1
2*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^
5))^(1/4) + (B*a + 3*A*b)*x) - (-I*a*b^2*x^4 - I*a^2*b)*(-(B^4*a^4 + 12*A*
B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^
(1/4)*log(I*a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A
^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*x) - (I*a*b^2*x^
4 + I*a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*
a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(-I*a^2*b*(-(B^4*a^4 + 12*A*B^3*a^
3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)
+ (B*a + 3*A*b)*x) - (a*b^2*x^4 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(-a^2*
b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*
A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*x) - 4*(B*a - A*b)*x)/(a*b^2*x^4
 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^4}{(a+bx^4)^2}\,dx = \frac{x(Ab-Ba)}{4a^2b+4ab^2x^4}$$
$$+\text{RootSum}\left(65536t^4a^7b^5+81A^4b^4+108A^3Bab^3+54A^2B^2a^2b^2+12AB^3a^3b+B^4a^4,\left(t\mapsto t\log\left(\frac{1\text{6}}{3A\text{b}}\right.\right.\right.$$

input
```
integrate((B*x**4+A)/(b*x**4+a)**2,x)
```

output
```
x*(A*b - B*a)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 81*A**4*b**4 + 108*A**3*B*a*b**3 + 54*A**2*B**2*a**2*b**2 + 12*A*B**3*a**
3*b + B**4*a**4, Lambda(_t, _t*log(16*_t*a**2*b/(3*A*b + B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^4}{(a+bx^4)^2}\,dx = -\frac{(Ba-Ab)x}{4\,(ab^2x^4+a^2b)} + \frac{\frac{2\sqrt{2}(Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba+3\,Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,ab}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(B*a - A*b)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(B*a + 3*A*b)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 3*A*b)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 3*A*b)*log(sqrt(b)*x^2 + sqr
t(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a + 3*A*b
)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4))
)/(a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx^4}{\left(a+bx^4\right)^2}\,dx = \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^2} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba+3\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^2} - \frac{Bax-Abx}{4\left(bx^4+a\right)ab}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*((a*b^3)
^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^
(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3
)^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32
*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/4*(B*a*x - A*b*x)/((b*x^4 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.98

$$\int \frac{A+Bx^4}{\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(a + b*x^4)^2,x)
```

output
```
(atan((((3*A*b + B*a)*((x*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/(4*a^2) -
 ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*b^2))/(16*(-a)^(7/4)*b^(5/4)))*1i)/(16*(
-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b
^2))/(4*a^2) + ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*b^2))/(16*(-a)^(7/4)*b^(5/
4)))*1i)/(16*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x*(9*A^2*b^3 + B^2*a^2
*b + 6*A*B*a*b^2))/(4*a^2) - ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*b^2))/(16*(-
a)^(7/4)*b^(5/4))))/(16*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*a)*((x*(9*A^2*b^
3 + B^2*a^2*b + 6*A*B*a*b^2))/(4*a^2) + ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*b
^2))/(16*(-a)^(7/4)*b^(5/4))))/(16*(-a)^(7/4)*b^(5/4))))*(3*A*b + B*a)*1i)
/(8*(-a)^(7/4)*b^(5/4)) + (atan((((3*A*b + B*a)*((x*(9*A^2*b^3 + B^2*a^2*b
 + 6*A*B*a*b^2))/(4*a^2) - ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*b^2)*1i)/(16*(
-a)^(7/4)*b^(5/4))))/(16*(-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x*(9*A^2*b
^3 + B^2*a^2*b + 6*A*B*a*b^2))/(4*a^2) + ((3*A*b + B*a)*(12*A*b^3 + 4*B*a*
b^2)*1i)/(16*(-a)^(7/4)*b^(5/4))))/(16*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a
)*((x*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/(4*a^2) - ((3*A*b + B*a)*(12*
A*b^3 + 4*B*a*b^2)*1i)/(16*(-a)^(7/4)*b^(5/4)))*1i)/(16*(-a)^(7/4)*b^(5/4)
) - ((3*A*b + B*a)*((x*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/(4*a^2) + ((
3*A*b + B*a)*(12*A*b^3 + 4*B*a*b^2)*1i)/(16*(-a)^(7/4)*b^(5/4)))*1i)/(16*(
-a)^(7/4)*b^(5/4))))*(3*A*b + B*a))/(8*(-a)^(7/4)*b^(5/4)) + (x*(A*b - B*a
))/(4*a*b*(a + b*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^4}{\left(a+bx^4\right)^2}\,dx$$
$$= \frac{\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\right.}{8b^{\frac{1}{4}}a^{\frac{3}{4}}}$$

input
```
int((B*x^4+A)/(b*x^4+a)^2,x)
```

output
```
(b**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2))) - log( - b**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(b)*x**2) + log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2)))/(8*a*b)
```

3.22 $\int \frac{A+Bx^4}{x^2(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 197

$$\int \frac{A+Bx^4}{x^2(a+bx^4)^2}\,dx = -\frac{A}{a^2x} - \frac{(Ab-aB)x^3}{4a^2(a+bx^4)} + \frac{(5Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}b^{3/4}} - \frac{(5Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}b^{3/4}} + \frac{(5Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{9/4}b^{3/4}}$$

output

```
-A/a^2/x-1/4*(A*b-B*a)*x^3/a^2/(b*x^4+a)-1/16*(5*A*b-B*a)*arctan(-1+2^(1/2
)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(9/4)/b^(3/4)-1/16*(5*A*b-B*a)*arctan(1+2^(
1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(9/4)/b^(3/4)+1/16*(5*A*b-B*a)*arctanh(2
^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(9/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \frac{-\frac{32\sqrt[4]{a}A}{x}+\frac{8\sqrt[4]{a}(-Ab+aB)x^3}{a+bx^4}+\frac{2\sqrt{2}(5Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}-\frac{2\sqrt{2}(5Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{\sqrt{2}(-5Ab+aB)\,\mathrm{l}}{}}{32a^{9/4}}$$

input
```
Integrate[(A + B*x^4)/(x^2*(a + b*x^4)^2),x]
```

output
```
((-32*a^(1/4)*A)/x + (8*a^(1/4)*(-(A*b) + a*B)*x^3)/(a + b*x^4) + (2*Sqrt[
2]*(5*A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) - (2*Sqr
t[2]*(5*A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(3/4) + (Sqr
t[2]*(-5*A*b + a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/b^(3/4) + (Sqrt[2]*(5*A*b - a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2])/b^(3/4))/(32*a^(9/4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5Ab-aB)\int \frac{1}{x^2(bx^4+a)}dx}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(5Ab-aB)\left(-\frac{b\int \frac{x^2}{bx^4+a}dx}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\downarrow$ 826

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\downarrow$ 1476

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\downarrow$ 1082

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx(a+bx^4)}$$

$\downarrow$ 217

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx(a+bx^4)}$$

$\downarrow$ 1479

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\right.}{4ab}$$

$$\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\big\downarrow$ 25

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}$$

$$\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\big\downarrow$ 27

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

$\big\downarrow$ 1103

$$\frac{(5Ab-aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{4ab}+\frac{Ab-aB}{4abx\left(a+bx^4\right)}$$

input
```
Int[(A + B*x^4)/(x^2*(a + b*x^4)^2),x]
```

output
```
(A*b - a*B)/(4*a*b*x*(a + b*x^4)) + ((5*A*b - a*B)*(-(1/(a*x)) - (b*((-(Ar
cTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) -
(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1
/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*S
qrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

method	result
default	$-\frac{\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^3}{b\,x^4+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^2}-\frac{A}{a^2x}$
risch	$\frac{-\frac{(5Ab-Ba)x^4}{4a^2}-\frac{A}{a}}{x(b\,x^4+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9b^3\_Z^4+625A^4b^4-500A^3Ba\,b^3+150A^2B^2a^2b^2-20A\,B^3a^3b+B^4a^4\right)}\_R\ln\left(\left(5\_R^4a^9b^3+2500\right.\right.\right.}{16}$

input
```
int((B*x^4+A)/x^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/4*A*b-1/4*B*a)*x^3/(b*x^4+a)+1/8*(5/4*A*b-1/4*B*a)/b/(a/b)^(1/4
)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a
/b)^(1/4)*x-1)))-A/a^2/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 778, normalized size of antiderivative = 3.95

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(B*a - 5*A*b)*x^4 - (a^2*b*x^5 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*
b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*
log(a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*
a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2
*B*a*b^2 - 125*A^3*b^3)*x) - (-I*a^2*b*x^5 - I*a^3*x)*(-(B^4*a^4 - 20*A*B^
3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^
(1/4)*log(I*a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 50
0*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b
+ 75*A^2*B*a*b^2 - 125*A^3*b^3)*x) - (I*a^2*b*x^5 + I*a^3*x)*(-(B^4*a^4 -
20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9
*b^3))^(1/4)*log(-I*a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*
b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^
2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*x) + (a^2*b*x^5 + a^3*x)*(-(B^4*a^
4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/
(a^9*b^3))^(1/4)*log(-a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^
2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*
B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*x) - 16*A*a)/(a^2*b*x^5 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \frac{-4Aa+x^4(-5Ab+Ba)}{4a^3x+4a^2bx^5}$$
$$+\mathrm{RootSum}\left(65536t^4a^9b^3+625A^4b^4-500A^3Bab^3+150A^2B^2a^2b^2-20AB^3a^3b+B^4a^4,\left(t\mapsto t\log\left(\right.\right.\right.$$

input
```
integrate((B*x**4+A)/x**2/(b*x**4+a)**2,x)
```

output
```
(-4*A*a + x**4*(-5*A*b + B*a))/(4*a**3*x + 4*a**2*b*x**5) + RootSum(65536*
_t**4*a**9*b**3 + 625*A**4*b**4 - 500*A**3*B*a*b**3 + 150*A**2*B**2*a**2*b
**2 - 20*A*B**3*a**3*b + B**4*a**4, Lambda(_t, _t*log(4096*_t**3*a**7*b**2
/(-125*A**3*b**3 + 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b + B**3*a**3) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^4}{x^2(a+bx^4)^2}\,dx = \frac{(Ba-5\,Ab)x^4-4\,Aa}{4\,(a^2bx^5+a^3x)} + \frac{(Ba-5\,Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}\,b^{\frac{1}{4}}\,x+\sqrt{a}\right)}{a^{\frac{1}{4}}\,b^{\frac{3}{4}}}+\right.}{32\,a^2}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*((B*a - 5*A*b)*x^4 - 4*A*a)/(a^2*b*x^5 + a^3*x) + 1/32*(B*a - 5*A*b)*(
2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^
(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \frac{Bax^4-5\,Abx^4-4\,Aa}{4\,(bx^5+ax)a^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b^3}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(B*a*x^4 - 5*A*b*x^4 - 4*A*a)/((b*x^5 + a*x)*a^2) + 1/16*sqrt(2)*((a*b
^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) + 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*
b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a^3*b^3) - 1/32*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(x
^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) + 1/32*sqrt(2)*((a*b^3)^
(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.49

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(5\,A\,b-B\,a)}{8\,(-a)^{9/4}\,b^{3/4}} - \frac{\operatorname{atan}\left(\frac{b^{1/4}\,x}{(-a)^{1/4}}\right)\,(5\,A\,b-B\,a)}{8\,(-a)^{9/4}\,b^{3/4}} - \frac{\frac{A}{a}+\frac{x^4\,(5\,A\,b-B\,a)}{4\,a^2}}{b\,x^5+a\,x}$$

input
```
int((A + B*x^4)/(x^2*(a + b*x^4)^2),x)
```

output
```
(atanh((b^(1/4)*x)/(-a)^(1/4))*(5*A*b - B*a))/(8*(-a)^(9/4)*b^(3/4)) - (at
an((b^(1/4)*x)/(-a)^(1/4))*(5*A*b - B*a))/(8*(-a)^(9/4)*b^(3/4)) - (A/a +
(x^4*(5*A*b - B*a))/(4*a^2))/(a*x + b*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)^2}\,dx = \frac{2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x-2b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x-b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{}\right.}{8a^2x}$$

input
```
int((B*x^4+A)/x^2/(b*x^4+a)^2,x)
```

output
```
(2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*x - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*x - b**(1
/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b
)*x**2)*x + b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sq
rt(a) + sqrt(b)*x**2)*x - 8*a)/(8*a**2*x)
```

3.23 $\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 197

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)^2}\,dx = -\frac{A}{3a^2x^3}-\frac{(Ab-aB)x}{4a^2\left(a+bx^4\right)}+\frac{(7Ab-3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(7Ab-3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(7Ab-3aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

output
```
-1/3*A/a^2/x^3-1/4*(A*b-B*a)*x/a^2/(b*x^4+a)-1/16*(7*A*b-3*B*a)*arctan(-1+
2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(1/4)-1/16*(7*A*b-3*B*a)*arc
tan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4)/b^(1/4)-1/16*(7*A*b-3*B*
a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/
4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)^2}\,dx$$
$$= \frac{-\frac{32a^{3/4}A}{x^3}+\frac{24a^{3/4}(-Ab+aB)x}{a+bx^4}+\frac{6\sqrt{2}(7Ab-3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}(7Ab-3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}(7Ab-3aB}{}}{96a^{11/4}}$$

input
```
Integrate[(A + B*x^4)/(x^4*(a + b*x^4)^2),x]
```

output
```
((-32*a^(3/4)*A)/x^3 + (24*a^(3/4)*(-(A*b) + a*B)*x)/(a + b*x^4) + (6*Sqrt
[2]*(7*A*b - 3*a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(1/4) - (6*
Sqrt[2]*(7*A*b - 3*a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(1/4) +
 (3*Sqrt[2]*(7*A*b - 3*a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/b^(1/4) + (3*Sqrt[2]*(-7*A*b + 3*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(1/4))/(96*a^(11/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {957, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)^2}\,dx$$
$$\Big\downarrow \text{957}$$
$$\frac{(7Ab-3aB)\int \frac{1}{x^4(bx^4+a)}dx}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$
$$\Big\downarrow \text{847}$$

$$\frac{(7Ab-3aB)\left(-\frac{b\int \frac{1}{bx^4+a}dx}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\downarrow$ 755

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\downarrow$ 1476

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\downarrow$ 1082

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\downarrow$ 217

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\downarrow$ 1479

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\cdots\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{\cdots}{3a\cdots}\right)}{4ab}+\frac{Ab-aB}{4abx^3\left(a+bx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3(a+bx^4)}$$

$\downarrow$ 1103

$$\frac{(7Ab-3aB)\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4ab}+\frac{Ab-aB}{4abx^3(a+bx^4)}$$

input
```
Int[(A + B*x^4)/(x^4*(a + b*x^4)^2),x]
```

output
```
(A*b - a*B)/(4*a*b*x^3*(a + b*x^4)) + ((7*A*b - 3*a*B)*(-1/3*1/(a*x^3) - (
b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) +
 ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqr
t[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt
[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/a))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.75

method	result
default	$-\frac{\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x}{b\,x^4+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}}{a^2}-\frac{A}{3a^2x^3}$
risch	$\frac{-\frac{(7Ab-3Ba)x^4}{12a^2}-\frac{A}{3a}}{x^3(b\,x^4+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}b\,\_Z^4+2401A^4b^4-4116A^3Ba\,b^3+2646A^2B^2a^2b^2-756A\,B^3a^3b+81B^4a^4\right)}\_R\ln\left(\left(-5\_R^4a\right.\right.\right.}{}$

input
```
int((B*x^4+A)/x^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*((1/4*A*b-1/4*B*a)*x/(b*x^4+a)+1/32*(7*A*b-3*B*a)*(a/b)^(1/4)/a*2^(
1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2
)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(
1/4)*x-1)))-1/3*A/a^2/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.44

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)^2}\,dx = \frac{4\,(3\,Ba-7\,Ab)x^4-3\left(a^2bx^7+a^3x^3\right)\left(-\frac{81\,B^4a^4-756\,AB^3a^3b+2646\,A^2B^2a^2b^2-4116\,A^3Bab^3+2401\,A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\right.\right.}{}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/48*(4*(3*B*a - 7*A*b)*x^4 - 3*(a^2*b*x^7 + a^3*x^3)*(-(81*B^4*a^4 - 756*
A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^1
1*b))^(1/4)*log(a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2
 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*x) -
 3*(I*a^2*b*x^7 + I*a^3*x^3)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*log(I*a^3*(-(
81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2
401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*x) - 3*(-I*a^2*b*x^7 - I*a^
3*x^3)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B
*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*log(-I*a^3*(-(81*B^4*a^4 - 756*A*B^
3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b)
)^(1/4) - (3*B*a - 7*A*b)*x) + 3*(a^2*b*x^7 + a^3*x^3)*(-(81*B^4*a^4 - 756
*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^
11*b))^(1/4)*log(-a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b
^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*x)
 - 16*A*a)/(a^2*b*x^7 + a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx^4}{x^4(a+bx^4)^2}\,dx = \frac{-4Aa+x^4(-7Ab+3Ba)}{12a^3x^3+12a^2bx^7}$$
$$+\mathrm{RootSum}\left(65536t^4a^{11}b+2401A^4b^4-4116A^3Bab^3+2646A^2B^2a^2b^2-756AB^3a^3b+81B^4a^4,\left(t\mapsto t\right.\right.$$

input
```
integrate((B*x**4+A)/x**4/(b*x**4+a)**2,x)
```

output
```
(-4*A*a + x**4*(-7*A*b + 3*B*a))/(12*a**3*x**3 + 12*a**2*b*x**7) + RootSum
(65536*_t**4*a**11*b + 2401*A**4*b**4 - 4116*A**3*B*a*b**3 + 2646*A**2*B**
2*a**2*b**2 - 756*A*B**3*a**3*b + 81*B**4*a**4, Lambda(_t, _t*log(16*_t*a*
*3/(-7*A*b + 3*B*a) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^4}{x^4(a+bx^4)^2}\,dx = \frac{(3\,Ba-7\,Ab)x^4-4\,Aa}{12\,(a^2bx^7+a^3x^3)} + \frac{\frac{2\,\sqrt{2}(3\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\,\sqrt{2}(3\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3\,Ba-7\,Ab)\log\left(\sqrt{b}x^2+\sqrt{}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\,a^2}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*((3*B*a - 7*A*b)*x^4 - 4*A*a)/(a^2*b*x^7 + a^3*x^3) + 1/32*(2*sqrt(2)
*(3*B*a - 7*A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*B*
a - 7*A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*B*a - 7*A*
b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)
) - sqrt(2)*(3*B*a - 7*A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx^4}{x^4(a+bx^4)^2}\,dx = \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,Ba-7\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,Ba-7\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,Ba-7\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}\,Ba-7\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b} + \frac{Bax-Abx}{4\,(bx^4+a)a^2} - \frac{A}{3\,a^2x^3}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b) + 1/16*sqrt(2)*(3*(a*b^
3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(a^3*b) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b
^3)^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b) - 1/32
*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*log(x^2 - sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^3*b) + 1/4*(B*a*x - A*b*x)/((b*x^4 + a)*a^2) -
1/3*A/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 843, normalized size of antiderivative = 4.28

$$\int \frac{A+Bx^4}{x^4 (a+bx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^4*(a + b*x^4)^2),x)
```

output
```
- (A/(3*a) + (x^4*(7*A*b - 3*B*a))/(12*a^2))/(a*x^3 + b*x^7) - (atan((((x*
(12544*A^2*a^6*b^5 + 2304*B^2*a^8*b^3 - 10752*A*B*a^7*b^4) - ((7*A*b - 3*B
*a)*(28672*A*a^9*b^4 - 12288*B*a^10*b^3))/(16*(-a)^(11/4)*b^(1/4)))*(7*A*b
 - 3*B*a)*1i)/(16*(-a)^(11/4)*b^(1/4)) + ((x*(12544*A^2*a^6*b^5 + 2304*B^2
*a^8*b^3 - 10752*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(28672*A*a^9*b^4 - 12288*
B*a^10*b^3))/(16*(-a)^(11/4)*b^(1/4)))*(7*A*b - 3*B*a)*1i)/(16*(-a)^(11/4)
*b^(1/4)))/((((x*(12544*A^2*a^6*b^5 + 2304*B^2*a^8*b^3 - 10752*A*B*a^7*b^4)
 - ((7*A*b - 3*B*a)*(28672*A*a^9*b^4 - 12288*B*a^10*b^3))/(16*(-a)^(11/4)*
b^(1/4)))*(7*A*b - 3*B*a))/(16*(-a)^(11/4)*b^(1/4)) - ((x*(12544*A^2*a^6*b
^5 + 2304*B^2*a^8*b^3 - 10752*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(28672*A*a^9
*b^4 - 12288*B*a^10*b^3))/(16*(-a)^(11/4)*b^(1/4)))*(7*A*b - 3*B*a))/(16*(
-a)^(11/4)*b^(1/4))))*(7*A*b - 3*B*a)*1i)/(8*(-a)^(11/4)*b^(1/4)) - (atan(
(((x*(12544*A^2*a^6*b^5 + 2304*B^2*a^8*b^3 - 10752*A*B*a^7*b^4) - ((7*A*b
- 3*B*a)*(28672*A*a^9*b^4 - 12288*B*a^10*b^3)*1i)/(16*(-a)^(11/4)*b^(1/4))
)*(7*A*b - 3*B*a))/(16*(-a)^(11/4)*b^(1/4)) + ((x*(12544*A^2*a^6*b^5 + 230
4*B^2*a^8*b^3 - 10752*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*(28672*A*a^9*b^4 - 1
2288*B*a^10*b^3)*1i)/(16*(-a)^(11/4)*b^(1/4)))*(7*A*b - 3*B*a))/(16*(-a)^(
11/4)*b^(1/4)))/((((x*(12544*A^2*a^6*b^5 + 2304*B^2*a^8*b^3 - 10752*A*B*a^7
*b^4) - ((7*A*b - 3*B*a)*(28672*A*a^9*b^4 - 12288*B*a^10*b^3)*1i)/(16*(-a)
^(11/4)*b^(1/4)))*(7*A*b - 3*B*a)*1i)/(16*(-a)^(11/4)*b^(1/4)) - ((x*(1...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^4}{x^4 (a+bx^4)^2} \, dx = \frac{6b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^3 - 6b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^3 + 3b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}\right.}{24a^2x^3}$$

input
```
int((B*x^4+A)/x^4/(b*x^4+a)^2,x)
```

output
```
(6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*x**3 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*x**3 +
 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(b)*x**2)*x**3 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*s
qrt(2)*x + sqrt(a) + sqrt(b)*x**2)*x**3 - 8*a)/(24*a**2*x**3)
```

3.24 $\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 216

$$\begin{aligned}\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2} dx = & -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x^3}{4a^3\left(a+bx^4\right)} \\ & -\frac{\sqrt[4]{b}(9Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{13/4}} \\ & +\frac{\sqrt[4]{b}(9Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{13/4}} \\ & -\frac{\sqrt[4]{b}(9Ab-5aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{13/4}}\end{aligned}$$

output
```
-1/5*A/a^2/x^5+(2*A*b-B*a)/a^3/x+1/4*b*(A*b-B*a)*x^3/a^3/(b*x^4+a)+1/16*b^
(1/4)*(9*A*b-5*B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(13/4)+
1/16*b^(1/4)*(9*A*b-5*B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
13/4)-1/16*b^(1/4)*(9*A*b-5*B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2
)+b^(1/2)*x^2))*2^(1/2)/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx$$

$$= \frac{-\frac{32a^{5/4}A}{x^5}-\frac{160\sqrt[4]{a}(-2Ab+aB)}{x}-\frac{40\sqrt[4]{a}b(-Ab+aB)x^3}{a+bx^4}-10\sqrt{2}\sqrt[4]{b}(9Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}\sqrt[4]{}}{}$$

input
```
Integrate[(A + B*x^4)/(x^6*(a + b*x^4)^2),x]
```

output
```
((-32*a^(5/4)*A)/x^5 - (160*a^(1/4)*(-2*A*b + a*B))/x - (40*a^(1/4)*b*(-(A
*b) + a*B)*x^3)/(a + b*x^4) - 10*Sqrt[2]*b^(1/4)*(9*A*b - 5*a*B)*ArcTan[1
- (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*b^(1/4)*(9*A*b - 5*a*B)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 5*Sqrt[2]*b^(1/4)*(9*A*b - 5*a*B)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*b^(1/4)*(-9
*A*b + 5*a*B)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(160
*a^(13/4))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.31, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {957, 847, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(9Ab-5aB)\int \frac{1}{x^6(bx^4+a)}dx}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \text{847}$$

$$\frac{(9Ab-5aB)\left(-\frac{b\int \frac{1}{x^2\left(bx^4+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{x^2}{bx^4+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$\downarrow$ 1082

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}$$

$$\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{a}}{a}}{4ab}$$

$$\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$\downarrow$ 25

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\cdots\right)}{a}\cdots\right)}{4ab}$$

$$\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$\downarrow$ 27

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{5ax^5}\right)}{4ab}+\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

$\downarrow$ 1103

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\cdots}{a}\right.}{4ab}$$

$$\frac{Ab-aB}{4abx^5\left(a+bx^4\right)}$$

input
```
Int[(A + B*x^4)/(x^6*(a + b*x^4)^2),x]
```

output
```
(A*b - a*B)/(4*a*b*x^5*(a + b*x^4)) + ((9*A*b - 5*a*B)*(-1/5*1/(a*x^5) - (
b*(-(1/(a*x)) - (b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^
(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/a))/(4
*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.76

method	result
default	$\frac{b\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{3}}{b\,x^{4}+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}-\frac{A}{5a^{2}x^{5}}-\frac{-2Ab+}{a^{3}x}$
risch	$\frac{\frac{b(9Ab-5Ba)x^{8}}{4a^{3}}+\frac{(9Ab-5Ba)x^{4}}{5a^{2}}-\frac{A}{5a}}{x^{5}(b\,x^{4}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^{4}+6561A^{4}b^{5}-14580A^{3}Ba\,b^{4}+12150A^{2}B^{2}a^{2}b^{3}-4500A\,B^{3}a^{3}b^{2}+625B^{4}a^{4}b\right)}\right.}{}$

input
```
int((B*x^4+A)/x^6/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*b*((1/4*A*b-1/4*B*a)*x^3/(b*x^4+a)+1/8*(9/4*A*b-5/4*B*a)/b/(a/b)^(1/
4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*
2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(
a/b)^(1/4)*x-1)))-1/5*A/a^2/x^5-(-2*A*b+B*a)/a^3/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 824, normalized size of antiderivative = 3.81

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/80*(20*(5*B*a*b - 9*A*b^2)*x^8 + 16*(5*B*a^2 - 9*A*a*b)*x^4 + 16*A*a^2
- 5*(a^3*b*x^9 + a^4*x^5)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^
2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(a^10*(-(
625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a
*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 12
15*A^2*B*a*b^3 - 729*A^3*b^4)*x) + 5*(I*a^3*b*x^9 + I*a^4*x^5)*(-(625*B^4*
a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6
561*A^4*b^5)/a^13)^(1/4)*log(I*a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2
+ 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) -
(125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*x) +
5*(-I*a^3*b*x^9 - I*a^4*x^5)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150
*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-I*a^
10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A
^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^
2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*x) + 5*(a^3*b*x^9 + a^4*x^5)*(-(625*B^
4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 +
 6561*A^4*b^5)/a^13)^(1/4)*log(-a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2
 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) -
 (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*x))/
(a^3*b*x^9 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx$$
$$= \text{RootSum}\left(65536t^4a^{13}+6561A^4b^5-14580A^3Bab^4+12150A^2B^2a^2b^3-4500AB^3a^3b^2+625B^4a^4b, \left(t\right.\right.$$
$$+\frac{-4Aa^2+x^8\cdot(45Ab^2-25Bab)+x^4\cdot(36Aab-20Ba^2)}{20a^4x^5+20a^3bx^9}$$

input
```
integrate((B*x**4+A)/x**6/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**13 + 6561*A**4*b**5 - 14580*A**3*B*a*b**4 + 12150*A
**2*B**2*a**2*b**3 - 4500*A*B**3*a**3*b**2 + 625*B**4*a**4*b, Lambda(_t, _
t*log(-4096*_t**3*a**10/(-729*A**3*b**4 + 1215*A**2*B*a*b**3 - 675*A*B**2*
a**2*b**2 + 125*B**3*a**3*b) + x))) + (-4*A*a**2 + x**8*(45*A*b**2 - 25*B*
a*b) + x**4*(36*A*a*b - 20*B*a**2))/(20*a**4*x**5 + 20*a**3*b*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx = -\frac{5\,(5\,Bab-9\,Ab^2)x^8+4\,(5\,Ba^2-9\,Aab)x^4+4\,Aa^2}{20\,(a^3bx^9+a^4x^5)}$$
$$-\frac{(5\,Bab-9\,Ab^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac14}b^{\frac14}x+\sqrt{a}\right.}{a^{\frac14}b^{\frac34}}\right.}{32\,a^3}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/20*(5*(5*B*a*b - 9*A*b^2)*x^8 + 4*(5*B*a^2 - 9*A*a*b)*x^4 + 4*A*a^2)/(a
^3*b*x^9 + a^4*x^5) - 1/32*(5*B*a*b - 9*A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt
(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4
)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a)
)/(a^(1/4)*b^(3/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx = -\frac{Babx^3-Ab^2x^3}{4\left(bx^4+a\right)a^3} - \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^4b^2} - \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^4b^2} + \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^4b^2} - \frac{\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-9\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^4b^2} - \frac{5\,Bax^4-10\,Abx^4+Aa}{5\,a^3x^5}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(B*a*b*x^3 - A*b^2*x^3)/((b*x^4 + a)*a^3) - 1/16*sqrt(2)*(5*(a*b^3)^(
3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(a^4*b^2) - 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3
)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^4*b^2) + 1/32*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(x^
2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^2) - 1/32*sqrt(2)*(5*(a*b^3)
^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(a^4*b^2) - 1/5*(5*B*a*x^4 - 10*A*b*x^4 + A*a)/(a^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx = \frac{\frac{x^4\,\left(9\,A\,b-5\,B\,a\right)}{5\,a^2}-\frac{A}{5\,a}+\frac{b\,x^8\,\left(9\,A\,b-5\,B\,a\right)}{4\,a^3}}{b\,x^9+a\,x^5} + \frac{{\left(-b\right)}^{1/4}\,\mathrm{atan}\left(\frac{{\left(-b\right)}^{1/4}\,x}{a^{1/4}}\right)\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^{13/4}} - \frac{{\left(-b\right)}^{1/4}\,\mathrm{atanh}\left(\frac{{\left(-b\right)}^{1/4}\,x}{a^{1/4}}\right)\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^{13/4}}$$

input
```
int((A + B*x^4)/(x^6*(a + b*x^4)^2),x)
```

output
```
((x^4*(9*A*b - 5*B*a))/(5*a^2) - A/(5*a) + (b*x^8*(9*A*b - 5*B*a))/(4*a^3)
)/(a*x^5 + b*x^9) + ((-b)^(1/4)*atan(((-b)^(1/4)*x)/a^(1/4))*(9*A*b - 5*B*
a))/(8*a^(13/4)) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x)/a^(1/4))*(9*A*b - 5*B*
a))/(8*a^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)^2}\,dx = \frac{-10b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^5+10b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^5+5b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\right.}{40a^3x^5}$$

input
```
int((B*x^4+A)/x^6/(b*x^4+a)^2,x)
```

output
```
( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**5 + 10*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*b*x**5 + 5*b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x +
 sqrt(a) + sqrt(b)*x**2)*b*x**5 - 5*b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)
*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*x**5 - 8*a**2 + 40*a*b*x**
4)/(40*a**3*x**5)
```

3.25 $\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = -\frac{5a(11bc-9ad)x\sqrt{a+bx^4}}{231b^3}+\frac{(11bc-9ad)x^5\sqrt{a+bx^4}}{77b^2}+\frac{dx^9\sqrt{a+bx^4}}{11b} + \frac{5a^{7/4}(11bc-9ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{462b^{13/4}\sqrt{a+bx^4}}$$

output

```
-5/231*a*(-9*a*d+11*b*c)*x*(b*x^4+a)^(1/2)/b^3+1/77*(-9*a*d+11*b*c)*x^5*(b
*x^4+a)^(1/2)/b^2+1/11*d*x^9*(b*x^4+a)^(1/2)/b+5/462*a^(7/4)*(-9*a*d+11*b*
c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Inverse
JacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{x\left((a+bx^4)\left(45a^2d+3b^2x^4(11c+7dx^4)-ab(55c+27dx^4)\right)+5a^2(11bc-9ad)\sqrt{1+\frac{bx^4}{a}}\text{Hypergeomet}\right.}{231b^3\sqrt{a+bx^4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(x*((a + b*x^4)*(45*a^2*d + 3*b^2*x^4*(11*c + 7*d*x^4) - a*b*(55*c + 27*d*
x^4)) + 5*a^2*(11*b*c - 9*a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4,
1/2, 5/4, -((b*x^4)/a)]))/(231*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 843, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(11bc-9ad)\int \frac{x^8}{\sqrt{bx^4+a}}dx}{11b}+\frac{dx^9\sqrt{a+bx^4}}{11b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(11bc-9ad)\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\int \frac{x^4}{\sqrt{bx^4+a}}dx}{7b}\right)}{11b}+\frac{dx^9\sqrt{a+bx^4}}{11b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(11bc-9ad)\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{7b}\right)}{11b}+\frac{dx^9\sqrt{a+bx^4}}{11b}$$

$$\Big\downarrow \text{761}$$

$$\frac{(11bc-9ad)\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{7b}\right)}{11b}+\frac{dx^9\sqrt{a+bx^4}}{11b}$$

input
```
Int[(x^8*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(d*x^9*Sqrt[a + b*x^4])/(11*b) + ((11*b*c - 9*a*d)*((x^5*Sqrt[a + b*x^4])/
(7*b) - (5*a*((x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])))/(7*b)))/(11*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.78

method	result
risch	$\frac{x(21 d b^2 x^8-27abd x^4+33b^2 c x^4+45a^2 d-55abc)\sqrt{b x^4+a}}{231 b^3}-\frac{5a^2(9ad-11cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{231 b^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}$
elliptic	$\frac{d x^9\sqrt{b x^4+a}}{11b}+\frac{\left(c-\frac{9ad}{11b}\right)x^5\sqrt{b x^4+a}}{7b}-\frac{5a\left(c-\frac{9ad}{11b}\right)x\sqrt{b x^4+a}}{21b^2}+\frac{5a^2\left(c-\frac{9ad}{11b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}$
default	$c\left(\frac{x^5\sqrt{b x^4+a}}{7b}-\frac{5ax\sqrt{b x^4+a}}{21b^2}+\frac{5a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\right)+d\left(\frac{x^9\sqrt{b x^4+a}}{11b}-\frac{9a x^5\sqrt{b x^4+a}}{77b^2}\right.$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/231*x*(21*b^2*d*x^8-27*a*b*d*x^4+33*b^2*c*x^4+45*a^2*d-55*a*b*c)/b^3*(b*
x^4+a)^(1/2)-5/231*a^2*(9*a*d-11*b*c)/b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a
^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.55

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{5\,(11\,abc-9\,a^2d)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+(21\,b^2dx^9+3\,(11\,b^2c-9\,abd)x^5-5\,(11\,abc-9\,a}{231\,b^3}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/231*(5*(11*a*b*c - 9*a^2*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b
)^(1/4)/x), -1) + (21*b^2*d*x^9 + 3*(11*b^2*c - 9*a*b*d)*x^5 - 5*(11*a*b*c
 - 9*a^2*d)*x)*sqrt(b*x^4 + a))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)} + \frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((1/2, 13/4), (17/4,), b*x
**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^8}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^8/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^8}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$
$$= \frac{45\sqrt{b\,x^4+a}\,a^2dx - 55\sqrt{b\,x^4+a}\,abcx - 27\sqrt{b\,x^4+a}\,abd\,x^5 + 33\sqrt{b\,x^4+a}\,b^2c\,x^5 + 21\sqrt{b\,x^4+a}\,b^2d\,x^9 -}{231b^3}$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
(45*sqrt(a + b*x**4)*a**2*d*x - 55*sqrt(a + b*x**4)*a*b*c*x - 27*sqrt(a +
b*x**4)*a*b*d*x**5 + 33*sqrt(a + b*x**4)*b**2*c*x**5 + 21*sqrt(a + b*x**4)
*b**2*d*x**9 - 45*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**3*d + 55*int(sqr
t(a + b*x**4)/(a + b*x**4),x)*a**2*b*c)/(231*b**3)
```

3.26 $\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\begin{aligned}\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx &= \frac{(7bc-5ad)x\sqrt{a+bx^4}}{21b^2}+\frac{dx^5\sqrt{a+bx^4}}{7b}\\ &\quad-\frac{a^{3/4}(7bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42b^{9/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/21*(-5*a*d+7*b*c)*x*(b*x^4+a)^(1/2)/b^2+1/7*d*x^5*(b*x^4+a)^(1/2)/b-1/42
*a^(3/4)*(-5*a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(
9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.60

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{x\left(-\left(\left(a+bx^4\right)\left(-7bc+5ad-3bdx^4\right)\right)+a(-7bc+5ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{21b^2\sqrt{a+bx^4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(x*(-((a + b*x^4)*(-7*b*c + 5*a*d - 3*b*d*x^4)) + a*(-7*b*c + 5*a*d)*Sqrt[
1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(21*b^2*Sq
rt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(7bc-5ad)\int \frac{x^4}{\sqrt{bx^4+a}}dx}{7b}+\frac{dx^5\sqrt{a+bx^4}}{7b}$$

$$\Big\downarrow \ 843$$

$$\frac{(7bc-5ad)\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{7b}+\frac{dx^5\sqrt{a+bx^4}}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bc-5ad)\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{7b}+\frac{dx^5\sqrt{a+bx^4}}{7b}$$

input
```
Int[(x^4*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(d*x^5*Sqrt[a + b*x^4])/(7*b) + ((7*b*c - 5*a*d)*((x*Sqrt[a + b*x^4])/(3*b
) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a +
b*x^4])))/(7*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{x(-3db\,x^4+5ad-7cb)\sqrt{b\,x^4+a}}{21b^2}+\frac{a(5ad-7cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$\frac{d\,x^5\sqrt{b\,x^4+a}}{7b}+\frac{\left(c-\frac{5ad}{7b}\right)x\sqrt{b\,x^4+a}}{3b}-\frac{a\left(c-\frac{5ad}{7b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{x^5\sqrt{b\,x^4+a}}{7b}-\frac{5ax\sqrt{b\,x^4+a}}{21b^2}+\frac{5a^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{21}\right.$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(-3*b*d*x^4+5*a*d-7*b*c)/b^2*(b*x^4+a)^(1/2)+1/21*a*(5*a*d-7*b*c)/
b^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx = -\frac{(7\,bc-5\,ad)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-(3\,bdx^5+(7\,bc-5\,ad)x)\sqrt{bx^4+a}}{21\,b^2}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*((7*b*c - 5*a*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)
/x), -1) - (3*b*d*x^5 + (7*b*c - 5*a*d)*x)*sqrt(b*x^4 + a))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx=\frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}+\frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx=\int \frac{(dx^4+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{-5\sqrt{b\,x^4+a}\,adx+7\sqrt{b\,x^4+a}\,bcx+3\sqrt{b\,x^4+a}\,bd\,x^5+5\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a^2d-7\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)abc}{21b^2}$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a + b*x**4)*a*d*x + 7*sqrt(a + b*x**4)*b*c*x + 3*sqrt(a + b*x**
4)*b*d*x**5 + 5*int(sqrt(a + b*x**4)/(a + b*x**4),x)*a**2*d - 7*int(sqrt(a
 + b*x**4)/(a + b*x**4),x)*a*b*c)/(21*b**2)
```

3.27 $\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{dx\sqrt{a+bx^4}}{3b} + \frac{(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/3*d*x*(b*x^4+a)^(1/2)/b+1/6*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4
)),1/2*2^(1/2))/a^(1/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{dx(a+bx^4)+(3bc-ad)x\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{3b\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/Sqrt[a + b*x^4],x]
```

output
```
(d*x*(a + b*x^4) + (3*b*c - a*d)*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^4)/a)])/(3*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {913, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(3bc-ad)\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}+\frac{dx\sqrt{a+bx^4}}{3b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(3bc-ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}+\frac{dx\sqrt{a+bx^4}}{3b}$$

input
```
Int[(c + d*x^4)/Sqrt[a + b*x^4],x]
```

output
```
(d*x*Sqrt[a + b*x^4])/(3*b) + ((3*b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(6*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

method	result	siz
elliptic	$\frac{d x \sqrt{b\,x^4+a}}{3 b}+\frac{\left(c-\frac{a d}{3 b}\right) \sqrt{1-\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	96
risch	$\frac{d x \sqrt{b\,x^4+a}}{3 b}-\frac{(a d-3 c b) \sqrt{1-\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	99
default	$\frac{c \sqrt{1-\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+d\left(\frac{x \sqrt{b\,x^4+a}}{3 b}-\frac{a \sqrt{1-\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{3 b \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)$	16

input
```
int((d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x*(b*x^4+a)^(1/2)/b+(c-1/3*a/b*d)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(
1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.49

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}adx + (3\,bc-ad)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)}{3\,ab}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(b*x^4 + a)*a*d*x + (3*b*c - a*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1))/(a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_p
olar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(1/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a}\,dx - \left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)ad + 3\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)bc}{3b}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*d*x - int(sqrt(a + b*x**4)/(a + b*x**4),x)*a*d + 3*int(s
qrt(a + b*x**4)/(a + b*x**4),x)*b*c)/(3*b)
```

3.28 $\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{3ax^3} - \frac{(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/3*c*(b*x^4+a)^(1/2)/a/x^3-1/6*(-3*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^
4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(
1/4)),1/2*2^(1/2))/a^(5/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \frac{-c(a+bx^4)+(-bc+3ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{3ax^3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
(-(c*(a + b*x^4)) + (-(b*c) + 3*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(3*a*x^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(bc-3ad)\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{c\sqrt{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{c\sqrt{a+bx^4}}{3ax^3}$$

input
```
Int[(c + d*x^4)/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
-1/3*(c*Sqrt[a + b*x^4])/(a*x^3) - ((b*c - 3*a*d)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(6*a^(5/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result	si
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{3a\,x^{3}}+\frac{\left(d-\frac{bc}{3a}\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	98
risch	$-\frac{c\sqrt{b\,x^{4}+a}}{3a\,x^{3}}+\frac{(3ad-cb)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	10
default	$\frac{d\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}+c\left(-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)$	16

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(b*x^4+a)^(1/2)/a/x^3+(d-1/3*b/a*c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \frac{(bc-3\,ad)\sqrt{a}x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-\sqrt{bx^4+a}bc}{3\,abx^3}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((b*c - 3*a*d)*sqrt(a)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/
4)), -1) - sqrt(b*x^4 + a)*b*c)/(a*b*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(1/2),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4\sqrt{a+bx^4}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-3\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^8+a\,x^4}dx\right)ad\,x^3+\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^8+a\,x^4}dx\right)bc\,x^3}{b\,x^3}$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 3*int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*a*d*x
**3 + int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)*b*c*x**3)/(b*x**3)
```

3.29 $\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{7ax^7}+\frac{(5bc-7ad)\sqrt{a+bx^4}}{21a^2x^3} + \frac{b^{3/4}(5bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42a^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/7*c*(b*x^4+a)^(1/2)/a/x^7+1/21*(-7*a*d+5*b*c)*(b*x^4+a)^(1/2)/a^2/x^3+1
/42*b^(3/4)*(-7*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/
a^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.52

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \frac{-3c(a+bx^4)+(5bc-7ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{21ax^7\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*Sqrt[a + b*x^4]),x]
```

output
```
(-3*c*(a + b*x^4) + (5*b*c - 7*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)])/(21*a*x^7*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5bc-7ad)\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{7a}-\frac{c\sqrt{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(5bc-7ad)\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(5bc-7ad)\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt{a+bx^4}}{7ax^7}$$

input
```
Int[(c + d*x^4)/(x^8*Sqrt[a + b*x^4]),x]
```

output
```
-1/7*(c*Sqrt[a + b*x^4])/(a*x^7) - ((5*b*c - 7*a*d)*(-1/3*Sqrt[a + b*x^4]/
(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqr
t[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqr
t[a + b*x^4])))/(7*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(7ad\,x^4-5bc\,x^4+3ac)}{21a^2x^7}-\frac{b(7ad-5cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{7a\,x^7}-\frac{(7ad-5cb)\sqrt{b\,x^4+a}}{21a^2x^3}-\frac{b(7ad-5cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{7a\,x^7}+\frac{5b\sqrt{b\,x^4+a}}{21a^2x^3}+\frac{5b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{\sqrt{b\,x^4+a}}{3a\,x^3}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \cdots}{\cdots}\right.$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(b*x^4+a)^(1/2)*(7*a*d*x^4-5*b*c*x^4+3*a*c)/a^2/x^7-1/21*b*(7*a*d-5*
b*c)/a^2/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(
1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.50

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = -\frac{(5\,bc-7\,ad)\sqrt{a}x^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-((5\,bc-7\,ad)x^4-3\,ac)\sqrt{bx^4+a}}{21\,a^2x^7}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*((5*b*c - 7*a*d)*sqrt(a)*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)
^(1/4)), -1) - ((5*b*c - 7*a*d)*x^4 - 3*a*c)*sqrt(b*x^4 + a))/(a^2*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.60

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \frac{c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(1/2),x)
```

output
```
c*gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*x**7*gamma(-3/4)) + d*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*sqrt(a)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+ax^8}}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+ax^8}}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^8\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8\sqrt{a+bx^4}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-7\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{12}+a\,x^8}dx\right)a d\,x^7+5\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{12}+a\,x^8}dx\right)bc\,x^7}{5b\,x^7}$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 7*int(sqrt(a + b*x**4)/(a*x**8 + b*x**12),x)*a*d*
x**7 + 5*int(sqrt(a + b*x**4)/(a*x**8 + b*x**12),x)*b*c*x**7)/(5*b*x**7)
```

3.30 $\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = -\frac{c\sqrt{a+bx^4}}{11ax^{11}}+\frac{(9bc-11ad)\sqrt{a+bx^4}}{77a^2x^7}-\frac{5b(9bc-11ad)\sqrt{a+bx^4}}{231a^3x^3} - \frac{5b^{7/4}(9bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{462a^{13/4}\sqrt{a+bx^4}}$$

output
```
-1/11*c*(b*x^4+a)^(1/2)/a/x^11+1/77*(-11*a*d+9*b*c)*(b*x^4+a)^(1/2)/a^2/x^
7-5/231*b*(-11*a*d+9*b*c)*(b*x^4+a)^(1/2)/a^3/x^3-5/462*b^(7/4)*(-11*a*d+9
*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(13/4)/(b*x^4+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.43

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \frac{-7c(a+bx^4)+(9bc-11ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)}{77ax^{11}\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^12*Sqrt[a + b*x^4]),x]
```

output
```
(-7*c*(a + b*x^4) + (9*b*c - 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometri
c2F1[-7/4, 1/2, -3/4, -((b*x^4)/a)])/(77*a*x^11*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 847, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9bc-11ad)\int \frac{1}{x^8\sqrt{bx^4+a}}dx}{11a}-\frac{c\sqrt{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(9bc-11ad)\left(-\frac{5b\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{11a}-\frac{c\sqrt{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(9bc-11ad)\left(-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{11a}-\frac{c\sqrt{a+bx^4}}{11ax^{11}}$$

$\Big\downarrow$ 761

$$-\frac{(9bc-11ad)\left(-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{11a}-\frac{c\sqrt{a+bx^4}}{11ax^{11}}$$

input
```
Int[(c + d*x^4)/(x^12*Sqrt[a + b*x^4]),x]
```

output
```
-1/11*(c*Sqrt[a + b*x^4])/(a*x^11) - ((9*b*c - 11*a*d)*(-1/7*Sqrt[a + b*x^
4]/(a*x^7) - (5*b*(-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-55abd\,x^{8}+45b^{2}c\,x^{8}+33a^{2}d\,x^{4}-27abc\,x^{4}+21a^{2}c\right)}{231a^{3}x^{11}}+\frac{5b^{2}\left(11ad-9cb\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{231a^{3}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{11a\,x^{11}}-\frac{\left(11ad-9cb\right)\sqrt{b\,x^{4}+a}}{77a^{2}x^{7}}+\frac{5b\left(11ad-9cb\right)\sqrt{b\,x^{4}+a}}{231a^{3}x^{3}}+\frac{5b^{2}\left(11ad-9cb\right)\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{231a^{3}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
default	$c\left(-\frac{\sqrt{b\,x^{4}+a}}{11a\,x^{11}}+\frac{9b\sqrt{b\,x^{4}+a}}{77a^{2}x^{7}}-\frac{15b^{2}\sqrt{b\,x^{4}+a}}{77a^{3}x^{3}}-\frac{15b^{3}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{77a^{3}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d\left(-\frac{\sqrt{b\,x^{4}+a}}{7a\,x^{7}}\right.$

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/231*(b*x^4+a)^(1/2)*(-55*a*b*d*x^8+45*b^2*c*x^8+33*a^2*d*x^4-27*a*b*c*x
^4+21*a^2*c)/a^3/x^11+5/231*b^2*(11*a*d-9*b*c)/a^3/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \frac{5\,(9\,b^2c-11\,abd)\sqrt{a}x^{11}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-(5\,(9\,b^2c-11\,abd)x^8-3\,(9\,abc-11\,a^2d)x^4}{231\,a^3x^{11}}$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/231*(5*(9*b^2*c - 11*a*b*d)*sqrt(a)*x^11*(-b/a)^(3/4)*elliptic_f(arcsin(
x*(-b/a)^(1/4)), -1) - (5*(9*b^2*c - 11*a*b*d)*x^8 - 3*(9*a*b*c - 11*a^2*d
)*x^4 + 21*a^2*c)*sqrt(b*x^4 + a))/(a^3*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.52

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \frac{c\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^{11}\Gamma\left(-\frac{7}{4}\right)} + \frac{d\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^{7}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**12/(b*x**4+a)**(1/2),x)
```

output
```
c*gamma(-11/4)*hyper((-11/4, 1/2), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*x**11*gamma(-7/4)) + d*gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^{12}}\,dx$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^12), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^{12}}\,dx$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^{12}\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(x^12*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x^4)/(x^12*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{12}\sqrt{a+bx^4}}\,dx$$
$$= \frac{-\sqrt{b\,x^4+a}\,d-11\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{16}+a\,x^{12}}dx\right)ad\,x^{11}+9\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{16}+a\,x^{12}}dx\right)bc\,x^{11}}{9b\,x^{11}}$$

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 11*int(sqrt(a + b*x**4)/(a*x**12 + b*x**16),x)*a*
d*x**11 + 9*int(sqrt(a + b*x**4)/(a*x**12 + b*x**16),x)*b*c*x**11)/(9*b*x*
*11)
```

3.31 $\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\begin{aligned}
\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx &= \frac{(9bc-7ad)x^3\sqrt{a+bx^4}}{45b^2}+\frac{dx^7\sqrt{a+bx^4}}{9b}-\frac{a(9bc-7ad)x\sqrt{a+bx^4}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
&+\frac{a^{5/4}(9bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^4}} \\
&-\frac{a^{5/4}(9bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{11/4}\sqrt{a+bx^4}}
\end{aligned}$$

output
```
1/45*(-7*a*d+9*b*c)*x^3*(b*x^4+a)^(1/2)/b^2+1/9*d*x^7*(b*x^4+a)^(1/2)/b-1/
15*a*(-7*a*d+9*b*c)*x*(b*x^4+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x^2)+1/15*a
^(5/4)*(-7*a*d+9*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^
2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(11/
4)/(b*x^4+a)^(1/2)-1/30*a^(5/4)*(-7*a*d+9*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x
^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^
(1/4)),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.31

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{x^3\left(-((a+bx^4)(-9bc+7ad-5bdx^4))+a(-9bc+7ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{45b^2\sqrt{a+bx^4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(x^3*(-((a + b*x^4)*(-9*b*c + 7*a*d - 5*b*d*x^4)) + a*(-9*b*c + 7*a*d)*Sqr
t[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)]))/(45*b^2*
Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(9bc-7ad)\int \frac{x^6}{\sqrt{bx^4+a}}dx}{9b}+\frac{dx^7\sqrt{a+bx^4}}{9b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(9bc-7ad)\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{9b}+\frac{dx^7\sqrt{a+bx^4}}{9b}$$

$\Big\downarrow$ 834

$$\frac{(9bc-7ad)\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{dx^7\sqrt{a+bx^4}}{9b}$$

$\Big\downarrow$ 27

$$\frac{(9bc-7ad)\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{dx^7\sqrt{a+bx^4}}{9b}$$

$\Big\downarrow$ 761

$$\frac{(9bc-7ad)\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{dx^7\sqrt{a+bx^4}}{9b}$$

$\Big\downarrow$ 1510

$$\frac{(9bc-7ad)\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\cdots\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}\cdots}{\sqrt{b}}\cdots\right)}{5b}\cdots\right)}{9b}$$

$$\frac{dx^7\sqrt{a+bx^4}}{9b}$$

input
```
Int[(x^6*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(d*x^7*Sqrt[a + b*x^4])/(9*b) + ((9*b*c - 7*a*d)*((x^3*Sqrt[a + b*x^4])/(5
*b) - (3*a*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]
) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a +
b*x^4])))/(5*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.47

method	result
risch	$-\frac{x^3(-5db\,x^4+7ad-9cb)\sqrt{b\,x^4+a}}{45b^2}+\frac{ia^{\frac{3}{2}}(7ad-9cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{15b^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
elliptic	$\frac{d\,x^7\sqrt{b\,x^4+a}}{9b}+\frac{\left(c-\frac{7ad}{9b}\right)x^3\sqrt{b\,x^4+a}}{5b}-\frac{3i\left(c-\frac{7ad}{9b}\right)a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x^3\sqrt{b\,x^4+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\left(\frac{x^7\sqrt{b\,x^4+a}}{9b}-\frac{7a}{}\right.$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*x^3*(-5*b*d*x^4+7*a*d-9*b*c)/b^2*(b*x^4+a)^(1/2)+1/15*I*a^(3/2)*(7*a
*d-9*b*c)/b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx =$$

$$-\frac{3\,(9\,abc-7\,a^2d)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-3\,(9\,abc-7\,a^2d)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right.}{45\,b^3x}$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/45*(3*(9*a*b*c - 7*a^2*d)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/
b)^(1/4)/x), -1) - 3*(9*a*b*c - 7*a^2*d)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) - (5*b^2*d*x^8 + (9*b^2*c - 7*a*b*d)*x^4 - 27
*a*b*c + 21*a^2*d)*sqrt(b*x^4 + a))/(b^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x
**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^6}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^6}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx=\int \frac{x^6\,(d\,x^4+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{-7\sqrt{b\,x^4+a}\,ad\,x^3+9\sqrt{b\,x^4+a}\,bc\,x^3+5\sqrt{b\,x^4+a}\,bd\,x^7+21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)a^2d-27\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)}{45b^2}$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
( - 7*sqrt(a + b*x**4)*a*d*x**3 + 9*sqrt(a + b*x**4)*b*c*x**3 + 5*sqrt(a +
 b*x**4)*b*d*x**7 + 21*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a**2*d
- 27*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a*b*c)/(45*b**2)
```

3.32 $\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 264

$$\begin{aligned}\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx &= \frac{dx^3\sqrt{a+bx^4}}{5b}+\frac{(5bc-3ad)x\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ &\quad - \frac{\sqrt[4]{a}(5bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\ &\quad + \frac{\sqrt[4]{a}(5bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output

```
1/5*d*x^3*(b*x^4+a)^(1/2)/b+1/5*(-3*a*d+5*b*c)*x*(b*x^4+a)^(1/2)/b^(3/2)/(
a^(1/2)+b^(1/2)*x^2)-1/5*a^(1/4)*(-3*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*
x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(
1/4))),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)+1/10*a^(1/4)*(-3*a*d+5*b*c)*(a
^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacob
iAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{x^3\left(3d(a+bx^4)+(5bc-3ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{15b\sqrt{a+bx^4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(x^3*(3*d*(a + b*x^4) + (5*b*c - 3*a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/2, 3/4, 7/4, -((b*x^4)/a)]))/(15*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(5bc-3ad)\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}+\frac{dx^3\sqrt{a+bx^4}}{5b}$$

$$\Big\downarrow \ 834$$

$$\frac{(5bc-3ad)\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}+\frac{dx^3\sqrt{a+bx^4}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5bc-3ad)\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}+\frac{dx^3\sqrt{a+bx^4}}{5b}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5bc-3ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}+\frac{dx^3\sqrt{a+bx^4}}{5b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(5bc-3ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right)}{5b}\frac{dx^3\sqrt{a+bx^4}}{5b}$$

input
```
Int[(x^2*(c + d*x^4))/Sqrt[a + b*x^4],x]
```

output
```
(d*x^3*Sqrt[a + b*x^4])/(5*b) + ((5*b*c - 3*a*d)*(-((-((x*Sqrt[a + b*x^4])
/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.47

method	result
risch	$\frac{d\,x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{i(3ad-5cb)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
elliptic	$\frac{d\,x^{3}\sqrt{b\,x^{4}+a}}{5b}+\frac{i\left(c-\frac{3ad}{5b}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
default	$\frac{ic\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}+d\left(\frac{x^{3}\sqrt{b\,x^{4}+a}}{5b}-\frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{}}{}}}{}\right.$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*d*x^3*(b*x^4+a)^(1/2)/b-1/5*I*(3*a*d-5*b*c)/b^(3/2)*a^(1/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^
(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(
x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.41

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx$$
$$=\frac{(5\,bc-3\,ad)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-(5\,bc-3\,ad)\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(}{5\,b^2x}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*((5*b*c - 3*a*d)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)
/x), -1) - (5*b*c - 3*a*d)*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)
^(1/4)/x), -1) + (b*d*x^4 + 5*b*c - 3*a*d)*sqrt(b*x^4 + a))/(b^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(1/2),x)
```

output
```
c*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(1/2),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a}\,d\,x^3-3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)ad+5\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)bc}{5b}$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*d*x**3 - 3*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a
*d + 5*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*b*c)/(5*b)
```

3.33 $\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 254

$$\begin{aligned}\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{ax}+\frac{(bc+ad)x\sqrt{a+bx^4}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ &\quad -\frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^4}} \\ &\quad +\frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-c*(b*x^4+a)^(1/2)/a/x+(a*d+b*c)*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1
/2)*x^2)-(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b
^(3/4)/(b*x^4+a)^(1/2)+1/2*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(
1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2
*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.30

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx = \frac{-3c(a+bx^4)+(bc+ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3ax\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
(-3*c*(a + b*x^4) + (b*c + a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[
1/2, 3/4, 7/4, -((b*x^4)/a)])/(3*a*x*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 955$$

$$\frac{(ad+bc)\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{c\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 834$$

$$\frac{(ad+bc)\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{c\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{(ad+bc)\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{c\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ad+bc)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{c\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(ad+bc)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-}{\sqrt{b}}\right)}{a}$$

$$\frac{c\sqrt{a+bx^4}}{ax}$$

input
```
Int[(c + d*x^4)/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-((c*Sqrt[a + b*x^4])/(a*x)) + ((b*c + a*d)*(-((-((x*Sqrt[a + b*x^4])/(Sqr
t[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{c\sqrt{b\,x^4+a}}{ax}+\frac{i(ad+cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{ax}+\frac{i\left(d+\frac{bc}{a}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{ax}+\frac{i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+\frac{id\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+}$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(b*x^4+a)^(1/2)/a/x+I*(a*d+b*c)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*
b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{dx^4+c}{\sqrt{bx^4+a}\,x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^4 + a)*(d*x^4 + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.32

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx = \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(1/2),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt
(a)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+a}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx=\int \frac{d\,x^4+c}{x^2\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\sqrt{a+bx^4}}\,dx=\frac{\sqrt{b\,x^4+a}\,d+\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^6+a\,x^2}dx\right)adx+\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^6+a\,x^2}dx\right)bcx}{bx}$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*d + int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*a*d*x + in
t(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)*b*c*x)/(b*x)
```

3.34 $\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\begin{aligned}\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx &= -\frac{c\sqrt{a+bx^4}}{5ax^5}+\frac{(3bc-5ad)\sqrt{a+bx^4}}{5a^2x}-\frac{\sqrt{b}(3bc-5ad)x\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)}\\ &+\frac{\sqrt[4]{b}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}}\\ &-\frac{\sqrt[4]{b}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/5*c*(b*x^4+a)^(1/2)/a/x^5+1/5*(-5*a*d+3*b*c)*(b*x^4+a)^(1/2)/a^2/x-1/5*
b^(1/2)*(-5*a*d+3*b*c)*x*(b*x^4+a)^(1/2)/a^2/(a^(1/2)+b^(1/2)*x^2)+1/5*b^(
1/4)*(-5*a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/
(b*x^4+a)^(1/2)-1/10*b^(1/4)*(-5*a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+
a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/
4)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx = \frac{-c(a+bx^4)+(3bc-5ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{5ax^5\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
(-(c*(a + b*x^4)) + (3*b*c - 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(5*a*x^5*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(3bc-5ad)\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(3bc-5ad)\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \text{ 834}$$

$$-\frac{(3bc-5ad)\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

$\big\downarrow$ 27

$$-\frac{(3bc-5ad)\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

$\big\downarrow$ 761

$$-\frac{(3bc-5ad)\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

$\big\downarrow$ 1510

$$-\frac{(3bc-5ad)\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\cdots\right)}{a}\cdots\right)}{5a}-\frac{c\sqrt{a+bx^4}}{5ax^5}$$

input
```
Int[(c + d*x^4)/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/5*(c*Sqrt[a + b*x^4])/(a*x^5) - ((3*b*c - 5*a*d)*(-(Sqrt[a + b*x^4]/(a*
x)) + (b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE
[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b])
 + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*
x^4])))/a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.48

method	result
risch	$-\frac{\sqrt{b\,x^4+a}\,(5ad\,x^4-3bc\,x^4+ac)}{5a^2x^5}+\frac{i\sqrt{b}\,(5ad-3cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a\,x^5}-\frac{(5ad-3cb)\sqrt{b\,x^4+a}}{5a^2x}+\frac{i\sqrt{b}\,(5ad-3cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5a\,x^5}+\frac{3b\sqrt{b\,x^4+a}}{5a^2x}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\right.$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/2)*(5*a*d*x^4-3*b*c*x^4+a*c)/a^2/x^5+1/5*I*b^(1/2)*(5*a*
d-3*b*c)/a^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.38

$$\int \frac{c + dx^4}{x^6\sqrt{a + bx^4}}\,dx$$
$$= \frac{(3\,bc - 5\,ad)\sqrt{a}x^5\left(-\frac{b}{a}\right)^{\frac{3}{4}} E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) | -1) - (3\,bc - 5\,ad)\sqrt{a}x^5\left(-\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) | -}{5\,a^2x^5}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*((3*b*c - 5*a*d)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(
1/4)), -1) - (3*b*c - 5*a*d)*sqrt(a)*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*
(-b/a)^(1/4)), -1) + ((3*b*c - 5*a*d)*x^4 - a*c)*sqrt(b*x^4 + a))/(a^2*x^5
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.30

$$\int \frac{c + dx^4}{x^6\sqrt{a + bx^4}}\,dx = \frac{c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{d\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(1/2),x)
```

output
```
c*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*x**5*gamma(-1/4)) + d*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*e
xp_polar(I*pi)/a)/(4*sqrt(a)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{dx^4+c}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(sqrt(b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(1/2)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{c + dx^4}{x^6\sqrt{a + bx^4}}\,dx = \frac{-\sqrt{b\,x^4 + a}\,d - 5\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{10}+a\,x^6}dx\right) ad\,x^5 + 3\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^{10}+a\,x^6}dx\right) bc\,x^5}{3b\,x^5}$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 5*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*a*d*
x**5 + 3*int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)*b*c*x**5)/(3*b*x**5)
```

3.35 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{a(bc-ad)x}{2b^3\sqrt{a+bx^4}} + \frac{(7bc-12ad)x\sqrt{a+bx^4}}{21b^3} + \frac{dx^5\sqrt{a+bx^4}}{7b^2} - \frac{5a^{3/4}(7bc-9ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{84b^{13/4}\sqrt{a+bx^4}}$$

output
```
1/2*a*(-a*d+b*c)*x/b^3/(b*x^4+a)^(1/2)+1/21*(-12*a*d+7*b*c)*x*(b*x^4+a)^(1
/2)/b^3+1/7*d*x^5*(b*x^4+a)^(1/2)/b^2-5/84*a^(3/4)*(-9*a*d+7*b*c)*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.58

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{x\left(-45a^2d+ab(35c-18dx^4)+2b^2x^4(7c+3dx^4)+5a(-7bc+9ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hyper}\right.}{42b^3\sqrt{a+bx^4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(-45*a^2*d + a*b*(35*c - 18*d*x^4) + 2*b^2*x^4*(7*c + 3*d*x^4) + 5*a*(-
7*b*c + 9*a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x
^4)/a)]))/(42*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {959, 817, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7bc-9ad)\int \frac{x^8}{(bx^4+a)^{3/2}}dx}{7b}+\frac{dx^9}{7b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-9ad)\left(\frac{5\int \frac{x^4}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{7b}+\frac{dx^9}{7b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 843$$

$$\frac{(7bc-9ad)\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{7b}+\frac{dx^9}{7b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{(7bc-9ad)\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{7b}+\frac{dx^9}{7b\sqrt{a+bx^4}}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(d*x^9)/(7*b*Sqrt[a + b*x^4]) + ((7*b*c - 9*a*d)*(-1/2*x^5/(b*Sqrt[a + b*x
^4]) + (5*((x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sq
rt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a
^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])))/(2*b)))/(7*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.79 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.11

method	result
elliptic	$-\frac{ax(ad-cb)}{2b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\,x^5\sqrt{b\,x^4+a}}{7b^2}+\frac{\left(-\frac{ad-cb}{b^2}-\frac{5da}{7b^2}\right)x\sqrt{b\,x^4+a}}{3b}+\frac{\left(\frac{a(ad-cb)}{2b^3}-\frac{\left(-\frac{ad-cb}{b^2}-\frac{5da}{7b^2}\right)a}{3b}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{ax}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{a^2x}{2b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x^5\sqrt{b\,x}}{7b^2}\right.$
risch	$-\frac{x\left(-3db\,x^4+12ad-7cb\right)\sqrt{b\,x^4+a}}{21b^3}+\frac{a\left(b(33ad-28cb)\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+12a^2d\right.}{}$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*a*x*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+1/7*d*x^5*(b*x^4+a)^(1/2)/b^2+1
/3*(-1/b^2*(a*d-b*c)-5/7/b^2*d*a)/b*x*(b*x^4+a)^(1/2)+(1/2*a*(a*d-b*c)/b^3
-1/3*(-1/b^2*(a*d-b*c)-5/7/b^2*d*a)/b*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.72

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{5\left((7\,b^2c-9\,abd)x^4+7\,abc-9\,a^2d\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-\left(6\,b^2dx^9+2\,(7\,b^2c-9\,abd)x^5\right.}{42\,(b^4x^4+ab^3)}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/42*(5*((7*b^2*c - 9*a*b*d)*x^4 + 7*a*b*c - 9*a^2*d)*sqrt(b)*(-a/b)^(3/4
)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (6*b^2*d*x^9 + 2*(7*b^2*c - 9*a
*b*d)*x^5 + 5*(7*a*b*c - 9*a^2*d)*x)*sqrt(b*x^4 + a))/(b^4*x^4 + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}+\frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{-45\sqrt{b\,x^4+a}\,a^2dx+35\sqrt{b\,x^4+a}\,abcx-9\sqrt{b\,x^4+a}\,abd\,x^5+7\sqrt{b\,x^4+a}\,b^2c\,x^5+3\sqrt{}}{}$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - 45*sqrt(a + b*x**4)*a**2*d*x + 35*sqrt(a + b*x**4)*a*b*c*x - 9*sqrt(a
+ b*x**4)*a*b*d*x**5 + 7*sqrt(a + b*x**4)*b**2*c*x**5 + 3*sqrt(a + b*x**4)
*b**2*d*x**9 + 45*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*
a**4*d - 35*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b
*c + 45*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*b*d*x
**4 - 35*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2
*c*x**4)/(21*b**3*(a + b*x**4))
```

3.36 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = -\frac{(bc-ad)x}{2b^2\sqrt{a+bx^4}} + \frac{dx\sqrt{a+bx^4}}{3b^2} + \frac{(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}b^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/2*(-a*d+b*c)*x/b^2/(b*x^4+a)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)/b^2+1/12*(-5
*a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2
)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(9/4)
/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{x\left(-3bc+5ad+2bdx^4+(3bc-5ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{6b^2\sqrt{a+bx^4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(x*(-3*b*c + 5*a*d + 2*b*d*x^4 + (3*b*c - 5*a*d)*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(6*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3bc-5ad)\int \frac{x^4}{(bx^4+a)^{3/2}}dx}{3b}+\frac{dx^5}{3b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(3bc-5ad)\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{3b}+\frac{dx^5}{3b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3bc-5ad)\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{3b}+\frac{dx^5}{3b\sqrt{a+bx^4}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(d*x^5)/(3*b*Sqrt[a + b*x^4]) + ((3*b*c - 5*a*d)*(-1/2*x/(b*Sqrt[a + b*x^4
]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*
EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(5/4)*Sqrt[a +
 b*x^4])))/(3*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

method	result
elliptic	$\frac{(ad-cb)x}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{dx\sqrt{b\,x^4+a}}{3b^2}+\frac{\left(-\frac{ad-cb}{2b^2}-\frac{da}{3b^2}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{ax}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}}{}}}{}\right.$
risch	$\frac{dx\sqrt{b\,x^4+a}}{3b^2}-\frac{a^2d\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+b(4ad-3cb)\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1-}}{2b}\right.}{3b^2}$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(a*d-b*c)*x/((x^4+a/b)*b)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)/b^2+(-1/2/
b^2*(a*d-b*c)-1/3/b^2*d*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.76

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx=\frac{((3\,b^2c-5\,abd)x^4+3\,abc-5\,a^2d)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)+(2\,abdx^5-(3}{6\,(ab^3x^4+a^2b^2)}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(((3*b^2*c - 5*a*b*d)*x^4 + 3*a*b*c - 5*a^2*d)*sqrt(b)*(-a/b)^(3/4)*el
liptic_f(arcsin((-a/b)^(1/4)/x), -1) + (2*a*b*d*x^5 - (3*a*b*c - 5*a^2*d)*
x)*sqrt(b*x^4 + a))/(a*b^3*x^4 + a^2*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.55

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/2)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{5\sqrt{b\,x^4+a}\,adx - 3\sqrt{b\,x^4+a}\,bcx + \sqrt{b\,x^4+a}\,bd\,x^5 - 5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^3d + 3\Big(}{3b^2\,(b}$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
(5*sqrt(a + b*x**4)*a*d*x - 3*sqrt(a + b*x**4)*b*c*x + sqrt(a + b*x**4)*b*
d*x**5 - 5*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*d
+ 3*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b*c - 5*i
nt(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b*d*x**4 + 3*i
nt(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b**2*c*x**4)/(3*b
**2*(a + b*x**4))
```

3.37 $\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx=\frac{(bc-ad)x}{2ab\sqrt{a+bx^4}}+\frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-a*d+b*c)*x/a/b/(b*x^4+a)^(1/2)+1/4*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*(
(b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*
x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx=\frac{x\left(bc-ad+(bc+ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{2ab\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
(x*(b*c - a*d + (b*c + a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2
, 5/4, -((b*x^4)/a)]))/(2*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {910, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+bc)\int \frac{1}{\sqrt{bx^4+a}}dx}{2ab}+\frac{x(bc-ad)}{2ab\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}(ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}b^{5/4}\sqrt{a+bx^4}}+\frac{x(bc-ad)}{2ab\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(3/2),x]
```

output
```
((b*c - a*d)*x)/(2*a*b*Sqrt[a + b*x^4]) + ((b*c + a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

method	result
elliptic	$-\frac{x(ad-cb)}{2ba\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\left(\frac{d}{b}-\frac{ad-cb}{2ab}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{E}}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/a*x*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+(d/b-1/2*(a*d-b*c)/a/b)/(I/a^(1/2
)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = -\frac{((b^2c+abd)x^4+abc+a^2d)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)-\sqrt{bx^4+a}(b^2c-abd)x}{2\left(ab^3x^4+a^2b^2\right)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((b^2*c + a*b*d)*x^4 + a*b*c + a^2*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_
f(arcsin(x*(-b/a)^(1/4)), -1) - sqrt(b*x^4 + a)*(b^2*c - a*b*d)*x)/(a*b^3*
x^4 + a^2*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(3/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,dx + \left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2d + \left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)abc + \left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4-}\right.}{b\,(b\,x^4+a)}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d*x + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*
x**8),x)*a**2*d + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*
a*b*c + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b*d*x**4
 + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*b**2*c*x**4)/(b
*(a + b*x**4))
```

3.38 $\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx = -\frac{c}{3ax^3\sqrt{a+bx^4}}-\frac{(5bc-3ad)x}{6a^2\sqrt{a+bx^4}} -\frac{(5bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(1/2)-1/6*(-3*a*d+5*b*c)*x/a^2/(b*x^4+a)^(1/2)-1/12
*(-3*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4)/b^(
1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.58

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx = \frac{-2ac-5bcx^4+3adx^4+(-5bc+3ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\right.}{6a^2x^3\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
(-2*a*c - 5*b*c*x^4 + 3*a*d*x^4 + (-5*b*c + 3*a*d)*x^4*Sqrt[1 + (b*x^4)/a]
*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(6*a^2*x^3*Sqrt[a + b*x^4
])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5bc-3ad)\int \frac{1}{(bx^4+a)^{3/2}}dx}{3a}-\frac{c}{3ax^3\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(5bc-3ad)\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{3a}-\frac{c}{3ax^3\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(5bc-3ad)\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{3a}-\frac{c}{3ax^3\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
-1/3*c/(a*x^3*Sqrt[a + b*x^4]) - ((5*b*c - 3*a*d)*(x/(2*a*Sqrt[a + b*x^4])
 + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b
*x^4])))/(3*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

method	result
elliptic	$\frac{x(ad-cb)}{2a^2\sqrt{(x^4+\frac{a}{b})b}}-\frac{c\sqrt{b\,x^4+a}}{3a^2x^3}+\frac{\left(\frac{ad-cb}{2a^2}-\frac{bc}{3a^2}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$d\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{bx}{2a^2\sqrt{(x^4+\frac{a}{b})b}}-\frac{\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{5b\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}}{}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{3a^2x^3}-\frac{c\,b^2\left(-\frac{x}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-3a^2d\left(\frac{x}{2a\sqrt{(x^4+\frac{a}{b})b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x}{\sqrt{a}}}}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}\right.}{3a^2}$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*x*(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/3/a^2*c*(b*x^4+a)^(1/2)/x^3+(1/2
/a^2*(a*d-b*c)-1/3*b/a^2*c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\frac{((5\,b^2c-3\,abd)x^7+(5\,abc-3\,a^2d)x^3)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-((5\,b}{6\,(a^2b^2x^7+a^3bx^3)}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(((5*b^2*c - 3*a*b*d)*x^7 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(a)*(-b/a)^(3
/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - ((5*b^2*c - 3*a*b*d)*x^4 + 2*
a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^7 + a^3*b*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(3/2),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{d\,x^4+c}{x^4\,{\left(b\,x^4+a\right)}^{3/2}}\,dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-3\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{12}+2ab\,x^8+a^2x^4}dx\right)a^2d\,x^3+5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{12}+2ab\,x^8+a^2x^4}dx\right)abc\,x^3-}{5b\,x^3\left(b\,x^4+a\right)}$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 3*int(sqrt(a + b*x**4)/(a**2*x**4 + 2*a*b*x**8 +
b**2*x**12),x)*a**2*d*x**3 + 5*int(sqrt(a + b*x**4)/(a**2*x**4 + 2*a*b*x**
8 + b**2*x**12),x)*a*b*c*x**3 - 3*int(sqrt(a + b*x**4)/(a**2*x**4 + 2*a*b*
x**8 + b**2*x**12),x)*a*b*d*x**7 + 5*int(sqrt(a + b*x**4)/(a**2*x**4 + 2*a
*b*x**8 + b**2*x**12),x)*b**2*c*x**7)/(5*b*x**3*(a + b*x**4))
```

3.39 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 180

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/2}}\,dx = -\frac{c}{7ax^7\sqrt{a+bx^4}}-\frac{9bc-7ad}{14a^2x^3\sqrt{a+bx^4}}+\frac{5(9bc-7ad)\sqrt{a+bx^4}}{42a^3x^3}+\frac{5b^{3/4}(9bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{84a^{13/4}\sqrt{a+bx^4}}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(1/2)-1/14*(-7*a*d+9*b*c)/a^2/x^3/(b*x^4+a)^(1/2)+5
/42*(-7*a*d+9*b*c)*(b*x^4+a)^(1/2)/a^3/x^3+5/84*b^(3/4)*(-7*a*d+9*b*c)*(a^
(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.40

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{3/2}}\,dx = \frac{-3ac+(9bc-7ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{21a^2x^7\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(3/2)),x]
```

output
```
(-3*a*c + (9*b*c - 7*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4,
3/2, 1/4, -((b*x^4)/a)])/(21*a^2*x^7*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(9bc-7ad)\int \frac{1}{x^4\left(bx^4+a\right)^{3/2}}dx}{7a}-\frac{c}{7ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(9bc-7ad)\left(\frac{5\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(9bc-7ad)\left(\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{(9bc-7ad)\left(\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(3/2)),x]
```

output
```
-1/7*c/(a*x^7*Sqrt[a + b*x^4]) - ((9*b*c - 7*a*d)*(1/(2*a*x^3*Sqrt[a + b*x
^4]) + (5*(-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/(2*a)))/(7*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

method	result
elliptic	$-\frac{bx(ad-cb)}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{7a^2x^7}-\frac{(7ad-12cb)\sqrt{b\,x^4+a}}{21a^3x^3}+\frac{\left(-\frac{b(ad-cb)}{2a^3}-\frac{b(7ad-12cb)}{21a^3}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{b^2x}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{7a^2x^7}+\frac{4b\sqrt{b\,x^4+a}}{7a^3x^3}+\frac{15b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{14a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{bx}{2a^2\sqrt{(x^4\cdots}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(7ad\,x^4-12bc\,x^4+3ac\right)}{21a^3x^7}-\frac{b\left(b(7ad-12cb)\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+28a^2d\left(\cdots\right.\right.}{\cdots}$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/a^3*x*(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/7/a^2*c*(b*x^4+a)^(1/2)/x^7-1
/21/a^3*(7*a*d-12*b*c)*(b*x^4+a)^(1/2)/x^3+(-1/2*b/a^3*(a*d-b*c)-1/21*b/a^
3*(7*a*d-12*b*c))/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/2}}\,dx = -\frac{5\left((9\,b^2c-7\,abd)x^{11}+(9\,abc-7\,a^2d)x^7\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-(5\,(9\,b^2c-7\,abd)x^8+2}{42\,(a^3bx^{11}+a^4x^7)}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/42*(5*((9*b^2*c - 7*a*b*d)*x^11 + (9*a*b*c - 7*a^2*d)*x^7)*sqrt(a)*(-b/
a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (5*(9*b^2*c - 7*a*b*d)*x
^8 + 2*(9*a*b*c - 7*a^2*d)*x^4 - 6*a^2*c)*sqrt(b*x^4 + a))/(a^3*b*x^11 + a
^4*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.50

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{7}{4},\frac{3}{2}\\-\frac{3}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{d\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{3}{2}\\\frac{1}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(3/2),x)
```

output
```
c*gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*x**7*gamma(-3/4)) + d*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{c + dx^4}{x^8 (a + bx^4)^{3/2}}\, dx = \int \frac{dx^4 + c}{(bx^4 + a)^{\frac{3}{2}} x^8}\, dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{c + dx^4}{x^8 (a + bx^4)^{3/2}}\, dx = \int \frac{dx^4 + c}{(bx^4 + a)^{\frac{3}{2}} x^8}\, dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^4}{x^8 (a + bx^4)^{3/2}}\, dx = \int \frac{d\, x^4 + c}{x^8\, (b\, x^4 + a)^{3/2}}\, dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-7\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{16}+2ab\,x^{12}+a^2x^8}dx\right)a^2d\,x^7+9\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{16}+2ab\,x^{12}+a^2x^8}dx\right)abc\,x^7}{9b\,x^7\left(b\,x^4+a\right.}$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 7*int(sqrt(a + b*x**4)/(a**2*x**8 + 2*a*b*x**12 +
 b**2*x**16),x)*a**2*d*x**7 + 9*int(sqrt(a + b*x**4)/(a**2*x**8 + 2*a*b*x*
*12 + b**2*x**16),x)*a*b*c*x**7 - 7*int(sqrt(a + b*x**4)/(a**2*x**8 + 2*a*
b*x**12 + b**2*x**16),x)*a*b*d*x**11 + 9*int(sqrt(a + b*x**4)/(a**2*x**8 +
 2*a*b*x**12 + b**2*x**16),x)*b**2*c*x**11)/(9*b*x**7*(a + b*x**4))
```

3.40 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 324

$$\begin{aligned}\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx &= -\frac{(bc-ad)x^7}{2b^2\sqrt{a+bx^4}}+\frac{7(9bc-11ad)x^3\sqrt{a+bx^4}}{90b^3}\\ &+\frac{dx^7\sqrt{a+bx^4}}{9b^2}-\frac{7a(9bc-11ad)x\sqrt{a+bx^4}}{30b^{7/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\ &+\frac{7a^{5/4}(9bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{a+bx^4}}\\ &-\frac{7a^{5/4}(9bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{60b^{15/4}\sqrt{a+bx^4}}\end{aligned}$$

output

```
-1/2*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(1/2)+7/90*(-11*a*d+9*b*c)*x^3*(b*x^4+a)
^(1/2)/b^3+1/9*d*x^7*(b*x^4+a)^(1/2)/b^2-7/30*a*(-11*a*d+9*b*c)*x*(b*x^4+a
)^(1/2)/b^(7/2)/(a^(1/2)+b^(1/2)*x^2)+7/30*a^(5/4)*(-11*a*d+9*b*c)*(a^(1/2
)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*a
rctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^4+a)^(1/2)-7/60*a^(5/
4)*(-11*a*d+9*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(15/4)
/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.32

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\left(77a^2d+b^2x^4(9c+5dx^4)-ab(63c+11dx^4)+7a(9bc-11ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hyperge}\right.}{45b^3\sqrt{a+bx^4}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*(77*a^2*d + b^2*x^4*(9*c + 5*d*x^4) - a*b*(63*c + 11*d*x^4) + 7*a*(9*
b*c - 11*a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^
4)/a)]))/(45*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 817, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(9bc-11ad)\int \frac{x^{10}}{(bx^4+a)^{3/2}}dx}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9bc-11ad)\left(\frac{7\int \frac{x^6}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{(9bc-11ad)\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(9bc-11ad)\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(9bc-11ad)\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(9bc-11ad)\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{9b}+\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

$\downarrow$ 1510

$$\frac{(9bc-11ad)\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\ldots\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\ldots\right)}{5b}\ldots\right)}{2b}\ldots\right)}{9b}\ldots$$

$$\frac{dx^{11}}{9b\sqrt{a+bx^4}}$$

input
```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(d*x^11)/(9*b*Sqrt[a + b*x^4]) + ((9*b*c - 11*a*d)*(-1/2*x^7/(b*Sqrt[a + b
*x^4]) + (7*((x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-((-((x*Sqrt[a + b*x^4])/
(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)))/(2*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.84 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.70

method	result
elliptic	$-\frac{a x^3(ad-cb)}{2b^3\sqrt{(x^4+\frac{a}{b})b}}+\frac{d x^7\sqrt{b x^4+a}}{9b^2}+\frac{\left(-\frac{ad-cb}{b^2}-\frac{7da}{9b^2}\right)x^3\sqrt{b x^4+a}}{5b}+\frac{i\left(\frac{3a(ad-cb)}{2b^3}-\frac{3\left(-\frac{ad-cb}{b^2}-\frac{7da}{9b^2}\right)a}{5b}\right)\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\cdots}}$
default	$c\left(\frac{a x^3}{2b^2\sqrt{(x^4+\frac{a}{b})b}}+\frac{x^3\sqrt{b x^4+a}}{5b^2}-\frac{21 i a^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{10b^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\right)+d\left(\cdots\right.$
risch	$-\frac{x^3(-5db x^4+16ad-9cb)\sqrt{b x^4+a}}{45b^3}+\frac{a\left(b(31ad-24cb)\left(-\frac{x^3}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{3i\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{Elli}\cdots\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b x^4+a}}\cdots\right.\right.}{\cdots}$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*a*x^3*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+1/9*d*x^7*(b*x^4+a)^(1/2)/b^2
+1/5*(-1/b^2*(a*d-b*c)-7/9/b^2*d*a)/b*x^3*(b*x^4+a)^(1/2)+I*(3/2*a*(a*d-b*
c)/b^3-3/5*(-1/b^2*(a*d-b*c)-7/9/b^2*d*a)/b*a)*a^(1/2)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x
*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.68

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = -\frac{21\left((9\,ab^2c-11\,a^2bd)x^5+(9\,a^2bc-11\,a^3d)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-21\left((9\,ab^2c-11\,a^2bd\right.}{}$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/90*(21*((9*a*b^2*c - 11*a^2*b*d)*x^5 + (9*a^2*b*c - 11*a^3*d)*x)*sqrt(b
)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) - 21*((9*a*b^2*c - 1
1*a^2*b*d)*x^5 + (9*a^2*b*c - 11*a^3*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f
(arcsin((-a/b)^(1/4)/x), -1) - (10*b^3*d*x^12 + 2*(9*b^3*c - 11*a*b^2*d)*x
^8 - 14*(9*a*b^2*c - 11*a^2*b*d)*x^4 - 189*a^2*b*c + 231*a^3*d)*sqrt(b*x^4
 + a))/(b^5*x^5 + a*b^4*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.25

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}+\frac{dx^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(3/2)*gamma(15/4)) + d*x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx=\int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx=\int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{77\sqrt{b\,x^4+a}\,a^2d\,x^3-63\sqrt{b\,x^4+a}\,abc\,x^3-11\sqrt{b\,x^4+a}\,abd\,x^7+9\sqrt{b\,x^4+a}\,b^2c\,x^7+}{}$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
(77*sqrt(a + b*x**4)*a**2*d*x**3 - 63*sqrt(a + b*x**4)*a*b*c*x**3 - 11*sqr
t(a + b*x**4)*a*b*d*x**7 + 9*sqrt(a + b*x**4)*b**2*c*x**7 + 5*sqrt(a + b*x
**4)*b**2*d*x**11 - 231*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b
**2*x**8),x)*a**4*d + 189*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
 b**2*x**8),x)*a**3*b*c - 231*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x*
*4 + b**2*x**8),x)*a**3*b*d*x**4 + 189*int((sqrt(a + b*x**4)*x**2)/(a**2 +
 2*a*b*x**4 + b**2*x**8),x)*a**2*b**2*c*x**4)/(45*b**3*(a + b*x**4))
```

3.41 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = -\frac{(bc-ad)x^3}{2b^2\sqrt{a+bx^4}}+\frac{dx^3\sqrt{a+bx^4}}{5b^2}+\frac{3(5bc-7ad)x\sqrt{a+bx^4}}{10b^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$-\frac{3\sqrt[4]{a}(5bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^4}}$$
$$+\frac{3\sqrt[4]{a}(5bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{20b^{11/4}\sqrt{a+bx^4}}$$

output
```
-1/2*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(1/2)+1/5*d*x^3*(b*x^4+a)^(1/2)/b^2+3/10
*(-7*a*d+5*b*c)*x*(b*x^4+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x^2)-3/10*a^(1/
4)*(-7*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(11/4)/(
b*x^4+a)^(1/2)+3/20*a^(1/4)*(-7*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4
)),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\left(5bc-7ad+bdx^4+(-5bc+7ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{5b^2\sqrt{a+bx^4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*(5*b*c - 7*a*d + b*d*x^4 + (-5*b*c + 7*a*d)*Sqrt[1 + (b*x^4)/a]*Hyper
geometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(5*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(5bc-7ad)\int \frac{x^6}{(bx^4+a)^{3/2}}dx}{5b}+\frac{dx^7}{5b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(5bc-7ad)\left(\frac{3\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{5b}+\frac{dx^7}{5b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{(5bc-7ad)\left(\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{5b}+\frac{dx^7}{5b\sqrt{a+bx^4}}$$

$\downarrow$ 27

$$\frac{(5bc-7ad)\left(\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{5b}+\frac{dx^7}{5b\sqrt{a+bx^4}}$$

$\downarrow$ 761

$$\frac{(5bc-7ad)\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{5b}+\frac{dx^7}{5b\sqrt{a+bx^4}}$$

$\downarrow$ 1510

$$\frac{(5bc-7ad)\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\cdots}{\sqrt{b}}\cdots\right)}{2b}\cdots\right)}{5b}$$

$$\frac{dx^7}{5b\sqrt{a+bx^4}}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(d*x^7)/(5*b*Sqrt[a + b*x^4]) + ((5*b*c - 7*a*d)*(-1/2*x^3/(b*Sqrt[a + b*x
^4]) + (3*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
E[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b])
 + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^
2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b
*x^4])))/(2*b)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
elliptic	$\frac{x^3(ad-cb)}{2b^2\sqrt{(x^4+\frac{a}{b})b}}+\frac{d\,x^3\sqrt{b\,x^4+a}}{5b^2}+\frac{i\left(-\frac{3(ad-cb)}{2b^2}-\frac{3da}{5b^2}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$c\left(-\frac{x^3}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{a\,x^3}{2b^2\sqrt{(x^4+\frac{a}{b}}}\right.$
risch	$\frac{d\,x^3\sqrt{b\,x^4+a}}{5b^2}-\frac{b(8ad-5cb)\left(-\frac{x^3}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+3a^2d\left(\right.}{5b^2}$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*x^3*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+1/5*d*x^3*(b*x^4+a)^(1/2)/b^2+I*
(-3/2/b^2*(a*d-b*c)-3/5/b^2*d*a)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^
(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^
(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.63

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{3\left((5\,b^2c-7\,abd)x^5+(5\,abc-7\,a^2d)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-3\left((5\,b^2c\right.}{}$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(3*((5*b^2*c - 7*a*b*d)*x^5 + (5*a*b*c - 7*a^2*d)*x)*sqrt(b)*(-a/b)^(
3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) - 3*((5*b^2*c - 7*a*b*d)*x^5 +
 (5*a*b*c - 7*a^2*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4
)/x), -1) + (2*b^2*d*x^8 + 2*(5*b^2*c - 7*a*b*d)*x^4 + 15*a*b*c - 21*a^2*d
)*sqrt(b*x^4 + a))/(b^4*x^5 + a*b^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.45 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}+\frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(3/2)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{-7\sqrt{b\,x^4+a}\,ad\,x^3+5\sqrt{b\,x^4+a}\,bc\,x^3+\sqrt{b\,x^4+a}\,bd\,x^7+21\left(\int\frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)a^3d}{5}$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
( - 7*sqrt(a + b*x**4)*a*d*x**3 + 5*sqrt(a + b*x**4)*b*c*x**3 + sqrt(a + b
*x**4)*b*d*x**7 + 21*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2
*x**8),x)*a**3*d - 15*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**
2*x**8),x)*a**2*b*c + 21*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
b**2*x**8),x)*a**2*b*d*x**4 - 15*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b
*x**4 + b**2*x**8),x)*a*b**2*c*x**4)/(5*b**2*(a + b*x**4))
```

3.42 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{(bc-ad)x^3}{2ab\sqrt{a+bx^4}} - \frac{(bc-3ad)x\sqrt{a+bx^4}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^4}} - \frac{(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^4}}$$

output
```
1/2*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(1/2)-1/2*(-3*a*d+b*c)*x*(b*x^4+a)^(1/2)/
a/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)+1/2*(-3*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b
*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))/a^(3/4)/b^(7/4)/(b*x^4+a)^(1/2)-1/4*(-3*a*d+b*c)*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.26

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\left(3ad+(bc-3ad)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{3ab\sqrt{a+bx^4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*(3*a*d + (b*c - 3*a*d)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2
, 7/4, -((b*x^4)/a)]))/(3*a*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^3(bc-ad)}{2ab\sqrt{a+bx^4}}-\frac{(bc-3ad)\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2ab}$$

$$\Big\downarrow \text{834}$$

$$\frac{x^3(bc-ad)}{2ab\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(bc-ad)}{2ab\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2ab}$$

$$\Big\downarrow \; 761$$

$$\frac{x^3(bc-ad)}{2ab\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2ab}$$

$$\Big\downarrow \; 1510$$

$$\frac{x^3(bc-ad)}{2ab\sqrt{a+bx^4}}-\frac{(bc-3ad)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right)}{2ab}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(3/2),x]
```

output
```
((b*c - a*d)*x^3)/(2*a*b*Sqrt[a + b*x^4]) - ((b*c - 3*a*d)*(-((-((x*Sqrt[a
 + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^
(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.55

method	result
elliptic	$-\frac{x^3(ad-cb)}{2ba\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{i\left(\frac{d}{b}+\frac{ad-cb}{2ab}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$c\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)+d\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\right.$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/a*x^3*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+I*(d/b+1/2*(a*d-b*c)/a/b)*a^(1/
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.61

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx =$$

$$-\frac{((b^2c-3\,abd)x^5+(abc-3\,a^2d)x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-((b^2c-3\,abd)x^5+(abc-3\,a^2d)}{2\,(ab^3x^5+a^2b^2x)}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((b^2*c - 3*a*b*d)*x^5 + (a*b*c - 3*a^2*d)*x)*sqrt(b)*(-a/b)^(3/4)*e
lliptic_e(arcsin((-a/b)^(1/4)/x), -1) - ((b^2*c - 3*a*b*d)*x^5 + (a*b*c -
3*a^2*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) -
(2*a*b*d*x^4 - a*b*c + 3*a^2*d)*sqrt(b*x^4 + a))/(a*b^3*x^5 + a^2*b^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(3/2),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/2)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(3/2),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{b\,x^4+a}\,d\,x^3 - 3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2d + \left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)abc - 3\left(\int \frac{\sqrt{b\,x^4+c}}{b^2x^8+2ab}\right.}{b\,(b\,x^4+a)}$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*d*x**3 - 3*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**
4 + b**2*x**8),x)*a**2*d + int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4
+ b**2*x**8),x)*a*b*c - 3*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
 b**2*x**8),x)*a*b*d*x**4 + int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4
 + b**2*x**8),x)*b**2*c*x**4)/(b*(a + b*x**4))
```

3.43 $\int \frac{c+dx^4}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 295

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/2}}\,dx = -\frac{c}{ax\sqrt{a+bx^4}} - \frac{(3bc-ad)x^3}{2a^2\sqrt{a+bx^4}} + \frac{(3bc-ad)x\sqrt{a+bx^4}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^4}} + \frac{(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
-c/a/x/(b*x^4+a)^(1/2)-1/2*(-a*d+3*b*c)*x^3/a^2/(b*x^4+a)^(1/2)+1/2*(-a*d+
3*b*c)*x*(b*x^4+a)^(1/2)/a^2/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-1/2*(-a*d+3*b*c
)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Elliptic
E(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/b^(3/4)/(b*x^4+a)^
(1/2)+1/4*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7
/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.24

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{3/2}}\,dx = \frac{-3ac+(-3bc+ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^2x\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
(-3*a*c + (-3*b*c + a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/
2, 7/4, -((b*x^4)/a)])/(3*a^2*x*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(3bc-ad)\int \frac{x^2}{(bx^4+a)^{3/2}}dx}{a}-\frac{c}{ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(3bc-ad)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 834$$

$$-\frac{(3bc-ad)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$-\frac{(3bc-ad)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$-\frac{(3bc-ad)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 1510$$

$$-\frac{(3bc-ad)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}}{2a}\cdots\right)}{a}\cdots\frac{c}{ax\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
-(c/(a*x*Sqrt[a + b*x^4])) - ((3*b*c - a*d)*(x^3/(2*a*Sqrt[a + b*x^4]) - (
-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4]))/(2*
a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.56

method	result
elliptic	$\frac{x^3(ad-cb)}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{a^2x}+\frac{i\left(-\frac{ad-cb}{2a^2}+\frac{bc}{a^2}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$c\left(-\frac{b\,x^3}{2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{a^2x}+\frac{3i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\Bigg($
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^2x}+\frac{a^2d\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)+c\,b^2\left(-\frac{x^3}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}\right.}{a^2}$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*x^3*(a*d-b*c)/((x^4+a/b)*b)^(1/2)-1/a^2*c*(b*x^4+a)^(1/2)/x+I*(-1/
2/a^2*(a*d-b*c)+b/a^2*c)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(
EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(
1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.57

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/2}}\,dx =$$
$$-\frac{((3\,b^2c-abd)x^5+(3\,abc-a^2d)x)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-((3\,b^2c-abd)x^5+(3\,abc-a^2 d}{2\left(a^2b^2x^5+a^3bx\right)}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(((3*b^2*c - a*b*d)*x^5 + (3*a*b*c - a^2*d)*x)*sqrt(a)*(-b/a)^(3/4)*e
lliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - ((3*b^2*c - a*b*d)*x^5 + (3*a*b*c
- a^2*d)*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) +
((3*b^2*c - a*b*d)*x^4 + 2*a*b*c)*sqrt(b*x^4 + a))/(a^2*b^2*x^5 + a^3*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(3/2),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/2}}\,dx = \int \frac{d\,x^4+c}{x^2\,(b\,x^4+a)^{3/2}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{10}+2ab\,x^6+a^2x^2}dx\right)a^2dx+3\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{10}+2ab\,x^6+a^2x^2}dx\right)abcx-\left(\int\right.}{3bx\left(b\,x^4+a\right)}$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b*
*2*x**10),x)*a**2*d*x + 3*int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b
**2*x**10),x)*a*b*c*x - int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**
2*x**10),x)*a*b*d*x**5 + 3*int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 +
b**2*x**10),x)*b**2*c*x**5)/(3*b*x*(a + b*x**4))
```

3.44 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/2}}\,dx =& -\frac{c}{5ax^5\sqrt{a+bx^4}}-\frac{7bc-5ad}{10a^2x\sqrt{a+bx^4}}\\&+\frac{3(7bc-5ad)\sqrt{a+bx^4}}{10a^3x}-\frac{3\sqrt{b}(7bc-5ad)x\sqrt{a+bx^4}}{10a^3\left(\sqrt{a}+\sqrt{b}x^2\right)}\\&+\frac{3\sqrt[4]{b}(7bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{a+bx^4}}\\&-\frac{3\sqrt[4]{b}(7bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{20a^{11/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(1/2)-1/10*(-5*a*d+7*b*c)/a^2/x/(b*x^4+a)^(1/2)+3/1
0*(-5*a*d+7*b*c)*(b*x^4+a)^(1/2)/a^3/x-3/10*b^(1/2)*(-5*a*d+7*b*c)*x*(b*x^
4+a)^(1/2)/a^3/(a^(1/2)+b^(1/2)*x^2)+3/10*b^(1/4)*(-5*a*d+7*b*c)*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(11/4)/(b*x^4+a)^(1/2)-3/20*b^(1/4)
*(-5*a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(11/4)/(b
*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/2}}\,dx = \frac{-ac+(7bc-5ad)x^4\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^2x^5\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(3/2)),x]
```

output
```
(-(a*c) + (7*b*c - 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4,
3/2, 3/4, -((b*x^4)/a)])/(5*a^2*x^5*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(7bc-5ad)\int \frac{1}{x^2(bx^4+a)^{3/2}}dx}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(7bc-5ad)\left(\frac{3\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 847$$

$$-\frac{(7bc-5ad)\left(\frac{3\left(\frac{b\int\frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$\big\downarrow$ 834

$$-\frac{(7bc-5ad)\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$\big\downarrow$ 27

$$-\frac{(7bc-5ad)\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$\big\downarrow$ 761

$$-\frac{(7bc-5ad)\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^4}}$$

$\downarrow$ 1510

$$-\frac{(7bc-5ad)\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right.}{a}\right.}{2a}\right.}{5a}$$

$$\frac{c}{5ax^5\sqrt{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(3/2)),x]
```

output
```
-1/5*c/(a*x^5*Sqrt[a + b*x^4]) - ((7*b*c - 5*a*d)*(1/(2*a*x*Sqrt[a + b*x^4
]) + (3*(-(Sqrt[a + b*x^4]/(a*x)) + (b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a]
+ Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/
4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a))/(2*a)))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.62

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a^2x^5}-\frac{(5ad-8cb)\sqrt{b\,x^4+a}}{5a^3x}-\frac{b\,x^3(ad-cb)}{2a^3\sqrt{(x^4+\frac{a}{b})b}}+\frac{i\left(\frac{b(5ad-8cb)}{5a^3}+\frac{b(ad-cb)}{2a^3}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,(\text{EllipticF}(x\sqrt{}\cdots}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(5ad\,x^4-8bc\,x^4+ac)}{5a^3x^5}+\frac{b^2\left((5ad-8cb)\left(-\frac{x^3}{2b\sqrt{(x^4+\frac{a}{b})b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}(\cdots\right.}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.\right.\cdots}{\cdots}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5a^2x^5}+\frac{8b\sqrt{b\,x^4+a}}{5a^3x}+\frac{b^2x^3}{2a^3\sqrt{(x^4+\frac{a}{b})b}}-\frac{21ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}(x\sqrt{\cdots}\right.}{10a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.\cdots$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^2*c*(b*x^4+a)^(1/2)/x^5-1/5/a^3*(5*a*d-8*b*c)*(b*x^4+a)^(1/2)/x-1/2
*b/a^3*x^3*(a*d-b*c)/((x^4+a/b)*b)^(1/2)+I*(1/5*b/a^3*(5*a*d-8*b*c)+1/2*b/
a^3*(a*d-b*c))*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.58

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/2}}\,dx = \frac{3\left((7\,b^2c-5\,abd)x^9+(7\,abc-5\,a^2d)x^5\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-3\,(\cdots}{\cdots}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(3*((7*b^2*c - 5*a*b*d)*x^9 + (7*a*b*c - 5*a^2*d)*x^5)*sqrt(a)*(-b/a)
^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 3*((7*b^2*c - 5*a*b*d)*x^9
 + (7*a*b*c - 5*a^2*d)*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a
)^(1/4)), -1) + (3*(7*b^2*c - 5*a*b*d)*x^8 + 2*(7*a*b*c - 5*a^2*d)*x^4 - 2
*a^2*c)*sqrt(b*x^4 + a))/(a^3*b*x^9 + a^4*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.27

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(3/2),x)
```

output
```
c*gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*x**5*gamma(-1/4)) + d*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/2)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/2}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/2}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/2}} \, dx = \int \frac{d\,x^4+c}{x^6\,(b\,x^4+a)^{3/2}} \, dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(3/2)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6\,(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{14}+2ab\,x^{10}+a^2x^6}dx\right)a^2d\,x^5+7\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^{14}+2ab\,x^{10}+a^2x^6}dx\right)abc\,x^5}{7b\,x^5\,(b\,x^4+a)}$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 5*int(sqrt(a + b*x**4)/(a**2*x**6 + 2*a*b*x**10 +
 b**2*x**14),x)*a**2*d*x**5 + 7*int(sqrt(a + b*x**4)/(a**2*x**6 + 2*a*b*x*
*10 + b**2*x**14),x)*a*b*c*x**5 - 5*int(sqrt(a + b*x**4)/(a**2*x**6 + 2*a*
b*x**10 + b**2*x**14),x)*a*b*d*x**9 + 7*int(sqrt(a + b*x**4)/(a**2*x**6 +
2*a*b*x**10 + b**2*x**14),x)*b**2*c*x**9)/(7*b*x**5*(a + b*x**4))
```

3.45 $\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = -\frac{a^2(bc-ad)x}{6b^4(a+bx^4)^{3/2}} + \frac{a(13bc-19ad)x}{12b^4\sqrt{a+bx^4}} + \frac{(7bc-19ad)x\sqrt{a+bx^4}}{21b^4} + \frac{dx^5\sqrt{a+bx^4}}{7b^3} - \frac{5a^{3/4}(7bc-13ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{56b^{17/4}\sqrt{a+bx^4}}$$

output
```
-1/6*a^2*(-a*d+b*c)*x/b^4/(b*x^4+a)^(3/2)+1/12*a*(-19*a*d+13*b*c)*x/b^4/(b
*x^4+a)^(1/2)+1/21*(-19*a*d+7*b*c)*x*(b*x^4+a)^(1/2)/b^4+1/7*d*x^5*(b*x^4+
a)^(1/2)/b^3-5/56*a^(3/4)*(-13*a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)
/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)
),1/2*2^(1/2))/b^(17/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.64

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-195a^3dx+ab^2x^5(147c-52dx^4)+21a^2bx(5c-13dx^4)+4b^3x^9(7c+3dx^4)+15a(\cdots}{84b^4(a+bx^4)^{3/\cdots}}$$

input
```
Integrate[(x^12*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(-195*a^3*d*x + a*b^2*x^5*(147*c - 52*d*x^4) + 21*a^2*b*x*(5*c - 13*d*x^4)
 + 4*b^3*x^9*(7*c + 3*d*x^4) + 15*a*(-7*b*c + 13*a*d)*x*(a + b*x^4)*Sqrt[1
 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(84*b^4*(a +
 b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 817, 817, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7bc-13ad)\int \frac{x^{12}}{(bx^4+a)^{5/2}}dx}{7b}+\frac{dx^{13}}{7b(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-13ad)\left(\frac{3\int \frac{x^8}{(bx^4+a)^{3/2}}dx}{2b}-\frac{x^9}{6b(a+bx^4)^{3/2}}\right)}{7b}+\frac{dx^{13}}{7b(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-13ad)\left(\frac{3\left(\frac{5\int \frac{x^4}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b(a+bx^4)^{3/2}}\right)}{7b}+\frac{dx^{13}}{7b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{(7bc-13ad)\left(\frac{3\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b(a+bx^4)^{3/2}}\right)}{7b}+\frac{dx^{13}}{7b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bc-13ad)\left(\frac{3\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b(a+bx^4)^{3/2}}\right)}{7b}+\frac{dx^{13}}{7b\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(x^12*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(d*x^13)/(7*b*(a + b*x^4)^(3/2)) + ((7*b*c - 13*a*d)*(-1/6*x^9/(b*(a + b*x
^4)^(3/2)) + (3*(-1/2*x^5/(b*Sqrt[a + b*x^4]) + (5*((x*Sqrt[a + b*x^4])/(3
*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a
+ b*x^4])))/(2*b)))/(2*b)))/(7*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.05 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.21

method	result
elliptic	$\frac{a^2 x(ad-cb)\sqrt{b\,x^4+a}}{6b^6\left(x^4+\frac{a}{b}\right)^2}-\frac{ax(19ad-13cb)}{12b^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{d\,x^5\sqrt{b\,x^4+a}}{7b^3}+\frac{\left(-\frac{2ad-cb}{b^3}-\frac{5da}{7b^3}\right)x\sqrt{b\,x^4+a}}{3b}+\frac{\left(\frac{a(3ad-2cb)}{b^4}-\frac{a(19ad-13cb)}{12b^4}-\right.}{}$
default	$c\left(-\frac{a^2 x\sqrt{b\,x^4+a}}{6b^5\left(x^4+\frac{a}{b}\right)^2}+\frac{13ax}{12b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^3}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{4b^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\left(\frac{a^3 x\sqrt{b\,x}}{6b^6\left(x^4+\right.}\right.$
risch	$-\frac{x(-3db\,x^4+19ad-7cb)\sqrt{b\,x^4+a}}{21b^4}+\frac{a\left(\frac{82ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-21a(4ad-3cb)\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-i}}{}\right.\right.}{}$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x/b^6*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12/b^4*a*x*(19*a*d-1
3*b*c)/((x^4+a/b)*b)^(1/2)+1/7*d*x^5*(b*x^4+a)^(1/2)/b^3+1/3*(-1/b^3*(2*a*
d-b*c)-5/7*d/b^3*a)/b*x*(b*x^4+a)^(1/2)+(a*(3*a*d-2*b*c)/b^4-1/12/b^4*a*(1
9*a*d-13*b*c)-1/3*(-1/b^3*(2*a*d-b*c)-5/7*d/b^3*a)/b*a)/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx =$$

$$-\frac{15\left((7\,b^3c-13\,ab^2d)x^8+2\,(7\,ab^2c-13\,a^2bd)x^4+7\,a^2bc-13\,a^3d\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-}{84\,(b^6x^8+2\,ab^5x^4}$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/84*(15*((7*b^3*c - 13*a*b^2*d)*x^8 + 2*(7*a*b^2*c - 13*a^2*b*d)*x^4 + 7
*a^2*b*c - 13*a^3*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x
), -1) - (12*b^3*d*x^13 + 4*(7*b^3*c - 13*a*b^2*d)*x^9 + 21*(7*a*b^2*c - 1
3*a^2*b*d)*x^5 + 15*(7*a^2*b*c - 13*a^3*d)*x)*sqrt(b*x^4 + a))/(b^6*x^8 +
2*a*b^5*x^4 + a^2*b^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 98.96 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.39

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^{13}\Gamma\left(\frac{13}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{17}{4}\right)} + \frac{dx^{17}\Gamma\left(\frac{17}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{17}{4}\\ \frac{21}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{21}{4}\right)}$$

input
```
integrate(x**12*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**13*gamma(13/4)*hyper((5/2, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(5/2)*gamma(17/4)) + d*x**17*gamma(17/4)*hyper((5/2, 17/4), (21/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(21/4))
```

Maxima [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^{12}\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-117\sqrt{b\,x^4+a}\,a^3dx+63\sqrt{b\,x^4+a}\,a^2bcx-117\sqrt{b\,x^4+a}\,a^2bd\,x^5+63\sqrt{b\,x^4+a}\,a\,b^2}{}$$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
( - 117*sqrt(a + b*x**4)*a**3*d*x + 63*sqrt(a + b*x**4)*a**2*b*c*x - 117*s
qrt(a + b*x**4)*a**2*b*d*x**5 + 63*sqrt(a + b*x**4)*a*b**2*c*x**5 - 13*sqr
t(a + b*x**4)*a*b**2*d*x**9 + 7*sqrt(a + b*x**4)*b**3*c*x**9 + 3*sqrt(a +
b*x**4)*b**3*d*x**13 + 117*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*
a*b**2*x**8 + b**3*x**12),x)*a**6*d - 63*int(sqrt(a + b*x**4)/(a**3 + 3*a*
*2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**5*b*c + 234*int(sqrt(a + b*x
**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**5*b*d*x**4
- 126*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x*
*12),x)*a**4*b**2*c*x**4 + 117*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4
+ 3*a*b**2*x**8 + b**3*x**12),x)*a**4*b**2*d*x**8 - 63*int(sqrt(a + b*x**4
)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*b**3*c*x**8)
/(21*b**4*(a**2 + 2*a*b*x**4 + b**2*x**8))
```

3.46 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}} \, dx = \frac{a(bc-ad)x}{6b^3(a+bx^4)^{3/2}} - \frac{(7bc-13ad)x}{12b^3\sqrt{a+bx^4}} + \frac{dx\sqrt{a+bx^4}}{3b^3} + \frac{5(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24\sqrt[4]{a}b^{13/4}\sqrt{a+bx^4}}$$

output
```
1/6*a*(-a*d+b*c)*x/b^3/(b*x^4+a)^(3/2)-1/12*(-13*a*d+7*b*c)*x/b^3/(b*x^4+a
)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)/b^3+5/24*(-3*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2
)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/
4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{x\left(15a^2d+b^2x^4(-7c+4dx^4)+ab(-5c+21dx^4)+5(bc-3ad)\,(a+bx^4)\,\sqrt{1+\frac{bx^4}{a}}\,\mathrm{H}\right.}{12b^3\,(a+bx^4)^{3/2}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(x*(15*a^2*d + b^2*x^4*(-7*c + 4*d*x^4) + a*b*(-5*c + 21*d*x^4) + 5*(b*c -
 3*a*d)*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((b*x^4)/a)]))/(12*b^3*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 817, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(bc-3ad)\int \frac{x^8}{(bx^4+a)^{5/2}}dx}{b}+\frac{dx^9}{3b\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(bc-3ad)\left(\frac{5\int \frac{x^4}{(bx^4+a)^{3/2}}dx}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{b}+\frac{dx^9}{3b\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(bc-3ad)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{b}+\frac{dx^9}{3b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(bc-3ad)\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{b}+\frac{dx^9}{3b\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(d*x^9)/(3*b*(a + b*x^4)^(3/2)) + ((b*c - 3*a*d)*(-1/6*x^5/(b*(a + b*x^4)^
(3/2)) + (5*(-1/2*x/(b*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4])))/(6*b)))/b
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.08

method	result
elliptic	$-\frac{ax(ad-cb)\sqrt{b\,x^4+a}}{6b^5\left(x^4+\frac{a}{b}\right)^2}+\frac{x(13ad-7cb)}{12b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{dx\sqrt{b\,x^4+a}}{3b^3}+\frac{\left(-\frac{2ad-cb}{b^3}+\frac{13ad-7cb}{12b^3}-\frac{da}{3b^3}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \cdots\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{ax\sqrt{b\,x^4+a}}{6b^4\left(x^4+\frac{a}{b}\right)^2}-\frac{7x}{12b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{5\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{a^2x\sqrt{b\,x^4+a}}{6b^5\left(x^4+\frac{a}{b}\right)^2}+\frac{13 \cdots}{12b^3\sqrt{\cdots}}\right.$
risch	$\frac{dx\sqrt{b\,x^4+a}}{3b^3}-\frac{\frac{7ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-3a(3ad-2cb)\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\cdots}\right.}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.}{\cdots}$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x/b^5*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/12/b^3*x*(13*a*d-7*b*
c)/((x^4+a/b)*b)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)/b^3+(-1/b^3*(2*a*d-b*c)+1/1
2/b^3*(13*a*d-7*b*c)-1/3*d/b^3*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.95

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{5\left((b^3c-3\,ab^2d)x^8+2\,(ab^2c-3\,a^2bd)x^4+a^2bc-3\,a^3d\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\right.}{12\,(ab^5x^8+2\,a^2b^4x^4}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(5*((b^3*c - 3*a*b^2*d)*x^8 + 2*(a*b^2*c - 3*a^2*b*d)*x^4 + a^2*b*c -
 3*a^3*d)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (4
*a*b^2*d*x^9 - 7*(a*b^2*c - 3*a^2*b*d)*x^5 - 5*(a^2*b*c - 3*a^3*d)*x)*sqrt
(b*x^4 + a))/(a*b^5*x^8 + 2*a^2*b^4*x^4 + a^3*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.46

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)} + \frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**9*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/2)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((5/2, 13/4), (17/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{9\sqrt{b\,x^4+a}\,a^2dx-3\sqrt{b\,x^4+a}\,abcx+9\sqrt{b\,x^4+a}\,abd\,x^5-3\sqrt{b\,x^4+a}\,b^2c\,x^5+\sqrt{b\,x^4}}{}$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
(9*sqrt(a + b*x**4)*a**2*d*x - 3*sqrt(a + b*x**4)*a*b*c*x + 9*sqrt(a + b*x
**4)*a*b*d*x**5 - 3*sqrt(a + b*x**4)*b**2*c*x**5 + sqrt(a + b*x**4)*b**2*d
*x**9 - 9*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**
3*x**12),x)*a**5*d + 3*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b*
*2*x**8 + b**3*x**12),x)*a**4*b*c - 18*int(sqrt(a + b*x**4)/(a**3 + 3*a**2
*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**4*b*d*x**4 + 6*int(sqrt(a + b*
x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*b**2*c*x
**4 - 9*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*
x**12),x)*a**3*b**2*d*x**8 + 3*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4
+ 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b**3*c*x**8)/(3*b**3*(a**2 + 2*a*b*x
**4 + b**2*x**8))
```

3.47 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = -\frac{(bc-ad)x}{6b^2(a+bx^4)^{3/2}} + \frac{(bc-7ad)x}{12ab^2\sqrt{a+bx^4}} + \frac{(bc+5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{5/4}b^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/6*(-a*d+b*c)*x/b^2/(b*x^4+a)^(3/2)+1/12*(-7*a*d+b*c)*x/a/b^2/(b*x^4+a)^
(1/2)+1/24*(5*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5
/4)/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-5a^2dx+b^2cx^5-abx(c+7dx^4)+(bc+5ad)x(a+bx^4)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2}}{12ab^2(a+bx^4)^{3/2}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(-5*a^2*d*x + b^2*c*x^5 - a*b*x*(c + 7*d*x^4) + (b*c + 5*a*d)*x*(a + b*x^4
)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(12*
a*b^2*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {957, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5ad+bc)\int \frac{x^4}{(bx^4+a)^{3/2}}dx}{6ab}+\frac{x^5(bc-ad)}{6ab(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5ad+bc)\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6ab}+\frac{x^5(bc-ad)}{6ab(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5ad+bc)\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6ab}+\frac{x^5(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
((b*c - a*d)*x^5)/(6*a*b*(a + b*x^4)^(3/2)) + ((b*c + 5*a*d)*(-1/2*x/(b*Sq
rt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(5
/4)*Sqrt[a + b*x^4])))/(6*a*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03

method	result
elliptic	$\frac{x(ad-cb)\sqrt{b\,x^4+a}}{6b^4\left(x^4+\frac{a}{b}\right)^2}-\frac{x(7ad-cb)}{12b^2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\left(\frac{d}{b^2}-\frac{7ad-cb}{12b^2a}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{x\sqrt{b\,x^4+a}}{6b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x}{12ba\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12ba\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(\frac{ax\sqrt{b\,x^4+a}}{6b^4\left(x^4+\frac{a}{b}\right)^2}-\frac{7}{12b^2\sqrt{(}}\right.$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x/b^4*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12/b^2/a*x*(7*a*d-b*c)/(
(x^4+a/b)*b)^(1/2)+(d/b^2-1/12/b^2/a*(7*a*d-b*c))/(I/a^(1/2)*b^(1/2))^(1/2
)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx =$$

$$-\frac{((b^3c+5\,ab^2d)x^8+2\,(ab^2c+5\,a^2bd)x^4+a^2bc+5\,a^3d)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)-((b^3c-7\,a}{12\,(ab^5x^8+2\,a^2b^4x^4+a^3b^3)}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(((b^3*c + 5*a*b^2*d)*x^8 + 2*(a*b^2*c + 5*a^2*b*d)*x^4 + a^2*b*c +
5*a^3*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - ((b
^3*c - 7*a*b^2*d)*x^5 - (a*b^2*c + 5*a^2*b*d)*x)*sqrt(b*x^4 + a))/(a*b^5*x
^8 + 2*a^2*b^4*x^4 + a^3*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.50 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**5*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(5/2)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-5\sqrt{b\,x^4+a}\,adx-\sqrt{b\,x^4+a}\,bcx-5\sqrt{b\,x^4+a}\,bd\,x^5+5\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}dx\right)}{}$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
( - 5*sqrt(a + b*x**4)*a*d*x - sqrt(a + b*x**4)*b*c*x - 5*sqrt(a + b*x**4)
*b*d*x**5 + 5*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 +
 b**3*x**12),x)*a**4*d + int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*
b**2*x**8 + b**3*x**12),x)*a**3*b*c + 10*int(sqrt(a + b*x**4)/(a**3 + 3*a*
*2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*b*d*x**4 + 2*int(sqrt(a +
b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b**2*c
*x**4 + 5*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**
3*x**12),x)*a**2*b**2*d*x**8 + int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4
+ 3*a*b**2*x**8 + b**3*x**12),x)*a*b**3*c*x**8)/(5*b**2*(a**2 + 2*a*b*x**4
 + b**2*x**8))
```

3.48 $\int \frac{c+dx^4}{(a+bx^4)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}} \, dx = \frac{(bc-ad)x}{6ab(a+bx^4)^{3/2}} + \frac{(5bc+ad)x}{12a^2b\sqrt{a+bx^4}} + \frac{(5bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{9/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/6*(-a*d+b*c)*x/a/b/(b*x^4+a)^(3/2)+1/12*(a*d+5*b*c)*x/a^2/b/(b*x^4+a)^(1
/2)+1/24*(a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2
)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4
)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}} \, dx = \frac{-a^2dx+5b^2cx^5+abx(7c+dx^4)+(5bc+ad)x(a+bx^4)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2}}{12a^2b(a+bx^4)^{3/2}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(5/2),x]
```

output
```
(-(a^2*d*x) + 5*b^2*c*x^5 + a*b*x*(7*c + d*x^4) + (5*b*c + a*d)*x*(a + b*x
^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(1
2*a^2*b*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+5bc)\int \frac{1}{(bx^4+a)^{3/2}}dx}{6ab}+\frac{x(bc-ad)}{6ab(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+5bc)\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6ab}+\frac{x(bc-ad)}{6ab(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ad+5bc)\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6ab}+\frac{x(bc-ad)}{6ab(a+bx^4)^{3/2}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(5/2),x]
```

output
```
((b*c - a*d)*x)/(6*a*b*(a + b*x^4)^(3/2)) + ((5*b*c + a*d)*(x/(2*a*Sqrt[a
+ b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x
^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(1/4)*S
qrt[a + b*x^4])))/(6*a*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

method	result
elliptic	$-\frac{x(ad-cb)\sqrt{b\,x^4+a}}{6a\,b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x(ad+5cb)}{12b\,a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{(ad+5cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{6a\,b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{5x}{12a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{5\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\left(-\frac{x\sqrt{b\,x^4+a}}{6b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{}{12ba}\right.$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a*x/b^3*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/12/b/a^2*x*(a*d+5*b*c
)/((x^4+a/b)*b)^(1/2)+1/12*(a*d+5*b*c)/a^2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-
I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx =$$
$$-\frac{((5\,b^3c+ab^2d)x^8+2\,(5\,ab^2c+a^2bd)x^4+5\,a^2bc+a^3d)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-((5\,b^3c+}{12\,(a^2b^4x^8+2\,a^3b^3x^4+a^4b^2)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(((5*b^3*c + a*b^2*d)*x^8 + 2*(5*a*b^2*c + a^2*b*d)*x^4 + 5*a^2*b*c
+ a^3*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - ((5
*b^3*c + a*b^2*d)*x^5 + (7*a*b^2*c - a^2*b*d)*x)*sqrt(b*x^4 + a))/(a^2*b^4
*x^8 + 2*a^3*b^3*x^4 + a^4*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/2)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(5/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(5/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,dx + \left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}dx\right) a^3d + 5\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}dx\right) a^2}{}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d*x + int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*
a*b**2*x**8 + b**3*x**12),x)*a**3*d + 5*int(sqrt(a + b*x**4)/(a**3 + 3*a**
2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b*c + 2*int(sqrt(a + b*x**4
)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b*d*x**4 + 1
0*int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12)
,x)*a*b**2*c*x**4 + int(sqrt(a + b*x**4)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*
x**8 + b**3*x**12),x)*a*b**2*d*x**8 + 5*int(sqrt(a + b*x**4)/(a**3 + 3*a**
2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*b**3*c*x**8)/(5*b*(a**2 + 2*a*b*
x**4 + b**2*x**8))
```

3.49 $\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 176

$$\int \frac{c+dx^4}{x^4 \left(a+bx^4\right)^{5/2}} \, dx = -\frac{c}{3ax^3 \left(a+bx^4\right)^{3/2}} - \frac{(3bc-ad)x}{6a^2 \left(a+bx^4\right)^{3/2}} - \frac{5(3bc-ad)x}{12a^3\sqrt{a+bx^4}} - \frac{5(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{24a^{13/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(3/2)-1/6*(-a*d+3*b*c)*x/a^2/(b*x^4+a)^(3/2)-5/12*(
-a*d+3*b*c)*x/a^3/(b*x^4+a)^(1/2)-5/24*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*
((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/a^(13/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^4}{x^4 \left(a+bx^4\right)^{5/2}} \, dx = \frac{-15b^2cx^8+a^2(-4c+7dx^4)+ab(-21cx^4+5dx^8)+5(-3bc+ad)x^4(a+bx^4)\sqrt{1}}{12a^3x^3\left(a+bx^4\right)^{3/2}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(5/2)),x]
```

output
```
(-15*b^2*c*x^8 + a^2*(-4*c + 7*d*x^4) + a*b*(-21*c*x^4 + 5*d*x^8) + 5*(-3*
b*c + a*d)*x^4*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^4)/a)])/(12*a^3*x^3*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 749, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3bc-ad)\int \frac{1}{(bx^4+a)^{5/2}}dx}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(3bc-ad)\left(\frac{5\int \frac{1}{(bx^4+a)^{3/2}}dx}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(3bc-ad)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(3bc-ad)\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{3ax^3\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(5/2)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(3/2)) - ((3*b*c - a*d)*(x/(6*a*(a + b*x^4)^(3/2
)) + (5*(x/(2*a*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b*x^4])))/(6*a)))/a
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00

method	result
elliptic	$\frac{x(ad-cb)\sqrt{b\,x^4+a}}{6a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x(5ad-11cb)}{12a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{3a^3x^3}+\frac{\left(\frac{5ad-11cb}{12a^3}-\frac{bc}{3a^3}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$d\left(\frac{x\sqrt{b\,x^4+a}}{6a\,b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{5x}{12a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{5\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{12a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+c\left(-\frac{x\sqrt{b\,x^4+a}}{6a^2b\left(x^4+\frac{a}{b}\right)^2}-\frac{\quad}{12a^3}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{3a^3x^3}-\frac{\frac{cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}-3a^2(ad-cb)\left(\frac{x\sqrt{b\,x^4+a}}{6a\,b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{5x}{12a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{5\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+i}}{12a^2\sqrt{}}\right.}{3a^3}$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*x/b^2*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/12/a^3*x*(5*a*d-11*b
*c)/((x^4+a/b)*b)^(1/2)-1/3/a^3*c*(b*x^4+a)^(1/2)/x^3+(1/12/a^3*(5*a*d-11*
b*c)-1/3*b/a^3*c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b
^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/2}}\,dx=\frac{5\left((3\,b^3c-ab^2d)x^{11}+2\,(3\,ab^2c-a^2bd)x^7+(3\,a^2bc-a^3d)x^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(\right.}{12\left(a^3b^3x^{11}+2\,a^4\right.}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(5*((3*b^3*c - a*b^2*d)*x^11 + 2*(3*a*b^2*c - a^2*b*d)*x^7 + (3*a^2*b
*c - a^3*d)*x^3)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -
1) - (5*(3*b^3*c - a*b^2*d)*x^8 + 7*(3*a*b^2*c - a^2*b*d)*x^4 + 4*a^2*b*c)
*sqrt(b*x^4 + a))/(a^3*b^3*x^11 + 2*a^4*b^2*x^7 + a^5*b*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.47

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}}\,dx = \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(5/2),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/2)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(5/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}}\,dx = \int \frac{d\,x^4+c}{x^4\,(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(5/2)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-3\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{16}+3a\,b^2x^{12}+3a^2b\,x^8+a^3x^4}dx\right)a^3d\,x^3+9\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{16}+3a\,b^2x^{12}+3a^2b\,x^8+\cdots}\right.}{\cdots}$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 3*int(sqrt(a + b*x**4)/(a**3*x**4 + 3*a**2*b*x**8
 + 3*a*b**2*x**12 + b**3*x**16),x)*a**3*d*x**3 + 9*int(sqrt(a + b*x**4)/(a
**3*x**4 + 3*a**2*b*x**8 + 3*a*b**2*x**12 + b**3*x**16),x)*a**2*b*c*x**3 -
 6*int(sqrt(a + b*x**4)/(a**3*x**4 + 3*a**2*b*x**8 + 3*a*b**2*x**12 + b**3
*x**16),x)*a**2*b*d*x**7 + 18*int(sqrt(a + b*x**4)/(a**3*x**4 + 3*a**2*b*x
**8 + 3*a*b**2*x**12 + b**3*x**16),x)*a*b**2*c*x**7 - 3*int(sqrt(a + b*x**
4)/(a**3*x**4 + 3*a**2*b*x**8 + 3*a*b**2*x**12 + b**3*x**16),x)*a*b**2*d*x
**11 + 9*int(sqrt(a + b*x**4)/(a**3*x**4 + 3*a**2*b*x**8 + 3*a*b**2*x**12
+ b**3*x**16),x)*b**3*c*x**11)/(9*b*x**3*(a**2 + 2*a*b*x**4 + b**2*x**8))
```

3.50 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 210

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx = -\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}-\frac{13bc-7ad}{42a^2x^3\left(a+bx^4\right)^{3/2}} -\frac{3(13bc-7ad)}{28a^3x^3\sqrt{a+bx^4}}+\frac{5(13bc-7ad)\sqrt{a+bx^4}}{28a^4x^3} +\frac{5b^{3/4}(13bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{56a^{17/4}\sqrt{a+bx^4}}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(3/2)-1/42*(-7*a*d+13*b*c)/a^2/x^3/(b*x^4+a)^(3/2)-
3/28*(-7*a*d+13*b*c)/a^3/x^3/(b*x^4+a)^(1/2)+5/28*(-7*a*d+13*b*c)*(b*x^4+a
)^(1/2)/a^4/x^3+5/56*b^(3/4)*(-7*a*d+13*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4
+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1
/4)),1/2*2^(1/2))/a^(17/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.39

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx=\frac{-3a^2c+(13bc-7ad)x^4(a+bx^4)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{21a^3x^7\left(a+bx^4\right)^{3/2}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(5/2)),x]
```

output
```
(-3*a^2*c + (13*b*c - 7*a*d)*x^4*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[-3/4, 5/2, 1/4, -((b*x^4)/a)])/(21*a^3*x^7*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 819, 819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13bc-7ad)\int \frac{1}{x^4(bx^4+a)^{5/2}}dx}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(13bc-7ad)\left(\frac{3\int \frac{1}{x^4\left(bx^4+a\right)^{3/2}}dx}{2a}+\frac{1}{6ax^3(a+bx^4)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(13bc-7ad)\left(\frac{3\left(\frac{5\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3(a+bx^4)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(13bc-7ad)\left(\frac{3\left(\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3(a+bx^4)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{(13bc-7ad)\left(\frac{3\left(\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3(a+bx^4)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(5/2)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(3/2)) - ((13*b*c - 7*a*d)*(1/(6*a*x^3*(a + b*x^
4)^(3/2)) + (3*(1/(2*a*x^3*Sqrt[a + b*x^4]) + (5*(-1/3*Sqrt[a + b*x^4]/(a*
x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a
 + b*x^4])))/(2*a)))/(2*a)))/(7*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

method	result
elliptic	$-\frac{x(ad-cb)\sqrt{b\,x^4+a}}{6a^3b\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(11ad-17cb)}{12a^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{7a^3x^7}-\frac{(7ad-19cb)\sqrt{b\,x^4+a}}{21a^4x^3}+\frac{\left(-\frac{b(11ad-17cb)}{12a^4}-\frac{b(7ad-19cb)}{21a^4}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{6a^3\left(x^4+\frac{a}{b}\right)^2}+\frac{17b^2x}{12a^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{7a^3x^7}+\frac{19b\sqrt{b\,x^4+a}}{21a^4x^3}+\frac{65b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{28a^4\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)-$
risch	$-\frac{\sqrt{b\,x^4+a}\,(7ad\,x^4-19bc\,x^4+3ac)}{21a^4x^7}-\frac{b\left(\frac{7ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}+21a(ad-2cb)\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{}}}}{}\right.\right.}{}$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^3*x/b*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/12*b/a^4*x*(11*a*d-17
*b*c)/((x^4+a/b)*b)^(1/2)-1/7/a^3*c*(b*x^4+a)^(1/2)/x^7-1/21/a^4*(7*a*d-19
*b*c)*(b*x^4+a)^(1/2)/x^3+(-1/12*b/a^4*(11*a*d-17*b*c)-1/21*b/a^4*(7*a*d-1
9*b*c))/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(
1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1
/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx = -\frac{15\left((13\,b^3c-7\,ab^2d)x^{15}+2\,(13\,ab^2c-7\,a^2bd)x^{11}+(13\,a^2bc-7\,a^3d)x^7\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,\middle|\right.}{84\left(a^4b^2x^{15}+2\,a^5\right.}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/84*(15*((13*b^3*c - 7*a*b^2*d)*x^15 + 2*(13*a*b^2*c - 7*a^2*b*d)*x^11 +
 (13*a^2*b*c - 7*a^3*d)*x^7)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/
a)^(1/4)), -1) - (15*(13*b^3*c - 7*a*b^2*d)*x^12 + 21*(13*a*b^2*c - 7*a^2*
b*d)*x^8 + 4*(13*a^2*b*c - 7*a^3*d)*x^4 - 12*a^3*c)*sqrt(b*x^4 + a))/(a^4*
b^2*x^15 + 2*a^5*b*x^11 + a^6*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 93.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx = \frac{c\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix} -\frac{7}{4},\frac{5}{2} \\ -\frac{3}{4} \end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{d\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{5}{2} \\ \frac{1}{4} \end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(5/2),x)
```

output
```
c*gamma(-7/4)*hyper((-7/4, 5/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/2)*x**7*gamma(-3/4)) + d*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/2)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/2}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{2}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{2}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/2}}\,dx = \int \frac{d\,x^4+c}{x^8\,(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(5/2)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-7\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{20}+3a\,b^2x^{16}+3a^2b\,x^{12}+a^3x^8}dx\right)a^3d\,x^7+13\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{20}+3a\,b^2x^{16}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 7*int(sqrt(a + b*x**4)/(a**3*x**8 + 3*a**2*b*x**1
2 + 3*a*b**2*x**16 + b**3*x**20),x)*a**3*d*x**7 + 13*int(sqrt(a + b*x**4)/
(a**3*x**8 + 3*a**2*b*x**12 + 3*a*b**2*x**16 + b**3*x**20),x)*a**2*b*c*x**
7 - 14*int(sqrt(a + b*x**4)/(a**3*x**8 + 3*a**2*b*x**12 + 3*a*b**2*x**16 +
 b**3*x**20),x)*a**2*b*d*x**11 + 26*int(sqrt(a + b*x**4)/(a**3*x**8 + 3*a*
*2*b*x**12 + 3*a*b**2*x**16 + b**3*x**20),x)*a*b**2*c*x**11 - 7*int(sqrt(a
 + b*x**4)/(a**3*x**8 + 3*a**2*b*x**12 + 3*a*b**2*x**16 + b**3*x**20),x)*a
*b**2*d*x**15 + 13*int(sqrt(a + b*x**4)/(a**3*x**8 + 3*a**2*b*x**12 + 3*a*
b**2*x**16 + b**3*x**20),x)*b**3*c*x**15)/(13*b*x**7*(a**2 + 2*a*b*x**4 +
b**2*x**8))
```

3.51 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 323

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = -\frac{(bc-ad)x^7}{6b^2(a+bx^4)^{3/2}} - \frac{(7bc-13ad)x^3}{12b^3\sqrt{a+bx^4}} + \frac{dx^3\sqrt{a+bx^4}}{5b^3} + \frac{7(5bc-11ad)x\sqrt{a+bx^4}}{20b^{7/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{7\sqrt[4]{a}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20b^{15/4}\sqrt{a+bx^4}} + \frac{7\sqrt[4]{a}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{40b^{15/4}\sqrt{a+bx^4}}$$

output

```
-1/6*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(3/2)-1/12*(-13*a*d+7*b*c)*x^3/b^3/(b*x^
4+a)^(1/2)+1/5*d*x^3*(b*x^4+a)^(1/2)/b^3+7/20*(-11*a*d+5*b*c)*x*(b*x^4+a)^
(1/2)/b^(7/2)/(a^(1/2)+b^(1/2)*x^2)-7/20*a^(1/4)*(-11*a*d+5*b*c)*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arc
tan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^4+a)^(1/2)+7/40*a^(1/4)
*(-11*a*d+5*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(15/4)/(
b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.34

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{x^3\left(-77a^2d+ab(35c-33dx^4)+3b^2x^4(5c+dx^4)+7(-5bc+11ad)\,(a+bx^4)\,\sqrt{1+\right.}{15b^3\,(a+bx^4)^{3/2}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(x^3*(-77*a^2*d + a*b*(35*c - 33*d*x^4) + 3*b^2*x^4*(5*c + d*x^4) + 7*(-5*
b*c + 11*a*d)*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 5/2,
7/4, -((b*x^4)/a)]))/(15*b^3*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {959, 817, 817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(5bc-11ad)\int \frac{x^{10}}{(bx^4+a)^{5/2}}dx}{5b}+\frac{dx^{11}}{5b\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(5bc-11ad)\left(\frac{7\int \frac{x^6}{(bx^4+a)^{3/2}}dx}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{5b}+\frac{dx^{11}}{5b\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5bc-11ad)\left(\frac{7\left(\frac{3\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{5b}+\frac{dx^{11}}{5b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(5bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{5b}+\frac{dx^{11}}{5b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{5b}+\frac{dx^{11}}{5b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{5b}+$$

$$\frac{dx^{11}}{5b\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(5bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right.}{2b}}{6b}\right.}{5b}$$

$$\frac{dx^{11}}{5b\left(a+bx^4\right)^{3/2}}$$

input

```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output

```
(d*x^11)/(5*b*(a + b*x^4)^(3/2)) + ((5*b*c - 11*a*d)*(-1/6*x^7/(b*(a + b*x
^4)^(3/2)) + (7*(-1/2*x^3/(b*Sqrt[a + b*x^4]) + (3*(-((-((x*Sqrt[a + b*x^4
])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^
2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)
*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(2*b)))/(6*b)))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.68

method	result
elliptic	$-\frac{a\,x^{3}(ad-cb)\sqrt{b\,x^{4}+a}}{6b^{5}\left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{x^{3}(5ad-3cb)}{4b^{3}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{d\,x^{3}\sqrt{b\,x^{4}+a}}{5b^{3}}+\frac{i\left(-\frac{2ad-cb}{b^{3}}-\frac{5ad-3cb}{4b^{3}}-\frac{3da}{5b^{3}}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{Ellipt}\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$
default	$c\left(\frac{a\,x^{3}\sqrt{b\,x^{4}+a}}{6b^{4}\left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{3x^{3}}{4b^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{7i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{4b^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+d$
risch	$\frac{d\,x^{3}\sqrt{b\,x^{4}+a}}{5b^{3}}-\frac{\frac{i(13ad-5cb)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}-5a(3ad-2cb)\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\right.}{}$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*x^3/b^5*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/4/b^3*x^3*(5*a*d-3*
b*c)/((x^4+a/b)*b)^(1/2)+1/5*d*x^3*(b*x^4+a)^(1/2)/b^3+I*(-1/b^3*(2*a*d-b*
c)-1/4/b^3*(5*a*d-3*b*c)-3/5*d/b^3*a)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1
-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/
2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx=\frac{21\left(\left(5\,b^3c-11\,ab^2d\right)x^9+2\left(5\,ab^2c-11\,a^2bd\right)x^5+\left(5\,a^2bc-11\,a^3d\right)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\text{arc}}{}$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/60*(21*((5*b^3*c - 11*a*b^2*d)*x^9 + 2*(5*a*b^2*c - 11*a^2*b*d)*x^5 + (5
*a^2*b*c - 11*a^3*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4
)/x), -1) - 21*((5*b^3*c - 11*a*b^2*d)*x^9 + 2*(5*a*b^2*c - 11*a^2*b*d)*x^
5 + (5*a^2*b*c - 11*a^3*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b
)^(1/4)/x), -1) + (12*b^3*d*x^12 + 12*(5*b^3*c - 11*a*b^2*d)*x^8 + 35*(5*a
*b^2*c - 11*a^2*b*d)*x^4 + 105*a^2*b*c - 231*a^3*d)*sqrt(b*x^4 + a))/(b^6*
x^9 + 2*a*b^5*x^5 + a^2*b^4*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 59.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.25

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{15}{4}\right)} + \frac{dx^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**11*gamma(11/4)*hyper((5/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(5/2)*gamma(15/4)) + d*x**15*gamma(15/4)*hyper((5/2, 15/4), (19/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-77\sqrt{b\,x^4+a}\,a^2d\,x^3+35\sqrt{b\,x^4+a}\,abc\,x^3-33\sqrt{b\,x^4+a}\,abd\,x^7+15\sqrt{b\,x^4+a}\,b^2c\,x^7}{}$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
( - 77*sqrt(a + b*x**4)*a**2*d*x**3 + 35*sqrt(a + b*x**4)*a*b*c*x**3 - 33*
sqrt(a + b*x**4)*a*b*d*x**7 + 15*sqrt(a + b*x**4)*b**2*c*x**7 + 3*sqrt(a +
 b*x**4)*b**2*d*x**11 + 231*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x
**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**5*d - 105*int((sqrt(a + b*x**4)*x*
*2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**4*b*c + 462*
int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x
**12),x)*a**4*b*d*x**4 - 210*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*
x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*b**2*c*x**4 + 231*int((sqrt(a +
 b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3
*b**2*d*x**8 - 105*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a
*b**2*x**8 + b**3*x**12),x)*a**2*b**3*c*x**8)/(15*b**3*(a**2 + 2*a*b*x**4
+ b**2*x**8))
```

3.52 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 303

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = -\frac{(bc-ad)x^3}{6b^2(a+bx^4)^{3/2}}+\frac{(bc-3ad)x^3}{4ab^2\sqrt{a+bx^4}}-\frac{(bc-7ad)x\sqrt{a+bx^4}}{4ab^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}$$
$$+\frac{(bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^4}}$$
$$-\frac{(bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8a^{3/4}b^{11/4}\sqrt{a+bx^4}}$$

output
```
-1/6*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(3/2)+1/4*(-3*a*d+b*c)*x^3/a/b^2/(b*x^4+
a)^(1/2)-1/4*(-7*a*d+b*c)*x*(b*x^4+a)^(1/2)/a/b^(5/2)/(a^(1/2)+b^(1/2)*x^2
)+1/4*(-7*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b
^(11/4)/(b*x^4+a)^(1/2)-1/8*(-7*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/
(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4))
,1/2*2^(1/2))/a^(3/4)/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.30

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{x^3\left(a(-bc+7ad+3bdx^4)+(bc-7ad)\,(a+bx^4)\,\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2}\right.\right.}{3ab^2\,(a+bx^4)^{3/2}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(x^3*(a*(-(b*c) + 7*a*d + 3*b*d*x^4) + (b*c - 7*a*d)*(a + b*x^4)*Sqrt[1 +
(b*x^4)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^4)/a)]))/(3*a*b^2*(a +
b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {957, 817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^7(bc-ad)}{6ab\,(a+bx^4)^{3/2}}-\frac{(bc-7ad)\int \frac{x^6}{(bx^4+a)^{3/2}}dx}{6ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^7(bc-ad)}{6ab\,(a+bx^4)^{3/2}}-\frac{(bc-7ad)\left(\frac{3\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6ab}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{x^7(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}-\frac{(bc-7ad)\left(\frac{3\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^7(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}-\frac{(bc-7ad)\left(\frac{3\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6ab}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{x^7(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}-\frac{(bc-7ad)\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6ab}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{x^7(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}} - \frac{(bc-7ad)\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}} - \frac{x}{\sqrt{}}}{\sqrt{b}}\right.}{2b}}{6ab}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
((b*c - a*d)*x^7)/(6*a*b*(a + b*x^4)^(3/2)) - ((b*c - 7*a*d)*(-1/2*x^3/(b*
Sqrt[a + b*x^4]) + (3*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) +
 (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)
^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4
]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
+ Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4
)*Sqrt[a + b*x^4])))/(2*b)))/(6*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.63

method	result
elliptic	$\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{6b^4\left(x^4+\frac{a}{b}\right)^2}-\frac{x^3(3ad-cb)}{4b^2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{i\left(\frac{d}{b^2}+\frac{3ad-cb}{4a\,b^2}\right)\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i}{\ }}\right.\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}$
default	$c\left(-\frac{x^3\sqrt{b\,x^4+a}}{6b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3}{4ba\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{4b^{\frac{3}{2}}\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\,\Bigg($

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3/b^4*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/4/b^2/a*x^3*(3*a*d-b*c
)/((x^4+a/b)*b)^(1/2)+I*(d/b^2+1/4*(3*a*d-b*c)/a/b^2)*a^(1/2)/(I/a^(1/2)*b
^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Elli
pticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.82

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx =$$

$$-\frac{3\left((b^3c-7\,ab^2d)x^9+2\,(ab^2c-7\,a^2bd)x^5+(a^2bc-7\,a^3d)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-3\left((b^3c\right.}{}$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*((b^3*c - 7*a*b^2*d)*x^9 + 2*(a*b^2*c - 7*a^2*b*d)*x^5 + (a^2*b*c
 - 7*a^3*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1)
 - 3*((b^3*c - 7*a*b^2*d)*x^9 + 2*(a*b^2*c - 7*a^2*b*d)*x^5 + (a^2*b*c - 7
*a^3*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (
12*a*b^2*d*x^8 - 5*(a*b^2*c - 7*a^2*b*d)*x^4 - 3*a^2*b*c + 21*a^3*d)*sqrt(
b*x^4 + a))/(a*b^5*x^9 + 2*a^2*b^4*x^5 + a^3*b^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**7*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/2)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((5/2, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{7\sqrt{b\,x^4+a}\,ad\,x^3-\sqrt{b\,x^4+a}\,bc\,x^3+3\sqrt{b\,x^4+a}\,bd\,x^7-21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}d\right.}{}$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
(7*sqrt(a + b*x**4)*a*d*x**3 - sqrt(a + b*x**4)*b*c*x**3 + 3*sqrt(a + b*x*
*4)*b*d*x**7 - 21*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*
b**2*x**8 + b**3*x**12),x)*a**4*d + 3*int((sqrt(a + b*x**4)*x**2)/(a**3 +
3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*b*c - 42*int((sqrt(a +
 b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3
*b*d*x**4 + 6*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2
*x**8 + b**3*x**12),x)*a**2*b**2*c*x**4 - 21*int((sqrt(a + b*x**4)*x**2)/(
a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b**2*d*x**8 + 3
*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*
x**12),x)*a*b**3*c*x**8)/(3*b**2*(a**2 + 2*a*b*x**4 + b**2*x**8))
```

3.53 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 302

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{(bc-ad)x^3}{6ab\,(a+bx^4)^{3/2}} + \frac{(bc+ad)x^3}{4a^2b\sqrt{a+bx^4}} - \frac{(bc+ad)x\sqrt{a+bx^4}}{4a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^4}} - \frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8a^{7/4}b^{7/4}\sqrt{a+bx^4}}$$

output
```
1/6*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(3/2)+1/4*(a*d+b*c)*x^3/a^2/b/(b*x^4+a)^(
1/2)-1/4*(a*d+b*c)*x*(b*x^4+a)^(1/2)/a^2/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)+1/4
*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/b^(7/4)/(
b*x^4+a)^(1/2)-1/8*(a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2)
)/a^(7/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.26

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{x^3\left(-a^2d+(bc+ad)\left(a+bx^4\right)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{3a^2b\left(a+bx^4\right)^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
(x^3*(-(a^2*d) + (b*c + a*d)*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeometri
c2F1[3/4, 5/2, 7/4, -((b*x^4)/a)]))/(3*a^2*b*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {957, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^4\right)}{\left(a+bx^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ad+bc)\int \frac{x^2}{\left(bx^4+a\right)^{3/2}}dx}{2ab}+\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(ad+bc)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}\right)}{2ab}+\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(ad+bc)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{2ab}+\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{(ad+bc)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{2ab}+\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{(ad+bc)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}\right)}{2ab}+\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{(ad+bc)\left(\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}}{2a}\right.}{2ab}$$

$$\frac{x^3(bc-ad)}{6ab\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(5/2),x]
```

output
```
((b*c - a*d)*x^3)/(6*a*b*(a + b*x^4)^(3/2)) + ((b*c + a*d)*(x^3/(2*a*Sqrt[
a + b*x^4]) - (-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt
[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a
 + b*x^4]))/(2*a)))/(2*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.58

method	result
elliptic	$-\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{6a\,b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3(ad+cb)}{4b\,a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i(ad+cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{4b^{\frac{3}{2}}a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x^3\sqrt{b\,x^4+a}}{6a\,b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3}{4a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{4a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)+d\Bigg($

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a*x^3/b^3*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/4/b/a^2*x^3*(a*d+b*
c)/((x^4+a/b)*b)^(1/2)-1/4*I/b^(3/2)/a^(3/2)*(a*d+b*c)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1
/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.74

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx=\frac{3\left((b^3c+ab^2d)x^8+2\left(ab^2c+a^2bd\right)x^4+a^2bc+a^3d\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-\right.}{}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(3*((b^3*c + a*b^2*d)*x^8 + 2*(a*b^2*c + a^2*b*d)*x^4 + a^2*b*c + a^3
*d)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 3*((b^3*
c + a*b^2*d)*x^8 + 2*(a*b^2*c + a^2*b*d)*x^4 + a^2*b*c + a^3*d)*sqrt(a)*(-
b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (3*(b^3*c + a*b^2*d)*x
^7 + (5*a*b^2*c + a^2*b*d)*x^3)*sqrt(b*x^4 + a))/(a^2*b^4*x^8 + 2*a^3*b^3*
x^4 + a^4*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.26

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(5/2),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(5/2)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{5/2}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(5/2),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d\,x^3+3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}dx\right)a^3d+3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3}dx\right)$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d*x**3 + 3*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2
*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**3*d + 3*int((sqrt(a + b*x**4)*
x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a**2*b*c + 6*
int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x
**12),x)*a**2*b*d*x**4 + 6*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x*
*4 + 3*a*b**2*x**8 + b**3*x**12),x)*a*b**2*c*x**4 + 3*int((sqrt(a + b*x**4
)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12),x)*a*b**2*d*x*
*8 + 3*int((sqrt(a + b*x**4)*x**2)/(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 +
 b**3*x**12),x)*b**3*c*x**8)/(3*b*(a**2 + 2*a*b*x**4 + b**2*x**8))
```

3.54 $\int \frac{c+dx^4}{x^2(a+bx^4)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 325

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{5/2}}\,dx = -\frac{c}{ax(a+bx^4)^{3/2}} - \frac{(7bc-ad)x^3}{6a^2(a+bx^4)^{3/2}} - \frac{(7bc-ad)x^3}{4a^3\sqrt{a+bx^4}} + \frac{(7bc-ad)x\sqrt{a+bx^4}}{4a^3\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{(7bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{11/4}b^{3/4}\sqrt{a+bx^4}} + \frac{(7bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8a^{11/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
-c/a/x/(b*x^4+a)^(3/2)-1/6*(-a*d+7*b*c)*x^3/a^2/(b*x^4+a)^(3/2)-1/4*(-a*d+
7*b*c)*x^3/a^3/(b*x^4+a)^(1/2)+1/4*(-a*d+7*b*c)*x*(b*x^4+a)^(1/2)/a^3/b^(1
/2)/(a^(1/2)+b^(1/2)*x^2)-1/4*(-a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a
)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))
),1/2*2^(1/2))/a^(11/4)/b^(3/4)/(b*x^4+a)^(1/2)+1/8*(-a*d+7*b*c)*(a^(1/2)+
b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(11/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.25

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{5/2}}\,dx = \frac{-3a^2c+(-7bc+ad)x^4(a+bx^4)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^3x\,(a+bx^4)^{3/2}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(5/2)),x]
```

output
```
(-3*a^2*c + (-7*b*c + a*d)*x^4*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeomet
ric2F1[3/4, 5/2, 7/4, -((b*x^4)/a)])/(3*a^3*x*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7bc-ad)\int \frac{x^2}{(bx^4+a)^{5/2}}dx}{a}-\frac{c}{ax\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(7bc-ad)\left(\frac{\int \frac{x^2}{(bx^4+a)^{3/2}}dx}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{ax\,(a+bx^4)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(7bc-ad)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 834

$$-\frac{(7bc-ad)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{(7bc-ad)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 761

$$-\frac{(7bc-ad)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$-\frac{(7bc-ad)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}}{2a}}{2a}\right.}{a}$$

$$\frac{c}{ax\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(5/2)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(3/2))) - ((7*b*c - a*d)*(x^3/(6*a*(a + b*x^4)^(3/2))
 + (x^3/(2*a*Sqrt[a + b*x^4]) - (-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[
b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
rt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt
[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(2*b^(3/4)*Sqrt[a + b*x^4]))/(2*a))/(2*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.63

method	result
elliptic	$\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{6a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3(ad-3cb)}{4a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{a^3x}+\frac{i\left(-\frac{ad-3cb}{4a^3}+\frac{bc}{a^3}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}$
default	$c\left(-\frac{x^3\sqrt{b\,x^4+a}}{6a^2b\left(x^4+\frac{a}{b}\right)^2}-\frac{3b\,x^3}{4a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{a^3x}+\frac{7i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i}{}}\right.\right.}{4a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^3x}+\frac{a^2(ad-cb)\left(\frac{x^3\sqrt{b\,x^4+a}}{6a\,b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3}{4a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{4a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.}{}$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*x^3/b^2*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/4/a^3*x^3*(a*d-3*b
*c)/((x^4+a/b)*b)^(1/2)-1/a^3*c*(b*x^4+a)^(1/2)/x+I*(-1/4/a^3*(a*d-3*b*c)+
b/a^3*c)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a
^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx =$$

$$-\frac{3\left(\left(7\,b^3c-ab^2d\right)x^9+2\left(7\,ab^2c-a^2bd\right)x^5+\left(7\,a^2bc-a^3d\right)x\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,\middle|\,-1\right)-3\left(\left(7\right.\right.}{}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(3*((7*b^3*c - a*b^2*d)*x^9 + 2*(7*a*b^2*c - a^2*b*d)*x^5 + (7*a^2*b
*c - a^3*d)*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1)
 - 3*((7*b^3*c - a*b^2*d)*x^9 + 2*(7*a*b^2*c - a^2*b*d)*x^5 + (7*a^2*b*c -
 a^3*d)*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (
3*(7*b^3*c - a*b^2*d)*x^8 + 5*(7*a*b^2*c - a^2*b*d)*x^4 + 12*a^2*b*c)*sqrt
(b*x^4 + a))/(a^3*b^3*x^9 + 2*a^4*b^2*x^5 + a^5*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.25

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx=\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x\Gamma\left(\frac{3}{4}\right)}+\frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(5/2),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/2)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(5/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{d\,x^4+c}{x^2\,\left(b\,x^4+a\right)^{5/2}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(5/2)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{14}+3a\,b^2x^{10}+3a^2b\,x^6+a^3x^2}dx\right)a^3dx+7\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{14}+3a\,b^2x^{10}+3a^2b\,x^6+a^3x}\right.}{}$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - int(sqrt(a + b*x**4)/(a**3*x**2 + 3*a**2*b*x**6 +
 3*a*b**2*x**10 + b**3*x**14),x)*a**3*d*x + 7*int(sqrt(a + b*x**4)/(a**3*x
**2 + 3*a**2*b*x**6 + 3*a*b**2*x**10 + b**3*x**14),x)*a**2*b*c*x - 2*int(s
qrt(a + b*x**4)/(a**3*x**2 + 3*a**2*b*x**6 + 3*a*b**2*x**10 + b**3*x**14),
x)*a**2*b*d*x**5 + 14*int(sqrt(a + b*x**4)/(a**3*x**2 + 3*a**2*b*x**6 + 3*
a*b**2*x**10 + b**3*x**14),x)*a*b**2*c*x**5 - int(sqrt(a + b*x**4)/(a**3*x
**2 + 3*a**2*b*x**6 + 3*a*b**2*x**10 + b**3*x**14),x)*a*b**2*d*x**9 + 7*in
t(sqrt(a + b*x**4)/(a**3*x**2 + 3*a**2*b*x**6 + 3*a*b**2*x**10 + b**3*x**1
4),x)*b**3*c*x**9)/(7*b*x*(a**2 + 2*a*b*x**4 + b**2*x**8))
```

3.55 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 357

$$\begin{aligned}\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx = &-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}-\frac{11bc-5ad}{30a^2x\left(a+bx^4\right)^{3/2}}\\&-\frac{7(11bc-5ad)}{60a^3x\sqrt{a+bx^4}}+\frac{7(11bc-5ad)\sqrt{a+bx^4}}{20a^4x}-\frac{7\sqrt{b}(11bc-5ad)x\sqrt{a+bx^4}}{20a^4\left(\sqrt{a}+\sqrt{b}x^2\right)}\\&+\frac{7\sqrt[4]{b}(11bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{20a^{15/4}\sqrt{a+bx^4}}\\&-\frac{7\sqrt[4]{b}(11bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{40a^{15/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(3/2)-1/30*(-5*a*d+11*b*c)/a^2/x/(b*x^4+a)^(3/2)-7/
60*(-5*a*d+11*b*c)/a^3/x/(b*x^4+a)^(1/2)+7/20*(-5*a*d+11*b*c)*(b*x^4+a)^(1
/2)/a^4/x-7/20*b^(1/2)*(-5*a*d+11*b*c)*x*(b*x^4+a)^(1/2)/a^4/(a^(1/2)+b^(1
/2)*x^2)+7/20*b^(1/4)*(-5*a*d+11*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/a^(15/4)/(b*x^4+a)^(1/2)-7/40*b^(1/4)*(-5*a*d+11*b*c)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(15/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.23

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx=\frac{-a^2c+(11bc-5ad)x^4(a+bx^4)\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^3x^5\left(a+bx^4\right)^{3/2}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(5/2)),x]
```

output
```
(-(a^2*c) + (11*b*c - 5*a*d)*x^4*(a + b*x^4)*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[-1/4, 5/2, 3/4, -((b*x^4)/a)])/(5*a^3*x^5*(a + b*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {955, 819, 819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(11bc-5ad)\int \frac{1}{x^2(bx^4+a)^{5/2}}dx}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(11bc-5ad)\left(\frac{7\int \frac{1}{x^2\left(bx^4+a\right)^{3/2}}dx}{6a}+\frac{1}{6ax(a+bx^4)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax(a+bx^4)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$\big\downarrow$ 847

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax(a+bx^4)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$\big\downarrow$ 834

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax(a+bx^4)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax(a+bx^4)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$\downarrow$ 761

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+ \cdots\right)}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

$\downarrow$ 1510

$$-\frac{(11bc-5ad)\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.\cdots}{\sqrt[4]{b}\sqrt{a+bx^4}}\cdots}{\sqrt{b}}\cdots\right)}{a}\cdots\right)}{2a}\cdots\right)}{6a}\cdots\right)}{5a}\cdots$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{3/2}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(5/2)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(3/2)) - ((11*b*c - 5*a*d)*(1/(6*a*x*(a + b*x^4)
^(3/2)) + (7*(1/(2*a*x*Sqrt[a + b*x^4]) + (3*(-(Sqrt[a + b*x^4]/(a*x)) + (
b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1
/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*El
lipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))
/a))/(2*a)))/(6*a)))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.68

method	result
elliptic	$-\frac{c\sqrt{b\,x^{4}+a}}{5a^{3}x^{5}}-\frac{(5ad-13cb)\sqrt{b\,x^{4}+a}}{5a^{4}x}-\frac{x^{3}(ad-cb)\sqrt{b\,x^{4}+a}}{6a^{3}b\left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{b\,x^{3}(3ad-5cb)}{4a^{4}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{i\left(\frac{b(5ad-13cb)}{5a^{4}}+\frac{b(3ad-5cb)}{4a^{4}}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\cdots}{\cdots}$
default	$c\left(-\frac{\sqrt{b\,x^{4}+a}}{5a^{3}x^{5}}+\frac{13b\sqrt{b\,x^{4}+a}}{5a^{4}x}+\frac{x^{3}\sqrt{b\,x^{4}+a}}{6a^{3}\left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{5b^{2}x^{3}}{4a^{4}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{77ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\cdots\right.}{20a^{\frac{7}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\cdots\right.$
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(5ad\,x^{4}-13bc\,x^{4}+ac\right)}{5a^{4}x^{5}}+\frac{b\left(\frac{i(5ad-13cb)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}-5a(ad\cdots\right.}{\cdots}$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^3*c*(b*x^4+a)^(1/2)/x^5-1/5/a^4*(5*a*d-13*b*c)*(b*x^4+a)^(1/2)/x-1/
6/a^3*x^3/b*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/4*b/a^4*x^3*(3*a*d-5*b
*c)/((x^4+a/b)*b)^(1/2)+I*(1/5*b/a^4*(5*a*d-13*b*c)+1/4*b/a^4*(3*a*d-5*b*c
))*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)
*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx = \frac{21\left((11\,b^3c-5\,ab^2d)x^{13}+2\,(11\,ab^2c-5\,a^2bd)x^9+(11\,a^2bc-5\,a^3d)x^5\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E}{}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/2),x, algorithm="fricas")
```

output
```
1/60*(21*((11*b^3*c - 5*a*b^2*d)*x^13 + 2*(11*a*b^2*c - 5*a^2*b*d)*x^9 + (
11*a^2*b*c - 5*a^3*d)*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)
^(1/4)), -1) - 21*((11*b^3*c - 5*a*b^2*d)*x^13 + 2*(11*a*b^2*c - 5*a^2*b*d
)*x^9 + (11*a^2*b*c - 5*a^3*d)*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin
(x*(-b/a)^(1/4)), -1) + (21*(11*b^3*c - 5*a*b^2*d)*x^12 + 35*(11*a*b^2*c -
 5*a^2*b*d)*x^8 + 12*(11*a^2*b*c - 5*a^3*d)*x^4 - 12*a^3*c)*sqrt(b*x^4 + a
))/(a^4*b^2*x^13 + 2*a^5*b*x^9 + a^6*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 78.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.25

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx = \frac{c\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{5}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x^5\Gamma\left(-\frac{1}{4}\right)}+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{2}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(5/2),x)
```

output
```
c*gamma(-5/4)*hyper((-5/4, 5/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/2)*x**5*gamma(-1/4)) + d*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/2)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx=\int \frac{d\,x^4+c}{x^6\,\left(b\,x^4+a\right)^{5/2}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(5/2)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{18}+3a\,b^2x^{14}+3a^2b\,x^{10}+a^3x^6}dx\right)a^3d\,x^5+11\left(\int \frac{\sqrt{b\,x^4+a}}{b^3x^{18}+3a\,b^2x^{14}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 5*int(sqrt(a + b*x**4)/(a**3*x**6 + 3*a**2*b*x**1
0 + 3*a*b**2*x**14 + b**3*x**18),x)*a**3*d*x**5 + 11*int(sqrt(a + b*x**4)/
(a**3*x**6 + 3*a**2*b*x**10 + 3*a*b**2*x**14 + b**3*x**18),x)*a**2*b*c*x**
5 - 10*int(sqrt(a + b*x**4)/(a**3*x**6 + 3*a**2*b*x**10 + 3*a*b**2*x**14 +
 b**3*x**18),x)*a**2*b*d*x**9 + 22*int(sqrt(a + b*x**4)/(a**3*x**6 + 3*a**
2*b*x**10 + 3*a*b**2*x**14 + b**3*x**18),x)*a*b**2*c*x**9 - 5*int(sqrt(a +
 b*x**4)/(a**3*x**6 + 3*a**2*b*x**10 + 3*a*b**2*x**14 + b**3*x**18),x)*a*b
**2*d*x**13 + 11*int(sqrt(a + b*x**4)/(a**3*x**6 + 3*a**2*b*x**10 + 3*a*b*
*2*x**14 + b**3*x**18),x)*b**3*c*x**13)/(11*b*x**5*(a**2 + 2*a*b*x**4 + b*
*2*x**8))
```

3.56 $\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 238

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{a^3(bc-ad)x}{10b^5(a+bx^4)^{5/2}} - \frac{a^2(31bc-41ad)x}{60b^5(a+bx^4)^{3/2}} + \frac{a(41bc-79ad)x}{24b^5\sqrt{a+bx^4}} + \frac{(7bc-26ad)x\sqrt{a+bx^4}}{21b^5} + \frac{dx^5\sqrt{a+bx^4}}{7b^4} - \frac{13a^{3/4}(7bc-17ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{112b^{21/4}\sqrt{a+bx^4}}$$

output

```
1/10*a^3*(-a*d+b*c)*x/b^5/(b*x^4+a)^(5/2)-1/60*a^2*(-41*a*d+31*b*c)*x/b^5/
(b*x^4+a)^(3/2)+1/24*a*(-79*a*d+41*b*c)*x/b^5/(b*x^4+a)^(1/2)+1/21*(-26*a*
d+7*b*c)*x*(b*x^4+a)^(1/2)/b^5+1/7*d*x^5*(b*x^4+a)^(1/2)/b^4-13/112*a^(3/4
)*(-17*a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2
)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(21/4)/
(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-3315a^4dx+13a^2b^2x^5(252c-425dx^4)+39a^3bx(35c-204dx^4)+5ab^3x^9(455c-13$$

input
```
Integrate[(x^16*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(-3315*a^4*d*x + 13*a^2*b^2*x^5*(252*c - 425*d*x^4) + 39*a^3*b*x*(35*c - 2
04*d*x^4) + 5*a*b^3*x^9*(455*c - 136*d*x^4) + 40*b^4*x^13*(7*c + 3*d*x^4)
+ 195*a*(-7*b*c + 17*a*d)*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(840*b^5*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 817, 817, 817, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(7bc-17ad)\int \frac{x^{16}}{(bx^4+a)^{7/2}}dx}{7b}+\frac{dx^{17}}{7b(a+bx^4)^{5/2}}$$

$$\Big\downarrow \ 817$$

$$\frac{(7bc-17ad)\left(\frac{13\int \frac{x^{12}}{(bx^4+a)^{5/2}}dx}{10b}-\frac{x^{13}}{10b(a+bx^4)^{5/2}}\right)}{7b}+\frac{dx^{17}}{7b(a+bx^4)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-17ad)\left(\frac{13\left(\frac{3\int \frac{x^8}{\left(bx^4+a\right)^{3/2}}dx}{2b}-\frac{x^9}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{13}}{10b(a+bx^4)^{5/2}}\right)}{7b}+\frac{dx^{17}}{7b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-17ad)\left(\frac{13\left(\frac{3\left(\frac{5\int \frac{x^4}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{13}}{10b(a+bx^4)^{5/2}}\right)}{7b}+\frac{dx^{17}}{7b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{(7bc-17ad)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{13}}{10b(a+bx^4)^{5/2}}\right)}{7b}+\frac{dx^{17}}{7b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7bc-17ad)\left(\frac{13\left(\frac{3\left(\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}\right)}{2b}-\frac{x^9}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{\cdots}{10\cdots}\right)}{7b}$$

$$\frac{dx^{17}}{7b\left(a+bx^4\right)^{5/2}}$$

input `Int[(x^16*(c + d*x^4))/(a + b*x^4)^(7/2),x]`

output
```
(d*x^17)/(7*b*(a + b*x^4)^(5/2)) + ((7*b*c - 17*a*d)*(-1/10*x^13/(b*(a + b
*x^4)^(5/2)) + (13*(-1/6*x^9/(b*(a + b*x^4)^(3/2)) + (3*(-1/2*x^5/(b*Sqrt[
a + b*x^4]) + (5*((x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])))/(2*b)))/(2*b)))/(10*b)
))/(7*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.21

method	result
elliptic	$-\frac{a^3 x(ad-cb)\sqrt{b x^4+a}}{10b^8\left(x^4+\frac{a}{b}\right)^3}+\frac{a^2x(41ad-31cb)\sqrt{bx^4+a}}{60b^7\left(x^4+\frac{a}{b}\right)^2}-\frac{ax(79ad-41cb)}{24b^5\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{dx^5\sqrt{bx^4+a}}{7b^4}+\frac{\left(-\frac{3ad-cb}{b^4}-\frac{5da}{7b^4}\right)x\sqrt{bx^4+a}}{3b}+\cdots$
default	$c\left(\frac{a^3x\sqrt{bx^4+a}}{10b^7\left(x^4+\frac{a}{b}\right)^3}-\frac{31a^2x\sqrt{bx^4+a}}{60b^6\left(x^4+\frac{a}{b}\right)^2}+\frac{41ax}{24b^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{bx^4+a}}{3b^4}-\frac{13a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{8b^4\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}\right)\cdots$
risch	$-\frac{x(-3dbx^4+26ad-7cb)\sqrt{bx^4+a}}{21b^5}+\frac{a\left(\frac{152ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}}-42a(5ad-3cb)\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\cdots\right)\right)}{\cdots}$

input

```
int(x^16*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-1/10*a^3*x/b^8*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60*a^2*x*(41*a*d-3
1*b*c)/b^7*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/24/b^5*a*x*(79*a*d-41*b*c)/((x^4+
a/b)*b)^(1/2)+1/7*d*x^5*(b*x^4+a)^(1/2)/b^4+1/3*(-1/b^4*(3*a*d-b*c)-5/7/b^
4*d*a)/b*x*(b*x^4+a)^(1/2)+(3*a*(2*a*d-b*c)/b^5-1/24/b^5*a*(79*a*d-41*b*c)
-1/3*(-1/b^4*(3*a*d-b*c)-5/7/b^4*d*a)/b*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/
a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx =$$

$$-\frac{195\left(\left(7\,b^4c-17\,ab^3d\right)x^{12}+3\left(7\,ab^3c-17\,a^2b^2d\right)x^8+7\,a^3bc-17\,a^4d+3\left(7\,a^2b^2c-17\,a^3bd\right)x^4\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}\cdots}{\cdots}$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/840*(195*((7*b^4*c - 17*a*b^3*d)*x^12 + 3*(7*a*b^3*c - 17*a^2*b^2*d)*x^
8 + 7*a^3*b*c - 17*a^4*d + 3*(7*a^2*b^2*c - 17*a^3*b*d)*x^4)*sqrt(b)*(-a/b
)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - (120*b^4*d*x^17 + 40*(7*b
^4*c - 17*a*b^3*d)*x^13 + 325*(7*a*b^3*c - 17*a^2*b^2*d)*x^9 + 468*(7*a^2*
b^2*c - 17*a^3*b*d)*x^5 + 195*(7*a^3*b*c - 17*a^4*d)*x)*sqrt(b*x^4 + a))/(
b^8*x^12 + 3*a*b^7*x^8 + 3*a^2*b^6*x^4 + a^3*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**16*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{16}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^16/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{16}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^16/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{x^{16}\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^16*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^16*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-1105\sqrt{b\,x^4+a}\,a^4dx+455\sqrt{b\,x^4+a}\,a^3bcx-1989\sqrt{b\,x^4+a}\,a^3bd\,x^5+819\sqrt{b\,x^4+a}$$

input
```
int(x^16*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - 1105*sqrt(a + b*x**4)*a**4*d*x + 455*sqrt(a + b*x**4)*a**3*b*c*x - 198
9*sqrt(a + b*x**4)*a**3*b*d*x**5 + 819*sqrt(a + b*x**4)*a**2*b**2*c*x**5 -
 1105*sqrt(a + b*x**4)*a**2*b**2*d*x**9 + 455*sqrt(a + b*x**4)*a*b**3*c*x*
*9 - 85*sqrt(a + b*x**4)*a*b**3*d*x**13 + 35*sqrt(a + b*x**4)*b**4*c*x**13
 + 15*sqrt(a + b*x**4)*b**4*d*x**17 + 1105*int(sqrt(a + b*x**4)/(a**4 + 4*
a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**8*d -
455*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b*
*3*x**12 + b**4*x**16),x)*a**7*b*c + 3315*int(sqrt(a + b*x**4)/(a**4 + 4*a
**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**7*b*d*x
**4 - 1365*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
 4*a*b**3*x**12 + b**4*x**16),x)*a**6*b**2*c*x**4 + 3315*int(sqrt(a + b*x*
*4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16
),x)*a**6*b**2*d*x**8 - 1365*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 +
6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*b**3*c*x**8 + 1105
*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*
x**12 + b**4*x**16),x)*a**5*b**3*d*x**12 - 455*int(sqrt(a + b*x**4)/(a**4
+ 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*
b**4*c*x**12)/(105*b**5*(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12
))
```

3.57 $\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = -\frac{a^2(bc-ad)x}{10b^4(a+bx^4)^{5/2}} + \frac{a(21bc-31ad)x}{60b^4(a+bx^4)^{3/2}} - \frac{(15bc-41ad)x}{24b^4\sqrt{a+bx^4}} + \frac{dx\sqrt{a+bx^4}}{3b^4} + \frac{(3bc-13ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{16\sqrt[4]{a}b^{17/4}\sqrt{a+bx^4}}$$

output

```
-1/10*a^2*(-a*d+b*c)*x/b^4/(b*x^4+a)^(5/2)+1/60*a*(-31*a*d+21*b*c)*x/b^4/(
b*x^4+a)^(3/2)-1/24*(-41*a*d+15*b*c)*x/b^4/(b*x^4+a)^(1/2)+1/3*d*x*(b*x^4+
a)^(1/2)/b^4+1/16*(-13*a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2
)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^
(1/2))/a^(1/4)/b^(17/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x\Big(195a^3d-9a^2b(5c-52dx^4)+5b^3x^8(-15c+8dx^4)+ab^2x^4(-108c+325dx^4)+15}{120b^4\,(a+bx^4)}$$

input
```
Integrate[(x^12*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x*(195*a^3*d - 9*a^2*b*(5*c - 52*d*x^4) + 5*b^3*x^8*(-15*c + 8*d*x^4) + a
*b^2*x^4*(-108*c + 325*d*x^4) + 15*(3*b*c - 13*a*d)*(a + b*x^4)^2*Sqrt[1 +
 (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)]))/(120*b^4*(a +
 b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 817, 817, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(3bc-13ad)\int \frac{x^{12}}{(bx^4+a)^{7/2}}dx}{3b}+\frac{dx^{13}}{3b\,(a+bx^4)^{5/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(3bc-13ad)\left(\frac{9\int \frac{x^8}{\left(bx^4+a\right)^{5/2}}dx}{10b}-\frac{x^9}{10b(a+bx^4)^{5/2}}\right)}{3b}+\frac{dx^{13}}{3b\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 817

$$\frac{(3bc-13ad)\left(\frac{9\left(\frac{5\int \frac{x^4}{\left(bx^4+a\right)^{3/2}}dx}{6b}-\frac{x^5}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^9}{10b(a+bx^4)^{5/2}}\right)}{3b}+\frac{dx^{13}}{3b\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 817

$$\frac{(3bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^9}{10b(a+bx^4)^{5/2}}\right)}{3b}+\frac{dx^{13}}{3b\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 761

$$\frac{(3bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^9}{10b(a+bx^4)^{5/2}}\right)}{3b}+\frac{dx^{13}}{3b\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(x^12*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(d*x^13)/(3*b*(a + b*x^4)^(5/2)) + ((3*b*c - 13*a*d)*(-1/10*x^9/(b*(a + b*
x^4)^(5/2)) + (9*(-1/6*x^5/(b*(a + b*x^4)^(3/2)) + (5*(-1/2*x/(b*Sqrt[a +
b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(5/4)*Sqr
t[a + b*x^4])))/(6*b)))/(10*b)))/(3*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.96 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{a^{2} x (a d-c b) \sqrt{b\,x^{4}+a}}{10 b^{7} \left(x^{4}+\frac{a}{b}\right)^{3}}-\frac{a x (31 a d-21 c b) \sqrt{b\,x^{4}+a}}{60 b^{6} \left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{x (41 a d-15 c b)}{24 b^{4} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{d x \sqrt{b\,x^{4}+a}}{3 b^{4}}+\frac{\left(-\frac{3 a d-c b}{b^{4}}+\frac{41 a d-15 c b}{24 b^{4}}-\frac{d a}{3 b^{4}}\right) \sqrt{1-\frac{i \sqrt{b}}{\sqrt{}}}}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\,\sqrt{}}$
default	$c \left(-\frac{a^{2} x \sqrt{b\,x^{4}+a}}{10 b^{6} \left(x^{4}+\frac{a}{b}\right)^{3}}+\frac{7 a x \sqrt{b\,x^{4}+a}}{20 b^{5} \left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{5 x}{8 b^{3} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{3 \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{8 b^{3} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}\right)+d \left(\frac{a^{3} x \sqrt{}}{10 b^{7} (}\right.$
risch	$\frac{d x \sqrt{b\,x^{4}+a}}{3 b^{4}}-\frac{\frac{10 a d \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}-9 a (2 a d-c b) \left(\frac{x}{2 a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{}\right.}{2 a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}\right.}{}$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*a^2*x/b^7*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60*a*x*(31*a*d-21*b
*c)/b^6*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/24/b^4*x*(41*a*d-15*b*c)/((x^4+a/b)*
b)^(1/2)+1/3*d*x*(b*x^4+a)^(1/2)/b^4+(-1/b^4*(3*a*d-b*c)+1/24/b^4*(41*a*d-
15*b*c)-1/3/b^4*d*a)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{15\left((3\,b^4c-13\,ab^3d)x^{12}+3\,(3\,ab^3c-13\,a^2b^2d)x^8+3\,a^3bc-13\,a^4d+3\,(3\,a^2b^2c-13\right.}{}$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(15*((3*b^4*c - 13*a*b^3*d)*x^12 + 3*(3*a*b^3*c - 13*a^2*b^2*d)*x^8
+ 3*a^3*b*c - 13*a^4*d + 3*(3*a^2*b^2*c - 13*a^3*b*d)*x^4)*sqrt(b)*(-a/b)^
(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (40*a*b^3*d*x^13 - 25*(3*a*
b^3*c - 13*a^2*b^2*d)*x^9 - 36*(3*a^2*b^2*c - 13*a^3*b*d)*x^5 - 15*(3*a^3*
b*c - 13*a^4*d)*x)*sqrt(b*x^4 + a))/(a*b^7*x^12 + 3*a^2*b^6*x^8 + 3*a^3*b^
5*x^4 + a^4*b^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 175.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.39

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{13}{4},\frac{7}{2}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{17}{4}\right)} + \frac{dx^{17}\Gamma\left(\frac{17}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{17}{4}\\ \frac{21}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{21}{4}\right)}$$

input
```
integrate(x**12*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**13*gamma(13/4)*hyper((13/4, 7/2), (17/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(7/2)*gamma(17/4)) + d*x**17*gamma(17/4)*hyper((7/2, 17/4), (21/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(21/4))
```

Maxima [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{x^{12}\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\frac{65\sqrt{b\,x^4+a}\,a^3dx-15\sqrt{b\,x^4+a}\,a^2bcx+117\sqrt{b\,x^4+a}\,a^2bd\,x^5-27\sqrt{b\,x^4+a}\,a\,b^2c\,x}{}$$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output

```
(65*sqrt(a + b*x**4)*a**3*d*x - 15*sqrt(a + b*x**4)*a**2*b*c*x + 117*sqrt(
a + b*x**4)*a**2*b*d*x**5 - 27*sqrt(a + b*x**4)*a*b**2*c*x**5 + 65*sqrt(a
+ b*x**4)*a*b**2*d*x**9 - 15*sqrt(a + b*x**4)*b**3*c*x**9 + 5*sqrt(a + b*x
**4)*b**3*d*x**13 - 65*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2
*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**7*d + 15*int(sqrt(a + b*x*
*4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16
),x)*a**6*b*c - 195*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b*
*2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**6*b*d*x**4 + 45*int(sqrt(a +
b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x
**16),x)*a**5*b**2*c*x**4 - 195*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4
 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*b**2*d*x**8 + 4
5*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3
*x**12 + b**4*x**16),x)*a**4*b**3*c*x**8 - 65*int(sqrt(a + b*x**4)/(a**4 +
 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b
**3*d*x**12 + 15*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*
x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**4*c*x**12)/(15*b**4*(a**3 +
 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12))
```

3.58 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{a(bc-ad)x}{10b^3(a+bx^4)^{5/2}} - \frac{(11bc-21ad)x}{60b^3(a+bx^4)^{3/2}} + \frac{(bc-15ad)x}{24ab^3\sqrt{a+bx^4}} + \frac{(bc+9ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{48a^{5/4}b^{13/4}\sqrt{a+bx^4}}$$

output
```
1/10*a*(-a*d+b*c)*x/b^3/(b*x^4+a)^(5/2)-1/60*(-21*a*d+11*b*c)*x/b^3/(b*x^4
+a)^(3/2)+1/24*(-15*a*d+b*c)*x/a/b^3/(b*x^4+a)^(1/2)+1/48*(9*a*d+b*c)*(a^(
1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/b^(13/4)/(b*x^4+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.70

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-x(45a^3d-5b^3cx^8+3ab^2x^4(4c+25dx^4)+a^2b(5c+108dx^4))+5(bc+9ad)x(a+b}{120ab^3\left(a+bx^4\right)^{5/2}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(-(x*(45*a^3*d - 5*b^3*c*x^8 + 3*a*b^2*x^4*(4*c + 25*d*x^4) + a^2*b*(5*c +
 108*d*x^4))) + 5*(b*c + 9*a*d)*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(120*a*b^3*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {957, 817, 817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(c+dx^4\right)}{\left(a+bx^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(9ad+bc)\int \frac{x^8}{(bx^4+a)^{5/2}}dx}{10ab}+\frac{x^9(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9ad+bc)\left(\frac{5\int \frac{x^4}{\left(bx^4+a\right)^{3/2}}dx}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^9(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9ad+bc)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^9(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(9ad+bc)\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^5}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^9(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x^9)/(10*a*b*(a + b*x^4)^(5/2)) + ((b*c + 9*a*d)*(-1/6*x^5/(b
*(a + b*x^4)^(3/2)) + (5*(-1/2*x/(b*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*b^(5/4)*Sqrt[a + b*x^4])))/(6*b)))/(10*a
*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.07

method	result
elliptic	$-\frac{ax(ad-cb)\sqrt{b\,x^4+a}}{10b^6\left(x^4+\frac{a}{b}\right)^3}+\frac{x(21ad-11cb)\sqrt{b\,x^4+a}}{60b^5\left(x^4+\frac{a}{b}\right)^2}-\frac{x(15ad-cb)}{24b^3a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\left(\frac{d}{b^3}-\frac{15ad-cb}{24b^3a}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$c\left(\frac{ax\sqrt{b\,x^4+a}}{10b^5\left(x^4+\frac{a}{b}\right)^3}-\frac{11x\sqrt{b\,x^4+a}}{60b^4\left(x^4+\frac{a}{b}\right)^2}+\frac{x}{24b^2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{24b^2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\left(-\frac{a^2x}{10b^6}\right.$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*a*x/b^6*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60*x*(21*a*d-11*b*c)
/b^5*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/24/b^3/a*x*(15*a*d-b*c)/((x^4+a/b)*b)^(
1/2)+(d/b^3-1/24/b^3/a*(15*a*d-b*c))/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2
)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx =$$
$$-\frac{5\left((b^4c+9\,ab^3d)x^{12}+3\,(ab^3c+9\,a^2b^2d)x^8+a^3bc+9\,a^4d+3\,(a^2b^2c+9\,a^3bd)x^4\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\right.\right.}{120\,(ab^7x^{12}+3\,a^2b^6x^8+3}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(5*((b^4*c + 9*a*b^3*d)*x^12 + 3*(a*b^3*c + 9*a^2*b^2*d)*x^8 + a^3*
b*c + 9*a^4*d + 3*(a^2*b^2*c + 9*a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)*ellipt
ic_f(arcsin(x*(-b/a)^(1/4)), -1) - (5*(b^4*c - 15*a*b^3*d)*x^9 - 12*(a*b^3
*c + 9*a^2*b^2*d)*x^5 - 5*(a^2*b^2*c + 9*a^3*b*d)*x)*sqrt(b*x^4 + a))/(a*b
^7*x^12 + 3*a^2*b^6*x^8 + 3*a^3*b^5*x^4 + a^4*b^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 128.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.44

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{7}{2}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{13}{4}\right)} + \frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{13}{4},\frac{7}{2}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**9*gamma(9/4)*hyper((9/4, 7/2), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(7/2)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((13/4, 7/2), (17/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-45\sqrt{b\,x^4+a}\,a^2dx-5\sqrt{b\,x^4+a}\,abcx-81\sqrt{b\,x^4+a}\,abd\,x^5-9\sqrt{b\,x^4+a}\,b^2c\,x^5-45}{}$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - 45*sqrt(a + b*x**4)*a**2*d*x - 5*sqrt(a + b*x**4)*a*b*c*x - 81*sqrt(a
+ b*x**4)*a*b*d*x**5 - 9*sqrt(a + b*x**4)*b**2*c*x**5 - 45*sqrt(a + b*x**4
)*b**2*d*x**9 + 45*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**
2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**6*d + 5*int(sqrt(a + b*x**4)/(
a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*
a**5*b*c + 135*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x*
*8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*b*d*x**4 + 15*int(sqrt(a + b*x**
4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16)
,x)*a**4*b**2*c*x**4 + 135*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*
a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b**2*d*x**8 + 15*int
(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**1
2 + b**4*x**16),x)*a**3*b**3*c*x**8 + 45*int(sqrt(a + b*x**4)/(a**4 + 4*a*
*3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**3*d
*x**12 + 5*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**4*c*x**12)/(45*b**3*(a**3 + 3*a**
2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12))
```

3.59 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}} dx = -\frac{(bc-ad)x}{10b^2(a+bx^4)^{5/2}} + \frac{(bc-11ad)x}{60ab^2(a+bx^4)^{3/2}} + \frac{(bc+ad)x}{24a^2b^2\sqrt{a+bx^4}} + \frac{(bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{48a^{9/4}b^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/10*(-a*d+b*c)*x/b^2/(b*x^4+a)^(5/2)+1/60*(-11*a*d+b*c)*x/a/b^2/(b*x^4+a
)^(3/2)+1/24*(a*d+b*c)*x/a^2/b^2/(b*x^4+a)^(1/2)+1/48*(a*d+b*c)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4)/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-5a^3dx+5b^3cx^9+ab^2x^5(12c+5dx^4)-a^2bx(5c+12dx^4)+5(bc+ad)x(a+bx^4)^2\sqrt{}}{120a^2b^2\left(a+bx^4\right)^{5/2}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(-5*a^3*d*x + 5*b^3*c*x^9 + a*b^2*x^5*(12*c + 5*d*x^4) - a^2*b*x*(5*c + 12
*d*x^4) + 5*(b*c + a*d)*x*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(120*a^2*b^2*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {957, 817, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \ 957$$

$$\frac{(ad+bc)\int \frac{x^4}{(bx^4+a)^{5/2}}dx}{2ab}+\frac{x^5(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \ 817$$

$$\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(bx^4+a\right)^{3/2}}dx}{6b}-\frac{x}{6b(a+bx^4)^{3/2}}\right)}{2ab}+\frac{x^5(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\big\downarrow \text{749}$$

$$\frac{(ad+bc)\left(\frac{\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}}{6b}-\frac{x}{6b(a+bx^4)^{3/2}}\right)}{2ab}+\frac{x^5(bc-ad)}{10ab\,(a+bx^4)^{5/2}}$$

$$\big\downarrow \text{761}$$

$$\frac{(ad+bc)\left(\frac{\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}}{6b}-\frac{x}{6b(a+bx^4)^{3/2}}\right)}{2ab}+\frac{x^5(bc-ad)}{10ab\,(a+bx^4)^{5/2}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x^5)/(10*a*b*(a + b*x^4)^(5/2)) + ((b*c + a*d)*(-1/6*x/(b*(a
+ b*x^4)^(3/2)) + (x/(2*a*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b*x^4]))/(6*b)))/(2*a*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

method	result
elliptic	$\frac{x(ad-cb)\sqrt{b\,x^4+a}}{10b^5\left(x^4+\frac{a}{b}\right)^3}-\frac{x(11ad-cb)\sqrt{b\,x^4+a}}{60a\,b^4\left(x^4+\frac{a}{b}\right)^2}+\frac{x(ad+cb)}{24b^2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{(ad+cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{24b^2a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{x\sqrt{b\,x^4+a}}{10b^4\left(x^4+\frac{a}{b}\right)^3}+\frac{x\sqrt{b\,x^4+a}}{60a\,b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x}{24b\,a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{24b\,a^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+d\Big(\frac{a\cdots}{10b\cdots}$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x/b^5*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60/a*x*(11*a*d-b*c)/b^4
*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/24/b^2/a^2*x*(a*d+b*c)/((x^4+a/b)*b)^(1/2)+
1/24/b^2/a^2*(a*d+b*c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1
/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = -\frac{5\left((b^4c+ab^3d)x^{12}+3\,(ab^3c+a^2b^2d)x^8+a^3bc+a^4d+3\,(a^2b^2c+a^3bd)x^4\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right.\right.}{120\,(a^2b^6x^{12}+3\,a^3b^5x^8+3\,a^4b^4x}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(5*((b^4*c + a*b^3*d)*x^12 + 3*(a*b^3*c + a^2*b^2*d)*x^8 + a^3*b*c
+ a^4*d + 3*(a^2*b^2*c + a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arc
sin(x*(-b/a)^(1/4)), -1) - (5*(b^4*c + a*b^3*d)*x^9 + 12*(a*b^3*c - a^2*b^
2*d)*x^5 - 5*(a^2*b^2*c + a^3*b*d)*x)*sqrt(b*x^4 + a))/(a^2*b^6*x^12 + 3*a
^3*b^5*x^8 + 3*a^4*b^4*x^4 + a^5*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 141.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.44

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{7}{2}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**5*gamma(5/4)*hyper((5/4, 7/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(7/2)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((9/4, 7/2), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-5\sqrt{b\,x^4+a}\,adx - 5\sqrt{b\,x^4+a}\,bcx - 9\sqrt{b\,x^4+a}\,bd\,x^5 + 5\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2x^8+4a^3}\right.}{}$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - 5*sqrt(a + b*x**4)*a*d*x - 5*sqrt(a + b*x**4)*b*c*x - 9*sqrt(a + b*x**
4)*b*d*x**5 + 5*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x
**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*d + 5*int(sqrt(a + b*x**4)/(a**
4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**
4*b*c + 15*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b*d*x**4 + 15*int(sqrt(a + b*x**4)/(
a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*
a**3*b**2*c*x**4 + 15*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*
b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**2*d*x**8 + 15*int(sqrt
(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b
**4*x**16),x)*a**2*b**3*c*x**8 + 5*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x
**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**3*d*x**12
 + 5*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b
**3*x**12 + b**4*x**16),x)*a*b**4*c*x**12)/(45*b**2*(a**3 + 3*a**2*b*x**4
+ 3*a*b**2*x**8 + b**3*x**12))
```

3.60 $\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 187

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx = \frac{(bc-ad)x}{10ab(a+bx^4)^{5/2}} + \frac{(9bc+ad)x}{60a^2b(a+bx^4)^{3/2}} + \frac{(9bc+ad)x}{24a^3b\sqrt{a+bx^4}} + \frac{(9bc+ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{48a^{13/4}b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/10*(-a*d+b*c)*x/a/b/(b*x^4+a)^(5/2)+1/60*(a*d+9*b*c)*x/a^2/b/(b*x^4+a)^(
3/2)+1/24*(a*d+9*b*c)*x/a^3/b/(b*x^4+a)^(1/2)+1/48*(a*d+9*b*c)*(a^(1/2)+b^
(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(13/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx = \frac{-40a^3dx+(9bc+ad)x\left(4a^2+6a(a+bx^4)+15(a+bx^4)^2+15(a+bx^4)^2\sqrt{1+\frac{bx^4}{a}}\right.}{360a^3b(a+bx^4)^{5/2}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(7/2),x]
```

output
```
(-40*a^3*d*x + (9*b*c + a*d)*x*(4*a^2 + 6*a*(a + b*x^4) + 15*(a + b*x^4)^2
 + 15*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -
((b*x^4)/a)]))/(360*a^3*b*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {910, 749, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+9bc)\int \frac{1}{(bx^4+a)^{5/2}}dx}{10ab}+\frac{x(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(ad+9bc)\left(\frac{5\int \frac{1}{\left(bx^4+a\right)^{3/2}}dx}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(ad+9bc)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(ad+9bc)\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x)/(10*a*b*(a + b*x^4)^(5/2)) + ((9*b*c + a*d)*(x/(6*a*(a + b
*x^4)^(3/2)) + (5*(x/(2*a*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b*x^4])))/(6*a)))/(10*a*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.02

method	result
elliptic	$-\frac{x(ad-cb)\sqrt{b\,x^4+a}}{10a\,b^4\left(x^4+\frac{a}{b}\right)^3}+\frac{x(ad+9cb)\sqrt{b\,x^4+a}}{60a^2b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x(ad+9cb)}{24b\,a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{(ad+9cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{24a^3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{10a\,b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{3x\sqrt{b\,x^4+a}}{20a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{3x}{8a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{8a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+d\left(-\frac{}{10}\right.$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/a*x/b^4*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60/a^2*x*(a*d+9*b*c)
/b^3*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/24/b/a^3*x*(a*d+9*b*c)/((x^4+a/b)*b)^(1
/2)+1/24*(a*d+9*b*c)/a^3/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx =$$

$$-\frac{5\left((9\,b^4c+ab^3d)x^{12}+3\,(9\,ab^3c+a^2b^2d)x^8+9\,a^3bc+a^4d+3\,(9\,a^2b^2c+a^3bd)x^4\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\right.\right.}{120\,(a^3b^5x^{12}+3\,a^4b^4x^8+3}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(5*((9*b^4*c + a*b^3*d)*x^12 + 3*(9*a*b^3*c + a^2*b^2*d)*x^8 + 9*a^
3*b*c + a^4*d + 3*(9*a^2*b^2*c + a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)*ellipt
ic_f(arcsin(x*(-b/a)^(1/4)), -1) - (5*(9*b^4*c + a*b^3*d)*x^9 + 12*(9*a*b^
3*c + a^2*b^2*d)*x^5 + 5*(15*a^2*b^2*c - a^3*b*d)*x)*sqrt(b*x^4 + a))/(a^3
*b^5*x^12 + 3*a^4*b^4*x^8 + 3*a^5*b^3*x^4 + a^6*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 79.77 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/2)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 7/2), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(7/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx=\int \frac{d\,x^4+c}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(7/2),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,dx+\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2x^8+4a^3b\,x^4+a^4}dx\right)a^4d+9\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2x^8\cdots}\right.}{}$$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - sqrt(a + b*x**4)*d*x + int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*
a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*d + 9*int(sqrt(a + b
*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x*
*16),x)*a**3*b*c + 3*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b
**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b*d*x**4 + 27*int(sqrt(a +
 b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*
x**16),x)*a**2*b**2*c*x**4 + 3*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4
+ 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**2*d*x**8 + 27
*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*
x**12 + b**4*x**16),x)*a*b**3*c*x**8 + int(sqrt(a + b*x**4)/(a**4 + 4*a**3
*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a*b**3*d*x**1
2 + 9*int(sqrt(a + b*x**4)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*
b**3*x**12 + b**4*x**16),x)*b**4*c*x**12)/(9*b*(a**3 + 3*a**2*b*x**4 + 3*a
*b**2*x**8 + b**3*x**12))
```

3.61 $\int \frac{c+dx^4}{x^4(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{7/2}}\,dx = -\frac{c}{3ax^3\left(a+bx^4\right)^{5/2}} - \frac{(13bc-3ad)x}{30a^2\left(a+bx^4\right)^{5/2}} - \frac{(13bc-3ad)x}{20a^3\left(a+bx^4\right)^{3/2}} - \frac{(13bc-3ad)x}{8a^4\sqrt{a+bx^4}} - \frac{(13bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{16a^{17/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(5/2)-1/30*(-3*a*d+13*b*c)*x/a^2/(b*x^4+a)^(5/2)-1/
20*(-3*a*d+13*b*c)*x/a^3/(b*x^4+a)^(3/2)-1/8*(-3*a*d+13*b*c)*x/a^4/(b*x^4+
a)^(1/2)-1/16*(-3*a*d+13*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^
(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2
))/a^(17/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/2}}\,dx = \frac{-40a^3c-(13bc-3ad)x^4\left(4a^2+6a(a+bx^4)+15(a+bx^4)^2+15(a+bx^4)^2\sqrt{1+\dots}\right.}{120a^4x^3(a+bx^4)^{5/2}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(7/2)),x]
```

output
```
(-40*a^3*c - (13*b*c - 3*a*d)*x^4*(4*a^2 + 6*a*(a + b*x^4) + 15*(a + b*x^4
)^2 + 15*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((b*x^4)/a)]))/(120*a^4*x^3*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 749, 749, 749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(13bc-3ad)\int \frac{1}{(bx^4+a)^{7/2}}dx}{3a}-\frac{c}{3ax^3(a+bx^4)^{5/2}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(13bc-3ad)\left(\frac{9\int \frac{1}{(bx^4+a)^{5/2}}dx}{10a}+\frac{x}{10a(a+bx^4)^{5/2}}\right)}{3a}-\frac{c}{3ax^3(a+bx^4)^{5/2}}$$

$$\big\downarrow \text{749}$$

$$-\frac{(13bc-3ad)\left(\frac{9\left(\frac{5\int \frac{1}{\left(bx^4+a\right)^{3/2}}dx}{6a}+\frac{x}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x}{10a(a+bx^4)^{5/2}}\right)}{3a}-\frac{c}{3ax^3\left(a+bx^4\right)^{5/2}}$$

$$\big\downarrow \text{749}$$

$$-\frac{(13bc-3ad)\left(\frac{9\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x}{10a(a+bx^4)^{5/2}}\right)}{3a}-\frac{c}{3ax^3\left(a+bx^4\right)^{5/2}}$$

$$\big\downarrow \text{761}$$

$$-\frac{(13bc-3ad)\left(\frac{9\left(\frac{5\left(\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}\right)}{6a}+\frac{x}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x}{10a(a+bx^4)^{5/2}}\right)}{3a}-\frac{c}{3ax^3\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(7/2)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(5/2)) - ((13*b*c - 3*a*d)*(x/(10*a*(a + b*x^4)^
(5/2)) + (9*(x/(6*a*(a + b*x^4)^(3/2)) + (5*(x/(2*a*Sqrt[a + b*x^4]) + ((S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*b^(1/4)*Sqrt[a + b*x^4])
))/(6*a)))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05

method	result
elliptic	$\frac{x(ad-cb)\sqrt{b\,x^4+a}}{10a^2b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{x(9ad-19cb)\sqrt{b\,x^4+a}}{60a^3b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x(9ad-31cb)}{24a^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{3a^4x^3}+\frac{\left(\frac{9ad-31cb}{24a^4}-\frac{bc}{3a^4}\right)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{E}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$d\left(\frac{x\sqrt{b\,x^4+a}}{10a\,b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{3x\sqrt{b\,x^4+a}}{20a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{3x}{8a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{3\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{8a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}\right)+c\left(-\frac{}{10}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{3a^4x^3}-\frac{\frac{cb\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}-3a^3(ad-cb)\left(\frac{x\sqrt{b\,x^4+a}}{10a\,b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{3x\sqrt{b\,x^4+a}}{20a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{3x}{8a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\right.}{}$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/a^2*x/b^3*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60/a^3*x*(9*a*d-19*
b*c)/b^2*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/24/a^4*x*(9*a*d-31*b*c)/((x^4+a/b)*
b)^(1/2)-1/3/a^4*c*(b*x^4+a)^(1/2)/x^3+(1/24/a^4*(9*a*d-31*b*c)-1/3*b/a^4*
c)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{7/2}}\,dx=\frac{15\left(\left(13\,b^4c-3\,ab^3d\right)x^{15}+3\left(13\,ab^3c-3\,a^2b^2d\right)x^{11}+3\left(13\,a^2b^2c-3\,a^3bd\right)x^7+\left(13\,a\right.\right.}{}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(15*((13*b^4*c - 3*a*b^3*d)*x^15 + 3*(13*a*b^3*c - 3*a^2*b^2*d)*x^11
 + 3*(13*a^2*b^2*c - 3*a^3*b*d)*x^7 + (13*a^3*b*c - 3*a^4*d)*x^3)*sqrt(a)*
(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (15*(13*b^4*c - 3*a*
b^3*d)*x^12 + 36*(13*a*b^3*c - 3*a^2*b^2*d)*x^8 + 40*a^3*b*c + 25*(13*a^2*
b^2*c - 3*a^3*b*d)*x^4)*sqrt(b*x^4 + a))/(a^4*b^4*x^15 + 3*a^5*b^3*x^11 +
3*a^6*b^2*x^7 + a^7*b*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 164.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.40

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/2}}\,dx = \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{7}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}x^3\Gamma\left(\frac{1}{4}\right)} + \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(7/2),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 7/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/2)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 7/2), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(7/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{2}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{d\,x^4+c}{x^4\,\left(b\,x^4+a\right)^{7/2}}\,dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(7/2)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(7/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{7/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-3\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{20}+4a\,b^3x^{16}+6a^2b^2x^{12}+4a^3b\,x^8+a^4x^4}dx\right)a^4d\,x^3+13\left(\int \frac{}{b^4x^{20}+4a\,b^3x}\right.}{}$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(7/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 3*int(sqrt(a + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8
 + 6*a**2*b**2*x**12 + 4*a*b**3*x**16 + b**4*x**20),x)*a**4*d*x**3 + 13*in
t(sqrt(a + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8 + 6*a**2*b**2*x**12 + 4*a*b*
*3*x**16 + b**4*x**20),x)*a**3*b*c*x**3 - 9*int(sqrt(a + b*x**4)/(a**4*x**
4 + 4*a**3*b*x**8 + 6*a**2*b**2*x**12 + 4*a*b**3*x**16 + b**4*x**20),x)*a*
*3*b*d*x**7 + 39*int(sqrt(a + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8 + 6*a**2*
b**2*x**12 + 4*a*b**3*x**16 + b**4*x**20),x)*a**2*b**2*c*x**7 - 9*int(sqrt
(a + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8 + 6*a**2*b**2*x**12 + 4*a*b**3*x**
16 + b**4*x**20),x)*a**2*b**2*d*x**11 + 39*int(sqrt(a + b*x**4)/(a**4*x**4
 + 4*a**3*b*x**8 + 6*a**2*b**2*x**12 + 4*a*b**3*x**16 + b**4*x**20),x)*a*b
**3*c*x**11 - 3*int(sqrt(a + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8 + 6*a**2*b
**2*x**12 + 4*a*b**3*x**16 + b**4*x**20),x)*a*b**3*d*x**15 + 13*int(sqrt(a
 + b*x**4)/(a**4*x**4 + 4*a**3*b*x**8 + 6*a**2*b**2*x**12 + 4*a*b**3*x**16
 + b**4*x**20),x)*b**4*c*x**15)/(13*b*x**3*(a**3 + 3*a**2*b*x**4 + 3*a*b**
2*x**8 + b**3*x**12))
```

3.62 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx &= -\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}-\frac{17bc-7ad}{70a^2x^3\left(a+bx^4\right)^{5/2}} \\ &\quad -\frac{13(17bc-7ad)}{420a^3x^3\left(a+bx^4\right)^{3/2}}-\frac{39(17bc-7ad)}{280a^4x^3\sqrt{a+bx^4}}+\frac{13(17bc-7ad)\sqrt{a+bx^4}}{56a^5x^3} \\ &\quad +\frac{13b^{3/4}(17bc-7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{112a^{21/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(5/2)-1/70*(-7*a*d+17*b*c)/a^2/x^3/(b*x^4+a)^(5/2)-
13/420*(-7*a*d+17*b*c)/a^3/x^3/(b*x^4+a)^(3/2)-39/280*(-7*a*d+17*b*c)/a^4/
x^3/(b*x^4+a)^(1/2)+13/56*(-7*a*d+17*b*c)*(b*x^4+a)^(1/2)/a^5/x^3+13/112*b
^(3/4)*(-7*a*d+17*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(2
1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.35

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx=\frac{-3a^3c+(17bc-7ad)x^4(a+bx^4)^2\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{21a^4x^7\left(a+bx^4\right)^{5/2}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(7/2)),x]
```

output
```
(-3*a^3*c + (17*b*c - 7*a*d)*x^4*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hyperge
ometric2F1[-3/4, 7/2, 1/4, -((b*x^4)/a)])/(21*a^4*x^7*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 819, 819, 819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(17bc-7ad)\int \frac{1}{x^4(bx^4+a)^{7/2}}dx}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(17bc-7ad)\left(\frac{13\int \frac{1}{x^4\left(bx^4+a\right)^{5/2}}dx}{10a}+\frac{1}{10ax^3(a+bx^4)^{5/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(17bc-7ad)\left(\frac{13\left(\frac{3\int \frac{1}{x^4\left(bx^4+a\right)^{3/2}}dx}{2a}+\frac{1}{6ax^3\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax^3(a+bx^4)^{5/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 819

$$-\frac{(17bc-7ad)\left(\frac{13\left(\frac{3\left(\frac{5\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax^3(a+bx^4)^{5/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 847

$$-\frac{(17bc-7ad)\left(\frac{13\left(\frac{3\left(\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax^3(a+bx^4)^{5/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 761

$$-\frac{(17bc-7ad)\left(\frac{13\left(\frac{3\left(\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^4\right)^{3/2}}\right)}{10a}\cdots\right)}{7a}$$

$$\frac{c}{7ax^7\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(7/2)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(5/2)) - ((17*b*c - 7*a*d)*(1/(10*a*x^3*(a + b*x
^4)^(5/2)) + (13*(1/(6*a*x^3*(a + b*x^4)^(3/2)) + (3*(1/(2*a*x^3*Sqrt[a +
b*x^4]) + (5*(-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x
^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/(2*a)))/(2*a)))/(10*a))
)/(7*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.04

method	result
elliptic	$-\frac{x(ad-cb)\sqrt{b\,x^4+a}}{10a^3b^2\left(x^4+\frac{a}{b}\right)^3}-\frac{x(19ad-29cb)\sqrt{b\,x^4+a}}{60a^4b\left(x^4+\frac{a}{b}\right)^2}-\frac{bx(31ad-65cb)}{24a^5\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{7a^4x^7}-\frac{(7ad-26cb)\sqrt{b\,x^4+a}}{21a^5x^3}+\frac{\left(-\frac{b(31ad-65cb)}{24a^5}\right.}{}$
default	$c\left(\frac{x\sqrt{b\,x^4+a}}{10a^3b\left(x^4+\frac{a}{b}\right)^3}+\frac{29x\sqrt{b\,x^4+a}}{60a^4\left(x^4+\frac{a}{b}\right)^2}+\frac{65b^2x}{24a^5\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{7a^4x^7}+\frac{26b\sqrt{b\,x^4+a}}{21a^5x^3}+\frac{221b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{Ellip}}{56a^5\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+}}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(7ad\,x^4-26bc\,x^4+3ac)}{21a^5x^7}-\frac{b\left(\frac{7ad\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}+21a(ad-3cb)\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}}{}}}{}\right.\right.}{}$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/a^3*x/b^2*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60/a^4*x*(19*a*d-2
9*b*c)/b*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/24*b/a^5*x*(31*a*d-65*b*c)/((x^4+a/
b)*b)^(1/2)-1/7/a^4*c*(b*x^4+a)^(1/2)/x^7-1/21/a^5*(7*a*d-26*b*c)*(b*x^4+a
)^(1/2)/x^3+(-1/24*b/a^5*(31*a*d-65*b*c)-1/21*b/a^5*(7*a*d-26*b*c))/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x
^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx =$$

$$-\frac{195\left((17\,b^4c-7\,ab^3d)x^{19}+3\,(17\,ab^3c-7\,a^2b^2d)x^{15}+3\,(17\,a^2b^2c-7\,a^3bd)x^{11}+(17\,a^3bc-7\,a^4d)x^7\right)\sqrt{a}(}{}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/840*(195*((17*b^4*c - 7*a*b^3*d)*x^19 + 3*(17*a*b^3*c - 7*a^2*b^2*d)*x^
15 + 3*(17*a^2*b^2*c - 7*a^3*b*d)*x^11 + (17*a^3*b*c - 7*a^4*d)*x^7)*sqrt(
a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - (195*(17*b^4*c -
7*a*b^3*d)*x^16 + 468*(17*a*b^3*c - 7*a^2*b^2*d)*x^12 + 325*(17*a^2*b^2*c
- 7*a^3*b*d)*x^8 - 120*a^4*c + 40*(17*a^3*b*c - 7*a^4*d)*x^4)*sqrt(b*x^4 +
 a))/(a^5*b^3*x^19 + 3*a^6*b^2*x^15 + 3*a^7*b*x^11 + a^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{2}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{2}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{d\,x^4+c}{x^8\,\left(b\,x^4+a\right)^{7/2}}\,dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(7/2)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(7/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-7\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{24}+4a\,b^3x^{20}+6a^2b^2x^{16}+4a^3b\,x^{12}+a^4x^8}dx\right)a^4d\,x^7+17\left(\int \frac{}{b^4x^{24}+4a\,b^3 \cdots}\right.}{\cdots}$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(7/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 7*int(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**1
2 + 6*a**2*b**2*x**16 + 4*a*b**3*x**20 + b**4*x**24),x)*a**4*d*x**7 + 17*i
nt(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**12 + 6*a**2*b**2*x**16 + 4*a*
b**3*x**20 + b**4*x**24),x)*a**3*b*c*x**7 - 21*int(sqrt(a + b*x**4)/(a**4*
x**8 + 4*a**3*b*x**12 + 6*a**2*b**2*x**16 + 4*a*b**3*x**20 + b**4*x**24),x
)*a**3*b*d*x**11 + 51*int(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**12 + 6
*a**2*b**2*x**16 + 4*a*b**3*x**20 + b**4*x**24),x)*a**2*b**2*c*x**11 - 21*
int(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**12 + 6*a**2*b**2*x**16 + 4*a
*b**3*x**20 + b**4*x**24),x)*a**2*b**2*d*x**15 + 51*int(sqrt(a + b*x**4)/(
a**4*x**8 + 4*a**3*b*x**12 + 6*a**2*b**2*x**16 + 4*a*b**3*x**20 + b**4*x**
24),x)*a*b**3*c*x**15 - 7*int(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**12
 + 6*a**2*b**2*x**16 + 4*a*b**3*x**20 + b**4*x**24),x)*a*b**3*d*x**19 + 17
*int(sqrt(a + b*x**4)/(a**4*x**8 + 4*a**3*b*x**12 + 6*a**2*b**2*x**16 + 4*
a*b**3*x**20 + b**4*x**24),x)*b**4*c*x**19)/(17*b*x**7*(a**3 + 3*a**2*b*x*
*4 + 3*a*b**2*x**8 + b**3*x**12))
```

3.63 $\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 384

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = -\frac{(bc-ad)x^{15}}{10b^2(a+bx^4)^{5/2}} - \frac{(3bc-5ad)x^{11}}{12b^3(a+bx^4)^{3/2}} - \frac{(33bc-67ad)x^7}{24b^4\sqrt{a+bx^4}} + \frac{77(9bc-19ad)x^3\sqrt{a+bx^4}}{360b^5} + \frac{dx^7\sqrt{a+bx^4}}{9b^4} - \frac{77a(9bc-19ad)x\sqrt{a+bx^4}}{120b^{11/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{77a^{5/4}(9bc-19ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{120b^{23/4}\sqrt{a+bx^4}} - \frac{77a^{5/4}(9bc-19ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{240b^{23/4}\sqrt{a+bx^4}}$$

output
```
-1/10*(-a*d+b*c)*x^15/b^2/(b*x^4+a)^(5/2)-1/12*(-5*a*d+3*b*c)*x^11/b^3/(b*
x^4+a)^(3/2)-1/24*(-67*a*d+33*b*c)*x^7/b^4/(b*x^4+a)^(1/2)+77/360*(-19*a*d
+9*b*c)*x^3*(b*x^4+a)^(1/2)/b^5+1/9*d*x^7*(b*x^4+a)^(1/2)/b^4-77/120*a*(-1
9*a*d+9*b*c)*x*(b*x^4+a)^(1/2)/b^(11/2)/(a^(1/2)+b^(1/2)*x^2)+77/120*a^(5/
4)*(-19*a*d+9*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(23/4)/
(b*x^4+a)^(1/2)-77/240*a^(5/4)*(-19*a*d+9*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x
^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^
(1/4)),1/2*2^(1/2))/b^(23/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.40

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x^3\Big(1045a^4d-55a^3b(9c-19dx^4)+b^4x^{12}(9c+5dx^4)+15a^2b^2x^4(-33c+19dx^4)-c$$

input
```
Integrate[(x^18*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x^3*(1045*a^4*d - 55*a^3*b*(9*c - 19*d*x^4) + b^4*x^12*(9*c + 5*d*x^4) +
15*a^2*b^2*x^4*(-33*c + 19*d*x^4) - a*b^3*x^8*(135*c + 19*d*x^4) - 55*a*(-
9*b*c + 19*a*d)*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 7
/2, 7/4, -((b*x^4)/a)]))/(45*b^5*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {959, 817, 817, 817, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{18}\left(c+dx^4\right)}{\left(a+bx^4\right)^{7/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(9bc-19ad)\int \frac{x^{18}}{\left(bx^4+a\right)^{7/2}}dx}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9bc-19ad)\left(\frac{3\int \frac{x^{14}}{\left(bx^4+a\right)^{5/2}}dx}{2b}-\frac{x^{15}}{10b\left(a+bx^4\right)^{5/2}}\right)}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9bc-19ad)\left(\frac{3\left(\frac{11\int \frac{x^{10}}{\left(bx^4+a\right)^{3/2}}dx}{6b}-\frac{x^{11}}{6b\left(a+bx^4\right)^{3/2}}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^4\right)^{5/2}}\right)}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(9bc-19ad)\left(\frac{3\left(\frac{11\left(\frac{7\int \frac{x^6}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^4\right)^{3/2}}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^4\right)^{5/2}}\right)}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{(9bc-19ad)\left(\frac{3\left(\frac{11\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int\frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^4\right)^{3/2}}\right)}{2b}-\frac{x^{15}}{10b(a+bx^4)^{5/2}}\right)}{9b}+$$

$$\frac{dx^{19}}{9b\,(a+bx^4)^{5/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{(9bc-19ad)\left(\frac{3\left(\frac{11\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^4\right)^{3/2}}\right)}{2b}-\frac{x^{15}}{10b(a+bx^4)^{5/2}}\right)}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{(9bc-19ad)\left(\frac{3\left(\frac{11\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^4\right)^{3/2}}\right)}{2b}-\frac{x^{15}}{10b(a+bx^4)^{5/2}}\right)}{9b}+\frac{dx^{19}}{9b\left(a+bx^4\right)^{5/2}}$$

$\big\downarrow$ 761

$$(9bc-19ad)\left(\frac{3\left(\frac{11\left(\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}\right)}{6b}-\cdots}{2b}\right.$$

↓ 1510

$$(9bc-19ad)\frac{3\frac{11\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\cdots}{\sqrt[4]{b}\sqrt{a+bx^4}\cdots}\cdots\right)}{5b}\cdots\right)}{2b}\cdots}{6b}\cdots}{2b}\cdots$$

input
```
Int[(x^18*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(d*x^19)/(9*b*(a + b*x^4)^(5/2)) + ((9*b*c - 19*a*d)*(-1/10*x^15/(b*(a + b
*x^4)^(5/2)) + (3*(-1/6*x^11/(b*(a + b*x^4)^(3/2)) + (11*(-1/2*x^7/(b*Sqrt
[a + b*x^4]) + (7*((x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-((-((x*Sqrt[a + b*
x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)
], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)))/(2*b)))/(6*b)
))/(2*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.99 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.84

method	result
elliptic	$-\frac{a^{3} x^{3} \left(a d -c b \right) \sqrt{b \,x^{4}+a}}{10 b^{8} \left(x^{4}+\frac{a}{b}\right)^{3}}+\frac{a^{2} x^{3} \left(43 a d -33 c b \right) \sqrt{b \,x^{4}+a}}{60 b^{7} \left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{a \,x^{3} \left(157 a d -87 c b \right)}{40 b^{5} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{d \,x^{7} \sqrt{b \,x^{4}+a}}{9 b^{4}}+\frac{\left(-\frac{3 a d -c b}{b^{4}}-\frac{7 d a}{9 b^{4}}\right) x^{3} \sqrt{b \,x^{4}+a}}{5 b}$
default	$c \left(\frac{a^{3} x^{3} \sqrt{b \,x^{4}+a}}{10 b^{7} \left(x^{4}+\frac{a}{b}\right)^{3}}-\frac{11 a^{2} x^{3} \sqrt{b \,x^{4}+a}}{20 b^{6} \left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{87 a \,x^{3}}{40 b^{4} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b^{4}}-\frac{231 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\cdots}\right.\right.}{40 b^{\frac{9}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+\cdots}}\right.$
risch	$-\frac{x^{3} \left(-5 d b \,x^{4}+34 a d -9 c b \right) \sqrt{b \,x^{4}+a}}{45 b^{5}}+\frac{a \left(\frac{i \left(124 a d -54 c b \right) \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}\, \sqrt{b}}+15\cdots\right.}{\cdots}$

input
```
int(x^18*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^3*x^3/b^8*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60*a^2*x^3*(43*a
*d-33*b*c)/b^7*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/40/b^5*a*x^3*(157*a*d-87*b*c)
/((x^4+a/b)*b)^(1/2)+1/9*d*x^7*(b*x^4+a)^(1/2)/b^4+1/5*(-1/b^4*(3*a*d-b*c)
-7/9/b^4*d*a)/b*x^3*(b*x^4+a)^(1/2)+I*(3*a*(2*a*d-b*c)/b^5+1/40/b^5*a*(157
*a*d-87*b*c)-3/5*(-1/b^4*(3*a*d-b*c)-7/9/b^4*d*a)/b*a)*a^(1/2)/(I/a^(1/2)*
b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(
1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.01

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx =$$

$$-\frac{231\left((9\,ab^4c-19\,a^2b^3d)x^{13}+3\,(9\,a^2b^3c-19\,a^3b^2d)x^9+3\,(9\,a^3b^2c-19\,a^4bd)x^5+(9\,a^4bc-19\,a^5d)x\right)\sqrt{b}}{}$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/360*(231*((9*a*b^4*c - 19*a^2*b^3*d)*x^13 + 3*(9*a^2*b^3*c - 19*a^3*b^2
*d)*x^9 + 3*(9*a^3*b^2*c - 19*a^4*b*d)*x^5 + (9*a^4*b*c - 19*a^5*d)*x)*sqr
t(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) - 231*((9*a*b^4*c
 - 19*a^2*b^3*d)*x^13 + 3*(9*a^2*b^3*c - 19*a^3*b^2*d)*x^9 + 3*(9*a^3*b^2*
c - 19*a^4*b*d)*x^5 + (9*a^4*b*c - 19*a^5*d)*x)*sqrt(b)*(-a/b)^(3/4)*ellip
tic_f(arcsin((-a/b)^(1/4)/x), -1) - (40*b^5*d*x^20 + 8*(9*b^5*c - 19*a*b^4
*d)*x^16 - 120*(9*a*b^4*c - 19*a^2*b^3*d)*x^12 - 517*(9*a^2*b^3*c - 19*a^3
*b^2*d)*x^8 - 2079*a^4*b*c + 4389*a^5*d - 616*(9*a^3*b^2*c - 19*a^4*b*d)*x
^4)*sqrt(b*x^4 + a))/(b^9*x^13 + 3*a*b^8*x^9 + 3*a^2*b^7*x^5 + a^3*b^6*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**18*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{18}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^18/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{18}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^18/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{x^{18}\,(d\,x^4+c)}{{(b\,x^4+a)}^{7/2}}\,dx$$

input
```
int((x^18*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^18*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\frac{1045\sqrt{b\,x^4+a}\,a^4d\,x^3-495\sqrt{b\,x^4+a}\,a^3bc\,x^3+1045\sqrt{b\,x^4+a}\,a^3bd\,x^7-495\sqrt{b\,x^4+}}{}$$

input
```
int(x^18*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
(1045*sqrt(a + b*x**4)*a**4*d*x**3 - 495*sqrt(a + b*x**4)*a**3*b*c*x**3 +
1045*sqrt(a + b*x**4)*a**3*b*d*x**7 - 495*sqrt(a + b*x**4)*a**2*b**2*c*x**
7 + 285*sqrt(a + b*x**4)*a**2*b**2*d*x**11 - 135*sqrt(a + b*x**4)*a*b**3*c
*x**11 - 19*sqrt(a + b*x**4)*a*b**3*d*x**15 + 9*sqrt(a + b*x**4)*b**4*c*x*
*15 + 5*sqrt(a + b*x**4)*b**4*d*x**19 - 3135*int((sqrt(a + b*x**4)*x**2)/(
a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*
a**8*d + 1485*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b
**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**7*b*c - 9405*int((sqrt(a + b
*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b
**4*x**16),x)*a**7*b*d*x**4 + 4455*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a
**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**6*b**2*
c*x**4 - 9405*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b
**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**6*b**2*d*x**8 + 4455*int((sq
rt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x
**12 + b**4*x**16),x)*a**5*b**3*c*x**8 - 3135*int((sqrt(a + b*x**4)*x**2)/
(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)
*a**5*b**3*d*x**12 + 1485*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**
4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b**4*c*x**12)/
(45*b**5*(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12))
```

3.64 $\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 350

$$\begin{aligned}\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx &= -\frac{(bc-ad)x^{11}}{10b^2(a+bx^4)^{5/2}}-\frac{(11bc-21ad)x^7}{60b^3(a+bx^4)^{3/2}} \\ &-\frac{(77bc-207ad)x^3}{120b^4\sqrt{a+bx^4}}+\frac{dx^3\sqrt{a+bx^4}}{5b^4}+\frac{77(bc-3ad)x\sqrt{a+bx^4}}{40b^{9/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ &-\frac{77\sqrt[4]{a}(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40b^{19/4}\sqrt{a+bx^4}} \\ &+\frac{77\sqrt[4]{a}(bc-3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80b^{19/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/10*(-a*d+b*c)*x^11/b^2/(b*x^4+a)^(5/2)-1/60*(-21*a*d+11*b*c)*x^7/b^3/(b
*x^4+a)^(3/2)-1/120*(-207*a*d+77*b*c)*x^3/b^4/(b*x^4+a)^(1/2)+1/5*d*x^3*(b
*x^4+a)^(1/2)/b^4+77/40*(-3*a*d+b*c)*x*(b*x^4+a)^(1/2)/b^(9/2)/(a^(1/2)+b^
(1/2)*x^2)-77/40*a^(1/4)*(-3*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^
(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2
*2^(1/2))/b^(19/4)/(b*x^4+a)^(1/2)+77/80*a^(1/4)*(-3*a*d+b*c)*(a^(1/2)+b^(
1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arct
an(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(19/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.37

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x^3\Big(-165a^3d+5ab^2x^4(11c-9dx^4)+55a^2b(c-3dx^4)+3b^3x^8(5c+dx^4)+55(-bc-}{15b^4\,(a+bx^4)^{5/2}}$$

input
```
Integrate[(x^14*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x^3*(-165*a^3*d + 5*a*b^2*x^4*(11*c - 9*d*x^4) + 55*a^2*b*(c - 3*d*x^4) +
 3*b^3*x^8*(5*c + d*x^4) + 55*(-(b*c) + 3*a*d)*(a + b*x^4)^2*Sqrt[1 + (b*x
^4)/a]*Hypergeometric2F1[3/4, 7/2, 7/4, -((b*x^4)/a)]))/(15*b^4*(a + b*x^4
)^(5/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {959, 817, 817, 817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(bc-3ad)\int \frac{x^{14}}{(bx^4+a)^{7/2}}dx}{b}+\frac{dx^{15}}{5b\,(a+bx^4)^{5/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(bc-3ad)\left(\frac{11\int \frac{x^{10}}{\left(bx^4+a\right)^{5/2}}dx}{10b}-\frac{x^{11}}{10b(a+bx^4)^{5/2}}\right)}{b}+\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$\Big\downarrow$ 817

$$\frac{(bc-3ad)\left(\frac{11\left(\frac{7\int \frac{x^6}{\left(bx^4+a\right)^{3/2}}dx}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{11}}{10b(a+bx^4)^{5/2}}\right)}{b}+\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$\Big\downarrow$ 817

$$\frac{(bc-3ad)\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{11}}{10b(a+bx^4)^{5/2}}\right)}{b}+\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$\Big\downarrow$ 834

$$\frac{(bc-3ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{11}}{10b(a+bx^4)^{5/2}}\right)}{b}+\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(bc-3ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}-\frac{x^{11}}{10b(a+bx^4)^{5/2}}\right)}{b}+\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \ 761$$

$$(bc-3ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10b}- \cdots\right)$$

$$\overline{\qquad\qquad\qquad\qquad b \qquad\qquad\qquad\qquad}$$

$$\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{(bc-3ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}}{2b}}{6b}}{10b}}{b}$$

$$\frac{dx^{15}}{5b\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(x^14*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(d*x^15)/(5*b*(a + b*x^4)^(5/2)) + ((b*c - 3*a*d)*(-1/10*x^11/(b*(a + b*x^
4)^(5/2)) + (11*(-1/6*x^7/(b*(a + b*x^4)^(3/2)) + (7*(-1/2*x^3/(b*Sqrt[a +
 b*x^4]) + (3*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4
)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt
[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a
 + b*x^4])))/(2*b)))/(6*b)))/(10*b)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.10 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

method	result
elliptic	$\frac{a^{2} x^{3} (a d - c b) \sqrt{b x^{4}+a}}{10 b^{7} \left(x^{4}+\frac{a}{b}\right)^{3}}-\frac{a x^{3} (33 a d - 23 c b) \sqrt{b x^{4}+a}}{60 b^{6} \left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{x^{3} (87 a d - 37 c b)}{40 b^{4} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{d x^{3} \sqrt{b x^{4}+a}}{5 b^{4}}+\frac{i \left(-\frac{3 a d - c b}{b^{4}}-\frac{87 a d - 37 c b}{40 b^{4}}-\frac{3 d a}{5 b^{4}}\right) \sqrt{a}}{}$
default	$c \left(-\frac{a^{2} x^{3} \sqrt{b x^{4}+a}}{10 b^{6} \left(x^{4}+\frac{a}{b}\right)^{3}}+\frac{23 a x^{3} \sqrt{b x^{4}+a}}{60 b^{5} \left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{37 x^{3}}{40 b^{3} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{77 i \sqrt{a} \sqrt{1-\frac{i \sqrt{b} x^{2}}{\sqrt{a}}} \sqrt{1+\frac{i \sqrt{b} x^{2}}{\sqrt{a}}} \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{Elliptic}\right.}{40 b^{\frac{7}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{b x^{4}+a}}\right.$
risch	$\frac{d x^{3} \sqrt{b x^{4}+a}}{5 b^{4}}-\frac{\frac{i (18 a d - 5 c b) \sqrt{a} \sqrt{1-\frac{i \sqrt{b} x^{2}}{\sqrt{a}}} \sqrt{1+\frac{i \sqrt{b} x^{2}}{\sqrt{a}}} \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{b}}{\sqrt{a}}} \sqrt{b x^{4}+a} \sqrt{b}}-15 a (2 a d - c b) \left(\frac{x^{3}}{2 a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}-\right.}{}$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*a^2*x^3/b^7*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60*a*x^3*(33*a*d-
23*b*c)/b^6*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/40/b^4*x^3*(87*a*d-37*b*c)/((x^4
+a/b)*b)^(1/2)+1/5*d*x^3*(b*x^4+a)^(1/2)/b^4+I*(-1/b^4*(3*a*d-b*c)-1/40/b^
4*(87*a*d-37*b*c)-3/5/b^4*d*a)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1
/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(
1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{231\left((b^4c-3\,ab^3d)x^{13}+3\,(ab^3c-3\,a^2b^2d)x^9+3\,(a^2b^2c-3\,a^3bd)x^5+(a^3bc-3\,a^4d)x\right.}{}$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(231*((b^4*c - 3*a*b^3*d)*x^13 + 3*(a*b^3*c - 3*a^2*b^2*d)*x^9 + 3*(
a^2*b^2*c - 3*a^3*b*d)*x^5 + (a^3*b*c - 3*a^4*d)*x)*sqrt(b)*(-a/b)^(3/4)*e
lliptic_e(arcsin((-a/b)^(1/4)/x), -1) - 231*((b^4*c - 3*a*b^3*d)*x^13 + 3*
(a*b^3*c - 3*a^2*b^2*d)*x^9 + 3*(a^2*b^2*c - 3*a^3*b*d)*x^5 + (a^3*b*c - 3
*a^4*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (
24*b^4*d*x^16 + 120*(b^4*c - 3*a*b^3*d)*x^12 + 517*(a*b^3*c - 3*a^2*b^2*d)
*x^8 + 231*a^3*b*c - 693*a^4*d + 616*(a^2*b^2*c - 3*a^3*b*d)*x^4)*sqrt(b*x
^4 + a))/(b^8*x^13 + 3*a*b^7*x^9 + 3*a^2*b^6*x^5 + a^3*b^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**14*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{x^{14}\,\left(d\,x^4+c\right)}{{\left(b\,x^4+a\right)}^{7/2}}\,dx$$

input
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-165\sqrt{b\,x^4+a}\,a^3d\,x^3+55\sqrt{b\,x^4+a}\,a^2bc\,x^3-165\sqrt{b\,x^4+a}\,a^2bd\,x^7+55\sqrt{b\,x^4+a}\,a}{}$$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - 165*sqrt(a + b*x**4)*a**3*d*x**3 + 55*sqrt(a + b*x**4)*a**2*b*c*x**3 -
 165*sqrt(a + b*x**4)*a**2*b*d*x**7 + 55*sqrt(a + b*x**4)*a*b**2*c*x**7 -
45*sqrt(a + b*x**4)*a*b**2*d*x**11 + 15*sqrt(a + b*x**4)*b**3*c*x**11 + 3*
sqrt(a + b*x**4)*b**3*d*x**15 + 495*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*
a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**7*d -
165*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
 4*a*b**3*x**12 + b**4*x**16),x)*a**6*b*c + 1485*int((sqrt(a + b*x**4)*x**
2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16)
,x)*a**6*b*d*x**4 - 495*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4
+ 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*b**2*c*x**4 + 14
85*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
4*a*b**3*x**12 + b**4*x**16),x)*a**5*b**2*d*x**8 - 495*int((sqrt(a + b*x**
4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*
x**16),x)*a**4*b**3*c*x**8 + 495*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**
3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b**3*d*
x**12 - 165*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**
2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**4*c*x**12)/(15*b**4*(a**3
 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x**12))
```

3.65 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 334

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = -\frac{(bc-ad)x^7}{10b^2(a+bx^4)^{5/2}} - \frac{(7bc-17ad)x^3}{60b^3(a+bx^4)^{3/2}} + \frac{(7bc-37ad)x^3}{40ab^3\sqrt{a+bx^4}} - \frac{7(bc-11ad)x\sqrt{a+bx^4}}{40ab^{7/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{7(bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40a^{3/4}b^{15/4}\sqrt{a+bx^4}} - \frac{7(bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80a^{3/4}b^{15/4}\sqrt{a+bx^4}}$$

output
```
-1/10*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(5/2)-1/60*(-17*a*d+7*b*c)*x^3/b^3/(b*x
^4+a)^(3/2)+1/40*(-37*a*d+7*b*c)*x^3/a/b^3/(b*x^4+a)^(1/2)-7/40*(-11*a*d+b
*c)*x*(b*x^4+a)^(1/2)/a/b^(7/2)/(a^(1/2)+b^(1/2)*x^2)+7/40*(-11*a*d+b*c)*(
a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(s
in(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(15/4)/(b*x^4+a)^(1
/2)-7/80*(-11*a*d+b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x
^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3
/4)/b^(15/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.34

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x^3\left(a(11a^2d-ab(c-11dx^4)+b^2x^4(-c+3dx^4))+(bc-11ad)\,(a+bx^4)^2\sqrt{1+\frac{bx^4}{a}}\right.}{3ab^3\,(a+bx^4)^{5/2}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x^3*(a*(11*a^2*d - a*b*(c - 11*d*x^4) + b^2*x^4*(-c + 3*d*x^4)) + (b*c -
11*a*d)*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 7/2, 7/4,
 -((b*x^4)/a)]))/(3*a*b^3*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 817, 817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{11}(bc-ad)}{10ab\,(a+bx^4)^{5/2}} - \frac{(bc-11ad)\int \frac{x^{10}}{(bx^4+a)^{5/2}}dx}{10ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{11}(bc-ad)}{10ab\,(a+bx^4)^{5/2}} - \frac{(bc-11ad)\left(\frac{7\int \frac{x^6}{(bx^4+a)^{3/2}}dx}{6b} - \frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{10ab}$$

$$\Big\downarrow \; 817$$

$$\frac{x^{11}(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}-\frac{(bc-11ad)\left(\frac{7\left(\frac{3\int\frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{10ab}$$

$$\Big\downarrow \; 834$$

$$\frac{x^{11}(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}-\frac{(bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{10ab}$$

$$\Big\downarrow \; 27$$

$$\frac{x^{11}(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}-\frac{(bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b(a+bx^4)^{3/2}}\right)}{10ab}$$

$$\Big\downarrow \; 761$$

$$\frac{x^{11}(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}-\frac{(bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}\right)}{6b}-\frac{x^7}{6b\left(a+bx^4\right)^{3/2}}\right)}{10ab}$$

$\downarrow$ 1510

$$\frac{x^{11}(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}-\frac{(bc-11ad)\left(\frac{7\left(\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right)}{2b}\right)}{6b}\right)}{10ab}$$

input
```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x^11)/(10*a*b*(a + b*x^4)^(5/2)) - ((b*c - 11*a*d)*(-1/6*x^7/
(b*(a + b*x^4)^(3/2)) + (7*(-1/2*x^3/(b*Sqrt[a + b*x^4]) + (3*(-((-((x*Sqr
t[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(2*b)))/(6*b)
))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.68

method	result
elliptic	$-\frac{a\,x^{3}(ad-cb)\sqrt{b\,x^{4}+a}}{10b^{6}\left(x^{4}+\frac{a}{b}\right)^{3}}+\frac{x^{3}(23ad-13cb)\sqrt{b\,x^{4}+a}}{60b^{5}\left(x^{4}+\frac{a}{b}\right)^{2}}-\frac{x^{3}(37ad-7cb)}{40b^{3}a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{i\left(\frac{d}{b^{3}}+\frac{37ad-7cb}{40b^{3}a}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{Ell}\ldots\right.}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}\ldots}}$
default	$c\left(\frac{a\,x^{3}\sqrt{b\,x^{4}+a}}{10b^{5}\left(x^{4}+\frac{a}{b}\right)^{3}}-\frac{13x^{3}\sqrt{b\,x^{4}+a}}{60b^{4}\left(x^{4}+\frac{a}{b}\right)^{2}}+\frac{7x^{3}}{40b^{2}a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{7i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\ldots}\right.\right.}{40b^{\frac{5}{2}}\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right.$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*a*x^3/b^6*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60*x^3*(23*a*d-13*
b*c)/b^5*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/40/b^3/a*x^3*(37*a*d-7*b*c)/((x^4+a
/b)*b)^(1/2)+I*(d/b^3+1/40*(37*a*d-7*b*c)/b^3/a)*a^(1/2)/(I/a^(1/2)*b^(1/2
))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(
b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE
(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.98

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx =$$

$$-\frac{21\left((b^4c-11\,ab^3d)x^{13}+3\,(ab^3c-11\,a^2b^2d)x^9+3\,(a^2b^2c-11\,a^3bd)x^5+(a^3bc-11\,a^4d)x\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(a}{}$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(21*((b^4*c - 11*a*b^3*d)*x^13 + 3*(a*b^3*c - 11*a^2*b^2*d)*x^9 + 3
*(a^2*b^2*c - 11*a^3*b*d)*x^5 + (a^3*b*c - 11*a^4*d)*x)*sqrt(b)*(-a/b)^(3/
4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) - 21*((b^4*c - 11*a*b^3*d)*x^13
+ 3*(a*b^3*c - 11*a^2*b^2*d)*x^9 + 3*(a^2*b^2*c - 11*a^3*b*d)*x^5 + (a^3*b
*c - 11*a^4*d)*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x),
-1) - (120*a*b^3*d*x^12 - 47*(a*b^3*c - 11*a^2*b^2*d)*x^8 - 21*a^3*b*c + 2
31*a^4*d - 56*(a^2*b^2*c - 11*a^3*b*d)*x^4)*sqrt(b*x^4 + a))/(a*b^7*x^13 +
 3*a^2*b^6*x^9 + 3*a^3*b^5*x^5 + a^4*b^4*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 173.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.24

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{11}{4},\frac{7}{2}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{15}{4}\right)} + \frac{dx^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**11*gamma(11/4)*hyper((11/4, 7/2), (15/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(7/2)*gamma(15/4)) + d*x**15*gamma(15/4)*hyper((7/2, 15/4), (19/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{11\sqrt{b\,x^4+a}\,a^2d\,x^3-\sqrt{b\,x^4+a}\,abc\,x^3+11\sqrt{b\,x^4+a}\,abd\,x^7-\sqrt{b\,x^4+a}\,b^2c\,x^7+3\sqrt{}}{}$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
(11*sqrt(a + b*x**4)*a**2*d*x**3 - sqrt(a + b*x**4)*a*b*c*x**3 + 11*sqrt(a
 + b*x**4)*a*b*d*x**7 - sqrt(a + b*x**4)*b**2*c*x**7 + 3*sqrt(a + b*x**4)*
b**2*d*x**11 - 33*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a*
*2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**6*d + 3*int((sqrt(a + b*
x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b*
*4*x**16),x)*a**5*b*c - 99*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x*
*4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*b*d*x**4 + 9*
int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a
*b**3*x**12 + b**4*x**16),x)*a**4*b**2*c*x**4 - 99*int((sqrt(a + b*x**4)*x
**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**1
6),x)*a**4*b**2*d*x**8 + 9*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x*
*4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**3*c*x**8 -
 33*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 +
 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**3*d*x**12 + 3*int((sqrt(a + b*x**
4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*
x**16),x)*a**2*b**4*c*x**12)/(3*b**3*(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8
 + b**3*x**12))
```

3.66 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 340

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = -\frac{(bc-ad)x^3}{10b^2(a+bx^4)^{5/2}} + \frac{(3bc-13ad)x^3}{60ab^2(a+bx^4)^{3/2}} + \frac{(3bc+7ad)x^3}{40a^2b^2\sqrt{a+bx^4}} - \frac{(3bc+7ad)x\sqrt{a+bx^4}}{40a^2b^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{(3bc+7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40a^{7/4}b^{11/4}\sqrt{a+bx^4}} - \frac{(3bc+7ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80a^{7/4}b^{11/4}\sqrt{a+bx^4}}$$

output
```
-1/10*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(5/2)+1/60*(-13*a*d+3*b*c)*x^3/a/b^2/(b
*x^4+a)^(3/2)+1/40*(7*a*d+3*b*c)*x^3/a^2/b^2/(b*x^4+a)^(1/2)-1/40*(7*a*d+3
*b*c)*x*(b*x^4+a)^(1/2)/a^2/b^(5/2)/(a^(1/2)+b^(1/2)*x^2)+1/40*(7*a*d+3*b*
c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ellipti
cE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(7/4)/b^(11/4)/(b*x^4+a
)^(1/2)-1/80*(7*a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/
a^(7/4)/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.29

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x^3\left(-a^2(3bc+7ad+7bdx^4)+(3bc+7ad)\left(a+bx^4\right)^2\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\,(\right.}{21a^2b^2\left(a+bx^4\right)^{5/2}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x^3*(-(a^2*(3*b*c + 7*a*d + 7*b*d*x^4)) + (3*b*c + 7*a*d)*(a + b*x^4)^2*S
qrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 7/2, 7/4, -((b*x^4)/a)]))/(21*a^
2*b^2*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 817, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^4\right)}{\left(a+bx^4\right)^{7/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7ad+3bc)\int \frac{x^6}{(bx^4+a)^{5/2}}dx}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7ad+3bc)\left(\frac{\int \frac{x^2}{\left(bx^4+a\right)^{3/2}}dx}{2b}-\frac{x^3}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 819$$

$$\frac{(7ad+3bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}}{2b}-\frac{x^3}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{(7ad+3bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2b}-\frac{x^3}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(7ad+3bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2b}-\frac{x^3}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{(7ad+3bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2b}-\frac{x^3}{6b(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{(7ad+3bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right.\right.\cdots}{\sqrt[4]{b}\sqrt{a+bx^4}}\cdots}{\sqrt{b}}\cdots}{2a}\cdots}{2b}\cdots\right.}{10ab\cdots}$$

$$\frac{x^7(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x^7)/(10*a*b*(a + b*x^4)^(5/2)) + ((3*b*c + 7*a*d)*(-1/6*x^3/
(b*(a + b*x^4)^(3/2)) + (x^3/(2*a*Sqrt[a + b*x^4]) - (-((-((x*Sqrt[a + b*x
^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
 b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4]))/(2*a))/(2*b)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.64

method	result
elliptic	$\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{10b^5\left(x^4+\frac{a}{b}\right)^3}-\frac{x^3(13ad-3cb)\sqrt{b\,x^4+a}}{60a\,b^4\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3(7ad+3cb)}{40b^2a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i(7ad+3cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.\right.}{40b^{\frac{5}{2}}a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
default	$c\left(-\frac{x^3\sqrt{b\,x^4+a}}{10b^4\left(x^4+\frac{a}{b}\right)^3}+\frac{x^3\sqrt{b\,x^4+a}}{20a\,b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{3x^3}{40b\,a^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{3i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\right.}{40b^{\frac{3}{2}}a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right.$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*x^3/b^5*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60/a*x^3*(13*a*d-3*b*
c)/b^4*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/40/b^2/a^2*x^3*(7*a*d+3*b*c)/((x^4+a/
b)*b)^(1/2)-1/40*I/b^(5/2)/a^(3/2)*(7*a*d+3*b*c)/(I/a^(1/2)*b^(1/2))^(1/2)
*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)
^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^
(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.95

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{3\left((3\,b^4c+7\,ab^3d)x^{12}+3\,(3\,ab^3c+7\,a^2b^2d)x^8+3\,a^3bc+7\,a^4d+3\,(3\,a^2b^2c+7\,a^3bd)x\right.}{}$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(3*((3*b^4*c + 7*a*b^3*d)*x^12 + 3*(3*a*b^3*c + 7*a^2*b^2*d)*x^8 + 3
*a^3*b*c + 7*a^4*d + 3*(3*a^2*b^2*c + 7*a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)
*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 3*((3*b^4*c + 7*a*b^3*d)*x^12 +
3*(3*a*b^3*c + 7*a^2*b^2*d)*x^8 + 3*a^3*b*c + 7*a^4*d + 3*(3*a^2*b^2*c + 7
*a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)
 + (3*(3*b^4*c + 7*a*b^3*d)*x^11 + 8*(3*a*b^3*c + 2*a^2*b^2*d)*x^7 + (3*a^
2*b^2*c + 7*a^3*b*d)*x^3)*sqrt(b*x^4 + a))/(a^2*b^6*x^12 + 3*a^3*b^5*x^8 +
 3*a^4*b^4*x^4 + a^5*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 114.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.24

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{11}{4},\frac{7}{2}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**7*gamma(7/4)*hyper((7/4, 7/2), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(7/2)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((11/4, 7/2), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{-7\sqrt{b\,x^4+a}\,ad\,x^3-3\sqrt{b\,x^4+a}\,bc\,x^3-7\sqrt{b\,x^4+a}\,bd\,x^7+21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2x^8\cdots}\right.}{\cdots}$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output
```
( - 7*sqrt(a + b*x**4)*a*d*x**3 - 3*sqrt(a + b*x**4)*b*c*x**3 - 7*sqrt(a +
 b*x**4)*b*d*x**7 + 21*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 +
 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**5*d + 9*int((sqrt(a
 + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12
 + b**4*x**16),x)*a**4*b*c + 63*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3
*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*b*d*x**4
 + 27*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8
 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*b**2*c*x**4 + 63*int((sqrt(a + b*x
**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**
4*x**16),x)*a**3*b**2*d*x**8 + 27*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a*
*3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**3*c
*x**8 + 21*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2
*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**3*d*x**12 + 9*int((sqrt(a
+ b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12
+ b**4*x**16),x)*a*b**4*c*x**12)/(21*b**2*(a**3 + 3*a**2*b*x**4 + 3*a*b**2
*x**8 + b**3*x**12))
```

3.67 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 343

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{(bc-ad)x^3}{10ab\,(a+bx^4)^{5/2}} + \frac{(7bc+3ad)x^3}{60a^2b\,(a+bx^4)^{3/2}} + \frac{(7bc+3ad)x^3}{40a^3b\sqrt{a+bx^4}} - \frac{(7bc+3ad)x\sqrt{a+bx^4}}{40a^3b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{(7bc+3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40a^{11/4}b^{7/4}\sqrt{a+bx^4}} - \frac{(7bc+3ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80a^{11/4}b^{7/4}\sqrt{a+bx^4}}$$

output
```
1/10*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(5/2)+1/60*(3*a*d+7*b*c)*x^3/a^2/b/(b*x^
4+a)^(3/2)+1/40*(3*a*d+7*b*c)*x^3/a^3/b/(b*x^4+a)^(1/2)-1/40*(3*a*d+7*b*c)
*x*(b*x^4+a)^(1/2)/a^3/b^(3/2)/(a^(1/2)+b^(1/2)*x^2)+1/40*(3*a*d+7*b*c)*(a
^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(11/4)/b^(7/4)/(b*x^4+a)^(1/
2)-1/80*(3*a*d+7*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^
2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(11
/4)/b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.36

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{x^3\left(-3a^3d+7bc(a+bx^4)^2\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},-\frac{bx^4}{a}\right)+3ad(a+b\right.}{21a^3b\,(a+bx^4)^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
(x^3*(-3*a^3*d + 7*b*c*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1
[3/4, 7/2, 7/4, -((b*x^4)/a)] + 3*a*d*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hy
pergeometric2F1[3/4, 7/2, 7/4, -((b*x^4)/a)]))/(21*a^3*b*(a + b*x^4)^(5/2)
)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 819, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{(3ad+7bc)\int \frac{x^2}{(bx^4+a)^{5/2}}dx}{10ab}+\frac{x^3(bc-ad)}{10ab\,(a+bx^4)^{5/2}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(3ad+7bc)\left(\frac{\int \frac{x^2}{\left(bx^4+a\right)^{3/2}}dx}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 819$$

$$\frac{(3ad+7bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{(3ad+7bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(3ad+7bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{(3ad+7bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a(a+bx^4)^{3/2}}\right)}{10ab}+\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{(3ad+7bc)\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}}{2a}}{2a}\right.}{10ab}$$

$$\frac{x^3(bc-ad)}{10ab\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(7/2),x]
```

output
```
((b*c - a*d)*x^3)/(10*a*b*(a + b*x^4)^(5/2)) + ((7*b*c + 3*a*d)*(x^3/(6*a*
(a + b*x^4)^(3/2)) + (x^3/(2*a*Sqrt[a + b*x^4]) - (-((-((x*Sqrt[a + b*x^4]
)/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2
)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4]))/(2*a))/(2*a)))/(10*a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.64

method	result
elliptic	$-\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{10a\,b^4\left(x^4+\frac{a}{b}\right)^3}+\frac{x^3(3ad+7cb)\sqrt{b\,x^4+a}}{60a^2b^3\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3(3ad+7cb)}{40b\,a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i(3ad+7cb)\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\right.\right.}{40b^{\frac{3}{2}}a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
default	$c\left(\frac{x^3\sqrt{b\,x^4+a}}{10a\,b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{7x^3\sqrt{b\,x^4+a}}{60a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{7x^3}{40a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{7i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(\right.\right.}{40a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}\right.$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/a*x^3/b^4*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60/a^2*x^3*(3*a*d+
7*b*c)/b^3*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/40/b/a^3*x^3*(3*a*d+7*b*c)/((x^4+
a/b)*b)^(1/2)-1/40*I/b^(3/2)/a^(5/2)*(3*a*d+7*b*c)/(I/a^(1/2)*b^(1/2))^(1/
2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*
b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{3\left((7\,b^4c+3\,ab^3d)x^{12}+3\,(7\,ab^3c+3\,a^2b^2d)x^8+7\,a^3bc+3\,a^4d+3\,(7\,a^2b^2c+3\,a^3bd)x\right.}{}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(3*((7*b^4*c + 3*a*b^3*d)*x^12 + 3*(7*a*b^3*c + 3*a^2*b^2*d)*x^8 + 7
*a^3*b*c + 3*a^4*d + 3*(7*a^2*b^2*c + 3*a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)
*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 3*((7*b^4*c + 3*a*b^3*d)*x^12 +
3*(7*a*b^3*c + 3*a^2*b^2*d)*x^8 + 7*a^3*b*c + 3*a^4*d + 3*(7*a^2*b^2*c + 3
*a^3*b*d)*x^4)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)
 + (3*(7*b^4*c + 3*a*b^3*d)*x^11 + 8*(7*a*b^3*c + 3*a^2*b^2*d)*x^7 + (47*a
^2*b^2*c + 3*a^3*b*d)*x^3)*sqrt(b*x^4 + a))/(a^3*b^5*x^12 + 3*a^4*b^4*x^8
+ 3*a^5*b^3*x^4 + a^6*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 81.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.23

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{7}{2} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(7/2),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(7/2)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((7/4, 7/2), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(7/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{7/2}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(7/2),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(7/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d\,x^3+3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2x^8+4a^3b\,x^4+a^4}dx\right)a^4d+7\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{16}+4a\,b^3x^{12}+6a^2b^2}\right.}{}$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(7/2),x)
```

output

```
( - sqrt(a + b*x**4)*d*x**3 + 3*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3
*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**4*d + 7*in
t((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b
**3*x**12 + b**4*x**16),x)*a**3*b*c + 9*int((sqrt(a + b*x**4)*x**2)/(a**4
+ 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**3*
b*d*x**4 + 21*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b
**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a**2*b**2*c*x**4 + 9*int((sqrt(
a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**1
2 + b**4*x**16),x)*a**2*b**2*d*x**8 + 21*int((sqrt(a + b*x**4)*x**2)/(a**4
 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a*b*
*3*c*x**8 + 3*int((sqrt(a + b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b
**2*x**8 + 4*a*b**3*x**12 + b**4*x**16),x)*a*b**3*d*x**12 + 7*int((sqrt(a
+ b*x**4)*x**2)/(a**4 + 4*a**3*b*x**4 + 6*a**2*b**2*x**8 + 4*a*b**3*x**12
+ b**4*x**16),x)*b**4*c*x**12)/(7*b*(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8
+ b**3*x**12))
```

3.68 $\int \frac{c+dx^4}{x^2(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 355

$$\begin{aligned}\int \frac{c+dx^4}{x^2(a+bx^4)^{7/2}}\,dx &= -\frac{c}{ax(a+bx^4)^{5/2}}-\frac{(11bc-ad)x^3}{10a^2(a+bx^4)^{5/2}}\\ &-\frac{7(11bc-ad)x^3}{60a^3(a+bx^4)^{3/2}}-\frac{7(11bc-ad)x^3}{40a^4\sqrt{a+bx^4}}+\frac{7(11bc-ad)x\sqrt{a+bx^4}}{40a^4\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)}\\ &-\frac{7(11bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40a^{15/4}b^{3/4}\sqrt{a+bx^4}}\\ &+\frac{7(11bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80a^{15/4}b^{3/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-c/a/x/(b*x^4+a)^(5/2)-1/10*(-a*d+11*b*c)*x^3/a^2/(b*x^4+a)^(5/2)-7/60*(-a
*d+11*b*c)*x^3/a^3/(b*x^4+a)^(3/2)-7/40*(-a*d+11*b*c)*x^3/a^4/(b*x^4+a)^(1
/2)+7/40*(-a*d+11*b*c)*x*(b*x^4+a)^(1/2)/a^4/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)
-7/40*(-a*d+11*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(15/4)
/b^(3/4)/(b*x^4+a)^(1/2)+7/80*(-a*d+11*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+
a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/
4)),1/2*2^(1/2))/a^(15/4)/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.24

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/2}}\,dx = -\frac{c}{ax(a+bx^4)^{5/2}} - \frac{(11bc-ad)x^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{7}{2},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^4\sqrt{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(7/2)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(5/2))) - ((11*b*c - a*d)*x^3*Sqrt[1 + (b*x^4)/a]*Hyp
ergeometric2F1[3/4, 7/2, 7/4, -((b*x^4)/a)])/(3*a^4*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {955, 819, 819, 819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(11bc-ad)\int \frac{x^2}{(bx^4+a)^{7/2}}dx}{a} - \frac{c}{ax(a+bx^4)^{5/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(11bc-ad)\left(\frac{7\int \frac{x^2}{(bx^4+a)^{5/2}}dx}{10a} + \frac{x^3}{10a(a+bx^4)^{5/2}}\right)}{a} - \frac{c}{ax(a+bx^4)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\int \frac{x^2}{\left(bx^4+a\right)^{3/2}}dx}{2a}+\frac{x^3}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x^3}{10a(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}}{2a}+\frac{x^3}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x^3}{10a(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{834}$$

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x^3}{10a(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{x^3}{10a(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 761

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}}{2a}+\frac{x^3}{6a\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{}{10a(a}\right.}{a}$$

$$\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 1510

$$-\frac{(11bc-ad)\left(\frac{7\left(\frac{\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\cdots\right)\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}\cdots}{\sqrt{b}}\cdots}{2a}}{2a}\cdots\right)}{10a}\cdots\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/2}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(7/2)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(5/2))) - ((11*b*c - a*d)*(x^3/(10*a*(a + b*x^4)^(5/2
)) + (7*(x^3/(6*a*(a + b*x^4)^(3/2)) + (x^3/(2*a*Sqrt[a + b*x^4]) - (-(((-(
(x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]
*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1
/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqr
t[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[
2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4]))/(2*a))/(
2*a)))/(10*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.69

method	result
elliptic	$\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{10a^2b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{x^3(7ad-17cb)\sqrt{b\,x^4+a}}{60a^3b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{x^3(7ad-37cb)}{40a^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{c\sqrt{b\,x^4+a}}{a^4x}+\frac{i\left(-\frac{7ad-37cb}{40a^4}+\frac{bc}{a^4}\right)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i}{}}}{\sqrt{\frac{i}{}}}$
default	$c\left(-\frac{x^3\sqrt{b\,x^4+a}}{10a^2b^2\left(x^4+\frac{a}{b}\right)^3}-\frac{17x^3\sqrt{b\,x^4+a}}{60a^3b\left(x^4+\frac{a}{b}\right)^2}-\frac{37b\,x^3}{40a^4\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{\sqrt{b\,x^4+a}}{a^4x}+\frac{77i\sqrt{b}\,\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{}\right.\right.}{40a^{\frac{7}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4}}\right.$
risch	$-\frac{c\sqrt{b\,x^4+a}}{a^4x}+\frac{a^3(ad-cb)\left(\frac{x^3\sqrt{b\,x^4+a}}{10a\,b^3\left(x^4+\frac{a}{b}\right)^3}+\frac{7x^3\sqrt{b\,x^4+a}}{60a^2b^2\left(x^4+\frac{a}{b}\right)^2}+\frac{7x^3}{40a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{7i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\right.}{40a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right.}{}$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/10/a^2*x^3/b^3*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3+1/60/a^3*x^3*(7*a*d
-17*b*c)/b^2*(b*x^4+a)^(1/2)/(x^4+a/b)^2+1/40/a^4*x^3*(7*a*d-37*b*c)/((x^4
+a/b)*b)^(1/2)-1/a^4*c*(b*x^4+a)^(1/2)/x+I*(-1/40/a^4*(7*a*d-37*b*c)+b/a^4
*c)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2
)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/2}}\,dx =$$

$$-\frac{21\left((11\,b^4c-ab^3d)x^{13}+3\,(11\,ab^3c-a^2b^2d)x^9+3\,(11\,a^2b^2c-a^3bd)x^5+(11\,a^3bc-a^4d)x\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\varepsilon}{}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
-1/120*(21*((11*b^4*c - a*b^3*d)*x^13 + 3*(11*a*b^3*c - a^2*b^2*d)*x^9 + 3
*(11*a^2*b^2*c - a^3*b*d)*x^5 + (11*a^3*b*c - a^4*d)*x)*sqrt(a)*(-b/a)^(3/
4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 21*((11*b^4*c - a*b^3*d)*x^13
+ 3*(11*a*b^3*c - a^2*b^2*d)*x^9 + 3*(11*a^2*b^2*c - a^3*b*d)*x^5 + (11*a^
3*b*c - a^4*d)*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) + (21*(11*b^4*c - a*b^3*d)*x^12 + 56*(11*a*b^3*c - a^2*b^2*d)*x^8 + 12
0*a^3*b*c + 47*(11*a^2*b^2*c - a^3*b*d)*x^4)*sqrt(b*x^4 + a))/(a^4*b^4*x^1
3 + 3*a^5*b^3*x^9 + 3*a^6*b^2*x^5 + a^7*b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 112.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.23

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{7/2}}\,dx = \frac{c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{7}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}x\Gamma\left(\frac{3}{4}\right)} + \frac{dx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(7/2),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 7/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/2)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 7/2), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(7/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{7/2}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{2}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{d\,x^4+c}{x^2\,\left(b\,x^4+a\right)^{7/2}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(7/2)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(7/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/2}}\,dx=\frac{-\sqrt{b\,x^4+a}\,d-\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{18}+4a\,b^3x^{14}+6a^2b^2x^{10}+4a^3b\,x^6+a^4x^2}dx\right)a^4dx+11\left(\int \frac{}{b^4x^{18}+4a\,b^3x^{14}+}\right.}{}$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(7/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - int(sqrt(a + b*x**4)/(a**4*x**2 + 4*a**3*b*x**6 +
 6*a**2*b**2*x**10 + 4*a*b**3*x**14 + b**4*x**18),x)*a**4*d*x + 11*int(sqr
t(a + b*x**4)/(a**4*x**2 + 4*a**3*b*x**6 + 6*a**2*b**2*x**10 + 4*a*b**3*x*
*14 + b**4*x**18),x)*a**3*b*c*x - 3*int(sqrt(a + b*x**4)/(a**4*x**2 + 4*a*
*3*b*x**6 + 6*a**2*b**2*x**10 + 4*a*b**3*x**14 + b**4*x**18),x)*a**3*b*d*x
**5 + 33*int(sqrt(a + b*x**4)/(a**4*x**2 + 4*a**3*b*x**6 + 6*a**2*b**2*x**
10 + 4*a*b**3*x**14 + b**4*x**18),x)*a**2*b**2*c*x**5 - 3*int(sqrt(a + b*x
**4)/(a**4*x**2 + 4*a**3*b*x**6 + 6*a**2*b**2*x**10 + 4*a*b**3*x**14 + b**
4*x**18),x)*a**2*b**2*d*x**9 + 33*int(sqrt(a + b*x**4)/(a**4*x**2 + 4*a**3
*b*x**6 + 6*a**2*b**2*x**10 + 4*a*b**3*x**14 + b**4*x**18),x)*a*b**3*c*x**
9 - int(sqrt(a + b*x**4)/(a**4*x**2 + 4*a**3*b*x**6 + 6*a**2*b**2*x**10 +
4*a*b**3*x**14 + b**4*x**18),x)*a*b**3*d*x**13 + 11*int(sqrt(a + b*x**4)/(
a**4*x**2 + 4*a**3*b*x**6 + 6*a**2*b**2*x**10 + 4*a*b**3*x**14 + b**4*x**1
8),x)*b**4*c*x**13)/(11*b*x*(a**3 + 3*a**2*b*x**4 + 3*a*b**2*x**8 + b**3*x
**12))
```

3.69 $\int \frac{c+dx^4}{x^6(a+bx^4)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 387

$$\begin{aligned}\int \frac{c+dx^4}{x^6(a+bx^4)^{7/2}}\,dx &= -\frac{c}{5ax^5(a+bx^4)^{5/2}} \\ &- \frac{3bc-ad}{10a^2x(a+bx^4)^{5/2}} - \frac{11(3bc-ad)}{60a^3x(a+bx^4)^{3/2}} - \frac{77(3bc-ad)}{120a^4x\sqrt{a+bx^4}} \\ &+ \frac{77(3bc-ad)\sqrt{a+bx^4}}{40a^5x} - \frac{77\sqrt{b}(3bc-ad)x\sqrt{a+bx^4}}{40a^5\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ &+ \frac{77\sqrt[4]{b}(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{40a^{19/4}\sqrt{a+bx^4}} \\ &- \frac{77\sqrt[4]{b}(3bc-ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{80a^{19/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(5/2)-1/10*(-a*d+3*b*c)/a^2/x/(b*x^4+a)^(5/2)-11/60
*(-a*d+3*b*c)/a^3/x/(b*x^4+a)^(3/2)-77/120*(-a*d+3*b*c)/a^4/x/(b*x^4+a)^(1
/2)+77/40*(-a*d+3*b*c)*(b*x^4+a)^(1/2)/a^5/x-77/40*b^(1/2)*(-a*d+3*b*c)*x*
(b*x^4+a)^(1/2)/a^5/(a^(1/2)+b^(1/2)*x^2)+77/40*b^(1/4)*(-a*d+3*b*c)*(a^(1
/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2
*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(19/4)/(b*x^4+a)^(1/2)-77/80*b^
(1/4)*(-a*d+3*b*c)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^
2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(19/4)
/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.21

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/2}} \, dx = \frac{-a^3c - 5(-3bc+ad)x^4(a+bx^4)^2 \sqrt{1+\frac{bx^4}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{7}{2}, \frac{3}{4}, -\frac{bx^4}{a}\right)}{5a^4x^5 (a+bx^4)^{5/2}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(7/2)),x]
```

output
```
(-(a^3*c) - 5*(-3*b*c + a*d)*x^4*(a + b*x^4)^2*Sqrt[1 + (b*x^4)/a]*Hyperge
ometric2F1[-1/4, 7/2, 3/4, -((b*x^4)/a)])/(5*a^4*x^5*(a + b*x^4)^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {955, 819, 819, 819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3bc-ad)\int \frac{1}{x^2(bx^4+a)^{7/2}}dx}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(3bc-ad)\left(\frac{11\int \frac{1}{x^2\left(bx^4+a\right)^{5/2}}dx}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(3bc-ad)\left(\frac{11\left(\frac{7\int \frac{1}{x^2\left(bx^4+a\right)^{3/2}}dx}{6a}+\frac{1}{6ax\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(3bc-ad)\left(\frac{11\left(\frac{7\left(\frac{3\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \text{847}$$

$$
-\frac{(3bc-ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}
$$

$\downarrow$ 834

$$-\frac{(3bc-ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{(3bc-ad)\left(\frac{11\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{1}{6ax\left(a+bx^4\right)^{3/2}}\right)}{10a}+\frac{1}{10ax(a+bx^4)^{5/2}}\right)}{c}-$$

$$a$$

$$\overline{5ax^5\left(a+bx^4\right)^{5/2}}$$

$\downarrow$ 761

$$(3bc-ad)\frac{11\left(\frac{7\left(\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}\right)}{6a}+\frac{\cdots}{6a\cdots}\cdots}{10a}$$

↓ 1510

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}} - \frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}$$

$$b\left(\ldots\right)$$

$$3\,\frac{\ldots}{a}$$

$$7\,\frac{\ldots}{2a}$$

$$11\,\frac{\ldots}{6a}$$

$$(3bc-ad)\,\frac{\ldots}{10a}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(7/2)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(5/2)) - ((3*b*c - a*d)*(1/(10*a*x*(a + b*x^4)^(
5/2)) + (11*(1/(6*a*x*(a + b*x^4)^(3/2)) + (7*(1/(2*a*x*Sqrt[a + b*x^4]) +
 (3*(-(Sqrt[a + b*x^4]/(a*x)) + (b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sq
rt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*S
qrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a))/(2*a)))/(6*a)))/(10*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.99 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.73

method	result
elliptic	$-\frac{c\sqrt{b\,x^4+a}}{5a^4x^5}-\frac{(5ad-18cb)\sqrt{b\,x^4+a}}{5a^5x}-\frac{x^3(ad-cb)\sqrt{b\,x^4+a}}{10a^3b^2\left(x^4+\frac{a}{b}\right)^3}-\frac{x^3(17ad-27cb)\sqrt{b\,x^4+a}}{60a^4b\left(x^4+\frac{a}{b}\right)^2}-\frac{b\,x^3(37ad-87cb)}{40a^5\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{i\left(\frac{b(5ad-18cb}{5a^5}\right.}{}$
default	$c\left(-\frac{\sqrt{b\,x^4+a}}{5a^4x^5}+\frac{18b\sqrt{b\,x^4+a}}{5a^5x}+\frac{x^3\sqrt{b\,x^4+a}}{10a^3b\left(x^4+\frac{a}{b}\right)^3}+\frac{9x^3\sqrt{b\,x^4+a}}{20a^4\left(x^4+\frac{a}{b}\right)^2}+\frac{87b^2x^3}{40a^5\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{231ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,(}{40a}\right.$
risch	$-\frac{\sqrt{b\,x^4+a}\,(5ad\,x^4-18bc\,x^4+ac)}{5a^5x^5}+\frac{b\left(\frac{i(5ad-18cb)\sqrt{a}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\,\sqrt{b}}-5a^2(ac\right.}{}$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^4*c*(b*x^4+a)^(1/2)/x^5-1/5/a^5*(5*a*d-18*b*c)*(b*x^4+a)^(1/2)/x-1/
10/a^3*x^3/b^2*(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+a/b)^3-1/60/a^4*x^3*(17*a*d-
27*b*c)/b*(b*x^4+a)^(1/2)/(x^4+a/b)^2-1/40*b/a^5*x^3*(37*a*d-87*b*c)/((x^4
+a/b)*b)^(1/2)+I*(1/5*b/a^5*(5*a*d-18*b*c)+1/40*b/a^5*(37*a*d-87*b*c))*a^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/
2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/2}}\,dx = \frac{231\left((3\,b^4c-ab^3d)x^{17}+3\,(3\,ab^3c-a^2b^2d)x^{13}+3\,(3\,a^2b^2c-a^3bd)x^9+(3\,a^3bc-a^4\right.}{}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/2),x, algorithm="fricas")
```

output
```
1/120*(231*((3*b^4*c - a*b^3*d)*x^17 + 3*(3*a*b^3*c - a^2*b^2*d)*x^13 + 3*
(3*a^2*b^2*c - a^3*b*d)*x^9 + (3*a^3*b*c - a^4*d)*x^5)*sqrt(a)*(-b/a)^(3/4
)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - 231*((3*b^4*c - a*b^3*d)*x^17 +
 3*(3*a*b^3*c - a^2*b^2*d)*x^13 + 3*(3*a^2*b^2*c - a^3*b*d)*x^9 + (3*a^3*b
*c - a^4*d)*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -
1) + (231*(3*b^4*c - a*b^3*d)*x^16 + 616*(3*a*b^3*c - a^2*b^2*d)*x^12 + 51
7*(3*a^2*b^2*c - a^3*b*d)*x^8 - 24*a^4*c + 120*(3*a^3*b*c - a^4*d)*x^4)*sq
rt(b*x^4 + a))/(a^5*b^3*x^17 + 3*a^6*b^2*x^13 + 3*a^7*b*x^9 + a^8*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/2}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{2}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/2}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{2}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/2}}\,dx=\int \frac{d\,x^4+c}{x^6\,\left(b\,x^4+a\right)^{7/2}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(7/2)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(7/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,d-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^4x^{22}+4a\,b^3x^{18}+6a^2b^2x^{14}+4a^3b\,x^{10}+a^4x^6}dx\right)a^4d\,x^5+15\left(\int \frac{}{b^4x^{22}+4a\,b^3}\right.}{}$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(7/2),x)
```

output
```
( - sqrt(a + b*x**4)*d - 5*int(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**1
0 + 6*a**2*b**2*x**14 + 4*a*b**3*x**18 + b**4*x**22),x)*a**4*d*x**5 + 15*i
nt(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**10 + 6*a**2*b**2*x**14 + 4*a*
b**3*x**18 + b**4*x**22),x)*a**3*b*c*x**5 - 15*int(sqrt(a + b*x**4)/(a**4*
x**6 + 4*a**3*b*x**10 + 6*a**2*b**2*x**14 + 4*a*b**3*x**18 + b**4*x**22),x
)*a**3*b*d*x**9 + 45*int(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**10 + 6*
a**2*b**2*x**14 + 4*a*b**3*x**18 + b**4*x**22),x)*a**2*b**2*c*x**9 - 15*in
t(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**10 + 6*a**2*b**2*x**14 + 4*a*b
**3*x**18 + b**4*x**22),x)*a**2*b**2*d*x**13 + 45*int(sqrt(a + b*x**4)/(a*
*4*x**6 + 4*a**3*b*x**10 + 6*a**2*b**2*x**14 + 4*a*b**3*x**18 + b**4*x**22
),x)*a*b**3*c*x**13 - 5*int(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**10 +
 6*a**2*b**2*x**14 + 4*a*b**3*x**18 + b**4*x**22),x)*a*b**3*d*x**17 + 15*i
nt(sqrt(a + b*x**4)/(a**4*x**6 + 4*a**3*b*x**10 + 6*a**2*b**2*x**14 + 4*a*
b**3*x**18 + b**4*x**22),x)*b**4*c*x**17)/(15*b*x**5*(a**3 + 3*a**2*b*x**4
 + 3*a*b**2*x**8 + b**3*x**12))
```

3.70 $\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{a^2(bc-ad)\left(a+bx^4\right)^{3/4}}{3b^4} - \frac{a(2bc-3ad)\left(a+bx^4\right)^{7/4}}{7b^4} + \frac{(bc-3ad)\left(a+bx^4\right)^{11/4}}{11b^4} + \frac{d\left(a+bx^4\right)^{15/4}}{15b^4}$$

output
```
1/3*a^2*(-a*d+b*c)*(b*x^4+a)^(3/4)/b^4-1/7*a*(-3*a*d+2*b*c)*(b*x^4+a)^(7/4
)/b^4+1/11*(-3*a*d+b*c)*(b*x^4+a)^(11/4)/b^4+1/15*d*(b*x^4+a)^(15/4)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{\left(a+bx^4\right)^{3/4}\left(160a^2bc-128a^3d-120ab^2cx^4+96a^2bdx^4+105b^3cx^8-84ab^2dx^8+77b^3dx^{12}\right)}{1155b^4}$$

input
```
Integrate[(x^11*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(160*a^2*b*c - 128*a^3*d - 120*a*b^2*c*x^4 + 96*a^2*b*d
*x^4 + 105*b^3*c*x^8 - 84*a*b^2*d*x^8 + 77*b^3*d*x^12))/(1155*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8(dx^4+c)}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{d(bx^4+a)^{11/4}}{b^3}+\frac{(bc-3ad)(bx^4+a)^{7/4}}{b^3}+\frac{a(3ad-2bc)(bx^4+a)^{3/4}}{b^3}-\frac{a^2(ad-bc)}{b^3\sqrt[4]{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^2(a+bx^4)^{3/4}(bc-ad)}{3b^4}+\frac{4(a+bx^4)^{11/4}(bc-3ad)}{11b^4}-\frac{4a(a+bx^4)^{7/4}(2bc-3ad)}{7b^4}+\frac{4d(a+bx^4)^{15/4}}{15b^4}\right)$$

input
```
Int[(x^11*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((4*a^2*(b*c - a*d)*(a + b*x^4)^(3/4))/(3*b^4) - (4*a*(2*b*c - 3*a*d)*(a +
 b*x^4)^(7/4))/(7*b^4) + (4*(b*c - 3*a*d)*(a + b*x^4)^(11/4))/(11*b^4) + (
4*d*(a + b*x^4)^(15/4))/(15*b^4))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{128\left(-\frac{105x^{8}\left(\frac{11d\,x^{4}}{15}+c\right)b^{3}}{128}+\frac{15\left(\frac{7d\,x^{4}}{10}+c\right)x^{4}a\,b^{2}}{16}-\frac{5\left(\frac{3d\,x^{4}}{5}+c\right)a^{2}b}{4}+a^{3}d\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{1155b^{4}}$	68
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-77b^{3}d\,x^{12}+84a\,b^{2}d\,x^{8}-105c\,b^{3}x^{8}-96a^{2}bd\,x^{4}+120a\,b^{2}c\,x^{4}+128a^{3}d-160a^{2}bc\right)}{1155b^{4}}$	77
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-77b^{3}d\,x^{12}+84a\,b^{2}d\,x^{8}-105c\,b^{3}x^{8}-96a^{2}bd\,x^{4}+120a\,b^{2}c\,x^{4}+128a^{3}d-160a^{2}bc\right)}{1155b^{4}}$	77
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-77b^{3}d\,x^{12}+84a\,b^{2}d\,x^{8}-105c\,b^{3}x^{8}-96a^{2}bd\,x^{4}+120a\,b^{2}c\,x^{4}+128a^{3}d-160a^{2}bc\right)}{1155b^{4}}$	77
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-77b^{3}d\,x^{12}+84a\,b^{2}d\,x^{8}-105c\,b^{3}x^{8}-96a^{2}bd\,x^{4}+120a\,b^{2}c\,x^{4}+128a^{3}d-160a^{2}bc\right)}{1155b^{4}}$	77

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-128/1155*(-105/128*x^8*(11/15*d*x^4+c)*b^3+15/16*(7/10*d*x^4+c)*x^4*a*b^2
-5/4*(3/5*d*x^4+c)*a^2*b+a^3*d)*(b*x^4+a)^(3/4)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$= \frac{(77\,b^3dx^{12}+21\,(5\,b^3c-4\,ab^2d)x^8-24\,(5\,ab^2c-4\,a^2bd)x^4+160\,a^2bc-128\,a^3d)(bx^4+a)^{\frac{3}{4}}}{1155\,b^4}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/1155*(77*b^3*d*x^12 + 21*(5*b^3*c - 4*a*b^2*d)*x^8 - 24*(5*a*b^2*c - 4*a
^2*b*d)*x^4 + 160*a^2*b*c - 128*a^3*d)*(b*x^4 + a)^(3/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.67

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$= \begin{cases} -\frac{128a^3d(a+bx^4)^{\frac{3}{4}}}{1155b^4}+\frac{32a^2c(a+bx^4)^{\frac{3}{4}}}{231b^3}+\frac{32a^2dx^4(a+bx^4)^{\frac{3}{4}}}{385b^3}-\frac{8acx^4(a+bx^4)^{\frac{3}{4}}}{77b^2}-\frac{4adx^8(a+bx^4)^{\frac{3}{4}}}{55b^2}+\frac{cx^8(a+bx^4)^{\frac{3}{4}}}{11b}+\frac{dx^{12}(a+}{15b} \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{16}}{16}}{\sqrt[4]{a}} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-128*a**3*d*(a + b*x**4)**(3/4)/(1155*b**4) + 32*a**2*c*(a + b*
x**4)**(3/4)/(231*b**3) + 32*a**2*d*x**4*(a + b*x**4)**(3/4)/(385*b**3) -
8*a*c*x**4*(a + b*x**4)**(3/4)/(77*b**2) - 4*a*d*x**8*(a + b*x**4)**(3/4)/
(55*b**2) + c*x**8*(a + b*x**4)**(3/4)/(11*b) + d*x**12*(a + b*x**4)**(3/4
)/(15*b), Ne(b, 0)), ((c*x**12/12 + d*x**16/16)/a**(1/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$=\frac{1}{1155}d\left(\frac{77\,(bx^4+a)^{\frac{15}{4}}}{b^4}-\frac{315\,(bx^4+a)^{\frac{11}{4}}a}{b^4}+\frac{495\,(bx^4+a)^{\frac{7}{4}}a^2}{b^4}-\frac{385\,(bx^4+a)^{\frac{3}{4}}a^3}{b^4}\right)$$
$$+\frac{1}{231}c\left(\frac{21\,(bx^4+a)^{\frac{11}{4}}}{b^3}-\frac{66\,(bx^4+a)^{\frac{7}{4}}a}{b^3}+\frac{77\,(bx^4+a)^{\frac{3}{4}}a^2}{b^3}\right)$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/1155*d*(77*(b*x^4 + a)^(15/4)/b^4 - 315*(b*x^4 + a)^(11/4)*a/b^4 + 495*(
b*x^4 + a)^(7/4)*a^2/b^4 - 385*(b*x^4 + a)^(3/4)*a^3/b^4) + 1/231*c*(21*(b
*x^4 + a)^(11/4)/b^3 - 66*(b*x^4 + a)^(7/4)*a/b^3 + 77*(b*x^4 + a)^(3/4)*a
^2/b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$=\frac{\frac{5\left(21\,(bx^4+a)^{\frac{11}{4}}-66\,(bx^4+a)^{\frac{7}{4}}a+77\,(bx^4+a)^{\frac{3}{4}}a^2\right)c}{b^2}+\frac{\left(77\,(bx^4+a)^{\frac{15}{4}}-315\,(bx^4+a)^{\frac{11}{4}}a+495\,(bx^4+a)^{\frac{7}{4}}a^2-385\,(bx^4+a)^{\frac{3}{4}}a^3\right)d}{b^3}}{1155\,b}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/1155*(5*(21*(b*x^4 + a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)
^(3/4)*a^2)*c/b^2 + (77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 49
5*(b*x^4 + a)^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)*d/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{d\,x^{12}}{15\,b} - \frac{128\,a^3\,d-160\,a^2\,b\,c}{1155\,b^4} + \frac{x^8\,\left(105\,b^3\,c-84\,a\,b^2\,d\right)}{1155\,b^4} + \frac{8\,a\,x^4\,\left(4\,a\,d-5\,b\,c\right)}{385\,b^3}\right)$$

input
```
int((x^11*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(3/4)*((d*x^12)/(15*b) - (128*a^3*d - 160*a^2*b*c)/(1155*b^4)
+ (x^8*(105*b^3*c - 84*a*b^2*d))/(1155*b^4) + (8*a*x^4*(4*a*d - 5*b*c))/(3
85*b^3))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^{15}}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**15/(a + b*x**4)**(1/4),x)*d + int(x**11/(a + b*x**4)**(1/4),x)*c
```

3.71 $\int \frac{x^7 (c+dx^4)}{\sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^7 (c+dx^4)}{\sqrt[4]{a+bx^4}} \, dx = -\frac{a(bc-ad) (a+bx^4)^{3/4}}{3b^3} + \frac{(bc-2ad) (a+bx^4)^{7/4}}{7b^3} + \frac{d(a+bx^4)^{11/4}}{11b^3}$$

output
```
-1/3*a*(-a*d+b*c)*(b*x^4+a)^(3/4)/b^3+1/7*(-2*a*d+b*c)*(b*x^4+a)^(7/4)/b^3
+1/11*d*(b*x^4+a)^(11/4)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^7 (c+dx^4)}{\sqrt[4]{a+bx^4}} \, dx = \frac{(a+bx^4)^{3/4} (-44abc+32a^2d+33b^2cx^4-24abdx^4+21b^2dx^8)}{231b^3}$$

input
```
Integrate[(x^7*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(-44*a*b*c + 32*a^2*d + 33*b^2*c*x^4 - 24*a*b*d*x^4 + 2
1*b^2*d*x^8))/(231*b^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (c+dx^4)}{\sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4 (dx^4+c)}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{d(bx^4+a)^{7/4}}{b^2}+\frac{(bc-2ad)(bx^4+a)^{3/4}}{b^2}+\frac{a(ad-bc)}{b^2\sqrt[4]{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4(a+bx^4)^{7/4}(bc-2ad)}{7b^3}-\frac{4a(a+bx^4)^{3/4}(bc-ad)}{3b^3}+\frac{4d(a+bx^4)^{11/4}}{11b^3}\right)$$

input
```
Int[(x^7*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((-4*a*(b*c - a*d)*(a + b*x^4)^(3/4))/(3*b^3) + (4*(b*c - 2*a*d)*(a + b*x^
4)^(7/4))/(7*b^3) + (4*d*(a + b*x^4)^(11/4))/(11*b^3))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{32\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(\frac{33\left(\frac{7d\,x^{4}}{11}+c\right)x^{4}b^{2}}{32}-\frac{11\left(\frac{6d\,x^{4}}{11}+c\right)ab}{8}+a^{2}d\right)}{231b^{3}}$	49
gosper	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(21d\,b^{2}x^{8}-24abd\,x^{4}+33b^{2}c\,x^{4}+32a^{2}d-44abc\right)}{231b^{3}}$	53
trager	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(21d\,b^{2}x^{8}-24abd\,x^{4}+33b^{2}c\,x^{4}+32a^{2}d-44abc\right)}{231b^{3}}$	53
risch	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(21d\,b^{2}x^{8}-24abd\,x^{4}+33b^{2}c\,x^{4}+32a^{2}d-44abc\right)}{231b^{3}}$	53
orering	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(21d\,b^{2}x^{8}-24abd\,x^{4}+33b^{2}c\,x^{4}+32a^{2}d-44abc\right)}{231b^{3}}$	53

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
32/231*(b*x^4+a)^(3/4)*(33/32*(7/11*d*x^4+c)*x^4*b^2-11/8*(6/11*d*x^4+c)*a
*b+a^2*d)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(21\,b^2dx^8+3\,(11\,b^2c-8\,abd)x^4-44\,abc+32\,a^2d)(bx^4+a)^{\frac{3}{4}}}{231\,b^3}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/231*(21*b^2*d*x^8 + 3*(11*b^2*c - 8*a*b*d)*x^4 - 44*a*b*c + 32*a^2*d)*(b
*x^4 + a)^(3/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} \frac{32a^2d(a+bx^4)^{\frac{3}{4}}}{231b^3} - \frac{4ac(a+bx^4)^{\frac{3}{4}}}{21b^2} - \frac{8adx^4(a+bx^4)^{\frac{3}{4}}}{77b^2} + \frac{cx^4(a+bx^4)^{\frac{3}{4}}}{7b} + \frac{dx^8(a+bx^4)^{\frac{3}{4}}}{11b} & \text{for } b \neq 0 \\ \frac{\frac{cx^8}{8}+\frac{dx^{12}}{12}}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((32*a**2*d*(a + b*x**4)**(3/4)/(231*b**3) - 4*a*c*(a + b*x**4)**
(3/4)/(21*b**2) - 8*a*d*x**4*(a + b*x**4)**(3/4)/(77*b**2) + c*x**4*(a + b
*x**4)**(3/4)/(7*b) + d*x**8*(a + b*x**4)**(3/4)/(11*b), Ne(b, 0)), ((c*x*
*8/8 + d*x**12/12)/a**(1/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{1}{231}\,d\left(\frac{21\,(bx^4+a)^{\frac{11}{4}}}{b^3} - \frac{66\,(bx^4+a)^{\frac{7}{4}}a}{b^3} + \frac{77\,(bx^4+a)^{\frac{3}{4}}a^2}{b^3}\right) + \frac{1}{21}\,c\left(\frac{3\,(bx^4+a)^{\frac{7}{4}}}{b^2} - \frac{7\,(bx^4+a)^{\frac{3}{4}}a}{b^2}\right)$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/231*d*(21*(b*x^4 + a)^(11/4)/b^3 - 66*(b*x^4 + a)^(7/4)*a/b^3 + 77*(b*x^
4 + a)^(3/4)*a^2/b^3) + 1/21*c*(3*(b*x^4 + a)^(7/4)/b^2 - 7*(b*x^4 + a)^(3
/4)*a/b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{\frac{11\left(3\,(bx^4+a)^{\frac{7}{4}} - 7\,(bx^4+a)^{\frac{3}{4}}a\right)c}{b} + \frac{\left(21\,(bx^4+a)^{\frac{11}{4}} - 66\,(bx^4+a)^{\frac{7}{4}}a + 77\,(bx^4+a)^{\frac{3}{4}}a^2\right)d}{b^2}}{231\,b}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/231*(11*(3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a)*c/b + (21*(b*x^4 +
 a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^(3/4)*a^2)*d/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{32\,a^2\,d-44\,a\,b\,c}{231\,b^3}+\frac{d\,x^8}{11\,b}+\frac{x^4\,\left(33\,b^2\,c-24\,a\,b\,d\right)}{231\,b^3}\right)$$

input
```
int((x^7*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(3/4)*((32*a^2*d - 44*a*b*c)/(231*b^3) + (d*x^8)/(11*b) + (x^4
*(33*b^2*c - 24*a*b*d))/(231*b^3))
```

Reduce [F]

$$\int \frac{x^7(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^{11}}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{x^7}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**11/(a + b*x**4)**(1/4),x)*d + int(x**7/(a + b*x**4)**(1/4),x)*c
```

3.72 $\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bc-ad)\left(a+bx^4\right)^{3/4}}{3b^2} + \frac{d(a+bx^4)^{7/4}}{7b^2}$$

output
```
1/3*(-a*d+b*c)*(b*x^4+a)^(3/4)/b^2+1/7*d*(b*x^4+a)^(7/4)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{\left(a+bx^4\right)^{3/4}(7bc-4ad+3bdx^4)}{21b^2}$$

input
```
Integrate[(x^3*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(7*b*c - 4*a*d + 3*b*d*x^4))/(21*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{dx^4+c}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{(bx^4+a)^{3/4}d}{b}+\frac{bc-ad}{b\sqrt[4]{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4(a+bx^4)^{3/4}(bc-ad)}{3b^2}+\frac{4d(a+bx^4)^{7/4}}{7b^2}\right)$$

input
```
Int[(x^3*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
((4*(b*c - a*d)*(a + b*x^4)^(3/4))/(3*b^2) + (4*d*(a + b*x^4)^(7/4))/(7*b^
2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-3db\,x^4+4ad-7cb)}{21b^2}$	31
trager	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-3db\,x^4+4ad-7cb)}{21b^2}$	31
risch	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-3db\,x^4+4ad-7cb)}{21b^2}$	31
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-3db\,x^4+4ad-7cb)}{21b^2}$	31
orering	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-3db\,x^4+4ad-7cb)}{21b^2}$	31

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(b*x^4+a)^(3/4)*(-3*b*d*x^4+4*a*d-7*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(3\,bdx^4+7\,bc-4\,ad)(bx^4+a)^{\frac{3}{4}}}{21\,b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/21*(3*b*d*x^4 + 7*b*c - 4*a*d)*(b*x^4 + a)^(3/4)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} -\frac{4ad(a+bx^4)^{\frac{3}{4}}}{21b^2}+\frac{c(a+bx^4)^{\frac{3}{4}}}{3b}+\frac{dx^4(a+bx^4)^{\frac{3}{4}}}{7b} & \text{for } b\neq 0 \\ \frac{\frac{cx^4}{4}+\frac{dx^8}{8}}{\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-4*a*d*(a + b*x**4)**(3/4)/(21*b**2) + c*(a + b*x**4)**(3/4)/(3
*b) + d*x**4*(a + b*x**4)**(3/4)/(7*b), Ne(b, 0)), ((c*x**4/4 + d*x**8/8)/
a**(1/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{1}{21}\,d\left(\frac{3\,(bx^4+a)^{\frac{7}{4}}}{b^2}-\frac{7\,(bx^4+a)^{\frac{3}{4}}a}{b^2}\right)+\frac{(bx^4+a)^{\frac{3}{4}}c}{3\,b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/21*d*(3*(b*x^4 + a)^(7/4)/b^2 - 7*(b*x^4 + a)^(3/4)*a/b^2) + 1/3*(b*x^4
+ a)^(3/4)*c/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{7\,(bx^4+a)^{\frac{3}{4}}c + \frac{\left(3\,(bx^4+a)^{\frac{7}{4}} - 7\,(bx^4+a)^{\frac{3}{4}}a\right)d}{b}}{21\,b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/21*(7*(b*x^4 + a)^(3/4)*c + (3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a
)*d/b)/b
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = -\left(b\,x^4+a\right)^{3/4}\left(\frac{4\,a\,d-7\,b\,c}{21\,b^2}-\frac{d\,x^4}{7\,b}\right)$$

input
```
int((x^3*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
-(a + b*x^4)^(3/4)*((4*a*d - 7*b*c)/(21*b^2) - (d*x^4)/(7*b))
```

Reduce [F]

$$\int \frac{x^3(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^3}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**7/(a + b*x**4)**(1/4),x)*d + int(x**3/(a + b*x**4)**(1/4),x)*c
```

3.73 $\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx=\frac{d(a+bx^4)^{3/4}}{3b}+\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

output
```
1/3*d*(b*x^4+a)^(3/4)/b+1/2*c*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-1/2*
c*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx=\frac{d(a+bx^4)^{3/4}}{3b}+\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

input
```
Integrate[(c + d*x^4)/(x*(a + b*x^4)^(1/4)),x]
```

output
```
(d*(a + b*x^4)^(3/4))/(3*b) + (c*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(
1/4)) - (c*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {948, 90, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^4\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{4}\left(c\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4+\frac{4d(a+bx^4)^{3/4}}{3b}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(\frac{4c\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}+\frac{4d(a+bx^4)^{3/4}}{3b}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b}-\frac{4c\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b}-4c\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}\right)$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b}-4c\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b}-4c\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b}-4c\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)\right)$$

input
```
Int[(c + d*x^4)/(x*(a + b*x^4)^(1/4)),x]
```

output
```
((4*d*(a + b*x^4)^(3/4))/(3*b) - 4*c*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4
)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{3}{4}} d\,a^{\frac{1}{4}}+6 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) c b-3 \ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) c b}{12 b\,a^{\frac{1}{4}}}$	79

input
```
int((d*x^4+c)/x/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*(b*x^4+a)^(3/4)*d*a^(1/4)+6*arctan((b*x^4+a)^(1/4)/a^(1/4))*c*b-3*
ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*c*b)/b/a^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.28

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = -\frac{3\left(\frac{c^4}{a}\right)^{\frac{1}{4}} b\log\left((bx^4+a)^{\frac{1}{4}}c^3+\left(\frac{c^4}{a}\right)^{\frac{3}{4}}a\right)-3i\left(\frac{c^4}{a}\right)^{\frac{1}{4}} b\log\left((bx^4+a)^{\frac{1}{4}}c^3+i\left(\frac{c^4}{a}\right)^{\frac{3}{4}}a\right)+3i\left(\frac{c^4}{a}\right)^{\frac{1}{4}} b\log}{12\,b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/12*(3*(c^4/a)^(1/4)*b*log((b*x^4 + a)^(1/4)*c^3 + (c^4/a)^(3/4)*a) - 3*
I*(c^4/a)^(1/4)*b*log((b*x^4 + a)^(1/4)*c^3 + I*(c^4/a)^(3/4)*a) + 3*I*(c^
4/a)^(1/4)*b*log((b*x^4 + a)^(1/4)*c^3 - I*(c^4/a)^(3/4)*a) - 3*(c^4/a)^(1
/4)*b*log((b*x^4 + a)^(1/4)*c^3 - (c^4/a)^(3/4)*a) - 4*(b*x^4 + a)^(3/4)*d
)/b
```

Sympy [A] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = d\left(\begin{cases} \frac{x^4}{4\sqrt[4]{a}} & \text{for}\ b=0 \\ \frac{(a+bx^4)^{\frac{3}{4}}}{3b} & \text{otherwise} \end{cases}\right) - \frac{c\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x/(b*x**4+a)**(1/4),x)
```

output
```
d*Piecewise((x**4/(4*a**(1/4)), Eq(b, 0)), ((a + b*x**4)**(3/4)/(3*b), Tru
e)) - c*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*exp_polar(I*pi)/(b*x**4))/(
4*b**(1/4)*x*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = \frac{1}{4}\,c\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)+\frac{(bx^4+a)^{\frac{3}{4}}d}{3\,b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/4*c*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4
) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)) + 1/3*(b*x^4 + a)^(3/
4)*d/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.58

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = \frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}} + \frac{\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\,c\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a} + \frac{\sqrt{2}c\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,(-a)^{\frac{1}{4}}} + \frac{(bx^4+a)^{\frac{3}{4}}d}{3\,b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(1/4)
)/(-a)^(1/4))/(-a)^(1/4) + 1/4*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)
^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 1/8*sqrt(2)*(-a)^(3
/4)*c*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a
))/a + 1/8*sqrt(2)*c*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^
4 + a) + sqrt(-a))/(-a)^(1/4) + 1/3*(b*x^4 + a)^(3/4)*d/b
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = \frac{d\,(b\,x^4+a)^{3/4}}{3\,b}+\frac{c\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}-\frac{c\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}$$

input
```
int((c + d*x^4)/(x*(a + b*x^4)^(1/4)),x)
```

output
```
(d*(a + b*x^4)^(3/4))/(3*b) + (c*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(1/
4)) - (c*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^3}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x}dx\right)c$$

input
```
int((d*x^4+c)/x/(b*x^4+a)^(1/4),x)
```

output
```
int(x**3/(a + b*x**4)**(1/4),x)*d + int(1/((a + b*x**4)**(1/4)*x),x)*c
```

3.74 $\int \frac{c+dx^4}{x^5 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{c+dx^4}{x^5 \sqrt[4]{a+bx^4}} \, dx = -\frac{c(a+bx^4)^{3/4}}{4ax^4} - \frac{(bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} + \frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

output
```
-1/4*c*(b*x^4+a)^(3/4)/a/x^4-1/8*(-4*a*d+b*c)*arctan((b*x^4+a)^(1/4)/a^(1/
4))/a^(5/4)+1/8*(-4*a*d+b*c)*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^4}{x^5 \sqrt[4]{a+bx^4}} \, dx = -\frac{c(a+bx^4)^{3/4}}{4ax^4} + \frac{(-bc+4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} + \frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^5*(a + b*x^4)^(1/4)),x]
```

output
```
-1/4*(c*(a + b*x^4)^(3/4))/(a*x^4) + ((-(b*c) + 4*a*d)*ArcTan[(a + b*x^4)^
(1/4)/a^(1/4)])/(8*a^(5/4)) + ((b*c - 4*a*d)*ArcTanh[(a + b*x^4)^(1/4)/a^(
1/4)])/(8*a^(5/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {948, 87, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^8\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{4}\left(-\frac{(bc-4ad)\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{4a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(-\frac{(bc-4ad)\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{(bc-4ad)\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{(bc-4ad)\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{(bc-4ad)\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{(bc-4ad)\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{(bc-4ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

input

```
Int[(c + d*x^4)/(x^5*(a + b*x^4)^(1/4)),x]
```

output

```
(-((c*(a + b*x^4)^(3/4))/(a*x^4)) + ((b*c - 4*a*d)*(-1/2*ArcTan[(a + b*x^4
)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4)))
)/a)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{c\left(b\,x^{4}+a\right)^{\frac{3}{4}} a^{\frac{1}{4}}+\left(\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)\right)\left(ad-\frac{cb}{4}\right)x^{4}}{4a^{\frac{5}{4}} x^{4}}$	85

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*(b*x^4+a)^(3/4)*a^(1/4)+(ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(
1/4)-a^(1/4)))-2*arctan((b*x^4+a)^(1/4)/a^(1/4)))*(a*d-1/4*c*b)*x^4)/a^(5/
4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 707, normalized size of antiderivative = 7.60

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/16*(a*x^4*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c
*d^3 + 256*a^4*d^4)/a^5)^(1/4)*log(a^4*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2
*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^5)^(3/4) - (b^3*c^3 - 12*a
*b^2*c^2*d + 48*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) - I*a*x^4*((b
^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d
^4)/a^5)^(1/4)*log(I*a^4*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 -
 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^5)^(3/4) - (b^3*c^3 - 12*a*b^2*c^2*d + 4
8*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + I*a*x^4*((b^4*c^4 - 16*a*
b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^5)^(1/4)
*log(-I*a^4*((b^4*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*
d^3 + 256*a^4*d^4)/a^5)^(3/4) - (b^3*c^3 - 12*a*b^2*c^2*d + 48*a^2*b*c*d^2
 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) - a*x^4*((b^4*c^4 - 16*a*b^3*c^3*d + 96*
a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4)/a^5)^(1/4)*log(-a^4*((b^4
*c^4 - 16*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 256*a^3*b*c*d^3 + 256*a^4*d^4
)/a^5)^(3/4) - (b^3*c^3 - 12*a*b^2*c^2*d + 48*a^2*b*c*d^2 - 64*a^3*d^3)*(b
*x^4 + a)^(1/4)) + 4*(b*x^4 + a)^(3/4)*c)/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}\,dx = -\frac{c\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x^5\Gamma\left(\frac{9}{4}\right)} - \frac{d\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**5/(b*x**4+a)**(1/4),x)
```

output
```
-c*gamma(5/4)*hyper((1/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**
(1/4)*x**5*gamma(9/4)) - d*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*exp_pola
r(I*pi)/(b*x**4))/(4*b**(1/4)*x*gamma(5/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(73) = 146$.

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.66

$$\int \frac{c + dx^4}{x^5 \sqrt[4]{a + bx^4}}\,dx = -\frac{1}{16}\,c\left(\frac{b\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{a} + \frac{4\,(bx^4+a)^{\frac{3}{4}}b}{(bx^4+a)a - a^2}\right) + \frac{1}{4}\,d\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/16*c*(b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)
^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a + 4*(b*x^4 + a
)^(3/4)*b/((b*x^4 + a)*a - a^2)) + 1/4*d*(2*arctan((b*x^4 + a)^(1/4)/a^(1/
4))/a^(1/4) + log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/
4)))/a^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.72

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}\,dx = -\frac{1}{32}b\left(\frac{2\sqrt{2}(bc-4ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}+2\left(bx^4+a\right)^{\frac14}\right)}{2(-a)^{\frac14}}\right)}{(-a)^{\frac14}ab}+\frac{2\sqrt{2}(bc-4ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac14}-2\left(b\right.\right.}{2(-a)^{\frac14}}\right.}{(-a)^{\frac14}ab}\right.$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/32*b*(2*sqrt(2)*(b*c - 4*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a*b) + 2*sqrt(2)*(b*c - 4*a*d
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4
))/((-a)^(1/4)*a*b) - sqrt(2)*(b*c - 4*a*d)*log(sqrt(2)*(b*x^4 + a)^(1/4)*
(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a*b) + sqrt(2)*(b*c -
 4*a*d)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt
(-a))/((-a)^(1/4)*a*b) + 8*(b*x^4 + a)^(3/4)*c/(a*b*x^4))
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}\,dx = \frac{d\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}-\frac{d\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}-\frac{c\,{\left(b\,x^4+a\right)}^{3/4}}{4\,a\,x^4}-\frac{b\,c\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}+\frac{b\,c\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}$$

input
```
int((c + d*x^4)/(x^5*(a + b*x^4)^(1/4)),x)
```

output
```
(d*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(1/4)) - (d*atanh((a + b*x^4)^(1/
4)/a^(1/4)))/(2*a^(1/4)) - (c*(a + b*x^4)^(3/4))/(4*a*x^4) - (b*c*atan((a
+ b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4)) + (b*c*atanh((a + b*x^4)^(1/4)/a^(1/4
)))/(8*a^(5/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^5\sqrt[4]{a+bx^4}}\,dx=\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^5}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x}dx\right)d$$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**5),x)*c + int(1/((a + b*x**4)**(1/4)*x),x)*d
```

3.75 $\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(8bc-5ad)x(a+bx^4)^{3/4}}{32b^2} + \frac{dx^5(a+bx^4)^{3/4}}{8b} - \frac{a(8bc-5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}} - \frac{a(8bc-5ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}$$

output
```
1/32*(-5*a*d+8*b*c)*x*(b*x^4+a)^(3/4)/b^2+1/8*d*x^5*(b*x^4+a)^(3/4)/b-1/64
*a*(-5*a*d+8*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)-1/64*a*(-5*a*d
+8*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$= \frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}(8bc-5ad+4bdx^4)+a(-8bc+5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+a(-8bc+5ad)\text{arctanh}\left(\right.}{64b^{9/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(8*b*c - 5*a*d + 4*b*d*x^4) + a*(-8*b*c + 5
*a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + a*(-8*b*c + 5*a*d)*ArcTanh[(
b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(8bc-5ad)\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$
$$\Big\downarrow \text{843}$$
$$\frac{(8bc-5ad)\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$

$$\Big\downarrow \text{770}$$

$$\frac{(8bc-5ad)\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$

$$\Big\downarrow \text{756}$$

$$\frac{(8bc-5ad)\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$

$$\Big\downarrow \text{216}$$

$$\frac{(8bc-5ad)\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8bc-5ad)\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}+\frac{dx^5(a+bx^4)^{3/4}}{8b}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(d*x^5*(a + b*x^4)^(3/4))/(8*b) + ((8*b*c - 5*a*d)*((x*(a + b*x^4)^(3/4))/
(4*b) - (a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b
^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$\frac{\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} x \left(\frac{d\,x^{4}}{2}+c\right) b^{\frac{5}{4}}}{4}+\frac{5\left(-4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x d\,b^{\frac{1}{4}}+\left(\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)\right)\left(a d-\frac{8 c b}{5}\right) a\right)}{128}}{b^{\frac{9}{4}}}$	113

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
5/128*(32/5*(b*x^4+a)^(3/4)*x*(1/2*d*x^4+c)*b^(5/4)+(-4*(b*x^4+a)^(3/4)*x*
d*b^(1/4)+(ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))-2*
arctan((b*x^4+a)^(1/4)/x/b^(1/4)))*(a*d-8/5*c*b))*a)/b^(9/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 809, normalized size of antiderivative = 6.37

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(b^2*((4096*a^4*b^4*c^4 - 10240*a^5*b^3*c^3*d + 9600*a^6*b^2*c^2*d^
2 - 4000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)^(1/4)*log(-(b^7*x*((4096*a^4*b^4*
c^4 - 10240*a^5*b^3*c^3*d + 9600*a^6*b^2*c^2*d^2 - 4000*a^7*b*c*d^3 + 625*
a^8*d^4)/b^9)^(3/4) + (512*a^3*b^3*c^3 - 960*a^4*b^2*c^2*d + 600*a^5*b*c*d
^2 - 125*a^6*d^3)*(b*x^4 + a)^(1/4))/x) - b^2*((4096*a^4*b^4*c^4 - 10240*a
^5*b^3*c^3*d + 9600*a^6*b^2*c^2*d^2 - 4000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)
^(1/4)*log((b^7*x*((4096*a^4*b^4*c^4 - 10240*a^5*b^3*c^3*d + 9600*a^6*b^2*
c^2*d^2 - 4000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)^(3/4) - (512*a^3*b^3*c^3 -
960*a^4*b^2*c^2*d + 600*a^5*b*c*d^2 - 125*a^6*d^3)*(b*x^4 + a)^(1/4))/x) +
 I*b^2*((4096*a^4*b^4*c^4 - 10240*a^5*b^3*c^3*d + 9600*a^6*b^2*c^2*d^2 - 4
000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)^(1/4)*log((I*b^7*x*((4096*a^4*b^4*c^4
- 10240*a^5*b^3*c^3*d + 9600*a^6*b^2*c^2*d^2 - 4000*a^7*b*c*d^3 + 625*a^8*
d^4)/b^9)^(3/4) - (512*a^3*b^3*c^3 - 960*a^4*b^2*c^2*d + 600*a^5*b*c*d^2 -
 125*a^6*d^3)*(b*x^4 + a)^(1/4))/x) - I*b^2*((4096*a^4*b^4*c^4 - 10240*a^5
*b^3*c^3*d + 9600*a^6*b^2*c^2*d^2 - 4000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)^(
1/4)*log((-I*b^7*x*((4096*a^4*b^4*c^4 - 10240*a^5*b^3*c^3*d + 9600*a^6*b^2
*c^2*d^2 - 4000*a^7*b*c*d^3 + 625*a^8*d^4)/b^9)^(3/4) - (512*a^3*b^3*c^3 -
 960*a^4*b^2*c^2*d + 600*a^5*b*c*d^2 - 125*a^6*d^3)*(b*x^4 + a)^(1/4))/x)
- 4*(4*b*d*x^5 + (8*b*c - 5*a*d)*x)*(b*x^4 + a)^(3/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/4)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(1/4)*gamma(13/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(103) = 206$.

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.09

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = -\frac{1}{128}\,d\left(\frac{5\,a^2\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}-\frac{4\left(\frac{9\,(bx^4+a)^{\frac{3}{4}}a^2b}{x^3}-\frac{5\,(bx^4+a)^{\frac{7}{4}}a^2}{x^7}\right)}{b^4-\frac{2\,(bx^4+a)b^3}{x^4}+\frac{(bx^4+a)^2b^2}{x^8}}\right)+\frac{1}{16}\,c\left(\frac{a\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b}-\frac{4\,(bx^4+a)^{\frac{3}{4}}a}{\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x^3}\right)$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/128*d*(5*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b
^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^
2 - 4*(9*(b*x^4 + a)^(3/4)*a^2*b/x^3 - 5*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^4 -
 2*(b*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8)) + 1/16*c*(a*(2*arctan((b*
x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)
/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 4*(b*x^4 + a)^(3/4)*a/((b^2
 - (b*x^4 + a)*b/x^4)*x^3))
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**8/(a + b*x**4)**(1/4),x)*d + int(x**4/(a + b*x**4)**(1/4),x)*c
```

3.76 $\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx=\frac{dx(a+bx^4)^{3/4}}{4b}+\frac{(4bc-ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}+\frac{(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

output
```
1/4*d*x*(b*x^4+a)^(3/4)/b+1/8*(-a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4
))/b^(5/4)+1/8*(-a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx=\frac{2\sqrt[4]{b}dx(a+bx^4)^{3/4}+(4bc-ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+(4bc-ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*d*x*(a + b*x^4)^(3/4) + (4*b*c - a*d)*ArcTan[(b^(1/4)*x)/(a + b
*x^4)^(1/4)] + (4*b*c - a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^
(5/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {913, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(4bc-ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \text{770}$$

$$\frac{(4bc-ad)\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \text{756}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \text{216}$$

$$\frac{(4bc-ad)\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bc-ad)\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}+\frac{dx\left(a+bx^4\right)^{3/4}}{4b}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(1/4),x]
```

output
```
(d*x*(a + b*x^4)^(3/4))/(4*b) + ((4*b*c - a*d)*(ArcTan[(b^(1/4)*x)/(a + b*
x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4
))))/(4*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

method	result
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x d\,b^{\frac{1}{4}}+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right) a d-8\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right) b c-\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a d+4\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}-\right.}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}\right.}\right.}{16 b^{\frac{5}{4}}}$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(b*x^4+a)^(3/4)*x*d*b^(1/4)+2*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*a*
d-8*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*b*c-ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(
-x*b^(1/4)+(b*x^4+a)^(1/4)))*a*d+4*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1
/4)+(b*x^4+a)^(1/4)))*b*c)/b^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 716, normalized size of antiderivative = 7.54

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(3/4)*d*x + b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2
*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(1/4)*log(-(b^4*x*((256*b^4*
c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5
)^(3/4) + (64*b^3*c^3 - 48*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4
+ a)^(1/4))/x) - b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 -
16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(1/4)*log((b^4*x*((256*b^4*c^4 - 256*a*b^3*
c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^
3*c^3 - 48*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x) +
 I*b*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3
 + a^4*d^4)/b^5)^(1/4)*log((I*b^4*x*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a
^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^3*c^3 - 48*a
*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x) - I*b*((256*b
^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/
b^5)^(1/4)*log((-I*b^4*x*((256*b^4*c^4 - 256*a*b^3*c^3*d + 96*a^2*b^2*c^2*
d^2 - 16*a^3*b*c*d^3 + a^4*d^4)/b^5)^(3/4) - (64*b^3*c^3 - 48*a*b^2*c^2*d
+ 12*a^2*b*c*d^2 - a^3*d^3)*(b*x^4 + a)^(1/4))/x))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.75 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(1/4)*gamma(9/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.91

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \frac{1}{16}\,d\left(\frac{a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b}-\frac{4\left(bx^4+a\right)^{\frac{3}{4}}a}{\left(b^2-\frac{\left(bx^4+a\right)b}{x^4}\right)x^3}\right) - \frac{1}{4}\,c\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/16*d*(a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 4*(b
*x^4 + a)^(3/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^3)) - 1/4*c*(2*arctan((b*x^
4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(
b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))
```

Giac [F]

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(1/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**4/(a + b*x**4)**(1/4),x)*d + int(1/(a + b*x**4)**(1/4),x)*c
```

3.77 $\int \frac{c+dx^4}{x^4 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{c+dx^4}{x^4 \sqrt[4]{a+bx^4}} \, dx = -\frac{c(a+bx^4)^{3/4}}{3ax^3} + \frac{d \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

output
```
-1/3*c*(b*x^4+a)^(3/4)/a/x^3+1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/
4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x^4 \sqrt[4]{a+bx^4}} \, dx = -\frac{c(a+bx^4)^{3/4}}{3ax^3} + \frac{d \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}} + \frac{d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(1/4)),x]
```

output
```
-1/3*(c*(a + b*x^4)^(3/4))/(a*x^3) + (d*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/
4)])/(2*b^(1/4)) + (d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*b^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {953, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{953}$$

$$d\int \frac{1}{\sqrt[4]{bx^4+a}}dx-\frac{c\left(a+bx^4\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{770}$$

$$d\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}-\frac{c\left(a+bx^4\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{756}$$

$$d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)-\frac{c\left(a+bx^4\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{216}$$

$$d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)-\frac{c\left(a+bx^4\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)-\frac{c\left(a+bx^4\right)^{3/4}}{3ax^3}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(1/4)),x]
```

output
```
-1/3*(c*(a + b*x^4)^(3/4))/(a*x^3) + d*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/
4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

method	result	size
pseudoelliptic	$-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)a d\,x^{3}-\frac{\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)a d\,x^{3}}{2}+\frac{2\left(b\,x^{4}+a\right)^{\frac{3}{4}}c\,b^{\frac{1}{4}}}{3}}{2b^{\frac{1}{4}}x^{3}a}$	93

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^(1/4)*(arctan((b*x^4+a)^(1/4)/x/b^(1/4))*a*d*x^3-1/2*ln((x*b^(1/4)+
(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*a*d*x^3+2/3*(b*x^4+a)^(3/4)
*c*b^(1/4))/x^3/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx=\frac{b^{\frac{3}{4}}c\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(1/4),x)
```

output
```
b**(3/4)*c*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4)) + d*x*gamm
a(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/4)*gam
ma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx = -\frac{1}{4}\,d\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)-\frac{(bx^4+a)^{\frac{3}{4}}c}{3\,ax^3}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/4*d*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
(b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/3*(b*x^
4 + a)^(3/4)*c/(a*x^3)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx = \frac{d\,x\,\left(\frac{b\,x^4}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{1/4}}-\frac{c\,\left(b\,x^4+a\right)^{3/4}}{3\,a\,x^3}$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(1/4)),x)
```

output
```
(d*x*((b*x^4)/a + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(b*x^4)/a))/(a + b*
x^4)^(1/4) - (c*(a + b*x^4)^(3/4))/(3*a*x^3)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(1/4),x)
```

output
```
int(1/(a + b*x**4)**(1/4),x)*d + int(1/((a + b*x**4)**(1/4)*x***4),x)*c
```

3.78 $\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}} \, dx = -\frac{c(a+bx^4)^{3/4}}{7ax^7} + \frac{(4bc-7ad)\,(a+bx^4)^{3/4}}{21a^2x^3}$$

output
```
-1/7*c*(b*x^4+a)^(3/4)/a/x^7+1/21*(-7*a*d+4*b*c)*(b*x^4+a)^(3/4)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}} \, dx = \frac{(a+bx^4)^{3/4}\,(-3ac+4bcx^4-7adx^4)}{21a^2x^7}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-3*a*c + 4*b*c*x^4 - 7*a*d*x^4))/(21*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(4bc-7ad)\int \frac{1}{x^4 \sqrt[4]{bx^4+a}}dx}{7a} - \frac{c(a+bx^4)^{3/4}}{7ax^7}$$

$$\Big\downarrow \; 796$$

$$\frac{(a+bx^4)^{3/4}(4bc-7ad)}{21a^2x^3} - \frac{c(a+bx^4)^{3/4}}{7ax^7}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(1/4)),x]
```

output
```
-1/7*(c*(a + b*x^4)^(3/4))/(a*x^7) + ((4*b*c - 7*a*d)*(a + b*x^4)^(3/4))/(
21*a^2*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7 d \,x^{4}}{3}+c \right) a -\frac{4 b c \,x^{4}}{3}\right) \left(b \,x^{4}+a \right)^{\frac{3}{4}}}{7 x^{7} a^{2}}$	36
gosper	$-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d \,x^{4}-4 b c \,x^{4}+3 a c \right)}{21 x^{7} a^{2}}$	37
trager	$-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d \,x^{4}-4 b c \,x^{4}+3 a c \right)}{21 x^{7} a^{2}}$	37
risch	$-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d \,x^{4}-4 b c \,x^{4}+3 a c \right)}{21 x^{7} a^{2}}$	37
orering	$-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d \,x^{4}-4 b c \,x^{4}+3 a c \right)}{21 x^{7} a^{2}}$	37

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/3*d*x^4+c)*a-4/3*b*c*x^4)*(b*x^4+a)^(3/4)/x^7/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}}\,dx = \frac{((4\,bc-7\,ad)x^4-3\,ac)(bx^4+a)^{\frac{3}{4}}}{21\,a^2x^7}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/21*((4*b*c - 7*a*d)*x^4 - 3*a*c)*(b*x^4 + a)^(3/4)/(a^2*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 3.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{c+dx^4}{x^8 \sqrt[4]{a+bx^4}}\,dx = -\frac{3b^{\frac{3}{4}}c\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{3}{4}}d\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{7}{4}}c\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(1/4),x)
```

output
```
-3*b**(3/4)*c*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamma(1/4)) +
 b**(3/4)*d*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4)) + b**(7/4
)*c*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^4}{x^8\sqrt[4]{a+bx^4}}\,dx = \frac{c\left(\frac{7\,(bx^4+a)^{\frac{3}{4}}b}{x^3} - \frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}\right)}{21\,a^2} - \frac{(bx^4+a)^{\frac{3}{4}}d}{3\,ax^3}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/21*c*(7*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(7/4)/x^7)/a^2 - 1/3*(b*
x^4 + a)^(3/4)*d/(a*x^3)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^4}{x^8\sqrt[4]{a+bx^4}}\,dx = -\frac{(b\,x^4+a)^{3/4}\,(3\,a\,c+7\,a\,d\,x^4-4\,b\,c\,x^4)}{21\,a^2\,x^7}$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(1/4)),x)
```

output
```
-((a + b*x^4)^(3/4)*(3*a*c + 7*a*d*x^4 - 4*b*c*x^4))/(21*a^2*x^7)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^8}dx\right)c+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^4}dx\right)d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**8),x)*c + int(1/((a + b*x**4)**(1/4)*x**4),x
)*d
```

3.79 $\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{c(a+bx^4)^{3/4}}{11ax^{11}}+\frac{(8bc-11ad)\,(a+bx^4)^{3/4}}{77a^2x^7} - \frac{4b(8bc-11ad)\,(a+bx^4)^{3/4}}{231a^3x^3}$$

output
```
-1/11*c*(b*x^4+a)^(3/4)/a/x^11+1/77*(-11*a*d+8*b*c)*(b*x^4+a)^(3/4)/a^2/x^
7-4/231*b*(-11*a*d+8*b*c)*(b*x^4+a)^(3/4)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}\,(-21a^2c+24abcx^4-33a^2dx^4-32b^2cx^8+44abdx^8)}{231a^3x^{11}}$$

input
```
Integrate[(c + d*x^4)/(x^12*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-21*a^2*c + 24*a*b*c*x^4 - 33*a^2*d*x^4 - 32*b^2*c*x^8
 + 44*a*b*d*x^8))/(231*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(8bc-11ad)\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx}{11a}-\frac{c\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{(8bc-11ad)\left(-\frac{4b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{c\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\left(\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)(8bc-11ad)}{11a}-\frac{c\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

input
```
Int[(c + d*x^4)/(x^12*(a + b*x^4)^(1/4)),x]
```

output
```
-1/11*(c*(a + b*x^4)^(3/4))/(a*x^11) - ((8*b*c - 11*a*d)*(-1/7*(a + b*x^4)
^(3/4)/(a*x^7) + (4*b*(a + b*x^4)^(3/4))/(21*a^2*x^3)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(\left(\frac{11 d\,x^{4}}{7}+c\right) a^{2}-\frac{8 b \left(\frac{11 d\,x^{4}}{6}+c\right) x^{4} a}{7}+\frac{32 b^{2} c\,x^{8}}{21}\right)}{11 x^{11} a^{3}}$	55
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-44 a b d\,x^{8}+32 b^{2} c\,x^{8}+33 a^{2} d\,x^{4}-24 a b c\,x^{4}+21 a^{2} c\right)}{231 x^{11} a^{3}}$	59
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-44 a b d\,x^{8}+32 b^{2} c\,x^{8}+33 a^{2} d\,x^{4}-24 a b c\,x^{4}+21 a^{2} c\right)}{231 x^{11} a^{3}}$	59
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-44 a b d\,x^{8}+32 b^{2} c\,x^{8}+33 a^{2} d\,x^{4}-24 a b c\,x^{4}+21 a^{2} c\right)}{231 x^{11} a^{3}}$	59
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-44 a b d\,x^{8}+32 b^{2} c\,x^{8}+33 a^{2} d\,x^{4}-24 a b c\,x^{4}+21 a^{2} c\right)}{231 x^{11} a^{3}}$	59

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/11*(b*x^4+a)^(3/4)*((11/7*d*x^4+c)*a^2-8/7*b*(11/6*d*x^4+c)*x^4*a+32/21
*b^2*c*x^8)/x^11/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{(4\,(8\,b^2c-11\,abd)x^8-3\,(8\,abc-11\,a^2d)x^4+21\,a^2c)(bx^4+a)^{\frac{3}{4}}}{231\,a^3x^{11}}$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/231*(4*(8*b^2*c - 11*a*b*d)*x^8 - 3*(8*a*b*c - 11*a^2*d)*x^4 + 21*a^2*c
)*(b*x^4 + a)^(3/4)/(a^3*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(78) = 156.

Time = 4.93 (sec) , antiderivative size = 490, normalized size of antiderivative = 5.83

$$\begin{aligned}\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx &= \frac{21a^4b^{\frac{19}{4}}c\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)} \\
&+ \frac{18a^3b^{\frac{23}{4}}cx^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)} \\
&+ \frac{5a^2b^{\frac{27}{4}}cx^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)} \\
&+ \frac{40ab^{\frac{31}{4}}cx^{12}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)} \\
&+ \frac{32b^{\frac{35}{4}}cx^{16}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)} \\
&- \frac{3b^{\frac{3}{4}}d\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{7}{4}}d\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)}\end{aligned}$$

input
```
integrate((d*x**4+c)/x**12/(b*x**4+a)**(1/4),x)
```

output
```
21*a**4*b**(19/4)*c*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**
8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1
/4)) + 18*a**3*b**(23/4)*c*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a
**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x
**16*gamma(1/4)) + 5*a**2*b**(27/4)*c*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-
11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*
a**3*b**6*x**16*gamma(1/4)) + 40*a*b**(31/4)*c*x**12*(a/(b*x**4) + 1)**(3/
4)*gamma(-11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(
1/4) + 64*a**3*b**6*x**16*gamma(1/4)) + 32*b**(35/4)*c*x**16*(a/(b*x**4) +
 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**1
2*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1/4)) - 3*b**(3/4)*d*(a/(b*x**4) +
 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamma(1/4)) + b**(7/4)*d*(a/(b*x**4) + 1
)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \frac{d\left(\frac{7\,(bx^4+a)^{\frac{3}{4}}b}{x^3}-\frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}\right)}{21\,a^2} - \frac{c\left(\frac{77\,(bx^4+a)^{\frac{3}{4}}b^2}{x^3}-\frac{66\,(bx^4+a)^{\frac{7}{4}}b}{x^7}+\frac{21\,(bx^4+a)^{\frac{11}{4}}}{x^{11}}\right)}{231\,a^3}$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/21*d*(7*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(7/4)/x^7)/a^2 - 1/231*c
*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 66*(b*x^4 + a)^(7/4)*b/x^7 + 21*(b*x^4 +
a)^(11/4)/x^11)/a^3
```

Giac [F]

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^{12}}\,dx$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{(b\,x^4+a)^{3/4}\,(33\,d\,a^2\,x^4+21\,c\,a^2-44\,d\,a\,b\,x^8-24\,c\,a\,b\,x^4+32\,c\,b^2\,x^8)}{231\,a^3\,x^{11}}$$

input
```
int((c + d*x^4)/(x^12*(a + b*x^4)^(1/4)),x)
```

output
```
-((a + b*x^4)^(3/4)*(21*a^2*c + 33*a^2*d*x^4 + 32*b^2*c*x^8 - 24*a*b*c*x^4
 - 44*a*b*d*x^8))/(231*a^3*x^11)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{12}}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{8}}dx\right)d$$

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**12),x)*c + int(1/((a + b*x**4)**(1/4)*x**8),
x)*d
```

3.80 $\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = -\frac{c(a+bx^4)^{3/4}}{15ax^{15}}+\frac{(4bc-5ad)\,(a+bx^4)^{3/4}}{55a^2x^{11}} -\frac{8b(4bc-5ad)\,(a+bx^4)^{3/4}}{385a^3x^7}+\frac{32b^2(4bc-5ad)\,(a+bx^4)^{3/4}}{1155a^4x^3}$$

output
```
-1/15*c*(b*x^4+a)^(3/4)/a/x^15+1/55*(-5*a*d+4*b*c)*(b*x^4+a)^(3/4)/a^2/x^1
1-8/385*b*(-5*a*d+4*b*c)*(b*x^4+a)^(3/4)/a^3/x^7+32/1155*b^2*(-5*a*d+4*b*c
)*(b*x^4+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}\,(-77a^3c+84a^2bcx^4-105a^3dx^4-96ab^2cx^8+120a^2bdx^8+128b^3cx^{12}-160ab^2dx^{12})}{1155a^4x^{15}}$$

input
```
Integrate[(c + d*x^4)/(x^16*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-77*a^3*c + 84*a^2*b*c*x^4 - 105*a^3*d*x^4 - 96*a*b^2*
c*x^8 + 120*a^2*b*d*x^8 + 128*b^3*c*x^12 - 160*a*b^2*d*x^12))/(1155*a^4*x^
15)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4bc-5ad)\int \frac{1}{x^{12}\sqrt[4]{bx^4+a}}dx}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-5ad)\left(-\frac{8b\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-5ad)\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)(4bc-5ad)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

input
```
Int[(c + d*x^4)/(x^16*(a + b*x^4)^(1/4)),x]
```

output
```
-1/15*(c*(a + b*x^4)^(3/4))/(a*x^15) - ((4*b*c - 5*a*d)*(-1/11*(a + b*x^4)
^(3/4)/(a*x^11) - (8*b*(-1/7*(a + b*x^4)^(3/4)/(a*x^7) + (4*b*(a + b*x^4)^
(3/4))/(21*a^2*x^3)))/(11*a)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{3}{4}}\left(\left(\frac{15d\,x^4}{11}+c\right)a^3-\frac{12b\,x^4\left(\frac{10d\,x^4}{7}+c\right)a^2}{11}+\frac{96\left(\frac{5d\,x^4}{3}+c\right)b^2x^8a}{77}-\frac{128b^3c\,x^{12}}{77}\right)}{15x^{15}a^4}$	74
gosper	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(160a\,b^2d\,x^{12}-128b^3c\,x^{12}-120a^2bd\,x^8+96a\,b^2c\,x^8+105a^3d\,x^4-84a^2bc\,x^4+77c\,a^3)}{1155x^{15}a^4}$	83
trager	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(160a\,b^2d\,x^{12}-128b^3c\,x^{12}-120a^2bd\,x^8+96a\,b^2c\,x^8+105a^3d\,x^4-84a^2bc\,x^4+77c\,a^3)}{1155x^{15}a^4}$	83
risch	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(160a\,b^2d\,x^{12}-128b^3c\,x^{12}-120a^2bd\,x^8+96a\,b^2c\,x^8+105a^3d\,x^4-84a^2bc\,x^4+77c\,a^3)}{1155x^{15}a^4}$	83
orering	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(160a\,b^2d\,x^{12}-128b^3c\,x^{12}-120a^2bd\,x^8+96a\,b^2c\,x^8+105a^3d\,x^4-84a^2bc\,x^4+77c\,a^3)}{1155x^{15}a^4}$	83

input
```
int((d*x^4+c)/x^16/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^4+a)^(3/4)*((15/11*d*x^4+c)*a^3-12/11*b*x^4*(10/7*d*x^4+c)*a^2+
96/77*(5/3*d*x^4+c)*b^2*x^8*a-128/77*b^3*c*x^12)/x^15/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{(32\,(4\,b^3c-5\,ab^2d)x^{12}-24\,(4\,ab^2c-5\,a^2bd)x^8+21\,(4\,a^2bc-5\,a^3d)x^4-77\,a^3c)(bx^4+a)^{\frac{3}{4}}}{1155\,a^4x^{15}}$$

input
```
integrate((d*x^4+c)/x^16/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/1155*(32*(4*b^3*c - 5*a*b^2*d)*x^12 - 24*(4*a*b^2*c - 5*a^2*b*d)*x^8 + 2
1*(4*a^2*b*c - 5*a^3*d)*x^4 - 77*a^3*c)*(b*x^4 + a)^(3/4)/(a^4*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(112) = 224$.

Time = 6.68 (sec) , antiderivative size = 1120, normalized size of antiderivative = 9.57

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**16/(b*x**4+a)**(1/4),x)
```

output
```
-231*a**6*b**(39/4)*c*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*
x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*
gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) - 441*a**5*b**(43/4)*c*x**4*
(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768
*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*
b**12*x**24*gamma(1/4)) - 225*a**4*b**(47/4)*c*x**8*(a/(b*x**4) + 1)**(3/4
)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamm
a(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)
) + 21*a**4*b**(19/4)*d*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4
*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gam
ma(1/4)) + 45*a**3*b**(51/4)*c*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/
(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a*
*5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 18*a**3*b**
(23/4)*d*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*gamm
a(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1/4)) +
 540*a**2*b**(55/4)*c*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7
*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*
x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 5*a**2*b**(27/4)*d*x
**8*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 1
28*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1/4)) + 864*a*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = -\frac{d\left(\frac{77\,(bx^4+a)^{\frac{3}{4}}b^2}{x^3} - \frac{66\,(bx^4+a)^{\frac{7}{4}}b}{x^7} + \frac{21\,(bx^4+a)^{\frac{11}{4}}}{x^{11}}\right)}{231\,a^3} + \frac{c\left(\frac{385\,(bx^4+a)^{\frac{3}{4}}b^3}{x^3} - \frac{495\,(bx^4+a)^{\frac{7}{4}}b^2}{x^7} + \frac{315\,(bx^4+a)^{\frac{11}{4}}b}{x^{11}} - \frac{77\,(bx^4+a)^{\frac{15}{4}}}{x^{15}}\right)}{1155\,a^4}$$

input
```
integrate((d*x^4+c)/x^16/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/231*d*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 66*(b*x^4 + a)^(7/4)*b/x^7 + 21*(
b*x^4 + a)^(11/4)/x^11)/a^3 + 1/1155*c*(385*(b*x^4 + a)^(3/4)*b^3/x^3 - 49
5*(b*x^4 + a)^(7/4)*b^2/x^7 + 315*(b*x^4 + a)^(11/4)*b/x^11 - 77*(b*x^4 +
a)^(15/4)/x^15)/a^4
```

Giac [F]

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^{16}}\,dx$$

input
```
integrate((d*x^4+c)/x^16/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^16), x)
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{(b\,x^4+a)^{3/4}\,(128\,b^3\,c-160\,a\,b^2\,d)}{1155\,a^4\,x^3} - \frac{(b\,x^4+a)^{3/4}\,(32\,b^2\,c-40\,a\,b\,d)}{385\,a^3\,x^7} - \frac{(b\,x^4+a)^{3/4}\,(5\,a\,d-4\,b\,c)}{55\,a^2\,x^{11}} - \frac{c\,(b\,x^4+a)^{3/4}}{15\,a\,x^{15}}$$

input
```
int((c + d*x^4)/(x^16*(a + b*x^4)^(1/4)),x)
```

output
```
((a + b*x^4)^(3/4)*(128*b^3*c - 160*a*b^2*d))/(1155*a^4*x^3) - ((a + b*x^4
)^(3/4)*(32*b^2*c - 40*a*b*d))/(385*a^3*x^7) - ((a + b*x^4)^(3/4)*(5*a*d -
 4*b*c))/(55*a^2*x^11) - (c*(a + b*x^4)^(3/4))/(15*a*x^15)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{16}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{12}}dx\right)d$$

input
```
int((d*x^4+c)/x^16/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**16),x)*c + int(1/((a + b*x**4)**(1/4)*x**12)
,x)*d
```

3.81 $\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 154

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = -\frac{a(10bc-7ad)x^3}{40b^2\sqrt[4]{a+bx^4}} + \frac{(10bc-7ad)x^3(a+bx^4)^{3/4}}{60b^2} + \frac{dx^7(a+bx^4)^{3/4}}{10b} - \frac{a^{3/2}(10bc-7ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/40*a*(-7*a*d+10*b*c)*x^3/b^2/(b*x^4+a)^(1/4)+1/60*(-7*a*d+10*b*c)*x^3*(
b*x^4+a)^(3/4)/b^2+1/10*d*x^7*(b*x^4+a)^(3/4)/b-1/40*a^(3/2)*(-7*a*d+10*b*
c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1
/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\left(-((a+bx^4)(7ad-2b(5c+3dx^4)))+a(-10bc+7ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right.\right.}{60b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(-((a + b*x^4)*(7*a*d - 2*b*(5*c + 3*d*x^4))) + a*(-10*b*c + 7*a*d)*(
1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^4)/a)]))/(60*
b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 843, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(10bc-7ad)\int \frac{x^6}{\sqrt[4]{bx^4+a}}dx}{10b}+\frac{dx^7(a+bx^4)^{3/4}}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(10bc-7ad)\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}\right)}{10b}+\frac{dx^7(a+bx^4)^{3/4}}{10b}$$

$$\Big\downarrow \text{839}$$

$$\frac{(10bc-7ad)\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)}{2b}\right)}{10b}+\frac{dx^7\left(a+bx^4\right)^{3/4}}{10b}$$

$\downarrow$ 813

$$\frac{(10bc-7ad)\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}+\frac{dx^7\left(a+bx^4\right)^{3/4}}{10b}$$

$\downarrow$ 858

$$\frac{(10bc-7ad)\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}+\frac{dx^7\left(a+bx^4\right)^{3/4}}{10b}$$

$\downarrow$ 807

$$\frac{(10bc-7ad)\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}+\frac{dx^7(a+bx^4)^{3/4}}{10b}$$

$\big\downarrow$ 212

$$\frac{(10bc-7ad)\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}+\frac{dx^7(a+bx^4)^{3/4}}{10b}$$

input

```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output

```
(d*x^7*(a + b*x^4)^(3/4))/(10*b) + ((10*b*c - 7*a*d)*((x^3*(a + b*x^4)^(3/
4))/(6*b) - (a*(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)
*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(
1/4))))/(2*b)))/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^6(d\,x^4+c)}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^6)/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(1/4)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(1/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**10/(a + b*x**4)**(1/4),x)*d + int(x**6/(a + b*x**4)**(1/4),x)*c
```

3.82 $\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(2bc-ad)x^3}{4b\sqrt[4]{a+bx^4}} + \frac{dx^3(a+bx^4)^{3/4}}{6b} + \frac{\sqrt{a}(2bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/4*(-a*d+2*b*c)*x^3/b/(b*x^4+a)^(1/4)+1/6*d*x^3*(b*x^4+a)^(3/4)/b+1/4*a^(
1/2)*(-a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2
/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\left(d(a+bx^4)+(2bc-ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{6b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(d*(a + b*x^4) + (2*b*c - a*d)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F
1[1/4, 3/4, 7/4, -((b*x^4)/a)]))/(6*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2bc-ad)\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}+\frac{dx^3(a+bx^4)^{3/4}}{6b}$$

$$\Big\downarrow \text{839}$$

$$\frac{(2bc-ad)\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{(bx^4+a)^{5/4}}dx\right)}{2b}+\frac{dx^3(a+bx^4)^{3/4}}{6b}$$

$$\Big\downarrow \text{813}$$

$$\frac{(2bc-ad)\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}+\frac{dx^3\left(a+bx^4\right)^{3/4}}{6b}$$

$\Big\downarrow$ 858

$$\frac{(2bc-ad)\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}+\frac{dx^3\left(a+bx^4\right)^{3/4}}{6b}$$

$\Big\downarrow$ 807

$$\frac{(2bc-ad)\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}+\frac{dx^3\left(a+bx^4\right)^{3/4}}{6b}$$

$\Big\downarrow$ 212

$$\frac{(2bc-ad)\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}+\frac{dx^3\left(a+bx^4\right)^{3/4}}{6b}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(d*x^3*(a + b*x^4)^(3/4))/(6*b) + ((2*b*c - a*d)*(x^3/(2*(a + b*x^4)^(1/4)
) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(d\,x^4+c)}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/4)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**6/(a + b*x**4)**(1/4),x)*d + int(x**2/(a + b*x**4)**(1/4),x)*c
```

3.83 $\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx = \frac{(2bc+ad)x^3}{2a\sqrt[4]{a+bx^4}} - \frac{c(a+bx^4)^{3/4}}{ax} + \frac{(2bc+ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
1/2*(a*d+2*b*c)*x^3/a/(b*x^4+a)^(1/4)-c*(b*x^4+a)^(3/4)/a/x+1/2*(a*d+2*b*c
)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/
2))/a^(1/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx = \frac{-3c(a+bx^4)+(2bc+ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3ax\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(1/4)),x]
```

output
```
(-3*c*(a + b*x^4) + (2*b*c + a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometric
2F1[1/4, 3/4, 7/4, -((b*x^4)/a)])/(3*a*x*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(ad+2bc)\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{a}-\frac{c(a+bx^4)^{3/4}}{ax}$$

$$\Big\downarrow \text{839}$$

$$\frac{(ad+2bc)\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{(bx^4+a)^{5/4}}dx\right)}{a}-\frac{c(a+bx^4)^{3/4}}{ax}$$

$$\Big\downarrow \text{813}$$

$$\frac{(ad+2bc)\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c(a+bx^4)^{3/4}}{ax}$$

$$\Big\downarrow \text{858}$$

$$\frac{(ad+2bc)\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax}$$

$$\Big\downarrow \; 807$$

$$\frac{(ad+2bc)\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax}$$

$$\Big\downarrow \; 212$$

$$\frac{(ad+2bc)\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c\left(a+bx^4\right)^{3/4}}{ax}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(1/4)),x]
```

output
```
-((c*(a + b*x^4)^(3/4))/(a*x)) + ((2*b*c + a*d)*(x^3/(2*(a + b*x^4)^(1/4))
 + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)
]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))))/a
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{2} \left(b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x)
```

output
```
int((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.81 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx = \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x\Gamma\left(\frac{3}{4}\right)} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(1/4),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
1/4)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx=\int \frac{d\,x^4+c}{x^2\,{\left(b\,x^4+a\right)}^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(1/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\sqrt[4]{a+bx^4}}\,dx=\left(\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^2}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(1/4),x)
```

output
```
int(x**2/(a + b*x**4)**(1/4),x)*d + int(1/((a + b*x**4)**(1/4)*x**2),x)*c
```

3.84 $\int \frac{c+dx^4}{x^6 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{c+dx^4}{x^6 \sqrt[4]{a+bx^4}} \, dx = \frac{2bc-5ad}{5ax\sqrt[4]{a+bx^4}} - \frac{c(a+bx^4)^{3/4}}{5ax^5} - \frac{\sqrt{b}(2bc-5ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-5*a*d+2*b*c)/a/x/(b*x^4+a)^(1/4)-1/5*c*(b*x^4+a)^(3/4)/a/x^5-1/5*b^(
1/2)*(-5*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x
^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{c+dx^4}{x^6 \sqrt[4]{a+bx^4}} \, dx = \frac{-c(a+bx^4)+(2bc-5ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{5ax^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(1/4)),x]
```

output
```
(-(c*(a + b*x^4)) + (2*b*c - 5*a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometr
ic2F1[-1/4, 1/4, 3/4, -((b*x^4)/a)])/(5*a*x^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-5ad)\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{841}$$

$$-\frac{(2bc-5ad)\left(-b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{813}$$

$$-\frac{(2bc-5ad)\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{858}$$

$$-\frac{(2bc-5ad)\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(2bc-5ad)\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{ 212}$$

$$-\frac{(2bc-5ad)\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c\left(a+bx^4\right)^{3/4}}{5ax^5}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(1/4)),x]
```

output
```
-1/5*(c*(a + b*x^4)^(3/4))/(a*x^5) - ((2*b*c - 5*a*d)*(-(1/(x*(a + b*x^4)^
(1/4))) + (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[
b]*x^2)]/2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{6} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x)
```

output
```
int((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b*x^10 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.58

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx=-\frac{c\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{6\sqrt[4]{b}x^6}+\frac{d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(1/4),x)
```

output
```
-c*hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*b**(1/4)*x**6)
 + d*gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(1/4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^6\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(1/4)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^6}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^2}dx\right)d$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**6),x)*c + int(1/((a + b*x**4)**(1/4)*x**2),x
)*d
```

3.85 $\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = -\frac{2a(3bc-2ad)x^2}{15b^2\sqrt[4]{a+bx^4}}+\frac{(3bc-2ad)x^2(a+bx^4)^{3/4}}{15b^2}+\frac{dx^6(a+bx^4)^{3/4}}{9b}+\frac{2a^{3/2}(3bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-2/15*a*(-2*a*d+3*b*c)*x^2/b^2/(b*x^4+a)^(1/4)+1/15*(-2*a*d+3*b*c)*x^2*(b*
x^4+a)^(3/4)/b^2+1/9*d*x^6*(b*x^4+a)^(3/4)/b+2/15*a^(3/2)*(-2*a*d+3*b*c)*(
1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b
^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.60

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^2\left(-((a+bx^4)(-9bc+6ad-5bdx^4))+3a(-3bc+2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right.\right.}{45b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^5*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(-((a + b*x^4)*(-9*b*c + 6*a*d - 5*b*d*x^4)) + 3*a*(-3*b*c + 2*a*d)*(
1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(45*
b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 807, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3bc-2ad)\int \frac{x^5}{\sqrt[4]{bx^4+a}}dx}{3b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3bc-2ad)\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx^2}{6b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(3bc-2ad)\left(\frac{2x^2(a+bx^4)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)}{6b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

$$\Big\downarrow \text{227}$$

$$\frac{(3bc-2ad)\left(\frac{2x^2(a+bx^4)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

$$\Big\downarrow \text{225}$$

$$\frac{(3bc-2ad)\left(\frac{2x^2(a+bx^4)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

$$\Big\downarrow \text{212}$$

$$\frac{(3bc-2ad)\left(\frac{2x^2(a+bx^4)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^6(a+bx^4)^{3/4}}{9b}$$

input

```
Int[(x^5*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(d*x^6*(a + b*x^4)^(3/4))/(9*b) + ((3*b*c - 2*a*d)*((2*x^2*(a + b*x^4)^(3/
4))/(5*b) - (2*a*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2
*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a
+ b*x^4)^(1/4))))/(6*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{5}\left(d\,x^{4}+c\right)}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx=\frac{cx^6\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6\sqrt[4]{a}}+\frac{dx^{10}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10\sqrt[4]{a}}$$

input
```
integrate(x**5*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4)) +
d*x**10*hyper((1/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(1/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^5\,(d\,x^4+c)}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**9/(a + b*x**4)**(1/4),x)*d + int(x**5/(a + b*x**4)**(1/4),x)*c
```

3.86 $\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{(5bc-2ad)x^2}{5b\sqrt[4]{a+bx^4}} + \frac{dx^2(a+bx^4)^{3/4}}{5b} - \frac{\sqrt{a}(5bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-2*a*d+5*b*c)*x^2/b/(b*x^4+a)^(1/4)+1/5*d*x^2*(b*x^4+a)^(3/4)/b-1/5*a
^(1/2)*(-2*a*d+5*b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x
^2/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^2\left(2d(a+bx^4)+(5bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{10b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(2*d*(a + b*x^4) + (5*b*c - 2*a*d)*(1 + (b*x^4)/a)^(1/4)*Hypergeometr
ic2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(10*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {959, 807, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \ 959$$

$$\frac{(5bc-2ad)\int \frac{x}{\sqrt[4]{bx^4+a}}dx}{5b}+\frac{dx^2(a+bx^4)^{3/4}}{5b}$$

$$\Big\downarrow \ 807$$

$$\frac{(5bc-2ad)\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{10b}+\frac{dx^2(a+bx^4)^{3/4}}{5b}$$

$$\Big\downarrow \ 227$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(5bc-2ad)\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{10b\sqrt[4]{a+bx^4}}+\frac{dx^2\left(a+bx^4\right)^{3/4}}{5b}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(5bc-2ad)\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{10b\sqrt[4]{a+bx^4}}+\frac{dx^2\left(a+bx^4\right)^{3/4}}{5b}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(5bc-2ad)\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{10b\sqrt[4]{a+bx^4}}+\frac{dx^2\left(a+bx^4\right)^{3/4}}{5b}$$

input

```
Int[(x*(c + d*x^4))/(a + b*x^4)^(1/4),x]
```

output

```
(d*x^2*(a + b*x^4)^(3/4))/(5*b) + ((5*b*c - 2*a*d)*(1 + (b*x^4)/a)^(1/4)*(
(2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]]/2, 2])/Sqrt[b]))/(10*b*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{1}{4}}}dx$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \frac{cx^2 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}} + \frac{dx^6 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6\sqrt[4]{a}}$$

input
```
integrate(x*(d*x**4+c)/(b*x**4+a)**(1/4),x)
```

output
```
c*x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4)) +
d*x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4))
```

Maxima [F]

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x\,(d\,x^4+c)}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((x*(c + d*x^4))/(a + b*x^4)^(1/4),x)
```

output
```
int((x*(c + d*x^4))/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x(c+dx^4)}{\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d + \left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(1/4),x)
```

output
```
int(x**5/(a + b*x**4)**(1/4),x)*d + int(x/(a + b*x**4)**(1/4),x)*c
```

3.87 $\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx=\frac{(bc+2ad)x^2}{2a\sqrt[4]{a+bx^4}}-\frac{c(a+bx^4)^{3/4}}{2ax^2}-\frac{(bc+2ad)\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
1/2*(2*a*d+b*c)*x^2/a/(b*x^4+a)^(1/4)-1/2*c*(b*x^4+a)^(3/4)/a/x^2-1/2*(2*a
*d+b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2
^(1/2))/a^(1/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx=\frac{-2c(a+bx^4)+(bc+2ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{4ax^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^3*(a + b*x^4)^(1/4)),x]
```

output
```
(-2*c*(a + b*x^4) + (b*c + 2*a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, -((b*x^4)/a)])/(4*a*x^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 807, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \ 955$$

$$\frac{(2ad+bc)\int \frac{x}{\sqrt[4]{bx^4+a}}dx}{2a}-\frac{c(a+bx^4)^{3/4}}{2ax^2}$$

$$\Big\downarrow \ 807$$

$$\frac{(2ad+bc)\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{4a}-\frac{c(a+bx^4)^{3/4}}{2ax^2}$$

$$\Big\downarrow \ 227$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(2ad+bc)\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{4a\sqrt[4]{a+bx^4}}-\frac{c(a+bx^4)^{3/4}}{2ax^2}$$

$$\Big\downarrow \ 225$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(2ad+bc)\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{4a\sqrt[4]{a+bx^4}}-\frac{c(a+bx^4)^{3/4}}{2ax^2}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(2ad+bc)\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{4a\sqrt[4]{a+bx^4}}-\frac{c\left(a+bx^4\right)^{3/4}}{2ax^2}$$

input
```
Int[(c + d*x^4)/(x^3*(a + b*x^4)^(1/4)),x]
```

output
```
-1/2*(c*(a + b*x^4)^(3/4))/(a*x^2) + ((b*c + 2*a*d)*(1 + (b*x^4)/a)^(1/4)*
((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)
/Sqrt[a]]/2, 2])/Sqrt[b]))/(4*a*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{3} \left(b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x)
```

output
```
int((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{1}{4}}x^{3}}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b*x^7 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.51

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}x^2} + \frac{dx^2\,{}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}}$$

input
```
integrate((d*x**4+c)/x**3/(b*x**4+a)**(1/4),x)
```

output
```
-c*hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4)*x**2)
+ d*x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^3\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(1/4)),x)
```

output
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^3\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^3}dx\right)c$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(1/4),x)
```

output
```
int(x/(a + b*x**4)**(1/4),x)*d + int(1/((a + b*x**4)**(1/4)*x**3),x)*c
```

3.88 $\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}} \, dx = -\frac{b(bc-2ad)x^2}{4a^2\sqrt[4]{a+bx^4}} - \frac{c(a+bx^4)^{3/4}}{6ax^6} + \frac{(bc-2ad)(a+bx^4)^{3/4}}{4a^2x^2} + \frac{\sqrt{b}(bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a+bx^4}}$$

output

```
-1/4*b*(-2*a*d+b*c)*x^2/a^2/(b*x^4+a)^(1/4)-1/6*c*(b*x^4+a)^(3/4)/a/x^6+1/
4*(-2*a*d+b*c)*(b*x^4+a)^(3/4)/a^2/x^2+1/4*b^(1/2)*(-2*a*d+b*c)*(1+b*x^4/a
)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b
*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.52

$$\int \frac{c+dx^4}{x^7\sqrt[4]{a+bx^4}}\,dx = \frac{-2c(a+bx^4)+3(bc-2ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{12ax^6\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^7*(a + b*x^4)^(1/4)),x]
```

output
```
(-2*c*(a + b*x^4) + 3*(b*c - 2*a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometr
ic2F1[-1/2, 1/4, 1/2, -((b*x^4)/a)])/(12*a*x^6*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 807, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^7\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(bc-2ad)\int \frac{1}{x^3\sqrt[4]{bx^4+a}}dx}{2a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(bc-2ad)\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^2}{4a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(bc-2ad)\left(\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{(a+bx^4)^{3/4}}{ax^2}\right)}{4a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

$\Big\downarrow$ 227

$$-\frac{(bc-2ad)\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2a\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{ax^2}\right)}{4a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

$\Big\downarrow$ 225

$$-\frac{(bc-2ad)\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{ax^2}\right)}{4a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

$\Big\downarrow$ 212

$$-\frac{(bc-2ad)\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{ax^2}\right)}{4a}-\frac{c(a+bx^4)^{3/4}}{6ax^6}$$

input

```
Int[(c + d*x^4)/(x^7*(a + b*x^4)^(1/4)),x]
```

output
```
-1/6*(c*(a + b*x^4)^(3/4))/(a*x^6) - ((b*c - 2*a*d)*(-((a + b*x^4)^(3/4)/(
a*x^2)) + (b*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqr
t[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a*(a + b*
x^4)^(1/4))))/(4*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{7} \left(b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x)
```

output
```
int((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}}\, dx = \int \frac{dx^{4} + c}{{\left(bx^{4} + a\right)}^{\frac{1}{4}} x^{7}}\, dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b*x^11 + a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.44

$$\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6\sqrt[4]{a}x^6} - \frac{d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}x^2}$$

input
```
integrate((d*x**4+c)/x**7/(b*x**4+a)**(1/4),x)
```

output
```
-c*hyper((-3/2, 1/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4)*x**6)
 - d*hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4)*x**2
)
```

Maxima [F]

$$\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^7), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(1/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^7\sqrt[4]{a+bx^4}}\,dx = \int \frac{d\,x^4+c}{x^7\,{\left(b\,x^4+a\right)}^{1/4}}\,dx$$

input
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(1/4)),x)
```

output
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^7\sqrt[4]{a+bx^4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^7}dx\right)c+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^3}dx\right)d$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**7),x)*c + int(1/((a + b*x**4)**(1/4)*x**3),x
)*d
```

3.89 $\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{a^2(bc-ad)\sqrt[4]{a+bx^4}}{b^4} - \frac{a(2bc-3ad)\,(a+bx^4)^{5/4}}{5b^4} + \frac{(bc-3ad)\,(a+bx^4)^{9/4}}{9b^4} + \frac{d(a+bx^4)^{13/4}}{13b^4}$$

output
```
a^2*(-a*d+b*c)*(b*x^4+a)^(1/4)/b^4-1/5*a*(-3*a*d+2*b*c)*(b*x^4+a)^(5/4)/b^
4+1/9*(-3*a*d+b*c)*(b*x^4+a)^(9/4)/b^4+1/13*d*(b*x^4+a)^(13/4)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}(416a^2bc-384a^3d-104ab^2cx^4+96a^2bdx^4+65b^3cx^8-60ab^2dx^8+45b^3d}{585b^4}$$

input
```
Integrate[(x^11*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(416*a^2*b*c - 384*a^3*d - 104*a*b^2*c*x^4 + 96*a^2*b*d
*x^4 + 65*b^3*c*x^8 - 60*a*b^2*d*x^8 + 45*b^3*d*x^12))/(585*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8(dx^4+c)}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{d(bx^4+a)^{9/4}}{b^3}+\frac{(bc-3ad)(bx^4+a)^{5/4}}{b^3}+\frac{a(3ad-2bc)\sqrt[4]{bx^4+a}}{b^3}-\frac{a^2(ad-bc)}{b^3(bx^4+a)^{3/4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt[4]{a+bx^4}(bc-ad)}{b^4}+\frac{4(a+bx^4)^{9/4}(bc-3ad)}{9b^4}-\frac{4a(a+bx^4)^{5/4}(2bc-3ad)}{5b^4}+\frac{4d(a+bx^4)^{13/4}}{13b^4}\right)$$

input
```
Int[(x^11*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((4*a^2*(b*c - a*d)*(a + b*x^4)^(1/4))/b^4 - (4*a*(2*b*c - 3*a*d)*(a + b*x
^4)^(5/4))/(5*b^4) + (4*(b*c - 3*a*d)*(a + b*x^4)^(9/4))/(9*b^4) + (4*d*(a
 + b*x^4)^(13/4))/(13*b^4))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{128(b\,x^4+a)^{\frac{1}{4}}\left(-\frac{65\left(\frac{9d\,x^4}{13}+c\right)x^8b^3}{384}+\frac{13x^4a\left(\frac{15d\,x^4}{26}+c\right)b^2}{48}-\frac{13a^2\left(\frac{3d\,x^4}{13}+c\right)b}{12}+a^3d\right)}{195b^4}$	68
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\left(-45b^3d\,x^{12}+60a\,b^2d\,x^8-65c\,b^3x^8-96a^2bd\,x^4+104a\,b^2c\,x^4+384a^3d-416a^2bc\right)}{585b^4}$	77
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\left(-45b^3d\,x^{12}+60a\,b^2d\,x^8-65c\,b^3x^8-96a^2bd\,x^4+104a\,b^2c\,x^4+384a^3d-416a^2bc\right)}{585b^4}$	77
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\left(-45b^3d\,x^{12}+60a\,b^2d\,x^8-65c\,b^3x^8-96a^2bd\,x^4+104a\,b^2c\,x^4+384a^3d-416a^2bc\right)}{585b^4}$	77
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\left(-45b^3d\,x^{12}+60a\,b^2d\,x^8-65c\,b^3x^8-96a^2bd\,x^4+104a\,b^2c\,x^4+384a^3d-416a^2bc\right)}{585b^4}$	77

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-128/195*(b*x^4+a)^(1/4)*(-65/384*(9/13*d*x^4+c)*x^8*b^3+13/48*x^4*a*(15/2
6*d*x^4+c)*b^2-13/12*a^2*(3/13*d*x^4+c)*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(45\,b^3dx^{12}+5\,(13\,b^3c-12\,ab^2d)x^8-8\,(13\,ab^2c-12\,a^2bd)x^4+416\,a^2bc-384\,a^3d)(b}{585\,b^4}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/585*(45*b^3*d*x^12 + 5*(13*b^3*c - 12*a*b^2*d)*x^8 - 8*(13*a*b^2*c - 12*
a^2*b*d)*x^4 + 416*a^2*b*c - 384*a^3*d)*(b*x^4 + a)^(1/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \begin{cases} -\frac{128a^3d\sqrt[4]{a+bx^4}}{195b^4}+\frac{32a^2c\sqrt[4]{a+bx^4}}{45b^3}+\frac{32a^2dx^4\sqrt[4]{a+bx^4}}{195b^3}-\frac{8acx^4\sqrt[4]{a+bx^4}}{45b^2}-\frac{4adx^8\sqrt[4]{a}}{39b} \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{16}}{16}}{a^{\frac{3}{4}}} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-128*a**3*d*(a + b*x**4)**(1/4)/(195*b**4) + 32*a**2*c*(a + b*x
**4)**(1/4)/(45*b**3) + 32*a**2*d*x**4*(a + b*x**4)**(1/4)/(195*b**3) - 8*
a*c*x**4*(a + b*x**4)**(1/4)/(45*b**2) - 4*a*d*x**8*(a + b*x**4)**(1/4)/(3
9*b**2) + c*x**8*(a + b*x**4)**(1/4)/(9*b) + d*x**12*(a + b*x**4)**(1/4)/(
13*b), Ne(b, 0)), ((c*x**12/12 + d*x**16/16)/a**(3/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{1}{195}\,d\left(\frac{15\,(bx^4+a)^{\frac{13}{4}}}{b^4} - \frac{65\,(bx^4+a)^{\frac{9}{4}}a}{b^4} + \frac{117\,(bx^4+a)^{\frac{5}{4}}a^2}{b^4} - \frac{195\,(bx^4+a)^{\frac{1}{4}}a^3}{b^4}\right) + \frac{1}{45}\,c\left(\frac{5\,(bx^4+a)^{\frac{9}{4}}}{b^3} - \frac{18\,(bx^4+a)^{\frac{5}{4}}a}{b^3} + \frac{45\,(bx^4+a)^{\frac{1}{4}}a^2}{b^3}\right)$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/195*d*(15*(b*x^4 + a)^(13/4)/b^4 - 65*(b*x^4 + a)^(9/4)*a/b^4 + 117*(b*x
^4 + a)^(5/4)*a^2/b^4 - 195*(b*x^4 + a)^(1/4)*a^3/b^4) + 1/45*c*(5*(b*x^4
+ a)^(9/4)/b^3 - 18*(b*x^4 + a)^(5/4)*a/b^3 + 45*(b*x^4 + a)^(1/4)*a^2/b^3
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}(a^2bc-a^3d)}{b^4} + \frac{65\,(bx^4+a)^{\frac{9}{4}}bc - 234\,(bx^4+a)^{\frac{5}{4}}abc + 45\,(bx^4+a)^{\frac{13}{4}}d - 195\,(bx^4+a)^{\frac{9}{4}}ad + 351\,(bx^4+a)^{\frac{5}{4}}a^2d}{585\,b^4}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
(b*x^4 + a)^(1/4)*(a^2*b*c - a^3*d)/b^4 + 1/585*(65*(b*x^4 + a)^(9/4)*b*c
- 234*(b*x^4 + a)^(5/4)*a*b*c + 45*(b*x^4 + a)^(13/4)*d - 195*(b*x^4 + a)^
(9/4)*a*d + 351*(b*x^4 + a)^(5/4)*a^2*d)/b^4
```

Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{d\,x^{12}}{13\,b}-\frac{384\,a^3\,d-416\,a^2\,b\,c}{585\,b^4}+\frac{x^8\,(65\,b^3\,c-60\,a\,b^2\,d)}{585\,b^4}+\frac{8\,a\,x^4\,(12\,a\,d-1}{585\,b^3}\right.$$

input
```
int((x^11*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(1/4)*((d*x^12)/(13*b) - (384*a^3*d - 416*a^2*b*c)/(585*b^4) +
 (x^8*(65*b^3*c - 60*a*b^2*d))/(585*b^4) + (8*a*x^4*(12*a*d - 13*b*c))/(58
5*b^3))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^{15}}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**15/(a + b*x**4)**(3/4),x)*d + int(x**11/(a + b*x**4)**(3/4),x)*c
```

3.90 $\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = -\frac{a(bc-ad)\sqrt[4]{a+bx^4}}{b^3}+\frac{(bc-2ad)\left(a+bx^4\right)^{5/4}}{5b^3}+\frac{d\left(a+bx^4\right)^{9/4}}{9b^3}$$

output
```
-a*(-a*d+b*c)*(b*x^4+a)^(1/4)/b^3+1/5*(-2*a*d+b*c)*(b*x^4+a)^(5/4)/b^3+1/9
*d*(b*x^4+a)^(9/4)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}(-36abc+32a^2d+9b^2cx^4-8abdx^4+5b^2dx^8)}{45b^3}$$

input
```
Integrate[(x^7*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(-36*a*b*c + 32*a^2*d + 9*b^2*c*x^4 - 8*a*b*d*x^4 + 5*b
^2*d*x^8))/(45*b^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (c+dx^4)}{(a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4 (dx^4+c)}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{d(bx^4+a)^{5/4}}{b^2}+\frac{(bc-2ad)\sqrt[4]{bx^4+a}}{b^2}+\frac{a(ad-bc)}{b^2 (bx^4+a)^{3/4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4(a+bx^4)^{5/4} (bc-2ad)}{5b^3}-\frac{4a\sqrt[4]{a+bx^4}(bc-ad)}{b^3}+\frac{4d(a+bx^4)^{9/4}}{9b^3}\right)$$

input
```
Int[(x^7*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((-4*a*(b*c - a*d)*(a + b*x^4)^(1/4))/b^3 + (4*(b*c - 2*a*d)*(a + b*x^4)^(
5/4))/(5*b^3) + (4*d*(a + b*x^4)^(9/4))/(9*b^3))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{32\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(\frac{9x^{4}\left(\frac{5d\,x^{4}}{9}+c\right)b^{2}}{32}-\frac{9\left(\frac{2d\,x^{4}}{9}+c\right)ab}{8}+a^{2}d\right)}{45b^{3}}$	49
gosper	$\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(5d\,b^{2}x^{8}-8abd\,x^{4}+9b^{2}c\,x^{4}+32a^{2}d-36abc\right)}{45b^{3}}$	53
trager	$\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(5d\,b^{2}x^{8}-8abd\,x^{4}+9b^{2}c\,x^{4}+32a^{2}d-36abc\right)}{45b^{3}}$	53
risch	$\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(5d\,b^{2}x^{8}-8abd\,x^{4}+9b^{2}c\,x^{4}+32a^{2}d-36abc\right)}{45b^{3}}$	53
orering	$\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(5d\,b^{2}x^{8}-8abd\,x^{4}+9b^{2}c\,x^{4}+32a^{2}d-36abc\right)}{45b^{3}}$	53

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
32/45*(b*x^4+a)^(1/4)*(9/32*x^4*(5/9*d*x^4+c)*b^2-9/8*(2/9*d*x^4+c)*a*b+a^
2*d)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(5\,b^2dx^8+(9\,b^2c-8\,abd)x^4-36\,abc+32\,a^2d)(bx^4+a)^{\frac{1}{4}}}{45\,b^3}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/45*(5*b^2*d*x^8 + (9*b^2*c - 8*a*b*d)*x^4 - 36*a*b*c + 32*a^2*d)*(b*x^4
+ a)^(1/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \begin{cases} \frac{32a^2d\sqrt[4]{a+bx^4}}{45b^3} - \frac{4ac\sqrt[4]{a+bx^4}}{5b^2} - \frac{8adx^4\sqrt[4]{a+bx^4}}{45b^2} + \frac{cx^4\sqrt[4]{a+bx^4}}{5b} + \frac{dx^8\sqrt[4]{a+bx^4}}{9b} & \text{fo} \\ \frac{\frac{cx^8}{8}+\frac{dx^{12}}{12}}{a^{\frac{3}{4}}} & \text{ot} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((32*a**2*d*(a + b*x**4)**(1/4)/(45*b**3) - 4*a*c*(a + b*x**4)**(
1/4)/(5*b**2) - 8*a*d*x**4*(a + b*x**4)**(1/4)/(45*b**2) + c*x**4*(a + b*x
**4)**(1/4)/(5*b) + d*x**8*(a + b*x**4)**(1/4)/(9*b), Ne(b, 0)), ((c*x**8/
8 + d*x**12/12)/a**(3/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{1}{45}\,d\left(\frac{5\,(bx^4+a)^{\frac{9}{4}}}{b^3}-\frac{18\,(bx^4+a)^{\frac{5}{4}}a}{b^3}+\frac{45\,(bx^4+a)^{\frac{1}{4}}a^2}{b^3}\right)+\frac{1}{5}\,c\left(\frac{(bx^4+a)^{\frac{5}{4}}}{b^2}-\frac{5\,(bx^4+a)^{\frac{1}{4}}a}{b^2}\right)$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/45*d*(5*(b*x^4 + a)^(9/4)/b^3 - 18*(b*x^4 + a)^(5/4)*a/b^3 + 45*(b*x^4 +
 a)^(1/4)*a^2/b^3) + 1/5*c*((b*x^4 + a)^(5/4)/b^2 - 5*(b*x^4 + a)^(1/4)*a/
b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = -\frac{(bx^4+a)^{\frac{1}{4}}(abc-a^2d)}{b^3}+\frac{9\,(bx^4+a)^{\frac{5}{4}}bc+5\,(bx^4+a)^{\frac{9}{4}}d-18\,(bx^4+a)^{\frac{5}{4}}ad}{45\,b^3}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(b*x^4 + a)^(1/4)*(a*b*c - a^2*d)/b^3 + 1/45*(9*(b*x^4 + a)^(5/4)*b*c + 5
*(b*x^4 + a)^(9/4)*d - 18*(b*x^4 + a)^(5/4)*a*d)/b^3
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{32\,a^2\,d-36\,a\,b\,c}{45\,b^3}+\frac{d\,x^8}{9\,b}+\frac{x^4\,\left(9\,b^2\,c-8\,a\,b\,d\right)}{45\,b^3}\right)$$

input
```
int((x^7*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(1/4)*((32*a^2*d - 36*a*b*c)/(45*b^3) + (d*x^8)/(9*b) + (x^4*(
9*b^2*c - 8*a*b*d))/(45*b^3))
```

Reduce [F]

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^{11}}{\left(b\,x^4+a\right)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{x^7}{\left(b\,x^4+a\right)^{\frac{3}{4}}}dx\right)c$$

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**11/(a + b*x**4)**(3/4),x)*d + int(x**7/(a + b*x**4)**(3/4),x)*c
```

3.91 $\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}} \, dx = \frac{(bc-ad)\sqrt[4]{a+bx^4}}{b^2} + \frac{d(a+bx^4)^{5/4}}{5b^2}$$

output
```
(-a*d+b*c)*(b*x^4+a)^(1/4)/b^2+1/5*d*(b*x^4+a)^(5/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}} \, dx = \frac{\sqrt[4]{a+bx^4}(5bc-4ad+bdx^4)}{5b^2}$$

input
```
Integrate[(x^3*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(5*b*c - 4*a*d + b*d*x^4))/(5*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}} dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{4}\int \frac{dx^4+c}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{4}\int \left(\frac{\sqrt[4]{bx^4+a}d}{b}+\frac{bc-ad}{b(bx^4+a)^{3/4}}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{4\sqrt[4]{a+bx^4}(bc-ad)}{b^2}+\frac{4d(a+bx^4)^{5/4}}{5b^2}\right)$$

input
```
Int[(x^3*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
((4*(b*c - a*d)*(a + b*x^4)^(1/4))/b^2 + (4*d*(a + b*x^4)^(5/4))/(5*b^2))/
4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-db\,x^4+4ad-5cb)}{5b^2}$	31
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-db\,x^4+4ad-5cb)}{5b^2}$	31
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-db\,x^4+4ad-5cb)}{5b^2}$	31
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-db\,x^4+4ad-5cb)}{5b^2}$	31
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-db\,x^4+4ad-5cb)}{5b^2}$	31

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/4)*(-b*d*x^4+4*a*d-5*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(bdx^4+5\,bc-4\,ad)(bx^4+a)^{\frac{1}{4}}}{5\,b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/5*(b*d*x^4 + 5*b*c - 4*a*d)*(b*x^4 + a)^(1/4)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \begin{cases} -\frac{4ad\sqrt[4]{a+bx^4}}{5b^2} + \frac{c\sqrt[4]{a+bx^4}}{b} + \frac{dx^4\sqrt[4]{a+bx^4}}{5b} & \text{for } b \neq 0 \\ \frac{\frac{cx^4}{4}+\frac{dx^8}{8}}{a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-4*a*d*(a + b*x**4)**(1/4)/(5*b**2) + c*(a + b*x**4)**(1/4)/b +
 d*x**4*(a + b*x**4)**(1/4)/(5*b), Ne(b, 0)), ((c*x**4/4 + d*x**8/8)/a**(3
/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{1}{5}\,d\left(\frac{(bx^4+a)^{\frac{5}{4}}}{b^2} - \frac{5\,(bx^4+a)^{\frac{1}{4}}a}{b^2}\right) + \frac{(bx^4+a)^{\frac{1}{4}}c}{b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*d*((b*x^4 + a)^(5/4)/b^2 - 5*(b*x^4 + a)^(1/4)*a/b^2) + (b*x^4 + a)^(1
/4)*c/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}d}{5\,b^2} + \frac{(bx^4+a)^{\frac{1}{4}}(bc-ad)}{b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/5*(b*x^4 + a)^(5/4)*d/b^2 + (b*x^4 + a)^(1/4)*(b*c - a*d)/b^2
```

Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = -\left(b\,x^4+a\right)^{1/4}\left(\frac{4\,a\,d-5\,b\,c}{5\,b^2}-\frac{d\,x^4}{5\,b}\right)$$

input
```
int((x^3*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
-(a + b*x^4)^(1/4)*((4*a*d - 5*b*c)/(5*b^2) - (d*x^4)/(5*b))
```

Reduce [F]

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,c+\left(\int \frac{x^7}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)bd}{b}$$

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(1/4)*c + int(x**7/(a + b*x**4)**(3/4),x)*b*d)/b
```

3.92 $\int \frac{c+dx^4}{x(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{c+dx^4}{x(a+bx^4)^{3/4}}\,dx = \frac{d\sqrt[4]{a+bx^4}}{b}-\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

output
```
d*(b*x^4+a)^(1/4)/b-1/2*c*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/2*c*ar
ctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x(a+bx^4)^{3/4}}\,dx = \frac{d\sqrt[4]{a+bx^4}}{b}-\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x*(a + b*x^4)^(3/4)),x]
```

output
```
(d*(a + b*x^4)^(1/4))/b - (c*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)
) - (c*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^4\left(bx^4+a\right)^{3/4}}dx^4$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{4}\left(c\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4+\frac{4d\sqrt[4]{a+bx^4}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{4c\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{b}+\frac{4d\sqrt[4]{a+bx^4}}{b}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{b}+\frac{4d\sqrt[4]{a+bx^4}}{b}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+\frac{4d\sqrt[4]{a+bx^4}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+\frac{4d\sqrt[4]{a+bx^4}}{b}\right)$$

input
```
Int[(c + d*x^4)/(x*(a + b*x^4)^(3/4)),x]
```

output
```
((4*d*(a + b*x^4)^(1/4))/b + (4*c*(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4
)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/b)/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{1}{4}} d\,a^{\frac{3}{4}}-\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) c b-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) c b}{4 b\,a^{\frac{3}{4}}}$	79

input
```
int((d*x^4+c)/x/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*(b*x^4+a)^(1/4)*d*a^(3/4)-ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(
1/4)-a^(1/4)))*c*b-2*arctan((b*x^4+a)^(1/4)/a^(1/4))*c*b)/b/a^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.25

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{b\left(\frac{c^4}{a^3}\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}c+a\left(\frac{c^4}{a^3}\right)^{\frac{1}{4}}\right)+i\,b\left(\frac{c^4}{a^3}\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}c+i\,a\left(\frac{c^4}{a^3}\right)^{\frac{1}{4}}\right)-i\,b\left(\frac{c^4}{a^3}\right)^{\frac{1}{4}}\log\left(\left(bx^4+\right.\right.}{4\,b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/4*(b*(c^4/a^3)^(1/4)*log((b*x^4 + a)^(1/4)*c + a*(c^4/a^3)^(1/4)) + I*b
*(c^4/a^3)^(1/4)*log((b*x^4 + a)^(1/4)*c + I*a*(c^4/a^3)^(1/4)) - I*b*(c^4
/a^3)^(1/4)*log((b*x^4 + a)^(1/4)*c - I*a*(c^4/a^3)^(1/4)) - b*(c^4/a^3)^(
1/4)*log((b*x^4 + a)^(1/4)*c - a*(c^4/a^3)^(1/4)) - 4*(b*x^4 + a)^(1/4)*d)
/b
```

Sympy [A] (verification not implemented)

Time = 29.88 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx = d\left(\begin{cases}\frac{x^4}{4a^{\frac{3}{4}}} & \text{for } b=0\\ \frac{\sqrt[4]{a+bx^4}}{b} & \text{otherwise}\end{cases}\right)-\frac{c\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x/(b*x**4+a)**(3/4),x)
```

output
```
d*Piecewise((x**4/(4*a**(3/4)), Eq(b, 0)), ((a + b*x**4)**(1/4)/b, True))
- c*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b*
*(3/4)*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx=-\frac{1}{4}\,c\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)+\frac{(bx^4+a)^{\frac{1}{4}}d}{b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/4*c*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - log(((b*x^4 + a)^(1/
4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4)) + (b*x^4 + a)^(1/4)*
d/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.67

$$\begin{aligned}\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx &= \frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{3}{4}}} \\ &+ \frac{\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{3}{4}}} \\ &+ \frac{\sqrt{2}c\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,(-a)^{\frac{3}{4}}} \\ &+ \frac{\sqrt{2}(-a)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a}+\frac{(bx^4+a)^{\frac{1}{4}}d}{b}\end{aligned}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(1/4)
)/(-a)^(1/4))/(-a)^(3/4) + 1/4*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)
^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 1/8*sqrt(2)*c*log(s
qrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/(-a)^(3/
4) + 1/8*sqrt(2)*(-a)^(1/4)*c*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) +
sqrt(b*x^4 + a) + sqrt(-a))/a + (b*x^4 + a)^(1/4)*d/b
```

Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx=\frac{d\,{\left(b\,x^4+a\right)}^{1/4}}{b}-\frac{c\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}-\frac{c\,\mathrm{atanh}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}$$

input
```
int((c + d*x^4)/(x*(a + b*x^4)^(3/4)),x)
```

output
```
(d*(a + b*x^4)^(1/4))/b - (c*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(3/4))
- (c*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(3/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{3/4}}\,dx=\frac{\left(b\,x^4+a\right)^{\frac{1}{4}} d+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}} x}d x\right) b c}{b}$$

input
```
int((d*x^4+c)/x/(b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(1/4)*d + int(1/((a + b*x**4)**(3/4)*x),x)*b*c)/b
```

3.93 $\int \frac{c+dx^4}{x^5(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{4ax^4}+\frac{(3bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}+\frac{(3bc-4ad)\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

output
```
-1/4*c*(b*x^4+a)^(1/4)/a/x^4+1/8*(-4*a*d+3*b*c)*arctan((b*x^4+a)^(1/4)/a^(
1/4))/a^(7/4)+1/8*(-4*a*d+3*b*c)*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{4ax^4}+\frac{(3bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}+\frac{(3bc-4ad)\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

input
```
Integrate[(c + d*x^4)/(x^5*(a + b*x^4)^(3/4)),x]
```

output
```
-1/4*(c*(a + b*x^4)^(1/4))/(a*x^4) + ((3*b*c - 4*a*d)*ArcTan[(a + b*x^4)^(
1/4)/a^(1/4)])/(8*a^(7/4)) + ((3*b*c - 4*a*d)*ArcTanh[(a + b*x^4)^(1/4)/a^
(1/4)])/(8*a^(7/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{dx^4+c}{x^8 \left(bx^4+a\right)^{3/4}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4} \left(-\frac{(3bc-4ad) \int \frac{1}{x^4 \left(bx^4+a\right)^{3/4}}dx^4}{4a}-\frac{c \sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{(3bc-4ad) \int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{ab}-\frac{c \sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4} \left(-\frac{(3bc-4ad) \left(-\frac{b \int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b \int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{ab}-\frac{c \sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\downarrow 216$$

$$\frac{1}{4}\left(-\frac{(3bc-4ad)\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}-\frac{c\sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\downarrow 219$$

$$\frac{1}{4}\left(-\frac{(3bc-4ad)\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}-\frac{c\sqrt[4]{a+bx^4}}{ax^4}\right)$$

input
```
Int[(c + d*x^4)/(x^5*(a + b*x^4)^(3/4)),x]
```

output
```
(-((c*(a + b*x^4)^(1/4))/(a*x^4)) - ((3*b*c - 4*a*d)*(-1/2*(b*ArcTan[(a +
b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2
*a^(3/4))))/(a*b))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{x^{4}\left(-4a^{2}d+3abc\right)\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)+2x^{4}\left(-4a^{2}d+3abc\right)\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)-4c\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{\frac{7}{4}}}{16a^{\frac{11}{4}}x^{4}}$	104

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(x^4*(-4*a^2*d+3*a*b*c)*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)
-a^(1/4)))+2*x^4*(-4*a^2*d+3*a*b*c)*arctan((b*x^4+a)^(1/4)/a^(1/4))-4*c*(b
*x^4+a)^(1/4)*a^(7/4))/a^(11/4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 607, normalized size of antiderivative = 6.39

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{3/4}}\,dx = -\frac{ax^4\left(\frac{81\,b^4c^4-432\,ab^3c^3d+864\,a^2b^2c^2d^2-768\,a^3bcd^3+256\,a^4d^4}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(\frac{81\,b^4c^4-432\,ab^3c^3d+864\,a^2b^2c^2d^2-768\,a^3bcd^3+256\,a^4d^4}{a^7}\right)^{\frac{1}{4}}\right.}{}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/16*(a*x^4*((81*b^4*c^4 - 432*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 768*a^
3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1/4)*log(a^2*((81*b^4*c^4 - 432*a*b^3*c^3*d
 + 864*a^2*b^2*c^2*d^2 - 768*a^3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1/4) - (b*x^
4 + a)^(1/4)*(3*b*c - 4*a*d)) + I*a*x^4*((81*b^4*c^4 - 432*a*b^3*c^3*d + 8
64*a^2*b^2*c^2*d^2 - 768*a^3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1/4)*log(I*a^2*(
(81*b^4*c^4 - 432*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 768*a^3*b*c*d^3 + 25
6*a^4*d^4)/a^7)^(1/4) - (b*x^4 + a)^(1/4)*(3*b*c - 4*a*d)) - I*a*x^4*((81*
b^4*c^4 - 432*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 768*a^3*b*c*d^3 + 256*a^
4*d^4)/a^7)^(1/4)*log(-I*a^2*((81*b^4*c^4 - 432*a*b^3*c^3*d + 864*a^2*b^2*
c^2*d^2 - 768*a^3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1/4) - (b*x^4 + a)^(1/4)*(3
*b*c - 4*a*d)) - a*x^4*((81*b^4*c^4 - 432*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^
2 - 768*a^3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1/4)*log(-a^2*((81*b^4*c^4 - 432*
a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 768*a^3*b*c*d^3 + 256*a^4*d^4)/a^7)^(1
/4) - (b*x^4 + a)^(1/4)*(3*b*c - 4*a*d)) + 4*(b*x^4 + a)^(1/4)*c)/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{3/4}}\,dx=-\frac{c\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}-\frac{d\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**5/(b*x**4+a)**(3/4),x)
```

output
```
-c*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b*
*(3/4)*x**7*gamma(11/4)) - d*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*exp_po
lar(I*pi)/(b*x**4))/(4*b**(3/4)*x**3*gamma(7/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.66

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{3/4}}\,dx=-\frac{1}{16}\,c\left(\frac{4\left(bx^4+a\right)^{\frac{1}{4}}b}{\left(bx^4+a\right)a-a^2}-\frac{3\left(\frac{2\,b\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{a}\right)-\frac{1}{4}\,d\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/16*c*(4*(b*x^4 + a)^(1/4)*b/((b*x^4 + a)*a - a^2) - 3*(2*b*arctan((b*x^
4 + a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^
4 + a)^(1/4) + a^(1/4)))/a^(3/4))/a) - 1/4*d*(2*arctan((b*x^4 + a)^(1/4)/a
^(1/4))/a^(3/4) - log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a
^(1/4)))/a^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.71

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{3/4}}\,dx = -\frac{1}{32}\,b\left(\frac{2\sqrt{2}(3\,bc-4\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}\,ab}+\frac{2\sqrt{2}(3\,bc-4\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(b\right.}{2\,(-a)^{\frac{1}{4}}}\right.}{(-a)^{\frac{3}{4}}\,ab}\right.$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-1/32*b*(2*sqrt(2)*(3*b*c - 4*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4)
+ 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a*b) + 2*sqrt(2)*(3*b*c - 4
*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^
(1/4))/((-a)^(3/4)*a*b) + sqrt(2)*(3*b*c - 4*a*d)*log(sqrt(2)*(b*x^4 + a)^
(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a*b) - sqrt(2)*
(3*b*c - 4*a*d)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a
) + sqrt(-a))/((-a)^(3/4)*a*b) + 8*(b*x^4 + a)^(1/4)*c/(a*b*x^4))
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{3/4}}\,dx = \frac{3\,b\,c\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}} - \frac{d\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}} - \frac{c\,\left(b\,x^4+a\right)^{1/4}}{4\,a\,x^4} - \frac{d\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}} + \frac{3\,b\,c\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}$$

input
```
int((c + d*x^4)/(x^5*(a + b*x^4)^(3/4)),x)
```

output
```
(3*b*c*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4)) - (d*atanh((a + b*x^4)
^(1/4)/a^(1/4)))/(2*a^(3/4)) - (c*(a + b*x^4)^(1/4))/(4*a*x^4) - (d*atan((
a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(3/4)) + (3*b*c*atanh((a + b*x^4)^(1/4)/a^
(1/4)))/(8*a^(7/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{3/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}} x^5}dx\right) c + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}} x}dx\right) d$$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**5),x)*c + int(1/((a + b*x**4)**(3/4)*x),x)*d
```

3.94 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}} \, dx = \frac{dx^3\sqrt[4]{a+bx^4}}{4b} - \frac{(4bc-3ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}} + \frac{(4bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}$$

output
```
1/4*d*x^3*(b*x^4+a)^(1/4)/b-1/8*(-3*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^
(1/4))/b^(7/4)+1/8*(-3*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/
4)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}} \, dx = \frac{2b^{3/4}dx^3\sqrt[4]{a+bx^4} + (-4bc+3ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + (4bc-3ad)\text{arctanh}\left(\frac{\sqrt[4]{\ }}{\sqrt[4]{a+\ }}\right.}{8b^{7/4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(3/4)*d*x^3*(a + b*x^4)^(1/4) + (-4*b*c + 3*a*d)*ArcTan[(b^(1/4)*x)/(
a + b*x^4)^(1/4)] + (4*b*c - 3*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
)/(8*b^(7/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \; 959$$

$$\frac{(4bc-3ad)\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b}$$

$$\Big\downarrow \; 854$$

$$\frac{(4bc-3ad)\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b}$$

$$\Big\downarrow \; 827$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b}$$

$$\Big\downarrow \; 216$$

$$\frac{(4bc-3ad)\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b}$$

$$\downarrow 219$$

$$\frac{(4bc-3ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b}$$

input

```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output

```
(d*x^3*(a + b*x^4)^(1/4))/(4*b) + ((4*b*c - 3*a*d)*(-1/2*ArcTan[(b^(1/4)*x
)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b
^(3/4))))/(4*b)
```

Defintions of rubi rules used

rule 216

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{d\,x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{4b}-\frac{3\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)\left(ad-\frac{4cb}{3}\right)}{16b^{\frac{7}{4}}}-\frac{3\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)\left(ad-\frac{4cb}{3}\right)}{8b^{\frac{7}{4}}}$	98

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*x^3*(b*x^4+a)^(1/4)/b-3/16/b^(7/4)*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-
x*b^(1/4)+(b*x^4+a)^(1/4)))*(a*d-4/3*c*b)-3/8/b^(7/4)*arctan((b*x^4+a)^(1/
4)/x/b^(1/4))*(a*d-4/3*c*b)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.33

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{4\,(bx^4+a)^{\frac{1}{4}}dx^3+b\left(\frac{256\,b^4c^4-768\,ab^3c^3d+864\,a^2b^2c^2d^2-432\,a^3bcd^3+81\,a^4d^4}{b^7}\right)^{\frac{1}{4}}\log\left(-\frac{b^2x\left(256\,b^4c^4-7\right.}{}\right.}{}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(1/4)*d*x^3 + b*((256*b^4*c^4 - 768*a*b^3*c^3*d + 864*
a^2*b^2*c^2*d^2 - 432*a^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1/4)*log(-(b^2*x*((2
56*b^4*c^4 - 768*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 432*a^3*b*c*d^3 + 81*
a^4*d^4)/b^7)^(1/4) + (b*x^4 + a)^(1/4)*(4*b*c - 3*a*d))/x) - b*((256*b^4*
c^4 - 768*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 432*a^3*b*c*d^3 + 81*a^4*d^4
)/b^7)^(1/4)*log((b^2*x*((256*b^4*c^4 - 768*a*b^3*c^3*d + 864*a^2*b^2*c^2*
d^2 - 432*a^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1/4) - (b*x^4 + a)^(1/4)*(4*b*c
- 3*a*d))/x) - I*b*((256*b^4*c^4 - 768*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 -
 432*a^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1/4)*log((I*b^2*x*((256*b^4*c^4 - 768
*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 432*a^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1
/4) - (b*x^4 + a)^(1/4)*(4*b*c - 3*a*d))/x) + I*b*((256*b^4*c^4 - 768*a*b^
3*c^3*d + 864*a^2*b^2*c^2*d^2 - 432*a^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1/4)*l
og((-I*b^2*x*((256*b^4*c^4 - 768*a*b^3*c^3*d + 864*a^2*b^2*c^2*d^2 - 432*a
^3*b*c*d^3 + 81*a^4*d^4)/b^7)^(1/4) - (b*x^4 + a)^(1/4)*(4*b*c - 3*a*d))/x
))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/4)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/4)*gamma(11/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.91

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{1}{4}\,c\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right) - \frac{1}{16}\,d\left(\frac{3\left(\frac{2\,a\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{b} + \frac{4\,(bx^4+a)^{\frac{1}{4}}a}{\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x}\right)$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/4*c*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - log(-(b^(1/4) - (
b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4)) - 1/16*d*(3*
(2*a*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a*log(-(b^(1/4) - (b*
x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b + 4*(b*x^4 +
 a)^(1/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x))
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**6/(a + b*x**4)**(3/4),x)*d + int(x**2/(a + b*x**4)**(3/4),x)*c
```

3.95 $\int \frac{c+dx^4}{x^2(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{ax} - \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

output
```
-c*(b*x^4+a)^(1/4)/a/x-1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)+1/2
*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{ax} - \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(3/4)),x]
```

output
```
-((c*(a + b*x^4)^(1/4))/(a*x)) - (d*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])
/(2*b^(3/4)) + (d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*b^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {953, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 \left(a+bx^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{953}$$

$$d\int \frac{x^2}{\left(bx^4+a\right)^{3/4}}dx-\frac{c\sqrt[4]{a+bx^4}}{ax}$$

$$\Big\downarrow \text{854}$$

$$d\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}-\frac{c\sqrt[4]{a+bx^4}}{ax}$$

$$\Big\downarrow \text{827}$$

$$d\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)-\frac{c\sqrt[4]{a+bx^4}}{ax}$$

$$\Big\downarrow \text{216}$$

$$d\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)-\frac{c\sqrt[4]{a+bx^4}}{ax}$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)-\frac{c\sqrt[4]{a+bx^4}}{ax}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(3/4)),x]
```

output
```
-((c*(a + b*x^4)^(1/4))/(a*x)) + d*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1
/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13

method	result	size
pseudoelliptic	$-\frac{c\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{\frac{3}{4}}-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right) a d x}{2}-\frac{\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a d x}{4}}{b^{\frac{3}{4}} x a}$	89

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/b^(3/4)*(c*(b*x^4+a)^(1/4)*b^(3/4)-1/2*arctan((b*x^4+a)^(1/4)/x/b^(1/4)
)*a*d*x-1/4*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*a
*d*x)/x/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/4}}\,dx = \frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} + \frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(3/4),x)
```

output
```
b**(1/4)*c*(a/(b*x**4) + 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4)) + d*x**3*g
amma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/4)*
gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{3/4}}\,dx = \frac{1}{4}d\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right) - \frac{\left(bx^4+a\right)^{\frac{1}{4}}c}{ax}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/4*d*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - log(-(b^(1/4) - (
b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4)) - (b*x^4 + a
)^(1/4)*c/(a*x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{3/4}}\,dx = \int \frac{d\,x^4+c}{x^2\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(3/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^2}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(3/4),x)
```

output
```
int(x**2/(a + b*x**4)**(3/4),x)*d + int(1/((a + b*x**4)**(3/4)*x**2),x)*c
```

3.96 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/4}}\,dx=-\frac{c\sqrt[4]{a+bx^4}}{5ax^5}+\frac{(4bc-5ad)\sqrt[4]{a+bx^4}}{5a^2x}$$

output
```
-1/5*c*(b*x^4+a)^(1/4)/a/x^5+1/5*(-5*a*d+4*b*c)*(b*x^4+a)^(1/4)/a^2/x
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/4}}\,dx=\frac{\sqrt[4]{a+bx^4}(-ac+4bcx^4-5adx^4)}{5a^2x^5}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-(a*c) + 4*b*c*x^4 - 5*a*d*x^4))/(5*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{3/4}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(4bc-5ad)\int \frac{1}{x^2\left(bx^4+a\right)^{3/4}}dx}{5a}-\frac{c\sqrt[4]{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \ 796$$

$$\frac{\sqrt[4]{a+bx^4}(4bc-5ad)}{5a^2x}-\frac{c\sqrt[4]{a+bx^4}}{5ax^5}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(3/4)),x]
```

output
```
-1/5*(c*(a + b*x^4)^(1/4))/(a*x^5) + ((4*b*c - 5*a*d)*(a + b*x^4)^(1/4))/(
5*a^2*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5ad\,x^4-4bc\,x^4+ac)}{5x^5a^2}$	36
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5ad\,x^4-4bc\,x^4+ac)}{5x^5a^2}$	36
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5ad\,x^4-4bc\,x^4+ac)}{5x^5a^2}$	36
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,((5d\,x^4+c)a-4bc\,x^4)}{5x^5a^2}$	36
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5ad\,x^4-4bc\,x^4+ac)}{5x^5a^2}$	36

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/4)*(5*a*d*x^4-4*b*c*x^4+a*c)/x^5/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = \frac{((4\,bc-5\,ad)x^4-ac)(bx^4+a)^{\frac{1}{4}}}{5\,a^2x^5}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/5*((4*b*c - 5*a*d)*x^4 - a*c)*(b*x^4 + a)^(1/4)/(a^2*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(42) = 84.

Time = 2.95 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.98

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{b}d\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}}c\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(3/4),x)
```

output
```
-b**(1/4)*c*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(16*a*x**4*gamma(3/4)) + b
**(1/4)*d*(a/(b*x**4) + 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4)) + b**(5/4)*
c*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*a**2*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = \frac{c\left(\frac{5\,(bx^4+a)^{\frac{1}{4}}b}{x}-\frac{(bx^4+a)^{\frac{5}{4}}}{x^5}\right)}{5\,a^2}-\frac{(bx^4+a)^{\frac{1}{4}}d}{ax}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*c*(5*(b*x^4 + a)^(1/4)*b/x - (b*x^4 + a)^(5/4)/x^5)/a^2 - (b*x^4 + a)^
(1/4)*d/(a*x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = -\frac{(b\,x^4+a)^{1/4}\,(a\,c+5\,a\,d\,x^4-4\,b\,c\,x^4)}{5\,a^2\,x^5}$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(3/4)),x)
```

output
```
-((a + b*x^4)^(1/4)*(a*c + 5*a*d*x^4 - 4*b*c*x^4))/(5*a^2*x^5)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{3/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^6}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^2}dx\right)d$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**6),x)*c + int(1/((a + b*x**4)**(3/4)*x**2),x
)*d
```

3.97 $\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=-\frac{c\sqrt[4]{a+bx^4}}{9ax^9}+\frac{(8bc-9ad)\sqrt[4]{a+bx^4}}{45a^2x^5}-\frac{4b(8bc-9ad)\sqrt[4]{a+bx^4}}{45a^3x}$$

output
```
-1/9*c*(b*x^4+a)^(1/4)/a/x^9+1/45*(-9*a*d+8*b*c)*(b*x^4+a)^(1/4)/a^2/x^5-4
/45*b*(-9*a*d+8*b*c)*(b*x^4+a)^(1/4)/a^3/x
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=\frac{\sqrt[4]{a+bx^4}\left(-5a^2c+8abcx^4-9a^2dx^4-32b^2cx^8+36abdx^8\right)}{45a^3x^9}$$

input
```
Integrate[(c + d*x^4)/(x^10*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-5*a^2*c + 8*a*b*c*x^4 - 9*a^2*d*x^4 - 32*b^2*c*x^8 +
36*a*b*d*x^8))/(45*a^3*x^9)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(8bc-9ad)\int \frac{1}{x^6\left(bx^4+a\right)^{3/4}}dx}{9a}-\frac{c\sqrt[4]{a+bx^4}}{9ax^9}$$

$$\Big\downarrow \ 803$$

$$-\frac{(8bc-9ad)\left(-\frac{4b\int \frac{1}{x^2\left(bx^4+a\right)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{c\sqrt[4]{a+bx^4}}{9ax^9}$$

$$\Big\downarrow \ 796$$

$$-\frac{\left(\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)(8bc-9ad)}{9a}-\frac{c\sqrt[4]{a+bx^4}}{9ax^9}$$

input
```
Int[(c + d*x^4)/(x^10*(a + b*x^4)^(3/4)),x]
```

output
```
-1/9*(c*(a + b*x^4)^(1/4))/(a*x^9) - ((8*b*c - 9*a*d)*(-1/5*(a + b*x^4)^(1
/4)/(a*x^5) + (4*b*(a + b*x^4)^(1/4))/(5*a^2*x)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(\frac{9 d\,x^{4}}{5}+c \right) a^{2}-\frac{8 \left(\frac{9 d\,x^{4}}{2}+c \right) b\,x^{4} a}{5}+\frac{32 b^{2} c\,x^{8}}{5}\right)}{9 x^{9} a^{3}}$	55
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(-36 a b d\,x^{8}+32 b^{2} c\,x^{8}+9 a^{2} d\,x^{4}-8 a b c\,x^{4}+5 a^{2} c \right)}{45 x^{9} a^{3}}$	59
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(-36 a b d\,x^{8}+32 b^{2} c\,x^{8}+9 a^{2} d\,x^{4}-8 a b c\,x^{4}+5 a^{2} c \right)}{45 x^{9} a^{3}}$	59
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(-36 a b d\,x^{8}+32 b^{2} c\,x^{8}+9 a^{2} d\,x^{4}-8 a b c\,x^{4}+5 a^{2} c \right)}{45 x^{9} a^{3}}$	59
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(-36 a b d\,x^{8}+32 b^{2} c\,x^{8}+9 a^{2} d\,x^{4}-8 a b c\,x^{4}+5 a^{2} c \right)}{45 x^{9} a^{3}}$	59

input
```
int((d*x^4+c)/x^10/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(b*x^4+a)^(1/4)*((9/5*d*x^4+c)*a^2-8/5*(9/2*d*x^4+c)*b*x^4*a+32/5*b^2
*c*x^8)/x^9/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{(4\,(8\,b^2c-9\,abd)x^8-(8\,abc-9\,a^2d)x^4+5\,a^2c)(bx^4+a)^{\frac{1}{4}}}{45\,a^3x^9}$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/45*(4*(8*b^2*c - 9*a*b*d)*x^8 - (8*a*b*c - 9*a^2*d)*x^4 + 5*a^2*c)*(b*x
^4 + a)^(1/4)/(a^3*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(76) = 152$.

Time = 4.14 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.81

$$\int \frac{c+dx^4}{x^{10}(a+bx^4)^{3/4}}\,dx = \frac{5a^4b^{\frac{17}{4}}c\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} + \frac{2a^3b^{\frac{21}{4}}cx^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} + \frac{21a^2b^{\frac{25}{4}}cx^8\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} + \frac{56ab^{\frac{29}{4}}cx^{12}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} + \frac{32b^{\frac{33}{4}}cx^{16}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} - \frac{\sqrt[4]{b}d\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}}d\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**10/(b*x**4+a)**(3/4),x)
```

output
```
5*a**4*b**(17/4)*c*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*
gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/4
)) + 2*a**3*b**(21/4)*c*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*
b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16
*gamma(3/4)) + 21*a**2*b**(25/4)*c*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4
)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3
*b**6*x**16*gamma(3/4)) + 56*a*b**(29/4)*c*x**12*(a/(b*x**4) + 1)**(1/4)*g
amma(-9/4)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4)
+ 64*a**3*b**6*x**16*gamma(3/4)) + 32*b**(33/4)*c*x**16*(a/(b*x**4) + 1)**
(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamm
a(3/4) + 64*a**3*b**6*x**16*gamma(3/4)) - b**(1/4)*d*(a/(b*x**4) + 1)**(1/
4)*gamma(-5/4)/(16*a*x**4*gamma(3/4)) + b**(5/4)*d*(a/(b*x**4) + 1)**(1/4)
*gamma(-5/4)/(4*a**2*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^4}{x^{10}(a+bx^4)^{3/4}}\,dx = \frac{d\left(\frac{5\,(bx^4+a)^{\frac{1}{4}}b}{x} - \frac{(bx^4+a)^{\frac{5}{4}}}{x^5}\right)}{5\,a^2} - \frac{\left(\frac{45\,(bx^4+a)^{\frac{1}{4}}b^2}{x} - \frac{18\,(bx^4+a)^{\frac{5}{4}}b}{x^5} + \frac{5\,(bx^4+a)^{\frac{9}{4}}}{x^9}\right)c}{45\,a^3}$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*d*(5*(b*x^4 + a)^(1/4)*b/x - (b*x^4 + a)^(5/4)/x^5)/a^2 - 1/45*(45*(b*
x^4 + a)^(1/4)*b^2/x - 18*(b*x^4 + a)^(5/4)*b/x^5 + 5*(b*x^4 + a)^(9/4)/x^
9)*c/a^3
```

Giac [F]

$$\int \frac{c+dx^4}{x^{10}(a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^{10}}\,dx$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^10), x)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\left(b\,x^4+a\right)^{1/4}\,\left(9\,d\,a^2\,x^4+5\,c\,a^2-36\,d\,a\,b\,x^8-8\,c\,a\,b\,x^4+32\,c\,b^2\,x^8\right)}{45\,a^3\,x^9}$$

input
```
int((c + d*x^4)/(x^10*(a + b*x^4)^(3/4)),x)
```

output
```
-((a + b*x^4)^(1/4)*(5*a^2*c + 9*a^2*d*x^4 + 32*b^2*c*x^8 - 8*a*b*c*x^4 -
36*a*b*d*x^8))/(45*a^3*x^9)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{10}}dx\right)c+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{6}}dx\right)d$$

input
```
int((d*x^4+c)/x^10/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**10),x)*c + int(1/((a + b*x**4)**(3/4)*x**6),
x)*d
```

3.98 $\int \frac{c+dx^4}{x^{14}(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{13ax^{13}}+\frac{(12bc-13ad)\sqrt[4]{a+bx^4}}{117a^2x^9} - \frac{8b(12bc-13ad)\sqrt[4]{a+bx^4}}{585a^3x^5}+\frac{32b^2(12bc-13ad)\sqrt[4]{a+bx^4}}{585a^4x}$$

output
```
-1/13*c*(b*x^4+a)^(1/4)/a/x^13+1/117*(-13*a*d+12*b*c)*(b*x^4+a)^(1/4)/a^2/
x^9-8/585*b*(-13*a*d+12*b*c)*(b*x^4+a)^(1/4)/a^3/x^5+32/585*b^2*(-13*a*d+1
2*b*c)*(b*x^4+a)^(1/4)/a^4/x
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}(-45a^3c+60a^2bcx^4-65a^3dx^4-96ab^2cx^8+104a^2bdx^8+384b^3cx^{12}-}{585a^4x^{13}}$$

input
```
Integrate[(c + d*x^4)/(x^14*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-45*a^3*c + 60*a^2*b*c*x^4 - 65*a^3*d*x^4 - 96*a*b^2*c
*x^8 + 104*a^2*b*d*x^8 + 384*b^3*c*x^12 - 416*a*b^2*d*x^12))/(585*a^4*x^13
)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(12bc-13ad)\int \frac{1}{x^{10}(bx^4+a)^{3/4}}dx}{13a}-\frac{c\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \ 803$$

$$-\frac{(12bc-13ad)\left(-\frac{8b\int \frac{1}{x^6\left(bx^4+a\right)^{3/4}}dx}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)}{13a}-\frac{c\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \ 803$$

$$-\frac{(12bc-13ad)\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^2\left(bx^4+a\right)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)}{13a}-\frac{c\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \ 796$$

$$-\frac{\left(-\frac{8b\left(\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)(12bc-13ad)}{13a}-\frac{c\sqrt[4]{a+bx^4}}{13ax^{13}}$$

input
```
Int[(c + d*x^4)/(x^14*(a + b*x^4)^(3/4)),x]
```

output
```
-1/13*(c*(a + b*x^4)^(1/4))/(a*x^13) - ((12*b*c - 13*a*d)*(-1/9*(a + b*x^4
)^(1/4)/(a*x^9) - (8*b*(-1/5*(a + b*x^4)^(1/4)/(a*x^5) + (4*b*(a + b*x^4)^
(1/4))/(5*a^2*x)))/(9*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\left(\left(\frac{13d\,x^4}{9}+c\right)a^3-\frac{4b\,x^4\left(\frac{26d\,x^4}{15}+c\right)a^2}{3}+\frac{32\left(\frac{13d\,x^4}{3}+c\right)b^2x^8a}{15}-\frac{128b^3c\,x^{12}}{15}\right)}{13x^{13}a^4}$	74
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(416a\,b^2d\,x^{12}-384b^3c\,x^{12}-104a^2bd\,x^8+96a\,b^2c\,x^8+65a^3d\,x^4-60a^2bc\,x^4+45c\,a^3)}{585x^{13}a^4}$	83
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(416a\,b^2d\,x^{12}-384b^3c\,x^{12}-104a^2bd\,x^8+96a\,b^2c\,x^8+65a^3d\,x^4-60a^2bc\,x^4+45c\,a^3)}{585x^{13}a^4}$	83
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(416a\,b^2d\,x^{12}-384b^3c\,x^{12}-104a^2bd\,x^8+96a\,b^2c\,x^8+65a^3d\,x^4-60a^2bc\,x^4+45c\,a^3)}{585x^{13}a^4}$	83
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(416a\,b^2d\,x^{12}-384b^3c\,x^{12}-104a^2bd\,x^8+96a\,b^2c\,x^8+65a^3d\,x^4-60a^2bc\,x^4+45c\,a^3)}{585x^{13}a^4}$	83

input
```
int((d*x^4+c)/x^14/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/13*(b*x^4+a)^(1/4)*((13/9*d*x^4+c)*a^3-4/3*b*x^4*(26/15*d*x^4+c)*a^2+32
/15*(13/3*d*x^4+c)*b^2*x^8*a-128/15*b^3*c*x^12)/x^13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \frac{(32\,(12\,b^3c-13\,ab^2d)x^{12}-8\,(12\,ab^2c-13\,a^2bd)x^8+5\,(12\,a^2bc-13\,a^3d)x^4-45\,c}{585\,a^4x^{13}}$$

input
```
integrate((d*x^4+c)/x^14/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/585*(32*(12*b^3*c - 13*a*b^2*d)*x^12 - 8*(12*a*b^2*c - 13*a^2*b*d)*x^8 +
 5*(12*a^2*b*c - 13*a^3*d)*x^4 - 45*a^3*c)*(b*x^4 + a)^(1/4)/(a^4*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(110) = 220$.

Time = 6.26 (sec) , antiderivative size = 1120, normalized size of antiderivative = 9.57

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**14/(b*x**4+a)**(3/4),x)
```

output
```
-45*a**6*b**(37/4)*c*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x
**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*g
amma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) - 75*a**5*b**(41/4)*c*x**4*(a
/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a
**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b*
*12*x**24*gamma(3/4)) - 51*a**4*b**(45/4)*c*x**8*(a/(b*x**4) + 1)**(1/4)*g
amma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3
/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) +
 5*a**4*b**(17/4)*d*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8
*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/
4)) + 231*a**3*b**(49/4)*c*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256
*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b
**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 2*a**3*b**(21/4
)*d*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*gamma(3/4)
 + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/4)) + 924*a
**2*b**(53/4)*c*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*
x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*
gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 21*a**2*b**(25/4)*d*x**8*(
a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**
4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/4)) + 1056*a*b**(5...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\left(\frac{45\,(bx^4+a)^{\frac{1}{4}}b^2}{x} - \frac{18\,(bx^4+a)^{\frac{5}{4}}b}{x^5} + \frac{5\,(bx^4+a)^{\frac{9}{4}}}{x^9}\right)d}{45\,a^3} + \frac{\left(\frac{195\,(bx^4+a)^{\frac{1}{4}}b^3}{x} - \frac{117\,(bx^4+a)^{\frac{5}{4}}b^2}{x^5} + \frac{65\,(bx^4+a)^{\frac{9}{4}}b}{x^9} - \frac{15\,(bx^4+a)^{\frac{13}{4}}}{x^{13}}\right)c}{195\,a^4}$$

input
```
integrate((d*x^4+c)/x^14/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/45*(45*(b*x^4 + a)^(1/4)*b^2/x - 18*(b*x^4 + a)^(5/4)*b/x^5 + 5*(b*x^4
+ a)^(9/4)/x^9)*d/a^3 + 1/195*(195*(b*x^4 + a)^(1/4)*b^3/x - 117*(b*x^4 +
a)^(5/4)*b^2/x^5 + 65*(b*x^4 + a)^(9/4)*b/x^9 - 15*(b*x^4 + a)^(13/4)/x^13
)*c/a^4
```

Giac [F]

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^{14}}\,dx$$

input
```
integrate((d*x^4+c)/x^14/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^14), x)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \frac{\left(b\,x^4+a\right)^{1/4}\left(384\,b^3\,c-416\,a\,b^2\,d\right)}{585\,a^4\,x} - \frac{\left(b\,x^4+a\right)^{1/4}\left(96\,b^2\,c-104\,a\,b\,d\right)}{585\,a^3\,x^5} - \frac{\left(b\,x^4+a\right)^{1/4}\left(13\,a\,d-12\,b\,c\right)}{117\,a^2\,x^9} - \frac{c\left(b\,x^4+a\right)^{1/4}}{13\,a\,x^{13}}$$

input
```
int((c + d*x^4)/(x^14*(a + b*x^4)^(3/4)),x)
```

output
```
((a + b*x^4)^(1/4)*(384*b^3*c - 416*a*b^2*d))/(585*a^4*x) - ((a + b*x^4)^(
1/4)*(96*b^2*c - 104*a*b*d))/(585*a^3*x^5) - ((a + b*x^4)^(1/4)*(13*a*d -
12*b*c))/(117*a^2*x^9) - (c*(a + b*x^4)^(1/4))/(13*a*x^13)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{14}}dx\right)c + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{10}}dx\right)d$$

input
```
int((d*x^4+c)/x^14/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**14),x)*c + int(1/((a + b*x**4)**(3/4)*x**10)
,x)*d
```

3.99 $\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(7bc-6ad)x^2\sqrt[4]{a+bx^4}}{21b^2} + \frac{dx^6\sqrt[4]{a+bx^4}}{7b} - \frac{2a^{3/2}(7bc-6ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21b^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
1/21*(-6*a*d+7*b*c)*x^2*(b*x^4+a)^(1/4)/b^2+1/7*d*x^6*(b*x^4+a)^(1/4)/b-2/
21*a^(3/2)*(-6*a*d+7*b*c)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(
1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{x^2\left(-\left(\left(a+bx^4\right)\left(-7bc+6ad-3bdx^4\right)\right)+a(-7bc+6ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometr}\right.}{21b^2\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(x^5*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(-((a + b*x^4)*(-7*b*c + 6*a*d - 3*b*d*x^4)) + a*(-7*b*c + 6*a*d)*(1
+ (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(21*b^
2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 807, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \; 959$$

$$\frac{(7bc-6ad)\int \frac{x^5}{(bx^4+a)^{3/4}}dx}{7b}+\frac{dx^6\sqrt[4]{a+bx^4}}{7b}$$

$$\Big\downarrow \; 807$$

$$\frac{(7bc-6ad)\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{14b}+\frac{dx^6\sqrt[4]{a+bx^4}}{7b}$$

$$\Big\downarrow \; 262$$

$$\frac{(7bc-6ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)}{14b}+\frac{dx^6\sqrt[4]{a+bx^4}}{7b}$$

$$\Big\downarrow \; 231$$

$$\frac{(7bc-6ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)}{14b}+\frac{dx^6\sqrt[4]{a+bx^4}}{7b}$$

$$\Big\downarrow \; 229$$

$$\frac{(7bc-6ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)}{14b}+\frac{dx^6\sqrt[4]{a+bx^4}}{7b}$$

input
```
Int[(x^5*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(d*x^6*(a + b*x^4)^(1/4))/(7*b) + ((7*b*c - 6*a*d)*((2*x^2*(a + b*x^4)^(1/
4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))/(14*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^5(d\,x^4+c)}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{cx^6{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}} + \frac{dx^{10}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{3}{4}}}$$

input
```
integrate(x**5*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4)) +
d*x**10*hyper((3/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(3/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx=\int \frac{x^5\,(d\,x^4+c)}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{3/4}}\,dx=\left(\int \frac{x^9}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**9/(a + b*x**4)**(3/4),x)*d + int(x**5/(a + b*x**4)**(3/4),x)*c
```

3.100 $\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{dx^2\sqrt[4]{a+bx^4}}{3b} + \frac{\sqrt{a}(3bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/3*d*x^2*(b*x^4+a)^(1/4)/b+1/3*a^(1/2)*(-2*a*d+3*b*c)*(1+b*x^4/a)^(3/4)*I
nverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^
(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{x^2\left(2d(a+bx^4)+(3bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{6b(a+bx^4)^{3/4}}$$

input
```
Integrate[(x*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(2*d*(a + b*x^4) + (3*b*c - 2*a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(6*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {959, 807, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3bc-2ad)\int \frac{x}{(bx^4+a)^{3/4}}dx}{3b}+\frac{dx^2\sqrt[4]{a+bx^4}}{3b}$$

$$\Big\downarrow \text{807}$$

$$\frac{(3bc-2ad)\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{6b}+\frac{dx^2\sqrt[4]{a+bx^4}}{3b}$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}(3bc-2ad)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{6b\left(a+bx^4\right)^{3/4}}+\frac{dx^2\sqrt[4]{a+bx^4}}{3b}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{a}\left(\frac{bx^4}{a}+1\right)^{3/4}(3bc-2ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}+\frac{dx^2\sqrt[4]{a+bx^4}}{3b}$$

input
```
Int[(x*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(d*x^2*(a + b*x^4)^(1/4))/(3*b) + (Sqrt[a]*(3*b*c - 2*a*d)*(1 + (b*x^4)/a)
^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^
4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(d\,x^4+c)}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx=\int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx=\frac{cx^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}}+\frac{dx^6\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}}$$

input
```
integrate(x*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4)) +
d*x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4))
```

Maxima [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx=\int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{x\,(d\,x^4+c)}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((x*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
int((x*(c + d*x^4))/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}}dx\right) d + \left(\int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}}dx\right) c$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**5/(a + b*x**4)**(3/4),x)*d + int(x/(a + b*x**4)**(3/4),x)*c
```

3.101 $\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{2ax^2} - \frac{(bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output

```
-1/2*c*(b*x^4+a)^(1/4)/a/x^2-1/2*(-2*a*d+b*c)*(1+b*x^4/a)^(3/4)*InverseJac
obiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^4+a)^(
3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx = \frac{-2c(a+bx^4)+(-bc+2ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)}{4ax^2\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^3*(a + b*x^4)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^4) + (-(b*c) + 2*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeomet
ric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)])/(4*a*x^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 807, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(bc-2ad)\int \frac{x}{\left(bx^4+a\right)^{3/4}}dx}{2a}-\frac{c\sqrt[4]{a+bx^4}}{2ax^2}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(bc-2ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{4a}-\frac{c\sqrt[4]{a+bx^4}}{2ax^2}$$

$$\Big\downarrow \text{231}$$

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}(bc-2ad)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{4a\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{2ax^2}$$

$$\Big\downarrow \text{229}$$

$$-\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}(bc-2ad)\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}\sqrt{b}\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{2ax^2}$$

input
```
Int[(c + d*x^4)/(x^3*(a + b*x^4)^(3/4)),x]
```

output
```
-1/2*(c*(a + b*x^4)^(1/4))/(a*x^2) - ((b*c - 2*a*d)*(1 + (b*x^4)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*Sqrt[a]*Sqrt[b]*(a + b*x
^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{3} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{3/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}} x^3} \, dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b*x^7 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.67

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{3/4}} \, dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2} + \frac{dx^2\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}}$$

input
```
integrate((d*x**4+c)/x**3/(b*x**4+a)**(3/4),x)
```

output
```
-c*hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4)*x**2)
+ d*x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{3/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}} x^3} \, dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{3}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{d\,x^4+c}{x^3\,\left(b\,x^4+a\right)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(3/4)),x)
```

output
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{3/4}}\,dx=\left(\int \frac{x}{\left(b\,x^4+a\right)^{\frac{3}{4}}}dx\right)d+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^3}dx\right)c$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(3/4),x)
```

output
```
int(x/(a + b*x**4)**(3/4),x)*d + int(1/((a + b*x**4)**(3/4)*x**3),x)*c
```

3.102 $\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{6ax^6}+\frac{(5bc-6ad)\sqrt[4]{a+bx^4}}{12a^2x^2} + \frac{\sqrt{b}(5bc-6ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/6*c*(b*x^4+a)^(1/4)/a/x^6+1/12*(-6*a*d+5*b*c)*(b*x^4+a)^(1/4)/a^2/x^2+1
/12*b^(1/2)*(-6*a*d+5*b*c)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^
(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{3/4}}\,dx = \frac{-2c(a+bx^4)+(5bc-6ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{12ax^6\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^7*(a + b*x^4)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^4) + (5*b*c - 6*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeometr
ic2F1[-1/2, 3/4, 1/2, -((b*x^4)/a)])/(12*a*x^6*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 807, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(5bc-6ad)\int \frac{1}{x^3(bx^4+a)^{3/4}}dx}{6a}-\frac{c\sqrt[4]{a+bx^4}}{6ax^6}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(5bc-6ad)\int \frac{1}{x^4(bx^4+a)^{3/4}}dx^2}{12a}-\frac{c\sqrt[4]{a+bx^4}}{6ax^6}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(5bc-6ad)\left(-\frac{b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{12a}-\frac{c\sqrt[4]{a+bx^4}}{6ax^6}$$

$$\Big\downarrow \text{231}$$

$$-\frac{(5bc-6ad)\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{12a}-\frac{c\sqrt[4]{a+bx^4}}{6ax^6}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(5bc-6ad)\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{12a}-\frac{c\sqrt[4]{a+bx^4}}{6ax^6}$$

input
```
Int[(c + d*x^4)/(x^7*(a + b*x^4)^(3/4)),x]
```

output
```
-1/6*(c*(a + b*x^4)^(1/4))/(a*x^6) - ((5*b*c - 6*a*d)*(-((a + b*x^4)^(1/4)
/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4))))/(12*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{7} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}dx$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{3/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{3}{4}} x^{7}}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b*x^11 + a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{3/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}x^6} - \frac{d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2}$$

input
```
integrate((d*x**4+c)/x**7/(b*x**4+a)**(3/4),x)
```

output
```
-c*hyper((-3/2, 3/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4)*x**6)
 - d*hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4)*x**2
)
```

Maxima [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^7), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^7\,(a+bx^4)^{3/4}}\,dx = \int \frac{d\,x^4+c}{x^7\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(3/4)),x)
```

output
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^7\,(a+bx^4)^{3/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^7}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^3}dx\right)d$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**7),x)*c + int(1/((a + b*x**4)**(3/4)*x**3),x
)*d
```

3.103 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{(6bc-5ad)x\sqrt[4]{a+bx^4}}{12b^2} + \frac{dx^5\sqrt[4]{a+bx^4}}{6b} + \frac{\sqrt{a}(6bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
1/12*(-5*a*d+6*b*c)*x*(b*x^4+a)^(1/4)/b^2+1/6*d*x^5*(b*x^4+a)^(1/4)/b+1/12
*a^(1/2)*(-5*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b
^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{x\left(-((a+bx^4)(5ad-2b(3c+dx^4)))+a(-6bc+5ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric}\right.}{12b^2\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output
```
(x*(-((a + b*x^4)*(5*a*d - 2*b*(3*c + d*x^4))) + a*(-6*b*c + 5*a*d)*(1 + (
b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)]))/(12*b^2*(
a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(6bc-5ad)\int \frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

$$\Big\downarrow \text{843}$$

$$\frac{(6bc-5ad)\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

$$\Big\downarrow \text{768}$$

$$\frac{(6bc-5ad)\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{(6bc-5ad)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

$\Big\downarrow$ 807

$$\frac{(6bc-5ad)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

$\Big\downarrow$ 229

$$\frac{(6bc-5ad)\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b}$$

input

```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(3/4),x]
```

output

```
(d*x^5*(a + b*x^4)^(1/4))/(6*b) + ((6*b*c - 5*a*d)*((x*(a + b*x^4)^(1/4))/
(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[
b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(6*b)
```

Defintions of rubi rules used

rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^4(d\,x^4+c)}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*x^8 + c*x^4)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(3/4)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(3/4),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**8/(a + b*x**4)**(3/4),x)*d + int(x**4/(a + b*x**4)**(3/4),x)*c
```

3.104 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/4}}\,dx=\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{(2bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
1/2*d*x*(b*x^4+a)^(1/4)/b-1/2*(-a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJa
cobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^4+a)^
(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/4}}\,dx=\frac{dx(a+bx^4)+(2bc-ad)x\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{2b\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(3/4),x]
```

output
```
(d*x*(a + b*x^4) + (2*b*c - a*d)*x*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1
[1/4, 3/4, 5/4, -((b*x^4)/a)])/(2*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {913, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(2bc-ad)\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}+\frac{dx\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}+\frac{dx\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{858}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{dx\sqrt[4]{a+bx^4}}{2b}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{a}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(3/4),x]
```

output
```
(d*x*(a + b*x^4)^(1/4))/(2*b) - ((2*b*c - a*d)*(1 + a/(b*x^4))^(3/4)*x^3*E
llipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[a]*Sqrt[b]*(a + b*x^
4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{dx^{4}+c}{\left(bx^{4}+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{3/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(3/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(3/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{3/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(3/4),x)
```

output
```
int(x**4/(a + b*x**4)**(3/4),x)*d + int(1/(a + b*x**4)**(3/4),x)*c
```

3.105 $\int \frac{c+dx^4}{x^4(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{3ax^3} + \frac{\sqrt{b}(2bc-3ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}$$

output
```
-1/3*c*(b*x^4+a)^(1/4)/a/x^3+1/3*b^(1/2)*(-3*a*d+2*b*c)*(1+a/b/x^4)^(3/4)*
x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^
4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{3/4}}\,dx = \frac{-c(a+bx^4)+(-2bc+3ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{3ax^3(a+bx^4)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(3/4)),x]
```

output
```
(-(c*(a + b*x^4)) + (-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeomet
ric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(3*a*x^3*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 \left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-3ad)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{c\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{768}$$

$$-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6a\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-3ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{c\sqrt[4]{a+bx^4}}{3ax^3}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(3/4)),x]
```

output
```
-1/3*(c*(a + b*x^4)^(1/4))/(a*x^3) + (Sqrt[b]*(2*b*c - 3*a*d)*(1 + a/(b*x^
4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a
 + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{4} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^{4} + c}{{\left(bx^{4} + a\right)}^{\frac{3}{4}} x^{4}}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b*x^8 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/4}}\,dx=\frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^3\Gamma\left(\frac{1}{4}\right)}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(3/4),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
3/4)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(3/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/4}} dx = \int \frac{d x^4 + c}{x^4 (b x^4 + a)^{3/4}} dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(3/4)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{3/4}} dx = \left(\int \frac{1}{(b x^4 + a)^{\frac{3}{4}}} dx\right) d + \left(\int \frac{1}{(b x^4 + a)^{\frac{3}{4}} x^4} dx\right) c$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(3/4),x)
```

output
```
int(1/(a + b*x**4)**(3/4),x)*d + int(1/((a + b*x**4)**(3/4)*x**4),x)*c
```

3.106 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx = -\frac{c\sqrt[4]{a+bx^4}}{7ax^7}+\frac{(6bc-7ad)\sqrt[4]{a+bx^4}}{21a^2x^3} - \frac{2b^{3/2}(6bc-7ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/7*c*(b*x^4+a)^(1/4)/a/x^7+1/21*(-7*a*d+6*b*c)*(b*x^4+a)^(1/4)/a^2/x^3-2
/21*b^(3/2)*(-7*a*d+6*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcco
t(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx = \frac{-3c(a+bx^4)+(6bc-7ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)}{21ax^7\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(3/4)),x]
```

output
```
(-3*c*(a + b*x^4) + (6*b*c - 7*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeometr
ic2F1[-3/4, 3/4, 1/4, -((b*x^4)/a)])/(21*a*x^7*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(6bc-7ad)\int \frac{1}{x^4(bx^4+a)^{3/4}}dx}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(6bc-7ad)\left(-\frac{2b\int \frac{1}{(bx^4+a)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{ 768}$$

$$-\frac{(6bc-7ad)\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{(6bc-7ad)\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \ 807$$

$$-\frac{(6bc-7ad)\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \ 229$$

$$-\frac{(6bc-7ad)\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{c\sqrt[4]{a+bx^4}}{7ax^7}$$

input

```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(3/4)),x]
```

output

```
-1/7*(c*(a + b*x^4)^(1/4))/(a*x^7) - ((6*b*c - 7*a*d)*(-1/3*(a + b*x^4)^(1
/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a
]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/(7*a)
```

Defintions of rubi rules used

rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{8} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x)
```

output
```
int((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b*x^12 + a*x^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx = \frac{c\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{4} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^7\Gamma\left(-\frac{3}{4}\right)} + \frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(3/4),x)
```

output
```
c*gamma(-7/4)*hyper((-7/4, 3/4), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*x**7*gamma(-3/4)) + d*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(3/4)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(3/4)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{d\,x^4+c}{x^8\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(3/4)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{3/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^8}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^4}dx\right)d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**8),x)*c + int(1/((a + b*x**4)**(3/4)*x**4),x
)*d
```

3.107 $\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = -\frac{a^2(bc-ad)}{b^4\sqrt[4]{a+bx^4}} - \frac{a(2bc-3ad)\left(a+bx^4\right)^{3/4}}{3b^4} + \frac{(bc-3ad)\left(a+bx^4\right)^{7/4}}{7b^4} + \frac{d\left(a+bx^4\right)^{11/4}}{11b^4}$$

output
```
-a^2*(-a*d+b*c)/b^4/(b*x^4+a)^(1/4)-1/3*a*(-3*a*d+2*b*c)*(b*x^4+a)^(3/4)/b
^4+1/7*(-3*a*d+b*c)*(b*x^4+a)^(7/4)/b^4+1/11*d*(b*x^4+a)^(11/4)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{384a^3d+3b^3x^8(11c+7dx^4)-4ab^2x^4(22c+9dx^4)+a^2b(-352c+96dx^4)}{231b^4\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^11*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(384*a^3*d + 3*b^3*x^8*(11*c + 7*d*x^4) - 4*a*b^2*x^4*(22*c + 9*d*x^4) + a
^2*b*(-352*c + 96*d*x^4))/(231*b^4*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{4}\int \frac{x^8(dx^4+c)}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \; 86$$

$$\frac{1}{4}\int \left(-\frac{(ad-bc)a^2}{b^3(bx^4+a)^{5/4}}+\frac{(3ad-2bc)a}{b^3\sqrt[4]{bx^4+a}}+\frac{d(bx^4+a)^{7/4}}{b^3}+\frac{(bc-3ad)(bx^4+a)^{3/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\frac{4a^2(bc-ad)}{b^4\sqrt[4]{a+bx^4}}+\frac{4(a+bx^4)^{7/4}(bc-3ad)}{7b^4}-\frac{4a(a+bx^4)^{3/4}(2bc-3ad)}{3b^4}+\frac{4d(a+bx^4)^{11/4}}{11b^4}\right)$$

input
```
Int[(x^11*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
((-4*a^2*(b*c - a*d))/(b^4*(a + b*x^4)^(1/4)) - (4*a*(2*b*c - 3*a*d)*(a +
b*x^4)^(3/4))/(3*b^4) + (4*(b*c - 3*a*d)*(a + b*x^4)^(7/4))/(7*b^4) + (4*d
*(a + b*x^4)^(11/4))/(11*b^4))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\frac{\left(\frac{7 d \,x^{4}}{11}+c \right) x^{8} b^{3}}{7}-\frac{8 \left(\frac{9 d \,x^{4}}{22}+c \right) x^{4} a \,b^{2}}{21}-\frac{32 \left(-\frac{3 d \,x^{4}}{11}+c \right) a^{2} b}{21}+\frac{128 a^{3} d}{77}}{\left(b \,x^{4}+a \right)^{\frac{1}{4}} b^{4}}$	68
gosper	$\frac{21 b^{3} d \,x^{12}-36 a \,b^{2} d \,x^{8}+33 c \,b^{3} x^{8}+96 a^{2} b d \,x^{4}-88 a \,b^{2} c \,x^{4}+384 a^{3} d -352 a^{2} b c}{231 \left(b \,x^{4}+a \right)^{\frac{1}{4}} b^{4}}$	77
trager	$\frac{21 b^{3} d \,x^{12}-36 a \,b^{2} d \,x^{8}+33 c \,b^{3} x^{8}+96 a^{2} b d \,x^{4}-88 a \,b^{2} c \,x^{4}+384 a^{3} d -352 a^{2} b c}{231 \left(b \,x^{4}+a \right)^{\frac{1}{4}} b^{4}}$	77
orering	$\frac{21 b^{3} d \,x^{12}-36 a \,b^{2} d \,x^{8}+33 c \,b^{3} x^{8}+96 a^{2} b d \,x^{4}-88 a \,b^{2} c \,x^{4}+384 a^{3} d -352 a^{2} b c}{231 \left(b \,x^{4}+a \right)^{\frac{1}{4}} b^{4}}$	77
risch	$\frac{\left(21 d \,b^{2} x^{8}-57 a b d \,x^{4}+33 b^{2} c \,x^{4}+153 a^{2} d -121 a b c \right) \left(b \,x^{4}+a \right)^{\frac{3}{4}}}{231 b^{4}}+\frac{a^{2} \left(a d -c b \right)}{\left(b \,x^{4}+a \right)^{\frac{1}{4}} b^{4}}$	78

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
128/77/(b*x^4+a)^(1/4)*(11/128*(7/11*d*x^4+c)*x^8*b^3-11/48*(9/22*d*x^4+c)
*x^4*a*b^2-11/12*(-3/11*d*x^4+c)*a^2*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{(21\,b^3dx^{12}+3\,(11\,b^3c-12\,ab^2d)x^8-8\,(11\,ab^2c-12\,a^2bd)x^4-352\,a^2bc+384\,a^3d)(b}{231\,(b^5x^4+ab^4)}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/231*(21*b^3*d*x^12 + 3*(11*b^3*c - 12*a*b^2*d)*x^8 - 8*(11*a*b^2*c - 12*
a^2*b*d)*x^4 - 352*a^2*b*c + 384*a^3*d)*(b*x^4 + a)^(3/4)/(b^5*x^4 + a*b^4
)
```

Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \begin{cases} \frac{128a^3d}{77b^4\sqrt[4]{a+bx^4}} - \frac{32a^2c}{21b^3\sqrt[4]{a+bx^4}} + \frac{32a^2dx^4}{77b^3\sqrt[4]{a+bx^4}} - \frac{8acx^4}{21b^2\sqrt[4]{a+bx^4}} - \frac{12adx^8}{77b^2\sqrt[4]{a+bx^4}} + \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{16}}{16}}{a^{\frac{5}{4}}} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((128*a**3*d/(77*b**4*(a + b*x**4)**(1/4)) - 32*a**2*c/(21*b**3*(
a + b*x**4)**(1/4)) + 32*a**2*d*x**4/(77*b**3*(a + b*x**4)**(1/4)) - 8*a*c
*x**4/(21*b**2*(a + b*x**4)**(1/4)) - 12*a*d*x**8/(77*b**2*(a + b*x**4)**(
1/4)) + c*x**8/(7*b*(a + b*x**4)**(1/4)) + d*x**12/(11*b*(a + b*x**4)**(1/
4)), Ne(b, 0)), ((c*x**12/12 + d*x**16/16)/a**(5/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{1}{77}\,d\left(\frac{7\,(bx^4+a)^{\frac{11}{4}}}{b^4} - \frac{33\,(bx^4+a)^{\frac{7}{4}}a}{b^4} + \frac{77\,(bx^4+a)^{\frac{3}{4}}a^2}{b^4} + \frac{77\,a^3}{(bx^4+a)^{\frac{1}{4}}b^4}\right) + \frac{1}{21}\,c\left(\frac{3\,(bx^4+a)^{\frac{7}{4}}}{b^3} - \frac{14\,(bx^4+a)^{\frac{3}{4}}a}{b^3} - \frac{21\,a^2}{(bx^4+a)^{\frac{1}{4}}b^3}\right)$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/77*d*(7*(b*x^4 + a)^(11/4)/b^4 - 33*(b*x^4 + a)^(7/4)*a/b^4 + 77*(b*x^4
+ a)^(3/4)*a^2/b^4 + 77*a^3/((b*x^4 + a)^(1/4)*b^4)) + 1/21*c*(3*(b*x^4 +
a)^(7/4)/b^3 - 14*(b*x^4 + a)^(3/4)*a/b^3 - 21*a^2/((b*x^4 + a)^(1/4)*b^3)
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = -\frac{a^2bc-a^3d}{(bx^4+a)^{\frac{1}{4}}b^4} + \frac{33\,(bx^4+a)^{\frac{7}{4}}b^{41}c - 154\,(bx^4+a)^{\frac{3}{4}}ab^{41}c + 21\,(bx^4+a)^{\frac{11}{4}}b^{40}d - 99\,(bx^4+a)^{\frac{7}{4}}ab^{40}d + 231\,(bx^4+a)^{\frac{3}{4}}a^2b^{40}}{231\,b^{44}}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-(a^2*b*c - a^3*d)/((b*x^4 + a)^(1/4)*b^4) + 1/231*(33*(b*x^4 + a)^(7/4)*b
^41*c - 154*(b*x^4 + a)^(3/4)*a*b^41*c + 21*(b*x^4 + a)^(11/4)*b^40*d - 99
*(b*x^4 + a)^(7/4)*a*b^40*d + 231*(b*x^4 + a)^(3/4)*a^2*b^40*d)/b^44
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{\frac{d\,(b\,x^4+a)^3}{11}+a^3\,d-\frac{3\,a\,d\,(b\,x^4+a)^2}{7}+\frac{b\,c\,(b\,x^4+a)^2}{7}+a^2\,d\,(b\,x^4+a)-a^2\,b\,c-\frac{2\,a\,b\,c\,(b\,x^4+a)}{3}}{b^4\,(b\,x^4+a)^{1/4}}$$

input
```
int((x^11*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
((d*(a + b*x^4)^3)/11 + a^3*d - (3*a*d*(a + b*x^4)^2)/7 + (b*c*(a + b*x^4)
^2)/7 + a^2*d*(a + b*x^4) - a^2*b*c - (2*a*b*c*(a + b*x^4))/3)/(b^4*(a + b
*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^{15}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**15/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(
x**11/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.108 $\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{a(bc-ad)}{b^3\sqrt[4]{a+bx^4}} + \frac{(bc-2ad)\left(a+bx^4\right)^{3/4}}{3b^3} + \frac{d(a+bx^4)^{7/4}}{7b^3}$$

output
```
a*(-a*d+b*c)/b^3/(b*x^4+a)^(1/4)+1/3*(-2*a*d+b*c)*(b*x^4+a)^(3/4)/b^3+1/7*
d*(b*x^4+a)^(7/4)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{28abc-32a^2d+7b^2cx^4-8abdx^4+3b^2dx^8}{21b^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^7*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(28*a*b*c - 32*a^2*d + 7*b^2*c*x^4 - 8*a*b*d*x^4 + 3*b^2*d*x^8)/(21*b^3*(a
 + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (c+dx^4)}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{x^4 (dx^4+c)}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \ 86$$

$$\frac{1}{4}\int \left(\frac{(bx^4+a)^{3/4} d}{b^2}+\frac{bc-2ad}{b^2 \sqrt[4]{bx^4+a}}+\frac{a(ad-bc)}{b^2 (bx^4+a)^{5/4}}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{4(a+bx^4)^{3/4} (bc-2ad)}{3b^3}+\frac{4a(bc-ad)}{b^3 \sqrt[4]{a+bx^4}}+\frac{4d(a+bx^4)^{7/4}}{7b^3}\right)$$

input
```
Int[(x^7*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
((4*a*(b*c - a*d))/(b^3*(a + b*x^4)^(1/4)) + (4*(b*c - 2*a*d)*(a + b*x^4)^
(3/4))/(3*b^3) + (4*d*(a + b*x^4)^(7/4))/(7*b^3))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$-\frac{32\left(-\frac{7\left(\frac{3d\,x^4}{7}+c\right)x^4b^2}{32}-\frac{7\left(-\frac{2d\,x^4}{7}+c\right)ab}{8}+a^2d\right)}{21\left(b\,x^4+a\right)^{\frac{1}{4}}b^3}$	49
gosper	$-\frac{-3d\,b^2x^8+8abd\,x^4-7b^2c\,x^4+32a^2d-28abc}{21\left(b\,x^4+a\right)^{\frac{1}{4}}b^3}$	53
trager	$-\frac{-3d\,b^2x^8+8abd\,x^4-7b^2c\,x^4+32a^2d-28abc}{21\left(b\,x^4+a\right)^{\frac{1}{4}}b^3}$	53
orering	$-\frac{-3d\,b^2x^8+8abd\,x^4-7b^2c\,x^4+32a^2d-28abc}{21\left(b\,x^4+a\right)^{\frac{1}{4}}b^3}$	53
risch	$-\frac{\left(-3db\,x^4+11ad-7cb\right)\left(b\,x^4+a\right)^{\frac{3}{4}}}{21b^3}-\frac{a(ad-cb)}{\left(b\,x^4+a\right)^{\frac{1}{4}}b^3}$	55

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-32/21*(-7/32*(3/7*d*x^4+c)*x^4*b^2-7/8*(-2/7*d*x^4+c)*a*b+a^2*d)/(b*x^4+a
)^(1/4)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{(3\,b^2dx^8 + (7\,b^2c - 8\,abd)x^4 + 28\,abc - 32\,a^2d)(bx^4+a)^{\frac{3}{4}}}{21\,(b^4x^4 + ab^3)}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/21*(3*b^2*d*x^8 + (7*b^2*c - 8*a*b*d)*x^4 + 28*a*b*c - 32*a^2*d)*(b*x^4
+ a)^(3/4)/(b^4*x^4 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \begin{cases} -\frac{32a^2d}{21b^3\sqrt[4]{a+bx^4}} + \frac{4ac}{3b^2\sqrt[4]{a+bx^4}} - \frac{8adx^4}{21b^2\sqrt[4]{a+bx^4}} + \frac{cx^4}{3b\sqrt[4]{a+bx^4}} + \frac{dx^8}{7b\sqrt[4]{a+bx^4}} & \text{for } b \\ \frac{\frac{cx^8}{8}+\frac{dx^{12}}{12}}{a^{\frac{5}{4}}} & \text{othe} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-32*a**2*d/(21*b**3*(a + b*x**4)**(1/4)) + 4*a*c/(3*b**2*(a + b
*x**4)**(1/4)) - 8*a*d*x**4/(21*b**2*(a + b*x**4)**(1/4)) + c*x**4/(3*b*(a
 + b*x**4)**(1/4)) + d*x**8/(7*b*(a + b*x**4)**(1/4)), Ne(b, 0)), ((c*x**8
/8 + d*x**12/12)/a**(5/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{1}{21}\,d\left(\frac{3\,(bx^4+a)^{\frac{7}{4}}}{b^3} - \frac{14\,(bx^4+a)^{\frac{3}{4}}a}{b^3} - \frac{21\,a^2}{(bx^4+a)^{\frac{1}{4}}b^3}\right) + \frac{1}{3}\,c\left(\frac{(bx^4+a)^{\frac{3}{4}}}{b^2} + \frac{3\,a}{(bx^4+a)^{\frac{1}{4}}b^2}\right)$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/21*d*(3*(b*x^4 + a)^(7/4)/b^3 - 14*(b*x^4 + a)^(3/4)*a/b^3 - 21*a^2/((b*
x^4 + a)^(1/4)*b^3)) + 1/3*c*((b*x^4 + a)^(3/4)/b^2 + 3*a/((b*x^4 + a)^(1/
4)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{abc-a^2d}{(bx^4+a)^{\frac{1}{4}}b^3} + \frac{7\,(bx^4+a)^{\frac{3}{4}}b^{19}c + 3\,(bx^4+a)^{\frac{7}{4}}b^{18}d - 14\,(bx^4+a)^{\frac{3}{4}}ab^{18}d}{21\,b^{21}}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
(a*b*c - a^2*d)/((b*x^4 + a)^(1/4)*b^3) + 1/21*(7*(b*x^4 + a)^(3/4)*b^19*c
 + 3*(b*x^4 + a)^(7/4)*b^18*d - 14*(b*x^4 + a)^(3/4)*a*b^18*d)/b^21
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx=\frac{3\,d\,\left(b\,x^4+a\right)^2-21\,a^2\,d-14\,a\,d\,\left(b\,x^4+a\right)+7\,b\,c\,\left(b\,x^4+a\right)+21\,a\,b\,c}{21\,b^3\,\left(b\,x^4+a\right)^{1/4}}$$

input
```
int((x^7*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
(3*d*(a + b*x^4)^2 - 21*a^2*d - 14*a*d*(a + b*x^4) + 7*b*c*(a + b*x^4) + 2
1*a*b*c)/(21*b^3*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{5/4}}\,dx=\left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d+\left(\int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**11/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(
x**7/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.109 $\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx=-\frac{bc-ad}{b^2\sqrt[4]{a+bx^4}}+\frac{d(a+bx^4)^{3/4}}{3b^2}$$

output
```
-(-a*d+b*c)/b^2/(b*x^4+a)^(1/4)+1/3*d*(b*x^4+a)^(3/4)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx=\frac{-3bc+4ad+bdx^4}{3b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^3*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(-3*b*c + 4*a*d + b*d*x^4)/(3*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^4)}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{4}\int \frac{dx^4+c}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{4}\int \left(\frac{d}{b\sqrt[4]{bx^4+a}}+\frac{bc-ad}{b\,(bx^4+a)^{5/4}}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{4d(a+bx^4)^{3/4}}{3b^2}-\frac{4(bc-ad)}{b^2\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[(x^3*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
((-4*(b*c - a*d))/(b^2*(a + b*x^4)^(1/4)) + (4*d*(a + b*x^4)^(3/4))/(3*b^2
))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
gosper	$\frac{db\,x^{4}+4ad-3cb}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	30
trager	$\frac{db\,x^{4}+4ad-3cb}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	30
orering	$\frac{db\,x^{4}+4ad-3cb}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	30
pseudoelliptic	$\frac{\left(d\,x^{4}-3c\right)b+4ad}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	31
risch	$\frac{d\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3b^{2}}+\frac{ad-cb}{\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	38

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*d*x^4+4*a*d-3*b*c)/(b*x^4+a)^(1/4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{(bdx^4-3\,bc+4\,ad)(bx^4+a)^{\frac{3}{4}}}{3\,(b^3x^4+ab^2)}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/3*(b*d*x^4 - 3*b*c + 4*a*d)*(b*x^4 + a)^(3/4)/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \begin{cases} \frac{4ad}{3b^2\sqrt[4]{a+bx^4}} - \frac{c}{b\sqrt[4]{a+bx^4}} + \frac{dx^4}{3b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{\frac{cx^4}{4}+\frac{dx^8}{8}}{a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((4*a*d/(3*b**2*(a + b*x**4)**(1/4)) - c/(b*(a + b*x**4)**(1/4))
+ d*x**4/(3*b*(a + b*x**4)**(1/4)), Ne(b, 0)), ((c*x**4/4 + d*x**8/8)/a**(
5/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{1}{3}\,d\left(\frac{(bx^4+a)^{\frac{3}{4}}}{b^2} + \frac{3\,a}{(bx^4+a)^{\frac{1}{4}}b^2}\right) - \frac{c}{(bx^4+a)^{\frac{1}{4}}b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/3*d*((b*x^4 + a)^(3/4)/b^2 + 3*a/((b*x^4 + a)^(1/4)*b^2)) - c/((b*x^4 +
a)^(1/4)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}d}{3\,b^2} - \frac{bc-ad}{(bx^4+a)^{\frac{1}{4}}b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/3*(b*x^4 + a)^(3/4)*d/b^2 - (b*c - a*d)/((b*x^4 + a)^(1/4)*b^2)
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{d\,(b\,x^4+a)+3\,a\,d-3\,b\,c}{3\,b^2\,(b\,x^4+a)^{1/4}}$$

input
```
int((x^3*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
(d*(a + b*x^4) + 3*a*d - 3*b*c)/(3*b^2*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^3}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**7/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(x
**3/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.110 $\int \frac{c+dx^4}{x(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{c+dx^4}{x(a+bx^4)^{5/4}}\,dx=\frac{bc-ad}{ab\sqrt[4]{a+bx^4}}+\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}$$

output
```
(-a*d+b*c)/a/b/(b*x^4+a)^(1/4)+1/2*c*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(5/
4)-1/2*c*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{x(a+bx^4)^{5/4}}\,dx=\frac{\frac{2\sqrt[4]{a}(bc-ad)}{b\sqrt[4]{a+bx^4}}+c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-c\,\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x*(a + b*x^4)^(5/4)),x]
```

output
```
((2*a^(1/4)*(b*c - a*d))/(b*(a + b*x^4)^(1/4)) + c*ArcTan[(a + b*x^4)^(1/4
)/a^(1/4)] - c*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(5/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {948, 87, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{5/4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^4\left(bx^4+a\right)^{5/4}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{4c\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}-\frac{4c\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}-\frac{4c\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}-\frac{4c\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}-\frac{4c\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{4(bc-ad)}{ab\sqrt[4]{a+bx^4}}-\frac{4c\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)$$

input
```
Int[(c + d*x^4)/(x*(a + b*x^4)^(5/4)),x]
```

output
```
((4*(b*c - a*d))/(a*b*(a + b*x^4)^(1/4)) - (4*c*(-1/2*ArcTan[(a + b*x^4)^(
1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a
)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$\frac{\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b c}{2 a^{\frac{5}{4}}}-\frac{\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) b c}{4 a^{\frac{5}{4}}}-\frac{a d -c b}{a \left(b\,x^{4}+a\right)^{\frac{1}{4}}}}{b}$	88

input
```
int((d*x^4+c)/x/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
(1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))*b/a^(5/4)*c-1/4*ln(((b*x^4+a)^(1/4)+a
^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*b/a^(5/4)*c-(a*d-b*c)/a/(b*x^4+a)^(1/4)
)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.08

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{5/4}}\,dx =$$

$$-\frac{(ab^2x^4+a^2b)\left(\frac{c^4}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{c^4}{a^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}c^3\right)-(i\,ab^2x^4+i\,a^2b)\left(\frac{c^4}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(\frac{c^4}{a^5}\right)^{\frac{3}{4}}+(bx^4+a\right.}{}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/4*((a*b^2*x^4 + a^2*b)*(c^4/a^5)^(1/4)*log(a^4*(c^4/a^5)^(3/4) + (b*x^4
 + a)^(1/4)*c^3) - (I*a*b^2*x^4 + I*a^2*b)*(c^4/a^5)^(1/4)*log(I*a^4*(c^4/
a^5)^(3/4) + (b*x^4 + a)^(1/4)*c^3) - (-I*a*b^2*x^4 - I*a^2*b)*(c^4/a^5)^(
1/4)*log(-I*a^4*(c^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)*c^3) - (a*b^2*x^4 + a^
2*b)*(c^4/a^5)^(1/4)*log(-a^4*(c^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)*c^3) - 4
*(b*x^4 + a)^(3/4)*(b*c - a*d))/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 33.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{5/4}}\,dx = d\left(\begin{cases} -\frac{1}{b\sqrt[4]{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{4}}} & \text{otherwise} \end{cases}\right) - \frac{c\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/x/(b*x**4+a)**(5/4),x)
```

output
```
d*Piecewise((-1/(b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**4/(4*a**(5/4)), Tr
ue)) - c*gamma(5/4)*hyper((5/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/
(4*b**(5/4)*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{4}c\left(\frac{\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{a} + \frac{4}{\left(bx^4+a\right)^{\frac{1}{4}}a}\right) - \frac{d}{\left(bx^4+a\right)^{\frac{1}{4}}b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/4*c*((2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/
4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a + 4/((b*x^4 + a)^(
1/4)*a)) - d/((b*x^4 + a)^(1/4)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.58

$$\int \frac{c+dx^4}{x(a+bx^4)^{5/4}}\,dx = \frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}\,a} + \frac{\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}\,a} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\,c\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a^2} + \frac{\sqrt{2}c\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,(-a)^{\frac{1}{4}}\,a} + \frac{bc-ad}{(bx^4+a)^{\frac{1}{4}}ab}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(1/4)
)/(-a)^(1/4))/((-a)^(1/4)*a) + 1/4*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a) + 1/8*sqrt(2)
*(-a)^(3/4)*c*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) +
 sqrt(-a))/a^2 + 1/8*sqrt(2)*c*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) +
 sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a) + (b*c - a*d)/((b*x^4 + a)^(1/
4)*a*b)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{x(a+bx^4)^{5/4}}\,dx = \frac{c}{a\,(b\,x^4+a)^{1/4}} - \frac{d}{b\,(b\,x^4+a)^{1/4}} + \frac{c\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{c\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}}$$

input
```
int((c + d*x^4)/(x*(a + b*x^4)^(5/4)),x)
```

output
```
c/(a*(a + b*x^4)^(1/4)) - d/(b*(a + b*x^4)^(1/4)) + (c*atan((a + b*x^4)^(1
/4)/a^(1/4)))/(2*a^(5/4)) - (c*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(5/4
))
```

Reduce [F]

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{5/4}}\,dx = \left(\int \frac{x^3}{\left(b\,x^4+a\right)^{\frac{1}{4}} a+\left(b\,x^4+a\right)^{\frac{1}{4}} b\,x^4}dx\right) d + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} ax+\left(b\,x^4+a\right)^{\frac{1}{4}} b\,x^5}dx\right) c$$

input
```
int((d*x^4+c)/x/(b*x^4+a)^(5/4),x)
```

output
```
int(x**3/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(1/4)*a*x + (a + b*x**4)**(1/4)*b*x**5),x)*c
```

3.111 $\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{5/4}}\,dx = -\frac{bc-ad}{a^2\sqrt[4]{a+bx^4}}-\frac{c\left(a+bx^4\right)^{3/4}}{4a^2x^4} - \frac{(5bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}+\frac{(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}$$

output
```
-(-a*d+b*c)/a^2/(b*x^4+a)^(1/4)-1/4*c*(b*x^4+a)^(3/4)/a^2/x^4-1/8*(-4*a*d+
5*b*c)*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)+1/8*(-4*a*d+5*b*c)*arctanh(
(b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{5/4}}\,dx = \frac{-\frac{2\sqrt[4]{a}\left(ac+5bcx^4-4adx^4\right)}{x^4\sqrt[4]{a+bx^4}}+(-5bc+4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+(5bc-4ad)\operatorname{arctanh}\Big(}{8a^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(x^5*(a + b*x^4)^(5/4)),x]
```

output
```
((-2*a^(1/4)*(a*c + 5*b*c*x^4 - 4*a*d*x^4))/(x^4*(a + b*x^4)^(1/4)) + (-5*
b*c + 4*a*d)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] + (5*b*c - 4*a*d)*ArcTanh[(
a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {948, 87, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^8 \left(bx^4+a\right)^{5/4}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\int \frac{1}{x^4(bx^4+a)^{5/4}}dx^4}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 827$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{(5bc-4ad)\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{c}{ax^4\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[(c + d*x^4)/(x^5*(a + b*x^4)^(5/4)),x]
```

output
```
(-(c/(a*x^4*(a + b*x^4)^(1/4))) - ((5*b*c - 4*a*d)*(4/(a*(a + b*x^4)^(1/4)
) - (4*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^
4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a))/(4*a))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{\left(2 d\,x^{4}-\frac{c}{2}\right) a^{\frac{5}{4}}+x^{4} \left(-\frac{5 b c\,a^{\frac{1}{4}}}{2}+\left(\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)-\frac{\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)}{2}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}} \left(a d-\frac{5 c b}{4}\right)\right)}{2 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{\frac{9}{4}} x^{4}}$	112

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/2/(b*x^4+a)^(1/4)/a^(9/4)*((2*d*x^4-1/2*c)*a^(5/4)+x^4*(-5/2*b*c*a^(1/4)
+(arctan((b*x^4+a)^(1/4)/a^(1/4))-1/2*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4
+a)^(1/4)-a^(1/4))))*(b*x^4+a)^(1/4)*(a*d-5/4*c*b)))/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 797, normalized size of antiderivative = 6.70

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{5/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/16*((a^2*b*x^8 + a^3*x^4)*((625*b^4*c^4 - 2000*a*b^3*c^3*d + 2400*a^2*b
^2*c^2*d^2 - 1280*a^3*b*c*d^3 + 256*a^4*d^4)/a^9)^(1/4)*log(a^7*((625*b^4*
c^4 - 2000*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 1280*a^3*b*c*d^3 + 256*a^4
*d^4)/a^9)^(3/4) - (125*b^3*c^3 - 300*a*b^2*c^2*d + 240*a^2*b*c*d^2 - 64*a
^3*d^3)*(b*x^4 + a)^(1/4)) + (-I*a^2*b*x^8 - I*a^3*x^4)*((625*b^4*c^4 - 20
00*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 1280*a^3*b*c*d^3 + 256*a^4*d^4)/a^
9)^(1/4)*log(I*a^7*((625*b^4*c^4 - 2000*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2
 - 1280*a^3*b*c*d^3 + 256*a^4*d^4)/a^9)^(3/4) - (125*b^3*c^3 - 300*a*b^2*c
^2*d + 240*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + (I*a^2*b*x^8 + I
*a^3*x^4)*((625*b^4*c^4 - 2000*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 1280*a
^3*b*c*d^3 + 256*a^4*d^4)/a^9)^(1/4)*log(-I*a^7*((625*b^4*c^4 - 2000*a*b^3
*c^3*d + 2400*a^2*b^2*c^2*d^2 - 1280*a^3*b*c*d^3 + 256*a^4*d^4)/a^9)^(3/4)
 - (125*b^3*c^3 - 300*a*b^2*c^2*d + 240*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 +
 a)^(1/4)) - (a^2*b*x^8 + a^3*x^4)*((625*b^4*c^4 - 2000*a*b^3*c^3*d + 2400
*a^2*b^2*c^2*d^2 - 1280*a^3*b*c*d^3 + 256*a^4*d^4)/a^9)^(1/4)*log(-a^7*((6
25*b^4*c^4 - 2000*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 1280*a^3*b*c*d^3 +
256*a^4*d^4)/a^9)^(3/4) - (125*b^3*c^3 - 300*a*b^2*c^2*d + 240*a^2*b*c*d^2
 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + 4*((5*b*c - 4*a*d)*x^4 + a*c)*(b*x^4 +
 a)^(3/4))/(a^2*b*x^8 + a^3*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 58.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{5/4}}\,dx=-\frac{c\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^9\Gamma\left(\frac{13}{4}\right)}-\frac{d\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**5/(b*x**4+a)**(5/4),x)
```

output
```
-c*gamma(9/4)*hyper((5/4, 9/4), (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b*
*(5/4)*x**9*gamma(13/4)) - d*gamma(5/4)*hyper((5/4, 5/4), (9/4,), a*exp_po
lar(I*pi)/(b*x**4))/(4*b**(5/4)*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{5/4}}\,dx = -\frac{1}{16}\,c\left(\frac{4\,(5\,(bx^4+a)b-4\,ab)}{(bx^4+a)^{\frac{5}{4}}a^2-(bx^4+a)^{\frac{1}{4}}a^3}+\frac{5\,b\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{a^2}\right) + \frac{1}{4}\,d\left(\frac{\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{a}+\frac{4}{(bx^4+a)^{\frac{1}{4}}a}\right)$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/16*c*(4*(5*(b*x^4 + a)*b - 4*a*b)/((b*x^4 + a)^(5/4)*a^2 - (b*x^4 + a)^
(1/4)*a^3) + 5*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^
4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2) + 1/4
*d*((2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4)
- a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a + 4/((b*x^4 + a)^(1/4
)*a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.39

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{5/4}}\,dx = -\frac{\sqrt{2}(5\,bc-4\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{1}{4}}\,a^2} - \frac{\sqrt{2}(5\,bc-4\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{1}{4}}\,a^2} + \frac{\sqrt{2}(5\,bc-4\,ad)\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{1}{4}}\,a^2} - \frac{\sqrt{2}(5\,bc-4\,ad)\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{1}{4}}\,a^2} - \frac{5\,(bx^4+a)bc-4\,abc-4\,(bx^4+a)ad+4\,a^2d}{4\left((bx^4+a)^{\frac{5}{4}}-(bx^4+a)^{\frac{1}{4}}a\right)a^2}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(5*b*c - 4*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2) - 1/16*sqrt(2)*(5*b*c - 4*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1
/4))/((-a)^(1/4)*a^2) + 1/32*sqrt(2)*(5*b*c - 4*a*d)*log(sqrt(2)*(b*x^4 +
a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^2) - 1/32*
sqrt(2)*(5*b*c - 4*a*d)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b
*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^2) - 1/4*(5*(b*x^4 + a)*b*c - 4*a*b*c
- 4*(b*x^4 + a)*a*d + 4*a^2*d)/(((b*x^4 + a)^(5/4) - (b*x^4 + a)^(1/4)*a)*
a^2)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{5/4}}\,dx = \frac{d}{a\,(b\,x^4+a)^{1/4}} - \frac{\frac{b\,c}{a}-\frac{5\,b\,c\,(b\,x^4+a)}{4\,a^2}}{a\,(b\,x^4+a)^{1/4}-(b\,x^4+a)^{5/4}} + \frac{d\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{d\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{5\,b\,c\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}} + \frac{5\,b\,c\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}}$$

input
```
int((c + d*x^4)/(x^5*(a + b*x^4)^(5/4)),x)
```

output
```
d/(a*(a + b*x^4)^(1/4)) - ((b*c)/a - (5*b*c*(a + b*x^4))/(4*a^2))/(a*(a +
b*x^4)^(1/4) - (a + b*x^4)^(5/4)) + (d*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2
*a^(5/4)) - (d*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(5/4)) - (5*b*c*atan
((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(9/4)) + (5*b*c*atanh((a + b*x^4)^(1/4)/
a^(1/4)))/(8*a^(9/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{5/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^5+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^9}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,ax+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^5}dx\right)d$$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**5 + (a + b*x**4)**(1/4)*b*x**9),x)*c + int
(1/((a + b*x**4)**(1/4)*a*x + (a + b*x**4)**(1/4)*b*x**5),x)*d
```

3.112 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = -\frac{(4bc-5ad)x}{4b^2\sqrt[4]{a+bx^4}} + \frac{dx^5}{4b\sqrt[4]{a+bx^4}} + \frac{(4bc-5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}} + \frac{(4bc-5ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

output
```
-1/4*(-5*a*d+4*b*c)*x/b^2/(b*x^4+a)^(1/4)+1/4*d*x^5/b/(b*x^4+a)^(1/4)+1/8*
(-5*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+1/8*(-5*a*d+4*b*c
)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(-4bc+5ad+bdx^4)}{\sqrt[4]{a+bx^4}} + (4bc-5ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + (4bc-5ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*x*(-4*b*c + 5*a*d + b*d*x^4))/(a + b*x^4)^(1/4) + (4*b*c - 5*a
*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + (4*b*c - 5*a*d)*ArcTanh[(b^(1/
4)*x)/(a + b*x^4)^(1/4)])/(8*b^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 817, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx^4)}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4bc-5ad)\int \frac{x^4}{(bx^4+a)^{5/4}}dx}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(4bc-5ad)\left(\frac{\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{(4bc-5ad)\left(\frac{\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{(4bc-5ad)\left(\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{(4bc-5ad)\left(\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bc-5ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{4b}+\frac{dx^5}{4b\sqrt[4]{a+bx^4}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(d*x^5)/(4*b*(a + b*x^4)^(1/4)) + ((4*b*c - 5*a*d)*(-(x/(b*(a + b*x^4)^(1/
4))) + (ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/
4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))/b))/(4*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.57

method	result
pseudoelliptic	$\frac{4b^{\frac{5}{4}} d\,x^{5}-16b^{\frac{5}{4}} c x+10\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right) a d \left(b\,x^{4}+a\right)^{\frac{1}{4}}-8\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right) c b \left(b\,x^{4}+a\right)^{\frac{1}{4}}-5\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right.}{16b^{\frac{9}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*b^(5/4)*d*x^5-16*b^(5/4)*c*x+10*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*
a*d*(b*x^4+a)^(1/4)-8*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*c*b*(b*x^4+a)^(1/4
)-5*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*a*d*(b*x^
4+a)^(1/4)+4*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*
c*b*(b*x^4+a)^(1/4)+20*a*d*x*b^(1/4))/b^(9/4)/(b*x^4+a)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 805, normalized size of antiderivative = 6.44

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/16*((b^3*x^4 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^
2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*log(-(b^7*x*((256*b^4*c
^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*
d^4)/b^9)^(3/4) + (64*b^3*c^3 - 240*a*b^2*c^2*d + 300*a^2*b*c*d^2 - 125*a^
3*d^3)*(b*x^4 + a)^(1/4))/x) - (b^3*x^4 + a*b^2)*((256*b^4*c^4 - 1280*a*b^
3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4
)*log((b^7*x*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 200
0*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(3/4) - (64*b^3*c^3 - 240*a*b^2*c^2*d +
300*a^2*b*c*d^2 - 125*a^3*d^3)*(b*x^4 + a)^(1/4))/x) - (-I*b^3*x^4 - I*a*b
^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c
*d^3 + 625*a^4*d^4)/b^9)^(1/4)*log((I*b^7*x*((256*b^4*c^4 - 1280*a*b^3*c^3
*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(3/4) - (
64*b^3*c^3 - 240*a*b^2*c^2*d + 300*a^2*b*c*d^2 - 125*a^3*d^3)*(b*x^4 + a)^
(1/4))/x) - (I*b^3*x^4 + I*a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*
a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)/b^9)^(1/4)*log((-I*b^7*x
*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^
3 + 625*a^4*d^4)/b^9)^(3/4) - (64*b^3*c^3 - 240*a*b^2*c^2*d + 300*a^2*b*c*
d^2 - 125*a^3*d^3)*(b*x^4 + a)^(1/4))/x) + 4*(b*d*x^5 - (4*b*c - 5*a*d)*x)
*(b*x^4 + a)^(3/4))/(b^3*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.84 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x**5*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(5/4)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/4)*gamma(13/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(101) = 202.

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.79

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{1}{16}\,d\left(\frac{4\left(4\,ab-\frac{5\,(bx^4+a)a}{x^4}\right)}{\frac{(bx^4+a)^{\frac{1}{4}}b^3}{x}-\frac{(bx^4+a)^{\frac{5}{4}}b^2}{x^5}} + \frac{5\,a\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right) - \frac{1}{4}\,c\left(\frac{\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{b} + \frac{4\,x}{(bx^4+a)^{\frac{1}{4}}b}\right)$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/16*d*(4*(4*a*b - 5*(b*x^4 + a)*a/x^4)/((b*x^4 + a)^(1/4)*b^3/x - (b*x^4
+ a)^(5/4)*b^2/x^5) + 5*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4)
 + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b
^(1/4))/b^2) - 1/4*c*((2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + l
og(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/
4))/b + 4*x/((b*x^4 + a)^(1/4)*b))
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(x
**4/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.113 $\int \frac{c+dx^4}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}} dx = \frac{(bc-ad)x}{ab\sqrt[4]{a+bx^4}} + \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}} + \frac{d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

output
```
(-a*d+b*c)*x/a/b/(b*x^4+a)^(1/4)+1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b
^(5/4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}} dx = \frac{\frac{2\sqrt[4]{b}(bc-ad)x}{a\sqrt[4]{a+bx^4}} + d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + d\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*(b*c - a*d)*x)/(a*(a + b*x^4)^(1/4)) + d*ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)] + d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {910, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{910}$$

$$\frac{d\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{d\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{b}+\frac{x(bc-ad)}{ab\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(5/4),x]
```

output
```
((b*c - a*d)*x)/(a*b*(a + b*x^4)^(1/4)) + (d*(ArcTan[(b^(1/4)*x)/(a + b*x^
4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))
))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{\frac{(-ad+cb)x}{b\left(b\,x^{4}+a\right)^{\frac{1}{4}}}-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)ad}{2b^{\frac{5}{4}}}+\frac{\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)ad}{4b^{\frac{5}{4}}}}{a}$	94

input
```
int((d*x^4+c)/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
((-a*d+b*c)/b*x/(b*x^4+a)^(1/4)-1/2*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*a/b^
(5/4)*d+1/4*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*a
/b^(5/4)*d)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.21

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{5/4}}\,dx = \frac{4\left(bx^4+a\right)^{\frac{3}{4}}(bc-ad)x+\left(ab^2x^4+a^2b\right)\left(\frac{d^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(\frac{d^4}{b^5}\right)^{\frac{3}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}d^3}{x}\right)-\left(ab^2x^4+\cdots\right.}{\cdots}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/4*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x + (a*b^2*x^4 + a^2*b)*(d^4/b^5)^(1/
4)*log((b^4*x*(d^4/b^5)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - (a*b^2*x^4 + a
^2*b)*(d^4/b^5)^(1/4)*log(-(b^4*x*(d^4/b^5)^(3/4) - (b*x^4 + a)^(1/4)*d^3)
/x) + (-I*a*b^2*x^4 - I*a^2*b)*(d^4/b^5)^(1/4)*log((I*b^4*x*(d^4/b^5)^(3/4
) + (b*x^4 + a)^(1/4)*d^3)/x) + (I*a*b^2*x^4 + I*a^2*b)*(d^4/b^5)^(1/4)*lo
g((-I*b^4*x*(d^4/b^5)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x))/(a*b^2*x^4 + a^2*
b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \frac{cx\Gamma\left(\frac{1}{4}\right)}{4a^{\frac{5}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x*gamma(1/4)/(4*a**(5/4)*(1 + b*x**4/a)**(1/4)*gamma(5/4)) + d*x**5*gamm
a(5/4)*hyper((5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/4)*gam
ma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{4}\,d\left(\frac{\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{b}+\frac{4\,x}{\left(bx^4+a\right)^{\frac{1}{4}}b}\right)+\frac{cx}{\left(bx^4+a\right)^{\frac{1}{4}}a}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/4*d*((2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
 (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 4*x/((
b*x^4 + a)^(1/4)*b)) + c*x/((b*x^4 + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx=\int \frac{d\,x^4+c}{{\left(b\,x^4+a\right)}^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(5/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{5/4}}\,dx=\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.114 $\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx=-\frac{c}{3ax^3\sqrt[4]{a+bx^4}}-\frac{(4bc-3ad)x}{3a^2\sqrt[4]{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(1/4)-1/3*(-3*a*d+4*b*c)*x/a^2/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx=\frac{-ac-4bcx^4+3adx^4}{3a^2x^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(5/4)),x]
```

output
```
(-(a*c) - 4*b*c*x^4 + 3*a*d*x^4)/(3*a^2*x^3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(4bc-3ad)\int \frac{1}{(bx^4+a)^{5/4}}dx}{3a} - \frac{c}{3ax^3\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 746$$

$$-\frac{x(4bc-3ad)}{3a^2\sqrt[4]{a+bx^4}} - \frac{c}{3ax^3\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(5/4)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(1/4)) - ((4*b*c - 3*a*d)*x)/(3*a^2*(a + b*x^4)^
(1/4))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
gosper	$-\frac{-3ad\,x^{4}+4bc\,x^{4}+ac}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	36
trager	$-\frac{-3ad\,x^{4}+4bc\,x^{4}+ac}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	36
pseudoelliptic	$-\frac{\left(-3d\,x^{4}+c\right)a+4bc\,x^{4}}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}}x^{3}a^{2}}$	36
orering	$-\frac{-3ad\,x^{4}+4bc\,x^{4}+ac}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	36
risch	$-\frac{c\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a^{2}x^{3}}+\frac{x\left(ad-cb\right)}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	42

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*a*d*x^4+4*b*c*x^4+a*c)/x^3/(b*x^4+a)^(1/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx=-\frac{\left(\left(4\,bc-3\,ad\right)x^4+ac\right)\left(bx^4+a\right)^{\frac{3}{4}}}{3\left(a^2bx^7+a^3x^3\right)}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/3*((4*b*c - 3*a*d)*x^4 + a*c)*(b*x^4 + a)^(3/4)/(a^2*b*x^7 + a^3*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.

Time = 16.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.00

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx = c\left(\frac{\Gamma\left(-\frac{3}{4}\right)}{16a\sqrt[4]{b}x^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)} + \frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a^2\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)}\right) + \frac{dx\Gamma\left(\frac{1}{4}\right)}{4a^{\frac{5}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(5/4),x)
```

output
```
c*(gamma(-3/4)/(16*a*b**(1/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(5/4)) + b
**(3/4)*gamma(-3/4)/(4*a**2*(a/(b*x**4) + 1)**(1/4)*gamma(5/4))) + d*x*gam
ma(1/4)/(4*a**(5/4)*(1 + b*x**4/a)**(1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{3}\,c\left(\frac{3\,bx}{\left(bx^4+a\right)^{\frac{1}{4}}a^2} + \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{a^2x^3}\right) + \frac{dx}{\left(bx^4+a\right)^{\frac{1}{4}}a}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/3*c*(3*b*x/((b*x^4 + a)^(1/4)*a^2) + (b*x^4 + a)^(3/4)/(a^2*x^3)) + d*x
/((b*x^4 + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}} x^4} \, dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/4}} \, dx = \frac{3\,a\,c-4\,c\,(b\,x^4+a)+3\,a\,d\,x^4}{3\,a^2\,x^3\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(5/4)),x)
```

output
```
(3*a*c - 4*c*(a + b*x^4) + 3*a*d*x^4)/(3*a^2*x^3*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{5/4}} \, dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^8}dx\right) c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right) d$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**4 + (a + b*x**4)**(1/4)*b*x**8),x)*c + int
(1/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d
```

3.115 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/4}}\,dx = -\frac{c}{7ax^7\sqrt[4]{a+bx^4}} - \frac{8bc-7ad}{7a^2x^3\sqrt[4]{a+bx^4}} + \frac{4(8bc-7ad)\left(a+bx^4\right)^{3/4}}{21a^3x^3}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(1/4)-1/7*(-7*a*d+8*b*c)/a^2/x^3/(b*x^4+a)^(1/4)+4/
21*(-7*a*d+8*b*c)*(b*x^4+a)^(3/4)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/4}}\,dx = \frac{-3a^2c+8abcx^4-7a^2dx^4+32b^2cx^8-28abdx^8}{21a^3x^7\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(5/4)),x]
```

output
```
(-3*a^2*c + 8*a*b*c*x^4 - 7*a^2*d*x^4 + 32*b^2*c*x^8 - 28*a*b*d*x^8)/(21*a
^3*x^7*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(8bc-7ad) \int \frac{1}{x^4 (bx^4+a)^{5/4}}dx}{7a}-\frac{c}{7ax^7 \sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(8bc-7ad) \left(-\frac{4b \int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{3a}-\frac{1}{3ax^3 \sqrt[4]{a+bx^4}}\right)}{7a}-\frac{c}{7ax^7 \sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(-\frac{4bx}{3a^2 \sqrt[4]{a+bx^4}}-\frac{1}{3ax^3 \sqrt[4]{a+bx^4}}\right) (8bc-7ad)}{7a}-\frac{c}{7ax^7 \sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(5/4)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(1/4)) - ((8*b*c - 7*a*d)*(-1/3*1/(a*x^3*(a + b*
x^4)^(1/4)) - (4*b*x)/(3*a^2*(a + b*x^4)^(1/4))))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{\left(\frac{7 d\,x^{4}}{3}+c \right) a^{2}-\frac{8 \left(-\frac{7 d\,x^{4}}{2}+c \right) b\,x^{4} a}{3}-\frac{32 b^{2} c\,x^{8}}{3}}{7 \left(b\,x^{4}+a \right)^{\frac{1}{4}} x^{7} a^{3}}$	55
gosper	$-\frac{28 a b d\,x^{8}-32 b^{2} c\,x^{8}+7 a^{2} d\,x^{4}-8 a b c\,x^{4}+3 a^{2} c}{21 x^{7} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{3}}$	59
trager	$-\frac{28 a b d\,x^{8}-32 b^{2} c\,x^{8}+7 a^{2} d\,x^{4}-8 a b c\,x^{4}+3 a^{2} c}{21 x^{7} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{3}}$	59
orering	$-\frac{28 a b d\,x^{8}-32 b^{2} c\,x^{8}+7 a^{2} d\,x^{4}-8 a b c\,x^{4}+3 a^{2} c}{21 x^{7} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{3}}$	59
risch	$-\frac{\left(b\,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d\,x^{4}-11 b c\,x^{4}+3 a c \right)}{21 a^{3} x^{7}}-\frac{x \left(a d -c b \right) b}{\left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{3}}$	62

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/7/(b*x^4+a)^(1/4)*((7/3*d*x^4+c)*a^2-8/3*(-7/2*d*x^4+c)*b*x^4*a-32/3*b^
2*c*x^8)/x^7/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/4}}\,dx = \frac{(4\,(8\,b^2c-7\,abd)x^8+(8\,abc-7\,a^2d)x^4-3\,a^2c)(bx^4+a)^{\frac{3}{4}}}{21\,(a^3bx^{11}+a^4x^7)}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/21*(4*(8*b^2*c - 7*a*b*d)*x^8 + (8*a*b*c - 7*a^2*d)*x^4 - 3*a^2*c)*(b*x^
4 + a)^(3/4)/(a^3*b*x^11 + a^4*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(75) = 150$.

Time = 30.63 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.77

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/4}}\,dx = c\left(-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)} + \frac{5a^2b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)} + \frac{40ab^{\frac{27}{4}}x^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)} + \frac{32b^{\frac{31}{4}}x^{12}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}\right) + d\left(\frac{\Gamma\left(-\frac{3}{4}\right)}{16a\sqrt[4]{b}x^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)} + \frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a^2\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)}\right)$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(5/4),x)
```

output
```
c*(-3*a**3*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x**
4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/
4)) + 5*a**2*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b
**4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*g
amma(5/4)) + 40*a*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a
**5*b**4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x*
*12*gamma(5/4)) + 32*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(
64*a**5*b**4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**
6*x**12*gamma(5/4))) + d*(gamma(-3/4)/(16*a*b**(1/4)*x**4*(a/(b*x**4) + 1)
**(1/4)*gamma(5/4)) + b**(3/4)*gamma(-3/4)/(4*a**2*(a/(b*x**4) + 1)**(1/4)
*gamma(5/4)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{3}\,d\left(\frac{3\,bx}{\left(bx^4+a\right)^{\frac{1}{4}}a^2}+\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{a^2x^3}\right) + \frac{1}{21}\,c\left(\frac{21\,b^2x}{\left(bx^4+a\right)^{\frac{1}{4}}a^3}+\frac{\frac{14\,\left(bx^4+a\right)^{\frac{3}{4}}b}{x^3}-\frac{3\,\left(bx^4+a\right)^{\frac{7}{4}}}{x^7}}{a^3}\right)$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/3*d*(3*b*x/((b*x^4 + a)^(1/4)*a^2) + (b*x^4 + a)^(3/4)/(a^2*x^3)) + 1/2
1*c*(21*b^2*x/((b*x^4 + a)^(1/4)*a^3) + (14*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b
*x^4 + a)^(7/4)/x^7)/a^3)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/4}}\,dx = -\frac{32\,c\,(b\,x^4+a)^2+21\,a^2\,c+21\,a^2\,d\,x^4-56\,a\,c\,(b\,x^4+a)-28\,a\,d\,x^4\,(b\,x^4+a)}{\left(\frac{21\,a^4\,x^3}{b}-\frac{21\,a^3\,x^3\,(b\,x^4+a)}{b}\right)\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(5/4)),x)
```

output
```
-(32*c*(a + b*x^4)^2 + 21*a^2*c + 21*a^2*d*x^4 - 56*a*c*(a + b*x^4) - 28*a
*d*x^4*(a + b*x^4))/(((21*a^4*x^3)/b - (21*a^3*x^3*(a + b*x^4))/b)*(a + b*
x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{5/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^8+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{12}}dx\right) c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^8}dx\right) d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**8 + (a + b*x**4)**(1/4)*b*x**12),x)*c + in
t(1/((a + b*x**4)**(1/4)*a*x**4 + (a + b*x**4)**(1/4)*b*x**8),x)*d
```

3.116 $\int \frac{c+dx^4}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{c+dx^4}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx=-\frac{c}{11ax^{11}\sqrt[4]{a+bx^4}}-\frac{12bc-11ad}{11a^2x^7\sqrt[4]{a+bx^4}}+\frac{8(12bc-11ad)\left(a+bx^4\right)^{3/4}}{77a^3x^7}-\frac{32b(12bc-11ad)\left(a+bx^4\right)^{3/4}}{231a^4x^3}$$

output
```
-1/11*c/a/x^11/(b*x^4+a)^(1/4)-1/11*(-11*a*d+12*b*c)/a^2/x^7/(b*x^4+a)^(1/
4)+8/77*(-11*a*d+12*b*c)*(b*x^4+a)^(3/4)/a^3/x^7-32/231*b*(-11*a*d+12*b*c)
*(b*x^4+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx=\frac{-21a^3c+36a^2bcx^4-33a^3dx^4-96ab^2cx^8+88a^2bdx^8-384b^3cx^{12}+352ab^2dx^{12}}{231a^4x^{11}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^12*(a + b*x^4)^(5/4)),x]
```

output
```
(-21*a^3*c + 36*a^2*b*c*x^4 - 33*a^3*d*x^4 - 96*a*b^2*c*x^8 + 88*a^2*b*d*x
^8 - 384*b^3*c*x^12 + 352*a*b^2*d*x^12)/(231*a^4*x^11*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{12}\left(a+bx^4\right)^{5/4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(12bc-11ad)\int \frac{1}{x^8(bx^4+a)^{5/4}}dx}{11a}-\frac{c}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(12bc-11ad)\left(-\frac{8b\int \frac{1}{x^4\left(bx^4+a\right)^{5/4}}dx}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{c}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(12bc-11ad)\left(-\frac{8b\left(-\frac{4b\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{3a}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{c}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{746}$$

$$
-\frac{\left(-\frac{8b\left(-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)(12bc-11ad)}{11a}-\frac{c}{11ax^{11}\sqrt[4]{a+bx^4}}
$$

input
```
Int[(c + d*x^4)/(x^12*(a + b*x^4)^(5/4)),x]
```

output
```
-1/11*c/(a*x^11*(a + b*x^4)^(1/4)) - ((12*b*c - 11*a*d)*(-1/7*1/(a*x^7*(a
+ b*x^4)^(1/4)) - (8*b*(-1/3*1/(a*x^3*(a + b*x^4)^(1/4)) - (4*b*x)/(3*a^2*
(a + b*x^4)^(1/4))))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\frac{11 d\,x^{4}}{7}+c \right) a^{3}-\frac{12 b\,x^{4} \left(\frac{22 d\,x^{4}}{9}+c \right) a^{2}}{7}+\frac{32 b^{2} x^{8} \left(-\frac{11 d\,x^{4}}{3}+c \right) a}{7}+\frac{128 b^{3} c\,x^{12}}{7}}{11 \left(b\,x^{4}+a \right)^{\frac{1}{4}} x^{11} a^{4}}$	74
gosper	$-\frac{-352 a\,b^{2} d\,x^{12}+384 b^{3} c\,x^{12}-88 a^{2} b d\,x^{8}+96 a\,b^{2} c\,x^{8}+33 a^{3} d\,x^{4}-36 a^{2} b c\,x^{4}+21 c\,a^{3}}{231 x^{11} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{4}}$	83
trager	$-\frac{-352 a\,b^{2} d\,x^{12}+384 b^{3} c\,x^{12}-88 a^{2} b d\,x^{8}+96 a\,b^{2} c\,x^{8}+33 a^{3} d\,x^{4}-36 a^{2} b c\,x^{4}+21 c\,a^{3}}{231 x^{11} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{4}}$	83
orering	$-\frac{-352 a\,b^{2} d\,x^{12}+384 b^{3} c\,x^{12}-88 a^{2} b d\,x^{8}+96 a\,b^{2} c\,x^{8}+33 a^{3} d\,x^{4}-36 a^{2} b c\,x^{4}+21 c\,a^{3}}{231 x^{11} \left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{4}}$	83
risch	$-\frac{\left(b\,x^{4}+a \right)^{\frac{3}{4}} \left(-121 a b d\,x^{8}+153 b^{2} c\,x^{8}+33 a^{2} d\,x^{4}-57 a b c\,x^{4}+21 a^{2} c \right)}{231 a^{4} x^{11}}+\frac{x\,b^{2} \left(a d -c b \right)}{\left(b\,x^{4}+a \right)^{\frac{1}{4}} a^{4}}$	85

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/11/(b*x^4+a)^(1/4)*((11/7*d*x^4+c)*a^3-12/7*b*x^4*(22/9*d*x^4+c)*a^2+32
/7*b^2*x^8*(-11/3*d*x^4+c)*a+128/7*b^3*c*x^12)/x^11/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^4}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx =$$
$$-\frac{(32\,(12\,b^3c-11\,ab^2d)x^{12}+8\,(12\,ab^2c-11\,a^2bd)x^8-3\,(12\,a^2bc-11\,a^3d)x^4+21\,a^3c)(bx^4+a)^{\frac{3}{4}}}{231\,(a^4bx^{15}+a^5x^{11})}$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/231*(32*(12*b^3*c - 11*a*b^2*d)*x^12 + 8*(12*a*b^2*c - 11*a^2*b*d)*x^8
- 3*(12*a^2*b*c - 11*a^3*d)*x^4 + 21*a^3*c)*(b*x^4 + a)^(3/4)/(a^4*b*x^15
+ a^5*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(107) = 214$.

Time = 48.90 (sec) , antiderivative size = 920, normalized size of antiderivative = 8.07

$$\int \frac{c + dx^4}{x^{12} (a + bx^4)^{5/4}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**12/(b*x**4+a)**(5/4),x)
```

output
```
c*(21*a**5*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x
**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*ga
mma(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 6*a**4*b**(43/4)*x**4*(a/(b*
x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b
**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x
**20*gamma(5/4)) + 45*a**3*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-1
1/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 76
8*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 540*a**
2*b**(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8
*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma
(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 864*a*b**(55/4)*x**16*(a/(b*x**
4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**1
0*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**2
0*gamma(5/4)) + 384*b**(59/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(
256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5
*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4))) + d*(-3*a**3*b
**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x**4*gamma(5/4)
 + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/4)) + 5*a**2
*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x**4*gam
ma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/4)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^4}{x^{12}(a+bx^4)^{5/4}}\,dx = \frac{1}{21}\,d\left(\frac{21\,b^2x}{(bx^4+a)^{\frac{1}{4}}a^3} + \frac{\frac{14\,(bx^4+a)^{\frac{3}{4}}b}{x^3} - \frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}}{a^3}\right) - \frac{1}{77}\,c\left(\frac{77\,b^3x}{(bx^4+a)^{\frac{1}{4}}a^4} + \frac{\frac{77\,(bx^4+a)^{\frac{3}{4}}b^2}{x^3} - \frac{33\,(bx^4+a)^{\frac{7}{4}}b}{x^7} + \frac{7\,(bx^4+a)^{\frac{11}{4}}}{x^{11}}}{a^4}\right)$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/21*d*(21*b^2*x/((b*x^4 + a)^(1/4)*a^3) + (14*(b*x^4 + a)^(3/4)*b/x^3 - 3
*(b*x^4 + a)^(7/4)/x^7)/a^3) - 1/77*c*(77*b^3*x/((b*x^4 + a)^(1/4)*a^4) +
(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 33*(b*x^4 + a)^(7/4)*b/x^7 + 7*(b*x^4 + a)
^(11/4)/x^11)/a^4)
```

Giac [F]

$$\int \frac{c+dx^4}{x^{12}(a+bx^4)^{5/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}}x^{12}}\,dx$$

input
```
integrate((d*x^4+c)/x^12/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^4}{x^{12}(a+bx^4)^{5/4}}\,dx = \frac{b\,(b\,x^4+a)^{3/4}\,(121\,a\,d-153\,b\,c)}{231\,a^4\,x^3} - \frac{(b\,x^4+a)^{3/4}\,(11\,a^2\,d-19\,a\,b\,c)}{77\,a^4\,x^7} - \frac{x\,(b^3\,c-a\,b^2\,d)}{a^4\,(b\,x^4+a)^{1/4}} - \frac{c\,(b\,x^4+a)^{3/4}}{11\,a^2\,x^{11}}$$

input
```
int((c + d*x^4)/(x^12*(a + b*x^4)^(5/4)),x)
```

output
```
(b*(a + b*x^4)^(3/4)*(121*a*d - 153*b*c))/(231*a^4*x^3) - ((a + b*x^4)^(3/
4)*(11*a^2*d - 19*a*b*c))/(77*a^4*x^7) - (x*(b^3*c - a*b^2*d))/(a^4*(a + b
*x^4)^(1/4)) - (c*(a + b*x^4)^(3/4))/(11*a^2*x^11)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{12}(a+bx^4)^{5/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{16}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^{8}+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{12}}dx\right)d$$

input
```
int((d*x^4+c)/x^12/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**12 + (a + b*x**4)**(1/4)*b*x**16),x)*c + i
nt(1/((a + b*x**4)**(1/4)*a*x**8 + (a + b*x**4)**(1/4)*b*x**12),x)*d
```

3.117 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}} dx = \frac{(6bc-7ad)x^3}{12b^2\sqrt[4]{a+bx^4}} + \frac{dx^7}{6b\sqrt[4]{a+bx^4}} + \frac{\sqrt{a}(6bc-7ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
1/12*(-7*a*d+6*b*c)*x^3/b^2/(b*x^4+a)^(1/4)+1/6*d*x^7/b/(b*x^4+a)^(1/4)+1/
4*a^(1/2)*(-7*a*d+6*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}} dx = \frac{x^3\left(6bc-7ad+2bdx^4+(-6bc+7ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{12b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(6*b*c - 7*a*d + 2*b*d*x^4 + (-6*b*c + 7*a*d)*(1 + (b*x^4)/a)^(1/4)*H
ypergeometric2F1[3/4, 5/4, 7/4, -((b*x^4)/a)]))/(12*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (c+dx^4)}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \; 959$$

$$\frac{(6bc-7ad)\int \frac{x^6}{(bx^4+a)^{5/4}}dx}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 815$$

$$\frac{(6bc-7ad)\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{2b}\right)}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 813$$

$$\frac{(6bc-7ad)\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 858$$

$$\frac{(6bc-7ad)\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 807$$

$$\frac{(6bc-7ad)\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 212$$

$$\frac{(6bc-7ad)\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}+\frac{dx^7}{6b\sqrt[4]{a+bx^4}}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(d*x^7)/(6*b*(a + b*x^4)^(1/4)) + ((6*b*c - 7*a*d)*(x^3/(2*b*(a + b*x^4)^(
1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[
b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))))/(6*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{6}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{5}{4}}}dx$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^6)*(b*x^4 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x**7*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(5/4)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(5/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**10/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(
x**6/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.118 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{dx^3}{2b\sqrt[4]{a+bx^4}} - \frac{(2bc-3ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/2*d*x^3/b/(b*x^4+a)^(1/4)-1/2*(-3*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*Ellipti
cE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/b^(3/2)/(b*x^4+a)
^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{x^3\left(3ad+(2bc-3ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{6ab\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(3*a*d + (2*b*c - 3*a*d)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4,
 5/4, 7/4, -((b*x^4)/a)]))/(6*a*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2bc-3ad)\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{2b}+\frac{dx^3}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}+\frac{dx^3}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{dx^3}{2b\sqrt[4]{a+bx^4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{dx^3}{2b\sqrt[4]{a+bx^4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{dx^3}{2b\sqrt[4]{a+bx^4}} - \frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-3ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(d*x^3)/(2*b*(a + b*x^4)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a/(b*x^4))^(1/4)*x
*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[a]*b^(3/2)*(a + b*
x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(d\,x^4+c)}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)*(b*x^4 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(5/4)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**6/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(x
**2/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.119 $\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=-\frac{c}{ax\sqrt[4]{a+bx^4}}+\frac{(2bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-c/a/x/(b*x^4+a)^(1/4)+(-a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*
arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\frac{-3ac+(-2bc+ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^2x\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(5/4)),x]
```

output
```
(-3*a*c + (-2*b*c + a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4,
5/4, 7/4, -((b*x^4)/a)])/(3*a^2*x*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-ad)\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{a}-\frac{c}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{813}$$

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{ab\sqrt[4]{a+bx^4}}-\frac{c}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{ab\sqrt[4]{a+bx^4}}-\frac{c}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2ab\sqrt[4]{a+bx^4}}-\frac{c}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(2bc-ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}-\frac{c}{ax\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(5/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(1/4))) + ((2*b*c - a*d)*(1 + a/(b*x^4))^(1/4)*x*Elli
pticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a^(3/2)*Sqrt[b]*(a + b*x^4)^(1
/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{2} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x)
```

output
```
int((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{5}{4}}x^{2}}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^2*x^10 + 2*a*b*x^6 + a^2*x^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\frac{c\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4},\frac{5}{4} \\ \frac{3}{4} \end{matrix}\middle| {\frac{bx^{4}e^{i\pi}}{a}} \right)}{4a^{\frac{5}{4}}x\Gamma\left(\frac{3}{4}\right)}+\frac{dx^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix} \frac{3}{4},\frac{5}{4} \\ \frac{7}{4} \end{matrix}\middle| {\frac{bx^{4}e^{i\pi}}{a}} \right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(5/4),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
5/4)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{d\,x^4+c}{x^2\,\left(b\,x^4+a\right)^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(5/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^2+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^6}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(5/4),x)
```

output
```
int(x**2/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(1/4)*a*x**2 + (a + b*x**4)**(1/4)*b*x**6),x)*c
```

3.120 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx = -\frac{c}{5ax^5\sqrt[4]{a+bx^4}}+\frac{6bc-5ad}{5a^2x\sqrt[4]{a+bx^4}} - \frac{2\sqrt{b}(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(1/4)+1/5*(-5*a*d+6*b*c)/a^2/x/(b*x^4+a)^(1/4)-2/5*
b^(1/2)*(-5*a*d+6*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2
)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx = \frac{-ac+(6bc-5ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^2x^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(5/4)),x]
```

output
```
(-(a*c) + (6*b*c - 5*a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4
, 5/4, 3/4, -((b*x^4)/a)])/(5*a^2*x^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(6bc-5ad)\int \frac{1}{x^2(bx^4+a)^{5/4}}dx}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 816$$

$$-\frac{(6bc-5ad)\left(-\frac{2b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 813$$

$$-\frac{(6bc-5ad)\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 858$$

$$-\frac{(6bc-5ad)\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

$\downarrow$ 807

$$-\frac{(6bc-5ad)\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

$\downarrow$ 212

$$-\frac{(6bc-5ad)\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{c}{5ax^5\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(5/4)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(1/4)) - ((6*b*c - 5*a*d)*(-(1/(a*x*(a + b*x^4)^
(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqr
t[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{6} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x)
```

output
```
int((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{5/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^2*x^14 + 2*a*b*x^10 + a^2*x^6),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{5/4}}\,dx = \frac{c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}-\frac{5}{4}, \frac{5}{4}\\ -\frac{1}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{d\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}-\frac{1}{4}, \frac{5}{4}\\ \frac{3}{4}\end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(5/4),x)
```

output
```
c*gamma(-5/4)*hyper((-5/4, 5/4), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*x**5*gamma(-1/4)) + d*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(5/4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{d\,x^4+c}{x^6\,\left(b\,x^4+a\right)^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(5/4)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{5/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^6+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{10}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^2+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^6}dx\right)d$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**6 + (a + b*x**4)**(1/4)*b*x**10),x)*c + in
t(1/((a + b*x**4)**(1/4)*a*x**2 + (a + b*x**4)**(1/4)*b*x**6),x)*d
```

3.121 $\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{(5bc-6ad)x^2}{5b^2\sqrt[4]{a+bx^4}} + \frac{dx^6}{5b\sqrt[4]{a+bx^4}} - \frac{2\sqrt{a}(5bc-6ad)\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-6*a*d+5*b*c)*x^2/b^2/(b*x^4+a)^(1/4)+1/5*d*x^6/b/(b*x^4+a)^(1/4)-2/5
*a^(1/2)*(-6*a*d+5*b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)
*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{x^2\left(-5bc+6ad+bdx^4+(5bc-6ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{5b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x^5*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(-5*b*c + 6*a*d + b*d*x^4 + (5*b*c - 6*a*d)*(1 + (b*x^4)/a)^(1/4)*Hyp
ergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(5*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 807, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5bc-6ad)\int \frac{x^5}{(bx^4+a)^{5/4}}dx}{5b}+\frac{dx^6}{5b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(5bc-6ad)\int \frac{x^4}{(bx^4+a)^{5/4}}dx^2}{10b}+\frac{dx^6}{5b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{250}$$

$$\frac{(5bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2a\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx^2}{b}\right)}{10b}+\frac{dx^6}{5b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{(5bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)}{10b}+\frac{dx^6}{5b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{(5bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)}{10b}+\frac{dx^6}{5b\sqrt[4]{a+bx^4}}$$

input
```
Int[(x^5*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(d*x^6)/(5*b*(a + b*x^4)^(1/4)) + ((5*b*c - 6*a*d)*((2*x^2)/(b*(a + b*x^4)
^(1/4)) - (4*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/
Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^4)^(1/4))))/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^5 (d\,x^4 + c)}{(b\,x^4 + a)^{\frac{5}{4}}}dx$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^5 (c + dx^4)}{(a + bx^4)^{5/4}}\,dx = \int \frac{(dx^4 + c)x^5}{(bx^4 + a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)*(b*x^4 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.49

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{cx^6\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}} + \frac{dx^{10}\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{5}{4}}}$$

input
```
integrate(x**5*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x**6*hyper((5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4)) +
d*x**10*hyper((5/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(5/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5\,(d\,x^4+c)}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**9/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(x
**5/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.122 $\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{dx^2}{b\sqrt[4]{a+bx^4}} + \frac{(bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
d*x^2/b/(b*x^4+a)^(1/4)+(-2*a*d+b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*a
rctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{x^2\left(2bc-2ad+(-bc+2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{2ab\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(x*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(2*b*c - 2*a*d + (-(b*c) + 2*a*d)*(1 + (b*x^4)/a)^(1/4)*Hypergeometri
c2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(2*a*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {959, 807, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(bc-2ad)\int \frac{x}{(bx^4+a)^{5/4}}dx}{b}+\frac{dx^2}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(bc-2ad)\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{2b}+\frac{dx^2}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(bc-2ad)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2ab\sqrt[4]{a+bx^4}}+\frac{dx^2}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(bc-2ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}+\frac{dx^2}{b\sqrt[4]{a+bx^4}}$$

input
```
Int[(x*(c + d*x^4))/(a + b*x^4)^(5/4),x]
```

output
```
(d*x^2)/(b*(a + b*x^4)^(1/4)) + ((b*c - 2*a*d)*(1 + (b*x^4)/a)^(1/4)*Ellip
ticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*b^(3/2)*(a + b*x^4)^(1/
4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(d\,x^4+c)}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)*(b*x^4 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \frac{cx^2{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}} + \frac{dx^6{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}}$$

input
```
integrate(x*(d*x**4+c)/(b*x**4+a)**(5/4),x)
```

output
```
c*x**2*hyper((1/2, 5/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4)) +
d*x**6*hyper((5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4))
```

Maxima [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \int \frac{x\,(d\,x^4+c)}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((x*(c + d*x^4))/(a + b*x^4)^(5/4),x)
```

output
```
int((x*(c + d*x^4))/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{5/4}}\,dx = \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(5/4),x)
```

output
```
int(x**5/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(x
/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*c
```

3.123 $\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx=-\frac{c}{2ax^2\sqrt[4]{a+bx^4}}-\frac{(3bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-1/2*c/a/x^2/(b*x^4+a)^(1/4)-1/2*(-2*a*d+3*b*c)*(1+b*x^4/a)^(1/4)*Elliptic
E(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/b^(1/2)/(b*x^4+a)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx=\frac{-2ac-6bcx^4+4adx^4+(3bc-2ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{b}{}\right.}{4a^2x^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^3*(a + b*x^4)^(5/4)),x]
```

output
```
(-2*a*c - 6*b*c*x^4 + 4*a*d*x^4 + (3*b*c - 2*a*d)*x^4*(1 + (b*x^4)/a)^(1/4
)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)])/(4*a^2*x^2*(a + b*x^4)^(
1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 807, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{5/4}} dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(3bc-2ad)\int \frac{x}{(bx^4+a)^{5/4}}dx}{2a}-\frac{c}{2ax^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(3bc-2ad)\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{4a}-\frac{c}{2ax^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{213}$$

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}(3bc-2ad)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{4a^2\sqrt[4]{a+bx^4}}-\frac{c}{2ax^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{\sqrt[4]{\frac{bx^4}{a}+1}(3bc-2ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\Big|2\right)}{2a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}-\frac{c}{2ax^2\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^3*(a + b*x^4)^(5/4)),x]
```

output
```
-1/2*c/(a*x^2*(a + b*x^4)^(1/4)) - ((3*b*c - 2*a*d)*(1 + (b*x^4)/a)^(1/4)*
EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*a^(3/2)*Sqrt[b]*(a + b*x
^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{3} \left(b\,x^{4}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x)
```

output
```
int((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{5/4}} dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}} x^3} dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^2*x^11 + 2*a*b*x^7 + a^2*x^3), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{5/4}} dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}} x^2} + \frac{dx^2\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}}$$

input
```
integrate((d*x**4+c)/x**3/(b*x**4+a)**(5/4),x)
```

output
```
-c*hyper((-1/2, 5/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4)*x**2)
+ d*x**2*hyper((1/2, 5/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{5/4}} dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}} x^3} dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{5}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{d\,x^4+c}{x^3\,{\left(b\,x^4+a\right)}^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(5/4)),x)
```

output
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{5/4}}\,dx=\left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^3+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^7}dx\right)c$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(5/4),x)
```

output
```
int(x/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)*d + int(1/((
a + b*x**4)**(1/4)*a*x**3 + (a + b*x**4)**(1/4)*b*x**7),x)*c
```

3.124 $\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{5/4}}\,dx = -\frac{c}{6ax^6\sqrt[4]{a+bx^4}}+\frac{7bc-6ad}{12a^2x^2\sqrt[4]{a+bx^4}} + \frac{\sqrt{b}(7bc-6ad)\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/6*c/a/x^6/(b*x^4+a)^(1/4)+1/12*(-6*a*d+7*b*c)/a^2/x^2/(b*x^4+a)^(1/4)+1
/4*b^(1/2)*(-6*a*d+7*b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/
2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{5/4}}\,dx = \frac{-2ac+(7bc-6ad)x^4\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{12a^2x^6\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^7*(a + b*x^4)^(5/4)),x]
```

output
```
(-2*a*c + (7*b*c - 6*a*d)*x^4*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2
, 5/4, 1/2, -((b*x^4)/a)])/(12*a^2*x^6*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 807, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(7bc-6ad)\int \frac{1}{x^3(bx^4+a)^{5/4}}dx}{6a}-\frac{c}{6ax^6\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(7bc-6ad)\int \frac{1}{x^4(bx^4+a)^{5/4}}dx^2}{12a}-\frac{c}{6ax^6\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 251}$$

$$-\frac{(7bc-6ad)\left(-\frac{3b\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{2a}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{12a}-\frac{c}{6ax^6\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 213}$$

$$-\frac{(7bc-6ad)\left(-\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2a^2\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{12a}-\frac{c}{6ax^6\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{(7bc-6ad)\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{12a}-\frac{c}{6ax^6\sqrt[4]{a+bx^4}}$$

input
```
Int[(c + d*x^4)/(x^7*(a + b*x^4)^(5/4)),x]
```

output
```
-1/6*c/(a*x^6*(a + b*x^4)^(1/4)) - ((7*b*c - 6*a*d)*(-(1/(a*x^2*(a + b*x^4
)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(12*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{7}\left(b\,x^{4}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x)
```

output
```
int((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{dx^{4}+c}{\left(bx^{4}+a\right)^{\frac{5}{4}}x^{7}}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^2*x^15 + 2*a*b*x^11 + a^2*x^7),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.52 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{5/4}} \, dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}x^6} - \frac{d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}x^2}$$

input
```
integrate((d*x**4+c)/x**7/(b*x**4+a)**(5/4),x)
```

output
```
-c*hyper((-3/2, 5/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4)*x**6)
 - d*hyper((-1/2, 5/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4)*x**2
)
```

Maxima [F]

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{5/4}} \, dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^7} \, dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^7), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{5/4}} \, dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{5}{4}}x^7} \, dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(5/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{5/4}}\,dx = \int \frac{d\,x^4+c}{x^7\,(b\,x^4+a)^{5/4}}\,dx$$

input
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(5/4)),x)
```

output
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{5/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^7+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{11}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^3+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^7}dx\right)d$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**7 + (a + b*x**4)**(1/4)*b*x**11),x)*c + in
t(1/((a + b*x**4)**(1/4)*a*x**3 + (a + b*x**4)**(1/4)*b*x**7),x)*d
```

3.125 $\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = -\frac{a^2(bc-ad)}{3b^4(a+bx^4)^{3/4}} - \frac{a(2bc-3ad)\sqrt[4]{a+bx^4}}{b^4} + \frac{(bc-3ad)(a+bx^4)^{5/4}}{5b^4} + \frac{d(a+bx^4)^{9/4}}{9b^4}$$

output
```
-1/3*a^2*(-a*d+b*c)/b^4/(b*x^4+a)^(3/4)-a*(-3*a*d+2*b*c)*(b*x^4+a)^(1/4)/b
^4+1/5*(-3*a*d+b*c)*(b*x^4+a)^(5/4)/b^4+1/9*d*(b*x^4+a)^(9/4)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = \frac{128a^3d - 96a^2b(c-dx^4) - 12ab^2x^4(6c+dx^4) + b^3x^8(9c+5dx^4)}{45b^4(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^11*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(128*a^3*d - 96*a^2*b*(c - d*x^4) - 12*a*b^2*x^4*(6*c + d*x^4) + b^3*x^8*(
9*c + 5*d*x^4))/(45*b^4*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8(dx^4+c)}{(bx^4+a)^{7/4}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(-\frac{(ad-bc)a^2}{b^3(bx^4+a)^{7/4}}+\frac{(3ad-2bc)a}{b^3(bx^4+a)^{3/4}}+\frac{d(bx^4+a)^{5/4}}{b^3}+\frac{(bc-3ad)\sqrt[4]{bx^4+a}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^2(bc-ad)}{3b^4(a+bx^4)^{3/4}}+\frac{4(a+bx^4)^{5/4}(bc-3ad)}{5b^4}-\frac{4a\sqrt[4]{a+bx^4}(2bc-3ad)}{b^4}+\frac{4d(a+bx^4)^{9/4}}{9b^4}\right)$$

input
```
Int[(x^11*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((-4*a^2*(b*c - a*d))/(3*b^4*(a + b*x^4)^(3/4)) - (4*a*(2*b*c - 3*a*d)*(a
+ b*x^4)^(1/4))/b^4 + (4*(b*c - 3*a*d)*(a + b*x^4)^(5/4))/(5*b^4) + (4*d*(
a + b*x^4)^(9/4))/(9*b^4))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\frac{x^{8}\left(\frac{5 d\,x^{4}}{9}+c \right) b^{3}}{5}-\frac{8 \left(\frac{d\,x^{4}}{6}+c \right) x^{4} a\,b^{2}}{5}-\frac{32 a^{2} \left(-d\,x^{4}+c \right) b}{15}+\frac{128 a^{3} d}{45}}{\left(b\,x^{4}+a \right)^{\frac{3}{4}} b^{4}}$	68
gosper	$\frac{5 b^{3} d\,x^{12}-12 a\,b^{2} d\,x^{8}+9 c\,b^{3} x^{8}+96 a^{2} b d\,x^{4}-72 a\,b^{2} c\,x^{4}+128 a^{3} d-96 a^{2} b c}{45 \left(b\,x^{4}+a \right)^{\frac{3}{4}} b^{4}}$	77
trager	$\frac{5 b^{3} d\,x^{12}-12 a\,b^{2} d\,x^{8}+9 c\,b^{3} x^{8}+96 a^{2} b d\,x^{4}-72 a\,b^{2} c\,x^{4}+128 a^{3} d-96 a^{2} b c}{45 \left(b\,x^{4}+a \right)^{\frac{3}{4}} b^{4}}$	77
orering	$\frac{5 b^{3} d\,x^{12}-12 a\,b^{2} d\,x^{8}+9 c\,b^{3} x^{8}+96 a^{2} b d\,x^{4}-72 a\,b^{2} c\,x^{4}+128 a^{3} d-96 a^{2} b c}{45 \left(b\,x^{4}+a \right)^{\frac{3}{4}} b^{4}}$	77
risch	$\frac{\left(5 d\,b^{2} x^{8}-17 a b d\,x^{4}+9 b^{2} c\,x^{4}+113 a^{2} d-81 a b c \right) \left(b\,x^{4}+a \right)^{\frac{1}{4}}}{45 b^{4}}+\frac{a^{2} \left(a d-c b \right)}{3 b^{4} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}$	79

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
128/45/(b*x^4+a)^(3/4)*(9/128*x^8*(5/9*d*x^4+c)*b^3-9/16*(1/6*d*x^4+c)*x^4
*a*b^2-3/4*a^2*(-d*x^4+c)*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{(5\,b^3dx^{12}+3\,(3\,b^3c-4\,ab^2d)x^8-24\,(3\,ab^2c-4\,a^2bd)x^4-96\,a^2bc+128\,a^3d)(bx^4+a}{45\,(b^5x^4+ab^4)}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/45*(5*b^3*d*x^12 + 3*(3*b^3*c - 4*a*b^2*d)*x^8 - 24*(3*a*b^2*c - 4*a^2*b
*d)*x^4 - 96*a^2*b*c + 128*a^3*d)*(b*x^4 + a)^(1/4)/(b^5*x^4 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.70

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \begin{cases} \frac{128a^3d}{45b^4(a+bx^4)^{\frac{3}{4}}} - \frac{32a^2c}{15b^3(a+bx^4)^{\frac{3}{4}}} + \frac{32a^2dx^4}{15b^3(a+bx^4)^{\frac{3}{4}}} - \frac{8acx^4}{5b^2(a+bx^4)^{\frac{3}{4}}} - \frac{4adx^8}{15b^2(a+bx^4)^{\frac{3}{4}}} + \frac{cx^8}{5b(a+bx^4)^{\frac{3}{4}}} + \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{16}}{16}}{a^{\frac{7}{4}}} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
Piecewise((128*a**3*d/(45*b**4*(a + b*x**4)**(3/4)) - 32*a**2*c/(15*b**3*(
a + b*x**4)**(3/4)) + 32*a**2*d*x**4/(15*b**3*(a + b*x**4)**(3/4)) - 8*a*c
*x**4/(5*b**2*(a + b*x**4)**(3/4)) - 4*a*d*x**8/(15*b**2*(a + b*x**4)**(3/
4)) + c*x**8/(5*b*(a + b*x**4)**(3/4)) + d*x**12/(9*b*(a + b*x**4)**(3/4))
, Ne(b, 0)), ((c*x**12/12 + d*x**16/16)/a**(7/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{1}{45}\,d\left(\frac{5\,(bx^4+a)^{\frac{9}{4}}}{b^4} - \frac{27\,(bx^4+a)^{\frac{5}{4}}a}{b^4} + \frac{135\,(bx^4+a)^{\frac{1}{4}}a^2}{b^4} + \frac{15\,a^3}{(bx^4+a)^{\frac{3}{4}}b^4}\right) + \frac{1}{15}\,c\left(\frac{3\,(bx^4+a)^{\frac{5}{4}}}{b^3} - \frac{30\,(bx^4+a)^{\frac{1}{4}}a}{b^3} - \frac{5\,a^2}{(bx^4+a)^{\frac{3}{4}}b^3}\right)$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/45*d*(5*(b*x^4 + a)^(9/4)/b^4 - 27*(b*x^4 + a)^(5/4)*a/b^4 + 135*(b*x^4
+ a)^(1/4)*a^2/b^4 + 15*a^3/((b*x^4 + a)^(3/4)*b^4)) + 1/15*c*(3*(b*x^4 +
a)^(5/4)/b^3 - 30*(b*x^4 + a)^(1/4)*a/b^3 - 5*a^2/((b*x^4 + a)^(3/4)*b^3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = -\frac{a^2bc-a^3d}{3\,(bx^4+a)^{\frac{3}{4}}b^4} + \frac{9\,(bx^4+a)^{\frac{5}{4}}b^{33}c - 90\,(bx^4+a)^{\frac{1}{4}}ab^{33}c + 5\,(bx^4+a)^{\frac{9}{4}}b^{32}d - 27\,(bx^4+a)^{\frac{5}{4}}ab^{32}d + 135\,(bx^4+a)^{\frac{1}{4}}a^2b^{32}d}{45\,b^{36}}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
-1/3*(a^2*b*c - a^3*d)/((b*x^4 + a)^(3/4)*b^4) + 1/45*(9*(b*x^4 + a)^(5/4)
*b^33*c - 90*(b*x^4 + a)^(1/4)*a*b^33*c + 5*(b*x^4 + a)^(9/4)*b^32*d - 27*
(b*x^4 + a)^(5/4)*a*b^32*d + 135*(b*x^4 + a)^(1/4)*a^2*b^32*d)/b^36
```

Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{5\,d\,(b\,x^4+a)^3+15\,a^3\,d-27\,a\,d\,(b\,x^4+a)^2+9\,b\,c\,(b\,x^4+a)^2+135\,a^2\,d\,(b\,x^4+a)-}{45\,b^4\,(b\,x^4+a)^{3/4}}$$

input
```
int((x^11*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
(5*d*(a + b*x^4)^3 + 15*a^3*d - 27*a*d*(a + b*x^4)^2 + 9*b*c*(a + b*x^4)^2
 + 135*a^2*d*(a + b*x^4) - 15*a^2*b*c - 90*a*b*c*(a + b*x^4))/(45*b^4*(a +
 b*x^4)^(3/4))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^{15}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**15/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(
x**11/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.126 $\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = \frac{a(bc-ad)}{3b^3(a+bx^4)^{3/4}} + \frac{(bc-2ad)\sqrt[4]{a+bx^4}}{b^3} + \frac{d(a+bx^4)^{5/4}}{5b^3}$$

output
```
1/3*a*(-a*d+b*c)/b^3/(b*x^4+a)^(3/4)+(-2*a*d+b*c)*(b*x^4+a)^(1/4)/b^3+1/5*
d*(b*x^4+a)^(5/4)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.80

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = \frac{20abc - 32a^2d + 15b^2cx^4 - 24abdx^4 + 3b^2dx^8}{15b^3(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^7*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(20*a*b*c - 32*a^2*d + 15*b^2*c*x^4 - 24*a*b*d*x^4 + 3*b^2*d*x^8)/(15*b^3*
(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{x^4(dx^4+c)}{(bx^4+a)^{7/4}}dx^4$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{4}\int \left(\frac{\sqrt[4]{bx^4+a}d}{b^2}+\frac{bc-2ad}{b^2(bx^4+a)^{3/4}}+\frac{a(ad-bc)}{b^2(bx^4+a)^{7/4}}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{4\sqrt[4]{a+bx^4}(bc-2ad)}{b^3}+\frac{4a(bc-ad)}{3b^3(a+bx^4)^{3/4}}+\frac{4d(a+bx^4)^{5/4}}{5b^3}\right)$$

input
```
Int[(x^7*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((4*a*(b*c - a*d))/(3*b^3*(a + b*x^4)^(3/4)) + (4*(b*c - 2*a*d)*(a + b*x^4
)^(1/4))/b^3 + (4*d*(a + b*x^4)^(5/4))/(5*b^3))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$-\frac{32\left(-\frac{15\left(\frac{d\,x^{4}}{5}+c\right)x^{4}b^{2}}{32}-\frac{5\left(-\frac{6d\,x^{4}}{5}+c\right)ab}{8}+a^{2}d\right)}{15\left(b\,x^{4}+a\right)^{\frac{3}{4}}b^{3}}$	49
gosper	$-\frac{-3d\,b^{2}x^{8}+24abd\,x^{4}-15b^{2}c\,x^{4}+32a^{2}d-20abc}{15\left(b\,x^{4}+a\right)^{\frac{3}{4}}b^{3}}$	53
trager	$-\frac{-3d\,b^{2}x^{8}+24abd\,x^{4}-15b^{2}c\,x^{4}+32a^{2}d-20abc}{15\left(b\,x^{4}+a\right)^{\frac{3}{4}}b^{3}}$	53
orering	$-\frac{-3d\,b^{2}x^{8}+24abd\,x^{4}-15b^{2}c\,x^{4}+32a^{2}d-20abc}{15\left(b\,x^{4}+a\right)^{\frac{3}{4}}b^{3}}$	53
risch	$-\frac{\left(-db\,x^{4}+9ad-5cb\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{5b^{3}}-\frac{a\left(ad-cb\right)}{3b^{3}\left(b\,x^{4}+a\right)^{\frac{3}{4}}}$	55

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-32/15/(b*x^4+a)^(3/4)*(-15/32*(1/5*d*x^4+c)*x^4*b^2-5/8*(-6/5*d*x^4+c)*a*
b+a^2*d)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{(3\,b^2dx^8+3\,(5\,b^2c-8\,abd)x^4+20\,abc-32\,a^2d)(bx^4+a)^{\frac{1}{4}}}{15\,(b^4x^4+ab^3)}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*d*x^8 + 3*(5*b^2*c - 8*a*b*d)*x^4 + 20*a*b*c - 32*a^2*d)*(b*x^
4 + a)^(1/4)/(b^4*x^4 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \begin{cases} -\frac{32a^2d}{15b^3(a+bx^4)^{\frac{3}{4}}}+\frac{4ac}{3b^2(a+bx^4)^{\frac{3}{4}}}-\frac{8adx^4}{5b^2(a+bx^4)^{\frac{3}{4}}}+\frac{cx^4}{b(a+bx^4)^{\frac{3}{4}}}+\frac{dx^8}{5b(a+bx^4)^{\frac{3}{4}}} & \text{for } b\neq 0 \\ \frac{\frac{cx^8}{8}+\frac{dx^{12}}{12}}{a^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
Piecewise((-32*a**2*d/(15*b**3*(a + b*x**4)**(3/4)) + 4*a*c/(3*b**2*(a + b
*x**4)**(3/4)) - 8*a*d*x**4/(5*b**2*(a + b*x**4)**(3/4)) + c*x**4/(b*(a +
b*x**4)**(3/4)) + d*x**8/(5*b*(a + b*x**4)**(3/4)), Ne(b, 0)), ((c*x**8/8
+ d*x**12/12)/a**(7/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{1}{15}\,d\left(\frac{3\,(bx^4+a)^{\frac{5}{4}}}{b^3} - \frac{30\,(bx^4+a)^{\frac{1}{4}}a}{b^3} - \frac{5\,a^2}{(bx^4+a)^{\frac{3}{4}}b^3}\right) + \frac{1}{3}\,c\left(\frac{3\,(bx^4+a)^{\frac{1}{4}}}{b^2} + \frac{a}{(bx^4+a)^{\frac{3}{4}}b^2}\right)$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/15*d*(3*(b*x^4 + a)^(5/4)/b^3 - 30*(b*x^4 + a)^(1/4)*a/b^3 - 5*a^2/((b*x
^4 + a)^(3/4)*b^3)) + 1/3*c*(3*(b*x^4 + a)^(1/4)/b^2 + a/((b*x^4 + a)^(3/4
)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{abc-a^2d}{3\,(bx^4+a)^{\frac{3}{4}}b^3} + \frac{5\,(bx^4+a)^{\frac{1}{4}}b^{13}c + (bx^4+a)^{\frac{5}{4}}b^{12}d - 10\,(bx^4+a)^{\frac{1}{4}}ab^{12}d}{5\,b^{15}}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
1/3*(a*b*c - a^2*d)/((b*x^4 + a)^(3/4)*b^3) + 1/5*(5*(b*x^4 + a)^(1/4)*b^1
3*c + (b*x^4 + a)^(5/4)*b^12*d - 10*(b*x^4 + a)^(1/4)*a*b^12*d)/b^15
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{3\,d\,(b\,x^4+a)^2-5\,a^2\,d-30\,a\,d\,(b\,x^4+a)+15\,b\,c\,(b\,x^4+a)+5\,a\,b\,c}{15\,b^3\,(b\,x^4+a)^{3/4}}$$

input
```
int((x^7*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
(3*d*(a + b*x^4)^2 - 5*a^2*d - 30*a*d*(a + b*x^4) + 15*b*c*(a + b*x^4) + 5
*a*b*c)/(15*b^3*(a + b*x^4)^(3/4))
```

Reduce [F]

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^7}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**11/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(
x**7/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.127 $\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{-bc+ad}{3b^2(a+bx^4)^{3/4}}+\frac{d\sqrt[4]{a+bx^4}}{b^2}$$

output
```
1/3*(a*d-b*c)/b^2/(b*x^4+a)^(3/4)+d*(b*x^4+a)^(1/4)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{-bc+4ad+3bdx^4}{3b^2(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^3*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(-(b*c) + 4*a*d + 3*b*d*x^4)/(3*b^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^4)}{(a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \; 946$$

$$\frac{1}{4}\int \frac{dx^4+c}{(bx^4+a)^{7/4}}dx^4$$

$$\Big\downarrow \; 53$$

$$\frac{1}{4}\int \left(\frac{d}{b (bx^4+a)^{3/4}}+\frac{bc-ad}{b (bx^4+a)^{7/4}}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{4d\sqrt[4]{a+bx^4}}{b^2}-\frac{4(bc-ad)}{3b^2 (a+bx^4)^{3/4}}\right)$$

input
```
Int[(x^3*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((-4*(b*c - a*d))/(3*b^2*(a + b*x^4)^(3/4)) + (4*d*(a + b*x^4)^(1/4))/b^2)
/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{3db\,x^{4}+4ad-cb}{3\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}}$	31
trager	$\frac{3db\,x^{4}+4ad-cb}{3\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}}$	31
orering	$\frac{3db\,x^{4}+4ad-cb}{3\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}}$	31
pseudoelliptic	$\frac{\left(3d\,x^{4}-c\right)b+4ad}{3\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}}$	32
risch	$\frac{ad-cb}{3b^{2}\left(b\,x^{4}+a\right)^{\frac{3}{4}}}+\frac{d\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{2}}$	38

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*b*d*x^4+4*a*d-b*c)/(b*x^4+a)^(3/4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{(3\,bdx^4-bc+4\,ad)(bx^4+a)^{\frac{1}{4}}}{3\,(b^3x^4+ab^2)}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/3*(3*b*d*x^4 - b*c + 4*a*d)*(b*x^4 + a)^(1/4)/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \begin{cases} \frac{4ad}{3b^2(a+bx^4)^{\frac{3}{4}}} - \frac{c}{3b(a+bx^4)^{\frac{3}{4}}} + \frac{dx^4}{b(a+bx^4)^{\frac{3}{4}}} & \text{for } b \neq 0 \\ \frac{\frac{cx^4}{4}+\frac{dx^8}{8}}{a^{\frac{7}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
Piecewise((4*a*d/(3*b**2*(a + b*x**4)**(3/4)) - c/(3*b*(a + b*x**4)**(3/4)
) + d*x**4/(b*(a + b*x**4)**(3/4)), Ne(b, 0)), ((c*x**4/4 + d*x**8/8)/a**(
7/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{1}{3}\,d\left(\frac{3\,(bx^4+a)^{\frac{1}{4}}}{b^2} + \frac{a}{(bx^4+a)^{\frac{3}{4}}b^2}\right) - \frac{c}{3\,(bx^4+a)^{\frac{3}{4}}b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/3*d*(3*(b*x^4 + a)^(1/4)/b^2 + a/((b*x^4 + a)^(3/4)*b^2)) - 1/3*c/((b*x^
4 + a)^(3/4)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}d}{b^2} - \frac{bc-ad}{3\,(bx^4+a)^{\frac{3}{4}}b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
(b*x^4 + a)^(1/4)*d/b^2 - 1/3*(b*c - a*d)/((b*x^4 + a)^(3/4)*b^2)
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{3\,d\,(b\,x^4+a)+a\,d-b\,c}{3\,b^2\,(b\,x^4+a)^{3/4}}$$

input
```
int((x^3*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
(3*d*(a + b*x^4) + a*d - b*c)/(3*b^2*(a + b*x^4)^(3/4))
```

Reduce [F]

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^7}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^3}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**7/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(x
**3/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.128 $\int \frac{c+dx^4}{x(a+bx^4)^{7/4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{c+dx^4}{x(a+bx^4)^{7/4}} dx = \frac{bc-ad}{3ab(a+bx^4)^{3/4}} - \frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{7/4}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{7/4}}$$

output
```
1/3*(-a*d+b*c)/a/b/(b*x^4+a)^(3/4)-1/2*c*arctan((b*x^4+a)^(1/4)/a^(1/4))/a
^(7/4)-1/2*c*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^4}{x(a+bx^4)^{7/4}} dx = \frac{\frac{2a^{3/4}(bc-ad)}{b(a+bx^4)^{3/4}} - 3c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - 3c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{6a^{7/4}}$$

input
```
Integrate[(c + d*x^4)/(x*(a + b*x^4)^(7/4)),x]
```

output
```
((2*a^(3/4)*(b*c - a*d))/(b*(a + b*x^4)^(3/4)) - 3*c*ArcTan[(a + b*x^4)^(1
/4)/a^(1/4)] - 3*c*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(6*a^(7/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{7/4}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^4\left(bx^4+a\right)^{7/4}}dx^4$$

$$\Big\downarrow \ 87$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4}{a}+\frac{4(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(\frac{4c\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \ 756$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{ab}+\frac{4(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}+\frac{4(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{4c\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}+\frac{4(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}\right)$$

input
```
Int[(c + d*x^4)/(x*(a + b*x^4)^(7/4)),x]
```

output
```
((4*(b*c - a*d))/(3*a*b*(a + b*x^4)^(3/4)) + (4*c*(-1/2*(b*ArcTan[(a + b*x
^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^
(3/4))))/(a*b))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$\frac{-\frac{\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) b c}{4 a^{\frac{7}{4}}}-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b c}{2 a^{\frac{7}{4}}}-\frac{a d -c b}{3 a \left(b\,x^{4}+a\right)^{\frac{3}{4}}}}{b}$	88

input
```
int((d*x^4+c)/x/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*b/a^(7/4)*c-
1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))*b/a^(7/4)*c-1/3*(a*d-b*c)/a/(b*x^4+a)^
(3/4))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.90

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{7/4}}\,dx = -\frac{3\,(ab^2x^4+a^2b)\left(\frac{c^4}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(\frac{c^4}{a^7}\right)^{\frac{1}{4}}+(bx^4+a)^{\frac{1}{4}}c\right)+3\,(i\,ab^2x^4+i\,a^2b)\left(\frac{c^4}{a^7}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(\frac{c^4}{a^7}\right)^{\frac{1}{4}}+(bx^4+\right.}{}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
-1/12*(3*(a*b^2*x^4 + a^2*b)*(c^4/a^7)^(1/4)*log(a^2*(c^4/a^7)^(1/4) + (b*
x^4 + a)^(1/4)*c) + 3*(I*a*b^2*x^4 + I*a^2*b)*(c^4/a^7)^(1/4)*log(I*a^2*(c
^4/a^7)^(1/4) + (b*x^4 + a)^(1/4)*c) + 3*(-I*a*b^2*x^4 - I*a^2*b)*(c^4/a^7
)^(1/4)*log(-I*a^2*(c^4/a^7)^(1/4) + (b*x^4 + a)^(1/4)*c) - 3*(a*b^2*x^4 +
 a^2*b)*(c^4/a^7)^(1/4)*log(-a^2*(c^4/a^7)^(1/4) + (b*x^4 + a)^(1/4)*c) -
4*(b*x^4 + a)^(1/4)*(b*c - a*d))/(a*b^2*x^4 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 23.74 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{7/4}}\,dx = d\left(\begin{cases} -\frac{1}{3b(a+bx^4)^{\frac{3}{4}}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{7}{4}}} & \text{otherwise} \end{cases}\right) - \frac{c\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{7}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x**4+c)/x/(b*x**4+a)**(7/4),x)
```

output
```
d*Piecewise((-1/(3*b*(a + b*x**4)**(3/4)), Ne(b, 0)), (x**4/(4*a**(7/4)),
True)) - c*gamma(7/4)*hyper((7/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4
))/(4*b**(7/4)*x**7*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{7/4}}\,dx = -\frac{1}{12}\,c\left(\frac{3\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{a}-\frac{4}{\left(bx^4+a\right)^{\frac{3}{4}}a}\right)-\frac{d}{3\left(bx^4+a\right)^{\frac{3}{4}}b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
-1/12*c*(3*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - log(((b*x^4 + a)
^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4))/a - 4/((b*x^4 +
a)^(3/4)*a)) - 1/3*d/((b*x^4 + a)^(3/4)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(66) = 132$.

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.50

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{7/4}}\,dx = \frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}c\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,(-a)^{\frac{3}{4}}\,a} + \frac{\sqrt{2}(-a)^{\frac{1}{4}}\,c\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a^2} + \frac{bc-ad}{3\left(bx^4+a\right)^{\frac{3}{4}}ab}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(1/4)
)/(-a)^(1/4))/((-a)^(3/4)*a) + 1/4*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a) + 1/8*sqrt(2)
*c*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/
((-a)^(3/4)*a) + 1/8*sqrt(2)*(-a)^(1/4)*c*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(
-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 + 1/3*(b*c - a*d)/((b*x^4 + a)
^(3/4)*a*b)
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{7/4}}\,dx = \frac{c}{3\,a\,\left(b\,x^4+a\right)^{3/4}}-\frac{d}{3\,b\,\left(b\,x^4+a\right)^{3/4}}-\frac{c\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{7/4}}-\frac{c\,\mathrm{atanh}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{7/4}}$$

input
```
int((c + d*x^4)/(x*(a + b*x^4)^(7/4)),x)
```

output
```
c/(3*a*(a + b*x^4)^(3/4)) - d/(3*b*(a + b*x^4)^(3/4)) - (c*atan((a + b*x^4
)^(1/4)/a^(1/4)))/(2*a^(7/4)) - (c*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^
(7/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{7/4}}\,dx = \left(\int \frac{x^{3}}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} a+\left(b\,x^{4}+a\right)^{\frac{3}{4}} b\,x^{4}}d x\right) d+\left(\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} a x+\left(b\,x^{4}+a\right)^{\frac{3}{4}} b\,x^{5}}d x\right) c$$

input
```
int((d*x^4+c)/x/(b*x^4+a)^(7/4),x)
```

output
```
int(x**3/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(3/4)*a*x + (a + b*x**4)**(3/4)*b*x**5),x)*c
```

3.129 $\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{7/4}}\,dx = -\frac{bc-ad}{3a^2\left(a+bx^4\right)^{3/4}} - \frac{c\sqrt[4]{a+bx^4}}{4a^2x^4} + \frac{(7bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{11/4}} + \frac{(7bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{11/4}}$$

output
```
-1/3*(-a*d+b*c)/a^2/(b*x^4+a)^(3/4)-1/4*c*(b*x^4+a)^(1/4)/a^2/x^4+1/8*(-4*
a*d+7*b*c)*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)+1/8*(-4*a*d+7*b*c)*arc
tanh((b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{7/4}}\,dx = \frac{\frac{2a^{3/4}\left(-3ac-7bcx^4+4adx^4\right)}{x^4\left(a+bx^4\right)^{3/4}} + 3(7bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + 3(7bc-4ad)\operatorname{arctanh}\left(\right.}{24a^{11/4}}$$

input
```
Integrate[(c + d*x^4)/(x^5*(a + b*x^4)^(7/4)),x]
```

output
```
((2*a^(3/4)*(-3*a*c - 7*b*c*x^4 + 4*a*d*x^4))/(x^4*(a + b*x^4)^(3/4)) + 3*
(7*b*c - 4*a*d)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] + 3*(7*b*c - 4*a*d)*ArcT
anh[(a + b*x^4)^(1/4)/a^(1/4)])/(24*a^(11/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 87, 61, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{dx^4+c}{x^8 (bx^4+a)^{7/4}} dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4} \left(-\frac{(7bc-4ad)\int \frac{1}{x^4(bx^4+a)^{7/4}}dx^4}{4a} - \frac{c}{ax^4 (a+bx^4)^{3/4}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(-\frac{(7bc-4ad)\left(\frac{\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4}{a} + \frac{4}{3a(a+bx^4)^{3/4}}\right)}{4a} - \frac{c}{ax^4 (a+bx^4)^{3/4}} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{(7bc-4ad)\left(\frac{4\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \; 756$$

$$\frac{1}{4}\left(-\frac{(7bc-4ad)\left(\frac{4\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{ab}+\frac{4}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{4}\left(-\frac{(7bc-4ad)\left(\frac{4\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}+\frac{4}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{3/4}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(-\frac{(7bc-4ad)\left(\frac{4\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{ab}+\frac{4}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{3/4}}\right)$$

input
```
Int[(c + d*x^4)/(x^5*(a + b*x^4)^(7/4)),x]
```

output
```
(-(c/(a*x^4*(a + b*x^4)^(3/4))) - ((7*b*c - 4*a*d)*(4/(3*a*(a + b*x^4)^(3/
4)) + (4*(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[
(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/(a*b)))/(4*a))/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{\left(ad-\frac{7cb}{4}\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}x^{4}a^{2}\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)+2\left(ad-\frac{7cb}{4}\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}x^{4}a^{2}\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)+a^{\frac{11}{4}}\left(\left(-\frac{4d\,x^{4}}{3}+c\right.\right.}{4\left(b\,x^{4}+a\right)^{\frac{3}{4}}x^{4}a^{\frac{19}{4}}}$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(b*x^4+a)^(3/4)*((a*d-7/4*c*b)*(b*x^4+a)^(3/4)*x^4*a^2*ln(((b*x^4+a)^
(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))+2*(a*d-7/4*c*b)*(b*x^4+a)^(3/4)*
x^4*a^2*arctan((b*x^4+a)^(1/4)/a^(1/4))+a^(11/4)*((-4/3*d*x^4+c)*a+7/3*b*c
*x^4))/x^4/a^(19/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 689, normalized size of antiderivative = 5.69

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{7/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
-1/48*(3*(a^2*b*x^8 + a^3*x^4)*((2401*b^4*c^4 - 5488*a*b^3*c^3*d + 4704*a^
2*b^2*c^2*d^2 - 1792*a^3*b*c*d^3 + 256*a^4*d^4)/a^11)^(1/4)*log(a^3*((2401
*b^4*c^4 - 5488*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 1792*a^3*b*c*d^3 + 25
6*a^4*d^4)/a^11)^(1/4) - (b*x^4 + a)^(1/4)*(7*b*c - 4*a*d)) + 3*(I*a^2*b*x
^8 + I*a^3*x^4)*((2401*b^4*c^4 - 5488*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 -
 1792*a^3*b*c*d^3 + 256*a^4*d^4)/a^11)^(1/4)*log(I*a^3*((2401*b^4*c^4 - 54
88*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 1792*a^3*b*c*d^3 + 256*a^4*d^4)/a^
11)^(1/4) - (b*x^4 + a)^(1/4)*(7*b*c - 4*a*d)) + 3*(-I*a^2*b*x^8 - I*a^3*x
^4)*((2401*b^4*c^4 - 5488*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 1792*a^3*b*
c*d^3 + 256*a^4*d^4)/a^11)^(1/4)*log(-I*a^3*((2401*b^4*c^4 - 5488*a*b^3*c^
3*d + 4704*a^2*b^2*c^2*d^2 - 1792*a^3*b*c*d^3 + 256*a^4*d^4)/a^11)^(1/4) -
 (b*x^4 + a)^(1/4)*(7*b*c - 4*a*d)) - 3*(a^2*b*x^8 + a^3*x^4)*((2401*b^4*c
^4 - 5488*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 1792*a^3*b*c*d^3 + 256*a^4*
d^4)/a^11)^(1/4)*log(-a^3*((2401*b^4*c^4 - 5488*a*b^3*c^3*d + 4704*a^2*b^2
*c^2*d^2 - 1792*a^3*b*c*d^3 + 256*a^4*d^4)/a^11)^(1/4) - (b*x^4 + a)^(1/4)
*(7*b*c - 4*a*d)) + 4*((7*b*c - 4*a*d)*x^4 + 3*a*c)*(b*x^4 + a)^(1/4))/(a^
2*b*x^8 + a^3*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{7/4}}\,dx = -\frac{c\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{7}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)} - \frac{d\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{7}{4}}x^{7}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**5/(b*x**4+a)**(7/4),x)
```

output
```
-c*gamma(11/4)*hyper((7/4, 11/4), (15/4,), a*exp_polar(I*pi)/(b*x**4))/(4*
b**(7/4)*x**11*gamma(15/4)) - d*gamma(7/4)*hyper((7/4, 7/4), (11/4,), a*ex
p_polar(I*pi)/(b*x**4))/(4*b**(7/4)*x**7*gamma(11/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.62

$$\int \frac{c + dx^4}{x^5 (a + bx^4)^{7/4}} \, dx = -\frac{1}{48} c \left(\frac{4 (7 (bx^4 + a)b - 4 ab)}{(bx^4 + a)^{\frac{7}{4}} a^2 - (bx^4 + a)^{\frac{3}{4}} a^3} - \frac{21 \left(\frac{2 b \arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}} \right)}{a^{\frac{3}{4}}} - \frac{b \log\left(\frac{(bx^4+a)^{\frac{1}{4}} - a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}} + a^{\frac{1}{4}}} \right)}{a^{\frac{3}{4}}} \right)}{a^2} \right) - \frac{1}{12} d \left(\frac{3 \left(\frac{2 \arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}} \right)}{a^{\frac{3}{4}}} - \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}} - a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}} + a^{\frac{1}{4}}} \right)}{a^{\frac{3}{4}}} \right)}{a} - \frac{4}{(bx^4 + a)^{\frac{3}{4}} a} \right)$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
-1/48*c*(4*(7*(b*x^4 + a)*b - 4*a*b)/((b*x^4 + a)^(7/4)*a^2 - (b*x^4 + a)^
(3/4)*a^3) - 21*(2*b*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b
*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4))/a^2) -
1/12*d*(3*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4))/a - 4/((b*x^4 + a
)^(3/4)*a))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.17

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{7/4}}\,dx = -\frac{\sqrt{2}(7\,bc-4\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}(7\,bc-4\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{3}{4}}\,a^2} - \frac{\sqrt{2}(7\,bc-4\,ad)\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{3}{4}}\,a^2} + \frac{\sqrt{2}(7\,bc-4\,ad)\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{3}{4}}\,a^2} - \frac{bc-ad}{3\,(bx^4+a)^{\frac{3}{4}}a^2} - \frac{(bx^4+a)^{\frac{1}{4}}c}{4\,a^2x^4}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(7*b*c - 4*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^2) - 1/16*sqrt(2)*(7*b*c - 4*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1
/4))/((-a)^(3/4)*a^2) - 1/32*sqrt(2)*(7*b*c - 4*a*d)*log(sqrt(2)*(b*x^4 +
a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a^2) + 1/32*
sqrt(2)*(7*b*c - 4*a*d)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b
*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a^2) - 1/3*(b*c - a*d)/((b*x^4 + a)^(3/4
)*a^2) - 1/4*(b*x^4 + a)^(1/4)*c/(a^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{7/4}}\,dx = \frac{d}{3a(bx^4+a)^{3/4}} - \frac{\frac{bc}{3a} - \frac{7bc(bx^4+a)}{12a^2}}{a(bx^4+a)^{3/4} - (bx^4+a)^{7/4}} - \frac{d\,\mathrm{atan}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{2a^{7/4}} - \frac{d\,\mathrm{atanh}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{2a^{7/4}} + \frac{7bc\,\mathrm{atan}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{8a^{11/4}} + \frac{7bc\,\mathrm{atanh}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{8a^{11/4}}$$

input
```
int((c + d*x^4)/(x^5*(a + b*x^4)^(7/4)),x)
```

output
```
d/(3*a*(a + b*x^4)^(3/4)) - ((b*c)/(3*a) - (7*b*c*(a + b*x^4))/(12*a^2))/(
a*(a + b*x^4)^(3/4) - (a + b*x^4)^(7/4)) - (d*atan((a + b*x^4)^(1/4)/a^(1/
4)))/(2*a^(7/4)) - (d*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(7/4)) + (7*b
*c*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(11/4)) + (7*b*c*atanh((a + b*x^4
)^(1/4)/a^(1/4)))/(8*a^(11/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{7/4}}\,dx = \left(\int \frac{1}{(bx^4+a)^{\frac{3}{4}}ax^5 + (bx^4+a)^{\frac{3}{4}}bx^9}dx\right)c + \left(\int \frac{1}{(bx^4+a)^{\frac{3}{4}}ax + (bx^4+a)^{\frac{3}{4}}bx^5}dx\right)d$$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**5 + (a + b*x**4)**(3/4)*b*x**9),x)*c + int
(1/((a + b*x**4)**(3/4)*a*x + (a + b*x**4)**(3/4)*b*x**5),x)*d
```

3.130 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = -\frac{(bc-ad)x^3}{3b^2(a+bx^4)^{3/4}}+\frac{dx^3\sqrt[4]{a+bx^4}}{4b^2} - \frac{(4bc-7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{11/4}}+\frac{(4bc-7ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{11/4}}$$

output
```
-1/3*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(3/4)+1/4*d*x^3*(b*x^4+a)^(1/4)/b^2-1/8*
(-7*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(11/4)+1/8*(-7*a*d+4*b*
c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{\frac{2b^{3/4}x^3(-4bc+7ad+3bdx^4)}{(a+bx^4)^{3/4}}+3(-4bc+7ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+3(4bc-7ad)\text{arctanh}\left(\right.}{24b^{11/4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((2*b^(3/4)*x^3*(-4*b*c + 7*a*d + 3*b*d*x^4))/(a + b*x^4)^(3/4) + 3*(-4*b*
c + 7*a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 3*(4*b*c - 7*a*d)*ArcTa
nh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(24*b^(11/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{x^7(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(4bc-7ad)\int \frac{x^6}{(bx^4+a)^{3/4}}dx}{3ab}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^7(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(4bc-7ad)\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}\right)}{3ab}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{x^7(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(4bc-7ad)\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{3ab}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{\frac{x^7(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}\right)}{3ab}}{}$$

$\Big\downarrow$ 216

$$\frac{x^7(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{3ab}$$

$\Big\downarrow$ 219

$$\frac{x^7(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{3ab}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^7)/(3*a*b*(a + b*x^4)^(3/4)) - ((4*b*c - 7*a*d)*((x^3*(a +
b*x^4)^(1/4))/(4*b) - (3*a*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(
3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b)))/(3*a*b
)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{\frac{21\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}\left(a d-\frac{4 c b}{7}\right)\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{16}+\frac{21\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{2}\left(a d-\frac{4 c b}{7}\right)\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)}{8}+b^{\frac{11}{4}}\left(\left(-\frac{3 d\,x^{4}}{4}+c\right)b-\right.}{3\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{19}{4}}}$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x^4+a)^(3/4)*(21/16*(b*x^4+a)^(3/4)*b^2*(a*d-4/7*c*b)*ln((x*b^(1/4
)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))+21/8*(b*x^4+a)^(3/4)*b^2*
(a*d-4/7*c*b)*arctan((b*x^4+a)^(1/4)/x/b^(1/4))+b^(11/4)*((-3/4*d*x^4+c)*b
-7/4*a*d)*x^3)/b^(19/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.53

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/48*(3*(b^3*x^4 + a*b^2)*((256*b^4*c^4 - 1792*a*b^3*c^3*d + 4704*a^2*b^2*
c^2*d^2 - 5488*a^3*b*c*d^3 + 2401*a^4*d^4)/b^11)^(1/4)*log(-(b^3*x*((256*b
^4*c^4 - 1792*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 5488*a^3*b*c*d^3 + 2401
*a^4*d^4)/b^11)^(1/4) + (b*x^4 + a)^(1/4)*(4*b*c - 7*a*d))/x) - 3*(b^3*x^4
 + a*b^2)*((256*b^4*c^4 - 1792*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 5488*a
^3*b*c*d^3 + 2401*a^4*d^4)/b^11)^(1/4)*log((b^3*x*((256*b^4*c^4 - 1792*a*b
^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 5488*a^3*b*c*d^3 + 2401*a^4*d^4)/b^11)^(
1/4) - (b*x^4 + a)^(1/4)*(4*b*c - 7*a*d))/x) - 3*(I*b^3*x^4 + I*a*b^2)*((2
56*b^4*c^4 - 1792*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 5488*a^3*b*c*d^3 +
2401*a^4*d^4)/b^11)^(1/4)*log((I*b^3*x*((256*b^4*c^4 - 1792*a*b^3*c^3*d +
4704*a^2*b^2*c^2*d^2 - 5488*a^3*b*c*d^3 + 2401*a^4*d^4)/b^11)^(1/4) - (b*x
^4 + a)^(1/4)*(4*b*c - 7*a*d))/x) - 3*(-I*b^3*x^4 - I*a*b^2)*((256*b^4*c^4
 - 1792*a*b^3*c^3*d + 4704*a^2*b^2*c^2*d^2 - 5488*a^3*b*c*d^3 + 2401*a^4*d
^4)/b^11)^(1/4)*log((-I*b^3*x*((256*b^4*c^4 - 1792*a*b^3*c^3*d + 4704*a^2*
b^2*c^2*d^2 - 5488*a^3*b*c*d^3 + 2401*a^4*d^4)/b^11)^(1/4) - (b*x^4 + a)^(
1/4)*(4*b*c - 7*a*d))/x) + 4*(3*b*d*x^7 - (4*b*c - 7*a*d)*x^3)*(b*x^4 + a)
^(1/4))/(b^3*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**7*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(7/4)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((7/4, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(7/4)*gamma(15/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(102) = 204$.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.83

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = -\frac{1}{12}\left(\frac{4\,x^3}{(bx^4+a)^{\frac{3}{4}}b} - \frac{3\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{b}\right)c + \frac{1}{48}\,d\left(\frac{4\left(4\,ab - \frac{7\,(bx^4+a)a}{x^4}\right)}{\frac{(bx^4+a)^{\frac{3}{4}}b^3}{x^3} - \frac{(bx^4+a)^{\frac{7}{4}}b^2}{x^7}} - \frac{21\left(\frac{2\,a\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{b^2}\right)$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
-1/12*(4*x^3/((b*x^4 + a)^(3/4)*b) - 3*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4
)*x))/b^(3/4) - log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a
)^(1/4)/x))/b^(3/4))/b)*c + 1/48*d*(4*(4*a*b - 7*(b*x^4 + a)*a/x^4)/((b*x^
4 + a)^(3/4)*b^3/x^3 - (b*x^4 + a)^(7/4)*b^2/x^7) - 21*(2*a*arctan((b*x^4
+ a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(
b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b^2)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**10/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(
x**6/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.131 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{(bc-ad)x^3}{3ab\,(a+bx^4)^{3/4}} - \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{7/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{7/4}}$$

output
```
1/3*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(3/4)-1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1
/4))/b^(7/4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{\frac{2b^{3/4}(bc-ad)x^3}{a(a+bx^4)^{3/4}} - 3d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + 3d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{6b^{7/4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((2*b^(3/4)*(b*c - a*d)*x^3)/(a*(a + b*x^4)^(3/4)) - 3*d*ArcTan[(b^(1/4)*x
)/(a + b*x^4)^(1/4)] + 3*d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(6*b^(7
/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {954, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^4)}{(a+bx^4)^{7/4}} dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{b}+\frac{x^3(bc-ad)}{3ab(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{854}$$

$$\frac{d\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}+\frac{x^3(bc-ad)}{3ab(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{d\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{b}+\frac{x^3(bc-ad)}{3ab(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{d\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{b}+\frac{x^3(bc-ad)}{3ab(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{b}+\frac{x^3(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^3)/(3*a*b*(a + b*x^4)^(3/4)) + (d*(-1/2*ArcTan[(b^(1/4)*x)/
(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(
3/4))))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{\frac{\ln\left(\frac{x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}{-x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}\right) a d}{4 b^{\frac{7}{4}}}+\frac{\arctan\left(\frac{\left(b x^{4}+a\right)^{\frac{1}{4}}}{x b^{\frac{1}{4}}}\right) a d}{2 b^{\frac{7}{4}}}-\frac{(a d-c b) x^{3}}{3 b \left(b x^{4}+a\right)^{\frac{3}{4}}}}{a}$	97

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
(1/4*ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))*a/b^(7/4
)*d+1/2*arctan((b*x^4+a)^(1/4)/x/b^(1/4))*a/b^(7/4)*d-1/3*(a*d-b*c)*x^3/b/
(b*x^4+a)^(3/4))/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.00

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{4\,(bx^4+a)^{\frac{1}{4}}(bc-ad)x^3+3\,(ab^2x^4+a^2b)\left(\frac{d^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{b^2x\left(\frac{d^4}{b^7}\right)^{\frac{1}{4}}+(bx^4+a)^{\frac{1}{4}}d}{x}\right)-3\,(ab^2x^4}{}$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/12*(4*(b*x^4 + a)^(1/4)*(b*c - a*d)*x^3 + 3*(a*b^2*x^4 + a^2*b)*(d^4/b^7
)^(1/4)*log((b^2*x*(d^4/b^7)^(1/4) + (b*x^4 + a)^(1/4)*d)/x) - 3*(a*b^2*x^
4 + a^2*b)*(d^4/b^7)^(1/4)*log(-(b^2*x*(d^4/b^7)^(1/4) - (b*x^4 + a)^(1/4)
*d)/x) - 3*(-I*a*b^2*x^4 - I*a^2*b)*(d^4/b^7)^(1/4)*log((I*b^2*x*(d^4/b^7)
^(1/4) + (b*x^4 + a)^(1/4)*d)/x) - 3*(I*a*b^2*x^4 + I*a^2*b)*(d^4/b^7)^(1/
4)*log((-I*b^2*x*(d^4/b^7)^(1/4) + (b*x^4 + a)^(1/4)*d)/x))/(a*b^2*x^4 + a
^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right)}{4a^{\frac{7}{4}}\left(1+\frac{bx^4}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**3*gamma(3/4)/(4*a**(7/4)*(1 + b*x**4/a)**(3/4)*gamma(7/4)) + d*x**7*g
amma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(7/4)
*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.24

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^3}{3\,(bx^4+a)^{\frac{3}{4}}a} - \frac{1}{12}\left(\frac{4\,x^3}{(bx^4+a)^{\frac{3}{4}}b} - \frac{3\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{b}\right)d$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/3*c*x^3/((b*x^4 + a)^(3/4)*a) - 1/12*(4*x^3/((b*x^4 + a)^(3/4)*b) - 3*(2
*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - log(-(b^(1/4) - (b*x^4 +
a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b)*d
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**6/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(x
**2/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.132 $\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}} dx = -\frac{c}{ax(a+bx^4)^{3/4}} - \frac{(4bc-ad)x^3}{3a^2(a+bx^4)^{3/4}}$$

output
```
-c/a/x/(b*x^4+a)^(3/4)-1/3*(-a*d+4*b*c)*x^3/a^2/(b*x^4+a)^(3/4)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}} dx = \frac{-3ac-4bcx^4+adx^4}{3a^2x(a+bx^4)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(7/4)),x]
```

output
```
(-3*a*c - 4*b*c*x^4 + a*d*x^4)/(3*a^2*x*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(4bc-ad)\int \frac{x^2}{(bx^4+a)^{7/4}}dx}{a}-\frac{c}{ax (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{x^3(4bc-ad)}{3a^2 (a+bx^4)^{3/4}}-\frac{c}{ax (a+bx^4)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(7/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(3/4))) - ((4*b*c - a*d)*x^3)/(3*a^2*(a + b*x^4)^(3/4
))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{\left(-\frac{d\,x^{4}}{3}+c\right)a+\frac{4bc\,x^{4}}{3}}{\left(b\,x^{4}+a\right)^{\frac{3}{4}}x\,a^{2}}$	36
gosper	$-\frac{-ad\,x^{4}+4bc\,x^{4}+3ac}{3x\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{2}}$	37
trager	$-\frac{-ad\,x^{4}+4bc\,x^{4}+3ac}{3x\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{2}}$	37
orering	$-\frac{-ad\,x^{4}+4bc\,x^{4}+3ac}{3x\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{2}}$	37
risch	$-\frac{c\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{2}x}+\frac{\left(ad-cb\right)x^{3}}{3a^{2}\left(b\,x^{4}+a\right)^{\frac{3}{4}}}$	45

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^4+a)^(3/4)*((-1/3*d*x^4+c)*a+4/3*b*c*x^4)/x/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/4}}\,dx = -\frac{((4\,bc-ad)x^4+3\,ac)(bx^4+a)^{\frac{1}{4}}}{3\,(a^2bx^5+a^3x)}$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
-1/3*((4*b*c - a*d)*x^4 + 3*a*c)*(b*x^4 + a)^(1/4)/(a^2*b*x^5 + a^3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(44) = 88.

Time = 11.75 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.06

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{7/4}}\,dx = c\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{16ab^{\frac{3}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\Gamma\left(-\frac{1}{4}\right)}{4a^2\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right) + \frac{dx^3\Gamma\left(\frac{3}{4}\right)}{4a^{\frac{7}{4}}\left(1+\frac{bx^4}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(7/4),x)
```

output
```
c*(3*gamma(-1/4)/(16*a*b**(3/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(7/4)) +
 b**(1/4)*gamma(-1/4)/(4*a**2*(a/(b*x**4) + 1)**(3/4)*gamma(7/4))) + d*x**
3*gamma(3/4)/(4*a**(7/4)*(1 + b*x**4/a)**(3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}}\,dx = \frac{dx^3}{3(bx^4+a)^{\frac{3}{4}}a} - \frac{1}{3}\left(\frac{bx^3}{(bx^4+a)^{\frac{3}{4}}a^2} + \frac{3(bx^4+a)^{\frac{1}{4}}}{a^2x}\right)c$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/3*d*x^3/((b*x^4 + a)^(3/4)*a) - 1/3*(b*x^3/((b*x^4 + a)^(3/4)*a^2) + 3*(
b*x^4 + a)^(1/4)/(a^2*x))*c
```

Giac [F]

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{7/4}}\,dx = -\frac{(b\,x^4+a)^{1/4}\left(\frac{c}{a} - x^4\left(\frac{d}{3\,a} - \frac{4\,b\,c}{3\,a^2}\right)\right)}{b\,x^5+a\,x}$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(7/4)),x)
```

output
```
-((a + b*x^4)^(1/4)*(c/a - x^4*(d/(3*a) - (4*b*c)/(3*a^2))))/(a*x + b*x^5)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^2+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^6}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(7/4),x)
```

output
```
int(x**2/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(3/4)*a*x**2 + (a + b*x**4)**(3/4)*b*x**6),x)*c
```

3.133 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/4}}\,dx=-\frac{c}{5ax^5\left(a+bx^4\right)^{3/4}}-\frac{8bc-5ad}{15a^2x\left(a+bx^4\right)^{3/4}}+\frac{4(8bc-5ad)\sqrt[4]{a+bx^4}}{15a^3x}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(3/4)-1/15*(-5*a*d+8*b*c)/a^2/x/(b*x^4+a)^(3/4)+4/1
5*(-5*a*d+8*b*c)*(b*x^4+a)^(1/4)/a^3/x
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/4}}\,dx=\frac{-3a^2c+24abcx^4-15a^2dx^4+32b^2cx^8-20abdx^8}{15a^3x^5\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(7/4)),x]
```

output
```
(-3*a^2*c + 24*a*b*c*x^4 - 15*a^2*d*x^4 + 32*b^2*c*x^8 - 20*a*b*d*x^8)/(15
*a^3*x^5*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/4}} dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(8bc-5ad)\int \frac{1}{x^2(bx^4+a)^{7/4}}dx}{5a} - \frac{c}{5ax^5 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(8bc-5ad)\left(-\frac{4b\int \frac{x^2}{(bx^4+a)^{7/4}}dx}{a} - \frac{1}{ax(a+bx^4)^{3/4}}\right)}{5a} - \frac{c}{5ax^5 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{4bx^3}{3a^2(a+bx^4)^{3/4}} - \frac{1}{ax(a+bx^4)^{3/4}}\right)(8bc-5ad)}{5a} - \frac{c}{5ax^5 (a+bx^4)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(7/4)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(3/4)) - ((8*b*c - 5*a*d)*(-(1/(a*x*(a + b*x^4)^
(3/4))) - (4*b*x^3)/(3*a^2*(a + b*x^4)^(3/4))))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{\left(-15 d \,x^{4}-3 c \right) a^{2}+24 b \left(-\frac{5 d \,x^{4}}{6}+c \right) x^{4} a +32 b^{2} c \,x^{8}}{15 \left(b \,x^{4}+a \right)^{\frac{3}{4}} x^{5} a^{3}}$	57
gosper	$-\frac{20 a b d \,x^{8}-32 b^{2} c \,x^{8}+15 a^{2} d \,x^{4}-24 a b c \,x^{4}+3 a^{2} c}{15 x^{5} \left(b \,x^{4}+a \right)^{\frac{3}{4}} a^{3}}$	59
trager	$-\frac{20 a b d \,x^{8}-32 b^{2} c \,x^{8}+15 a^{2} d \,x^{4}-24 a b c \,x^{4}+3 a^{2} c}{15 x^{5} \left(b \,x^{4}+a \right)^{\frac{3}{4}} a^{3}}$	59
orering	$-\frac{20 a b d \,x^{8}-32 b^{2} c \,x^{8}+15 a^{2} d \,x^{4}-24 a b c \,x^{4}+3 a^{2} c}{15 x^{5} \left(b \,x^{4}+a \right)^{\frac{3}{4}} a^{3}}$	59
risch	$-\frac{\left(b \,x^{4}+a \right)^{\frac{1}{4}} \left(5 a d \,x^{4}-9 b c \,x^{4}+a c \right)}{5 a^{3} x^{5}}-\frac{b \left(a d -c b \right) x^{3}}{3 a^{3} \left(b \,x^{4}+a \right)^{\frac{3}{4}}}$	63

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-15*d*x^4-3*c)*a^2+24*b*(-5/6*d*x^4+c)*x^4*a+32*b^2*c*x^8)/(b*x^4+a
)^(3/4)/x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/4}}\,dx = \frac{(4\,(8\,b^2c-5\,abd)x^8+3\,(8\,abc-5\,a^2d)x^4-3\,a^2c)(bx^4+a)^{\frac{1}{4}}}{15\,(a^3bx^9+a^4x^5)}$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
1/15*(4*(8*b^2*c - 5*a*b*d)*x^8 + 3*(8*a*b*c - 5*a^2*d)*x^4 - 3*a^2*c)*(b*
x^4 + a)^(1/4)/(a^3*b*x^9 + a^4*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(73) = 146.

Time = 27.98 (sec) , antiderivative size = 398, normalized size of antiderivative = 4.80

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{7/4}}\,dx = c\left(-\frac{3a^3b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{7}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{7}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{7}{4}\right)} + \frac{21a^2b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{7}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{7}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{7}{4}\right)} + \frac{56ab^{\frac{25}{4}}x^8\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{7}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{7}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{7}{4}\right)} + \frac{32b^{\frac{29}{4}}x^{12}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{7}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{7}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{7}{4}\right)}\right) + d\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{16ab^{\frac{3}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{\sqrt[4]{b}\Gamma\left(-\frac{1}{4}\right)}{4a^2\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(7/4),x)
```

output
```
c*(-3*a**3*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(64*a**5*b**4*x**
4*gamma(7/4) + 128*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b**6*x**12*gamma(7/
4)) + 21*a**2*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(64*a**5*
b**4*x**4*gamma(7/4) + 128*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b**6*x**12*
gamma(7/4)) + 56*a*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(64*
a**5*b**4*x**4*gamma(7/4) + 128*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b**6*x
**12*gamma(7/4)) + 32*b**(29/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/
(64*a**5*b**4*x**4*gamma(7/4) + 128*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b*
*6*x**12*gamma(7/4))) + d*(3*gamma(-1/4)/(16*a*b**(3/4)*x**4*(a/(b*x**4) +
 1)**(3/4)*gamma(7/4)) + b**(1/4)*gamma(-1/4)/(4*a**2*(a/(b*x**4) + 1)**(3
/4)*gamma(7/4)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/4}}\,dx = \frac{1}{15}\left(\frac{5\,b^2x^3}{(bx^4+a)^{\frac{3}{4}}a^3} + \frac{3\left(\frac{10\,(bx^4+a)^{\frac{1}{4}}b}{x} - \frac{(bx^4+a)^{\frac{5}{4}}}{x^5}\right)}{a^3}\right)c - \frac{1}{3}\left(\frac{bx^3}{(bx^4+a)^{\frac{3}{4}}a^2} + \frac{3\,(bx^4+a)^{\frac{1}{4}}}{a^2x}\right)d$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
1/15*(5*b^2*x^3/((b*x^4 + a)^(3/4)*a^3) + 3*(10*(b*x^4 + a)^(1/4)*b/x - (b
*x^4 + a)^(5/4)/x^5)/a^3)*c - 1/3*(b*x^3/((b*x^4 + a)^(3/4)*a^2) + 3*(b*x^
4 + a)^(1/4)/(a^2*x))*d
```

Giac [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/4}}\,dx = -\frac{15\,d\,a^2\,x^4+3\,c\,a^2+20\,d\,a\,b\,x^8-24\,c\,a\,b\,x^4-32\,c\,b^2\,x^8}{15\,a^3\,x^5\,(b\,x^4+a)^{3/4}}$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(7/4)),x)
```

output
```
-(3*a^2*c + 15*a^2*d*x^4 - 32*b^2*c*x^8 - 24*a*b*c*x^4 + 20*a*b*d*x^8)/(15
*a^3*x^5*(a + b*x^4)^(3/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{7/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^6+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^{10}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^2+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^6}dx\right)d$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**6 + (a + b*x**4)**(3/4)*b*x**10),x)*c + in
t(1/((a + b*x**4)**(3/4)*a*x**2 + (a + b*x**4)**(3/4)*b*x**6),x)*d
```

3.134 $\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx=-\frac{c}{9ax^9\left(a+bx^4\right)^{3/4}}-\frac{4bc-3ad}{9a^2x^5\left(a+bx^4\right)^{3/4}}+\frac{8(4bc-3ad)\sqrt[4]{a+bx^4}}{45a^3x^5}-\frac{32b(4bc-3ad)\sqrt[4]{a+bx^4}}{45a^4x}$$

output
```
-1/9*c/a/x^9/(b*x^4+a)^(3/4)-1/9*(-3*a*d+4*b*c)/a^2/x^5/(b*x^4+a)^(3/4)+8/
45*(-3*a*d+4*b*c)*(b*x^4+a)^(1/4)/a^3/x^5-32/45*b*(-3*a*d+4*b*c)*(b*x^4+a)
^(1/4)/a^4/x
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx=\frac{-128b^3cx^{12}+96ab^2x^8(-c+dx^4)+12a^2bx^4(c+6dx^4)-a^3(5c+9dx^4)}{45a^4x^9\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^10*(a + b*x^4)^(7/4)),x]
```

output
```
(-128*b^3*c*x^12 + 96*a*b^2*x^8*(-c + d*x^4) + 12*a^2*b*x^4*(c + 6*d*x^4)
- a^3*(5*c + 9*d*x^4))/(45*a^4*x^9*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4bc-3ad)\int \frac{1}{x^6(bx^4+a)^{7/4}}dx}{3a}-\frac{c}{9ax^9\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-3ad)\left(-\frac{8b\int \frac{1}{x^2\left(bx^4+a\right)^{7/4}}dx}{5a}-\frac{1}{5ax^5(a+bx^4)^{3/4}}\right)}{3a}-\frac{c}{9ax^9\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-3ad)\left(-\frac{8b\left(-\frac{4b\int \frac{x^2}{\left(bx^4+a\right)^{7/4}}dx}{a}-\frac{1}{ax\left(a+bx^4\right)^{3/4}}\right)}{5a}-\frac{1}{5ax^5(a+bx^4)^{3/4}}\right)}{3a}-\frac{c}{9ax^9\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{8b\left(-\frac{4bx^3}{3a^2\left(a+bx^4\right)^{3/4}}-\frac{1}{ax\left(a+bx^4\right)^{3/4}}\right)}{5a}-\frac{1}{5ax^5(a+bx^4)^{3/4}}\right)(4bc-3ad)}{3a}-\frac{c}{9ax^9\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^10*(a + b*x^4)^(7/4)),x]
```

output
```
-1/9*c/(a*x^9*(a + b*x^4)^(3/4)) - ((4*b*c - 3*a*d)*(-1/5*1/(a*x^5*(a + b*
x^4)^(3/4)) - (8*b*(-(1/(a*x*(a + b*x^4)^(3/4))) - (4*b*x^3)/(3*a^2*(a + b
*x^4)^(3/4))))/(5*a)))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\left(-9d\,x^{4}-5c\right)a^{3}+12b\,x^{4}\left(6d\,x^{4}+c\right)a^{2}-96b^{2}x^{8}\left(-d\,x^{4}+c\right)a-128b^{3}c\,x^{12}}{45\left(b\,x^{4}+a\right)^{\frac{3}{4}}x^{9}a^{4}}$	76
gosper	$-\frac{-96a\,b^{2}d\,x^{12}+128b^{3}c\,x^{12}-72a^{2}bd\,x^{8}+96a\,b^{2}c\,x^{8}+9a^{3}d\,x^{4}-12a^{2}bc\,x^{4}+5c\,a^{3}}{45x^{9}\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{4}}$	83
trager	$-\frac{-96a\,b^{2}d\,x^{12}+128b^{3}c\,x^{12}-72a^{2}bd\,x^{8}+96a\,b^{2}c\,x^{8}+9a^{3}d\,x^{4}-12a^{2}bc\,x^{4}+5c\,a^{3}}{45x^{9}\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{4}}$	83
orering	$-\frac{-96a\,b^{2}d\,x^{12}+128b^{3}c\,x^{12}-72a^{2}bd\,x^{8}+96a\,b^{2}c\,x^{8}+9a^{3}d\,x^{4}-12a^{2}bc\,x^{4}+5c\,a^{3}}{45x^{9}\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{4}}$	83
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(-81abd\,x^{8}+113b^{2}c\,x^{8}+9a^{2}d\,x^{4}-17abc\,x^{4}+5a^{2}c\right)}{45a^{4}x^{9}}+\frac{b^{2}\left(ad-cb\right)x^{3}}{3a^{4}\left(b\,x^{4}+a\right)^{\frac{3}{4}}}$	88

input
```
int((d*x^4+c)/x^10/(b*x^4+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
1/45*((-9*d*x^4-5*c)*a^3+12*b*x^4*(6*d*x^4+c)*a^2-96*b^2*x^8*(-d*x^4+c)*a-
128*b^3*c*x^12)/(b*x^4+a)^(3/4)/x^9/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx =$$
$$-\frac{\left(32\left(4\,b^3c-3\,ab^2d\right)x^{12}+24\left(4\,ab^2c-3\,a^2bd\right)x^8-3\left(4\,a^2bc-3\,a^3d\right)x^4+5\,a^3c\right)\left(bx^4+a\right)^{\frac{1}{4}}}{45\left(a^4bx^{13}+a^5x^9\right)}$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
-1/45*(32*(4*b^3*c - 3*a*b^2*d)*x^12 + 24*(4*a*b^2*c - 3*a^2*b*d)*x^8 - 3*
(4*a^2*b*c - 3*a^3*d)*x^4 + 5*a^3*c)*(b*x^4 + a)^(1/4)/(a^4*b*x^13 + a^5*x
^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(107) = 214$.

Time = 68.28 (sec) , antiderivative size = 920, normalized size of antiderivative = 8.07

$$\int \frac{c + dx^4}{x^{10} (a + bx^4)^{7/4}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**10/(b*x**4+a)**(7/4),x)
```

output
```
c*(15*a**5*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(256*a**7*b**9*x*
*8*gamma(7/4) + 768*a**6*b**10*x**12*gamma(7/4) + 768*a**5*b**11*x**16*gam
ma(7/4) + 256*a**4*b**12*x**20*gamma(7/4)) - 6*a**4*b**(41/4)*x**4*(a/(b*x
**4) + 1)**(1/4)*gamma(-9/4)/(256*a**7*b**9*x**8*gamma(7/4) + 768*a**6*b**
10*x**12*gamma(7/4) + 768*a**5*b**11*x**16*gamma(7/4) + 256*a**4*b**12*x**
20*gamma(7/4)) + 231*a**3*b**(45/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-9/
4)/(256*a**7*b**9*x**8*gamma(7/4) + 768*a**6*b**10*x**12*gamma(7/4) + 768*
a**5*b**11*x**16*gamma(7/4) + 256*a**4*b**12*x**20*gamma(7/4)) + 924*a**2*
b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(256*a**7*b**9*x**8*ga
mma(7/4) + 768*a**6*b**10*x**12*gamma(7/4) + 768*a**5*b**11*x**16*gamma(7/
4) + 256*a**4*b**12*x**20*gamma(7/4)) + 1056*a*b**(53/4)*x**16*(a/(b*x**4)
 + 1)**(1/4)*gamma(-9/4)/(256*a**7*b**9*x**8*gamma(7/4) + 768*a**6*b**10*x
**12*gamma(7/4) + 768*a**5*b**11*x**16*gamma(7/4) + 256*a**4*b**12*x**20*g
amma(7/4)) + 384*b**(57/4)*x**20*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(256*
a**7*b**9*x**8*gamma(7/4) + 768*a**6*b**10*x**12*gamma(7/4) + 768*a**5*b**
11*x**16*gamma(7/4) + 256*a**4*b**12*x**20*gamma(7/4))) + d*(-3*a**3*b**(1
7/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(64*a**5*b**4*x**4*gamma(7/4) + 1
28*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b**6*x**12*gamma(7/4)) + 21*a**2*b*
*(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(64*a**5*b**4*x**4*gamma(
7/4) + 128*a**4*b**5*x**8*gamma(7/4) + 64*a**3*b**6*x**12*gamma(7/4)) +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx = -\frac{1}{45}\left(\frac{15\,b^3x^3}{\left(bx^4+a\right)^{\frac{3}{4}}a^4}+\frac{\frac{135\,\left(bx^4+a\right)^{\frac{1}{4}}b^2}{x}-\frac{27\,\left(bx^4+a\right)^{\frac{5}{4}}b}{x^5}+\frac{5\,\left(bx^4+a\right)^{\frac{9}{4}}}{x^9}}{a^4}\right)c + \frac{1}{15}\left(\frac{5\,b^2x^3}{\left(bx^4+a\right)^{\frac{3}{4}}a^3}+\frac{3\left(\frac{10\,\left(bx^4+a\right)^{\frac{1}{4}}b}{x}-\frac{\left(bx^4+a\right)^{\frac{5}{4}}}{x^5}\right)}{a^3}\right)d$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
-1/45*(15*b^3*x^3/((b*x^4 + a)^(3/4)*a^4) + (135*(b*x^4 + a)^(1/4)*b^2/x -
 27*(b*x^4 + a)^(5/4)*b/x^5 + 5*(b*x^4 + a)^(9/4)/x^9)/a^4)*c + 1/15*(5*b^
2*x^3/((b*x^4 + a)^(3/4)*a^3) + 3*(10*(b*x^4 + a)^(1/4)*b/x - (b*x^4 + a)^
(5/4)/x^5)/a^3)*d
```

Giac [F]

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^{10}}\,dx$$

input
```
integrate((d*x^4+c)/x^10/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^10), x)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx = -\frac{9\,d\,a^3\,x^4+5\,c\,a^3-72\,d\,a^2\,b\,x^8-12\,c\,a^2\,b\,x^4-96\,d\,a\,b^2\,x^{12}+96\,c\,a\,b^2\,x^8+128\,c\,b^3\,x^{12}}{45\,a^4\,x^9\,\left(b\,x^4+a\right)^{3/4}}$$

input
```
int((c + d*x^4)/(x^10*(a + b*x^4)^(7/4)),x)
```

output
```
-(5*a^3*c + 9*a^3*d*x^4 + 128*b^3*c*x^12 - 12*a^2*b*c*x^4 + 96*a*b^2*c*x^8
 - 72*a^2*b*d*x^8 - 96*a*b^2*d*x^12)/(45*a^4*x^9*(a + b*x^4)^(3/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^{10}\left(a+bx^4\right)^{7/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}a\,x^{10}+\left(b\,x^4+a\right)^{\frac{3}{4}}b\,x^{14}}dx\right)c + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}a\,x^{6}+\left(b\,x^4+a\right)^{\frac{3}{4}}b\,x^{10}}dx\right)d$$

input
```
int((d*x^4+c)/x^10/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**10 + (a + b*x**4)**(3/4)*b*x**14),x)*c + i
nt(1/((a + b*x**4)**(3/4)*a*x**6 + (a + b*x**4)**(3/4)*b*x**10),x)*d
```

3.135 $\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{a(bc-ad)x^2}{3b^3(a+bx^4)^{3/4}} + \frac{(7bc-13ad)x^2\sqrt[4]{a+bx^4}}{21b^3} + \frac{dx^6\sqrt[4]{a+bx^4}}{7b^2} - \frac{4a^{3/2}(7bc-10ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21b^{7/2}(a+bx^4)^{3/4}}$$

output
```
1/3*a*(-a*d+b*c)*x^2/b^3/(b*x^4+a)^(3/4)+1/21*(-13*a*d+7*b*c)*x^2*(b*x^4+a
)^(1/4)/b^3+1/7*d*x^6*(b*x^4+a)^(1/4)/b^2-4/21*a^(3/2)*(-10*a*d+7*b*c)*(1+
b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^
(7/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.68

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{x^2\left(-20a^2d+2ab(7c-5dx^4)+b^2x^4(7c+3dx^4)+2a(-7bc+10ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hy}\right.}{21b^3(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^9*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(x^2*(-20*a^2*d + 2*a*b*(7*c - 5*d*x^4) + b^2*x^4*(7*c + 3*d*x^4) + 2*a*(-
7*b*c + 10*a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((
b*x^4)/a)]))/(21*b^3*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {957, 807, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{10}(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(7bc-10ad)\int \frac{x^9}{(bx^4+a)^{3/4}}dx}{3ab}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{10}(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(7bc-10ad)\int \frac{x^8}{(bx^4+a)^{3/4}}dx^2}{6ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^{10}(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(7bc-10ad)\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{7b}\right)}{6ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^{10}(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(7bc-10ad)\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{3b}\right)}{7b}\right)}{6ab}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{x^{10}(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(7bc-10ad)\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b\left(a+bx^4\right)^{3/4}}\right)}{7b}\right)}{6ab}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{x^{10}(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(7bc-10ad)\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}\right)}{7b}\right)}{6ab}$$

input
```
Int[(x^9*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^10)/(3*a*b*(a + b*x^4)^(3/4)) - ((7*b*c - 10*a*d)*((2*x^6*(
a + b*x^4)^(1/4))/(7*b) - (6*a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^(3/
2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3
*b^(3/2)*(a + b*x^4)^(3/4))))/(7*b)))/(6*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^9(d\,x^4+c)}{(b\,x^4+a)^{\frac{7}{4}}}dx$$

input
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*x^13 + c*x^9)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^{10}{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{7}{4}}} + \frac{dx^{14}{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{7}{4}}}$$

input
```
integrate(x**9*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**10*hyper((7/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(7/4))
+ d*x**14*hyper((7/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(7/4)
)
```

Maxima [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^9/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^9/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^9\,(d\,x^4+c)}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((x^9*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^9*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^{13}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**13/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(
x**9/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.136 $\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=-\frac{(bc-ad)x^2}{3b^2(a+bx^4)^{3/4}}+\frac{dx^2\sqrt[4]{a+bx^4}}{3b^2}+\frac{2\sqrt{a}(bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{5/2}(a+bx^4)^{3/4}}$$

output

```
-1/3*(-a*d+b*c)*x^2/b^2/(b*x^4+a)^(3/4)+1/3*d*x^2*(b*x^4+a)^(1/4)/b^2+2/3*
a^(1/2)*(-2*a*d+b*c)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*
x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{x^2\left(-bc+2ad+bdx^4+(bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{3b^2(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^5*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(x^2*(-(b*c) + 2*a*d + b*d*x^4 + (b*c - 2*a*d)*(1 + (b*x^4)/a)^(3/4)*Hyper
geometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(3*b^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {957, 807, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c+dx^4)}{(a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^6(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(bc-2ad)\int \frac{x^5}{(bx^4+a)^{3/4}}dx}{ab}$$

$$\Big\downarrow \; 807$$

$$\frac{x^6(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(bc-2ad)\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{2ab}$$

$$\Big\downarrow \; 262$$

$$\frac{x^6(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(bc-2ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)}{2ab}$$

$$\Big\downarrow \; 231$$

$$\frac{x^6(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(bc-2ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)}{2ab}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{x^6(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(bc-2ad)\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)}{2ab}$$

input
```
Int[(x^5*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^6)/(3*a*b*(a + b*x^4)^(3/4)) - ((b*c - 2*a*d)*((2*x^2*(a +
b*x^4)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(S
qrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))/(2*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{5}\left(d\,x^{4}+c\right)}{\left(b\,x^{4}+a\right)^{\frac{7}{4}}}d x$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^6\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{7}{4}}} + \frac{dx^{10}\,{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{7}{4}}}$$

input
```
integrate(x**5*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**6*hyper((3/2, 7/4), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(7/4)) +
d*x**10*hyper((7/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(7/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^5\,(d\,x^4+c)}{{\left(b\,x^4+a\right)}^{7/4}}\,dx$$

input
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**9/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(x
**5/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.137 $\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}} dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}} dx = \frac{(bc-ad)x^2}{3ab(a+bx^4)^{3/4}} + \frac{(bc+2ad)\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{a}b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/3*(-a*d+b*c)*x^2/a/b/(b*x^4+a)^(3/4)+1/3*(2*a*d+b*c)*(1+b*x^4/a)^(3/4)*I
nverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(3/2)/(b
*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}} dx = \frac{x^2\left(2bc-2ad+(bc+2ad)\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{6ab(a+bx^4)^{3/4}}$$

input
```
Integrate[(x*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(x^2*(2*b*c - 2*a*d + (b*c + 2*a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F
1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(6*a*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {957, 807, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2ad+bc)\int \frac{x}{(bx^4+a)^{3/4}}dx}{3ab}+\frac{x^2(bc-ad)}{3ab\,(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2ad+bc)\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{6ab}+\frac{x^2(bc-ad)}{3ab\,(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}(2ad+bc)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{6ab\,(a+bx^4)^{3/4}}+\frac{x^2(bc-ad)}{3ab\,(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}(2ad+bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{a}b^{3/2}\,(a+bx^4)^{3/4}}+\frac{x^2(bc-ad)}{3ab\,(a+bx^4)^{3/4}}$$

input
```
Int[(x*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^2)/(3*a*b*(a + b*x^4)^(3/4)) + ((b*c + 2*a*d)*(1 + (b*x^4)/
a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*b^(3/2)
*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(d\,x^4+c)}{(b\,x^4+a)^{\frac{7}{4}}}dx$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}} + \frac{dx^6\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{7}{4}}}$$

input
```
integrate(x*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**2*hyper((1/2, 7/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(7/4)) +
d*x**6*hyper((3/2, 7/4), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(7/4))
```

Maxima [F]

$$\int \frac{x(c + dx^4)}{(a + bx^4)^{7/4}}\,dx = \int \frac{(dx^4 + c)x}{(bx^4 + a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x(c + dx^4)}{(a + bx^4)^{7/4}}\,dx = \int \frac{(dx^4 + c)x}{(bx^4 + a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c + dx^4)}{(a + bx^4)^{7/4}}\,dx = \int \frac{x\,(d\,x^4 + c)}{(b\,x^4 + a)^{7/4}}\,dx$$

input
```
int((x*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**5/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(x
/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.138 $\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{7/4}}\,dx = -\frac{c}{2ax^2\left(a+bx^4\right)^{3/4}}-\frac{(5bc-2ad)x^2}{6a^2\left(a+bx^4\right)^{3/4}} -\frac{(5bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{6a^{3/2}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/2*c/a/x^2/(b*x^4+a)^(3/4)-1/6*(-2*a*d+5*b*c)*x^2/a^2/(b*x^4+a)^(3/4)-1/
6*(-2*a*d+5*b*c)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/
a^(1/2)),2^(1/2))/a^(3/2)/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{7/4}}\,dx = \frac{-6ac-10bcx^4+4adx^4+(-5bc+2ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.}{12a^2x^2\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^3*(a + b*x^4)^(7/4)),x]
```

output
```
(-6*a*c - 10*b*c*x^4 + 4*a*d*x^4 + (-5*b*c + 2*a*d)*x^4*(1 + (b*x^4)/a)^(3
/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)])/(12*a^2*x^2*(a + b*x^4
)^(3/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 807, 215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(5bc-2ad)\int \frac{x}{(bx^4+a)^{7/4}}dx}{2a}-\frac{c}{2ax^2 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(5bc-2ad)\int \frac{1}{(bx^4+a)^{7/4}}dx^2}{4a}-\frac{c}{2ax^2 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{ 215}$$

$$-\frac{(5bc-2ad)\left(\frac{\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3a}+\frac{2x^2}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{2ax^2 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{ 231}$$

$$-\frac{(5bc-2ad)\left(\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3a(a+bx^4)^{3/4}}+\frac{2x^2}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(5bc-2ad)\left(\frac{2\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^4)^{3/4}}+\frac{2x^2}{3a(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^3*(a + b*x^4)^(7/4)),x]
```

output
```
-1/2*c/(a*x^2*(a + b*x^4)^(3/4)) - ((5*b*c - 2*a*d)*((2*x^2)/(3*a*(a + b*x
^4)^(3/4)) + (2*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[
a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^4)^(3/4))))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{3} \left(b\,x^{4}+a \right)^{\frac{7}{4}}}d x$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^3 \left(a+bx^4\right)^{7/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{7}{4}} x^{3}}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^11 + 2*a*b*x^7 + a^2*x^3), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{c+dx^4}{x^3 \left(a+bx^4\right)^{7/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}x^2} + \frac{dx^2\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}}$$

input
```
integrate((d*x**4+c)/x**3/(b*x**4+a)**(7/4),x)
```

output
```
-c*hyper((-1/2, 7/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(7/4)*x**2)
+ d*x**2*hyper((1/2, 7/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^3 \left(a+bx^4\right)^{7/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^3 \left(a+bx^4\right)^{7/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{7/4}}\,dx=\int \frac{d\,x^4+c}{x^3\,{\left(b\,x^4+a\right)}^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(7/4)),x)
```

output
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(7/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{7/4}}\,dx=\left(\int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d+\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^3+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^7}dx\right)c$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(7/4),x)
```

output
```
int(x/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(1/((
a + b*x**4)**(3/4)*a*x**3 + (a + b*x**4)**(3/4)*b*x**7),x)*c
```

3.139 $\int \frac{c+dx^4}{x^7(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{c+dx^4}{x^7(a+bx^4)^{7/4}}\,dx = -\frac{c}{6ax^6(a+bx^4)^{3/4}} - \frac{3bc-2ad}{6a^2x^2(a+bx^4)^{3/4}} + \frac{5(3bc-2ad)\sqrt[4]{a+bx^4}}{12a^3x^2} + \frac{5\sqrt{b}(3bc-2ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{5/2}(a+bx^4)^{3/4}}$$

output
```
-1/6*c/a/x^6/(b*x^4+a)^(3/4)-1/6*(-2*a*d+3*b*c)/a^2/x^2/(b*x^4+a)^(3/4)+5/
12*(-2*a*d+3*b*c)*(b*x^4+a)^(1/4)/a^3/x^2+5/12*b^(1/2)*(-2*a*d+3*b*c)*(1+b
*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(
5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.48

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{7/4}}\, dx = \frac{-2ac+3(3bc-2ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{12a^2x^6\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^7*(a + b*x^4)^(7/4)),x]
```

output
```
(-2*a*c + 3*(3*b*c - 2*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-1
/2, 7/4, 1/2, -((b*x^4)/a)])/(12*a^2*x^6*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 807, 253, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{7/4}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(3bc-2ad)\int \frac{1}{x^3\left(bx^4+a\right)^{7/4}}dx}{2a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(3bc-2ad)\int \frac{1}{x^4\left(bx^4+a\right)^{7/4}}dx^2}{4a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 253}$$

$$-\frac{(3bc-2ad)\left(\frac{5\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^2}{3a}+\frac{2}{3ax^2(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

$\Big\downarrow$ 264

$$-\frac{(3bc-2ad)\left(\frac{5\left(-\frac{b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{3a}+\frac{2}{3ax^2(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

$\Big\downarrow$ 231

$$-\frac{(3bc-2ad)\left(\frac{5\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{3a}+\frac{2}{3ax^2(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

$\Big\downarrow$ 229

$$-\frac{(3bc-2ad)\left(\frac{5\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{3a}+\frac{2}{3ax^2(a+bx^4)^{3/4}}\right)}{4a}-\frac{c}{6ax^6\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^7*(a + b*x^4)^(7/4)),x]
```

output
```
-1/6*c/(a*x^6*(a + b*x^4)^(3/4)) - ((3*b*c - 2*a*d)*(2/(3*a*x^2*(a + b*x^4
)^(3/4)) + (5*(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/
4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/
4))))/(3*a)))/(4*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{7} \left(b\,x^{4}+a \right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^{4} + c}{{\left(bx^{4} + a\right)}^{\frac{7}{4}} x^{7}}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^15 + 2*a*b*x^11 + a^2*x^7),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{7/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{7}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{7}{4}}x^6} - \frac{d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}x^2}$$

input
```
integrate((d*x**4+c)/x**7/(b*x**4+a)**(7/4),x)
```

output
```
-c*hyper((-3/2, 7/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(7/4)*x**6)
 - d*hyper((-1/2, 7/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(7/4)*x**2
)
```

Maxima [F]

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{7/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^7), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{7/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{7/4}}\,dx = \int \frac{d\,x^4+c}{x^7\,(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(7/4)),x)
```

output
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(7/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{7/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^7+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^{11}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^3+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^7}dx\right)d$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**7 + (a + b*x**4)**(3/4)*b*x**11),x)*c + in
t(1/((a + b*x**4)**(3/4)*a*x**3 + (a + b*x**4)**(3/4)*b*x**7),x)*d
```

3.140 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{a(bc-ad)x}{3b^3(a+bx^4)^{3/4}}+\frac{(6bc-11ad)x\sqrt[4]{a+bx^4}}{12b^3}+\frac{dx^5\sqrt[4]{a+bx^4}}{6b^2}+\frac{5\sqrt{a}(2bc-3ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12b^{5/2}(a+bx^4)^{3/4}}$$

output
```
1/3*a*(-a*d+b*c)*x/b^3/(b*x^4+a)^(3/4)+1/12*(-11*a*d+6*b*c)*x*(b*x^4+a)^(1
/4)/b^3+1/6*d*x^5*(b*x^4+a)^(1/4)/b^2+5/12*a^(1/2)*(-3*a*d+2*b*c)*(1+a/b/x
^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(
5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.67

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{x\left(-15a^2d+ab(10c-9dx^4)+2b^2x^4(3c+dx^4)+5a(-2bc+3ad)\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hype}\right.}{12b^3(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(x*(-15*a^2*d + a*b*(10*c - 9*d*x^4) + 2*b^2*x^4*(3*c + d*x^4) + 5*a*(-2*b
*c + 3*a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^
4)/a)]))/(12*b^3*(a + b*x^4)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {957, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^9(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-3ad)\int \frac{x^8}{(bx^4+a)^{3/4}}dx}{ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int \frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}\right)}{ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{768}$$

$$\frac{\frac{x^9(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}\right)}{6b}\right)}{ab}}{}$$

$\downarrow$ 858

$$\frac{\frac{x^9(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{ab}}{}$$

$\downarrow$ 807

$$\frac{\frac{x^9(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{ab}}{}$$

$\downarrow$ 229

$$\frac{x^9(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{ab}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^9)/(3*a*b*(a + b*x^4)^(3/4)) - ((2*b*c - 3*a*d)*((x^5*(a +
b*x^4)^(1/4))/(6*b) - (5*a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/
(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[
b]*(a + b*x^4)^(3/4))))/(6*b)))/(a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{8} \left(d\,x^{4}+c \right)}{\left(b\,x^{4}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^8 (c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*x^12 + c*x^8)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\frac{cx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}+\frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{7}{4},\frac{13}{4}\\\frac{17}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**9*gamma(9/4)*hyper((7/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(7/4)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((7/4, 13/4), (17/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(7/4)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx=\int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**12/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(
x**8/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.141 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = -\frac{(bc-ad)x}{3b^2(a+bx^4)^{3/4}} + \frac{dx\sqrt[4]{a+bx^4}}{2b^2} - \frac{(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{6\sqrt{a}b^{3/2}(a+bx^4)^{3/4}}$$

output
```
-1/3*(-a*d+b*c)*x/b^2/(b*x^4+a)^(3/4)+1/2*d*x*(b*x^4+a)^(1/4)/b^2-1/6*(-5*
a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^
(1/2)),2^(1/2))/a^(1/2)/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}} \, dx = \frac{x\left(-2bc+5ad+3bdx^4+(2bc-5ad)\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right.\right.}{6b^2(a+bx^4)^{3/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
(x*(-2*b*c + 5*a*d + 3*b*d*x^4 + (2*b*c - 5*a*d)*(1 + (b*x^4)/a)^(3/4)*Hyp
ergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)]))/(6*b^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx^4)}{(a+bx^4)^{7/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^5(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\int \frac{x^4}{(bx^4+a)^{3/4}}dx}{3ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{3ab}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^5(bc-ad)}{3ab(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)}{3ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^5(bc-ad)}{3ab\,(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{3ab}$$

$\Big\downarrow$ 807

$$\frac{x^5(bc-ad)}{3ab\,(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{3ab}$$

$\Big\downarrow$ 229

$$\frac{x^5(bc-ad)}{3ab\,(a+bx^4)^{3/4}}-\frac{(2bc-5ad)\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{3ab}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x^5)/(3*a*b*(a + b*x^4)^(3/4)) - ((2*b*c - 5*a*d)*((x*(a + b*
x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sq
rt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(3*a*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))    Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^4(d\,x^4+c)}{(b\,x^4+a)^{\frac{7}{4}}}dx$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*x^8 + c*x^4)*(b*x^4 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x**5*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(7/4)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((7/4, 9/4), (13/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(7/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(7/4),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**8/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(x
**4/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.142 $\int \frac{c+dx^4}{(a+bx^4)^{7/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}} \, dx = \frac{(bc-ad)x}{3ab\,(a+bx^4)^{3/4}} - \frac{(2bc+ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
1/3*(-a*d+b*c)*x/a/b/(b*x^4+a)^(3/4)-1/3*(a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3
*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/b^(1/2)/
(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}} \, dx = \frac{x\left(bc-ad+(2bc+ad)\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{3ab\,(a+bx^4)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(7/4),x]
```

output
```
(x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4,
 3/4, 5/4, -((b*x^4)/a)]))/(3*a*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {910, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{7/4}}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+2bc)\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3ab}+\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3ab\left(a+bx^4\right)^{3/4}}+\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6ab\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{x(bc-ad)}{3ab\left(a+bx^4\right)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(ad+2bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(7/4),x]
```

output
```
((b*c - a*d)*x)/(3*a*b*(a + b*x^4)^(3/4)) - ((2*b*c + a*d)*(1 + a/(b*x^4))
^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*Sqrt[
b]*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{\left(b\,x^{4}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx=\int \frac{dx^{4}+c}{\left(bx^{4}+a\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx=\frac{cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{5}{4}\right)}+\frac{dx^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(7/4),x)
```

output
```
c*x*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/4)*gamma(5/4)) + d*x**5*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**4*exp_
polar(I*pi)/a)/(4*a**(7/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \int \frac{d\,x^4+c}{(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(a + b*x^4)^(7/4),x)
```

output
```
int((c + d*x^4)/(a + b*x^4)^(7/4), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{7/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(7/4),x)
```

output
```
int(x**4/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d + int(1
/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*c
```

3.143 $\int \frac{c+dx^4}{x^4(a+bx^4)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{7/4}}\,dx = -\frac{c}{3ax^3(a+bx^4)^{3/4}} - \frac{(2bc-ad)x}{3a^2(a+bx^4)^{3/4}} + \frac{2\sqrt{b}(2bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{5/2}(a+bx^4)^{3/4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(3/4)-1/3*(-a*d+2*b*c)*x/a^2/(b*x^4+a)^(3/4)+2/3*b^
(1/2)*(-a*d+2*b*c)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2
)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{7/4}}\,dx = \frac{-ac-2bcx^4+adx^4+2(-2bc+ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\right.}{3a^2x^3(a+bx^4)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(7/4)),x]
```

output
```
(-(a*c) - 2*b*c*x^4 + a*d*x^4 + 2*(-2*b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)
*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(3*a^2*x^3*(a + b*x^4)^(3
/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 749, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-ad)\int \frac{1}{(bx^4+a)^{7/4}}dx}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(2bc-ad)\left(\frac{2\int \frac{1}{(bx^4+a)^{3/4}}dx}{3a}+\frac{x}{3a(a+bx^4)^{3/4}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{(2bc-ad)\left(\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a(a+bx^4)^{3/4}}+\frac{x}{3a(a+bx^4)^{3/4}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{(2bc-ad)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a(a+bx^4)^{3/4}}\right)}{a}-\frac{c}{3ax^3\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(2bc-ad)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}\right)}{a}-\frac{c}{3ax^3\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(2bc-ad)\left(\frac{x}{3a(a+bx^4)^{3/4}}-\frac{2\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}\right)}{a}-\frac{c}{3ax^3\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(7/4)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(3/4)) - ((2*b*c - a*d)*(x/(3*a*(a + b*x^4)^(3/4
)) - (2*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b
]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/a
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{4}\left(b\,x^{4}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}} x^4} \, dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^12 + 2*a*b*x^8 + a^2*x^4), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}} \, dx = \frac{c\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{7}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}} x^3 \Gamma\left(\frac{1}{4}\right)} + \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}} \Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(7/4),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 7/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
7/4)*x**3*gamma(1/4)) + d*x*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(7/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^4), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}}\,dx = \int \frac{d\,x^4+c}{x^4\,(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(7/4)),x)
```

output
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(7/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{7/4}} \, dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^8} dx \right) c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4} dx \right) d$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**4 + (a + b*x**4)**(3/4)*b*x**8),x)*c + int
(1/((a + b*x**4)**(3/4)*a + (a + b*x**4)**(3/4)*b*x**4),x)*d
```

3.144 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/4}}\,dx = -\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}} - \frac{10bc-7ad}{21a^2x^3\left(a+bx^4\right)^{3/4}} + \frac{2(10bc-7ad)\sqrt[4]{a+bx^4}}{21a^3x^3} - \frac{4b^{3/2}(10bc-7ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{7/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(3/4)-1/21*(-7*a*d+10*b*c)/a^2/x^3/(b*x^4+a)^(3/4)+
2/21*(-7*a*d+10*b*c)*(b*x^4+a)^(1/4)/a^3/x^3-4/21*b^(3/2)*(-7*a*d+10*b*c)*
(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1
/2))/a^(7/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.46

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{7/4}}\,dx = \frac{-3ac+(10bc-7ad)x^4\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{1}{4},-\frac{bx^4}{a}\right)}{21a^2x^7\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(7/4)),x]
```

output
```
(-3*a*c + (10*b*c - 7*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/
4, 7/4, 1/4, -((b*x^4)/a)])/(21*a^2*x^7*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{7/4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(10bc-7ad)\int \frac{1}{x^4\left(bx^4+a\right)^{7/4}}dx}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(10bc-7ad)\left(\frac{2\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx}{a}+\frac{1}{3ax^3\left(a+bx^4\right)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(-\frac{2b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{a}+\frac{1}{3ax^3(a+bx^4)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$\downarrow$ 768

$$-\frac{(10bc-7ad)\left(\frac{2\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{a}+\frac{1}{3ax^3(a+bx^4)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$\downarrow$ 858

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{a}+\frac{1}{3ax^3(a+bx^4)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$\downarrow$ 807

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{a}+\frac{1}{3ax^3(a+bx^4)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{a}+\frac{1}{3ax^3\left(a+bx^4\right)^{3/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(7/4)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(3/4)) - ((10*b*c - 7*a*d)*(1/(3*a*x^3*(a + b*x^
4)^(3/4)) + (2*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4)
)^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a +
 b*x^4)^(3/4))))/a))/(7*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{8} \left(b\,x^{4}+a \right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x)
```

output
```
int((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{7}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*(d*x^4 + c)/(b^2*x^16 + 2*a*b*x^12 + a^2*x^8),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.58

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{7/4}}\,dx=\frac{c\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{7}{4}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}x^7\Gamma\left(-\frac{3}{4}\right)}+\frac{d\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{7}{4}\\\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{7}{4}}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(7/4),x)
```

output
```
c*gamma(-7/4)*hyper((-7/4, 7/4), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(7/4)*x**7*gamma(-3/4)) + d*gamma(-3/4)*hyper((-3/4, 7/4), (1/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(7/4)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{7/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{7}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(7/4)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{7/4}}\,dx = \int \frac{d\,x^4+c}{x^8\,(b\,x^4+a)^{7/4}}\,dx$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(7/4)),x)
```

output
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(7/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8(a+bx^4)^{7/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^8+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^{12}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,a\,x^4+(b\,x^4+a)^{\frac{3}{4}}\,b\,x^8}dx\right)d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(7/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*a*x**8 + (a + b*x**4)**(3/4)*b*x**12),x)*c + in
t(1/((a + b*x**4)**(3/4)*a*x**4 + (a + b*x**4)**(3/4)*b*x**8),x)*d
```

3.145 $\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{a^2(bc-ad)}{5b^4(a+bx^4)^{5/4}}+\frac{a(2bc-3ad)}{b^4\sqrt[4]{a+bx^4}} + \frac{(bc-3ad)(a+bx^4)^{3/4}}{3b^4}+\frac{d(a+bx^4)^{7/4}}{7b^4}$$

output
```
-1/5*a^2*(-a*d+b*c)/b^4/(b*x^4+a)^(5/4)+a*(-3*a*d+2*b*c)/b^4/(b*x^4+a)^(1/
4)+1/3*(-3*a*d+b*c)*(b*x^4+a)^(3/4)/b^4+1/7*d*(b*x^4+a)^(7/4)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{224a^2bc-384a^3d+280ab^2cx^4-480a^2bdx^4+35b^3cx^8-60ab^2dx^8+15b^3dx^{12}}{105b^4(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^11*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(224*a^2*b*c - 384*a^3*d + 280*a*b^2*c*x^4 - 480*a^2*b*d*x^4 + 35*b^3*c*x^
8 - 60*a*b^2*d*x^8 + 15*b^3*d*x^12)/(105*b^4*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8(dx^4+c)}{(bx^4+a)^{9/4}}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(-\frac{(ad-bc)a^2}{b^3(bx^4+a)^{9/4}}+\frac{(3ad-2bc)a}{b^3(bx^4+a)^{5/4}}+\frac{d(bx^4+a)^{3/4}}{b^3}+\frac{bc-3ad}{b^3\sqrt[4]{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^2(bc-ad)}{5b^4(a+bx^4)^{5/4}}+\frac{4a(2bc-3ad)}{b^4\sqrt[4]{a+bx^4}}+\frac{4(a+bx^4)^{3/4}(bc-3ad)}{3b^4}+\frac{4d(a+bx^4)^{7/4}}{7b^4}\right)$$

input
```
Int[(x^11*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((-4*a^2*(b*c - a*d))/(5*b^4*(a + b*x^4)^(5/4)) + (4*a*(2*b*c - 3*a*d))/(b
^4*(a + b*x^4)^(1/4)) + (4*(b*c - 3*a*d)*(a + b*x^4)^(3/4))/(3*b^4) + (4*d
*(a + b*x^4)^(7/4))/(7*b^4))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{128\left(-\frac{35\left(\frac{3 d\,x^{4}}{7}+c\right) x^{8} b^{3}}{384}-\frac{35\left(-\frac{3 d\,x^{4}}{14}+c\right) x^{4} a\,b^{2}}{48}-\frac{7\left(-\frac{15 d\,x^{4}}{7}+c\right) a^{2} b}{12}+a^{3} d\right)}{35\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{4}}$	68
gosper	$-\frac{-15 b^{3} d\,x^{12}+60 a\,b^{2} d\,x^{8}-35 c\,b^{3} x^{8}+480 a^{2} b d\,x^{4}-280 a\,b^{2} c\,x^{4}+384 a^{3} d-224 a^{2} b c}{105\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{4}}$	77
trager	$-\frac{-15 b^{3} d\,x^{12}+60 a\,b^{2} d\,x^{8}-35 c\,b^{3} x^{8}+480 a^{2} b d\,x^{4}-280 a\,b^{2} c\,x^{4}+384 a^{3} d-224 a^{2} b c}{105\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{4}}$	77
orering	$-\frac{-15 b^{3} d\,x^{12}+60 a\,b^{2} d\,x^{8}-35 c\,b^{3} x^{8}+480 a^{2} b d\,x^{4}-280 a\,b^{2} c\,x^{4}+384 a^{3} d-224 a^{2} b c}{105\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{4}}$	77
risch	$-\frac{\left(-3 d b\,x^{4}+18 a d-7 c b\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{21 b^{4}}-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(15 a b d\,x^{4}-10 b^{2} c\,x^{4}+14 a^{2} d-9 a b c\right) a}{5 b^{4}\left(x^{8} b^{2}+2 a\,x^{4} b+a^{2}\right)}$	96

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-128/35*(-35/384*(3/7*d*x^4+c)*x^8*b^3-35/48*(-3/14*d*x^4+c)*x^4*a*b^2-7/1
2*(-15/7*d*x^4+c)*a^2*b+a^3*d)/(b*x^4+a)^(5/4)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{(15\,b^3dx^{12}+5\,(7\,b^3c-12\,ab^2d)x^8+40\,(7\,ab^2c-12\,a^2bd)x^4+224\,a^2bc-384\,a^3d)(bx}{105\,(b^6x^8+2\,ab^5x^4+a^2b^4)}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/105*(15*b^3*d*x^12 + 5*(7*b^3*c - 12*a*b^2*d)*x^8 + 40*(7*a*b^2*c - 12*a
^2*b*d)*x^4 + 224*a^2*b*c - 384*a^3*d)*(b*x^4 + a)^(3/4)/(b^6*x^8 + 2*a*b^
5*x^4 + a^2*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(88) = 176.

Time = 1.48 (sec) , antiderivative size = 338, normalized size of antiderivative = 3.38

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \begin{cases} -\frac{384a^3d}{105ab^4\sqrt[4]{a+bx^4}+105b^5x^4\sqrt[4]{a+bx^4}} + \frac{224a^2bc}{105ab^4\sqrt[4]{a+bx^4}+105b^5x^4\sqrt[4]{a+bx^4}} - \frac{}{105ab^4\sqrt[4]{a+} \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{16}}{16}}{a^{\frac{9}{4}}} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
Piecewise((-384*a**3*d/(105*a*b**4*(a + b*x**4)**(1/4) + 105*b**5*x**4*(a
+ b*x**4)**(1/4)) + 224*a**2*b*c/(105*a*b**4*(a + b*x**4)**(1/4) + 105*b**
5*x**4*(a + b*x**4)**(1/4)) - 480*a**2*b*d*x**4/(105*a*b**4*(a + b*x**4)**
(1/4) + 105*b**5*x**4*(a + b*x**4)**(1/4)) + 280*a*b**2*c*x**4/(105*a*b**4
*(a + b*x**4)**(1/4) + 105*b**5*x**4*(a + b*x**4)**(1/4)) - 60*a*b**2*d*x*
*8/(105*a*b**4*(a + b*x**4)**(1/4) + 105*b**5*x**4*(a + b*x**4)**(1/4)) +
35*b**3*c*x**8/(105*a*b**4*(a + b*x**4)**(1/4) + 105*b**5*x**4*(a + b*x**4
)**(1/4)) + 15*b**3*d*x**12/(105*a*b**4*(a + b*x**4)**(1/4) + 105*b**5*x**
4*(a + b*x**4)**(1/4)), Ne(b, 0)), ((c*x**12/12 + d*x**16/16)/a**(9/4), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{1}{35}\,d\left(\frac{5\,(bx^4+a)^{\frac{7}{4}}}{b^4} - \frac{35\,(bx^4+a)^{\frac{3}{4}}a}{b^4} - \frac{105\,a^2}{(bx^4+a)^{\frac{1}{4}}b^4} + \frac{7\,a^3}{(bx^4+a)^{\frac{5}{4}}b^4}\right) + \frac{1}{15}\,c\left(\frac{5\,(bx^4+a)^{\frac{3}{4}}}{b^3} + \frac{30\,a}{(bx^4+a)^{\frac{1}{4}}b^3} - \frac{3\,a^2}{(bx^4+a)^{\frac{5}{4}}b^3}\right)$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
1/35*d*(5*(b*x^4 + a)^(7/4)/b^4 - 35*(b*x^4 + a)^(3/4)*a/b^4 - 105*a^2/((b
*x^4 + a)^(1/4)*b^4) + 7*a^3/((b*x^4 + a)^(5/4)*b^4)) + 1/15*c*(5*(b*x^4 +
 a)^(3/4)/b^3 + 30*a/((b*x^4 + a)^(1/4)*b^3) - 3*a^2/((b*x^4 + a)^(5/4)*b^
3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{10\,(bx^4+a)abc - a^2bc - 15\,(bx^4+a)a^2d + a^3d}{5\,(bx^4+a)^{\frac{5}{4}}b^4} + \frac{7\,(bx^4+a)^{\frac{3}{4}}b^{25}c + 3\,(bx^4+a)^{\frac{7}{4}}b^{24}d - 21\,(bx^4+a)^{\frac{3}{4}}ab^{24}d}{21\,b^{28}}$$

input
```
integrate(x^11*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
1/5*(10*(b*x^4 + a)*a*b*c - a^2*b*c - 15*(b*x^4 + a)*a^2*d + a^3*d)/((b*x^
4 + a)^(5/4)*b^4) + 1/21*(7*(b*x^4 + a)^(3/4)*b^25*c + 3*(b*x^4 + a)^(7/4)
*b^24*d - 21*(b*x^4 + a)^(3/4)*a*b^24*d)/b^28
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{\frac{d\,(b\,x^4+a)^3}{7}+\frac{a^3\,d}{5}-a\,d\,(b\,x^4+a)^2+\frac{b\,c\,(b\,x^4+a)^2}{3}-3\,a^2\,d\,(b\,x^4+a)-\frac{a^2\,b\,c}{5}+2\,a\,b\,c\,(b\,x^4}{b^4\,(b\,x^4+a)^{5/4}}$$

input
```
int((x^11*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
((d*(a + b*x^4)^3)/7 + (a^3*d)/5 - a*d*(a + b*x^4)^2 + (b*c*(a + b*x^4)^2)
/3 - 3*a^2*d*(a + b*x^4) - (a^2*b*c)/5 + 2*a*b*c*(a + b*x^4))/(b^4*(a + b*
x^4)^(5/4))
```

Reduce [F]

$$\int \frac{x^{11}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{15}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^11*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**15/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**11/((a + b*x**4)**(1/4)*a**2 + 2
*(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.146 $\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{a(bc-ad)}{5b^3(a+bx^4)^{5/4}} - \frac{bc-2ad}{b^3\sqrt[4]{a+bx^4}} + \frac{d(a+bx^4)^{3/4}}{3b^3}$$

output
```
1/5*a*(-a*d+b*c)/b^3/(b*x^4+a)^(5/4)-(-2*a*d+b*c)/b^3/(b*x^4+a)^(1/4)+1/3*
d*(b*x^4+a)^(3/4)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{-12abc+32a^2d-15b^2cx^4+40abdx^4+5b^2dx^8}{15b^3(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^7*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(-12*a*b*c + 32*a^2*d - 15*b^2*c*x^4 + 40*a*b*d*x^4 + 5*b^2*d*x^8)/(15*b^3
*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (c+dx^4)}{(a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{x^4 (dx^4+c)}{(bx^4+a)^{9/4}}dx^4$$

$$\Big\downarrow \ 86$$

$$\frac{1}{4}\int \left(\frac{d}{b^2 \sqrt[4]{bx^4+a}}+\frac{bc-2ad}{b^2 (bx^4+a)^{5/4}}+\frac{a(ad-bc)}{b^2 (bx^4+a)^{9/4}}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{4(bc-2ad)}{b^3 \sqrt[4]{a+bx^4}}+\frac{4a(bc-ad)}{5b^3 (a+bx^4)^{5/4}}+\frac{4d(a+bx^4)^{3/4}}{3b^3}\right)$$

input
```
Int[(x^7*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((4*a*(b*c - a*d))/(5*b^3*(a + b*x^4)^(5/4)) - (4*(b*c - 2*a*d))/(b^3*(a +
 b*x^4)^(1/4)) + (4*d*(a + b*x^4)^(3/4))/(3*b^3))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{-\left(-\frac{d\,x^{4}}{3}+c\right)x^{4} b^{2}-\frac{4\left(-\frac{10 d\,x^{4}}{3}+c\right)a b}{5}+\frac{32 a^{2} d}{15}}{\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{3}}$	49
gosper	$\frac{5 d\,b^{2} x^{8}+40 a b d\,x^{4}-15 b^{2} c\,x^{4}+32 a^{2} d-12 a b c}{15 \left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{3}}$	53
trager	$\frac{5 d\,b^{2} x^{8}+40 a b d\,x^{4}-15 b^{2} c\,x^{4}+32 a^{2} d-12 a b c}{15 \left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{3}}$	53
orering	$\frac{5 d\,b^{2} x^{8}+40 a b d\,x^{4}-15 b^{2} c\,x^{4}+32 a^{2} d-12 a b c}{15 \left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{3}}$	53
risch	$\frac{d \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3 b^{3}}+\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(10 a b d\,x^{4}-5 b^{2} c\,x^{4}+9 a^{2} d-4 a b c\right)}{5 b^{3} \left(x^{8} b^{2}+2 a\,x^{4} b+a^{2}\right)}$	80

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
32/15/(b*x^4+a)^(5/4)*(-15/32*(-1/3*d*x^4+c)*x^4*b^2-3/8*(-10/3*d*x^4+c)*a
*b+a^2*d)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{(5\,b^2dx^8 - 5\,(3\,b^2c - 8\,abd)x^4 - 12\,abc + 32\,a^2d)(bx^4+a)^{\frac{3}{4}}}{15\,(b^5x^8 + 2\,ab^4x^4 + a^2b^3)}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/15*(5*b^2*d*x^8 - 5*(3*b^2*c - 8*a*b*d)*x^4 - 12*a*b*c + 32*a^2*d)*(b*x^
4 + a)^(3/4)/(b^5*x^8 + 2*a*b^4*x^4 + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(61) = 122.

Time = 1.01 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.38

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \begin{cases} \frac{32a^2d}{15ab^3\sqrt[4]{a+bx^4}+15b^4x^4\sqrt[4]{a+bx^4}} - \frac{12abc}{15ab^3\sqrt[4]{a+bx^4}+15b^4x^4\sqrt[4]{a+bx^4}} + \frac{40abd}{15ab^3\sqrt[4]{a+bx^4}+1} \\ \frac{\frac{cx^8}{8}+\frac{dx^{12}}{12}}{a^{\frac{9}{4}}} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
Piecewise((32*a**2*d/(15*a*b**3*(a + b*x**4)**(1/4) + 15*b**4*x**4*(a + b*
x**4)**(1/4)) - 12*a*b*c/(15*a*b**3*(a + b*x**4)**(1/4) + 15*b**4*x**4*(a
+ b*x**4)**(1/4)) + 40*a*b*d*x**4/(15*a*b**3*(a + b*x**4)**(1/4) + 15*b**4
*x**4*(a + b*x**4)**(1/4)) - 15*b**2*c*x**4/(15*a*b**3*(a + b*x**4)**(1/4)
 + 15*b**4*x**4*(a + b*x**4)**(1/4)) + 5*b**2*d*x**8/(15*a*b**3*(a + b*x**
4)**(1/4) + 15*b**4*x**4*(a + b*x**4)**(1/4)), Ne(b, 0)), ((c*x**8/8 + d*x
**12/12)/a**(9/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{1}{15}\,d\left(\frac{5\,(bx^4+a)^{\frac{3}{4}}}{b^3}+\frac{30\,a}{(bx^4+a)^{\frac{1}{4}}b^3}-\frac{3\,a^2}{(bx^4+a)^{\frac{5}{4}}b^3}\right) - \frac{1}{5}\,c\left(\frac{5}{(bx^4+a)^{\frac{1}{4}}b^2}-\frac{a}{(bx^4+a)^{\frac{5}{4}}b^2}\right)$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
1/15*d*(5*(b*x^4 + a)^(3/4)/b^3 + 30*a/((b*x^4 + a)^(1/4)*b^3) - 3*a^2/((b
*x^4 + a)^(5/4)*b^3)) - 1/5*c*(5/((b*x^4 + a)^(1/4)*b^2) - a/((b*x^4 + a)^
(5/4)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}d}{3\,b^3} - \frac{5\,(bx^4+a)bc-abc-10\,(bx^4+a)ad+a^2d}{5\,(bx^4+a)^{\frac{5}{4}}b^3}$$

input
```
integrate(x^7*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
1/3*(b*x^4 + a)^(3/4)*d/b^3 - 1/5*(5*(b*x^4 + a)*b*c - a*b*c - 10*(b*x^4 +
 a)*a*d + a^2*d)/((b*x^4 + a)^(5/4)*b^3)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{5\,d\,\left(b\,x^4+a\right)^2-3\,a^2\,d+30\,a\,d\,\left(b\,x^4+a\right)-15\,b\,c\,\left(b\,x^4+a\right)+3\,a\,b\,c}{15\,b^3\,\left(b\,x^4+a\right)^{5/4}}$$

input
```
int((x^7*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
(5*d*(a + b*x^4)^2 - 3*a^2*d + 30*a*d*(a + b*x^4) - 15*b*c*(a + b*x^4) + 3
*a*b*c)/(15*b^3*(a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{x^7(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{11}}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^8}dx\right) d + \left(\int \frac{x^7}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^8}dx\right) c$$

input
```
int(x^7*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**11/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**7/((a + b*x**4)**(1/4)*a**2 + 2*
(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.147 $\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{-bc+ad}{5b^2(a+bx^4)^{5/4}} - \frac{d}{b^2\sqrt[4]{a+bx^4}}$$

output
```
1/5*(a*d-b*c)/b^2/(b*x^4+a)^(5/4)-d/b^2/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{-bc-4ad-5bdx^4}{5b^2(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^3*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(-(b*c) - 4*a*d - 5*b*d*x^4)/(5*b^2*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^4\right)}{\left(a+bx^4\right)^{9/4}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{4}\int \frac{dx^4+c}{\left(bx^4+a\right)^{9/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{d}{b\left(bx^4+a\right)^{5/4}}+\frac{bc-ad}{b\left(bx^4+a\right)^{9/4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4(bc-ad)}{5b^2\left(a+bx^4\right)^{5/4}}-\frac{4d}{b^2\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[(x^3*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((-4*(b*c - a*d))/(5*b^2*(a + b*x^4)^(5/4)) - (4*d)/(b^2*(a + b*x^4)^(1/4)
))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{5db\,x^{4}+4ad+cb}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{2}}$	30
trager	$-\frac{5db\,x^{4}+4ad+cb}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{2}}$	30
pseudoelliptic	$-\frac{4\left(\frac{\left(5d\,x^{4}+c\right)b}{4}+ad\right)}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{2}}$	30
orering	$-\frac{5db\,x^{4}+4ad+cb}{5\left(b\,x^{4}+a\right)^{\frac{5}{4}} b^{2}}$	30

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(5*b*d*x^4+4*a*d+b*c)/(b*x^4+a)^(5/4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{(5\,bdx^4+bc+4\,ad)(bx^4+a)^{\frac{3}{4}}}{5\,(b^4x^8+2\,ab^3x^4+a^2b^2)}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
-1/5*(5*b*d*x^4 + b*c + 4*a*d)*(b*x^4 + a)^(3/4)/(b^4*x^8 + 2*a*b^3*x^4 +
a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(37) = 74.

Time = 1.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.25

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \begin{cases} -\frac{4ad}{5ab^2\sqrt[4]{a+bx^4}+5b^3x^4\sqrt[4]{a+bx^4}} - \frac{bc}{5ab^2\sqrt[4]{a+bx^4}+5b^3x^4\sqrt[4]{a+bx^4}} - \frac{5bdx^4}{5ab^2\sqrt[4]{a+bx^4}+5b^3x} \\ \frac{\frac{cx^4}{4}+\frac{dx^8}{8}}{a^{\frac{9}{4}}} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
Piecewise((-4*a*d/(5*a*b**2*(a + b*x**4)**(1/4) + 5*b**3*x**4*(a + b*x**4)
**(1/4)) - b*c/(5*a*b**2*(a + b*x**4)**(1/4) + 5*b**3*x**4*(a + b*x**4)**(
1/4)) - 5*b*d*x**4/(5*a*b**2*(a + b*x**4)**(1/4) + 5*b**3*x**4*(a + b*x**4
)**(1/4)), Ne(b, 0)), ((c*x**4/4 + d*x**8/8)/a**(9/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{1}{5}\,d\left(\frac{5}{(bx^4+a)^{\frac{1}{4}}b^2} - \frac{a}{(bx^4+a)^{\frac{5}{4}}b^2}\right) - \frac{c}{5\,(bx^4+a)^{\frac{5}{4}}b}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/5*d*(5/((b*x^4 + a)^(1/4)*b^2) - a/((b*x^4 + a)^(5/4)*b^2)) - 1/5*c/((b
*x^4 + a)^(5/4)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{bc+5\,(bx^4+a)d-ad}{5\,(bx^4+a)^{\frac{5}{4}}b^2}$$

input
```
integrate(x^3*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
-1/5*(b*c + 5*(b*x^4 + a)*d - a*d)/((b*x^4 + a)^(5/4)*b^2)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{5\,d\,(b\,x^4+a)-a\,d+b\,c}{5\,b^2\,(b\,x^4+a)^{5/4}}$$

input
```
int((x^3*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
-(5*d*(a + b*x^4) - a*d + b*c)/(5*b^2*(a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{x^3(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^3}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^3*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**7/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**3/((a + b*x**4)**(1/4)*a**2 + 2*(
a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.148 $\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx=\frac{bc-ad}{5ab\left(a+bx^4\right)^{5/4}}+\frac{c}{a^2\sqrt[4]{a+bx^4}}+\frac{c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{9/4}}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{9/4}}$$

output
```
1/5*(-a*d+b*c)/a/b/(b*x^4+a)^(5/4)+c/a^2/(b*x^4+a)^(1/4)+1/2*c*arctan((b*x
^4+a)^(1/4)/a^(1/4))/a^(9/4)-1/2*c*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(9/4
)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx=\frac{\frac{2\sqrt[4]{a}\left(6abc-a^2d+5b^2cx^4\right)}{b\left(a+bx^4\right)^{5/4}}+5c\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-5c\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{10a^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(x*(a + b*x^4)^(9/4)),x]
```

output
```
((2*a^(1/4)*(6*a*b*c - a^2*d + 5*b^2*c*x^4))/(b*(a + b*x^4)^(5/4)) + 5*c*A
rcTan[(a + b*x^4)^(1/4)/a^(1/4)] - 5*c*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])
/(10*a^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {948, 87, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{dx^4+c}{x^4\left(bx^4+a\right)^{9/4}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{x^4\left(bx^4+a\right)^{5/4}}dx^4}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\frac{c\left(\frac{\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \; 827$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{a}+\frac{4(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(\frac{c\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{a}+\frac{4(bc-ad)}{5ab\,(a+bx^4)^{5/4}}\right)$$

input
```
Int[(c + d*x^4)/(x*(a + b*x^4)^(9/4)),x]
```

output
```
((4*(b*c - a*d))/(5*a*b*(a + b*x^4)^(5/4)) + (c*(4/(a*(a + b*x^4)^(1/4)) -
 (4*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^
(1/4)/a^(1/4)]/(2*a^(1/4))))/a))/a)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$-\frac{-\frac{24cb\,a^{\frac{5}{4}}}{5}+\frac{4d\,a^{\frac{9}{4}}}{5}+b\left(-4b\,x^{4}a^{\frac{1}{4}}+\left(b\,x^{4}+a\right)^{\frac{5}{4}}\left(\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)\right)\right)c}{4a^{\frac{9}{4}}\left(b\,x^{4}+a\right)^{\frac{5}{4}}b}$	104

input
```
int((d*x^4+c)/x/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(9/4)*(-24/5*c*b*a^(5/4)+4/5*d*a^(9/4)+b*(-4*b*x^4*a^(1/4)+(b*x^4+a
)^(5/4)*(ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))-2*arctan(
(b*x^4+a)^(1/4)/a^(1/4)))*c)/(b*x^4+a)^(5/4)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.28

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{9/4}}\,dx =$$

$$-\frac{5\left(a^2b^3x^8+2\,a^3b^2x^4+a^4b\right)\left(\frac{c^4}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(\frac{c^4}{a^9}\right)^{\frac{3}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}c^3\right)+5\left(-i\,a^2b^3x^8-2i\,a^3b^2x^4-i\,a^4b\right)\left(\frac{c^4}{a^9}\right.}{}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
-1/20*(5*(a^2*b^3*x^8 + 2*a^3*b^2*x^4 + a^4*b)*(c^4/a^9)^(1/4)*log(a^7*(c^
4/a^9)^(3/4) + (b*x^4 + a)^(1/4)*c^3) + 5*(-I*a^2*b^3*x^8 - 2*I*a^3*b^2*x^
4 - I*a^4*b)*(c^4/a^9)^(1/4)*log(I*a^7*(c^4/a^9)^(3/4) + (b*x^4 + a)^(1/4)
*c^3) + 5*(I*a^2*b^3*x^8 + 2*I*a^3*b^2*x^4 + I*a^4*b)*(c^4/a^9)^(1/4)*log(
-I*a^7*(c^4/a^9)^(3/4) + (b*x^4 + a)^(1/4)*c^3) - 5*(a^2*b^3*x^8 + 2*a^3*b
^2*x^4 + a^4*b)*(c^4/a^9)^(1/4)*log(-a^7*(c^4/a^9)^(3/4) + (b*x^4 + a)^(1/
4)*c^3) - 4*(5*b^2*c*x^4 + 6*a*b*c - a^2*d)*(b*x^4 + a)^(3/4))/(a^2*b^3*x^
8 + 2*a^3*b^2*x^4 + a^4*b)
```

Sympy [A] (verification not implemented)

Time = 43.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx = d\left(\begin{cases} -\frac{1}{5ab\sqrt[4]{a+bx^4}+5b^2x^4\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{9}{4}}} & \text{otherwise} \end{cases}\right) - \frac{c\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{9}{4}}x^9\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)/x/(b*x**4+a)**(9/4),x)
```

output
```
d*Piecewise((-1/(5*a*b*(a + b*x**4)**(1/4) + 5*b**2*x**4*(a + b*x**4)**(1/
4)), Ne(b, 0)), (x**4/(4*a**(9/4)), True)) - c*gamma(9/4)*hyper((9/4, 9/4)
, (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(9/4)*x**9*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx = \frac{1}{20}\,c\left(\frac{5\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{a^2}+\frac{4\left(5\,bx^4+6\,a\right)}{\left(bx^4+a\right)^{\frac{5}{4}}a^2}\right) - \frac{d}{5\left(bx^4+a\right)^{\frac{5}{4}}b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
1/20*c*(5*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2 + 4*(5*b*x^4
+ 6*a)/((b*x^4 + a)^(5/4)*a^2)) - 1/5*d/((b*x^4 + a)^(5/4)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.25

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{9/4}}\,dx = \frac{\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}\,a^2} + \frac{\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,(-a)^{\frac{1}{4}}\,a^2} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\,c\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a^3} + \frac{\sqrt{2}c\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,(-a)^{\frac{1}{4}}\,a^2} + \frac{5\,(bx^4+a)bc+abc-a^2d}{5\,(bx^4+a)^{\frac{5}{4}}a^2b}$$

input
```
integrate((d*x^4+c)/x/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^(1/4)
)/(-a)^(1/4))/((-a)^(1/4)*a^2) + 1/4*sqrt(2)*c*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2) + 1/8*sqr
t(2)*(-a)^(3/4)*c*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 +
a) + sqrt(-a))/a^3 + 1/8*sqrt(2)*c*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/
4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^2) + 1/5*(5*(b*x^4 + a)*b*c
 + a*b*c - a^2*d)/((b*x^4 + a)^(5/4)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^4}{x\,(a+bx^4)^{9/4}}\,dx = \frac{\frac{c}{5\,a}+\frac{c\,(b\,x^4+a)}{a^2}}{(b\,x^4+a)^{5/4}} - \frac{d}{5\,b\,(b\,x^4+a)^{5/4}} + \frac{c\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{9/4}} - \frac{c\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{9/4}}$$

input
```
int((c + d*x^4)/(x*(a + b*x^4)^(9/4)),x)
```

output
```
(c/(5*a) + (c*(a + b*x^4))/a^2)/(a + b*x^4)^(5/4) - d/(5*b*(a + b*x^4)^(5/
4)) + (c*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(9/4)) - (c*atanh((a + b*x^
4)^(1/4)/a^(1/4)))/(2*a^(9/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x\left(a+bx^4\right)^{9/4}}\,dx = \left(\int \frac{x^3}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^8}dx\right) d + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2x+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^5+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^9}dx\right) c$$

input
```
int((d*x^4+c)/x/(b*x^4+a)^(9/4),x)
```

output
```
int(x**3/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(1/4)*a**2*x + 2*(a
 + b*x**4)**(1/4)*a*b*x**5 + (a + b*x**4)**(1/4)*b**2*x**9),x)*c
```

3.149 $\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{9/4}}\,dx=-\frac{bc-ad}{5a^2\left(a+bx^4\right)^{5/4}}-\frac{2bc-ad}{a^3\sqrt[4]{a+bx^4}}-\frac{c\left(a+bx^4\right)^{3/4}}{4a^3x^4}-\frac{(9bc-4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{13/4}}+\frac{(9bc-4ad)\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{13/4}}$$

output
```
-1/5*(-a*d+b*c)/a^2/(b*x^4+a)^(5/4)-(-a*d+2*b*c)/a^3/(b*x^4+a)^(1/4)-1/4*c
*(b*x^4+a)^(3/4)/a^3/x^4-1/8*(-4*a*d+9*b*c)*arctan((b*x^4+a)^(1/4)/a^(1/4)
)/a^(13/4)+1/8*(-4*a*d+9*b*c)*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{9/4}}\,dx=\frac{-\frac{2\sqrt[4]{a}\left(45b^2cx^8+a^2\left(5c-24dx^4\right)+ab\left(54cx^4-20dx^8\right)\right)}{x^4\left(a+bx^4\right)^{5/4}}+5(-9bc+4ad)\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)+}{40a^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(x^5*(a + b*x^4)^(9/4)),x]
```

output
```
((-2*a^(1/4)*(45*b^2*c*x^8 + a^2*(5*c - 24*d*x^4) + a*b*(54*c*x^4 - 20*d*x
^8)))/(x^4*(a + b*x^4)^(5/4)) + 5*(-9*b*c + 4*a*d)*ArcTan[(a + b*x^4)^(1/4
)/a^(1/4)] + 5*(9*b*c - 4*a*d)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(40*a^(
13/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {948, 87, 61, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{dx^4+c}{x^8 (bx^4+a)^{9/4}} dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4} \left(-\frac{(9bc-4ad) \int \frac{1}{x^4 (bx^4+a)^{9/4}} dx^4}{4a} - \frac{c}{ax^4 (a+bx^4)^{5/4}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(-\frac{(9bc-4ad) \left(\frac{\int \frac{1}{x^4 \left(bx^4+a\right)^{5/4}} dx^4}{a} + \frac{4}{5a(a+bx^4)^{5/4}} \right)}{4a} - \frac{c}{ax^4 (a+bx^4)^{5/4}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4}{a\sqrt[4]{a+bx^4}}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{a\sqrt[4]{a+bx^4}}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \textbf{25}$$

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/4}}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/4}}\right)$$

$\downarrow$ 827

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/}}\right.$$

$\downarrow$ 216

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5/4}}\right.$$

$\downarrow$ 219

$$\frac{1}{4}\left(-\frac{(9bc-4ad)\left(\frac{\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}}{a}+\frac{4}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{ax^4\left(a+bx^4\right)^{5}}\right.$$

input
```
Int[(c + d*x^4)/(x^5*(a + b*x^4)^(9/4)),x]
```

output
```
(-(c/(a*x^4*(a + b*x^4)^(5/4))) - ((9*b*c - 4*a*d)*(4/(5*a*(a + b*x^4)^(5/
4)) + (4/(a*(a + b*x^4)^(1/4)) - (4*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)
]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a)/a))/(4*a))
/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{\frac{54\left(-\frac{10 d x^4}{27}+c\right) b x^4 a^{\frac{5}{4}}}{5}+\left(-\frac{24 d x^4}{5}+c\right) a^{\frac{9}{4}}+\left(9 b^2 c x^4 a^{\frac{1}{4}}+\left(b x^4+a\right)^{\frac{5}{4}}\left(a d-\frac{9 c b}{4}\right)\left(\ln\left(\frac{\left(b x^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b x^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)-2 \arctan\left(\frac{(b x}{}\right)\right)\right)}{4 a^{\frac{13}{4}}\left(b x^4+a\right)^{\frac{5}{4}} x^4}$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(54/5*(-10/27*d*x^4+c)*b*x^4*a^(5/4)+(-24/5*d*x^4+c)*a^(9/4)+(9*b^2*c
*x^4*a^(1/4)+(b*x^4+a)^(5/4)*(a*d-9/4*c*b)*(ln(((b*x^4+a)^(1/4)+a^(1/4))/(
(b*x^4+a)^(1/4)-a^(1/4)))-2*arctan((b*x^4+a)^(1/4)/a^(1/4))))*x^4)/a^(13/4
)/(b*x^4+a)^(5/4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.02

$$\int \frac{c+dx^4}{x^5 \left(a+bx^4\right)^{9/4}}\, dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
-1/80*(5*(a^3*b^2*x^12 + 2*a^4*b*x^8 + a^5*x^4)*((6561*b^4*c^4 - 11664*a*b
^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 2304*a^3*b*c*d^3 + 256*a^4*d^4)/a^13)^(1
/4)*log(a^10*((6561*b^4*c^4 - 11664*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 2
304*a^3*b*c*d^3 + 256*a^4*d^4)/a^13)^(3/4) - (729*b^3*c^3 - 972*a*b^2*c^2*
d + 432*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + 5*(-I*a^3*b^2*x^12
- 2*I*a^4*b*x^8 - I*a^5*x^4)*((6561*b^4*c^4 - 11664*a*b^3*c^3*d + 7776*a^2
*b^2*c^2*d^2 - 2304*a^3*b*c*d^3 + 256*a^4*d^4)/a^13)^(1/4)*log(I*a^10*((65
61*b^4*c^4 - 11664*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 2304*a^3*b*c*d^3 +
 256*a^4*d^4)/a^13)^(3/4) - (729*b^3*c^3 - 972*a*b^2*c^2*d + 432*a^2*b*c*d
^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + 5*(I*a^3*b^2*x^12 + 2*I*a^4*b*x^8 +
I*a^5*x^4)*((6561*b^4*c^4 - 11664*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 230
4*a^3*b*c*d^3 + 256*a^4*d^4)/a^13)^(1/4)*log(-I*a^10*((6561*b^4*c^4 - 1166
4*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 2304*a^3*b*c*d^3 + 256*a^4*d^4)/a^1
3)^(3/4) - (729*b^3*c^3 - 972*a*b^2*c^2*d + 432*a^2*b*c*d^2 - 64*a^3*d^3)*
(b*x^4 + a)^(1/4)) - 5*(a^3*b^2*x^12 + 2*a^4*b*x^8 + a^5*x^4)*((6561*b^4*c
^4 - 11664*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 2304*a^3*b*c*d^3 + 256*a^4
*d^4)/a^13)^(1/4)*log(-a^10*((6561*b^4*c^4 - 11664*a*b^3*c^3*d + 7776*a^2*
b^2*c^2*d^2 - 2304*a^3*b*c*d^3 + 256*a^4*d^4)/a^13)^(3/4) - (729*b^3*c^3 -
 972*a*b^2*c^2*d + 432*a^2*b*c*d^2 - 64*a^3*d^3)*(b*x^4 + a)^(1/4)) + 4*(5
*(9*b^2*c - 4*a*b*d)*x^8 + 6*(9*a*b*c - 4*a^2*d)*x^4 + 5*a^2*c)*(b*x^4 ...
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 132.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{9/4}}\,dx = -\frac{c\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{9}{4}}x^{13}\Gamma\left(\frac{17}{4}\right)} - \frac{d\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{9}{4}}x^{9}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**5/(b*x**4+a)**(9/4),x)
```

output
```
-c*gamma(13/4)*hyper((9/4, 13/4), (17/4,), a*exp_polar(I*pi)/(b*x**4))/(4*
b**(9/4)*x**13*gamma(17/4)) - d*gamma(9/4)*hyper((9/4, 9/4), (13/4,), a*ex
p_polar(I*pi)/(b*x**4))/(4*b**(9/4)*x**9*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^4}{x^5 (a+bx^4)^{9/4}} dx = -\frac{1}{80} c \left(\frac{4 \left(45 (bx^4+a)^2 b - 36 (bx^4+a)ab - 4 a^2 b\right)}{(bx^4+a)^{\frac{9}{4}} a^3 - (bx^4+a)^{\frac{5}{4}} a^4} + \frac{45 b \left(\frac{2 \arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}} - a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}} + a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} \right)}{a^3} \right) + \frac{1}{20} d \left(\frac{5 \left(\frac{2 \arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}} - a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}} + a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} \right)}{a^2} + \frac{4 (5 bx^4 + 6 a)}{(bx^4+a)^{\frac{5}{4}} a^2} \right)$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/80*c*(4*(45*(b*x^4 + a)^2*b - 36*(b*x^4 + a)*a*b - 4*a^2*b)/((b*x^4 + a
)^(9/4)*a^3 - (b*x^4 + a)^(5/4)*a^4) + 45*b*(2*arctan((b*x^4 + a)^(1/4)/a^
(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^
(1/4)))/a^(1/4))/a^3) + 1/20*d*(5*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(
1/4) + log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^
(1/4))/a^2 + 4*(5*b*x^4 + 6*a)/((b*x^4 + a)^(5/4)*a^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(120) = 240$.

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.97

$$\int \frac{c+dx^4}{x^5\left(a+bx^4\right)^{9/4}}\,dx = -\frac{\sqrt{2}(9\,bc-4\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{1}{4}}\,a^3} - \frac{\sqrt{2}(9\,bc-4\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,(-a)^{\frac{1}{4}}\,a^3} + \frac{\sqrt{2}(9\,bc-4\,ad)\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{1}{4}}\,a^3} - \frac{\sqrt{2}(9\,bc-4\,ad)\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{1}{4}}\,a^3} - \frac{10\,(bx^4+a)bc+abc-5\,(bx^4+a)ad-a^2d}{5\,(bx^4+a)^{\frac{5}{4}}a^3} - \frac{(bx^4+a)^{\frac{3}{4}}c}{4\,a^3x^4}$$

input
```
integrate((d*x^4+c)/x^5/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(9*b*c - 4*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^3) - 1/16*sqrt(2)*(9*b*c - 4*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1
/4))/((-a)^(1/4)*a^3) + 1/32*sqrt(2)*(9*b*c - 4*a*d)*log(sqrt(2)*(b*x^4 +
a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^3) - 1/32*
sqrt(2)*(9*b*c - 4*a*d)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b
*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^3) - 1/5*(10*(b*x^4 + a)*b*c + a*b*c -
 5*(b*x^4 + a)*a*d - a^2*d)/((b*x^4 + a)^(5/4)*a^3) - 1/4*(b*x^4 + a)^(3/4
)*c/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{9/4}}\,dx = \frac{\frac{d}{5a}+\frac{d(bx^4+a)}{a^2}}{(bx^4+a)^{5/4}} - \frac{\frac{bc}{5a}+\frac{9bc(bx^4+a)}{5a^2}-\frac{9bc(bx^4+a)^2}{4a^3}}{a(bx^4+a)^{5/4}-(bx^4+a)^{9/4}} + \frac{d\,\mathrm{atan}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{2a^{9/4}} - \frac{d\,\mathrm{atanh}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{2a^{9/4}} - \frac{9bc\,\mathrm{atan}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{8a^{13/4}} + \frac{9bc\,\mathrm{atanh}\left(\frac{(bx^4+a)^{1/4}}{a^{1/4}}\right)}{8a^{13/4}}$$

input
```
int((c + d*x^4)/(x^5*(a + b*x^4)^(9/4)),x)
```

output
```
(d/(5*a) + (d*(a + b*x^4))/a^2)/(a + b*x^4)^(5/4) - ((b*c)/(5*a) + (9*b*c*
(a + b*x^4))/(5*a^2) - (9*b*c*(a + b*x^4)^2)/(4*a^3))/(a*(a + b*x^4)^(5/4)
 - (a + b*x^4)^(9/4)) + (d*atan((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(9/4)) -
(d*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^(9/4)) - (9*b*c*atan((a + b*x^4)
^(1/4)/a^(1/4)))/(8*a^(13/4)) + (9*b*c*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(
8*a^(13/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^5(a+bx^4)^{9/4}}\,dx = \left(\int \frac{1}{(bx^4+a)^{\frac{1}{4}}a^2x^5+2(bx^4+a)^{\frac{1}{4}}ab\,x^9+(bx^4+a)^{\frac{1}{4}}b^2x^{13}}dx\right)c + \left(\int \frac{1}{(bx^4+a)^{\frac{1}{4}}a^2x+2(bx^4+a)^{\frac{1}{4}}ab\,x^5+(bx^4+a)^{\frac{1}{4}}b^2x^9}dx\right)d$$

input
```
int((d*x^4+c)/x^5/(b*x^4+a)^(9/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**2*x**5 + 2*(a + b*x**4)**(1/4)*a*b*x**9 + (a
 + b*x**4)**(1/4)*b**2*x**13),x)*c + int(1/((a + b*x**4)**(1/4)*a**2*x + 2
*(a + b*x**4)**(1/4)*a*b*x**5 + (a + b*x**4)**(1/4)*b**2*x**9),x)*d
```

3.150 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{a(bc-ad)x}{5b^3(a+bx^4)^{5/4}} - \frac{(6bc-11ad)x}{5b^3\sqrt[4]{a+bx^4}} + \frac{dx(a+bx^4)^{3/4}}{4b^3} + \frac{(4bc-9ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{13/4}} + \frac{(4bc-9ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{13/4}}$$

output
```
1/5*a*(-a*d+b*c)*x/b^3/(b*x^4+a)^(5/4)-1/5*(-11*a*d+6*b*c)*x/b^3/(b*x^4+a)
^(1/4)+1/4*d*x*(b*x^4+a)^(3/4)/b^3+1/8*(-9*a*d+4*b*c)*arctan(b^(1/4)*x/(b*
x^4+a)^(1/4))/b^(13/4)+1/8*(-9*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4
))/b^(13/4)
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.83

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(45a^2d+b^2x^4(-24c+5dx^4)+ab(-20c+54dx^4))}{(a+bx^4)^{5/4}} + 5(4bc-9ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + 5(4bc}{40b^{13/4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((2*b^(1/4)*x*(45*a^2*d + b^2*x^4*(-24*c + 5*d*x^4) + a*b*(-20*c + 54*d*x^
4)))/(a + b*x^4)^(5/4) + 5*(4*b*c - 9*a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^
(1/4)] + 5*(4*b*c - 9*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(40*b^(
13/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {957, 817, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (c+dx^4)}{(a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \ 957$$

$$\frac{x^9(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(4bc-9ad)\int \frac{x^8}{(bx^4+a)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \ 817$$

$$\frac{x^9(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$$\Big\downarrow \ 843$$

$$\frac{x^9(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$$\Big\downarrow \text{770}$$

$$\frac{x^9(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$$\Big\downarrow \text{756}$$

$$\frac{x^9(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^9(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$\Big\downarrow$ 219

$$\frac{x^9(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^9)/(5*a*b*(a + b*x^4)^(5/4)) - ((4*b*c - 9*a*d)*(-(x^5/(b*(
a + b*x^4)^(1/4))) + (5*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan[(b^(1/4)
*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)
]/(2*b^(1/4))))/(4*b)))/b))/(5*a*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{9\left(\frac{16\left(-\frac{27 d x^4}{10}+c\right) x a b^{\frac{5}{4}}}{9}+\left(-\frac{4}{9} d x^9+\frac{32}{15} c x^5\right) b^{\frac{9}{4}}-4 a^2 d x b^{\frac{1}{4}}+\left(a d-\frac{4 c b}{9}\right)\left(b x^4+a\right)^{\frac{5}{4}}\left(\ln\left(\frac{x b^{\frac{1}{4}}+\left(b x^4+a\right)^{\frac{1}{4}}}{-x b^{\frac{1}{4}}+\left(b x^4+a\right)^{\frac{1}{4}}}\right)-2 \arctan\right.\right.}{16 b^{\frac{13}{4}}\left(b x^4+a\right)^{\frac{5}{4}}}$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-9/16/b^(13/4)*(16/9*(-27/10*d*x^4+c)*x*a*b^(5/4)+(-4/9*d*x^9+32/15*c*x^5)
*b^(9/4)-4*a^2*d*x*b^(1/4)+(a*d-4/9*c*b)*(b*x^4+a)^(5/4)*(ln((x*b^(1/4)+(b
*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))-2*arctan((b*x^4+a)^(1/4)/x/b^
(1/4))))/(b*x^4+a)^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 884, normalized size of antiderivative = 5.85

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/80*(5*(b^5*x^8 + 2*a*b^4*x^4 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d
 + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(1/4)*lo
g(-(b^10*x*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664
*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(3/4) + (64*b^3*c^3 - 432*a*b^2*c^2*d +
 972*a^2*b*c*d^2 - 729*a^3*d^3)*(b*x^4 + a)^(1/4))/x) - 5*(b^5*x^8 + 2*a*b
^4*x^4 + a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2
- 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(1/4)*log((b^10*x*((256*b^4*c^4
- 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d
^4)/b^13)^(3/4) - (64*b^3*c^3 - 432*a*b^2*c^2*d + 972*a^2*b*c*d^2 - 729*a^
3*d^3)*(b*x^4 + a)^(1/4))/x) - 5*(-I*b^5*x^8 - 2*I*a*b^4*x^4 - I*a^2*b^3)*
((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^
3 + 6561*a^4*d^4)/b^13)^(1/4)*log((I*b^10*x*((256*b^4*c^4 - 2304*a*b^3*c^3
*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(3/4)
- (64*b^3*c^3 - 432*a*b^2*c^2*d + 972*a^2*b*c*d^2 - 729*a^3*d^3)*(b*x^4 +
a)^(1/4))/x) - 5*(I*b^5*x^8 + 2*I*a*b^4*x^4 + I*a^2*b^3)*((256*b^4*c^4 - 2
304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)
/b^13)^(1/4)*log((-I*b^10*x*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^
2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)/b^13)^(3/4) - (64*b^3*c^3 -
432*a*b^2*c^2*d + 972*a^2*b*c*d^2 - 729*a^3*d^3)*(b*x^4 + a)^(1/4))/x) + 4
*(5*b^2*d*x^9 - 6*(4*b^2*c - 9*a*b*d)*x^5 - 5*(4*a*b*c - 9*a^2*d)*x)*(b...
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{13}{4}\right)} + \frac{dx^{13}\Gamma\left(\frac{13}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**9*gamma(9/4)*hyper((9/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(9/4)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((9/4, 13/4), (17/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(9/4)*gamma(17/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(123) = 246$.

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{1}{20}\left(\frac{4\left(b+\frac{5\,(bx^4+a)}{x^4}\right)x^5}{(bx^4+a)^{\frac{5}{4}}b^2}+\frac{5\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)c + \frac{1}{80}d\left(\frac{4\left(4\,ab^2+\frac{36\,(bx^4+a)ab}{x^4}-\frac{45\,(bx^4+a)^2a}{x^8}\right)}{\frac{(bx^4+a)^{\frac{5}{4}}b^4}{x^5}-\frac{(bx^4+a)^{\frac{9}{4}}b^3}{x^9}}+\frac{45\,a\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^3}\right)$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/20*(4*(b + 5*(b*x^4 + a)/x^4)*x^5/((b*x^4 + a)^(5/4)*b^2) + 5*(2*arctan
((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4
)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2)*c + 1/80*d*(4*(4*a*b^2
 + 36*(b*x^4 + a)*a*b/x^4 - 45*(b*x^4 + a)^2*a/x^8)/((b*x^4 + a)^(5/4)*b^4
/x^5 - (b*x^4 + a)^(9/4)*b^3/x^9) + 45*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1
/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 +
 a)^(1/4)/x))/b^(1/4))/b^3)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**12/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**8/((a + b*x**4)**(1/4)*a**2 + 2*
(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.151 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{(bc-ad)x^5}{5ab\,(a+bx^4)^{5/4}} - \frac{dx}{b^2\sqrt[4]{a+bx^4}} + \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{9/4}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{9/4}}$$

output
```
1/5*(-a*d+b*c)*x^5/a/b/(b*x^4+a)^(5/4)-d*x/b^2/(b*x^4+a)^(1/4)+1/2*d*arcta
n(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/
4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(-5a^2d+b^2cx^4-6abdx^4)}{a(a+bx^4)^{5/4}} + 5d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + 5d\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{10b^{9/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((2*b^(1/4)*x*(-5*a^2*d + b^2*c*x^4 - 6*a*b*d*x^4))/(a*(a + b*x^4)^(5/4))
+ 5*d*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 5*d*ArcTanh[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)])/(10*b^(9/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {954, 817, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c + dx^4)}{(a + bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \; 954$$

$$\frac{d \int \frac{x^4}{(bx^4+a)^{5/4}} dx}{b} + \frac{x^5 (bc - ad)}{5ab (a + bx^4)^{5/4}}$$

$$\Big\downarrow \; 817$$

$$\frac{d \left(\frac{\int \frac{1}{\sqrt[4]{bx^4+a}} dx}{b} - \frac{x}{b \sqrt[4]{a + bx^4}} \right)}{b} + \frac{x^5 (bc - ad)}{5ab (a + bx^4)^{5/4}}$$

$$\Big\downarrow \; 770$$

$$\frac{d \left(\frac{\int \frac{1}{1 - \frac{bx^4}{bx^4+a}} d \frac{x}{\sqrt[4]{bx^4+a}}}{b} - \frac{x}{b \sqrt[4]{a + bx^4}} \right)}{b} + \frac{x^5 (bc - ad)}{5ab (a + bx^4)^{5/4}}$$

$$\Big\downarrow \; 756$$

$$\frac{d \left(\frac{\frac{1}{2} \int \frac{1}{1 - \frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}} d \frac{x}{\sqrt[4]{bx^4+a}} + \frac{1}{2} \int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}} + 1} d \frac{x}{\sqrt[4]{bx^4+a}}}{b} - \frac{x}{b \sqrt[4]{a + bx^4}} \right)}{b} + \frac{x^5 (bc - ad)}{5ab (a + bx^4)^{5/4}}$$

$$\Big\downarrow \; 216$$

$$\frac{d\left(\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{b}+\frac{x^5(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}$$

$\big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}\right)}{b}+\frac{x^5(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^5)/(5*a*b*(a + b*x^4)^(5/4)) + (d*(-(x/(b*(a + b*x^4)^(1/4)
)) + (ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)
*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))/b))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$-\frac{\frac{12 a b^{\frac{5}{4}} d x^{5}}{5}-\frac{2 b^{\frac{9}{4}} c x^{5}}{5}+d \left(2 a x b^{\frac{1}{4}}+\frac{\left(b x^{4}+a\right)^{\frac{5}{4}} \left(2 \arctan\left(\frac{\left(b x^{4}+a\right)^{\frac{1}{4}}}{x b^{\frac{1}{4}}}\right)-\ln\left(\frac{x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}{-x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}\right)\right)}{2}\right) a}{2 b^{\frac{9}{4}} \left(b x^{4}+a\right)^{\frac{5}{4}} a}$	117

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^(9/4)/(b*x^4+a)^(5/4)*(12/5*a*b^(5/4)*d*x^5-2/5*b^(9/4)*c*x^5+d*(2*
a*x*b^(1/4)+1/2*(b*x^4+a)^(5/4)*(2*arctan((b*x^4+a)^(1/4)/x/b^(1/4))-ln((x
*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))))*a)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.28

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{5\left(ab^4x^8+2\,a^2b^3x^4+a^3b^2\right)\left(\frac{d^4}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{b^7x\left(\frac{d^4}{b^9}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}d^3}{x}\right)-5\left(ab^4x^8+2\,a^2b^3x^4+\right.}{}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/20*(5*(a*b^4*x^8 + 2*a^2*b^3*x^4 + a^3*b^2)*(d^4/b^9)^(1/4)*log((b^7*x*(
d^4/b^9)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - 5*(a*b^4*x^8 + 2*a^2*b^3*x^4
+ a^3*b^2)*(d^4/b^9)^(1/4)*log(-(b^7*x*(d^4/b^9)^(3/4) - (b*x^4 + a)^(1/4)
*d^3)/x) - 5*(I*a*b^4*x^8 + 2*I*a^2*b^3*x^4 + I*a^3*b^2)*(d^4/b^9)^(1/4)*l
og((I*b^7*x*(d^4/b^9)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - 5*(-I*a*b^4*x^8
- 2*I*a^2*b^3*x^4 - I*a^3*b^2)*(d^4/b^9)^(1/4)*log((-I*b^7*x*(d^4/b^9)^(3/
4) + (b*x^4 + a)^(1/4)*d^3)/x) + 4*((b^2*c - 6*a*b*d)*x^5 - 5*a^2*d*x)*(b*
x^4 + a)^(3/4))/(a*b^4*x^8 + 2*a^2*b^3*x^4 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{4}\right)}{4a^{\frac{9}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+4a^{\frac{5}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)} + \frac{dx^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**5*gamma(5/4)/(4*a**(9/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 4*a**(5/4
)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4)) + d*x**9*gamma(9/4)*hyper((9/4,
 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(9/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^5}{5\,(bx^4+a)^{\frac{5}{4}}a} - \frac{1}{20}\left(\frac{4\left(b+\frac{5\,(bx^4+a)}{x^4}\right)x^5}{(bx^4+a)^{\frac{5}{4}}b^2} + \frac{5\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^2}\right)d$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
1/5*c*x^5/((b*x^4 + a)^(5/4)*a) - 1/20*(4*(b + 5*(b*x^4 + a)/x^4)*x^5/((b*
x^4 + a)^(5/4)*b^2) + 5*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) +
 log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(
1/4))/b^2)*d
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\int \frac{x^4\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d+\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**4/((a + b*x**4)**(1/4)*a**2 + 2*(
a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.152 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}} \, dx = \frac{(bc-ad)x}{5ab\left(a+bx^4\right)^{5/4}} + \frac{(4bc+ad)x}{5a^2b\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-a*d+b*c)*x/a/b/(b*x^4+a)^(5/4)+1/5*(a*d+4*b*c)*x/a^2/b/(b*x^4+a)^(1/
4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{9/4}} \, dx = \frac{x(5ac+4bcx^4+adx^4)}{5a^2\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(9/4),x]
```

output
```
(x*(5*a*c + 4*b*c*x^4 + a*d*x^4))/(5*a^2*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {903, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \text{903}$$

$$\frac{4c\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{4cx}{5a^2\sqrt[4]{a+bx^4}}+\frac{x(c+dx^4)}{5a(a+bx^4)^{5/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(9/4),x]
```

output
```
(4*c*x)/(5*a^2*(a + b*x^4)^(1/4)) + (x*(c + d*x^4))/(5*a*(a + b*x^4)^(5/4)
)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 903
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Simp[
c*(q/(a*(p + 1)))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
 0] && GtQ[q, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5 \left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
trager	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5 \left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
pseudoelliptic	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5 \left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34
orering	$\frac{x\left(a d\,x^{4}+4 b c\,x^{4}+5 a c\right)}{5 \left(b\,x^{4}+a\right)^{\frac{5}{4}} a^{2}}$	34

input
```
int((d*x^4+c)/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/5*x*(a*d*x^4+4*b*c*x^4+5*a*c)/(b*x^4+a)^(5/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = \frac{((4\,bc+ad)x^5+5\,acx)(bx^4+a)^{\frac{3}{4}}}{5\,(a^2b^2x^8+2\,a^3bx^4+a^4)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/5*((4*b*c + a*d)*x^5 + 5*a*c*x)*(b*x^4 + a)^(3/4)/(a^2*b^2*x^8 + 2*a^3*b
*x^4 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(53) = 106$.

Time = 18.88 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.11

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = c\left(\frac{5ax\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)} + \frac{4bx^5\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}\right) + \frac{dx^5\Gamma\left(\frac{5}{4}\right)}{4a^{\frac{9}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+4a^{\frac{5}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*(5*a*x*gamma(1/4)/(16*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a*
*(9/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4)) + 4*b*x**5*gamma(1/4)/(16*
a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/4)*b*x**4*(1 + b*x**
4/a)**(1/4)*gamma(9/4))) + d*x**5*gamma(5/4)/(4*a**(9/4)*(1 + b*x**4/a)**(
1/4)*gamma(9/4) + 4*a**(5/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx = -\frac{\left(b-\frac{5\,(bx^4+a)}{x^4}\right)cx^5}{5\,(bx^4+a)^{\frac{5}{4}}a^2} + \frac{dx^5}{5\,(bx^4+a)^{\frac{5}{4}}a}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/5*(b - 5*(b*x^4 + a)/x^4)*c*x^5/((b*x^4 + a)^(5/4)*a^2) + 1/5*d*x^5/((b
*x^4 + a)^(5/4)*a)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(9/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\frac{5\,a\,c\,x+a\,d\,x^5+4\,b\,c\,x^5}{5\,a^2\,\left(b\,x^4+a\right)^{5/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(9/4),x)
```

output
```
(5*a*c*x + a*d*x^5 + 4*b*c*x^5)/(5*a^2*(a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{9/4}}\,dx=\left(\int \frac{x^4}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^8}dx\right)d+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^8}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
b*x**4)**(1/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(1/4)*a**2 + 2*(a +
b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.153 $\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx=-\frac{c}{3ax^3\left(a+bx^4\right)^{5/4}}-\frac{(8bc-3ad)x}{15a^2\left(a+bx^4\right)^{5/4}}-\frac{4(8bc-3ad)x}{15a^3\sqrt[4]{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(5/4)-1/15*(-3*a*d+8*b*c)*x/a^2/(b*x^4+a)^(5/4)-4/1
5*(-3*a*d+8*b*c)*x/a^3/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx=\frac{-5a^2c-40abcx^4+15a^2dx^4-32b^2cx^8+12abdx^8}{15a^3x^3\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(9/4)),x]
```

output
```
(-5*a^2*c - 40*a*b*c*x^4 + 15*a^2*d*x^4 - 32*b^2*c*x^8 + 12*a*b*d*x^8)/(15
*a^3*x^3*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(8bc-3ad)\int \frac{1}{(bx^4+a)^{9/4}}dx}{3a}-\frac{c}{3ax^3 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(8bc-3ad)\left(\frac{4\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{3a}-\frac{c}{3ax^3 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(\frac{4x}{5a^2 \sqrt[4]{a+bx^4}}+\frac{x}{5a(a+bx^4)^{5/4}}\right)(8bc-3ad)}{3a}-\frac{c}{3ax^3 (a+bx^4)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(9/4)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(5/4)) - ((8*b*c - 3*a*d)*(x/(5*a*(a + b*x^4)^(5
/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(3*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$\frac{(15d\,x^4-5c)a^2-40\left(-\frac{3d\,x^4}{10}+c\right)b\,x^4a-32b^2c\,x^8}{15(b\,x^4+a)^{\frac{5}{4}}x^3a^3}$	57
gosper	$-\frac{-12abd\,x^8+32b^2c\,x^8-15a^2d\,x^4+40abc\,x^4+5a^2c}{15x^3(b\,x^4+a)^{\frac{5}{4}}a^3}$	59
trager	$-\frac{-12abd\,x^8+32b^2c\,x^8-15a^2d\,x^4+40abc\,x^4+5a^2c}{15x^3(b\,x^4+a)^{\frac{5}{4}}a^3}$	59
orering	$-\frac{-12abd\,x^8+32b^2c\,x^8-15a^2d\,x^4+40abc\,x^4+5a^2c}{15x^3(b\,x^4+a)^{\frac{5}{4}}a^3}$	59
risch	$-\frac{c(b\,x^4+a)^{\frac{3}{4}}}{3a^3x^3}+\frac{(b\,x^4+a)^{\frac{3}{4}}x(4abd\,x^4-9b^2c\,x^4+5a^2d-10abc)}{5a^3(x^8b^2+2a\,x^4b+a^2)}$	84

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/15*((15*d*x^4-5*c)*a^2-40*(-3/10*d*x^4+c)*b*x^4*a-32*b^2*c*x^8)/(b*x^4+a
)^(5/4)/x^3/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx = -\frac{(4\,(8\,b^2c-3\,abd)x^8+5\,(8\,abc-3\,a^2d)x^4+5\,a^2c)(bx^4+a)^{\frac{3}{4}}}{15\,(a^3b^2x^{11}+2\,a^4bx^7+a^5x^3)}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
-1/15*(4*(8*b^2*c - 3*a*b*d)*x^8 + 5*(8*a*b*c - 3*a^2*d)*x^4 + 5*a^2*c)*(b
*x^4 + a)^(3/4)/(a^3*b^2*x^11 + 2*a^4*b*x^7 + a^5*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(75) = 150.

Time = 39.94 (sec) , antiderivative size = 360, normalized size of antiderivative = 4.56

$$\begin{aligned}\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx = {}& c\left(\frac{5a^2b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{64a^5b^4\Gamma\left(\frac{9}{4}\right)+128a^4b^5x^4\Gamma\left(\frac{9}{4}\right)+64a^3b^6x^8\Gamma\left(\frac{9}{4}\right)}\right.\\
&+\frac{40ab^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{64a^5b^4\Gamma\left(\frac{9}{4}\right)+128a^4b^5x^4\Gamma\left(\frac{9}{4}\right)+64a^3b^6x^8\Gamma\left(\frac{9}{4}\right)}\\
&\left.+\frac{32b^{\frac{27}{4}}x^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{64a^5b^4\Gamma\left(\frac{9}{4}\right)+128a^4b^5x^4\Gamma\left(\frac{9}{4}\right)+64a^3b^6x^8\Gamma\left(\frac{9}{4}\right)}\right)\\
&+d\left(\frac{5ax\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}\right.\\
&\left.+\frac{4bx^5\Gamma\left(\frac{1}{4}\right)}{16a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)+16a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{9}{4}\right)}\right)\end{aligned}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(9/4),x)
```

output
```
c*(5*a**2*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4*gamm
a(9/4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4)) + 4
0*a*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4*gamma
(9/4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4)) + 32
*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4*gamma(9/
4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4))) + d*(5
*a*x*gamma(1/4)/(16*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/
4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(9/4)) + 4*b*x**5*gamma(1/4)/(16*a**(
13/4)*(1 + b*x**4/a)**(1/4)*gamma(9/4) + 16*a**(9/4)*b*x**4*(1 + b*x**4/a)
**(1/4)*gamma(9/4)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{9/4}}\,dx = -\frac{\left(b-\frac{5\left(bx^4+a\right)}{x^4}\right)dx^5}{5\left(bx^4+a\right)^{\frac{5}{4}}a^2} + \frac{1}{15}\left(\frac{3\left(b^2-\frac{10\left(bx^4+a\right)b}{x^4}\right)x^5}{\left(bx^4+a\right)^{\frac{5}{4}}a^3} - \frac{5\left(bx^4+a\right)^{\frac{3}{4}}}{a^3x^3}\right)c$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
-1/5*(b - 5*(b*x^4 + a)/x^4)*d*x^5/((b*x^4 + a)^(5/4)*a^2) + 1/15*(3*(b^2
- 10*(b*x^4 + a)*b/x^4)*x^5/((b*x^4 + a)^(5/4)*a^3) - 5*(b*x^4 + a)^(3/4)/
(a^3*x^3))*c
```

Giac [F]

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{9/4}}\,dx = \frac{3\,a^2\,c-32\,c\,(b\,x^4+a)^2+3\,a^2\,d\,x^4+24\,a\,c\,(b\,x^4+a)+12\,a\,d\,x^4\,(b\,x^4+a)}{15\,a^3\,x^3\,(b\,x^4+a)^{5/4}}$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(9/4)),x)
```

output
```
(3*a^2*c - 32*c*(a + b*x^4)^2 + 3*a^2*d*x^4 + 24*a*c*(a + b*x^4) + 12*a*d*
x^4*(a + b*x^4))/(15*a^3*x^3*(a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{9/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2x^4+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{12}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(9/4),x)
```

output

```
int(1/((a + b*x**4)**(1/4)*a**2*x**4 + 2*(a + b*x**4)**(1/4)*a*b*x**8 + (a
 + b*x**4)**(1/4)*b**2*x**12),x)*c + int(1/((a + b*x**4)**(1/4)*a**2 + 2*(
a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*d
```

3.154 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{9/4}}\,dx = -\frac{c}{7ax^7\left(a+bx^4\right)^{5/4}} - \frac{12bc-7ad}{35a^2x^3\left(a+bx^4\right)^{5/4}} - \frac{8(12bc-7ad)}{35a^3x^3\sqrt[4]{a+bx^4}} + \frac{32(12bc-7ad)\left(a+bx^4\right)^{3/4}}{105a^4x^3}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(5/4)-1/35*(-7*a*d+12*b*c)/a^2/x^3/(b*x^4+a)^(5/4)-
8/35*(-7*a*d+12*b*c)/a^3/x^3/(b*x^4+a)^(1/4)+32/105*(-7*a*d+12*b*c)*(b*x^4
+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{9/4}}\,dx = \frac{-15a^3c+60a^2bcx^4-35a^3dx^4+480ab^2cx^8-280a^2bdx^8+384b^3cx^{12}-224ab^2dx^{1}}{105a^4x^7\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(9/4)),x]
```

output
```
(-15*a^3*c + 60*a^2*b*c*x^4 - 35*a^3*d*x^4 + 480*a*b^2*c*x^8 - 280*a^2*b*d
*x^8 + 384*b^3*c*x^12 - 224*a*b^2*d*x^12)/(105*a^4*x^7*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(12bc-7ad)\int \frac{1}{x^4(bx^4+a)^{9/4}}dx}{7a}-\frac{c}{7ax^7 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(12bc-7ad)\left(-\frac{8b\int \frac{1}{(bx^4+a)^{9/4}}dx}{3a}-\frac{1}{3ax^3(a+bx^4)^{5/4}}\right)}{7a}-\frac{c}{7ax^7 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(12bc-7ad)\left(-\frac{8b\left(\frac{4\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{5/4}}\right)}{7a}-\frac{c}{7ax^7 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(-\frac{8b\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{5/4}}\right)(12bc-7ad)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(9/4)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(5/4)) - ((12*b*c - 7*a*d)*(-1/3*1/(a*x^3*(a + b
*x^4)^(5/4)) - (8*b*(x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^
(1/4))))/(3*a)))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{(-35 d x^{4}-15 c) a^{3}+60 b x^{4} \left(-\frac{14 d x^{4}}{3}+c \right) a^{2}+480 b^{2} x^{8} \left(-\frac{7 d x^{4}}{15}+c \right) a+384 b^{3} c x^{12}}{105 (b x^{4}+a)^{\frac{5}{4}} x^{7} a^{4}}$	76
gosper	$-\frac{224 a b^{2} d x^{12}-384 b^{3} c x^{12}+280 a^{2} b d x^{8}-480 a b^{2} c x^{8}+35 a^{3} d x^{4}-60 a^{2} b c x^{4}+15 c a^{3}}{105 x^{7} (b x^{4}+a)^{\frac{5}{4}} a^{4}}$	83
trager	$-\frac{224 a b^{2} d x^{12}-384 b^{3} c x^{12}+280 a^{2} b d x^{8}-480 a b^{2} c x^{8}+35 a^{3} d x^{4}-60 a^{2} b c x^{4}+15 c a^{3}}{105 x^{7} (b x^{4}+a)^{\frac{5}{4}} a^{4}}$	83
orering	$-\frac{224 a b^{2} d x^{12}-384 b^{3} c x^{12}+280 a^{2} b d x^{8}-480 a b^{2} c x^{8}+35 a^{3} d x^{4}-60 a^{2} b c x^{4}+15 c a^{3}}{105 x^{7} (b x^{4}+a)^{\frac{5}{4}} a^{4}}$	83
risch	$-\frac{(b x^{4}+a)^{\frac{3}{4}} (7 a d x^{4}-18 b c x^{4}+3 a c)}{21 a^{4} x^{7}}-\frac{(b x^{4}+a)^{\frac{3}{4}} x (9 a b d x^{4}-14 b^{2} c x^{4}+10 a^{2} d-15 a b c) b}{5 a^{4} (x^{8} b^{2}+2 a x^{4} b+a^{2})}$	103

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-35*d*x^4-15*c)*a^3+60*b*x^4*(-14/3*d*x^4+c)*a^2+480*b^2*x^8*(-7/1
5*d*x^4+c)*a+384*b^3*c*x^12)/(b*x^4+a)^(5/4)/x^7/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{c + dx^4}{x^8 \left(a + bx^4\right)^{9/4}}\, dx = \frac{(32\,(12\,b^3c - 7\,ab^2d)x^{12} + 40\,(12\,ab^2c - 7\,a^2bd)x^8 + 5\,(12\,a^2bc - 7\,a^3d)x^4 - 15\,a^3c)}{105\,(a^4b^2x^{15} + 2\,a^5bx^{11} + a^6x^7)}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
1/105*(32*(12*b^3*c - 7*a*b^2*d)*x^12 + 40*(12*a*b^2*c - 7*a^2*b*d)*x^8 +
5*(12*a^2*b*c - 7*a^3*d)*x^4 - 15*a^3*c)*(b*x^4 + a)^(3/4)/(a^4*b^2*x^15 +
 2*a^5*b*x^11 + a^6*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(107) = 214.

Time = 79.39 (sec) , antiderivative size = 726, normalized size of antiderivative = 6.42

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{9/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(9/4),x)
```

output
```
c*(-15*a**4*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(256*a**7*b**9*x
**4*gamma(9/4) + 768*a**6*b**10*x**8*gamma(9/4) + 768*a**5*b**11*x**12*gam
ma(9/4) + 256*a**4*b**12*x**16*gamma(9/4)) + 45*a**3*b**(43/4)*x**4*(a/(b*
x**4) + 1)**(3/4)*gamma(-7/4)/(256*a**7*b**9*x**4*gamma(9/4) + 768*a**6*b*
*10*x**8*gamma(9/4) + 768*a**5*b**11*x**12*gamma(9/4) + 256*a**4*b**12*x**
16*gamma(9/4)) + 540*a**2*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-7/
4)/(256*a**7*b**9*x**4*gamma(9/4) + 768*a**6*b**10*x**8*gamma(9/4) + 768*a
**5*b**11*x**12*gamma(9/4) + 256*a**4*b**12*x**16*gamma(9/4)) + 864*a*b**(
51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(256*a**7*b**9*x**4*gamma(
9/4) + 768*a**6*b**10*x**8*gamma(9/4) + 768*a**5*b**11*x**12*gamma(9/4) +
256*a**4*b**12*x**16*gamma(9/4)) + 384*b**(55/4)*x**16*(a/(b*x**4) + 1)**(
3/4)*gamma(-7/4)/(256*a**7*b**9*x**4*gamma(9/4) + 768*a**6*b**10*x**8*gamm
a(9/4) + 768*a**5*b**11*x**12*gamma(9/4) + 256*a**4*b**12*x**16*gamma(9/4)
)) + d*(5*a**2*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4
*gamma(9/4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4)
) + 40*a*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4*
gamma(9/4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4))
 + 32*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(64*a**5*b**4*gam
ma(9/4) + 128*a**4*b**5*x**4*gamma(9/4) + 64*a**3*b**6*x**8*gamma(9/4)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{9/4}}\,dx = \frac{1}{15}\left(\frac{3\left(b^2-\frac{10\,(bx^4+a)b}{x^4}\right)x^5}{(bx^4+a)^{\frac{5}{4}}a^3}-\frac{5\,(bx^4+a)^{\frac{3}{4}}}{a^3x^3}\right)d - \frac{1}{35}c\left(\frac{7\left(b^3-\frac{15\,(bx^4+a)b^2}{x^4}\right)x^5}{(bx^4+a)^{\frac{5}{4}}a^4}-\frac{5\left(\frac{7\,(bx^4+a)^{\frac{3}{4}}b}{x^3}-\frac{(bx^4+a)^{\frac{7}{4}}}{x^7}\right)}{a^4}\right)$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
1/15*(3*(b^2 - 10*(b*x^4 + a)*b/x^4)*x^5/((b*x^4 + a)^(5/4)*a^3) - 5*(b*x^
4 + a)^(3/4)/(a^3*x^3))*d - 1/35*c*(7*(b^3 - 15*(b*x^4 + a)*b^2/x^4)*x^5/(
(b*x^4 + a)^(5/4)*a^4) - 5*(7*(b*x^4 + a)^(3/4)*b/x^3 - (b*x^4 + a)^(7/4)/
x^7)/a^4)
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.30

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{9/4}}\,dx = \frac{x\left(\frac{b\left(2\,a\,d-3\,b\,c\right)}{2\,a^3}-\frac{a\left(\frac{2\,b^3\,c-a\,b^2\,d}{5\,a^4}+\frac{7\,b^2\,\left(2\,a\,d-3\,b\,c\right)}{10\,a^4}\right)}{b}\right)}{\left(b\,x^4+a\right)^{5/4}} - \frac{c\,\left(b\,x^4+a\right)^{3/4}}{7\,a^3\,x^7}-\frac{\left(b\,x^4+a\right)^{3/4}\,\left(7\,a^3\,d-18\,a^2\,b\,c\right)}{21\,a^6\,x^3}+\frac{x\,\left(14\,b^2\,c-9\,a\,b\,d\right)}{5\,a^4\,\left(b\,x^4+a\right)^{1/4}}$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(9/4)),x)
```

output
```
(x*((b*(2*a*d - 3*b*c))/(2*a^3) - (a*((2*b^3*c - a*b^2*d)/(5*a^4) + (7*b^2
*(2*a*d - 3*b*c))/(10*a^4)))/b))/(a + b*x^4)^(5/4) - (c*(a + b*x^4)^(3/4))
/(7*a^3*x^7) - ((a + b*x^4)^(3/4)*(7*a^3*d - 18*a^2*b*c))/(21*a^6*x^3) + (
x*(14*b^2*c - 9*a*b*d))/(5*a^4*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{9/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,a^2x^8+2\left(b\,x^4+a\right)^{\frac{1}{4}}\,ab\,x^{12}+\left(b\,x^4+a\right)^{\frac{1}{4}}\,b^2x^{16}}dx\right)c + \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,a^2x^4+2\left(b\,x^4+a\right)^{\frac{1}{4}}\,ab\,x^8+\left(b\,x^4+a\right)^{\frac{1}{4}}\,b^2x^{12}}dx\right)d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(9/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**2*x**8 + 2*(a + b*x**4)**(1/4)*a*b*x**12 + (
a + b*x**4)**(1/4)*b**2*x**16),x)*c + int(1/((a + b*x**4)**(1/4)*a**2*x**4
 + 2*(a + b*x**4)**(1/4)*a*b*x**8 + (a + b*x**4)**(1/4)*b**2*x**12),x)*d
```

3.155 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{(bc-ad)x^7}{5b^2(a+bx^4)^{5/4}} + \frac{7(6bc-11ad)x^3}{60b^3\sqrt[4]{a+bx^4}} + \frac{dx^7}{6b^2\sqrt[4]{a+bx^4}} + \frac{7\sqrt{a}(6bc-11ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(5/4)+7/60*(-11*a*d+6*b*c)*x^3/b^3/(b*x^
4+a)^(1/4)+1/6*d*x^7/b^2/(b*x^4+a)^(1/4)+7/20*a^(1/2)*(-11*a*d+6*b*c)*(1+a
/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^
(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{x^3\left(-77a^2d+ab(42c-22dx^4)+4b^2x^4(3c+dx^4)+7(-6bc+11ad)\,(a+bx^4)\,\sqrt[4]{1+\right.}{24b^3\,(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^3*(-77*a^2*d + a*b*(42*c - 22*d*x^4) + 4*b^2*x^4*(3*c + d*x^4) + 7*(-6*
b*c + 11*a*d)*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4
, 7/4, -((b*x^4)/a)]))/(24*b^3*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 815, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{11}(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(6bc-11ad)\int \frac{x^{10}}{(bx^4+a)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\int \frac{x^6}{(bx^4+a)^{5/4}}dx}{6b}\right)}{5ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{2b}\right)}{6b}\right)}{5ab}$$

$$\Big\downarrow \text{813}$$

$$\frac{\frac{x^{11}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5ab}$$

$\Big\downarrow$ 858

$$\frac{\frac{x^{11}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5ab}$$

$\Big\downarrow$ 807

$$\frac{\frac{x^{11}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5ab}$$

$\Big\downarrow$ 212

$$\frac{\frac{x^{11}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-(6bc-11ad)\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5ab}$$

input
```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^11)/(5*a*b*(a + b*x^4)^(5/4)) - ((6*b*c - 11*a*d)*(x^7/(6*b
*(a + b*x^4)^(1/4)) - (7*a*(x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 +
a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/
2)*(a + b*x^4)^(1/4))))/(6*b)))/(5*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{10}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{9}{4}}}dx$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^14 + c*x^10)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a
^2*b*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 63.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{15}{4}\right)} + \frac{dx^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{15}{4}\\\frac{19}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**11*gamma(11/4)*hyper((9/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/
(4*a**(9/4)*gamma(15/4)) + d*x**15*gamma(15/4)*hyper((9/4, 15/4), (19/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(9/4)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**14/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**10/((a + b*x**4)**(1/4)*a**2 + 2
*(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.156 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{(bc-ad)x^3}{5b^2(a+bx^4)^{5/4}} + \frac{dx^3}{2b^2\sqrt[4]{a+bx^4}} - \frac{3(2bc-7ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{10\sqrt{a}b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(5/4)+1/2*d*x^3/b^2/(b*x^4+a)^(1/4)-3/10
*(-7*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a
^(1/2))),2^(1/2))/a^(1/2)/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{x^3\left(a(-2bc+7ad+2bdx^4) + (2bc-7ad)(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\right(}{4ab^2(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^3*(a*(-2*b*c + 7*a*d + 2*b*d*x^4) + (2*b*c - 7*a*d)*(a + b*x^4)*(1 + (b
*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^4)/a)]))/(4*a*b^2*(
a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^7(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(2bc-7ad)\int \frac{x^6}{(bx^4+a)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^7(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(2bc-7ad)\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{2b}\right)}{5ab}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^7(bc-ad)}{5ab(a+bx^4)^{5/4}}-\frac{(2bc-7ad)\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^7(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(2bc-7ad)\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$\downarrow$ 807

$$\frac{x^7(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(2bc-7ad)\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

$\downarrow$ 212

$$\frac{x^7(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(2bc-7ad)\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5ab}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^7)/(5*a*b*(a + b*x^4)^(5/4)) - ((2*b*c - 7*a*d)*(x^3/(2*b*(
a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sq
rt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))))/(5*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^6(d\,x^4+c)}{(b\,x^4+a)^{\frac{9}{4}}}dx$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^6)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^
2*b*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**7*gamma(7/4)*hyper((7/4, 9/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(9/4)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((9/4, 11/4), (15/4,), b*
x**4*exp_polar(I*pi)/a)/(4*a**(9/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**10/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**6/((a + b*x**4)**(1/4)*a**2 + 2*
(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.157 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\frac{(bc-ad)x^3}{5ab\,(a+bx^4)^{5/4}}-\frac{(2bc+3ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(5/4)-1/5*(3*a*d+2*b*c)*(1+a/b/x^4)^(1/4)
*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/b^(3/2)
/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\frac{x^3\left(-3a^2d+(2bc+3ad)\,(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{6a^2b\,(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^3*(-3*a^2*d + (2*b*c + 3*a*d)*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hyperge
ometric2F1[3/4, 9/4, 7/4, -((b*x^4)/a)]))/(6*a^2*b*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {957, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(3ad+2bc)\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{5ab}+\frac{x^3(bc-ad)}{5ab\,(a+bx^4)^{5/4}}$$

$$\Big\downarrow \; 813$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(3ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab^2\sqrt[4]{a+bx^4}}+\frac{x^3(bc-ad)}{5ab\,(a+bx^4)^{5/4}}$$

$$\Big\downarrow \; 858$$

$$\frac{x^3(bc-ad)}{5ab\,(a+bx^4)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(3ad+2bc)\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 807$$

$$\frac{x^3(bc-ad)}{5ab\,(a+bx^4)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(3ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{10ab^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 212$$

$$\frac{x^3(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}(3ad+2bc)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^4}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^3)/(5*a*b*(a + b*x^4)^(5/4)) - ((2*b*c + 3*a*d)*(1 + a/(b*x
^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*b^(
3/2)*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{2} \left(d\,x^{4}+c\right)}{\left(b\,x^{4}+a\right)^{\frac{9}{4}}}d x$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2
*b*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{9}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{9}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 9/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a
**(9/4)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((7/4, 9/4), (11/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(9/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**6/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**2/((a + b*x**4)**(1/4)*a**2 + 2*(
a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.158 $\int \frac{c+dx^4}{x^2(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{9/4}}\,dx = -\frac{c}{ax(a+bx^4)^{5/4}} - \frac{(6bc-ad)x^3}{5a^2(a+bx^4)^{5/4}} + \frac{2(6bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-c/a/x/(b*x^4+a)^(5/4)-1/5*(-a*d+6*b*c)*x^3/a^2/(b*x^4+a)^(5/4)+2/5*(-a*d+
6*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),
2^(1/2))/a^(5/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{9/4}}\,dx = \frac{-3a^2c+(-6bc+ad)x^4(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^3x(a+bx^4)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(9/4)),x]
```

output
```
(-3*a^2*c + (-6*b*c + a*d)*x^4*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeom
etric2F1[3/4, 9/4, 7/4, -((b*x^4)/a)])/(3*a^3*x*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(6bc-ad)\int \frac{x^2}{(bx^4+a)^{9/4}}dx}{a}-\frac{c}{ax(a+bx^4)^{5/4}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(6bc-ad)\left(\frac{2\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x^3}{5a(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{ax(a+bx^4)^{5/4}}$$

$$\Big\downarrow \; 813$$

$$-\frac{(6bc-ad)\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{ax(a+bx^4)^{5/4}}$$

$$\Big\downarrow \; 858$$

$$-\frac{(6bc-ad)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \textbf{807}$$

$$-\frac{(6bc-ad)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \textbf{212}$$

$$-\frac{(6bc-ad)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(9/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(5/4))) - ((6*b*c - a*d)*(x^3/(5*a*(a + b*x^4)^(5/4))
 - (2*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2
])/(5*a^(3/2)*Sqrt[b]*(a + b*x^4)^(1/4))))/a
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{2} \left(b\,x^{4}+a \right)^{\frac{9}{4}}}dx$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x)
```

output
```
int((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{9/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{9}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^3*x^14 + 3*a*b^2*x^10 + 3*a^2*b*
x^6 + a^3*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.57 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{9/4}}\,dx=\frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}x\Gamma\left(\frac{3}{4}\right)}+\frac{dx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(9/4),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 9/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(
9/4)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 9/4), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(4*a**(9/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{9/4}}\,dx = \int \frac{d\,x^4+c}{x^2\,(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(9/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(9/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2x^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^6+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{10}}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(9/4),x)
```

output
```
int(x**2/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(1/4)*a**2*x**2 + 2
*(a + b*x**4)**(1/4)*a*b*x**6 + (a + b*x**4)**(1/4)*b**2*x**10),x)*c
```

3.159 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{9/4}}\,dx = -\frac{c}{5ax^5\left(a+bx^4\right)^{5/4}} - \frac{2bc-ad}{5a^2x\left(a+bx^4\right)^{5/4}} + \frac{6(2bc-ad)}{5a^3x\sqrt[4]{a+bx^4}} - \frac{12\sqrt{b}(2bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(5/4)-1/5*(-a*d+2*b*c)/a^2/x/(b*x^4+a)^(5/4)+6/5*(-
a*d+2*b*c)/a^3/x/(b*x^4+a)^(1/4)-12/5*b^(1/2)*(-a*d+2*b*c)*(1+a/b/x^4)^(1/
4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^
4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{9/4}}\,dx = \frac{-a^2c-5(-2bc+ad)x^4(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^3x^5\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(9/4)),x]
```

output
```
(-(a^2*c) - 5*(-2*b*c + a*d)*x^4*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hyperge
ometric2F1[-1/4, 9/4, 3/4, -((b*x^4)/a)])/(5*a^3*x^5*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{9/4}} dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(2bc-ad)\int \frac{1}{x^2(bx^4+a)^{9/4}}dx}{a}-\frac{c}{5ax^5 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(2bc-ad)\left(\frac{6\int \frac{1}{x^2\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{ 816}$$

$$-\frac{(2bc-ad)\left(\frac{6\left(-\frac{2b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{(2bc-ad)\left(\frac{6\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/4}}$$

$\big\downarrow$ 858

$$-\frac{(2bc-ad)\left(\frac{6\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/4}}$$

$\big\downarrow$ 807

$$-\frac{(2bc-ad)\left(\frac{6\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/4}}$$

$\big\downarrow$ 212

$$-\frac{(2bc-ad)\left(\frac{6\left(\frac{\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}}{}\right)}{5a}+\frac{1}{5ax(a+bx^4)^{5/4}}\right)}{a}-\frac{c}{5ax^5\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(9/4)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(5/4)) - ((2*b*c - a*d)*(1/(5*a*x*(a + b*x^4)^(5
/4)) + (6*(-(1/(a*x*(a + b*x^4)^(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)
*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/
4))))/(5*a)))/a
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{6} \left(b\,x^{4}+a\right)^{\frac{9}{4}}}dx$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x)
```

output
```
int((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^3*x^18 + 3*a*b^2*x^14 + 3*a^2*b*
x^10 + a^3*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{9/4}}\,dx = \frac{c\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{9}{4} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}x^5\Gamma\left(-\frac{1}{4}\right)} + \frac{d\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{9}{4}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(9/4),x)
```

output
```
c*gamma(-5/4)*hyper((-5/4, 9/4), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(9/4)*x**5*gamma(-1/4)) + d*gamma(-1/4)*hyper((-1/4, 9/4), (3/4,), b*x**4*
exp_polar(I*pi)/a)/(4*a**(9/4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6 \left(a+bx^4\right)^{9/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{9}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6 \left(a+bx^4\right)^{9/4}}\,dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{9}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6 \left(a+bx^4\right)^{9/4}}\,dx = \int \frac{d\,x^4+c}{x^6\,\left(b\,x^4+a\right)^{9/4}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(9/4)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(9/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{9/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2x^6+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^{10}+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{14}}dx\right)c + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2x^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^6+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{10}}dx\right)d$$

input

```
int((d*x^4+c)/x^6/(b*x^4+a)^(9/4),x)
```

output

```
int(1/((a + b*x**4)**(1/4)*a**2*x**6 + 2*(a + b*x**4)**(1/4)*a*b*x**10 + (
a + b*x**4)**(1/4)*b**2*x**14),x)*c + int(1/((a + b*x**4)**(1/4)*a**2*x**2
 + 2*(a + b*x**4)**(1/4)*a*b*x**6 + (a + b*x**4)**(1/4)*b**2*x**10),x)*d
```

3.160 $\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{a(bc-ad)x^2}{5b^3(a+bx^4)^{5/4}} + \frac{(5bc-12ad)x^2}{5b^3\sqrt[4]{a+bx^4}} + \frac{dx^2(a+bx^4)^{3/4}}{5b^3} - \frac{12\sqrt{a}(bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*a*(-a*d+b*c)*x^2/b^3/(b*x^4+a)^(5/4)+1/5*(-12*a*d+5*b*c)*x^2/b^3/(b*x^
4+a)^(1/4)+1/5*d*x^2*(b*x^4+a)^(3/4)/b^3-12/5*a^(1/2)*(-2*a*d+b*c)*(1+b*x^
4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(7/2)
/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.71

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{x^2\left(12a^2d+b^2x^4(-7c+dx^4)+ab(-6c+14dx^4)+6(bc-2ad)(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\right.}{5b^3(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^9*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^2*(12*a^2*d + b^2*x^4*(-7*c + d*x^4) + a*b*(-6*c + 14*d*x^4) + 6*(b*c -
 2*a*d)*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 -((b*x^4)/a)]))/(5*b^3*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 807, 250, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 (c + dx^4)}{(a + bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{10}(bc - ad)}{5ab\,(a + bx^4)^{5/4}} - \frac{(bc - 2ad)\int \frac{x^9}{(bx^4+a)^{5/4}}dx}{ab}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^{10}(bc - ad)}{5ab\,(a + bx^4)^{5/4}} - \frac{(bc - 2ad)\int \frac{x^8}{(bx^4+a)^{5/4}}dx^2}{2ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{x^{10}(bc - ad)}{5ab\,(a + bx^4)^{5/4}} - \frac{(bc - 2ad)\left(\frac{2x^6}{5b\sqrt[4]{a + bx^4}} - \frac{6a\int \frac{x^4}{\left(bx^4+a\right)^{5/4}}dx^2}{5b}\right)}{2ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{x^{10}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(bc-2ad)\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2a\int\frac{1}{\left(bx^4+a\right)^{5/4}}dx^2}{b}\right)}{5b}\right)}{2ab}$$

$\Big\downarrow$ 213

$$\frac{x^{10}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(bc-2ad)\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)}{5b}\right)}{2ab}$$

$\Big\downarrow$ 212

$$\frac{x^{10}(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(bc-2ad)\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)}{5b}\right)}{2ab}$$

input
```
Int[(x^9*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^10)/(5*a*b*(a + b*x^4)^(5/4)) - ((b*c - 2*a*d)*((2*x^6)/(5*
b*(a + b*x^4)^(1/4)) - (6*a*((2*x^2)/(b*(a + b*x^4)^(1/4)) - (4*Sqrt[a]*(1
 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(b^(3/2
)*(a + b*x^4)^(1/4))))/(5*b)))/(2*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{9} \left(d\,x^{4}+c \right)}{\left(b\,x^{4}+a \right)^{\frac{9}{4}}}d x$$

input
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^13 + c*x^9)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^
2*b*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.40

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^{10}\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{9}{4}}} + \frac{dx^{14}\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{9}{4}}}$$

input
```
integrate(x**9*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**10*hyper((9/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(9/4))
+ d*x**14*hyper((9/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(9/4)
)
```

Maxima [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^9/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^9}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^9/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^9\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^9*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^9*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^9(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^{13}}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^9*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**13/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a
+ b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**9/((a + b*x**4)**(1/4)*a**2 + 2*
(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.161 $\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = -\frac{(bc-ad)x^2}{5b^2(a+bx^4)^{5/4}} + \frac{dx^2}{b^2\sqrt[4]{a+bx^4}} + \frac{2(bc-6ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*(-a*d+b*c)*x^2/b^2/(b*x^4+a)^(5/4)+d*x^2/b^2/(b*x^4+a)^(1/4)+2/5*(-6*
a*d+b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),
2^(1/2))/a^(1/2)/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{x^2\left(-6a^2d+2b^2cx^4+ab(c-7dx^4)+(-bc+6ad)(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\text{ Hypergeomet}\right.}{5ab^2(a+bx^4)^{5/4}}$$

input
```
Integrate[(x^5*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^2*(-6*a^2*d + 2*b^2*c*x^4 + a*b*(c - 7*d*x^4) + (-(b*c) + 6*a*d)*(a + b
*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]
))/(5*a*b^2*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 807, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^6(bc-ad)}{5ab(a+bx^4)^{5/4}} - \frac{(bc-6ad)\int \frac{x^5}{(bx^4+a)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^6(bc-ad)}{5ab(a+bx^4)^{5/4}} - \frac{(bc-6ad)\int \frac{x^4}{(bx^4+a)^{5/4}}dx^2}{10ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{x^6(bc-ad)}{5ab(a+bx^4)^{5/4}} - \frac{(bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}} - \frac{2a\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{b}\right)}{10ab}$$

$$\Big\downarrow \text{213}$$

$$\frac{x^6(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)}{10ab}$$

$$\Big\downarrow \text{212}$$

$$\frac{x^6(bc-ad)}{5ab\left(a+bx^4\right)^{5/4}}-\frac{(bc-6ad)\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)}{10ab}$$

input
```
Int[(x^5*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^6)/(5*a*b*(a + b*x^4)^(5/4)) - ((b*c - 6*a*d)*((2*x^2)/(b*(
a + b*x^4)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqr
t[b]*x^2)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^4)^(1/4))))/(10*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{5}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{9}{4}}}dx$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^9 + c*x^5)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2
*b*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^6\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{9}{4}}} + \frac{dx^{10}\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{9}{4}}}$$

input
```
integrate(x**5*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**6*hyper((3/2, 9/4), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(9/4)) +
d*x**10*hyper((9/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(9/4))
```

Maxima [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x^5}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^5/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x^5\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x^5*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x^5(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x^5*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**9/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(x**5/((a + b*x**4)**(1/4)*a**2 + 2*(
a + b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.162 $\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\frac{(bc-ad)x^2}{5ab\,(a+bx^4)^{5/4}}+\frac{(3bc+2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/5*(-a*d+b*c)*x^2/a/b/(b*x^4+a)^(5/4)+1/5*(2*a*d+3*b*c)*(1+b*x^4/a)^(1/4)
*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/b^(3/2)/(
b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx=\frac{x^2\left(2a^2d+6b^2cx^4+4ab(2c+dx^4)-(3bc+2ad)\,(a+bx^4)\,\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometr}\right.}{10a^2b\,(a+bx^4)^{5/4}}$$

input
```
Integrate[(x*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
(x^2*(2*a^2*d + 6*b^2*c*x^4 + 4*a*b*(2*c + d*x^4) - (3*b*c + 2*a*d)*(a + b
*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]
))/(10*a^2*b*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {957, 807, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2ad+3bc)\int \frac{x}{(bx^4+a)^{5/4}}dx}{5ab}+\frac{x^2(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2ad+3bc)\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{10ab}+\frac{x^2(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(2ad+3bc)\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{10a^2b\sqrt[4]{a+bx^4}}+\frac{x^2(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}(2ad+3bc)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^2(bc-ad)}{5ab(a+bx^4)^{5/4}}$$

input
```
Int[(x*(c + d*x^4))/(a + b*x^4)^(9/4),x]
```

output
```
((b*c - a*d)*x^2)/(5*a*b*(a + b*x^4)^(5/4)) + ((3*b*c + 2*a*d)*(1 + (b*x^4
)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*a^(3/2)*b^(3/
2)*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(d\,x^4 + c)}{(b\,x^4 + a)^{\frac{9}{4}}}dx$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*x^5 + c*x)*(b*x^4 + a)^(3/4)/(b^3*x^12 + 3*a*b^2*x^8 + 3*a^2*b
*x^4 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \frac{cx^2\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}} + \frac{dx^6\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{9}{4}}}$$

input
```
integrate(x*(d*x**4+c)/(b*x**4+a)**(9/4),x)
```

output
```
c*x**2*hyper((1/2, 9/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(9/4)) +
d*x**6*hyper((3/2, 9/4), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(9/4))
```

Maxima [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(9/4), x)
```

Giac [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{(dx^4+c)x}{(bx^4+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(x*(d*x^4+c)/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x/(b*x^4 + a)^(9/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \int \frac{x\,(d\,x^4+c)}{(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((x*(c + d*x^4))/(a + b*x^4)^(9/4),x)
```

output
```
int((x*(c + d*x^4))/(a + b*x^4)^(9/4), x)
```

Reduce [F]

$$\int \frac{x(c+dx^4)}{(a+bx^4)^{9/4}}\,dx = \left(\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)c$$

input
```
int(x*(d*x^4+c)/(b*x^4+a)^(9/4),x)
```

output
```
int(x**5/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a +
 b*x**4)**(1/4)*b**2*x**8),x)*d + int(x/((a + b*x**4)**(1/4)*a**2 + 2*(a +
 b*x**4)**(1/4)*a*b*x**4 + (a + b*x**4)**(1/4)*b**2*x**8),x)*c
```

3.163 $\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{9/4}}\,dx=-\frac{c}{2ax^2\left(a+bx^4\right)^{5/4}}-\frac{(7bc-2ad)x^2}{10a^2\left(a+bx^4\right)^{5/4}}-\frac{3(7bc-2ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{10a^{5/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-1/2*c/a/x^2/(b*x^4+a)^(5/4)-1/10*(-2*a*d+7*b*c)*x^2/a^2/(b*x^4+a)^(5/4)-3
/10*(-2*a*d+7*b*c)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/
a^(1/2))),2^(1/2))/a^(5/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^4}{x^3\left(a+bx^4\right)^{9/4}}\,dx=\frac{-2(21b^2cx^8+a^2(5c-8dx^4)+ab(28cx^4-6dx^8))+3(7bc-2ad)x^4(a+bx^4)\sqrt[4]{1-}}{20a^3x^2\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^3*(a + b*x^4)^(9/4)),x]
```

output
```
(-2*(21*b^2*c*x^8 + a^2*(5*c - 8*d*x^4) + a*b*(28*c*x^4 - 6*d*x^8)) + 3*(7
*b*c - 2*a*d)*x^4*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4,
 1/2, 3/2, -((b*x^4)/a)])/(20*a^3*x^2*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 807, 215, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(7bc-2ad)\int \frac{x}{(bx^4+a)^{9/4}}dx}{2a}-\frac{c}{2ax^2 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(7bc-2ad)\int \frac{1}{(bx^4+a)^{9/4}}dx^2}{4a}-\frac{c}{2ax^2 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{215}$$

$$-\frac{(7bc-2ad)\left(\frac{3\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{5a}+\frac{2x^2}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{2ax^2 (a+bx^4)^{5/4}}$$

$$\Big\downarrow \text{213}$$

$$-\frac{(7bc-2ad)\left(\frac{3\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{5a^2\sqrt[4]{a+bx^4}}+\frac{2x^2}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \text{ 212}$$

$$-\frac{(7bc-2ad)\left(\frac{6\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{2x^2}{5a(a+bx^4)^{5/4}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^3*(a + b*x^4)^(9/4)),x]
```

output
```
-1/2*c/(a*x^2*(a + b*x^4)^(5/4)) - ((7*b*c - 2*a*d)*((2*x^2)/(5*a*(a + b*x
^4)^(5/4)) + (6*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[
a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^4)^(1/4))))/(4*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{3} \left(b\,x^{4}+a \right)^{\frac{9}{4}}}dx$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x)
```

output
```
int((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^3 \left(a+bx^4\right)^{9/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{9}{4}}x^{3}}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^3*x^15 + 3*a*b^2*x^11 + 3*a^2*b*
x^7 + a^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}x^2} + \frac{dx^2\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{9}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}}$$

input
```
integrate((d*x**4+c)/x**3/(b*x**4+a)**(9/4),x)
```

output
```
-c*hyper((-1/2, 9/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(9/4)*x**2)
+ d*x**2*hyper((1/2, 9/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(9/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^3}\,dx$$

input
```
integrate((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}}\,dx = \int \frac{d\,x^4+c}{x^3\,(b\,x^4+a)^{9/4}}\,dx$$

input
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(9/4)),x)
```

output
```
int((c + d*x^4)/(x^3*(a + b*x^4)^(9/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^3 (a+bx^4)^{9/4}}\,dx = \left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^4+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^8}dx\right)d + \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^2x^3+2\,(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^7+(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{11}}dx\right)c$$

input
```
int((d*x^4+c)/x^3/(b*x^4+a)^(9/4),x)
```

output
```
int(x/((a + b*x**4)**(1/4)*a**2 + 2*(a + b*x**4)**(1/4)*a*b*x**4 + (a + b*
x**4)**(1/4)*b**2*x**8),x)*d + int(1/((a + b*x**4)**(1/4)*a**2*x**3 + 2*(a
 + b*x**4)**(1/4)*a*b*x**7 + (a + b*x**4)**(1/4)*b**2*x**11),x)*c
```

3.164 $\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx = -\frac{c}{6ax^6\left(a+bx^4\right)^{5/4}}-\frac{11bc-6ad}{30a^2x^2\left(a+bx^4\right)^{5/4}}+\frac{7(11bc-6ad)}{60a^3x^2\sqrt[4]{a+bx^4}}+\frac{7\sqrt{b}(11bc-6ad)\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/6*c/a/x^6/(b*x^4+a)^(5/4)-1/30*(-6*a*d+11*b*c)/a^2/x^2/(b*x^4+a)^(5/4)+
7/60*(-6*a*d+11*b*c)/a^3/x^2/(b*x^4+a)^(1/4)+7/20*b^(1/2)*(-6*a*d+11*b*c)*
(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/
a^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.53

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx = \frac{-2a^2c+(11bc-6ad)x^4(a+bx^4)\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{12a^3x^6\left(a+bx^4\right)^{5/4}}$$

input
```
Integrate[(c + d*x^4)/(x^7*(a + b*x^4)^(9/4)),x]
```

output
```
(-2*a^2*c + (11*b*c - 6*a*d)*x^4*(a + b*x^4)*(1 + (b*x^4)/a)^(1/4)*Hyperge
ometric2F1[-1/2, 9/4, 1/2, -((b*x^4)/a)])/(12*a^3*x^6*(a + b*x^4)^(5/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 807, 253, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^7 \left(a+bx^4\right)^{9/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(11bc-6ad)\int \frac{1}{x^3 (bx^4+a)^{9/4}}dx}{6a}-\frac{c}{6ax^6 \left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(11bc-6ad)\int \frac{1}{x^4 (bx^4+a)^{9/4}}dx^2}{12a}-\frac{c}{6ax^6 \left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(11bc-6ad)\left(\frac{7\int \frac{1}{x^4 \left(bx^4+a\right)^{5/4}}dx^2}{5a}+\frac{2}{5ax^2 (a+bx^4)^{5/4}}\right)}{12a}-\frac{c}{6ax^6 \left(a+bx^4\right)^{5/4}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{(11bc-6ad)\left(\frac{7\left(-\frac{3b\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{2a}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{2}{5ax^2(a+bx^4)^{5/4}}\right)}{12a}-\frac{c}{6ax^6\left(a+bx^4\right)^{5/4}}$$

$\Big\downarrow$ 213

$$-\frac{(11bc-6ad)\left(\frac{7\left(-\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2a^2\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{2}{5ax^2(a+bx^4)^{5/4}}\right)}{12a}-\frac{c}{6ax^6\left(a+bx^4\right)^{5/4}}$$

$\Big\downarrow$ 212

$$-\frac{(11bc-6ad)\left(\frac{7\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{2}{5ax^2(a+bx^4)^{5/4}}\right)}{12a}-\frac{c}{6ax^6\left(a+bx^4\right)^{5/4}}$$

input
```
Int[(c + d*x^4)/(x^7*(a + b*x^4)^(9/4)),x]
```

output
```
-1/6*c/(a*x^6*(a + b*x^4)^(5/4)) - ((11*b*c - 6*a*d)*(2/(5*a*x^2*(a + b*x^
4)^(5/4)) + (7*(-(1/(a*x^2*(a + b*x^4)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^4)/a
)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^4
)^(1/4))))/(5*a)))/(12*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{7}\left(b\,x^{4}+a\right)^{\frac{9}{4}}}dx$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x)
```

output
```
int((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{9}{4}}x^7}\,dx$$

input
```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^3*x^19 + 3*a*b^2*x^15 + 3*a^2*b*
x^11 + a^3*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 78.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{9/4}} \, dx = -\frac{c\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{9}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{9}{4}}x^6} - \frac{d\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}x^2}$$

input

```
integrate((d*x**4+c)/x**7/(b*x**4+a)**(9/4),x)
```

output

```
-c*hyper((-3/2, 9/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(9/4)*x**6)
 - d*hyper((-1/2, 9/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(9/4)*x**2
)
```

Maxima [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{9/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^7} \, dx$$

input

```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x, algorithm="maxima")
```

output

```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^7), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^7 (a+bx^4)^{9/4}} \, dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{9}{4}}x^7} \, dx$$

input

```
integrate((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(9/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx=\int \frac{d\,x^4+c}{x^7\,\left(b\,x^4+a\right)^{9/4}}\,dx$$

input
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(9/4)),x)
```

output
```
int((c + d*x^4)/(x^7*(a + b*x^4)^(9/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^7\left(a+bx^4\right)^{9/4}}\,dx=\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2x^7+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^{11}+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^{15}}dx\right) c$$
$$+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^2x^3+2\left(b\,x^4+a\right)^{\frac{1}{4}} ab\,x^7+\left(b\,x^4+a\right)^{\frac{1}{4}} b^2x^{11}}dx\right) d$$

input
```
int((d*x^4+c)/x^7/(b*x^4+a)^(9/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**2*x**7 + 2*(a + b*x**4)**(1/4)*a*b*x**11 + (
a + b*x**4)**(1/4)*b**2*x**15),x)*c + int(1/((a + b*x**4)**(1/4)*a**2*x**3
 + 2*(a + b*x**4)**(1/4)*a*b*x**7 + (a + b*x**4)**(1/4)*b**2*x**11),x)*d
```

3.165 $\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{a^2(bc-ad)x}{9b^4\left(a+bx^4\right)^{9/4}} + \frac{a(19bc-28ad)x}{45b^4\left(a+bx^4\right)^{5/4}} - \frac{(59bc-158ad)x}{45b^4\sqrt[4]{a+bx^4}} + \frac{dx(a+bx^4)^{3/4}}{4b^4} + \frac{(4bc-13ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{17/4}} + \frac{(4bc-13ad)\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{17/4}}$$

output
```
-1/9*a^2*(-a*d+b*c)*x/b^4/(b*x^4+a)^(9/4)+1/45*a*(-28*a*d+19*b*c)*x/b^4/(b
*x^4+a)^(5/4)-1/45*(-158*a*d+59*b*c)*x/b^4/(b*x^4+a)^(1/4)+1/4*d*x*(b*x^4+
a)^(3/4)/b^4+1/8*(-13*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(17/4
)+1/8*(-13*a*d+4*b*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(17/4)
```

Mathematica [A] (verified)

Time = 6.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.81

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(585a^3d-9a^2b(20c-143dx^4)+b^3x^8(-236c+45dx^4)+ab^2x^4(-396c+767dx^4))}{(a+bx^4)^{9/4}}+45(4bc-13ad)\,\mathrm{arc}}{360b^{17/4}}$$

input
```
Integrate[(x^12*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((2*b^(1/4)*x*(585*a^3*d - 9*a^2*b*(20*c - 143*d*x^4) + b^3*x^8*(-236*c +
45*d*x^4) + a*b^2*x^4*(-396*c + 767*d*x^4)))/(a + b*x^4)^(9/4) + 45*(4*b*c
 - 13*a*d)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 45*(4*b*c - 13*a*d)*Arc
Tanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(360*b^(17/4))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {957, 817, 817, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{13}(bc-ad)}{9ab(a+bx^4)^{9/4}}-\frac{(4bc-13ad)\int \frac{x^{12}}{(bx^4+a)^{9/4}}dx}{9ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{13}(bc-ad)}{9ab(a+bx^4)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\int \frac{x^8}{(bx^4+a)^{5/4}}dx}{5b}-\frac{x^9}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$\Big\downarrow$ 817

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\int\frac{x^4}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$\Big\downarrow$ 843

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$\Big\downarrow$ 770

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$\Big\downarrow$ 756

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b(a+bx^4)^{5}}\right)}{9ab}$$

$\downarrow$ 216

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9ab}$$

$\downarrow$ 219

$$\frac{x^{13}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(4bc-13ad)\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9ab}$$

input
```
Int[(x^12*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^13)/(9*a*b*(a + b*x^4)^(9/4)) - ((4*b*c - 13*a*d)*(-1/5*x^9
/(b*(a + b*x^4)^(5/4)) + (9*(-(x^5/(b*(a + b*x^4)^(1/4))) + (5*((x*(a + b*
x^4)^(3/4))/(4*b) - (a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))
+ ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/b))/(5*b)))
/(9*a*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{13\left(\left(-\frac{4}{13} d x^{13}+\frac{944}{585} c x^{9}\right) b^{\frac{13}{4}}+\frac{16\left(-\frac{143 d x^{4}}{20}+c\right) x a^{2} b^{\frac{5}{4}}}{13}+\frac{176\left(-\frac{767 d x^{4}}{396}+c\right) x^{5} a b^{\frac{9}{4}}}{65}-4 a^{3} d x b^{\frac{1}{4}}+\left(\ln\left(\frac{x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}{-x b^{\frac{1}{4}}+\left(b x^{4}+a\right)^{\frac{1}{4}}}\right.\right.\right.}{16 b^{\frac{17}{4}} \left(b x^{4}+a\right)^{\frac{9}{4}}}$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
-13/16/b^(17/4)*((-4/13*d*x^13+944/585*c*x^9)*b^(13/4)+16/13*(-143/20*d*x^
4+c)*x*a^2*b^(5/4)+176/65*(-767/396*d*x^4+c)*x^5*a*b^(9/4)-4*a^3*d*x*b^(1/
4)+(ln((x*b^(1/4)+(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))-2*arctan(
(b*x^4+a)^(1/4)/x/b^(1/4)))*(a*d-4/13*c*b)*(b*x^4+a)^(9/4))/(b*x^4+a)^(9/4
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 963, normalized size of antiderivative = 5.29

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/720*(45*(b^7*x^12 + 3*a*b^6*x^8 + 3*a^2*b^5*x^4 + a^3*b^4)*((256*b^4*c^4
 - 3328*a*b^3*c^3*d + 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^
4*d^4)/b^17)^(1/4)*log(-(b^13*x*((256*b^4*c^4 - 3328*a*b^3*c^3*d + 16224*a
^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^4*d^4)/b^17)^(3/4) + (64*b^3*
c^3 - 624*a*b^2*c^2*d + 2028*a^2*b*c*d^2 - 2197*a^3*d^3)*(b*x^4 + a)^(1/4)
)/x) - 45*(b^7*x^12 + 3*a*b^6*x^8 + 3*a^2*b^5*x^4 + a^3*b^4)*((256*b^4*c^4
 - 3328*a*b^3*c^3*d + 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^
4*d^4)/b^17)^(1/4)*log((b^13*x*((256*b^4*c^4 - 3328*a*b^3*c^3*d + 16224*a^
2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^4*d^4)/b^17)^(3/4) - (64*b^3*c
^3 - 624*a*b^2*c^2*d + 2028*a^2*b*c*d^2 - 2197*a^3*d^3)*(b*x^4 + a)^(1/4))
/x) - 45*(-I*b^7*x^12 - 3*I*a*b^6*x^8 - 3*I*a^2*b^5*x^4 - I*a^3*b^4)*((256
*b^4*c^4 - 3328*a*b^3*c^3*d + 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 +
28561*a^4*d^4)/b^17)^(1/4)*log((I*b^13*x*((256*b^4*c^4 - 3328*a*b^3*c^3*d
+ 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^4*d^4)/b^17)^(3/4) -
 (64*b^3*c^3 - 624*a*b^2*c^2*d + 2028*a^2*b*c*d^2 - 2197*a^3*d^3)*(b*x^4 +
 a)^(1/4))/x) - 45*(I*b^7*x^12 + 3*I*a*b^6*x^8 + 3*I*a^2*b^5*x^4 + I*a^3*b
^4)*((256*b^4*c^4 - 3328*a*b^3*c^3*d + 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b
*c*d^3 + 28561*a^4*d^4)/b^17)^(1/4)*log((-I*b^13*x*((256*b^4*c^4 - 3328*a*
b^3*c^3*d + 16224*a^2*b^2*c^2*d^2 - 35152*a^3*b*c*d^3 + 28561*a^4*d^4)/b^1
7)^(3/4) - (64*b^3*c^3 - 624*a*b^2*c^2*d + 2028*a^2*b*c*d^2 - 2197*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**12*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.63

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{1}{180}\left(\frac{4\left(5\,b^2+\frac{9\,(bx^4+a)b}{x^4}+\frac{45\,(bx^4+a)^2}{x^8}\right)x^9}{(bx^4+a)^{\frac{9}{4}}b^3}+\frac{45\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^3}\right)c + \frac{1}{720}\,d\left(\frac{4\left(20\,ab^3+\frac{52\,(bx^4+a)ab^2}{x^4}+\frac{468\,(bx^4+a)^2ab}{x^8}-\frac{585\,(bx^4+a)^3a}{x^{12}}\right)}{\frac{(bx^4+a)^{\frac{9}{4}}b^5}{x^9}-\frac{(bx^4+a)^{\frac{13}{4}}b^4}{x^{13}}}+\frac{585\,a\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^4}\right)$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
-1/180*(4*(5*b^2 + 9*(b*x^4 + a)*b/x^4 + 45*(b*x^4 + a)^2/x^8)*x^9/((b*x^4
 + a)^(9/4)*b^3) + 45*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + l
og(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/
4))/b^3)*c + 1/720*d*(4*(20*a*b^3 + 52*(b*x^4 + a)*a*b^2/x^4 + 468*(b*x^4
+ a)^2*a*b/x^8 - 585*(b*x^4 + a)^3*a/x^12)/((b*x^4 + a)^(9/4)*b^5/x^9 - (b
*x^4 + a)^(13/4)*b^4/x^13) + 585*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)
)/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1
/4)/x))/b^(1/4))/b^4)
```

Giac [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{x^{12}\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^{16}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**16/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
 int(x**12/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
 3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.166 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{(bc-ad)x^9}{9ab\,(a+bx^4)^{9/4}} - \frac{dx^5}{5b^2\,(a+bx^4)^{5/4}} - \frac{dx}{b^3\sqrt[4]{a+bx^4}} + \frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{13/4}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{13/4}}$$

output
```
1/9*(-a*d+b*c)*x^9/a/b/(b*x^4+a)^(9/4)-1/5*d*x^5/b^2/(b*x^4+a)^(5/4)-d*x/b
^3/(b*x^4+a)^(1/4)+1/2*d*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)+1/2*d*
arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)
```

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{-\frac{2\sqrt[4]{b}x(45a^3d+99a^2bdx^4-5b^3cx^8+59ab^2dx^8)}{a(a+bx^4)^{9/4}} + 45d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + 45d\operatorname{arctanh}\left(\frac{\sqrt[4]{}}{\sqrt[4]{a}}\right.}{90b^{13/4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((-2*b^(1/4)*x*(45*a^3*d + 99*a^2*b*d*x^4 - 5*b^3*c*x^8 + 59*a*b^2*d*x^8))
/(a*(a + b*x^4)^(9/4)) + 45*d*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 45*d
*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(90*b^(13/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {954, 817, 817, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (c+dx^4)}{(a+bx^4)^{13/4}}\,dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int \frac{x^8}{(bx^4+a)^{9/4}}dx}{b}+\frac{x^9 (bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{d\left(\frac{\int \frac{x^4}{\left(bx^4+a\right)^{5/4}}dx}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9 (bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{d\left(\frac{\frac{\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9 (bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{d\left(\frac{\frac{\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 756

$$\frac{d\left(\frac{\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 216

$$\frac{d\left(\frac{\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 219

$$\frac{d\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b(a+bx^4)^{5/4}}\right)}{b}+\frac{x^9(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^9)/(9*a*b*(a + b*x^4)^(9/4)) + (d*(-1/5*x^5/(b*(a + b*x^4)^
(5/4)) + (-(x/(b*(a + b*x^4)^(1/4))) + (ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/
4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))/b)/b
))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94

method	result	si
pseudoelliptic	$\frac{\frac{4b^{\frac{13}{4}}c\,x^9}{9}+d\left(-\frac{44a\,b^{\frac{5}{4}}x^5}{5}-\frac{236b^{\frac{9}{4}}x^9}{45}-4a^2x\,b^{\frac{1}{4}}+\left(b\,x^4+a\right)^{\frac{9}{4}}\left(\ln\left(\frac{x\,b^{\frac{1}{4}}+\left(b\,x^4+a\right)^{\frac{1}{4}}}{-x\,b^{\frac{1}{4}}+\left(b\,x^4+a\right)^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x\,b^{\frac{1}{4}}}\right)\right)\right)a}{4b^{\frac{13}{4}}\left(b\,x^4+a\right)^{\frac{9}{4}}a}$	1

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^(13/4)/(b*x^4+a)^(9/4)*(4/9*b^(13/4)*c*x^9+d*(-44/5*a*b^(5/4)*x^5-23
6/45*b^(9/4)*x^9-4*a^2*x*b^(1/4)+(b*x^4+a)^(9/4)*(ln((x*b^(1/4)+(b*x^4+a)^
(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))-2*arctan((b*x^4+a)^(1/4)/x/b^(1/4)))
*a)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{45\left(ab^6x^{12}+3\,a^2b^5x^8+3\,a^3b^4x^4+a^4b^3\right)\left(\frac{d^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(\frac{b^{10}x\left(\frac{d^4}{b^{13}}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}d^3}{x}\right)-45\,(ab^6}{}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/180*(45*(a*b^6*x^12 + 3*a^2*b^5*x^8 + 3*a^3*b^4*x^4 + a^4*b^3)*(d^4/b^13
)^(1/4)*log((b^10*x*(d^4/b^13)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - 45*(a*b
^6*x^12 + 3*a^2*b^5*x^8 + 3*a^3*b^4*x^4 + a^4*b^3)*(d^4/b^13)^(1/4)*log(-(
b^10*x*(d^4/b^13)^(3/4) - (b*x^4 + a)^(1/4)*d^3)/x) - 45*(I*a*b^6*x^12 + 3
*I*a^2*b^5*x^8 + 3*I*a^3*b^4*x^4 + I*a^4*b^3)*(d^4/b^13)^(1/4)*log((I*b^10
*x*(d^4/b^13)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - 45*(-I*a*b^6*x^12 - 3*I*
a^2*b^5*x^8 - 3*I*a^3*b^4*x^4 - I*a^4*b^3)*(d^4/b^13)^(1/4)*log((-I*b^10*x
*(d^4/b^13)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) + 4*((5*b^3*c - 59*a*b^2*d)*
x^9 - 99*a^2*b*d*x^5 - 45*a^3*d*x)*(b*x^4 + a)^(3/4))/(a*b^6*x^12 + 3*a^2*
b^5*x^8 + 3*a^3*b^4*x^4 + a^4*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 126.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{4}\right)}{4a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+8a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+4a^{\frac{5}{4}}b^2x^8\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)} + \frac{dx^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{13}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*x**9*gamma(9/4)/(4*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 8*a**(9
/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 4*a**(5/4)*b**2*x**8*(1 + b
*x**4/a)**(1/4)*gamma(13/4)) + d*x**13*gamma(13/4)*hyper((13/4, 13/4), (17
/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(13/4)*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{cx^9}{9\,(bx^4+a)^{\frac{9}{4}}a} - \frac{1}{180}\left(\frac{4\left(5\,b^2+\frac{9\,(bx^4+a)b}{x^4}+\frac{45\,(bx^4+a)^2}{x^8}\right)x^9}{(bx^4+a)^{\frac{9}{4}}b^3} + \frac{45\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{b^3}\right)d$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
1/9*c*x^9/((b*x^4 + a)^(9/4)*a) - 1/180*(4*(5*b^2 + 9*(b*x^4 + a)*b/x^4 +
45*(b*x^4 + a)^2/x^8)*x^9/((b*x^4 + a)^(9/4)*b^3) + 45*(2*arctan((b*x^4 +
a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1
/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^3)*d
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{x^8\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**12/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
 int(x**8/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.167 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\frac{(bc-ad)x^5}{9ab\,(a+bx^4)^{9/4}}+\frac{(4bc+5ad)x^5}{45a^2b\,(a+bx^4)^{5/4}}$$

output
```
1/9*(-a*d+b*c)*x^5/a/b/(b*x^4+a)^(9/4)+1/45*(5*a*d+4*b*c)*x^5/a^2/b/(b*x^4
+a)^(5/4)
```

Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\frac{9acx^5+4bcx^9+5adx^9}{45a^2\,(a+bx^4)^{9/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
(9*a*c*x^5 + 4*b*c*x^9 + 5*a*d*x^9)/(45*a^2*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5ad+4bc)\int \frac{x^4}{(bx^4+a)^{9/4}}dx}{9ab}+\frac{x^5(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^5(5ad+4bc)}{45a^2b\,(a+bx^4)^{5/4}}+\frac{x^5(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^5)/(9*a*b*(a + b*x^4)^(9/4)) + ((4*b*c + 5*a*d)*x^5)/(45*a^
2*b*(a + b*x^4)^(5/4))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^{5}\left(5 a d \,x^{4}+4 b c \,x^{4}+9 a c \right)}{45 \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{2}}$	37
trager	$\frac{x^{5}\left(5 a d \,x^{4}+4 b c \,x^{4}+9 a c \right)}{45 \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{2}}$	37
pseudoelliptic	$\frac{x^{5}\left(5 a d \,x^{4}+4 b c \,x^{4}+9 a c \right)}{45 \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{2}}$	37
orering	$\frac{x^{5}\left(5 a d \,x^{4}+4 b c \,x^{4}+9 a c \right)}{45 \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{2}}$	37

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^5*(5*a*d*x^4+4*b*c*x^4+9*a*c)/(b*x^4+a)^(9/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{((4\,bc+5\,ad)x^9+9\,acx^5)(bx^4+a)^{\frac{3}{4}}}{45\,(a^2b^3x^{12}+3\,a^3b^2x^8+3\,a^4bx^4+a^5)}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/45*((4*b*c + 5*a*d)*x^9 + 9*a*c*x^5)*(b*x^4 + a)^(3/4)/(a^2*b^3*x^12 + 3
*a^3*b^2*x^8 + 3*a^4*b*x^4 + a^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(56) = 112$.

Time = 131.63 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.30

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = c\left(\frac{9ax^5\Gamma\left(\frac{5}{4}\right)}{16a^{\frac{17}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+32a^{\frac{13}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+16a^{\frac{9}{4}}b^2x^8\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)} + \frac{4bx^9\Gamma\left(\frac{5}{4}\right)}{16a^{\frac{17}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+32a^{\frac{13}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+16a^{\frac{9}{4}}b^2x^8\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)}\right) + \frac{dx^9\Gamma\left(\frac{9}{4}\right)}{4a^{\frac{13}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+8a^{\frac{9}{4}}bx^4\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)+4a^{\frac{5}{4}}b^2x^8\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*(9*a*x**5*gamma(5/4)/(16*a**(17/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 3
2*a**(13/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 16*a**(9/4)*b**2*x*
*8*(1 + b*x**4/a)**(1/4)*gamma(13/4)) + 4*b*x**9*gamma(5/4)/(16*a**(17/4)*
(1 + b*x**4/a)**(1/4)*gamma(13/4) + 32*a**(13/4)*b*x**4*(1 + b*x**4/a)**(1
/4)*gamma(13/4) + 16*a**(9/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4))
) + d*x**9*gamma(9/4)/(4*a**(13/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 8*a
**(9/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 4*a**(5/4)*b**2*x**8*(1
 + b*x**4/a)**(1/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{\left(5\,b - \frac{9\,(bx^4+a)}{x^4}\right)cx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^2} + \frac{dx^9}{9\,(bx^4+a)^{\frac{9}{4}}a}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
-1/45*(5*b - 9*(b*x^4 + a)/x^4)*c*x^9/((b*x^4 + a)^(9/4)*a^2) + 1/9*d*x^9/
((b*x^4 + a)^(9/4)*a)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(13/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{c\,x^5}{5\,a\,(b\,x^4+a)^{9/4}} + \frac{d\,x^9}{9\,a\,(b\,x^4+a)^{9/4}} + \frac{4\,b\,c\,x^9}{45\,a^2\,(b\,x^4+a)^{9/4}}$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
(c*x^5)/(5*a*(a + b*x^4)^(9/4)) + (d*x^9)/(9*a*(a + b*x^4)^(9/4)) + (4*b*c
*x^9)/(45*a^2*(a + b*x^4)^(9/4))
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
int(x**4/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.168 $\int \frac{c+dx^4}{\left(a+bx^4\right)^{13/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{13/4}} \, dx=\frac{(bc-ad)x}{9ab\left(a+bx^4\right)^{9/4}}+\frac{(8bc+ad)x}{45a^2b\left(a+bx^4\right)^{5/4}}+\frac{4(8bc+ad)x}{45a^3b\sqrt[4]{a+bx^4}}$$

output
```
1/9*(-a*d+b*c)*x/a/b/(b*x^4+a)^(9/4)+1/45*(a*d+8*b*c)*x/a^2/b/(b*x^4+a)^(5
/4)+4/45*(a*d+8*b*c)*x/a^3/b/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{13/4}} \, dx=\frac{45a^2cx+72abcx^5+9a^2dx^5+32b^2cx^9+4abdx^9}{45a^3\left(a+bx^4\right)^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(13/4),x]
```

output
```
(45*a^2*c*x + 72*a*b*c*x^5 + 9*a^2*d*x^5 + 32*b^2*c*x^9 + 4*a*b*d*x^9)/(45
*a^3*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {910, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+8bc)\int \frac{1}{(bx^4+a)^{9/4}}dx}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+8bc)\left(\frac{4\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a(a+bx^4)^{5/4}}\right)(ad+8bc)}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x)/(9*a*b*(a + b*x^4)^(9/4)) + ((8*b*c + a*d)*(x/(5*a*(a + b*
x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(9*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^{4}}{5}+c\right)a^{2}+\frac{8b\,x^{4}\left(\frac{d\,x^{4}}{18}+c\right)a}{5}+\frac{32b^{2}c\,x^{8}}{45}\right)}{\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	52
gosper	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57
trager	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57
orering	$\frac{x\left(4abd\,x^{8}+32b^{2}c\,x^{8}+9a^{2}d\,x^{4}+72abc\,x^{4}+45a^{2}c\right)}{45\left(b\,x^{4}+a\right)^{\frac{9}{4}}a^{3}}$	57

input
```
int((d*x^4+c)/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^4+a)^(9/4)*x*((1/5*d*x^4+c)*a^2+8/5*b*x^4*(1/18*d*x^4+c)*a+32/45*b^
2*c*x^8)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{13/4}}\,dx = \frac{\left(4\left(8\,b^2c+abd\right)x^9+9\left(8\,abc+a^2d\right)x^5+45\,a^2cx\right)\left(bx^4+a\right)^{\frac{3}{4}}}{45\left(a^3b^3x^{12}+3\,a^4b^2x^8+3\,a^5bx^4+a^6\right)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/45*(4*(8*b^2*c + a*b*d)*x^9 + 9*(8*a*b*c + a^2*d)*x^5 + 45*a^2*c*x)*(b*x
^4 + a)^(3/4)/(a^3*b^3*x^12 + 3*a^4*b^2*x^8 + 3*a^5*b*x^4 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(83) = 166.

Time = 86.38 (sec) , antiderivative size = 709, normalized size of antiderivative = 7.79

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{13/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*(45*a**5*x*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) +
192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**
2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*
x**4/a)**(1/4)*gamma(13/4)) + 117*a**4*b*x**5*gamma(1/4)/(64*a**(33/4)*(1
+ b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4
)*gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4)
+ 64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4)) + 104*a**3*b*
*2*x**9*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a
**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**2*x**
8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*x**4/
a)**(1/4)*gamma(13/4)) + 32*a**2*b**3*x**13*gamma(1/4)/(64*a**(33/4)*(1 +
b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*
gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) +
64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4))) + d*(9*a*x**5*
gamma(5/4)/(16*a**(17/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 32*a**(13/4)*
b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 16*a**(9/4)*b**2*x**8*(1 + b*x*
*4/a)**(1/4)*gamma(13/4)) + 4*b*x**9*gamma(5/4)/(16*a**(17/4)*(1 + b*x**4/
a)**(1/4)*gamma(13/4) + 32*a**(13/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13
/4) + 16*a**(9/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = -\frac{\left(5\,b-\frac{9\,(bx^4+a)}{x^4}\right)dx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^2} + \frac{\left(5\,b^2-\frac{18\,(bx^4+a)b}{x^4}+\frac{45\,(bx^4+a)^2}{x^8}\right)cx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^3}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
-1/45*(5*b - 9*(b*x^4 + a)/x^4)*d*x^9/((b*x^4 + a)^(9/4)*a^2) + 1/45*(5*b^
2 - 18*(b*x^4 + a)*b/x^4 + 45*(b*x^4 + a)^2/x^8)*c*x^9/((b*x^4 + a)^(9/4)*
a^3)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(13/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \frac{4\,a\,d\,x\,(b\,x^4+a)^2-5\,a^3\,d\,x+32\,b\,c\,x\,(b\,x^4+a)^2+a^2\,d\,x\,(b\,x^4+a)+5\,a^2\,b\,c\,x+8\,a}{45\,a^3\,b\,(b\,x^4+a)^{9/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(13/4),x)
```

output
```
(4*a*d*x*(a + b*x^4)^2 - 5*a^3*d*x + 32*b*c*x*(a + b*x^4)^2 + a^2*d*x*(a +
 b*x^4) + 5*a^2*b*c*x + 8*a*b*c*x*(a + b*x^4))/(45*a^3*b*(a + b*x^4)^(9/4)
)
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int((d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
int(1/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3*(a
 + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.169 $\int \frac{c+dx^4}{x^4(a+bx^4)^{13/4}} dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{13/4}} dx = -\frac{c}{3ax^3(a+bx^4)^{9/4}} - \frac{(4bc-ad)x}{9a^2(a+bx^4)^{9/4}} - \frac{8(4bc-ad)x}{45a^3(a+bx^4)^{5/4}} - \frac{32(4bc-ad)x}{45a^4\sqrt[4]{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(9/4)-1/9*(-a*d+4*b*c)*x/a^2/(b*x^4+a)^(9/4)-8/45*(
-a*d+4*b*c)*x/a^3/(b*x^4+a)^(5/4)-32/45*(-a*d+4*b*c)*x/a^4/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{13/4}} dx = \frac{-128b^3cx^{12}-15a^3(c-3dx^4)+32ab^2x^8(-9c+dx^4)+36a^2bx^4(-5c+2dx^4)}{45a^4x^3(a+bx^4)^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(13/4)),x]
```

output
```
(-128*b^3*c*x^12 - 15*a^3*(c - 3*d*x^4) + 32*a*b^2*x^8*(-9*c + d*x^4) + 36
*a^2*b*x^4*(-5*c + 2*d*x^4))/(45*a^4*x^3*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{13/4}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(4bc-ad)\int \frac{1}{(bx^4+a)^{13/4}}dx}{a}-\frac{c}{3ax^3 (a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{(4bc-ad)\left(\frac{8\int \frac{1}{(bx^4+a)^{9/4}}dx}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{(4bc-ad)\left(\frac{8\left(\frac{4\int \frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{3ax^3 (a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{ 746}$$

$$-\frac{\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)(4bc-ad)}{a}-\frac{c}{3ax^3\left(a+bx^4\right)^{9/4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(13/4)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(9/4)) - ((4*b*c - a*d)*(x/(9*a*(a + b*x^4)^(9/4
)) + (8*(x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(9*
a)))/a
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$\frac{(45d\,x^4-15c)a^3-180\left(-\frac{2d\,x^4}{5}+c\right)b\,x^4a^2-288b^2x^8\left(-\frac{d\,x^4}{9}+c\right)a-128b^3c\,x^{12}}{45(b\,x^4+a)^{\frac{9}{4}}x^3a^4}$	76
gosper	$-\frac{-32a\,b^2d\,x^{12}+128b^3c\,x^{12}-72a^2bd\,x^8+288a\,b^2c\,x^8-45a^3d\,x^4+180a^2bc\,x^4+15c\,a^3}{45x^3(b\,x^4+a)^{\frac{9}{4}}a^4}$	83
trager	$-\frac{-32a\,b^2d\,x^{12}+128b^3c\,x^{12}-72a^2bd\,x^8+288a\,b^2c\,x^8-45a^3d\,x^4+180a^2bc\,x^4+15c\,a^3}{45x^3(b\,x^4+a)^{\frac{9}{4}}a^4}$	83
orering	$-\frac{-32a\,b^2d\,x^{12}+128b^3c\,x^{12}-72a^2bd\,x^8+288a\,b^2c\,x^8-45a^3d\,x^4+180a^2bc\,x^4+15c\,a^3}{45x^3(b\,x^4+a)^{\frac{9}{4}}a^4}$	83
risch	$-\frac{c(b\,x^4+a)^{\frac{3}{4}}}{3a^4x^3}+\frac{(b\,x^4+a)^{\frac{3}{4}}x(32a\,b^2d\,x^8-113c\,b^3x^8+72a^2bd\,x^4-243a\,b^2c\,x^4+45a^3d-135a^2bc)}{45a^4(b^3x^{12}+3a\,b^2x^8+3a^2b\,x^4+a^3)}$	119

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
1/45*((45*d*x^4-15*c)*a^3-180*(-2/5*d*x^4+c)*b*x^4*a^2-288*b^2*x^8*(-1/9*d
*x^4+c)*a-128*b^3*c*x^12)/(b*x^4+a)^(9/4)/x^3/a^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{13/4}}\,dx = -\frac{(32\,(4\,b^3c-ab^2d)x^{12}+72\,(4\,ab^2c-a^2bd)x^8+45\,(4\,a^2bc-a^3d)x^4+15\,a^3c)(bx^4+a)^{\frac{3}{4}}}{45\,(a^4b^3x^{15}+3\,a^5b^2x^{11}+3\,a^6bx^7+a^7x^3)}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
-1/45*(32*(4*b^3*c - a*b^2*d)*x^12 + 72*(4*a*b^2*c - a^2*b*d)*x^8 + 45*(4*
a^2*b*c - a^3*d)*x^4 + 15*a^3*c)*(b*x^4 + a)^(3/4)/(a^4*b^3*x^15 + 3*a^5*b
^2*x^11 + 3*a^6*b*x^7 + a^7*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(104) = 208$.

Time = 165.83 (sec) , antiderivative size = 898, normalized size of antiderivative = 8.39

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{13/4}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(13/4),x)
```

output
```
c*(45*a**3*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(256*a**7*b**9*ga
mma(13/4) + 768*a**6*b**10*x**4*gamma(13/4) + 768*a**5*b**11*x**8*gamma(13
/4) + 256*a**4*b**12*x**12*gamma(13/4)) + 540*a**2*b**(43/4)*x**4*(a/(b*x*
*4) + 1)**(3/4)*gamma(-3/4)/(256*a**7*b**9*gamma(13/4) + 768*a**6*b**10*x*
*4*gamma(13/4) + 768*a**5*b**11*x**8*gamma(13/4) + 256*a**4*b**12*x**12*ga
mma(13/4)) + 864*a*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(256
*a**7*b**9*gamma(13/4) + 768*a**6*b**10*x**4*gamma(13/4) + 768*a**5*b**11*
x**8*gamma(13/4) + 256*a**4*b**12*x**12*gamma(13/4)) + 384*b**(51/4)*x**12
*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(256*a**7*b**9*gamma(13/4) + 768*a**6
*b**10*x**4*gamma(13/4) + 768*a**5*b**11*x**8*gamma(13/4) + 256*a**4*b**12
*x**12*gamma(13/4))) + d*(45*a**5*x*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a
)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13
/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(2
1/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gamma(13/4)) + 117*a**4*b*x**5*gamma
(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(29/4)*b*x*
*4*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 192*a**(25/4)*b**2*x**8*(1 + b*x**4
/a)**(1/4)*gamma(13/4) + 64*a**(21/4)*b**3*x**12*(1 + b*x**4/a)**(1/4)*gam
ma(13/4)) + 104*a**3*b**2*x**9*gamma(1/4)/(64*a**(33/4)*(1 + b*x**4/a)**(1
/4)*gamma(13/4) + 192*a**(29/4)*b*x**4*(1 + b*x**4/a)**(1/4)*gamma(13/4) +
 192*a**(25/4)*b**2*x**8*(1 + b*x**4/a)**(1/4)*gamma(13/4) + 64*a**(21/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{13/4}}\,dx = \frac{\left(5\,b^2-\frac{18\,(bx^4+a)b}{x^4}+\frac{45\,(bx^4+a)^2}{x^8}\right)dx^9}{45\,(bx^4+a)^{\frac{9}{4}}a^3} - \frac{1}{45}\left(\frac{\left(5\,b^3-\frac{27\,(bx^4+a)b^2}{x^4}+\frac{135\,(bx^4+a)^2b}{x^8}\right)x^9}{(bx^4+a)^{\frac{9}{4}}a^4}+\frac{15\,(bx^4+a)^{\frac{3}{4}}}{a^4x^3}\right)c$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
1/45*(5*b^2 - 18*(b*x^4 + a)*b/x^4 + 45*(b*x^4 + a)^2/x^8)*d*x^9/((b*x^4 +
 a)^(9/4)*a^3) - 1/45*((5*b^3 - 27*(b*x^4 + a)*b^2/x^4 + 135*(b*x^4 + a)^2
*b/x^8)*x^9/((b*x^4 + a)^(9/4)*a^4) + 15*(b*x^4 + a)^(3/4)/(a^4*x^3))*c
```

Giac [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{c+dx^4}{x^4 \left(a+bx^4\right)^{13/4}}\,dx = -\frac{-45\,d\,a^3\,x^4+15\,c\,a^3-72\,d\,a^2\,b\,x^8+180\,c\,a^2\,b\,x^4-32\,d\,a\,b^2\,x^{12}+288\,c\,a\,b^2\,x^8+128\,c\,b^3\,x^{12}}{45\,a^4\,x^3\,{\left(b\,x^4+a\right)}^{9/4}}$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(13/4)),x)
```

output
```
-(15*a^3*c - 45*a^3*d*x^4 + 128*b^3*c*x^12 + 180*a^2*b*c*x^4 + 288*a*b^2*c
*x^8 - 72*a^2*b*d*x^8 - 32*a*b^2*d*x^12)/(45*a^4*x^3*(a + b*x^4)^(9/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 \left(a+bx^4\right)^{13/4}}\,dx = \left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^3x^4+3\left(b\,x^4+a\right)^{\frac{1}{4}} a^2b\,x^8+3\left(b\,x^4+a\right)^{\frac{1}{4}} a\,b^2x^{12}+\left(b\,x^4+a\right)^{\frac{1}{4}} b^3 \right.$$
$$+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a^3+3\left(b\,x^4+a\right)^{\frac{1}{4}} a^2b\,x^4+3\left(b\,x^4+a\right)^{\frac{1}{4}} a\,b^2x^8+\left(b\,x^4+a\right)^{\frac{1}{4}} b^3x^{12}}dx\right) d$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(13/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**3*x**4 + 3*(a + b*x**4)**(1/4)*a**2*b*x**8 +
 3*(a + b*x**4)**(1/4)*a*b**2*x**12 + (a + b*x**4)**(1/4)*b**3*x**16),x)*c
 + int(1/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d
```

3.170 $\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{13/4}}\,dx = -\frac{c}{7ax^7\left(a+bx^4\right)^{9/4}}-\frac{16bc-7ad}{63a^2x^3\left(a+bx^4\right)^{9/4}} -\frac{4(16bc-7ad)}{105a^3x^3\left(a+bx^4\right)^{5/4}}-\frac{32(16bc-7ad)}{105a^4x^3\sqrt[4]{a+bx^4}}+\frac{128(16bc-7ad)\left(a+bx^4\right)^{3/4}}{315a^5x^3}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(9/4)-1/63*(-7*a*d+16*b*c)/a^2/x^3/(b*x^4+a)^(9/4)-
4/105*(-7*a*d+16*b*c)/a^3/x^3/(b*x^4+a)^(5/4)-32/105*(-7*a*d+16*b*c)/a^4/x
^3/(b*x^4+a)^(1/4)+128/315*(-7*a*d+16*b*c)*(b*x^4+a)^(3/4)/a^5/x^3
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.73

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{13/4}}\,dx = \frac{2048b^4cx^{16}+60a^3bx^4(4c-21dx^4)+288a^2b^2x^8(10c-7dx^4)+128ab^3x^{12}(36c-7}{315a^5x^7\left(a+bx^4\right)^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(13/4)),x]
```

output
```
(2048*b^4*c*x^16 + 60*a^3*b*x^4*(4*c - 21*d*x^4) + 288*a^2*b^2*x^8*(10*c -
 7*d*x^4) + 128*a*b^3*x^12*(36*c - 7*d*x^4) - 15*a^4*(3*c + 7*d*x^4))/(315
*a^5*x^7*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{13/4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(16bc-7ad)\int \frac{1}{x^4(bx^4+a)^{13/4}}dx}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(16bc-7ad)\left(-\frac{4b\int \frac{1}{\left(bx^4+a\right)^{13/4}}dx}{a}-\frac{1}{3ax^3(a+bx^4)^{9/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(16bc-7ad)\left(-\frac{4b\left(\frac{8\int \frac{1}{\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{a}-\frac{1}{3ax^3(a+bx^4)^{9/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(16bc-7ad)\left(-\frac{4b\left(\frac{8\left(\frac{4\int\frac{1}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{1}{3ax^3(a+bx^4)^{9/4}}\right)}{7a}-\frac{c}{7ax^7(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{746}$$

$$-\frac{\left(-\frac{4b\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a(a+bx^4)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{1}{3ax^3(a+bx^4)^{9/4}}\right)(16bc-7ad)}{7a}-\frac{c}{7ax^7(a+bx^4)^{9/4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(13/4)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(9/4)) - ((16*b*c - 7*a*d)*(-1/3*1/(a*x^3*(a + b
*x^4)^(9/4)) - (4*b*(x/(9*a*(a + b*x^4)^(9/4)) + (8*(x/(5*a*(a + b*x^4)^(5
/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(9*a)))/a))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(-105 d \,x^{4}-45 c \right) a^{4}+240 \left(-\frac{21 d \,x^{4}}{4}+c \right) b \,x^{4} a^{3}+2880 \left(-\frac{7 d \,x^{4}}{10}+c \right) b^{2} x^{8} a^{2}+4608 \left(-\frac{7 d \,x^{4}}{36}+c \right) b^{3} x^{12} a +2048 b^{4} c \,x^{16}}{315 \left(b \,x^{4}+a \right)^{\frac{9}{4}} x^{7} a^{5}}$
gosper	$-\frac{896 a \,b^{3} d \,x^{16}-2048 b^{4} c \,x^{16}+2016 a^{2} b^{2} d \,x^{12}-4608 a \,b^{3} c \,x^{12}+1260 a^{3} b d \,x^{8}-2880 a^{2} b^{2} c \,x^{8}+105 a^{4} d \,x^{4}-240 a^{3} b c \,x^{4}+45 c \,a^{4}}{315 x^{7} \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{5}}$
trager	$-\frac{896 a \,b^{3} d \,x^{16}-2048 b^{4} c \,x^{16}+2016 a^{2} b^{2} d \,x^{12}-4608 a \,b^{3} c \,x^{12}+1260 a^{3} b d \,x^{8}-2880 a^{2} b^{2} c \,x^{8}+105 a^{4} d \,x^{4}-240 a^{3} b c \,x^{4}+45 c \,a^{4}}{315 x^{7} \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{5}}$
orering	$-\frac{896 a \,b^{3} d \,x^{16}-2048 b^{4} c \,x^{16}+2016 a^{2} b^{2} d \,x^{12}-4608 a \,b^{3} c \,x^{12}+1260 a^{3} b d \,x^{8}-2880 a^{2} b^{2} c \,x^{8}+105 a^{4} d \,x^{4}-240 a^{3} b c \,x^{4}+45 c \,a^{4}}{315 x^{7} \left(b \,x^{4}+a \right)^{\frac{9}{4}} a^{5}}$
risch	$-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} \left(7 a d \,x^{4}-25 b c \,x^{4}+3 a c \right)}{21 a^{5} x^{7}}-\frac{\left(b \,x^{4}+a \right)^{\frac{3}{4}} x \left(113 a \,b^{2} d \,x^{8}-239 c \,b^{3} x^{8}+243 a^{2} b d \,x^{4}-504 a \,b^{2} c \,x^{4}+135 a^{3} d -270 a^{2} b}{45 a^{5} \left(b^{3} x^{12}+3 a \,b^{2} x^{8}+3 a^{2} b \,x^{4}+a^{3}\right)}$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(13/4),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-105*d*x^4-45*c)*a^4+240*(-21/4*d*x^4+c)*b*x^4*a^3+2880*(-7/10*d*x
^4+c)*b^2*x^8*a^2+4608*(-7/36*d*x^4+c)*b^3*x^12*a+2048*b^4*c*x^16)/(b*x^4+
a)^(9/4)/x^7/a^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{13/4}}\,dx = \frac{(128\,(16\,b^4c-7\,ab^3d)x^{16}+288\,(16\,ab^3c-7\,a^2b^2d)x^{12}+180\,(16\,a^2b^2c-7\,a^3bd)x^8}{315\,(a^5b^3x^{19}+3\,a^6b^2x^{15}+3\,a^7bx^{11}+a^8}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
1/315*(128*(16*b^4*c - 7*a*b^3*d)*x^16 + 288*(16*a*b^3*c - 7*a^2*b^2*d)*x^
12 + 180*(16*a^2*b^2*c - 7*a^3*b*d)*x^8 - 45*a^4*c + 15*(16*a^3*b*c - 7*a^
4*d)*x^4)*(b*x^4 + a)^(3/4)/(a^5*b^3*x^19 + 3*a^6*b^2*x^15 + 3*a^7*b*x^11
+ a^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8 \left(a+bx^4\right)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(13/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{13/4}}\,dx = \frac{1}{315}\left(\frac{7\left(5\,b^4-\frac{36\,(bx^4+a)b^3}{x^4}+\frac{270\,(bx^4+a)^2b^2}{x^8}\right)x^9}{(bx^4+a)^{\frac{9}{4}}a^5}+\frac{15\left(\frac{28\,(bx^4+a)^{\frac{3}{4}}b}{x^3}-\frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}\right)}{a^5}\right)c - \frac{1}{45}\left(\frac{\left(5\,b^3-\frac{27\,(bx^4+a)b^2}{x^4}+\frac{135\,(bx^4+a)^2b}{x^8}\right)x^9}{(bx^4+a)^{\frac{9}{4}}a^4}+\frac{15\,(bx^4+a)^{\frac{3}{4}}}{a^4x^3}\right)d$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
1/315*(7*(5*b^4 - 36*(b*x^4 + a)*b^3/x^4 + 270*(b*x^4 + a)^2*b^2/x^8)*x^9/
((b*x^4 + a)^(9/4)*a^5) + 15*(28*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(
7/4)/x^7)/a^5)*c - 1/45*((5*b^3 - 27*(b*x^4 + a)*b^2/x^4 + 135*(b*x^4 + a)
^2*b/x^8)*x^9/((b*x^4 + a)^(9/4)*a^4) + 15*(b*x^4 + a)^(3/4)/(a^4*x^3))*d
```

Giac [F]

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{13/4}}\,dx = \frac{x\left(\frac{b(224\,a\,d-521\,b\,c)}{126\,a^4}+\frac{a\left(\frac{4\,b^3\,c}{35\,a^5}-\frac{b^2\,(224\,a\,d-521\,b\,c)}{90\,a^5}\right)}{b}\right)}{(b\,x^4+a)^{5/4}} - \frac{x^4\left(\frac{2\,b^2\,c}{7\,a^3}+\frac{4\,b\,(7\,a^4\,d-25\,a^3\,b\,c)}{63\,a^6}\right)+\frac{7\,a^4\,d-25\,a^3\,b\,c}{21\,a^5}}{x^3\,(b\,x^4+a)^{9/4}} - \frac{c\,(b\,x^4+a)^{3/4}}{7\,a^4\,x^7} + \frac{x\,(2048\,b^2\,c-896\,a\,b\,d)}{315\,a^5\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(13/4)),x)
```

output
```
(x*((b*(224*a*d - 521*b*c))/(126*a^4) + (a*((4*b^3*c)/(35*a^5) - (b^2*(224
*a*d - 521*b*c))/(90*a^5)))/b))/(a + b*x^4)^(5/4) - (x^4*((2*b^2*c)/(7*a^3
) + (4*b*(7*a^4*d - 25*a^3*b*c))/(63*a^6)) + (7*a^4*d - 25*a^3*b*c)/(21*a^
5))/(x^3*(a + b*x^4)^(9/4)) - (c*(a + b*x^4)^(3/4))/(7*a^4*x^7) + (x*(2048
*b^2*c - 896*a*b*d))/(315*a^5*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{13/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3x^8+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^{12}+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{16}+(b\,x^4+a)^{\frac{1}{4}}\,b \cdots}\right.$$

$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^8+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{16}}dx\right)d$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(13/4),x)
```

output

```
int(1/((a + b*x**4)**(1/4)*a**3*x**8 + 3*(a + b*x**4)**(1/4)*a**2*b*x**12
+ 3*(a + b*x**4)**(1/4)*a*b**2*x**16 + (a + b*x**4)**(1/4)*b**3*x**20),x)*
c + int(1/((a + b*x**4)**(1/4)*a**3*x**4 + 3*(a + b*x**4)**(1/4)*a**2*b*x*
*8 + 3*(a + b*x**4)**(1/4)*a*b**2*x**12 + (a + b*x**4)**(1/4)*b**3*x**16),
x)*d
```

3.171 $\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{(bc-ad)x^{11}}{9b^2(a+bx^4)^{9/4}} - \frac{(11bc-20ad)x^7}{45b^3(a+bx^4)^{5/4}} + \frac{77(2bc-5ad)x^3}{180b^4\sqrt[4]{a+bx^4}} + \frac{dx^7}{6b^3\sqrt[4]{a+bx^4}} + \frac{77\sqrt{a}(2bc-5ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{60b^{9/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/9*(-a*d+b*c)*x^11/b^2/(b*x^4+a)^(9/4)-1/45*(-20*a*d+11*b*c)*x^7/b^3/(b*
x^4+a)^(5/4)+77/180*(-5*a*d+2*b*c)*x^3/b^4/(b*x^4+a)^(1/4)+1/6*d*x^7/b^3/(
b*x^4+a)^(1/4)+77/60*a^(1/2)*(-5*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(
sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(9/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{x^3\Big(-385a^3d+22a^2b(7c-15dx^4)+12ab^2x^4(11c-5dx^4)+8b^3x^8(3c+dx^4)+77(-}{48b^4(a+bx^4)^{9}}$$

input
```
Integrate[(x^14*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
(x^3*(-385*a^3*d + 22*a^2*b*(7*c - 15*d*x^4) + 12*a*b^2*x^4*(11*c - 5*d*x^
4) + 8*b^3*x^8*(3*c + d*x^4) + 77*(-2*b*c + 5*a*d)*(a + b*x^4)^2*(1 + (b*x
^4)/a)^(1/4)*Hypergeometric2F1[3/4, 13/4, 7/4, -((b*x^4)/a)]))/(48*b^4*(a
+ b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {957, 817, 815, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{15}(bc-ad)}{9ab(a+bx^4)^{9/4}}-\frac{(2bc-5ad)\int \frac{x^{14}}{(bx^4+a)^{9/4}}dx}{3ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{15}(bc-ad)}{9ab(a+bx^4)^{9/4}}-\frac{(2bc-5ad)\left(\frac{11\int \frac{x^{10}}{(bx^4+a)^{5/4}}dx}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{15}(bc-ad)}{9ab(a+bx^4)^{9/4}}-\frac{(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\int \frac{x^6}{(bx^4+a)^{5/4}}dx}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{\frac{x^{15}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}$$

$$\Big\downarrow \text{813}$$

$$\frac{\frac{x^{15}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{x^{15}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}}{}$$

$\downarrow$ 807

$$\frac{\frac{x^{15}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}}{}$$

$$\Big\downarrow \; 212$$

$$\frac{x^{15}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-5ad)\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b(a+bx^4)^{5/4}}\right)}{3ab}$$

input
```
Int[(x^14*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^15)/(9*a*b*(a + b*x^4)^(9/4)) - ((2*b*c - 5*a*d)*(-1/5*x^11
/(b*(a + b*x^4)^(5/4)) + (11*(x^7/(6*b*(a + b*x^4)^(1/4)) - (7*a*(x^3/(2*b
*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[
Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))))/(6*b)))/(5*b
)))/(3*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{14}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{13}{4}}}dx$$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((d*x^18 + c*x^14)*(b*x^4 + a)^(3/4)/(b^4*x^16 + 4*a*b^3*x^12 + 6*
a^2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**14*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\int \frac{x^{14}\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\left(\int \frac{x^{18}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**18/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
 int(x**14/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
 3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.172 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{(bc-ad)x^7}{9b^2(a+bx^4)^{9/4}} - \frac{(7bc-16ad)x^3}{45b^3(a+bx^4)^{5/4}} + \frac{dx^3}{2b^3\sqrt[4]{a+bx^4}} - \frac{7(2bc-11ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{30\sqrt{a}b^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/9*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(9/4)-1/45*(-16*a*d+7*b*c)*x^3/b^3/(b*x^
4+a)^(5/4)+1/2*d*x^3/b^3/(b*x^4+a)^(1/4)-7/30*(-11*a*d+2*b*c)*(1+a/b/x^4)^
(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/b^
(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{x^3\left(a(77a^2d+12b^2x^4(-c+dx^4)+ab(-14c+66dx^4))+7(2bc-11ad)\left(a+bx^4\right)^2 \right.}{24ab^3\left(a+bx^4\right)^{9/4}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
(x^3*(a*(77*a^2*d + 12*b^2*x^4*(-c + d*x^4) + a*b*(-14*c + 66*d*x^4)) + 7*
(2*b*c - 11*a*d)*(a + b*x^4)^2*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4
, 13/4, 7/4, -((b*x^4)/a)]))/(24*a*b^3*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 817, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-11ad)\int \frac{x^{10}}{(bx^4+a)^{9/4}}dx}{9ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-11ad)\left(\frac{7\int \frac{x^6}{\left(bx^4+a\right)^{5/4}}dx}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-11ad)\left(\frac{7\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-11ad)\left(\frac{7\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-\frac{(2bc-11ad)\left(\frac{7\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$$\Big\downarrow \text{807}$$

$$\frac{\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-(2bc-11ad)\left(\frac{7\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

$\downarrow$ 212

$$\frac{\frac{x^{11}(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}-(2bc-11ad)\left(\frac{7\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b(a+bx^4)^{5/4}}\right)}{9ab}$$

input
```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^11)/(9*a*b*(a + b*x^4)^(9/4)) - ((2*b*c - 11*a*d)*(-1/5*x^7
/(b*(a + b*x^4)^(5/4)) + (7*(x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 +
 a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3
/2)*(a + b*x^4)^(1/4))))/(5*b)))/(9*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{10}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{13}{4}}}dx$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((d*x^14 + c*x^10)*(b*x^4 + a)^(3/4)/(b^4*x^16 + 4*a*b^3*x^12 + 6*
a^2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 148.98 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{11}{4},\frac{13}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{15}{4}\right)} + \frac{dx^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{13}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*x**11*gamma(11/4)*hyper((11/4, 13/4), (15/4,), b*x**4*exp_polar(I*pi)/a)
/(4*a**(13/4)*gamma(15/4)) + d*x**15*gamma(15/4)*hyper((13/4, 15/4), (19/4
,), b*x**4*exp_polar(I*pi)/a)/(4*a**(13/4)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)d + \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**14/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
 int(x**10/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
 3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.173 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = -\frac{(bc-ad)x^3}{9b^2(a+bx^4)^{9/4}} + \frac{(bc-4ad)x^3}{15ab^2(a+bx^4)^{5/4}} - \frac{(2bc+7ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{3/2}b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/9*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(9/4)+1/15*(-4*a*d+b*c)*x^3/a/b^2/(b*x^4
+a)^(5/4)-1/15*(7*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(
b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{x^3\left(-a^2(7ad+2b(c+3dx^4)) + (2bc+7ad)(a+bx^4)^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F}\right.}{12a^2b^2(a+bx^4)^{9/4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
(x^3*(-(a^2*(7*a*d + 2*b*(c + 3*d*x^4))) + (2*b*c + 7*a*d)*(a + b*x^4)^2*(
1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 13/4, 7/4, -((b*x^4)/a)]))/(12
*a^2*b^2*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {957, 817, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (c+dx^4)}{(a+bx^4)^{13/4}} dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7ad+2bc)\int \frac{x^6}{(bx^4+a)^{9/4}}dx}{9ab}+\frac{x^7(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7ad+2bc)\left(\frac{3\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{5b}-\frac{x^3}{5b(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{(7ad+2bc)\left(\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(7ad+2bc)\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \; 807$$

$$\frac{(7ad+2bc)\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{10b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{(7ad+2bc)\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}-\frac{x^3}{5b(a+bx^4)^{5/4}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab\left(a+bx^4\right)^{9/4}}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^7)/(9*a*b*(a + b*x^4)^(9/4)) + ((2*b*c + 7*a*d)*(-1/5*x^3/(
b*(a + b*x^4)^(5/4)) - (3*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]
/(Sqrt[b]*x^2)]/2, 2])/(5*Sqrt[a]*b^(3/2)*(a + b*x^4)^(1/4))))/(9*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^6(d\,x^4+c)}{(b\,x^4+a)^{\frac{13}{4}}}dx$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^6)*(b*x^4 + a)^(3/4)/(b^4*x^16 + 4*a*b^3*x^12 + 6*a
^2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 132.77 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx=\frac{cx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{13}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{11}{4}\right)}+\frac{dx^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{11}{4},\frac{13}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*x**7*gamma(7/4)*hyper((7/4, 13/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(13/4)*gamma(11/4)) + d*x**11*gamma(11/4)*hyper((11/4, 13/4), (15/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*a**(13/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)d$$
$$+\left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**10/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
 int(x**6/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 +
3*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.174 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{(bc-ad)x^3}{9ab\,(a+bx^4)^{9/4}} + \frac{(2bc+ad)x^3}{15a^2b\,(a+bx^4)^{5/4}} - \frac{2(2bc+ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/9*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(9/4)+1/15*(a*d+2*b*c)*x^3/a^2/b/(b*x^4+a
)^(5/4)-2/15*(a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{x^3\left(-a^3d+2bc(a+bx^4)^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{13}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+ad(a+\right.}{6a^3b\,(a+bx^4)^{9/4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
(x^3*(-(a^3*d) + 2*b*c*(a + b*x^4)^2*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2
F1[3/4, 13/4, 7/4, -((b*x^4)/a)] + a*d*(a + b*x^4)^2*(1 + (b*x^4)/a)^(1/4)
*Hypergeometric2F1[3/4, 13/4, 7/4, -((b*x^4)/a)]))/(6*a^3*b*(a + b*x^4)^(9
/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^4)}{(a+bx^4)^{13/4}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ad+2bc)\int \frac{x^2}{(bx^4+a)^{9/4}}dx}{3ab}+\frac{x^3(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(ad+2bc)\left(\frac{2\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x^3}{5a(a+bx^4)^{5/4}}\right)}{3ab}+\frac{x^3(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{813}$$

$$\frac{(ad+2bc)\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a(a+bx^4)^{5/4}}\right)}{3ab}+\frac{x^3(bc-ad)}{9ab(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(ad+2bc)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}\right)}{3ab}+\frac{x^3(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(ad+2bc)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}\right)}{3ab}+\frac{x^3(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{(ad+2bc)\left(\frac{x^3}{5a(a+bx^4)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}\right)}{3ab}+\frac{x^3(bc-ad)}{9ab\,(a+bx^4)^{9/4}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(13/4),x]
```

output
```
((b*c - a*d)*x^3)/(9*a*b*(a + b*x^4)^(9/4)) + ((2*b*c + a*d)*(x^3/(5*a*(a
+ b*x^4)^(5/4)) - (2*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqr
t[b]*x^2)]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^4)^(1/4))))/(3*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^2(d\,x^4+c)}{(b\,x^4+a)^{\frac{13}{4}}}dx$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)*(b*x^4 + a)^(3/4)/(b^4*x^16 + 4*a*b^3*x^12 + 6*a^
2*b^2*x^8 + 4*a^3*b*x^4 + a^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 88.59 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \frac{cx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{13}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{dx^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{13}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(13/4),x)
```

output
```
c*x**3*gamma(3/4)*hyper((3/4, 13/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*
a**(13/4)*gamma(7/4)) + d*x**7*gamma(7/4)*hyper((7/4, 13/4), (11/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*a**(13/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(13/4), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{13}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(13/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(13/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(13/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right.$$
$$+\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{12}}dx\right)c$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(13/4),x)
```

output
```
int(x**6/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
int(x**2/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*c
```

3.175 $\int \frac{c+dx^4}{x^2 (a+bx^4)^{13/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{13/4}} \, dx = -\frac{c}{ax (a+bx^4)^{9/4}} - \frac{(10bc-ad)x^3}{9a^2 (a+bx^4)^{9/4}} - \frac{2(10bc-ad)x^3}{15a^3 (a+bx^4)^{5/4}} + \frac{4(10bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\, xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output

```
-c/a/x/(b*x^4+a)^(9/4)-1/9*(-a*d+10*b*c)*x^3/a^2/(b*x^4+a)^(9/4)-2/15*(-a*
d+10*b*c)*x^3/a^3/(b*x^4+a)^(5/4)+4/15*(-a*d+10*b*c)*(1+a/b/x^4)^(1/4)*x*E
llipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/b^(1/2)/(b*
x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{13/4}}\,dx = -\frac{c}{ax(a+bx^4)^{9/4}} - \frac{(10bc-ad)x^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{13}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^4\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(13/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(9/4))) - ((10*b*c - a*d)*x^3*(1 + (b*x^4)/a)^(1/4)*H
ypergeometric2F1[3/4, 13/4, 7/4, -((b*x^4)/a)])/(3*a^4*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{13/4}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(10bc-ad)\int \frac{x^2}{(bx^4+a)^{13/4}}dx}{a} - \frac{c}{ax(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(10bc-ad)\left(\frac{2\int \frac{x^2}{(bx^4+a)^{9/4}}dx}{3a} + \frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a} - \frac{c}{ax(a+bx^4)^{9/4}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(10bc-ad)\left(\frac{2\left(\frac{2\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{9/4}}$$

$\big\downarrow$ 813

$$-\frac{(10bc-ad)\left(\frac{2\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{9/4}}$$

$\big\downarrow$ 858

$$-\frac{(10bc-ad)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{9/4}}$$

$\big\downarrow$ 807

$$-\frac{(10bc-ad)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 212

$$-\frac{(10bc-ad)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{9/4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(13/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(9/4))) - ((10*b*c - a*d)*(x^3/(9*a*(a + b*x^4)^(9/4)
) + (2*(x^3/(5*a*(a + b*x^4)^(5/4)) - (2*(1 + a/(b*x^4))^(1/4)*x*EllipticE
[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^4)^(1/4)
)))/(3*a)))/a
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^4 + c}{x^2\,(b\,x^4 + a)^{\frac{13}{4}}}dx$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x)
```

output
```
int((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2 \left(a+bx^4\right)^{13/4}} \, dx = \int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{13}{4}}x^2} \, dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^4*x^18 + 4*a*b^3*x^14 + 6*a^2*b^
2*x^10 + 4*a^3*b*x^6 + a^4*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 110.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^4}{x^2 \left(a+bx^4\right)^{13/4}} \, dx = \frac{c\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{13}{4} \\ \frac{3}{4} \end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}x\Gamma\left(\frac{3}{4}\right)} + \frac{dx^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{3}{4}, \frac{13}{4} \\ \frac{7}{4} \end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{13}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(13/4),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 13/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(13/4)*x*gamma(3/4)) + d*x**3*gamma(3/4)*hyper((3/4, 13/4), (7/4,), b*x**4
*exp_polar(I*pi)/a)/(4*a**(13/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2(a+bx^4)^{13/4}}\,dx = \int \frac{d\,x^4+c}{x^2\,(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(13/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(13/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{13/4}}\,dx = \left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a^3+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2 b\,x^4+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2 x^8+(b\,x^4+a)^{\frac{1}{4}}\,b^3 x^{12}}\right.$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3 x^2+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2 b\,x^6+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2 x^{10}+(b\,x^4+a)^{\frac{1}{4}}\,b^3 x^{14}}dx\right)c$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(13/4),x)
```

output
```
int(x**2/((a + b*x**4)**(1/4)*a**3 + 3*(a + b*x**4)**(1/4)*a**2*b*x**4 + 3
*(a + b*x**4)**(1/4)*a*b**2*x**8 + (a + b*x**4)**(1/4)*b**3*x**12),x)*d +
int(1/((a + b*x**4)**(1/4)*a**3*x**2 + 3*(a + b*x**4)**(1/4)*a**2*b*x**6 +
 3*(a + b*x**4)**(1/4)*a*b**2*x**10 + (a + b*x**4)**(1/4)*b**3*x**14),x)*c
```

3.176 $\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{13/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 183

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{13/4}}\,dx=-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}-\frac{14bc-5ad}{45a^2x\left(a+bx^4\right)^{9/4}}-\frac{2(14bc-5ad)}{45a^3x\left(a+bx^4\right)^{5/4}}+\frac{4(14bc-5ad)}{15a^4x\sqrt[4]{a+bx^4}}-\frac{8\sqrt{b}(14bc-5ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(9/4)-1/45*(-5*a*d+14*b*c)/a^2/x/(b*x^4+a)^(9/4)-2/
45*(-5*a*d+14*b*c)/a^3/x/(b*x^4+a)^(5/4)+4/15*(-5*a*d+14*b*c)/a^4/x/(b*x^4
+a)^(1/4)-8/15*b^(1/2)*(-5*a*d+14*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1
/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(9/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.45

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \frac{-a^3c+(14bc-5ad)x^4(a+bx^4)^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^4x^5(a+bx^4)^{9/4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(13/4)),x]
```

output
```
(-(a^3*c) + (14*b*c - 5*a*d)*x^4*(a + b*x^4)^2*(1 + (b*x^4)/a)^(1/4)*Hyper
geometric2F1[-1/4, 13/4, 3/4, -((b*x^4)/a)])/(5*a^4*x^5*(a + b*x^4)^(9/4))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {955, 819, 819, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(14bc-5ad)\int \frac{1}{x^2(bx^4+a)^{13/4}}dx}{5a}-\frac{c}{5ax^5(a+bx^4)^{9/4}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(14bc-5ad)\left(\frac{10\int \frac{1}{x^2\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5(a+bx^4)^{9/4}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\int\frac{1}{x^2\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 816

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\left(-\frac{2b\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 813

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

$$\downarrow 858$$

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 807

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

$\Big\downarrow$ 212

$$-\frac{(14bc-5ad)\left(\frac{10\left(\frac{6\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax(a+bx^4)^{9/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{9/4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(13/4)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(9/4)) - ((14*b*c - 5*a*d)*(1/(9*a*x*(a + b*x^4)
^(9/4)) + (10*(1/(5*a*x*(a + b*x^4)^(5/4)) + (6*(-(1/(a*x*(a + b*x^4)^(1/4
))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]
*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(5*a)))/(9*a)))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{6} \left(b\,x^{4}+a\right)^{\frac{13}{4}}}d x$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x)
```

output
```
int((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{13}{4}}x^{6}}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^4*x^22 + 4*a*b^3*x^18 + 6*a^2*b^
2*x^14 + 4*a^3*b*x^10 + a^4*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(13/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{13}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(13/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \int \frac{d\,x^4+c}{x^6\,(b\,x^4+a)^{13/4}}\,dx$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(13/4)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(13/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{13/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3x^6+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^{10}+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{14}+(b\,x^4+a)^{\frac{1}{4}}\,b$$

$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^3x^2+3\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^6+3\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{10}+(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{14}}dx\right)d$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(13/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**3*x**6 + 3*(a + b*x**4)**(1/4)*a**2*b*x**10
+ 3*(a + b*x**4)**(1/4)*a*b**2*x**14 + (a + b*x**4)**(1/4)*b**3*x**18),x)*
c + int(1/((a + b*x**4)**(1/4)*a**3*x**2 + 3*(a + b*x**4)**(1/4)*a**2*b*x*
*6 + 3*(a + b*x**4)**(1/4)*a*b**2*x**10 + (a + b*x**4)**(1/4)*b**3*x**14),
x)*d
```

3.177 $\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{a^3(bc-ad)x}{13b^5(a+bx^4)^{13/4}} - \frac{a^2(40bc-53ad)x}{117b^5(a+bx^4)^{9/4}} + \frac{2a(191bc-373ad)x}{585b^5(a+bx^4)^{5/4}} - \frac{2(406bc-1433ad)x}{585b^5\sqrt[4]{a+bx^4}} + \frac{dx(a+bx^4)^{3/4}}{4b^5} + \frac{(4bc-17ad)\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{21/4}} + \frac{(4bc-17ad)\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{21/4}}$$

output
```
1/13*a^3*(-a*d+b*c)*x/b^5/(b*x^4+a)^(13/4)-1/117*a^2*(-53*a*d+40*b*c)*x/b^
5/(b*x^4+a)^(9/4)+2/585*a*(-373*a*d+191*b*c)*x/b^5/(b*x^4+a)^(5/4)-2/585*(
-1433*a*d+406*b*c)*x/b^5/(b*x^4+a)^(1/4)+1/4*d*x*(b*x^4+a)^(3/4)/b^5+1/8*(
-17*a*d+4*b*c)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(21/4)+1/8*(-17*a*d+4*b
*c)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(21/4)
```

Mathematica [A] (verified)

Time = 6.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(9945a^4d-468a^3b(5c-68dx^4)+b^4x^{12}(-3248c+585dx^4)+26a^2b^2x^4(-288c+1343dx^4)+4ab^3x^8(-2054c+34}{(a+bx^4)^{13/4}}}{468}$$

input
```
Integrate[(x^16*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((2*b^(1/4)*x*(9945*a^4*d - 468*a^3*b*(5*c - 68*d*x^4) + b^4*x^12*(-3248*c
 + 585*d*x^4) + 26*a^2*b^2*x^4*(-288*c + 1343*d*x^4) + 4*a*b^3*x^8*(-2054*
c + 3451*d*x^4)))/(a + b*x^4)^(13/4) + 585*(4*b*c - 17*a*d)*ArcTan[(b^(1/4
)*x)/(a + b*x^4)^(1/4)] + 585*(4*b*c - 17*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*
x^4)^(1/4)])/(4680*b^(21/4))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {957, 817, 817, 817, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{x^{17}(bc-ad)}{13ab(a+bx^4)^{13/4}} - \frac{(4bc-17ad)\int \frac{x^{16}}{(bx^4+a)^{13/4}}dx}{13ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{17}(bc-ad)}{13ab(a+bx^4)^{13/4}} - \frac{(4bc-17ad)\left(\frac{13\int \frac{x^{12}}{(bx^4+a)^{9/4}}dx}{9b} - \frac{x^{13}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(4bc-17ad)\left(\frac{13\left(\frac{9\int\frac{x^8}{\left(bx^4+a\right)^{5/4}}dx}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{13}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\int\frac{x^4}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{13}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{13}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\big\downarrow$ 770

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{13}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \; 756$$

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)\cdots}}{9b}\cdots\right)}{13ab}$$

$\downarrow$ 216

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-$$

$$(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}\right.$$

↓ 219

$$\frac{x^{17}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-$$

$$(4bc-17ad)\left(\frac{13\left(\frac{9\left(\frac{5\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^9}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\right.$$

input
```
Int[(x^16*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^17)/(13*a*b*(a + b*x^4)^(13/4)) - ((4*b*c - 17*a*d)*(-1/9*x
^13/(b*(a + b*x^4)^(9/4)) + (13*(-1/5*x^9/(b*(a + b*x^4)^(5/4)) + (9*(-(x^
5/(b*(a + b*x^4)^(1/4))) + (5*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)
^(1/4)]/(2*b^(1/4))))/(4*b)))/b))/(5*b)))/(9*b)))/(13*a*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{158\left(-\frac{3451 d\,x^{4}}{2054}+c\right)x^{9} a\,b^{\frac{13}{4}}}{45}+\frac{\left(d\,x^{17}-\frac{3248}{585} c\,x^{13}\right)b^{\frac{17}{4}}}{4}-\left(-\frac{68 d\,x^{4}}{5}+c\right)x\,a^{3} b^{\frac{5}{4}}-\frac{16 x^{5} a^{2}\left(-\frac{1343 d\,x^{4}}{288}+c\right)b^{\frac{9}{4}}}{5}+\frac{17 a^{4} d x\,b^{\frac{1}{4}}}{4}+\frac{17\left(a \cdots\right.}{\cdots}}{b^{\frac{21}{4}}\left(b\,x^{4}+a\right)^{\frac{13}{4}}}$

input
```
int(x^16*(d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
17/8/b^(21/4)*(-1264/765*(-3451/2054*d*x^4+c)*x^9*a*b^(13/4)+2/17*(d*x^17-
3248/585*c*x^13)*b^(17/4)-8/17*(-68/5*d*x^4+c)*x*a^3*b^(5/4)-128/85*x^5*a^
2*(-1343/288*d*x^4+c)*b^(9/4)+2*a^4*d*x*b^(1/4)+(a*d-4/17*c*b)*(b*x^4+a)^(
13/4)*(arctan((b*x^4+a)^(1/4)/x/b^(1/4))-1/2*ln((x*b^(1/4)+(b*x^4+a)^(1/4)
)/(-x*b^(1/4)+(b*x^4+a)^(1/4)))))/(b*x^4+a)^(13/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 1042, normalized size of antiderivative = 4.89

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/9360*(585*(b^9*x^16 + 4*a*b^8*x^12 + 6*a^2*b^7*x^8 + 4*a^3*b^6*x^4 + a^4
*b^5)*((256*b^4*c^4 - 4352*a*b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^3
*b*c*d^3 + 83521*a^4*d^4)/b^21)^(1/4)*log(-(b^16*x*((256*b^4*c^4 - 4352*a*
b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^3*b*c*d^3 + 83521*a^4*d^4)/b^2
1)^(3/4) + (64*b^3*c^3 - 816*a*b^2*c^2*d + 3468*a^2*b*c*d^2 - 4913*a^3*d^3
)*(b*x^4 + a)^(1/4))/x) - 585*(b^9*x^16 + 4*a*b^8*x^12 + 6*a^2*b^7*x^8 + 4
*a^3*b^6*x^4 + a^4*b^5)*((256*b^4*c^4 - 4352*a*b^3*c^3*d + 27744*a^2*b^2*c
^2*d^2 - 78608*a^3*b*c*d^3 + 83521*a^4*d^4)/b^21)^(1/4)*log((b^16*x*((256*
b^4*c^4 - 4352*a*b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^3*b*c*d^3 + 8
3521*a^4*d^4)/b^21)^(3/4) - (64*b^3*c^3 - 816*a*b^2*c^2*d + 3468*a^2*b*c*d
^2 - 4913*a^3*d^3)*(b*x^4 + a)^(1/4))/x) - 585*(-I*b^9*x^16 - 4*I*a*b^8*x^
12 - 6*I*a^2*b^7*x^8 - 4*I*a^3*b^6*x^4 - I*a^4*b^5)*((256*b^4*c^4 - 4352*a
*b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^3*b*c*d^3 + 83521*a^4*d^4)/b^
21)^(1/4)*log((I*b^16*x*((256*b^4*c^4 - 4352*a*b^3*c^3*d + 27744*a^2*b^2*c
^2*d^2 - 78608*a^3*b*c*d^3 + 83521*a^4*d^4)/b^21)^(3/4) - (64*b^3*c^3 - 81
6*a*b^2*c^2*d + 3468*a^2*b*c*d^2 - 4913*a^3*d^3)*(b*x^4 + a)^(1/4))/x) - 5
85*(I*b^9*x^16 + 4*I*a*b^8*x^12 + 6*I*a^2*b^7*x^8 + 4*I*a^3*b^6*x^4 + I*a^
4*b^5)*((256*b^4*c^4 - 4352*a*b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^
3*b*c*d^3 + 83521*a^4*d^4)/b^21)^(1/4)*log((-I*b^16*x*((256*b^4*c^4 - 4352
*a*b^3*c^3*d + 27744*a^2*b^2*c^2*d^2 - 78608*a^3*b*c*d^3 + 83521*a^4*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**16*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.55

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx =$$

$$-\frac{1}{2340}\left(\frac{4\left(45\,b^3+\frac{65\,(bx^4+a)b^2}{x^4}+\frac{117\,(bx^4+a)^2 b}{x^8}+\frac{585\,(bx^4+a)^3}{x^{12}}\right)x^{13}}{(bx^4+a)^{\frac{13}{4}}b^4}+\frac{585\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+}{x}}\right.}{b^{\frac{1}{4}}}\right.}{b^4}\right.$$

$$+\frac{1}{9360}\,d\left(\frac{4\left(180\,ab^4+\frac{340\,(bx^4+a)ab^3}{x^4}+\frac{884\,(bx^4+a)^2ab^2}{x^8}+\frac{7956\,(bx^4+a)^3ab}{x^{12}}-\frac{9945\,(bx^4+a)^4a}{x^{16}}\right)}{\frac{(bx^4+a)^{\frac{13}{4}}b^6}{x^{13}}-\frac{(bx^4+a)^{\frac{17}{4}}b^5}{x^{17}}}+\frac{9945\,a\left(\frac{2\,\arctan\left(\frac{(b}{}\right.}{b^{\frac{1}{4}}}\right.}{}\right.$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/2340*(4*(45*b^3 + 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4 + a)^2*b/x^8 + 58
5*(b*x^4 + a)^3/x^12)*x^13/((b*x^4 + a)^(13/4)*b^4) + 585*(2*arctan((b*x^4
 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b
^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^4)*c + 1/9360*d*(4*(180*a*b^4 +
340*(b*x^4 + a)*a*b^3/x^4 + 884*(b*x^4 + a)^2*a*b^2/x^8 + 7956*(b*x^4 + a)
^3*a*b/x^12 - 9945*(b*x^4 + a)^4*a/x^16)/((b*x^4 + a)^(13/4)*b^6/x^13 - (b
*x^4 + a)^(17/4)*b^5/x^17) + 9945*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x
))/b^(1/4) + log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(
1/4)/x))/b^(1/4))/b^5)
```

Giac [F]

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{16}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^16*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^16/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{x^{16}\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^16*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^16*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^{16}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{20}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1}\right.$$
$$+\left(\int \frac{x^{16}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int(x^16*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**20/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**16/((a + b*x**4)**(1/4)*a*
*4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16)
,x)*c
```

3.178 $\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\frac{(bc-ad)x^{13}}{13ab\,(a+bx^4)^{13/4}}-\frac{dx^9}{9b^2\,(a+bx^4)^{9/4}}-\frac{dx^5}{5b^3\,(a+bx^4)^{5/4}}-\frac{dx}{b^4\sqrt[4]{a+bx^4}}+\frac{d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{17/4}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{17/4}}$$

output
```
1/13*(-a*d+b*c)*x^13/a/b/(b*x^4+a)^(13/4)-1/9*d*x^9/b^2/(b*x^4+a)^(9/4)-1/
5*d*x^5/b^3/(b*x^4+a)^(5/4)-d*x/b^4/(b*x^4+a)^(1/4)+1/2*d*arctan(b^(1/4)*x
/(b*x^4+a)^(1/4))/b^(17/4)+1/2*d*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(17/
4)
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\frac{-\frac{2\sqrt[4]{b}x(585a^4d+1872a^3bdx^4+2054a^2b^2dx^8-45b^4cx^{12}+812ab^3dx^{12})}{a(a+bx^4)^{13/4}}+585d\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+}{1170b^{17/4}}$$

input
```
Integrate[(x^12*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((-2*b^(1/4)*x*(585*a^4*d + 1872*a^3*b*d*x^4 + 2054*a^2*b^2*d*x^8 - 45*b^4
*c*x^12 + 812*a*b^3*d*x^12))/(a*(a + b*x^4)^(13/4)) + 585*d*ArcTan[(b^(1/4
)*x)/(a + b*x^4)^(1/4)] + 585*d*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(1
170*b^(17/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {954, 817, 817, 817, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int \frac{x^{12}}{(bx^4+a)^{13/4}}dx}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{d\left(\frac{\int \frac{x^8}{\left(bx^4+a\right)^{9/4}}dx}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{d\left(\frac{\frac{\int \frac{x^4}{\left(bx^4+a\right)^{5/4}}dx}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{d\left(\frac{\frac{\frac{\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 770

$$\frac{d\left(\frac{\frac{\frac{\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 756

$$\frac{d\left(\frac{\frac{\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 216

$$\frac{d\left(\frac{\frac{\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}}{b}-\frac{x^5}{5b\left(a+bx^4\right)^{5/4}}}{b}-\frac{x^9}{9b(a+bx^4)^{9/4}}\right)}{b}+\frac{x^{13}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(x^12*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^13)/(13*a*b*(a + b*x^4)^(13/4)) + (d*(-1/9*x^9/(b*(a + b*x^
4)^(9/4)) + (-1/5*x^5/(b*(a + b*x^4)^(5/4)) + (-(x/(b*(a + b*x^4)^(1/4)))
+ (ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)
/(a + b*x^4)^(1/4)]/(2*b^(1/4)))/b)/b)/b))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{-\frac{3248 a\, b^{\frac{13}{4}} d\, x^{13}}{585}+\frac{4 b^{\frac{17}{4}} c\, x^{13}}{13}+d\left(-\frac{64 a^{2} b^{\frac{5}{4}} x^{5}}{5}-\frac{632 a\, b^{\frac{9}{4}} x^{9}}{45}-4 a^{3} x\, b^{\frac{1}{4}}+\left(b\, x^{4}+a\right)^{\frac{13}{4}}\left(\ln\left(\frac{x\, b^{\frac{1}{4}}+\left(b\, x^{4}+a\right)^{\frac{1}{4}}}{-x\, b^{\frac{1}{4}}+\left(b\, x^{4}+a\right)^{\frac{1}{4}}}\right)-2 \arctan\left(\right.\right.\right.}{4 b^{\frac{17}{4}} \left(b\, x^{4}+a\right)^{\frac{13}{4}} a}$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-3248/585*a*b^(13/4)*d*x^13+4/13*b^(17/4)*c*x^13+d*(-64/5*a^2*b^(5/4)
*x^5-632/45*a*b^(9/4)*x^9-4*a^3*x*b^(1/4)+(b*x^4+a)^(13/4)*(ln((x*b^(1/4)+
(b*x^4+a)^(1/4))/(-x*b^(1/4)+(b*x^4+a)^(1/4)))-2*arctan((b*x^4+a)^(1/4)/x/
b^(1/4)))*a)/b^(17/4)/(b*x^4+a)^(13/4)/a
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.23

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{585\,(ab^8x^{16}+4\,a^2b^7x^{12}+6\,a^3b^6x^8+4\,a^4b^5x^4+a^5b^4)\left(\frac{d^4}{b^{17}}\right)^{\frac{1}{4}}\log\left(\frac{b^{13}x\left(\frac{d^4}{b^{17}}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}}{x}\right.}{}$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/2340*(585*(a*b^8*x^16 + 4*a^2*b^7*x^12 + 6*a^3*b^6*x^8 + 4*a^4*b^5*x^4 +
 a^5*b^4)*(d^4/b^17)^(1/4)*log((b^13*x*(d^4/b^17)^(3/4) + (b*x^4 + a)^(1/4
)*d^3)/x) - 585*(a*b^8*x^16 + 4*a^2*b^7*x^12 + 6*a^3*b^6*x^8 + 4*a^4*b^5*x
^4 + a^5*b^4)*(d^4/b^17)^(1/4)*log(-(b^13*x*(d^4/b^17)^(3/4) - (b*x^4 + a)
^(1/4)*d^3)/x) - 585*(I*a*b^8*x^16 + 4*I*a^2*b^7*x^12 + 6*I*a^3*b^6*x^8 +
4*I*a^4*b^5*x^4 + I*a^5*b^4)*(d^4/b^17)^(1/4)*log((I*b^13*x*(d^4/b^17)^(3/
4) + (b*x^4 + a)^(1/4)*d^3)/x) - 585*(-I*a*b^8*x^16 - 4*I*a^2*b^7*x^12 - 6
*I*a^3*b^6*x^8 - 4*I*a^4*b^5*x^4 - I*a^5*b^4)*(d^4/b^17)^(1/4)*log((-I*b^1
3*x*(d^4/b^17)^(3/4) + (b*x^4 + a)^(1/4)*d^3)/x) - 4*(2054*a^2*b^2*d*x^9 -
 (45*b^4*c - 812*a*b^3*d)*x^13 + 1872*a^3*b*d*x^5 + 585*a^4*d*x)*(b*x^4 +
a)^(3/4))/(a*b^8*x^16 + 4*a^2*b^7*x^12 + 6*a^3*b^6*x^8 + 4*a^4*b^5*x^4 + a
^5*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**12*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{cx^{13}}{13\,(bx^4+a)^{\frac{13}{4}}a} - \frac{1}{2340}\left(\frac{4\left(45\,b^3+\frac{65\,(bx^4+a)b^2}{x^4}+\frac{117\,(bx^4+a)^2b}{x^8}+\frac{585\,(bx^4+a)^3}{x^{12}}\right)x^{13}}{(bx^4+a)^{\frac{13}{4}}b^4} + \frac{585\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+}{x}}\right.}{b^{\frac{1}{4}}}\right.}{b^4}\right.$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
1/13*c*x^13/((b*x^4 + a)^(13/4)*a) - 1/2340*(4*(45*b^3 + 65*(b*x^4 + a)*b^
2/x^4 + 117*(b*x^4 + a)^2*b/x^8 + 585*(b*x^4 + a)^3/x^12)*x^13/((b*x^4 + a
)^(13/4)*b^4) + 585*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log
(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)
)/b^4)*d
```

Giac [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{12}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^12*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^12/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{x^{12}\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^12*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^{12}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{16}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1}\right.$$
$$+\left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int(x^12*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**16/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**12/((a + b*x**4)**(1/4)*a*
*4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16)
,x)*c
```

3.179 $\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{(bc-ad)x^9}{13ab\,(a+bx^4)^{13/4}} + \frac{(4bc+9ad)x^9}{117a^2b\,(a+bx^4)^{9/4}}$$

output
```
1/13*(-a*d+b*c)*x^9/a/b/(b*x^4+a)^(13/4)+1/117*(9*a*d+4*b*c)*x^9/a^2/b/(b*
x^4+a)^(9/4)
```

Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^9(13ac+4bcx^4+9adx^4)}{117a^2\,(a+bx^4)^{13/4}}$$

input
```
Integrate[(x^8*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^9*(13*a*c + 4*b*c*x^4 + 9*a*d*x^4))/(117*a^2*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (c+dx^4)}{(a+bx^4)^{17/4}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(9ad+4bc)\int \frac{x^8}{(bx^4+a)^{13/4}}dx}{13ab} + \frac{x^9(bc-ad)}{13ab(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^9(9ad+4bc)}{117a^2b(a+bx^4)^{9/4}} + \frac{x^9(bc-ad)}{13ab(a+bx^4)^{13/4}}$$

input
```
Int[(x^8*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^9)/(13*a*b*(a + b*x^4)^(13/4)) + ((4*b*c + 9*a*d)*x^9)/(117
*a^2*b*(a + b*x^4)^(9/4))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^{9} \left(9 a d \,x^{4}+4 b c \,x^{4}+13 a c \right)}{117 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{2}}$	37
trager	$\frac{x^{9} \left(9 a d \,x^{4}+4 b c \,x^{4}+13 a c \right)}{117 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{2}}$	37
pseudoelliptic	$\frac{x^{9} \left(9 a d \,x^{4}+4 b c \,x^{4}+13 a c \right)}{117 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{2}}$	37
orering	$\frac{x^{9} \left(9 a d \,x^{4}+4 b c \,x^{4}+13 a c \right)}{117 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{2}}$	37

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/117*x^9*(9*a*d*x^4+4*b*c*x^4+13*a*c)/(b*x^4+a)^(13/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{((4\,bc+9\,ad)x^{13}+13\,acx^9)(bx^4+a)^{\frac{3}{4}}}{117\,(a^2b^4x^{16}+4\,a^3b^3x^{12}+6\,a^4b^2x^8+4\,a^5bx^4+a^6)}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/117*((4*b*c + 9*a*d)*x^13 + 13*a*c*x^9)*(b*x^4 + a)^(3/4)/(a^2*b^4*x^16
+ 4*a^3*b^3*x^12 + 6*a^4*b^2*x^8 + 4*a^5*b*x^4 + a^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = -\frac{\left(9\,b-\frac{13\,(bx^4+a)}{x^4}\right)cx^{13}}{117\,(bx^4+a)^{\frac{13}{4}}a^2}+\frac{dx^{13}}{13\,(bx^4+a)^{\frac{13}{4}}a}$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/117*(9*b - 13*(b*x^4 + a)/x^4)*c*x^13/((b*x^4 + a)^(13/4)*a^2) + 1/13*d
*x^13/((b*x^4 + a)^(13/4)*a)
```

Giac [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^8}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^8/(b*x^4 + a)^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.36

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x\left(\frac{a^2\,d-a\,b\,c}{117\,a\,b^3}+\frac{a\left(\frac{d}{9\,b^2}-\frac{13\,b^2\,c-13\,a\,b\,d}{117\,a\,b^3}\right)}{b}\right)}{(b\,x^4+a)^{9/4}} - \frac{x\left(\frac{d}{5\,b^3}+\frac{18\,a\,d-5\,b\,c}{585\,a\,b^3}\right)}{(b\,x^4+a)^{5/4}} + \frac{a\,x\left(\frac{c}{13\,b}-\frac{a\,d}{13\,b^2}\right)}{b\,(b\,x^4+a)^{13/4}} + \frac{x\,(9\,a\,d+4\,b\,c)}{117\,a^2\,b^3\,(b\,x^4+a)^{1/4}}$$

input
```
int((x^8*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
(x*((a^2*d - a*b*c)/(117*a*b^3) + (a*(d/(9*b^2) - (13*b^2*c - 13*a*b*d)/(1
17*a*b^3)))/b))/(a + b*x^4)^(9/4) - (x*(d/(5*b^3) + (18*a*d - 5*b*c)/(585*
a*b^3)))/(a + b*x^4)^(5/4) + (a*x*(c/(13*b) - (a*d)/(13*b^2)))/(b*(a + b*x
^4)^(13/4)) + (x*(9*a*d + 4*b*c))/(117*a^2*b^3*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^8(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1}\right.$$
$$+\left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int(x^8*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**12/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**8/((a + b*x**4)**(1/4)*a**
4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x*
*8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),
x)*c
```

3.180 $\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}} \, dx = \frac{(bc-ad)x^5}{13ab\,(a+bx^4)^{13/4}} + \frac{(8bc+5ad)x^5}{117a^2b\,(a+bx^4)^{9/4}} + \frac{4(8bc+5ad)x^5}{585a^3b\,(a+bx^4)^{5/4}}$$

output
```
1/13*(-a*d+b*c)*x^5/a/b/(b*x^4+a)^(13/4)+1/117*(5*a*d+8*b*c)*x^5/a^2/b/(b*
x^4+a)^(9/4)+4/585*(5*a*d+8*b*c)*x^5/a^3/b/(b*x^4+a)^(5/4)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}} \, dx = \frac{x^5(117a^2c+104abcx^4+65a^2dx^4+32b^2cx^8+20abdx^8)}{585a^3\,(a+bx^4)^{13/4}}$$

input
```
Integrate[(x^4*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^5*(117*a^2*c + 104*a*b*c*x^4 + 65*a^2*d*x^4 + 32*b^2*c*x^8 + 20*a*b*d*x
^8))/(585*a^3*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {957, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx^4)}{(a+bx^4)^{17/4}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5ad+8bc)\int \frac{x^4}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x^5(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{803}$$

$$\frac{(5ad+8bc)\left(\frac{4b\int \frac{x^8}{\left(bx^4+a\right)^{13/4}}dx}{5a}+\frac{x^5}{5a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^5(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{4bx^9}{45a^2(a+bx^4)^{9/4}}+\frac{x^5}{5a(a+bx^4)^{9/4}}\right)(5ad+8bc)}{13ab}+\frac{x^5(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(x^4*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^5)/(13*a*b*(a + b*x^4)^(13/4)) + ((8*b*c + 5*a*d)*(x^5/(5*a
*(a + b*x^4)^(9/4)) + (4*b*x^9)/(45*a^2*(a + b*x^4)^(9/4))))/(13*a*b)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$\frac{\left(\left(\frac{5 d \,x^{4}}{9}+c \right) a^{2}+\frac{8 b \left(\frac{5 d \,x^{4}}{26}+c \right) x^{4} a}{9}+\frac{32 b^{2} c \,x^{8}}{117}\right) x^{5}}{5 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{3}}$	55
gosper	$\frac{x^{5} \left(20 a b d \,x^{8}+32 b^{2} c \,x^{8}+65 a^{2} d \,x^{4}+104 a b c \,x^{4}+117 a^{2} c \right)}{585 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{3}}$	59
trager	$\frac{x^{5} \left(20 a b d \,x^{8}+32 b^{2} c \,x^{8}+65 a^{2} d \,x^{4}+104 a b c \,x^{4}+117 a^{2} c \right)}{585 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{3}}$	59
orering	$\frac{x^{5} \left(20 a b d \,x^{8}+32 b^{2} c \,x^{8}+65 a^{2} d \,x^{4}+104 a b c \,x^{4}+117 a^{2} c \right)}{585 \left(b \,x^{4}+a \right)^{\frac{13}{4}} a^{3}}$	59

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/5*((5/9*d*x^4+c)*a^2+8/9*b*(5/26*d*x^4+c)*x^4*a+32/117*b^2*c*x^8)/(b*x^4
+a)^(13/4)*x^5/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{(4\,(8\,b^2c+5\,abd)x^{13}+13\,(8\,abc+5\,a^2d)x^9+117\,a^2cx^5)(bx^4+a)^{\frac{3}{4}}}{585\,(a^3b^4x^{16}+4\,a^4b^3x^{12}+6\,a^5b^2x^8+4\,a^6bx^4+a^7)}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/585*(4*(8*b^2*c + 5*a*b*d)*x^13 + 13*(8*a*b*c + 5*a^2*d)*x^9 + 117*a^2*c
*x^5)*(b*x^4 + a)^(3/4)/(a^3*b^4*x^16 + 4*a^4*b^3*x^12 + 6*a^5*b^2*x^8 + 4
*a^6*b*x^4 + a^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = -\frac{\left(9\,b-\frac{13\,(bx^4+a)}{x^4}\right)dx^{13}}{117\,(bx^4+a)^{\frac{13}{4}}a^2}+\frac{\left(45\,b^2-\frac{130\,(bx^4+a)b}{x^4}+\frac{117\,(bx^4+a)^2}{x^8}\right)cx^{13}}{585\,(bx^4+a)^{\frac{13}{4}}a^3}$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/117*(9*b - 13*(b*x^4 + a)/x^4)*d*x^13/((b*x^4 + a)^(13/4)*a^2) + 1/585*
(45*b^2 - 130*(b*x^4 + a)*b/x^4 + 117*(b*x^4 + a)^2/x^8)*c*x^13/((b*x^4 +
a)^(13/4)*a^3)
```

Giac [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^4}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^4*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^4/(b*x^4 + a)^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x\,(5\,a\,d+8\,b\,c)}{585\,a^2\,b^2\,(b\,x^4+a)^{5/4}} - \frac{x\left(\frac{d}{9\,b^2}+\frac{a\,d-b\,c}{117\,a\,b^2}\right)}{(b\,x^4+a)^{9/4}} - \frac{x\left(\frac{c}{13\,b}-\frac{a\,d}{13\,b^2}\right)}{(b\,x^4+a)^{13/4}} + \frac{x\,(20\,a\,d+32\,b\,c)}{585\,a^3\,b^2\,(b\,x^4+a)^{1/4}}$$

input
```
int((x^4*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
(x*(5*a*d + 8*b*c))/(585*a^2*b^2*(a + b*x^4)^(5/4)) - (x*(d/(9*b^2) + (a*d
 - b*c)/(117*a*b^2)))/(a + b*x^4)^(9/4) - (x*(c/(13*b) - (a*d)/(13*b^2)))/
(a + b*x^4)^(13/4) + (x*(20*a*d + 32*b*c))/(585*a^3*b^2*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^4(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1$$
$$+\left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}$$

input
```
int(x^4*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**4/((a + b*x**4)**(1/4)*a**4
 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**
8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x
)*c
```

3.181 $\int \frac{c+dx^4}{(a+bx^4)^{17/4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}} dx = \frac{(bc-ad)x}{13ab(a+bx^4)^{13/4}} + \frac{(12bc+ad)x}{117a^2b(a+bx^4)^{9/4}} + \frac{8(12bc+ad)x}{585a^3b(a+bx^4)^{5/4}} + \frac{32(12bc+ad)x}{585a^4b\sqrt[4]{a+bx^4}}$$

output
```
1/13*(-a*d+b*c)*x/a/b/(b*x^4+a)^(13/4)+1/117*(a*d+12*b*c)*x/a^2/b/(b*x^4+a
)^(9/4)+8/585*(a*d+12*b*c)*x/a^3/b/(b*x^4+a)^(5/4)+32/585*(a*d+12*b*c)*x/a
^4/b/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}} dx = \frac{384b^3cx^{13}+32ab^2x^9(39c+dx^4)+52a^2bx^5(27c+2dx^4)+117a^3(5cx+dx^5)}{585a^4(a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(a + b*x^4)^(17/4),x]
```

output
```
(384*b^3*c*x^13 + 32*a*b^2*x^9*(39*c + d*x^4) + 52*a^2*b*x^5*(27*c + 2*d*x
^4) + 117*a^3*(5*c*x + d*x^5))/(585*a^4*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {910, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(ad+12bc)\int \frac{1}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{8\int \frac{1}{\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$\frac{(ad+12bc)\left(\frac{8\left(\frac{4\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)(ad+12bc)}{13ab}+\frac{x(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x)/(13*a*b*(a + b*x^4)^(13/4)) + ((12*b*c + a*d)*(x/(9*a*(a +
 b*x^4)^(9/4)) + (8*(x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^
(1/4))))/(9*a)))/(13*a*b)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{x\left(\left(\frac{d\,x^4}{5}+c\right)a^3+\frac{12\left(\frac{2d\,x^4}{27}+c\right)b\,x^4a^2}{5}+\frac{32\left(\frac{d\,x^4}{39}+c\right)b^2x^8a}{15}+\frac{128b^3c\,x^{12}}{195}\right)}{\left(b\,x^4+a\right)^{\frac{13}{4}}a^4}$	71
gosper	$\frac{x\left(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585c\,a^3\right)}{585\left(b\,x^4+a\right)^{\frac{13}{4}}a^4}$	81
trager	$\frac{x\left(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585c\,a^3\right)}{585\left(b\,x^4+a\right)^{\frac{13}{4}}a^4}$	81
orering	$\frac{x\left(32a\,b^2d\,x^{12}+384b^3c\,x^{12}+104a^2bd\,x^8+1248a\,b^2c\,x^8+117a^3d\,x^4+1404a^2bc\,x^4+585c\,a^3\right)}{585\left(b\,x^4+a\right)^{\frac{13}{4}}a^4}$	81

input
```
int((d*x^4+c)/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^4+a)^(13/4)*x*((1/5*d*x^4+c)*a^3+12/5*(2/27*d*x^4+c)*b*x^4*a^2+32/1
5*(1/39*d*x^4+c)*b^2*x^8*a+128/195*b^3*c*x^12)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^{17/4}}\,dx = \frac{\left(32\left(12\,b^3c+ab^2d\right)x^{13}+104\left(12\,ab^2c+a^2bd\right)x^9+117\left(12\,a^2bc+a^3d\right)x^5+585\,a^3cx\right)(}{585\left(a^4b^4x^{16}+4\,a^5b^3x^{12}+6\,a^6b^2x^8+4\,a^7bx^4+a^8\right)}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/585*(32*(12*b^3*c + a*b^2*d)*x^13 + 104*(12*a*b^2*c + a^2*b*d)*x^9 + 117
*(12*a^2*b*c + a^3*d)*x^5 + 585*a^3*c*x)*(b*x^4 + a)^(3/4)/(a^4*b^4*x^16 +
 4*a^5*b^3*x^12 + 6*a^6*b^2*x^8 + 4*a^7*b*x^4 + a^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \frac{\left(45\,b^2 - \frac{130\,(bx^4+a)b}{x^4} + \frac{117\,(bx^4+a)^2}{x^8}\right)dx^{13}}{585\,(bx^4+a)^{\frac{13}{4}}a^3} - \frac{\left(15\,b^3 - \frac{65\,(bx^4+a)b^2}{x^4} + \frac{117\,(bx^4+a)^2b}{x^8} - \frac{195\,(bx^4+a)^3}{x^{12}}\right)cx^{13}}{195\,(bx^4+a)^{\frac{13}{4}}a^4}$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
1/585*(45*b^2 - 130*(b*x^4 + a)*b/x^4 + 117*(b*x^4 + a)^2/x^8)*d*x^13/((b*
x^4 + a)^(13/4)*a^3) - 1/195*(15*b^3 - 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4
 + a)^2*b/x^8 - 195*(b*x^4 + a)^3/x^12)*c*x^13/((b*x^4 + a)^(13/4)*a^4)
```

Giac [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate((d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/(b*x^4 + a)^(17/4), x)
```

Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \frac{x\left(\frac{c}{13\,a}-\frac{d}{13\,b}\right)}{(b\,x^4+a)^{13/4}} + \frac{x\,(a\,d+12\,b\,c)}{117\,a^2\,b\,(b\,x^4+a)^{9/4}} + \frac{x\,(8\,a\,d+96\,b\,c)}{585\,a^3\,b\,(b\,x^4+a)^{5/4}} + \frac{x\,(32\,a\,d+384\,b\,c)}{585\,a^4\,b\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(a + b*x^4)^(17/4),x)
```

output
```
(x*(c/(13*a) - d/(13*b)))/(a + b*x^4)^(13/4) + (x*(a*d + 12*b*c))/(117*a^2
*b*(a + b*x^4)^(9/4)) + (x*(8*a*d + 96*b*c))/(585*a^3*b*(a + b*x^4)^(5/4))
 + (x*(32*a*d + 384*b*c))/(585*a^4*b*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{1}\right.$$
$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}\right.$$

input
```
int((d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(1/((a + b*x**4)**(1/4)*a**4 +
4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 +
 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x)*c
```

3.182 $\int \frac{c+dx^4}{x^4(a+bx^4)^{17/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{17/4}} \, dx = -\frac{c}{3ax^3 (a+bx^4)^{13/4}} - \frac{(16bc-3ad)x}{39a^2 (a+bx^4)^{13/4}} - \frac{4(16bc-3ad)x}{117a^3 (a+bx^4)^{9/4}} - \frac{32(16bc-3ad)x}{585a^4 (a+bx^4)^{5/4}} - \frac{128(16bc-3ad)x}{585a^5 \sqrt[4]{a+bx^4}}$$

output
```
-1/3*c/a/x^3/(b*x^4+a)^(13/4)-1/39*(-3*a*d+16*b*c)*x/a^2/(b*x^4+a)^(13/4)-
4/117*(-3*a*d+16*b*c)*x/a^3/(b*x^4+a)^(9/4)-32/585*(-3*a*d+16*b*c)*x/a^4/(
b*x^4+a)^(5/4)-128/585*(-3*a*d+16*b*c)*x/a^5/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{17/4}} \, dx = \frac{-2048b^4cx^{16} - 195a^4(c-3dx^4) + 1248a^2b^2x^8(-6c+dx^4) + 128ab^3x^{12}(-52c+3}{585a^5x^3 (a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(x^4*(a + b*x^4)^(17/4)),x]
```

output
```
(-2048*b^4*c*x^16 - 195*a^4*(c - 3*d*x^4) + 1248*a^2*b^2*x^8*(-6*c + d*x^4
) + 128*a*b^3*x^12*(-52*c + 3*d*x^4) + 156*a^3*b*x^4*(-20*c + 9*d*x^4))/(5
85*a^5*x^3*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {955, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{17/4}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(16bc-3ad)\int \frac{1}{(bx^4+a)^{17/4}}dx}{3a}-\frac{c}{3ax^3 (a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(16bc-3ad)\left(\frac{12\int \frac{1}{(bx^4+a)^{13/4}}dx}{13a}+\frac{x}{13a(a+bx^4)^{13/4}}\right)}{3a}-\frac{c}{3ax^3 (a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(16bc-3ad)\left(\frac{12\left(\frac{8\int \frac{1}{(bx^4+a)^{9/4}}dx}{9a}+\frac{x}{9a(a+bx^4)^{9/4}}\right)}{13a}+\frac{x}{13a(a+bx^4)^{13/4}}\right)}{3a}-\frac{c}{3ax^3 (a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(16bc-3ad)\left(\frac{12\left(\frac{8\left(\frac{4\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x}{13a(a+bx^4)^{13/4}}\right)}{3a}-\frac{c}{3ax^3\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 746

$$-\frac{\left(\frac{12\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x}{13a(a+bx^4)^{13/4}}\right)(16bc-3ad)}{3a}-\frac{c}{3ax^3\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(x^4*(a + b*x^4)^(17/4)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^4)^(13/4)) - ((16*b*c - 3*a*d)*(x/(13*a*(a + b*x^4)
^(13/4)) + (12*(x/(9*a*(a + b*x^4)^(9/4)) + (8*(x/(5*a*(a + b*x^4)^(5/4))
+ (4*x)/(5*a^2*(a + b*x^4)^(1/4))))/(9*a)))/(13*a)))/(3*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\left(585 d\,x^{4}-195 c\right) a^{4}-3120 \left(-\frac{9 d\,x^{4}}{20}+c\right) b\,x^{4} a^{3}-7488 \left(-\frac{d\,x^{4}}{6}+c\right) b^{2} x^{8} a^{2}-6656 \left(-\frac{3 d\,x^{4}}{52}+c\right) b^{3} x^{12} a-2048 b^{4} c\,x^{16}}{585 \left(b\,x^{4}+a\right)^{\frac{13}{4}} x^{3} a^{5}}$
gosper	$-\frac{-384 a\,b^{3} d\,x^{16}+2048 b^{4} c\,x^{16}-1248 a^{2} b^{2} d\,x^{12}+6656 a\,b^{3} c\,x^{12}-1404 a^{3} b d\,x^{8}+7488 a^{2} b^{2} c\,x^{8}-585 a^{4} d\,x^{4}+3120 a^{3} b c\,x^{4}+195\cdots}{585 x^{3} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{5}}$
trager	$-\frac{-384 a\,b^{3} d\,x^{16}+2048 b^{4} c\,x^{16}-1248 a^{2} b^{2} d\,x^{12}+6656 a\,b^{3} c\,x^{12}-1404 a^{3} b d\,x^{8}+7488 a^{2} b^{2} c\,x^{8}-585 a^{4} d\,x^{4}+3120 a^{3} b c\,x^{4}+195\cdots}{585 x^{3} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{5}}$
orering	$-\frac{-384 a\,b^{3} d\,x^{16}+2048 b^{4} c\,x^{16}-1248 a^{2} b^{2} d\,x^{12}+6656 a\,b^{3} c\,x^{12}-1404 a^{3} b d\,x^{8}+7488 a^{2} b^{2} c\,x^{8}-585 a^{4} d\,x^{4}+3120 a^{3} b c\,x^{4}+195\cdots}{585 x^{3} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{5}}$
risch	$-\frac{c \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3 a^{5} x^{3}}+\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} x \left(384 a\,b^{3} d\,x^{12}-1853 c\,b^{4} x^{12}+1248 a^{2} b^{2} d\,x^{8}-5876 a\,b^{3} c\,x^{8}+1404 a^{3} b d\,x^{4}-6318 a^{2} b^{2} c\,x^{4}+58\cdots\right.}{585 a^{5} \left(b^{4} x^{16}+4 a\,b^{3} x^{12}+6 a^{2} x^{8} b^{2}+4 x^{4} a^{3} b+a^{4}\right)}$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/585*((585*d*x^4-195*c)*a^4-3120*(-9/20*d*x^4+c)*b*x^4*a^3-7488*(-1/6*d*x
^4+c)*b^2*x^8*a^2-6656*(-3/52*d*x^4+c)*b^3*x^12*a-2048*b^4*c*x^16)/(b*x^4+
a)^(13/4)/x^3/a^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{17/4}}\,dx =$$

$$-\frac{(128\,(16\,b^4c-3\,ab^3d)x^{16}+416\,(16\,ab^3c-3\,a^2b^2d)x^{12}+468\,(16\,a^2b^2c-3\,a^3bd)x^8+195\,a^4c+195\,(16\,a^3}{585\,(a^5b^4x^{19}+4\,a^6b^3x^{15}+6\,a^7b^2x^{11}+4\,a^8bx^7+a^9x^3)}$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
-1/585*(128*(16*b^4*c - 3*a*b^3*d)*x^16 + 416*(16*a*b^3*c - 3*a^2*b^2*d)*x
^12 + 468*(16*a^2*b^2*c - 3*a^3*b*d)*x^8 + 195*a^4*c + 195*(16*a^3*b*c - 3
*a^4*d)*x^4)*(b*x^4 + a)^(3/4)/(a^5*b^4*x^19 + 4*a^6*b^3*x^15 + 6*a^7*b^2*
x^11 + 4*a^8*b*x^7 + a^9*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^4\left(a+bx^4\right)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)/x**4/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{17/4}}\,dx = -\frac{\left(15\,b^3-\frac{65\,(bx^4+a)b^2}{x^4}+\frac{117\,(bx^4+a)^2b}{x^8}-\frac{195\,(bx^4+a)^3}{x^{12}}\right)dx^{13}}{195\,(bx^4+a)^{\frac{13}{4}}a^4} + \frac{1}{585}\left(\frac{\left(45\,b^4-\frac{260\,(bx^4+a)b^3}{x^4}+\frac{702\,(bx^4+a)^2b^2}{x^8}-\frac{2340\,(bx^4+a)^3b}{x^{12}}\right)x^{13}}{(bx^4+a)^{\frac{13}{4}}a^5}-\frac{195\,(bx^4+a)^{\frac{3}{4}}}{a^5x^3}\right)c$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/195*(15*b^3 - 65*(b*x^4 + a)*b^2/x^4 + 117*(b*x^4 + a)^2*b/x^8 - 195*(b
*x^4 + a)^3/x^12)*d*x^13/((b*x^4 + a)^(13/4)*a^4) + 1/585*((45*b^4 - 260*(
b*x^4 + a)*b^3/x^4 + 702*(b*x^4 + a)^2*b^2/x^8 - 2340*(b*x^4 + a)^3*b/x^12
)*x^13/((b*x^4 + a)^(13/4)*a^5) - 195*(b*x^4 + a)^(3/4)/(a^5*x^3))*c
```

Giac [F]

$$\int \frac{c+dx^4}{x^4(a+bx^4)^{17/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{17}{4}}x^4}\,dx$$

input
```
integrate((d*x^4+c)/x^4/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{17/4}}\,dx = \frac{x^4 \left(\frac{12\,a\,d-12\,b\,c}{117\,a^3}-\frac{4\,b\,c}{9\,a^3}\right)-\frac{c}{3\,a^2}}{x^3\,(b\,x^4+a)^{9/4}} + \frac{x\left(\frac{d}{13\,a}-\frac{b\,c}{13\,a^2}\right)}{(b\,x^4+a)^{13/4}} + \frac{x\,(96\,a\,d-512\,b\,c)}{585\,a^4\,(b\,x^4+a)^{5/4}} + \frac{x\,(384\,a\,d-2048\,b\,c)}{585\,a^5\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(x^4*(a + b*x^4)^(17/4)),x)
```

output
```
(x^4*((12*a*d - 12*b*c)/(117*a^3) - (4*b*c)/(9*a^3)) - c/(3*a^2))/(x^3*(a
+ b*x^4)^(9/4)) + (x*(d/(13*a) - (b*c)/(13*a^2)))/(a + b*x^4)^(13/4) + (x*
(96*a*d - 512*b*c))/(585*a^4*(a + b*x^4)^(5/4)) + (x*(384*a*d - 2048*b*c))
/(585*a^5*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^4 (a+bx^4)^{17/4}}\,dx = \left(\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4x^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^8+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^{12}+4\,(b\,x^4+a)^{\frac{1}{4}}\right.\right.$$
$$+\left(\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.\right.$$

input
```
int((d*x^4+c)/x^4/(b*x^4+a)^(17/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**4*x**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**8 +
 6*(a + b*x**4)**(1/4)*a**2*b**2*x**12 + 4*(a + b*x**4)**(1/4)*a*b**3*x**1
6 + (a + b*x**4)**(1/4)*b**4*x**20),x)*c + int(1/((a + b*x**4)**(1/4)*a**4
 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**
8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x
)*d
```

3.183 $\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 173

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}} \, dx = -\frac{c}{7ax^7 (a+bx^4)^{13/4}} - \frac{20bc-7ad}{91a^2x^3 (a+bx^4)^{13/4}} - \frac{16(20bc-7ad)}{819a^3x^3 (a+bx^4)^{9/4}} - \frac{64(20bc-7ad)}{1365a^4x^3 (a+bx^4)^{5/4}} - \frac{512(20bc-7ad)}{1365a^5x^3 \sqrt[4]{a+bx^4}} + \frac{2048(20bc-7ad)(a+bx^4)^{3/4}}{4095a^6x^3}$$

output
```
-1/7*c/a/x^7/(b*x^4+a)^(13/4)-1/91*(-7*a*d+20*b*c)/a^2/x^3/(b*x^4+a)^(13/4
)-16/819*(-7*a*d+20*b*c)/a^3/x^3/(b*x^4+a)^(9/4)-64/1365*(-7*a*d+20*b*c)/a
^4/x^3/(b*x^4+a)^(5/4)-512/1365*(-7*a*d+20*b*c)/a^5/x^3/(b*x^4+a)^(1/4)+20
48/4095*(-7*a*d+20*b*c)*(b*x^4+a)^(3/4)/a^6/x^3
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.73

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx = \frac{40960b^5cx^{20}+780a^4bx^4(5c-28dx^4)+2496a^3b^2x^8(25c-21dx^4)+3328a^2b^3x^{12}(4}{4095a^6x^7(a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(x^8*(a + b*x^4)^(17/4)),x]
```

output
```
(40960*b^5*c*x^20 + 780*a^4*b*x^4*(5*c - 28*d*x^4) + 2496*a^3*b^2*x^8*(25*
c - 21*d*x^4) + 3328*a^2*b^3*x^12*(45*c - 14*d*x^4) + 2048*a*b^4*x^16*(65*
c - 7*d*x^4) - 195*a^5*(3*c + 7*d*x^4))/(4095*a^6*x^7*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 803, 749, 749, 749, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(20bc-7ad)\int \frac{1}{x^4(bx^4+a)^{17/4}}dx}{7a}-\frac{c}{7ax^7(a+bx^4)^{13/4}}$$

$$\Big\downarrow \; 803$$

$$-\frac{(20bc-7ad)\left(-\frac{16b\int \frac{1}{(bx^4+a)^{17/4}}dx}{3a}-\frac{1}{3ax^3(a+bx^4)^{13/4}}\right)}{7a}-\frac{c}{7ax^7(a+bx^4)^{13/4}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(20bc-7ad)\left(-\frac{16b\left(\frac{12\int\frac{1}{\left(bx^4+a\right)^{13/4}}dx}{13a}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{13/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(20bc-7ad)\left(-\frac{16b\left(\frac{12\left(\frac{8\int\frac{1}{\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{13/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(20bc-7ad)\left(-\frac{16b\left(\frac{12\left(\frac{8\left(\frac{4\int\frac{1}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{13/4}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 746$$

$$-\frac{\left(-\frac{16b\left(\frac{12\left(\frac{8\left(\frac{4x}{5a^2\sqrt[4]{a+bx^4}}+\frac{x}{5a\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{x}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x}{13a\left(a+bx^4\right)^{13/4}}\right)}{3a}-\frac{1}{3ax^3(a+bx^4)^{13/4}}\right)(20bc-7ad)}{7a}-\frac{c}{7ax^7\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(x^8*(a + b*x^4)^(17/4)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^4)^(13/4)) - ((20*b*c - 7*a*d)*(-1/3*1/(a*x^3*(a +
b*x^4)^(13/4)) - (16*b*(x/(13*a*(a + b*x^4)^(13/4)) + (12*(x/(9*a*(a + b*x
^4)^(9/4)) + (8*(x/(5*a*(a + b*x^4)^(5/4)) + (4*x)/(5*a^2*(a + b*x^4)^(1/4
))))/(9*a)))/(13*a)))/(3*a)))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(-1365 d\,x^{4}-585 c\right) a^{5}+3900 b\,x^{4} \left(-\frac{28 d\,x^{4}}{5}+c\right) a^{4}+62400 \left(-\frac{21 d\,x^{4}}{25}+c\right) b^{2} x^{8} a^{3}+149760 \left(-\frac{14 d\,x^{4}}{45}+c\right) b^{3} x^{12} a^{2}+133120 \left(-\frac{7}{}\right.\cdots}{4095 \left(b\,x^{4}+a\right)^{\frac{13}{4}} x^{7} a^{6}}$
gosper	$-\frac{14336 a\,b^{4} d\,x^{20}-40960 b^{5} c\,x^{20}+46592 a^{2} b^{3} d\,x^{16}-133120 a\,b^{4} c\,x^{16}+52416 a^{3} b^{2} d\,x^{12}-149760 a^{2} b^{3} c\,x^{12}+21840 a^{4} b d\,x^{8}-6\cdots}{4095 x^{7} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{6}}$
trager	$-\frac{14336 a\,b^{4} d\,x^{20}-40960 b^{5} c\,x^{20}+46592 a^{2} b^{3} d\,x^{16}-133120 a\,b^{4} c\,x^{16}+52416 a^{3} b^{2} d\,x^{12}-149760 a^{2} b^{3} c\,x^{12}+21840 a^{4} b d\,x^{8}-6\cdots}{4095 x^{7} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{6}}$
orering	$-\frac{14336 a\,b^{4} d\,x^{20}-40960 b^{5} c\,x^{20}+46592 a^{2} b^{3} d\,x^{16}-133120 a\,b^{4} c\,x^{16}+52416 a^{3} b^{2} d\,x^{12}-149760 a^{2} b^{3} c\,x^{12}+21840 a^{4} b d\,x^{8}-6\cdots}{4095 x^{7} \left(b\,x^{4}+a\right)^{\frac{13}{4}} a^{6}}$
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(7 a d\,x^{4}-32 b c\,x^{4}+3 a c\right)}{21 a^{6} x^{7}}-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} x \left(1853 a\,b^{3} d\,x^{12}-4960 c\,b^{4} x^{12}+5876 a^{2} b^{2} d\,x^{8}-15535 a\,b^{3} c\,x^{8}+6318 a^{3}\cdots\right.}{585 a^{6} \left(b^{4} x^{16}+4 a\,b^{3} x^{12}+6 a^{2} x^{8} b^{2}+4 x^{4} a^{3}\cdots\right.}$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(17/4),x,method=_RETURNVERBOSE)
```

output
```
1/4095*((-1365*d*x^4-585*c)*a^5+3900*b*x^4*(-28/5*d*x^4+c)*a^4+62400*(-21/
25*d*x^4+c)*b^2*x^8*a^3+149760*(-14/45*d*x^4+c)*b^3*x^12*a^2+133120*(-7/65
*d*x^4+c)*b^4*x^16*a+40960*b^5*c*x^20)/(b*x^4+a)^(13/4)/x^7/a^6
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{17/4}}\,dx=\frac{(2048\,(20\,b^5c-7\,ab^4d)x^{20}+6656\,(20\,ab^4c-7\,a^2b^3d)x^{16}+7488\,(20\,a^2b^3c-7\,a^3b^2}{4095\,(a^6b^4x^{23}+4\,a^7b^3x^{19}+}$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
1/4095*(2048*(20*b^5*c - 7*a*b^4*d)*x^20 + 6656*(20*a*b^4*c - 7*a^2*b^3*d)
*x^16 + 7488*(20*a^2*b^3*c - 7*a^3*b^2*d)*x^12 + 3120*(20*a^3*b^2*c - 7*a^
4*b*d)*x^8 - 585*a^5*c + 195*(20*a^4*b*c - 7*a^5*d)*x^4)*(b*x^4 + a)^(3/4)
/(a^6*b^4*x^23 + 4*a^7*b^3*x^19 + 6*a^8*b^2*x^15 + 4*a^9*b*x^11 + a^10*x^7
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^8\left(a+bx^4\right)^{17/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**4+c)/x**8/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx = -\frac{1}{819}\left(\frac{7\left(9\,b^5-\frac{65\,(bx^4+a)b^4}{x^4}+\frac{234\,(bx^4+a)^2b^3}{x^8}-\frac{1170\,(bx^4+a)^3b^2}{x^{12}}\right)x^{13}}{(bx^4+a)^{\frac{13}{4}}a^6}-\frac{39\left(\frac{35\,(bx^4+a)^{\frac{3}{4}}b}{x^3}-\frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}\right)}{a^6}\right)c + \frac{1}{585}\left(\frac{\left(45\,b^4-\frac{260\,(bx^4+a)b^3}{x^4}+\frac{702\,(bx^4+a)^2b^2}{x^8}-\frac{2340\,(bx^4+a)^3b}{x^{12}}\right)x^{13}}{(bx^4+a)^{\frac{13}{4}}a^5}-\frac{195\,(bx^4+a)^{\frac{3}{4}}}{a^5x^3}\right)d$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
-1/819*(7*(9*b^5 - 65*(b*x^4 + a)*b^4/x^4 + 234*(b*x^4 + a)^2*b^3/x^8 - 11
70*(b*x^4 + a)^3*b^2/x^12)*x^13/((b*x^4 + a)^(13/4)*a^6) - 39*(35*(b*x^4 +
 a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(7/4)/x^7)/a^6)*c + 1/585*((45*b^4 - 260*(
b*x^4 + a)*b^3/x^4 + 702*(b*x^4 + a)^2*b^2/x^8 - 2340*(b*x^4 + a)^3*b/x^12
)*x^13/((b*x^4 + a)^(13/4)*a^5) - 195*(b*x^4 + a)^(3/4)/(a^5*x^3))*d
```

Giac [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx = \int \frac{dx^4+c}{(bx^4+a)^{\frac{17}{4}}x^8}\,dx$$

input
```
integrate((d*x^4+c)/x^8/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx = \frac{x\left(\frac{b\,(3584\,a\,d-10357\,b\,c)}{1638\,a^5}+\frac{a\left(\frac{4\,b^3\,c}{35\,a^6}-\frac{b^2\,(3584\,a\,d-10357\,b\,c)}{1170\,a^6}\right)}{b}\right)}{(b\,x^4+a)^{5/4}} - \frac{\frac{7\,a^5\,d-32\,a^4\,b\,c}{28\,a^6}-\frac{a\left(\frac{25\,b^2\,c}{91\,a^3}+\frac{17\,b\,(7\,a^5\,d-32\,a^4\,b\,c)}{364\,a^7}\right)}{b}}{x^3\,(b\,x^4+a)^{13/4}} - \frac{x^4\left(\frac{2\,b^2\,c}{7\,a^4}+\frac{4\,b\,(112\,a^4\,d-437\,a^3\,b\,c)}{819\,a^7}\right)+\frac{112\,a^4\,d-437\,a^3\,b\,c}{273\,a^6}}{x^3\,(b\,x^4+a)^{9/4}} - \frac{c\,(b\,x^4+a)^{3/4}}{7\,a^5\,x^7} - \frac{2048\,b\,x\,(7\,a\,d-20\,b\,c)}{4095\,a^6\,(b\,x^4+a)^{1/4}}$$

input
```
int((c + d*x^4)/(x^8*(a + b*x^4)^(17/4)),x)
```

output
```
(x*((b*(3584*a*d - 10357*b*c))/(1638*a^5) + (a*((4*b^3*c)/(35*a^6) - (b^2*
(3584*a*d - 10357*b*c))/(1170*a^6)))/b))/(a + b*x^4)^(5/4) - ((7*a^5*d - 3
2*a^4*b*c)/(28*a^6) - (a*((25*b^2*c)/(91*a^3) + (17*b*(7*a^5*d - 32*a^4*b*
c))/(364*a^7)))/b)/(x^3*(a + b*x^4)^(13/4)) - (x^4*((2*b^2*c)/(7*a^4) + (4
*b*(112*a^4*d - 437*a^3*b*c))/(819*a^7)) + (112*a^4*d - 437*a^3*b*c)/(273*
a^6))/(x^3*(a + b*x^4)^(9/4)) - (c*(a + b*x^4)^(3/4))/(7*a^5*x^7) - (2048*
b*x*(7*a*d - 20*b*c))/(4095*a^6*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{c+dx^4}{x^8 (a+bx^4)^{17/4}}\,dx = \left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^{12}+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^{16}+4\,(b\,x^4+a)^{\frac{1}{4}}\right.$$

$$+\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a^4x^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^8+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^{12}+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{16}+(b\,x^4+a)^{\frac{1}{4}}\,b^4\right.$$

input
```
int((d*x^4+c)/x^8/(b*x^4+a)^(17/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**4*x**8 + 4*(a + b*x**4)**(1/4)*a**3*b*x**12
+ 6*(a + b*x**4)**(1/4)*a**2*b**2*x**16 + 4*(a + b*x**4)**(1/4)*a*b**3*x**
20 + (a + b*x**4)**(1/4)*b**4*x**24),x)*c + int(1/((a + b*x**4)**(1/4)*a**
4*x**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**8 + 6*(a + b*x**4)**(1/4)*a**2*b*
*2*x**12 + 4*(a + b*x**4)**(1/4)*a*b**3*x**16 + (a + b*x**4)**(1/4)*b**4*x
**20),x)*d
```

3.184 $\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 212

$$\begin{aligned}\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx &= -\frac{(bc-ad)x^{15}}{13b^2(a+bx^4)^{13/4}}-\frac{(15bc-28ad)x^{11}}{117b^3(a+bx^4)^{9/4}}\\ &-\frac{(33bc-85ad)x^7}{117b^4(a+bx^4)^{5/4}}+\frac{77(6bc-19ad)x^3}{468b^5\sqrt[4]{a+bx^4}}+\frac{dx^7}{6b^4\sqrt[4]{a+bx^4}}\\ &+\frac{77\sqrt{a}(6bc-19ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{156b^{11/2}\sqrt[4]{a+bx^4}}\end{aligned}$$

output

```
-1/13*(-a*d+b*c)*x^15/b^2/(b*x^4+a)^(13/4)-1/117*(-28*a*d+15*b*c)*x^11/b^3
/(b*x^4+a)^(9/4)-1/117*(-85*a*d+33*b*c)*x^7/b^4/(b*x^4+a)^(5/4)+77/468*(-1
9*a*d+6*b*c)*x^3/b^5/(b*x^4+a)^(1/4)+1/6*d*x^7/b^4/(b*x^4+a)^(1/4)+77/156*
a^(1/2)*(-19*a*d+6*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/
2)*x^2/a^(1/2))),2^(1/2))/b^(11/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.73

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^3\Big(-1463a^4d+22a^3b(21c-95dx^4)+60a^2b^2x^4(11c-19dx^4)+8ab^3x^8(45c-19dx}{}$$

input
```
Integrate[(x^18*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^3*(-1463*a^4*d + 22*a^3*b*(21*c - 95*d*x^4) + 60*a^2*b^2*x^4*(11*c - 19
*d*x^4) + 8*a*b^3*x^8*(45*c - 19*d*x^4) + 16*b^4*x^12*(3*c + d*x^4) + 77*(
-6*b*c + 19*a*d)*(a + b*x^4)^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4
, 17/4, 7/4, -((b*x^4)/a)]))/(96*b^5*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {957, 817, 817, 815, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{x^{19}(bc-ad)}{13ab\,(a+bx^4)^{13/4}}-\frac{(6bc-19ad)\int \frac{x^{18}}{(bx^4+a)^{13/4}}dx}{13ab}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\int\frac{x^{14}}{\left(bx^4+a\right)^{9/4}}dx}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\Big\downarrow$ 817

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\int\frac{x^{10}}{\left(bx^4+a\right)^{5/4}}dx}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\Big\downarrow$ 815

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\int\frac{x^6}{\left(bx^4+a\right)^{5/4}}dx}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\Big\downarrow$ 815

$$\frac{\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}}{}$$

$\downarrow$ 813

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b\left(a+bx^4\right)^{9/4}}\right)}{13ab}$$

$\downarrow$ 858

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\downarrow$ 807

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$\downarrow$ 212

$$\frac{x^{19}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(6bc-19ad)\left(\frac{5\left(\frac{11\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{5b}-\frac{x^{11}}{5b\left(a+bx^4\right)^{5/4}}\right)}{3b}-\frac{x^{15}}{9b(a+bx^4)\cdots}\right)}{13ab}$$

input
```
Int[(x^18*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^19)/(13*a*b*(a + b*x^4)^(13/4)) - ((6*b*c - 19*a*d)*(-1/9*x
^15/(b*(a + b*x^4)^(9/4)) + (5*(-1/5*x^11/(b*(a + b*x^4)^(5/4)) + (11*(x^7
/(6*b*(a + b*x^4)^(1/4)) - (7*a*(x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*
(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*
b^(3/2)*(a + b*x^4)^(1/4))))/(6*b)))/(5*b)))/(3*b)))/(13*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{18}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{17}{4}}}dx$$

input
```
int(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\int \frac{(dx^4+c)x^{18}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((d*x^22 + c*x^18)*(b*x^4 + a)^(3/4)/(b^5*x^20 + 5*a*b^4*x^16 + 10
*a^2*b^3*x^12 + 10*a^3*b^2*x^8 + 5*a^4*b*x^4 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**18*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{18}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^18/(b*x^4 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{18}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^18/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\int \frac{x^{18}\,\left(d\,x^4+c\right)}{{\left(b\,x^4+a\right)}^{17/4}}\,dx$$

input
```
int((x^18*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^18*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^{18}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\left(\int \frac{x^{22}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1\right.$$
$$+\left(\int \frac{x^{18}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}\right.$$

input
```
int(x^18*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**22/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**18/((a + b*x**4)**(1/4)*a*
*4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16)
,x)*c
```

3.185 $\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = -\frac{(bc-ad)x^{11}}{13b^2(a+bx^4)^{13/4}} - \frac{(11bc-24ad)x^7}{117b^3(a+bx^4)^{9/4}} - \frac{(77bc-285ad)x^3}{585b^4(a+bx^4)^{5/4}} + \frac{dx^3}{2b^4\sqrt[4]{a+bx^4}} - \frac{77(2bc-15ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{390\sqrt{a}b^{9/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/13*(-a*d+b*c)*x^11/b^2/(b*x^4+a)^(13/4)-1/117*(-24*a*d+11*b*c)*x^7/b^3/
(b*x^4+a)^(9/4)-1/585*(-285*a*d+77*b*c)*x^3/b^4/(b*x^4+a)^(5/4)+1/2*d*x^3/
b^4/(b*x^4+a)^(1/4)-77/390*(-15*a*d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(s
in(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/b^(9/2)/(b*x^4+a)^(1/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^3\Big(a(1155a^3d-22a^2b(7c-75dx^4)+120b^3x^8(-c+dx^4)+20ab^2x^4(-11c+45dx^4)}{240ab^4\,(a+}$$

input
```
Integrate[(x^14*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^3*(a*(1155*a^3*d - 22*a^2*b*(7*c - 75*d*x^4) + 120*b^3*x^8*(-c + d*x^4)
 + 20*a*b^2*x^4*(-11*c + 45*d*x^4)) - 77*(-2*b*c + 15*a*d)*(a + b*x^4)^3*(
1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 17/4, 7/4, -((b*x^4)/a)]))/(24
0*a*b^4*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {957, 817, 817, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}dx$$

$$\Big\downarrow \; 957$$

$$\frac{x^{15}(bc-ad)}{13ab\,(a+bx^4)^{13/4}}-\frac{(2bc-15ad)\int \frac{x^{14}}{(bx^4+a)^{13/4}}dx}{13ab}$$

$$\Big\downarrow \; 817$$

$$\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(2bc-15ad)\left(\frac{11\int\frac{x^{10}}{\left(bx^4+a\right)^{9/4}}dx}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \ 817$$

$$\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(2bc-15ad)\left(\frac{11\left(\frac{7\int\frac{x^6}{\left(bx^4+a\right)^{5/4}}dx}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \ 815$$

$$\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(2bc-15ad)\left(\frac{11\left(\frac{7\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \ 813$$

$$\frac{\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\left(2bc-15ad\right)\left(\frac{11\left(\frac{7\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\left(2bc-15ad\right)\left(\frac{11\left(\frac{7\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\downarrow \text{807}$$

$$\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(2bc-15ad)\left(\frac{11\left(\frac{7\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

$$\downarrow \text{212}$$

$$\frac{x^{15}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}-\frac{(2bc-15ad)\left(\frac{11\left(\frac{7\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{5b}-\frac{x^7}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^4)^{9/4}}\right)}{13ab}$$

input
```
Int[(x^14*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^15)/(13*a*b*(a + b*x^4)^(13/4)) - ((2*b*c - 15*a*d)*(-1/9*x
^11/(b*(a + b*x^4)^(9/4)) + (11*(-1/5*x^7/(b*(a + b*x^4)^(5/4)) + (7*(x^3/
(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[Arc
Tan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))))/(5*b)))/
(9*b)))/(13*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{14}(d\,x^{4}+c)}{(b\,x^{4}+a)^{\frac{17}{4}}}dx$$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((d*x^18 + c*x^14)*(b*x^4 + a)^(3/4)/(b^5*x^20 + 5*a*b^4*x^16 + 10
*a^2*b^3*x^12 + 10*a^3*b^2*x^8 + 5*a^4*b*x^4 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**14*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{14}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^14/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{x^{14}\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^14*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^{14}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{18}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{1}}\right.$$
$$+\left(\int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int(x^14*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**18/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**14/((a + b*x**4)**(1/4)*a*
*4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16)
,x)*c
```

3.186 $\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = -\frac{(bc-ad)x^7}{13b^2(a+bx^4)^{13/4}} - \frac{(7bc-20ad)x^3}{117b^3(a+bx^4)^{9/4}} + \frac{(7bc-59ad)x^3}{195ab^3(a+bx^4)^{5/4}} - \frac{7(2bc+11ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{195a^{3/2}b^{7/2}\sqrt[4]{a+bx^4}}$$

output

```
-1/13*(-a*d+b*c)*x^7/b^2/(b*x^4+a)^(13/4)-1/117*(-20*a*d+7*b*c)*x^3/b^3/(b
*x^4+a)^(9/4)+1/195*(-59*a*d+7*b*c)*x^3/a/b^3/(b*x^4+a)^(5/4)-7/195*(11*a*
d+2*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))
),2^(1/2))/a^(3/2)/b^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.74

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^3\left(-a^2(77a^2d+20b^2x^4(c+3dx^4)+2ab(7c+55dx^4))+7(2bc+11ad)\,(a+bx^4)^3\right.}{120a^2b^3\,(a+bx^4)^{13/4}}$$

input
```
Integrate[(x^10*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^3*(-(a^2*(77*a^2*d + 20*b^2*x^4*(c + 3*d*x^4) + 2*a*b*(7*c + 55*d*x^4))
) + 7*(2*b*c + 11*a*d)*(a + b*x^4)^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2
F1[3/4, 17/4, 7/4, -((b*x^4)/a)]))/(120*a^2*b^3*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 817, 817, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(11ad+2bc)\int \frac{x^{10}}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x^{11}(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(11ad+2bc)\left(\frac{7\int \frac{x^6}{(bx^4+a)^{9/4}}dx}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$\Big\downarrow$ 817

$$\frac{(11ad+2bc)\left(\frac{7\left(\frac{3\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{5b}-\frac{x^3}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 813

$$\frac{(11ad+2bc)\left(\frac{7\left(\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 858

$$\frac{(11ad+2bc)\left(\frac{7\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 807

$$\frac{(11ad+2bc)\left(\frac{7\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{10b^2\sqrt[4]{a+bx^4}}-\frac{x^3}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{(11ad+2bc)\left(\frac{7\left(-\frac{3x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5\sqrt{a}b^{3/2}\sqrt[4]{a+bx^4}}-\frac{x^3}{5b\left(a+bx^4\right)^{5/4}}\right)}{9b}-\frac{x^7}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^{11}(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(x^10*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^11)/(13*a*b*(a + b*x^4)^(13/4)) + ((2*b*c + 11*a*d)*(-1/9*x
^7/(b*(a + b*x^4)^(9/4)) + (7*(-1/5*x^3/(b*(a + b*x^4)^(5/4)) - (3*(1 + a/
(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*Sqrt[a]
*b^(3/2)*(a + b*x^4)^(1/4))))/(9*b)))/(13*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{10}(d\,x^4+c)}{(b\,x^4+a)^{\frac{17}{4}}}dx$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((d*x^14 + c*x^10)*(b*x^4 + a)^(3/4)/(b^5*x^20 + 5*a*b^4*x^16 + 10
*a^2*b^3*x^12 + 10*a^3*b^2*x^8 + 5*a^4*b*x^4 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**10*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\int \frac{(dx^4+c)x^{10}}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^10/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx=\int \frac{x^{10}\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^10*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1}\right.$$
$$+\left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}}\right.$$

input
```
int(x^10*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**14/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**10/((a + b*x**4)**(1/4)*a*
*4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16)
,x)*c
```

3.187 $\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = -\frac{(bc-ad)x^3}{13b^2(a+bx^4)^{13/4}} + \frac{(3bc-16ad)x^3}{117ab^2(a+bx^4)^{9/4}} + \frac{(6bc+7ad)x^3}{195a^2b^2(a+bx^4)^{5/4}} - \frac{2(6bc+7ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{195a^{5/2}b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/13*(-a*d+b*c)*x^3/b^2/(b*x^4+a)^(13/4)+1/117*(-16*a*d+3*b*c)*x^3/a/b^2/
(b*x^4+a)^(9/4)+1/195*(7*a*d+6*b*c)*x^3/a^2/b^2/(b*x^4+a)^(5/4)-2/195*(7*a
*d+6*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2)
)),2^(1/2))/a^(5/2)/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^3\left(-6bc-7ad-10bdx^4+\frac{6bc(a+bx^4)^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{a^3}+\frac{7d(a+\cdots}{\cdots}\right)}{60b^2\left(a+bx^4\right)^{13/4}}$$

input
```
Integrate[(x^6*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^3*(-6*b*c - 7*a*d - 10*b*d*x^4 + (6*b*c*(a + b*x^4)^3*(1 + (b*x^4)/a)^(
1/4)*Hypergeometric2F1[3/4, 17/4, 7/4, -((b*x^4)/a)])/a^3 + (7*d*(a + b*x^
4)^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 17/4, 7/4, -((b*x^4)/a)]
)/a^2))/(60*b^2*(a + b*x^4)^(13/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {957, 817, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7ad+6bc)\int \frac{x^6}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7ad+6bc)\left(\frac{\int \frac{x^2}{(bx^4+a)^{9/4}}dx}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(7ad+6bc)\left(\frac{\frac{2\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{5a}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{(7ad+6bc)\left(\frac{\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{(7ad+6bc)\left(\frac{\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{(7ad+6bc)\left(\frac{\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{(7ad+6bc)\left(\frac{\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}}{3b}-\frac{x^3}{9b(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^7(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(x^6*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^7)/(13*a*b*(a + b*x^4)^(13/4)) + ((6*b*c + 7*a*d)*(-1/9*x^3
/(b*(a + b*x^4)^(9/4)) + (x^3/(5*a*(a + b*x^4)^(5/4)) - (2*(1 + a/(b*x^4))
^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*Sqrt[b]
*(a + b*x^4)^(1/4)))/(3*b)))/(13*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^6(d\,x^4+c)}{(b\,x^4+a)^{\frac{17}{4}}}dx$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^6)*(b*x^4 + a)^(3/4)/(b^5*x^20 + 5*a*b^4*x^16 + 10*
a^2*b^3*x^12 + 10*a^3*b^2*x^8 + 5*a^4*b*x^4 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^6}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^6/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{x^6\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^6*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1\right.$$
$$+\left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}\right.$$

input
```
int(x^6*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**10/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 +
6*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12
+ (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**6/((a + b*x**4)**(1/4)*a**
4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x*
*8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),
x)*c
```

3.188 $\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}} \, dx = \frac{(bc-ad)x^3}{13ab\,(a+bx^4)^{13/4}} + \frac{(10bc+3ad)x^3}{117a^2b\,(a+bx^4)^{9/4}} + \frac{2(10bc+3ad)x^3}{195a^3b\,(a+bx^4)^{5/4}} - \frac{4(10bc+3ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{195a^{7/2}b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/13*(-a*d+b*c)*x^3/a/b/(b*x^4+a)^(13/4)+1/117*(3*a*d+10*b*c)*x^3/a^2/b/(b
*x^4+a)^(9/4)+2/195*(3*a*d+10*b*c)*x^3/a^3/b/(b*x^4+a)^(5/4)-4/195*(3*a*d+
10*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2)))
,2^(1/2))/a^(7/2)/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \frac{x^3\left(-3a^4d+10bc(a+bx^4)^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{4},\frac{7}{4},-\frac{bx^4}{a}\right)+3ad(a\right.}{30a^4b\,(a+bx^4)^{13/4}}$$

input
```
Integrate[(x^2*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
(x^3*(-3*a^4*d + 10*b*c*(a + b*x^4)^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric
2F1[3/4, 17/4, 7/4, -((b*x^4)/a)] + 3*a*d*(a + b*x^4)^3*(1 + (b*x^4)/a)^(1
/4)*Hypergeometric2F1[3/4, 17/4, 7/4, -((b*x^4)/a)]))/(30*a^4*b*(a + b*x^4
)^(13/4))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {957, 819, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(3ad+10bc)\int \frac{x^2}{(bx^4+a)^{13/4}}dx}{13ab}+\frac{x^3(bc-ad)}{13ab\,(a+bx^4)^{13/4}}$$

$$\Big\downarrow \; 819$$

$$\frac{(3ad+10bc)\left(\frac{2\int \frac{x^2}{\left(bx^4+a\right)^{9/4}}dx}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 819

$$\frac{(3ad+10bc)\left(\frac{2\left(\frac{2\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 813

$$\frac{(3ad+10bc)\left(\frac{2\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 858

$$\frac{(3ad+10bc)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 807

$$\frac{(3ad+10bc)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 212

$$\frac{(3ad+10bc)\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a(a+bx^4)^{9/4}}\right)}{13ab}+\frac{x^3(bc-ad)}{13ab\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(x^2*(c + d*x^4))/(a + b*x^4)^(17/4),x]
```

output
```
((b*c - a*d)*x^3)/(13*a*b*(a + b*x^4)^(13/4)) + ((10*b*c + 3*a*d)*(x^3/(9*
a*(a + b*x^4)^(9/4)) + (2*(x^3/(5*a*(a + b*x^4)^(5/4)) - (2*(1 + a/(b*x^4)
)^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*Sqrt[b
]*(a + b*x^4)^(1/4))))/(3*a)))/(13*a*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^2(d\,x^4+c)}{(b\,x^4+a)^{\frac{17}{4}}}dx$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((d*x^6 + c*x^2)*(b*x^4 + a)^(3/4)/(b^5*x^20 + 5*a*b^4*x^16 + 10*a
^2*b^3*x^12 + 10*a^3*b^2*x^8 + 5*a^4*b*x^4 + a^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x**4+c)/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(17/4), x)
```

Giac [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{(dx^4+c)x^2}{(bx^4+a)^{\frac{17}{4}}}\,dx$$

input
```
integrate(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*x^2/(b*x^4 + a)^(17/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \int \frac{x^2\,(d\,x^4+c)}{(b\,x^4+a)^{17/4}}\,dx$$

input
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(17/4),x)
```

output
```
int((x^2*(c + d*x^4))/(a + b*x^4)^(17/4), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx^4)}{(a+bx^4)^{17/4}}\,dx = \left(\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^1\right.$$
$$+\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a^4+4\,(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^4+6\,(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^8+4\,(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{12}+(b\,x^4+a)^{\frac{1}{4}}\,b^4x^{16}\right.$$

input
```
int(x^2*(d*x^4+c)/(b*x^4+a)^(17/4),x)
```

output
```
int(x**6/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(x**2/((a + b*x**4)**(1/4)*a**4
 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x**
8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 + (a + b*x**4)**(1/4)*b**4*x**16),x
)*c
```

3.189 $\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=-\frac{c}{ax\left(a+bx^4\right)^{13/4}}-\frac{(14bc-ad)x^3}{13a^2\left(a+bx^4\right)^{13/4}}-\frac{10(14bc-ad)x^3}{117a^3\left(a+bx^4\right)^{9/4}}-\frac{4(14bc-ad)x^3}{39a^4\left(a+bx^4\right)^{5/4}}+\frac{8(14bc-ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{9/2}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-c/a/x/(b*x^4+a)^(13/4)-1/13*(-a*d+14*b*c)*x^3/a^2/(b*x^4+a)^(13/4)-10/117
*(-a*d+14*b*c)*x^3/a^3/(b*x^4+a)^(9/4)-4/39*(-a*d+14*b*c)*x^3/a^4/(b*x^4+a
)^(5/4)+8/39*(-a*d+14*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^
(1/2)*x^2/a^(1/2))),2^(1/2))/a^(9/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.46

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{17/4}}\,dx = -\frac{c}{ax (a+bx^4)^{13/4}} - \frac{(14bc-ad)x^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{17}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(c + d*x^4)/(x^2*(a + b*x^4)^(17/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(13/4))) - ((14*b*c - a*d)*x^3*(1 + (b*x^4)/a)^(1/4)*
Hypergeometric2F1[3/4, 17/4, 7/4, -((b*x^4)/a)])/(3*a^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {955, 819, 819, 819, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^2 (a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(14bc-ad)\int \frac{x^2}{(bx^4+a)^{17/4}}dx}{a} - \frac{c}{ax (a+bx^4)^{13/4}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(14bc-ad)\left(\frac{10\int \frac{x^2}{(bx^4+a)^{13/4}}dx}{13a} + \frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{a} - \frac{c}{ax (a+bx^4)^{13/4}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\int \frac{x^2}{\left(bx^4+a\right)^{9/4}}dx}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 819

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\left(\frac{2\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{a}-\frac{c}{ax\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 813

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{5ab\sqrt[4]{a+bx^4}}+\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}\right)}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{c^a}-\frac{c^a}{ax\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 858

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{c^a}-\frac{}{ax\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 807$$

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{5ab\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{c^a}-\frac{a}{ax\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 212$$

$$-\frac{(14bc-ad)\left(\frac{10\left(\frac{2\left(\frac{x^3}{5a\left(a+bx^4\right)^{5/4}}-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^4}}\right)}{3a}+\frac{x^3}{9a\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{x^3}{13a(a+bx^4)^{13/4}}\right)}{c^a}-\frac{a}{ax\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(x^2*(a + b*x^4)^(17/4)),x]
```

output
```
-(c/(a*x*(a + b*x^4)^(13/4))) - ((14*b*c - a*d)*(x^3/(13*a*(a + b*x^4)^(13
/4)) + (10*(x^3/(9*a*(a + b*x^4)^(9/4)) + (2*(x^3/(5*a*(a + b*x^4)^(5/4))
- (2*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(5*a^(3/2)*Sqrt[b]*(a + b*x^4)^(1/4))))/(3*a)))/(13*a)))/a
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{2} \left(b\,x^{4}+a \right)^{\frac{17}{4}}}dx$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x)
```

output
```
int((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^2 \left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^{4}+c}{{\left(bx^{4}+a\right)}^{\frac{17}{4}}x^{2}}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^5*x^22 + 5*a*b^4*x^18 + 10*a^2*b
^3*x^14 + 10*a^3*b^2*x^10 + 5*a^4*b*x^6 + a^5*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**4+c)/x**2/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{17}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{17}{4}}x^2}\,dx$$

input
```
integrate((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{d\,x^4+c}{x^2\,{\left(b\,x^4+a\right)}^{17/4}}\,dx$$

input
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(17/4)),x)
```

output
```
int((c + d*x^4)/(x^2*(a + b*x^4)^(17/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^2\left(a+bx^4\right)^{17/4}}\,dx=\left(\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}a^4+4\left(b\,x^4+a\right)^{\frac{1}{4}}a^3b\,x^4+6\left(b\,x^4+a\right)^{\frac{1}{4}}a^2b^2x^8+4\left(b\,x^4+a\right)^{\frac{1}{4}}a\,b^3\right.$$
$$+\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}a^4x^2+4\left(b\,x^4+a\right)^{\frac{1}{4}}a^3b\,x^6+6\left(b\,x^4+a\right)^{\frac{1}{4}}a^2b^2x^{10}+4\left(b\,x^4+a\right)^{\frac{1}{4}}a\,b^3x^{14}+\left(b\,x^4+a\right)^{\frac{1}{4}}b^4\right.$$

input
```
int((d*x^4+c)/x^2/(b*x^4+a)^(17/4),x)
```

output
```
int(x**2/((a + b*x**4)**(1/4)*a**4 + 4*(a + b*x**4)**(1/4)*a**3*b*x**4 + 6
*(a + b*x**4)**(1/4)*a**2*b**2*x**8 + 4*(a + b*x**4)**(1/4)*a*b**3*x**12 +
 (a + b*x**4)**(1/4)*b**4*x**16),x)*d + int(1/((a + b*x**4)**(1/4)*a**4*x*
*2 + 4*(a + b*x**4)**(1/4)*a**3*b*x**6 + 6*(a + b*x**4)**(1/4)*a**2*b**2*x
**10 + 4*(a + b*x**4)**(1/4)*a*b**3*x**14 + (a + b*x**4)**(1/4)*b**4*x**18
),x)*c
```

3.190 $\int \frac{c+dx^4}{x^6(a+bx^4)^{17/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 213

$$\int \frac{c+dx^4}{x^6(a+bx^4)^{17/4}}\,dx = -\frac{c}{5ax^5(a+bx^4)^{13/4}} - \frac{18bc-5ad}{65a^2x(a+bx^4)^{13/4}} - \frac{14(18bc-5ad)}{585a^3x(a+bx^4)^{9/4}} - \frac{28(18bc-5ad)}{585a^4x(a+bx^4)^{5/4}} + \frac{56(18bc-5ad)}{195a^5x\sqrt[4]{a+bx^4}} - \frac{112\sqrt{b}(18bc-5ad)\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{195a^{11/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5*c/a/x^5/(b*x^4+a)^(13/4)-1/65*(-5*a*d+18*b*c)/a^2/x/(b*x^4+a)^(13/4)-
14/585*(-5*a*d+18*b*c)/a^3/x/(b*x^4+a)^(9/4)-28/585*(-5*a*d+18*b*c)/a^4/x/
(b*x^4+a)^(5/4)+56/195*(-5*a*d+18*b*c)/a^5/x/(b*x^4+a)^(1/4)-112/195*b^(1/
2)*(-5*a*d+18*b*c)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^
2/a^(1/2))),2^(1/2))/a^(11/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.39

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{17/4}}\,dx = \frac{-a^4c+(18bc-5ad)x^4(a+bx^4)^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{17}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{5a^5x^5(a+bx^4)^{13/4}}$$

input
```
Integrate[(c + d*x^4)/(x^6*(a + b*x^4)^(17/4)),x]
```

output
```
(-(a^4*c) + (18*b*c - 5*a*d)*x^4*(a + b*x^4)^3*(1 + (b*x^4)/a)^(1/4)*Hyper
geometric2F1[-1/4, 17/4, 3/4, -((b*x^4)/a)])/(5*a^5*x^5*(a + b*x^4)^(13/4)
)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {955, 819, 819, 819, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^4}{x^6 (a+bx^4)^{17/4}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(18bc-5ad)\int \frac{1}{x^2(bx^4+a)^{17/4}}dx}{5a}-\frac{c}{5ax^5(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(18bc-5ad)\left(\frac{14\int \frac{1}{x^2\left(bx^4+a\right)^{13/4}}dx}{13a}+\frac{1}{13ax(a+bx^4)^{13/4}}\right)}{5a}-\frac{c}{5ax^5(a+bx^4)^{13/4}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\int\frac{1}{x^2\left(bx^4+a\right)^{9/4}}dx}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx^4)^{13/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\int\frac{1}{x^2\left(bx^4+a\right)^{5/4}}dx}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx^4)^{13/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \text{816}$$

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\left(-\frac{2b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx^4)^{13/4}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$\Big\downarrow$ 813

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx}\right.}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 858

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx^4}\right.}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$\downarrow$ 807

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{1}{13ax(a+bx^4)}\right.}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

$$\Big\downarrow \; 212$$

$$-\frac{(18bc-5ad)\left(\frac{14\left(\frac{10\left(\frac{6\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}+\frac{1}{5ax\left(a+bx^4\right)^{5/4}}\right)}{9a}+\frac{1}{9ax\left(a+bx^4\right)^{9/4}}\right)}{13a}+\frac{}{13ax(a}}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^4\right)^{13/4}}$$

input
```
Int[(c + d*x^4)/(x^6*(a + b*x^4)^(17/4)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^4)^(13/4)) - ((18*b*c - 5*a*d)*(1/(13*a*x*(a + b*x^
4)^(13/4)) + (14*(1/(9*a*x*(a + b*x^4)^(9/4)) + (10*(1/(5*a*x*(a + b*x^4)^
(5/4)) + (6*(-(1/(a*x*(a + b*x^4)^(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/
4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(
1/4))))/(5*a)))/(9*a)))/(13*a)))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{4}+c}{x^{6}\left(b\,x^{4}+a\right)^{\frac{17}{4}}}dx$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x)
```

output
```
int((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x)
```

Fricas [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^{4}+c}{\left(bx^{4}+a\right)^{\frac{17}{4}}x^{6}}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*(d*x^4 + c)/(b^5*x^26 + 5*a*b^4*x^22 + 10*a^2*b
^3*x^18 + 10*a^3*b^2*x^14 + 5*a^4*b*x^10 + a^5*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**4+c)/x**6/(b*x**4+a)**(17/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{17}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{dx^4+c}{\left(bx^4+a\right)^{\frac{17}{4}}x^6}\,dx$$

input
```
integrate((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)/((b*x^4 + a)^(17/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\int \frac{d\,x^4+c}{x^6\,{\left(b\,x^4+a\right)}^{17/4}} \,d x$$

input
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(17/4)),x)
```

output
```
int((c + d*x^4)/(x^6*(a + b*x^4)^(17/4)), x)
```

Reduce [F]

$$\int \frac{c+dx^4}{x^6\left(a+bx^4\right)^{17/4}}\,dx=\left(\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{4} x^{6}+4 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{3} b\,x^{10}+6 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{2} b^{2} x^{14}+4 \left(b\,x^{4}+a\right)^{\frac{1}{4}}\right. \\ +\left(\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{4} x^{2}+4 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{3} b\,x^{6}+6 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{2} b^{2} x^{10}+4 \left(b\,x^{4}+a\right)^{\frac{1}{4}} a\,b^{3} x^{14}+\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{4}\right.$$

input
```
int((d*x^4+c)/x^6/(b*x^4+a)^(17/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a**4*x**6 + 4*(a + b*x**4)**(1/4)*a**3*b*x**10
+ 6*(a + b*x**4)**(1/4)*a**2*b**2*x**14 + 4*(a + b*x**4)**(1/4)*a*b**3*x**
18 + (a + b*x**4)**(1/4)*b**4*x**22),x)*c + int(1/((a + b*x**4)**(1/4)*a**
4*x**2 + 4*(a + b*x**4)**(1/4)*a**3*b*x**6 + 6*(a + b*x**4)**(1/4)*a**2*b*
*2*x**10 + 4*(a + b*x**4)**(1/4)*a*b**3*x**14 + (a + b*x**4)**(1/4)*b**4*x
**18),x)*d
```

3.191 $\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)\,dx = \frac{d(ex)^{1+m}\left(a+bx^4\right)^{1+p}}{be(5+m+4p)} + \frac{\left(\frac{c}{1+m}-\frac{ad}{b(5+m+4p)}\right)(ex)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{e}$$

output
```
d*(e*x)^(1+m)*(b*x^4+a)^(p+1)/b/e/(5+m+4*p)+(c/(1+m)-a*d/b/(5+m+4*p))*(e*x
)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/e/((1+
b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)\,dx = \frac{x(ex)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(c(5+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)+d(1+m)x^4\,\text{Hyp}\right.}{(1+m)(5+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(c*(5 + m)*Hypergeometric2F1[(1 + m)/4, -p, (5 +
m)/4, -((b*x^4)/a)] + d*(1 + m)*x^4*Hypergeometric2F1[(5 + m)/4, -p, (9 +
m)/4, -((b*x^4)/a)]))/((1 + m)*(5 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^4)(ex)^m (a+bx^4)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\int (ex)^m (bx^4+a)^p\,dx+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\int (ex)^m\left(\frac{bx^4}{a}+1\right)^p dx+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{e(m+1)}+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^4)^(1 + p))/(b*e*(5 + m + 4*p)) + ((c - (a*d*(1
+ m))/(b*(5 + m + 4*p)))*(e*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1
+ m)/4, -p, (5 + m)/4, -((b*x^4)/a)])/(e*(1 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right) dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx=\int \left(dx^4+c\right)\left(bx^4+a\right)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx=\int \left(dx^4+c\right)\left(bx^4+a\right)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx = \int \left(dx^4+c\right)\left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx = \int (e\,x)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right) dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4),x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

output
```
(e**m*(4*x**m*(a + b*x**4)**p*a*d*p*x + x**m*(a + b*x**4)**p*b*c*m*x + 4*x
**m*(a + b*x**4)**p*b*c*p*x + 5*x**m*(a + b*x**4)**p*b*c*x + x**m*(a + b*x
**4)**p*b*d*m*x**5 + 4*x**m*(a + b*x**4)**p*b*d*p*x**5 + x**m*(a + b*x**4)
**p*b*d*x**5 - 4*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16
*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p*
*2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m**3*p - 32*int((x**m*(a + b*x
**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**
4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x
)*a**2*d*m**2*p**2 - 28*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a
*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 +
16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m**2*p - 64*int((x**m*(
a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m
**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*
x**4),x)*a**2*d*m*p**3 - 128*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p
+ 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x*
*4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m*p**2 - 44*int((x
**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a
+ b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 +
 5*b*x**4),x)*a**2*d*m*p - 64*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p
 + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*...
```

3.192 $\int x^{-1-4(1+p)}(a+bx^4)^p(c+dx^4)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int x^{-1-4(1+p)}(a+bx^4)^p(c+dx^4)\,dx = -\frac{cx^{-4(1+p)}(a+bx^4)^{1+p}}{4a(1+p)} - \frac{dx^{-4p}(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^4}{a}\right)}{4p}$$

output

```
-1/4*c*(b*x^4+a)^(p+1)/a/(p+1)/(x^(4*p+4))-1/4*d*(b*x^4+a)^p*hypergeom([-p
, -p],[1-p],-b*x^4/a)/p/(x^(4*p))/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int x^{-1-4(1+p)}\left(a+bx^4\right)^p\left(c+dx^4\right)\,dx$$
$$=\frac{1}{4}x^{-4p}\left(a+bx^4\right)^p\left(-\frac{c(a+bx^4)}{a(1+p)x^4}-\frac{d\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^4}{a}\right)}{p}\right)$$

input
```
Integrate[x^(-1 - 4*(1 + p))*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
((a + b*x^4)^p*(-((c*(a + b*x^4))/(a*(1 + p)*x^4)) - (d*Hypergeometric2F1[
-p, -p, 1 - p, -((b*x^4)/a)])/(p*(1 + (b*x^4)/a)^p)))/(4*x^(4*p))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-4(p+1)-1}\left(c+dx^4\right)\left(a+bx^4\right)^p\,dx$$
$$\Big\downarrow \text{954}$$
$$\frac{d\int x^{-4p-5}\left(bx^4+a\right)^{p+1}dx}{b}-\frac{x^{-4(p+1)}(bc-ad)\left(a+bx^4\right)^{p+1}}{4ab(p+1)}$$
$$\Big\downarrow \text{882}$$

$$\frac{dx^{-4(p+1)}\left(\frac{x^4}{a+bx^4}\right)^{p+1}(a+bx^4)^{p+1}\int\frac{\left(\frac{x^4}{bx^4+a}\right)^{-p-2}}{1-\frac{bx^4}{bx^4+a}}d\frac{x^4}{bx^4+a}}{4b}-\frac{x^{-4(p+1)}(bc-ad)(a+bx^4)^{p+1}}{4ab(p+1)}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-4(p+1)}(bc-ad)(a+bx^4)^{p+1}}{4ab(p+1)}-\frac{dx^{-4(p+1)}(a+bx^4)^{p+1}\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^4}{bx^4+a}\right)}{4b(p+1)}$$

input
```
Int[x^(-1 - 4*(1 + p))*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
-1/4*((b*c - a*d)*(a + b*x^4)^(1 + p))/(a*b*(1 + p)*x^(4*(1 + p))) - (d*(a
 + b*x^4)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*x^4)/(a + b*x^4)])/(
4*b*(1 + p)*x^(4*(1 + p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-5-4p} \left(b\,x^{4}+a\right)^{p} \left(d\,x^{4}+c\right)dx$$

input
```
int(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x)
```

output
```
int(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x)
```

Fricas [F]

$$\int x^{-1-4(1+p)} \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \int \left(dx^4+c\right) \left(bx^4+a\right)^p x^{-4\,p-5} \, dx$$

input
```
integrate(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^4 + a)^p*x^(-4*p - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-4(1+p)} \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \text{Timed out}$$

input
```
integrate(x**(-5-4*p)*(b*x**4+a)**p*(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-4(1+p)}\left(a+bx^4\right)^p\left(c+dx^4\right)\,dx=\int\left(dx^4+c\right)\left(bx^4+a\right)^px^{-4p-5}\,dx$$

input
```
integrate(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x, algorithm="maxima")
```

output
```
d*integrate(e^(p*log(b*x^4 + a) - 4*p*log(x))/x, x) - 1/4*(b*x^4 + a)*c*e^
(p*log(b*x^4 + a) - 4*p*log(x))/(a*(p + 1)*x^4)
```

Giac [F]

$$\int x^{-1-4(1+p)}\left(a+bx^4\right)^p\left(c+dx^4\right)\,dx=\int\left(dx^4+c\right)\left(bx^4+a\right)^px^{-4p-5}\,dx$$

input
```
integrate(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p*x^(-4*p - 5), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-4(1+p)}\left(a+bx^4\right)^p\left(c+dx^4\right)\,dx=\int\frac{\left(b\,x^4+a\right)^p\,\left(d\,x^4+c\right)}{x^{4\,p+5}}\,dx$$

input
```
int(((a + b*x^4)^p*(c + d*x^4))/x^(4*p + 5),x)
```

output
```
int(((a + b*x^4)^p*(c + d*x^4))/x^(4*p + 5), x)
```

Reduce [F]

$$\int x^{-1-4(1+p)}\left(a+bx^4\right)^p\left(c+dx^4\right)\,dx$$
$$=\frac{-(b\,x^4+a)^p\,ac-(b\,x^4+a)^p\,bc\,x^4+4x^{4p}\left(\int\frac{(b\,x^4+a)^p}{x^{4p}x}dx\right)adp\,x^4+4x^{4p}\left(\int\frac{(b\,x^4+a)^p}{x^{4p}x}dx\right)ad\,x^4}{4x^{4p}a\,x^4\,(p+1)}$$

input
```
int(x^(-5-4*p)*(b*x^4+a)^p*(d*x^4+c),x)
```

output
```
( - (a + b*x**4)**p*a*c - (a + b*x**4)**p*b*c*x**4 + 4*x**(4*p)*int((a + b
*x**4)**p/(x**(4*p)*x),x)*a*d*p*x**4 + 4*x**(4*p)*int((a + b*x**4)**p/(x**
(4*p)*x),x)*a*d*x**4)/(4*x**(4*p)*a*x**4*(p + 1))
```

3.193 $\int (ex)^m (a+bx^4)^p (a(1+m)+b(1+m+4(1+p))$

Optimal result

Integrand size = 34, antiderivative size = 22

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right)\,dx=\frac{(ex)^{1+m}\left(a+bx^4\right)^{1+p}}{e}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^(p+1)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right)\,dx$$
$$=\frac{x(ex)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)+b(5+m+4p)x\right.}{5+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(a*(1 + m) + b*(1 + m + 4*(1 + p))*x^4),x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(a*(5 + m)*Hypergeometric2F1[(1 + m)/4, -p, (5 +
m)/4, -((b*x^4)/a)] + b*(5 + m + 4*p)*x^4*Hypergeometric2F1[(5 + m)/4, -p,
 (9 + m)/4, -((b*x^4)/a)]))/((5 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(m+1)+bx^4(m+4(p+1)+1)\right) \, dx$$

$$\Big\downarrow \text{ 951}$$

$$\frac{(ex)^{m+1} \left(a+bx^4\right)^{p+1}}{e}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(a*(1 + m) + b*(1 + m + 4*(1 + p))*x^4),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^(1 + p))/e
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$x(ex)^m (b x^4 + a)^{p+1}$	19
parallelrisch	$\frac{x^5(ex)^m (b x^4+a)^p b^2+x(ex)^m (b x^4+a)^p ab}{b}$	45
risch	$(b x^4 + a)^p x(b x^4 + a) x^m e^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}$	65
orering	$\frac{(b x^4+a)x(ex)^m (b x^4+a)^p (a(1+m)+b(5+m+4p)x^4)}{bm x^4+4bp x^4+5b x^4+am+a}$	67

input
```
int((e*x)^m*(b*x^4+a)^p*(a*(1+m)+b*(5+m+4*p)*x^4),x,method=_RETURNVERBOSE)
```

output
```
x*(e*x)^m*(b*x^4+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right) dx = \left(bx^5+ax\right)\left(bx^4+a\right)^p (ex)^m$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(a*(1+m)+b*(5+m+4*p)*x^4),x, algorithm="fric
as")
```

output
```
(b*x^5 + a*x)*(b*x^4 + a)^p*(e*x)^m
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(a*(1+m)+b*(5+m+4*p)*x**4),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right)\,dx = \left(be^m x^5+ae^m x\right)e^{\left(p\log\left(bx^4+a\right)+m\log\left(x\right)\right)}$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(a*(1+m)+b*(5+m+4*p)*x^4),x, algorithm="maxi
ma")
```

output
```
(b*e^m*x^5 + a*e^m*x)*e^(p*log(b*x^4 + a) + m*log(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right)\,dx = \left(bx^4+a\right)^p (ex)^m bx^5+\left(bx^4+a\right)^p (ex)^m ax$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(a*(1+m)+b*(5+m+4*p)*x^4),x, algorithm="giac
")
```

output
```
(b*x^4 + a)^p*(e*x)^m*b*x^5 + (b*x^4 + a)^p*(e*x)^m*a*x
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right) \, dx = x\,(e\,x)^m\,\left(b\,x^4+a\right)^{p+1}$$

input
```
int((a*(m + 1) + b*x^4*(m + 4*p + 5))*(e*x)^m*(a + b*x^4)^p,x)
```

output
```
x*(e*x)^m*(a + b*x^4)^(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^m \left(a+bx^4\right)^p \left(a(1+m)+b(1+m+4(1+p))x^4\right) \, dx = x^m e^m \left(b\,x^4+a\right)^p x \left(b\,x^4+a\right)$$

input
```
int((e*x)^m*(b*x^4+a)^p*(a*(1+m)+b*(5+m+4*p)*x^4),x)
```

output
```
x**m*e**m*(a + b*x**4)**p*x*(a + b*x**4)
```

3.194 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx = -\frac{(bc+ad)x^4}{4b^2d^2} + \frac{x^8}{8bd} - \frac{a^3\log(a+bx^4)}{4b^3(bc-ad)} + \frac{c^3\log(c+dx^4)}{4d^3(bc-ad)}$$

output
```
-1/4*(a*d+b*c)*x^4/b^2/d^2+1/8*x^8/b/d-1/4*a^3*ln(b*x^4+a)/b^3/(-a*d+b*c)+
1/4*c^3*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)} dx = \frac{bd(bc-ad)x^4(-2bc-2ad+bdx^4) - 2a^3d^3\log(a+bx^4) + 2b^3c^3\log(c+dx^4)}{8b^3d^3(bc-ad)}$$

input
```
Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*d*(b*c - a*d)*x^4*(-2*b*c - 2*a*d + b*d*x^4) - 2*a^3*d^3*Log[a + b*x^4]
 + 2*b^3*c^3*Log[c + d*x^4])/(8*b^3*d^3*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$$
$$\Big\downarrow \ 948$$
$$\frac{1}{4}\int \frac{x^{12}}{(bx^4+a)(dx^4+c)}dx^4$$
$$\Big\downarrow \ 93$$
$$\frac{1}{4}\int \left(\frac{x^4}{bd}+\frac{-bc-ad}{b^2d^2}-\frac{a^3}{b^2(bc-ad)(bx^4+a)}-\frac{c^3}{d^2(ad-bc)(dx^4+c)}\right)dx^4$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{4}\left(-\frac{a^3\log(a+bx^4)}{b^3(bc-ad)}-\frac{x^4(ad+bc)}{b^2d^2}+\frac{c^3\log(c+dx^4)}{d^3(bc-ad)}+\frac{x^8}{2bd}\right)$$

input
```
Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(((b*c + a*d)*x^4)/(b^2*d^2)) + x^8/(2*b*d) - (a^3*Log[a + b*x^4])/(b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(d^3*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{(-db\,x^4+ad+cb)^2}{8b^3d^3}+\frac{a^3\ln(b\,x^4+a)}{4b^3(ad-cb)}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-cb)}$	78
norman	$\frac{x^8}{8bd}-\frac{(ad+cb)x^4}{4b^2d^2}+\frac{a^3\ln(b\,x^4+a)}{4b^3(ad-cb)}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-cb)}$	83
parallelrisch	$\frac{x^8a\,b^2d^3-x^8b^3c\,d^2-2x^4a^2b\,d^3+2x^4b^3c^2d+2a^3\ln(b\,x^4+a)d^3-2c^3\ln(d\,x^4+c)b^3}{8b^3d^3(ad-cb)}$	99
risch	$\frac{x^8}{8bd}-\frac{a\,x^4}{4b^2d}-\frac{c\,x^4}{4b\,d^2}+\frac{a^2}{8b^3d}+\frac{ac}{4b^2d^2}+\frac{c^2}{8b\,d^3}+\frac{a^3\ln(-b\,x^4-a)}{4b^3(ad-cb)}-\frac{c^3\ln(d\,x^4+c)}{4d^3(ad-cb)}$	124

input
```
int(x^15/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-b*d*x^4+a*d+b*c)^2/b^3/d^3+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)-1/4*c^3
/d^3/(a*d-b*c)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \frac{(b^3cd^2-ab^2d^3)x^8-2\,a^3d^3\log(bx^4+a)+2\,b^3c^3\log(dx^4+c)-2\,(b^3c^2d-a^2bd^3)x^4}{8\,(b^4cd^3-ab^3d^4)}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*lo
g(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3\log(bx^4+a)}{4(b^4c-ab^3d)} + \frac{c^3\log(dx^4+c)}{4(bcd^3-ad^4)} + \frac{bdx^8-2(bc+ad)x^4}{8b^2d^2}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^
3 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3\log(|bx^4+a|)}{4(b^4c-ab^3d)} + \frac{c^3\log(|dx^4+c|)}{4(bcd^3-ad^4)} + \frac{bdx^8-2\,bcx^4-2\,adx^4}{8\,b^2d^2}$$

input
```
integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c
))/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{x^{15}}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{x^8}{8\,b\,d} - \frac{c^3\,\ln\left(d\,x^4+c\right)}{4\,\left(a\,d^4-b\,c\,d^3\right)} - \frac{a^3\,\ln\left(b\,x^4+a\right)}{4\,\left(b^4\,c-a\,b^3\,d\right)} - \frac{x^4\,\left(a\,d+b\,c\right)}{4\,b^2\,d^2}$$

input
```
int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*
x^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{x^{15}}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{2\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)a^3d^3-2\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)b^3c^3+2\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\right.}{8b^3d^3\,(ac}$$

input
```
int(x^15/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(2*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**3*d**3
- 2*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b**3*c**3
 + 2*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**3*d**3 -
 2*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b**3*c**3 - 2
*a**2*b*d**3*x**4 + a*b**2*d**3*x**8 + 2*b**3*c**2*d*x**4 - b**3*c*d**2*x*
*8)/(8*b**3*d**3*(a*d - b*c))
```

3.195 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx = \frac{x^4}{4bd} + \frac{a^2 \log(a+bx^4)}{4b^2(bc-ad)} - \frac{c^2 \log(c+dx^4)}{4d^2(bc-ad)}$$

output
```
1/4*x^4/b/d+1/4*a^2*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c^2*ln(d*x^4+c)/d^2/(-a
*d+b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)} dx = \frac{a^2d^2 \log(a+bx^4) - b(d(-bc+ad)x^4 + bc^2 \log(c+dx^4))}{4b^2d^2(bc-ad)}$$

input
```
Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))
/(4*b^2*d^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{ 93}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b(bc-ad)(bx^4+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{a^2\log(a+bx^4)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^4)}{d^2(bc-ad)}+\frac{x^4}{bd}\right)$$

input
```
Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(x^4/(b*d) + (a^2*Log[a + b*x^4])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4])
/(d^2*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^4}{4bd}-\frac{a^2\ln(b\,x^4+a)}{4b^2(ad-cb)}+\frac{c^2\ln(d\,x^4+c)}{4d^2(ad-cb)}$	65
norman	$\frac{x^4}{4bd}-\frac{a^2\ln(b\,x^4+a)}{4b^2(ad-cb)}+\frac{c^2\ln(d\,x^4+c)}{4d^2(ad-cb)}$	65
risch	$\frac{x^4}{4bd}+\frac{c^2\ln(d\,x^4+c)}{4d^2(ad-cb)}-\frac{a^2\ln(-b\,x^4-a)}{4b^2(ad-cb)}$	68
parallelrisch	$-\frac{-x^4ab\,d^2+x^4b^2cd+a^2\ln(b\,x^4+a)\,d^2-c^2\ln(d\,x^4+c)\,b^2}{4b^2d^2(ad-cb)}$	70

input
```
int(x^11/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/b/d-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^4+a)+1/4*c^2/d^2/(a*d-b*c)*ln(d*x
^4+c)
```

Fricas [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{(a+bx^4)\,(c+dx^4)}\,dx=\frac{(b^2cd-abd^2)x^4+a^2d^2\log\left(bx^4+a\right)-b^2c^2\log\left(dx^4+c\right)}{4\,(b^3cd^2-ab^2d^3)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4
+ c))/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd} + \frac{a^2\log(bx^4+a)}{4\,(b^3c-ab^2d)} - \frac{c^2\log(dx^4+c)}{4\,(bcd^2-ad^3)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x
^4 + c)/(b*c*d^2 - a*d^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^4}{4\,bd} + \frac{a^2\log(|bx^4+a|)}{4\,(b^3c-ab^2d)} - \frac{c^2\log(|dx^4+c|)}{4\,(bcd^2-ad^3)}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*lo
g(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\,\ln\left(b\,x^4+a\right)}{4\,b^3\,c-4\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^4+c\right)}{4\,a\,d^3-4\,b\,c\,d^2}+\frac{x^4}{4\,b\,d}$$

input
```
int(x^11/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
 - 4*b*c*d^2) + x^4/(4*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.14

$$\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx = \frac{-\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)a^2d^2+\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)b^2c^2-\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\right.}{4b^2d^2\,(ad-bc)}$$

input
```
int(x^11/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*d**2
 + log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b**2*c**2
- log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a**2*d**2 + lo
g(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b**2*c**2 + a*b*d*
*2*x**4 - b**2*c*d*x**4)/(4*b**2*d**2*(a*d - b*c))
```

3.196 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(a+bx^4)}{4b(bc-ad)}+\frac{c\log(c+dx^4)}{4d(bc-ad)}$$

output
```
-1/4*a*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*c*ln(d*x^4+c)/d/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{ad\log(a+bx^4)-bc\log(c+dx^4)}{4b^2cd-4abd^2}$$

input
```
Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
-((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{4}\int \left(\frac{c}{(bc-ad)(dx^4+c)}-\frac{a}{(bc-ad)(bx^4+a)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{c\log(c+dx^4)}{d(bc-ad)}-\frac{a\log(a+bx^4)}{b(bc-ad)}\right)$$

input
```
Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((a*Log[a + b*x^4])/(b*(b*c - a*d))) + (c*Log[c + d*x^4])/(d*(b*c - a*d)
))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{a\ln(b\,x^4+a)d-c\ln(d\,x^4+c)b}{4b(ad-cb)d}$	43
default	$\frac{a\ln(b\,x^4+a)}{4(ad-cb)b}-\frac{c\ln(d\,x^4+c)}{4(ad-cb)d}$	50
norman	$\frac{a\ln(b\,x^4+a)}{4(ad-cb)b}-\frac{c\ln(d\,x^4+c)}{4(ad-cb)d}$	50
risch	$\frac{a\ln(b\,x^4+a)}{4(ad-cb)b}-\frac{c\ln(-d\,x^4-c)}{4d(ad-cb)}$	53

input
```
int(x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*ln(b*x^4+a)*d-c*ln(d*x^4+c)*b)/b/(a*d-b*c)/d
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{ad\log(bx^4+a)-bc\log(dx^4+c)}{4(b^2cd-abd^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 40.79 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.72

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = \frac{a\log\left(x^4+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{4b(ad-bc)} - \frac{c\log\left(x^4+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{4d(ad-bc)}$$

input
```
integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

output
```
a*log(x**4 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**
2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(4*b*(a*d - b*c)) - c*log(x**4 + (-a**
2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b
*c)))/(a*d + b*c))/(4*d*(a*d - b*c))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(bx^4+a)}{4(b^2c-abd)} + \frac{c\log(dx^4+c)}{4(bcd-ad^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^
2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\log(|bx^4+a|)}{4(b^2c-abd)}+\frac{c\log(|dx^4+c|)}{4(bcd-ad^2)}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*
c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\,\ln(b\,x^4+a)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\,\ln(d\,x^4+c)}{4\,a\,d^2-4\,b\,c\,d}$$

input
```
int(x^7/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4
*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.17

$$\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)ad-\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)bc+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x\right.}{4bd(ad-bc)}$$

input
```
int(x^7/(b*x^4+a)/(d*x^4+c),x)
```

output

```
(log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*d - log( -
 d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b*c + log(b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*d - log(d**(1/4)*c**(1/4)*s
qrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b*c)/(4*b*d*(a*d - b*c))
```

3.197 $\int \frac{x^3}{(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)} dx = \frac{\log(a+bx^4)}{4(bc-ad)} - \frac{\log(c+dx^4)}{4(bc-ad)}$$

output
```
ln(b*x^4+a)/(-4*a*d+4*b*c)-ln(d*x^4+c)/(-4*a*d+4*b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)} dx = \frac{\log(a+bx^4) - \log(c+dx^4)}{4bc-4ad}$$

input
```
Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}dx$$
$$\Big\downarrow \text{946}$$
$$\frac{1}{4}\int \frac{1}{(bx^4+a)(dx^4+c)}dx^4$$
$$\Big\downarrow \text{47}$$
$$\frac{1}{4}\left(\frac{b\int \frac{1}{bx^4+a}dx^4}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx^4}{bc-ad}\right)$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{4}\left(\frac{\log(a+bx^4)}{bc-ad}-\frac{\log(c+dx^4)}{bc-ad}\right)$$

input
```
Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[a + b*x^4]/(b*c - a*d) - Log[c + d*x^4]/(b*c - a*d))/4
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b\,x^4+a)-\ln(d\,x^4+c)}{4(ad-cb)}$	32
default	$-\frac{\ln(b\,x^4+a)}{4(ad-cb)}+\frac{\ln(d\,x^4+c)}{4ad-4cb}$	42
norman	$-\frac{\ln(b\,x^4+a)}{4(ad-cb)}+\frac{\ln(d\,x^4+c)}{4ad-4cb}$	42
risch	$-\frac{\ln(-b\,x^4-a)}{4(ad-cb)}+\frac{\ln(d\,x^4+c)}{4ad-4cb}$	45

input
```
int(x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(ln(b*x^4+a)-ln(d*x^4+c))/(a*d-b*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(bx^4+a)-\log(dx^4+c)}{4(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)} - \frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}$$

input
```
integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

output
```
log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b
*d))/(4*(a*d - b*c))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{\log(bx^4+a)}{4(bc-ad)} - \frac{\log(dx^4+c)}{4(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\log\left(|bx^4+a|\right)}{4(b^2c-abd)} - \frac{d\log\left(|dx^4+c|\right)}{4(bcd-ad^2)}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 1012, normalized size of antiderivative = 22.49

$$\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^3/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
-(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 51
2*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*
d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(38
4*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 5
12*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^
4*d^4*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x
^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b
^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024
*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c
) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^
4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*
d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^
6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6
+ 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c
^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d
- 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c
) + ((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*
c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 153
6*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*
c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.20

$$\int \frac{x^3}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{-\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4ad-4bc}$$

input
```
int(x^3/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2) + log( -
d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2) - log(b**(1/4)*a**(1
/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2) + log(d**(1/4)*c**(1/4)*sqrt(2)*x
+ sqrt(c) + sqrt(d)*x**2))/(4*(a*d - b*c))
```

3.198 $\int \frac{1}{x(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x(a+bx^4)(c+dx^4)} dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^4)}{4a(bc-ad)} + \frac{d\log(c+dx^4)}{4c(bc-ad)}$$

output
```
ln(x)/a/c-1/4*b*ln(b*x^4+a)/a/(-a*d+b*c)+1/4*d*ln(d*x^4+c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^4)(c+dx^4)} dx = \frac{4bc\log(x) - 4ad\log(x) - bc\log(a+bx^4) + ad\log(c+dx^4)}{4abc^2 - 4a^2cd}$$

input
```
Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4
*a*b*c^2 - 4*a^2*c*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^4) (c+dx^4)} \, dx$$
$$\Big\downarrow \text{ 948}$$
$$\frac{1}{4} \int \frac{1}{x^4 (bx^4+a) (dx^4+c)}dx^4$$
$$\Big\downarrow \text{ 93}$$
$$\frac{1}{4} \int \left(\frac{b^2}{a(ad-bc) (bx^4+a)}+\frac{d^2}{c(bc-ad) (dx^4+c)}+\frac{1}{acx^4}\right)dx^4$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{4} \left(-\frac{b \log (a+bx^4)}{a(bc-ad)}+\frac{d \log (c+dx^4)}{c(bc-ad)}+\frac{\log (x^4)}{ac}\right)$$

input
```
Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(Log[x^4]/(a*c) - (b*Log[a + b*x^4])/(a*(b*c - a*d)) + (d*Log[c + d*x^4])/
(c*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{4\ln(x)ad-4\ln(x)bc+b\ln(b\,x^4+a)c-d\ln(d\,x^4+c)a}{4ac(ad-cb)}$	55
default	$\frac{b\ln(b\,x^4+a)}{4a(ad-cb)}+\frac{\ln(x)}{ac}-\frac{d\ln(d\,x^4+c)}{4c(ad-cb)}$	59
norman	$\frac{b\ln(b\,x^4+a)}{4a(ad-cb)}+\frac{\ln(x)}{ac}-\frac{d\ln(d\,x^4+c)}{4c(ad-cb)}$	59
risch	$\frac{b\ln(b\,x^4+a)}{4a(ad-cb)}+\frac{\ln(x)}{ac}-\frac{d\ln(d\,x^4+c)}{4c(ad-cb)}$	59

input
```
int(1/x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*ln(x)*a*d-4*ln(x)*b*c+b*ln(b*x^4+a)*c-d*ln(d*x^4+c)*a)/a/c/(a*d-b*c
)
```

Fricas [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{bc\log\left(bx^4+a\right)-ad\log\left(dx^4+c\right)-4\left(bc-ad\right)\log\left(x\right)}{4\left(abc^2-a^2cd\right)}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b
*c^2 - a^2*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = -\frac{b\log(bx^4+a)}{4(abc-a^2d)} + \frac{d\log(dx^4+c)}{4(bc^2-acd)} + \frac{\log(x^4)}{4\,ac}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*
d) + 1/4*log(x^4)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx^4)(c+dx^4)}\,dx = -\frac{b^2\log(|bx^4+a|)}{4(ab^2c-a^2bd)} + \frac{d^2\log(|dx^4+c|)}{4(bc^2d-acd^2)} + \frac{\log(x^4)}{4\,ac}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
 c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\,\ln\left(b\,x^4+a\right)}{4\,a^2\,d-4\,a\,b\,c}+\frac{d\,\ln\left(d\,x^4+c\right)}{4\,b\,c^2-4\,a\,c\,d}+\frac{\ln\left(x\right)}{a\,c}$$

input
```
int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a
*c*d) + log(x)/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.05

$$\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \frac{\log\left(-b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)bc-\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)ad+\log\left(b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{b}\,x\right.}{4ac\left(ad-bc\right)}$$

input
```
int(1/x/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c - log( -
 d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a*d + log(b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c - log(d**(1/4)*c**(1/4)*s
qrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a*d + 4*log(x)*a*d - 4*log(x)*b*c)/(4*a
*c*(a*d - b*c))
```

3.199 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{4acx^4} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2\log(a+bx^4)}{4a^2(bc-ad)} - \frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}$$

output
```
-1/4/a/c/x^4-(a*d+b*c)*ln(x)/a^2/c^2+1/4*b^2*ln(b*x^4+a)/a^2/(-a*d+b*c)-1/
4*d^2*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{4acx^4} + \frac{(-bc-ad)\log(x)}{a^2c^2} - \frac{b^2\log(a+bx^4)}{4a^2(-bc+ad)} - \frac{d^2\log(c+dx^4)}{4c^2(bc-ad)}$$

input
```
Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4]
)/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{1}{x^8 (bx^4+a) (dx^4+c)} dx^4$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{4} \int \left(-\frac{b^3}{a^2 (ad-bc) (bx^4+a)} - \frac{d^3}{c^2 (bc-ad) (dx^4+c)} + \frac{-bc-ad}{a^2 c^2 x^4} + \frac{1}{acx^8}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{b^2 \log (a+bx^4)}{a^2 (bc-ad)} - \frac{\log (x^4) (ad+bc)}{a^2 c^2} - \frac{d^2 \log (c+dx^4)}{c^2 (bc-ad)} - \frac{1}{acx^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(1/(a*c*x^4)) - ((b*c + a*d)*Log[x^4])/(a^2*c^2) + (b^2*Log[a + b*x^4])/
(a^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(c^2*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{4ac\,x^{4}}-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-cb\right)}+\frac{d^{2}\ln\left(d\,x^{4}+c\right)}{4c^{2}\left(ad-cb\right)}-\frac{\left(ad+cb\right)\ln\left(x\right)}{a^{2}c^{2}}$	82
default	$-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-cb\right)}-\frac{1}{4ac\,x^{4}}+\frac{\left(-ad-cb\right)\ln\left(x\right)}{a^{2}c^{2}}+\frac{d^{2}\ln\left(d\,x^{4}+c\right)}{4c^{2}\left(ad-cb\right)}$	83
risch	$-\frac{1}{4ac\,x^{4}}-\frac{\ln\left(x\right)d}{a\,c^{2}}-\frac{\ln\left(x\right)b}{a^{2}c}+\frac{d^{2}\ln\left(-d\,x^{4}-c\right)}{4c^{2}\left(ad-cb\right)}-\frac{b^{2}\ln\left(b\,x^{4}+a\right)}{4a^{2}\left(ad-cb\right)}$	90
parallelrisch	$-\frac{4\ln\left(x\right)x^{4}a^{2}d^{2}-4\ln\left(x\right)x^{4}b^{2}c^{2}+b^{2}\ln\left(b\,x^{4}+a\right)c^{2}x^{4}-d^{2}\ln\left(d\,x^{4}+c\right)a^{2}x^{4}+a^{2}cd-ab\,c^{2}}{4a^{2}c^{2}x^{4}\left(ad-cb\right)}$	99

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c/x^4-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)+1/4*d^2/c^2/(a*d-b*c)*ln(d*
x^4+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

Fricas [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = \frac{b^2c^2x^4\log(bx^4+a) - a^2d^2x^4\log(dx^4+c) - 4(b^2c^2-a^2d^2)x^4\log(x) - abc^2 + a^2cd}{4(a^2bc^3-a^3c^2d)x^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2
- a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = \frac{b^2\log(bx^4+a)}{4(a^2bc-a^3d)} - \frac{d^2\log(dx^4+c)}{4(bc^3-ac^2d)} - \frac{(bc+ad)\log(x^4)}{4a^2c^2} - \frac{1}{4acx^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
 a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = \frac{b^3\log(|bx^4+a|)}{4(a^2b^2c-a^3bd)} - \frac{d^3\log(|dx^4+c|)}{4(bc^3d-ac^2d^2)} - \frac{(bc+ad)\log(x^4)}{4a^2c^2} + \frac{bcx^4+adx^4-ac}{4a^2c^2x^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4
+ c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*
c*x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{b^2\ln(b\,x^4+a)}{4\,(a^3\,d-a^2\,b\,c)} - \frac{d^2\ln(d\,x^4+c)}{4\,(b\,c^3-a\,c^2\,d)} - \frac{1}{4\,a\,c\,x^4} - \frac{\ln(x)\,(a\,d+b\,c)}{a^2\,c^2}$$

input
```
int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*
c^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^5 (a + bx^4) (c + dx^4)} \, dx$$
$$= \frac{-\log\left(-b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \sqrt{b}\, x^2\right) b^2 c^2 x^4 + \log\left(-d^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{c} + \sqrt{d}\, x^2\right) a^2 d^2 x^4 - \log\left(b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x +\right.}{4a^2 c^2}$$

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b**2*c**2
*x**4 + log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**2*
d**2*x**4 - log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b**2
*c**2*x**4 + log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**
2*d**2*x**4 - 4*log(x)*a**2*d**2*x**4 + 4*log(x)*b**2*c**2*x**4 - a**2*c*d
 + a*b*c**2)/(4*a**2*c**2*x**4*(a*d - b*c))
```

3.200 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(bc+ad)x^2}{2b^2d^2}+\frac{x^6}{6bd}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}+\frac{c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

output
```
-1/2*(a*d+b*c)*x^2/b^2/d^2+1/6*x^6/b/d-1/2*a^(5/2)*arctan(b^(1/2)*x^2/a^(1
/2))/b^(5/2)/(-a*d+b*c)+1/2*c^(5/2)*arctan(d^(1/2)*x^2/c^(1/2))/d^(5/2)/(-
a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{6}\left(\frac{x^2(-3bc-3ad+bdx^4)}{b^2d^2}+\frac{3a^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(-bc+ad)}+\frac{3c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

input
```
Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x
^2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/S
qrt[c]])/(d^(5/2)*(b*c - a*d)))/6
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {965, 381, 27, 444, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{381}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\int \frac{3x^4((bc+ad)x^4+ac)}{(bx^4+a)(dx^4+c)}dx^2}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\int \frac{x^4((bc+ad)x^4+ac)}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\int \frac{(b^2c^2+abdc+a^2d^2)x^4+ac(bc+ad)}{(bx^4+a)(dx^4+c)}dx^2}{bd}}{bd}\right)$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\frac{b^2c^3\int\frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a^3d^2\int\frac{1}{bx^4+a}dx^2}{bc-ad}}{bd}}{bd}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^6}{3bd}-\frac{\frac{x^2(ad+bc)}{bd}-\frac{\frac{b^2c^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{a^{5/2}d^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}}{bd}}{bd}\right)$$

input
```
Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(x^6/(3*b*d) - (((b*c + a*d)*x^2)/(b*d) - (-((a^(5/2)*d^2*ArcTan[(Sqrt[b]*
x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (b^2*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/
Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/(b*d))/(b*d))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

method	result
default	$-\frac{-\frac{x^{6} b d}{6}+\frac{(a d+c b) x^{2}}{2}}{b^{2} d^{2}}+\frac{a^{3} \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2 b^{2}(a d-c b)\sqrt{a b}}-\frac{c^{3} \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2 d^{2}(a d-c b)\sqrt{c d}}$
risch	$\frac{x^{6}}{6 b d}-\frac{x^{2} a}{2 b^{2} d}-\frac{x^{2} c}{2 b\,d^{2}}+\frac{\sqrt{-c d}\,c^{2} \ln\left(\left(-a^{6} d^{8}+a\,d^{3} c^{5} b^{5}\right) x^{2}+\left(-c d\right)^{\frac{3}{2}} a\,b^{5} c^{4} d+\left(-c d\right)^{\frac{3}{2}} b^{6} c^{5}+a^{6} d^{7}\sqrt{-c d}+b^{6} c^{6}\sqrt{-c d}\,d\right)}{4 d^{3}(a d-c b)}-\frac{\sqrt{-cd}$

input
```
int(x^13/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/d^2*(-1/6*x^6*b*d+1/2*(a*d+b*c)*x^2)+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1
/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x
^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 576, normalized size of antiderivative = 5.14

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \left[\frac{2\,(b^2cd-abd^2)x^6-3\,a^2d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-3\,b^2c^2\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-6\,(b^2c^2-}{12\,(b^3cd^2-ab^2d^3)}\right.$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x
^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^
2*sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a
*b^2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*
x^2*sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) -
c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12
*(2*(b^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c
) - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a))
 - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b
*d^2)*x^6 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt
(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 -
a*b^2*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd^2-ad^3)\sqrt{cd}} + \frac{bdx^6-3(bc+ad)x^2}{6b^2d^2}$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*a
rctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b
*c + a*d)*x^2)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(b^3c-ab^2d)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bcd^2-ad^3)\sqrt{cd}} + \frac{b^2d^2x^6-3\,b^2cdx^2-3\,abd^2x^2}{6\,b^3d^3}$$

input
```
integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*a
rctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 -
3*b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.75

$$
\begin{aligned}
&\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx \\
&= \frac{\ln\left(d^{10}\left(-a^5b^5\right)^{5/2} + b^{20}c^{10}\sqrt{-a^5b^5} - a^2b^{23}c^{10}x^2 - a^{12}b^{13}d^{10}x^2 + 2b^{10}c^5d^5\left(-a^5b^5\right)^{3/2} + 2a^7b^{18}c^5d^5\right.}{4b^6c - 4ab^5d} \\
&\quad - \frac{\ln\left(d^{10}\left(-a^5b^5\right)^{5/2} + b^{20}c^{10}\sqrt{-a^5b^5} + a^2b^{23}c^{10}x^2 + a^{12}b^{13}d^{10}x^2 + 2b^{10}c^5d^5\left(-a^5b^5\right)^{3/2} - 2a^7b^{18}c^5\right.}{4\left(b^6c - ab^5d\right)} \\
&\quad - \frac{\ln\left(b^{10}\left(-c^5d^5\right)^{5/2} + a^{10}d^{20}\sqrt{-c^5d^5} + a^{10}c^2d^{23}x^2 + b^{10}c^{12}d^{13}x^2 + 2a^5b^5d^{10}\left(-c^5d^5\right)^{3/2} - 2a^5b^5c^7\right.}{4\left(ad^6 - bcd^5\right)} \\
&\quad + \frac{\ln\left(b^{10}\left(-c^5d^5\right)^{5/2} + a^{10}d^{20}\sqrt{-c^5d^5} - a^{10}c^2d^{23}x^2 - b^{10}c^{12}d^{13}x^2 + 2a^5b^5d^{10}\left(-c^5d^5\right)^{3/2} + 2a^5b^5c^7\right.}{4ad^6 - 4bcd^5} \\
&\quad + \frac{x^6}{6bd} - \frac{x^2(ad+bc)}{2b^2d^2}
\end{aligned}
$$

input
```
int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^
2 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*
d^5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5
/2) + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2
+ 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/
2))/(4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d
^5)^(1/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*
d^5)^(3/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5
)) + (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^
23*x^2 - b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*
c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2
*(a*d + b*c))/(2*b^2*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.98

$$\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2d^3-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2d^3+3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b}{6b^3d^3(ad-bc)}$$

input
```
int(x^13/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**2*d**3 - 3*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*d**3 + 3*sqrt
(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1
/4)*sqrt(2)))*b**3*c**2 + 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2
) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**3*c**2 - 3*a**2*b*d**3*x*
*2 + a*b**2*d**3*x**6 + 3*b**3*c**2*d*x**2 - b**3*c*d**2*x**6)/(6*b**3*d**
3*(a*d - b*c))
```

3.201 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{x^2}{2bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

output
```
1/2*x^2/b/d+1/2*a^(3/2)*arctan(b^(1/2)*x^2/a^(1/2))/b^(3/2)/(-a*d+b*c)-1/2
*c^(3/2)*arctan(d^(1/2)*x^2/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{\left(-\frac{a}{b}+\frac{c}{d}\right)x^2 + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

input
```
Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c
^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {965, 381, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{ 381}$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\int \frac{(bc+ad)x^4+ac}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$

$$\Big\downarrow \text{ 397}$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a^2d\int \frac{1}{bx^4+a}dx^2}{bc-ad}}{bd}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{x^2}{bd}-\frac{\frac{bc^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{a^{3/2}d\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}}{bd}\right)$$

input
```
Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(x^2/(b*d) - (-((a^(3/2)*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c -
a*d))) + (b*c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/
(b*d))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{x^{2}}{2bd}-\frac{a^{2} \arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2b(ad-cb)\sqrt{ab}}+\frac{c^{2} \arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2d(ad-cb)\sqrt{cd}}$
risch	$\frac{x^{2}}{2bd}+\frac{\sqrt{-ab}\,a\ln\left(\left(b^{3}c\,d^{3}a^{3}-b^{6}c^{4}\right)x^{2}+\left(-ab\right)^{\frac{3}{2}}a^{3}d^{4}+\left(-ab\right)^{\frac{3}{2}}a^{2}bc\,d^{3}+a^{4}d^{4}\sqrt{-ab}\,b+b^{5}c^{4}\sqrt{-ab}\right)}{4b^{2}(ad-cb)}-\frac{\sqrt{-ab}\,a\ln\left(\left(b^{3}c\,d^{3}a^{3}-b^{6}c^{4}\right)\right.}{}$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/d-1/2*a^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*
c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.52

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\cdots}{\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-2\,(bc-ad)x^2}{4\,(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\cdots}{2\cdots}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) +
b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c
- a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/
b)/a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) +
 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^
2*sqrt(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^
4 + a)) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arcta
n(b*x^2*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*
d)*x^2)/(b^2*c*d - a*b*d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(75) = 150$.

Time = 21.42 (sec) , antiderivative size = 932, normalized size of antiderivative = 10.13

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

output
```
-sqrt(-a**3/b**3)*log(x**2 + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a*
*3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/
b**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d -
b*c)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sq
rt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(
4*(a*d - b*c)) + sqrt(-a**3/b**3)*log(x**2 + (a**4*d**4*sqrt(-a**3/b**3)/(
a*d - b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4
*c*d**5*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)
**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3
 + b**4*c**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d +
a*b**2*c**3))/(4*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x**2 + (-a**4*d**4*sq
rt(-c**3/d**3)/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c
)**3 + a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d
**4*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2
)/(a*d - b*c)**3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 +
a**2*b*c**2*d + a*b**2*c**3))/(4*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x**2
+ (a**4*d**4*sqrt(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(
3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3
- a*b**5*c**2*d**4*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c
**3/d**3)**(3/2)/(a*d - b*c)**3 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arct
an(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arct
an(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.63

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\ln\left(b^9\,c^6\,\sqrt{-a^3\,b^3}-a^3\,d^6\,(-a^3\,b^3)^{3/2}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2+2\,b^3\,c^3\,d^3\,(-a^3\,b^3)^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)}{4\,b^4\,c-4\,a\,b^3\,d}$$
$$-\frac{\ln\left(a^3\,d^6\,(-a^3\,b^3)^{3/2}-b^9\,c^6\,\sqrt{-a^3\,b^3}+a\,b^{11}\,c^6\,x^2+a^7\,b^5\,d^6\,x^2-2\,b^3\,c^3\,d^3\,(-a^3\,b^3)^{3/2}-2\,a^4\,b^8\,c^3\,d^3\,x^2\right)}{4\,(b^4\,c-a\,b^3\,d)}$$
$$-\frac{\ln\left(b^6\,c^3\,(-c^3\,d^3)^{3/2}-a^6\,d^9\,\sqrt{-c^3\,d^3}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2-2\,a^3\,b^3\,d^3\,(-c^3\,d^3)^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)}{4\,(a\,d^4-b\,c\,d^3)}$$
$$+\frac{\ln\left(a^6\,d^9\,\sqrt{-c^3\,d^3}-b^6\,c^3\,(-c^3\,d^3)^{3/2}+a^6\,c\,d^{11}\,x^2+b^6\,c^7\,d^5\,x^2+2\,a^3\,b^3\,d^3\,(-c^3\,d^3)^{3/2}-2\,a^3\,b^3\,c^4\,d^8\,x^2\right)}{4\,a\,d^4-4\,b\,c\,d^3}$$
$$+\frac{x^2}{2\,b\,d}$$

input
```
int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2
+ a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2
)*(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2)
- b^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*
d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c
- a*b^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a
^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b
^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c
^3*d^3)^(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^
2 + 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/
2))/(4*a*d^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.00

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a\,d^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a\,d^2-\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b^2c-\sqrt{}}{2b^2d^2\,(ad-bc)}$$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a*d**2 + sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d**2 - sqrt(d)*sqrt(c)*ata
n((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b
**2*c - sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d*
*(1/4)*c**(1/4)*sqrt(2)))*b**2*c + a*b*d**2*x**2 - b**2*c*d*x**2)/(2*b**2*
d**2*(a*d - b*c))
```

3.202 $\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}$$

output
```
-1/2*a^(1/2)*arctan(b^(1/2)*x^2/a^(1/2))/b^(1/2)/(-a*d+b*c)+1/2*c^(1/2)*ar
ctan(d^(1/2)*x^2/c^(1/2))/d^(1/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}}}{2bc-2ad}$$

input
```
Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqr
t[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {965, 383, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{ 383}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{a\int \frac{1}{bx^4+a}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[
c]*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$\frac{a \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2(a d-c b)\sqrt{a b}}-\frac{c \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2(a d-c b)\sqrt{c d}}$
risch	$\frac{\sqrt{-c d}\,\ln\left(\left(-a^{2} d^{4}+a b c\,d^{3}\right)x^{2}+\left(-c d\right)^{\frac{3}{2}} a b d+\left(-c d\right)^{\frac{3}{2}} b^{2} c+a^{2} d^{3}\sqrt{-c d}+\sqrt{-c d}\,b^{2} c^{2} d\right)}{4 d(a d-c b)}-\frac{\sqrt{-c d}\,\ln\left(\left(-a^{2} d^{4}+a b c\,d^{3}\right)x^{2}-\left(-c d\right)^{\frac{3}{2}} a b d$ ... $4d(ad$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*c/(a*d-b*c)/(c*d
)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.11

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\ -\frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\ \frac{2\,\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)}{4\,(bc-ad)},\ -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx^2\sqrt{\frac{c}{d}}}{c}\right)}{2\,(bc-ad)}\right]$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt
(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/
4*(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2
*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*
sqrt(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
))/(b*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c))/(b*c - a*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^
2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^
2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.80

$$\int \frac{x^5}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}+b^4\,c^2\,\sqrt{-a\,b}-b^5\,c^2\,x^2+2\,b^2\,c\,d\,{\left(-a\,b\right)}^{3/2}-a^2\,b^3\,d^2\,x^2+2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,b^2\,c-4\,a\,b\,d} - \frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}+b^4\,c^2\,\sqrt{-a\,b}+b^5\,c^2\,x^2+2\,b^2\,c\,d\,{\left(-a\,b\right)}^{3/2}+a^2\,b^3\,d^2\,x^2-2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,\left(b^2\,c-a\,b\,d\right)} - \frac{\ln\left(b^2\,{\left(-c\,d\right)}^{5/2}+a^2\,d^4\,\sqrt{-c\,d}+a^2\,d^5\,x^2+2\,a\,b\,d^2\,{\left(-c\,d\right)}^{3/2}+b^2\,c^2\,d^3\,x^2-2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,\left(a\,d^2-b\,c\,d\right)} + \frac{\ln\left(b^2\,{\left(-c\,d\right)}^{5/2}+a^2\,d^4\,\sqrt{-c\,d}-a^2\,d^5\,x^2+2\,a\,b\,d^2\,{\left(-c\,d\right)}^{3/2}-b^2\,c^2\,d^3\,x^2+2\,a\,b\,c\,d^4\,x^2\right)\,\sqrt{-c\,d}}{4\,a\,d^2-4\,b\,c\,d}$$

input
```
int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a
*b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*
a*b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^
2*c*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(
b^2*c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x
^2 + 2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1
/2))/(4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) -
a^2*d^5*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*
(-c*d)^(1/2))/(4*a*d^2 - 4*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \frac{x^5}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b+\sqrt{d}\,\sqrt{c}}{2bd\,(ad-bc)}$$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*d - sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + sqrt(d)*sqrt(c)*atan((d**(
1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b + sqrt
(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1
/4)*sqrt(2)))*b)/(2*b*d*(a*d - b*c))
```

3.203 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)}$$

output
```
1/2*b^(1/2)*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/(-a*d+b*c)-1/2*d^(1/2)*arc
tan(d^(1/2)*x^2/c^(1/2))/c^(1/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx=\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

input
```
Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d
]*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {965, 303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{303}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{bx^4+a}dx^2}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

method	result
default	$-\frac{b \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2\left(a d-c b\right)\sqrt{a b}}+\frac{d \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2\left(a d-c b\right)\sqrt{c d}}$
risch	$\frac{\sqrt{-c d}\,\ln\left(\left(-c\,d^{3} a+b\,c^{2} d^{2}\right) x^{2}+\left(-c d\right)^{\frac{3}{2}} a d+\left(-c d\right)^{\frac{3}{2}} b c+2\sqrt{-c d}\,b\,c^{2} d\right)}{4c\left(a d-c b\right)}-\frac{\sqrt{-c d}\,\ln\left(\left(-c\,d^{3} a+b\,c^{2} d^{2}\right) x^{2}-\left(-c d\right)^{\frac{3}{2}} a d-\left(-c d\right)^{\frac{3}{2}} b c-2\right.}{4c\left(a d-c b\right)}$

input
```
int(x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*d/(a*d-b*c)/(c*
d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.91

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)}, -\frac{2\sqrt{\frac{d}{c}}\arctan\left(x^2\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)}{4\,(bc-ad)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\right.}}{dx^4+c}\right.}{4\,(bc-ad)}\right.$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt
(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), -1/
4*(2*sqrt(d/c)*arctan(x^2*sqrt(d/c)) + sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqr
t(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), 1/4*(2*sqrt(b/a)*arctan(x^2*sqrt(b
/a)) - sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c
- a*d), 1/2*(sqrt(b/a)*arctan(x^2*sqrt(b/a)) - sqrt(d/c)*arctan(x^2*sqrt(d
/c)))/(b*c - a*d)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx = \frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc-ad)\sqrt{cd}}$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 399, normalized size of antiderivative = 5.05

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\ln\left(a^2 d^2 (-a b)^{5/2} + b^2 c^2 (-a b)^{5/2} + 2 c d (-a b)^{7/2} - a^2 b^5 c^2 x^2 - a^4 b^3 d^2 x^2 + 2 a^3 b^4 c d x^2\right) \sqrt{-a b}}{4 a^2 d - 4 a b c}$$
$$- \frac{\ln\left(a^2 d^2 (-a b)^{5/2} + b^2 c^2 (-a b)^{5/2} + 2 c d (-a b)^{7/2} + a^2 b^5 c^2 x^2 + a^4 b^3 d^2 x^2 - 2 a^3 b^4 c d x^2\right) \sqrt{-a b}}{4 (a^2 d - a b c)}$$
$$- \frac{\ln\left(a^2 d^2 (-c d)^{5/2} + b^2 c^2 (-c d)^{5/2} + 2 a b (-c d)^{7/2} + a^2 c^2 d^5 x^2 + b^2 c^4 d^3 x^2 - 2 a b c^3 d^4 x^2\right) \sqrt{-c d}}{4 (b c^2 - a c d)}$$
$$+ \frac{\ln\left(a^2 d^2 (-c d)^{5/2} + b^2 c^2 (-c d)^{5/2} + 2 a b (-c d)^{7/2} - a^2 c^2 d^5 x^2 - b^2 c^4 d^3 x^2 + 2 a b c^3 d^4 x^2\right) \sqrt{-c d}}{4 b c^2 - 4 a c d}$$

input
```
int(x/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^
2*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*
d - 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-
a*b)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)
^(1/2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(
5/2) + 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*
d^4*x^2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b
^2*c^2*(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x
^2 + 2*a*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.94

$$\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\, atan\left(\frac{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{b}\,x}{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) c + \sqrt{b}\,\sqrt{a}\, atan\left(\frac{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} + 2\sqrt{b}\,x}{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) c - \sqrt{d}\,\sqrt{c}\, atan\left(\frac{d^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{2} - 2\sqrt{d}\,x}{d^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{2}}\right) a - \sqrt{d}\,\sqrt{c}\, a}{2ac\,(ad - bc)}$$

input
```
int(x/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*c + sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - sqrt(d)*sqrt(c)*atan((d**(1/4
)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a - sqrt(d)
*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)
*sqrt(2)))*a)/(2*a*c*(a*d - b*c))
```

3.204 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx=-\frac{1}{2acx^2}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

output
```
-1/2/a/c/x^2-1/2*b^(3/2)*arctan(b^(1/2)*x^2/a^(1/2))/a^(3/2)/(-a*d+b*c)+1/
2*d^(3/2)*arctan(d^(1/2)*x^2/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$$
$$=\frac{\frac{b}{a}-\frac{d}{c}-\frac{b^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}-\frac{b^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}}+\frac{d^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right)}{c^{3/2}}}{2(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
 - (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2
)*x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcT
an[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {965, 382, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^4+a) (dx^4+c)} dx^2$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{2} \left(\frac{\int -\frac{bdx^4+bc+ad}{(bx^4+a)(dx^4+c)} dx^2}{ac} - \frac{1}{acx^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^4+bc+ad}{(bx^4+a)(dx^4+c)} dx^2}{ac} - \frac{1}{acx^2} \right)$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2} \left(-\frac{\frac{b^2 c \int \frac{1}{bx^4+a} dx^2}{bc-ad} - \frac{ad^2 \int \frac{1}{dx^4+c} dx^2}{bc-ad}}{ac} - \frac{1}{acx^2} \right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{ad^{3/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{1}{acx^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(1/(a*c*x^2)) - ((b^(3/2)*c*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c
 - a*d)) - (a*d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
)/(a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

method	result
default	$\frac{b^2 \arctan\left(\frac{b x^2}{\sqrt{ab}}\right)}{2a(ad-cb)\sqrt{ab}}-\frac{1}{2ac\,x^2}-\frac{d^2 \arctan\left(\frac{d x^2}{\sqrt{cd}}\right)}{2c(ad-cb)\sqrt{cd}}$
risch	$-\frac{1}{2ac\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^2a^5-2a^4bcd+c^2a^3b^2\right)\_Z^2+b^3\right)}\_R\ln\left(\left(\left(-5c^3a^7d^4+18c^4a^6b\,d^3-26a^5c^5b^2d^2+18c^6a^4b^3d-5c^7a^3b^4\right)\right.\right.\right.}{}$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2/a/c/x^2-1/2*
d^2/c/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.52

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2\,bc-2\,ad}{4\,(abc^2-a^2cd)x^2}, \frac{2\,adx^2\sqrt{\frac{d}{c}}\arctan\right.$$

$$-\frac{2\,bcx^2\sqrt{\frac{b}{a}}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)+adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2\,bc-2\,ad}{4\,(abc^2-a^2cd)x^2},$$

$$\left.-\frac{bcx^2\sqrt{\frac{b}{a}}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)-adx^2\sqrt{\frac{d}{c}}\arctan\left(x^2\sqrt{\frac{d}{c}}\right)+bc-ad}{2\,(abc^2-a^2cd)x^2}\right]$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)
) + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) +
 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*a*d*x^2*sqrt(d/c)*arctan
(x^2*sqrt(d/c)) - b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/
(b*x^4 + a)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*b*c*x^2*s
qrt(b/a)*arctan(x^2*sqrt(b/a)) + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*s
qrt(-d/c) - c)/(d*x^4 + c)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1
/2*(b*c*x^2*sqrt(b/a)*arctan(x^2*sqrt(b/a)) - a*d*x^2*sqrt(d/c)*arctan(x^2
*sqrt(d/c)) + b*c - a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. 2(76) = 152.

Time = 72.16 (sec) , antiderivative size = 1103, normalized size of antiderivative = 11.99

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

output
```
-sqrt(-b**3/a**3)*log(x**2 + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b
*c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b*
*2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**
3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a
**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a*
*3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*
d - b*c)) + sqrt(-b**3/a**3)*log(x**2 + (a**7*c**3*d**4*(-b**3/a**3)**(3/2
)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 +
 2*a**5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt
(-b**3/a**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b
*c)**3 + a**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqr
t(-b**3/a**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**
2))/(4*(a*d - b*c)) - sqrt(-d**3/c**3)*log(x**2 + (-a**7*c**3*d**4*(-d**3/
c**3)**(3/2)/(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d
- b*c)**3 - 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**
5*d**5*sqrt(-d**3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/
2)/(a*d - b*c)**3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b*
*5*c**5*sqrt(-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b*
*4*c**2*d**2))/(4*(a*d - b*c)) + sqrt(-d**3/c**3)*log(x**2 + (a**7*c**3*d*
*4*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a + bx^4) (c + dx^4)} dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (abc - a^2 d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^2 - acd)\sqrt{cd}} - \frac{1}{2 acx^2}$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arc
tan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a + bx^4) (c + dx^4)} dx = -\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (abc - a^2 d)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^2 - acd)\sqrt{cd}} - \frac{1}{2 acx^2}$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arc
tan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.85

$$\int \frac{1}{x^3 (a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\ln\left(c^3 x^2 (-a^3 b^3)^{3/2} - a^8 b d^3 + a^5 b^4 c^3 + a^6 d^3 x^2 \sqrt{-a^3 b^3}\right)\sqrt{-a^3 b^3}}{4 a^4 d - 4 a^3 b c} - \frac{\ln\left(c^3 x^2 (-a^3 b^3)^{3/2} + a^8 b d^3 - a^5 b^4 c^3 + a^6 d^3 x^2 \sqrt{-a^3 b^3}\right)\sqrt{-a^3 b^3}}{4 (a^4 d - a^3 b c)} - \frac{1}{2 a c x^2} - \frac{\ln\left(a^3 x^2 (-c^3 d^3)^{3/2} + b^3 c^8 d - a^3 c^5 d^4 + b^3 c^6 x^2 \sqrt{-c^3 d^3}\right)\sqrt{-c^3 d^3}}{4 (b c^4 - a c^3 d)} + \frac{\ln\left(a^3 x^2 (-c^3 d^3)^{3/2} - b^3 c^8 d + a^3 c^5 d^4 + b^3 c^6 x^2 \sqrt{-c^3 d^3}\right)\sqrt{-c^3 d^3}}{4 b c^4 - 4 a c^3 d}$$

input
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^
3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3
*b^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^
3*b^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d
^3)^(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*
d^3)^(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^
8*d + a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c
^4 - 4*a*c^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^3 (a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{b}\,x}{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) b c^2 x^2 - \sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} + 2\sqrt{b}\,x}{b^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) b c^2 x^2 + \sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{2} - 2\sqrt{d}\,x}{d^{\frac{1}{4}} c^{\frac{1}{4}} \sqrt{2}}\right) a^2}{2a^2 c^2 x^2 (ad - bc)}$$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/
4)*a**(1/4)*sqrt(2)))*b*c**2*x**2 - sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4
)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c**2*x**2 + sqrt(d
)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4
)*sqrt(2)))*a**2*d*x**2 + sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2)
+ 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d*x**2 - a**2*c*d + a*b*c
**2)/(2*a**2*c**2*x**2*(a*d - b*c))
```

3.205 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

output
```
-1/6/a/c/x^6+1/2*(a*d+b*c)/a^2/c^2/x^2+1/2*b^(5/2)*arctan(b^(1/2)*x^2/a^(1
/2))/a^(5/2)/(-a*d+b*c)-1/2*d^(5/2)*arctan(d^(1/2)*x^2/c^(1/2))/c^(5/2)/(-
a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$=\frac{\frac{b}{a}-\frac{d}{c}-\frac{3b^2x^4}{a^2}+\frac{3d^2x^4}{c^2}+\frac{3b^{5/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/2}}-\frac{3d^{5/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/2}}-\frac{3d}{}}{6(-bc+ad)x^6}$$

input
```
Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2
]*b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4
)*x)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {965, 382, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^4+a) (dx^4+c)}dx^2$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{2} \left(\frac{\int -\frac{3(bdx^4+bc+ad)}{x^4(bx^4+a)(dx^4+c)}dx^2}{3ac} - \frac{1}{3acx^6} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^4+bc+ad}{x^4(bx^4+a)(dx^4+c)}dx^2}{ac} - \frac{1}{3acx^6} \right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2} \left(-\frac{-\frac{\int \frac{bd(bc+ad)x^4+b^2c^2+a^2d^2+abcd}{(bx^4+a)(dx^4+c)}dx^2}{ac} - \frac{ad+bc}{acx^2}}{ac} - \frac{1}{3acx^6} \right)$$

$$\Big\downarrow \; 397$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{b^3c^2\int\frac{1}{bx^4+a}dx^2}{bc-ad}-\frac{a^2d^3\int\frac{1}{dx^4+c}dx^2}{bc-ad}}{ac}-\frac{ad+bc}{acx^2}}{ac}-\frac{1}{3acx^6}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(-\frac{-\frac{\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{a^2d^{5/2}\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{ad+bc}{acx^2}}{ac}-\frac{1}{3acx^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-1/3*1/(a*c*x^6) - (-((b*c + a*d)/(a*c*x^2)) - ((b^(5/2)*c^2*ArcTan[(Sqrt
[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a^2*d^(5/2)*ArcTan[(Sqrt[d]*x^
2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

method	result
default	$-\frac{b^{3} \arctan\left(\frac{b\,x^{2}}{\sqrt{a b}}\right)}{2 a^{2} \left(a d -c b \right) \sqrt{a b}}-\frac{1}{6 a c\,x^{6}}-\frac{-a d -c b}{2 a^{2} c^{2} x^{2}}+\frac{d^{3} \arctan\left(\frac{d\,x^{2}}{\sqrt{c d}}\right)}{2 c^{2} \left(a d -c b \right) \sqrt{c d}}$
risch	$\frac{\frac{\left(a d +c b \right) x^{4}}{2 a^{2} c^{2}}-\frac{1}{6 a c}}{x^{6}}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(d^{2} c^{5} a^{2}-2 c^{6} d a b +b^{2} c^{7}\right) \_Z^{2}+d^{5}\right)}\_R \ln\left(\left(\left(\left(5 c^{5} a^{9} d^{4}-18 c^{6} a^{8} b\,d^{3}+26 c^{7} a^{7} b^{2} d^{2}-18 c^{8} a^{6} b^{3} d +5 c^{9}\right.\right.\right.\right.}{}$

input
```
int(1/x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/6/a/c/x^6-1
/2/a^2/c^2*(-a*d-b*c)/x^2+1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(
c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 576, normalized size of antiderivative = 5.14

$$\int \frac{1}{x^7 (a + bx^4)(c + dx^4)} dx$$
$$= \left[-\frac{3 b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + 3a^2d^2x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) - 6(b^2c^2-a^2d^2)x^4 + 2a}{12(a^2bc^3-a^3c^2d)x^6} \right.$$
$$- \frac{6a^2d^2x^6\sqrt{\frac{d}{c}}\arctan\left(x^2\sqrt{\frac{d}{c}}\right) + 3b^2c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) - 6(b^2c^2-a^2d^2)x^4 + 2abc^2 - 2a}{12(a^2bc^3-a^3c^2d)x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x
^4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(
d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c
^3 - a^3*c^2*d)*x^6), -1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(x^2*sqrt(d/c))
 + 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3
*c^2*d)*x^6), 1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(x^2*sqrt(b/a)) - 3*a^2*
d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 6*(
b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x
^6), 1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(x^2*sqrt(b/a)) - 3*a^2*d^2*x^6*sq
rt(d/c)*arctan(x^2*sqrt(d/c)) + 3*(b^2*c^2 - a^2*d^2)*x^4 - a*b*c^2 + a^2*
c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} \, dx = \frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^3 - ac^2d)\sqrt{cd}} + \frac{3 (bc + ad)x^4 - ac}{6 a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x
^4 - a*c)/(a^2*c^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (a + bx^4) (c + dx^4)} \, dx = \frac{b^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 (a^2bc - a^3d)\sqrt{ab}} - \frac{d^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 (bc^3 - ac^2d)\sqrt{cd}} + \frac{3 bcx^4 + 3 adx^4 - ac}{6 a^2c^2x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a
*d*x^4 - a*c)/(a^2*c^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.78

$$\int \frac{1}{x^7 (a+bx^4)(c+dx^4)} dx$$
$$= \frac{\ln \left(c^{10} (-a^5 b^5)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} - a^{12} b^{13} c^{10} x^2 - a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 (-a^5 b^5)^{3/2} + 2 a^{17} b^8 c^5 d\right.}{4 a^6 d - 4 a^5 b c}$$
$$- \frac{\ln \left(c^{10} (-a^5 b^5)^{5/2} + a^{20} d^{10} \sqrt{-a^5 b^5} + a^{12} b^{13} c^{10} x^2 + a^{22} b^3 d^{10} x^2 + 2 a^{10} c^5 d^5 (-a^5 b^5)^{3/2} - 2 a^{17} b^8 c\right.}{4 (a^6 d - a^5 b c)}$$
$$- \frac{\frac{1}{6 a c} - \frac{x^4 (a d + b c)}{2 a^2 c^2}}{x^6}$$
$$- \frac{\ln \left(a^{10} (-c^5 d^5)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} + a^{10} c^{12} d^{13} x^2 + b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} (-c^5 d^5)^{3/2} - 2 a^5 b^5 c^{17}\right.}{4 (b c^6 - a c^5 d)}$$
$$+ \frac{\ln \left(a^{10} (-c^5 d^5)^{5/2} + b^{10} c^{20} \sqrt{-c^5 d^5} - a^{10} c^{12} d^{13} x^2 - b^{10} c^{22} d^3 x^2 + 2 a^5 b^5 c^{10} (-c^5 d^5)^{3/2} + 2 a^5 b^5 c^{17}\right.}{4 b c^6 - 4 a c^5 d}$$

input
```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x
^2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*
d^5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5
/2) + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2
+ 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/
2))/(4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^
6 - (log(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^
13*x^2 + b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c
^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5
)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*
x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)
^(1/2))/(4*b*c^6 - 4*a*c^5*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)b^2c^3x^6+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)b^2c^3x^6-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)}{6a^3c^3x^6(ad-bc)}$$

input
```
int(1/x^7/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4
)*a**(1/4)*sqrt(2)))*b**2*c**3*x**6 + 3*sqrt(b)*sqrt(a)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c**3*x**6 -
 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4
)*c**(1/4)*sqrt(2)))*a**3*d**2*x**6 - 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**
(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*d**2*x**6 -
 a**3*c**2*d + 3*a**3*c*d**2*x**4 + a**2*b*c**3 - 3*a*b**2*c**3*x**4)/(6*a
**3*c**3*x**6*(a*d - b*c))
```

3.206 $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 335

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{x}{bd} - \frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} + \frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} - \frac{c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} + \frac{a^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} - \frac{c^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}d^{5/4}(bc-ad)}$$

output
```
x/b/d+1/4*a^(5/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(5/4)/(-a
*d+b*c)+1/4*a^(5/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(5/4)/(-
a*d+b*c)-1/4*c^(5/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(5/4)/
(-a*d+b*c)-1/4*c^(5/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(5/4)
/(-a*d+b*c)+1/4*a^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)
*x^2))*2^(1/2)/b^(5/4)/(-a*d+b*c)-1/4*c^(5/4)*arctanh(2^(1/2)*c^(1/4)*d^(1
/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(5/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.13

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-\frac{8ax}{b}+\frac{8cx}{d}-\frac{2\sqrt{2}a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}}+\frac{2\sqrt{2}c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{d^{5/4}}-\frac{2\sqrt{2}c^{5/4}\text{arcta}}{d}}{}$$

input
```
Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x
)/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/
d^(5/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {979, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$
$$\Big\downarrow \ 979$$
$$\frac{x}{bd}-\frac{\int \frac{(bc+ad)x^4+ac}{(bx^4+a)(dx^4+c)}dx}{bd}$$

$\Big\downarrow$ 1020

$$\frac{x}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^4+c}dx}{bc-ad}-\frac{a^2d\int \frac{1}{bx^4+a}dx}{bc-ad}}{bd}$$

$\Big\downarrow$ 755

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

$\Big\downarrow$ 1476

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

$\Big\downarrow$ 1082

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\cdots\right)}{\cdots}}{bd}$$

$\Big\downarrow$ 217

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

$\Big\downarrow$ 1479

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\left(\cdots\right)}}{\cdots}}{\cdots}\cdots\right.}{\cdots}}{bd}\cdots$$

$\Big\downarrow$ 25

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-\cdots}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\cdots}{\cdots}\cdots\right)}}{2\sqrt{2}\sqrt[4]{\cdots}}\cdots}{\cdots}\cdots\right.}{\cdots}}{bd}\cdots$$

$\Big\downarrow$ 27

$$\frac{x}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\cdots}{x^2+\cdots}}{\cdots}}{2\sqrt{a}}\cdots\right.}{\cdots}}{bd}\cdots$$

$\Big\downarrow$ 1103

$$\frac{x}{bd} - \frac{\frac{bc^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad} - a^2d\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{bd}$$

input
```
Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
x/(b*d) - (-((a^2*d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4
)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*
d)) + (b*c^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*
d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/
4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*
d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

method	result
default	$\frac{x}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b(ad-cb)}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}}\right.\right.}{}$
risch	$\frac{x}{bd}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(d^5a^4-4a^3bc\,d^4+6a^2b^2c^2d^3-4a\,b^3c^3d^2+b^4c^4d\right)\_Z^4+b^4c^5\right)}\_R\ln\left(\left(\left(-a^5b\,d^6+3a^4b^2c\,d^5-2a^3b^3c^2d^4-2a^2b^4c^3d^3\right.\right.\right.}{4db}$

input
```
int(x^8/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d-1/8/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)
+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a
/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/d*c/(a*d-b*c)*(c/d)^
(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)
*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2
)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 1196, normalized size of antiderivative = 3.57

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*((-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3
+ a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - (-
a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^
2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) - I*(-a^5/(
b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^
4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(I*b^2*c - I*a*b*d)) + I*(-a^5/(b
^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2
- 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(-I*b^2*c + I*a*b*d)) - (-c^5/(b^4
*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))
^(1/4)*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*
d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + (-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*
b*d*log(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) + I*(-c^5/(b^4*c^4*d^5 - 4
*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*l
og(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.12

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{\right.}{}}{8\left(b^2c-abd\right)}$$

$$- \frac{\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}} - \frac{\sqrt{2}c^{\frac{5}{4}}\log(}{}}{8\left(bcd-ad^2\right)}$$

$$+ \frac{x}{bd}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(
2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt
(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sq
rt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(
sqrt(c)*sqrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/
4)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.40

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} - \frac{(cd^3)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{(ab^3)^{\frac{1}{4}}a\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}a\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}c\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{(cd^3)^{\frac{1}{4}}c\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{x}{bd}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a
*b^2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4
))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2
 - sqrt(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) -
1/4*(c*d^3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c*d^2 - sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 6361, normalized size of antiderivative = 18.99

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)} dx = \text{Too large to display}$$

input
```
int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

output

```
atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^
(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 5
12*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2
*d^2 - 1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))
*1i - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^
(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 5
12*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2
*d^2 - 1024*a*b^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))
*1i)/((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
 a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^2-2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d^2-2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b^2}{}$$

input

```
int(x^8/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d**2 - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*d*
*2 - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**2*c + 2*d**(3/4)*c**(1/4)*sqrt(2)*at
an((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*
b**2*c + b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqrt(2)*x + sq
rt(a) + sqrt(b)*x**2)*a*d**2 - b**(3/4)*a**(1/4)*sqrt(2)*log(b**(1/4)*a**(
1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*a*d**2 - d**(3/4)*c**(1/4)*sqrt(2
)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b**2*c + d*
*(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d
)*x**2)*b**2*c + 8*a*b*d**2*x - 8*b**2*c*d*x)/(8*b**2*d**2*(a*d - b*c))
```

3.207 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{3/4}(bc-ad)} + \frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}d^{3/4}(bc-ad)}$$

output
```
-1/4*a^(3/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(3/4)/(-a*d+b*
c)-1/4*a^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(3/4)/(-a*d+b
*c)+1/4*c^(3/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(3/4)/(-a*d
+b*c)+1/4*c^(3/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(3/4)/(-a*
d+b*c)+1/4*a^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2)
)*2^(1/2)/b^(3/4)/(-a*d+b*c)-1/4*c^(3/4)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x
/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(3/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \frac{2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2a^{3/4}d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2b}{}$$

input
```
Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \text{ 981}$$

$$\frac{c\int \frac{x^2}{dx^4+c}dx}{bc-ad}-\frac{a\int \frac{x^2}{bx^4+a}dx}{bc-ad}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$\Big\downarrow$ 1476

$$\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$\Big\downarrow$ 1082

$$\frac{c\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$\Big\downarrow$ 217

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1479$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$\downarrow$ 27

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

$\downarrow$ 1103

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-$$

$$\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}$$

input
```
Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))
) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*
Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(S
qrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(b*c - a*d)) + (c*((-(Ar
cTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) -
(-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1
/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*S
qrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

method	result
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.}{8(ad-cb)d}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^4d^7-4a^3bc\,d^6+6a^2b^2c^2d^5-4a\,b^3c^3d^4+b^4c^4d^3\right)\_Z^4+c^3\right)}\_R\ln\left(\left(\left(2a^4b^3d^7-8a^3b^4c\,d^6+12a^2b^5c^2d^5-8a\,b^6c^3d^4+2b^7\right.\right.\right.}{4}$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*
2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d
)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1427, normalized size of antiderivative = 4.36

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
+ a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d
^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a
^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d +
 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (b^
5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*
a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) -
 1/4*I*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^
3 + a^4*b^3*d^4))^(1/4)*log(a^2*x - (I*b^5*c^3 - 3*I*a*b^4*c^2*d + 3*I*a^2
*b^3*c*d^2 - I*a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2
*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*I*(-a^3/(b^7*c^4 - 4*a
*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log
(a^2*x - (-I*b^5*c^3 + 3*I*a*b^4*c^2*d - 3*I*a^2*b^3*c*d^2 + I*a^3*b^2*d^3
)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a
^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2
*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*a*
b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^
4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*
c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^
(1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.11

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = -\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\right.}{a}\right.}{8(bc-ad)} + \frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2-\right.}{c}\right.}{8(bc-ad)}$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^
(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*
d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.39

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^
3*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c
/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt
(2)*a*d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b)
)/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)
^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqr
t(2)*a*d^4) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d
))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 2553, normalized size of antiderivative = 7.81

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*
c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-
a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2
*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 +
256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c
^3*d))^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1
024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192
*a*b^7*c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3
 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*
(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c
^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7
*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*
a*b^6*c^3*d))^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4
 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^
6*c^3*d))^(1/4) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1
024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a
^3*b*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 153
6*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256
*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\sqrt{2}\left(-2b^{\frac{1}{4}}a^{\frac{3}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2b^{\frac{1}{4}}a^{\frac{3}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2d^{\frac{1}{4}}c^{\frac{3}{4}}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b-2d^{\frac{1}{4}}c\cdots}{}$$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(2)*( - 2*b**(1/4)*a**(3/4)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 2*b**(1/4)*a**(3/4)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 2*d**(1/4
)*c**(3/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/
4)*sqrt(2)))*b - 2*d**(1/4)*c**(3/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*s
qrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b + b**(1/4)*a**(3/4)*log( - b**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*d - b**(1/4)*a**(3/4)*log(
b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*d - d**(1/4)*c**(3/4
)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b + d**(1/4
)*c**(3/4)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b))/(
8*b*d*(a*d - b*c))
```

3.208 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\begin{aligned}\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = {} & \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)} \\ & -\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)} \\ & -\frac{\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}+\frac{\sqrt[4]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}\end{aligned}$$

output
```
-1/4*a^(1/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(1/4)/(-a*d+b*
c)-1/4*a^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(1/4)/(-a*d+b
*c)+1/4*c^(1/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(1/4)/(-a*d
+b*c)+1/4*c^(1/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(1/4)/(-a*
d+b*c)-1/4*a^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2)
)*2^(1/2)/b^(1/4)/(-a*d+b*c)+1/4*c^(1/4)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x
/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(1/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt[4]{b}\sqrt[4]{\ }}{}$$

input
```
Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1
- (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {981, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \ 981$$

$$\frac{c\int \frac{1}{dx^4+c}dx}{bc-ad}-\frac{a\int \frac{1}{bx^4+a}dx}{bc-ad}$$

$$\Big\downarrow \ 755$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$\Big\downarrow$ 1479

$$\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$\Big\downarrow$ 25

$$\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$\downarrow$ 27

$$\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

$\downarrow$ 1103

$$\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}$$

input
```
Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))
) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*
Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(S
qrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d)) + (c*((-(Ar
cTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) +
(-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1
/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*S
qrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.67

method	result
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8ad-8cb}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+}{8}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{4}b\,d^{4}-4c\,d^{3}a^{3}b^{2}+6c^{2}d^{2}a^{2}b^{3}-4c^{3}da\,b^{4}+b^{5}c^{4}\right)\_Z^{4}+a\right)}\_R\ln\left(\left(\left(-a^{5}b\,d^{6}+3a^{4}b^{2}c\,d^{5}-2a^{3}b^{3}c^{2}d^{4}-2a^{2}b^{4}c^{3}d^{3}+3a\,b^{5}\right.\right.\right.\right.}{}$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*
(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/
d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1067, normalized size of antiderivative = 3.26

$$\int \frac{x^4}{(a+bx^4)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log
(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(I*b*c - I*a*d)*
(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4))^(1/4) + x) - 1/4*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(-I*b*c + I*a*d)*(-a/(b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)
+ x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*
c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-c/(b^4*c
^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/
4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(I*b*c - I
*a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4
 + a^4*d^5))^(1/4) + x) + 1/4*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log}{}}{8(bc-ad)}$$

$$+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log}{}}{8(bc-ad)}$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*
b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sq
rt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*
sqrt(c)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sq
rt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(
2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqr
t(c)*sqrt(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*
x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/d^(1/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.34

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} - \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d)
 + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^
2) - 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x
^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) -
1/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*
c*d - sqrt(2)*a*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 5889, normalized size of antiderivative = 18.01

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 25
6*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d
) - (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3
 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*
b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024
*a*b^4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 10
24*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3
*d^3*x*512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*
b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 +
256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3
*d))/((-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3
*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d -
 6144*a*b^6*c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a
^3*b^4*c^3*d^4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 10
24*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 -
4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*
d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)
*2i - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5
+ 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c
*d^4) - (a^5*b*c*d^6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{2}\left(-2b^{\frac{3}{4}}a^{\frac{1}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2b^{\frac{3}{4}}a^{\frac{1}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)d+2d^{\frac{3}{4}}c^{\frac{1}{4}}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b-2d^{\frac{3}{4}}c\cdots\right)}{\cdots}$$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c),x)
```

output

```
(sqrt(2)*( - 2*b**(3/4)*a**(1/4)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 2*b**(3/4)*a**(1/4)*atan((b**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*d + 2*d**(3/4
)*c**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/
4)*sqrt(2)))*b - 2*d**(3/4)*c**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*s
qrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b - b**(3/4)*a**(1/4)*log( - b**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*d + b**(3/4)*a**(1/4)*log(
b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*d + d**(3/4)*c**(1/4
)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b - d**(3/4
)*c**(1/4)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*b))/(
8*b*d*(a*d - b*c))
```

3.209 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

output
```
1/4*b^(1/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/(-a*d+b*c
)+1/4*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/(-a*d+b*
c)-1/4*d^(1/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(1/4)/(-a*d+
b*c)-1/4*d^(1/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(1/4)/(-a*d
+b*c)-1/4*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))
*2^(1/2)/a^(1/4)/(-a*d+b*c)+1/4*d^(1/4)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/
(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(1/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{-2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{b}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt[4]{a}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\sqrt[4]{a}$$

input
```
Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^
(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {982, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \text{982}$$

$$\frac{b\int \frac{x^2}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{x^2}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{826}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{b\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{217}$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1479}$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$\downarrow$ 27

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

$\downarrow$ 1103

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}$$

input
```
Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.67

method	result
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\right.\right.}{8(ad-cb)}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^4a^5-4c\,d^3a^4b+6c^2d^2a^3b^2-4c^3d\,a^2b^3+a\,b^4c^4\right)\_Z^4+b\right)}\_R\ln\left(\left(\left(a^6d^6-4a^5bc\,d^5+7a^4b^2c^2d^4-8a^3b^3c^3d^3+7a^2b^4c^4d^2\right.\right.\right.\right.}{4}$

input
```
int(x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1
/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)
*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)
*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1331, normalized size of antiderivative = 4.07

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(1/4)*log(b*x + (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*
d^3 + a^5*d^4))^(3/4)) - 1/4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*
c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2
*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3
*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) + 1/4*I*(-b/(a*b^4*c^4 - 4
*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b
*x - (I*a*b^3*c^3 - 3*I*a^2*b^2*c^2*d + 3*I*a^3*b*c*d^2 - I*a^4*d^3)*(-b/(
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)
)^(3/4)) - 1/4*I*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*
a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x - (-I*a*b^3*c^3 + 3*I*a^2*b^2*c^2*d
- 3*I*a^3*b*c*d^2 + I*a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^
2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*
c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x +
(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a
*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/
4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4
*c*d^4))^(1/4)*log(d*x - (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*
c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}\right.}{a^{\frac{1}{4}}b}\right.}{8(bc-ad)}$$
$$-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x^2\right.}{c}\right.}{8(bc-ad)}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(
1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sq
rt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*
(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt
(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(233) = 466$.

Time = 0.14 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)}$$

input

```
integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)
*a^2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4
)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c
^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1
/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)
*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a
^2*b^2*d) + 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 6633, normalized size of antiderivative = 20.28

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^
4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/
4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2
*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*
d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
+ 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a
^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3
*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^
6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2
*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^
6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^
4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*
d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 7
68*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b
^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*
a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^
2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 153
6*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{2}\left(2b^{\frac{1}{4}}a^{\frac{3}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2b^{\frac{1}{4}}a^{\frac{3}{4}}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2d^{\frac{1}{4}}c^{\frac{3}{4}}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a+2d^{\frac{1}{4}}c^{\frac{3}{4}}\cdots}{\cdots}$$

input
```
int(x^2/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(2)*(2*b**(1/4)*a**(3/4)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*b**(1/4)*a**(3/4)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*d**(1/4)*c
**(3/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*
sqrt(2)))*a + 2*d**(1/4)*c**(3/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt
(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a - b**(1/4)*a**(3/4)*log( - b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c + b**(1/4)*a**(3/4)*log(b**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c + d**(1/4)*c**(3/4)*l
og( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a - d**(1/4)*c
**(3/4)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a))/(8*a
*c*(a*d - b*c))
```

3.210 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 327

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = -\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}+\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}-\frac{d^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

output
```
1/4*b^(3/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*c
)+1/4*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/(-a*d+b*
c)-1/4*d^(3/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d+
b*c)-1/4*d^(3/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(3/4)/(-a*d
+b*c)+1/4*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))
*2^(1/2)/a^(3/4)/(-a*d+b*c)-1/4*d^(3/4)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/
(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(3/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{-2b^{3/4}c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2a^{3/4}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2\cdots}{\cdots}$$

input
```
Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^
(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
 + Sqrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {917, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \; 917$$

$$\frac{b\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \; 755$$

$$\frac{b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{b\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1479$$

$$\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

input
```
Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.69

method	result
default	$-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)a}+\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^{4}a^{7}-4c\,d^{3}a^{6}b+6c^{2}d^{2}a^{5}b^{2}-4c^{3}d\,a^{4}b^{3}+a^{3}b^{4}c^{4}\right)\_Z^{4}+b^{3}\right)}\_R\ln\left(\left(\left(-a^{7}d^{7}+4c\,a^{6}b\,d^{6}-6c^{2}a^{5}b^{2}d^{5}+3a^{4}b^{3}c^{3}d^{4}+3a^{3}b^{4}\right.\right.\right.\right.$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/(a*d-b*c)*(c/d)^(1/4)/c
*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/
d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 1171, normalized size of antiderivative = 3.58

$$\int \frac{1}{(a+bx^4)\,(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*
b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b
^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^
7*d^4))^(1/4)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*I*(-b^3/(a
^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4
))^(1/4)*log(b*x - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) + 1/4*I*(-b^3/(a^
3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)
)^(1/4)*log(b*x - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*
d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*
c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^
(1/4)*log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2
*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*l
og(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^
2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*I*(-d^3/(b^4*c^7 - 4*a*b^
3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*
x - (I*b*c^2 - I*a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\cdots\right.}{a\cdots}}{8(bc-ad)}$$
$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{\cdots}\right.}{\cdots}}{8(bc-ad)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*s
qrt(2)*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sq
rt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.34

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d)
- 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d
) + 1/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2
)*a*b*c - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^
2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1
/4*(c*d^3)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
^2 - sqrt(2)*a*c*d)
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 6153, normalized size of antiderivative = 18.82

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*
a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c
^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3
/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*
b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4
*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2
*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 -
4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a
^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6
+ 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 10
24*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1
024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3
/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^
2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 -
20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 2
0480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x
*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{\sqrt{2}\left(2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2b^{\frac{3}{4}}a^{\frac{1}{4}}atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c-2d^{\frac{3}{4}}c^{\frac{1}{4}}atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a+2d^{\frac{3}{4}}c^{\frac{1}{4}}\cdots}{\cdots}$$

input
```
int(1/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(2)*(2*b**(3/4)*a**(1/4)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*b**(3/4)*a**(1/4)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*c - 2*d**(3/4)*c
**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*
sqrt(2)))*a + 2*d**(3/4)*c**(1/4)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt
(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a + b**(3/4)*a**(1/4)*log( - b**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c - b**(3/4)*a**(1/4)*log(b**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*c - d**(3/4)*c**(1/4)*l
og( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a + d**(3/4)*c
**(1/4)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a))/(8*a
*c*(a*d - b*c))
```

3.211 $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 338

$$\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{acx} + \frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} + \frac{d^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} + \frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} - \frac{d^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}$$

output
```
-1/a/c/x-1/4*b^(5/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/
(-a*d+b*c)-1/4*b^(5/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)
/(-a*d+b*c)+1/4*d^(5/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(5/
4)/(-a*d+b*c)+1/4*d^(5/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(5
/4)/(-a*d+b*c)+1/4*b^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1
/2)*x^2))*2^(1/2)/a^(5/4)/(-a*d+b*c)-1/4*d^(5/4)*arctanh(2^(1/2)*c^(1/4)*d
^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(5/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (a + bx^4) (c + dx^4)} \, dx$$
$$= \frac{\frac{8b}{a} - \frac{8d}{c} - \frac{2\sqrt{2}b^{5/4}x \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}b^{5/4}x \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}d^{5/4}x \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}} - \frac{2\sqrt{2}d^{5/4}x \operatorname{arcta}}{c^{5}}}{}$$

input
```
Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(
1/4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/
4)])/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2
]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*
x)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + bx^4) (c + dx^4)} \, dx$$
$$\Big\downarrow \; 980$$

$$\frac{\int -\frac{x^2(bdx^4+bc+ad)}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{x^2(bdx^4+bc+ad)}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \; 1054$$

$$-\frac{\int \left(\frac{b^2cx^2}{(bc-ad)(bx^4+a)}+\frac{ad^2x^2}{(ad-bc)(dx^4+c)}\right)dx}{ac}-\frac{1}{acx}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{ad^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{b^{5/4}c\log(}{}}{}$$

$$\frac{1}{acx}$$

input
```
Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-(1/(a*c*x)) - (-1/2*(b^(5/4)*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(5/4)*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a*d^(5/4)*ArcTan[1 - (Sqrt[2]*
d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a*d^(5/4)*ArcTan[1
 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(5/4
)*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(
1/4)*(b*c - a*d)) - (b^(5/4)*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) +
(a*d^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt
[2]*c^(1/4)*(b*c - a*d)))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.70

method	result
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a(ad-cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{1}{acx}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\right.}{8c}$
risch	Expression too large to display

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)
^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/a/c/x-1/8*d/c/(a*d-b*c)/(c/d
)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/
4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1
/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1461, normalized size of antiderivative = 4.32

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d +
3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^
3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^
4*x - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5
*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(3/4)) + I*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a
^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (I*a^4*b^3*c^3 - 3*I*a^5*b^
2*c^2*d + 3*I*a^6*b*c*d^2 - I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - I*(-b^5/(a^5*b^
4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1
/4)*a*c*x*log(b^4*x - (-I*a^4*b^3*c^3 + 3*I*a^5*b^2*c^2*d - 3*I*a^6*b*c*d^
2 + I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 -
4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*
b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x + (b^3
*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a
*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) +
(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4
*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2\right.}{}\right.}{8\left(abc-a^2d\right)}+\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{d}x\right.}{}\right.}{8\left(bc^2-acd\right)}-\frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*
a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) +
 1/8*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1
/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 -
 sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) -
 1/(a*c*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx = -\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt
(2)*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*
c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4
)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2
)*a^3*b*d) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d)
)/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(
a*c*x)
```

Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 5962, normalized size of antiderivative = 17.64

$$\int \frac{1}{x^2 (a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a
^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12
*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3
 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 +
 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*
c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^
14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*
a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 10
24*a^20*b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^
5 - 768*a^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9
 - 768*a^17*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^1
2)*1i) + (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*
a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^1
2*b^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^
3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9
+ 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3
*c^8*d))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a
^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096
*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1
024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^1...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2 (a+bx^4) (c+dx^4)} dx$$
$$= \frac{-2b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x + 2b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x + 2d^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{2}\, atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right.}{}$$

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c**2*x + 2*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))
*b*c**2*x + 2*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) -
2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d*x - 2*d**(1/4)*c**(3/4)*s
qrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*s
qrt(2)))*a**2*d*x + b**(1/4)*a**(3/4)*sqrt(2)*log( - b**(1/4)*a**(1/4)*sqr
t(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c**2*x - b**(1/4)*a**(3/4)*sqrt(2)*log(
b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c**2*x - d**(1/4)*
c**(3/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x*
*2)*a**2*d*x + d**(1/4)*c**(3/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x +
 sqrt(c) + sqrt(d)*x**2)*a**2*d*x - 8*a**2*c*d + 8*a*b*c**2)/(8*a**2*c**2*
x*(a*d - b*c))
```

3.212 $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 340

$$\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{3acx^3} + \frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} - \frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} + \frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} - \frac{b^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} + \frac{d^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}$$

output
```
-1/3/a/c/x^3-1/4*b^(7/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(7
/4)/(-a*d+b*c)-1/4*b^(7/4)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(
7/4)/(-a*d+b*c)+1/4*d^(7/4)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c
^(7/4)/(-a*d+b*c)+1/4*d^(7/4)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/
c^(7/4)/(-a*d+b*c)-1/4*b^(7/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+
b^(1/2)*x^2))*2^(1/2)/a^(7/4)/(-a*d+b*c)+1/4*d^(7/4)*arctanh(2^(1/2)*c^(1/
4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4 (a+bx^4)(c+dx^4)} dx$$
$$= \frac{\frac{8b}{a} - \frac{8d}{c} - \frac{6\sqrt{2}b^{7/4}x^3 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}b^{7/4}x^3 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}} + \frac{6\sqrt{2}d^{7/4}x^3 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}} - \frac{6\sqrt{2}d^{7/4}x^3 a}{}}{}$$

input
```
Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)
/c^(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)
/c^(1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[
2]*d^(7/4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(
7/4))/(24*(-(b*c) + a*d)*x^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {980, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4)(c+dx^4)} dx$$
$$\Big\downarrow \text{980}$$

$$\frac{\int -\frac{3(bdx^4+bc+ad)}{(bx^4+a)(dx^4+c)}dx}{3ac}-\frac{1}{3acx^3}$$

$\big\downarrow$ 27

$$-\frac{\int \frac{bdx^4+bc+ad}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{3acx^3}$$

$\big\downarrow$ 1020

$$-\frac{\frac{b^2c\int \frac{1}{bx^4+a}dx}{bc-ad}-\frac{ad^2\int \frac{1}{dx^4+c}dx}{bc-ad}}{ac}-\frac{1}{3acx^3}$$

$\big\downarrow$ 755

$$-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{1}{3acx^3}$$

$\big\downarrow$ 1476

$$-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}\cdots}{2\sqrt{c}}\cdots\right.}{bc-ad}}{ac}$$

$$\frac{1}{3acx^3}$$

$\big\downarrow$ 1082

$$-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\sqrt{2}\cdots\right.}\cdots}{\cdots}}{\cdots}\right.}{\cdots}}{ac}$$

$$\frac{1}{3acx^3}$$

$\big\downarrow$ 217

$$-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}}{ac}$$

$$\frac{1}{3acx^3}$$

$\downarrow$ 1479

$$-\frac{\frac{b^2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{-\frac{\int -\ldots}{\ldots}}{\ldots}\right)}{\ldots}}{ac}$$

$$\frac{1}{3acx^3}$$

$\downarrow$ 25

$$-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{d}\left(x^2-\ldots\right)}}{2\sqrt{2}\ldots}}{\ldots}\right)}{\ldots}}{ac}$$

$$\frac{1}{3acx^3}$$

$\downarrow$ 27

$$-\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\cdots}{2\sqrt{c}}\cdots\right.}{ac}$$

$$\frac{1}{3acx^3}$$

$$\Big\downarrow \ 1103$$

$$-\frac{b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\cdots\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{}\right.}{ac}$$

$$\frac{1}{3acx^3}$$

input

```
Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

output

```
-1/3*1/(a*c*x^3) - ((b^2*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b
*c - a*d) - (a*d^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^
(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)
*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + S
qrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d
))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.71

method	result
default	$\frac{b^2 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} \left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2(ad-cb)}-\frac{1}{3ac\,x^3}-\frac{d^2 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} \left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right) \cdots}{x^2-\left(\frac{c}{d}\right) \cdots}\right.\right. \cdots}{\cdots}$
risch	$-\frac{1}{3ac\,x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(d^4a^{11}-4a^{10}bc\,d^3+6a^9b^2c^2d^2-4a^8b^3c^3d+a^7b^4c^4\right)\_Z^4+b^7\right)} \_R\ln\left(\left(\left(-5a^{15}c^7d^8+38a^{14}b\,c^8d^7-128a^{13} \cdots\right.\right.\right.\right.}{\cdots}$

input
```
int(1/x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/3/a/c/x^3-1/8/c^2*d^2/(a
*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x
^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2
*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 1255, normalized size of antiderivative = 3.69

$$\int \frac{1}{x^4 (a + bx^4) (c + dx^4)} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/12*(3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10
*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x + (-b^7/(a^7*b^4*c^4 - 4*a^8
*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a^2*b*
c - a^3*d)) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 -
 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7*b^4*c^4
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*
(a^2*b*c - a^3*d)) - 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x - (-b^7/(a^7
*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4
))^(1/4)*(I*a^2*b*c - I*a^3*d)) + 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d
 + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x
 - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(-I*a^2*b*c + I*a^3*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^
3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x
^3*log(d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^
3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*(-d^7/(b^4*c^11 -
 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4
)*a*c*x^3*log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2
 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) + 3*I*(-d^7/(b
^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\right.}{}}{8\left(abc-a^2d\right)} + \frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\right.}{}}{8\left(bc^2-acd\right)} - \frac{1}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output

```
-1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^
2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^
2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqr
t(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) +
 1/8*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1
/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^
2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^
2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqr
t(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) -
 1/3/(a*c*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^4 (a+bx^4)(c+dx^4)} dx = -\frac{(ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} - \frac{(ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} + \frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{(ab^3)^{\frac{1}{4}} b \log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(ab^3)^{\frac{1}{4}} b \log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{1}{3\,acx^3}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^
(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*
sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2
)*a^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*a
rctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3
- sqrt(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
 sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) +
 1/4*(c*d^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3 - sqrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 7459, normalized size of antiderivative = 21.94

$$\int \frac{1}{x^4 (a+bx^4) (c+dx^4)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*
a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a
^5*b^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^
3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i
)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*
d^2 - 1024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a
^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3
) + (a^8*b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*
b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x
*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^
2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*
b*c*d^3) - (a^11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 102
4*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^
10*d*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*
a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c
^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)
*((b^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (a+bx^4)(c+dx^4)}\,dx = \frac{-6b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x^3+6b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x^3+6d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right.}{}$$

input
```
int(1/x^4/(b*x^4+a)/(d*x^4+c),x)
```

output

```
( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b*c**2*x**3 + 6*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2
)))*b*c**2*x**3 + 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt
(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*d*x**3 - 6*d**(3/4)*c
**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c
**(1/4)*sqrt(2)))*a**2*d*x**3 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - b**(1/4
)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b*c**2*x**3 + 3*b**(3/4)*a*
*(1/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b
*c**2*x**3 + 3*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*
x + sqrt(c) + sqrt(d)*x**2)*a**2*d*x**3 - 3*d**(3/4)*c**(1/4)*sqrt(2)*log(
d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**2*d*x**3 - 8*a**2
*c*d + 8*a*b*c**2)/(24*a**2*c**2*x**3*(a*d - b*c))
```

3.213 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)} dx$

Optimal result

Integrand size = 22, antiderivative size = 357

$$\begin{aligned}\int \frac{1}{x^6(a+bx^4)(c+dx^4)} dx &= -\frac{1}{5acx^5}+\frac{bc+ad}{a^2c^2x}-\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}\\
&+\frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}+\frac{d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}\\
&-\frac{d^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}-\frac{b^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{9/4}(bc-ad)}\\
&+\frac{d^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}\end{aligned}$$

output
```
-1/5/a/c/x^5+(a*d+b*c)/a^2/c^2/x+1/4*b^(9/4)*arctan(-1+2^(1/2)*b^(1/4)*x/a
^(1/4))*2^(1/2)/a^(9/4)/(-a*d+b*c)+1/4*b^(9/4)*arctan(1+2^(1/2)*b^(1/4)*x/
a^(1/4))*2^(1/2)/a^(9/4)/(-a*d+b*c)-1/4*d^(9/4)*arctan(-1+2^(1/2)*d^(1/4)*
x/c^(1/4))*2^(1/2)/c^(9/4)/(-a*d+b*c)-1/4*d^(9/4)*arctan(1+2^(1/2)*d^(1/4)
*x/c^(1/4))*2^(1/2)/c^(9/4)/(-a*d+b*c)-1/4*b^(9/4)*arctanh(2^(1/2)*a^(1/4)
*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(9/4)/(-a*d+b*c)+1/4*d^(9/4)*a
rctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(9/4)/(-
a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$= \frac{\frac{8b}{a}-\frac{8d}{c}-\frac{40b^2x^4}{a^2}+\frac{40d^2x^4}{c^2}+\frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}b^{9/4}x^5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}}-\frac{10\sqrt{2}d^{9/4}x^5\arctan\left(1-\cdots\right.}{c^{9/4}}}{}$$

input
```
Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(
9/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^
(9/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d
^(9/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*
d^(9/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*
b^(9/4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4
) + (5*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {980, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^4) (c+dx^4)} \, dx$$

$$\Big\downarrow \ 980$$

$$\frac{\int -\frac{5(bdx^4+bc+ad)}{x^2(bx^4+a)(dx^4+c)}dx}{5ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{bdx^4+bc+ad}{x^2(bx^4+a)(dx^4+c)}dx}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \ 1053$$

$$-\frac{-\frac{\int \frac{x^2\left(bd(bc+ad)x^4+b^2c^2+a^2d^2+abcd\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \ 1054$$

$$-\frac{-\frac{\int \left(\frac{c^2x^2b^3}{(bc-ad)\left(bx^4+a\right)}+\frac{a^2d^3x^2}{(ad-bc)\left(dx^4+c\right)}\right)dx}{ac}-\frac{ad+bc}{acx}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{\frac{a^2d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}-}{ac}}{}$$

$$\frac{1}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/5*1/(a*c*x^5) - (-((b*c + a*d)/(a*c*x)) - (-1/2*(b^(9/4)*c^2*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(9/4)*c^
2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d))
 + (a^2*d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d)) - (a^2*d^(9/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*
Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(9/4)*c^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(9/4)*c^2*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*
(b*c - a*d)) - (a^2*d^(9/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a
*c))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.73

method	result
default	$-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2(ad-cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{1}{5ac\,x^5}-\frac{-ad-cb}{a^2c^2x}+\frac{d^2\sqrt{2}\left(\ln\left(\frac{x^2-}{x^2+}\right.\right.}{}$
risch	Expression too large to display

input
```
int(1/x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/
b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/5/a/c/x^5-1/a^2/c^2*(-a
*d-b*c)/x+1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^
(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.64 (sec) , antiderivative size = 1526, normalized size of antiderivative = 4.27

$$\int \frac{1}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/20*(5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*
b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*
d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9
*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d
^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*
b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2
*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) + 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^1
0*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c
^2*x^5*log(b^7*x - (I*a^7*b^3*c^3 - 3*I*a^8*b^2*c^2*d + 3*I*a^9*b*c*d^2 -
I*a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4
*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*I*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^
3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*l
og(b^7*x - (-I*a^7*b^3*c^3 + 3*I*a^8*b^2*c^2*d - 3*I*a^9*b*c*d^2 + I*a^10*
d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b
*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^
2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x
+ (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^
13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)
)^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^4) (c+dx^4)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^6 (a + bx^4)(c + dx^4)}\,dx$$
$$= \frac{b^3 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{}\right.}{a^{\frac{1}{4}}} \right.}{8\,(a^2bc - a^3d)}$$
$$- \frac{d^3 \left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{d}x\right.}{} \right.}{8\,(bc^3 - ac^2d)}$$
$$+ \frac{5\,(bc + ad)x^4 - ac}{5\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 -
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d)
- 1/8*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*a
rctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2
- sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d
) + 1/5*(5*(b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^6 (a + bx^4)(c + dx^4)} dx = \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc - \sqrt{2}a^4d\right)} + \frac{(ab^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc - \sqrt{2}a^4d\right)} - \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4 - \sqrt{2}ac^3d\right)} - \frac{(cd^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4 - \sqrt{2}ac^3d\right)} - \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc - \sqrt{2}a^4d\right)} + \frac{(ab^3)^{\frac{3}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}a^3bc - \sqrt{2}a^4d\right)} + \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4 - \sqrt{2}ac^3d\right)} - \frac{(cd^3)^{\frac{3}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^4 - \sqrt{2}ac^3d\right)} + \frac{5\,bcx^4 + 5\,adx^4 - ac}{5\,a^2c^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4
*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/
d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)
*a*c^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^
(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2
)*a*c^3*d) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d)
)/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a
^2*c^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 4547, normalized size of antiderivative = 12.74

$$\int \frac{1}{x^6 (a + bx^4) (c + dx^4)} dx = \text{Too large to display}$$

input
```
int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 10
24*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*
a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d
 + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*
x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*
b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(25
6*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2
 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 +
 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12
*b*c*d^3))^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c
^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/
4) + 6144*a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024
*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096
*a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3
*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6
*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1
536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x
*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b
^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(25
6*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*...
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^6 (a+bx^4)(c+dx^4)}\,dx = \frac{10b^{\frac{9}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^3x^5-10b^{\frac{9}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{b}\,x}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^3x^5-10d^{\frac{9}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\ldots}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\ldots}\right.\ldots}{\ldots}$$

input
```
int(1/x^6/(b*x^4+a)/(d*x^4+c),x)
```

output

```
(10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*c**3*x**5 - 10*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*b**2*c**3*x**5 - 10*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4
)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*d**2*x**5 + 10*
d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(
d**(1/4)*c**(1/4)*sqrt(2)))*a**3*d**2*x**5 - 5*b**(1/4)*a**(3/4)*sqrt(2)*l
og( - b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(b)*x**2)*b**2*c**3*x**5
 + 5*b**(1/4)*a**(3/4)*sqrt(2)*log(b**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(b)*x**2)*b**2*c**3*x**5 + 5*d**(1/4)*c**(3/4)*sqrt(2)*log( - d**(1/4
)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*a**3*d**2*x**5 - 5*d**(1/4)
*c**(3/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2
)*a**3*d**2*x**5 - 8*a**3*c**2*d + 40*a**3*c*d**2*x**4 + 8*a**2*b*c**3 - 4
0*a*b**2*c**3*x**4)/(40*a**3*c**3*x**5*(a*d - b*c))
```

3.214 $\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 255

$$\begin{aligned}\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx = & -\frac{b^2(2bc-3ad)x^3}{3d^3}+\frac{b^3x^7}{7d^2}-\frac{(bc-ad)^3x^3}{4cd^3(c+dx^4)} \\ & -\frac{(bc-ad)^2(11bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{5/4}d^{15/4}} \\ & +\frac{(bc-ad)^2(11bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{5/4}d^{15/4}} \\ & -\frac{(bc-ad)^2(11bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{5/4}d^{15/4}}\end{aligned}$$

output

```
-1/3*b^2*(-3*a*d+2*b*c)*x^3/d^3+1/7*b^3*x^7/d^2-1/4*(-a*d+b*c)^3*x^3/c/d^3
/(d*x^4+c)+1/16*(-a*d+b*c)^2*(a*d+11*b*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1
/4))*2^(1/2)/c^(5/4)/d^(15/4)+1/16*(-a*d+b*c)^2*(a*d+11*b*c)*arctan(1+2^(1
/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(5/4)/d^(15/4)-1/16*(-a*d+b*c)^2*(a*d+11*
b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(5
/4)/d^(15/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.20

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx$$

$$= \frac{-224b^2d^{3/4}(2bc-3ad)x^3+96b^3d^{7/4}x^7+\frac{168d^{3/4}(-bc+ad)^3x^3}{c(c+dx^4)}-\frac{42\sqrt{2}(bc-ad)^2(11bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{5/4}}+\frac{42\sqrt{2}(\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(x^2*(a + b*x^4)^3)/(c + d*x^4)^2,x]
```

output
```
(-224*b^2*d^(3/4)*(2*b*c - 3*a*d)*x^3 + 96*b^3*d^(7/4)*x^7 + (168*d^(3/4)*
(-(b*c) + a*d)^3*x^3)/(c*(c + d*x^4)) - (42*Sqrt[2]*(b*c - a*d)^2*(11*b*c
+ a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/4) + (42*Sqrt[2]*(b*c
 - a*d)^2*(11*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(5/4)
+ (21*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) - (21*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d
)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(672*d^
(15/4))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^4\right)^3}{\left(c+dx^4\right)^2}\,dx$$

$$\Big\downarrow \; 968$$

$$-\frac{\int -\frac{x^2\left(bx^4+a\right)\left(b(11bc-7ad)x^4+a(3bc+ad)\right)}{dx^4+c}dx}{4cd}-\frac{x^3\left(a+bx^4\right)^2(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2(bx^4+a)\left(b(11bc-7ad)x^4+a(3bc+ad)\right)}{dx^4+c}dx}{4cd}-\frac{x^3\left(a+bx^4\right)^2(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 1040$$

$$\frac{\int \left(\frac{b^2(11bc-7ad)x^6}{d}-\frac{b\left(11b^2c^2-21abdc+6a^2d^2\right)x^2}{d^2}+\frac{\left(11b^3c^3-21ab^2dc^2+9a^2bd^2c+a^3d^3\right)x^2}{d^2(dx^4+c)}\right)dx}{4cd}-\frac{x^3\left(a+bx^4\right)^2(bc-ad)}{4cd\left(c+dx^4\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{bx^3\left(6a^2d^2-21abcd+11b^2c^2\right)}{3d^2}-\frac{(bc-ad)^2(ad+11bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2(ad+11bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{b^2x^7(11bc-7a\ldots}{7d}}{4cd}$$

$$\frac{x^3\left(a+bx^4\right)^2(bc-ad)}{4cd\left(c+dx^4\right)}$$

input

```
Int[(x^2*(a + b*x^4)^3)/(c + d*x^4)^2,x]
```

output

```
-1/4*((b*c - a*d)*x^3*(a + b*x^4)^2)/(c*d*(c + d*x^4)) + (-1/3*(b*(11*b^2*
c^2 - 21*a*b*c*d + 6*a^2*d^2)*x^3)/d^2 + (b^2*(11*b*c - 7*a*d)*x^7)/(7*d)
- ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(
2*Sqrt[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*(
11*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*S
qrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*d^(11/4)))/(4*c*
d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.62

method	result
risch	$\frac{b^3x^7}{7d^2}+\frac{b^2a\,x^3}{d^2}-\frac{2b^3c\,x^3}{3d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x^3}{4c\,d^3(d\,x^4+c)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{\left(a^3d^3+9a^2bc\,d^2-21a\,b^2c^2d+11b^3c^3\right)}{\_R}}{16d^4c}$
default	$\frac{b^2\left(\frac{bd\,x^7}{7}+\frac{(3ad-2cb)x^3}{3}\right)}{d^3}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^3}{4c\left(d\,x^4+c\right)}+\frac{\left(a^3d^3+9a^2bc\,d^2-21a\,b^2c^2d+11b^3c^3\right)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{32cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^3}$

input
```
int(x^2*(b*x^4+a)^3/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*x^7/d^2+b^2/d^2*a*x^3-2/3*b^3/d^3*c*x^3+1/4*(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/c*x^3/d^3/(d*x^4+c)+1/16/d^4/c*sum((a^3*d^3+9*a^2*
b*c*d^2-21*a*b^2*c^2*d+11*b^3*c^3)/_R*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 2079, normalized size of antiderivative = 8.15

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/336*(48*b^3*c*d^2*x^11 - 16*(11*b^3*c^2*d - 21*a*b^2*c*d^2)*x^7 - 28*(11
*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3)*x^3 + 21*(c*d^4*x^4
 + c^2*d^3)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^
10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*
c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^
4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a
^12*d^12)/(c^5*d^15))^(1/4)*log(c^4*d^11*(-(14641*b^12*c^12 - 111804*a*b^1
1*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*
b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b
^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^
2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(c^5*d^15))^(3/4) + (1331*b^9*c^9 -
 7623*a*b^8*c^8*d + 17820*a^2*b^7*c^7*d^2 - 21372*a^3*b^6*c^6*d^3 + 13194*
a^4*b^5*c^5*d^4 - 3186*a^5*b^4*c^4*d^5 - 372*a^6*b^3*c^3*d^6 + 180*a^7*b^2
*c^2*d^7 + 27*a^8*b*c*d^8 + a^9*d^9)*x) - 21*(I*c*d^4*x^4 + I*c^2*d^3)*(-(
14641*b^12*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588
*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 16018
8*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9
*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(c^5*
d^15))^(1/4)*log(I*c^4*d^11*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d + 36
8082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*d^...
```

Sympy [A] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.69

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx = \frac{b^3x^7}{7d^2} + x^3\left(\frac{ab^2}{d^2} - \frac{2b^3c}{3d^3}\right) + \frac{x^3(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{4c^2d^3 + 4cd^4x^4}$$
$$+\mathrm{RootSum}\left(65536t^4c^5d^{15} + a^{12}d^{12} + 36a^{11}bcd^{11} + 402a^{10}b^2c^2d^{10} + 692a^9b^3c^3d^9 - 10017a^8b^4c^4d^8 - 568\right.$$

input
```
integrate(x**2*(b*x**4+a)**3/(d*x**4+c)**2,x)
```

output
```
b**3*x**7/(7*d**2) + x**3*(a*b**2/d**2 - 2*b**3*c/(3*d**3)) + x**3*(a**3*d
**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(4*c**2*d**3 + 4*c*d*
*4*x**4) + RootSum(65536*_t**4*c**5*d**15 + a**12*d**12 + 36*a**11*b*c*d**
11 + 402*a**10*b**2*c**2*d**10 + 692*a**9*b**3*c**3*d**9 - 10017*a**8*b**4
*c**4*d**8 - 5688*a**7*b**5*c**5*d**7 + 160188*a**6*b**6*c**6*d**6 - 48664
8*a**5*b**7*c**7*d**5 + 746703*a**4*b**8*c**8*d**4 - 676588*a**3*b**9*c**9
*d**3 + 368082*a**2*b**10*c**10*d**2 - 111804*a*b**11*c**11*d + 14641*b**1
2*c**12, Lambda(_t, _t*log(4096*_t**3*c**4*d**11/(a**9*d**9 + 27*a**8*b*c*
d**8 + 180*a**7*b**2*c**2*d**7 - 372*a**6*b**3*c**3*d**6 - 3186*a**5*b**4*
c**4*d**5 + 13194*a**4*b**5*c**5*d**4 - 21372*a**3*b**6*c**6*d**3 + 17820*
a**2*b**7*c**7*d**2 - 7623*a*b**8*c**8*d + 1331*b**9*c**9) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.20

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx$$
$$= -\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^3}{4\,(cd^4x^4 + c^2d^3)} + \frac{3\,b^3dx^7 - 7\,(2\,b^3c - 3\,ab^2d)x^3}{21\,d^3}$$
$$+ \frac{(11\,b^3c^3 - 21\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \right.}{32\,cd^3}$$

input
```
integrate(x^2*(b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3/(c*d^4*x^4 +
c^2*d^3) + 1/21*(3*b^3*d*x^7 - 7*(2*b^3*c - 3*a*b^2*d)*x^3)/d^3 + 1/32*(11
*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*(2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
- sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*s
qrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(
c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(1/4)*d^(3/4)))/(c*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(202) = 404$.

Time = 0.13 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.01

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx = -\frac{b^3c^3x^3 - 3\,ab^2c^2dx^3 + 3\,a^2bcd^2x^3 - a^3d^3x^3}{4\,(dx^4+c)cd^3}$$
$$+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^3c^3 - 21\,(cd^3)^{\frac{3}{4}}\,ab^2c^2d + 9\,(cd^3)^{\frac{3}{4}}\,a^2bcd^2 + (cd^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^6}$$
$$+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^3c^3 - 21\,(cd^3)^{\frac{3}{4}}\,ab^2c^2d + 9\,(cd^3)^{\frac{3}{4}}\,a^2bcd^2 + (cd^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^6}$$
$$-\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^3c^3 - 21\,(cd^3)^{\frac{3}{4}}\,ab^2c^2d + 9\,(cd^3)^{\frac{3}{4}}\,a^2bcd^2 + (cd^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32\,c^2d^6}$$
$$+\frac{\sqrt{2}\left(11\,(cd^3)^{\frac{3}{4}}\,b^3c^3 - 21\,(cd^3)^{\frac{3}{4}}\,ab^2c^2d + 9\,(cd^3)^{\frac{3}{4}}\,a^2bcd^2 + (cd^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{32\,c^2d^6}$$
$$+\frac{3\,b^3d^{12}x^7 - 14\,b^3cd^{11}x^3 + 21\,ab^2d^{12}x^3}{21\,d^{14}}$$

input
```
integrate(x^2*(b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
-1/4*(b^3*c^3*x^3 - 3*a*b^2*c^2*d*x^3 + 3*a^2*b*c*d^2*x^3 - a^3*d^3*x^3)/(
(d*x^4 + c)*c*d^3) + 1/16*sqrt(2)*(11*(c*d^3)^(3/4)*b^3*c^3 - 21*(c*d^3)^(
3/4)*a*b^2*c^2*d + 9*(c*d^3)^(3/4)*a^2*b*c*d^2 + (c*d^3)^(3/4)*a^3*d^3)*ar
ctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^6) + 1/16
*sqrt(2)*(11*(c*d^3)^(3/4)*b^3*c^3 - 21*(c*d^3)^(3/4)*a*b^2*c^2*d + 9*(c*d
^3)^(3/4)*a^2*b*c*d^2 + (c*d^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^6) - 1/32*sqrt(2)*(11*(c*d^3)^(3/4
)*b^3*c^3 - 21*(c*d^3)^(3/4)*a*b^2*c^2*d + 9*(c*d^3)^(3/4)*a^2*b*c*d^2 + (
c*d^3)^(3/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^
6) + 1/32*sqrt(2)*(11*(c*d^3)^(3/4)*b^3*c^3 - 21*(c*d^3)^(3/4)*a*b^2*c^2*d
 + 9*(c*d^3)^(3/4)*a^2*b*c*d^2 + (c*d^3)^(3/4)*a^3*d^3)*log(x^2 - sqrt(2)*
x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^6) + 1/21*(3*b^3*d^12*x^7 - 14*b^3*c*d^1
1*x^3 + 21*a*b^2*d^12*x^3)/d^14
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.39

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx$$

$$= x^3\left(\frac{a\,b^2}{d^2}-\frac{2\,b^3\,c}{3\,d^3}\right)+\frac{b^3\,x^7}{7\,d^2}+\frac{x^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{4\,c\,(d^4\,x^4+c\,d^3)}$$

$$-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,x\,(a\,d-b\,c)^2\,(a\,d+11\,b\,c)\,(a^6\,d^6+18\,a^5\,b\,c\,d^5+39\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+639\,a^2\,b^4\,c^4\,d^2-462\,a\,b^5\,c^5\,d+1}{(-c)^{1/4}\,(a^9\,d^9+27\,a^8\,b\,c\,d^8+180\,a^7\,b^2\,c^2\,d^7-372\,a^6\,b^3\,c^3\,d^6-3186\,a^5\,b^4\,c^4\,d^5+13194\,a^4\,b^5\,c^5\,d^4-21372\,a^3\,b^6\,c^6\,d^3+17820\,a^2\,b^7\,c^7\,d^2}}{8\,(-c)^{5/4}\,d^{15/4}}$$

$$-\frac{\mathrm{atan}\left(\frac{d^{1/4}\,x\,(a\,d-b\,c)^2\,(a\,d+11\,b\,c)\,(a^6\,d^6+18\,a^5\,b\,c\,d^5+39\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+639\,a^2\,b^4\,c^4\,d^2-462\,a\,b^5\,c^5\,d+12}{(-c)^{1/4}\,(a^9\,d^9+27\,a^8\,b\,c\,d^8+180\,a^7\,b^2\,c^2\,d^7-372\,a^6\,b^3\,c^3\,d^6-3186\,a^5\,b^4\,c^4\,d^5+13194\,a^4\,b^5\,c^5\,d^4-21372\,a^3\,b^6\,c^6\,d^3+17820\,a^2\,b^7\,c^7\,d^2}}{8\,(-c)^{5/4}\,d^{15/4}}$$

input
```
int((x^2*(a + b*x^4)^3)/(c + d*x^4)^2,x)
```

output
```
x^3*((a*b^2)/d^2 - (2*b^3*c)/(3*d^3)) + (b^3*x^7)/(7*d^2) + (x^3*(a^3*d^3
- b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*c*(c*d^3 + d^4*x^4)) - (ata
n((d^(1/4)*x*(a*d - b*c)^2*(a*d + 11*b*c)*(a^6*d^6 + 121*b^6*c^6 + 639*a^2
*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 39*a^4*b^2*c^2*d^4 - 462*a*b^5*c^5*d
+ 18*a^5*b*c*d^5))/((-c)^(1/4)*(a^9*d^9 + 1331*b^9*c^9 + 17820*a^2*b^7*c^7
*d^2 - 21372*a^3*b^6*c^6*d^3 + 13194*a^4*b^5*c^5*d^4 - 3186*a^5*b^4*c^4*d^
5 - 372*a^6*b^3*c^3*d^6 + 180*a^7*b^2*c^2*d^7 - 7623*a*b^8*c^8*d + 27*a^8*
b*c*d^8)))*(a*d - b*c)^2*(a*d + 11*b*c))/(8*(-c)^(5/4)*d^(15/4)) - (atan((
d^(1/4)*x*(a*d - b*c)^2*(a*d + 11*b*c)*(a^6*d^6 + 121*b^6*c^6 + 639*a^2*b^
4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 39*a^4*b^2*c^2*d^4 - 462*a*b^5*c^5*d + 1
8*a^5*b*c*d^5)*1i)/((-c)^(1/4)*(a^9*d^9 + 1331*b^9*c^9 + 17820*a^2*b^7*c^7
*d^2 - 21372*a^3*b^6*c^6*d^3 + 13194*a^4*b^5*c^5*d^4 - 3186*a^5*b^4*c^4*d^
5 - 372*a^6*b^3*c^3*d^6 + 180*a^7*b^2*c^2*d^7 - 7623*a*b^8*c^8*d + 27*a^8*
b*c*d^8)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-c)^(5/4)*d^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1355, normalized size of antiderivative = 5.31

$$\int \frac{x^2(a+bx^4)^3}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^4+a)^3/(d*x^4+c)^2,x)
```

output

```
( - 42*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*c*d**3 - 42*d**(1/4)*c**(3/4)*sqrt
(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt
(2)))*a**3*d**4*x**4 - 378*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/
4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c**2*d**2 -
378*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*
x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c*d**3*x**4 + 882*d**(1/4)*c**(3/4)
*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)
*sqrt(2)))*a*b**2*c**3*d + 882*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c*
*(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b**2*c**2*d**
2*x**4 - 462*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2
*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**3*c**4 - 462*d**(1/4)*c**(3/4)
*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)
*sqrt(2)))*b**3*c**3*d*x**4 + 42*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*
c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*c*d**3 +
 42*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*
x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*d**4*x**4 + 378*d**(1/4)*c**(3/4)*sqr
t(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqr
t(2)))*a**2*b*c**2*d**2 + 378*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**
(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c*d**3...
```

3.215 $\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 249

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^5}{5d^2}-\frac{(bc-ad)^3x}{4cd^3(c+dx^4)} - \frac{3(bc-ad)^2(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{13/4}} + \frac{3(bc-ad)^2(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{13/4}} + \frac{3(bc-ad)^2(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{8\sqrt{2}c^{7/4}d^{13/4}}$$

output
```
-b^2*(-3*a*d+2*b*c)*x/d^3+1/5*b^3*x^5/d^2-1/4*(-a*d+b*c)^3*x/c/d^3/(d*x^4+
c)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1
/2)/c^(7/4)/d^(13/4)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1+2^(1/2)*d^(1/4
)*x/c^(1/4))*2^(1/2)/c^(7/4)/d^(13/4)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arctan
h(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/d^(13/4
)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx$$

$$= \frac{-160b^2\sqrt[4]{d}(2bc-3ad)x+32b^3d^{5/4}x^5+\frac{40\sqrt[4]{d}(-bc+ad)^3x}{c(c+dx^4)}-\frac{30\sqrt{2}(bc-ad)^2(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{7/4}}+\frac{30\sqrt{2}(bc-a}{}}{}$$

input
```
Integrate[(a + b*x^4)^3/(c + d*x^4)^2,x]
```

output
```
(-160*b^2*d^(1/4)*(2*b*c - 3*a*d)*x + 32*b^3*d^(5/4)*x^5 + (40*d^(1/4)*(-(
b*c) + a*d)^3*x)/(c*(c + d*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (30*Sqrt[2]*(b*c - a*d
)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) - (15*S
qrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2])/c^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(160*d^(13/4))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx$$

$$\Big\downarrow \text{915}$$

$$\int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{3bdx^4(bc-ad)^2+(bc-ad)^2(ad+2bc)}{d^3(c+dx^4)^2}+\frac{b^3x^4}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(bc-ad)^2(ad+3bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{13/4}}+\frac{3(bc-ad)^2(ad+3bc)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}d^{13/4}}-\frac{b^2x(2bc-3ad)}{d^3}-\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{13/4}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{13/4}}-\frac{x(bc-ad)^3}{4cd^3\left(c+dx^4\right)}+\frac{b^3x^5}{5d^2}$$

input
```
Int[(a + b*x^4)^3/(c + d*x^4)^2,x]
```

output
```
-((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^5)/(5*d^2) - ((b*c - a*d)^3*x)/(4*
c*d^3*(c + d*x^4)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*d^
(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(13/4)) + (3*(b*c - a*d)^2*(3*b*c
+ a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(13/4
)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(13/4)) + (3*(b*c - a*d)^2*(3*b*c
+ a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]
*c^(7/4)*d^(13/4))
```

Defintions of rubi rules used

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.61

method	result
risch	$\frac{b^3x^5}{5d^2}+\frac{3b^2ax}{d^2}-\frac{2b^3cx}{d^3}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x}{4c\,d^3(d\,x^4+c)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(d\_Z^4+c\right)}\frac{\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)\ln\left(\right.}{\_R^3}\right.}{16d^4c}$
default	$\frac{b^2\left(\frac{1}{5}x^5bd+3adx-2cbx\right)}{d^3}+\frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{4c\left(d\,x^4+c\right)}+\frac{3\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{32c^2}}{d^3}$

input
```
int((b*x^4+a)^3/(d*x^4+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3*x^5/d^2+3*b^2/d^2*a*x-2*b^3/d^3*c*x+1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a
*b^2*c^2*d-b^3*c^3)/c*x/d^3/(d*x^4+c)+3/16/d^4/c*sum((a^3*d^3+a^2*b*c*d^2-
5*a*b^2*c^2*d+3*b^3*c^3)/_R^3*ln(x-_R),_R=RootOf(_Z^4*d+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1742, normalized size of antiderivative = 7.00

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
1/80*(16*b^3*c*d^2*x^9 - 48*(3*b^3*c^2*d - 5*a*b^2*c*d^2)*x^5 + 15*(c*d^4*
x^4 + c^2*d^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^
2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 64
4*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3
*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(c^7*d^13))
^(1/4)*log(3*c^2*d^3*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c
^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^
5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a
^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(c^7*
d^13))^(1/4) + 3*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*x) -
15*(-I*c*d^4*x^4 - I*c^2*d^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a
^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b
^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d
^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^
12)/(c^7*d^13))^(1/4)*log(3*I*c^2*d^3*(-(81*b^12*c^12 - 540*a*b^11*c^11*d
+ 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 3
28*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b
^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 +
 a^12*d^12)/(c^7*d^13))^(1/4) + 3*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2
 + a^3*d^3)*x) - 15*(I*c*d^4*x^4 + I*c^2*d^3)*(-(81*b^12*c^12 - 540*a*b...
```

Sympy [A] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = \frac{b^3x^5}{5d^2} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{x(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{4c^2d^3 + 4cd^4x^4}$$
$$+\mathrm{RootSum}\left(65536t^4c^7d^{13} + 81a^{12}d^{12} + 324a^{11}bcd^{11} - 1134a^{10}b^2c^2d^{10} - 3564a^9b^3c^3d^9 + 10287a^8b^4c^4d^8\right.$$

input
```
integrate((b*x**4+a)**3/(d*x**4+c)**2,x)
```

output
```
b**3*x**5/(5*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*
a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(4*c**2*d**3 + 4*c*d**4*x**4)
 + RootSum(65536*_t**4*c**7*d**13 + 81*a**12*d**12 + 324*a**11*b*c*d**11 -
 1134*a**10*b**2*c**2*d**10 - 3564*a**9*b**3*c**3*d**9 + 10287*a**8*b**4*c
**4*d**8 + 11016*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 26568*a
**5*b**7*c**7*d**5 + 84159*a**4*b**8*c**8*d**4 - 156492*a**3*b**9*c**9*d**
3 + 118098*a**2*b**10*c**10*d**2 - 43740*a*b**11*c**11*d + 6561*b**12*c**1
2, Lambda(_t, _t*log(16*_t*c**2*d**3/(3*a**3*d**3 + 3*a**2*b*c*d**2 - 15*a
*b**2*c**2*d + 9*b**3*c**3) + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(198) = 396$.

Time = 0.12 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{4\,(cd^4x^4+c^2d^3)} + \frac{b^3dx^5-5\,(2\,b^3c-3\,ab^2d)x}{5\,d^3} + \frac{3\left(\frac{2\sqrt{2}(3\,b^3c^3-5\,ab^2c^2d+a^2bcd^2+a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(3\,b^3c^3-5\,ab^2c^2d+a^2bcd^2+a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\cdots\right)}{2\sqrt{\sqrt{c}\cdots}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}\cdots\right)}{\cdots}$$

input
```
integrate((b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^4 + c^
2*d^3) + 1/5*(b^3*d*x^5 - 5*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3/32*(2*sqrt(2)
*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*arctan(1/2*sqrt(2)*(2
*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 +
a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b^3*c^3 - 5*
a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^3*c^3 - 5*a*b^2*c^2*d +
 a^2*b*c*d^2 + a^3*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt
(c))/(c^(3/4)*d^(1/4)))/(c*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(198) = 396$.

Time = 0.13 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx$$
$$= \frac{3\sqrt{2}\left(3\,(cd^3)^{\frac{1}{4}}b^3c^3 - 5\,(cd^3)^{\frac{1}{4}}ab^2c^2d + (cd^3)^{\frac{1}{4}}a^2bcd^2 + (cd^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^4}$$
$$+ \frac{3\sqrt{2}\left(3\,(cd^3)^{\frac{1}{4}}b^3c^3 - 5\,(cd^3)^{\frac{1}{4}}ab^2c^2d + (cd^3)^{\frac{1}{4}}a^2bcd^2 + (cd^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^4}$$
$$+ \frac{3\sqrt{2}\left(3\,(cd^3)^{\frac{1}{4}}b^3c^3 - 5\,(cd^3)^{\frac{1}{4}}ab^2c^2d + (cd^3)^{\frac{1}{4}}a^2bcd^2 + (cd^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^4}$$
$$- \frac{3\sqrt{2}\left(3\,(cd^3)^{\frac{1}{4}}b^3c^3 - 5\,(cd^3)^{\frac{1}{4}}ab^2c^2d + (cd^3)^{\frac{1}{4}}a^2bcd^2 + (cd^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^4}$$
$$- \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{4\,(dx^4+c)cd^3} + \frac{b^3d^8x^5 - 10\,b^3cd^7x + 15\,ab^2d^8x}{5\,d^{10}}$$

input
```
integrate((b*x^4+a)^3/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*(3*(c*d^3)^(1/4)*b^3*c^3 - 5*(c*d^3)^(1/4)*a*b^2*c^2*d + (c*d
^3)^(1/4)*a^2*b*c*d^2 + (c*d^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + s
qrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^4) + 3/16*sqrt(2)*(3*(c*d^3)^(1/4)
*b^3*c^3 - 5*(c*d^3)^(1/4)*a*b^2*c^2*d + (c*d^3)^(1/4)*a^2*b*c*d^2 + (c*d^
3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/
4))/(c^2*d^4) + 3/32*sqrt(2)*(3*(c*d^3)^(1/4)*b^3*c^3 - 5*(c*d^3)^(1/4)*a*
b^2*c^2*d + (c*d^3)^(1/4)*a^2*b*c*d^2 + (c*d^3)^(1/4)*a^3*d^3)*log(x^2 + s
qrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^4) - 3/32*sqrt(2)*(3*(c*d^3)^(1/4
)*b^3*c^3 - 5*(c*d^3)^(1/4)*a*b^2*c^2*d + (c*d^3)^(1/4)*a^2*b*c*d^2 + (c*d
^3)^(1/4)*a^3*d^3)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^4)
- 1/4*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((d*x^4
+ c)*c*d^3) + 1/5*(b^3*d^8*x^5 - 10*b^3*c*d^7*x + 15*a*b^2*d^8*x)/d^10
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1615, normalized size of antiderivative = 6.49

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^4)^3/(c + d*x^4)^2,x)
```

output
```
x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^5)/(5*d^2) + (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*c*(c*d^3 + d^4*x^4)) + (atan((((a
*d - b*c)^2*(a*d + 3*b*c)*((9*x*(a^6*d^6 + 9*b^6*c^6 + 31*a^2*b^4*c^4*d^2
- 4*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d + 2*a^5*b*c*d^5))
/(4*c^2*d^3) - (3*(a*d - b*c)^2*(a*d + 3*b*c)*(12*a^3*d^3 + 36*b^3*c^3 - 6
0*a*b^2*c^2*d + 12*a^2*b*c*d^2))/(16*(-c)^(7/4)*d^(13/4)))*3i)/(16*(-c)^(7
/4)*d^(13/4)) + ((a*d - b*c)^2*(a*d + 3*b*c)*((9*x*(a^6*d^6 + 9*b^6*c^6 +
31*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*
d + 2*a^5*b*c*d^5))/(4*c^2*d^3) + (3*(a*d - b*c)^2*(a*d + 3*b*c)*(12*a^3*d
^3 + 36*b^3*c^3 - 60*a*b^2*c^2*d + 12*a^2*b*c*d^2))/(16*(-c)^(7/4)*d^(13/4
)))*3i)/(16*(-c)^(7/4)*d^(13/4)))/((3*(a*d - b*c)^2*(a*d + 3*b*c)*((9*x*(a
^6*d^6 + 9*b^6*c^6 + 31*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^
2*d^4 - 30*a*b^5*c^5*d + 2*a^5*b*c*d^5))/(4*c^2*d^3) - (3*(a*d - b*c)^2*(a
*d + 3*b*c)*(12*a^3*d^3 + 36*b^3*c^3 - 60*a*b^2*c^2*d + 12*a^2*b*c*d^2))/(
16*(-c)^(7/4)*d^(13/4))))/(16*(-c)^(7/4)*d^(13/4)) - (3*(a*d - b*c)^2*(a*d
 + 3*b*c)*((9*x*(a^6*d^6 + 9*b^6*c^6 + 31*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*
d^3 - 9*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d + 2*a^5*b*c*d^5))/(4*c^2*d^3) + (
3*(a*d - b*c)^2*(a*d + 3*b*c)*(12*a^3*d^3 + 36*b^3*c^3 - 60*a*b^2*c^2*d +
12*a^2*b*c*d^2))/(16*(-c)^(7/4)*d^(13/4))))/(16*(-c)^(7/4)*d^(13/4))))*(a*
d - b*c)^2*(a*d + 3*b*c)*3i)/(8*(-c)^(7/4)*d^(13/4)) + (3*atan(((3*(a*d...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1347, normalized size of antiderivative = 5.41

$$\int \frac{(a+bx^4)^3}{(c+dx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^4+a)^3/(d*x^4+c)^2,x)
```

output
```
( - 30*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*c*d**3 - 30*d**(3/4)*c**(1/4)*sqrt
(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt
(2)))*a**3*d**4*x**4 - 30*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4
)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c**2*d**2 - 3
0*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)
/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c*d**3*x**4 + 150*d**(3/4)*c**(1/4)*s
qrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*s
qrt(2)))*a*b**2*c**3*d + 150*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(
1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a*b**2*c**2*d**2*
x**4 - 90*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sq
rt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*b**3*c**4 - 90*d**(3/4)*c**(1/4)*sqr
t(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqr
t(2)))*b**3*c**3*d*x**4 + 30*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(
1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**3*c*d**3 + 30*
d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(
d**(1/4)*c**(1/4)*sqrt(2)))*a**3*d**4*x**4 + 30*d**(3/4)*c**(1/4)*sqrt(2)*
atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))
)*a**2*b*c**2*d**2 + 30*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*
sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*a**2*b*c*d**3*x**4 ...
```

3.216 $\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\begin{aligned}\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx &= -\frac{c^3}{a^2x}+\frac{d^3x^3}{3b^2}-\frac{(bc-ad)^3x^3}{4a^2b^2(a+bx^4)}\\ &+\frac{(bc-ad)^2(5bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}b^{11/4}}\\ &-\frac{(bc-ad)^2(5bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}b^{11/4}}\\ &+\frac{(bc-ad)^2(5bc+7ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{9/4}b^{11/4}}\end{aligned}$$

output
```
-c^3/a^2/x+1/3*d^3*x^3/b^2-1/4*(-a*d+b*c)^3*x^3/a^2/b^2/(b*x^4+a)-1/16*(-a
*d+b*c)^2*(7*a*d+5*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(9/
4)/b^(11/4)-1/16*(-a*d+b*c)^2*(7*a*d+5*b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(
1/4))*2^(1/2)/a^(9/4)/b^(11/4)+1/16*(-a*d+b*c)^2*(7*a*d+5*b*c)*arctanh(2^(
1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(9/4)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \frac{1}{96}\left(-\frac{96c^3}{a^2x}+\frac{32d^3x^3}{b^2}+\frac{24(-bc+ad)^3x^3}{a^2b^2(a+bx^4)} + \frac{6\sqrt{2}(bc-ad)^2(5bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}b^{11/4}} - \frac{6\sqrt{2}(bc-ad)^2(5bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}b^{11/4}} - \frac{3\sqrt{2}(bc-ad)^2(5bc+7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{9/4}b^{11/4}} + \frac{3\sqrt{2}(bc-ad)^2(5bc+7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{9/4}b^{11/4}}\right)$$

input
```
Integrate[(c + d*x^4)^3/(x^2*(a + b*x^4)^2),x]
```

output
```
((-96*c^3)/(a^2*x) + (32*d^3*x^3)/b^2 + (24*(-(b*c) + a*d)^3*x^3)/(a^2*b^2
*(a + b*x^4)) + (6*Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*x)/a^(1/4)])/(a^(9/4)*b^(11/4)) - (6*Sqrt[2]*(b*c - a*d)^2*(5*b
*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(9/4)*b^(11/4)) -
(3*Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/(a^(9/4)*b^(11/4)) + (3*Sqrt[2]*(b*c - a*d)^2*(5*b*
c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(9/4
)*b^(11/4)))/96
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx$$

$$\Big\downarrow \text{968}$$

$$\frac{(c+dx^4)^2(bc-ad)}{4abx(a+bx^4)} - \frac{\int -\frac{(dx^4+c)(c(5bc-ad)-d(3bc-7ad)x^4)}{x^2(bx^4+a)}dx}{4ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^4+c)(c(5bc-ad)-d(3bc-7ad)x^4)}{x^2(bx^4+a)}dx}{4ab} + \frac{(c+dx^4)^2(bc-ad)}{4abx(a+bx^4)}$$

$$\Big\downarrow \text{1040}$$

$$\frac{\int \left(-\frac{(ad-5bc)c^2}{ax^2} - \frac{d^2(3bc-7ad)x^2}{b} - \frac{(ad-bc)^2(5bc+7ad)x^2}{ab(bx^4+a)}\right)dx}{4ab} + \frac{(c+dx^4)^2(bc-ad)}{4abx(a+bx^4)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(7ad+5bc)}{2\sqrt{2}a^{5/4}b^{7/4}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(7ad+5bc)}{2\sqrt{2}a^{5/4}b^{7/4}} - \frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^{2}\right.}{4\sqrt{2}a^{5/4}b^{7/4}}}{4ab}$$
$$\frac{(c+dx^4)^2(bc-ad)}{4abx(a+bx^4)}$$

input
```
Int[(c + d*x^4)^3/(x^2*(a + b*x^4)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^4)^2)/(4*a*b*x*(a + b*x^4)) + (-((c^2*(5*b*c - a*d))
/(a*x)) - (d^2*(3*b*c - 7*a*d)*x^3)/(3*b) + ((b*c - a*d)^2*(5*b*c + 7*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*b^(7/4)) - ((
b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(5/4)*b^(7/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((b*c
 - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(4*Sqrt[2]*a^(5/4)*b^(7/4)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

method	result
default	$\frac{d^3x^3}{3b^2} - \frac{\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x^3}{b\,x^4+a} + \frac{\left(\frac{7}{4}a^3d^3-\frac{3}{4}a\,b^2c^2d+\frac{5}{4}b^3c^3-\frac{9}{4}a^2bc\,d^2\right)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{}}{\left(\frac{a}{b}\right.}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^2b^2}$
risch	$\frac{d^3x^3}{3b^2} + \frac{\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-5b^3c^3\right)x^4}{4a^2}-\frac{c^3b^2}{a}}{b^2x\left(b\,x^4+a\right)} + \frac{\_R=\mathrm{RootOf}\left(2401a^{12}d^{12}-12348a^{11}bc\,d^{11}+19698a^{10}b^2c^2d^{10}+2324a^9b^3c^3d^9-37\right.}{}$

input
```
int((d*x^4+c)^3/x^2/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^3*x^3/b^2-1/a^2/b^2*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/4*a*b^2*c^2*d+1
/4*b^3*c^3)*x^3/(b*x^4+a)+1/8*(7/4*a^3*d^3-3/4*a*b^2*c^2*d+5/4*b^3*c^3-9/4
*a^2*b*c*d^2)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(
1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1)))-c^3/a^2/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 2090, normalized size of antiderivative = 8.46

$$\int \frac{\left(c+dx^4\right)^3}{x^2\left(a+bx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/x^2/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/48*(16*a^2*b*d^3*x^8 - 48*a*b^2*c^3 - 4*(15*b^3*c^3 - 9*a*b^2*c^2*d + 9*
a^2*b*c*d^2 - 7*a^3*d^3)*x^4 - 3*(a^2*b^3*x^5 + a^3*b^2*x)*(-(625*b^12*c^1
2 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 +
1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 271
44*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*
a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4
)*log(a^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d
^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5
+ 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 +
2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 240
1*a^12*d^12)/(a^9*b^11))^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*
b^7*c^7*d^2 + 1308*a^3*b^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^
4*d^5 + 1140*a^6*b^3*c^3*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 3
43*a^9*d^9)*x) - 3*(-I*a^2*b^3*x^5 - I*a^3*b^2*x)*(-(625*b^12*c^12 - 1500*
a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*
b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^
5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*
c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(I*a
^7*b^8*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11
060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19...
```

Sympy [A] (verification not implemented)

Time = 21.43 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \frac{-4ab^2c^3+x^4(a^3d^3-3a^2bcd^2+3ab^2c^2d-5b^3c^3)}{4a^3b^2x+4a^2b^3x^5}$$
$$+\mathrm{RootSum}\Big(65536t^4a^9b^{11}+2401a^{12}d^{12}-12348a^{11}bcd^{11}+19698a^{10}b^2c^2d^{10}+2324a^9b^3c^3d^9-37665a^8b$$
$$+\frac{d^3x^3}{3b^2}$$

input
```
integrate((d*x**4+c)**3/x**2/(b*x**4+a)**2,x)
```

output
```
(-4*a*b**2*c**3 + x**4*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 5*
b**3*c**3))/(4*a**3*b**2*x + 4*a**2*b**3*x**5) + RootSum(65536*_t**4*a**9*
b**11 + 2401*a**12*d**12 - 12348*a**11*b*c*d**11 + 19698*a**10*b**2*c**2*d
**10 + 2324*a**9*b**3*c**3*d**9 - 37665*a**8*b**4*c**4*d**8 + 27144*a**7*b
**5*c**5*d**7 + 19068*a**6*b**6*c**6*d**6 - 28728*a**5*b**7*c**7*d**5 + 10
71*a**4*b**8*c**8*d**4 + 11060*a**3*b**9*c**9*d**3 - 3150*a**2*b**10*c**10
*d**2 - 1500*a*b**11*c**11*d + 625*b**12*c**12, Lambda(_t, _t*log(-4096*_t
**3*a**7*b**8/(343*a**9*d**9 - 1323*a**8*b*c*d**8 + 1260*a**7*b**2*c**2*d*
*7 + 1140*a**6*b**3*c**3*d**6 - 2430*a**5*b**4*c**4*d**5 + 342*a**4*b**5*c
**5*d**4 + 1308*a**3*b**6*c**6*d**3 - 540*a**2*b**7*c**7*d**2 - 225*a*b**8
*c**8*d + 125*b**9*c**9) + x))) + d**3*x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \frac{d^3x^3}{3\,b^2} - \frac{4\,ab^2c^3 + (5\,b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^4}{4\,(a^2b^3x^5 + a^3b^2x)} - \frac{(5\,b^3c^3 - 3\,ab^2c^2d - 9\,a^2bcd^2 + 7\,a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \right.}{32\,a^2b^2}$$

input
```
integrate((d*x^4+c)^3/x^2/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/3*d^3*x^3/b^2 - 1/4*(4*a*b^2*c^3 + (5*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*x^4)/(a^2*b^3*x^5 + a^3*b^2*x) - 1/32*(5*b^3*c^3 - 3*a*b^
2*c^2*d - 9*a^2*b*c*d^2 + 7*a^3*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt
(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4
)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2
)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)
*b^(3/4)))/(a^2*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(196) = 392$.

Time = 0.13 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \frac{d^3x^3}{3\,b^2} - \frac{5\,b^3c^3x^4 - 3\,ab^2c^2dx^4 + 3\,a^2bcd^2x^4 - a^3d^3x^4 + 4\,ab^2c^3}{4\,(bx^5+ax)a^2b^2} - \frac{\sqrt{2}\Big(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b^5} - \frac{\sqrt{2}\Big(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^3b^5} + \frac{\sqrt{2}\Big(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b^5} - \frac{\sqrt{2}\Big(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^3b^5}$$

input
```
integrate((d*x^4+c)^3/x^2/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/3*d^3*x^3/b^2 - 1/4*(5*b^3*c^3*x^4 - 3*a*b^2*c^2*d*x^4 + 3*a^2*b*c*d^2*x
^4 - a^3*d^3*x^4 + 4*a*b^2*c^3)/((b*x^5 + a*x)*a^2*b^2) - 1/16*sqrt(2)*(5*
(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*
b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)
^(1/4))/(a/b)^(1/4))/(a^3*b^5) - 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3
*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3
*b^5) + 1/32*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*
d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^5) - 1/32*sqrt(2)*(5*(a*b^3)^(3/4)*b^
3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b
^3)^(3/4)*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.64

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \frac{\frac{x^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3)}{4\,a^2}-\frac{b^2\,c^3}{a}}{b^3\,x^5+a\,b^2\,x}+\frac{d^3\,x^3}{3\,b^2}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,(a\,d-b\,c)^2\,(7\,a\,d+5\,b\,c)\,(12544\,a^{13}\,b^8\,d^6-32256\,a^{12}\,b^9\,c\,d^5+9984\,a^{11}\,b^{10}\,c^2\,d^4+31744\,a^{10}\,b^{11}\,c^3\,d^3-20736\cdots}{8\,(-a)^{9/4}\,b^{11/4}\,(10976\,a^{14}\,b^5\,d^9-42336\,a^{13}\,b^6\,c\,d^8+40320\,a^{12}\,b^7\,c^2\,d^7+36480\,a^{11}\,b^8\,c^3\,d^6-77760\,a^{10}\,b^9\,c^4\,d^5+10944\,a^9\,b^{10}\,c^5\,d^4+4\cdots}\cdots}{8\,(-a)^{9/4}\,b^{11/4}}$$
$$-\frac{\mathrm{atan}\left(\frac{x\,(a\,d-b\,c)^2\,(7\,a\,d+5\,b\,c)\,(12544\,a^{13}\,b^8\,d^6-32256\,a^{12}\,b^9\,c\,d^5+9984\,a^{11}\,b^{10}\,c^2\,d^4+31744\,a^{10}\,b^{11}\,c^3\,d^3-20736\,a\cdots}{8\,(-a)^{9/4}\,b^{11/4}\,(10976\,a^{14}\,b^5\,d^9-42336\,a^{13}\,b^6\,c\,d^8+40320\,a^{12}\,b^7\,c^2\,d^7+36480\,a^{11}\,b^8\,c^3\,d^6-77760\,a^{10}\,b^9\,c^4\,d^5+10944\,a^9\,b^{10}\,c^5\,d^4+4\cdots}\cdots}{8\,(-a)^{9/4}\,b^{11/4}}$$

input
```
int((c + d*x^4)^3/(x^2*(a + b*x^4)^2),x)
```

output
```
((x^4*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a^2) - (b^
2*c^3)/a)/(b^3*x^5 + a*b^2*x) + (d^3*x^3)/(3*b^2) - (atan((x*(a*d - b*c)^2
*(7*a*d + 5*b*c)*(6400*a^7*b^14*c^6 + 12544*a^13*b^8*d^6 - 7680*a^8*b^13*c
^5*d - 32256*a^12*b^9*c*d^5 - 20736*a^9*b^12*c^4*d^2 + 31744*a^10*b^11*c^3
*d^3 + 9984*a^11*b^10*c^2*d^4))/(8*(-a)^(9/4)*b^(11/4)*(4000*a^5*b^14*c^9
+ 10976*a^14*b^5*d^9 - 7200*a^6*b^13*c^8*d - 42336*a^13*b^6*c*d^8 - 17280*
a^7*b^12*c^7*d^2 + 41856*a^8*b^11*c^6*d^3 + 10944*a^9*b^10*c^5*d^4 - 77760
*a^10*b^9*c^4*d^5 + 36480*a^11*b^8*c^3*d^6 + 40320*a^12*b^7*c^2*d^7)))*(a*
d - b*c)^2*(7*a*d + 5*b*c))/(8*(-a)^(9/4)*b^(11/4)) - (atan((x*(a*d - b*c)
^2*(7*a*d + 5*b*c)*(6400*a^7*b^14*c^6 + 12544*a^13*b^8*d^6 - 7680*a^8*b^13
*c^5*d - 32256*a^12*b^9*c*d^5 - 20736*a^9*b^12*c^4*d^2 + 31744*a^10*b^11*c
^3*d^3 + 9984*a^11*b^10*c^2*d^4)*1i)/(8*(-a)^(9/4)*b^(11/4)*(4000*a^5*b^14
*c^9 + 10976*a^14*b^5*d^9 - 7200*a^6*b^13*c^8*d - 42336*a^13*b^6*c*d^8 - 1
7280*a^7*b^12*c^7*d^2 + 41856*a^8*b^11*c^6*d^3 + 10944*a^9*b^10*c^5*d^4 -
77760*a^10*b^9*c^4*d^5 + 36480*a^11*b^8*c^3*d^6 + 40320*a^12*b^7*c^2*d^7))
)*(a*d - b*c)^2*(7*a*d + 5*b*c)*1i)/(8*(-a)^(9/4)*b^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1356, normalized size of antiderivative = 5.49

$$\int \frac{(c+dx^4)^3}{x^2(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^3/x^2/(b*x^4+a)^2,x)
```

output
```
(42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3*x - 54*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)
))*a**3*b*c*d**2*x + 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x**5 - 18*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(
b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d*x - 54*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**2*b**2*c*d**2*x**5 + 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**3*x
 - 18*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**2*d*x**5 + 30*b**(1/4)*a**(3/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4
)*sqrt(2)))*b**4*c**3*x**5 - 42*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a
**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3*x +
54*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c*d**2*x - 42*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**3*b*d**3*x**5 + 18*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1
/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*...
```

3.217 $\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = -\frac{c^3}{3a^2x^3}+\frac{d^3x}{b^2}-\frac{(bc-ad)^3x}{4a^2b^2(a+bx^4)} + \frac{(bc-ad)^2(7bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}b^{9/4}} - \frac{(bc-ad)^2(7bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}b^{9/4}} - \frac{(bc-ad)^2(7bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{11/4}b^{9/4}}$$

output

```
-1/3*c^3/a^2/x^3+d^3*x/b^2-1/4*(-a*d+b*c)^3*x/a^2/b^2/(b*x^4+a)-1/16*(-a*d
+b*c)^2*(5*a*d+7*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(11/4
)/b^(9/4)-1/16*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/
4))*2^(1/2)/a^(11/4)/b^(9/4)-1/16*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctanh(2^(1/
2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(11/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \frac{1}{96}\left(-\frac{32c^3}{a^2x^3}+\frac{96d^3x}{b^2}+\frac{24(-bc+ad)^3x}{a^2b^2(a+bx^4)} + \frac{6\sqrt{2}(bc-ad)^2(7bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{11/4}b^{9/4}} - \frac{6\sqrt{2}(bc-ad)^2(7bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{11/4}b^{9/4}} + \frac{3\sqrt{2}(bc-ad)^2(7bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{11/4}b^{9/4}} - \frac{3\sqrt{2}(bc-ad)^2(7bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{a^{11/4}b^{9/4}}\right)$$

input
```
Integrate[(c + d*x^4)^3/(x^4*(a + b*x^4)^2),x]
```

output
```
((-32*c^3)/(a^2*x^3) + (96*d^3*x)/b^2 + (24*(-(b*c) + a*d)^3*x)/(a^2*b^2*(
a + b*x^4)) + (6*Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*x)/a^(1/4)])/(a^(11/4)*b^(9/4)) - (6*Sqrt[2]*(b*c - a*d)^2*(7*b*c
 + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(11/4)*b^(9/4)) + (3
*Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(a^(11/4)*b^(9/4)) - (3*Sqrt[2]*(b*c - a*d)^2*(7*b*c
+ 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(11/4)
*b^(9/4)))/96
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^4)^3}{x^4 (a+bx^4)^2} \, dx$$

$$\Big\downarrow \text{ 968}$$

$$\frac{(c+dx^4)^2 (bc-ad)}{4abx^3 (a+bx^4)}-\frac{\int -\frac{(dx^4+c)(c(7bc-3ad)-d(bc-5ad)x^4)}{x^4(bx^4+a)}dx}{4ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(dx^4+c)(c(7bc-3ad)-d(bc-5ad)x^4)}{x^4(bx^4+a)}dx}{4ab}+\frac{(c+dx^4)^2 (bc-ad)}{4abx^3 (a+bx^4)}$$

$$\Big\downarrow \text{ 1040}$$

$$\frac{\int \left(-\frac{(3ad-7bc)c^2}{ax^4}-\frac{d^2(bc-5ad)}{b}-\frac{(ad-bc)^2(7bc+5ad)}{ab(bx^4+a)}\right)dx}{4ab}+\frac{(c+dx^4)^2 (bc-ad)}{4abx^3 (a+bx^4)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(5ad+7bc)}{2\sqrt{2}a^{7/4}b^{5/4}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(5ad+7bc)}{2\sqrt{2}a^{7/4}b^{5/4}}+\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x\right.}{4\sqrt{2}a^{7/4}b^{5/4}}}{4ab}$$

$$\frac{(c+dx^4)^2 (bc-ad)}{4abx^3 (a+bx^4)}$$

input
```
Int[(c + d*x^4)^3/(x^4*(a + b*x^4)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^4)^2)/(4*a*b*x^3*(a + b*x^4)) + (-1/3*(c^2*(7*b*c -
3*a*d))/(a*x^3) - (d^2*(b*c - 5*a*d)*x)/b + ((b*c - a*d)^2*(7*b*c + 5*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*b^(5/4)) - ((
b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(7/4)*b^(5/4)) + ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*b^(5/4)) - ((b*c
 - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(4*Sqrt[2]*a^(7/4)*b^(5/4)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.89

method	result
default	$\frac{d^3x}{b^2}-\frac{\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x}{b\,x^4+a}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{32a}}{a^2b^2}$
risch	$\frac{d^3x}{b^2}+\frac{\frac{\left(3a^3d^3-9a^2bc\,d^2+9a\,b^2c^2d-7b^3c^3\right)x^4}{12a^2}-\frac{c^3b^2}{3a}}{b^2x^3(b\,x^4+a)}+\frac{\_R=\text{RootOf}\left(625a^{12}d^{12}-1500a^{11}bc\,d^{11}-3150a^{10}b^2c^2d^{10}+11060a^9b^3c^3d^9+1071\right.}{}$

input
```
int((d*x^4+c)^3/x^4/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*x/b^2-1/a^2/b^2*((-1/4*a^3*d^3+3/4*a^2*b*c*d^2-3/4*a*b^2*c^2*d+1/4*b^3
*c^3)*x/(b*x^4+a)+1/32*(5*a^3*d^3-3*a^2*b*c*d^2-9*a*b^2*c^2*d+7*b^3*c^3)*(
a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b
)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)))-1/3*c^3/a^2/x^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 1764, normalized size of antiderivative = 7.29

$$\int \frac{(c+dx^4)^3}{x^4\,(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/x^4/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/48*(48*a^2*b*d^3*x^8 - 16*a*b^2*c^3 - 4*(7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 - 15*a^3*d^3)*x^4 - 3*(a^2*b^3*x^7 + a^3*b^2*x^3)*(-(2401*b^12*
c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^
3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6
- 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3
150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1
/4)*log(a^3*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c
^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7
*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^
8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 +
625*a^12*d^12)/(a^11*b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + 5*a^3*d^3)*x) - 3*(I*a^2*b^3*x^7 + I*a^3*b^2*x^3)*(-(2401*b^12*c^12
- 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 3
7665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 287
28*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a
^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*l
og(I*a^3*b^2*(-(2401*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10
*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^
5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 +
 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**4+c)**3/x**4/(b*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(191) = 382$.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \frac{d^3x}{b^2} - \frac{4\,ab^2c^3 + (7\,b^3c^3 - 9\,ab^2c^2d + 9\,a^2bcd^2 - 3\,a^3d^3)x^4}{12\,(a^2b^3x^7 + a^3b^2x^3)} - \frac{\frac{2\sqrt{2}(7\,b^3c^3 - 9\,ab^2c^2d - 3\,a^2bcd^2 + 5\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(7\,b^3c^3 - 9\,ab^2c^2d - 3\,a^2bcd^2 + 5\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x -\right.}{2\sqrt{\right.}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input

```
integrate((d*x^4+c)^3/x^4/(b*x^4+a)^2,x, algorithm="maxima")
```

output

```
d^3*x/b^2 - 1/12*(4*a*b^2*c^3 + (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 - 3*a^3*d^3)*x^4)/(a^2*b^3*x^7 + a^3*b^2*x^3) - 1/32*(2*sqrt(2)*(7*b^3*c^
3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(
b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 2*sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^
3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(7*b^3*c^3 - 9*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b
^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(191) = 382$.

Time = 0.13 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \frac{d^3x}{b^2} - \frac{c^3}{3a^2x^3} - \frac{\sqrt{2}\left(7(ab^3)^{\frac{1}{4}}b^3c^3 - 9(ab^3)^{\frac{1}{4}}ab^2c^2d - 3(ab^3)^{\frac{1}{4}}a^2bcd^2 + 5(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^3b^3} - \frac{\sqrt{2}\left(7(ab^3)^{\frac{1}{4}}b^3c^3 - 9(ab^3)^{\frac{1}{4}}ab^2c^2d - 3(ab^3)^{\frac{1}{4}}a^2bcd^2 + 5(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^3b^3} - \frac{\sqrt{2}\left(7(ab^3)^{\frac{1}{4}}b^3c^3 - 9(ab^3)^{\frac{1}{4}}ab^2c^2d - 3(ab^3)^{\frac{1}{4}}a^2bcd^2 + 5(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^3b^3} + \frac{\sqrt{2}\left(7(ab^3)^{\frac{1}{4}}b^3c^3 - 9(ab^3)^{\frac{1}{4}}ab^2c^2d - 3(ab^3)^{\frac{1}{4}}a^2bcd^2 + 5(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^3b^3} - \frac{b^3c^3x - 3ab^2c^2dx + 3a^2bcd^2x - a^3d^3x}{4(bx^4+a)a^2b^2}$$

input
```
integrate((d*x^4+c)^3/x^4/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
d^3*x/b^2 - 1/3*c^3/(a^2*x^3) - 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*
(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*
a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*
b^3) - 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d
 - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(
2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^3*b^3) - 1/32*sqrt(2)*(7*(a
*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*
c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(a^3*b^3) + 1/32*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*
b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^3*b^3) - 1/4*(b^3*c^3*x - 3*a*b^2
*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1740, normalized size of antiderivative = 7.19

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^4)^3/(x^4*(a + b*x^4)^2),x)
```

output
```
((x^4*(3*a^3*d^3 - 7*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(12*a^2) -
(b^2*c^3)/(3*a))/(b^3*x^7 + a*b^2*x^3) + (d^3*x)/b^2 - (atan((((a*d - b*c)
^2*(5*a*d + 7*b*c)*(x*(12544*a^6*b^15*c^6 + 6400*a^12*b^9*d^6 - 32256*a^7*
b^14*c^5*d - 7680*a^11*b^10*c*d^5 + 9984*a^8*b^13*c^4*d^2 + 31744*a^9*b^12
*c^3*d^3 - 20736*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(2867
2*a^9*b^14*c^3 + 20480*a^12*b^11*d^3 - 36864*a^10*b^13*c^2*d - 12288*a^11*
b^12*c*d^2))/(16*(-a)^(11/4)*b^(9/4)))*1i)/(16*(-a)^(11/4)*b^(9/4)) + ((a*
d - b*c)^2*(5*a*d + 7*b*c)*(x*(12544*a^6*b^15*c^6 + 6400*a^12*b^9*d^6 - 32
256*a^7*b^14*c^5*d - 7680*a^11*b^10*c*d^5 + 9984*a^8*b^13*c^4*d^2 + 31744*
a^9*b^12*c^3*d^3 - 20736*a^10*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*a*d + 7*b*
c)*(28672*a^9*b^14*c^3 + 20480*a^12*b^11*d^3 - 36864*a^10*b^13*c^2*d - 122
88*a^11*b^12*c*d^2))/(16*(-a)^(11/4)*b^(9/4)))*1i)/(16*(-a)^(11/4)*b^(9/4)
))/(((a*d - b*c)^2*(5*a*d + 7*b*c)*(x*(12544*a^6*b^15*c^6 + 6400*a^12*b^9*
d^6 - 32256*a^7*b^14*c^5*d - 7680*a^11*b^10*c*d^5 + 9984*a^8*b^13*c^4*d^2
+ 31744*a^9*b^12*c^3*d^3 - 20736*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*
d + 7*b*c)*(28672*a^9*b^14*c^3 + 20480*a^12*b^11*d^3 - 36864*a^10*b^13*c^2
*d - 12288*a^11*b^12*c*d^2))/(16*(-a)^(11/4)*b^(9/4))))/(16*(-a)^(11/4)*b^
(9/4)) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(x*(12544*a^6*b^15*c^6 + 6400*a^12
*b^9*d^6 - 32256*a^7*b^14*c^5*d - 7680*a^11*b^10*c*d^5 + 9984*a^8*b^13*c^4
*d^2 + 31744*a^9*b^12*c^3*d^3 - 20736*a^10*b^11*c^2*d^4) + ((a*d - b*c)...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1388, normalized size of antiderivative = 5.74

$$\int \frac{(c+dx^4)^3}{x^4(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^3/x^4/(b*x^4+a)^2,x)
```

output

```
(30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*
x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3*x**3 - 18*b**(3/4)*a**(1/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt
(2)))*a**3*b*c*d**2*x**3 + 30*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x**7
 - 54*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b
)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d*x**3 - 18*b**(3/4)*a**(
1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(
1/4)*sqrt(2)))*a**2*b**2*c*d**2*x**7 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((
b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*
*3*c**3*x**3 - 54*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**2*d*x**7 + 42*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*b**4*c**3*x**7 - 30*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*
*4*d**3*x**3 + 18*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c*d**2*x**3 - 30*b**(
3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(
1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x**7 + 54*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)...
```

3.218 $\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 252

$$\begin{aligned}\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx =& -\frac{c^3}{5a^2x^5}+\frac{c^2(2bc-3ad)}{a^3x}+\frac{(bc-ad)^3x^3}{4a^3b(a+bx^4)}\\ &-\frac{3(bc-ad)^2(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{13/4}b^{7/4}}\\ &+\frac{3(bc-ad)^2(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{13/4}b^{7/4}}\\ &-\frac{3(bc-ad)^2(3bc+ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{13/4}b^{7/4}}\end{aligned}$$

output
```
-1/5*c^3/a^2/x^5+c^2*(-3*a*d+2*b*c)/a^3/x+1/4*(-a*d+b*c)^3*x^3/a^3/b/(b*x^
4+a)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^
(1/2)/a^(13/4)/b^(7/4)+3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1+2^(1/2)*b^(1
/4)*x/a^(1/4))*2^(1/2)/a^(13/4)/b^(7/4)-3/16*(-a*d+b*c)^2*(a*d+3*b*c)*arct
anh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(13/4)/b^(7
/4)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.21

$$\int \frac{\left(c+dx^4\right)^3}{x^6\left(a+bx^4\right)^2}\,dx$$

$$= \frac{-\frac{32a^{5/4}c^3}{x^5} - \frac{160\sqrt[4]{a}c^2(-2bc+3ad)}{x} - \frac{40\sqrt[4]{a}(-bc+ad)^3x^3}{b(a+bx^4)} - \frac{30\sqrt{2}(bc-ad)^2(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{7/4}} + \frac{30\sqrt{2}(bc-ad)^2(3bc+a}{}}{16}$$

input
```
Integrate[(c + d*x^4)^3/(x^6*(a + b*x^4)^2),x]
```

output
```
((-32*a^(5/4)*c^3)/x^5 - (160*a^(1/4)*c^2*(-2*b*c + 3*a*d))/x - (40*a^(1/4
)*(-(b*c) + a*d)^3*x^3)/(b*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(3*b*c
 + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(7/4) + (30*Sqrt[2]*(b*
c - a*d)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/b^(7/4)
+ (15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*x + Sqrt[b]*x^2])/b^(7/4) - (15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/b^(7/4))/(160*a^(1
3/4))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.43, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^4\right)^3}{x^6\left(a+bx^4\right)^2}\,dx$$

$$\Big\downarrow \; 968$$

$$\frac{\left(c+dx^4\right)^2(bc-ad)}{4abx^5\left(a+bx^4\right)} - \frac{\int -\frac{\left(dx^4+c\right)\left(d(bc+3ad)x^4+c(9bc-5ad)\right)}{x^6\left(bx^4+a\right)}dx}{4ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(dx^4+c)(d(bc+3ad)x^4+c(9bc-5ad))}{x^6(bx^4+a)}dx}{4ab}+\frac{(c+dx^4)^2(bc-ad)}{4abx^5(a+bx^4)}$$

$$\Big\downarrow \; 1040$$

$$\frac{\int \left(-\frac{(5ad-9bc)c^2}{ax^6}-\frac{(9b^2c^2-15abdc+2a^2d^2)c}{a^2x^2}+\frac{3(ad-bc)^2(3bc+ad)x^2}{a^2(bx^4+a)}\right)dx}{4ab}+\frac{(c+dx^4)^2(bc-ad)}{4abx^5(a+bx^4)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)(bc-ad)^2(ad+3bc)}{2\sqrt{2}a^{9/4}b^{3/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)(bc-ad)^2(ad+3bc)}{2\sqrt{2}a^{9/4}b^{3/4}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{\ldots}\right.}{4\sqrt{2}a^{9/4}b^{3/4}}}{4ab}$$

$$\frac{(c+dx^4)^2(bc-ad)}{4abx^5(a+bx^4)}$$

input
```
Int[(c + d*x^4)^3/(x^6*(a + b*x^4)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^4)^2)/(4*a*b*x^5*(a + b*x^4)) + (-1/5*(c^2*(9*b*c -
5*a*d))/(a*x^5) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*a^2*d^2))/(a^2*x) - (3*(b
*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt
[2]*a^(9/4)*b^(3/4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*b^(3/4)) + (3*(b*c - a*d)^2*(3*b*c
 + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(9/4)*b^(3/4)) - (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*b^(3/4)))/(4*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.90

method	result
default	$\frac{-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) x^{3}}{4 b \left(b \,x^{4}+a \right)}+\frac{3 \left(a^{3} d^{3}+a^{2} b c \,d^{2}-5 a \,b^{2} c^{2} d +3 b^{3} c^{3}\right) \sqrt{2}\, \left(\ln \left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\right.}{32 b^{2} \left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^{3}}$
risch	$\frac{-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+15 a \,b^{2} c^{2} d -9 b^{3} c^{3}\right) x^{8}}{4 a^{3} b}-\frac{3 c^{2} \left(5 a d -3 c b \right) x^{4}}{5 a^{2}}-\frac{c^{3}}{5 a}}{x^{5} \left(b \,x^{4}+a \right)}+\frac{3 \left(\underset{\textit{\_R} =\operatorname{RootOf}\left(a^{13} b^{7} \textit{\_Z}^{4}+a^{12} d^{12}+4 a^{11} b c \,d^{11}-14 a^{10} b^{2} c^{2} d^{10}-44 a^{9}\right.}{}\right.}{}$

input
```
int((d*x^4+c)^3/x^6/(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b*x^3/(b*x^4+a)+
3/32*(a^3*d^3+a^2*b*c*d^2-5*a*b^2*c^2*d+3*b^3*c^3)/b^2/(a/b)^(1/4)*2^(1/2)
*(ln((x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x-1)))-1/5*c^3/a^2/x^5-c^2*(3*a*d-2*b*c)/a^3/x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 2096, normalized size of antiderivative = 8.32

$$\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^4+c)^3/x^6/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/80*(20*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^8 - 16*a
^2*b*c^3 + 48*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4 + 15*(a^3*b^2*x^9 + a^4*b*
x^5)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a
^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*
c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 -
 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log
(27*a^10*b^5*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2
- 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*
a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c
^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(
3/4) + 27*(27*b^9*c^9 - 135*a*b^8*c^8*d + 252*a^2*b^7*c^7*d^2 - 188*a^3*b^
6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*a^5*b^4*c^4*d^5 - 20*a^6*b^3*c^3*d^6 -
12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 + a^9*d^9)*x) - 15*(I*a^3*b^2*x^9 + I*a
^4*b*x^5)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1
932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6
*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*
d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4
)*log(27*I*a^10*b^5*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1458*a^2*b^10*c^
10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5
 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44...
```

Sympy [A] (verification not implemented)

Time = 175.52 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx$$
$$= \text{RootSum}\Big(65536t^4a^{13}b^7 + 81a^{12}d^{12} + 324a^{11}bcd^{11} - 1134a^{10}b^2c^2d^{10} - 3564a^9b^3c^3d^9 + 10287a^8b^4c^4d^8 +$$
$$+ \frac{-4a^2bc^3 + x^8(-5a^3d^3 + 15a^2bcd^2 - 75ab^2c^2d + 45b^3c^3) + x^4(-60a^2bc^2d + 36ab^2c^3)}{20a^4bx^5 + 20a^3b^2x^9}$$

input
```
integrate((d*x**4+c)**3/x**6/(b*x**4+a)**2,x)
```

output
```
RootSum(65536*_t**4*a**13*b**7 + 81*a**12*d**12 + 324*a**11*b*c*d**11 - 11
34*a**10*b**2*c**2*d**10 - 3564*a**9*b**3*c**3*d**9 + 10287*a**8*b**4*c**4
*d**8 + 11016*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 26568*a**5
*b**7*c**7*d**5 + 84159*a**4*b**8*c**8*d**4 - 156492*a**3*b**9*c**9*d**3 +
 118098*a**2*b**10*c**10*d**2 - 43740*a*b**11*c**11*d + 6561*b**12*c**12,
Lambda(_t, _t*log(4096*_t**3*a**10*b**5/(27*a**9*d**9 + 81*a**8*b*c*d**8 -
 324*a**7*b**2*c**2*d**7 - 540*a**6*b**3*c**3*d**6 + 2106*a**5*b**4*c**4*d
**5 - 162*a**4*b**5*c**5*d**4 - 5076*a**3*b**6*c**6*d**3 + 6804*a**2*b**7*
c**7*d**2 - 3645*a*b**8*c**8*d + 729*b**9*c**9) + x))) + (-4*a**2*b*c**3 +
 x**8*(-5*a**3*d**3 + 15*a**2*b*c*d**2 - 75*a*b**2*c**2*d + 45*b**3*c**3)
+ x**4*(-60*a**2*b*c**2*d + 36*a*b**2*c**3))/(20*a**4*b*x**5 + 20*a**3*b**
2*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx = \frac{5\,(9\,b^3c^3-15\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^8-4\,a^2bc^3+12\,(3\,ab^2c^3-5\,a^2bc^2d)x^4}{20\,(a^3b^2x^9+a^4bx^5)} + \frac{3\,(3\,b^3c^3-5\,ab^2c^2d+a^2bcd^2+a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}}{}\right.}{32\,a^3b}$$

input
```
integrate((d*x^4+c)^3/x^6/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/20*(5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^8 - 4*a^2
*b*c^3 + 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)/(a^3*b^2*x^9 + a^4*b*x^5) +
 3/32*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(
b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(
b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(
a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*
x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(201) = 402$.

Time = 0.13 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx = \frac{b^3c^3x^3 - 3\,ab^2c^2dx^3 + 3\,a^2bcd^2x^3 - a^3d^3x^3}{4\,(bx^4+a)a^3b} + \frac{3\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^4b^4} + \frac{3\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^4b^4} - \frac{3\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^4b^4} + \frac{3\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32\,a^4b^4} + \frac{10\,bc^3x^4 - 15\,ac^2dx^4 - ac^3}{5\,a^3x^5}$$

input
```
integrate((d*x^4+c)^3/x^6/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*(b^3*c^3*x^3 - 3*a*b^2*c^2*d*x^3 + 3*a^2*b*c*d^2*x^3 - a^3*d^3*x^3)/((
b*x^4 + a)*a^3*b) + 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4
)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^4*b^4) + 3/16*sqrt
(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)
*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a
/b)^(1/4))/(a/b)^(1/4))/(a^4*b^4) - 3/32*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3
- 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*
a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^4*b^4) + 3/32*sqr
t(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4
)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(a^4*b^4) + 1/5*(10*b*c^3*x^4 - 15*a*c^2*d*x^4 - a*c^3)/(a^3*x^5
)
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 652, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx^4)^3}{x^6(a+bx^4)^2}\,dx$$
$$= \frac{3\,\mathrm{atan}\left(\frac{3\,x\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(2304\,a^{16}\,b^5\,d^6+4608\,a^{15}\,b^6\,c\,d^5-20736\,a^{14}\,b^7\,c^2\,d^4-9216\,a^{13}\,b^8\,c^3\,d^3+71424\,a^{12}\,b}{8\,(-a)^{13/4}\,b^{7/4}\,(864\,a^{16}\,b^3\,d^9+2592\,a^{15}\,b^4\,c\,d^8-10368\,a^{14}\,b^5\,c^2\,d^7-17280\,a^{13}\,b^6\,c^3\,d^6+67392\,a^{12}\,b^7\,c^4\,d^5-5184\,a^{11}\,b^8\,c^5\,d^4-162432}\right.}{8\,(-a)^{13/4}\,b^{7/4}}$$
$$-\frac{\frac{c^3}{5\,a}+\frac{x^8\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d-9\,b^3\,c^3)}{4\,a^3\,b}+\frac{3\,c^2\,x^4\,(5\,a\,d-3\,b\,c)}{5\,a^2}}{b\,x^9+a\,x^5}$$
$$-\frac{3\,\mathrm{atanh}\left(\frac{3\,x\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(2304\,a^{16}\,b^5\,d^6+4608\,a^{15}\,b^6\,c\,d^5-20736\,a^{14}\,b^7\,c^2\,d^4-9216\,a^{13}\,b^8\,c^3\,d^3+71424}{8\,(-a)^{13/4}\,b^{7/4}\,(864\,a^{16}\,b^3\,d^9+2592\,a^{15}\,b^4\,c\,d^8-10368\,a^{14}\,b^5\,c^2\,d^7-17280\,a^{13}\,b^6\,c^3\,d^6+67392\,a^{12}\,b^7\,c^4\,d^5-5184\,a^{11}\,b^8\,c^5\,d^4-16}\right.}{8\,(-a)^{13/4}\,b^{7/4}}$$

input
```
int((c + d*x^4)^3/(x^6*(a + b*x^4)^2),x)
```

output
```
(3*atan((3*x*(a*d - b*c)^2*(a*d + 3*b*c)*(20736*a^10*b^11*c^6 + 2304*a^16*
b^5*d^6 - 69120*a^11*b^10*c^5*d + 4608*a^15*b^6*c*d^5 + 71424*a^12*b^9*c^4
*d^2 - 9216*a^13*b^8*c^3*d^3 - 20736*a^14*b^7*c^2*d^4))/(8*(-a)^(13/4)*b^(
7/4)*(23328*a^7*b^12*c^9 + 864*a^16*b^3*d^9 - 116640*a^8*b^11*c^8*d + 2592
*a^15*b^4*c*d^8 + 217728*a^9*b^10*c^7*d^2 - 162432*a^10*b^9*c^6*d^3 - 5184
*a^11*b^8*c^5*d^4 + 67392*a^12*b^7*c^4*d^5 - 17280*a^13*b^6*c^3*d^6 - 1036
8*a^14*b^5*c^2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(8*(-a)^(13/4)*b^(7/4))
 - (c^3/(5*a) + (x^8*(a^3*d^3 - 9*b^3*c^3 + 15*a*b^2*c^2*d - 3*a^2*b*c*d^2
))/(4*a^3*b) + (3*c^2*x^4*(5*a*d - 3*b*c))/(5*a^2))/(a*x^5 + b*x^9) - (3*a
tanh((3*x*(a*d - b*c)^2*(a*d + 3*b*c)*(20736*a^10*b^11*c^6 + 2304*a^16*b^5
*d^6 - 69120*a^11*b^10*c^5*d + 4608*a^15*b^6*c*d^5 + 71424*a^12*b^9*c^4*d^
2 - 9216*a^13*b^8*c^3*d^3 - 20736*a^14*b^7*c^2*d^4))/(8*(-a)^(13/4)*b^(7/4
)*(23328*a^7*b^12*c^9 + 864*a^16*b^3*d^9 - 116640*a^8*b^11*c^8*d + 2592*a^
15*b^4*c*d^8 + 217728*a^9*b^10*c^7*d^2 - 162432*a^10*b^9*c^6*d^3 - 5184*a^
11*b^8*c^5*d^4 + 67392*a^12*b^7*c^4*d^5 - 17280*a^13*b^6*c^3*d^6 - 10368*a
^14*b^5*c^2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(8*(-a)^(13/4)*b^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1403, normalized size of antiderivative = 5.57

$$\int \frac{(c + dx^4)^3}{x^6 (a + bx^4)^2} \, dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^3/x^6/(b*x^4+a)^2,x)
```

output
```
( - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**4*d**3*x**5 - 30*b**(1/4)*a**(3/4)*s
qrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*s
qrt(2)))*a**3*b*c*d**2*x**5 - 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*
a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x
**9 + 150*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sq
rt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b**2*c**2*d*x**5 - 30*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*
a**(1/4)*sqrt(2)))*a**2*b**2*c*d**2*x**9 - 90*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*
a*b**3*c**3*x**5 + 150*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**3*c**2*d*x**9 - 90
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(b)*x)/
(b**(1/4)*a**(1/4)*sqrt(2)))*b**4*c**3*x**9 + 30*b**(1/4)*a**(3/4)*sqrt(2)
*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)
))*a**4*d**3*x**5 + 30*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*c*d**2*x**5 + 30
*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/
(b**(1/4)*a**(1/4)*sqrt(2)))*a**3*b*d**3*x**9 - 150*b**(1/4)*a**(3/4)*sqrt
(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(b)*x)/(b**(1/4)*a**(1/4)*s...
```

3.219 $\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{a^2\sqrt{c+dx^4}}{2b^3} - \frac{(bc+ad)\left(c+dx^4\right)^{3/2}}{6b^2d^2} + \frac{\left(c+dx^4\right)^{5/2}}{10bd^2} - \frac{a^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

output
```
1/2*a^2*(d*x^4+c)^(1/2)/b^3-1/6*(a*d+b*c)*(d*x^4+c)^(3/2)/b^2/d^2+1/10*(d*
x^4+c)^(5/2)/b/d^2-1/2*a^2*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^4+c)^(1/2
)/(-a*d+b*c)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{c+dx^4}\left(15a^2d^2-5abd(c+dx^4)+b^2(-2c^2+cdx^4+3d^2x^8)\right)}{30b^3d^2} - \frac{a^2\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^11*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(Sqrt[c + d*x^4]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^4) + b^2*(-2*c^2 + c*d*x^4
 + 3*d^2*x^8)))/(30*b^3*d^2) - (a^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqr
t[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{x^8\sqrt{dx^4+c}}{bx^4+a}dx^4$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{4}\int \left(\frac{\sqrt{dx^4+c}a^2}{b^2\left(bx^4+a\right)}+\frac{\left(dx^4+c\right)^{3/2}}{bd}+\frac{(-bc-ad)\sqrt{dx^4+c}}{b^2d}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{2a^2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{c+dx^4}}{b^3}-\frac{2\left(c+dx^4\right)^{3/2}(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^4\right)^{5/2}}{5bd^2}\right)$$

input
```
Int[(x^11*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((2*a^2*Sqrt[c + d*x^4])/b^3 - (2*(b*c + a*d)*(c + d*x^4)^(3/2))/(3*b^2*d^
2) + (2*(c + d*x^4)^(5/2))/(5*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/b^(7/2))/4
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{-\left(\left(-\frac{2\left(-\frac{3 d x^4}{2}+c\right)\left(d x^4+c\right) b^2}{15}-\frac{\left(d x^4+c\right) a b d}{3}+a^2 d^2\right)\sqrt{(ad-cb)b}\,\sqrt{d x^4+c}+a^2 d^2 (ad-cb)\arctan\left(\frac{\sqrt{d x^4+c}\, b}{\sqrt{(ad-cb)b}}\right)\right)}{2\sqrt{(ad-cb)b}\, d^2 b^3}$
risch	$\frac{\left(3b^2d^2x^8-5x^4ab\,d^2+x^4b^2cd+15a^2d^2-5abcd-2b^2c^2\right)\sqrt{d\,x^4+c}}{30d^2b^3}-\frac{a^2(ad-cb)\left(-\frac{\ln\left(-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
default	$-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}\left(-3d\,x^4+2c\right)}{30b\,d^2}+\frac{a^2\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{b\left(\frac{x^4\left(d\,x^4+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^4+c\right)^{\frac{3}{2}}}{15d^2}\right)-\frac{a\left(d\,x^4+c\right)^{\frac{3}{2}}}{3d}}{2b^2}+\frac{a^2\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input

```
int(x^11*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output

```
-1/2/((a*d-b*c)*b)^(1/2)*(-(-2/15*(-3/2*d*x^4+c)*(d*x^4+c)*b^2-1/3*(d*x^4+
c)*a*b*d+a^2*d^2)*((a*d-b*c)*b)^(1/2)*(d*x^4+c)^(1/2)+a^2*d^2*(a*d-b*c)*ar
ctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))/d^2/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.24

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

$$= \left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,(3\,b^2d^2x^8+(b^2cd-5\,abd^2)x^4-2\,b^2c^2-5\,abcd+}{60\,b^3d^2}\right.$$

$$-\frac{15\,a^2d^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(3\,b^2d^2x^8+(b^2cd-5\,abd^2)x^4-2\,b^2c^2-5\,abcd+15\,a}{30\,b^3d^2}$$

input
```
integrate(x^11*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/60*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(
d*x^4 + c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(3*b^2*d^2*x^8 + (b^2*c
*d - 5*a*b*d^2)*x^4 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2)*sqrt(d*x^4 + c))
/(b^3*d^2), -1/30*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^8 + (b^2*c*d - 5*a*b*d
^2)*x^4 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2)*sqrt(d*x^4 + c))/(b^3*d^2)]
```

Sympy [A] (verification not implemented)

Time = 12.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.34

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(\frac{a^2d^3\sqrt{c+dx^4}}{4b^3} - \frac{a^2d^3(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^4\sqrt{\frac{ad-bc}{b}}} + \frac{d(c+dx^4)^{\frac{5}{2}}}{20b} + \frac{(c+dx^4)^{\frac{3}{2}}(-ad^2-bcd)}{12b^2}\right)}{d^3} & \text{for } d \neq 0 \\ \sqrt{c}\left(\frac{a^2\left(\begin{cases} \frac{x^4}{a} & \text{for } b=0 \\ \frac{\log(a+bx^4)}{b} & \text{otherwise} \end{cases}\right)}{4b^2} - \frac{ax^4}{4b^2} + \frac{x^8}{8b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Piecewise((2*(a**2*d**3*sqrt(c + d*x**4)/(4*b**3) - a**2*d**3*(a*d - b*c)*
atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*b**4*sqrt((a*d - b*c)/b)) +
d*(c + d*x**4)**(5/2)/(20*b) + (c + d*x**4)**(3/2)*(-a*d**2 - b*c*d)/(12*b
**2))/d**3, Ne(d, 0)), (sqrt(c)*(a**2*Piecewise((x**4/a, Eq(b, 0)), (log(a
 + b*x**4)/b, True))/(4*b**2) - a*x**4/(4*b**2) + x**8/(8*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(a^2bc-a^3d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^3} + \frac{3\,(dx^4+c)^{\frac{5}{2}}b^4d^8-5\,(dx^4+c)^{\frac{3}{2}}b^4cd^8-5\,(dx^4+c)^{\frac{3}{2}}ab^3d^9+15\sqrt{dx^4+c}a^2b^2d^{10}}{30\,b^5d^{10}}$$

input
```
integrate(x^11*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt
(-b^2*c + a*b*d)*b^3) + 1/30*(3*(d*x^4 + c)^(5/2)*b^4*d^8 - 5*(d*x^4 + c)^
(3/2)*b^4*c*d^8 - 5*(d*x^4 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^4 + c)*a^2*b
^2*d^10)/(b^5*d^10)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.57

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \left(\frac{c^2}{2\,b\,d^2}+\frac{\left(\frac{c}{b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\,(2\,a\,d^3-2\,b\,c\,d^2)}{2\,b\,d^2}\right)\sqrt{d\,x^4+c} - \left(\frac{c}{3\,b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{12\,b^2\,d^4}\right)\,{\left(d\,x^4+c\right)}^{3/2} + \frac{{\left(d\,x^4+c\right)}^{5/2}}{10\,b\,d^2} - \frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^3\,d-a^2\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{7/2}}$$

input
```
int((x^11*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
(c^2/(2*b*d^2) + ((c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4))*(2*a*d^3
 - 2*b*c*d^2))/(2*b*d^2))*(c + d*x^4)^(1/2) - (c/(3*b*d^2) + (2*a*d^3 - 2*
b*c*d^2)/(12*b^2*d^4))*(c + d*x^4)^(3/2) + (c + d*x^4)^(5/2)/(10*b*d^2) -
(a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^3*d - a^2*b
*c))*(a*d - b*c)^(1/2))/(2*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1108, normalized size of antiderivative = 8.86

$$\int \frac{x^{11}\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int(x^11*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
( - 15*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt
(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d
 - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*c**2*d**2 - 180*sqrt(b)*sqrt
(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 +
 sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*s
qrt(a*d - b*c)*x**2))*a**2*c*d**3*x**4 - 240*sqrt(b)*sqrt(c + d*x**4)*sqrt
(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt
(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x*
*2))*a**2*d**4*x**8 - 75*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqr
t(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)
*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*c**2*d**2*x**2 - 30
0*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x
**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt
(d)*sqrt(a*d - b*c)*x**2))*a**2*c*d**3*x**6 - 240*sqrt(d)*sqrt(b)*sqrt(a*d
 - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*
d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))
*a**2*d**4*x**10 + 75*sqrt(d)*sqrt(c + d*x**4)*a**2*b*c**2*d**2*x**2 + 300
*sqrt(d)*sqrt(c + d*x**4)*a**2*b*c*d**3*x**6 + 240*sqrt(d)*sqrt(c + d*x**4
)*a**2*b*d**4*x**10 - 25*sqrt(d)*sqrt(c + d*x**4)*a*b**2*c**3*d*x**2 - 125
*sqrt(d)*sqrt(c + d*x**4)*a*b**2*c**2*d**2*x**6 - 180*sqrt(d)*sqrt(c + ...
```

3.220 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx=-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{(c+dx^4)^{3/2}}{6bd}+\frac{a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

output
```
-1/2*a*(d*x^4+c)^(1/2)/b^2+1/6*(d*x^4+c)^(3/2)/b/d+1/2*a*(-a*d+b*c)^(1/2)*
arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{c+dx^4}(-3ad+b(c+dx^4))}{6b^2d}+\frac{a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

input
```
Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[-(b*c) + a*
d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4\sqrt{dx^4+c}}{bx^4+a}dx^4$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^4}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b}+\frac{2\sqrt{c+dx^4}}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}+\frac{2\sqrt{c+dx^4}}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2\left(c+dx^4\right)^{3/2}}{3bd}-\frac{a\left(\frac{2\sqrt{c+dx^4}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}\right)$$

input
```
Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((2*(c + d*x^4)^(3/2))/(3*b*d) - (a*((2*Sqrt[c + d*x^4])/b - (2*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/b^(3/2)))/b)/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}\,\left(-d b\,x^4+3 a d-c b\right)}{3}+\frac{a d (a d-c b) \arctan\left(\frac{\sqrt{d\,x^4+c}\, b}{\sqrt{(a d-c b) b}}\right)}{\sqrt{(a d-c b) b}}}{2 b^2 d}$
risch	$-\frac{\left(-d b\,x^4+3 a d-c b\right)\sqrt{d\,x^4+c}}{6 d\,b^2}+\frac{a(ad-cb)\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$
elliptic	$\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6bd}-\frac{a\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)\cdots}\right)}{b}\cdots\right)}{\cdots}$
default	$\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6bd}-\frac{a\left(\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2\cdots}\right)}{b}\cdots}{\cdots}\cdots\right)}{\cdots}$

input
```
int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(-1/3*(d*x^4+c)^(1/2)*(-b*d*x^4+3*a*d-b*c)+a*d*(a*d-b*c)/((a*d-b*c
)*b)^(1/2)*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,(bdx^4+bc-3\,ad)\sqrt{dx^4+c}}{12\,b^2d}, \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\arctan\left(\right.}{}\right.$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
 + c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt
(d*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x
^4 + c))/(b^2*d)]
```

Sympy [A] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^4}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^4\right)}{b} & \text{otherwise}\end{cases}\right)}{4b}+\frac{x^4}{4b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Piecewise((2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c
+ d*x**4)**(3/2)/(12*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**4/a,
Eq(b, 0)), (log(a + b*x**4)/b, True))/(4*b) + x**4/(4*b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^4+c}abd^3}{6\,b^3d^3}$$

input
```
integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*
a*b*d^3)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\left(d\,x^4+c\right)^{3/2}}{6\,b\,d} - \frac{a\,\sqrt{d\,x^4+c}}{2\,b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

input
```
int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(
1/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/
2))/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 583, normalized size of antiderivative = 6.27

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{3\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{b}\,c+\sqrt{b}\,d\,x^4}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}\,\sqrt{ad-bc}\,x^2}\right)acd+12\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{a}}{\sqrt{d\,x^4+c}}\right.}{}$$

input
```
int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output

```
(3*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c +
 d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b
*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*c*d + 12*sqrt(b)*sqrt(c + d*x**4)*s
qrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + s
qrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)
*x**2))*a*d**2*x**4 + 9*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt
(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*
sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*c*d*x**2 + 12*sqrt(d)*s
qrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(
b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*
d - b*c)*x**2))*a*d**2*x**6 - 9*sqrt(d)*sqrt(c + d*x**4)*a*b*c*d*x**2 - 12
*sqrt(d)*sqrt(c + d*x**4)*a*b*d**2*x**6 + 3*sqrt(d)*sqrt(c + d*x**4)*b**2*
c**2*x**2 + 7*sqrt(d)*sqrt(c + d*x**4)*b**2*c*d*x**6 + 4*sqrt(d)*sqrt(c +
d*x**4)*b**2*d**2*x**10 - 3*a*b*c**2*d - 15*a*b*c*d**2*x**4 - 12*a*b*d**3*
x**8 + b**2*c**3 + 6*b**2*c**2*d*x**4 + 9*b**2*c*d**2*x**8 + 4*b**2*d**3*x
**12)/(6*b**3*d*(sqrt(c + d*x**4)*c + 4*sqrt(c + d*x**4)*d*x**4 + 3*sqrt(d
)*c*x**2 + 4*sqrt(d)*d*x**6))
```

3.221 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

output

```
1/2*(d*x^4+c)^(1/2)/b-1/2*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b}-\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}\right)$$

input

```
Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output

```
(Sqrt[c + d*x^4]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[-(b*c) + a*d]])/b^(3/2))/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^4$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{4}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{b}+\frac{2\sqrt{c+dx^4}}{b}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}+\frac{2\sqrt{c+dx^4}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((2*Sqrt[c + d*x^4])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[b*c - a*d]])/b^(3/2))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{\sqrt{d\,x^{4}+c}-\frac{(ad-cb)\arctan\left(\frac{\sqrt{d\,x^{4}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{2b}$
risch	$\frac{\sqrt{d\,x^{4}+c}}{2b}-\frac{(ad-cb)\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-c}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\right)}{b}$
default	$\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\sqrt{}\right.}{b}}\right.}{b}$
elliptic	$\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\sqrt{}\right.}{b}}\right.}{b}$

input
```
int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*((d*x^4+c)^(1/2)-(a*d-b*c)*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1
/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.23

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b},\right.$$
$$\left.-\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*
c - a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
sqrt(d*x^4 + c))/b]
```

Sympy [A] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for}\ d\neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^4}{4a} & \text{for}\ b=0 \\ \frac{\log\left(4a+4bx^4\right)}{4b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Piecewise((2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**
4)/sqrt((a*d - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(
c)*Piecewise((x**4/(4*a), Eq(b, 0)), (log(4*a + 4*b*x**4)/(4*b), True)), T
rue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b} + \frac{\sqrt{dx^4+c}}{2b}$$

input
```
integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

input
```
int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
(c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.11

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{-\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{b}\,c+\sqrt{b}\,d\,x^4}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}\,\sqrt{ad-bc}\,x^2}\right) - \sqrt{d}\,\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{b}\,c+}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}\,\sqrt{a}}\right.}{2b^2\left(\sqrt{d\,x^4+c}+\sqrt{d}\,x^2\right)}$$

input
```
int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
( - sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c
+ d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d -
b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2)) - sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*at
an((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(s
qrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*x**2 + sq
rt(d)*sqrt(c + d*x**4)*b*x**2 + b*c + b*d*x**4)/(2*b**2*(sqrt(c + d*x**4)
+ sqrt(d)*x**2))
```

3.222 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}+\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

output
```
-1/2*c^(1/2)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a+1/2*(-a*d+b*c)^(1/2)*arcta
nh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx=\frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}}-\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

output
```
((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])
/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{x^4\left(bx^4+a\right)}dx^4$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2c\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

output
```
((-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{\sqrt{c}}\right)+\frac{(a d-c b)\arctan\left(\frac{\sqrt{d\,x^{4}+c}\,b}{\sqrt{(a d-c b) b}}\right)}{\sqrt{(a d-c b) b}}}{2 a}$
elliptic	$\frac{\sqrt{d\,x^{4}+c}-\sqrt{c}\,\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{2 a}-\frac{\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}+\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{\frac{d\sqrt{-a b}}{b}+\left(x^{2}-\frac{\sqrt{-}}{}\right.}{\sqrt{d}}\right.}{}}{}$
default	$\frac{\frac{\sqrt{d\,x^{4}+c}}{2}-\frac{\sqrt{c}\,\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{2}}{a}-\frac{b\left(\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}+\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{\frac{d\sqrt{-a b}}{b}+\left(x^{2}-\right.}{\sqrt{d}}\right.}{}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-c^(1/2)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+(a*d-b*c)*arctan((d*x^4+c
)^(1/2)*b/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.44

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx$$
$$=\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,a},\ \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}b}{bc-}\right.}{}\right.$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b
*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c
)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^
4 + c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^4
+ c)) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)
*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^4 + c)))/a]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.

Time = 7.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx = \begin{cases} \frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for}\ d\neq 0 \\ \sqrt{c}\left(-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^4}{a} & \text{for}\ b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{2a}-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^4}{a} & \text{for}\ b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^4\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{2a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

output
```
Piecewise((2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d
- b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/
b)))/d, Ne(d, 0)), (sqrt(c)*(-b*Piecewise(((a/(2*b) + x**4)/a, Eq(b, 0)),
(-log(a - 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a) - b*Piecewise(((a/(2*b
) + x**4)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**4))/(2*b), True))/(2*a)
), True))
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a}+\frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

input
```
integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x\,(a+bx^4)}\,dx = \frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a}+\frac{\mathrm{atanh}\left(\frac{a\,b^2\,c\,d^3\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b}$$

input
```
int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
```

output
```
(c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 -
a*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(1
6*a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c
*d^3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2
*b^2*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 802, normalized size of antiderivative = 9.44

$$\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^(1/2)/x/(b*x^4+a),x)
```

output
```
(2*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d
- b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x*
*2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)) - 2*sqrt(b)*sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*atan((sqrt(b)*sqrt(c +
d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2
*a*d - b*c))*a*d + 2*sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*
d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*b*c + sqrt(d)*sqrt(b)*sqrt(a)
*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*sqrt(a*d - b*c)*log
( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c
 + d*x**4) + sqrt(d)*sqrt(b)*x**2) - sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt
(d)*sqrt(b)*x**2) + sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d
 + b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqr
t(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*d - sqrt(b)*sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt
(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x*
*2)*b*c - sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*lo
g(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(...
```

3.223 $\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx = -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

output
```
-1/4*(d*x^4+c)^(1/2)/a/x^4+1/4*(-a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2
))/a^2/c^(1/2)-1/2*b^(1/2)*(-a*d+b*c)^(1/2)*arctanh(b^(1/2)*(d*x^4+c)^(1/2
)/(-a*d+b*c)^(1/2))/a^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)} dx = \frac{-\frac{a\sqrt{c+dx^4}}{x^4} - 2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right) + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

output
```
(-((a*Sqrt[c + d*x^4])/x^4) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*
x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^5 (a+bx^4)} dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{\sqrt{dx^4+c}}{x^8 (bx^4+a)}dx^4$$

$$\Big\downarrow \ 110$$

$$\frac{1}{4}\left(\frac{\int -\frac{bdx^4+2bc-ad}{2x^4(bx^4+a)\sqrt{dx^4+c}}dx^4}{a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(-\frac{\int \frac{bdx^4+2bc-ad}{x^4(bx^4+a)\sqrt{dx^4+c}}dx^4}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{4}\left(-\frac{\frac{(2bc-ad)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{2b(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(-\frac{\frac{2(2bc-ad)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{4b(bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(-\frac{\frac{4\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a}-\frac{2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a}-\frac{\sqrt{c+dx^4}}{ax^4}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*x^4)) - ((-2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/S
qrt[c]])/(a*Sqrt[c]) + (4*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^4])/Sqrt[b*c - a*d]])/a)/(2*a))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{4}+c}}{x^{4}}-\frac{(ad-2cb)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2(ad-cb)b\arctan\left(\frac{\sqrt{d\,x^{4}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{4a^{2}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{4a\,x^{4}}-\frac{(ad-2cb)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{2a\sqrt{c}}+\frac{2(ad-cb)b\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-\cdots}}{\cdots}\right.\cdots}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\cdots\right.}{\cdots}$
elliptic	$\frac{-\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{2c\,x^{4}}+\frac{d\left(\sqrt{d\,x^{4}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)\right)}{2c}}{2a}-\frac{b\left(\sqrt{d\,x^{4}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)\right)}{2a^{2}}+\frac{b\left(\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)\cdots}\cdots\right.}{\cdots}$
default	$\frac{-\frac{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}{4c\,x^{4}}-\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{4}+c}}{x^{2}}\right)}{4\sqrt{c}}+\frac{d\sqrt{d\,x^{4}+c}}{4c}}{a}+\frac{b^{2}\left(\frac{\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$

input
```
int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-a*(d*x^4+c)^(1/2)/x^4-(a*d-2*b*c)/c^(1/2)*arctanh((d*x^4+c)^(1/2
)/c^(1/2))-2*(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan((d*x^4+c)^(1/2)*b/((a*
d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.41

$$\int \frac{\sqrt{c+dx^4}}{x^5 (a+bx^4)}\,dx$$
$$= \left[\frac{2\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-(2\,bc-ad)\sqrt{c}x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{dx}}{8\,a^2cx^4}\right.$$
$$-\frac{(2\,bc-ad)\sqrt{-c}x^4\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^4+c}}\right)-\sqrt{b^2c-abd}cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+\sqrt{dx^4+c}ac}{4\,a^2cx^4}$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^
4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((
d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^
2*c*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*
sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4),
-1/4*((2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^4 + c)) - sqrt(b
^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^
2*c - a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt
(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
4 + b*c)) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^4 + c)) -
sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5 (a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^5 (a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^5}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\frac{(b^2c-abd)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}-\frac{\sqrt{dx^4+c}}{4\,ax^4}$$

input
```
integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(
a^2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx=\frac{\mathrm{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{16\,\left(\frac{a\,b^3\,d^5}{16}-\frac{b^4\,c\,d^4}{16}\right)}\right)\,\sqrt{b^2\,c-a\,b\,d}}{2\,a^2}-\frac{\sqrt{d\,x^4+c}}{4\,a\,x^4}-\frac{\mathrm{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^4+c}}{16\,\left(\frac{b^4\,c\,d^4}{16}-\frac{3\,a\,b^3\,d^5}{32}+\frac{a^2\,b^2\,d^6}{32\,c}\right)}-\frac{3\,b^3\,d^5\,\sqrt{d\,x^4+c}}{32\,\sqrt{c}\,\left(\frac{a\,b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32}+\frac{b^4\,c\,d^4}{16\,a}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}}{32\,c^{3/2}\,\left(\frac{b^2\,d^6}{32\,c}-\frac{3\,b^3\,d^5}{32\,a}+\frac{b^4\,c\,d^4}{16\,a^2}\right)}\right)\,\left(a\,d-2\,b\,c\right)}{4\,a^2\,\sqrt{c}}$$

input
```
int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
```

output
```
(atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/
16 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/
(4*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16
 - (3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2
))/(32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a)))
 + (b^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)
/(32*a) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2930, normalized size of antiderivative = 25.48

$$\int \frac{\sqrt{c+dx^4}}{x^5\,(a+bx^4)}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x)
```

output

```
( - 4*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b
*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*d*x
**6 - 2*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2
*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(
b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*c*x**4 - 4
*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d -
b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2
)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*d*x**8 + 4*sqrt(d
)*sqrt(b)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d
- b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*d*x**6 - 4*sqrt(d)*sqrt(b)*sq
rt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*atan(
(sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) + 2*a*d - b*c))*b*c*x**6 + 2*sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)
*sqrt(a*d - b*c) + 2*a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*s
qrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*c*d*
x**4 + 4*sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*ata
n((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)
*sqrt(a*d - b*c) + 2*a*d - b*c))*a*d**2*x**8 - 2*sqrt(b)*sqrt(2*sqrt(d)...
```

3.224 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{x^2\sqrt{c+dx^4}}{4b}-\frac{\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

output
```
1/4*x^2*(d*x^4+c)^(1/2)/b-1/2*a^(1/2)*(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(
1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/b^2+1/4*(-2*a*d+b*c)*arctanh(d^(1/2)*x^2
/(d*x^4+c)^(1/2))/b^2/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$=\frac{b\sqrt{d}x^2\sqrt{c+dx^4}-2\sqrt{a}\sqrt{d}\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)+(bc-2ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx\cdots}\right.}{4b^2\sqrt{d}}$$

input
```
Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(b*Sqrt[d]*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[
(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*
d])] + (b*c - 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 380, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4\sqrt{dx^4+c}}{bx^4+a}dx^2$$

$$\Big\downarrow \text{380}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\int \frac{ac-(bc-2ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2b}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}}{2b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int\frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2a(bc-ad)\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2b}-\frac{\frac{2\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b}\right)$$

input
```
Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((x^2*Sqrt[c + d*x^4])/(2*b) - ((2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*
c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/b - ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(2*b))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{-\frac{2a(ad-cb)\,\mathrm{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\sqrt{d\,x^4+c}\,b\,x^2+\frac{(2ad-cb)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{\sqrt{d}}}{4b^2}$
risch	$\frac{x^2\sqrt{d\,x^4+c}}{4b}-\frac{(2ad-cb)\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}}-\frac{2a(ad-cb)\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-a}}{b}\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{4\sqrt{-ab}\sqrt{-\frac{ad-cb}{b}}}\right.}{}$
default	$\frac{\frac{x^2\sqrt{d\,x^4+c}}{4}+\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{4\sqrt{d}}}{b}-\frac{a\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-}}{}\right.}{\sqrt{d}}\right.}{}\right.}{}$
elliptic	$\frac{\frac{x^2\sqrt{d\,x^4+c}}{2}+\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2\sqrt{d}}}{2b}-\frac{a\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\sqrt{}\right.}{\sqrt{d}}\right.}{}\right.}{}$

input
```
int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*(-2*a*(a*d-b*c)/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2
/(a*(a*d-b*c))^(1/2))-(d*x^4+c)^(1/2)*b*x^2+(2*a*d-b*c)/d^(1/2)*arctanh((d
*x^4+c)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 720, normalized size of antiderivative = 6.00

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}bdx^2-(bc-2ad)\sqrt{d}\log\left(-2dx^4+2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8abc}{}\right.}{8b^2d}\right.$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*s
qrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((
b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8
 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2
*a*d)*sqrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)) + sqrt(-a*b*c + a^
2*d)*d*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c
*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-
a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 +
 c)*b*d*x^2 - 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 -
 a^2*c*d)*x^2)) - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*s
qrt(d)*x^2 - c))/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - sqrt(a*b*c - a^2*
d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2
*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - (b*c - 2*a*d)*s
qrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)))/(b^2*d)]
```

Sympy [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}x^5}{bx^4+a}\,dx$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^5\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 870, normalized size of antiderivative = 7.25

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Too large to display}$$

input
```
int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
(2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)*d*x**2 + 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c
)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt
(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*d*x**2 - 2*sqrt(d)*sqrt(a)*sqrt(c + d
*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*s
qrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*d*x**2 + sqrt(a)*sqrt(a*d - b
*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)
*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*c*d + 2*sqrt(a)*sqrt(a*d - b*c)*
log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqr
t(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*d**2*x**4 + sqrt(a)*sqrt(a*d - b*c)*
log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c
 + d*x**4) + sqrt(d)*sqrt(b)*x**2)*c*d + 2*sqrt(a)*sqrt(a*d - b*c)*log(sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x*
*4) + sqrt(d)*sqrt(b)*x**2)*d**2*x**4 - sqrt(a)*sqrt(a*d - b*c)*log(2*sqrt
(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d +
2*b*d*x**4)*c*d - 2*sqrt(a)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d
 - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*d**2*x**
4 - 4*sqrt(c + d*x**4)*log((sqrt(c + d*x**4) + sqrt(d)*x**2)/sqrt(c))*a*d*
*2*x**2 + 2*sqrt(c + d*x**4)*log((sqrt(c + d*x**4) + sqrt(d)*x**2)/sqrt...
```

3.225 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

output
```
1/2*(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/
a^(1/2)/b+1/2*d^(1/2)*arctanh(d^(1/2)*x^2/(d*x^4+c)^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{2b}$$

input
```
Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]
))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*Log[Sqrt[d]*x^2 + Sqrt[c
+ d*x^4]])/(2*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {965, 301, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{bx^4+a}dx^2$$

$$\Big\downarrow \text{301}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{d\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b}\right)$$

input
```
Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])
/(Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/b)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{\frac{(ad-cb)\,\mathrm{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{2b}$
default	$\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\sqrt{}\right.}{b}}\right.}{b}}{4}$
elliptic	$\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\sqrt{}\right.}{b}}\right.}{b}}{4}$

input
```
int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*((a*d-b*c)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2))/(a*(a
*d-b*c))^(1/2)-d^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 617, normalized size of antiderivative = 6.78

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \left[\frac{2\sqrt{d}\log\left(-2\,dx^4-2\sqrt{dx^4+c}\sqrt{d}x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\right.}{b^2x^8+2\,abx^4+a}\right.}{8\,b}\right.$$
$$-\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-d}}{dx^2}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((abc-2\,a^2d)x^6-a^2\right.}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b}$$

input
```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-
(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x
^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqr
t(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)) - sqrt(-(b*c - a*d)/a)*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c
- a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arcta
n(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*
d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x
^4 + c)*sqrt(d)*x^2 - c))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2
*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6
+ (b*c^2 - a*c*d)*x^2)) - 2*sqrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^
2)))/b]
```

Sympy [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x}{bx^4+a}\,dx$$

input
```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.40

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{-\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}\log\left(-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{b}\,\sqrt{d\,x^4+c}+\sqrt{d}\,\sqrt{b}\,x^2\right)-\sqrt{d}\,\sqrt{a}\,\sqrt{ad-}}{}$$

input
```
int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
( - sqrt(d)*sqrt(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
 - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)
- sqrt(d)*sqrt(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*
c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2) + sqr
t(d)*sqrt(a)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqr
t(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4) + 2*log((sqrt(c + d*x**
4) + sqrt(d)*x**2)/sqrt(c))*a*d)/(4*sqrt(d)*a*b)
```

3.226 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}$$

output
```
-1/2*(d*x^4+c)^(1/2)/a/x^2-1/2*(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(1/2)*x^
2/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

output
```
-1/2*Sqrt[c + d*x^4]/(a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x^2*
(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 377, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{x^4\left(bx^4+a\right)}dx^2$$

$$\Big\downarrow \; 377$$

$$\frac{1}{2}\left(\frac{\int -\frac{bc-ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{bc-ad}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(-\frac{(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(-\frac{\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^4}}{ax^2}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*x^2)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2
)/(Sqrt[a]*Sqrt[c + d*x^4])])/a^(3/2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}}{x^2}+\frac{(ad-cb)\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{2a}$
risch	$-\frac{\sqrt{d\,x^4+c}}{2a\,x^2}+\frac{(ad-cb)\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$
default	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{2c\,x^2}+\frac{d\,x^2\sqrt{d\,x^4+c}}{2c}+\frac{\sqrt{d}\,\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2}}{a}-\frac{b\left(\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\cdots\right)}{\cdots}}{\cdots}\cdots\right)}{\cdots}$
elliptic	$\frac{-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{c\,x^2}+\frac{2d\left(\frac{x^2\sqrt{d\,x^4+c}}{2}+\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2\sqrt{d}}\right)}{c}}{2a}-\frac{b\left(\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\cdots}{\cdots}\cdots\right)}{\cdots}$

input
```
int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-(d*x^4+c)^(1/2)/x^2+(a*d-b*c)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*
d-b*c))^(1/2))/(a*(a*d-b*c))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\sqrt{dx}}{8\,ax^2}\right.$$

$$\left.-\frac{x^2\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)+2\sqrt{dx^4+c}}{4\,ax^2}\right]$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2))
 - 4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^
2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^3}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)}\,dx = \frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a} + \frac{c\sqrt{d}}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)a}$$

input
```
integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) +
c*sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 455, normalized size of antiderivative = 5.99

$$\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx$$
$$=\frac{\sqrt{d}\,\sqrt{a}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\,\log\left(-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{b}\,\sqrt{d\,x^4+c}+\sqrt{d}\,\sqrt{b}\,x^2\right)x^2+\sqrt{}$$

input
```
int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x)
```

output
```
(sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*
sqrt(b)*x**2)*x**2 + sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(
sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d
*x**4) + sqrt(d)*sqrt(b)*x**2)*x**2 - sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqr
t(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*
x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*x**2 + sqrt(a)*sqrt(a*d - b*c)*log( - s
qrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*
x**4) + sqrt(d)*sqrt(b)*x**2)*d*x**4 + sqrt(a)*sqrt(a*d - b*c)*log(sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4)
+ sqrt(d)*sqrt(b)*x**2)*d*x**4 - sqrt(a)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x
**4)*d*x**4 - 4*sqrt(c + d*x**4)*a*d*x**2 - 2*sqrt(d)*a*c - 4*sqrt(d)*a*d*
x**4)/(4*a**2*x**2*(sqrt(d)*sqrt(c + d*x**4) + d*x**2))
```

3.227 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}}$$

output
```
-1/6*(d*x^4+c)^(1/2)/a/x^6+1/6*(-a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/x^2+1/2*
b*(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^
(5/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = \frac{\sqrt{c+dx^4}(3bcx^4-a(c+dx^4))}{6a^2cx^6}+\frac{b\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

output
```
(Sqrt[c + d*x^4]*(3*b*c*x^4 - a*(c + d*x^4)))/(6*a^2*c*x^6) + (b*Sqrt[b*c
- a*d]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 377, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^7 (a+bx^4)} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{\sqrt{dx^4+c}}{x^8 (bx^4+a)} dx^2$$

$$\Big\downarrow \text{377}$$

$$\frac{1}{2} \left(\frac{\int -\frac{2bdx^4+3bc-ad}{x^4(bx^4+a)\sqrt{dx^4+c}} dx^2}{3a} - \frac{\sqrt{c+dx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{2bdx^4+3bc-ad}{x^4(bx^4+a)\sqrt{dx^4+c}} dx^2}{3a} - \frac{\sqrt{c+dx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2} \left(-\frac{-\frac{\int \frac{3bc(bc-ad)}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a} - \frac{\sqrt{c+dx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{-\frac{3b(bc-ad) \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{a} - \frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a} - \frac{\sqrt{c+dx^4}}{3ax^6} \right)$$

$$\big\downarrow 291$$

$$\frac{1}{2}\left(-\frac{-\frac{3b(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

$$\big\downarrow 218$$

$$\frac{1}{2}\left(-\frac{-\frac{3b\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^4}(3bc-ad)}{acx^2}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^6}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

output
```
(-1/3*Sqrt[c + d*x^4]/(a*x^6) - (-(((3*b*c - a*d)*Sqrt[c + d*x^4])/(a*c*x^
2)) - (3*b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c +
d*x^4])])/a^(3/2))/(3*a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^4+c}\,\left(a d\,x^4-3 b c\,x^4+a c\right)}{3 x^6}-\frac{c b (a d-c b) \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^4+c}}{x^2 \sqrt{a (a d-c b)}}\right)}{\sqrt{a (a d-c b)}}}{2 a^2 c}$
risch	$-\frac{\sqrt{d\,x^4+c}\,\left(a d\,x^4-3 b c\,x^4+a c\right)}{6 c\,a^2 x^6}-\frac{(a d-c b) b \left(-\frac{\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d\sqrt{-a b}\left(x^2-\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^2-\frac{\sqrt{-a b}}{b}\right)^2 d+\frac{2 d \sqrt{\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-a b}}{b}}\right)}{4\sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}} \cdots\right)}{\cdots}$
default	$-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6 a\,x^6 c}+\frac{b^2 \left(\frac{\sqrt{\left(x^2-\frac{\sqrt{-a b}}{b}\right)^2 d+\frac{2 d\sqrt{-a b}\left(x^2-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}+\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{\frac{d\sqrt{-a b}}{b}+\left(x^2-\frac{\sqrt{-a b}}{b}\right) d}{\sqrt{d}}+\sqrt{\left(x^2-\frac{\sqrt{-a}}{b}\cdots\right.}\right)}{b}}{\cdots} \cdots\right)}{\cdots}$
elliptic	$-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{6 a\,x^6 c}-\frac{b\left(-\frac{\left(d\,x^4+c\right)^{\frac{3}{2}}}{c\,x^2}+\frac{2 d\left(\frac{x^2\sqrt{d\,x^4+c}}{2}+\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2\sqrt{d}}\right)}{c}\right)}{2 a^2}+\frac{b^2\left(\sqrt{\left(x^2-\frac{\sqrt{-a b}}{b}\right)^2 d+\frac{2 d\sqrt{-a b}\left(x^2-\cdots\right)}{b}} \cdots\right)}{\cdots}$

input
```
int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(-1/3*(d*x^4+c)^(1/2)*(a*d*x^4-3*b*c*x^4+a*c)/x^6-c*b*(a*d-b*c)/(a
*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))/c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$$
$$= \left[\frac{3\,bcx^6\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)+4\,(}{24\,a^2cx^6}\right.$$

input
```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^
6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(
3*b*c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*
x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2))
+ 2*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^7\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^7}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(90) = 180.

Time = 0.27 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{c+dx^4}}{x^7\,(a+bx^4)}\,dx = -\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2} - \frac{3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}}{3\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2}$$

input
```
integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*
a^2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x
^2 - sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c^2*sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4
 + c))^2 - c)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{x^7\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 1091, normalized size of antiderivative = 9.92

$$\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx=\text{Too large to display}$$

input
```
int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x)
```

output
```
( - 3*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqr
t(d)*sqrt(b)*x**2)*b*c**2*x**6 - 12*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(
a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*d*x**10 - 3*sqrt(d)*s
qrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)
*b*c**2*x**6 - 12*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqr
t(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x*
*4) + sqrt(d)*sqrt(b)*x**2)*b*c*d*x**10 + 3*sqrt(d)*sqrt(a)*sqrt(c + d*x**
4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(
c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b*c**2*x**6 + 12*sqrt(d)*sqrt(a)*
sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2
*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b*c*d*x**10 - 9*sqr
t(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d
 + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c**2*d*x**8 -
 12*sqrt(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
- 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*d**2
*x**12 - 9*sqrt(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b
*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b...
```

3.228 $\int \frac{\sqrt{c+dx^4}}{x^{11}(a+bx^4)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{\sqrt{c+dx^4}}{x^{11}(a+bx^4)} \, dx = -\frac{\sqrt{c+dx^4}}{10ax^{10}} + \frac{(5bc-ad)\sqrt{c+dx^4}}{30a^2cx^6} - \frac{(15b^2c^2-5abcd-2a^2d^2)\sqrt{c+dx^4}}{30a^3c^2x^2} - \frac{b^2\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{7/2}}$$

output
```
-1/10*(d*x^4+c)^(1/2)/a/x^10+1/30*(-a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/x^6-1
/30*(-2*a^2*d^2-5*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/x^2-1/2*b^2*
(-a*d+b*c)^(1/2)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(7
/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}\left(15b^2c^2x^8-5abcx^4\left(c+dx^4\right)+a^2\left(3c^2+cdx^4-2d^2x^8\right)\right)}{30a^3c^2x^{10}} - \frac{b^2\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^11*(a + b*x^4)),x]
```

output
```
-1/30*(Sqrt[c + d*x^4]*(15*b^2*c^2*x^8 - 5*a*b*c*x^4*(c + d*x^4) + a^2*(3*
c^2 + c*d*x^4 - 2*d^2*x^8)))/(a^3*c^2*x^10) - (b^2*Sqrt[b*c - a*d]*ArcTan[
(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*
d])])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {965, 377, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^4+c}}{x^{12}\left(bx^4+a\right)}dx^2$$

$$\Big\downarrow \text{377}$$

$$\frac{1}{2}\left(\frac{\int -\frac{4bdx^4+5bc-ad}{x^8(bx^4+a)\sqrt{dx^4+c}}dx^2}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{\int \frac{4bdx^4+5bc-ad}{x^8(bx^4+a)\sqrt{dx^4+c}}dx^2}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 445

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{2bd(5bc-ad)x^4+15b^2c^2-2a^2d^2-5abcd}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^4}(5bc-ad)}{3acx^6}}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 445

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{\int \frac{15b^2c^2(bc-ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{2ad^2}{c}-5bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-ad)}{3acx^6}}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{15b^2c(bc-ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{2ad^2}{c}-5bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-ad)}{3acx^6}}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{15b^2c(bc-ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{2ad^2}{c}-5bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-ad)}{3acx^6}}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(-\frac{-\frac{-\frac{15b^2c\sqrt{bc-ad}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{2ad^2}{c}-5bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-ad)}{3acx^6}}{5a}-\frac{\sqrt{c+dx^4}}{5ax^{10}}\right)$$

input
```
Int[Sqrt[c + d*x^4]/(x^11*(a + b*x^4)),x]
```

output
```
(-1/5*Sqrt[c + d*x^4]/(a*x^10) - (-1/3*((5*b*c - a*d)*Sqrt[c + d*x^4])/(a*
c*x^6) - (-((((15*b^2*c)/a - 5*b*d - (2*a*d^2)/c)*Sqrt[c + d*x^4])/x^2) -
(15*b^2*c*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d
*x^4])])/a^(3/2))/(3*a*c))/(5*a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\left(\left(-\frac{2d\,x^4}{3}+c\right)(d\,x^4+c)a^2-\frac{5a(d\,x^4+c)bc\,x^4}{3}+5b^2c^2x^8\right)\sqrt{a(ad-cb)}\,\sqrt{d\,x^4+c}-5b^2c^2x^{10}(ad-cb)\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right.}{10\sqrt{a(ad-cb)}\,x^{10}a^3c^2}$
risch	$-\frac{\sqrt{d\,x^4+c}\,(-2a^2d^2x^8-5abcd\,x^8+15b^2c^2x^8+a^2cd\,x^4-5ab\,c^2x^4+3a^2c^2)}{30a^3x^{10}c^2}+\frac{b^2(ad-cb)\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\right.}{b}}{}\right.}{}\right.}{}$
default	$-\frac{(d\,x^4+c)^{\frac{3}{2}}(-2d\,x^4+3c)}{30a\,x^{10}c^2}+\frac{b^2\left(-\frac{(d\,x^4+c)^{\frac{3}{2}}}{2c\,x^2}+\frac{d\,x^2\sqrt{d\,x^4+c}}{2c}+\frac{\sqrt{d}\,\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2}\right)}{a^3}+\frac{b(d\,x^4+c)^{\frac{3}{2}}}{6a^2x^6c}-\frac{b^3\left(\frac{\sqrt{\left(x^2\right.}}{}\right.}{}$
elliptic	$\frac{-\frac{(d\,x^4+c)^{\frac{3}{2}}}{5c\,x^{10}}+\frac{2d(d\,x^4+c)^{\frac{3}{2}}}{15c^2x^6}}{2a}+\frac{b^2\left(-\frac{(d\,x^4+c)^{\frac{3}{2}}}{c\,x^2}+\frac{2d\left(\frac{x^2\sqrt{d\,x^4+c}}{2}+\frac{c\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2\sqrt{d}}\right)}{c}\right)}{2a^3}+\frac{b(d\,x^4+c)^{\frac{3}{2}}}{6a^2x^6c}-\frac{b^3\left(\sqrt{\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^11/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/10/(a*(a*d-b*c))^(1/2)*(((-2/3*d*x^4+c)*(d*x^4+c)*a^2-5/3*a*(d*x^4+c)*b
*c*x^4+5*b^2*c^2*x^8)*(a*(a*d-b*c))^(1/2)*(d*x^4+c)^(1/2)-5*b^2*c^2*x^10*(
a*d-b*c)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))/x^10/a^3/c^2
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{c+dx^4}}{x^{11}(a+bx^4)}\,dx$$
$$= \left[\frac{15\,b^2c^2x^{10}\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((abc-2\,a^2d)x^6-a^2cx^2)\sqrt{dx^4+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-}{120\,a^3c^2x^{10}}\right.$$
$$-\frac{15\,b^2c^2x^{10}\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^6+(bc^2-acd)x^2)}\right)+2\,((15\,b^2c^2-5\,abcd-2\,a^2d^2)x^8-(5\,abc^2-}{60\,a^3c^2x^{10}}$$

input
```
integrate((d*x^4+c)^(1/2)/x^11/(b*x^4+a),x, algorithm="fricas")
```

output
```
[1/120*(15*b^2*c^2*x^10*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8
*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^
2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a
*b*x^4 + a^2)) - 4*((15*b^2*c^2 - 5*a*b*c*d - 2*a^2*d^2)*x^8 - (5*a*b*c^2
- a^2*c*d)*x^4 + 3*a^2*c^2)*sqrt(d*x^4 + c))/(a^3*c^2*x^10), -1/60*(15*b^2
*c^2*x^10*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*
x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2))
+ 2*((15*b^2*c^2 - 5*a*b*c*d - 2*a^2*d^2)*x^8 - (5*a*b*c^2 - a^2*c*d)*x^4
+ 3*a^2*c^2)*sqrt(d*x^4 + c))/(a^3*c^2*x^10)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^{11}(a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^{11}(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**11/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**11*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^{11}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^11/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^11), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(135) = 270$.

Time = 0.34 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx=\frac{\left(b^3c\sqrt{d}-ab^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^3}+\frac{15\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^8b^2c\sqrt{d}-15\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^8abd^{\frac{3}{2}}-60\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^6b^2c^2\sqrt{d}+3}{}$$

input
```
integrate((d*x^4+c)^(1/2)/x^11/(b*x^4+a),x, algorithm="giac")
```

output
```
1/2*(b^3*c*sqrt(d) - a*b^2*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)
*a^3) + 1/15*(15*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^8*b^2*c*sqrt(d) - 15*(sqr
t(d)*x^2 - sqrt(d*x^4 + c))^8*a*b*d^(3/2) - 60*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^6*b^2*c^2*sqrt(d) + 30*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^6*a*b*c*d^(3/2
) + 30*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^6*a^2*d^(5/2) + 90*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^4*b^2*c^3*sqrt(d) - 20*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*a
*b*c^2*d^(3/2) + 10*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*a^2*c*d^(5/2) - 60*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^2*c^4*sqrt(d) + 10*(sqrt(d)*x^2 - sqrt(
d*x^4 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*c
^2*d^(5/2) + 15*b^2*c^5*sqrt(d) - 5*a*b*c^4*d^(3/2) - 2*a^2*c^3*d^(5/2))/(
((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)^5*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{x^{11}\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^11*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^11*(a + b*x^4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 1815, normalized size of antiderivative = 11.42

$$\int \frac{\sqrt{c+dx^4}}{x^{11}\left(a+bx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((d*x^4+c)^(1/2)/x^11/(b*x^4+a),x)
```

output

```
(15*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)*b**2*c**3*x**10 + 180*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sq
rt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
 + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c**2*d*x**14 + 24
0*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*s
qrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)
*sqrt(b)*x**2)*b**2*c*d**2*x**18 + 15*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqr
t(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + s
qrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c**3*x**10 + 180*sqrt
(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sq
rt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*
x**2)*b**2*c**2*d*x**14 + 240*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d -
b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*s
qrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c*d**2*x**18 - 15*sqrt(d)*sqr
t(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c
) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b**2*c**3*x**1
0 - 180*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x
**4)*b**2*c**2*d*x**14 - 240*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d ...
```

3.229 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 671

$$
\begin{aligned}
\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{x\sqrt{c+dx^4}}{3b}+\frac{\sqrt[4]{-a}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}} \\
&+\frac{\sqrt[4]{-a}\sqrt{-bc+ad}\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}} \\
&+\frac{c^{3/4}(bc-2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3b\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
1/3*x*(d*x^4+c)^(1/2)/b+1/4*(-a)^(1/4)*(a*d-b*c)^(1/2)*arctan((a*d-b*c)^(1
/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)+1/4*(-a)^(1/4)*(a*d-b*c)
^(1/2)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/
4)+1/3*c^(3/4)*(-2*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))
/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/
2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(
1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a
)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^2/c^(
1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+
c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4
))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/
d^(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1
/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.36

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\left(\frac{(2bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+5\left(c+dx^4+\frac{5a^2c^2\,\text{Appe}}{(a+bx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\right.\right.}\right.\right.}{15b\sqrt{c+dx^4}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2,
 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d
*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.64 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {978, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \ 978$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\int \frac{ac-(2bc-3ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{3b}$$

$$\Big\downarrow \ 1021$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{(2bc-3ad)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{3b}$$

$$\Big\downarrow \ 761$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(2bc-3ad)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{3b}$$

$$\Big\downarrow \ 925$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{\frac{3a(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(2bc-3ad)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{3b}$$

$$\Big\downarrow \ 1541$$

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{3a(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}\right.}{}}{2a}}{b}}{3b}$$

$\downarrow$ 27

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{3a(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)}{}}{2a}}{b}}{3b}$$

$\downarrow$ 761

$$\frac{x\sqrt{c+dx^4}}{3b}-\frac{3a(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\right.}{}}{b}}{}$$

$\downarrow$ 2221

$$\frac{x\sqrt{dx^4+c}}{3b}-\frac{3a(bc-ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{}{\sqrt[4]{}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.}{}$$

$$\Big\downarrow \ 2223$$

$$\frac{x\sqrt{dx^4+c}}{3b}-\frac{}{3a(bc-ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{}{\sqrt[4]{}}\right)}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{2a}\right.}$$

input
```
Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x*Sqrt[c + d*x^4])/(3*b) - (-1/2*((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) + (3*a*(b*c - a*d)*(
((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] -
 Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]
)/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.45

method	result
risch	$\frac{x\sqrt{dx^4+c}}{3b} - \frac{\frac{(3ad-2cb)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}} - \frac{3a(ad-cb)\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b+a)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2dx^2\_\alpha^2+2c \cdots}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{dx^4 \cdots}}\right)}{\sqrt{\frac{-ad+cb}{b}}} \cdots}{\cdots}\right.}{\cdots}}{3b}$
elliptic	$\frac{x\sqrt{dx^4+c}}{3b} + \frac{\left(-\frac{ad-cb}{b^2}-\frac{c}{3b}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}} + \frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b+a)} \frac{(ad-cb)\left(-\frac{\operatorname{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\right)}{\sqrt{\cdots}} \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{\frac{x\sqrt{dx^4+c}}{3} + \frac{2c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}}}{b} - \frac{a\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}} - \frac{\sum_{\_\alpha=\mathrm{RootOf}(\_Z^4b \cdots}{\cdots}\right.}{\cdots}$

input

```
int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(d*x^4+c)^(1/2)/b-1/3/b*((3*a*d-2*b*c)/b/(I/c^(1/2)*d^(1/2))^(1/2)*(
1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(
1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-3/8*a*(a*d-b*c)/b^2*sum(1/_a
lpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+
b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1
-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1
/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,
(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*
b+a)))
```

Fricas [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}x^4}{bx^4+a}\,dx$$

input
```
integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^4\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{\sqrt{d\,x^4+c}\,x-\left(\int \frac{\sqrt{d\,x^4+c}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)ac-3\left(\int \frac{\sqrt{d\,x^4+c}\,x^4}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)ad+2\left(\int \frac{\sqrt{d\,x^4+c}\,x^4}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)b}{3b}$$

input
```
int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
(sqrt(c + d*x**4)*x - int(sqrt(c + d*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*
d*x**8),x)*a*c - 3*int((sqrt(c + d*x**4)*x**4)/(a*c + a*d*x**4 + b*c*x**4
+ b*d*x**8),x)*a*d + 2*int((sqrt(c + d*x**4)*x**4)/(a*c + a*d*x**4 + b*c*x
**4 + b*d*x**8),x)*b*c)/(3*b)
```

3.230 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 749

$$
\begin{aligned}
&\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx \\
&= \frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{\sqrt{-bc+ad}\,\text{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} \\
&+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} \\
&+ \frac{d^{3/4}(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\right.\right.}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{c}\ldots}\right.\right.}{8ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
1/4*(a*d-b*c)^(1/2)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^
(1/2))/(-a)^(3/4)/b^(3/4)+1/4*(a*d-b*c)^(1/2)*arctanh((a*d-b*c)^(1/2)*x/(-
a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)+1/2*d^(3/4)*(c^(1/2)+
d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b/c^(1/4)/(d*x^4+c)^(1/2)+1/2*d^(3/4
)*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/
2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b/c^(1/4)/(a*d
+b*c)/(d*x^4+c)^(1/2)+1/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*
(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi
(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2)
)^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/b/c^(1/4)/(b^(1/2)*c
^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)+1/8*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(
1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2
)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))/a/b/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(
1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{5acx\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(-2bc\,\text{AppellF1}\left(\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\,A\right.\right.}$$

input
```
Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

output
```
(5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^
4)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x
^4)/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Rubi [A] (warning: unable to verify)

Time = 2.16 (sec) , antiderivative size = 964, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {922, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4} \, dx$$

$$\Big\downarrow \; 922$$

$$\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \; 761$$

$$\frac{(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 925$$

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 1541$$

$$(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-\cdots\right.}{\cdots}}{2a}\right.$$

$$b$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 27$$

$$(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{\cdots}\right.}{\cdots}}{2a}\right.$$

$$b$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 761$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.$$

$$b$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$$\Big\downarrow \; 2221$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\right.$$

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}$$

$\downarrow$ 2223

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{dx^4+c}}+$$

$$(bc-ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\ldots\right.}{2\sqrt[4]{b}\sqrt{bc-\ldots}}\right.}{\ldots}}{2a}\right.$$

input
```
Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

output
```
(d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*
x^4]) + ((b*c - a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*
x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[
b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(
1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a
]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt
[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(
1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*S
qrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^
(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c -
a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sq
rt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.36

method	result
default	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-cb)\left(-\frac{\text{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\cdots}}{\cdots}\right.}{\cdots}}{8b^2}$
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right)}\frac{(ad-cb)\left(-\frac{\text{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\cdots}}{\cdots}\right.}{\cdots}}{8b^2}$

input
```
int((d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1
/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(a + b*x^4),x)
```

output
```
int((c + d*x^4)^(1/2)/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}dx$$

input
```
int((d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int(sqrt(c + d*x**4)/(a + b*x**4),x)
```

3.231 $\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 674

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{3ax^3} + \frac{\sqrt[4]{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}$$

$$+ \frac{\sqrt[4]{b}\sqrt{-bc+ad}\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}$$

$$- \frac{d^{3/4}(2bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
-1/3*(d*x^4+c)^(1/2)/a/x^3+1/4*b^(1/4)*(a*d-b*c)^(1/2)*arctan((a*d-b*c)^(1
/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)+1/4*b^(1/4)*(a*d-b*c)
^(1/2)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^
(7/4)-1/3*d^(3/4)*(-a*d+2*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d
^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/
2))/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2
)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-
(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^2/
c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x
^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(
1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d
^(1/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.20 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx$$
$$= \frac{-bdx^8\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac(ac+4bcx^4-adx^4+bdx^8)\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10\right.}{(a+bx^4)\left(-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2\right.}}{15a^2x^3\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

output
```
(-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Ap
pellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a
*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*A
ppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4,
3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {975, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^4 \left(a+bx^4\right)} \, dx$$

$$\Big\downarrow \text{ 975}$$

$$\frac{\int -\frac{bdx^4+3bc-2ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{bdx^4+3bc-2ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 1021}$$

$$-\frac{3(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx+d\int \frac{1}{\sqrt{dx^4+c}}dx}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{3(bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 925}$$

$$-\frac{3(bc-ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3a}-\frac{\sqrt{c+dx^4}}{3ax^3}$$

$$\Big\downarrow \text{ 1541}$$

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\cdots}{\cdots}}{\cdots}\cdots\right)}{3a\cdots}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$\big\downarrow$ 27

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{c}\cdots\right)}{\cdots}}{2a}\cdots\right)}{3a\cdots}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$\big\downarrow$ 761

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}\cdots\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$\big\downarrow$ 2221

$$-\frac{3(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}\cdots\right)}{\cdots}$$

$$\frac{\sqrt{c+dx^4}}{3ax^3}$$

$\Big\downarrow$ 2223

$$-\frac{\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3(bc-ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}$$

$$\frac{\sqrt{dx^4+c}}{3ax^3}$$

input
```
Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

output
```
-1/3*Sqrt[c + d*x^4]/(a*x^3) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*(b*c - a*d)*(((a*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*
c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d)
)/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[
b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt
[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{\sqrt{d\,x^{4}+c}}{3a\,x^{3}}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}+\frac{(-3ad+3cb)\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}}{}\right.}{}}{3a}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{3a\,x^{3}}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b+a\right)}\frac{(-ad+cb)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^{2}\_\alpha^{2}+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^{4}}}\right.}{\sqrt{\frac{-ad+cb}{b}}}\right.}{}$
default	$\frac{-\frac{\sqrt{d\,x^{4}+c}}{3x^{3}}+\frac{2d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{a}-\frac{b\left(\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^{4}b\right.}\right.}{}$

input
```
int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^4+c)^(1/2)/a/x^3-1/3/a*(d/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*(-3*a*d+3*b*c)/b*sum(1/_alpha^3*(-1
/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1
/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2
)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^4}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{x^4\,(b\,x^4+a)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{x^4(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{b\,x^8+a\,x^4}dx$$

input
```
int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x)
```

output
```
int(sqrt(c + d*x**4)/(a*x**4 + b*x**8),x)
```

3.232 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 857

$$\begin{aligned}\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx &= \frac{x^3\sqrt{c+dx^4}}{5b}+\frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)}\\
&-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b^2}\\
&-\frac{\sqrt[4]{c}(2bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{5b^2d^{3/4}\sqrt{c+dx^4}}\\
&+\frac{\sqrt[4]{c}(b^2c^2+abcd-5a^2d^2)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{5b^2d^{3/4}(bc+ad)\sqrt{c+dx^4}}\\
&+\frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\ldots\right.\right.}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}\\
&-\frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\ldots\right.\right.}{8b^{5/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}\end{aligned}$$

output
```
1/5*x^3*(d*x^4+c)^(1/2)/b+1/5*(-5*a*d+2*b*c)*x*(d*x^4+c)^(1/2)/b^2/d^(1/2)
/(c^(1/2)+d^(1/2)*x^2)-1/4*a*(-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan
((-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/b^2-1/4*a*((-a*
d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan(((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1
/2)*x/(d*x^4+c)^(1/2))/b^2-1/5*c^(1/4)*(-5*a*d+2*b*c)*(c^(1/2)+d^(1/2)*x^2
)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/2*2^(1/2))/b^2/d^(3/4)/(d*x^4+c)^(1/2)+1/5*c^(1/4)*(-5*a^2*
d^2+a*b*c*d+b^2*c^2)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2
)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b^2/d^
(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2
)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(5/2)/c^(
1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*a*
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*
x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^
(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(
1/2)/d^(1/2),1/2*2^(1/2))/b^(5/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^
(1/2))/d^(1/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.16

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{7ax^3(c+dx^4) - 7acx^3\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + (2bc-5ad)x^7\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\,(}{35ab\sqrt{c+dx^4}}$$

input
```
Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
 7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c
]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c
+ d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.65 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {978, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{978}$$

$$\frac{x^3\sqrt{c+dx^4}}{5b}-\frac{\int \frac{x^2(3ac-(2bc-5ad)x^4)}{(bx^4+a)\sqrt{dx^4+c}}dx}{5b}$$

$$\Big\downarrow \text{1054}$$

$$\frac{x^3\sqrt{c+dx^4}}{5b}-\frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{dx^4+c}}-\frac{5(a^2d-abc)x^2}{b(bx^4+a)\sqrt{dx^4+c}}\right)dx}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\sqrt{dx^4+c}}{5b}-$$

$$\frac{5\sqrt{-a}(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{5(-a)^{3/4}\sqrt{bc-ad}\,\text{arcta}}{4b}$$

input
```
Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output

```
(x^3*Sqrt[c + d*x^4])/(5*b) - (-(((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(b*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2))) - (5*(-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sq
rt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)) + (5*(
-a)^(3/4)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*
Sqrt[c + d*x^4])])/(4*b^(5/4)) + (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) - (c^(1/4)*(2*b*c - 5
*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4
]) - (5*a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4]) - (5*a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^4]) + (5*Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) - (5*Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d...
```

Defintions of rubi rules used

rule 978

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.39

method	result
risch	$\frac{x^3\sqrt{d\,x^4+c}}{5b}-\frac{\frac{i(5ad-2cb)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}-5a(ad-cb)\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right)}\cdots\right.}{5b}$
elliptic	$\frac{x^3\sqrt{d\,x^4+c}}{5b}+\frac{i\left(-\frac{ad-cb}{b^2}-\frac{3c}{5b}\right)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}+\frac{a\left(\sum_{\_\alpha=\text{RootOf}\left(\cdots\right.}\cdots\right.}{\cdots}$
default	$\frac{\frac{x^3\sqrt{d\,x^4+c}}{5}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,\sqrt{d}}}{b}-\frac{a\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{Elliptic}\cdots\right.}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{\cdots}}\cdots\right.}{\cdots}$

input
```
int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(d*x^4+c)^(1/2)/b-1/5/b*(I*(5*a*d-2*b*c)/b*c^(1/2)/(I/c^(1/2)*d^(1
/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-Ellipti
cE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-5/8*a*(a*d-b*c)/b^2*sum(1/_alpha*(-1/((
-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)
/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^
(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticP
i(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d
^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

input
```
integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^6\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$
$$= \frac{\sqrt{d\,x^4+c}\,x^3-5\left(\int \frac{\sqrt{d\,x^4+c}\,x^6}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)ad+2\left(\int \frac{\sqrt{d\,x^4+c}\,x^6}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)bc-3\left(\int \frac{\sqrt{d\,x^4+c}\,x^2}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)}{5b}$$

input
```
int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
(sqrt(c + d*x**4)*x**3 - 5*int((sqrt(c + d*x**4)*x**6)/(a*c + a*d*x**4 + b
*c*x**4 + b*d*x**8),x)*a*d + 2*int((sqrt(c + d*x**4)*x**6)/(a*c + a*d*x**4
 + b*c*x**4 + b*d*x**8),x)*b*c - 3*int((sqrt(c + d*x**4)*x**2)/(a*c + a*d*
x**4 + b*c*x**4 + b*d*x**8),x)*a*c)/(5*b)
```

3.233 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 786

$$\begin{aligned}
&\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx \\
&= \frac{\sqrt{d}x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b} + \frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b} \\
&\quad - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}} \\
&\quad + \frac{a\sqrt[4]{c}d^{5/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{b(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
d^(1/2)*x*(d*x^4+c)^(1/2)/b/(c^(1/2)+d^(1/2)*x^2)+1/4*(-(-a*d+b*c)/(-a)^(1
/2)/b^(1/2))^(1/2)*arctan((-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+
c)^(1/2))/b+1/4*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan(((-a*d+b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/b-c^(1/4)*d^(1/4)*(c^(1/2)+d^(
1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan
(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/b/(d*x^4+c)^(1/2)+a*c^(1/4)*d^(5/4)*(c^(
1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8
*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d
*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c
^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(
1/2)/d^(1/2),1/2*2^(1/2))/b^(3/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^
(1/2))/d^(1/4)/(d*x^4+c)^(1/2)+1/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-
a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*E
llipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2
)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))/b^(3/2)/c^(1/
4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{x^3\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a
)])/(3*a*Sqrt[(c + d*x^4)/c])
```

Rubi [A] (warning: unable to verify)

Time = 2.55 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {994, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \ 994$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\int \frac{x^2}{\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \ 834$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{c}\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \ 761$$

$$\frac{(bc-ad)\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}+\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$$\Big\downarrow \ 993$$

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}} - \frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b} + \frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}} - \frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}\right)}{b}$$

$\big\downarrow$ 1510

$$\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}} - \frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b} + \frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}} - \frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$\big\downarrow$ 1541

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc} - \frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}} - \frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc} + \frac{\sqrt{b}\sqrt{c}(\cdots}{\cdots}}{\cdots}\right.}{b} + \frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}} - \frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$\big\downarrow$ 27

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\right.}{}}{2\sqrt{b}}\right.}{b}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$\downarrow$ 761

$$\frac{(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\right.}{b}$$

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}\right)}{b}$$

$\downarrow$ 2221

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}\cdots\right)}{b}\cdots$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\cdots\right)}{4\sqrt{-\cdots}}\cdots\right)}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\cdots\right)}{bc+ad}\cdots}{2\sqrt{b}}\cdots\right)\cdots$$

$\downarrow$ 2223

$$\frac{d\left(\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}\cdots\right)}{b}\cdots$$

$$(bc-ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\cdots\right)}{4\sqrt{-\cdots}}\cdots\right)}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\cdots\right)}{bc+ad}\cdots}{2\sqrt{b}}\cdots\right)\cdots$$

input
```
Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(d*(-((-((x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + (c^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^4]))/Sqrt[d]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^4]))
)/b + ((b*c - a*d)*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] +
Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/
2) + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[
-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1
/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 994
```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Simp[d/b   Int[x^2/Sqrt[c + d*x^4], x], x] + Simp[(b*c - a*d)/b   Int[x^2/(
(a + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.38

method	result
default	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{(ad-cb)\left(-\frac{\operatorname{arctanh}\left(\frac{\ldots}{2\ldots}\right)}{\ldots}\ldots\right.}{\ldots}}{\ldots}$
elliptic	$\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{(ad-cb)\left(-\frac{\operatorname{arctanh}\left(\frac{\ldots}{2\ldots}\right)}{\ldots}\ldots\right.}{\ldots}}{\ldots}$

input
```
int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1
/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2
)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/b^2*sum(
(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*
c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alph
a^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d
*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_al
pha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

input
```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

input
```
integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{x^2\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}dx$$

input
```
int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int((sqrt(c + d*x**4)*x**2)/(a + b*x**4),x)
```

3.234 $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 809

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx = -\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)}$$

$$-\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a}-\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a}$$

$$-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}}$$

$$+\frac{bc^{5/4}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{a(bc+ad)\sqrt{c+dx^4}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8\sqrt{b}\sqrt[4]{c}\left((-a)^{3/2}\sqrt{b}\sqrt{c}+a^2\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{8a\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
-(d*x^4+c)^(1/2)/a/x+d^(1/2)*x*(d*x^4+c)^(1/2)/a/(c^(1/2)+d^(1/2)*x^2)-1/4
*(-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan((-(-a*d+b*c)/(-a)^(1/2)/b^(
1/2))^(1/2)*x/(d*x^4+c)^(1/2))/a-1/4*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)
*arctan(((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/a-c^(1/4)
*d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*E
llipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/a/(d*x^4+c)^(1/2)+b
*c^(5/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a/(a*d+b*c
)/(d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(
1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin
(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(1/2)/c^(1/4)/((-a)^(3/2)
*b^(1/2)*c^(1/2)+a^2*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)
+(-a)^(1/2)*d^(1/2))*(-a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+
d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(
1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2
*2^(1/2))/a/b^(1/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)
/(d*x^4+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$$
$$= \frac{-21a(c+dx^4)-7(bc-2ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}}{21a^2x\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

output
```
(-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c
+ d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.28 (sec) , antiderivative size = 1021, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {975, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}dx$$

$$\Big\downarrow \text{975}$$

$$\frac{\int -\frac{x^2(-bdx^4+bc-2ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{x^2(-bdx^4+bc-2ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \text{1054}$$

$$-\frac{\int \left(\frac{(bc-ad)x^2}{(bx^4+a)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{a}-\frac{\sqrt{c+dx^4}}{ax}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{\ldots}}{\sqrt[4]{-a}\ldots}\right.}{4\sqrt[4]{-a}\sqrt[4]{\ldots}}\ldots}{\ldots}$$

$$\frac{\sqrt{dx^4+c}}{ax}$$

input
```
Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

output
```
-(Sqrt[c + d*x^4]/(a*x)) - (-((Sqrt[d]*x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[
d]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)
*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(1/4)) - (Sqrt[b*c - a*d]*ArcTanh[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(1/
4)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt[c + d
*x^4] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*Sqrt[
c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.72 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{\sqrt{d x^4+c}}{a x}+\frac{\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{(ad-cb)\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctan}\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}}{a}\cdots$
elliptic	$-\frac{\sqrt{d x^4+c}}{a x}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b+a\right)}\frac{(-ad\cdots}{\cdots}}{\cdots}\cdots$
default	$\frac{-\frac{\sqrt{d x^4+c}}{x}+\frac{2i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}}{a}-\frac{b\left(\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\operatorname{Elli}\cdots\right.}{b\sqrt{\frac{i\sqrt{\cdots}}{\sqrt{c}}}\cdots}\cdots\right.}{\cdots}$

input
```
int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-(d*x^4+c)^(1/2)/a/x+1/a*(I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I
/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2)
)^(1/2),I))+1/8*(a*d-b*c)/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(
1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1
/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/
2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/
2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^
(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{x^2\,(b\,x^4+a)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx=\int \frac{\sqrt{d\,x^4+c}}{b\,x^6+a\,x^2}dx$$

input
```
int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x)
```

output
```
int(sqrt(c + d*x**4)/(a*x**2 + b*x**6),x)
```

3.235 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx=-\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2}+\frac{(c+dx^4)^{3/2}}{6bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

output
```
-1/2*(a*d+b*c)*(d*x^4+c)^(1/2)/b^2/d^2+1/6*(d*x^4+c)^(3/2)/b/d^2-1/2*a^2*a
rctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{\sqrt{c+dx^4}(-2bc-3ad+bdx^4)}{6b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(2*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2(bx^4+a)\sqrt{dx^4+c}}+\frac{\sqrt{dx^4+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^4+c}}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^4}(ad+bc)}{b^2d^2}+\frac{2\left(c+dx^4\right)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^4])/(b^2*d^2) + (2*(c + d*x^4)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/4
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{-\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)a^2d^2+\sqrt{(ad-cb)b}\,\sqrt{d\,x^4+c}\left(\frac{\left(-d\,x^4+2c\right)b}{3}+ad\right)}{2\sqrt{(ad-cb)b}\,b^2d^2}$
risch	$-\frac{\left(-db\,x^4+3ad+2cb\right)\sqrt{d\,x^4+c}}{6d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\cdots\right)}{b}\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)\cdots}{4b^3\sqrt{-\frac{ad-cb}{b}}}$
default	$-\frac{\sqrt{d\,x^4+c}\,\left(-d\,x^4+2c\right)}{6b\,d^2}+\frac{a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-a\cdots}}{b}\right)}{b}\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$
elliptic	$\frac{x^4\sqrt{d\,x^4+c}}{6bd}-\frac{c\sqrt{d\,x^4+c}}{3b\,d^2}-\frac{a\sqrt{d\,x^4+c}}{2b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)\cdots}{4b^3\sqrt{-\frac{ad-cb}{b}}}$

input
```
int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*b)^(1/2)*(-arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))*a
^2*d^2+((a*d-b*c)*b)^(1/2)*(d*x^4+c)^(1/2)*(1/3*(-d*x^4+2*c)*b+a*d))/b^2/d
^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.78

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$=\left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2d^2)x^4\right)\cdots}{12\left(b^4cd^2-ab^3d^3\right)}\right.$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3
*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b
^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\frac{3\,a^2d^2\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2-3\sqrt{dx^4+c}b^2c-3\sqrt{dx^4+c}abd}{b^3}}{6\,d^2}$$

input
```
integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*(3*a^2*d^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b^2) + ((d*x^4 + c)^(3/2)*b^2 - 3*sqrt(d*x^4 + c)*b^2*c - 3*sqrt
(d*x^4 + c)*a*b*d)/b^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b\,d^2} - \left(\frac{c}{b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^
4))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^
(1/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 794, normalized size of antiderivative = 7.63

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(3*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c +
 d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b
*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*c*d**2 + 12*sqrt(b)*sqrt(c + d*x
**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)
*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d
- b*c)*x**2))*a**2*d**3*x**4 + 9*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqr
t(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c +
 d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*c*d**2*x**2
 + 12*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**
4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) +
sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*d**3*x**6 - 9*sqrt(d)*sqrt(c + d*x**4)
*a**2*b*c*d**2*x**2 - 12*sqrt(d)*sqrt(c + d*x**4)*a**2*b*d**3*x**6 + 3*sqr
t(d)*sqrt(c + d*x**4)*a*b**2*c**2*d*x**2 + 7*sqrt(d)*sqrt(c + d*x**4)*a*b*
*2*c*d**2*x**6 + 4*sqrt(d)*sqrt(c + d*x**4)*a*b**2*d**3*x**10 + 6*sqrt(d)*
sqrt(c + d*x**4)*b**3*c**3*x**2 + 5*sqrt(d)*sqrt(c + d*x**4)*b**3*c**2*d*x
**6 - 4*sqrt(d)*sqrt(c + d*x**4)*b**3*c*d**2*x**10 - 3*a**2*b*c**2*d**2 -
15*a**2*b*c*d**3*x**4 - 12*a**2*b*d**4*x**8 + a*b**2*c**3*d + 6*a*b**2*c**
2*d**2*x**4 + 9*a*b**2*c*d**3*x**8 + 4*a*b**2*d**4*x**12 + 2*b**3*c**4 + 9
*b**3*c**3*d*x**4 + 3*b**3*c**2*d**2*x**8 - 4*b**3*c*d**3*x**12)/(6*b**3*d
**2*(sqrt(c + d*x**4)*a*c*d + 4*sqrt(c + d*x**4)*a*d**2*x**4 - sqrt(c +...
```

3.236 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2bd} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

output
```
1/2*(d*x^4+c)^(1/2)/b/d+1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(
1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input
```
Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \ 90$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{bd}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{b}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(\frac{2\sqrt{c+dx^4}}{bd}-\frac{2a\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^4}}{bd}\right)$$

input
```
Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((2*Sqrt[c + d*x^4])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^4+c}}{d}-\frac{a\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{2b}$
risch	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-cb}{b}}}+ \cdots$
elliptic	$\frac{\sqrt{d\,x^4+c}}{2bd}+\frac{a\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-cb}{b}}}+ \cdots$
default	$\frac{\sqrt{d\,x^4+c}}{2bd}-\frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}- \cdots\right)}{b}$

input
```
int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(1/d*(d*x^4+c)^(1/2)-a/((a*d-b*c)*b)^(1/2)*arctan((d*x^4+c)^(1/2)*b/
((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)},\ -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-\sqrt{dx^4+c}(b^2c-abd)}{2\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^4+c}}{b}}{2d}$$

input
```
integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^4 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c
)^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.99

$$\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{b}\,c+\sqrt{b}\,d\,x^4}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}\,\sqrt{ad-bc}\,x^2}\right)ad-\sqrt{d}\,\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{}}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}}\right.}{2b^2d\left(\sqrt{d\,x^4+c}\,ad-\sqrt{d\,x^4+c}\,bc\right.}$$

input
```
int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
( - sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c
+ d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d -
b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*d - sqrt(d)*sqrt(b)*sqrt(a*d - b*c
)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4
)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*d*x
**2 + sqrt(d)*sqrt(c + d*x**4)*a*b*d*x**2 - sqrt(d)*sqrt(c + d*x**4)*b**2*
c*x**2 + a*b*c*d + a*b*d**2*x**4 - b**2*c**2 - b**2*c*d*x**4)/(2*b**2*d*(s
qrt(c + d*x**4)*a*d - sqrt(c + d*x**4)*b*c + sqrt(d)*a*d*x**2 - sqrt(d)*b*
c*x**2))
```

3.237 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{2\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]]/(2*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{4}\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{2d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
 - a*d])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^{4}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{2\sqrt{(ad-cb)b}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-cb}{b}}{}\right.}{}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-cb}{b}}{}\right.}{}$

input
```
int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\,(b^2c-abd)}\right]$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(
b*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 7.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}} & \text{for } d \neq 0 \\ \begin{cases} \frac{x^4}{4a\sqrt{c}} & \text{for } b = 0 \\ \tilde{\infty}x^4 & \text{for } \sqrt{c} = 0 \\ \frac{\log\left(4a\sqrt{c}+4b\sqrt{c}x^4\right)}{4b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**4/(4*a*sqrt(c)), Eq(b, 0)), (zoo*x**4, Eq
(sqrt(c), 0)), (log(4*a*sqrt(c) + 4*b*sqrt(c)*x**4)/(4*b*sqrt(c)), True)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}}$$

input
```
integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\mathrm{atan}\left(\frac{b\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{d}\,\sqrt{b}\,\sqrt{d\,x^4+c}\,x^2+\sqrt{b}\,c+\sqrt{b}\,d\,x^4}{\sqrt{d\,x^4+c}\,\sqrt{ad-bc}+\sqrt{d}\,\sqrt{ad-bc}\,x^2}\right)}{2b\,(ad-bc)}$$

input
```
int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqr
t(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(
a*d - b*c)*x**2)))/(2*b*(a*d - b*c))
```

3.238 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

output
```
-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/2*b^(1/2)*arctanh(b^(1/2
)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

input
```
Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c])/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{4} \int \frac{1}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^4$$

$$\Big\downarrow \; 97$$

$$\frac{1}{4} \left(\frac{\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a} - \frac{b\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{a} \right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4} \left(\frac{2\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad} - \frac{2b\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad} \right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4} \left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}} \right)$$

input
```
Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{2a}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-\frac{ad-cb}{b}}}$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\right.}{}$

input
```
int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-arctanh((d*x^4+c)^(1/2)/c^(1/2))/c^(1/2)-b/((a*d-b*c)*b)^(1/2)*arc
tan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.53

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,ac},\right.$$
$$-\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^4+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)}{4\,ac},\ \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{}}{b}\right.}{}$$
$$\left.-\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^4+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^4+c}}\right)}{2\,ac}\right]$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sq
rt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), -1/4*(2*c*sqrt(-b/(b*c - a*d))*a
rctan(sqrt(d*x^4 + c)*sqrt(-b/(b*c - a*d))) - sqrt(c)*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^4 + c)))/(a*c), -1/2*(c*s
qrt(-b/(b*c - a*d))*arctan(sqrt(d*x^4 + c)*sqrt(-b/(b*c - a*d))) - sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^4 + c)))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^4\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{2b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**4)/sqrt(-a**2/b**2))/(2*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

Maxima [F]

$$\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x}\,dx$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{2\,a\,\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}+\cdots\right.}{\cdots}}\right.\cdots}{2\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output

```
- atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1
/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)) +
((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)
*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b
^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^
2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*
c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c +
 d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b
*c))))/(4*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(
1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^
3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4
*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^
2*d - a*b*c))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 874, normalized size of antiderivative = 10.28

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input

```
int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output

```
( - 2*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a
*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)
*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)) + 2*sqrt(b)*
sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*atan((sqrt(b)*sqrt(c
 + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
+ 2*a*d - b*c))*a*d - 2*sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2
*a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*b*c - sqrt(d)*sqrt(b)*sqrt
(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*sqrt(a*d - b*c)*
log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqr
t(c + d*x**4) + sqrt(d)*sqrt(b)*x**2) + sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*sqrt(a*d - b*c)*log(sqrt(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + s
qrt(d)*sqrt(b)*x**2) - sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*
a*d + b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*d + sqrt(b)*sqrt(2*sqrt
(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)
*x**2)*b*c + sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sq...
```

3.239 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{c+dx^4}}{4acx^4}+\frac{(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

output
```
-1/4*(d*x^4+c)^(1/2)/a/c/x^4+1/4*(a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/
2))/a^2/c^(3/2)-1/2*b^(3/2)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/
2))/a^2/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{-\frac{a\sqrt{c+dx^4}}{cx^4}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^2}$$

input
```
Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((a*Sqrt[c + d*x^4])/(c*x^4)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
4])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4} \int \frac{1}{x^8 (bx^4+a) \sqrt{dx^4+c}} dx^4$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4} \left(-\frac{\int \frac{bdx^4+2bc+ad}{2x^4 (bx^4+a) \sqrt{dx^4+c}} dx^4}{ac} - \frac{\sqrt{c+dx^4}}{acx^4} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(-\frac{\int \frac{bdx^4+2bc+ad}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^4}{2ac} - \frac{\sqrt{c+dx^4}}{acx^4} \right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{4} \left(-\frac{\frac{(ad+2bc) \int \frac{1}{x^4 \sqrt{dx^4+c}} dx^4}{a} - \frac{2b^2 c \int \frac{1}{(bx^4+a) \sqrt{dx^4+c}} dx^4}{a}}{2ac} - \frac{\sqrt{c+dx^4}}{acx^4} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\frac{2(ad+2bc)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{4b^2c\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{\frac{4b^{3/2}c\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*c*x^4)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^4+c}}{c\,x^4}+\frac{(ad+2cb)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2b^2\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{4a^2}$
risch	$-\frac{\sqrt{d\,x^4+c}}{4ac\,x^4}-\frac{-\frac{(ad+2cb)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a\sqrt{c}}-\frac{2b^2c\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}}{\cdots}$
elliptic	$-\frac{\sqrt{d\,x^4+c}}{4ac\,x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4a\,c^{\frac{3}{2}}}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-\cdots}{b}}\cdots}{x^2\cdots}\right)}{4a^2\sqrt{\cdots}}\cdots$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{4c\,x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a}+\frac{b^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-a\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-a/c*(d*x^4+c)^(1/2)/x^4+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^4+c)^(1
/2)/c^(1/2))+2*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.43

$$\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\,bc^2x^4\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+(2\,bc+ad)\sqrt{c}x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\sqrt{c}+2\,c}{x^4}\right)-2\,\sqrt{}}{8\,a^2c^2x^4}\right.$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(
d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*s
qrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x
^4 + c)*a*c)/(a^2*c^2*x^4), 1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(s
qrt(d*x^4 + c)*sqrt(-b/(b*c - a*d))) + (2*b*c + a*d)*sqrt(c)*x^4*log((d*x^
4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c^
2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*
sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) - (2*b*c + a
*d)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^4 + c)) - sqrt(d*x^4 + c)*a*c)/(
a^2*c^2*x^4), 1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^4 + c)
*sqrt(-b/(b*c - a*d))) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d
*x^4 + c)) - sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^4) \sqrt{c + dx^4}} dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^5} dx$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (a + bx^4) \sqrt{c + dx^4}} dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c + abd}a^2} - \frac{(2\,bc + ad) \arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^4 + c}}{4\,acx^4}$$

input
```
integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{\ln\left(\sqrt{d\,x^4+c}\,(b^4\,c-a\,b^3\,d)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c} - \frac{\ln\left(\sqrt{d\,x^4+c}\,(b^4\,c-a\,b^3\,d)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\sqrt{b^4\,c-a\,b^3\,d}}{4\,(a^3\,d-a^2\,b\,c)} - \frac{\sqrt{d\,x^4+c}}{4\,a\,c\,x^4} - \frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^4+c}\,3\mathrm{i}}{16\,\sqrt{c^3}\left(\frac{3\,b^4\,d^4}{16\,c}+\frac{5\,a\,b^3\,d^5}{32\,c^2}+\frac{a^2\,b^2\,d^6}{32\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^4+c}\,1\mathrm{i}}{32\,\sqrt{c^3}\left(\frac{5\,b^3\,d^5}{32\,a}+\frac{b^2\,d^6}{32\,c}+\frac{3\,b^4\,c\,d^4}{16\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^4+c}\,5\mathrm{i}}{32\,\sqrt{c^3}\left(\frac{3\,b^4\,d^4}{16\,a}+\frac{5\,b^3\,d^5}{32\,c}+\frac{a\,b^2\,d^6}{32\,c^2}\right)}\right)(a\,d+2\,b}{4\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
(log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x
^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x
^4) - (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(1
6*c) + (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d
*x^4)^(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (
3*b^4*c*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*(
(3*b^4*d^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2
*b*c)*1i)/(4*a^2*(c^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 3329, normalized size of antiderivative = 28.45

$$\int \frac{1}{x^5 (a + bx^4) \sqrt{c + dx^4}} \, dx = \text{Too large to display}$$

input
```
int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(4*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
 + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*s
qrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*c*d*x*
*6 + 2*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*
a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b
)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*c**2*x**4 +
 4*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d
- b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x*
*2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*c*d*x**8 - 4*sq
rt(d)*sqrt(b)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*
a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*s
qrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*c*d*x**6 + 4*sqrt(d)*sqrt
(b)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)
*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*b*c**2*x**6 - 2*sqrt(b)*sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) +
sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c
))*a*c**2*d*x**4 - 4*sqrt(b)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*
d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*c*d**2*x**8 + 2*sqrt(b)*...
```

3.240 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}$$

output
```
1/4*x^2*(d*x^4+c)^(1/2)/b/d+1/2*a^(3/2)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2
)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(1/2)-1/4*(2*a*d+b*c)*arctanh(d^(1/2)*x^
2/(d*x^4+c)^(1/2))/b^2/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\frac{bx^2\sqrt{c+dx^4}}{d}+\frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^2}$$

input
```
Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]
*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{381}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\int \frac{(bc+2ad)x^4+ac}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2bd}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{2a^2d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{2a^2d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{2bd}\right)$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2bd}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^4}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input
```
Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((x^2*Sqrt[c + d*x^4])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(2*b*d))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{-\frac{2a^{2} \operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\frac{\sqrt{d\,x^{4}+c}\,b\,x^{2}}{d}+\frac{(2ad+cb)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{4b^{2}}$
risch	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4bd}-\frac{(2ad+cb)\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{2b\sqrt{d}}-\frac{2a^{2}d\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d\cdots}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$
default	$\frac{\frac{x^{2}\sqrt{d\,x^{4}+c}}{4d}-\frac{c\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{4d^{\frac{3}{2}}}}{b}+\frac{a^{2}\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\cdots}{}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{\cdots}$
elliptic	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4bd}-\frac{c\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{4b\,d^{\frac{3}{2}}}-\frac{a\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{2b^{2}\sqrt{d}}-\frac{a^{2}\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-\cdots}{b}}\cdots}{\cdots}\right)}{4b^{2}\sqrt{-a\cdots}}$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*(-2*a^2/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d
-b*c))^(1/2))-(d*x^4+c)^(1/2)*b/d*x^2+(2*a*d+b*c)/d^(3/2)*arctanh((d*x^4+c
)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 745, normalized size of antiderivative = 6.06

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{2\sqrt{dx^4+c}bdx^2 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b^2d^2}\right.$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d
*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1
/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*
d)*sqrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)))/(b^2*d^2), 1/8*(2*sq
rt(d*x^4 + c)*b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*
a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) +
 (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/
(b^2*d^2), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan
(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x
^6 + a*c*x^2)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d
*x^2)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1128, normalized size of antiderivative = 9.17

$$\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output

```
(2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)*a*d**2*x**2 + 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d
- b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)
*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*d**2*x**2 - 2*sqrt(d)*sqrt(a)*
sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2
*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*a*d**2*x**2 + sqrt(
a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*c*d**2 + 2*sqrt
(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d
+ b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*d**3*x**4 + sq
rt(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*c*d**2 + 2*sqrt
(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b
*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*d**3*x**4 - sqrt(
a)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(
c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*a*c*d**2 - 2*sqrt(a)*sqrt(a*d - b
*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x
**2 + 2*a*d + 2*b*d*x**4)*a*d**3*x**4 - 4*sqrt(c + d*x**4)*log((sqrt(c + d
*x**4) + sqrt(d)*x**2)/sqrt(c))*a**2*d**3*x**2 + 2*sqrt(c + d*x**4)*log...
```

3.241 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}$$

output
```
-1/2*a^(1/2)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/b/(-a*d+
b*c)^(1/2)+1/2*arctanh(d^(1/2)*x^2/(d*x^4+c)^(1/2))/b/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{2b}$$

input
```
Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^2 + Sqrt[c + d*x
^4]]/Sqrt[d])/(2*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{385}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}\right)$$

input
```
Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(b*Sqrt[d]))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{\frac{a \operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{\sqrt{d}}}{2b}$
default	$\frac{\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}}-\frac{a\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}\right)}{}$
elliptic	$\frac{\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2b\sqrt{d}}+\frac{a\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-ad}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,b\sqrt{-\frac{ad-cb}{b}}}$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*(a/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^
(1/2))-1/d^(1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 638, normalized size of antiderivative = 7.01

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\sqrt{-\frac{a}{bc-}}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,bd}\right.$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(d*x^4 + c)*sqrt(-d)/(d*x^2)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan
(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x
^6 + a*c*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c)
)/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*
sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arct
an(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.75

$$\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{-\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}\log\left(-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{b}\,\sqrt{d\,x^4+c}+\sqrt{d}\,\sqrt{b}\,x^2\right)-\sqrt{d}\,\sqrt{a}\,\sqrt{ad-}}{}$$

input
```
int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
( - sqrt(d)*sqrt(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
 - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)
- sqrt(d)*sqrt(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*
c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2) + sqr
t(d)*sqrt(a)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqr
t(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4) + 2*log((sqrt(c + d*x**
4) + sqrt(d)*x**2)/sqrt(c))*a*d - 2*log((sqrt(c + d*x**4) + sqrt(d)*x**2)/
sqrt(c))*b*c)/(4*sqrt(d)*b*(a*d - b*c))
```

3.242 $\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/2*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(2*Sqrt[a]*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{2}\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*
c - a*d])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{4}+c}}{x^{2} \sqrt{a (a d-c b)}}\right)}{2 \sqrt{a (a d-c b)}}$
default	$-\frac{\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d \sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d \sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}}{x^{2}-\frac{\sqrt{-a b}}{b}}\right)}{4 \sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}}+\frac{\ln\left(\frac{-\frac{2(a d-c b}{b}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d \sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d \sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}}{x^{2}-\frac{\sqrt{-a b}}{b}}\right)}{4 \sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}}+\frac{\ln\left(\frac{-\frac{2(a d-c b}{b}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{8\,(abc-a^2d)}, \text{arctan}\right.$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c -
a^2*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c
 - a^2*d)]
```

Sympy [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.06

$$\int \frac{x}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{ad-bc}\left(\log\left(-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{b}\,\sqrt{d\,x^4+c}+\sqrt{d}\,\sqrt{b}\,x^2\right)+\log\left(\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{\ldots}}\right.\right.}{}$$

input
```
int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(sqrt(a)*sqrt(a*d - b*c)*(log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) -
2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2) + log(sqrt
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**
4) + sqrt(d)*sqrt(b)*x**2) - log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqr
t(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)))/(4*a*(a*d - b*c))
```

3.243 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/2*(d*x^4+c)^(1/2)/a/c/x^2-1/2*b*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*
x^4+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*Sqrt[c + d*x^4]/(a*c*x^2) - (b*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2
 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c - a*d
])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^2$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{2} \left(\frac{\int -\frac{bc}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4}}{acx^2} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bc}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4}}{acx^2} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{b \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{a} - \frac{\sqrt{c+dx^4}}{acx^2} \right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2} \left(-\frac{b \int \frac{1}{a-(ad-bc)x^4} d\frac{x^2}{\sqrt{dx^4+c}}}{a} - \frac{\sqrt{c+dx^4}}{acx^2} \right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{b\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{acx^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*c*x^2)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*S
qrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^4+c}}{x^2}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)bc}{\sqrt{a(ad-cb)}}}{2ac}$
default	$-\frac{\sqrt{d x^4+c}}{2ac\, x^2}-\frac{b\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}\cdots\right)}{a\cdots}$
risch	$-\frac{\sqrt{d x^4+c}}{2ac\, x^2}+\frac{b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}-\cdots$
elliptic	$-\frac{\sqrt{d x^4+c}}{2ac\, x^2}+\frac{b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}-\cdots$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-(d*x^4+c)^(1/2)/x^2-1/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2
)/x^2/(a*(a*d-b*c))^(1/2))*b*c)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(64) = 128$.

Time = 0.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}}\,dx$$
$$= \left[-\frac{\sqrt{-abc + a^2d}bcx^2 \log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^8 - 2\,(3\,abc^2 - 4\,a^2cd)x^4 + a^2c^2 + 4\,((bc - 2\,ad)x^6 - acx^2)\sqrt{dx^4 + c}\sqrt{-abc + a^2d}}{b^2x^8 + 2\,abx^4 + a^2}\right) + 4}{8\,(a^2bc^2 - a^3cd)x^2}, -\frac{\sqrt{abc - a^2d}bcx^2 \arctan\left(\frac{((bc - 2\,ad)x^4 - ac)\sqrt{dx^4 + c}\sqrt{abc - a^2d}}{2\,((abcd - a^2d^2)x^6 + (abc^2 - a^2cd)x^2)}\right) + 2\,\sqrt{dx^4 + c}(abc - a^2d)}{4\,(a^2bc^2 - a^3cd)x^2} \right]$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*
c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))
 + 4*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sq
rt(a*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^
2)) + 2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}}\,dx = \int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}} dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^3} dx$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (a + bx^4) \sqrt{c + dx^4}} dx = \frac{1}{2} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{\left(\sqrt{d}x^2 - \sqrt{dx^4 + c}\right)^2 b - bc + 2\, ad}{2\sqrt{abcd - a^2d^2}} \right)}{\sqrt{abcd - a^2d^2} ad} + \frac{2}{\left(\left(\sqrt{d}x^2 - \sqrt{dx^4 + c}\right)^2 - c\right) ad} \right)$$

input
```
integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 547, normalized size of antiderivative = 6.84

$$\int \frac{1}{x^3 (a+bx^4) \sqrt{c+dx^4}}\,dx = \frac{-\sqrt{d}\,\sqrt{a}\,\sqrt{d\,x^4+c}\,\sqrt{ad-bc}\log\left(-\sqrt{2\sqrt{d}\,\sqrt{a}\,\sqrt{ad-bc}-2ad+bc}+\sqrt{b}\,\sqrt{d\,x^4+c}+\sqrt{d}\,\sqrt{b}\,x^2\right)bc\,x^2}{}$$

input
```
int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output

```
( - sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)*b*c*x**2 - sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*
c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqr
t(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*x**2 + sqrt(d)*sqrt(a)*sqrt(c +
d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*
sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b*c*x**2 - sqrt(a)*sqrt(a*d
- b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt
(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*d*x**4 - sqrt(a)*sqrt(a*d
 - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b
)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*d*x**4 + sqrt(a)*sqrt(a*d -
 b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b
*x**2 + 2*a*d + 2*b*d*x**4)*b*c*d*x**4 - 4*sqrt(c + d*x**4)*a**2*d**2*x**2
 + 4*sqrt(c + d*x**4)*a*b*c*d*x**2 - 2*sqrt(d)*a**2*c*d - 4*sqrt(d)*a**2*d
**2*x**4 + 2*sqrt(d)*a*b*c**2 + 4*sqrt(d)*a*b*c*d*x**4)/(4*a**2*c*x**2*(sq
rt(d)*sqrt(c + d*x**4)*a*d - sqrt(d)*sqrt(c + d*x**4)*b*c + a*d**2*x**2 -
b*c*d*x**2))
```

3.244 $\int \frac{1}{x^7(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

output
```
-1/6*(d*x^4+c)^(1/2)/a/c/x^6+1/6*(2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c^2/x^2
+1/2*b^2*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*
d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}\left(-ac+3bcx^4+2adx^4\right)}{6a^2c^2x^6}+\frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(6*a^2*c^2*x^6) + (b^2*
ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^4+a) \sqrt{dx^4+c}} dx^2$$

$$\Big\downarrow \; 382$$

$$\frac{1}{2} \left(\frac{\int -\frac{2bdx^4+3bc+2ad}{x^4(bx^4+a)\sqrt{dx^4+c}} dx^2}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2} \left(-\frac{\int \frac{2bdx^4+3bc+2ad}{x^4(bx^4+a)\sqrt{dx^4+c}} dx^2}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \; 445$$

$$\frac{1}{2} \left(-\frac{-\frac{\int \frac{3b^2c^2}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4}(2ad+3bc)}{acx^2}}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^6} \right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{-\frac{3b^2c\int\frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}(2ad+3bc)}{acx^2}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^6}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(-\frac{-\frac{3b^2c\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}(2ad+3bc)}{acx^2}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^6}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(2ad+3bc)}{acx^2}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-1/3*Sqrt[c + d*x^4]/(a*c*x^6) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(a*
c*x^2)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]
)/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}\,\left(-2 a d\,x^{4}-3 b c\,x^{4}+a c\right)}{3 x^{6}}+\frac{b^{2} c^{2} \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{4}+c}}{x^{2} \sqrt{a(a d-c b)}}\right)}{\sqrt{a(a d-c b)}}}{2 a^{2} c^{2}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}\,(-2 a d\,x^{4}-3 b c\,x^{4}+a c)}{6 c^{2} a^{2} x^{6}}+\frac{b^{2}\left(-\frac{\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}}{}}}{x^{2}-\frac{\sqrt{-a b}}{b}}\right)}{4\sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}}\right.}{}$
default	$-\frac{\sqrt{d\,x^{4}+c}\,(-2 d\,x^{4}+c)}{6 a\,x^{6} c^{2}}+\frac{b^{2}\left(-\frac{\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a}}{b}\right.}{b}}}{x^{2}-\frac{\sqrt{-a b}}{b}}\right)}{4\sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}}\right.}{}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{6 a c\,x^{6}}+\frac{d\sqrt{d\,x^{4}+c}}{3 a\,c^{2} x^{2}}+\frac{b\sqrt{d\,x^{4}+c}}{2 a^{2} x^{2} c}-\frac{b^{2}\ln\left(\frac{-\frac{2(a d-c b)}{b}+\frac{2 d\sqrt{-a b}\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-c b}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{}}{}}}{x^{2}-\frac{\sqrt{-a b}}{b}}\right)}{4 a^{2}\sqrt{-a b}\,\sqrt{-\frac{a d-c b}{b}}}$

input

```
int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/2/a^2*(-1/3*(d*x^4+c)^(1/2)*(-2*a*d*x^4-3*b*c*x^4+a*c)/x^6+b^2*c^2/(a*(a
*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

$$= \left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^6\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((bc-2\,ad)x^6-acx^2)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{24\,(a^3bc^3-a^4c^2d)x^6}\right.$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 +
 a^2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^
4)*sqrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(
a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^
4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}}\,dx =$$

$$-\frac{1}{6}d^{\frac{5}{2}}\left(\frac{3\,b^2 \arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2} + \frac{2\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4 b - 6\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+\right.\right.}}\right.$$

input
```
integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b*c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqr
t(d)*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1240, normalized size of antiderivative = 10.78

$$\int \frac{1}{x^7 (a+bx^4) \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

input
```
int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(3*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)*b**2*c**2*x**6 + 12*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(
a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c*d*x**10 + 3*sqrt(d
)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt
(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x*
*2)*b**2*c**2*x**6 + 12*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*l
og(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c
+ d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c*d*x**10 - 3*sqrt(d)*sqrt(a)*sqrt(
c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt
(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b**2*c**2*x**6 - 12*sqrt
(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b**2*c*
d*x**10 + 9*sqrt(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
 - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*
b**2*c**2*d*x**8 + 12*sqrt(a)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt
(b)*x**2)*b**2*c*d**2*x**12 + 9*sqrt(a)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sq...
```

3.245 $\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 660

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}} - \frac{(-a)^{5/4}\text{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}}$$
$$- \frac{c^{3/4}(bc+4ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6bd^{5/4}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
1/3*x*(d*x^4+c)^(1/2)/b/d-1/4*(-a)^(5/4)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/
4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)-1/4*(-a)^(5/4)*arctanh
((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(
1/2)-1/6*c^(3/4)*(4*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2)
)/b/d^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Ell
ipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)+1/8*a*(b^(1/2)*c
^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/
2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*
c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/
2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/
2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.25 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.38

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x\Bigg(-\frac{(bc+3ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ad}+5\Bigg(\frac{c}{d}+x^4+\frac{5a^2c^2\,\mathrm{Appe}}{d(a+bx^4)\Big(-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\Big(2b}}{15b\sqrt{c+dx^4}}$$

input
```
Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF
1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/
4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/
4, -((d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.52, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {979, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 979}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\int \frac{(bc+3ad)x^4+ac}{(bx^4+a)\sqrt{dx^4+c}}dx}{3bd}$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{3a^2d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}}{3bd}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}}{3bd}$$

$$\Big\downarrow \text{ 925}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}}{3bd}$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \cdots}{ad+b\cdots}}{2a}\cdots\right.}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\cdots}{\left(1-\cdots\right.}}{ad+bc}}{2a}\cdots\right.}{3bd}$$

$$\Big\downarrow \; 761$$

$$\frac{x\sqrt{c+dx^4}}{3bd}-$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\cdots}{\cdots}}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \; 2221$$

$$\frac{x\sqrt{dx^4+c}}{3bd}-$$

$$\frac{(bc+3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{3a^2d\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\,\text{arct}\cdots\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\cdots}{\cdots}\right.}{\cdots}$$

$$\Big\downarrow \text{2223}$$

$$\frac{x\sqrt{dx^4+c}}{3bd}-$$

$$\frac{\frac{(bc+3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-3a^2d\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\,\text{arct}\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.}{}$$

input
```
Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x*Sqrt[c + d*x^4])/(3*b*d) - (((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(
1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (3*a^2*d*(((a*((Sqrt[
b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*A
rcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4
)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sq
rt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(
b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt
[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sq
rt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*
Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.45

method	result
elliptic	$\frac{x\sqrt{d\,x^4+c}}{3bd}+\frac{\left(-\frac{a}{b^2}-\frac{c}{3db}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2\cdots}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{\cdots}}\right)}{\sqrt{\frac{-ad+cb}{b}}}\cdots}{\cdots}\right)}{\cdots}$
risch	$\frac{x\sqrt{d\,x^4+c}}{3bd}-\frac{\frac{(3ad+cb)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{3a^2d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}\cdots}{\cdots}\right)}{\cdots}}{3bd}$
default	$\frac{\frac{x\sqrt{d\,x^4+c}}{3d}-\frac{c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3d\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{b}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\,\_\alpha^3b\sqrt{\cdots}}{\cdots}}{\cdots}\right)}{8b^3\cdots}$

input
```
int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(d*x^4+c)^(1/2)/b/d+(-a/b^2-1/3/d/b*c)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-
I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/
2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/8*a^2/b^3*sum(1/_alpha^3*(-1
/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1
/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)
*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Ellipt
icPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2
)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}\,x-\left(\int \frac{\sqrt{d\,x^4+c}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)ac-3\left(\int \frac{\sqrt{d\,x^4+c}\,x^4}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)ad-\left(\int \frac{\sqrt{d\,x^4+c}\,x^4}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)bc}{3bd}$$

input
```
int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**4)*x - int(sqrt(c + d*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*
d*x**8),x)*a*c - 3*int((sqrt(c + d*x**4)*x**4)/(a*c + a*d*x**4 + b*c*x**4
+ b*d*x**8),x)*a*d - int((sqrt(c + d*x**4)*x**4)/(a*c + a*d*x**4 + b*c*x**
4 + b*d*x**8),x)*b*c)/(3*b*d)
```

3.246 $\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 625

$$\begin{aligned}
&\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx \\
&= -\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{-a}\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}} \\
&\quad + \frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-1/4*(-a)^(1/4)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/b^(3/4)/(a*d-b*c)^(1/2)-1/4*(-a)^(1/4)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(
1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/2*c^(3/4)*(c^(1/2)
+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1
/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(
c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),
-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(
1/2),1/2*2^(1/2))/b/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(
d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x
^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1
/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))/d^(1/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{x^5\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

input
```
Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4
)/a)])/(5*a*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.18 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.53, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {983, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{925}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}$$

$$\Big\downarrow \text{1541}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\cdots\right.}{\cdots}}{2a}\cdots\right.}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{(\sqrt{b}}{(\sqrt{}}}{ad+bc}}{2a}}{b}$$

$\downarrow$ 761

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{}\right.}{}\right.}{b}$$

$\downarrow$ 2221

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-$$

$$a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{a\sqrt[4]{d}\left(\sqrt{c}+\right.}{}\right.$$

$\downarrow$ 2223

$$\frac{\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-$$

$$a\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-a}}{\sqrt[4]{-a}\sqrt[4]{b}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{}}{2a}\right.$$

input
```
Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*
x^4]) - (a*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (S
qrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])
/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/
4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a
*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(
((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((
Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.42

method	result
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\,\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{}}{\_\alpha}\right)}{8b^2}$
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\,\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\,\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}}{}}{\_\alpha}\right)}{8b^2}$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),
I)-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha
^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^
(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^
2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/
d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))
),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^4/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^4}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.247 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 625

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}}$$
$$+ \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}$$

output
```
-1/4*b^(1/4)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/
(-a)^(3/4)/(a*d-b*c)^(1/2)-1/4*b^(1/4)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4
)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/2*d^(3/4)*(c^(1/2)
+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1
/8*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(
c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),
-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(
1/2),1/2*2^(1/2))/a/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(
d*x^4+c)^(1/2)+1/8*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x
^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1
/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))/d^(1/4)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.26

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx =$$

$$-\frac{5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*
x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4
)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Rubi [A] (warning: unable to verify)

Time = 1.99 (sec) , antiderivative size = 860, normalized size of antiderivative = 1.38, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 925}$$

$$\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{a\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}$$

$$\Big\downarrow \; 2221$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+$$

$$\frac{\frac{a\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\cdots}{\sqrt[4]{-\cdots}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{\cdots}}{2a}$$

$$\Big\downarrow \; 2223$$

$$\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\cdots}{\sqrt[4]{-\cdots}}\right.}{2\sqrt[4]{b}\sqrt{bc-ad}}\right.}{\cdots}}{2a}$$

$$\frac{\frac{\left(\sqrt{d}a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\frac{\sqrt[4]{-a}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\text{arct}\cdots}{2\sqrt[4]{b}\sqrt{\cdots}}\right.}{\cdots}}{2a}$$

input
```
Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] -
 Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]
)/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*
c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.31

method	result	s
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4} b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\, x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d+c b}{b}}\, \sqrt{d\, x^{4}+c}}\right)}{\sqrt{\frac{-a d+c b}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \mathrm{EllipticPi}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, \frac{i \sqrt{c}\, \_\alpha^{2} b}{\sqrt{d}\, a}, \frac{\sqrt{-\frac{i \sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, a \sqrt{d\, x^{4}+c}}}{\_\alpha^{3}}}{8 b}$	1
elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^{4} b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2 d\, x^{2} \_\alpha^{2}+2 c}{2 \sqrt{\frac{-a d+c b}{b}}\, \sqrt{d\, x^{4}+c}}\right)}{\sqrt{\frac{-a d+c b}{b}}}+\frac{2 \_\alpha^{3} b \sqrt{1-\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \sqrt{1+\frac{i \sqrt{d}\, x^{2}}{\sqrt{c}}}\, \mathrm{EllipticPi}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, \frac{i \sqrt{c}\, \_\alpha^{2} b}{\sqrt{d}\, a}, \frac{\sqrt{-\frac{i \sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\, a \sqrt{d\, x^{4}+c}}}{\_\alpha^{3}}}{8 b}$	1

input
```
int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^
2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)
*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**4)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.248 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 662

$$\begin{aligned}
&\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx \\
&= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{-bc+ad}}-\frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{-bc+ad}} \\
&-\frac{d^{3/4}(4bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6ac^{5/4}(bc+ad)\sqrt{c+dx^4}} \\
&-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-1/3*(d*x^4+c)^(1/2)/a/c/x^3-1/4*b^(5/4)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/
4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/4*b^(5/4)*arctanh
((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*c
)^(1/2)-1/6*d^(3/4)*(a*d+4*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+
d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1
/2))/a/c^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*b*(b^(1/2)*c^(1/2)+(-a)^(1/2)
*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*
EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/
2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/
(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*b*(b^(1/2
)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^
(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^4+c)^
(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 (a+bx^4) \sqrt{c+dx^4}} \, dx = \frac{-bdx^8\sqrt{1+\frac{dx^4}{c}} \text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + \frac{5a\left(-5ac(ac+4bcx^4+2adx^4+bdx^8)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - \right.}{(a+bx^4)\left(5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - 2\right.}}{15a^2cx^3\sqrt{c+dx^4}}$$

input
```
Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*Appell
F1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c +
d*x^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d
*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5
*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c
*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4
, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^
4])
```

Rubi [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {980, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{bdx^4+3bc+ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{bdx^4+3bc+ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{ 1021}$$

$$-\frac{3bc\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx + d\int \frac{1}{\sqrt{dx^4+c}}dx}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{3bc\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac} - \frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{925}$$

$$-\frac{3bc\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{1541}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\cdots\right.}{\cdots}}{2a}\cdots\right)}{3ac}\cdots$$

$$\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\cdots}{a\cdots}}{2a}\cdots\right)}{3ac}\cdots$$

$$\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{\cdots}\right.}{\cdots}\cdots\right)}{\cdots}\cdots$$

$$\frac{\sqrt{c+dx^4}}{3acx^3}$$

$$\Big\downarrow \text{2221}$$

$$-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{a\sqrt[4]{d}\cdots}{\cdots}\right)}{\cdots}-\frac{\sqrt{c+dx^4}}{3acx^3}$$

$\Big\downarrow$ 2223

$$-\frac{\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\cdots\right)}{\cdots}-\frac{\sqrt{dx^4+c}}{3acx^3}$$

input
```
Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/3*Sqrt[c + d*x^4]/(a*c*x^3) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*b*c*(((a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
+ Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c
- a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] -
Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a
) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])
])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.44

method	result
default	$\frac{-\frac{\sqrt{d x^4+c}}{3 c x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}}{a}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2 d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i}{\cdots}}\cdots}{\cdots}}{\cdots}}{8a}$
risch	$-\frac{\sqrt{d x^4+c}}{3ac\,x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}+\frac{3c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2 d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3 b\cdots}{\cdots}}{\cdots}\cdots\right)}{8\cdots}}{3ac}$
elliptic	$-\frac{\sqrt{d x^4+c}}{3ac\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3ca\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2 d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha\cdots}{\cdots}}{\cdots}}{\cdots}$

input
```
int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3*d/c/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(
1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*
c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4
+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x
^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/
c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))
^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx$$

input
```
integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}}\, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}}\, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + c}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}}\, dx = \int \frac{1}{x^4 \left(b\,x^4 + a\right) \sqrt{d\,x^4 + c}}\, dx$$

input
```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{\sqrt{d\,x^4 + c}}{bd\,x^{12} + ad\,x^8 + bc\,x^8 + ac\,x^4} dx$$

input
```
int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**4)/(a*c*x**4 + a*d*x**8 + b*c*x**8 + b*d*x**12),x)
```

3.249 $\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 804

$$\begin{aligned}\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}x^2\right)} \\ &- \frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} - \frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4b(bc-ad)} \\ &- \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}} \\ &+ \frac{\sqrt[4]{c}(bc+2ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2bd^{3/4}(bc+ad)\sqrt{c+dx^4}} \\ &+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\ &- \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}\end{aligned}$$

output
```
x*(d*x^4+c)^(1/2)/b/d^(1/2)/(c^(1/2)+d^(1/2)*x^2)-1/4*a*(-(-a*d+b*c)/(-a)^
(1/2)/b^(1/2))^(1/2)*arctan((-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^
4+c)^(1/2))/b/(-a*d+b*c)-1/4*a*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arcta
n(((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/b/(-a*d+b*c)-c^
(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/b/d^(3/4)/(d*x^4+c)^(1
/2)+1/2*c^(1/4)*(2*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))
/b/d^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(
1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(
(-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)-1/8*a*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d
^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1
/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*
2^(1/2))/b^(3/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(d
*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x^7\sqrt{\frac{c+dx^4}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

input
```
Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^
4)/a)])/(7*a*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.47 (sec) , antiderivative size = 1089, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {983, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{983}$$

$$\frac{\int \frac{x^2}{\sqrt{dx^4+c}}dx}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{c}\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^4+c}}dx}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}$$

$$\Big\downarrow \text{993}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{\sqrt{dx^4+c}}dx}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int\frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int\frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}\right)}{b}$$

$\downarrow$ 1541

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-\cdots}\right.\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{b}$$

$\downarrow$ 27

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\cdots}{ad\cdots}}{2\sqrt{b}}\cdots\right)}{b}$$

$\downarrow$ 761

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^4}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{x\sqrt{c+dx^4}}{\sqrt{c}+\sqrt{d}x^2}}{\sqrt{d}}}{b}-$$

$$\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}\cdots\right.}{\cdots}\cdots\right)}{b\cdots}$$

$\downarrow$ 2221

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}}{b}-$$

$$\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},\cdots\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\cdots\right)}{bc+ad}\cdots}{2\sqrt{b}}\cdots\right)}{\cdots}$$

$\downarrow$ 2223

$$\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^4+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^4+c}}-\frac{x\sqrt{dx^4+c}}{\sqrt{d}x^2+\sqrt{c}}}{\sqrt{d}}}{b}-$$

$$a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

input
```
Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-((-((x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + (c^(1/4)*(Sqrt[c] + S
qrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^4]))/Sqrt[d]) + (c^(1/4
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^4]))/b
 - (a*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b
]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[
d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*
(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/S
qrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sq
rt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sq
rt[c + d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(c^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)
*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + ((...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.36

method	result
default	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\sqrt{d}}-\frac{a\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^4 b+a\right)}-\frac{\frac{\text{arctanh}\left(\frac{2d\,x^2\_c\cdots}{2\sqrt{\frac{-ad+cb}{b}}\cdots}\right)}{\sqrt{\frac{-ad+cb}{b}}}\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{i\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d\,x^4+c}\sqrt{d}}-\frac{a\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^4 b+a\right)}-\frac{\frac{\text{arctanh}\left(\frac{2d\,x^2\_c\cdots}{2\sqrt{\frac{-ad+cb}{b}}\cdots}\right)}{\sqrt{\frac{-ad+cb}{b}}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I
/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2
)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/8*a/b^2*su
m(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-
a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/
a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c
)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/
a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*x^6/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{x^6}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{\sqrt{d\,x^4+c}\,x^6}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**6)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.250 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 656

$$
\begin{aligned}
\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx
&= \frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)} + \frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{c+dx^4}}\right)}{4(bc-ad)} \\
&\quad - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2(bc+ad)\sqrt{c+dx^4}} \\
&\quad - \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
(-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan((-(-a*d+b*c)/(-a)^(1/2)/b^(1
/2))^(1/2)*x/(d*x^4+c)^(1/2))/(-4*a*d+4*b*c)+((-a*d+b*c)/(-a)^(1/2)/b^(1/2
))^(1/2)*arctan(((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/(
-4*a*d+4*b*c)-1/2*c^(1/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2
)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^
(1/2))/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*
(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))
^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(1/2)/c^(1/4)/((-a)^(
1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/d^(1/4)/(d*x^4+c)^(1/2)+1/8*(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x
^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(
1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))
/b^(1/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(d*x^4+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\frac{x^3\sqrt{\frac{c+dx^4}{c}}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

input
```
Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4
)/a)])/(3*a*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 1.87 (sec) , antiderivative size = 865, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {993, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 993}$$

$$\frac{\int \frac{1}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{b}x^2+\sqrt{-a}\right)\sqrt{dx^4+c}}dx}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}-\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}$$

$\big\downarrow$ 2221

$$\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}\right.}{ad+bc}$$

$$\overline{\qquad 2\sqrt{b} \qquad}$$

$$\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2\sqrt{b}}$$

$\big\downarrow$ 2223

$$\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}\right.}{bc+ad}$$

$$\overline{\qquad 2\sqrt{b} \qquad}$$

$$\frac{\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\text{arctanh}\left(\dots\right.}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{b\dots}}\right.}{}}{2\sqrt{b}}$$

input
```
Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqr
t[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*Ar
cTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*(-a)^(
1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/Sqrt[b]
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
 d*x^4])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[
d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(c^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((Sqrt[b
]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]/S
qrt[-a] + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*(Sqrt[b] - (Sqrt[-a]*Sqrt[d]
)/Sqrt[c])^2)/(Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.29

method	result	s
default	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2d x^2 \_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d x^4+c}}}{\_\alpha}}{8b}$	1
elliptic	$\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)} \frac{-\frac{\operatorname{arctanh}\left(\frac{2d x^2 \_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3 b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2 b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d x^4+c}}}{\_\alpha}}{8b}$	1

input
```
int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+
2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_
alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^2}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx$$

input
```
int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**2)/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.251 $\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 833

$$\begin{aligned}
\int \frac{1}{x^2(a+bx^4)\sqrt{c+dx^4}}\,dx &= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}x^2\right)} \\
&- \frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}-\frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\arctan\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}x}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)} \\
&- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{ac^{3/4}\sqrt{c+dx^4}} \\
&+ \frac{\sqrt[4]{d}(2bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2ac^{3/4}(bc+ad)\sqrt{c+dx^4}} \\
&+ \frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}-\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}-a\sqrt{d}\right)\sqrt{c+dx^4}} \\
&- \frac{\sqrt{b}\left(\frac{\sqrt{b}\sqrt[4]{c}}{\sqrt[4]{d}}+\frac{\sqrt{-a}\sqrt[4]{d}}{\sqrt[4]{c}}\right)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8a\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt{c+dx^4}}
\end{aligned}$$

output
```
-(d*x^4+c)^(1/2)/a/c/x+d^(1/2)*x*(d*x^4+c)^(1/2)/a/c/(c^(1/2)+d^(1/2)*x^2)
-1/4*b*(-(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*arctan((-(-a*d+b*c)/(-a)^(1/
2)/b^(1/2))^(1/2)*x/(d*x^4+c)^(1/2))/a/(-a*d+b*c)-1/4*b*((-a*d+b*c)/(-a)^(
1/2)/b^(1/2))^(1/2)*arctan(((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)*x/(d*x^4+
c)^(1/2))/a/(-a*d+b*c)-d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d
^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2
))/a/c^(3/4)/(d*x^4+c)^(1/2)+1/2*d^(1/4)*(a*d+2*b*c)*(c^(1/2)+d^(1/2)*x^2)
*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4
)*x/c^(1/4)),1/2*2^(1/2))/a/c^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*b^(1/2)*
(b^(1/2)*c^(1/4)/d^(1/4)-(-a)^(1/2)*d^(1/4)/c^(1/4))*(c^(1/2)+d^(1/2)*x^2)
*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)
/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*
x^4+c)^(1/2)-1/8*b^(1/2)*(b^(1/2)*c^(1/4)/d^(1/4)+(-a)^(1/2)*d^(1/4)/c^(1/
4))*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Ellipt
icPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(
1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))/a/((-a)^(1/2)*b^(1
/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}} \, dx$$
$$= \frac{-21a(c+dx^4)+7(-bc+ad)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}}{21a^2cx\sqrt{c+dx^4}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4
, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]
*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt
[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.31 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4) \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{ 980}$$

$$\frac{\int -\frac{x^2(-bdx^4+bc-ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{x^2(-bdx^4+bc-ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{ 1054}$$

$$-\frac{\int \left(\frac{bcx^2}{(bx^4+a)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{ac}-\frac{\sqrt{c+dx^4}}{acx}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{\sqrt{b}c^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{b^{3/4}c\arctan\left(\frac{\sqrt{bc-a}}{\sqrt[4]{-a}\sqrt[4]{b}}\right.}{4\sqrt[4]{-a}\sqrt{bc-ad}}}{}$$

$$\frac{\sqrt{dx^4+c}}{acx}$$

input
```
Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
-(Sqrt[c + d*x^4]/(a*c*x)) - (-((Sqrt[d]*x*Sqrt[c + d*x^4])/(Sqrt[c] + Sqr
t[d]*x^2)) + (b^(3/4)*c*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) - (b^(3/4)*c*ArcTanh[(Sqrt[
b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*
c - a*d]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt
[c + d*x^4] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2
*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^4
]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*c
^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt
[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt
[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)
/c^(1/4)], 1/2])/(8*Sqrt[-a]*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[
b]*c^(3/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*(S...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 3.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.37

method	result
default	$\frac{-\frac{\sqrt{d x^4+c}}{c x}+\frac{i\sqrt{d}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}}{a}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{2 d x^2 \cdots}{2\sqrt{\frac{-ad+c\cdots}{b}}}\right)}{\sqrt{\frac{-ad+c\cdots}{b}}}}{\cdots}}{\cdots}$
elliptic	$-\frac{\sqrt{d x^4+c}}{a c x}+\frac{i\sqrt{d}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\,a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\cdots}{\cdots}}{\cdots}}{\cdots}$
risch	$-\frac{\sqrt{d x^4+c}}{a c x}+\frac{\frac{i\sqrt{d}\sqrt{c}\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{d x^4+c}}-\frac{c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\mathrm{arctanh}\left(\frac{\cdots}{2\sqrt{\cdots}}\right)}{\sqrt{\cdots}}}{\cdots}\cdots\right)}{\cdots}}{a c\cdots}$

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/c*(d*x^4+c)^(1/2)/x+I*d^(1/2)/c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1
-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1
/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/
2))^(1/2),I)))-1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1
/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^
(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx$$

input
```
integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + cx^2}} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{(bx^4 + a)\sqrt{dx^4 + cx^2}} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{1}{x^2 \left(b\,x^4 + a\right) \sqrt{d\,x^4 + c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^4) \sqrt{c + dx^4}} \, dx = \int \frac{\sqrt{d\, x^4 + c}}{bd\, x^{10} + ad\, x^6 + bc\, x^6 + ac\, x^2} dx$$

input
```
int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**4)/(a*c*x**2 + a*d*x**6 + b*c*x**6 + b*d*x**10),x)
```

3.252 $\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 154

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{(bc+2ad)\sqrt{c+dx^4}}{2b^3d^2} + \frac{a^3\sqrt{c+dx^4}}{4b^3(bc-ad)\,(a+bx^4)} + \frac{(c+dx^4)^{3/2}}{6b^2d^2} - \frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/2*(2*a*d+b*c)*(d*x^4+c)^(1/2)/b^3/d^2+1/4*a^3*(d*x^4+c)^(1/2)/b^3/(-a*d
+b*c)/(b*x^4+a)+1/6*(d*x^4+c)^(3/2)/b^2/d^2-1/4*a^2*(-5*a*d+6*b*c)*arctanh
(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}(-15a^3d^2+2a^2bd(4c-5dx^4)+2b^3cx^4(2c-dx^4)+2ab^2(2c^2+3cdx^4+d^2x^8))}{12b^3d^2(bc-ad)\,(a+bx^4)} + \frac{a^2(-6bc+5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{4b^{7/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*
x^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c
- a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[-(b*c) + a*d]])/(4*b^(7/2)*(-(b*c) + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 109, 27, 164, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{4}\int \frac{x^{12}}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \ 109$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{x^4(4ac-(2bc-5ad)x^4)}{2(bx^4+a)\sqrt{dx^4+c}}dx^4}{b(bc-ad)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{x^4(4ac-(2bc-5ad)x^4)}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2b(bc-ad)}\right)$$

$$\Big\downarrow \ 164$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}-\frac{a^2(6bc-5ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{b^2}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \, 73$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}-\frac{2a^2(6bc-5ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{b^2d}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \, 221$$

$$\frac{1}{4}\left(\frac{ax^8\sqrt{c+dx^4}}{b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{2a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{3b^2d^2}}{2b(bc-ad)}\right)$$

input
```
Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*x^8*Sqrt[c + d*x^4])/(b*(b*c - a*d)*(a + b*x^4)) - ((2*Sqrt[c + d*x^4]
*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(3*b^2*d^
2) + (2*a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(b^(5/2)*Sqrt[b*c - a*d]))/(2*b*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{5\left(-\left(ad-\frac{6cb}{5}\right)d^2(b\,x^4+a)a^2\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)+\left(-\frac{4\left(-\frac{d\,x^4}{2}+c\right)x^4c\,b^3}{15}-\frac{4a\left(\frac{d\,x^4}{2}+c\right)(d\,x^4+c)b^2}{15}-\frac{8d\left(-\frac{5d\,x^4}{4}+c\right)a^2}{15}\right.}{4\sqrt{(ad-cb)b}\,d^2b^3(ad-cb)(b\,x^4+a)}$
risch	$-\frac{(-db\,x^4+6ad+2cb)\sqrt{d\,x^4+c}}{6d^2b^3}-\frac{3a^2\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\right.}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^4\sqrt{-\frac{ad-cb}{b}}}$
default	$-\frac{\sqrt{d\,x^4+c}\,(-d\,x^4+2c)}{6b^2d^2}-\frac{a\sqrt{d\,x^4+c}}{b^3d}+\frac{3a^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2c}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}\right.}{}$
elliptic	$\frac{x^4\sqrt{d\,x^4+c}}{6b^2d}-\frac{c\sqrt{d\,x^4+c}}{3b^2d^2}-\frac{a\sqrt{d\,x^4+c}}{b^3d}-\frac{3a^2\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d}{}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^4\sqrt{-\frac{ad-cb}{b}}}$

input
```
int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4*(-(a*d-6/5*c*b)*d^2*(b*x^4+a)*a^2*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2))+(-4/15*(-1/2*d*x^4+c)*x^4*c*b^3-4/15*a*(1/2*d*x^4+c)*(d*x^4+c)*
b^2-8/15*d*(-5/4*d*x^4+c)*a^2*b+a^3*d^2)*(d*x^4+c)^(1/2)*((a*d-b*c)*b)^(1/
2))/((a*d-b*c)*b)^(1/2)/d^2/b^3/(a*d-b*c)/(b*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(130) = 260$.

Time = 0.12 (sec) , antiderivative size = 622, normalized size of antiderivative = 4.04

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{3\,(6\,a^3bcd^2-5\,a^4d^3+(6\,a^2b^2cd^2-5\,a^3bd^3)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,(2\,(b^5c\right.}{24\,(ab^6c^2d^2-2\,a^2b^5c}$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)
*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b
^2*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^
3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 -
 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d
*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 -
 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (
6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^
2 + a^2*b^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 -
15*a^4*b*d^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^
3)*x^4)*sqrt(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 +
(b^7*c^2*d^2 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{dx^4+c}a^3d}{4\,(b^4c-ab^3d)((dx^4+c)b-bc+ad)} + \frac{(6\,a^2bc-5\,a^3d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(b^4c-ab^3d)\sqrt{-b^2c+abd}} + \frac{(dx^4+c)^{\frac{3}{2}}b^4d^4-3\,\sqrt{dx^4+c}b^4cd^4-6\,\sqrt{dx^4+c}ab^3d^5}{6\,b^6d^6}$$

input
```
integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d))
+ 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))
/((b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4
 - 3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.21

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(d\,x^4+c)^{3/2}}{6\,b^2\,d^2} - \left(\frac{3\,c}{2\,b^2\,d^2} + \frac{a\,d-b\,c}{b^3\,d^2}\right)\sqrt{d\,x^4+c} + \frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(5\,a\,d-6\,b\,c)}{\sqrt{a\,d-b\,c}\,(5\,a^3\,d-6\,a^2\,b\,c)}\right)(5\,a\,d-6\,b\,c)}{4\,b^{7/2}\,(a\,d-b\,c)^{3/2}} - \frac{a^3\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b^4\,(d\,x^4+c)-2\,b^4\,c+2\,a\,b^3\,d\right)}$$

input
```
int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2)
)*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*
b*c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/
2)*(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c
 + d*x^4) - 2*b^4*c + 2*a*b^3*d))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3265, normalized size of antiderivative = 21.20

$$\int \frac{x^{15}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
(15*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c
+ d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d -
b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**4*c**2*d**3 + 180*sqrt(b)*sqrt(c
+ d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sq
rt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt
(a*d - b*c)*x**2))*a**4*c*d**4*x**4 + 240*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*
d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)
*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2)
)*a**4*d**5*x**8 - 18*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(
d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d
*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**3*b*c**3*d**2 -
 201*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c
 + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d -
 b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**3*b*c**2*d**3*x**4 - 108*sqrt(b)
*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x
**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt
(d)*sqrt(a*d - b*c)*x**2))*a**3*b*c*d**4*x**8 + 240*sqrt(b)*sqrt(c + d*x**
4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c
 + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d -
b*c)*x**2))*a**3*b*d**5*x**12 - 18*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - ...
```

3.253 $\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{c+dx^4}}{2b^2d} - \frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)(a+bx^4)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/2*(d*x^4+c)^(1/2)/b^2/d-1/4*a^2*(d*x^4+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^4+a)
+1/4*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^4}(-3a^2d+2b^2cx^4+2ab(c-dx^4))}{d(bc-ad)(a+bx^4)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{5/2}}$$

input
```
Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^4]*(-3*a^2*d + 2*b^2*c*x^4 + 2*a*b*(c - d*x^4)))/(d
*(b*c - a*d)*(a + b*x^4)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{4}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^4}{2(bx^4+a)\sqrt{dx^4+c}}dx^4}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{4}\left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{d}-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^4}}{b^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{a^2\sqrt{c+dx^4}}{b^2\left(a+bx^4\right)\left(bc-ad\right)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^4}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^4])/(b^2*(b*c - a*d)*(a + b*x^4))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^4])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\frac{3d\left(ad-\frac{4cb}{3}\right)\left(b x^4+a\right)a\arctan\left(\frac{\sqrt{d x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{4}+\frac{3\left(-\frac{2b^2 c x^4}{3}-\frac{2a\left(-d x^4+c\right)b}{3}+a^2 d\right)\sqrt{d x^4+c}\,\sqrt{(ad-cb)b}}{4}}{b^2(ad-cb)d(b x^4+a)\sqrt{(ad-cb)b}}$
risch	$\frac{\sqrt{d x^4+c}}{2b^2 d}+\frac{a\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{2b^3\sqrt{-\frac{ad-cb}{b}}}+ \cdots$
elliptic	$\frac{\sqrt{d x^4+c}}{2b^2 d}+\frac{a\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{2b^3\sqrt{-\frac{ad-cb}{b}}}+ \cdots$
default	$\frac{\sqrt{d x^4+c}}{2b^2 d}+\frac{a^2\left(-\frac{\sqrt{-ab}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{8b(ad\cdots}\cdots\right)}{\cdots}$

input
```
int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/((a*d-b*c)*b)^(1/2)*(-d*(a*d-4/3*c*b)*(b*x^4+a)*a*arctan((d*x^4+c)^(1/
2)*b/((a*d-b*c)*b)^(1/2))+(-2/3*b^2*c*x^4-2/3*a*(-d*x^4+c)*b+a^2*d)*(d*x^4
+c)^(1/2)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^4+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.12 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^4)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2bc-ad+2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2(2ab^3c^2-}{8(ab^5c^2d-2a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2ab^5cd^2+}\right.$$
$$-\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^4)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)-(2ab^3c^2-5a^2b^2c}{4(ab^5c^2d-2a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2ab^5cd^2+a^2b}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2
*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*
b*d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4
*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a
^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4
), -1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c
)) - (2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3
*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\frac{\sqrt{dx^4+c}a^2d^3}{(b^3c-ab^2d)((dx^4+c)b-bc+ad)}+\frac{(4\,abcd^2-3\,a^2d^3)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c-ab^2d)\sqrt{-b^2c+abd}}-\frac{2\sqrt{dx^4+c}d}{b^2}}{4\,d^2}$$

input
```
integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(sqrt(d*x^4 + c)*a^2*d^3/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*
d)) + (4*a*b*c*d^2 - 3*a^2*d^3)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b
*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) - 2*sqrt(d*x^4 + c)*d/b^2)/d
^2
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\sqrt{d\,x^4+c}}{2\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{4\,b^{5/2}\,(a\,d-b\,c)^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

input
```
int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*
(c + d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2072, normalized size of antiderivative = 16.85

$$\int \frac{x^{11}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
( - 3*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(
c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d
- b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**3*c*d**2 - 12*sqrt(b)*sqrt(c +
d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt
(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a
*d - b*c)*x**2))*a**3*d**3*x**4 + 4*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*
c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**
4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2
*b*c**2*d + 13*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt
(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*
sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*b*c*d**2*x**4 - 12*s
qrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x
**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c)
+ sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*b*d**3*x**8 + 4*sqrt(b)*sqrt(c + d*x
**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)
*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d
- b*c)*x**2))*a*b**2*c**2*d*x**4 + 16*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d -
b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x
**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*
b**2*c*d**2*x**8 - 9*sqrt(d)*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt...
```

3.254 $\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{a\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

output
```
1/4*a*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-1/4*(-a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{4b^{3/2}}$$

input
```
Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{4}\left(\frac{(2bc-ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2b(bc-ad)}+\frac{a\sqrt{c+dx^4}}{b(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^4}}{b(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{a\sqrt{c+dx^4}}{b(a+bx^4)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[c + d*x^4])/(b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/4
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^4+c}}{b\,x^4+a}+\frac{(ad-2cb)\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{4(ad-cb)b}$
elliptic	$-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b^2\sqrt{-\frac{ad-cb}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-cb}{b}}{}\right.}{}$
default	$\frac{-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4b\sqrt{-\frac{ad-cb}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}-}{}\right.}{}}{b}$

input
```
int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/b*(-a*(d*x^4+c)^(1/2)/(b*x^4+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \left[\frac{((2\,b^2c-abd)x^4+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(ab^2c-a^2b}{8\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^4)}\right.$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a))
 + 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3
*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*
b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))
/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{\sqrt{dx^4+c}ad^2}{(b^2c-abd)((dx^4+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{4\,d}$$

input
```
integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)(a\,d-2\,b\,c)}{4\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\sqrt{d\,x^4+c}}{2\,b\,(a\,d-b\,c)\,(2\,b\,(d\,x^4+c)+2\,a\,d-2\,b\,c)}$$

input
```
int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c
 + d*x^4) + 2*a*d - 2*b*c))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1344, normalized size of antiderivative = 13.58

$$\int \frac{x^7}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*
sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt
(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*d*x**2 - 4*sqrt(d)*sqrt(
b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)
*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sq
rt(d)*sqrt(a*d - b*c)*x**2))*a*b*c*x**2 + 2*sqrt(d)*sqrt(b)*sqrt(c + d*x**
4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c
 + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d -
b*c)*x**2))*a*b*d*x**6 - 4*sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c
)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4
)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*b**2*
c*x**6 + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x*
*2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(
d)*sqrt(a*d - b*c)*x**2))*a**2*c*d + 2*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(
d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d
*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a**2*d**2*x**4 - 2
*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqr
t(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(
a*d - b*c)*x**2))*a*b*c**2 - 3*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(
b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4...
```

3.255 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{4(bc-ad)\,(a+bx^4)} + \frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}$$

output
```
-1/4*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*d*arctanh(b^(1/2)*(d*x^4+c)^
(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\left(-\frac{\sqrt{c+dx^4}}{(bc-ad)\,(a+bx^4)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{4}\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4}\left(-\frac{d\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^4}{2(bc-ad)}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(-\frac{\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{bc-ad}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{4}\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/((b*c - a*d)*(a + b*x^4))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^4+c}}{b\,x^4+a}+\frac{d\arctan\left(\frac{\sqrt{d\,x^4+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{4ad-4cb}$
default	$-\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\cdots\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8b(ad-cb)\sqrt{-\frac{ad-\cdots}{b}}}$
elliptic	$-\frac{\sqrt{-ab}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\cdots\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8b(ad-cb)\sqrt{-\frac{ad-\cdots}{b}}}$

input
```
int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)*((d*x^4+c)^(1/2)/(b*x^4+a)+d/((a*d-b*c)*b)^(1/2)*arctan((d*x
^4+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$=\left[-\frac{(bdx^4+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}(b^2c-abd)}{8\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)},\right.$$
$$\left.-\frac{(bdx^4+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^4+c}\sqrt{-b^2c+abd}}{bdx^4+bc}\right)+\sqrt{dx^4+c}(b^2c-abd)}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4)}\right]$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*
sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2
*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(
sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b
^2*c - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^4+c}d}{4\left((dx^4+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{d\sqrt{d\,x^4+c}}{2\,(a\,d-b\,c)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{(a\,d-b\,c)}^{3/2}}$$

input
```
int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) +
(d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d -
b*c)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 872, normalized size of antiderivative = 10.02

$$\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
(2*sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*
sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt
(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*d*x**2 + 2*sqrt(d)*sqrt(b)*
sqrt(c + d*x**4)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x*
*2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(
d)*sqrt(a*d - b*c)*x**2))*b*d*x**6 + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)
*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x
**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*a*c*d + 2*sqrt(b)*sq
rt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sq
rt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*
x**2))*a*d**2*x**4 + sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(d)*sqrt(b)*sqrt(c
+ d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**4)/(sqrt(c + d*x**4)*sqrt(a*d -
b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*b*c*d*x**4 + 2*sqrt(b)*sqrt(a*d - b*
c)*atan((sqrt(d)*sqrt(b)*sqrt(c + d*x**4)*x**2 + sqrt(b)*c + sqrt(b)*d*x**
4)/(sqrt(c + d*x**4)*sqrt(a*d - b*c) + sqrt(d)*sqrt(a*d - b*c)*x**2))*b*d*
*2*x**8 + sqrt(c + d*x**4)*a*b*c*d + 2*sqrt(c + d*x**4)*a*b*d**2*x**4 - sq
rt(c + d*x**4)*b**2*c**2 - 2*sqrt(c + d*x**4)*b**2*c*d*x**4 + 2*sqrt(d)*a*
b*c*d*x**2 + 2*sqrt(d)*a*b*d**2*x**6 - 2*sqrt(d)*b**2*c**2*x**2 - 2*sqrt(d
)*b**2*c*d*x**6)/(4*b*(2*sqrt(d)*sqrt(c + d*x**4)*a**3*d**2*x**2 - 4*sqrt(
d)*sqrt(c + d*x**4)*a**2*b*c*d*x**2 + 2*sqrt(d)*sqrt(c + d*x**4)*a**2*b...
```

3.256 $\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}$$

output
```
1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/2*arctanh((d*x^4+c)^(1/2)/c
^(1/2))/a^2/c^(1/2)+1/4*b^(1/2)*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(
1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{-\frac{ab\sqrt{c+dx^4}}{(-bc+ad)(a+bx^4)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

input
```
Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^4])/((-(b*c) + a*d)*(a + b*x^4))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(bx^4+a\right)^2\sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{4}\left(\frac{\int \frac{bdx^4+2bc-2ad}{2x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{\int \frac{bdx^4+2(bc-ad)}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{4}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{a\left(a+bx^4\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(a*(b*c - a*d)*(a + b*x^4)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$\frac{(bx^4+a)\left(cb-\frac{3ad}{2}\right)\sqrt{c}\,b\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{(ad-cb)b}}\right)-\frac{\left(2(ad-cb)\left(bx^4+a\right)\operatorname{arctanh}\left(\frac{\sqrt{dx^4+c}}{\sqrt{c}}\right)+\sqrt{dx^4+c}\sqrt{c}\,ab\right)\sqrt{(ad-cb)b}}{2}}{2\sqrt{c}\sqrt{(ad-cb)b}\,a^2(ad-cb)(bx^4+a)}$
elliptic	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\cdots}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4a^2\sqrt{-\frac{ad-cb}{b}}}\cdots$
default	$-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^4+c}}{x^2}\right)}{2a^2\sqrt{c}}-\frac{b\left(-\frac{\sqrt{-ab}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\cdots\right)}{b}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$

input
```
int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x^4+a)*(c*b-3/2*a*d)*c^(1/2)*b*arctan((d*x^4+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2))-1/2*(2*(a*d-b*c)*(b*x^4+a)*arctanh((d*x^4+c)^(1/2)/c^(1/2))+(d*
x^4+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^4+a)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.18

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^
4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x
^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^
4 + c)*a*b*c - ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(
-b/(b*c - a*d))*arctan(sqrt(d*x^4 + c)*sqrt(-b/(b*c - a*d))) + ((b^2*c - a
*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c)
+ 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/8*(2
*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*
arctan(sqrt(-c)/sqrt(d*x^4 + c)) + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^
2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x
^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(a^3*b*c^2 - a^4*c*
d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*a*b*c - ((2*b^2*c
^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan(s
qrt(d*x^4 + c)*sqrt(-b/(b*c - a*d))) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^
2*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^4 + c)))/(a^3*b*c^2 - a^4*c*d + (a^
2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{\sqrt{dx^4+c}bd}{4\left(abc-a^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/
4*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Too large to display}$$

input
```
int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*
b^3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c +
 d*x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 2
56*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a
^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
 b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c
*d^2)) + ((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c
*d^3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^
5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c
 + d*x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 -
 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*
d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d
- 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 -
 a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d
 - b*c)^3)^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b
*c*d^2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a
^4*b*c*d) - ((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a...
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 9123, normalized size of antiderivative = 69.11

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
( - 12*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d -
b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*
*2*d**2*x**2 + 8*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**
4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d
- b*c))*a*b*c*d*x**2 - 12*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(
c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
 + 2*a*d - b*c))*a*b*d**2*x**6 + 8*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2
*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt
(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a
*d - b*c) + 2*a*d - b*c))*b**2*c*d*x**6 - 6*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2
*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt
(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a
*d - b*c) + 2*a*d - b*c))*a**2*c*d - 12*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sqr
t(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*
sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d -
 b*c) + 2*a*d - b*c))*a**2*d**2*x**4 + 4*sqrt(d)*sqrt(b)*sqrt(a)*sqrt(2*sq
rt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt...
```

3.257 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)(a+bx^4)} - \frac{\sqrt{c+dx^4}}{4acx^4(a+bx^4)} + \frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}$$

output
```
-1/4*b*(-a*d+2*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^4+a)-1/4*(d*x^4+
c)^(1/2)/a/c/x^4/(b*x^4+a)+1/4*(a*d+4*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2)
)/a^3/c^(3/2)-1/4*b^(3/2)*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(
-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{\frac{a\sqrt{c+dx^4}(-a^2d+2b^2cx^4+ab(c-dx^4))}{c(-bc+ad)x^4(a+bx^4)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{c^{3/2}}}{4a^3}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*Sqrt[c + d*x^4]*(-(a^2*d) + 2*b^2*c*x^4 + a*b*(c - d*x^4)))/(c*(-(b*c)
 + a*d)*x^4*(a + b*x^4)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^4])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^4]/Sqrt[c]])/c^(3/2))/(4*a^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{4}\int \frac{1}{x^8 (bx^4+a)^2 \sqrt{dx^4+c}}dx^4$$

$$\Big\downarrow \; 114$$

$$\frac{1}{4}\left(-\frac{\int \frac{3bdx^4+4bc+ad}{2x^4(bx^4+a)^2\sqrt{dx^4+c}}dx^4}{ac} - \frac{\sqrt{c+dx^4}}{acx^4(a+bx^4)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\left(-\frac{\int \frac{3bdx^4+4bc+ad}{x^4(bx^4+a)^2\sqrt{dx^4+c}}dx^4}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 168$$

$$\frac{1}{4}\left(-\frac{\frac{\int \frac{bd(2bc-ad)x^4+(bc-ad)(4bc+ad)}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^4\sqrt{dx^4+c}}dx^4}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^4}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^8}{d}-\frac{c}{d}}d\sqrt{dx^4+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^8}{d}+a-\frac{bc}{d}}d\sqrt{dx^4+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^4}(2bc-ad)}{a(a+bx^4)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^4}}{acx^4\left(a+bx^4\right)}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-(Sqrt[c + d*x^4]/(a*c*x^4*(a + b*x^4))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^4])/(a*(b*c - a*d)*(a + b*x^4)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^4]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{(b x^4+a)\left(cb-\frac{5ad}{4}\right) b^2 x^4 c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{d x^4+c}\, b}{\sqrt{(ad-cb)b}}\right)-\frac{\left(c x^4 \left(b x^4+a\right)(ad+4cb)(ad-cb)\,\mathrm{arctanh}\left(\frac{\sqrt{d x^4+c}}{\sqrt{c}}\right)+\left(2b^2 c x^4+a\left(-d x^4+\cdots\right.\right.\right.}{4}}{\sqrt{(ad-cb)b}\, c^{\frac{5}{2}} x^4 a^3 (ad-cb)(b x^4+a)}$
risch	$-\frac{\sqrt{d x^4+c}}{4c a^2 x^4}-\frac{-\frac{(ad+4cb)\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{2a\sqrt{c}}-2b^2c\left(-\frac{\sqrt{-ab}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
elliptic	$-\frac{\sqrt{d x^4+c}}{4c a^2 x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{4a^2c^{\frac{3}{2}}}-\frac{b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-a\cdots}}{\cdots}}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{2a^3\sqrt{-\frac{ad-cb}{b}}}$
default	$\frac{-\frac{\sqrt{d x^4+c}}{4c x^4}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^4+c}}{x^2}\right)}{4c^{\frac{3}{2}}}}{a^2}+\frac{b^2\left(-\frac{\sqrt{-ab}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8ab(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2\cdots}{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((b*x^4+a)*(c*b-5/4*a*d)*b^2*x^4*c^(5/2)*arctan((d*x^4+c)^(1/2)*b/((a*d-b
*c)*b)^(1/2))-1/4*(c*x^4*(b*x^4+a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^4+c)
^(1/2)/c^(1/2))+(2*b^2*c*x^4+a*(-d*x^4+c)*b-a^2*d)*(d*x^4+c)^(1/2)*a*c^(3/
2))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/c^(5/2)/x^4/a^3/(a*d-b*c)/(b*
x^4+a)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.43

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4
)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2
*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^
4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*
b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^
8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), 1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^
4 + c)*sqrt(-b/(b*c - a*d))) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8
+ (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 + 2*sqrt(d
*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^
2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c
^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 +
 (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(
d*x^4 + c)) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^
2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^
2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), 1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(s...
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

input
```
integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\,x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^4+c}b}{\sqrt{-b^2c+abd}}\right)}{4\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^4+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^4+c}b^2c^2d-(dx^4+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^4+c}abcd^2-\sqrt{dx^4+c}a^2d^3}{4\,(a^2bc^2-a^3cd)\left((dx^4+c)^2b-2\,(dx^4+c)bc+bc^2+(dx^4+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4
+ c)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 +
 c)^2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b
*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 3822, normalized size of antiderivative = 20.66

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
(((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d
 - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a
^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^
2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c
 + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^
3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^
6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^
2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*
a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c...
```

Reduce [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 15292, normalized size of antiderivative = 82.66

$$\int \frac{1}{x^5 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

input
```
int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
(40*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c
) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*
sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a**2*
c**2*d**2*x**6 + 80*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*
x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a
*d - b*c))*a**2*c*d**3*x**10 - 32*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*
sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(
b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) + 2*a*d - b*c))*a*b*c**3*d*x**6 - 24*sqrt(b)*sqrt(a)*sqrt(c + d*x
**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)*sqrt(a*d - b*c)
*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*b*c**2*d**2*x**10 + 80*sqrt(b)*sqrt
(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c)
*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)/sq
rt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*a*b*c*d**3*x**14 - 32
*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) +
 2*a*d - b*c)*sqrt(a*d - b*c)*atan((sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqr
t(b)*x**2)/sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*a*d - b*c))*b**2*c**
3*d*x**10 - 64*sqrt(b)*sqrt(a)*sqrt(c + d*x**4)*sqrt(2*sqrt(d)*sqrt(a)*...
```

3.258 $\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 191

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}$$

output
```
1/4*(-2*a*d+b*c)*x^2*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a*x^6*(d*x^4+c)^
(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/4*a^(3/2)*(-4*a*d+5*b*c)*arctan((-a*d+b*c)^
(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/b^3/(-a*d+b*c)^(3/2)-1/4*(4*a*d+b*c)*ar
ctanh(d^(1/2)*x^2/(d*x^4+c)^(1/2))/b^3/d^(3/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{bx^2\sqrt{c+dx^4}(-2a^2d+b^2cx^4+ab(c-dx^4))}{d(bc-ad)(a+bx^4)} + \frac{a^{3/2}(5bc-4ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{(bc+4ad)\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{d^{3/2}}}{4b^3}$$

input

```
Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
((b*x^2*Sqrt[c + d*x^4]*(-2*a^2*d + b^2*c*x^4 + a*b*(c - d*x^4)))/(d*(b*c
- a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(
Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(3
/2) - ((b*c + 4*a*d)*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^4]])/d^(3/2))/(4*b^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {965, 372, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)} - \frac{\int \frac{x^4(3ac-2(bc-2ad)x^4)}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 444$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{-\frac{\int -\frac{2\left((bc-ad)(bc+4ad)x^4+ac(bc-2ad)\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{2bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\int \frac{(bc-ad)(bc+4ad)x^4+ac(bc-2ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 398$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}-\frac{a^2d(5bc-4ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{a^2d(5bc-4ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^2d(5bc-4ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 291$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^2d(5bc-4ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{ax^6\sqrt{c+dx^4}}{2b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{(bc-ad)(4ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}-\frac{a^{3/2}d(5bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}}{bd}-\frac{x^2\sqrt{c+dx^4}(bc-2ad)}{bd}}{2b(bc-ad)}\right)$$

input
```
Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*x^6*Sqrt[c + d*x^4])/(2*b*(b*c - a*d)*(a + b*x^4)) - (-(((b*c - 2*a*d)
*x^2*Sqrt[c + d*x^4])/(b*d)) + (-((a^(3/2)*d*(5*b*c - 4*a*d)*ArcTan[(Sqrt[
b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[b*c - a*d])) + ((b*c -
 a*d)*(b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sqrt[d]))/(
b*d))/(2*b*(b*c - a*d)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{\frac{a^{2}\left(-\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(4ad-5cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{ad-cb}-\frac{\sqrt{d\,x^{4}+c}\,b\,x^{2}}{d}+\frac{(4ad+cb)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{4b^{3}}$
risch	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4b^{2}d}-\frac{a\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{b^{3}\sqrt{d}}-\frac{\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)c}{4b^{2}d^{\frac{3}{2}}}-\frac{5a^{2}\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad}{}}}{}\right.}{8b^{3}\sqrt{-a}}$
elliptic	$\frac{x^{2}\sqrt{d\,x^{4}+c}}{4b^{2}d}-\frac{a\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)}{b^{3}\sqrt{d}}-\frac{\ln\left(\sqrt{d}\,x^{2}+\sqrt{d\,x^{4}+c}\right)c}{4b^{2}d^{\frac{3}{2}}}-\frac{5a^{2}\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad}{}}}{}\right.}{8b^{3}\sqrt{-a}}$
default	Expression too large to display

input
```
int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^3*(a^2/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(4*a*d-5*b*c)/(a
*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))-(d*x
^4+c)^(1/2)*b/d*x^2+(4*a*d+b*c)/d^(3/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1391, normalized size of antiderivative = 7.28

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\text{Too large to display}$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4
*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c)
 + (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 -
4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b
*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*
b*x^4 + a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x
^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4), 1/16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*
d - 4*a^2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(d*x^4 + c)*sqrt(-d)/(d*x^2)) +
(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^
2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)
*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4
), -1/8*((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*s
qrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*s
qrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3
*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 ...
```

Sympy [F]

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(163) = 326.

Time = 0.22 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.74

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= -\frac{\left(5\,a^2bc\sqrt{d}-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,(b^4c-ab^3d)\sqrt{abcd-a^2d^2}}+\frac{\sqrt{dx^4+c}x^2}{4\,b^2d}$$
$$+\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2bcd-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^3d^2-a^2bc^2d}{2\left(b^4c\sqrt{d}-ab^3d^{\frac{3}{2}}\right)\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad\right.}$$
$$+\frac{(bc+4\,ad)\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{8\,b^3d^{\frac{3}{2}}}$$

input
```
integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d
*x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*
sqrt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2*a^2*b*c*d - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
a^3*d^2 - a^2*b*c^2*d)/((b^4*c*sqrt(d) - a*b^3*d^(3/2))*((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) + 1/8*(b*c + 4*a*d)*log((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2)/(b^3*d^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{13}}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 5868, normalized size of antiderivative = 30.72

$$\int \frac{x^{13}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\text{Too large to display}$$

input
```
int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
(16*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)*a**3*c*d**3*x**2 + 32*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sq
rt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
 + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**3*d**4*x**6 - 20*sq
rt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(
a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqr
t(b)*x**2)*a**2*b*c**2*d**2*x**2 - 24*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqr
t(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
+ sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*b*c*d**3*x**6 + 32
*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sq
rt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*
sqrt(b)*x**2)*a**2*b*d**4*x**10 - 20*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt
(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) +
 sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b**2*c**2*d**2*x**6 -
40*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)*a*b**2*c*d**3*x**10 + 16*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*
sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c)
+ sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**3*c*d**3*x**2 + 3...
```

3.259 $\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

output
```
1/4*a*x^2*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-1/4*a^(1/2)*(-2*a*d+3*b*c
)*arctan((-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(3/2
)+1/2*arctanh(d^(1/2)*x^2/(d*x^4+c)^(1/2))/b^2/d^(1/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{\frac{abx^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{d}}}{4b^2}$$

input
```
Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^2 + Sqrt[c + d*x^
4]])/Sqrt[d])/(4*b^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^4+c}}dx^2}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b(a+bx^4)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}}{2b(bc-ad)}\right)$$

$$\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\,(a+bx^4)\,(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\,(a+bx^4)\,(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^4}}{2b\,(a+bx^4)\,(bc-ad)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((a*x^2*Sqrt[c + d*x^4])/(2*b*(b*c - a*d)*(a + b*x^4)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}-\frac{(2ad-3cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{ad-cb}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^4+c}}{x^2\sqrt{d}}\right)}{\sqrt{d}}}{4b^2}$
elliptic	$\frac{\ln\left(\sqrt{d}\,x^2+\sqrt{d\,x^4+c}\right)}{2b^2\sqrt{d}}-\frac{a\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8b^2(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+ ad\sqrt{-ab}\,\ln\Bigg(-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\,(x^2-$
default	Expression too large to display

input
```
int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*(-a/(a*d-b*c)*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(2*a*d-3*b*c)/(a*
(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))-2/d^(
1/2)*arctanh((d*x^4+c)^(1/2)/x^2/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1083, normalized size of antiderivative = 7.68

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output

```
[1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2
 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqr
t(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*
d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*
d*x^2 - 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(d*x^4
 + c)*sqrt(-d)/(d*x^2)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*
(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b
^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^
2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 +
 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4
 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2
*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(d)*x^2 - c))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1
/8*(2*sqrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d
 - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)...
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input

```
integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output

```
Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.21 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.11

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)} - \frac{\log\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2\right)}{4\,b^2\sqrt{d}}$$

input
```
integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*s
qrt(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqr
t(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/
(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)
) - 1/4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^9}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 3238, normalized size of antiderivative = 22.96

$$\int \frac{x^9}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - 4*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(
a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqr
t(b)*x**2)*a**2*d**2*x**2 + 6*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log
( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c
 + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*c*d*x**2 - 4*sqrt(a)*sqrt(c + d*x**
4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d +
 b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*d**2*x**6 + 6
*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sq
rt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*
x**2)*b**2*c*d*x**6 - 4*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(
2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4
) + sqrt(d)*sqrt(b)*x**2)*a**2*d**2*x**2 + 6*sqrt(a)*sqrt(c + d*x**4)*sqrt
(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sq
rt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*c*d*x**2 - 4*sqrt(a)*sq
rt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c)
- 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*d**2
*x**6 + 6*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt
(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sq
rt(b)*x**2)*b**2*c*d*x**6 + 4*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*x**2 ...
```

3.260 $\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)}+\frac{c\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}(bc-ad)^{3/2}}$$

output
```
-1/4*x^2*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*c*arctan((-a*d+b*c)^(1/2
)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\left(-\frac{x^2\sqrt{c+dx^4}}{(bc-ad)(a+bx^4)}+\frac{c\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-((x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/4
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{2}\left(\frac{\int \frac{c}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{2(bc-ad)}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{c\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{2(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-1/2*(x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^4+c}\,x^2}{c\left(b\,x^4+a\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{4(ad-cb)}$
elliptic	$\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8b(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\sqrt{\right.}}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8b^2(ad-cb)\sqrt{-\frac{ad-cb}{b}}}$
default	Expression too large to display

input
```
int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*c/(a*d-b*c)*(-(d*x^4+c)^(1/2)*x^2/c/(b*x^4+a)+1/(a*(a*d-b*c))^(1/2)*a
rctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(77) = 154.

Time = 0.15 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$=\left[-\frac{4\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2+4\,(}{b^2x^8+2\,abx^4+a^2}\right.}{16\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\right.$$
$$\left.-\frac{2\sqrt{dx^4+c}(abc-a^2d)x^2-(bcx^4+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^4-ac)\sqrt{dx^4+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^6+(abc^2-a^2cd)x^2)}\right)}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^4)}\right]$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sq
rt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c
*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(
d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*
c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d
 + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(77) = 154.

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2ad+bc^2\right)(b^2c-abd}$$

input
```
integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*(
(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b
 - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*a*d + b*c^2)*(b^2*c - a*b*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^5}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1561, normalized size of antiderivative = 16.78

$$\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqr
t(d)*sqrt(b)*x**2)*a*c*x**2 - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d
- b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt
(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c*x**6 - 2*sqrt(d)*sqrt(a)*
sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c
) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*c*x*
*2 - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)*b*c*x**6 + 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d -
b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c + d*x**4)*b*
x**2 + 2*a*d + 2*b*d*x**4)*a*c*x**2 + 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*s
qrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) + 2*sqrt(d)*sqrt(c +
d*x**4)*b*x**2 + 2*a*d + 2*b*d*x**4)*b*c*x**6 - sqrt(a)*sqrt(a*d - b*c)*lo
g( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(
c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*c**2 - 2*sqrt(a)*sqrt(a*d - b*c)*log
( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c
 + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*c*d*x**4 - sqrt(a)*sqrt(a*d - b*c)*lo
g( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(
c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b*c**2*x**4 - 2*sqrt(a)*sqrt(a*d - ...
```

3.261 $\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)(a+bx^4)} + \frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/4*b*x^2*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)+1/4*(-2*a*d+b*c)*arctan((
-a*d+b*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{bx^2\sqrt{c+dx^4}}{4a(-bc+ad)(a+bx^4)} + \frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/4*(b*x^2*Sqrt[c + d*x^4])/(a*(-(b*c) + a*d)*(a + b*x^4)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(4*a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{296}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{2a(bc-ad)}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{2a(bc-ad)}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{(bc-2ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{2a(a+bx^4)(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^4+c}\,x^2}{b\,x^4+a}+\frac{(2ad-cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^4+c}}{x^2\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{4(ad-cb)a}$
default	$-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8a(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-cb)\sqrt{-\frac{ad-cb}{b}}}$
elliptic	$-\frac{\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{8a(ad-cb)\left(x^2-\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^2-\right.}}{x^2-\frac{\sqrt{-ab}}{b}}\right.}{8ba(ad-cb)\sqrt{-\frac{ad-cb}{b}}}$

input
```
int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/a*(-b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)+(2*a*d-b*c)/(a*(a*d-b*c)
)^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.17 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \left[\frac{4\sqrt{dx^4+c}(ab^2c-a^2bd)x^2-((b^2c-2\,abd)x^4+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\right.}{}\right.}{16\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2}\right.$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
- a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d +
 a^4*b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c
 - 2*a*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c -
2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)
*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(88) = 176.

Time = 0.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx =$$

$$-\frac{1}{4}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2bc\right.}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}x^2-\sqrt{dx^4+\right.\right.}\right.$$

input
```
integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*a*d - b*c^2)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqr
t(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x}{{\left(b\,x^4+a\right)}^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 2444, normalized size of antiderivative = 23.50

$$\int \frac{x}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
(4*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*
sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d
)*sqrt(b)*x**2)*a**2*d*x**2 - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d
- b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt
(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*c*x**2 + 4*sqrt(d)*sqrt(a
)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d
- b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a
*b*d*x**6 - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt
(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**
4) + sqrt(d)*sqrt(b)*x**2)*b**2*c*x**6 + 4*sqrt(d)*sqrt(a)*sqrt(c + d*x**4
)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c
) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*d*x**2 - 2*sqrt(
d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqr
t(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x
**2)*a*b*c*x**2 + 4*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(s
qrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*
x**4) + sqrt(d)*sqrt(b)*x**2)*a*b*d*x**6 - 2*sqrt(d)*sqrt(a)*sqrt(c + d*x*
*4)*sqrt(a*d - b*c)*log(sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b
*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**2*c*x**6 - 4*sqr
t(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(2*sqrt(d)*sqrt(a)*sqr...
```

3.262 $\int \frac{1}{x^3(a+bx^4)^2\sqrt{c+dx^4}} dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} dx = -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2(a+bx^4)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/4*(-2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1/4*b*(d*x^4+c)^(
1/2)/a/(-a*d+b*c)/x^2/(b*x^4+a)-1/4*b*(-4*a*d+3*b*c)*arctan((-a*d+b*c)^(1/
2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} dx = \frac{\sqrt{c+dx^4}(2abc-2a^2d+3b^2cx^4-2abdx^4)}{4a^2c(-bc+ad)x^2(a+bx^4)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^4+bx^2\sqrt{c+dx^4}}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(Sqrt[c + d*x^4]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^4 - 2*a*b*d*x^4))/(4*a^2*c
*(-(b*c) + a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^4 + b*x^2*Sqrt[c + d*x^4])/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(5
/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^4+a)^2 \sqrt{dx^4+c}} dx^2$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{2} \left(\frac{b\sqrt{c+dx^4}}{2ax^2 (a+bx^4) (bc-ad)} - \frac{\int -\frac{2bdx^4+3bc-2ad}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^2}{2a(bc-ad)} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{2bdx^4+3bc-2ad}{x^4 (bx^4+a) \sqrt{dx^4+c}} dx^2}{2a(bc-ad)} + \frac{b\sqrt{c+dx^4}}{2ax^2 (a+bx^4) (bc-ad)} \right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2} \left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{(bx^4+a)\sqrt{dx^4+c}} dx^2}{ac} - \frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)} + \frac{b\sqrt{c+dx^4}}{2ax^2 (a+bx^4) (bc-ad)} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2(a+bx^4)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}(3bc-2ad)}{acx^2}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^2(a+bx^4)(bc-ad)}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*x^2*(a + b*x^4)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^4])/(a*c*x^2)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}}{x^{2}}+\frac{bc\left(\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(4ad-3cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{2ad-2cb}}{2a^{2}c}$
risch	$-\frac{\sqrt{d\,x^{4}+c}}{2a^{2}x^{2}c}+\frac{3b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8a^{2}\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}-$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{2a^{2}x^{2}c}+\frac{3b\ln\left(\frac{-\frac{2(ad-cb)}{b}+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-cb}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2d\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-cb}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8a^{2}\sqrt{-ab}\,\sqrt{-\frac{ad-cb}{b}}}-$
default	Expression too large to display

input
```
int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(-(d*x^4+c)^(1/2)/x^2+1/2*b*c/(a*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*
x^4+a)-(4*a*d-3*b*c)/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*
(a*d-b*c))^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.25 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^{3}\left(a+bx^{4}\right)^{2}\sqrt{c+dx^{4}}}\,dx$$

$$= \Bigg[-\frac{((3\,b^{3}c^{2}-4\,ab^{2}cd)x^{6}+(3\,ab^{2}c^{2}-4\,a^{2}bcd)x^{2})\sqrt{-abc+a^{2}d}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{8}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{4}+c}{b^{2}x^{8}+2\,c}\right.}{16\,((a^{3}b^{3}c^{3}-2\,a^{4}b^{2}c^{2}d+a^{5}}$$

$$-\frac{((3\,b^{3}c^{2}-4\,ab^{2}cd)x^{6}+(3\,ab^{2}c^{2}-4\,a^{2}bcd)x^{2})\sqrt{abc-a^{2}d}\arctan\left(\frac{((bc-2\,ad)x^{4}-ac)\sqrt{dx^{4}+c}\sqrt{abc-a^{2}d}}{2\,((abcd-a^{2}d^{2})x^{6}+(abc^{2}-a^{2}cd)x^{2})}\right)+2\,(2}{8\,((a^{3}b^{3}c^{3}-2\,a^{4}b^{2}c^{2}d+a^{5}bcd^{2})x^{6}+(a^{4}b^{2}c^{3}-2}$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x
^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*
c^2 - 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2
)*x^4)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6
 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a
*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arcta
n(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*
c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*
b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(
d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*
c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(129) = 258$.

Time = 0.38 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{1}{4}d^{\frac{5}{2}}\left(\frac{(3\,b^2c-4\,abd)\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^2bcd^2-a^3d^3)\sqrt{abcd-a^2d^2}} + \frac{2\left(3\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^4b^2c-4\left(\sqrt{d}x^2-\sqrt{\right.\right.}}{\left(\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^6b-3\left(\sqrt{d}x^2-\sqrt{\right.\right.}}\right.$$

input
```
integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c
))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sq
rt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*
(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^2 -
 sqrt(d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(
d)*x^2 - sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^
2 - 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*
d^3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^3\,(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 3526, normalized size of antiderivative = 23.66

$$\int \frac{1}{x^3 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input

```
int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
( - 4*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(
d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqr
t(d)*sqrt(b)*x**2)*a**2*b*c*d*x**2 - 16*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*s
qrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c
) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*b*d**2*x**6 + 3*
sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqr
t(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*s
qrt(b)*x**2)*a*b**2*c**2*x**2 + 8*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*
d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sq
rt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a*b**2*c*d*x**6 - 16*sqrt(d
)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*s
qrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)
*x**2)*a*b**2*d**2*x**10 + 3*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b
*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)
*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*b**3*c**2*x**6 + 12*sqrt(d)*sqrt
(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*
d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)
*b**3*c*d*x**10 - 4*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log(s
qrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*
x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*b*c*d*x**2 - 16*sqrt(d)*sqrt(a)*sqrt...
```

3.263 $\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^7(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6(a+bx^4)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

output

```
-1/12*(-2*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6+1/12*(-4*a^2*d^2
-8*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(-a*d+b*c)/x^2+1/4*b*(d*x^4
+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^4+a)+1/4*b^2*(-6*a*d+5*b*c)*arctan((-a*d+b
*c)^(1/2)*x^2/a^(1/2)/(d*x^4+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx = -\frac{\sqrt{c+dx^4}(15b^3c^2x^8+2ab^2cx^4(5c-4dx^4)+2a^3d(c-2dx^4)-2a^2b(c^2+3cdx^4+2d^2x^8))}{12a^3c^2(-bc+ad)x^6(a+bx^4)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^2\left(\sqrt{d}x^2+\sqrt{c+dx^4}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/12*(Sqrt[c + d*x^4]*(15*b^3*c^2*x^8 + 2*a*b^2*c*x^4*(5*c - 4*d*x^4) + 2
*a^3*d*(c - 2*d*x^4) - 2*a^2*b*(c^2 + 3*c*d*x^4 + 2*d^2*x^8)))/(a^3*c^2*(-
(b*c) + a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b
*x^2*(Sqrt[d]*x^2 + Sqrt[c + d*x^4]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(7/
2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{x^8 (bx^4+a)^2 \sqrt{dx^4+c}}dx^2$$

$$\Big\downarrow \text{ 374}$$

$$\frac{1}{2}\left(\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}-\frac{\int -\frac{4bdx^4+5bc-2ad}{x^8(bx^4+a)\sqrt{dx^4+c}}dx^2}{2a(bc-ad)}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \frac{4bdx^4+5bc-2ad}{x^8(bx^4+a)\sqrt{dx^4+c}}dx^2}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$\Big\downarrow$ 445

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{2bd(5bc-2ad)x^4+15b^2c^2-4a^2d^2-8abcd}{x^4\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$\Big\downarrow$ 445

$$\frac{1}{2}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{ac}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx^2}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^4}d\frac{x^2}{\sqrt{dx^4+c}}}{a}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^2}}{3ac}-\frac{\sqrt{c+dx^4}(5bc-2ad)}{3acx^6}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{2ax^6\left(a+bx^4\right)(bc-ad)}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((b*Sqrt[c + d*x^4])/(2*a*(b*c - a*d)*x^6*(a + b*x^4)) + (-1/3*((5*b*c - 2
*a*d)*Sqrt[c + d*x^4])/(a*c*x^6) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)/c
)*Sqrt[c + d*x^4])/x^2) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]
*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*a
*(b*c - a*d)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{4}+c}\,\left(-2 a d\,x^{4}-6 b c\,x^{4}+a c\right)}{3 x^{6}}-\frac{b^{2} c^{2} \left(\frac{b\sqrt{d\,x^{4}+c}\,x^{2}}{b\,x^{4}+a}-\frac{(6 a d-5 c b) \operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{4}+c}}{x^{2}\sqrt{a(a d-c b)}}\right)}{\sqrt{a(a d-c b)}}\right)}{2(a d-c b)}}{2 a^{3} c^{2}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}\,\left(-2 a d\,x^{4}-6 b c\,x^{4}+a c\right)}{6 c^{2} a^{3} x^{6}}-\frac{b^{2}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}\,\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}}{8 a^{3}(a d-c b)\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}+\frac{b d\sqrt{-a b}\,\ln\left(-\frac{2(a d-c b)}{b}+\cdots\right.}{\cdots}$
elliptic	$-\frac{\sqrt{d\,x^{4}+c}}{6 a^{2} c\,x^{6}}+\frac{d\sqrt{d\,x^{4}+c}}{3 a^{2} c^{2} x^{2}}+\frac{b\sqrt{d\,x^{4}+c}}{a^{3} x^{2} c}-\frac{b^{2}\sqrt{\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)^{2} d+\frac{2 d\sqrt{-a b}\,\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-c b}{b}}}{8 a^{3}(a d-c b)\left(x^{2}-\frac{\sqrt{-a b}}{b}\right)}+\frac{b d\sqrt{-a b}\,\ln\left(-\frac{2(a d-\cdots}{b}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(-1/3*(d*x^4+c)^(1/2)*(-2*a*d*x^4-6*b*c*x^4+a*c)/x^6-1/2*b^2*c^2/(
a*d-b*c)*(b*(d*x^4+c)^(1/2)*x^2/(b*x^4+a)-(6*a*d-5*b*c)/(a*(a*d-b*c))^(1/2
)*arctanh(a*(d*x^4+c)^(1/2)/x^2/(a*(a*d-b*c))^(1/2))))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^7 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)
*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a
^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^
2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5
*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*
d^2)*x^6), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^
2*b^2*c^2*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 -
 a^2*c*d)*x^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 +
4*a^4*b*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*
b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))
/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*
a^6*b*c^3*d + a^7*c^2*d^2)*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2 \sqrt{dx^4+c}\, x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(184) = 368$.

Time = 0.47 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{1}{12} d^{\frac{7}{2}} \left(\frac{3\,(5\,b^3c - 6\,ab^2d)\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^3bcd^3 - a^4d^4)\sqrt{abcd-a^2d^2}} - \frac{6\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+\right.\right.}{(a^3bcd^3 - a^4d^4)\left(\left(\sqrt{d}x^2 - \sqrt{dx^4+c}\right)^4 b\right.} \right.$$

input
```
integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d
^4)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c
- 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a
^4*d^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4
 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*
x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^7\,(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 7920, normalized size of antiderivative = 38.08

$$\int \frac{1}{x^7 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx = \text{Too large to display}$$

input

```
int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output

```
(72*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)
*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(
d)*sqrt(b)*x**2)*a**3*b**2*c**2*d**2*x**6 + 864*sqrt(d)*sqrt(a)*sqrt(c + d
*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a
*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**3*b**2*c*d
**3*x**10 + 1152*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - s
qrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*
x**4) + sqrt(d)*sqrt(b)*x**2)*a**3*b**2*d**4*x**14 - 150*sqrt(d)*sqrt(a)*s
qrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b
*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2
*b**3*c**3*d*x**6 - 1728*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*
log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqr
t(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*b**3*c**2*d**2*x**10 - 1536*sqr
t(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a
)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt
(b)*x**2)*a**2*b**3*c*d**3*x**14 + 1152*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*s
qrt(a*d - b*c)*log( - sqrt(2*sqrt(d)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c
) + sqrt(b)*sqrt(c + d*x**4) + sqrt(d)*sqrt(b)*x**2)*a**2*b**3*d**4*x**18
+ 75*sqrt(d)*sqrt(a)*sqrt(c + d*x**4)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(d
)*sqrt(a)*sqrt(a*d - b*c) - 2*a*d + b*c) + sqrt(b)*sqrt(c + d*x**4) + s...
```

3.264 $\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 852

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= -\frac{(20b^2c^2+36abcd-77a^2d^2)\,x\sqrt{c+dx^4}}{84b^3d^2(bc-ad)} + \frac{(4bc-11ad)x^5\sqrt{c+dx^4}}{28b^2d(bc-ad)}$$

$$+ \frac{ax^9\sqrt{c+dx^4}}{4b(bc-ad)\,(a+bx^4)} + \frac{(-a)^{9/4}(13bc-11ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{15/4}(-bc+ad)^{3/2}}$$

$$+ \frac{(-a)^{9/4}(13bc-11ad)\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{15/4}(-bc+ad)^{3/2}}$$

$$+ \frac{c^{3/4}(5b^2c^2+19abcd+77a^2d^2)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{42b^3d^{9/4}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{a^2\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(13bc-11ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{arct}\right.}{32b^4\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{a^2\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(13bc-11ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{arctan}\right.}{32b^4\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

output
```
-1/84*(-77*a^2*d^2+36*a*b*c*d+20*b^2*c^2)*x*(d*x^4+c)^(1/2)/b^3/d^2/(-a*d+
b*c)+1/28*(-11*a*d+4*b*c)*x^5*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a*x^9*(
d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/16*(-a)^(9/4)*(-11*a*d+13*b*c)*arc
tan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(15/4)/(a*d-b*
c)^(3/2)+1/16*(-a)^(9/4)*(-11*a*d+13*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(
1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(15/4)/(a*d-b*c)^(3/2)+1/42*c^(3/4)*(77*a^
2*d^2+19*a*b*c*d+5*b^2*c^2)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))
/b^3/d^(9/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/32*a^2*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))*(-11*a*d+13*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1
/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2
)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))/b^4/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/
(d*x^4+c)^(1/2)-1/32*a^2*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-11*a*d+13*
b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Ellip
ticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^4/c^(1/4)/(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.68 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.47

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{x\left(-\frac{(20b^3c^3+36ab^2c^2d+196a^2bcd^2-231a^3d^3)x^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ad^2(-bc+ad)}+5\left(-\frac{20bc^2}{d^2}-\frac{56ac}{d}-56ax^4-\frac{8bcx}{d}\right.\right.}{}$$

input
```
Integrate[x^16/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*(-(((20*b^3*c^3 + 36*a*b^2*c^2*d + 196*a^2*b*c*d^2 - 231*a^3*d^3)*x^4*S
qrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])
/(a*d^2*(-(b*c) + a*d))) + 5*((-20*b*c^2)/d^2 - (56*a*c)/d - 56*a*x^4 - (8
*b*c*x^4)/d + 12*b*x^8 + (21*a^3*c)/((b*c - a*d)*(a + b*x^4)) + (21*a^3*d*
x^4)/((b*c - a*d)*(a + b*x^4)) - (5*a^2*c^2*(-20*b^2*c^2 - 36*a*b*c*d + 77
*a^2*d^2)*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d^2*(-(
b*c) + a*d)*(a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -
((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))))
)/(420*b^3*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.97 (sec) , antiderivative size = 1162, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {970, 1052, 25, 1052, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\int \frac{x^8\left(9ac-(4bc-11ad)x^4\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{4b(bc-ad)}$$

$$\Big\downarrow \text{1052}$$

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{-\frac{\int -\frac{x^4\left(\left(20b^2c^2+36abdc-77a^2d^2\right)x^4+5ac(4bc-11ad)\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{7bd}-\frac{x^5\sqrt{c+dx^4}(4bc-11ad)}{7bd}}{4b(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\int \frac{x^4\left(\left(20b^2c^2+36abdc-77a^2d^2\right)x^4+5ac(4bc-11ad)\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{7bd}-\frac{x^5\sqrt{c+dx^4}(4bc-11ad)}{7bd}}{4b(bc-ad)}$$

$$\Big\downarrow \text{1052}$$

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\int\frac{\left(20b^3c^3+36ab^2dc^2+196a^2bd^2c-231a^3d^3\right)x^4+ac\left(20b^2c^2+36abdc-77a^2d^2\right)}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3bd}}{7bd}-\frac{x^5\sqrt{c+dx^4}(4bc-11ad)}{7bd}}{4b(bc-ad)}$$

$\downarrow$ 1021

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{21a^3d^2(13bc-11ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}}{3bd}}{7bd}-\frac{x^5\sqrt{c+\cdots}}{\cdots}}{4b(bc-ad)}$$

$\downarrow$ 761

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{21a^3d\cdots}{\cdots}}{3bd}}{7bd}\cdots}{4b(bc-ad)}$$

$\downarrow$ 925

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{21a^3d\cdots}{\cdots}}{3bd}}{7bd}\cdots}{4b(bc-ad)}$$

$\downarrow$ 1541

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-$$

$$\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{21a^3d\cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-$$

$$\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{21a^3d\cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 761

$$\frac{ax^9\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-$$

$$\frac{\frac{1}{3}x\sqrt{c+dx^4}\left(-\frac{77a^2d}{b}+36ac+\frac{20bc^2}{d}\right)-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-231a^3d^3+196a^2bcd^2+36ab^2c^2d+20b^3c^3\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{21a^3d\cdots}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 2221

$$\frac{ax^9\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-$$

$21a^3d^2($

$$\frac{1}{3}\left(-\frac{77da^2}{b}+36ca+\frac{20bc^2}{d}\right)x\sqrt{dx^4+c}-\frac{\left(20b^3c^3+36ab^2dc^2+196a^2bd^2c-231a^3d^3\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-$$

$\downarrow$ 2223

$$\frac{ax^9\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-$$

$21a^3d^2($

$$\frac{1}{3}\left(-\frac{77da^2}{b}+36ca+\frac{20bc^2}{d}\right)x\sqrt{dx^4+c}-\frac{\left(20b^3c^3+36ab^2dc^2+196a^2bd^2c-231a^3d^3\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}}-$$

input

```
Int[x^16/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(a*x^9*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (-1/7*((4*b*c - 11
*a*d)*x^5*Sqrt[c + d*x^4])/(b*d) + (((36*a*c + (20*b*c^2)/d - (77*a^2*d)/b
)*x*Sqrt[c + d*x^4])/3 - (((20*b^3*c^3 + 36*a*b^2*c^2*d + 196*a^2*b*c*d^2
- 231*a^3*d^3)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/
4)*Sqrt[c + d*x^4]) - (21*a^3*d^2*(13*b*c - 11*a*d)*(((a*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*
c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)
*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^
(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d)
)/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[
b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.64 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.49

method	result
elliptic	$-\frac{a^{3} x \sqrt{d\,x^{4}+c}}{4 b^{3} \left(a d -c b \right) \left(b\,x^{4}+a \right)}+\frac{x^{5} \sqrt{d\,x^{4}+c}}{7 b^{2} d}+\frac{\left(-\frac{2 a}{b^{3}}-\frac{5 c}{7 b^{2} d}\right) x \sqrt{d\,x^{4}+c}}{3 d}+\frac{\left(\frac{3 a^{2}}{b^{4}}-\frac{d\,a^{3}}{4 b^{4} \left(a d -c b \right)}-\frac{\left(-\frac{2 a}{b^{3}}-\frac{5 c}{7 b^{2} d}\right) c}{3 d}\right) \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i}{}}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}$
risch	$-\frac{x \left(-3 d b\,x^{4}+14 a d +5 c b \right) \sqrt{d\,x^{4}+c}}{21 d^{2} b^{3}}+\frac{\frac{\left(63 a^{2} d^{2}+14 a b c d +5 b^{2} c^{2}\right) \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\text{EllipticF}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{b \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}-\frac{21 a^{3} d^{2} \left(\sum_{\_\alpha =\text{RootOf}\left(\_\right)}\right)}{}}{}$
default	$\frac{\frac{x^{5} \sqrt{d\,x^{4}+c}}{7 d}-\frac{5 c x \sqrt{d\,x^{4}+c}}{21 d^{2}}+\frac{5 c^{2} \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\text{EllipticF}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{21 d^{2} \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{b^{2}}+\frac{a^{4} \left(-\frac{b x \sqrt{d\,x^{4}+c}}{4 a \left(a d -c b \right) \left(b\,x^{4}+a \right)}-\frac{d \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}}{4 \left(a d -c b \right) a \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right)}{}$

input

```
int(x^16/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^3*a^3/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)+1/7/b^2/d*x^5*(d*x^4+c)
^(1/2)+1/3*(-2*a/b^3-5/7/b^2/d*c)/d*x*(d*x^4+c)^(1/2)+(3*a^2/b^4-1/4/b^4*d
*a^3/(a*d-b*c)-1/3*(-2*a/b^3-5/7/b^2/d*c)/d*c)/(I/c^(1/2)*d^(1/2))^(1/2)*(
1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(
1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32*a^3/b^5*sum((11*a*d-13*
b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d
*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2
)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(
1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1
/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_a
lpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^16/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**16/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**16/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{16}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^16/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^16/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{16}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^16/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^16/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^{16}}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^16/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^16/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{16}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{too large to display}$$

input
```
int(x^16/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
(55*sqrt(c + d*x**4)*a**2*c*d*x - 33*sqrt(c + d*x**4)*a**2*d**2*x**5 + 25*
sqrt(c + d*x**4)*a*b*c**2*x + 18*sqrt(c + d*x**4)*a*b*c*d*x**5 + 9*sqrt(c
+ d*x**4)*a*b*d**2*x**9 + 15*sqrt(c + d*x**4)*b**2*c**2*x**5 - 9*sqrt(c +
d*x**4)*b**2*c*d*x**9 - 55*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4
 - a**2*b*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4
 - a*b**2*c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),
x)*a**5*c**2*d**2 + 30*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a
**2*b*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a
*b**2*c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a
**4*b*c**3*d - 55*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*b
*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2
*c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**4*b
*c**2*d**2*x**4 + 25*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**
2*b*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b
**2*c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**
3*b**2*c**4 + 30*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*
c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*
c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**3*b*
*2*c**3*d*x**4 + 25*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2
*b*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a...
```

3.265 $\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 776

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(4bc-7ad)x\sqrt{c+dx^4}}{12b^2d(bc-ad)} + \frac{ax^5\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{(-a)^{5/4}(9bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{11/4}(-bc+ad)^{3/2}} + \frac{(-a)^{5/4}(9bc-7ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{11/4}(-bc+ad)^{3/2}} - \frac{c^{3/4}(bc+7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6b^2d^{5/4}(bc+ad)\sqrt{c+dx^4}} + \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(9bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{32b^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} + \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(9bc-7ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

output
```
1/12*(-7*a*d+4*b*c)*x*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a*x^5*(d*x^4+c)
^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/16*(-a)^(5/4)*(-7*a*d+9*b*c)*arctan((a*d-b
*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(11/4)/(a*d-b*c)^(3/2)+1
/16*(-a)^(5/4)*(-7*a*d+9*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)
/(d*x^4+c)^(1/2))/b^(11/4)/(a*d-b*c)^(3/2)-1/6*c^(3/4)*(7*a*d+b*c)*(c^(1/2
)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b^2/d^(5/4)/(a*d+b*c)/(d*x^4+c)^(1
/2)+1/32*a*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-7*a*d+9*b*c)*(c^(1/2)+d^
(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arct
an(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^3/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*a*(b^(1/2)*c^(1/2)
-(-a)^(1/2)*d^(1/2))*(-7*a*d+9*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1
/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),
1/2*2^(1/2))/b^3/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*
d+b*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.45 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.52

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{x^5\left(\frac{(4b^2c^2+20abcd-21a^2d^2)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a(-bc+ad)}+\frac{5\left(5ac\left(7a^2d^2+4abd^2x^4-4b^2c(c+dx^4)\right)\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c}\right.\right.}{(bc-ad)(a+bx^4)\left(-5ac\,\text{AppellF1}\left(\frac{1}{4},\right.\right.}\right.}{60b^2d\sqrt{c+}}$$

input
```
Integrate[x^12/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x^5*(((4*b^2*c^2 + 20*a*b*c*d - 21*a^2*d^2)*Sqrt[1 + (d*x^4)/c]*AppellF1[
5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])/(a*(-(b*c) + a*d)) + (5*(5*
a*c*(7*a^2*d^2 + 4*a*b*d^2*x^4 - 4*b^2*c*(c + d*x^4))*AppellF1[1/4, 1/2, 1
, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*(c + d*x^4)*(-7*a^2*d + 4*b^2*c*x^4
 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b
*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/
((b*c - a*d)*(a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c),
-((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b
*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))
)/(60*b^2*d*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.72 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.41, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {970, 1052, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax^5\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\int \frac{x^4(5ac-(4bc-7ad)x^4)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4b(bc-ad)}$$

$$\Big\downarrow \text{1052}$$

$$\frac{ax^5\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{-\frac{\int -\frac{(4b^2c^2+20abdc-21a^2d^2)x^4+ac(4bc-7ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{3bd}-\frac{x\sqrt{c+dx^4}(4bc-7ad)}{3bd}}{4b(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{ax^5\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\frac{\int \frac{(4b^2c^2+20abdc-21a^2d^2)x^4+ac(4bc-7ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{3bd}-\frac{x\sqrt{c+dx^4}(4bc-7ad)}{3bd}}{4b(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{\left(-21a^2d^2+20abcd+4b^2c^2\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{b}-\frac{3a^2d(9bc-7ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}}{3bd}-\frac{x\sqrt{c+dx^4}(4bc-7ad)}{3bd}}{4b(bc-ad)}$$

$\downarrow$ 761

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d(9bc-7ad)\int\frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{b}}{3bd}-\frac{x\sqrt{c+dx^4}(4bc-7ad)}{3bd}}{4b(bc-ad)}$$

$\downarrow$ 925

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d(9bc-7ad)\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int\frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4 \cdots}}}{2a}\cdots\right.}{b}}{3bd}\cdots}{4b(bc-ad)}$$

$\downarrow$ 1541

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}-\frac{3a^2d(9bc-7ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{\cdots}\right.}{\cdots}}{2a}\cdots\right.}{\cdots}}{3bd}\cdots}{4b(b\cdots}$$

$\downarrow$ 27

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)} -$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{3a^2d(9bc-7ad)\left(\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{\ldots}\right.}{2a}\ldots\right.}{\ldots}$$

$$\ldots 3bd \ldots \qquad 4b(bc-\ldots$$

$\Big\downarrow$ 761

$$\frac{ax^5\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)} -$$

$$\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}} - \frac{3a^2d(9bc-7ad)\left(\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}\ldots\right.}{\ldots}$$

$\Big\downarrow$ 2221

$$\frac{ax^5\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)} -$$

$$\frac{\left(4b^2c^2+20abdc-21a^2d^2\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}} - \frac{3a^2d(9bc-7ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\ldots}{2\sqrt[4]{c}(bc+ad)\sqrt{dx\ldots}}\ldots\right.}{\ldots}$$

$\Big\downarrow$ 2223

$$\frac{ax^5\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)} -$$

$$\frac{\frac{\left(4b^2c^2+20abdc-21a^2d^2\right)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^4+c}} - \frac{3a^2d(9bc-7ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx}}\right.}{}}{}$$

input

```
Int[x^12/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(a*x^5*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (-1/3*((4*b*c - 7*
a*d)*x*Sqrt[c + d*x^4])/(b*d) + (((4*b^2*c^2 + 20*a*b*c*d - 21*a^2*d^2)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4
]) - (3*a^2*d*(9*b*c - 7*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*
d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4
)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] +
 (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d]
)^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*
Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] -
Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTan
h[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sq
rt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqr...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.80 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.48

method	result
elliptic	$\frac{a^2 x\sqrt{d x^4+c}}{4b^2(ad-cb)(b x^4+a)}+\frac{x\sqrt{d x^4+c}}{3b^2 d}+\frac{\left(-\frac{2a}{b^3}+\frac{d a^2}{4b^3(ad-cb)}-\frac{c}{3b^2 d}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{a^2\left(\sum_{\_\alpha=\mathrm{Ro}} \right.}{}$
risch	$\frac{x\sqrt{d x^4+c}}{3b^2 d}-\frac{\frac{(6ad+cb)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}-\frac{9a^2 d\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}}{}\right.}{}}{}$
default	$\frac{\frac{x\sqrt{d x^4+c}}{3d}-\frac{c\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3d\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}}{b^2}-\frac{2a\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^3\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}}+\frac{3a^2\left(\sum_{\_\alpha=\mathrm{Root}} \right.}{}$

input
```
int(x^12/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*a^2/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)+1/3/b^2/d*x*(d*x^4+c)^(1
/2)+(-2*a/b^3+1/4/b^3*d*a^2/(a*d-b*c)-1/3/b^2/d*c)/(I/c^(1/2)*d^(1/2))^(1/
2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+
c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/32*a^2/b^4*sum((7*a*d-
9*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2
*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1
/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)
^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^
(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),
_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^12/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**12/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**12/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^12/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^12/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^12/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^12/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{12}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

input
```
int(x^12/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - 5*sqrt(c + d*x**4)*a*c*x + 3*sqrt(c + d*x**4)*a*d*x**5 - 3*sqrt(c + d*
x**4)*b*c*x**5 + 5*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*
b*c**2 + a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**
2*c*d*x**8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**4*
c**2*d - 5*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2 +
 a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*c*d*x*
*8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**3*b*c**3 +
 5*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2 + a**2*b*
c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*c*d*x**8 + a*b
**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**3*b*c**2*d*x**4 -
5*int(sqrt(c + d*x**4)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2 + a**2*b*c
*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*c*d*x**8 + a*b*
*2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**2*b**2*c**3*x**4 -
21*int((sqrt(c + d*x**4)*x**8)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2 +
a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*c*d*x**
8 + a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**4*d**3 + 34
*int((sqrt(c + d*x**4)*x**8)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2 + a*
*2*b*c*d*x**4 + 2*a**2*b*d**2*x**8 - 2*a*b**2*c**2*x**4 - a*b**2*c*d*x**8
+ a*b**2*d**2*x**12 - b**3*c**2*x**8 - b**3*c*d*x**12),x)*a**3*b*c*d**2 -
21*int((sqrt(c + d*x**4)*x**8)/(a**3*c*d + a**3*d**2*x**4 - a**2*b*c**2...
```

3.266 $\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 723

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)(a+bx^4)} + \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(-bc+ad)^{3/2}}$$

$$+ \frac{\sqrt[4]{-a}(5bc-3ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16b^{7/4}(-bc+ad)^{3/2}}$$

$$+ \frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{d}(bc+ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

$$- \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

output
```
1/4*a*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/16*(-a)^(1/4)*(-3*a*d+5*b
*c)*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(
a*d-b*c)^(3/2)+1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-
a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(3/2)+1/2*c^(3/4)*(c^(
1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(
1/2)-1/32*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(c^(1/2)+d^(
1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arcta
n(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/
2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/32*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)
+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b
*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.35

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{x\left(\frac{(4bc-3ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{ab}+\frac{5a\left(c+dx^4+\frac{5ac^2\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\right.}{-5ac\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4}\right.\right.}\right.}{b(a+bx^4)}\right.}{20(bc-ad)\sqrt{c+dx^4}}$$

input
```
Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 1032, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {970, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{970}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\int \frac{ac-(4bc-3ad)x^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{4b(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{(4bc-3ad)\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{4b(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{ax\sqrt{c+dx^4}}{4b(a+bx^4)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(4bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{4b(bc-ad)}$$

$$\Big\downarrow \text{925}$$

$$\frac{\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{a(5bc-3ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(4bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}}}{4b(bc-ad)}$$

$\downarrow$ 1541

$$\frac{\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{\cdots}\right.}{\cdots}}{2a}\cdots\right)}{b}\cdots}{4b(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\cdots\right.}{\cdots}}{2a}\cdots\right)}{b}\cdots}{4b(bc-ad)}$$

$\downarrow$ 761

$$\frac{ax\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\cdots}{\cdots}\cdots\right)}{b}\cdots$$

$\downarrow$ 2221

$$\frac{ax\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-\frac{a(5bc-3ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\cdots\right)}{2\sqrt[4]{b}\sqrt{bc-ad}\cdots}\cdots\right)}{2a}\cdots\right)}{\cdots}$$

$\downarrow$ 2223

$$\frac{ax\sqrt{dx^4+c}}{4b(bc-ad)\left(bx^4+a\right)}-\frac{a(5bc-3ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\cdots\right)}{2\sqrt[4]{b}\sqrt{bc-ad}\cdots}\cdots\right)}{2a}\cdots\right)}{\cdots}$$

input

```
Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (-1/2*((4*b*c - 3*a*
d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*c^(1/4)*d^(1/4)*Sqrt[c + d*
x^4]) + (a*(5*b*c - 3*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[
c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*(
(Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (S
qrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(
Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[
b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.47

method	result
elliptic	$-\frac{ax\sqrt{d\,x^4+c}}{4(ad-cb)b(b\,x^4+a)}+\frac{\left(\frac{1}{b^2}-\frac{ad}{4(ad-cb)b^2}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{a\left(\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right)}\frac{(3ad-5c}{}\right.}{}$
default	$\frac{\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{b^2\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{a^2\left(-\frac{bx\sqrt{d\,x^4+c}}{4a(ad-cb)\left(b\,x^4+a\right)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\_\alpha=}{}\right.}{}$

input
```
int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/(a*d-b*c)/b*x*(d*x^4+c)^(1/2)/(b*x^4+a)+(1/b^2-1/4*a*d/(a*d-b*c)/b^
2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*
d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I
)-1/32*a/b^3*sum((3*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)
*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+
2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(
1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(
1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c
^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^8}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**8)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d
*x**8 + b**2*c*x**8 + b**2*d*x**12),x)
```

3.267 $\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 617

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)(a+bx^4)}$$
$$+\frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{32ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$
$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right.\right.}{32ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

output
```
-1/4*x*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/16*(a*d+b*c)*arctan((a*d-b*c
)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)
^(3/2)+1/16*(a*d+b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+
c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/2)+1/32*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))*(a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x
^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
/a/b/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^
4+c)^(1/2)+1/32*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(a*d+b*c)*(c^(1/2)+d^
(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arct
an(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/
2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/b/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^
(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{x\left(\frac{dx^4\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}+\frac{5\left(c+dx^4+\frac{5ac^2\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{-5ac\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+2x^4\left(2bc\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right.\right.}\right.}{a+bx^4}\right.}{20(-bc+ad)\sqrt{c+dx^4}}$$

input
```
Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -(
(b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*
x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -
((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/
(a + b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.64, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {971, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \ 971$$

$$\frac{\int \frac{c-dx^4}{(bx^4+a)\sqrt{dx^4+c}}dx}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 1021$$

$$\frac{\frac{(ad+bc)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{d\int \frac{1}{\sqrt{dx^4+c}}dx}{b}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{(ad+bc)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 925$$

$$\frac{\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}}}{4(bc-ad)}-\frac{x\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 1541$$

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\right.}{}}{2a}\right.}{b}$$

$$4(bc-ad)$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 27

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int}{ad-}}{2a}\right.}{b}$$

$$4(bc-ad)$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 761

$$\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}+\frac{\sqrt{b}\left(\sqrt{-\right.}{}\right.}{b}$$

$$\frac{x\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 2221

$$\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\cdots}{\sqrt[4]{-\cdots}}\right)}{2\sqrt[4]{b}\sqrt{bc-ad}}\cdots\right.}{2a}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{x\sqrt{dx^4+c}}{4(bc-ad)\left(bx^4+a\right)}$$

$\Big\downarrow$ 2223

$$\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\frac{\cdots}{\sqrt[4]{-\cdots}}\right)}{2\sqrt[4]{b}\sqrt{bc-ad}}\cdots\right.}{2a}\cdots}{\cdots}\right.}{\cdots}$$

$$\frac{x\sqrt{dx^4+c}}{4(bc-ad)\left(bx^4+a\right)}$$

input
```
Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/4*(x*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (-1/2*(d^(3/4)*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*c^(1/4)*Sqrt[c + d*x^4]) + ((b*c + a
*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[
-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b
])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt
[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d
])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2
)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(
1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a
)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
 - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d]...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.25 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.53

method	result
elliptic	$\frac{x\sqrt{d\,x^4+c}}{4(ad-cb)(b\,x^4+a)}+\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-cb)b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{(ad+cb)\left(-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2\cdots}{2\sqrt{\frac{-ad+cb}{b}}\cdots}\right)}{\sqrt{\frac{-ad+cb}{b}}\cdots}\cdots\right)}{\cdots}$
default	$\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha^3}}{8b^2}-\cdots$

input
```
int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*d/(a*d-b*c)/b/(I/c^(1/2)*d^(
1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2
)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b^2*sum((a
*d+b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^
2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(
1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2
)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d
^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)))
,_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^4}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}\,dx$$

input
```
int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d
*x**8 + b**2*c*x**8 + b**2*d*x**12),x)
```

3.268 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 723

$$
\begin{aligned}
&\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx \\
&= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(-bc+ad)^{3/2}} \\
&\quad - \frac{\sqrt[4]{b}(3bc-5ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(-bc+ad)^{3/2}} \\
&\quad + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
1/4*b*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/16*b^(1/4)*(-5*a*d+3*b*c)
*arctan((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(
a*d-b*c)^(3/2)-1/16*b^(1/4)*(-5*a*d+3*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^
(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(3/2)+1/2*d^(3/4)*(c^(
1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(
1/2)+1/32*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-5*a*d+3*b*c)*(c^(1/2)+d^(
1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arcta
n(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/
2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))*(-5*a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)
+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b
*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.30 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.54

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{-5acx\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\left(5a(4bc-4ad+bdx^4)+bdx^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4}\right.\right.}{20a^2(bc-ad)(a+bx^4)\sqrt{c+dx^4}\left(-5a\right.}$$

input
```
Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b
*c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5
/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) +
 d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4
)/c), -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(2
0*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/
4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/
c), -((b*x^4)/a)])))
```

Rubi [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 1016, normalized size of antiderivative = 1.41, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}-\frac{\int -\frac{bdx^4+3bc-4ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^4+3bc-4ad}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{(3bc-5ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx+d\int \frac{1}{\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3bc-5ad)\int \frac{1}{(bx^4+a)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$\big\downarrow$ 925

$$\frac{(3bc-5ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}\cdots}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$\big\downarrow$ 1541

$$\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}\cdots\right.}{\cdots}}{2a}\cdots\right.}{4a(bc-ad)}+\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$\big\downarrow$ 761

$$(3bc-5ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}\right.$$

$$\frac{bx\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

$\downarrow$ 2221

$$\frac{b\sqrt{dx^4+c}x}{4a(bc-ad)\left(bx^4+a\right)}+$$

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

$\downarrow$ 2223

$$\frac{b\sqrt{dx^4+c}x}{4a(bc-ad)\left(bx^4+a\right)}+$$

$$\frac{d^{3/4}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

input

```
Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((d^(3/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + (3*b*c - 5*a*
d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(
Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-
a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b]
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[
c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[
c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d]
)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1
/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)
^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.46

method	result
default	$-\frac{bx\sqrt{dx^4+c}}{4a(ad-cb)(bx^4+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}(-5ad+3cb)\left(-\frac{\operatorname{arctanh}\left(\frac{2}{2\sqrt{-}}\right.}{\sqrt{-}}\right.}{}$
elliptic	$-\frac{bx\sqrt{dx^4+c}}{4a(ad-cb)(bx^4+a)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\sqrt{dx^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}(-5ad+3cb)\left(-\frac{\operatorname{arctanh}\left(\frac{2}{2\sqrt{-}}\right.}{\sqrt{-}}\right.}{}$

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/a/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2
)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)
^(1/2)/(d*x^4+c)^(1/2)*EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/b/a*s
um((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*
(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*
d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d
^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I
*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2
))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 +
 b**2*c*x**8 + b**2*d*x**12),x)
```

3.269 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 778

$$\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx = -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3}$$
$$+\frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3(a+bx^4)} - \frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$-\frac{b^{5/4}(7bc-9ad)\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{11/4}(-bc+ad)^{3/2}}$$
$$-\frac{d^{3/4}(7bc+ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{6a^2c^{5/4}(bc+ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$
$$-\frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{32a^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}$$

output
```
-1/12*(-4*a*d+7*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/4*b*(d*x^4+c)^
(1/2)/a/(-a*d+b*c)/x^3/(b*x^4+a)-1/16*b^(5/4)*(-9*a*d+7*b*c)*arctan((a*d-b
*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a*d-b*c)^(3/2
)-1/16*b^(5/4)*(-9*a*d+7*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)
/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a*d-b*c)^(3/2)-1/6*d^(3/4)*(a*d+7*b*c)*(c^(
1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a^2/c^(5/4)/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/32*b*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-9*a*d+7*b*c)*(c^(1/2)
+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*a
rctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)
^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^3/c^(1/4)/(b^(1/2)*c^(1/2)-(
-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(b^(1/2)*c^(1
/2)-(-a)^(1/2)*d^(1/2))*(-9*a*d+7*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c
^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1
/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/
2),1/2*2^(1/2))/a^3/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(
-a*d+b*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \frac{bd(7bc-4ad)x^8\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{a\left(25ac\left(-7b^2cx^4(4c+dx^4)+4a^2d(c+2dx^4)+4ab(-c^2+5\cdots\right.\right.}{(a+bx^4)\cdots}}{60a\cdots}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2
*d*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2
*c*x^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a
)])))/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a
^3*c*(-(b*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 1074, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {972, 25, 1053, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3 (a+bx^4) (bc-ad)}-\frac{\int -\frac{5bdx^4+7bc-4ad}{x^4(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^4+7bc-4ad}{x^4(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^3 (a+bx^4) (bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{bd(7bc-4ad)x^4+21b^2c^2-4a^2d^2-20abcd}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}(7bc-4ad)}{3acx^3}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^3 (a+bx^4) (bc-ad)}$$

$$\Big\downarrow \text{1021}$$

$$\frac{-\frac{d(7bc-4ad)\int \frac{1}{\sqrt{dx^4+c}}dx+3bc(7bc-9ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}(7bc-4ad)}{3acx^3}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 761$$

$$\frac{-\frac{3bc(7bc-9ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}(7bc-4ad)}{3acx^3}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 925$$

$$\frac{-\frac{3bc(7bc-9ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}\cdots}{4a(bc-ad)}$$

$$+\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 1541$$

$$\frac{-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{\cdots}\right.}{\cdots}}{2\cdots}\cdots\right.}{3ac}\cdots}{4a(bc-\cdots}$$

$$+\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{\cdots}\right.}{}}{2a}\right.}{3ac}$$

$$4a(bc-ad)$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 761$$

$$-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}\right.\cdots}{}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^3\left(a+bx^4\right)(bc-ad)}$$

$$\Big\downarrow \ 2221$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x^3\left(bx^4+a\right)}+$$

$$\frac{-\frac{\sqrt{dx^4+c}(7bc-4ad)}{3acx^3}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc(7bc-9ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{\cdots}\right.}{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\Big\downarrow \ 2223$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x^3\left(bx^4+a\right)}+$$

$$-\frac{\sqrt{dx^4+c}(7bc-4ad)}{3acx^3}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^4+c}}+3bc(7bc-9ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right.}{2\ldots}\right.}{\ldots}$$

input
```
Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (-1/3*((7*b*c - 4*
a*d)*Sqrt[c + d*x^4])/(a*c*x^3) - ((d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4]) + 3*b*c*(7*b*c - 9*
a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d
]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(
1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]
*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[
b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqr
t[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[
d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d
)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^
(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x)/((-
a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.44 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.47

method	result
elliptic	$\frac{b^2 x\sqrt{d\,x^4+c}}{4a^2(ad-cb)(b\,x^4+a)}-\frac{\sqrt{d\,x^4+c}}{3c\,a^2x^3}+\frac{\left(\frac{bd}{4a^2(ad-cb)}-\frac{d}{3c\,a^2}\right)\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4 b\cdots\right.}\cdots}{\cdots}$
default	$\frac{-\frac{\sqrt{d\,x^4+c}}{3c\,x^3}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{3c\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}}{a^2}-\frac{b\left(-\frac{bx\sqrt{d\,x^4+c}}{4a(ad-cb)\left(b\,x^4+a\right)}-\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\cdots\right.}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{d\,x^4+c}}{3c\,a^2x^3}-\frac{\frac{d\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}+\frac{3c\left(\sum_{\_\alpha=\operatorname{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3b\cdots}{\cdots}}{\cdots}\cdots\right.}{8\cdots}}{\cdots}$

input
```
int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/a^2/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/3/c/a^2*(d*x^4+c)^(1/2
)/x^3+(1/4*b*d/a^2/(a*d-b*c)-1/3/c*d/a^2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*
EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-1/32/a^2*sum((9*a*d-7*b*c)/(a*d-b
*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*
b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^
2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx=\int \frac{1}{x^4\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{-\sqrt{d\,x^4+c}-\left(\int \frac{\sqrt{d\,x^4+c}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx\right)a^2d\,x^3-7\left(\int \frac{\sqrt{d\,x^4+c}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+c}\right.}{}$$

input
```
int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**4) - int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*
c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a**2*d*x**3 - 7*int
(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b*
*2*c*x**8 + b**2*d*x**12),x)*a*b*c*x**3 - int(sqrt(c + d*x**4)/(a**2*c + a
**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*
a*b*d*x**7 - 7*int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 +
 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*b**2*c*x**7 - 5*int((sqrt(c
 + d*x**4)*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**
2*c*x**8 + b**2*d*x**12),x)*a*b*d*x**3 - 5*int((sqrt(c + d*x**4)*x**4)/(a*
*2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x*
*12),x)*b**2*d*x**7)/(3*a*c*x**3*(a + b*x**4))
```

3.270 $\int \frac{1}{x^8(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 854

$$
\begin{aligned}
&\int \frac{1}{x^8(a+bx^4)^2\sqrt{c+dx^4}}\,dx \\
&= -\frac{(11bc-4ad)\sqrt{c+dx^4}}{28a^2c(bc-ad)x^7} + \frac{(77b^2c^2-36abcd-20a^2d^2)\sqrt{c+dx^4}}{84a^3c^2(bc-ad)x^3} \\
&\quad + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^7(a+bx^4)} - \frac{b^{9/4}(11bc-13ad)\arctan\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{15/4}(-bc+ad)^{3/2}} \\
&\quad - \frac{b^{9/4}(11bc-13ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{15/4}(-bc+ad)^{3/2}} \\
&\quad + \frac{d^{3/4}(77b^2c^2+19abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{42a^3c^{9/4}(bc+ad)\sqrt{c+dx^4}} \\
&\quad + \frac{b^2\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(11bc-13ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{arct}\right.}{32a^4\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}} \\
&\quad + \frac{b^2\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(11bc-13ad)\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\,\text{arctan}\right.}{32a^4\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^4}}
\end{aligned}
$$

output
```
-1/28*(-4*a*d+11*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^7+1/84*(-20*a^2*d
^2-36*a*b*c*d+77*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(-a*d+b*c)/x^3+1/4*b*(d*
x^4+c)^(1/2)/a/(-a*d+b*c)/x^7/(b*x^4+a)-1/16*b^(9/4)*(-13*a*d+11*b*c)*arct
an((a*d-b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(15/4)/(a*d-
b*c)^(3/2)-1/16*b^(9/4)*(-13*a*d+11*b*c)*arctanh((a*d-b*c)^(1/2)*x/(-a)^(1
/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(15/4)/(a*d-b*c)^(3/2)+1/42*d^(3/4)*(5*a
^2*d^2+19*a*b*c*d+77*b^2*c^2)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^
(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2
))/a^3/c^(9/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/32*b^2*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))*(-13*a*d+11*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^
(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2
^(1/2))/a^4/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c
)/(d*x^4+c)^(1/2)+1/32*b^2*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-13*a*d+1
1*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Ell
ipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^4/c^(1/4)/(b^(
1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.67 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^8\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

$$= \frac{bd\left(77b^2c^2-36abcd-20a^2d^2\right)x^{12}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+\frac{5a\left(\left(c+dx^4\right)\left(77b^3c^2x^8+4ab^2cx^4\cdots\right.\right.}{\cdots}}{42\cdots}$$

input
```
Integrate[1/(x^8*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*d*(77*b^2*c^2 - 36*a*b*c*d - 20*a^2*d^2)*x^12*Sqrt[1 + (d*x^4)/c]*Appel
lF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + (5*a*((c + d*x^4)*(77*
b^3*c^2*x^8 + 4*a*b^2*c*x^4*(11*c - 9*d*x^4) + 4*a^3*d*(3*c - 5*d*x^4) - 4
*a^2*b*(3*c^2 + 6*c*d*x^4 + 5*d^2*x^8)) + (5*a*c*(-231*b^3*c^3 + 196*a*b^2
*c^2*d + 36*a^2*b*c*d^2 + 20*a^3*d^3)*x^8*AppellF1[1/4, 1/2, 1, 5/4, -((d*
x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -
((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*
x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/
(a + b*x^4))/(420*a^4*c^2*(b*c - a*d)*x^7*Sqrt[c + d*x^4])
```

Rubi [A] (warning: unable to verify)

Time = 2.79 (sec) , antiderivative size = 1144, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {972, 25, 1053, 1053, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+bx^4\right)^2 \sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{972}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7 \left(a+bx^4\right) (bc-ad)}-\frac{\int -\frac{9bdx^4+11bc-4ad}{x^8 \left(bx^4+a\right) \sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{9bdx^4+11bc-4ad}{x^8 \left(bx^4+a\right) \sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^7 \left(a+bx^4\right) (bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$\frac{-\frac{\int \frac{5bd(11bc-4ad)x^4+77b^2c^2-20a^2d^2-36abcd}{x^4 \left(bx^4+a\right) \sqrt{dx^4+c}}dx}{7ac}-\frac{\sqrt{c+dx^4}(11bc-4ad)}{7acx^7}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^7 \left(a+bx^4\right) (bc-ad)}$$

$$\Big\downarrow \text{1053}$$

$$-\frac{-\frac{-\frac{\int \frac{bd\left(77b^2c^2-36abdc-20a^2d^2\right)x^4+231b^3c^3-20a^3d^3-36a^2bcd^2-196ab^2c^2d}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}\left(\frac{77b^2c}{a}-\frac{20ad^2}{c}-36bd\right)}{3x^3}}{7ac}-\frac{\sqrt{c+dx^4}(11bc-4ad)}{7acx^7}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^7\left(a+bx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1021

$$-\frac{-\frac{-\frac{d\left(-20a^2d^2-36abcd+77b^2c^2\right)\int \frac{1}{\sqrt{dx^4+c}}dx+21b^2c^2(11bc-13ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx}{3ac}-\frac{\sqrt{c+dx^4}\left(\frac{77b^2c}{a}-\frac{20ad^2}{c}-36bd\right)}{3x^3}}{7ac}-\frac{\sqrt{c+dx^4}(11bc-4ad)}{7acx^7}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^7\left(a+bx^4\right)(bc-ad)}$$

$\Big\downarrow$ 761

$$-\frac{-\frac{-\frac{21b^2c^2(11bc-13ad)\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}}dx+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-20a^2d^2-36abcd+77b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}-\frac{\sqrt{c+dx^4}\left(\cdots\right.}{\cdots}}{7ac}\cdots}{4a(bc-ad)}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7\left(a+bx^4\right)(bc-ad)}$$

$\Big\downarrow$ 925

$$-\frac{-\frac{-\frac{21b^2c^2(11bc-13ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^2}{\sqrt{-a}}+1\right)\sqrt{dx^4+c}}dx}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(-20a^2d^2-36abcd+77b^2c^2\right)\text{EllipticF}\left(2\cdots\right.}{2\sqrt[4]{c}\sqrt{c+dx^4}}}{3ac}\cdots}{7ac}\cdots}{4a(bc-ad)}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7\left(a+bx^4\right)(bc-ad)}$$

$\Big\downarrow$ 1541

$$-\frac{-\frac{21b^2c^2(11bc-13ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\cdots}{\cdots}\right.}{3ac\cdots}}{\cdots}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{21b^2c^2(11bc-13ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int\frac{1}{\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\cdots\right.}{\cdots}}{2a}\right.}{3ac\cdots}}{\cdots}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 761$$

$$-\frac{-\frac{21b^2c^2(11bc-13ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^2+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{dx^4+c}}dx}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\cdots\right.}{2\sqrt[4]{c}\sqrt{c+dx^4}(ad+bc)}}{2a}\right.}{\cdots}}{\cdots}$$

$$\frac{b\sqrt{c+dx^4}}{4ax^7(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 2221$$

$$\frac{\sqrt{dx^4+c}b}{4a(bc-ad)x^7(bx^4+a)}+$$

$$21b^2(11bc-13ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

$$-\frac{\sqrt{dx^4+c}(11bc-4ad)}{7acx^7}-\frac{-\frac{\sqrt{dx^4+c}\left(\frac{77cb^2}{a}-36db-\frac{20ad^2}{c}\right)}{3x^3}-\cdots}{\cdots}$$

$\downarrow$ 2223

$$\frac{\sqrt{dx^4+c}b}{4a(bc-ad)x^7(bx^4+a)}+$$

$$21b^2(11bc-13ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\cdots\right.\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^4+c}}\right.$$

$$-\frac{\sqrt{dx^4+c}(11bc-4ad)}{7acx^7}-\frac{-\frac{\sqrt{dx^4+c}\left(\frac{77cb^2}{a}-36db-\frac{20ad^2}{c}\right)}{3x^3}-\cdots}{\cdots}$$

input
```
Int[1/(x^8*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^7*(a + b*x^4)) + (-1/7*((11*b*c - 4
*a*d)*Sqrt[c + d*x^4])/(a*c*x^7) - (-1/3*(((77*b^2*c)/a - 36*b*d - (20*a*d
^2)/c)*Sqrt[c + d*x^4])/x^3 - ((d^(3/4)*(77*b^2*c^2 - 36*a*b*c*d - 20*a^2*
d^2)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*E
llipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^4])
+ 21*b^2*c^2*(11*b*c - 13*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])
*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*S
qrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/
4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)
^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
+ (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]
*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTa
nh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(2*b^(1/4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.47

method	result
elliptic	$-\frac{b^{3} x \sqrt{d\,x^{4}+c}}{4 a^{3} \left(a d-c b\right) \left(b\,x^{4}+a\right)}-\frac{\sqrt{d\,x^{4}+c}}{7 c\,a^{2} x^{7}}+\frac{\left(5 a d+14 c b\right) \sqrt{d\,x^{4}+c}}{21 c^{2} a^{3} x^{3}}+\frac{\left(-\frac{b^{2} d}{4 a^{3} \left(a d-c b\right)}+\frac{d \left(5 a d+14 c b\right)}{21 c^{2} a^{3}}\right) \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{Ellipticl}}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}$
risch	$-\frac{\sqrt{d\,x^{4}+c}\,\left(-5 a d\,x^{4}-14 b c\,x^{4}+3 a c\right)}{21 c^{2} a^{3} x^{7}}+\frac{\frac{5 a\,d^{2} \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}+\frac{14 b c d \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticF}\left(\right.}{\sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{}$
default	$\frac{-\frac{\sqrt{d\,x^{4}+c}}{7 c\,x^{7}}+\frac{5 d \sqrt{d\,x^{4}+c}}{21 c^{2} x^{3}}+\frac{5 d^{2} \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}, i\right)}{21 c^{2} \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}}}{a^{2}}+\frac{b^{2} \left(-\frac{b x \sqrt{d\,x^{4}+c}}{4 a \left(a d-c b\right) \left(b\,x^{4}+a\right)}-\frac{d \sqrt{1-\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\sqrt{1+\frac{i \sqrt{d}\,x^{2}}{\sqrt{c}}}\,\mathrm{El}}{4 \left(a d-c b\right) a \sqrt{\frac{i \sqrt{d}}{\sqrt{c}}}}\right.}{}$

input
```
int(1/x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/a^3/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^4+a)-1/7/c/a^2*(d*x^4+c)^(1/
2)/x^7+1/21/c^2*(5*a*d+14*b*c)/a^3*(d*x^4+c)^(1/2)/x^3+(-1/4*b^2*d/a^3/(a*
d-b*c)+1/21/c^2*d*(5*a*d+14*b*c)/a^3)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/
2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elli
pticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)+1/32*b/a^3*sum((13*a*d-11*b*c)/(a*d-b
*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/(
(-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*
b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^
2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**8*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+cx^8}}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{(bx^4+a)^2\sqrt{dx^4+cx^8}}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^8\,(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(1/(x^8*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^8*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (a+bx^4)^2 \sqrt{c+dx^4}}\, dx$$
$$= \frac{-3\sqrt{d\,x^4+c}\,ac+5\sqrt{d\,x^4+c}\,ad\,x^4+11\sqrt{d\,x^4+c}\,bc\,x^4+5\left(\int \frac{\sqrt{d\,x^4+c}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx\right) a^{3}}{}$$

input
```
int(1/x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - 3*sqrt(c + d*x**4)*a*c + 5*sqrt(c + d*x**4)*a*d*x**4 + 11*sqrt(c + d*x
**4)*b*c*x**4 + 5*int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**
4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a**3*d**2*x**7 + 19*int(
sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**
2*c*x**8 + b**2*d*x**12),x)*a**2*b*c*d*x**7 + 5*int(sqrt(c + d*x**4)/(a**2
*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**1
2),x)*a**2*b*d**2*x**11 + 77*int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 +
2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b**2*c**2*x
**7 + 19*int(sqrt(c + d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b
*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b**2*c*d*x**11 + 77*int(sqrt(c
+ d*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**
8 + b**2*d*x**12),x)*b**3*c**2*x**11 + 25*int((sqrt(c + d*x**4)*x**4)/(a**
2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**
12),x)*a**2*b*d**2*x**7 + 55*int((sqrt(c + d*x**4)*x**4)/(a**2*c + a**2*d*
x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b**2
*c*d*x**7 + 25*int((sqrt(c + d*x**4)*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c
*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b**2*d**2*x**11 +
55*int((sqrt(c + d*x**4)*x**4)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*
b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*b**3*c*d*x**11)/(21*a**2*c**2*x*
*7*(a + b*x**4))
```

3.271 $\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1159

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

output
```
1/4*d^(1/2)*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^2)-1/4*x^3*(
d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/16*(-a*d+3*b*c)*arctan((-a*d+b*c)^(1
/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3
/2)-1/16*(-a*d+3*b*c)*arctanh((-a*d+b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4
+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)-1/4*c^(1/4)*d^(1/4)*(c^(1/2
)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*a
rctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/b/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*c^
(1/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1
/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/b/(-a*d+b*c)/
(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)
/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)
),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(-a*d+
b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x
^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^
(1/4)),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(
-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-a*d+
3*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Ell
ipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(3/2)/c^(1/4)
/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{-7ax^3(c+dx^4)+7cx^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+dx^7(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\,\text{A}}{28a(bc-ad)\,(a+bx^4)\sqrt{c+dx^4}}$$

input
```
Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3
/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 +
(d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*
(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 1085, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \ 971$$

$$\frac{\int \frac{x^2(dx^4+3c)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4(bc-ad)}-\frac{x^3\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 1054$$

$$\frac{\int \left(\frac{dx^2}{b\sqrt{dx^4+c}}+\frac{(3bc-ad)x^2}{b(bx^4+a)\sqrt{dx^4+c}}\right)dx}{4(bc-ad)}-\frac{x^3\sqrt{c+dx^4}}{4(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{(3bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{(3bc-ad)\arctan\left(\frac{\sqrt{b}}{\sqrt[4]{-a}}\right.}{4\sqrt[4]{-a}b^{5/4}\sqrt{bc}}}{}$$

$$\frac{x^3\sqrt{dx^4+c}}{4(bc-ad)(bx^4+a)}$$

input
```
Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
-1/4*(x^3*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((Sqrt[d]*x*Sqrt[c
+ d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[
b*c - a*d]) - ((3*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[b*c - a*d]) - (c^(1/4)*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1
/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^
2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4])
 - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
- ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*
Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d...
```

Defintions of rubi rules used

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.58 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.30

method	result
elliptic	$\frac{x^3\sqrt{d\,x^4+c}}{4(ad-cb)(b\,x^4+a)} - \frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-cb)b\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right.}}{}$
default	$\frac{\sum_{\_\alpha=\text{RootOf}\left(\_Z^4b+a\right)}\frac{-\frac{\operatorname{arctanh}\left(\frac{2d\,x^2\_\alpha^2+2c}{2\sqrt{\frac{-ad+cb}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+cb}{b}}}+\frac{2\_\alpha^3b\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\text{EllipticPi}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},\frac{i\sqrt{c}\,\_\alpha^2b}{\sqrt{d}\,a},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,a\sqrt{d\,x^4+c}}}{\_\alpha}}{8b^2} -$

input

```
int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/4/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/4*I*d^(1/2)/(a*d-b*c)/b*c^(1
/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)
*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2)
,I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b^2*sum((-a*d+3*b*c)/(a
*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)
/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^
3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x
^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alph
a^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Root
Of(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**6/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^6}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**6)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d
*x**8 + b**2*c*x**8 + b**2*d*x**12),x)
```

3.272 $\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1169

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

output
```
-1/4*d^(1/2)*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^2)+1/4*b*x^
3*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/16*(-3*a*d+b*c)*arctan((-a*d+b*
c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*
c)^(3/2)+1/16*(-3*a*d+b*c)*arctanh((-a*d+b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)+1/4*c^(1/4)*d^(1/4)*(c
^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticE(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/a/(-a*d+b*c)/(d*x^4+c)^(1/2)-1
/8*c^(1/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^
2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a/(-a*d+
b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x
^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x/c^
(1/4)),1/2*2^(1/2))/a/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2
)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/
4)*x/c^(1/4)),1/2*2^(1/2))/a/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))*(-3*a*d+b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^
(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/b^(1
/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(-a*d+b*c)/(...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{21abx^3(c+dx^4)+7(bc-4ad)x^3(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-3bdx^7(a+bx^4)}{84a^2(bc-ad)\,(a+bx^4)\sqrt{c+dx^4}}$$

input
```
Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)
/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((
b*x^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 1071, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {972, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 972}$$

$$\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}-\frac{\int -\frac{x^2(-bdx^4+bc-4ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^2(-bdx^4+bc-4ad)}{(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}+\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{\int \left(\frac{(bc-3ad)x^2}{(bx^4+a)\sqrt{dx^4+c}}-\frac{dx^2}{\sqrt{dx^4+c}}\right)dx}{4a(bc-ad)}+\frac{bx^3\sqrt{c+dx^4}}{4a(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\sqrt{dx^4+c}x^3}{4a(bc-ad)(bx^4+a)}+$$

$$-\frac{(bc-3ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{(bc-3ad)\arctan\left(\frac{\sqrt{b}}{\sqrt[4]{-a}}\right.}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc}}$$

input
```
Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(b*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (-((Sqrt[d]*x*Sqrt
[c + d*x^4])/(Sqrt[c] + Sqrt[d]*x^2)) + ((b*c - 3*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[
b*c - a*d]) - ((b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])]])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + (c^(1/4)*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]])/Sqrt[c + d*x^4] - (c^(1/4)*
d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]])/(2*Sqrt[c + d*x^4]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt
[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2]])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt
[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2]])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2]])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.31

method	result
default	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4a(ad-cb)(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4\right.}}{}$
elliptic	$-\frac{b\,x^3\sqrt{d\,x^4+c}}{4a(ad-cb)(b\,x^4+a)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{4(ad-cb)a\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4\right.}}{}$

input
```
int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/a/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I*d^(1/2)/(a*d-b*c)/a
*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^
(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^
(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/2),I))-1/32/b/a*sum((-3*a*d+b*
c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_a
lpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)
/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*
_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx=\int \frac{x^2}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{\sqrt{d\,x^4+c}\,x^2}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx$$

input
```
int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**4)*x**2)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d
*x**8 + b**2*c*x**8 + b**2*d*x**12),x)
```

3.273 $\int \frac{1}{x^2(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1248

$$\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \text{Too large to display}$$

output
```
-1/4*(-4*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x+1/4*d^(1/2)*(-4*a*d
+5*b*c)*x*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^2)+1/4*b*(d*
x^4+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^4+a)-1/16*b^(3/4)*(-7*a*d+5*b*c)*arctan((
-a*d+b*c)^(1/2)*x/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c
)^(3/2)+1/16*b^(3/4)*(-7*a*d+5*b*c)*arctanh((-a*d+b*c)^(1/2)*x/(-a)^(1/4)/
b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/4*d^(1/4)*(-4*a*d+5
*b*c)*(c^(1/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*Elli
pticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))/a^2/c^(3/4)/(-a*d+b*c)
/(d*x^4+c)^(1/2)+1/16*b^(1/2)*d^(1/4)*(-7*a*d+5*b*c)*(c^(1/2)+d^(1/2)*x^2)
*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4
)*x/c^(1/4)),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b^(1/2)*d^(1/4)*(-7*a*d+5*b*c)*(c^(1/2)+d
^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/
2)*d^(1/2))/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*d^(1/4)*(-4*a*d+5*b*c)*(c^(1/2)
+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(d^(1/4)*x/c^(1/4)),1/2*2^(1/2))/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)^(1
/2)-1/32*b^(1/2)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-7*a*d+5*b*c)*(c^(1
/2)+d^(1/2)*x^2)*((d*x^4+c)/(c^(1/2)+d^(1/2)*x^2)^2)^(1/2)*EllipticPi(sin(
2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{21a(c+dx^4)(4a^2d-5b^2cx^4-4ab(c-dx^4)) - 7(5b^2c^2-12abcd+4a^2d^2)x^4(a+bx^4)\sqrt{1+\frac{dx^4}{c}}\operatorname{Appell}}{84a^3c(bc-ad)x(a}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c
^2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[
3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*
(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c),
-((b*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 1148, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {972, 25, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}} \, dx$$

$$\Big\downarrow \; 972$$

$$\frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)} - \frac{\int -\frac{3bdx^4+5bc-4ad}{x^2(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3bdx^4+5bc-4ad}{x^2(bx^4+a)\sqrt{dx^4+c}}dx}{4a(bc-ad)} + \frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 1053$$

$$\frac{-\frac{\int \frac{x^2\left((bc-2ad)(5bc-2ad)-bd(5bc-4ad)x^4\right)}{(bx^4+a)\sqrt{dx^4+c}}dx}{ac} - \frac{\sqrt{c+dx^4}(5bc-4ad)}{acx}}{4a(bc-ad)} + \frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 1054$$

$$\frac{-\frac{\int \left(\frac{(5b^2c^2-7abcd)x^2}{(bx^4+a)\sqrt{dx^4+c}} - \frac{d(5bc-4ad)x^2}{\sqrt{dx^4+c}}\right)dx}{ac} - \frac{\sqrt{c+dx^4}(5bc-4ad)}{acx}}{4a(bc-ad)} + \frac{b\sqrt{c+dx^4}}{4ax(a+bx^4)(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{dx^4+c}\,b}{4a(bc-ad)x\left(bx^4+a\right)}+$$

$$-\frac{\sqrt{dx^4+c}(5bc-4ad)}{acx}-\frac{-\frac{\sqrt{b}c^{3/4}(5bc-7ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\ldots b^{3/4}$$

input

```
Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output

```
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) + (-(((5*b*c - 4*a*d)*
Sqrt[c + d*x^4])/(a*c*x)) - (-((Sqrt[d]*(5*b*c - 4*a*d)*x*Sqrt[c + d*x^4])
/(Sqrt[c] + Sqrt[d]*x^2)) + (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*
d]) - (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b
^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) + (c^(1/4)*d^(1/4
)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt
[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/Sqrt[c + d*x^4]
 - (c^(1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2
])/(2*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])
*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c
+ a*d)*Sqrt[c + d*x^4]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b]
)*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(b*c
 + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.73 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.31

method	result
elliptic	$\frac{b^2x^3\sqrt{dx^4+c}}{4a^2(ad-cb)(bx^4+a)} - \frac{\sqrt{dx^4+c}}{c\,a^2x} + \frac{i\left(-\frac{bd}{4a^2(ad-cb)}+\frac{d}{c\,a^2}\right)\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\right.\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}\,\sqrt{d}}$
default	$\frac{-\frac{\sqrt{dx^4+c}}{cx}+\frac{i\sqrt{d}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{c}\,\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{dx^4+c}}}{a^2} - \frac{b\left(-\frac{b\,x^3\sqrt{d\,x^4+c}}{4a(ad-cb)\left(b\,x^4+a\right)}+\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x}{\sqrt{c}}}}{}\right.}{}$ $\frac{i\sqrt{d}\,\sqrt{c}\,\sqrt{1-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\sqrt{1+\frac{i\sqrt{d}\,x^2}{\sqrt{c}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}},i\right)\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}} - \frac{c\left(\sum_{\_\alpha=\mathrm{RootOf}\left(\_Z^4b+a\right)} -\frac{\mathrm{arctanh}\left(\frac{}{2\sqrt{}}\right.}{\sqrt{}}\right.}{}$
risch	$-\frac{\sqrt{d\,x^4+c}}{c\,a^2x}+\frac{}{}$

input
```
int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/a^2/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)-1/c/a^2*(d*x^4+c)^(1/2
)/x+I*(-1/4*b*d/a^2/(a*d-b*c)+1/c*d/a^2)*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)
*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)
^(1/2)/d^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^
(1/2)*d^(1/2))^(1/2),I))-1/32/a^2*sum((7*a*d-5*b*c)/(a*d-b*c)/_alpha*(-1/(
(-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2
)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d
^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*Elliptic
Pi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*
d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx = \int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$

input
```
integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4)^2 \sqrt{c + dx^4}}\, dx = \int \frac{1}{(bx^4 + a)^2 \sqrt{dx^4 + c}x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4)^2 \sqrt{c + dx^4}}\, dx = \int \frac{1}{(bx^4 + a)^2 \sqrt{dx^4 + c}x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^4)^2 \sqrt{c + dx^4}}\, dx = \int \frac{1}{x^2 (b\, x^4 + a)^2 \sqrt{d\, x^4 + c}}\, dx$$

input
```
int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^4)^2 \sqrt{c+dx^4}}\,dx$$
$$= \frac{-\sqrt{d\,x^4+c}-3\left(\int \frac{\sqrt{d\,x^4+c}\,x^6}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx\right)abdx-3\left(\int \frac{\sqrt{d\,x^4+c}\,x^6}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a}\right.}{}$$

input
```
int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**4) - 3*int((sqrt(c + d*x**4)*x**6)/(a**2*c + a**2*d*x**4
 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b*d*x -
3*int((sqrt(c + d*x**4)*x**6)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b
*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*b**2*d*x**5 + int((sqrt(c + d*x**
4)*x**2)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8
 + b**2*d*x**12),x)*a**2*d*x - 5*int((sqrt(c + d*x**4)*x**2)/(a**2*c + a**
2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*
b*c*x + int((sqrt(c + d*x**4)*x**2)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 +
 2*a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)*a*b*d*x**5 - 5*int((sqrt(c
+ d*x**4)*x**2)/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*a*b*d*x**8 + b**2
*c*x**8 + b**2*d*x**12),x)*b**2*c*x**5)/(a*c*x*(a + b*x**4))
```

3.274 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=-\frac{1}{4}\arctan\left(\frac{1+x^2}{x\sqrt{-1+x^4}}\right)-\frac{1}{4}\text{arctanh}\left(\frac{1-x^2}{x\sqrt{-1+x^4}}\right)$$

output
```
-1/4*arctan((x^2+1)/x/(x^4-1)^(1/2))-1/4*arctanh((-x^2+1)/x/(x^4-1)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\left(-\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-1+x^4}}{x}\right)$$

input
```
Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

output
```
(-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2
 + I/2)*Sqrt[-1 + x^4])/x]
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.62 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {993, 1535, 763, 2213, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}dx$$

$$\Big\downarrow \text{ 993}$$

$$\frac{1}{2}\int \frac{1}{(x^2+i)\sqrt{x^4-1}}dx-\frac{1}{2}\int \frac{1}{(i-x^2)\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{ 1535}$$

$$\frac{1}{2}\left(-\frac{1}{2}i\int \frac{1}{\sqrt{x^4-1}}dx-\frac{1}{2}i\int \frac{i-x^2}{(x^2+i)\sqrt{x^4-1}}dx\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{1}{\sqrt{x^4-1}}dx+\frac{1}{2}i\int \frac{x^2+i}{(i-x^2)\sqrt{x^4-1}}dx\right)$$

$$\Big\downarrow \text{ 763}$$

$$\frac{1}{2}\left(-\frac{1}{2}i\int \frac{i-x^2}{(x^2+i)\sqrt{x^4-1}}dx-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{x^2+i}{(i-x^2)\sqrt{x^4-1}}dx+\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)$$

$$\Big\downarrow \text{ 2213}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{i-\frac{2x^2}{x^4-1}}d\frac{x}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}-\frac{1}{2}\int \frac{1}{\frac{2x^2}{x^4-1}+i}d\frac{x}{\sqrt{x^4-1}}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{i-\frac{2x^2}{x^4-1}}d\frac{x}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}+\left(\frac{1}{4}+\frac{i}{4}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\left(-\frac{1}{4}-\frac{i}{4}\right)\arctan\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}+\left(\frac{1}{4}+\frac{i}{4}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)\right)$$

input
```
Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

output
```
((-1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] - ((I/2)*Sqrt[-1 + x^2]*S
qrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*
Sqrt[-1 + x^4]))/2 + ((1/4 + I/4)*ArcTanh[((1 + I)*x)/Sqrt[-1 + x^4]] + ((
I/2)*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x
^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4]))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(\ln\left(2\right)+\ln\left(\frac{(1+i)\sqrt{x^4-1}+2x}{ix^2-1}\right)+\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right)\right)$
default	$\frac{\ln\left(\frac{\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}+1}{\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}+1}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
elliptic	$\frac{\ln\left(\frac{\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}+1}{\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}+1}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
trager	$\frac{\ln\left(-\frac{-8\,\mathrm{RootOf}\left(32\_Z^2-8\_Z+1\right)x+\sqrt{x^4-1}+2x}{8x^2\,\mathrm{RootOf}\left(32\_Z^2-8\_Z+1\right)-x^2-1}\right)}{4}-\ln\left(-\frac{-8\,\mathrm{RootOf}\left(32\_Z^2-8\_Z+1\right)x+\sqrt{x^4-1}+2x}{8x^2\,\mathrm{RootOf}\left(32\_Z^2-8\_Z+1\right)-x^2-1}\right)\mathrm{Roo}$

input
```
int(x^2/(x^4-1)^(1/2)/(x^4+1),x,method=_RETURNVERBOSE)
```

output
```
(1/8-1/8*I)*(ln(2)+ln(((1+I)*(x^4-1)^(1/2)+2*x)/(I*x^2-1))+arctan((1/2+1/2
*I)*(x^4-1)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\,\sqrt{x^4-1}x-1}{x^4+1}\right)$$

input
```
integrate(x^2/(x^4-1)^(1/2)/(x^4+1),x, algorithm="fricas")
```

output
```
1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4
- 1)*x - 1)/(x^4 + 1))
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,(x^4+1)}\,dx$$

input
```
integrate(x**2/(x**4-1)**(1/2)/(x**4+1),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4-1)^(1/2)/(x^4+1),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4-1)^(1/2)/(x^4+1),x, algorithm="giac")
```

output
```
integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

input
```
int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
```

output
```
int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{\sqrt{x^4-1}\,x^2}{x^8-1}dx$$

input
```
int(x^2/(x^4-1)^(1/2)/(x^4+1),x)
```

output
```
int((sqrt(x**4 - 1)*x**2)/(x**8 - 1),x)
```

3.275 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

output
```
2/5*(e*x)^(5/2)*(d*x^4+c)^(1/2)*AppellF1(5/8,1,-1/2,13/8,-b*x^4/a,-d*x^4/c
)/a/e/(1+d*x^4/c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\frac{2x(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},-\frac{1}{2},1,\frac{13}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c)
, -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \text{966}$$

$$\frac{2\int \frac{e^6x^2\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^2x^2\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \text{1013}$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{e^2x^2\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{5}{8},1,-\frac{1}{2},\frac{13}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a),
-((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} \sqrt{d\,x^{4}+c}}{b\,x^{4}+a}dx$$

input
```
int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*sqrt(e*x)*e*x/(b*x^4 + a), x)
```

Sympy [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{(ex)^{\frac{3}{2}}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{\sqrt{dx^4+c}(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\int \frac{(e\,x)^{3/2}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx=\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{d\,x^4+c}\,x}{b\,x^4+a}dx\right)e$$

input
```
int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(c + d*x**4)*x)/(a + b*x**4),x)*e
```

3.276 $\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}$$

output
```
2/3*(e*x)^(3/2)*(d*x^4+c)^(1/2)*AppellF1(3/8,1,-1/2,11/8,-b*x^4/a,-d*x^4/c
)/a/e/(1+d*x^4/c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2x\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},-\frac{1}{2},1,\frac{11}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c),
-((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}dx$$

$$\Big\downarrow \ 966$$

$$\frac{2\int \frac{e^5x\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3\int \frac{ex\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \ 1013$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{ex\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \ 1012$$

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{3}{8},1,-\frac{1}{2},\frac{11}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

output
```
(2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a),
-((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{ex}\,\sqrt{d\,x^{4}+c}}{b\,x^{4}+a}dx$$

input
```
int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \int \frac{\sqrt{e\,x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}dx\right)$$

input
```
int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(c + d*x**4))/(a + b*x**4),x)
```

3.277 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}(a+bx^4)} dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx=\frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

output
```
2*(e*x)^(1/2)*(d*x^4+c)^(1/2)*AppellF1(1/8,1,-1/2,9/8,-b*x^4/a,-d*x^4/c)/a
/e/(1+d*x^4/c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx=\frac{2x\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},-\frac{1}{2},1,\frac{9}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\sqrt{\frac{c+dx^4}{c}}}$$

input
```
Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

output
```
(2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a
)])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {966, 27, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}dx$$

$$\Big\downarrow \ 966$$

$$\frac{2\int \frac{e^4\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3\int \frac{\sqrt{dx^4+c}}{bx^4e^4+ae^4}d\sqrt{ex}$$

$$\Big\downarrow \ 937$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{\sqrt{\frac{dx^4}{c}+1}}{bx^4e^4+ae^4}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \ 936$$

$$\frac{2\sqrt{ex}\sqrt{c+dx^4}\,\text{AppellF1}\left(\frac{1}{8},1,-\frac{1}{2},\frac{9}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

output
```
(2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((
d*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\sqrt{e x}\,\left(b\,x^{4}+a\right)}dx$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

output
```
int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \text{Timed out}$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\sqrt{ex}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{\sqrt{e\,x}\,\left(b\,x^4+a\right)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,(a+bx^4)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{d\,x^4+c}}{\sqrt{x}\,a+\sqrt{x}\,b\,x^4}dx\right)}{e}$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

output
```
(sqrt(e)*int(sqrt(c + d*x**4)/(sqrt(x)*a + sqrt(x)*b*x**4),x))/e
```

3.278 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=-\frac{2\sqrt{c+dx^4}\,\operatorname{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

output
```
-2*(d*x^4+c)^(1/2)*AppellF1(-1/8,1,-1/2,7/8,-b*x^4/a,-d*x^4/c)/a/e/(e*x)^(
1/2)/(1+d*x^4/c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.

Time = 11.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx=\frac{x\left(-70a(c+dx^4)-10(bc-4ad)x^4\sqrt{1+\frac{dx^4}{c}}\operatorname{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right.}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

input
```
Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

output
```
(x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[
7/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^
4)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(
e*x)^(3/2)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {966, 27, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}dx$$

$$\Big\downarrow \; 966$$

$$\frac{2\int \frac{e^3\sqrt{dx^4+c}}{x(bx^4e^4+ae^4)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{\sqrt{dx^4+c}}{ex\,(bx^4e^4+ae^4)}d\sqrt{ex}$$

$$\Big\downarrow \; 1013$$

$$\frac{2e^3\sqrt{c+dx^4}\int \frac{\sqrt{\frac{dx^4}{c}+1}}{ex(bx^4e^4+ae^4)}d\sqrt{ex}}{\sqrt{\frac{dx^4}{c}+1}}$$

$$\Big\downarrow \; 1012$$

$$-\frac{2\sqrt{c+dx^4}\,\text{AppellF1}\left(-\frac{1}{8},1,-\frac{1}{2},\frac{7}{8},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

input
```
Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

output
```
(-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c
)])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 966
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(
m + 1) - 1)*(a + b*(x^(k*n)/e^n))^p*(c + d*(x^(k*n)/e^n))^q, x], x, (e*x)^(
1/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && FractionQ[m] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{4}+c}}{\left(ex\right)^{\frac{3}{2}}\left(b\,x^{4}+a\right)}dx$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

output
```
int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*sqrt(e*x)/(b*e^2*x^6 + a*e^2*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx = \int \frac{\sqrt{c+dx^4}}{(ex)^{\frac{3}{2}}(a+bx^4)}\,dx$$

input
```
integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
```

output
```
Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \int \frac{\sqrt{dx^4+c}}{(bx^4+a)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \int \frac{\sqrt{d\,x^4+c}}{(e\,x)^{3/2}\,(b\,x^4+a)}\,dx$$

input
```
int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)),x)
```

output
```
int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\,(a+bx^4)}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{d\,x^4+c}}{\sqrt{x}\,a x+\sqrt{x}\,b\,x^5}dx\right)}{e^2}$$

input
```
int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

output
```
(sqrt(e)*int(sqrt(c + d*x**4)/(sqrt(x)*a*x + sqrt(x)*b*x**5),x))/e**2
```

3.279 $\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}} dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}} dx = -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\left(\frac{a^2}{1+m}+\frac{bc(bc(5+m)-2ad(7+m))}{d^2(3+m)(7+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}$$

output

```
-b*(b*c*(5+m)-2*a*d*(7+m))*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d^2/e/(3+m)/(7+m)+b
^2*(e*x)^(5+m)*(d*x^4+c)^(1/2)/d/e^5/(7+m)+(a^2/(1+m)+b*c*(b*c*(5+m)-2*a*d
*(7+m))/d^2/(3+m)/(7+m))*(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*hypergeom([1/2, 1/4
+1/4*m],[5/4+1/4*m],-d*x^4/c)/e/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 9.44 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx$$
$$= \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\left(2a(9+m\right.\right.}{(1+m)(5+m)(}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/
2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hyperg
eometric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hyper
geometric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)
*(9 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {964, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2 (ex)^m}{\sqrt{c+dx^4}}\,dx$$
$$\Big\downarrow \text{964}$$
$$\frac{\int \frac{(ex)^m (a^2d(m+7)-b(bc(m+5)-2ad(m+7))x^4)}{\sqrt{dx^4+c}}dx}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$
$$\Big\downarrow \text{959}$$

$$\frac{\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))}{de(m+3)}}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\frac{\sqrt{\frac{dx^4}{c}+1}\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(m+7))}{de(m+3)}}{d(m+7)}+\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(a^2d(m+7)+\frac{bc(m+1)(bc(m+5)-2ad(m+7))}{d(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}(ex)^{m+1}(bc(m+5)-2ad(}{de(m+3)}}{d(m+7)}$$
$$\frac{b^2\sqrt{c+dx^4}(ex)^{m+5}}{de^5(m+7)}$$

input
```
Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

output
```
(b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (-((b*(b*c*(5 + m) -
 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m))) + ((a^2*d*(7
 + m) + (b*c*(1 + m)*(b*c*(5 + m) - 2*a*d*(7 + m)))/(d*(3 + m)))*(e*x)^(1
+ m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d
*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4]))/(d*(7 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 964
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Simp[1/(b*(m + n*(p + 2) + 1))   Int[(e*x)
^m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) - d*(a*d*(m + n + 1) - 2*b*
c*(m + n*(p + 2) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (b x^4 + a)^2}{\sqrt{d x^4 + c}}dx$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \frac{a^2 e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{abe^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)} + \frac{b^2 e^m x^{m+9}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4}+\frac{9}{4} \\ \frac{m}{4}+\frac{13}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{13}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
a**2*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,)
, d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**(m
+ 5)*gamma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**(m + 9)*gamma(m/4
+ 9/4)*hyper((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4
*sqrt(c)*gamma(m/4 + 13/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^2}{\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^2}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a + bx^4)^2}{\sqrt{c + dx^4}} \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
(e**m*(2*x**m*sqrt(c + d*x**4)*a*b*d*m*x + 14*x**m*sqrt(c + d*x**4)*a*b*d*
x - x**m*sqrt(c + d*x**4)*b**2*c*m*x - 5*x**m*sqrt(c + d*x**4)*b**2*c*x +
x**m*sqrt(c + d*x**4)*b**2*d*m*x**5 + 3*x**m*sqrt(c + d*x**4)*b**2*d*x**5
+ int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d
*m*x**4 + 21*d*x**4),x)*a**2*d**2*m**4 + 20*int((x**m*sqrt(c + d*x**4))/(c
*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4),x)*a**2*d**
2*m**3 + 142*int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*
x**4 + 10*d*m*x**4 + 21*d*x**4),x)*a**2*d**2*m**2 + 420*int((x**m*sqrt(c +
 d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4)
,x)*a**2*d**2*m + 441*int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*c
+ d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4),x)*a**2*d**2 - 2*int((x**m*sqrt(c
 + d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**
4),x)*a*b*c*d*m**4 - 36*int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*
c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4),x)*a*b*c*d*m**3 - 216*int((x**m
*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d*m*x**4 + 2
1*d*x**4),x)*a*b*c*d*m**2 - 476*int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c
*m + 21*c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4),x)*a*b*c*d*m - 294*int(
(x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m + 21*c + d*m**2*x**4 + 10*d*m*x**
4 + 21*d*x**4),x)*a*b*c*d + int((x**m*sqrt(c + d*x**4))/(c*m**2 + 10*c*m +
 21*c + d*m**2*x**4 + 10*d*m*x**4 + 21*d*x**4),x)*b**2*c**2*m**4 + 16*i...
```

3.280 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}} \, dx = \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} + \frac{\left(\frac{a}{1+m}-\frac{bc}{d(3+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}$$

output
```
b*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d/e/(3+m)+(a/(1+m)-b*c/d/(3+m))*(e*x)^(1+m)*
(1+d*x^4/c)^(1/2)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)/e/(d*x^
4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)x^4\,\text{Hypergeometric2F1}\right.}{(1+m)(5+m)\sqrt{c+dx^4}}$$

input
```
Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[1/2, (1 + m)/4
, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[1/2, (5 + m)/
4, (9 + m)/4, -((d*x^4)/c)]))/((1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(ex)^m}{\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(a-\frac{bc(m+1)}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\left(a-\frac{bc(m+1)}{d(m+3)}\right)\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(a-\frac{bc(m+1)}{d(m+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}(ex)^{m+1}}{de(m+3)}$$

input
```
Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

output
```
(b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a - (b*c*(1 + m))/(d*(
3 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4 + a)}{\sqrt{d\,x^4 + c}}dx$$

input
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a + bx^4)}{\sqrt{c + dx^4}}\,dx = \int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

$$\int \frac{(ex)^m (a + bx^4)}{\sqrt{c + dx^4}}\,dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4} + \frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4} + \frac{1}{4} \\ \frac{m}{4} + \frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4} + \frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4} + \frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{4} + \frac{5}{4} \\ \frac{m}{4} + \frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4} + \frac{9}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d
*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)*
gamma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*
pi)/c)/(4*sqrt(c)*gamma(m/4 + 9/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \int \frac{(bx^4+a)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^4)}{\sqrt{c+dx^4}}\,dx$$
$$= \frac{e^m \left(x^m \sqrt{d\,x^4+c}\,bx + \left(\int \frac{x^m \sqrt{d\,x^4+c}}{dm\,x^4+3d\,x^4+cm+3c}dx\right) ad\,m^2 + 6\left(\int \frac{x^m \sqrt{d\,x^4+c}}{dm\,x^4+3d\,x^4+cm+3c}dx\right) adm + 9\left(\int \frac{x^m \sqrt{d\,x^4}}{dm\,x^4+3d\,x^4}\right.\right.}{d}$$

input
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
(e**m*(x**m*sqrt(c + d*x**4)*b*x + int((x**m*sqrt(c + d*x**4))/(c*m + 3*c
+ d*m*x**4 + 3*d*x**4),x)*a*d*m**2 + 6*int((x**m*sqrt(c + d*x**4))/(c*m +
3*c + d*m*x**4 + 3*d*x**4),x)*a*d*m + 9*int((x**m*sqrt(c + d*x**4))/(c*m +
 3*c + d*m*x**4 + 3*d*x**4),x)*a*d - int((x**m*sqrt(c + d*x**4))/(c*m + 3*
c + d*m*x**4 + 3*d*x**4),x)*b*c*m**2 - 4*int((x**m*sqrt(c + d*x**4))/(c*m
+ 3*c + d*m*x**4 + 3*d*x**4),x)*b*c*m - 3*int((x**m*sqrt(c + d*x**4))/(c*m
 + 3*c + d*m*x**4 + 3*d*x**4),x)*b*c))/(d*(m + 3))
```

3.281 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^
4/c)/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 +
m)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 +
m)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x)^m/sqrt(d*x^4 + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right| \frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((e*x)**m/(d*x**4+c)**(1/2),x)
```

output
```
e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/(c + d*x^4)^(1/2),x)
```

output
```
int((e*x)^m/(c + d*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx = e^m\left(\int \frac{x^m\sqrt{d\,x^4+c}}{d\,x^4+c}dx\right)$$

input
```
int((e*x)^m/(d*x^4+c)^(1/2),x)
```

output
```
e**m*int((x**m*sqrt(c + d*x**4))/(c + d*x**4),x)
```

3.282 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},1,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,1,1/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{c+dx^4}\left(bc\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{4},\frac{5+m}{4},-\right.\right.}{ac(bc-ad)(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -(
(d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/
4, -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},1,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,\sqrt{c+dx^4}}\,dx=\int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx=\int \frac{(e\,x)^m}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx=e^m\left(\int \frac{x^m\sqrt{d\,x^4+c}}{bd\,x^8+ad\,x^4+bc\,x^4+ac}dx\right)$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

output
```
e**m*int((x**m*sqrt(c + d*x**4))/(a*c + a*d*x**4 + b*c*x**4 + b*d*x**8),x)
```

3.283 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},2,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,2,1/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a^2/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.

Time = 10.01 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$
$$= \frac{x(ex)^m\sqrt{c+dx^4}\left(-abcd\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)\,\text{AppellF1}\left(\frac{1+m}{4},2,-\frac{1}{2},\right.\right.}{a^2c(bc-ad)^2(1+m)\sqrt{1+\frac{dx^4}{c}}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)
/4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2,
-1/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1
/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqr
t[1 + (d*x^4)/c])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)^2\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},2,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*
a*b*c + a^2*d)*x^4 + a^2*c), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^2\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx = e^m\left(\int \frac{x^m\sqrt{d\,x^4+c}}{b^2d\,x^{12}+2abd\,x^8+b^2c\,x^8+a^2d\,x^4+2abc\,x^4+a^2c}dx\right)$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

output
```
e**m*int((x**m*sqrt(c + d*x**4))/(a**2*c + a**2*d*x**4 + 2*a*b*c*x**4 + 2*
a*b*d*x**8 + b**2*c*x**8 + b**2*d*x**12),x)
```

3.284 $\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 81

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,3,1/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a^3/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},3,\frac{1}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b
*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)^3\sqrt{\frac{dx^4}{c}+1}}dx}{\sqrt{c+dx^4}}$$

$$\Big\downarrow \ 1012$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},3,\frac{1}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\,\sqrt{d\,x^4+c}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\,\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 +
3*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3\sqrt{dx^4+c}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^3\,\sqrt{d\,x^4+c}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\sqrt{c+dx^4}}\,dx = \text{too large to display}$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

output

```
(e**m*(x**m*sqrt(c + d*x**4)*a*d*m*x + 7*x**m*sqrt(c + d*x**4)*a*d*x + x**
m*sqrt(c + d*x**4)*b*c*m*x - 3*x**m*sqrt(c + d*x**4)*b*c*x - x**m*sqrt(c +
 d*x**4)*b*d*m*x**5 + 5*x**m*sqrt(c + d*x**4)*b*d*x**5 + int((x**m*sqrt(c
+ d*x**4)*x**12)/(a**4*c*d*m**2 + 8*a**4*c*d*m + 7*a**4*c*d + a**4*d**2*m*
*2*x**4 + 8*a**4*d**2*m*x**4 + 7*a**4*d**2*x**4 + a**3*b*c**2*m**2 - 2*a**
3*b*c**2*m - 3*a**3*b*c**2 + 4*a**3*b*c*d*m**2*x**4 + 22*a**3*b*c*d*m*x**4
 + 18*a**3*b*c*d*x**4 + 3*a**3*b*d**2*m**2*x**8 + 24*a**3*b*d**2*m*x**8 +
21*a**3*b*d**2*x**8 + 3*a**2*b**2*c**2*m**2*x**4 - 6*a**2*b**2*c**2*m*x**4
 - 9*a**2*b**2*c**2*x**4 + 6*a**2*b**2*c*d*m**2*x**8 + 18*a**2*b**2*c*d*m*
x**8 + 12*a**2*b**2*c*d*x**8 + 3*a**2*b**2*d**2*m**2*x**12 + 24*a**2*b**2*
d**2*m*x**12 + 21*a**2*b**2*d**2*x**12 + 3*a*b**3*c**2*m**2*x**8 - 6*a*b**
3*c**2*m*x**8 - 9*a*b**3*c**2*x**8 + 4*a*b**3*c*d*m**2*x**12 + 2*a*b**3*c*
d*m*x**12 - 2*a*b**3*c*d*x**12 + a*b**3*d**2*m**2*x**16 + 8*a*b**3*d**2*m*
x**16 + 7*a*b**3*d**2*x**16 + b**4*c**2*m**2*x**12 - 2*b**4*c**2*m*x**12 -
 3*b**4*c**2*x**12 + b**4*c*d*m**2*x**16 - 2*b**4*c*d*m*x**16 - 3*b**4*c*d
*x**16),x)*a**3*b**2*d**3*m**4 + 2*int((x**m*sqrt(c + d*x**4)*x**12)/(a**4
*c*d*m**2 + 8*a**4*c*d*m + 7*a**4*c*d + a**4*d**2*m**2*x**4 + 8*a**4*d**2*
m*x**4 + 7*a**4*d**2*x**4 + a**3*b*c**2*m**2 - 2*a**3*b*c**2*m - 3*a**3*b*
c**2 + 4*a**3*b*c*d*m**2*x**4 + 22*a**3*b*c*d*m*x**4 + 18*a**3*b*c*d*x**4
+ 3*a**3*b*d**2*m**2*x**8 + 24*a**3*b*d**2*m*x**8 + 21*a**3*b*d**2*x**8...
```

3.285 $\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} + \frac{(a^2d^2(3-2m-m^2)+2abcd(3+4m+m^2)-b^2c^2(5+6m+m^2))\,(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\ \text{Hypergeometric2F}}{2cd^2e(1+m)(3+m)\sqrt{c+dx^4}}$$

output
```
1/2*(-a*d+b*c)^2*(e*x)^(1+m)/c/d^2/e/(d*x^4+c)^(1/2)+b^2*(e*x)^(1+m)*(d*x^
4+c)^(1/2)/d^2/e/(3+m)+1/2*(a^2*d^2*(-m^2-2*m+3)+2*a*b*c*d*(m^2+4*m+3)-b^2
*c^2*(m^2+6*m+5))*(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*hypergeom([1/2, 1/4+1/4*m]
,[5/4+1/4*m],-d*x^4/c)/c/d^2/e/(1+m)/(3+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a^2(45+14m+m^2)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)\right.}{}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/
2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hyperg
eometric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hyper
geometric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 +
m)*(9 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {963, 25, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2 (ex)^m}{(c+dx^4)^{3/2}}dx$$

$$\Big\downarrow \text{963}$$

$$\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}-\frac{\int -\frac{(ex)^m(2b^2cdx^4+2a^2d^2-(bc-ad)^2(m+1))}{\sqrt{dx^4+c}}dx}{2cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(2b^2cdx^4+2a^2d^2-(bc-ad)^2(m+1))}{\sqrt{dx^4+c}}dx}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{(2b^2c^2(m+1)-(m+3)(2a^2d^2-(m+1)(bc-ad)^2))\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx}{m+3}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\int\frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{(m+3)\sqrt{c+dx^4}}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\frac{2b^2c\sqrt{c+dx^4}(ex)^{m+1}}{e(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{e(m+1)(m+3)\sqrt{c+dx^4}}}{2cd^2}+\frac{(ex)^{m+1}(bc-ad)^2}{2cd^2e\sqrt{c+dx^4}}$$

input
```
Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + ((2*b^2*c*(e*x
)^(1 + m)*Sqrt[c + d*x^4])/(e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*
a^2*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hyperg
eometric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(e*(1 + m)*(3 + m)*S
qrt[c + d*x^4]))/(2*c*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 963
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Simp[1/(a*b^2*n*(p + 1))   Int[(e*x)^m*(a + b*x
^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)^2}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*
c*d*x^4 + c^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

output
```
Integral((e*x)**m*(a + b*x**4)**2/(c + d*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)^2(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^2}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output

```
(e**m*(2*x**m*sqrt(c + d*x**4)*a*b*d*m*x + 6*x**m*sqrt(c + d*x**4)*a*b*d*x
 - x**m*sqrt(c + d*x**4)*b**2*c*m*x - 5*x**m*sqrt(c + d*x**4)*b**2*c*x + x
**m*sqrt(c + d*x**4)*b**2*d*m*x**5 - x**m*sqrt(c + d*x**4)*b**2*d*x**5 + i
nt((x**m*sqrt(c + d*x**4))/(c**2*m**2 + 2*c**2*m - 3*c**2 + 2*c*d*m**2*x**
4 + 4*c*d*m*x**4 - 6*c*d*x**4 + d**2*m**2*x**8 + 2*d**2*m*x**8 - 3*d**2*x*
*8),x)*a**2*c*d**2*m**4 + 4*int((x**m*sqrt(c + d*x**4))/(c**2*m**2 + 2*c**
2*m - 3*c**2 + 2*c*d*m**2*x**4 + 4*c*d*m*x**4 - 6*c*d*x**4 + d**2*m**2*x**
8 + 2*d**2*m*x**8 - 3*d**2*x**8),x)*a**2*c*d**2*m**3 - 2*int((x**m*sqrt(c
+ d*x**4))/(c**2*m**2 + 2*c**2*m - 3*c**2 + 2*c*d*m**2*x**4 + 4*c*d*m*x**4
 - 6*c*d*x**4 + d**2*m**2*x**8 + 2*d**2*m*x**8 - 3*d**2*x**8),x)*a**2*c*d*
*2*m**2 - 12*int((x**m*sqrt(c + d*x**4))/(c**2*m**2 + 2*c**2*m - 3*c**2 +
2*c*d*m**2*x**4 + 4*c*d*m*x**4 - 6*c*d*x**4 + d**2*m**2*x**8 + 2*d**2*m*x*
*8 - 3*d**2*x**8),x)*a**2*c*d**2*m + 9*int((x**m*sqrt(c + d*x**4))/(c**2*m
**2 + 2*c**2*m - 3*c**2 + 2*c*d*m**2*x**4 + 4*c*d*m*x**4 - 6*c*d*x**4 + d*
*2*m**2*x**8 + 2*d**2*m*x**8 - 3*d**2*x**8),x)*a**2*c*d**2 + int((x**m*sqr
t(c + d*x**4))/(c**2*m**2 + 2*c**2*m - 3*c**2 + 2*c*d*m**2*x**4 + 4*c*d*m*
x**4 - 6*c*d*x**4 + d**2*m**2*x**8 + 2*d**2*m*x**8 - 3*d**2*x**8),x)*a**2*
d**3*m**4*x**4 + 4*int((x**m*sqrt(c + d*x**4))/(c**2*m**2 + 2*c**2*m - 3*c
**2 + 2*c*d*m**2*x**4 + 4*c*d*m*x**4 - 6*c*d*x**4 + d**2*m**2*x**8 + 2*d**
2*m*x**8 - 3*d**2*x**8),x)*a**2*d**3*m**3*x**4 - 2*int((x**m*sqrt(c + d...
```

3.286 $\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx = -\frac{b(ex)^{1+m}}{de(1-m)\sqrt{c+dx^4}} + \frac{\left(\frac{a}{c+cm}+\frac{b}{d-dm}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}$$

output
```
-b*(e*x)^(1+m)/d/e/(1-m)/(d*x^4+c)^(1/2)+(a/(c*m+c)+b/(-d*m+d))*(e*x)^(1+m
)*(1+d*x^4/c)^(1/2)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)/e/(d*
x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 4.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\left(a(5+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)+b(1+m)\right.}{c(1+m)(5+m)\sqrt{c+dx^4}}$$

input
```
Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[3/2, (1 + m)/4
, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[3/2, (5 + m)/
4, (9 + m)/4, -((d*x^4)/c)]))/(c*(1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)(ex)^m}{(c+dx^4)^{3/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ad(1-m)+bc(m+1))\int \frac{(ex)^m}{\sqrt{dx^4+c}}dx}{2cd}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ad(1-m)+bc(m+1))\int \frac{(ex)^m}{\sqrt{\frac{dx^4}{c}+1}}dx}{2cd\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}(ad(1-m)+bc(m+1))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

input
```
Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

output
```
-1/2*((b*c - a*d)*(e*x)^(1 + m))/(c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) +
 b*c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1
+ m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{ae^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\right| \frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^{m+5}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\right| \frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

output
```
a*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d
*x**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**(m + 5)
*gamma(m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I
*pi)/c)/(4*c**(3/2)*gamma(m/4 + 9/4))
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(bx^4+a)(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x)
```

output
```
int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^4)}{(c+dx^4)^{3/2}}\,dx = \frac{e^m \left(x^m \sqrt{d\,x^4+c}\,bx + \left(\int \frac{x^m \sqrt{d\,x^4+c}}{d^2 m\,x^8-d^2x^8+2cdm\,x^4-2cd\,x^4+c^2m-c^2}dx\right) acd\,m^2 - 2\left(\int \frac{}{d^2 m\,x^8}\right.\right.}{}$$

input

```
int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output

```
(e**m*(x**m*sqrt(c + d*x**4)*b*x + int((x**m*sqrt(c + d*x**4))/(c**2*m - c
**2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x)*a*c*d*m**2 -
 2*int((x**m*sqrt(c + d*x**4))/(c**2*m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4
+ d**2*m*x**8 - d**2*x**8),x)*a*c*d*m + int((x**m*sqrt(c + d*x**4))/(c**2*
m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x)*a*c*d +
 int((x**m*sqrt(c + d*x**4))/(c**2*m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4 +
d**2*m*x**8 - d**2*x**8),x)*a*d**2*m**2*x**4 - 2*int((x**m*sqrt(c + d*x**4
))/(c**2*m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x
)*a*d**2*m*x**4 + int((x**m*sqrt(c + d*x**4))/(c**2*m - c**2 + 2*c*d*m*x**
4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x)*a*d**2*x**4 - int((x**m*sqrt(
c + d*x**4))/(c**2*m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d*
*2*x**8),x)*b*c**2*m**2 + int((x**m*sqrt(c + d*x**4))/(c**2*m - c**2 + 2*c
*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x)*b*c**2 - int((x**m*sq
rt(c + d*x**4))/(c**2*m - c**2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 -
 d**2*x**8),x)*b*c*d*m**2*x**4 + int((x**m*sqrt(c + d*x**4))/(c**2*m - c**
2 + 2*c*d*m*x**4 - 2*c*d*x**4 + d**2*m*x**8 - d**2*x**8),x)*b*c*d*x**4))/(
d*(c*m - c + d*m*x**4 - d*x**4))
```

3.287 $\int \frac{(ex)^m}{(c+dx^4)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}} dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},\frac{5+m}{4},-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^
4/c)/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}} dx = \frac{x(ex)^m\sqrt{1+\frac{dx^4}{c}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{4},1+\frac{1+m}{4},-\frac{dx^4}{c}\right)}{c(1+m)\sqrt{c+dx^4}}$$

input
```
Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
```

output
```
(x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 +
m)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{\left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{4},\frac{m+5}{4},-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 +
m)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \frac{e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((e*x)**m/(d*x**4+c)**(3/2),x)
```

output
```
e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(d\,x^4+c)^{3/2}}\,dx$$

input
```
int((e*x)^m/(c + d*x^4)^(3/2),x)
```

output
```
int((e*x)^m/(c + d*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx = e^m\left(\int \frac{x^m\sqrt{d\,x^4+c}}{d^2x^8+2cd\,x^4+c^2}dx\right)$$

input
```
int((e*x)^m/(d*x^4+c)^(3/2),x)
```

output
```
e**m*int((x**m*sqrt(c + d*x**4))/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.288 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},1,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,1,3/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(84) = 168.

Time = 11.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.01

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx=\frac{x(ex)^m\sqrt{c+dx^4}\Big(b^2c^2\,\text{AppellF1}\left(\frac{1+m}{4},-\frac{1}{2},1,\frac{5+m}{4},-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\Big(-bc\,\text{F}}{ac^2(}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4
, -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 +
m)/4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d
*x^4)/c])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)\left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},1,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*
c^2 + 2*a*c*d)*x^4 + a*c^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

output
```
Integral((e*x)**m/((a + b*x**4)*(c + d*x**4)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = \int \frac{(e\,x)^m}{(b\,x^4+a)\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{3/2}}\,dx = e^m \left(\int \frac{x^m\sqrt{d\,x^4+c}}{b\,d^2x^{12}+a\,d^2x^8+2bcd\,x^8+2acd\,x^4+b\,c^2x^4+a\,c^2}dx\right)$$

input
```
int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

output
```
e**m*int((x**m*sqrt(c + d*x**4))/(a*c**2 + 2*a*c*d*x**4 + a*d**2*x**8 + b*
c**2*x**4 + 2*b*c*d*x**8 + b*d**2*x**12),x)
```

3.289 $\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx=\frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,2,3/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a^2/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx=\frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\text{AppellF1}\left(\frac{1+m}{4},2,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2(1+m)\left(c+dx^4\right)^{3/2}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -(
(b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{\frac{dx^4}{c}+1} \int \frac{(ex)^m}{(bx^4+a)^2 \left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1} \text{AppellF1}\left(\frac{m+1}{4},2,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^2\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^1
2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*
c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^2(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^2\,(c+dx^4)^{3/2}}\,dx=\int \frac{(e\,x)^m}{(b\,x^4+a)^2\,(d\,x^4+c)^{3/2}}\,dx$$

input
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)^2\,(c+dx^4)^{3/2}}\,dx=\text{too large to display}$$

input
```
int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

output

```
(e**m*(x**m*sqrt(c + d*x**4)*a*d*m*x + 3*x**m*sqrt(c + d*x**4)*a*d*x + x**
m*sqrt(c + d*x**4)*b*c*m*x + x**m*sqrt(c + d*x**4)*b*c*x - x**m*sqrt(c + d
*x**4)*b*d*m*x**5 + 5*x**m*sqrt(c + d*x**4)*b*d*x**5 + int((x**m*sqrt(c +
d*x**4)*x**12)/(a**3*c**2*d*m**2 + 4*a**3*c**2*d*m + 3*a**3*c**2*d + 2*a**
3*c*d**2*m**2*x**4 + 8*a**3*c*d**2*m*x**4 + 6*a**3*c*d**2*x**4 + a**3*d**3
*m**2*x**8 + 4*a**3*d**3*m*x**8 + 3*a**3*d**3*x**8 + a**2*b*c**3*m**2 + 2*
a**2*b*c**3*m + a**2*b*c**3 + 4*a**2*b*c**2*d*m**2*x**4 + 12*a**2*b*c**2*d
*m*x**4 + 8*a**2*b*c**2*d*x**4 + 5*a**2*b*c*d**2*m**2*x**8 + 18*a**2*b*c*d
**2*m*x**8 + 13*a**2*b*c*d**2*x**8 + 2*a**2*b*d**3*m**2*x**12 + 8*a**2*b*d
**3*m*x**12 + 6*a**2*b*d**3*x**12 + 2*a*b**2*c**3*m**2*x**4 + 4*a*b**2*c**
3*m*x**4 + 2*a*b**2*c**3*x**4 + 5*a*b**2*c**2*d*m**2*x**8 + 12*a*b**2*c**2
*d*m*x**8 + 7*a*b**2*c**2*d*x**8 + 4*a*b**2*c*d**2*m**2*x**12 + 12*a*b**2*
c*d**2*m*x**12 + 8*a*b**2*c*d**2*x**12 + a*b**2*d**3*m**2*x**16 + 4*a*b**2
*d**3*m*x**16 + 3*a*b**2*d**3*x**16 + b**3*c**3*m**2*x**8 + 2*b**3*c**3*m*
x**8 + b**3*c**3*x**8 + 2*b**3*c**2*d*m**2*x**12 + 4*b**3*c**2*d*m*x**12 +
 2*b**3*c**2*d*x**12 + b**3*c*d**2*m**2*x**16 + 2*b**3*c*d**2*m*x**16 + b*
*3*c*d**2*x**16),x)*a**2*b**2*c*d**3*m**4 - 2*int((x**m*sqrt(c + d*x**4)*x
**12)/(a**3*c**2*d*m**2 + 4*a**3*c**2*d*m + 3*a**3*c**2*d + 2*a**3*c*d**2*
m**2*x**4 + 8*a**3*c*d**2*m*x**4 + 6*a**3*c*d**2*x**4 + a**3*d**3*m**2*x**
8 + 4*a**3*d**3*m*x**8 + 3*a**3*d**3*x**8 + a**2*b*c**3*m**2 + 2*a**2*b...
```

3.290 $\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,\text{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}$$

output
```
(e*x)^(1+m)*(1+d*x^4/c)^(1/2)*AppellF1(1/4+1/4*m,3,3/2,5/4+1/4*m,-b*x^4/a,
-d*x^4/c)/a^3/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Mathematica [A] (verified)

Time = 11.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \frac{x(ex)^m\left(1+\frac{dx^4}{c}\right)^{3/2}\text{AppellF1}\left(\frac{1+m}{4},3,\frac{3}{2},\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(1+m)\left(c+dx^4\right)^{3/2}}$$

input
```
Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

output
```
(x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -(
(b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{1013}$$
$$\frac{\sqrt{\frac{dx^4}{c}+1}\int \frac{(ex)^m}{(bx^4+a)^3\left(\frac{dx^4}{c}+1\right)^{3/2}}dx}{c\sqrt{c+dx^4}}$$
$$\Big\downarrow \text{1012}$$
$$\frac{\sqrt{\frac{dx^4}{c}+1}(ex)^{m+1}\operatorname{AppellF1}\left(\frac{m+1}{4},3,\frac{3}{2},\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

input
```
Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

output
```
((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4,
-((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m}{(b\,x^4+a)^3\,(d\,x^4+c)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

output
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3\,(c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)
*x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*
b*c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}}\,dx = \int \frac{(ex)^m}{(bx^4+a)^3(dx^4+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \int \frac{(e\,x)^m}{(b\,x^4+a)^3 \,(d\,x^4+c)^{3/2}} \, dx$$

input
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x)
```

output
```
int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^4)^3 (c+dx^4)^{3/2}} \, dx = \text{too large to display}$$

input
```
int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

output

```
(e**m*(x**m*sqrt(c + d*x**4)*a*d*m*x + 3*x**m*sqrt(c + d*x**4)*a*d*x + x**
m*sqrt(c + d*x**4)*b*c*m*x - 3*x**m*sqrt(c + d*x**4)*b*c*x - x**m*sqrt(c +
 d*x**4)*b*d*m*x**5 + 9*x**m*sqrt(c + d*x**4)*b*d*x**5 + int((x**m*sqrt(c
+ d*x**4)*x**12)/(a**4*c**2*d*m**2 + 4*a**4*c**2*d*m + 3*a**4*c**2*d + 2*a
**4*c*d**2*m**2*x**4 + 8*a**4*c*d**2*m*x**4 + 6*a**4*c*d**2*x**4 + a**4*d*
*3*m**2*x**8 + 4*a**4*d**3*m*x**8 + 3*a**4*d**3*x**8 + a**3*b*c**3*m**2 -
2*a**3*b*c**3*m - 3*a**3*b*c**3 + 5*a**3*b*c**2*d*m**2*x**4 + 8*a**3*b*c**
2*d*m*x**4 + 3*a**3*b*c**2*d*x**4 + 7*a**3*b*c*d**2*m**2*x**8 + 22*a**3*b*
c*d**2*m*x**8 + 15*a**3*b*c*d**2*x**8 + 3*a**3*b*d**3*m**2*x**12 + 12*a**3
*b*d**3*m*x**12 + 9*a**3*b*d**3*x**12 + 3*a**2*b**2*c**3*m**2*x**4 - 6*a**
2*b**2*c**3*m*x**4 - 9*a**2*b**2*c**3*x**4 + 9*a**2*b**2*c**2*d*m**2*x**8
- 9*a**2*b**2*c**2*d*x**8 + 9*a**2*b**2*c*d**2*m**2*x**12 + 18*a**2*b**2*c
*d**2*m*x**12 + 9*a**2*b**2*c*d**2*x**12 + 3*a**2*b**2*d**3*m**2*x**16 + 1
2*a**2*b**2*d**3*m*x**16 + 9*a**2*b**2*d**3*x**16 + 3*a*b**3*c**3*m**2*x**
8 - 6*a*b**3*c**3*m*x**8 - 9*a*b**3*c**3*x**8 + 7*a*b**3*c**2*d*m**2*x**12
 - 8*a*b**3*c**2*d*m*x**12 - 15*a*b**3*c**2*d*x**12 + 5*a*b**3*c*d**2*m**2
*x**16 + 2*a*b**3*c*d**2*m*x**16 - 3*a*b**3*c*d**2*x**16 + a*b**3*d**3*m**
2*x**20 + 4*a*b**3*d**3*m*x**20 + 3*a*b**3*d**3*x**20 + b**4*c**3*m**2*x**
12 - 2*b**4*c**3*m*x**12 - 3*b**4*c**3*x**12 + 2*b**4*c**2*d*m**2*x**16 -
4*b**4*c**2*d*m*x**16 - 6*b**4*c**2*d*x**16 + b**4*c*d**2*m**2*x**20 - ...
```

3.291 $\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 369

$$\begin{aligned}
&\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx \\
&= \frac{d(a^2d^2(45+14m+m^2)-3abcd(5+m)(13+m+4p)+3b^2c^2(117+m^2+88p+16p^2+m(22+8p)))}{b^3e(5+m+4p)(9+m+4p)(13+m+4p)} \\
&\quad -\frac{d^2(ad(9+m)-3bc(13+m+4p))(ex)^{5+m}\left(a+bx^4\right)^{1+p}}{b^2e^5(9+m+4p)(13+m+4p)}+\frac{d^3(ex)^{9+m}\left(a+bx^4\right)^{1+p}}{be^9(13+m+4p)} \\
&\quad +\frac{\left(\frac{c^3}{1+m}-\frac{ad(a^2d^2(45+14m+m^2)-3abcd(5+m)(13+m+4p)+3b^2c^2(117+m^2+88p+16p^2+m(22+8p)))}{b^3(5+m+4p)(9+m+4p)(13+m+4p)}\right)(ex)^{1+m}\left(a+bx^4\right)^p \left(1\right.}{e}
\end{aligned}$$

output
```
d*(a^2*d^2*(m^2+14*m+45)-3*a*b*c*d*(5+m)*(13+m+4*p)+3*b^2*c^2*(117+m^2+88*
p+16*p^2+m*(22+8*p)))*(e*x)^(1+m)*(b*x^4+a)^(p+1)/b^3/e/(5+m+4*p)/(9+m+4*p
)/(13+m+4*p)-d^2*(a*d*(9+m)-3*b*c*(13+m+4*p))*(e*x)^(5+m)*(b*x^4+a)^(p+1)/
b^2/e^5/(9+m+4*p)/(13+m+4*p)+d^3*(e*x)^(9+m)*(b*x^4+a)^(p+1)/b/e^9/(13+m+4
*p)+(c^3/(1+m)-a*d*(a^2*d^2*(m^2+14*m+45)-3*a*b*c*d*(5+m)*(13+m+4*p)+3*b^2
*c^2*(117+m^2+88*p+16*p^2+m*(22+8*p)))/b^3/(5+m+4*p)/(9+m+4*p)/(13+m+4*p))
*(e*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/e
/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 6.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.51

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 dx$$
$$= x(ex)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(\frac{c^3 \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{1+m} + dx^4 \left(\frac{3c^2 \text{Hypergeometric2F1}\left(\frac{5+m}{4},-p,\frac{9+m}{4},-\frac{bx^4}{a}\right)}{5+m} + dx^4 \left(\frac{3c \text{Hypergeometric2F1}\left(\frac{9+m}{4},-p,\frac{13+m}{4},-\frac{bx^4}{a}\right)}{9+m} + \frac{dx^4 \text{Hypergeometric2F1}\left(\frac{13+m}{4},-p,\frac{17+m}{4},-\frac{bx^4}{a}\right)}{13+m}\right.\right.\right.$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^3,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*((c^3*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4,
 -((b*x^4)/a)])/(1 + m) + d*x^4*((3*c^2*Hypergeometric2F1[(5 + m)/4, -p, (
9 + m)/4, -((b*x^4)/a)])/(5 + m) + d*x^4*((3*c*Hypergeometric2F1[(9 + m)/4
, -p, (13 + m)/4, -((b*x^4)/a)])/(9 + m) + (d*x^4*Hypergeometric2F1[(13 +
m)/4, -p, (17 + m)/4, -((b*x^4)/a)])/(13 + m)))))/(1 + (b*x^4)/a)^p
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {977, 25, 1051, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)^3 (ex)^m \left(a+bx^4\right)^p dx$$

$$\Big\downarrow \text{977}$$

$$\frac{\int -(ex)^m \left(bx^4+a\right)^p \left(dx^4+c\right) \left(d(ad(m+9)-bc(m+4p+21))x^4+c(ad(m+1)-bc(m+4p+13))\right) dx}{b(m+4p+13)}+$$
$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}$$

$\Big\downarrow$ 25

$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}-$$
$$\frac{\int (ex)^m \left(bx^4+a\right)^p \left(dx^4+c\right) \left(d(ad(m+9)-bc(m+4p+21))x^4+c(ad(m+1)-bc(m+4p+13))\right) dx}{b(m+4p+13)}$$

$\Big\downarrow$ 1051

$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}-$$
$$\frac{\frac{\int (ex)^m \left(bx^4+a\right)^p \left(c(4bc(p+2)(ad(m+1)-bc(m+4p+13))+(bc-ad)(m+1)(ad(m+9)-bc(m+4p+13)))-d\left(b^2\left(m^2+(8p+26)m+16p^2+104p+201\right.\right.\right.}{b(m+4p+9)}}{b(m+4p+1}$$

$\Big\downarrow$ 959

$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}-$$
$$\left(\frac{ad(m+1)\left(a^2d^2\left(m^2+14m+45\right)-2abcd\left(m^2+4m(p+5)+12p+75\right)+b^2c^2\left(m^2+m(8p+26)+16p^2+104p+201\right)\right)}{b(m+4p+5)}+c(4bc(p+2)(ad(m+1)-bc(m+4p+13))+(m\right.$$

$\Big\downarrow$ 889

$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}-$$
$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(\frac{ad(m+1)\left(a^2d^2\left(m^2+14m+45\right)-2abcd\left(m^2+4m(p+5)+12p+75\right)+b^2c^2\left(m^2+m(8p+26)+16p^2+104p+201\right)\right)}{b(m+4p+5)}+c(4bc(p+2)(ad(m+1\right.$$

$\Big\downarrow$ 888

$$\frac{d\left(c+dx^4\right)^2 (ex)^{m+1} \left(a+bx^4\right)^{p+1}}{be(m+4p+13)}-$$
$$\frac{(ex)^{m+1} \left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(\frac{ad(m+1)\left(a^2d^2\left(m^2+14m+45\right)-2abcd\left(m^2+4m(p+5)+12p+75\right)+b^2c^2\left(m^2+m(8p+26)+16p^2+104p+201\right)\right)}{b(m+4p+5)}+c(4bc(p+2)(ad\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^3,x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^4)^(1 + p)*(c + d*x^4)^2)/(b*e*(13 + m + 4*p)) -
 ((d*(a*d*(9 + m) - b*c*(21 + m + 4*p))*(e*x)^(1 + m)*(a + b*x^4)^(1 + p)*
(c + d*x^4))/(b*e*(9 + m + 4*p)) + (-((d*(a^2*d^2*(45 + 14*m + m^2) - 2*a*
b*c*d*(75 + m^2 + 12*p + 4*m*(5 + p)) + b^2*c^2*(201 + m^2 + 104*p + 16*p^
2 + m*(26 + 8*p)))*(e*x)^(1 + m)*(a + b*x^4)^(1 + p))/(b*e*(5 + m + 4*p)))
 + ((c*(4*b*c*(2 + p)*(a*d*(1 + m) - b*c*(13 + m + 4*p)) + (b*c - a*d)*(1
+ m)*(a*d*(9 + m) - b*c*(13 + m + 4*p))) + (a*d*(1 + m)*(a^2*d^2*(45 + 14*
m + m^2) - 2*a*b*c*d*(75 + m^2 + 12*p + 4*m*(5 + p)) + b^2*c^2*(201 + m^2
+ 104*p + 16*p^2 + m*(26 + 8*p))))/(b*(5 + m + 4*p)))*(e*x)^(1 + m)*(a + b
*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -((b*x^4)/a)])/(e*(1 +
 m)*(1 + (b*x^4)/a)^p))/(b*(9 + m + 4*p)))/(b*(13 + m + 4*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^3 dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int \left(dx^4+c\right)^3 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^12 + 3*c*d^2*x^8 + 3*c^2*d*x^4 + c^3)*(b*x^4 + a)^p*(e*x)^
m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(d*x**4+c)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int \left(dx^4+c\right)^3 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^3*(b*x^4 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int \left(dx^4+c\right)^3 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^3*(b*x^4 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \int (e\,x)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^3 \, dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^3,x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^3, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^3 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^3,x)
```

output

```
(e**m*(4*x**m*(a + b*x**4)**p*a**3*d**3*m**2*p*x + 56*x**m*(a + b*x**4)**p
*a**3*d**3*m*p*x + 180*x**m*(a + b*x**4)**p*a**3*d**3*p*x - 12*x**m*(a + b
*x**4)**p*a**2*b*c*d**2*m**2*p*x - 48*x**m*(a + b*x**4)**p*a**2*b*c*d**2*m
*p**2*x - 216*x**m*(a + b*x**4)**p*a**2*b*c*d**2*m*p*x - 240*x**m*(a + b*x
**4)**p*a**2*b*c*d**2*p**2*x - 780*x**m*(a + b*x**4)**p*a**2*b*c*d**2*p*x
- 4*x**m*(a + b*x**4)**p*a**2*b*d**3*m**2*p*x**5 - 16*x**m*(a + b*x**4)**p
*a**2*b*d**3*m*p**2*x**5 - 40*x**m*(a + b*x**4)**p*a**2*b*d**3*m*p*x**5 -
144*x**m*(a + b*x**4)**p*a**2*b*d**3*p**2*x**5 - 36*x**m*(a + b*x**4)**p*a
**2*b*d**3*p*x**5 + 12*x**m*(a + b*x**4)**p*a*b**2*c**2*d*m**2*p*x + 96*x*
*m*(a + b*x**4)**p*a*b**2*c**2*d*m*p**2*x + 264*x**m*(a + b*x**4)**p*a*b**
2*c**2*d*m*p*x + 192*x**m*(a + b*x**4)**p*a*b**2*c**2*d*p**3*x + 1056*x**m
*(a + b*x**4)**p*a*b**2*c**2*d*p**2*x + 1404*x**m*(a + b*x**4)**p*a*b**2*c
**2*d*p*x + 12*x**m*(a + b*x**4)**p*a*b**2*c*d**2*m**2*p*x**5 + 96*x**m*(a
 + b*x**4)**p*a*b**2*c*d**2*m*p**2*x**5 + 168*x**m*(a + b*x**4)**p*a*b**2*
c*d**2*m*p*x**5 + 192*x**m*(a + b*x**4)**p*a*b**2*c*d**2*p**3*x**5 + 672*x
**m*(a + b*x**4)**p*a*b**2*c*d**2*p**2*x**5 + 156*x**m*(a + b*x**4)**p*a*b
**2*c*d**2*p*x**5 + 4*x**m*(a + b*x**4)**p*a*b**2*d**3*m**2*p*x**9 + 32*x*
*m*(a + b*x**4)**p*a*b**2*d**3*m*p**2*x**9 + 24*x**m*(a + b*x**4)**p*a*b**
2*d**3*m*p*x**9 + 64*x**m*(a + b*x**4)**p*a*b**2*d**3*p**3*x**9 + 96*x**m*
(a + b*x**4)**p*a*b**2*d**3*p**2*x**9 + 20*x**m*(a + b*x**4)**p*a*b**2*...
```

3.292 $\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 207

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx$$
$$= -\frac{d(ad(5+m)-2bc(9+m+4p))(ex)^{1+m}\left(a+bx^4\right)^{1+p}}{b^2e(5+m+4p)(9+m+4p)} + \frac{d^2(ex)^{5+m}\left(a+bx^4\right)^{1+p}}{be^5(9+m+4p)}$$
$$+ \frac{\left(\frac{c^2}{1+m}+\frac{ad(ad(5+m)-2bc(9+m+4p))}{b^2(5+m+4p)(9+m+4p)}\right)(ex)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},\right.}{e}$$

output

```
-d*(a*d*(5+m)-2*b*c*(9+m+4*p))*(e*x)^(1+m)*(b*x^4+a)^(p+1)/b^2/e/(5+m+4*p)
/(9+m+4*p)+d^2*(e*x)^(5+m)*(b*x^4+a)^(p+1)/b/e^5/(9+m+4*p)+(c^2/(1+m)+a*d*
(a*d*(5+m)-2*b*c*(9+m+4*p))/b^2/(5+m+4*p)/(9+m+4*p))*(e*x)^(1+m)*(b*x^4+a)
^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/e/((1+b*x^4/a)^p)
```

Mathematica [A] (warning: unable to verify)

Time = 97.51 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.28

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx$$
$$= \frac{x(ex)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \left(a(13+m) \left(c^2(45+14m+m^2)+2cd(9+10m+m^2)\, x^4+d^2(5+6m\right.\right.}{}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(a*(13 + m)*(c^2*(45 + 14*m + m^2) + 2*c*d*(9 + 1
0*m + m^2)*x^4 + d^2*(5 + 6*m + m^2)*x^8)*Gamma[-p]*Hypergeometric2F1[(1 +
 m)/4, -p, (13 + m)/4, -((b*x^4)/a)] - 8*b*(1 + m)*x^4*(c + d*x^4)*(c*(7 +
 m) + d*(3 + m)*x^4)*Gamma[1 - p]*Hypergeometric2F1[(5 + m)/4, 1 - p, (17
+ m)/4, -((b*x^4)/a)] - 16*b*(1 + m)*x^4*(c + d*x^4)^2*Gamma[1 - p]*Hyperg
eometricPFQ[{2, 5/4 + m/4, 1 - p}, {1, 17/4 + m/4}, -((b*x^4)/a)]))/(a*(1
+ m)*(5 + m)*(9 + m)*(13 + m)*(1 + (b*x^4)/a)^p*Gamma[-p])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {964, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^4\right)^2 (ex)^m \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \text{ 964}$$

$$\frac{\int (ex)^m \left(bx^4+a\right)^p \left(bc^2(m+4p+9)-d(ad(m+5)-2bc(m+4p+9))x^4\right) dx}{b(m+4p+9)}+\frac{d^2(ex)^{m+5} \left(a+bx^4\right)^{p+1}}{be^5(m+4p+9)}$$

$$\Big\downarrow \text{ 959}$$

$$\frac{\left(\frac{ad(m+1)(ad(m+5)-2bc(m+4p+9))}{b(m+4p+5)}+bc^2(m+4p+9)\right)\int (ex)^m\left(bx^4+a\right)^p dx-\frac{d(ex)^{m+1}\left(a+bx^4\right)^{p+1}(ad(m+5)-2bc(m+4p+\ldots}{be(m+4p+5)}}{b(m+4p+9)}$$
$$\frac{d^2(ex)^{m+5}\left(a+bx^4\right)^{p+1}}{be^5(m+4p+9)}$$

$\big\downarrow$ 889

$$\frac{\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(\frac{ad(m+1)(ad(m+5)-2bc(m+4p+9))}{b(m+4p+5)}+bc^2(m+4p+9)\right)\int (ex)^m\left(\frac{bx^4}{a}+1\right)^p dx-\frac{d(ex)^{m+1}(a+\ldots}{\ldots}}{b(m+4p+9)}$$
$$\frac{d^2(ex)^{m+5}\left(a+bx^4\right)^{p+1}}{be^5(m+4p+9)}$$

$\big\downarrow$ 888

$$\frac{\frac{(ex)^{m+1}\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(\frac{ad(m+1)(ad(m+5)-2bc(m+4p+9))}{b(m+4p+5)}+bc^2(m+4p+9)\right)\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{e(m+1)}-\frac{d(ex)^{m+1}(a\ldots}{\ldots}}{b(m+4p+9)}$$
$$\frac{d^2(ex)^{m+5}\left(a+bx^4\right)^{p+1}}{be^5(m+4p+9)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^2,x]
```

output
```
(d^2*(e*x)^(5 + m)*(a + b*x^4)^(1 + p))/(b*e^5*(9 + m + 4*p)) + (-((d*(a*d
*(5 + m) - 2*b*c*(9 + m + 4*p))*(e*x)^(1 + m)*(a + b*x^4)^(1 + p))/(b*e*(5
 + m + 4*p))) + ((b*c^2*(9 + m + 4*p) + (a*d*(1 + m)*(a*d*(5 + m) - 2*b*c*
(9 + m + 4*p)))/(b*(5 + m + 4*p)))*(e*x)^(1 + m)*(a + b*x^4)^p*Hypergeomet
ric2F1[(1 + m)/4, -p, (5 + m)/4, -((b*x^4)/a)])/(e*(1 + m)*(1 + (b*x^4)/a)
^p))/(b*(9 + m + 4*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 964
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[d^2*(e*x)^(m + n + 1)*((a + b*x^n)^(p + 1)/(b*e^(n
+ 1)*(m + n*(p + 2) + 1))), x] + Simp[1/(b*(m + n*(p + 2) + 1))   Int[(e*x)
^m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) - d*(a*d*(m + n + 1) - 2*b*
c*(m + n*(p + 2) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^2 dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \int \left(dx^4+c\right)^2 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^8 + 2*c*d*x^4 + c^2)*(b*x^4 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \int \left(dx^4+c\right)^2 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)^2*(b*x^4 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \int \left(dx^4+c\right)^2 \left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)^2*(b*x^4 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \int (e\,x)^m \,\left(b\,x^4+a\right)^p \,\left(d\,x^4+c\right)^2 \,dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^2,x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^2, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^2 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^2,x)
```

output

```
(e**m*( - 4*x**m*(a + b*x**4)**p*a**2*d**2*m*p*x - 20*x**m*(a + b*x**4)**p
*a**2*d**2*p*x + 8*x**m*(a + b*x**4)**p*a*b*c*d*m*p*x + 32*x**m*(a + b*x**
4)**p*a*b*c*d*p**2*x + 72*x**m*(a + b*x**4)**p*a*b*c*d*p*x + 4*x**m*(a + b
*x**4)**p*a*b*d**2*m*p*x**5 + 16*x**m*(a + b*x**4)**p*a*b*d**2*p**2*x**5 +
 4*x**m*(a + b*x**4)**p*a*b*d**2*p*x**5 + x**m*(a + b*x**4)**p*b**2*c**2*m
**2*x + 8*x**m*(a + b*x**4)**p*b**2*c**2*m*p*x + 14*x**m*(a + b*x**4)**p*b
**2*c**2*m*x + 16*x**m*(a + b*x**4)**p*b**2*c**2*p**2*x + 56*x**m*(a + b*x
**4)**p*b**2*c**2*p*x + 45*x**m*(a + b*x**4)**p*b**2*c**2*x + 2*x**m*(a +
b*x**4)**p*b**2*c*d*m**2*x**5 + 16*x**m*(a + b*x**4)**p*b**2*c*d*m*p*x**5
+ 20*x**m*(a + b*x**4)**p*b**2*c*d*m*x**5 + 32*x**m*(a + b*x**4)**p*b**2*c
*d*p**2*x**5 + 80*x**m*(a + b*x**4)**p*b**2*c*d*p*x**5 + 18*x**m*(a + b*x*
*4)**p*b**2*c*d*x**5 + x**m*(a + b*x**4)**p*b**2*d**2*m**2*x**9 + 8*x**m*(
a + b*x**4)**p*b**2*d**2*m*p*x**9 + 6*x**m*(a + b*x**4)**p*b**2*d**2*m*x**
9 + 16*x**m*(a + b*x**4)**p*b**2*d**2*p**2*x**9 + 24*x**m*(a + b*x**4)**p*
b**2*d**2*p*x**9 + 5*x**m*(a + b*x**4)**p*b**2*d**2*x**9 + 4*int((x**m*(a
+ b*x**4)**p)/(a*m**3 + 12*a*m**2*p + 15*a*m**2 + 48*a*m*p**2 + 120*a*m*p
+ 59*a*m + 64*a*p**3 + 240*a*p**2 + 236*a*p + 45*a + b*m**3*x**4 + 12*b*m*
*2*p*x**4 + 15*b*m**2*x**4 + 48*b*m*p**2*x**4 + 120*b*m*p*x**4 + 59*b*m*x*
*4 + 64*b*p**3*x**4 + 240*b*p**2*x**4 + 236*b*p*x**4 + 45*b*x**4),x)*a**3*
d**2*m**5*p + 48*int((x**m*(a + b*x**4)**p)/(a*m**3 + 12*a*m**2*p + 15*...
```

3.293 $\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx = \frac{d(ex)^{1+m}\left(a+bx^4\right)^{1+p}}{be(5+m+4p)} + \frac{\left(\frac{c}{1+m}-\frac{ad}{b(5+m+4p)}\right)(ex)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{e}$$

output
```
d*(e*x)^(1+m)*(b*x^4+a)^(p+1)/b/e/(5+m+4*p)+(c/(1+m)-a*d/b/(5+m+4*p))*(e*x
)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/e/((1+
b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) dx = \frac{x(ex)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(c(5+m)\text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)+d(1+m)x^4\,\text{Hyp}\right.}{(1+m)(5+m)}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(c*(5 + m)*Hypergeometric2F1[(1 + m)/4, -p, (5 +
m)/4, -((b*x^4)/a)] + d*(1 + m)*x^4*Hypergeometric2F1[(5 + m)/4, -p, (9 +
m)/4, -((b*x^4)/a)]))/((1 + m)*(5 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^4)(ex)^m(a+bx^4)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\int (ex)^m(bx^4+a)^p\,dx+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\int (ex)^m\left(\frac{bx^4}{a}+1\right)^p dx+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+4p+5)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{e(m+1)}+\frac{d(ex)^{m+1}(a+bx^4)^{p+1}}{be(m+4p+5)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^4)^(1 + p))/(b*e*(5 + m + 4*p)) + ((c - (a*d*(1
+ m))/(b*(5 + m + 4*p)))*(e*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1
+ m)/4, -p, (5 + m)/4, -((b*x^4)/a)])/(e*(1 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right) dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \int \left(dx^4+c\right)\left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \int \left(dx^4+c\right)\left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \int \left(dx^4+c\right)\left(bx^4+a\right)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((d*x^4 + c)*(b*x^4 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \int (e\,x)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right) \, dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4),x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4), x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right) \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c),x)
```

output

```
(e**m*(4*x**m*(a + b*x**4)**p*a*d*p*x + x**m*(a + b*x**4)**p*b*c*m*x + 4*x
**m*(a + b*x**4)**p*b*c*p*x + 5*x**m*(a + b*x**4)**p*b*c*x + x**m*(a + b*x
**4)**p*b*d*m*x**5 + 4*x**m*(a + b*x**4)**p*b*d*p*x**5 + x**m*(a + b*x**4)
**p*b*d*x**5 - 4*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16
*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p*
*2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m**3*p - 32*int((x**m*(a + b*x
**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**
4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x
)*a**2*d*m**2*p**2 - 28*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a
*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 +
16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m**2*p - 64*int((x**m*(
a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m
**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*
x**4),x)*a**2*d*m*p**3 - 128*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p
+ 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x*
*4 + 16*b*p**2*x**4 + 24*b*p*x**4 + 5*b*x**4),x)*a**2*d*m*p**2 - 44*int((x
**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p + 6*a*m + 16*a*p**2 + 24*a*p + 5*a
+ b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*m*x**4 + 16*b*p**2*x**4 + 24*b*p*x**4 +
 5*b*x**4),x)*a**2*d*m*p - 64*int((x**m*(a + b*x**4)**p)/(a*m**2 + 8*a*m*p
 + 6*a*m + 16*a*p**2 + 24*a*p + 5*a + b*m**2*x**4 + 8*b*m*p*x**4 + 6*b*...
```

3.294 $\int (ex)^m \left(a+bx^4\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int (ex)^m \left(a+bx^4\right)^p \, dx = \frac{(ex)^{1+m} \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,\frac{5+m}{4},-\frac{bx^4}{a}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^p*hypergeom([-p, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/e/
(1+m)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (ex)^m \left(a+bx^4\right)^p \, dx = \frac{x(ex)^m \left(a+bx^4\right)^p \left(1+\frac{bx^4}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{4},-p,1+\frac{1+m}{4},-\frac{bx^4}{a}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, 1 + (1 + m)/4, -
((b*x^4)/a)])/((1 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^4}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} \left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{4},-p,\frac{m+5}{4},-\frac{bx^4}{a}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^p*Hypergeometric2F1[(1 + m)/4, -p, (5 + m)/4, -
((b*x^4)/a)])/(e*(1 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p dx$$

input
```
int((e*x)^m*(b*x^4+a)^p,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \,dx=\int \left(bx^4+a\right)^p(ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 85.95 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (ex)^m \left(a+bx^4\right)^p\,dx = \frac{a^p e^m x^{m+1}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-p,\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p,x)
```

output
```
a**p*e**m*x**(m + 1)*gamma(m/4 + 1/4)*hyper((-p, m/4 + 1/4), (m/4 + 5/4,),
 b*x**4*exp_polar(I*pi)/a)/(4*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p\,dx = \int (bx^4+a)^p (ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p\,dx = \int (bx^4+a)^p (ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \, dx = \int (e\,x)^m \left(b\,x^4+a\right)^p \, dx$$

input
```
int((e*x)^m*(a + b*x^4)^p,x)
```

output
```
int((e*x)^m*(a + b*x^4)^p, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \, dx$$
$$= \frac{e^m \left(x^m (b\,x^4+a)^p\,x + 4\left(\int \frac{x^m (b\,x^4+a)^p}{bm\,x^4+4bp\,x^4+b\,x^4+am+4ap+a}dx\right) amp + 16\left(\int \frac{x^m (b\,x^4+a)^p}{bm\,x^4+4bp\,x^4+b\,x^4+am+4ap+a}dx\right) a\,p^2 -}{m+4p+1}$$

input
```
int((e*x)^m*(b*x^4+a)^p,x)
```

output
```
(e**m*(x**m*(a + b*x**4)**p*x + 4*int((x**m*(a + b*x**4)**p)/(a*m + 4*a*p
+ a + b*m*x**4 + 4*b*p*x**4 + b*x**4),x)*a*m*p + 16*int((x**m*(a + b*x**4)
**p)/(a*m + 4*a*p + a + b*m*x**4 + 4*b*p*x**4 + b*x**4),x)*a*p**2 + 4*int(
(x**m*(a + b*x**4)**p)/(a*m + 4*a*p + a + b*m*x**4 + 4*b*p*x**4 + b*x**4),
x)*a*p))/(m + 4*p + 1)
```

3.295 $\int \frac{(ex)^m(a+bx^4)^p}{c+dx^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{(ex)^m\left(a+bx^4\right)^p}{c+dx^4}\,dx = \frac{(ex)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{4},-p,1,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ce(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^p*AppellF1(1/4+1/4*m,-p,1,5/4+1/4*m,-b*x^4/a,-d*x^4/
c)/c/e/(1+m)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m\left(a+bx^4\right)^p}{c+dx^4}\,dx = \frac{x(ex)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{4},-p,1,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^p)/(c + d*x^4),x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*AppellF1[(1 + m)/4, -p, 1, (5 + m)/4, -((b*x^4)/a
), -((d*x^4)/c)])/(c*(1 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4} \, dx$$

$$\Big\downarrow \; 1013$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \frac{(ex)^m \left(\frac{bx^4}{a}+1\right)^p}{dx^4+c}dx$$

$$\Big\downarrow \; 1012$$

$$\frac{(ex)^{m+1} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{4},-p,1,\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ce(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^4)^p)/(c + d*x^4),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^p*AppellF1[(1 + m)/4, -p, 1, (5 + m)/4, -((b*x^
4)/a), -((d*x^4)/c)])/(c*e*(1 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4 + a)^p}{d\,x^4 + c}dx$$

input
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x)
```

output
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a + bx^4)^p}{c + dx^4}\,dx = \int \frac{(bx^4 + a)^p (ex)^m}{dx^4 + c}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(e*x)^m/(d*x^4 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(bx^4+a)^p (ex)^m}{dx^4+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m/(d*x^4 + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(bx^4+a)^p (ex)^m}{dx^4+c}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m/(d*x^4 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^p}{d\,x^4+c}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^p)/(c + d*x^4),x)
```

output
```
int(((e*x)^m*(a + b*x^4)^p)/(c + d*x^4), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{c+dx^4}\,dx = e^m \left(\int \frac{x^m (b\,x^4+a)^p}{d\,x^4+c}dx\right)$$

input
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c),x)
```

output
```
e**m*int((x**m*(a + b*x**4)**p)/(c + d*x**4),x)
```

3.296 $\int \frac{(ex)^m(a+bx^4)^p}{(c+dx^4)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int \frac{(ex)^m\left(a+bx^4\right)^p}{\left(c+dx^4\right)^2}\,dx = \frac{(ex)^{1+m}\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{4},-p,2,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^p*AppellF1(1/4+1/4*m,-p,2,5/4+1/4*m,-b*x^4/a,-d*x^4/
c)/c^2/e/(1+m)/((1+b*x^4/a)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(ex)^m\left(a+bx^4\right)^p}{\left(c+dx^4\right)^2}\,dx = \frac{x(ex)^m\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{4},-p,2,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^4)^p)/(c + d*x^4)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*AppellF1[(1 + m)/4, -p, 2, (5 + m)/4, -((b*x^4)/a
), -((d*x^4)/c)])/(c^2*(1 + m)*(1 + (b*x^4)/a)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2} dx$$

$$\Big\downarrow \text{ 1013}$$

$$(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int \frac{(ex)^m \left(\frac{bx^4}{a}+1\right)^p}{(dx^4+c)^2} dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(ex)^{m+1} (a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{4},-p,2,\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2 e(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^4)^p)/(c + d*x^4)^2,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^p*AppellF1[(1 + m)/4, -p, 2, (5 + m)/4, -((b*x^
4)/a), -((d*x^4)/c)])/(c^2*e*(1 + m)*(1 + (b*x^4)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m (b\,x^4+a)^p}{(d\,x^4+c)^2}dx$$

input
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2}\,dx=\int \frac{(bx^4+a)^p(ex)^m}{(dx^4+c)^2}\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p/(d*x**4+c)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2} \, dx = \int \frac{(bx^4+a)^p (ex)^m}{(dx^4+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m/(d*x^4 + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2} \, dx = \int \frac{(bx^4+a)^p (ex)^m}{(dx^4+c)^2} \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(e*x)^m/(d*x^4 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2}\,dx = \int \frac{(e\,x)^m\,(b\,x^4+a)^p}{(d\,x^4+c)^2}\,dx$$

input
```
int(((e*x)^m*(a + b*x^4)^p)/(c + d*x^4)^2,x)
```

output
```
int(((e*x)^m*(a + b*x^4)^p)/(c + d*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^4)^p}{(c+dx^4)^2}\,dx = e^m \left(\int \frac{x^m (b\,x^4+a)^p}{d^2x^8+2cd\,x^4+c^2}dx\right)$$

input
```
int((e*x)^m*(b*x^4+a)^p/(d*x^4+c)^2,x)
```

output
```
e**m*int((x**m*(a + b*x**4)**p)/(c**2 + 2*c*d*x**4 + d**2*x**8),x)
```

3.297 $\int (ex)^m (a+bx^4)^p (c+dx^4)^p \, dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int (ex)^m (a+bx^4)^p (c+dx^4)^p \, dx = \frac{(ex)^{1+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^p \left(1+\frac{dx^4}{c}\right)^{-p} \text{AppellF1}\left(\frac{1+m}{4},-p,-p,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^p*(d*x^4+c)^p*AppellF1(1/4+1/4*m,-p,-p,5/4+1/4*m,-b*
x^4/a,-d*x^4/c)/e/(1+m)/((1+b*x^4/a)^p)/((1+d*x^4/c)^p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int (ex)^m (a+bx^4)^p (c+dx^4)^p \, dx = \frac{x(ex)^m (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^p \left(1+\frac{dx^4}{c}\right)^{-p} \text{AppellF1}\left(\frac{1+m}{4},-p,-p,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^p,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^p*AppellF1[(1 + m)/4, -p, -p, (5 + m)
/4, -((b*x^4)/a), -((d*x^4)/c)])/((1 + m)*(1 + (b*x^4)/a)^p*(1 + (d*x^4)/c
)^p)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^4}{a}+1\right)^p \left(dx^4+c\right)^p dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^p \left(\frac{dx^4}{c}+1\right)^{-p} \int (ex)^m \left(\frac{bx^4}{a}+1\right)^p \left(\frac{dx^4}{c}+1\right)^p dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(ex)^{m+1} \left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^p \left(\frac{dx^4}{c}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{4},-p,-p,\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^p,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^p*(c + d*x^4)^p*AppellF1[(1 + m)/4, -p, -p, (5
+ m)/4, -((b*x^4)/a), -((d*x^4)/c)])/(e*(1 + m)*(1 + (b*x^4)/a)^p*(1 + (d*
x^4)/c)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m \left(b\,x^4+a\right)^p \left(d\,x^4+c\right)^p dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x)
```

Fricas [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \,dx=\int (bx^4+a)^p (dx^4+c)^p (ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(d*x^4 + c)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x***4+a)**p*(d*x***4+c)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx = \int (bx^4+a)^p(dx^4+c)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx = \int (bx^4+a)^p(dx^4+c)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx = \int (e\,x)^m\,\left(b\,x^4+a\right)^p\,\left(d\,x^4+c\right)^p \,dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^p,x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^p, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^p \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^p,x)
```

output

```
(e**m*(x**m*(c + d*x**4)**p*(a + b*x**4)**p*x + 4*int((x**m*(c + d*x**4)**
p*(a + b*x**4)**p*x**4)/(a*c*m + 8*a*c*p + a*c + a*d*m*x**4 + 8*a*d*p*x**4
 + a*d*x**4 + b*c*m*x**4 + 8*b*c*p*x**4 + b*c*x**4 + b*d*m*x**8 + 8*b*d*p*
x**8 + b*d*x**8),x)*a*d*m*p + 32*int((x**m*(c + d*x**4)**p*(a + b*x**4)**p
*x**4)/(a*c*m + 8*a*c*p + a*c + a*d*m*x**4 + 8*a*d*p*x**4 + a*d*x**4 + b*c
*m*x**4 + 8*b*c*p*x**4 + b*c*x**4 + b*d*m*x**8 + 8*b*d*p*x**8 + b*d*x**8),
x)*a*d*p**2 + 4*int((x**m*(c + d*x**4)**p*(a + b*x**4)**p*x**4)/(a*c*m + 8
*a*c*p + a*c + a*d*m*x**4 + 8*a*d*p*x**4 + a*d*x**4 + b*c*m*x**4 + 8*b*c*p
*x**4 + b*c*x**4 + b*d*m*x**8 + 8*b*d*p*x**8 + b*d*x**8),x)*a*d*p + 4*int(
(x**m*(c + d*x**4)**p*(a + b*x**4)**p*x**4)/(a*c*m + 8*a*c*p + a*c + a*d*m
*x**4 + 8*a*d*p*x**4 + a*d*x**4 + b*c*m*x**4 + 8*b*c*p*x**4 + b*c*x**4 + b
*d*m*x**8 + 8*b*d*p*x**8 + b*d*x**8),x)*b*c*m*p + 32*int((x**m*(c + d*x**4
)**p*(a + b*x**4)**p*x**4)/(a*c*m + 8*a*c*p + a*c + a*d*m*x**4 + 8*a*d*p*x
**4 + a*d*x**4 + b*c*m*x**4 + 8*b*c*p*x**4 + b*c*x**4 + b*d*m*x**8 + 8*b*d
*p*x**8 + b*d*x**8),x)*b*c*p**2 + 4*int((x**m*(c + d*x**4)**p*(a + b*x**4)
**p*x**4)/(a*c*m + 8*a*c*p + a*c + a*d*m*x**4 + 8*a*d*p*x**4 + a*d*x**4 +
b*c*m*x**4 + 8*b*c*p*x**4 + b*c*x**4 + b*d*m*x**8 + 8*b*d*p*x**8 + b*d*x**
8),x)*b*c*p + 8*int((x**m*(c + d*x**4)**p*(a + b*x**4)**p)/(a*c*m + 8*a*c*
p + a*c + a*d*m*x**4 + 8*a*d*p*x**4 + a*d*x**4 + b*c*m*x**4 + 8*b*c*p*x**4
 + b*c*x**4 + b*d*m*x**8 + 8*b*d*p*x**8 + b*d*x**8),x)*a*c*m*p + 64*int...
```

3.298 $\int x^m(2+bx^4)^p(3+dx^4)^q\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int x^m(2+bx^4)^p(3+dx^4)^q\,dx=\frac{2^p3^qx^{1+m}\operatorname{AppellF1}\left(\frac{1+m}{4},-p,-q,\frac{5+m}{4},-\frac{bx^4}{2},-\frac{dx^4}{3}\right)}{1+m}$$

output
```
2^p*3^q*x^(1+m)*AppellF1(1/4+1/4*m,-p,-q,5/4+1/4*m,-1/2*b*x^4,-1/3*d*x^4)/
(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int x^m(2+bx^4)^p(3+dx^4)^q\,dx=\frac{2^p3^qx^{1+m}\operatorname{AppellF1}\left(\frac{1+m}{4},-p,-q,1+\frac{1+m}{4},-\frac{bx^4}{2},-\frac{dx^4}{3}\right)}{1+m}$$

input
```
Integrate[x^m*(2 + b*x^4)^p*(3 + d*x^4)^q,x]
```

output
```
(2^p*3^q*x^(1 + m)*AppellF1[(1 + m)/4, -p, -q, 1 + (1 + m)/4, -1/2*(b*x^4)
, -1/3*(d*x^4)])/(1 + m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (bx^4+2)^p (dx^4+3)^q \, dx$$

$$\Big\downarrow \ 1012$$

$$\frac{2^p 3^q x^{m+1} \operatorname{AppellF1}\left(\frac{m+1}{4},-p,-q,\frac{m+5}{4},-\frac{bx^4}{2},-\frac{dx^4}{3}\right)}{m+1}$$

input
```
Int[x^m*(2 + b*x^4)^p*(3 + d*x^4)^q,x]
```

output
```
(2^p*3^q*x^(1 + m)*AppellF1[(1 + m)/4, -p, -q, (5 + m)/4, -1/2*(b*x^4), -1
/3*(d*x^4)])/(1 + m)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Maple [F]

$$\int x^{m} \left(b\,x^{4}+2\right)^{p} \left(d\,x^{4}+3\right)^{q} dx$$

input
```
int(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x)
```

output
```
int(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x)
```

Fricas [F]

$$\int x^m \left(2+bx^4\right)^p \left(3+dx^4\right)^q \,dx = \int \left(bx^4+2\right)^p \left(dx^4+3\right)^q x^m \,dx$$

input
```
integrate(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x, algorithm="fricas")
```

output
```
integral((b*x^4 + 2)^p*(d*x^4 + 3)^q*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \left(2+bx^4\right)^p \left(3+dx^4\right)^q \,dx = \text{Timed out}$$

input
```
integrate(x**m*(b*x**4+2)**p*(d*x**4+3)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \left(2+bx^4\right)^p \left(3+dx^4\right)^q \, dx = \int \left(bx^4+2\right)^p \left(dx^4+3\right)^q x^m \, dx$$

input
```
integrate(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + 2)^p*(d*x^4 + 3)^q*x^m, x)
```

Giac [F]

$$\int x^m \left(2+bx^4\right)^p \left(3+dx^4\right)^q \, dx = \int \left(bx^4+2\right)^p \left(dx^4+3\right)^q x^m \, dx$$

input
```
integrate(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x, algorithm="giac")
```

output
```
integrate((b*x^4 + 2)^p*(d*x^4 + 3)^q*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(2+bx^4\right)^p \left(3+dx^4\right)^q \, dx = \int x^m \left(b\,x^4+2\right)^p \left(d\,x^4+3\right)^q \, dx$$

input
```
int(x^m*(b*x^4 + 2)^p*(d*x^4 + 3)^q,x)
```

output
```
int(x^m*(b*x^4 + 2)^p*(d*x^4 + 3)^q, x)
```

Reduce [F]

$$\int x^m (2+bx^4)^p (3+dx^4)^q \, dx = \text{too large to display}$$

input
```
int(x^m*(b*x^4+2)^p*(d*x^4+3)^q,x)
```

output
```
(x**m*(d*x**4 + 3)**q*(b*x**4 + 2)**p*x + 12*int((x**m*(d*x**4 + 3)**q*(b*
x**4 + 2)**p*x**4)/(b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8 +
3*b*m*x**4 + 12*b*p*x**4 + 12*b*q*x**4 + 3*b*x**4 + 2*d*m*x**4 + 8*d*p*x**
4 + 8*d*q*x**4 + 2*d*x**4 + 6*m + 24*p + 24*q + 6),x)*b*m*q + 48*int((x**m
*(d*x**4 + 3)**q*(b*x**4 + 2)**p*x**4)/(b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*
q*x**8 + b*d*x**8 + 3*b*m*x**4 + 12*b*p*x**4 + 12*b*q*x**4 + 3*b*x**4 + 2*
d*m*x**4 + 8*d*p*x**4 + 8*d*q*x**4 + 2*d*x**4 + 6*m + 24*p + 24*q + 6),x)*
b*p*q + 48*int((x**m*(d*x**4 + 3)**q*(b*x**4 + 2)**p*x**4)/(b*d*m*x**8 + 4
*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8 + 3*b*m*x**4 + 12*b*p*x**4 + 12*b*q*
x**4 + 3*b*x**4 + 2*d*m*x**4 + 8*d*p*x**4 + 8*d*q*x**4 + 2*d*x**4 + 6*m +
24*p + 24*q + 6),x)*b*q**2 + 12*int((x**m*(d*x**4 + 3)**q*(b*x**4 + 2)**p*
x**4)/(b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8 + 3*b*m*x**4 +
12*b*p*x**4 + 12*b*q*x**4 + 3*b*x**4 + 2*d*m*x**4 + 8*d*p*x**4 + 8*d*q*x**
4 + 2*d*x**4 + 6*m + 24*p + 24*q + 6),x)*b*q + 8*int((x**m*(d*x**4 + 3)**q
*(b*x**4 + 2)**p*x**4)/(b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**
8 + 3*b*m*x**4 + 12*b*p*x**4 + 12*b*q*x**4 + 3*b*x**4 + 2*d*m*x**4 + 8*d*p
*x**4 + 8*d*q*x**4 + 2*d*x**4 + 6*m + 24*p + 24*q + 6),x)*d*m*p + 32*int((
x**m*(d*x**4 + 3)**q*(b*x**4 + 2)**p*x**4)/(b*d*m*x**8 + 4*b*d*p*x**8 + 4*
b*d*q*x**8 + b*d*x**8 + 3*b*m*x**4 + 12*b*p*x**4 + 12*b*q*x**4 + 3*b*x**4
+ 2*d*m*x**4 + 8*d*p*x**4 + 8*d*q*x**4 + 2*d*x**4 + 6*m + 24*p + 24*q +...
```

3.299 $\int (ex)^m (a+bx^4)^p (c+dx^4)^q \, dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int (ex)^m (a+bx^4)^p (c+dx^4)^q \, dx = \frac{(ex)^{1+m}(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1+m}{4},-p,-q,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4+1/4*m,-p,-q,5/4+1/4*m,-b*
x^4/a,-d*x^4/c)/e/(1+m)/((1+b*x^4/a)^p)/((1+d*x^4/c)^q)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int (ex)^m (a+bx^4)^p (c+dx^4)^q \, dx = \frac{x(ex)^m (a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} (c+dx^4)^q \left(1+\frac{dx^4}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1+m}{4},-p,-q,\frac{5+m}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
(x*(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[(1 + m)/4, -p, -q, (5 + m)
/4, -((b*x^4)/a), -((d*x^4)/c)])/((1 + m)*(1 + (b*x^4)/a)^p*(1 + (d*x^4)/c
)^q)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^4}{a}+1\right)^p \left(dx^4+c\right)^q dx$$

$$\Big\downarrow \text{1013}$$

$$\left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^q \left(\frac{dx^4}{c}+1\right)^{-q} \int (ex)^m \left(\frac{bx^4}{a}+1\right)^p \left(\frac{dx^4}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1} \left(a+bx^4\right)^p \left(\frac{bx^4}{a}+1\right)^{-p} \left(c+dx^4\right)^q \left(\frac{dx^4}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{4},-p,-q,\frac{m+5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^4)^p*(c + d*x^4)^q,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[(1 + m)/4, -p, -q, (5
+ m)/4, -((b*x^4)/a), -((d*x^4)/c)])/(e*(1 + m)*(1 + (b*x^4)/a)^p*(1 + (d*
x^4)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m\,(b\,x^4+a)^p\,(d\,x^4+c)^q\,dx$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x)
```

output
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x)
```

Fricas [F]

$$\int (ex)^m\,(a+bx^4)^p\,(c+dx^4)^q\,dx=\int (bx^4+a)^p(dx^4+c)^q(ex)^m\,dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p*(d*x^4 + c)^q*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**4+a)**p*(d*x**4+c)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int \left(bx^4+a\right)^p \left(dx^4+c\right)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p*(d*x^4 + c)^q*(e*x)^m, x)
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{Timed out}$$

input
```
integrate((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \int (e\,x)^m \,\left(b\,x^4+a\right)^p \,\left(d\,x^4+c\right)^q \, dx$$

input
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^q,x)
```

output
```
int((e*x)^m*(a + b*x^4)^p*(c + d*x^4)^q, x)
```

Reduce [F]

$$\int (ex)^m \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(b*x^4+a)^p*(d*x^4+c)^q,x)
```

output

```
(e**m*(x**m*(c + d*x**4)**q*(a + b*x**4)**p*x + 4*int((x**m*(c + d*x**4)**
q*(a + b*x**4)**p*x**4)/(a*c*m + 4*a*c*p + 4*a*c*q + a*c + a*d*m*x**4 + 4*
a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + b*c*m*x**4 + 4*b*c*p*x**4 + 4*b*c*q
*x**4 + b*c*x**4 + b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)
*a*d*m*p + 16*int((x**m*(c + d*x**4)**q*(a + b*x**4)**p*x**4)/(a*c*m + 4*a
*c*p + 4*a*c*q + a*c + a*d*m*x**4 + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4
 + b*c*m*x**4 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 + b*d*m*x**8 + 4*b*
d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*a*d*p**2 + 16*int((x**m*(c + d*x**4
)**q*(a + b*x**4)**p*x**4)/(a*c*m + 4*a*c*p + 4*a*c*q + a*c + a*d*m*x**4 +
 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + b*c*m*x**4 + 4*b*c*p*x**4 + 4*b*
c*q*x**4 + b*c*x**4 + b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8)
,x)*a*d*p*q + 4*int((x**m*(c + d*x**4)**q*(a + b*x**4)**p*x**4)/(a*c*m + 4
*a*c*p + 4*a*c*q + a*c + a*d*m*x**4 + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x*
*4 + b*c*m*x**4 + 4*b*c*p*x**4 + 4*b*c*q*x**4 + b*c*x**4 + b*d*m*x**8 + 4*
b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x)*a*d*p + 4*int((x**m*(c + d*x**4)*
*q*(a + b*x**4)**p*x**4)/(a*c*m + 4*a*c*p + 4*a*c*q + a*c + a*d*m*x**4 + 4
*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*x**4 + b*c*m*x**4 + 4*b*c*p*x**4 + 4*b*c*
q*x**4 + b*c*x**4 + b*d*m*x**8 + 4*b*d*p*x**8 + 4*b*d*q*x**8 + b*d*x**8),x
)*b*c*m*q + 16*int((x**m*(c + d*x**4)**q*(a + b*x**4)**p*x**4)/(a*c*m + 4*
a*c*p + 4*a*c*q + a*c + a*d*m*x**4 + 4*a*d*p*x**4 + 4*a*d*q*x**4 + a*d*...
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
      return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:  #isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:  #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file