Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.4/56-1.1.3.4-c

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**610**]. This is test number [56].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.67 (608)	0.33 (2)
Mathematica	99.51 (607)	0.49 (3)
Sympy	50.66 (309)	49.34 (301)
Fricas	49.02 (299)	50.98 (311)
Maple	42.95 (262)	57.05 (348)
Reduce	36.89 (225)	63.11 (385)
Mupad	32.13 (196)	67.87 (414)
Maxima	31.97 (195)	68.03 (415)
Giac	31.15 (190)	68.85 (420)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

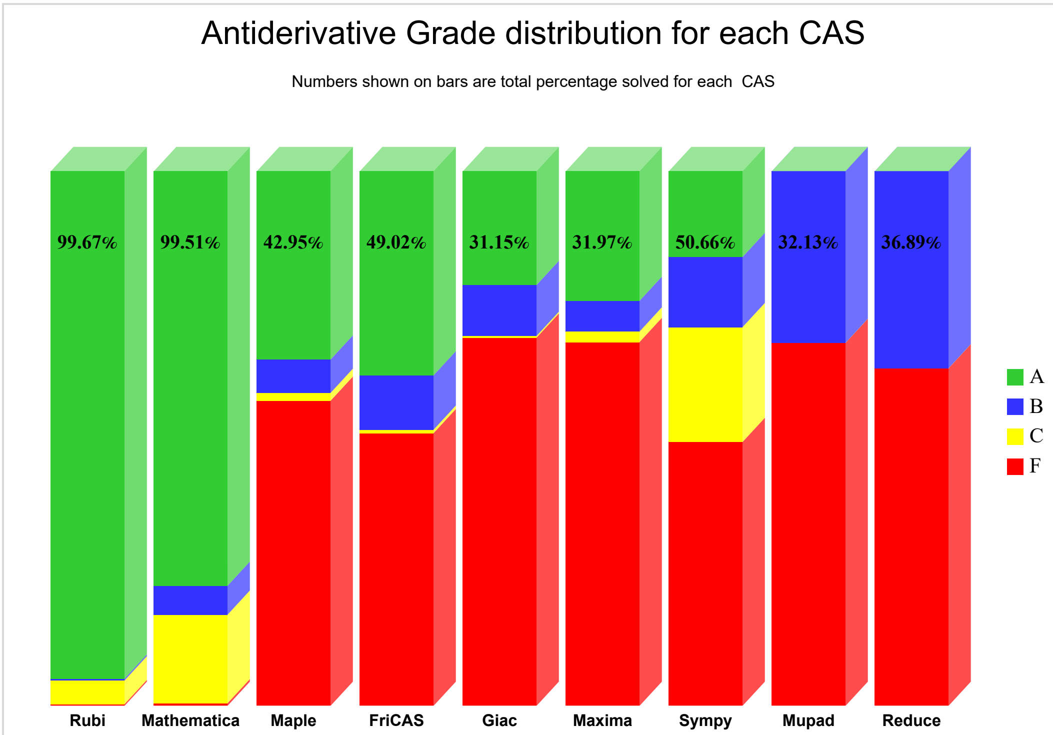
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

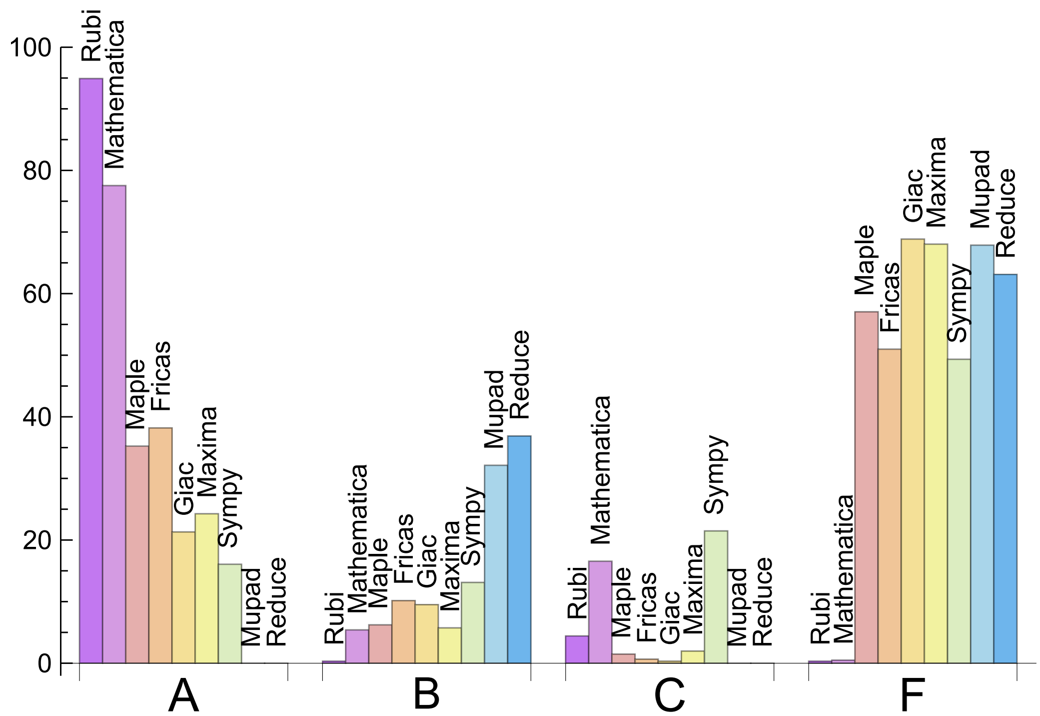
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.918	0.328	4.426	0.328
Mathematica	77.541	5.410	16.557	0.492
Fricas	38.197	10.164	0.656	50.984
Maple	35.246	6.230	1.475	57.049
Maxima	24.262	5.738	1.967	68.033
Giac	21.311	9.508	0.328	68.852
Sympy	16.066	13.115	21.475	49.344
Mupad	0.000	32.131	0.000	67.869
Reduce	0.000	36.885	0.000	63.115

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Sympy	301	46.18	37.21	16.61
Fricas	311	64.31	12.22	23.47
Maple	348	100.00	0.00	0.00
Reduce	385	100.00	0.00	0.00
Mupad	414	0.00	100.00	0.00
Maxima	415	93.49	0.00	6.51
Giac	420	87.14	1.43	11.43

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.11
Reduce	0.22
Giac	0.22
Rubi	0.53
Mathematica	1.98
Maple	2.22
Mupad	4.45
Sympy	20.10

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	99.22	1.69	81.00	1.14
Mathematica	111.78	1.18	97.00	0.97
Reduce	118.21	1.67	80.00	1.40
Rubi	154.18	1.04	97.00	1.00
Maple	155.16	1.73	74.00	0.95
Mupad	185.40	1.98	68.00	1.00
Giac	187.60	2.22	104.00	1.29
Fricas	202.56	2.19	133.00	1.70
Sympy	351.85	3.90	144.00	1.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

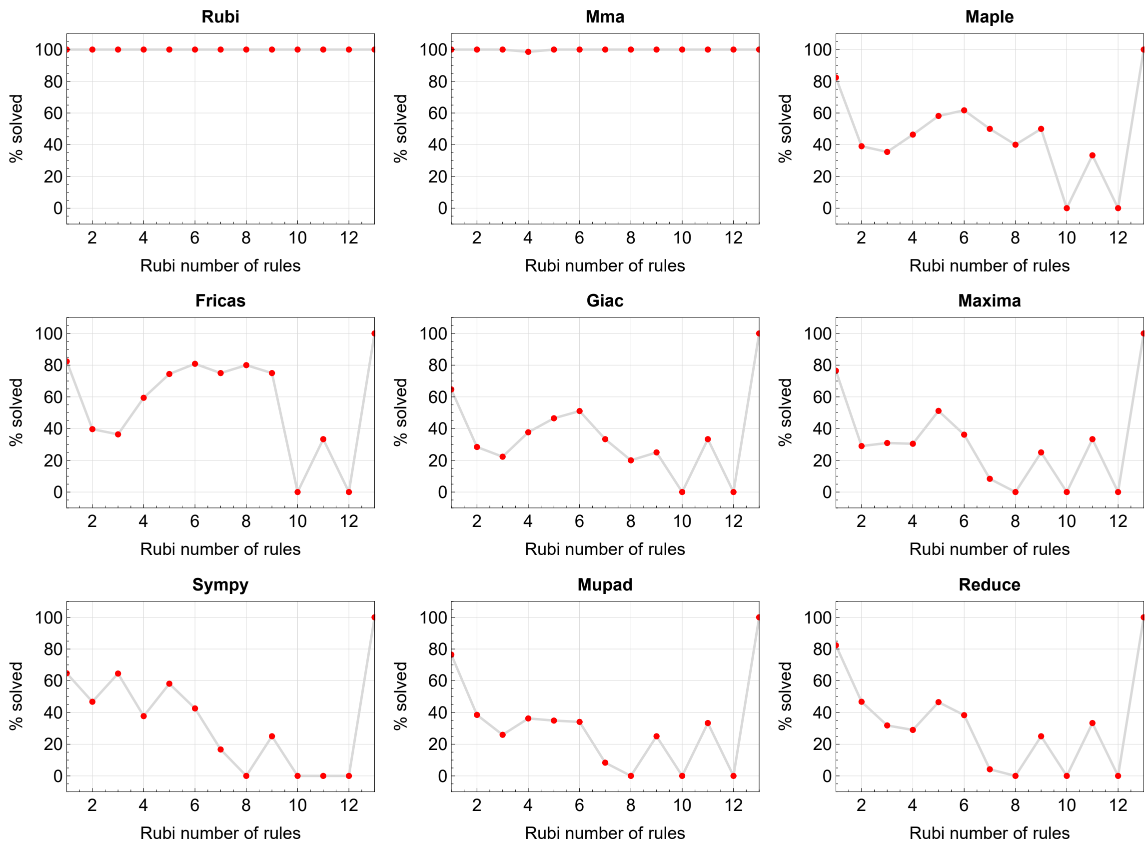


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

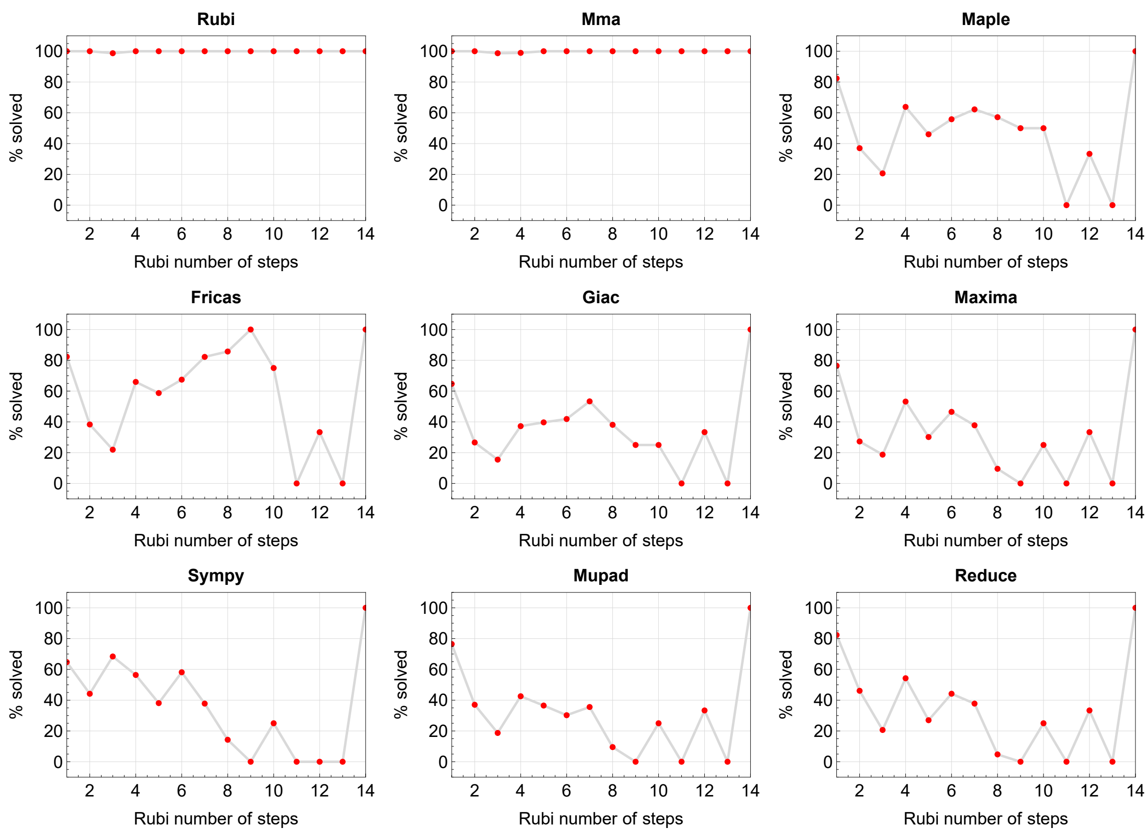


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

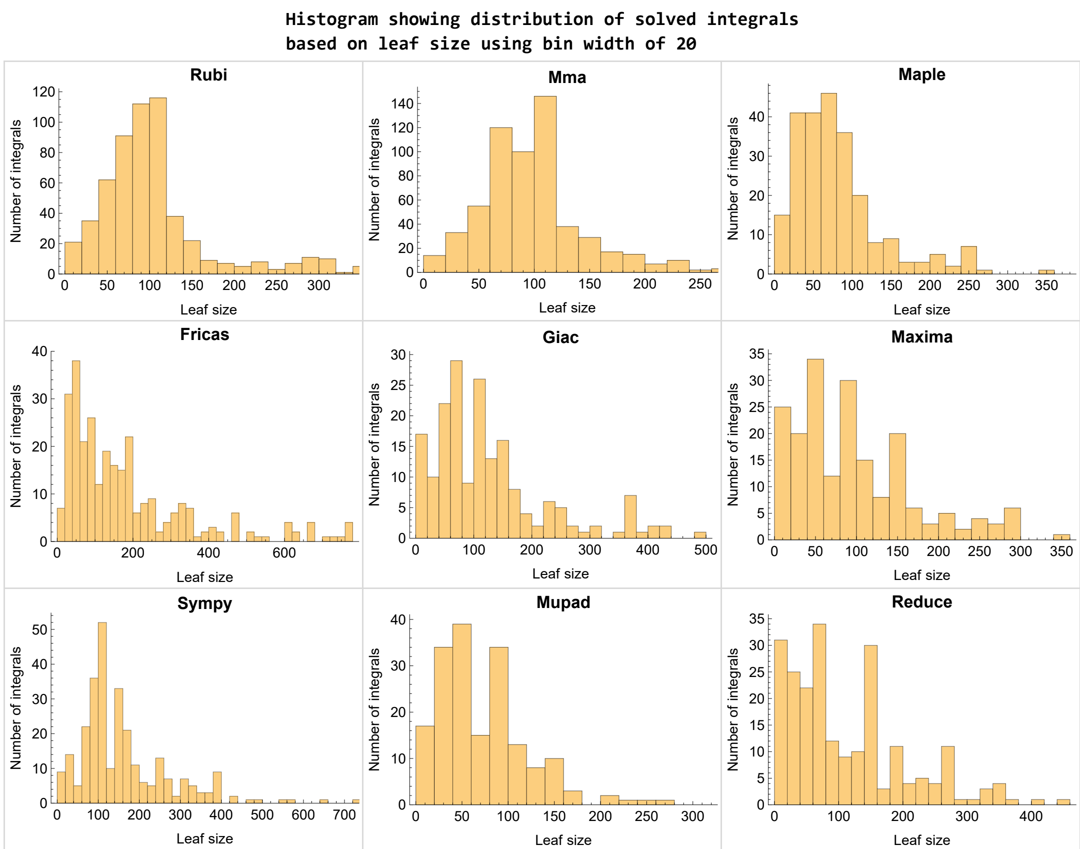


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

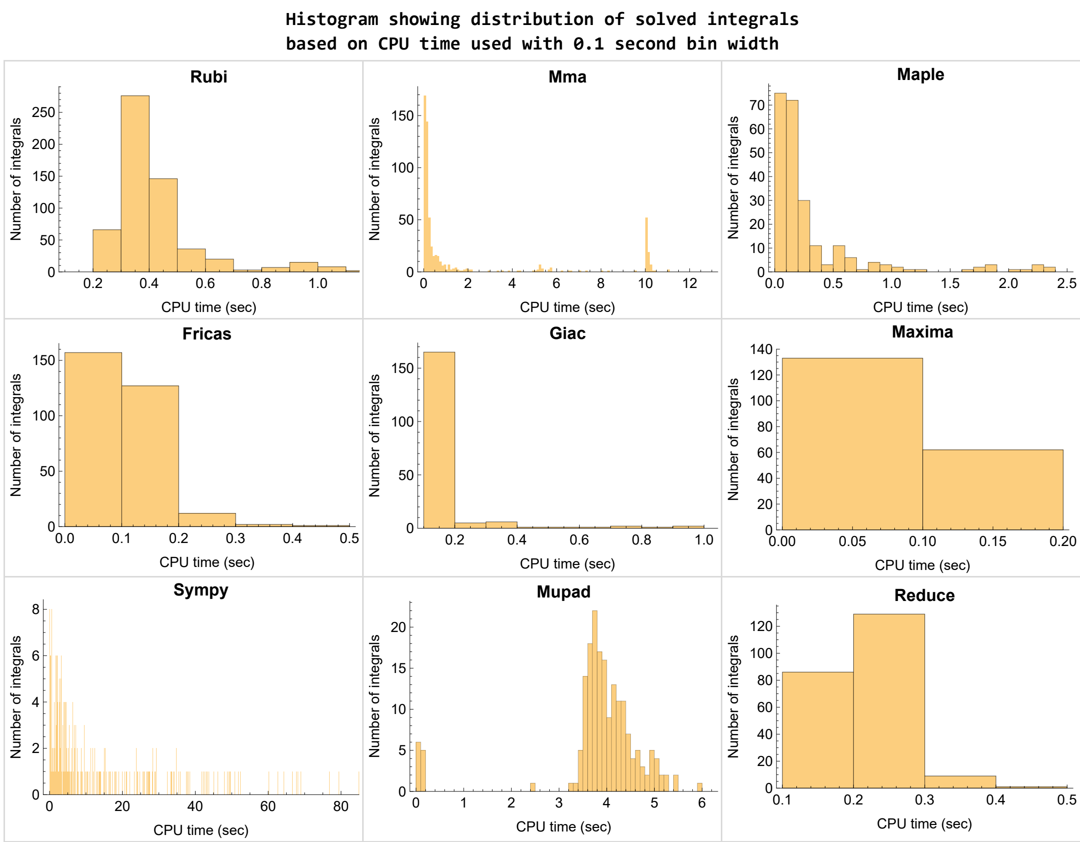


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

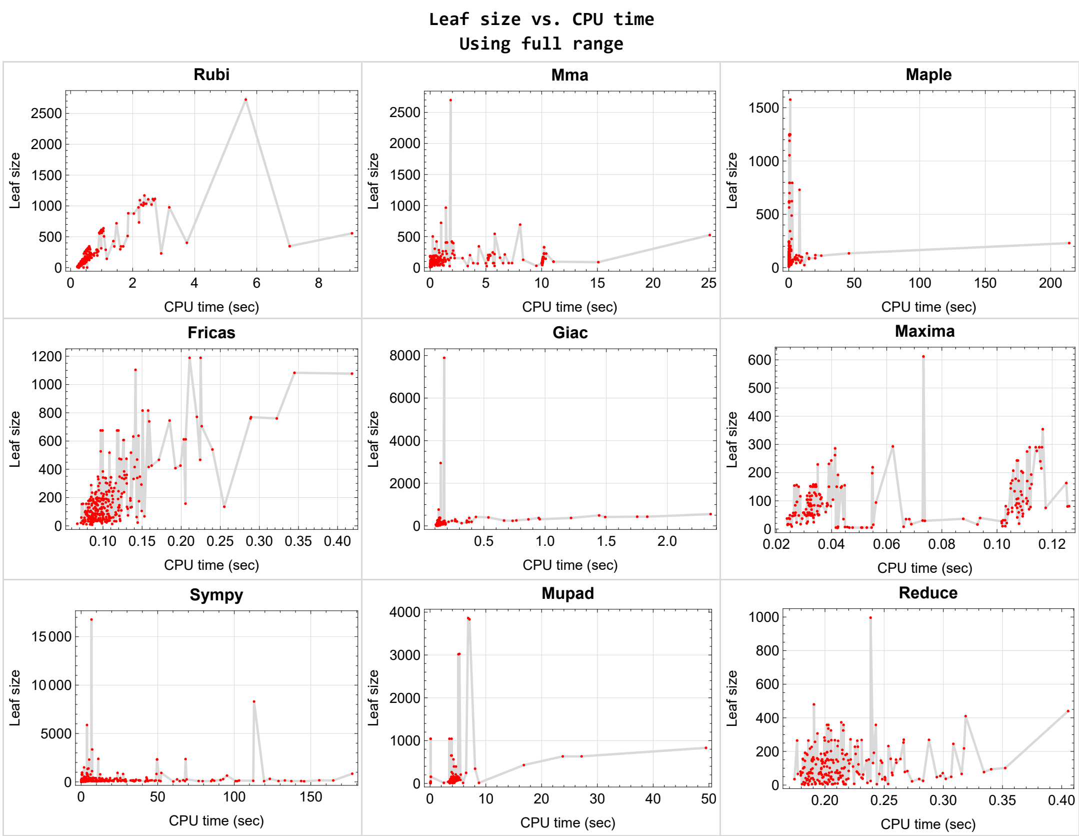


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {23, 24, 25, 26, 52, 53, 54, 55, 56, 101, 102, 103, 104, 123, 124, 125, 126, 127, 204, 211, 212, 231, 232, 233, 517, 518, 523, 524, 529, 530, 531, 536, 538, 543, 551, 560, 562, 567, 570, 571}

Mathematica {70, 71, 84, 85, 87, 88, 89, 90, 110, 111, 112, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 202, 204, 211, 234, 235, 243, 514, 517, 518, 523, 524, 529, 530, 531, 536, 543, 551, 560, 562, 567, 570}

Maple {257, 264, 403, 404, 405, 585, 586, 587, 588}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

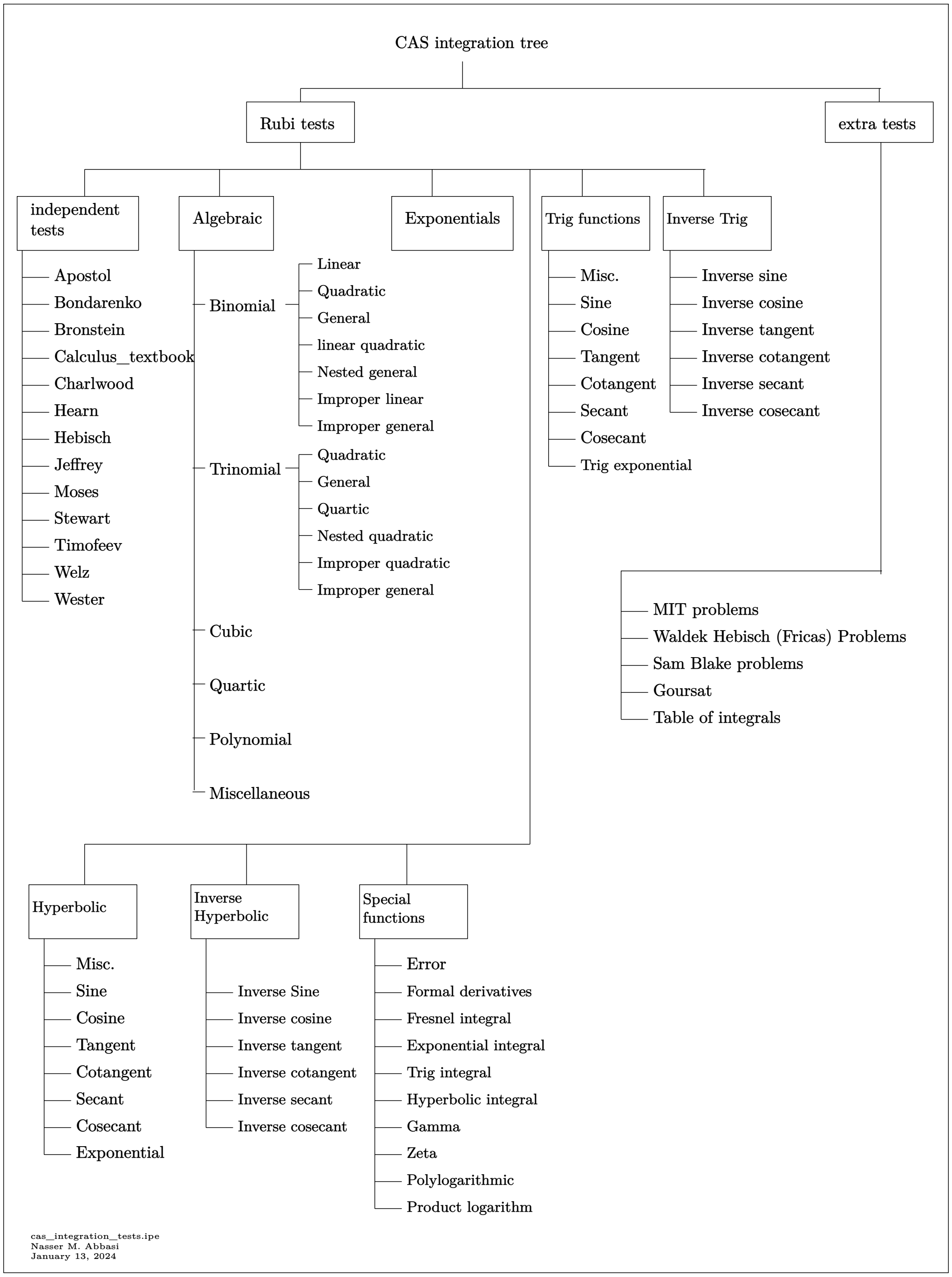
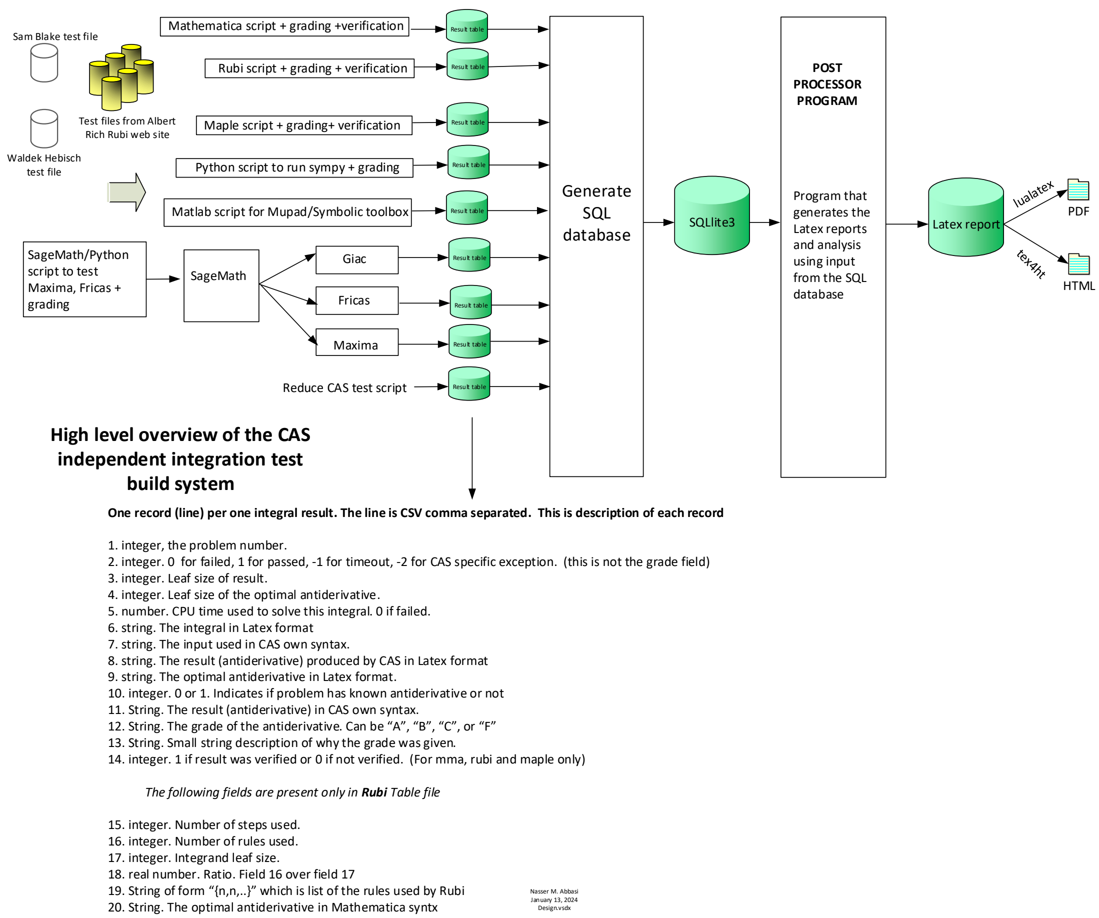


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26,
27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51,
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B grade { 397, 398 }

C grade { 211, 212, 213, 220, 221, 495, 517, 518, 523, 524, 529, 530, 531, 537, 538, 542, 544, 545, 549, 550, 552, 553, 557, 558, 559, 561, 562 }

F normal fail { 565, 566 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 225, 234, 235, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 512, 513, 519, 520, 525, 526, 527, 528, 532, 533, 534, 535, 536, 539, 540, 541, 543, 546, 547, 548, 551, 554, 555, 556, 560, 563, 564, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609 }

B grade { 70, 71, 72, 73, 84, 85, 86, 87, 88, 89, 90, 110, 111, 112, 131, 132, 133, 134, 135, 224, 236, 237, 238, 239, 240, 244, 245, 246, 247, 248, 464, 465, 466 }

C grade { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 101, 102, 103, 104, 105, 106, 107, 123, 124, 125, 126, 127, 128, 129, 130, 211, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 401, 402, 435, 468, 495, 509, 510, 511, 514, 515, 516, 517, 518, 521, 522, 523, 524, 529, 530, 531, 537, 538, 542, 544, 545, 549, 550, 552, 553, 557, 558, 559, 561, 562, 585, 586, 587, 588, 610 }

F normal fail { 565, 566, 595 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 214, 215, 216, 217, 222, 223, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 278, 285, 333, 340, 347, 354, 361, 368, 398, 399, 400, 401, 402, 406, 429, 439, 445, 448, 449, 450, 452, 453, 454, 455, 456, 457, 459, 460, 461, 462, 463, 464, 465, 466, 469, 470, 471, 472, 473, 474, 475, 476, 477, 508, 509, 510, 581, 582, 583, 584, 589, 590, 591, 592, 593, 594, 604, 605, 606, 607, 608, 609, 610 }

B grade { 218, 219, 237, 246, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 397, 435, 458, 467, 468, 478, 511, 514, 515, 521, 528 }

C grade { 224, 403, 404, 405, 451, 585, 586, 587, 588 }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 87, 88, 89, 90, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 225, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 268, 269, 270, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 286, 287, 288, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 436, 437, 438, 440, 441, 442, 443, 444, 446, 447, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 595, 596, 597, 598, 599, 600, 601, 602, 603 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 75, 77, 78, 79, 83, 91, 92, 93, 94, 95, 96, 97, 100, 114, 116, 117, 118, 122, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 264, 271, 278, 285, 289, 290, 291, 292, 293, 294, 295, 296, 333, 340, 347, 354, 361, 368, 399, 400, 401, 402, 406, 429, 435, 439, 450, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 462, 469, 470, 471, 472, 477, 478, 479, 480, 481, 482, 484, 485, 486, 487, 488, 508, 509, 510, 511, 514, 515, 516, 521, 522, 528, 589, 590, 591, 592, 593, 594, 604, 605, 606, 608, 609, 610 }

B grade { 64, 65, 74, 76, 80, 81, 82, 98, 99, 113, 115, 119, 120, 121, 157, 183, 238, 246, 261, 262, 263, 265, 266, 267, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 397, 398, 403, 404, 405, 445, 448, 449, 457, 463, 464, 465, 466, 467, 468, 473, 474, 475, 476, 483, 489, 490, 607 }

C grade { 581, 582, 583, 584 }

F normal fail { 15, 16, 17, 18, 19, 20, 21, 22, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 71, 84, 88, 104, 105, 111, 134, 136, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 268, 269, 270, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 286, 287, 288, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 407, 408, 409, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 436, 437, 438, 440, 441, 442, 443, 444, 446, 447, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 517, 518, 519, 520, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 595, 596, 597, 598, 599, 600, 601, 602, 603 }

F(-1) timedout fail { 68, 69, 72, 73, 85, 86, 87, 89, 90, 101, 102, 103, 106, 107, 108, 109, 110, 112, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 212, 213, 220, 221, 585, 586, 587, 588 }

F(-2) exception fail { 67, 70, 211, 250, 251, 252, 253, 330, 331, 332, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 410, 411, 412, 413, 414 }

Maxima

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 27, 28, 29, 30, 31, 33, 34, 35, 36, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 172, 173, 174, 175, 176, 177, 178, 183, 184, 185, 186, 187, 188, 189, 190, 191, 215, 216, 217, 222, 223, 224, 225, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 254, 255, 256, 257, 261, 262, 263, 264, 271, 278, 285, 289, 290, 291, 292, 293, 297, 298, 299, 300, 304, 305, 306, 307, 308, 333, 340, 347, 354, 361, 368, 400, 401, 402, 403, 404, 405, 406, 429, 435, 439, 445, 448, 449, 450, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 462, 468, 469, 470, 471, 477, 508, 509, 510, 511, 581, 582, 583, 584 }

B grade { 6, 7, 32, 137, 138, 151, 152, 153, 154, 167, 168, 169, 170, 171, 179, 180, 181, 182, 192, 193, 246, 397, 398, 399, 457, 463, 464, 465, 466, 467, 472, 473, 474, 475, 476 }

C grade { 589, 590, 591, 592, 593, 594, 604, 605, 606, 607, 608, 609 }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 268, 269, 270, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 286, 287, 288, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 436, 437, 438, 440, 441, 442, 443, 444, 446, 447, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 585, 586, 587, 588, 595, 596, 597, 598, 599, 600, 601, 602, 603, 610 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 58, 59, 74, 75, 76, 91, 92, 93, 113, 114, 115, 258, 259, 260, 265, 266, 267, 294, 295, 296, 301, 302, 303, 309, 310, 311 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 8, 27, 28, 29, 30, 31, 34, 57, 58, 59, 60, 61, 64, 74, 75, 76, 77, 78, 91, 92, 93, 94, 95, 98, 99, 113, 114, 115, 116, 117, 137, 138, 139, 145, 146, 147, 148, 150, 151, 152, 153, 154, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 176, 177, 179, 180, 181, 182, 183, 184, 188, 189, 190, 192, 193, 214, 215, 216, 217, 218, 219, 222, 223, 234, 235, 241, 242, 243, 244, 245, 247, 248, 249, 289, 290, 291, 292, 293, 297, 298, 299, 300, 304, 305, 306, 307, 308, 399, 429, 448, 449, 450, 451, 464, 469, 470, 471, 473, 474, 475, 477, 508, 581, 582, 583, 584, 590, 591, 592, 593, 594, 605 }

B grade { 9, 10, 62, 63, 79, 81, 100, 118, 119, 120, 121, 122, 140, 141, 142, 143, 144, 149, 155, 156, 157, 158, 159, 160, 173, 174, 175, 178, 185, 186, 187, 191, 225, 236, 237, 239, 240, 246, 254, 255, 256, 261, 262, 263, 397, 398, 403, 404, 405, 406, 435, 465, 466, 467, 468, 509, 510, 511 }

C grade { 607, 609 }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 67, 68, 69, 70, 71, 72, 73, 84, 85, 86, 87, 88, 89, 90, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 257, 258, 259, 260, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 295, 296, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 400, 401, 402, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 418, 419, 420, 426, 427, 431, 432, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 472, 476, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 516, 517, 518, 522, 523, 524, 528, 529, 530, 531, 535, 536, 537, 538, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 576, 577, 578, 579, 580, 585, 586, 587, 588, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 608, 610 }

F(-1) timedout fail { 65, 66, 80, 82, 83, 238 }

F(-2) exception fail { 7, 32, 33, 35, 36, 96, 97, 224, 417, 421, 422, 423, 424, 425, 428, 430, 433, 434, 496, 497, 498, 502, 503, 505, 506, 507, 512, 513, 514, 515, 519, 520, 521, 525, 526, 527, 532, 533, 534, 539, 540, 546, 547, 554, 555, 574, 575, 589 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 9, 10, 27, 28, 29, 30, 31, 34, 35, 36, 57, 58, 59, 60, 61, 74, 75, 76, 77, 78, 91, 92, 93, 94, 95, 113, 114, 115, 116, 117, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 215, 216, 217, 222, 223, 224, 225, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 271, 278, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 429, 435, 439, 448, 449, 450, 451, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 508, 509, 510, 511, 581, 582, 583, 584, 589, 590, 591, 592, 593, 594, 599, 604, 605, 606, 608, 609 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 151, 152, 165, 166, 168, 169, 180, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 236, 245, 250, 251, 252, 253, 268, 269, 270, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 286, 287, 288, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 436, 437, 438, 440, 441, 442, 443, 444, 445, 446, 447, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 585, 586, 587, 588, 595, 596, 597, 598, 600, 601, 602, 603, 607, 610 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 23, 24, 25, 26, 30, 39, 54, 59, 60, 93, 94, 138, 139, 140, 141, 142, 143, 144, 149, 150, 151, 155, 156, 157, 158, 159, 160, 166, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 190, 193, 215, 216, 217, 226, 227, 228, 229, 231, 232, 233, 237, 257, 264, 292, 293, 299, 300, 301, 307, 308, 309, 333, 340, 354, 361, 368, 399, 400, 401, 402, 418, 429, 448, 449, 450, 469, 470, 471, 477, 528 }

B grade { 6, 10, 27, 28, 29, 34, 137, 145, 146, 147, 148, 152, 153, 154, 161, 162, 163, 164, 165, 167, 168, 169, 176, 182, 188, 189, 191, 192, 254, 255, 256, 258, 259, 260, 261, 262, 263, 265, 266, 267, 271, 278, 285, 291, 294, 295, 296, 298, 302, 347, 397, 398, 403, 404, 405, 406, 435, 439, 452, 453, 454, 458, 459, 460, 464, 465, 466, 467, 468, 472, 473, 474, 475, 476, 508, 509, 515, 516, 521, 522 }

C grade { 15, 16, 17, 18, 19, 20, 21, 22, 44, 49, 136, 222, 223, 224, 268, 269, 270, 275, 276, 277, 279, 280, 281, 282, 283, 284, 312, 313, 319, 320, 330, 331, 332, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 369, 370, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 393, 394, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 419, 420, 422, 423, 424, 426, 427, 428, 430, 431, 432, 433, 434, 451, 498, 499, 502, 503, 504, 505, 506, 507, 511, 527, 534, 535, 540, 541, 547, 548, 556, 596, 597, 598, 599 }

F normal fail { 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 211, 212, 213, 214, 218, 219, 220, 221, 234, 235, 236, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 272, 273, 274, 314, 315, 316, 317, 436, 437, 438, 440, 478, 481, 482, 483, 487, 488, 489, 491, 493, 494, 538, 568, 573, 576, 577, 578, 579, 580, 581, 582, 583, 584, 589, 590, 591, 592, 593, 594, 595, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610 }

F(-1) timedout fail { 31, 32, 33, 35, 36, 37, 38, 40, 41, 42, 43, 45, 46, 47, 48, 50, 51, 52, 53, 55, 56, 74, 75, 76, 79, 113, 114, 115, 123, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 225, 230, 241, 242, 250, 251, 286, 289, 290, 297, 303, 304, 305, 306, 310, 311, 318, 321, 322, 325, 329, 371, 377, 391, 392, 395, 396, 421, 425, 479, 480, 484, 485, 486, 490, 496, 497, 510, 512, 513, 514, 519, 520, 525, 526, 531, 532, 533, 539, 544, 545, 546, 551, 552, 553, 554, 555, 559, 560, 561, 562, 563, 585, 586, 587, 588 }

F(-2) exception fail { 252, 253, 287, 288, 323, 324, 326, 327, 328, 441, 442, 443, 444, 445, 446, 447, 455, 456, 457, 461, 462, 463, 492, 495, 500, 501, 517, 518, 523, 524, 529, 530, 536, 537, 542, 543, 549, 550, 557, 558, 564, 565, 566, 567, 569, 570, 571, 572, 574, 575 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 214, 215, 216, 217, 218, 219, 222, 223, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 278, 285, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 429, 435, 439, 445, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 478, 484, 508, 509, 510, 511, 514, 515, 516, 521, 522, 528, 581, 582, 583, 584, 589, 590, 591, 592, 593, 594, 604, 605, 606, 607, 608, 609 }

C grade { }

F normal fail { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 250, 251, 252, 253, 275, 276, 277, 279, 280, 281, 282, 283, 284, 286, 287, 288, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 434, 436, 437, 438, 440, 441, 442, 443, 444, 446, 447, 477, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 512, 513, 517, 518, 519, 520, 523, 524, 525, 526, 527, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 585, 586, 587, 588, 595, 596, 597, 598, 599, 600, 601, 602, 603, 610 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	15	12	15	11	10	16	35	11
N.S.	1	1.27	1.00	0.80	1.00	0.73	0.67	1.07	2.33	0.73
time (sec)	N/A	0.261	0.005	0.107	0.024	0.079	0.080	0.118	0.248	0.093

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	25	19	14	17	15	17	17	62	13
N.S.	1	1.32	1.00	0.74	0.89	0.79	0.89	0.89	3.26	0.68
time (sec)	N/A	0.274	0.006	0.072	0.029	0.067	0.063	0.122	0.232	0.104

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	49	81	62	121	77	59	59
N.S.	1	1.00	0.75	0.67	1.11	0.85	1.66	1.05	0.81	0.81
time (sec)	N/A	0.358	0.052	0.547	0.032	0.084	1.015	0.124	0.257	3.798

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	47	41	71	38	37	31
N.S.	1	1.00	0.72	0.65	1.02	0.89	1.54	0.83	0.80	0.67
time (sec)	N/A	0.314	0.034	0.128	0.026	0.077	0.596	0.127	0.302	3.861

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	50	70	166	73	53	131	51
N.S.	1	1.00	1.00	0.86	1.21	2.86	1.26	0.91	2.26	0.88
time (sec)	N/A	0.307	0.079	0.158	0.112	0.088	11.292	0.121	0.241	4.371

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	83	77	77	144	228	262	99	253	91
N.S.	1	0.98	0.91	0.91	1.69	2.68	3.08	1.16	2.98	1.07
time (sec)	N/A	0.342	0.152	0.276	0.105	0.124	62.679	0.130	0.266	4.971

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	86	77	74	172	224	121	0	269	0
N.S.	1	0.95	0.85	0.81	1.89	2.46	1.33	0.00	2.96	0.00
time (sec)	N/A	0.365	0.598	5.276	0.106	0.101	43.382	0.000	0.288	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	55	84	176	70	55	151	0
N.S.	1	1.00	1.02	0.86	1.31	2.75	1.09	0.86	2.36	0.00
time (sec)	N/A	0.322	0.539	2.230	0.107	0.094	10.494	0.141	0.260	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	56	46	75	72	43	37
N.S.	1	1.00	0.74	0.68	1.06	0.87	1.42	1.36	0.81	0.70
time (sec)	N/A	0.299	0.704	1.665	0.032	0.086	24.095	0.166	0.251	3.499

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	55	96	69	287	112	66	96
N.S.	1	1.00	0.75	0.66	1.16	0.83	3.46	1.35	0.80	1.16
time (sec)	N/A	0.341	0.934	3.118	0.027	0.082	92.751	0.181	0.315	3.793

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	80	0	0	99	80	0	220	0
N.S.	1	1.00	0.28	0.00	0.00	0.35	0.28	0.00	0.77	0.00
time (sec)	N/A	0.547	10.091	0.000	0.000	0.080	13.111	0.000	0.622	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	75	0	0	86	80	0	182	0
N.S.	1	1.00	0.28	0.00	0.00	0.32	0.30	0.00	0.68	0.00
time (sec)	N/A	0.536	10.057	0.000	0.000	0.079	7.897	0.000	0.583	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	287	86	0	0	104	83	0	203	0
N.S.	1	1.00	0.30	0.00	0.00	0.36	0.29	0.00	0.70	0.00
time (sec)	N/A	0.505	10.036	0.000	0.000	0.077	28.375	0.000	0.677	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	316	72	0	0	121	90	0	203	0
N.S.	1	0.99	0.23	0.00	0.00	0.38	0.28	0.00	0.64	0.00
time (sec)	N/A	0.573	10.032	0.000	0.000	0.096	128.870	0.000	0.896	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	276	78	0	0	0	80	0	213	0
N.S.	1	1.01	0.28	0.00	0.00	0.00	0.29	0.00	0.78	0.00
time (sec)	N/A	0.531	10.072	0.000	0.000	0.000	23.857	0.000	0.648	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	72	0	0	0	78	0	177	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.30	0.00	0.69	0.00
time (sec)	N/A	0.457	10.029	0.000	0.000	0.000	7.284	0.000	0.562	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	273	87	0	0	0	82	0	203	0
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.30	0.00	0.74	0.00
time (sec)	N/A	0.494	10.028	0.000	0.000	0.000	34.739	0.000	0.740	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	307	305	72	0	0	0	90	0	203	0
N.S.	1	0.99	0.23	0.00	0.00	0.00	0.29	0.00	0.66	0.00
time (sec)	N/A	0.552	10.029	0.000	0.000	0.000	143.669	0.000	1.061	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	578	563	80	0	0	0	80	0	229	0
N.S.	1	0.97	0.14	0.00	0.00	0.00	0.14	0.00	0.40	0.00
time (sec)	N/A	1.008	10.076	0.000	0.000	0.000	79.363	0.000	0.717	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	562	563	72	0	0	0	80	0	192	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.14	0.00	0.34	0.00
time (sec)	N/A	0.952	10.065	0.000	0.000	0.000	14.996	0.000	0.619	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	578	559	71	0	0	0	82	0	199	0
N.S.	1	0.97	0.12	0.00	0.00	0.00	0.14	0.00	0.34	0.00
time (sec)	N/A	0.938	10.025	0.000	0.000	0.000	15.524	0.000	0.623	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	612	589	72	0	0	0	88	0	203	0
N.S.	1	0.96	0.12	0.00	0.00	0.00	0.14	0.00	0.33	0.00
time (sec)	N/A	0.999	10.034	0.000	0.000	0.000	67.797	0.000	0.803	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	582	79	0	0	108	80	0	228	0
N.S.	1	1.01	0.14	0.00	0.00	0.19	0.14	0.00	0.40	0.00
time (sec)	N/A	0.977	10.084	0.000	0.000	0.081	17.233	0.000	0.706	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	557	565	71	0	0	94	80	0	189	0
N.S.	1	1.01	0.13	0.00	0.00	0.17	0.14	0.00	0.34	0.00
time (sec)	N/A	0.922	10.049	0.000	0.000	0.086	8.605	0.000	0.564	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	582	72	0	0	112	83	0	203	0
N.S.	1	1.00	0.12	0.00	0.00	0.19	0.14	0.00	0.35	0.00
time (sec)	N/A	0.967	10.030	0.000	0.000	0.093	21.056	0.000	0.632	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	72	0	0	129	90	0	203	0
N.S.	1	1.00	0.12	0.00	0.00	0.21	0.15	0.00	0.33	0.00
time (sec)	N/A	1.011	10.032	0.000	0.000	0.081	76.733	0.000	0.854	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	69	116	96	337	104	94	89
N.S.	1	1.00	0.71	0.67	1.13	0.93	3.27	1.01	0.91	0.86
time (sec)	N/A	0.403	0.065	0.187	0.029	0.077	2.273	0.126	0.340	3.712

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	50	84	74	240	63	71	59
N.S.	1	1.00	0.77	0.68	1.15	1.01	3.29	0.86	0.97	0.81
time (sec)	N/A	0.368	0.044	0.155	0.027	0.090	1.222	0.125	0.300	3.511

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	34	30	49	52	143	32	49	32
N.S.	1	1.04	0.74	0.65	1.07	1.13	3.11	0.70	1.07	0.70
time (sec)	N/A	0.330	0.036	0.122	0.027	0.087	0.778	0.118	0.307	3.343

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	81	71	73	81	240	94	67	218	66
N.S.	1	1.05	0.92	0.95	1.05	3.12	1.22	0.87	2.83	0.86
time (sec)	N/A	0.349	0.101	0.241	0.103	0.091	26.021	0.125	0.317	4.104

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	106	99	97	170	348	0	101	410	127
N.S.	1	0.95	0.88	0.87	1.52	3.11	0.00	0.90	3.66	1.13
time (sec)	N/A	0.374	0.138	0.483	0.111	0.094	0.000	0.121	0.319	4.954

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	112	100	101	205	342	0	0	440	0
N.S.	1	0.92	0.82	0.83	1.68	2.80	0.00	0.00	3.61	0.00
time (sec)	N/A	0.413	0.808	9.240	0.109	0.100	0.000	0.000	0.405	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	88	80	82	99	252	0	0	245	0
N.S.	1	0.95	0.86	0.88	1.06	2.71	0.00	0.00	2.63	0.00
time (sec)	N/A	0.366	0.700	4.754	0.111	0.095	0.000	0.000	0.308	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	39	36	51	56	146	42	54	35
N.S.	1	1.00	0.60	0.55	0.78	0.86	2.25	0.65	0.83	0.54
time (sec)	N/A	0.327	0.645	2.178	0.033	0.083	164.672	0.139	0.297	3.921

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	62	57	87	81	0	0	77	68
N.S.	1	0.98	0.75	0.69	1.05	0.98	0.00	0.00	0.93	0.82
time (sec)	N/A	0.351	0.498	1.764	0.032	0.095	0.000	0.000	0.334	3.768

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	109	79	74	129	105	0	0	101	151
N.S.	1	0.96	0.70	0.65	1.14	0.93	0.00	0.00	0.89	1.34
time (sec)	N/A	0.405	1.109	3.177	0.034	0.093	0.000	0.000	0.352	4.757

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	313	109	0	0	156	0	0	426	0
N.S.	1	0.99	0.35	0.00	0.00	0.50	0.00	0.00	1.35	0.00
time (sec)	N/A	0.579	10.096	0.000	0.000	0.090	0.000	0.000	1.105	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	296	101	0	0	145	0	0	386	0
N.S.	1	1.00	0.34	0.00	0.00	0.49	0.00	0.00	1.30	0.00
time (sec)	N/A	0.563	10.111	0.000	0.000	0.086	0.000	0.000	1.025	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	301	300	103	0	0	149	80	0	350	0
N.S.	1	1.00	0.34	0.00	0.00	0.50	0.27	0.00	1.16	0.00
time (sec)	N/A	0.538	10.110	0.000	0.000	0.092	101.899	0.000	0.998	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	318	316	116	0	0	165	0	0	374	0
N.S.	1	0.99	0.36	0.00	0.00	0.52	0.00	0.00	1.18	0.00
time (sec)	N/A	0.556	10.064	0.000	0.000	0.090	0.000	0.000	1.118	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	348	345	81	0	0	180	0	0	374	0
N.S.	1	0.99	0.23	0.00	0.00	0.52	0.00	0.00	1.07	0.00
time (sec)	N/A	0.600	10.048	0.000	0.000	0.085	0.000	0.000	1.606	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	303	305	107	0	0	0	0	0	416	0
N.S.	1	1.01	0.35	0.00	0.00	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.573	10.086	0.000	0.000	0.000	0.000	0.000	1.093	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	289	101	0	0	0	0	0	376	0
N.S.	1	1.02	0.36	0.00	0.00	0.00	0.00	0.00	1.32	0.00
time (sec)	N/A	0.529	10.069	0.000	0.000	0.000	0.000	0.000	1.082	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	283	102	0	0	0	78	0	340	0
N.S.	1	0.99	0.36	0.00	0.00	0.00	0.27	0.00	1.19	0.00
time (sec)	N/A	0.525	10.059	0.000	0.000	0.000	119.487	0.000	0.910	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	300	116	0	0	0	0	0	374	0
N.S.	1	0.99	0.38	0.00	0.00	0.00	0.00	0.00	1.24	0.00
time (sec)	N/A	0.538	10.068	0.000	0.000	0.000	0.000	0.000	1.217	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	334	81	0	0	0	0	0	374	0
N.S.	1	0.99	0.24	0.00	0.00	0.00	0.00	0.00	1.11	0.00
time (sec)	N/A	0.600	10.037	0.000	0.000	0.000	0.000	0.000	1.776	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	592	109	0	0	0	0	0	438	0
N.S.	1	0.97	0.18	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	1.050	10.096	0.000	0.000	0.000	0.000	0.000	1.101	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	592	592	93	0	0	0	0	0	398	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.998	10.085	0.000	0.000	0.000	0.000	0.000	1.050	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	595	576	81	0	0	0	80	0	362	0
N.S.	1	0.97	0.14	0.00	0.00	0.00	0.13	0.00	0.61	0.00
time (sec)	N/A	0.945	10.089	0.000	0.000	0.000	137.270	0.000	0.934	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	588	80	0	0	0	0	0	370	0
N.S.	1	0.96	0.13	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.979	10.043	0.000	0.000	0.000	0.000	0.000	0.944	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	642	618	81	0	0	0	0	0	374	0
N.S.	1	0.96	0.13	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	1.034	10.036	0.000	0.000	0.000	0.000	0.000	1.372	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	606	611	109	0	0	165	0	0	437	0
N.S.	1	1.01	0.18	0.00	0.00	0.27	0.00	0.00	0.72	0.00
time (sec)	N/A	0.991	10.093	0.000	0.000	0.091	0.000	0.000	1.231	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	587	594	92	0	0	152	0	0	398	0
N.S.	1	1.01	0.16	0.00	0.00	0.26	0.00	0.00	0.68	0.00
time (sec)	N/A	0.968	10.079	0.000	0.000	0.091	0.000	0.000	1.048	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	593	595	81	0	0	156	80	0	362	0
N.S.	1	1.00	0.14	0.00	0.00	0.26	0.13	0.00	0.61	0.00
time (sec)	N/A	0.973	10.061	0.000	0.000	0.086	100.834	0.000	0.925	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	611	81	0	0	173	0	0	374	0
N.S.	1	1.00	0.13	0.00	0.00	0.28	0.00	0.00	0.61	0.00
time (sec)	N/A	0.997	10.046	0.000	0.000	0.096	0.000	0.000	1.016	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	640	639	81	0	0	188	0	0	374	0
N.S.	1	1.00	0.13	0.00	0.00	0.29	0.00	0.00	0.58	0.00
time (sec)	N/A	1.060	10.062	0.000	0.000	0.083	0.000	0.000	1.445	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	93	0	288	0	102	77	103
N.S.	1	1.00	0.88	0.89	0.00	2.77	0.00	0.98	0.74	0.99
time (sec)	N/A	0.462	0.187	0.945	0.000	0.085	0.000	0.121	1.445	3.938

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	59	0	205	0	64	38	58
N.S.	1	1.00	0.99	0.80	0.00	2.77	0.00	0.86	0.51	0.78
time (sec)	N/A	0.327	0.101	0.231	0.000	0.085	0.000	0.121	1.015	3.906

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	38	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.75	0.78
time (sec)	N/A	0.296	0.049	0.128	0.000	0.083	12.344	0.125	1.012	4.201

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	65	0	385	114	71	84	652
N.S.	1	1.00	0.94	0.76	0.00	4.53	1.34	0.84	0.99	7.67
time (sec)	N/A	0.356	0.119	0.180	0.000	0.100	10.398	0.123	1.983	3.882

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	518	0	104	198	396
N.S.	1	1.09	0.93	0.79	0.00	4.43	0.00	0.89	1.69	3.38
time (sec)	N/A	0.429	0.277	0.299	0.000	0.108	0.000	0.120	8.779	4.280

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	136	98	0	739	0	237	159	0
N.S.	1	1.09	1.11	0.80	0.00	6.01	0.00	1.93	1.29	0.00
time (sec)	N/A	0.479	0.950	4.839	0.000	0.159	0.000	0.239	3.218	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	108	70	0	632	0	156	91	0
N.S.	1	0.99	1.19	0.77	0.00	6.95	0.00	1.71	1.00	0.00
time (sec)	N/A	0.381	0.483	2.204	0.000	0.139	0.000	0.133	1.135	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	38	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.70	0.00
time (sec)	N/A	0.311	0.602	1.843	0.000	0.113	0.000	0.131	0.394	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	67	0	332	0	0	38	0
N.S.	1	1.00	1.25	0.84	0.00	4.15	0.00	0.00	0.48	0.00
time (sec)	N/A	0.380	0.522	3.096	0.000	0.127	0.000	0.000	0.425	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	121	88	0	416	0	0	84	0
N.S.	1	1.10	1.05	0.77	0.00	3.62	0.00	0.00	0.73	0.00
time (sec)	N/A	0.495	1.028	6.496	0.000	0.140	0.000	0.000	1.422	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	38	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.346	10.036	0.000	0.000	0.000	0.000	0.000	0.387	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	38	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.369	10.029	0.000	0.000	0.000	0.000	0.000	0.404	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	36	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.340	10.030	0.000	0.000	0.000	0.000	0.000	0.372	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	35	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.306	10.136	0.000	0.000	0.000	0.000	0.000	0.400	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	38	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.344	10.079	0.000	0.000	0.000	0.000	0.000	0.458	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	38	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.391	10.090	0.000	0.000	0.000	0.000	0.000	0.415	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	38	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.395	10.093	0.000	0.000	0.000	0.000	0.000	0.452	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	142	803	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.15	6.53	1.17
time (sec)	N/A	0.475	0.359	0.324	0.000	0.121	0.000	0.124	7.867	4.330

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	62	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.63	0.96
time (sec)	N/A	0.387	0.194	0.220	0.000	0.121	0.000	0.121	4.133	4.016

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	86	71	0	302	0	93	62	84
N.S.	1	0.99	0.99	0.82	0.00	3.47	0.00	1.07	0.71	0.97
time (sec)	N/A	0.361	0.166	0.181	0.000	0.096	0.000	0.119	2.962	3.867

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	816	0	139	951	3025
N.S.	1	1.17	0.94	1.11	0.00	6.18	0.00	1.05	7.20	22.92
time (sec)	N/A	0.496	0.364	0.365	0.000	0.151	0.000	0.130	11.403	5.259

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1189	0	257	1165	3860
N.S.	1	1.15	0.88	1.03	0.00	6.43	0.00	1.39	6.30	20.86
time (sec)	N/A	0.646	0.881	0.539	0.000	0.211	0.000	0.120	36.298	6.833

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1077	0	343	1002	0
N.S.	1	1.18	1.09	0.83	0.00	7.64	0.00	2.43	7.11	0.00
time (sec)	N/A	0.550	2.196	5.783	0.000	0.418	0.000	0.141	7.668	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	0	532	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	0.00	5.72	0.00
time (sec)	N/A	0.389	1.002	4.545	0.000	0.162	0.000	0.000	3.048	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	62	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	0.60	0.00
time (sec)	N/A	0.401	1.188	4.687	0.000	0.172	0.000	0.136	0.527	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	0	832	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	0.00	5.58	0.00
time (sec)	N/A	0.552	1.277	7.796	0.000	0.203	0.000	0.000	5.439	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	0	920	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	0.00	4.42	0.00
time (sec)	N/A	0.707	2.173	13.877	0.000	0.289	0.000	0.000	8.658	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0	62	0
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.349	10.129	0.000	0.000	0.000	0.000	0.000	0.597	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0	62	0
N.S.	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.372	10.136	0.000	0.000	0.000	0.000	0.000	0.541	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0	60	0
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.339	10.119	0.000	0.000	0.000	0.000	0.000	0.590	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0	59	0
N.S.	1	1.00	5.58	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.308	10.216	0.000	0.000	0.000	0.000	0.000	0.568	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	447	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	7.21	0.00
time (sec)	N/A	0.346	10.207	0.000	0.000	0.000	0.000	0.000	9.072	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	452	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	7.06	0.00
time (sec)	N/A	0.390	10.197	0.000	0.000	0.000	0.000	0.000	9.958	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0	446	0
N.S.	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	6.97	0.00
time (sec)	N/A	0.391	10.191	0.000	0.000	0.000	0.000	0.000	10.637	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	93	0	288	0	102	77	103
N.S.	1	1.00	0.88	0.89	0.00	2.77	0.00	0.98	0.74	0.99
time (sec)	N/A	0.444	0.194	0.665	0.000	0.109	0.000	0.120	20.529	4.165

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	59	0	205	0	64	38	58
N.S.	1	1.00	0.99	0.80	0.00	2.77	0.00	0.86	0.51	0.78
time (sec)	N/A	0.346	0.103	0.141	0.000	0.123	0.000	0.123	13.111	3.794

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	39	0	130	87	40	38	40
N.S.	1	1.00	1.00	0.76	0.00	2.55	1.71	0.78	0.75	0.78
time (sec)	N/A	0.299	0.046	0.078	0.000	0.120	17.997	0.119	13.426	3.531

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	65	0	385	114	71	84	652
N.S.	1	1.00	0.94	0.76	0.00	4.53	1.34	0.84	0.99	7.67
time (sec)	N/A	0.353	0.116	0.114	0.000	0.127	12.950	0.132	32.768	3.787

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	109	92	0	518	0	104	24	396
N.S.	1	1.09	0.93	0.79	0.00	4.43	0.00	0.89	0.21	3.38
time (sec)	N/A	0.420	0.287	0.191	0.000	0.136	0.000	0.138	200.039	4.644

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	136	98	0	745	0	0	159	0
N.S.	1	1.09	1.11	0.80	0.00	6.06	0.00	0.00	1.29	0.00
time (sec)	N/A	0.460	1.593	10.085	0.000	0.185	0.000	0.000	47.928	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	90	108	70	0	638	0	0	91	0
N.S.	1	0.99	1.19	0.77	0.00	7.01	0.00	0.00	1.00	0.00
time (sec)	N/A	0.380	0.759	7.342	0.000	0.145	0.000	0.000	14.689	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	74	42	0	245	0	72	38	0
N.S.	1	1.00	1.37	0.78	0.00	4.54	0.00	1.33	0.70	0.00
time (sec)	N/A	0.295	0.653	5.652	0.000	0.120	0.000	0.144	2.811	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	100	67	0	332	0	116	38	0
N.S.	1	1.00	1.25	0.84	0.00	4.15	0.00	1.45	0.48	0.00
time (sec)	N/A	0.373	0.617	9.749	0.000	0.139	0.000	0.141	6.944	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	121	88	0	416	0	205	84	0
N.S.	1	1.10	1.05	0.77	0.00	3.62	0.00	1.78	0.73	0.00
time (sec)	N/A	0.492	1.686	20.177	0.000	0.158	0.000	0.279	168.089	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	672	1022	140	0	0	0	0	0	145	0
N.S.	1	1.52	0.21	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	2.608	10.106	0.000	0.000	0.000	0.000	0.000	25.254	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	976	65	0	0	0	0	0	38	0
N.S.	1	1.54	0.10	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.187	10.048	0.000	0.000	0.000	0.000	0.000	2.626	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	635	876	65	0	0	0	0	0	36	0
N.S.	1	1.38	0.10	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.038	10.034	0.000	0.000	0.000	0.000	0.000	2.599	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	672	1006	141	0	0	0	0	0	38	0
N.S.	1	1.50	0.21	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.336	10.117	0.000	0.000	0.000	0.000	0.000	5.233	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1017	1111	65	0	0	0	0	0	38	0
N.S.	1	1.09	0.06	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.649	10.059	0.000	0.000	0.000	0.000	0.000	2.643	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	788	881	65	0	0	0	0	0	38	0
N.S.	1	1.12	0.08	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.858	10.040	0.000	0.000	0.000	0.000	0.000	2.578	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1053	1021	141	0	0	0	0	0	38	0
N.S.	1	0.97	0.13	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.276	10.096	0.000	0.000	0.000	0.000	0.000	4.624	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	38	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.337	10.042	0.000	0.000	0.000	0.000	0.000	2.710	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	38	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.341	10.064	0.000	0.000	0.000	0.000	0.000	2.570	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	35	0
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.303	10.171	0.000	0.000	0.000	0.000	0.000	2.570	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	38	0
N.S.	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.335	10.091	0.000	0.000	0.000	0.000	0.000	3.970	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	38	0
N.S.	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.343	10.169	0.000	0.000	0.000	0.000	0.000	4.781	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	144	130	133	0	475	0	142	803	144
N.S.	1	1.17	1.06	1.08	0.00	3.86	0.00	1.15	6.53	1.17
time (sec)	N/A	0.465	0.366	0.334	0.000	0.130	0.000	0.127	170.305	4.072

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	348	0	116	62	95
N.S.	1	0.99	1.01	0.84	0.00	3.52	0.00	1.17	0.63	0.96
time (sec)	N/A	0.383	0.185	0.212	0.000	0.147	0.000	0.131	79.515	3.863

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	86	86	71	0	302	0	93	62	84
N.S.	1	0.99	0.99	0.82	0.00	3.47	0.00	1.07	0.71	0.97
time (sec)	N/A	0.351	0.183	0.188	0.000	0.116	0.000	0.123	56.951	3.647

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	155	124	146	0	816	0	139	24	3017
N.S.	1	1.17	0.94	1.11	0.00	6.18	0.00	1.05	0.18	22.86
time (sec)	N/A	0.503	0.370	0.451	0.000	0.158	0.000	0.130	200.035	5.084

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	213	163	191	0	1189	0	257	24	3832
N.S.	1	1.15	0.88	1.03	0.00	6.43	0.00	1.39	0.13	20.71
time (sec)	N/A	0.601	0.712	0.644	0.000	0.225	0.000	0.122	200.036	7.068

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	153	117	0	1083	0	298	1002	0
N.S.	1	1.18	1.09	0.83	0.00	7.68	0.00	2.11	7.11	0.00
time (sec)	N/A	0.531	2.908	20.279	0.000	0.345	0.000	0.266	155.045	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	112	81	0	426	0	244	532	0
N.S.	1	1.04	1.20	0.87	0.00	4.58	0.00	2.62	5.72	0.00
time (sec)	N/A	0.385	1.387	15.375	0.000	0.199	0.000	0.361	58.283	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	124	90	0	467	0	237	62	0
N.S.	1	1.04	1.19	0.87	0.00	4.49	0.00	2.28	0.60	0.00
time (sec)	N/A	0.395	1.355	15.422	0.000	0.224	0.000	0.178	7.752	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	157	157	112	0	612	0	418	832	0
N.S.	1	1.05	1.05	0.75	0.00	4.11	0.00	2.81	5.58	0.00
time (sec)	N/A	0.525	1.841	24.820	0.000	0.206	0.000	0.435	112.723	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	212	201	134	0	760	0	395	24	0
N.S.	1	1.02	0.97	0.64	0.00	3.65	0.00	1.90	0.12	0.00
time (sec)	N/A	0.684	3.573	45.935	0.000	0.322	0.000	0.537	200.037	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	788	1114	226	0	0	0	0	0	24	0
N.S.	1	1.41	0.29	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.717	10.219	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	735	1052	173	0	0	0	0	0	62	0
N.S.	1	1.43	0.24	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.352	10.149	0.000	0.000	0.000	0.000	0.000	83.577	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	627	1033	159	0	0	0	0	0	62	0
N.S.	1	1.65	0.25	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.377	10.115	0.000	0.000	0.000	0.000	0.000	76.597	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	735	1036	169	0	0	0	0	0	60	0
N.S.	1	1.41	0.23	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.436	10.141	0.000	0.000	0.000	0.000	0.000	7.908	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	788	1092	225	0	0	0	0	0	24	0
N.S.	1	1.39	0.29	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.680	10.223	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1177	1107	159	0	0	0	0	0	62	0
N.S.	1	0.94	0.14	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.501	10.122	0.000	0.000	0.000	0.000	0.000	84.857	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1187	1093	169	0	0	0	0	0	62	0
N.S.	1	0.92	0.14	0.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.232	10.147	0.000	0.000	0.000	0.000	0.000	7.843	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1266	1170	226	0	0	0	0	0	24	0
N.S.	1	0.92	0.18	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	2.378	10.229	0.000	0.000	0.000	0.000	0.000	200.034	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	62	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.331	10.140	0.000	0.000	0.000	0.000	0.000	7.523	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	62	0
N.S.	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	0.97	0.00
time (sec)	N/A	0.333	10.144	0.000	0.000	0.000	0.000	0.000	7.608	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0	59	0
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.302	10.220	0.000	0.000	0.000	0.000	0.000	8.209	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	24	0
N.S.	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.347	10.409	0.000	0.000	0.000	0.000	0.000	200.042	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	24	0
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.347	10.233	0.000	0.000	0.000	0.000	0.000	200.043	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	83	0	0	0	80	0	73	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.90	0.00	0.82	0.00
time (sec)	N/A	0.352	10.085	0.000	0.000	0.000	51.880	0.000	0.299	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	112	118	116	243	242	226	143	145	134
N.S.	1	0.94	0.99	0.97	2.04	2.03	1.90	1.20	1.22	1.13
time (sec)	N/A	0.373	0.711	0.120	0.107	0.112	35.871	0.140	0.189	4.946

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	97	91	159	191	144	105	104	93
N.S.	1	1.00	1.13	1.06	1.85	2.22	1.67	1.22	1.21	1.08
time (sec)	N/A	0.338	0.129	0.062	0.113	0.101	33.320	0.137	0.182	4.481

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	78	80	77	108	155	107	87	102	68
N.S.	1	1.13	1.16	1.12	1.57	2.25	1.55	1.26	1.48	0.99
time (sec)	N/A	0.335	0.184	0.074	0.108	0.099	33.986	0.182	0.204	4.300

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	66	82	84	67	166	85	163	96	57
N.S.	1	1.12	1.39	1.42	1.14	2.81	1.44	2.76	1.63	0.97
time (sec)	N/A	0.316	0.133	0.077	0.109	0.112	9.008	0.359	0.197	4.164

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	47	48	49	60	73	250	110	91
N.S.	1	1.09	1.02	1.04	1.07	1.30	1.59	5.43	2.39	1.98
time (sec)	N/A	0.315	0.101	0.082	0.032	0.082	1.988	0.665	0.202	3.664

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	69	70	84	85	109	310	152	126
N.S.	1	1.05	0.93	0.95	1.14	1.15	1.47	4.19	2.05	1.70
time (sec)	N/A	0.353	0.134	0.093	0.034	0.087	2.716	0.956	0.202	3.824

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	93	94	118	109	143	370	192	168
N.S.	1	1.04	0.89	0.90	1.13	1.05	1.38	3.56	1.85	1.62
time (sec)	N/A	0.410	0.153	0.108	0.033	0.120	2.233	0.948	0.241	4.180

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	113	118	152	133	177	430	232	210
N.S.	1	1.03	0.84	0.88	1.13	0.99	1.32	3.21	1.73	1.57
time (sec)	N/A	0.458	0.182	0.141	0.034	0.137	3.086	1.835	0.254	4.608

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	108	113	158	131	1386	175	123	117
N.S.	1	0.95	0.72	0.75	1.05	0.87	9.24	1.17	0.82	0.78
time (sec)	N/A	0.450	0.068	0.099	0.034	0.111	4.785	0.130	0.192	3.844

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	88	89	124	107	910	140	99	97
N.S.	1	0.97	0.75	0.76	1.06	0.91	7.78	1.20	0.85	0.83
time (sec)	N/A	0.397	0.060	0.080	0.036	0.091	3.708	0.132	0.182	4.539

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	64	65	90	82	422	105	75	77
N.S.	1	0.99	0.76	0.77	1.07	0.98	5.02	1.25	0.89	0.92
time (sec)	N/A	0.335	0.051	0.072	0.032	0.079	2.690	0.121	0.209	3.932

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	55	57	119	72	50	54
N.S.	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	0.94	1.02
time (sec)	N/A	0.297	0.032	0.066	0.032	0.076	2.428	0.126	0.205	3.553

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	68	83	75	150	107	116	99	80
N.S.	1	1.03	1.03	1.26	1.14	2.27	1.62	1.76	1.50	1.21
time (sec)	N/A	0.328	0.087	0.062	0.105	0.109	3.117	0.130	0.190	3.725

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	81	75	93	133	158	107	78	159	97
N.S.	1	0.95	0.88	1.09	1.56	1.86	1.26	0.92	1.87	1.14
time (sec)	N/A	0.334	0.131	0.085	0.109	0.092	4.467	0.138	0.193	4.101

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	88	91	102	193	188	144	130	185	0
N.S.	1	0.97	1.00	1.12	2.12	2.07	1.58	1.43	2.03	0.00
time (sec)	N/A	0.354	0.171	0.085	0.112	0.105	5.845	0.144	0.226	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	124	127	277	238	226	139	228	0
N.S.	1	0.93	1.01	1.03	2.25	1.93	1.84	1.13	1.85	0.00
time (sec)	N/A	0.404	0.189	0.088	0.114	0.127	13.006	0.153	0.225	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	110	123	116	240	243	253	144	145	130
N.S.	1	0.96	1.07	1.01	2.09	2.11	2.20	1.25	1.26	1.13
time (sec)	N/A	0.360	0.166	0.072	0.116	0.108	68.925	0.137	0.247	5.025

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	98	96	171	203	216	121	144	105
N.S.	1	1.05	1.00	0.98	1.74	2.07	2.20	1.23	1.47	1.07
time (sec)	N/A	0.356	0.322	0.081	0.108	0.085	85.697	0.186	0.201	4.871

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	103	99	134	195	187	225	128	95
N.S.	1	1.04	1.11	1.06	1.44	2.10	2.01	2.42	1.38	1.02
time (sec)	N/A	0.352	0.234	0.088	0.106	0.096	23.711	0.265	0.221	4.772

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	84	91	108	80	213	114	254	139	72
N.S.	1	1.11	1.20	1.42	1.05	2.80	1.50	3.34	1.83	0.95
time (sec)	N/A	0.341	0.234	0.089	0.109	0.112	15.060	0.765	0.198	5.002

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	49	48	49	84	189	370	152	122
N.S.	1	1.09	1.07	1.04	1.07	1.83	4.11	8.04	3.30	2.65
time (sec)	N/A	0.326	0.156	0.096	0.033	0.107	4.653	1.212	0.181	4.502

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	71	70	84	109	258	430	192	164
N.S.	1	1.05	0.96	0.95	1.14	1.47	3.49	5.81	2.59	2.22
time (sec)	N/A	0.362	0.185	0.118	0.032	0.123	4.697	1.753	0.183	4.951

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	94	94	118	134	326	490	232	206
N.S.	1	1.04	0.90	0.90	1.13	1.29	3.13	4.71	2.23	1.98
time (sec)	N/A	0.429	0.238	0.151	0.034	0.131	5.419	1.442	0.211	5.495

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	138	115	118	152	157	393	550	272	248
N.S.	1	1.03	0.86	0.88	1.13	1.17	2.93	4.10	2.03	1.85
time (sec)	N/A	0.453	0.292	0.230	0.032	0.205	5.316	2.353	0.222	6.464

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	143	110	115	158	155	3351	175	147	137
N.S.	1	0.95	0.73	0.77	1.05	1.03	22.34	1.17	0.98	0.91
time (sec)	N/A	0.459	0.088	0.111	0.033	0.082	7.225	0.125	0.220	4.017

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	113	89	91	124	132	2304	140	123	118
N.S.	1	0.97	0.76	0.78	1.06	1.13	19.69	1.20	1.05	1.01
time (sec)	N/A	0.397	0.072	0.088	0.034	0.101	5.269	0.129	0.228	3.845

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	83	66	67	90	106	1340	105	99	97
N.S.	1	0.99	0.79	0.80	1.07	1.26	15.95	1.25	1.18	1.15
time (sec)	N/A	0.356	0.053	0.082	0.032	0.105	4.176	0.126	0.216	3.855

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	55	80	498	72	74	77
N.S.	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.40	1.45
time (sec)	N/A	0.311	0.036	0.075	0.033	0.079	3.183	0.123	0.181	3.911

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	81	99	91	197	184	140	141	0
N.S.	1	0.99	0.90	1.10	1.01	2.19	2.04	1.56	1.57	0.00
time (sec)	N/A	0.366	0.118	0.073	0.107	0.093	3.063	0.129	0.189	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	107	105	147	163	184	202	124	187	0
N.S.	1	0.91	0.89	1.25	1.38	1.56	1.71	1.05	1.58	0.00
time (sec)	N/A	0.371	0.146	0.094	0.125	0.099	4.140	0.160	0.215	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	105	92	115	207	210	216	145	202	78
N.S.	1	0.94	0.82	1.03	1.85	1.88	1.93	1.29	1.80	0.70
time (sec)	N/A	0.359	0.213	0.097	0.106	0.100	6.827	0.147	0.205	5.113

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	112	117	127	275	240	253	156	228	0
N.S.	1	0.91	0.95	1.03	2.24	1.95	2.06	1.27	1.85	0.00
time (sec)	N/A	0.374	0.247	0.092	0.111	0.104	14.051	0.155	0.228	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	140	150	151	354	292	287	214	268	0
N.S.	1	0.88	0.94	0.95	2.23	1.84	1.81	1.35	1.69	0.00
time (sec)	N/A	0.428	0.239	0.108	0.117	0.149	43.042	0.169	0.231	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	105	99	178	192	150	111	105	99
N.S.	1	0.98	1.17	1.10	1.98	2.13	1.67	1.23	1.17	1.10
time (sec)	N/A	0.337	0.297	0.079	0.107	0.092	24.727	0.130	0.217	5.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	80	82	109	146	66	79	68	59
N.S.	1	0.97	1.36	1.39	1.85	2.47	1.12	1.34	1.15	1.00
time (sec)	N/A	0.313	0.070	0.062	0.111	0.121	29.293	0.127	0.185	4.726

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	76	69	54	130	71	66	55	35
N.S.	1	1.12	1.77	1.60	1.26	3.02	1.65	1.53	1.28	0.81
time (sec)	N/A	0.298	0.082	0.069	0.109	0.135	6.932	0.132	0.213	4.346

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	39	44	48	39	65	124	68	35
N.S.	1	1.12	0.91	1.02	1.12	0.91	1.51	2.88	1.58	0.81
time (sec)	N/A	0.314	0.083	0.075	0.032	0.075	1.507	0.318	0.180	3.914

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	60	67	83	62	100	180	110	58
N.S.	1	1.06	0.83	0.93	1.15	0.86	1.39	2.50	1.53	0.81
time (sec)	N/A	0.355	0.103	0.083	0.033	0.104	1.836	0.378	0.224	3.884

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	93	91	118	86	133	236	152	102
N.S.	1	1.05	0.92	0.90	1.17	0.85	1.32	2.34	1.50	1.01
time (sec)	N/A	0.395	0.138	0.092	0.031	0.089	3.045	0.735	0.237	3.721

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	56	62	85	59	338	99	51	53
N.S.	1	0.99	0.68	0.76	1.04	0.72	4.12	1.21	0.62	0.65
time (sec)	N/A	0.336	0.056	0.074	0.039	0.072	2.793	0.118	0.186	4.399

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	39	49	36	70	66	28	67
N.S.	1	1.00	0.67	0.76	0.96	0.71	1.37	1.29	0.55	1.31
time (sec)	N/A	0.280	0.033	0.065	0.035	0.077	2.134	0.125	0.185	4.078

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	58	125	39	80	74	65
N.S.	1	1.00	1.51	1.55	1.23	2.66	0.83	1.70	1.57	1.38
time (sec)	N/A	0.289	0.050	0.062	0.104	0.102	2.054	0.116	0.228	4.146

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	80	93	121	138	66	64	144	94
N.S.	1	1.00	1.31	1.52	1.98	2.26	1.08	1.05	2.36	1.54
time (sec)	N/A	0.321	0.081	0.079	0.109	0.106	3.107	0.135	0.197	4.657

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	92	101	112	200	195	150	125	186	0
N.S.	1	0.99	1.09	1.20	2.15	2.10	1.61	1.34	2.00	0.00
time (sec)	N/A	0.356	0.177	0.079	0.109	0.091	7.521	0.146	0.197	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	113	159	140	215	304	177	144	257	134
N.S.	1	0.99	1.39	1.23	1.89	2.67	1.55	1.26	2.25	1.18
time (sec)	N/A	0.367	0.355	0.112	0.116	0.104	66.641	0.139	0.192	5.217

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	139	114	144	249	264	105	209	90
N.S.	1	0.99	1.67	1.37	1.73	3.00	3.18	1.27	2.52	1.08
time (sec)	N/A	0.337	0.221	0.092	0.112	0.108	28.970	0.128	0.215	4.665

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	58	75	75	69	200	75	69	133	54
N.S.	1	1.12	1.44	1.44	1.33	3.85	1.44	1.33	2.56	1.04
time (sec)	N/A	0.303	0.088	0.069	0.104	0.093	7.254	0.128	0.207	4.265

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	36	46	46	46	68	66	101	46
N.S.	1	1.10	0.86	1.10	1.10	1.10	1.62	1.57	2.40	1.10
time (sec)	N/A	0.316	0.077	0.082	0.026	0.081	0.795	0.187	0.197	3.477

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	60	69	81	73	85	188	140	66
N.S.	1	1.09	0.88	1.01	1.19	1.07	1.25	2.76	2.06	0.97
time (sec)	N/A	0.355	0.110	0.092	0.033	0.100	3.699	0.400	0.211	3.731

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	81	94	116	98	114	303	185	91
N.S.	1	1.04	0.81	0.94	1.16	0.98	1.14	3.03	1.85	0.91
time (sec)	N/A	0.396	0.141	0.097	0.032	0.083	4.332	0.864	0.218	4.025

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	104	118	151	121	146	414	227	154
N.S.	1	1.05	0.83	0.94	1.20	0.96	1.16	3.29	1.80	1.22
time (sec)	N/A	0.443	0.157	0.114	0.032	0.091	5.057	1.492	0.189	4.174

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	107	80	91	128	95	561	146	84	79
N.S.	1	0.96	0.71	0.81	1.14	0.85	5.01	1.30	0.75	0.71
time (sec)	N/A	0.388	0.067	0.100	0.032	0.116	4.644	0.118	0.220	4.933

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	77	57	66	90	70	267	106	59	81
N.S.	1	0.97	0.72	0.84	1.14	0.89	3.38	1.34	0.75	1.03
time (sec)	N/A	0.334	0.048	0.088	0.037	0.121	4.182	0.127	0.201	4.224

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	53	47	65	62	36	38
N.S.	1	1.00	0.73	0.96	1.18	1.04	1.44	1.38	0.80	0.84
time (sec)	N/A	0.296	0.033	0.079	0.035	0.081	4.006	0.117	0.185	3.988

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	80	189	206	108	165	60
N.S.	1	1.00	1.20	1.34	1.36	3.20	3.49	1.83	2.80	1.02
time (sec)	N/A	0.326	0.068	0.069	0.126	0.092	6.005	0.123	0.190	4.167

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	91	127	162	242	262	107	325	0
N.S.	1	0.97	1.02	1.43	1.82	2.72	2.94	1.20	3.65	0.00
time (sec)	N/A	0.363	0.164	0.098	0.105	0.108	9.796	0.139	0.205	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	117	113	155	243	308	180	148	373	0
N.S.	1	0.98	0.94	1.29	2.02	2.57	1.50	1.23	3.11	0.00
time (sec)	N/A	0.406	0.189	0.108	0.108	0.109	18.221	0.151	0.214	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.403	0.207	0.000	0.000	0.000	0.000	0.000	0.335	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	1199	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	14.27	0.00
time (sec)	N/A	0.430	0.178	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	419	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	5.30	0.00
time (sec)	N/A	0.376	0.054	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.358	0.201	0.000	0.000	0.000	0.000	0.000	5.253	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	0.326	0.000	0.000	0.000	0.000	0.000	8.099	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	100	0	0	0	0	0	1179	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	11.91	0.00
time (sec)	N/A	0.391	0.178	0.000	0.000	0.000	0.000	0.000	0.344	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	100	0	0	0	0	0	0	0
N.S.	1	1.01	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	0.171	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	95	0	0	0	0	0	35	0
N.S.	1	1.01	0.99	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.395	0.174	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	85	110	0	0	0	0	0	0	0
N.S.	1	1.21	1.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	0.068	0.000	0.000	0.000	0.000	0.000	0.277	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	145	125	0	0	0	0	0	0	0
N.S.	1	1.08	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.425	5.209	0.000	0.000	0.000	0.000	0.000	1.211	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	230	232	206	0	0	0	0	0	24	0
N.S.	1	1.01	0.90	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.633	5.345	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.504	0.412	0.000	0.000	0.000	0.000	0.000	5.839	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	432	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	4.75	0.00
time (sec)	N/A	0.494	0.321	0.000	0.000	0.000	0.000	0.000	1.208	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.488	0.341	0.000	0.000	0.000	0.000	0.000	3.417	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.411	0.000	0.000	0.000	0.000	0.000	12.803	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.399	0.000	0.000	0.000	0.000	0.000	47.495	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0	1332	0
N.S.	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	12.69	0.00
time (sec)	N/A	0.456	0.237	0.000	0.000	0.000	0.000	0.000	0.470	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	67	68	0	0	0	0	0	39	0
N.S.	1	0.13	0.13	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.356	10.039	0.000	0.000	0.000	0.000	0.000	0.281	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	67	126	0	0	0	0	0	39	0
N.S.	1	0.41	0.78	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.360	8.339	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	67	218	0	0	0	0	0	37	0
N.S.	1	0.31	1.02	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.344	5.773	0.000	0.000	0.000	0.000	0.000	0.273	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	47	29	0	121	0	35	78	0
N.S.	1	1.00	1.52	0.94	0.00	3.90	0.00	1.13	2.52	0.00
time (sec)	N/A	0.253	0.065	0.587	0.000	0.132	0.000	0.130	0.267	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	27	38	27	29	41	25	33
N.S.	1	1.00	1.00	0.71	1.00	0.71	0.76	1.08	0.66	0.87
time (sec)	N/A	0.304	0.009	0.112	0.025	0.086	0.203	0.118	0.283	3.984

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	61	61	46	49	47	37	54	47	38
N.S.	1	1.30	1.30	0.98	1.04	1.00	0.79	1.15	1.00	0.81
time (sec)	N/A	0.358	0.009	0.067	0.031	0.122	0.145	0.125	0.294	0.112

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	46	49	47	37	54	47	38
N.S.	1	1.00	1.00	0.75	0.80	0.77	0.61	0.89	0.77	0.62
time (sec)	N/A	0.316	0.009	0.085	0.025	0.104	0.161	0.122	0.251	3.242

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	176	0	174	0	48	133	0
N.S.	1	1.00	1.29	2.98	0.00	2.95	0.00	0.81	2.25	0.00
time (sec)	N/A	0.374	0.086	0.286	0.000	0.146	0.000	0.134	0.248	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	176	0	174	0	48	133	0
N.S.	1	1.00	1.29	2.98	0.00	2.95	0.00	0.81	2.25	0.00
time (sec)	N/A	0.410	0.063	0.086	0.000	0.107	0.000	0.135	0.260	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	67	96	0	0	0	0	0	68	0
N.S.	1	0.25	0.36	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.413	11.057	0.000	0.000	0.000	0.000	0.000	0.299	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	67	96	0	0	0	0	0	68	0
N.S.	1	0.25	0.36	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.460	11.072	0.000	0.000	0.000	0.000	0.000	0.257	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	47	50	45	37	119	59	36	58
N.S.	1	1.05	0.63	0.67	0.60	0.49	1.59	0.79	0.48	0.77
time (sec)	N/A	0.317	0.684	0.184	0.109	0.093	35.451	0.146	0.280	4.489

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	25	19	24	102	24	22	36
N.S.	1	1.00	1.00	0.74	0.56	0.71	3.00	0.71	0.65	1.06
time (sec)	N/A	0.267	0.389	0.374	0.108	0.105	2.871	0.142	0.274	4.113

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	A	A	C	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	79	67	35	56	73	0	36	82
N.S.	1	1.00	2.03	1.72	0.90	1.44	1.87	0.00	0.92	2.10
time (sec)	N/A	0.300	0.279	0.160	0.044	0.108	18.060	0.000	0.280	5.445

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	77	0	61	94	0	370	72	344
N.S.	1	1.05	0.96	0.00	0.76	1.18	0.00	4.62	0.90	4.30
time (sec)	N/A	0.353	0.477	0.000	0.105	0.099	0.000	0.380	0.464	8.069

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	309	317	73	0	0	67	141	0	93	0
N.S.	1	1.03	0.24	0.00	0.00	0.22	0.46	0.00	0.30	0.00
time (sec)	N/A	0.617	7.346	0.000	0.000	0.153	103.154	0.000	0.462	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	73	0	0	53	138	0	64	0
N.S.	1	1.00	0.27	0.00	0.00	0.20	0.51	0.00	0.24	0.00
time (sec)	N/A	0.503	5.729	0.000	0.000	0.089	5.960	0.000	0.366	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	66	0	0	23	114	0	34	0
N.S.	1	1.00	0.29	0.00	0.00	0.10	0.50	0.00	0.15	0.00
time (sec)	N/A	0.429	4.269	0.000	0.000	0.144	3.366	0.000	0.311	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	73	0	0	51	107	0	38	0
N.S.	1	1.00	0.27	0.00	0.00	0.19	0.39	0.00	0.14	0.00
time (sec)	N/A	0.506	5.646	0.000	0.000	0.083	15.342	0.000	0.334	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	319	73	0	0	69	0	0	38	0
N.S.	1	1.03	0.23	0.00	0.00	0.22	0.00	0.00	0.12	0.00
time (sec)	N/A	0.602	7.044	0.000	0.000	0.108	0.000	0.000	0.347	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	579	601	73	0	0	65	141	0	68	0
N.S.	1	1.04	0.13	0.00	0.00	0.11	0.24	0.00	0.12	0.00
time (sec)	N/A	1.025	6.540	0.000	0.000	0.091	15.348	0.000	0.432	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	533	554	73	0	0	33	121	0	35	0
N.S.	1	1.04	0.14	0.00	0.00	0.06	0.23	0.00	0.07	0.00
time (sec)	N/A	0.933	3.861	0.000	0.000	0.115	2.957	0.000	0.266	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	573	599	71	0	0	59	107	0	38	0
N.S.	1	1.05	0.12	0.00	0.00	0.10	0.19	0.00	0.07	0.00
time (sec)	N/A	1.027	4.992	0.000	0.000	0.100	5.405	0.000	0.361	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	143	99	75	57	62	0	162	83	831
N.S.	1	1.06	0.73	0.56	0.42	0.46	0.00	1.20	0.61	6.16
time (sec)	N/A	0.438	1.376	0.145	0.026	0.143	0.000	0.140	0.270	49.472

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	87	65	47	57	0	127	66	632
N.S.	1	1.03	0.84	0.62	0.45	0.55	0.00	1.22	0.63	6.08
time (sec)	N/A	0.370	1.151	0.085	0.031	0.085	0.000	0.139	0.246	27.174

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	404	52	37	52	0	92	49	0
N.S.	1	1.01	5.53	0.71	0.51	0.71	0.00	1.26	0.67	0.00
time (sec)	N/A	0.323	1.406	0.081	0.029	0.100	0.000	0.140	0.199	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	264	72	26	46	61	57	33	41
N.S.	1	1.00	7.14	1.95	0.70	1.24	1.65	1.54	0.89	1.11
time (sec)	N/A	0.271	1.136	0.088	0.031	0.091	1.743	0.129	0.218	3.994

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	69	184	47	27	55	0	0	44	129
N.S.	1	1.82	4.84	1.24	0.71	1.45	0.00	0.00	1.16	3.39
time (sec)	N/A	0.343	0.958	0.091	0.102	0.090	0.000	0.000	0.198	5.141

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	421	23	10	30	0	48	38	31
N.S.	1	1.00	13.58	0.74	0.32	0.97	0.00	1.55	1.23	1.00
time (sec)	N/A	0.249	1.945	0.093	0.102	0.103	0.000	0.120	0.220	3.899

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	693	28	21	37	0	90	56	43
N.S.	1	1.00	11.00	0.44	0.33	0.59	0.00	1.43	0.89	0.68
time (sec)	N/A	0.279	8.071	0.092	0.102	0.109	0.000	0.133	0.215	3.771

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	41	33	31	44	0	111	72	55
N.S.	1	1.05	0.44	0.35	0.33	0.47	0.00	1.18	0.77	0.59
time (sec)	N/A	0.330	10.041	0.093	0.103	0.086	0.000	0.139	0.211	3.868

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	135	46	38	41	49	0	132	87	67
N.S.	1	1.08	0.37	0.30	0.33	0.39	0.00	1.06	0.70	0.54
time (sec)	N/A	0.379	10.057	0.105	0.107	0.086	0.000	0.139	0.210	3.911

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	88	65	47	57	0	76	66	632
N.S.	1	1.03	0.85	0.62	0.45	0.55	0.00	0.73	0.63	6.08
time (sec)	N/A	0.373	1.163	0.138	0.031	0.087	0.000	0.123	0.177	23.791

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	71	406	55	37	52	0	59	49	429
N.S.	1	0.97	5.56	0.75	0.51	0.71	0.00	0.81	0.67	5.88
time (sec)	N/A	0.324	1.518	0.102	0.024	0.094	0.000	0.107	0.205	16.816

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	265	41	24	46	0	39	33	0
N.S.	1	1.00	7.57	1.17	0.69	1.31	0.00	1.11	0.94	0.00
time (sec)	N/A	0.267	1.407	0.095	0.034	0.101	0.000	0.119	0.212	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	40	16	27	0	20	19	6
N.S.	1	1.00	4.75	5.00	2.00	3.38	0.00	2.50	2.38	0.75
time (sec)	N/A	0.223	0.860	0.100	0.025	0.088	0.000	0.110	0.200	4.068

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	146	20	10	25	0	25	20	19
N.S.	1	1.00	5.03	0.69	0.34	0.86	0.00	0.86	0.69	0.66
time (sec)	N/A	0.235	0.906	0.106	0.103	0.084	0.000	0.112	0.204	4.224

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	407	25	21	34	0	48	39	33
N.S.	1	1.00	6.46	0.40	0.33	0.54	0.00	0.76	0.62	0.52
time (sec)	N/A	0.275	1.976	0.106	0.104	0.092	0.000	0.118	0.188	4.283

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	41	30	31	39	0	69	57	43
N.S.	1	1.05	0.44	0.32	0.33	0.41	0.00	0.73	0.61	0.46
time (sec)	N/A	0.327	1.126	0.103	0.102	0.092	0.000	0.124	0.203	4.312

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	0	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.291	0.145	0.000	0.000	0.000	0.000	0.000	16.236	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	45	47	0	0	0	0	0	26	0
N.S.	1	0.92	0.96	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.333	0.195	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.176	0.000	0.000	0.000	0.000	0.000	42.809	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	105	105	0	0	0	0	0	0	0
N.S.	1	0.96	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.417	0.211	0.000	0.000	0.000	0.000	0.000	41.908	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	46	51	82	274	107	72	45
N.S.	1	1.00	1.00	1.02	1.13	1.82	6.09	2.38	1.60	1.00
time (sec)	N/A	0.313	0.081	0.165	0.030	0.093	0.530	0.132	0.195	3.506

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	45	50	75	255	103	70	45
N.S.	1	1.00	0.89	0.98	1.09	1.63	5.54	2.24	1.52	0.98
time (sec)	N/A	0.316	0.070	0.129	0.035	0.096	0.365	0.126	0.189	3.493

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	39	48	69	236	83	66	38
N.S.	1	1.00	0.92	0.98	1.20	1.72	5.90	2.08	1.65	0.95
time (sec)	N/A	0.290	0.069	0.116	0.028	0.087	0.301	0.122	0.186	3.435

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	33	44	0	30	32
N.S.	1	1.00	1.06	1.00	1.06	0.97	1.29	0.00	0.88	0.94
time (sec)	N/A	0.288	0.031	0.123	0.025	0.105	0.159	0.000	0.251	4.217

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	45	0	70	248	0	71	45
N.S.	1	1.00	0.85	0.96	0.00	1.49	5.28	0.00	1.51	0.96
time (sec)	N/A	0.322	0.084	0.139	0.000	0.080	0.347	0.000	0.187	3.510

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	44	0	69	313	0	70	45
N.S.	1	1.00	0.79	0.83	0.00	1.30	5.91	0.00	1.32	0.85
time (sec)	N/A	0.308	0.082	0.144	0.000	0.082	0.320	0.000	0.195	3.935

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	45	45	0	74	332	0	73	45
N.S.	1	1.00	0.92	0.92	0.00	1.51	6.78	0.00	1.49	0.92
time (sec)	N/A	0.315	0.085	0.141	0.000	0.086	0.548	0.000	0.204	3.537

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	76	96	189	770	275	151	80
N.S.	1	1.00	0.92	1.00	1.26	2.49	10.13	3.62	1.99	1.05
time (sec)	N/A	0.378	0.121	0.256	0.027	0.113	0.686	0.126	0.202	3.734

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	76	93	189	767	275	151	80
N.S.	1	1.00	0.92	1.00	1.22	2.49	10.09	3.62	1.99	1.05
time (sec)	N/A	0.373	0.101	0.240	0.031	0.087	0.531	0.130	0.210	3.677

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	68	94	175	726	232	146	71
N.S.	1	1.00	1.00	0.97	1.34	2.50	10.37	3.31	2.09	1.01
time (sec)	N/A	0.348	0.105	0.211	0.056	0.088	0.552	0.129	0.235	3.754

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	52	59	62	70	60	80	0	44	61
N.S.	1	0.95	1.07	1.13	1.27	1.09	1.45	0.00	0.80	1.11
time (sec)	N/A	0.316	0.059	0.210	0.031	0.113	0.177	0.000	0.213	3.702

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	75	0	177	785	0	148	80
N.S.	1	1.00	0.87	0.96	0.00	2.27	10.06	0.00	1.90	1.03
time (sec)	N/A	0.378	0.128	0.231	0.000	0.081	0.483	0.000	0.207	3.698

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	76	0	179	955	0	152	80
N.S.	1	1.00	0.85	0.90	0.00	2.13	11.37	0.00	1.81	0.95
time (sec)	N/A	0.382	0.123	0.233	0.000	0.084	0.599	0.000	0.222	3.679

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	76	0	179	959	0	152	80
N.S.	1	1.00	0.85	0.90	0.00	2.13	11.42	0.00	1.81	0.95
time (sec)	N/A	0.389	0.124	0.230	0.000	0.093	0.621	0.000	0.191	3.793

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	0	0	0	162	0	5	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	2.89	0.00	0.09	0.00
time (sec)	N/A	0.307	0.078	0.000	0.000	0.000	1.789	0.000	0.189	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	0	0	0	162	0	5	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	2.89	0.00	0.09	0.00
time (sec)	N/A	0.298	0.064	0.000	0.000	0.000	1.720	0.000	0.192	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	110	0	1	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.62	0.00	0.02	0.00
time (sec)	N/A	0.280	0.090	0.000	0.000	0.000	1.219	0.000	0.186	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	38	38	38	50	34	83	0	2	35
N.S.	1	1.09	1.09	1.09	1.43	0.97	2.37	0.00	0.06	1.00
time (sec)	N/A	0.323	0.043	0.104	0.034	0.091	0.562	0.000	0.183	3.621

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	73	0	0	0	0	0	5	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.307	0.082	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	0	0	0	0	0	5	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.315	0.074	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	76	0	0	0	0	0	5	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.319	0.082	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	767	0	15	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	9.13	0.00	0.18	0.00
time (sec)	N/A	0.361	0.075	0.000	0.000	0.000	6.193	0.000	0.191	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	767	0	13	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	9.13	0.00	0.15	0.00
time (sec)	N/A	0.355	0.067	0.000	0.000	0.000	4.266	0.000	0.199	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	72	56	0	0	0	741	0	11	0
N.S.	1	1.01	0.79	0.00	0.00	0.00	10.44	0.00	0.15	0.00
time (sec)	N/A	0.335	0.051	0.000	0.000	0.000	3.844	0.000	0.216	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	47	54	63	73	248	0	22	52
N.S.	1	1.00	0.90	1.04	1.21	1.40	4.77	0.00	0.42	1.00
time (sec)	N/A	0.350	0.077	0.122	0.035	0.086	1.107	0.000	0.187	3.612

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	0	780	0	18	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	9.63	0.00	0.22	0.00
time (sec)	N/A	0.373	0.080	0.000	0.000	0.000	12.449	0.000	0.210	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	821	0	18	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	9.66	0.00	0.21	0.00
time (sec)	N/A	0.352	0.087	0.000	0.000	0.000	27.407	0.000	0.219	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	0	821	0	18	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	9.66	0.00	0.21	0.00
time (sec)	N/A	0.353	0.078	0.000	0.000	0.000	49.214	0.000	0.208	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	75	0	0	0	2360	0	28	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	27.13	0.00	0.32	0.00
time (sec)	N/A	0.351	0.081	0.000	0.000	0.000	68.190	0.000	0.268	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	75	0	0	0	2315	0	26	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	26.92	0.00	0.30	0.00
time (sec)	N/A	0.348	0.069	0.000	0.000	0.000	49.539	0.000	0.187	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	78	58	0	0	0	2319	0	24	0
N.S.	1	1.28	0.95	0.00	0.00	0.00	38.02	0.00	0.39	0.00
time (sec)	N/A	0.335	0.061	0.000	0.000	0.000	49.664	0.000	0.192	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	69	64	68	106	133	542	0	61	83
N.S.	1	0.96	0.89	0.94	1.47	1.85	7.53	0.00	0.85	1.15
time (sec)	N/A	0.364	0.102	0.154	0.036	0.090	2.041	0.000	0.197	3.715

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0	34	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.361	0.088	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	76	0	0	0	0	0	34	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.356	0.086	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	0	0	0	0	0	34	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.352	0.087	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	243	58	83	0	71	75	50
N.S.	1	1.00	1.00	4.42	1.05	1.51	0.00	1.29	1.36	0.91
time (sec)	N/A	0.314	0.173	0.198	0.030	0.093	0.000	0.137	0.213	3.970

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	243	58	83	0	71	75	50
N.S.	1	1.00	1.00	4.42	1.05	1.51	0.00	1.29	1.36	0.91
time (sec)	N/A	0.322	0.163	0.165	0.031	0.098	0.000	0.134	0.188	3.958

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	243	58	83	301	71	75	50
N.S.	1	1.00	1.00	4.42	1.05	1.51	5.47	1.29	1.36	0.91
time (sec)	N/A	0.312	0.203	0.138	0.032	0.096	16.262	0.130	0.187	4.555

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	209	58	83	126	71	73	50
N.S.	1	1.00	1.00	3.80	1.05	1.51	2.29	1.29	1.33	0.91
time (sec)	N/A	0.319	0.154	0.144	0.027	0.101	1.397	0.126	0.257	3.988

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	240	58	77	134	58	69	50
N.S.	1	1.00	0.98	4.53	1.09	1.45	2.53	1.09	1.30	0.94
time (sec)	N/A	0.320	0.130	0.139	0.030	0.116	1.806	0.121	0.197	4.183

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	243	0	83	366	0	74	52
N.S.	1	1.00	1.04	4.58	0.00	1.57	6.91	0.00	1.40	0.98
time (sec)	N/A	0.320	0.262	0.139	0.000	0.106	1.289	0.000	0.209	4.418

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	57	243	0	87	389	0	77	52
N.S.	1	1.00	1.04	4.42	0.00	1.58	7.07	0.00	1.40	0.95
time (sec)	N/A	0.317	0.175	0.142	0.000	0.092	6.349	0.000	0.188	4.407

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	57	243	0	87	389	0	77	52
N.S.	1	1.00	1.04	4.42	0.00	1.58	7.07	0.00	1.40	0.95
time (sec)	N/A	0.316	0.176	0.143	0.000	0.090	25.563	0.000	0.196	4.328

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	621	104	191	0	123	154	85
N.S.	1	1.00	0.99	7.06	1.18	2.17	0.00	1.40	1.75	0.97
time (sec)	N/A	0.392	0.281	0.447	0.031	0.104	0.000	0.139	0.189	4.198

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	87	621	104	191	842	123	154	85
N.S.	1	1.00	0.99	7.06	1.18	2.17	9.57	1.40	1.75	0.97
time (sec)	N/A	0.379	0.305	0.284	0.032	0.098	177.029	0.135	0.216	4.222

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	564	104	190	218	123	152	85
N.S.	1	1.00	0.97	6.27	1.16	2.11	2.42	1.37	1.69	0.94
time (sec)	N/A	0.361	0.266	0.285	0.032	0.119	1.957	0.139	0.189	4.043

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	614	104	184	233	104	148	85
N.S.	1	1.00	0.98	7.14	1.21	2.14	2.71	1.21	1.72	0.99
time (sec)	N/A	0.374	0.213	0.279	0.031	0.107	3.032	0.131	0.203	4.132

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	85	611	0	189	190	0	150	99
N.S.	1	1.00	0.99	7.10	0.00	2.20	2.21	0.00	1.74	1.15
time (sec)	N/A	0.364	0.329	0.279	0.000	0.100	1.323	0.000	0.205	4.437

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	619	0	194	1112	0	156	99
N.S.	1	1.00	0.98	6.88	0.00	2.16	12.36	0.00	1.73	1.10
time (sec)	N/A	0.366	0.281	0.282	0.000	0.099	7.118	0.000	0.256	4.317

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	621	0	195	0	0	156	99
N.S.	1	1.00	1.02	7.06	0.00	2.22	0.00	0.00	1.77	1.12
time (sec)	N/A	0.378	0.273	0.283	0.000	0.135	0.000	0.000	0.181	4.358

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	1249	150	339	0	175	265	115
N.S.	1	1.00	0.98	10.32	1.24	2.80	0.00	1.45	2.19	0.95
time (sec)	N/A	0.464	0.368	1.029	0.034	0.129	0.000	0.177	0.176	4.283

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	1249	150	339	0	175	265	115
N.S.	1	1.00	0.99	10.50	1.26	2.85	0.00	1.47	2.23	0.97
time (sec)	N/A	0.417	0.299	1.042	0.028	0.144	0.000	0.148	0.186	4.419

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	1054	150	339	0	175	265	115
N.S.	1	1.00	0.99	8.86	1.26	2.85	0.00	1.47	2.23	0.97
time (sec)	N/A	0.426	0.362	0.550	0.042	0.103	0.000	0.144	0.200	4.218

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	1190	150	338	309	175	263	115
N.S.	1	1.00	0.98	9.83	1.24	2.79	2.55	1.45	2.17	0.95
time (sec)	N/A	0.417	0.317	0.559	0.034	0.124	2.155	0.141	0.211	4.244

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	115	1241	150	330	332	150	259	115
N.S.	1	1.00	0.98	10.61	1.28	2.82	2.84	1.28	2.21	0.98
time (sec)	N/A	0.432	0.261	0.523	0.033	0.094	3.611	0.128	0.203	4.330

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	116	1239	0	337	275	0	264	145
N.S.	1	1.00	0.99	10.59	0.00	2.88	2.35	0.00	2.26	1.24
time (sec)	N/A	0.429	0.417	0.534	0.000	0.104	1.715	0.000	0.227	4.668

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	120	1249	0	343	0	0	267	145
N.S.	1	1.00	0.99	10.32	0.00	2.83	0.00	0.00	2.21	1.20
time (sec)	N/A	0.431	0.672	0.537	0.000	0.110	0.000	0.000	0.208	4.531

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	121	1249	0	343	0	0	267	145
N.S.	1	1.00	1.02	10.50	0.00	2.88	0.00	0.00	2.24	1.22
time (sec)	N/A	0.430	0.358	0.539	0.000	0.110	0.000	0.000	0.205	4.452

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	196	0	7	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	3.06	0.00	0.11	0.00
time (sec)	N/A	0.301	0.252	0.000	0.000	0.000	6.369	0.000	0.225	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	196	0	5	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	3.06	0.00	0.08	0.00
time (sec)	N/A	0.308	0.170	0.000	0.000	0.000	3.148	0.000	0.216	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0	4	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.306	0.151	0.000	0.000	0.000	0.000	0.000	0.218	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	0	0	0	0	0	6	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.305	0.240	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	9	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.311	0.206	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	0	0	0	0	0	9	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.329	0.228	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	0	0	0	0	0	15	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.387	0.287	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	0	0	0	932	0	14	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	10.13	0.00	0.15	0.00
time (sec)	N/A	0.362	0.200	0.000	0.000	0.000	23.769	0.000	0.185	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	0	0	0	925	0	16	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	10.51	0.00	0.18	0.00
time (sec)	N/A	0.359	0.191	0.000	0.000	0.000	52.372	0.000	0.187	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	0	0	0	0	0	18	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.355	0.280	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0	22	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.364	0.214	0.000	0.000	0.000	0.000	0.000	0.264	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	0	0	0	0	0	22	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.361	0.221	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	113	0	0	0	0	0	28	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.367	0.344	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	113	0	0	0	0	0	27	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.363	0.225	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	110	0	0	0	0	0	31	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.351	0.214	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	112	0	0	0	0	0	34	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.357	0.322	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	114	0	0	0	0	0	40	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.359	0.261	0.000	0.000	0.000	0.000	0.000	0.189	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	114	0	0	0	0	0	40	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.366	0.270	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	101	0	0	0	116	0	252	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	1.17	0.00	2.55	0.00
time (sec)	N/A	0.397	0.093	0.000	0.000	0.000	2.130	0.000	0.199	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	0	0	0	116	0	252	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.15	0.00	2.50	0.00
time (sec)	N/A	0.391	0.079	0.000	0.000	0.000	1.859	0.000	0.208	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	0	0	0	114	0	241	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	1.27	0.00	2.68	0.00
time (sec)	N/A	0.370	0.072	0.000	0.000	0.000	1.608	0.000	0.197	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	65	65	55	75	130	110	0	60	0
N.S.	1	0.94	0.94	0.80	1.09	1.88	1.59	0.00	0.87	0.00
time (sec)	N/A	0.319	0.082	0.231	0.118	0.121	13.573	0.000	0.202	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	0	0	0	116	0	297	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	1.14	0.00	2.91	0.00
time (sec)	N/A	0.411	0.102	0.000	0.000	0.000	2.272	0.000	0.201	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	0	0	0	117	0	303	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.12	0.00	2.91	0.00
time (sec)	N/A	0.410	0.102	0.000	0.000	0.000	2.042	0.000	0.187	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	0	0	0	117	0	296	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.10	0.00	2.79	0.00
time (sec)	N/A	0.396	0.095	0.000	0.000	0.000	2.157	0.000	0.193	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	250	0	496	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.45	0.00	4.86	0.00
time (sec)	N/A	0.405	0.096	0.000	0.000	0.000	7.514	0.000	0.213	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	0	0	0	250	0	488	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.45	0.00	4.78	0.00
time (sec)	N/A	0.397	0.091	0.000	0.000	0.000	5.376	0.000	0.221	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	0	0	0	248	0	466	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.73	0.00	5.12	0.00
time (sec)	N/A	0.368	0.114	0.000	0.000	0.000	3.980	0.000	0.222	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	83	83	69	90	181	153	0	81	0
N.S.	1	0.93	0.93	0.78	1.01	2.03	1.72	0.00	0.91	0.00
time (sec)	N/A	0.328	0.111	0.084	0.108	0.101	20.647	0.000	0.196	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	0	0	0	246	0	560	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	2.39	0.00	5.44	0.00
time (sec)	N/A	0.412	0.102	0.000	0.000	0.000	4.346	0.000	0.216	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	103	0	0	0	248	0	568	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.36	0.00	5.41	0.00
time (sec)	N/A	0.416	0.112	0.000	0.000	0.000	4.735	0.000	0.206	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	103	0	0	0	248	0	569	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	2.36	0.00	5.42	0.00
time (sec)	N/A	0.410	0.102	0.000	0.000	0.000	6.383	0.000	0.190	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	396	0	818	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	3.81	0.00	7.87	0.00
time (sec)	N/A	0.409	0.112	0.000	0.000	0.000	18.915	0.000	0.188	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	396	0	808	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	3.81	0.00	7.77	0.00
time (sec)	N/A	0.388	0.109	0.000	0.000	0.000	12.032	0.000	0.209	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	0	0	394	0	771	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	4.24	0.00	8.29	0.00
time (sec)	N/A	0.380	0.092	0.000	0.000	0.000	8.450	0.000	0.196	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	101	101	85	106	235	206	0	102	0
N.S.	1	0.91	0.91	0.77	0.95	2.12	1.86	0.00	0.92	0.00
time (sec)	N/A	0.351	0.133	0.099	0.107	0.090	28.374	0.000	0.201	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	389	0	913	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	3.70	0.00	8.70	0.00
time (sec)	N/A	0.413	0.133	0.000	0.000	0.000	8.642	0.000	0.197	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	391	0	923	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	3.65	0.00	8.63	0.00
time (sec)	N/A	0.416	0.131	0.000	0.000	0.000	9.582	0.000	0.230	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	391	0	924	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	3.65	0.00	8.64	0.00
time (sec)	N/A	0.410	0.122	0.000	0.000	0.000	11.817	0.000	0.193	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	101	0	0	0	114	0	99	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	1.18	0.00	1.02	0.00
time (sec)	N/A	0.403	0.097	0.000	0.000	0.000	2.121	0.000	0.187	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	101	0	0	0	114	0	95	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	1.18	0.00	0.98	0.00
time (sec)	N/A	0.396	0.086	0.000	0.000	0.000	1.922	0.000	0.199	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	0	112	0	91	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.30	0.00	1.06	0.00
time (sec)	N/A	0.378	0.071	0.000	0.000	0.000	1.476	0.000	0.223	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	50	42	59	109	80	0	41	0
N.S.	1	0.96	1.00	0.84	1.18	2.18	1.60	0.00	0.82	0.00
time (sec)	N/A	0.291	0.064	0.088	0.108	0.091	3.823	0.000	0.192	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	0	0	0	112	0	119	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	1.10	0.00	1.17	0.00
time (sec)	N/A	0.399	0.103	0.000	0.000	0.000	1.714	0.000	0.182	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	0	0	0	114	0	123	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.10	0.00	1.18	0.00
time (sec)	N/A	0.396	0.101	0.000	0.000	0.000	1.932	0.000	0.173	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	0	0	0	114	0	123	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.10	0.00	1.18	0.00
time (sec)	N/A	0.413	0.097	0.000	0.000	0.000	2.214	0.000	0.253	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	114	0	23	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.10	0.00	0.22	0.00
time (sec)	N/A	0.402	0.112	0.000	0.000	0.000	2.954	0.000	0.181	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	114	0	21	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.10	0.00	0.20	0.00
time (sec)	N/A	0.389	0.095	0.000	0.000	0.000	2.684	0.000	0.189	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	103	77	0	0	0	112	0	20	0
N.S.	1	1.11	0.83	0.00	0.00	0.00	1.20	0.00	0.22	0.00
time (sec)	N/A	0.379	0.073	0.000	0.000	0.000	2.673	0.000	0.185	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	60	53	78	179	78	0	23	0
N.S.	1	0.97	1.00	0.88	1.30	2.98	1.30	0.00	0.38	0.00
time (sec)	N/A	0.309	0.100	0.088	0.112	0.135	21.127	0.000	0.188	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	112	0	27	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	1.11	0.00	0.27	0.00
time (sec)	N/A	0.397	0.118	0.000	0.000	0.000	4.352	0.000	0.173	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	0	0	0	114	0	27	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	1.11	0.00	0.26	0.00
time (sec)	N/A	0.401	0.117	0.000	0.000	0.000	5.318	0.000	0.176	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	0	0	0	114	0	27	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	1.11	0.00	0.26	0.00
time (sec)	N/A	0.386	0.120	0.000	0.000	0.000	6.412	0.000	0.181	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	104	0	0	0	114	0	36	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.00	0.00	0.32	0.00
time (sec)	N/A	0.406	0.118	0.000	0.000	0.000	11.826	0.000	0.216	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	114	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.10	0.00	0.33	0.00
time (sec)	N/A	0.386	0.166	0.000	0.000	0.000	9.505	0.000	0.197	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	108	87	0	0	0	112	0	33	0
N.S.	1	1.16	0.94	0.00	0.00	0.00	1.20	0.00	0.35	0.00
time (sec)	N/A	0.377	0.102	0.000	0.000	0.000	9.587	0.000	0.203	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	81	76	68	90	268	102	0	37	0
N.S.	1	0.99	0.93	0.83	1.10	3.27	1.24	0.00	0.45	0.00
time (sec)	N/A	0.329	0.147	0.085	0.111	0.114	29.030	0.000	0.209	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	0	0	0	112	0	43	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	1.07	0.00	0.41	0.00
time (sec)	N/A	0.407	0.145	0.000	0.000	0.000	47.988	0.000	0.194	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	105	0	0	0	114	0	43	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.07	0.00	0.40	0.00
time (sec)	N/A	0.383	0.141	0.000	0.000	0.000	112.401	0.000	0.186	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	105	105	0	0	0	0	0	43	0
N.S.	1	0.95	0.95	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.384	0.145	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	0	0	150	0	269	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.26	0.00	2.26	0.00
time (sec)	N/A	0.426	0.570	0.000	0.000	0.000	33.532	0.000	0.196	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	118	0	0	0	150	0	251	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	1.29	0.00	2.16	0.00
time (sec)	N/A	0.416	0.460	0.000	0.000	0.000	3.616	0.000	0.227	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	150	0	268	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.30	0.00	2.33	0.00
time (sec)	N/A	0.408	0.401	0.000	0.000	0.000	2.750	0.000	0.195	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	0	151	0	306	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.31	0.00	2.66	0.00
time (sec)	N/A	0.439	0.552	0.000	0.000	0.000	4.154	0.000	0.195	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	118	0	0	0	151	0	311	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.18	0.00	2.43	0.00
time (sec)	N/A	0.433	0.528	0.000	0.000	0.000	24.168	0.000	0.209	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	117	119	0	0	0	0	0	517	0
N.S.	1	0.95	0.97	0.00	0.00	0.00	0.00	0.00	4.20	0.00
time (sec)	N/A	0.393	0.639	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	0	318	0	494	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	2.65	0.00	4.12	0.00
time (sec)	N/A	0.420	0.527	0.000	0.000	0.000	22.005	0.000	0.233	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	0	0	0	318	0	512	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.74	0.00	4.41	0.00
time (sec)	N/A	0.417	0.436	0.000	0.000	0.000	13.920	0.000	0.210	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	116	0	0	0	316	0	578	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	2.72	0.00	4.98	0.00
time (sec)	N/A	0.431	0.944	0.000	0.000	0.000	16.643	0.000	0.229	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	0	316	0	586	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	2.63	0.00	4.88	0.00
time (sec)	N/A	0.428	0.616	0.000	0.000	0.000	123.232	0.000	0.208	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	0	0	0	148	0	104	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.29	0.00	0.90	0.00
time (sec)	N/A	0.428	0.584	0.000	0.000	0.000	19.020	0.000	0.195	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	0	0	0	148	0	99	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.29	0.00	0.86	0.00
time (sec)	N/A	0.420	0.464	0.000	0.000	0.000	2.628	0.000	0.208	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	0	0	0	148	0	104	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.32	0.00	0.93	0.00
time (sec)	N/A	0.414	0.399	0.000	0.000	0.000	2.672	0.000	0.230	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	115	115	0	0	0	148	0	131	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	1.26	0.00	1.12	0.00
time (sec)	N/A	0.424	0.553	0.000	0.000	0.000	6.901	0.000	0.210	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	0	0	148	0	136	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.24	0.00	1.14	0.00
time (sec)	N/A	0.432	0.521	0.000	0.000	0.000	45.106	0.000	0.208	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	0	0	0	148	0	27	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.21	0.00	0.22	0.00
time (sec)	N/A	0.429	0.709	0.000	0.000	0.000	32.372	0.000	0.263	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	121	0	0	0	148	0	25	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.21	0.00	0.20	0.00
time (sec)	N/A	0.419	0.566	0.000	0.000	0.000	5.492	0.000	0.196	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	118	116	0	0	0	148	0	31	0
N.S.	1	0.98	0.97	0.00	0.00	0.00	1.23	0.00	0.26	0.00
time (sec)	N/A	0.415	0.469	0.000	0.000	0.000	10.832	0.000	0.200	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	0	0	0	148	0	35	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	1.29	0.00	0.30	0.00
time (sec)	N/A	0.412	0.757	0.000	0.000	0.000	37.891	0.000	0.213	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	121	0	0	0	0	0	35	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.404	0.664	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	122	121	0	0	0	0	0	40	0
N.S.	1	0.94	0.93	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.428	0.801	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	121	0	0	0	148	0	38	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	1.13	0.00	0.29	0.00
time (sec)	N/A	0.425	0.822	0.000	0.000	0.000	48.802	0.000	0.208	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	118	118	0	0	0	148	0	46	0
N.S.	1	0.93	0.93	0.00	0.00	0.00	1.17	0.00	0.36	0.00
time (sec)	N/A	0.425	0.585	0.000	0.000	0.000	133.066	0.000	0.200	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	0	0	0	49	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.418	0.971	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	119	121	0	0	0	0	0	55	0
N.S.	1	0.95	0.97	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.405	0.791	0.000	0.000	0.000	0.000	0.000	0.236	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	65	16	59	293	78	60	185	41	44
N.S.	1	3.25	0.80	2.95	14.65	3.90	3.00	9.25	2.05	2.20
time (sec)	N/A	0.398	0.149	0.733	0.062	0.086	0.307	0.142	0.212	3.681

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	45	16	40	198	54	39	102	28	30
N.S.	1	2.25	0.80	2.00	9.90	2.70	1.95	5.10	1.40	1.50
time (sec)	N/A	0.329	0.088	0.319	0.055	0.096	0.243	0.131	0.187	3.694

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	25	14	21	104	30	19	43	15	17
N.S.	1	0.89	0.50	0.75	3.71	1.07	0.68	1.54	0.54	0.61
time (sec)	N/A	0.281	0.050	0.148	0.040	0.084	0.178	0.126	0.201	3.551

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	7	8	8	7	5	0	8	7
N.S.	1	1.00	0.64	0.73	0.73	0.64	0.45	0.00	0.73	0.64
time (sec)	N/A	0.227	0.003	0.113	0.066	0.086	0.241	0.000	0.186	3.574

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	77	17	17	21	12	0	17	16
N.S.	1	1.00	3.85	0.85	0.85	1.05	0.60	0.00	0.85	0.80
time (sec)	N/A	0.255	0.164	0.188	0.069	0.072	0.660	0.000	0.191	3.646

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	78	17	30	34	26	0	30	29
N.S.	1	1.00	3.90	0.85	1.50	1.70	1.30	0.00	1.50	1.45
time (sec)	N/A	0.258	0.166	0.364	0.073	0.097	1.448	0.000	0.194	3.677

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	137	106	1576	219	1104	16781	7893	996	563
N.S.	1	0.92	0.71	10.58	1.47	7.41	112.62	52.97	6.68	3.78
time (sec)	N/A	0.499	0.199	1.128	0.055	0.142	6.764	0.176	0.239	4.194

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	102	78	699	155	527	5882	2951	480	265
N.S.	1	0.92	0.70	6.30	1.40	4.75	52.99	26.59	4.32	2.39
time (sec)	N/A	0.424	0.133	0.533	0.045	0.097	3.832	0.146	0.191	3.872

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	66	49	229	91	185	1498	763	180	91
N.S.	1	0.92	0.68	3.18	1.26	2.57	20.81	10.60	2.50	1.26
time (sec)	N/A	0.347	0.063	0.241	0.038	0.107	1.930	0.130	0.185	3.668

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	36	26	38	40	57	194	117	41	28
N.S.	1	0.92	0.67	0.97	1.03	1.46	4.97	3.00	1.05	0.72
time (sec)	N/A	0.290	0.027	0.121	0.033	0.109	0.816	0.122	0.184	3.590

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	57	0	0	0	377	0	13	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	4.90	0.00	0.17	0.00
time (sec)	N/A	0.355	0.088	0.000	0.000	0.000	2.159	0.000	0.236	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	83	0	0	0	2382	0	19	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	22.47	0.00	0.18	0.00
time (sec)	N/A	0.394	0.092	0.000	0.000	0.000	11.208	0.000	0.193	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	83	0	0	0	8303	0	32	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	74.13	0.00	0.29	0.00
time (sec)	N/A	0.403	0.087	0.000	0.000	0.000	112.937	0.000	0.205	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	118	0	0	0	352	0	0	0
N.S.	1	1.11	0.92	0.00	0.00	0.00	2.75	0.00	0.00	0.00
time (sec)	N/A	0.477	0.122	0.000	0.000	0.000	23.225	0.000	0.219	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	141	117	0	0	0	167	0	728	0
N.S.	1	1.11	0.92	0.00	0.00	0.00	1.31	0.00	5.73	0.00
time (sec)	N/A	0.471	0.099	0.000	0.000	0.000	3.877	0.000	0.214	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	125	117	0	0	0	165	0	179	0
N.S.	1	1.02	0.95	0.00	0.00	0.00	1.34	0.00	1.46	0.00
time (sec)	N/A	0.457	0.114	0.000	0.000	0.000	2.704	0.000	0.197	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	139	120	0	0	0	165	0	27	0
N.S.	1	0.97	0.83	0.00	0.00	0.00	1.15	0.00	0.19	0.00
time (sec)	N/A	0.482	0.134	0.000	0.000	0.000	5.549	0.000	0.201	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	120	0	0	0	165	0	40	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	1.15	0.00	0.28	0.00
time (sec)	N/A	0.484	0.194	0.000	0.000	0.000	46.213	0.000	0.223	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	99	0	0	0	109	0	1466	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.09	0.00	14.66	0.00
time (sec)	N/A	0.400	0.087	0.000	0.000	0.000	6.728	0.000	0.211	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	99	0	0	0	109	0	1438	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	1.09	0.00	14.38	0.00
time (sec)	N/A	0.390	0.071	0.000	0.000	0.000	4.364	0.000	0.243	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	94	0	0	0	107	0	1336	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	1.20	0.00	15.01	0.00
time (sec)	N/A	0.393	0.026	0.000	0.000	0.000	3.001	0.000	0.202	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	52	0	0	0	88	0	132	0
N.S.	1	0.97	0.76	0.00	0.00	0.00	1.29	0.00	1.94	0.00
time (sec)	N/A	0.310	0.044	0.000	0.000	0.000	12.335	0.000	0.199	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	0	0	0	110	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.427	0.080	0.000	0.000	0.000	4.256	0.000	0.215	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	100	0	0	0	112	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.431	0.084	0.000	0.000	0.000	6.380	0.000	0.217	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	116	0	0	0	0	0	0	0
N.S.	1	1.10	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.469	0.639	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	136	116	0	0	0	143	0	1519	0
N.S.	1	1.10	0.94	0.00	0.00	0.00	1.15	0.00	12.25	0.00
time (sec)	N/A	0.464	0.424	0.000	0.000	0.000	24.283	0.000	0.263	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	113	0	0	0	143	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.472	0.697	0.000	0.000	0.000	16.523	0.000	0.223	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	113	0	0	0	146	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.448	0.680	0.000	0.000	0.000	38.597	0.000	0.227	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	124	116	0	0	0	0	0	0	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	0.549	0.000	0.000	0.000	0.000	0.000	0.314	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	100	0	0	0	175	0	1313	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	1.77	0.00	13.26	0.00
time (sec)	N/A	0.404	0.119	0.000	0.000	0.000	9.872	0.000	0.204	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	0	0	0	175	0	1099	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	1.79	0.00	11.21	0.00
time (sec)	N/A	0.398	0.113	0.000	0.000	0.000	6.722	0.000	0.193	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	172	0	1193	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	1.89	0.00	13.11	0.00
time (sec)	N/A	0.377	0.070	0.000	0.000	0.000	4.695	0.000	0.204	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	31	16	21	85	16	21	16
N.S.	1	1.00	1.00	1.94	1.00	1.31	5.31	1.00	1.31	1.00
time (sec)	N/A	0.251	0.007	0.825	0.024	0.086	13.634	0.122	0.215	4.373

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	108	94	0	0	0	170	0	1468	0
N.S.	1	1.20	1.04	0.00	0.00	0.00	1.89	0.00	16.31	0.00
time (sec)	N/A	0.431	0.131	0.000	0.000	0.000	21.226	0.000	0.208	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	97	0	0	0	117	0	1619	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.12	0.00	15.57	0.00
time (sec)	N/A	0.398	0.089	0.000	0.000	0.000	6.019	0.000	0.214	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	100	0	0	0	177	0	1652	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	1.65	0.00	15.44	0.00
time (sec)	N/A	0.420	0.134	0.000	0.000	0.000	15.017	0.000	0.211	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	115	0	0	0	160	0	0	0
N.S.	1	1.02	0.94	0.00	0.00	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.465	0.102	0.000	0.000	0.000	34.223	0.000	0.212	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	101	87	0	0	0	82	0	118	0
N.S.	1	1.09	0.94	0.00	0.00	0.00	0.88	0.00	1.27	0.00
time (sec)	N/A	0.423	0.214	0.000	0.000	0.000	64.375	0.000	0.187	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	110	46	36	40	36	48	24	31
N.S.	1	1.00	5.00	2.09	1.64	1.82	1.64	2.18	1.09	1.41
time (sec)	N/A	0.257	0.160	8.722	0.088	0.099	2.571	0.145	0.239	3.610

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	32	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.352	0.081	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	30	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.346	0.073	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	28	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.39	0.00
time (sec)	N/A	0.326	0.021	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	69	58	348	0	57	162
N.S.	1	1.00	0.92	0.92	1.10	0.92	5.52	0.00	0.90	2.57
time (sec)	N/A	0.384	0.063	0.191	0.033	0.084	1.783	0.000	0.180	4.830

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0	40	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.366	0.084	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0	40	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.378	0.086	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	160	135	0	0	0	0	0	60	0
N.S.	1	1.13	0.95	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	0.677	0.147	0.000	0.000	0.000	0.000	0.000	0.199	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	161	134	0	0	0	0	0	58	0
N.S.	1	1.13	0.94	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.618	0.141	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	142	108	0	0	0	0	0	56	0
N.S.	1	1.16	0.89	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.549	0.096	0.000	0.000	0.000	0.000	0.000	0.201	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	98	107	100	151	224	0	0	270	0
N.S.	1	0.97	1.06	0.99	1.50	2.22	0.00	0.00	2.67	0.00
time (sec)	N/A	0.470	0.141	0.258	0.044	0.090	0.000	0.000	0.267	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	158	133	0	0	0	0	0	74	0
N.S.	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.659	0.160	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	163	136	0	0	0	0	0	74	0
N.S.	1	1.12	0.94	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.664	0.155	0.000	0.000	0.000	0.000	0.000	0.183	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	24	21	20	26	19	21	35	12
N.S.	1	1.00	1.50	1.31	1.25	1.62	1.19	1.31	2.19	0.75
time (sec)	N/A	0.253	0.007	0.073	0.031	0.076	0.059	0.116	0.174	0.044

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	24	23	34	20	30	41	17
N.S.	1	1.00	1.42	1.26	1.21	1.79	1.05	1.58	2.16	0.89
time (sec)	N/A	0.247	0.013	0.086	0.027	0.074	0.057	0.118	0.190	0.050

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	117	85	82	75	106	92	77	153	103
N.S.	1	1.21	0.88	0.85	0.77	1.09	0.95	0.79	1.58	1.06
time (sec)	N/A	0.512	0.038	0.122	0.107	0.084	0.229	0.122	0.194	3.729

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	108	47	98	122	61	100	187	47
N.S.	1	1.39	1.23	0.53	1.11	1.39	0.69	1.14	2.12	0.53
time (sec)	N/A	0.600	0.063	0.167	0.109	0.088	145.627	0.120	0.200	0.098

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	119	134	188	231	177	347	0	217	0
N.S.	1	0.92	1.03	1.45	1.78	1.36	2.67	0.00	1.67	0.00
time (sec)	N/A	0.519	0.134	0.979	0.039	0.083	5.131	0.000	0.187	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	82	87	118	150	108	221	0	131	0
N.S.	1	0.91	0.97	1.31	1.67	1.20	2.46	0.00	1.46	0.00
time (sec)	N/A	0.426	0.090	0.380	0.039	0.106	3.217	0.000	0.200	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	55	50	65	83	56	116	0	63	0
N.S.	1	0.92	0.83	1.08	1.38	0.93	1.93	0.00	1.05	0.00
time (sec)	N/A	0.360	0.055	0.187	0.035	0.110	1.906	0.000	0.200	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	44	59	60	45	0	0	44	0
N.S.	1	0.96	0.81	1.09	1.11	0.83	0.00	0.00	0.81	0.00
time (sec)	N/A	0.361	0.059	0.384	0.038	0.087	0.000	0.000	0.231	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	70	75	107	121	120	0	0	128	0
N.S.	1	0.93	1.00	1.43	1.61	1.60	0.00	0.00	1.71	0.00
time (sec)	N/A	0.391	0.094	0.826	0.036	0.098	0.000	0.000	0.198	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	100	157	243	267	0	0	325	0
N.S.	1	0.92	0.95	1.50	2.31	2.54	0.00	0.00	3.10	0.00
time (sec)	N/A	0.436	0.141	1.826	0.040	0.111	0.000	0.000	0.215	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	144	185	342	286	230	428	0	283	0
N.S.	1	0.91	1.17	2.16	1.81	1.46	2.71	0.00	1.79	0.00
time (sec)	N/A	0.573	0.165	0.968	0.041	0.098	7.749	0.000	0.200	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	107	125	157	192	146	277	0	178	0
N.S.	1	0.91	1.06	1.33	1.63	1.24	2.35	0.00	1.51	0.00
time (sec)	N/A	0.472	0.110	0.391	0.042	0.093	5.125	0.000	0.200	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	78	76	91	112	82	150	0	92	0
N.S.	1	0.91	0.88	1.06	1.30	0.95	1.74	0.00	1.07	0.00
time (sec)	N/A	0.401	0.080	0.195	0.036	0.117	3.194	0.000	0.188	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	66	66	78	81	74	0	0	71	0
N.S.	1	0.93	0.93	1.10	1.14	1.04	0.00	0.00	1.00	0.00
time (sec)	N/A	0.388	0.079	0.396	0.036	0.132	0.000	0.000	0.212	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	90	93	125	147	166	0	0	194	0
N.S.	1	0.95	0.98	1.32	1.55	1.75	0.00	0.00	2.04	0.00
time (sec)	N/A	0.443	0.113	0.838	0.043	0.104	0.000	0.000	0.202	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	112	94	169	262	301	0	0	336	0
N.S.	1	0.93	0.78	1.41	2.18	2.51	0.00	0.00	2.80	0.00
time (sec)	N/A	0.486	0.181	1.810	0.041	0.105	0.000	0.000	0.203	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	172	13	154	154	175	13	153	154
N.S.	1	1.00	12.29	0.93	11.00	11.00	12.50	0.93	10.93	11.00
time (sec)	N/A	0.224	0.008	0.102	0.026	0.073	0.063	0.123	0.198	0.149

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	15	156	156	182	156	155	156
N.S.	1	1.00	11.38	0.94	9.75	9.75	11.38	9.75	9.69	9.75
time (sec)	N/A	0.245	0.004	0.209	0.034	0.074	0.071	0.124	0.264	3.824

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	15	156	156	185	156	155	156
N.S.	1	1.00	11.62	0.94	9.75	9.75	11.56	9.75	9.69	9.75
time (sec)	N/A	0.254	0.004	0.280	0.027	0.079	0.071	0.124	0.202	0.145

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	360	189	186	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	17.14	9.00	8.86	10.90
time (sec)	N/A	0.254	0.044	214.131	0.035	0.101	13.793	0.211	0.184	4.068

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	107	50	16	32	39	70	13	25
N.S.	1	1.00	8.23	3.85	1.23	2.46	3.00	5.38	1.00	1.92
time (sec)	N/A	0.251	0.177	8.842	0.093	0.097	2.877	0.162	0.183	3.617

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	9	10	8	11	9	8
N.S.	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.12	1.00
time (sec)	N/A	0.248	0.005	0.092	0.026	0.084	0.099	0.116	0.211	3.517

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	17	13	12	18	13	13
N.S.	1	1.13	1.00	0.93	1.13	0.87	0.80	1.20	0.87	0.87
time (sec)	N/A	0.264	0.006	0.092	0.024	0.112	0.118	0.125	0.201	0.070

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	15	14	17	13	12	15	38	13
N.S.	1	1.13	1.00	0.93	1.13	0.87	0.80	1.00	2.53	0.87
time (sec)	N/A	0.266	0.006	0.085	0.024	0.075	0.213	0.130	0.205	3.542

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	17	19	17	47	17	29	0	17	15
N.S.	1	1.13	1.27	1.13	3.13	1.13	1.93	0.00	1.13	1.00
time (sec)	N/A	0.278	0.029	0.111	0.033	0.099	0.387	0.000	0.182	3.504

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	81	81	87	13	78	12
N.S.	1	1.00	1.00	0.93	5.79	5.79	6.21	0.93	5.57	0.86
time (sec)	N/A	0.215	0.016	0.153	0.126	0.087	0.547	0.121	0.206	5.920

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	80	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	5.00	0.88
time (sec)	N/A	0.239	0.020	0.217	0.038	0.080	0.957	0.123	0.179	2.470

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	81	81	87	15	80	14
N.S.	1	1.00	1.00	0.94	5.06	5.06	5.44	0.94	5.00	0.88
time (sec)	N/A	0.241	0.025	0.193	0.040	0.078	1.099	0.127	0.227	8.768

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	23	612	105	129	0	99	105
N.S.	1	1.00	1.00	1.10	29.14	5.00	6.14	0.00	4.71	5.00
time (sec)	N/A	0.252	0.069	11.903	0.073	0.150	50.850	0.000	0.214	3.648

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	69	53	46	58	56	41	37
N.S.	1	1.00	0.88	1.33	1.02	0.88	1.12	1.08	0.79	0.71
time (sec)	N/A	0.289	0.055	0.686	0.104	0.086	34.768	0.127	21.262	3.510

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	142	143	0	705	0	0	179	0
N.S.	1	1.00	1.53	1.54	0.00	7.58	0.00	0.00	1.92	0.00
time (sec)	N/A	0.396	10.360	0.178	0.000	0.226	0.000	0.000	0.217	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	252	226	223	0	0	607	0	0	1009	0
N.S.	1	0.90	0.88	0.00	0.00	2.41	0.00	0.00	4.00	0.00
time (sec)	N/A	0.504	1.495	0.000	0.000	0.127	0.000	0.000	0.259	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	181	178	0	0	469	0	0	739	0
N.S.	1	0.91	0.89	0.00	0.00	2.36	0.00	0.00	3.71	0.00
time (sec)	N/A	0.468	0.663	0.000	0.000	0.144	0.000	0.000	0.246	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	136	141	0	0	359	0	0	511	0
N.S.	1	0.93	0.97	0.00	0.00	2.46	0.00	0.00	3.50	0.00
time (sec)	N/A	0.408	0.382	0.000	0.000	0.128	0.000	0.000	0.228	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	123	0	0	281	0	0	54	0
N.S.	1	0.98	1.38	0.00	0.00	3.16	0.00	0.00	0.61	0.00
time (sec)	N/A	0.360	0.227	0.000	0.000	0.094	0.000	0.000	0.192	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	89	122	0	0	408	0	0	84	0
N.S.	1	0.98	1.34	0.00	0.00	4.48	0.00	0.00	0.92	0.00
time (sec)	N/A	0.369	0.707	0.000	0.000	0.193	0.000	0.000	0.201	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	93	57	0	0	135	0	0	144	0
N.S.	1	0.98	0.60	0.00	0.00	1.42	0.00	0.00	1.52	0.00
time (sec)	N/A	0.360	0.702	0.000	0.000	0.255	0.000	0.000	0.183	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	356	295	274	0	0	771	0	0	1317	0
N.S.	1	0.83	0.77	0.00	0.00	2.17	0.00	0.00	3.70	0.00
time (sec)	N/A	0.631	2.060	0.000	0.000	0.220	0.000	0.000	0.289	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	288	251	241	0	0	607	0	0	1012	0
N.S.	1	0.87	0.84	0.00	0.00	2.11	0.00	0.00	3.51	0.00
time (sec)	N/A	0.606	0.818	0.000	0.000	0.126	0.000	0.000	0.266	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	205	191	0	0	471	0	0	742	0
N.S.	1	0.94	0.88	0.00	0.00	2.16	0.00	0.00	3.40	0.00
time (sec)	N/A	0.540	0.415	0.000	0.000	0.123	0.000	0.000	0.278	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	151	157	0	0	361	0	0	515	0
N.S.	1	0.98	1.02	0.00	0.00	2.34	0.00	0.00	3.34	0.00
time (sec)	N/A	0.477	0.276	0.000	0.000	0.130	0.000	0.000	0.245	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	131	185	0	0	540	0	0	84	0
N.S.	1	0.98	1.39	0.00	0.00	4.06	0.00	0.00	0.63	0.00
time (sec)	N/A	0.458	0.489	0.000	0.000	0.240	0.000	0.000	0.199	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	163	217	0	0	769	0	0	114	0
N.S.	1	1.11	1.48	0.00	0.00	5.23	0.00	0.00	0.78	0.00
time (sec)	N/A	0.478	1.108	0.000	0.000	0.289	0.000	0.000	0.206	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0	36	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.404	0.200	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	195	141	0	0	0	0	0	64	0
N.S.	1	1.11	0.81	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.753	0.289	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	76	0	0	0	0	0	40	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.385	0.096	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	40	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.375	0.076	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	78	78	0	0	0	0	0	36	0
N.S.	1	0.64	0.64	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.398	0.119	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	357	513	212	0	0	0	0	0	0	0
N.S.	1	1.44	0.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.838	6.694	0.000	0.000	0.000	0.000	0.000	0.673	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	243	160	0	0	0	0	0	0	0
N.S.	1	1.08	0.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.840	6.264	0.000	0.000	0.000	0.000	0.000	0.307	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	124	115	0	0	0	160	0	0	0
N.S.	1	1.02	0.94	0.00	0.00	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.469	0.020	0.000	0.000	0.000	37.596	0.000	0.210	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	0	0	0	73	0	176	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	1.09	0.00	2.63	0.00
time (sec)	N/A	0.318	0.010	0.000	0.000	0.000	8.834	0.000	0.185	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	0	0	0	0	0	28	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.380	0.167	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	78	0	0	0	0	0	41	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.393	0.235	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	133	134	0	0	0	165	0	0	0
N.S.	1	1.04	1.05	0.00	0.00	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.472	0.217	0.000	0.000	0.000	27.099	0.000	0.377	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	123	126	0	0	0	163	0	0	0
N.S.	1	1.02	1.05	0.00	0.00	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.449	0.155	0.000	0.000	0.000	27.352	0.000	0.275	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	114	114	0	0	0	153	0	0	0
N.S.	1	1.02	1.02	0.00	0.00	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.419	0.070	0.000	0.000	0.000	38.168	0.000	0.208	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	120	0	0	0	150	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.479	0.113	0.000	0.000	0.000	26.854	0.000	0.203	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	131	0	0	0	160	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.494	0.115	0.000	0.000	0.000	27.203	0.000	0.257	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	139	134	0	0	0	165	0	0	0
N.S.	1	0.98	0.94	0.00	0.00	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.491	0.109	0.000	0.000	0.000	26.735	0.000	0.326	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	29	25	46	35	22	22
N.S.	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10	1.10
time (sec)	N/A	0.228	0.035	0.346	0.074	0.099	1.313	0.127	0.195	3.695

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	80	52	32	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	2.96	1.93	1.19	1.74
time (sec)	N/A	0.243	0.087	0.811	0.068	0.117	43.673	0.131	0.203	3.755

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	0	56	32	47
N.S.	1	1.00	3.59	0.96	1.30	1.19	0.00	2.07	1.19	1.74
time (sec)	N/A	0.260	0.092	1.789	0.067	0.097	0.000	0.130	0.207	4.097

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	101	60	39	35	116	66	32	54
N.S.	1	1.00	3.74	2.22	1.44	1.30	4.30	2.44	1.19	2.00
time (sec)	N/A	0.263	0.134	2.332	0.094	0.093	28.267	0.151	0.190	3.915

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	1	245	0	0	0	0	0	1192	0
N.S.	1	0.00	0.78	0.00	0.00	0.00	0.00	0.00	3.81	0.00
time (sec)	N/A	0.533	5.613	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	225	1	190	0	0	0	0	0	664	0
N.S.	1	0.00	0.84	0.00	0.00	0.00	0.00	0.00	2.95	0.00
time (sec)	N/A	0.272	5.384	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	232	103	730	0	300	0	0	307	0
N.S.	1	1.45	0.64	4.56	0.00	1.88	0.00	0.00	1.92	0.00
time (sec)	N/A	0.764	0.153	8.196	0.000	0.130	0.000	0.000	0.194	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	164	106	488	0	221	648	0	196	0
N.S.	1	1.20	0.77	3.56	0.00	1.61	4.73	0.00	1.43	0.00
time (sec)	N/A	0.510	0.189	2.205	0.000	0.105	95.260	0.000	0.202	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	114	64	0	0	155	464	0	106	0
N.S.	1	0.98	0.55	0.00	0.00	1.34	4.00	0.00	0.91	0.00
time (sec)	N/A	0.397	0.013	0.000	0.000	0.100	42.669	0.000	0.183	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	347	343	0	0	0	0	0	1143	0
N.S.	1	1.15	1.14	0.00	0.00	0.00	0.00	0.00	3.78	0.00
time (sec)	N/A	7.059	4.371	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	507	556	544	0	0	0	0	0	0	0
N.S.	1	1.10	1.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	9.068	5.788	0.000	0.000	0.000	0.000	0.000	0.564	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	1	194	0	0	0	0	0	698	0
N.S.	1	0.01	1.04	0.00	0.00	0.00	0.00	0.00	3.75	0.00
time (sec)	N/A	0.418	5.271	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	1	147	0	0	0	0	0	308	0
N.S.	1	0.01	1.01	0.00	0.00	0.00	0.00	0.00	2.12	0.00
time (sec)	N/A	0.268	5.218	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	B	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	114	93	269	0	131	184	0	110	0
N.S.	1	1.41	1.15	3.32	0.00	1.62	2.27	0.00	1.36	0.00
time (sec)	N/A	0.419	0.185	2.079	0.000	0.098	91.045	0.000	0.199	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	0	0	99	109	0	66	0
N.S.	1	1.00	0.91	0.00	0.00	1.41	1.56	0.00	0.94	0.00
time (sec)	N/A	0.309	0.014	0.000	0.000	0.098	42.269	0.000	0.195	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	230	227	0	0	0	0	0	662	0
N.S.	1	1.32	1.30	0.00	0.00	0.00	0.00	0.00	3.80	0.00
time (sec)	N/A	2.920	1.994	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	403	395	0	0	0	0	0	0	0
N.S.	1	1.28	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.746	2.080	0.000	0.000	0.000	0.000	0.000	0.327	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	263	195	0	0	0	0	0	583	0
N.S.	1	1.25	0.93	0.00	0.00	0.00	0.00	0.00	2.78	0.00
time (sec)	N/A	0.892	5.389	0.000	0.000	0.000	0.000	0.000	0.196	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	154	149	0	0	0	0	0	349	0
N.S.	1	1.15	1.11	0.00	0.00	0.00	0.00	0.00	2.60	0.00
time (sec)	N/A	0.557	5.210	0.000	0.000	0.000	0.000	0.000	0.219	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	101	87	0	0	0	82	0	118	0
N.S.	1	1.09	0.94	0.00	0.00	0.00	0.88	0.00	1.27	0.00
time (sec)	N/A	0.392	0.071	0.000	0.000	0.000	84.949	0.000	0.207	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	70	0	53	31	0	38	0
N.S.	1	1.00	1.00	2.19	0.00	1.66	0.97	0.00	1.19	0.00
time (sec)	N/A	0.243	0.011	0.532	0.000	0.100	41.899	0.000	0.232	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	143	140	0	0	0	0	0	351	0
N.S.	1	1.49	1.46	0.00	0.00	0.00	0.00	0.00	3.66	0.00
time (sec)	N/A	1.166	0.659	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	297	283	0	0	0	0	0	0	0
N.S.	1	1.55	1.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.601	0.778	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	308	2724	2698	0	0	0	0	0	0	0
N.S.	1	8.84	8.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.646	1.841	0.000	0.000	0.000	0.000	0.000	6.415	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	250	264	203	0	0	0	0	0	570	0
N.S.	1	1.06	0.81	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.884	5.202	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	160	148	0	0	0	0	0	299	0
N.S.	1	0.94	0.87	0.00	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.614	5.177	0.000	0.000	0.000	0.000	0.000	0.202	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	84	98	0	0	0	100	0	136	0
N.S.	1	0.88	1.03	0.00	0.00	0.00	1.05	0.00	1.43	0.00
time (sec)	N/A	0.385	0.124	0.000	0.000	0.000	25.860	0.000	0.206	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	47	58	0	0	0	37	0	22	0
N.S.	1	0.81	1.00	0.00	0.00	0.00	0.64	0.00	0.38	0.00
time (sec)	N/A	0.295	0.008	0.000	0.000	0.000	3.176	0.000	0.211	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	96	88	0	0	0	0	0	175	0
N.S.	1	1.60	1.47	0.00	0.00	0.00	0.00	0.00	2.92	0.00
time (sec)	N/A	0.392	0.087	0.000	0.000	0.000	0.000	0.000	0.246	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	114	110	0	0	0	0	0	0	0
N.S.	1	0.92	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.207	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	196	192	0	0	0	0	0	0	0
N.S.	1	0.87	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.862	0.324	0.000	0.000	0.000	0.000	0.000	2.692	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	186	156	0	0	0	0	0	474	0
N.S.	1	0.99	0.83	0.00	0.00	0.00	0.00	0.00	2.52	0.00
time (sec)	N/A	0.615	5.170	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	109	106	0	0	0	122	0	212	0
N.S.	1	0.92	0.90	0.00	0.00	0.00	1.03	0.00	1.80	0.00
time (sec)	N/A	0.417	0.124	0.000	0.000	0.000	33.492	0.000	0.196	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	57	63	0	0	0	39	0	70	0
N.S.	1	0.86	0.95	0.00	0.00	0.00	0.59	0.00	1.06	0.00
time (sec)	N/A	0.302	0.019	0.000	0.000	0.000	10.065	0.000	0.214	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	78	78	0	0	0	0	0	36	0
N.S.	1	0.64	0.64	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.370	0.055	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	105	103	0	0	0	0	0	0	0
N.S.	1	1.48	1.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	0.090	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	166	162	0	0	0	0	0	0	0
N.S.	1	1.20	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.556	0.164	0.000	0.000	0.000	0.000	0.000	1.667	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	507	502	0	0	0	0	0	0	0
N.S.	1	2.09	2.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.076	0.235	0.000	0.000	0.000	0.000	0.000	35.158	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	191	156	0	0	0	0	0	785	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	4.18	0.00
time (sec)	N/A	0.630	5.217	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	113	106	0	0	0	124	0	384	0
N.S.	1	0.96	0.90	0.00	0.00	0.00	1.05	0.00	3.25	0.00
time (sec)	N/A	0.426	0.205	0.000	0.000	0.000	86.897	0.000	0.229	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	61	64	0	0	0	41	0	144	0
N.S.	1	0.92	0.97	0.00	0.00	0.00	0.62	0.00	2.18	0.00
time (sec)	N/A	0.310	0.077	0.000	0.000	0.000	29.250	0.000	0.273	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	79	78	0	0	0	0	0	356	0
N.S.	1	0.41	0.41	0.00	0.00	0.00	0.00	0.00	1.85	0.00
time (sec)	N/A	0.377	0.179	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	79	78	0	0	0	0	0	56	0
N.S.	1	0.40	0.40	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.389	0.176	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	106	103	0	0	0	0	0	0	0
N.S.	1	1.45	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	0.156	0.000	0.000	0.000	0.000	0.000	0.928	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	313	304	0	0	0	0	0	0	0
N.S.	1	2.24	2.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.987	0.416	0.000	0.000	0.000	0.000	0.000	30.257	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	733	722	0	0	0	0	0	0	0
N.S.	1	2.99	2.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.205	0.978	0.000	0.000	0.000	0.000	0.000	152.068	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	191	156	0	0	0	0	0	1177	0
N.S.	1	1.02	0.83	0.00	0.00	0.00	0.00	0.00	6.26	0.00
time (sec)	N/A	0.623	5.248	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	113	106	0	0	0	0	0	602	0
N.S.	1	0.96	0.90	0.00	0.00	0.00	0.00	0.00	5.10	0.00
time (sec)	N/A	0.428	0.200	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	61	64	0	0	0	41	0	237	0
N.S.	1	0.92	0.97	0.00	0.00	0.00	0.62	0.00	3.59	0.00
time (sec)	N/A	0.305	0.070	0.000	0.000	0.000	60.180	0.000	0.182	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	79	78	0	0	0	0	0	765	0
N.S.	1	0.30	0.29	0.00	0.00	0.00	0.00	0.00	2.87	0.00
time (sec)	N/A	0.376	0.191	0.000	0.000	0.000	0.000	0.000	0.237	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	79	78	0	0	0	0	0	0	0
N.S.	1	0.30	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.181	0.000	0.000	0.000	0.000	0.000	0.275	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	79	78	0	0	0	0	0	74	0
N.S.	1	0.29	0.29	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.383	0.216	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	106	103	0	0	0	0	0	0	0
N.S.	1	1.45	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.432	0.162	0.000	0.000	0.000	0.000	0.000	2.519	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	430	421	0	0	0	0	0	0	0
N.S.	1	3.07	3.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.378	0.575	0.000	0.000	0.000	0.000	0.000	168.113	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	977	966	0	0	0	0	0	31	0
N.S.	1	3.99	3.94	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	3.180	1.410	0.000	0.000	0.000	0.000	0.000	200.046	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	0.162	0.000	0.000	0.000	0.000	0.000	0.283	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.254	0.000	0.000	0.000	0.000	0.000	0.333	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.445	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	112	107	0	0	0	0	0	0	0
N.S.	1	1.06	1.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.477	0.167	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	97	0	0	0	0	0	32	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.445	0.195	0.000	0.000	0.000	0.000	0.000	0.215	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	102	98	0	0	0	0	0	998	0
N.S.	1	0.99	0.95	0.00	0.00	0.00	0.00	0.00	9.69	0.00
time (sec)	N/A	0.503	0.188	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	231	232	195	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.652	0.520	0.000	0.000	0.000	0.000	0.000	0.831	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	145	118	0	0	0	0	0	0	0
N.S.	1	1.07	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.437	0.240	0.000	0.000	0.000	0.000	0.000	0.423	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	85	84	0	0	0	0	0	0	0
N.S.	1	1.21	1.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.360	0.153	0.000	0.000	0.000	0.000	0.000	0.269	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	103	0	0	0	0	0	937	0
N.S.	1	1.01	1.06	0.00	0.00	0.00	0.00	0.00	9.66	0.00
time (sec)	N/A	0.392	0.145	0.000	0.000	0.000	0.000	0.000	0.235	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	98	110	0	0	0	0	0	0	0
N.S.	1	1.01	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.402	0.354	0.000	0.000	0.000	0.000	0.000	0.417	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	110	0	0	0	0	0	0	0
N.S.	1	1.01	1.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	0.522	0.000	0.000	0.000	0.000	0.000	0.817	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	178	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	2.28	0.00
time (sec)	N/A	0.379	0.070	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	170	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	2.43	0.00
time (sec)	N/A	0.346	0.052	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	168	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.30	0.00
time (sec)	N/A	0.339	0.046	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	73	0	0	0	0	0	79	0
N.S.	1	1.01	1.01	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.357	0.063	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0	196	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	2.58	0.00
time (sec)	N/A	0.352	0.056	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	290	183	203	290	675	0	376	358	1044
N.S.	1	1.14	0.72	0.80	1.14	2.65	0.00	1.47	1.40	4.09
time (sec)	N/A	0.835	0.310	1.299	0.115	0.118	0.000	0.139	0.203	3.858

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	344	183	203	290	675	0	376	358	1044
N.S.	1	1.35	0.72	0.80	1.14	2.65	0.00	1.47	1.40	4.09
time (sec)	N/A	1.695	0.007	0.263	0.114	0.099	0.000	0.135	0.202	0.100

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	344	183	203	290	675	0	376	358	1044
N.S.	1	1.35	0.72	0.80	1.14	2.65	0.00	1.47	1.40	4.09
time (sec)	N/A	1.623	0.006	0.260	0.116	0.097	0.000	0.141	0.216	0.094

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	344	183	203	290	675	0	376	358	1044
N.S.	1	1.35	0.72	0.80	1.14	2.65	0.00	1.47	1.40	4.09
time (sec)	N/A	1.408	0.006	0.254	0.112	0.120	0.000	0.141	0.243	3.469

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	235	88	795	0	0	0	0	334	0
N.S.	1	0.96	0.36	3.23	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.461	15.080	2.652	0.000	0.000	0.000	0.000	0.503	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	289	88	795	0	0	0	0	334	0
N.S.	1	1.17	0.36	3.23	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.858	0.059	0.671	0.000	0.000	0.000	0.000	0.482	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	289	88	795	0	0	0	0	334	0
N.S.	1	1.17	0.36	3.23	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	0.846	0.058	0.619	0.000	0.000	0.000	0.000	0.501	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	246	289	88	795	0	0	0	0	334	0
N.S.	1	1.17	0.36	3.23	0.00	0.00	0.00	0.00	1.36	0.00
time (sec)	N/A	1.132	0.060	0.617	0.000	0.000	0.000	0.000	0.510	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	32	15	44	0	0	66	32
N.S.	1	1.00	0.97	0.89	0.42	1.22	0.00	0.00	1.83	0.89
time (sec)	N/A	0.304	1.203	0.129	0.055	0.095	0.000	0.000	0.201	3.787

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	5	35	0	12	8	23
N.S.	1	1.00	1.00	0.83	0.17	1.21	0.00	0.41	0.28	0.79
time (sec)	N/A	0.286	9.524	0.094	0.055	0.122	0.000	0.123	0.202	3.896

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	5	35	0	10	6	23
N.S.	1	1.00	0.93	0.83	0.17	1.21	0.00	0.34	0.21	0.79
time (sec)	N/A	0.269	5.778	0.092	0.051	0.107	0.000	0.123	0.253	3.797

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	22	5	33	0	9	5	21
N.S.	1	1.00	0.96	0.88	0.20	1.32	0.00	0.36	0.20	0.84
time (sec)	N/A	0.253	5.074	0.094	0.048	0.078	0.000	0.127	0.206	3.731

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	5	32	0	9	8	20
N.S.	1	1.00	1.00	0.88	0.21	1.33	0.00	0.38	0.33	0.83
time (sec)	N/A	0.278	1.749	0.096	0.053	0.081	0.000	0.114	0.198	3.828

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	24	5	35	0	12	8	23
N.S.	1	1.00	0.90	0.83	0.17	1.21	0.00	0.41	0.28	0.79
time (sec)	N/A	0.273	3.370	0.095	0.051	0.076	0.000	0.120	0.196	3.836

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	0.000	0.000	0.000	0.000	0.000	0.000	0.241	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	237	175	0	0	0	162	0	664	0
N.S.	1	1.27	0.94	0.00	0.00	0.00	0.87	0.00	3.55	0.00
time (sec)	N/A	0.652	0.193	0.000	0.000	0.000	155.514	0.000	0.209	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	143	112	0	0	0	117	0	351	0
N.S.	1	1.07	0.84	0.00	0.00	0.00	0.87	0.00	2.62	0.00
time (sec)	N/A	0.469	0.154	0.000	0.000	0.000	19.931	0.000	0.221	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	73	0	150	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.92	0.00	1.90	0.00
time (sec)	N/A	0.334	0.108	0.000	0.000	0.000	4.887	0.000	0.204	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	46	51
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.85	0.00	1.15	1.28
time (sec)	N/A	0.267	0.050	0.000	0.000	0.000	1.389	0.000	0.235	3.659

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	24	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.397	0.143	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	1528	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	27.29	0.00
time (sec)	N/A	0.328	0.120	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	112	99	0	0	0	0	0	0	0
N.S.	1	0.99	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	0.150	0.000	0.000	0.000	0.000	0.000	1.015	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	197	155	0	0	0	0	0	0	0
N.S.	1	1.14	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	0.215	0.000	0.000	0.000	0.000	0.000	14.750	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	8	44	0	0	64	29
N.S.	1	1.00	1.00	0.88	0.24	1.33	0.00	0.00	1.94	0.88
time (sec)	N/A	0.351	0.068	0.147	0.042	0.089	0.000	0.000	0.229	3.767

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	26	5	41	0	44	6	25
N.S.	1	1.00	1.03	0.84	0.16	1.32	0.00	1.42	0.19	0.81
time (sec)	N/A	0.314	0.033	0.106	0.046	0.106	0.000	0.124	0.217	3.617

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	7	41	0	0	4	27
N.S.	1	1.00	1.00	0.89	0.25	1.46	0.00	0.00	0.14	0.96
time (sec)	N/A	0.281	0.048	0.112	0.046	0.097	0.000	0.000	0.198	3.596

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	4	57	0	17	47	0
N.S.	1	1.00	1.00	1.07	0.14	2.04	0.00	0.61	1.68	0.00
time (sec)	N/A	0.286	0.056	0.097	0.042	0.110	0.000	0.128	0.205	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	7	41	0	0	5	22
N.S.	1	1.00	1.00	0.88	0.27	1.58	0.00	0.00	0.19	0.85
time (sec)	N/A	0.308	0.054	0.095	0.045	0.086	0.000	0.000	0.244	3.796

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	5	44	0	19	17	25
N.S.	1	1.00	1.00	0.84	0.16	1.42	0.00	0.61	0.55	0.81
time (sec)	N/A	0.308	0.063	0.101	0.042	0.085	0.000	0.119	0.215	3.905

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	719	525	623	0	235	0	0	263	0
N.S.	1	1.24	0.90	1.07	0.00	0.40	0.00	0.00	0.45	0.00
time (sec)	N/A	1.478	25.075	2.360	0.000	0.086	0.000	0.000	1.486	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [451] had the largest ratio of [.65000000000000022]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.27	18	0.167
2	A	5	4	1.32	20	0.200
3	A	4	3	1.00	22	0.136
4	A	4	3	1.00	22	0.136
5	A	5	4	1.00	22	0.182
6	A	6	5	0.98	22	0.227
7	A	6	5	0.95	22	0.227
8	A	5	4	1.00	22	0.182
9	A	2	2	1.00	22	0.091
10	A	3	3	1.00	22	0.136
11	A	5	4	1.00	22	0.182
12	A	4	3	1.00	20	0.150
13	A	5	4	1.00	22	0.182
14	A	6	5	0.99	22	0.227
15	A	3	3	1.01	22	0.136
16	A	2	2	1.00	19	0.105
17	A	3	3	1.00	22	0.136
18	A	4	4	0.99	22	0.182
19	A	6	6	0.97	22	0.273
20	A	6	6	1.00	22	0.273
21	A	6	6	0.97	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	7	7	0.96	22	0.318
23	A	7	6	1.01	22	0.273
24	A	6	5	1.01	22	0.227
25	A	7	6	1.00	22	0.273
26	A	8	7	1.00	22	0.318
27	A	4	3	1.00	22	0.136
28	A	4	3	1.00	22	0.136
29	A	4	3	1.04	22	0.136
30	A	6	5	1.05	22	0.227
31	A	7	6	0.95	22	0.273
32	A	7	6	0.92	22	0.273
33	A	6	5	0.95	22	0.227
34	A	2	2	1.00	22	0.091
35	A	3	3	0.98	22	0.136
36	A	4	4	0.96	22	0.182
37	A	6	5	0.99	22	0.227
38	A	5	4	1.00	22	0.182
39	A	5	4	1.00	20	0.200
40	A	6	5	0.99	22	0.227
41	A	7	6	0.99	22	0.273
42	A	4	4	1.01	22	0.182
43	A	3	3	1.02	22	0.136
44	A	3	3	0.99	19	0.158
45	A	4	4	0.99	22	0.182
46	A	5	5	0.99	22	0.227
47	A	7	7	0.97	22	0.318
48	A	7	7	1.00	22	0.318
49	A	6	6	0.97	22	0.273
50	A	7	7	0.96	22	0.318
51	A	8	8	0.96	22	0.364
52	A	8	7	1.01	22	0.318
53	A	7	6	1.01	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	7	6	1.00	22	0.273
55	A	8	7	1.00	22	0.318
56	A	9	8	1.00	22	0.364
57	A	4	3	1.00	24	0.125
58	A	5	4	1.00	24	0.167
59	A	4	3	1.00	24	0.125
60	A	5	4	1.00	24	0.167
61	A	7	6	1.09	24	0.250
62	A	8	7	1.09	24	0.292
63	A	7	6	0.99	24	0.250
64	A	4	3	1.00	24	0.125
65	A	7	6	1.00	24	0.250
66	A	8	7	1.10	24	0.292
67	A	2	2	1.00	24	0.083
68	A	4	3	1.00	24	0.125
69	A	4	3	1.00	22	0.136
70	A	2	2	1.00	21	0.095
71	A	2	2	1.00	24	0.083
72	A	4	3	1.00	24	0.125
73	A	4	3	1.00	24	0.125
74	A	7	6	1.17	24	0.250
75	A	5	4	0.99	24	0.167
76	A	5	4	0.99	24	0.167
77	A	7	6	1.17	24	0.250
78	A	8	7	1.15	24	0.292
79	A	8	7	1.18	24	0.292
80	A	6	5	1.04	24	0.208
81	A	5	4	1.04	24	0.167
82	A	8	7	1.05	24	0.292
83	A	9	8	1.02	24	0.333
84	A	2	2	1.00	24	0.083
85	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	22	0.136
87	A	2	2	1.00	21	0.095
88	A	2	2	1.00	24	0.083
89	A	4	3	1.00	24	0.125
90	A	4	3	1.00	24	0.125
91	A	4	3	1.00	24	0.125
92	A	5	4	1.00	24	0.167
93	A	4	3	1.00	24	0.125
94	A	5	4	1.00	24	0.167
95	A	7	6	1.09	24	0.250
96	A	8	7	1.09	24	0.292
97	A	7	6	0.99	24	0.250
98	A	4	3	1.00	24	0.125
99	A	7	6	1.00	24	0.250
100	A	8	7	1.10	24	0.292
101	A	11	10	1.52	24	0.417
102	A	10	9	1.54	24	0.375
103	A	8	7	1.38	22	0.318
104	A	12	11	1.50	24	0.458
105	A	13	12	1.09	24	0.500
106	A	8	7	1.12	24	0.292
107	A	6	5	0.97	24	0.208
108	A	2	2	1.00	24	0.083
109	A	2	2	1.00	24	0.083
110	A	2	2	1.00	21	0.095
111	A	2	2	1.00	24	0.083
112	A	2	2	1.00	24	0.083
113	A	7	6	1.17	24	0.250
114	A	5	4	0.99	24	0.167
115	A	5	4	0.99	24	0.167
116	A	7	6	1.17	24	0.250
117	A	8	7	1.15	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	7	1.18	24	0.292
119	A	6	5	1.04	24	0.208
120	A	5	4	1.04	24	0.167
121	A	8	7	1.05	24	0.292
122	A	9	8	1.02	24	0.333
123	A	13	12	1.41	24	0.500
124	A	11	10	1.43	24	0.417
125	A	11	10	1.65	24	0.417
126	A	12	11	1.41	22	0.500
127	A	13	12	1.39	24	0.500
128	A	5	4	0.94	24	0.167
129	A	6	5	0.92	24	0.208
130	A	7	6	0.92	24	0.250
131	A	2	2	1.00	24	0.083
132	A	2	2	1.00	24	0.083
133	A	2	2	1.00	21	0.095
134	A	2	2	1.00	24	0.083
135	A	2	2	1.00	24	0.083
136	A	3	3	1.00	22	0.136
137	A	7	6	0.94	22	0.273
138	A	6	5	1.00	22	0.227
139	A	6	5	1.13	20	0.250
140	A	6	5	1.12	22	0.227
141	A	4	3	1.09	22	0.136
142	A	4	3	1.05	22	0.136
143	A	4	3	1.04	22	0.136
144	A	4	3	1.03	22	0.136
145	A	5	5	0.95	22	0.227
146	A	4	4	0.97	22	0.182
147	A	3	3	0.99	22	0.136
148	A	2	2	1.00	22	0.091
149	A	6	5	1.03	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	5	0.95	19	0.263
151	A	6	5	0.97	22	0.227
152	A	7	6	0.93	22	0.273
153	A	7	6	0.96	22	0.273
154	A	7	6	1.05	22	0.273
155	A	7	6	1.04	20	0.300
156	A	7	6	1.11	22	0.273
157	A	4	3	1.09	22	0.136
158	A	4	3	1.05	22	0.136
159	A	4	3	1.04	22	0.136
160	A	4	3	1.03	22	0.136
161	A	5	5	0.95	22	0.227
162	A	4	4	0.97	22	0.182
163	A	3	3	0.99	22	0.136
164	A	2	2	1.00	22	0.091
165	A	7	6	0.99	22	0.273
166	A	7	6	0.91	22	0.273
167	A	7	6	0.94	19	0.316
168	A	7	6	0.91	22	0.273
169	A	8	7	0.88	22	0.318
170	A	6	5	0.98	22	0.227
171	A	5	4	0.97	20	0.200
172	A	5	4	1.12	22	0.182
173	A	4	3	1.12	22	0.136
174	A	4	3	1.06	22	0.136
175	A	4	3	1.05	22	0.136
176	A	3	3	0.99	22	0.136
177	A	2	2	1.00	22	0.091
178	A	5	4	1.00	19	0.211
179	A	5	4	1.00	22	0.182
180	A	6	5	0.99	22	0.227
181	A	7	6	0.99	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	5	0.99	20	0.250
183	A	5	4	1.12	22	0.182
184	A	4	3	1.10	22	0.136
185	A	4	3	1.09	22	0.136
186	A	4	3	1.04	22	0.136
187	A	4	3	1.05	22	0.136
188	A	6	5	0.96	22	0.227
189	A	5	4	0.97	22	0.182
190	A	4	3	1.00	19	0.158
191	A	5	4	1.00	22	0.182
192	A	6	5	0.97	22	0.227
193	A	7	6	0.98	22	0.273
194	A	5	4	1.00	22	0.182
195	A	5	4	1.00	22	0.182
196	A	5	4	1.00	19	0.211
197	A	5	4	1.00	22	0.182
198	A	5	4	1.00	22	0.182
199	A	4	3	1.01	22	0.136
200	A	4	3	1.01	20	0.150
201	A	4	3	1.01	22	0.136
202	A	4	3	1.21	22	0.136
203	A	5	4	1.08	22	0.182
204	A	7	6	1.01	22	0.273
205	A	5	4	1.00	26	0.154
206	A	5	4	1.00	26	0.154
207	A	5	4	1.00	26	0.154
208	A	5	4	1.00	26	0.154
209	A	5	4	1.00	26	0.154
210	A	5	4	0.96	24	0.167
211	C	3	3	0.13	28	0.107
212	C	3	3	0.41	28	0.107
213	C	3	3	0.31	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	2	1.00	25	0.080
215	A	2	2	1.00	22	0.091
216	A	3	3	1.30	24	0.125
217	A	2	2	1.00	20	0.100
218	A	6	5	1.00	28	0.179
219	A	7	6	1.00	28	0.214
220	C	5	4	0.25	28	0.143
221	C	6	5	0.25	28	0.179
222	A	5	4	1.05	31	0.129
223	A	1	1	1.00	31	0.032
224	A	4	3	1.00	31	0.097
225	A	7	6	1.05	31	0.194
226	A	4	4	1.03	31	0.129
227	A	3	3	1.00	31	0.097
228	A	2	2	1.00	28	0.071
229	A	3	3	1.00	31	0.097
230	A	4	4	1.03	31	0.129
231	A	5	5	1.04	31	0.161
232	A	4	4	1.04	29	0.138
233	A	5	5	1.05	31	0.161
234	A	7	6	1.06	28	0.214
235	A	6	5	1.03	28	0.179
236	A	5	4	1.01	28	0.143
237	A	4	3	1.00	28	0.107
238	A	5	4	1.82	28	0.143
239	A	1	1	1.00	28	0.036
240	A	2	2	1.00	28	0.071
241	A	3	3	1.05	28	0.107
242	A	4	4	1.08	28	0.143
243	A	6	5	1.03	28	0.179
244	A	5	4	0.97	28	0.143
245	A	4	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
246	A	3	2	1.00	28	0.071
247	A	1	1	1.00	28	0.036
248	A	2	2	1.00	28	0.071
249	A	3	3	1.05	28	0.107
250	A	3	2	1.00	26	0.077
251	A	4	3	0.92	28	0.107
252	A	5	4	1.00	26	0.154
253	A	6	5	0.96	28	0.179
254	A	2	2	1.00	18	0.111
255	A	2	2	1.00	16	0.125
256	A	2	2	1.00	15	0.133
257	A	4	3	1.00	18	0.167
258	A	2	2	1.00	18	0.111
259	A	2	2	1.00	18	0.111
260	A	2	2	1.00	18	0.111
261	A	2	2	1.00	20	0.100
262	A	2	2	1.00	18	0.111
263	A	2	2	1.00	17	0.118
264	A	5	4	0.95	20	0.200
265	A	2	2	1.00	20	0.100
266	A	2	2	1.00	20	0.100
267	A	2	2	1.00	20	0.100
268	A	2	2	1.00	20	0.100
269	A	2	2	1.00	18	0.111
270	A	2	2	1.00	17	0.118
271	A	4	3	1.09	20	0.150
272	A	2	2	1.00	20	0.100
273	A	2	2	1.00	20	0.100
274	A	2	2	1.00	20	0.100
275	A	2	2	1.00	20	0.100
276	A	2	2	1.00	18	0.111
277	A	2	2	1.01	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	4	3	1.00	20	0.150
279	A	2	2	1.00	20	0.100
280	A	2	2	1.00	20	0.100
281	A	2	2	1.00	20	0.100
282	A	2	2	1.00	20	0.100
283	A	2	2	1.00	18	0.111
284	A	2	2	1.28	17	0.118
285	A	4	3	0.96	20	0.150
286	A	2	2	1.00	20	0.100
287	A	2	2	1.00	20	0.100
288	A	2	2	1.00	20	0.100
289	A	2	2	1.00	20	0.100
290	A	2	2	1.00	20	0.100
291	A	2	2	1.00	20	0.100
292	A	2	2	1.00	20	0.100
293	A	2	2	1.00	20	0.100
294	A	2	2	1.00	20	0.100
295	A	2	2	1.00	20	0.100
296	A	2	2	1.00	20	0.100
297	A	2	2	1.00	22	0.091
298	A	2	2	1.00	22	0.091
299	A	2	2	1.00	22	0.091
300	A	2	2	1.00	22	0.091
301	A	2	2	1.00	22	0.091
302	A	2	2	1.00	22	0.091
303	A	2	2	1.00	22	0.091
304	A	2	2	1.00	22	0.091
305	A	2	2	1.00	22	0.091
306	A	2	2	1.00	22	0.091
307	A	2	2	1.00	22	0.091
308	A	2	2	1.00	22	0.091
309	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	2	2	1.00	22	0.091
311	A	2	2	1.00	22	0.091
312	A	2	2	1.00	22	0.091
313	A	2	2	1.00	22	0.091
314	A	2	2	1.00	22	0.091
315	A	2	2	1.00	22	0.091
316	A	2	2	1.00	22	0.091
317	A	2	2	1.00	22	0.091
318	A	2	2	1.00	22	0.091
319	A	2	2	1.00	22	0.091
320	A	2	2	1.00	22	0.091
321	A	2	2	1.00	22	0.091
322	A	2	2	1.00	22	0.091
323	A	2	2	1.00	22	0.091
324	A	2	2	1.00	22	0.091
325	A	2	2	1.00	22	0.091
326	A	2	2	1.00	22	0.091
327	A	2	2	1.00	22	0.091
328	A	2	2	1.00	22	0.091
329	A	2	2	1.00	22	0.091
330	A	3	3	1.00	22	0.136
331	A	3	3	1.00	20	0.150
332	A	3	3	1.00	19	0.158
333	A	6	5	0.94	22	0.227
334	A	3	3	1.00	22	0.136
335	A	3	3	1.00	22	0.136
336	A	3	3	1.00	22	0.136
337	A	3	3	1.00	22	0.136
338	A	3	3	1.00	20	0.150
339	A	3	3	1.00	19	0.158
340	A	7	6	0.93	22	0.273
341	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	3	3	1.00	22	0.136
343	A	3	3	1.00	22	0.136
344	A	3	3	1.00	22	0.136
345	A	3	3	1.00	20	0.150
346	A	3	3	1.00	19	0.158
347	A	8	7	0.91	22	0.318
348	A	3	3	1.00	22	0.136
349	A	3	3	1.00	22	0.136
350	A	3	3	1.00	22	0.136
351	A	3	3	1.00	22	0.136
352	A	3	3	1.00	20	0.150
353	A	3	3	1.00	19	0.158
354	A	5	4	0.96	22	0.182
355	A	3	3	1.00	22	0.136
356	A	3	3	1.00	22	0.136
357	A	3	3	1.00	22	0.136
358	A	3	3	1.00	22	0.136
359	A	3	3	1.00	20	0.150
360	A	3	3	1.11	19	0.158
361	A	5	4	0.97	22	0.182
362	A	3	3	1.00	22	0.136
363	A	3	3	1.00	22	0.136
364	A	3	3	1.00	22	0.136
365	A	3	3	1.00	22	0.136
366	A	3	3	1.00	20	0.150
367	A	3	3	1.16	19	0.158
368	A	6	5	0.99	22	0.227
369	A	3	3	1.00	22	0.136
370	A	3	3	1.00	22	0.136
371	A	3	3	0.95	22	0.136
372	A	3	3	1.00	26	0.115
373	A	3	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	3	3	1.00	26	0.115
375	A	3	3	1.00	26	0.115
376	A	3	3	1.00	26	0.115
377	A	3	3	0.95	26	0.115
378	A	3	3	1.00	26	0.115
379	A	3	3	1.00	26	0.115
380	A	3	3	1.00	26	0.115
381	A	3	3	1.00	26	0.115
382	A	3	3	1.00	26	0.115
383	A	3	3	1.00	26	0.115
384	A	3	3	1.00	26	0.115
385	A	3	3	0.98	26	0.115
386	A	3	3	1.00	26	0.115
387	A	3	3	1.00	26	0.115
388	A	3	3	1.00	26	0.115
389	A	3	3	0.98	26	0.115
390	A	3	3	1.00	26	0.115
391	A	3	3	1.00	26	0.115
392	A	3	3	0.94	26	0.115
393	A	3	3	1.00	26	0.115
394	A	3	3	0.93	26	0.115
395	A	3	3	1.00	26	0.115
396	A	3	3	0.95	26	0.115
397	B	2	2	3.25	32	0.062
398	B	2	2	2.25	30	0.067
399	A	2	2	0.89	21	0.095
400	A	2	2	1.00	29	0.069
401	A	1	1	1.00	32	0.031
402	A	1	1	1.00	32	0.031
403	A	2	2	0.92	22	0.091
404	A	2	2	0.92	22	0.091
405	A	2	2	0.92	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	2	2	0.92	13	0.154
407	A	2	2	1.00	22	0.091
408	A	2	2	1.00	22	0.091
409	A	2	2	1.00	22	0.091
410	A	3	3	1.11	24	0.125
411	A	3	3	1.11	24	0.125
412	A	3	3	1.02	24	0.125
413	A	3	3	0.97	24	0.125
414	A	3	3	1.00	24	0.125
415	A	3	3	1.00	20	0.150
416	A	3	3	1.00	18	0.167
417	A	3	3	1.00	17	0.176
418	A	4	3	0.97	20	0.150
419	A	3	3	1.00	20	0.150
420	A	3	3	1.00	20	0.150
421	A	3	3	1.10	24	0.125
422	A	3	3	1.10	24	0.125
423	A	3	3	1.00	24	0.125
424	A	3	3	1.00	24	0.125
425	A	3	3	1.01	24	0.125
426	A	3	3	1.00	27	0.111
427	A	3	3	1.00	25	0.120
428	A	3	3	1.00	22	0.136
429	A	2	2	1.00	21	0.095
430	A	3	3	1.20	29	0.103
431	A	3	3	1.00	24	0.125
432	A	3	3	1.00	27	0.111
433	A	3	3	1.02	22	0.136
434	A	4	3	1.09	27	0.111
435	A	1	1	1.00	34	0.029
436	A	2	2	1.00	22	0.091
437	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	2	2	1.00	19	0.105
439	A	4	3	1.00	22	0.136
440	A	2	2	1.00	22	0.091
441	A	2	2	1.00	22	0.091
442	A	3	3	1.13	22	0.136
443	A	3	3	1.13	20	0.150
444	A	3	3	1.16	19	0.158
445	A	4	3	0.97	22	0.136
446	A	3	3	1.11	22	0.136
447	A	3	3	1.12	22	0.136
448	A	2	2	1.00	14	0.143
449	A	2	2	1.00	20	0.100
450	A	10	9	1.21	20	0.450
451	A	14	13	1.39	20	0.650
452	A	4	3	0.92	26	0.115
453	A	4	3	0.91	26	0.115
454	A	4	3	0.92	24	0.125
455	A	4	3	0.96	26	0.115
456	A	4	3	0.93	26	0.115
457	A	4	3	0.92	26	0.115
458	A	4	3	0.91	26	0.115
459	A	4	3	0.91	26	0.115
460	A	4	3	0.91	24	0.125
461	A	4	3	0.93	26	0.115
462	A	4	3	0.95	26	0.115
463	A	4	3	0.93	26	0.115
464	A	1	1	1.00	17	0.059
465	A	3	2	1.00	21	0.095
466	A	3	2	1.00	21	0.095
467	A	3	2	1.00	25	0.080
468	A	1	1	1.00	31	0.032
469	A	2	2	1.12	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
470	A	4	3	1.13	21	0.143
471	A	4	3	1.13	21	0.143
472	A	4	3	1.13	21	0.143
473	A	1	1	1.00	17	0.059
474	A	3	2	1.00	21	0.095
475	A	3	2	1.00	21	0.095
476	A	3	2	1.00	25	0.080
477	A	6	5	1.00	22	0.227
478	A	7	6	1.00	26	0.231
479	A	8	7	0.90	30	0.233
480	A	7	6	0.91	30	0.200
481	A	6	5	0.93	30	0.167
482	A	5	4	0.98	30	0.133
483	A	5	4	0.98	30	0.133
484	A	4	3	0.98	30	0.100
485	A	10	9	0.83	30	0.300
486	A	9	8	0.87	30	0.267
487	A	8	7	0.94	30	0.233
488	A	7	6	0.98	30	0.200
489	A	8	7	0.98	30	0.233
490	A	7	6	1.11	30	0.200
491	A	2	2	1.00	24	0.083
492	A	3	3	1.11	24	0.125
493	A	2	2	1.00	24	0.083
494	A	2	2	1.00	24	0.083
495	C	2	2	0.64	29	0.069
496	A	6	6	1.44	24	0.250
497	A	5	5	1.08	24	0.208
498	A	3	3	1.02	22	0.136
499	A	2	2	1.00	15	0.133
500	A	2	2	1.00	24	0.083
501	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
502	A	3	3	1.04	24	0.125
503	A	3	3	1.02	22	0.136
504	A	3	3	1.02	20	0.150
505	A	3	3	1.00	24	0.125
506	A	3	3	1.00	24	0.125
507	A	3	3	0.98	24	0.125
508	A	1	1	1.00	17	0.059
509	A	1	1	1.00	27	0.037
510	A	1	1	1.00	27	0.037
511	A	1	1	1.00	27	0.037
512	A	2	2	0.00	29	0.069
513	A	1	1	0.00	29	0.034
514	A	6	6	1.45	29	0.207
515	A	4	4	1.20	27	0.148
516	A	3	3	0.98	20	0.150
517	C	2	2	1.15	29	0.069
518	C	2	2	1.10	29	0.069
519	A	2	2	0.01	29	0.069
520	A	1	1	0.01	29	0.034
521	A	3	3	1.41	27	0.111
522	A	2	2	1.00	20	0.100
523	C	2	2	1.32	29	0.069
524	C	2	2	1.28	29	0.069
525	A	6	5	1.25	29	0.172
526	A	5	4	1.15	29	0.138
527	A	4	3	1.09	27	0.111
528	A	1	1	1.00	20	0.050
529	C	2	2	1.49	29	0.069
530	C	2	2	1.55	29	0.069
531	C	2	2	8.84	29	0.069
532	A	6	5	1.06	27	0.185
533	A	5	4	0.94	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	4	3	0.88	25	0.120
535	A	3	2	0.81	18	0.111
536	A	2	2	1.60	27	0.074
537	C	2	2	0.92	27	0.074
538	C	2	2	0.87	27	0.074
539	A	5	4	0.99	29	0.138
540	A	4	3	0.92	27	0.111
541	A	3	2	0.86	20	0.100
542	C	2	2	0.64	29	0.069
543	A	2	2	1.48	29	0.069
544	C	2	2	1.20	29	0.069
545	C	2	2	2.09	29	0.069
546	A	5	4	1.02	29	0.138
547	A	4	3	0.96	27	0.111
548	A	3	2	0.92	20	0.100
549	C	2	2	0.41	29	0.069
550	C	2	2	0.40	29	0.069
551	A	2	2	1.45	29	0.069
552	C	2	2	2.24	29	0.069
553	C	2	2	2.99	29	0.069
554	A	5	4	1.02	29	0.138
555	A	4	3	0.96	27	0.111
556	A	3	2	0.92	20	0.100
557	C	2	2	0.30	29	0.069
558	C	2	2	0.30	29	0.069
559	C	2	2	0.29	29	0.069
560	A	2	2	1.45	29	0.069
561	C	2	2	3.07	29	0.069
562	C	2	2	3.99	29	0.069
563	A	1	1	1.00	22	0.045
564	A	3	3	1.00	24	0.125
565	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	F	0	0	N/A	0.000	N/A
567	A	3	3	1.06	31	0.097
568	A	3	3	1.00	27	0.111
569	A	3	3	0.99	31	0.097
570	A	7	6	1.00	26	0.231
571	A	5	4	1.07	26	0.154
572	A	4	3	1.21	24	0.125
573	A	4	3	1.01	22	0.136
574	A	4	3	1.01	26	0.115
575	A	4	3	1.01	26	0.115
576	A	3	3	1.00	24	0.125
577	A	3	3	1.00	22	0.136
578	A	3	3	1.00	21	0.143
579	A	5	4	1.01	24	0.167
580	A	3	3	1.00	24	0.125
581	A	12	11	1.14	37	0.297
582	A	3	3	1.35	47	0.064
583	A	3	3	1.35	47	0.064
584	A	2	2	1.35	57	0.035
585	A	3	2	0.96	39	0.051
586	A	4	3	1.17	61	0.049
587	A	4	3	1.17	61	0.049
588	A	4	3	1.17	83	0.036
589	A	3	3	1.00	26	0.115
590	A	3	3	1.00	26	0.115
591	A	3	3	1.00	24	0.125
592	A	3	3	1.00	23	0.130
593	A	3	3	1.00	26	0.115
594	A	3	3	1.00	26	0.115
595	A	4	4	1.00	17	0.235
596	A	6	5	1.27	17	0.294
597	A	6	5	1.07	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	5	4	1.00	15	0.267
599	A	3	2	1.00	9	0.222
600	A	6	5	1.00	17	0.294
601	A	4	3	1.00	17	0.176
602	A	5	4	0.99	17	0.235
603	A	7	6	1.14	17	0.353
604	A	3	3	1.00	28	0.107
605	A	3	3	1.00	28	0.107
606	A	3	3	1.00	26	0.115
607	A	3	3	1.00	25	0.120
608	A	3	3	1.00	28	0.107
609	A	3	3	1.00	28	0.107
610	A	10	9	1.24	21	0.429

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1+x^6}{x(1-x^6)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

output
```
ln(x)-1/3*ln(-x^6+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx=\log(x)-\frac{1}{3}\log\left(1-x^6\right)$$

input
```
Integrate[(1 + x^6)/(x*(1 - x^6)),x]
```

output
```
Log[x] - Log[1 - x^6]/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6+1}{x\left(1-x^6\right)}\,dx$$
$$\Big\downarrow \ 948$$
$$\frac{1}{6}\int \frac{x^6+1}{x^6\left(1-x^6\right)}dx^6$$
$$\Big\downarrow \ 86$$
$$\frac{1}{6}\int \left(\frac{1}{x^6}-\frac{2}{x^6-1}\right)dx^6$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{6}\left(\log\left(x^6\right)-2\log\left(1-x^6\right)\right)$$

input
```
Int[(1 + x^6)/(x*(1 - x^6)),x]
```

output
```
(Log[x^6] - 2*Log[1 - x^6])/6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-1)}{3}$	12
meijerg	$-\frac{\ln(-x^6+1)}{3}+\ln(x)+\frac{i\pi}{6}$	18
default	$-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(x^2+x+1)}{3}+\ln(x)-\frac{\ln(x+1)}{3}-\frac{\ln(x-1)}{3}$	36
norman	$-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(x^2+x+1)}{3}+\ln(x)-\frac{\ln(x+1)}{3}-\frac{\ln(x-1)}{3}$	36
parallelrisch	$-\frac{\ln(x^2-x+1)}{3}-\frac{\ln(x^2+x+1)}{3}+\ln(x)-\frac{\ln(x+1)}{3}-\frac{\ln(x-1)}{3}$	36

input
```
int((x^6+1)/x/(-x^6+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/3*ln(x^6-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x(1-x^6)}\,dx=-\frac{1}{3}\log\left(x^6-1\right)+\log(x)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
```

output
```
-1/3*log(x^6 - 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \log\left(x\right) - \frac{\log\left(x^6-1\right)}{3}$$

input
```
integrate((x**6+1)/x/(-x**6+1),x)
```

output
```
log(x) - log(x**6 - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = -\frac{1}{3}\log\left(x^6-1\right) + \frac{1}{6}\log\left(x^6\right)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
```

output
```
-1/3*log(x^6 - 1) + 1/6*log(x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx = \frac{1}{6}\log\left(x^6\right) - \frac{1}{3}\log\left(\left|x^6-1\right|\right)$$

input
```
integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
```

output
```
1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = \ln(x) - \frac{\ln(x^6-1)}{3}$$

input
```
int(-(x^6 + 1)/(x*(x^6 - 1)),x)
```

output
```
log(x) - log(x^6 - 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \frac{1+x^6}{x(1-x^6)}\,dx = -\frac{\log(x^2-x+1)}{3} - \frac{\log(x^2+x+1)}{3} - \frac{\log(x-1)}{3} - \frac{\log(x+1)}{3} + \log(x)$$

input
```
int((x^6+1)/x/(-x^6+1),x)
```

output
```
( - log(x**2 - x + 1) - log(x**2 + x + 1) - log(x - 1) - log(x + 1) + 3*lo
g(x))/3
```

3.2 $\int \frac{-2+3x^6}{x(5+2x^6)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx=-\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

output
```
-2/5*ln(x)+19/60*ln(2*x^6+5)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2+3x^6}{x\,(5+2x^6)}\,dx=-\frac{2\log(x)}{5}+\frac{19}{60}\log\left(5+2x^6\right)$$

input
```
Integrate[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

output
```
(-2*Log[x])/5 + (19*Log[5 + 2*x^6])/60
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^6-2}{x(2x^6+5)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6}\int -\frac{2-3x^6}{x^6(2x^6+5)}dx^6$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{6}\int \frac{2-3x^6}{x^6(2x^6+5)}dx^6$$

$$\Big\downarrow \ 86$$

$$-\frac{1}{6}\int \left(\frac{2}{5x^6}-\frac{19}{5(2x^6+5)}\right)dx^6$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\left(\frac{19}{10}\log\left(2x^6+5\right)-\frac{2\log\left(x^6\right)}{5}\right)$$

input
```
Int[(-2 + 3*x^6)/(x*(5 + 2*x^6)),x]
```

output
```
((-2*Log[x^6])/5 + (19*Log[5 + 2*x^6])/10)/6
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(x^6+\frac{5}{2}\right)}{60}$	14
default	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
norman	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
risch	$-\frac{2\ln(x)}{5}+\frac{19\ln\left(2x^6+5\right)}{60}$	16
meijerg	$\frac{19\ln\left(1+\frac{2x^6}{5}\right)}{60}-\frac{2\ln(x)}{5}-\frac{\ln(2)}{15}+\frac{\ln(5)}{15}$	24

input
```
int((3*x^6-2)/x/(2*x^6+5),x,method=_RETURNVERBOSE)
```

output
```
-2/5*ln(x)+19/60*ln(x^6+5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\log\left(2\,x^6+5\right) - \frac{2}{5}\log\left(x\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="fricas")
```

output
```
19/60*log(2*x^6 + 5) - 2/5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = -\frac{2\log\left(x\right)}{5} + \frac{19\log\left(2x^6+5\right)}{60}$$

input
```
integrate((3*x**6-2)/x/(2*x**6+5),x)
```

output
```
-2*log(x)/5 + 19*log(2*x**6 + 5)/60
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\log\left(2\,x^6+5\right) - \frac{1}{15}\log\left(x^6\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="maxima")
```

output
```
19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19}{60}\log\left(2x^6+5\right) - \frac{1}{15}\log\left(x^6\right)$$

input
```
integrate((3*x^6-2)/x/(2*x^6+5),x, algorithm="giac")
```

output
```
19/60*log(2*x^6 + 5) - 1/15*log(x^6)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19\ln\left(x^6+\frac{5}{2}\right)}{60} - \frac{2\ln(x)}{5}$$

input
```
int((3*x^6 - 2)/(x*(2*x^6 + 5)),x)
```

output
```
(19*log(x^6 + 5/2))/60 - (2*log(x))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.26

$$\int \frac{-2+3x^6}{x(5+2x^6)}\,dx = \frac{19\log\left(5^{\frac{1}{3}}+2^{\frac{1}{3}}x^2\right)}{60} + \frac{19\log\left(-10^{\frac{1}{6}}\sqrt{3}\,x+5^{\frac{1}{3}}+2^{\frac{1}{3}}x^2\right)}{60} + \frac{19\log\left(10^{\frac{1}{6}}\sqrt{3}\,x+5^{\frac{1}{3}}+2^{\frac{1}{3}}x^2\right)}{60} - \frac{2\log(x)}{5}$$

input
```
int((3*x^6-2)/x/(2*x^6+5),x)
```

output
```
(19*log(5**(1/3) + 2**(1/3)*x**2) + 19*log( - 10**(1/6)*sqrt(3)*x + 5**(1/
3) + 2**(1/3)*x**2) + 19*log(10**(1/6)*sqrt(3)*x + 5**(1/3) + 2**(1/3)*x**
2) - 24*log(x))/60
```

3.3 $\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{a(bc-ad)}{3b^3\sqrt{a+bx^6}}+\frac{(bc-2ad)\sqrt{a+bx^6}}{3b^3}+\frac{d(a+bx^6)^{3/2}}{9b^3}$$

output
```
1/3*a*(-a*d+b*c)/b^3/(b*x^6+a)^(1/2)+1/3*(-2*a*d+b*c)*(b*x^6+a)^(1/2)/b^3+
1/9*d*(b*x^6+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{6abc-8a^2d+3b^2cx^6-4abdx^6+b^2dx^{12}}{9b^3\sqrt{a+bx^6}}$$

input
```
Integrate[(x^11*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(6*a*b*c - 8*a^2*d + 3*b^2*c*x^6 - 4*a*b*d*x^6 + b^2*d*x^12)/(9*b^3*Sqrt[a
 + b*x^6])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(c+dx^6\right)}{\left(a+bx^6\right)^{3/2}}dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6}\int \frac{x^6\left(dx^6+c\right)}{\left(bx^6+a\right)^{3/2}}dx^6$$

$$\Big\downarrow \ 86$$

$$\frac{1}{6}\int \left(\frac{\sqrt{bx^6+a}d}{b^2}+\frac{bc-2ad}{b^2\sqrt{bx^6+a}}+\frac{a(ad-bc)}{b^2\left(bx^6+a\right)^{3/2}}\right)dx^6$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\left(\frac{2\sqrt{a+bx^6}(bc-2ad)}{b^3}+\frac{2a(bc-ad)}{b^3\sqrt{a+bx^6}}+\frac{2d\left(a+bx^6\right)^{3/2}}{3b^3}\right)$$

input
```
Int[(x^11*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
((2*a*(b*c - a*d))/(b^3*Sqrt[a + b*x^6]) + (2*(b*c - 2*a*d)*Sqrt[a + b*x^6
])/b^3 + (2*d*(a + b*x^6)^(3/2))/(3*b^3))/6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{8\left(-\frac{3\left(\frac{d\,x^{6}}{3}+c\right)x^{6}b^{2}}{8}-\frac{3a\left(-\frac{2d\,x^{6}}{3}+c\right)b}{4}+a^{2}d\right)}{9\sqrt{b\,x^{6}+a}\,b^{3}}$	49
gosper	$-\frac{-d\,x^{12}b^{2}+4abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-6abc}{9\sqrt{b\,x^{6}+a}\,b^{3}}$	53
trager	$-\frac{-d\,x^{12}b^{2}+4abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-6abc}{9\sqrt{b\,x^{6}+a}\,b^{3}}$	53
orering	$-\frac{-d\,x^{12}b^{2}+4abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-6abc}{9\sqrt{b\,x^{6}+a}\,b^{3}}$	53
risch	$-\frac{\left(-x^{6}bd+5ad-3cb\right)\sqrt{b\,x^{6}+a}}{9b^{3}}-\frac{a(ad-cb)}{3\sqrt{b\,x^{6}+a}\,b^{3}}$	55

input
```
int(x^11*(d*x^6+c)/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8/9/(b*x^6+a)^(1/2)*(-3/8*(1/3*d*x^6+c)*x^6*b^2-3/4*a*(-2/3*d*x^6+c)*b+a^
2*d)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{(b^2dx^{12}+(3\,b^2c-4\,abd)x^6+6\,abc-8\,a^2d)\sqrt{bx^6+a}}{9\,(b^4x^6+ab^3)}$$

input
```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(b^2*d*x^12 + (3*b^2*c - 4*a*b*d)*x^6 + 6*a*b*c - 8*a^2*d)*sqrt(b*x^6
+ a)/(b^4*x^6 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \begin{cases} -\frac{8a^2d}{9b^3\sqrt{a+bx^6}} + \frac{2ac}{3b^2\sqrt{a+bx^6}} - \frac{4adx^6}{9b^2\sqrt{a+bx^6}} + \frac{cx^6}{3b\sqrt{a+bx^6}} + \frac{dx^{12}}{9b\sqrt{a+bx^6}} & \text{for } b \neq 0 \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{18}}{18}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
Piecewise((-8*a**2*d/(9*b**3*sqrt(a + b*x**6)) + 2*a*c/(3*b**2*sqrt(a + b*
x**6)) - 4*a*d*x**6/(9*b**2*sqrt(a + b*x**6)) + c*x**6/(3*b*sqrt(a + b*x**
6)) + d*x**12/(9*b*sqrt(a + b*x**6)), Ne(b, 0)), ((c*x**12/12 + d*x**18/18
)/a**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{1}{9}\,d\left(\frac{(bx^6+a)^{\frac{3}{2}}}{b^3} - \frac{6\,\sqrt{bx^6+a}a}{b^3} - \frac{3\,a^2}{\sqrt{bx^6+a}b^3}\right) + \frac{1}{3}\,c\left(\frac{\sqrt{bx^6+a}}{b^2} + \frac{a}{\sqrt{bx^6+a}b^2}\right)$$

input

```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output

```
1/9*d*((b*x^6 + a)^(3/2)/b^3 - 6*sqrt(b*x^6 + a)*a/b^3 - 3*a^2/(sqrt(b*x^6
 + a)*b^3)) + 1/3*c*(sqrt(b*x^6 + a)/b^2 + a/(sqrt(b*x^6 + a)*b^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{abc-a^2d}{3\,\sqrt{bx^6+a}b^3} + \frac{3\,\sqrt{bx^6+a}b^7c + (bx^6+a)^{\frac{3}{2}}b^6d - 6\,\sqrt{bx^6+a}ab^6d}{9\,b^9}$$

input

```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output

```
1/3*(a*b*c - a^2*d)/(sqrt(b*x^6 + a)*b^3) + 1/9*(3*sqrt(b*x^6 + a)*b^7*c +
 (b*x^6 + a)^(3/2)*b^6*d - 6*sqrt(b*x^6 + a)*a*b^6*d)/b^9
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{d\,(b\,x^6+a)^2-3\,a^2\,d-6\,a\,d\,(b\,x^6+a)+3\,b\,c\,(b\,x^6+a)+3\,a\,b\,c}{9\,b^3\,\sqrt{b\,x^6+a}}$$

input
```
int((x^11*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
(d*(a + b*x^6)^2 - 3*a^2*d - 6*a*d*(a + b*x^6) + 3*b*c*(a + b*x^6) + 3*a*b
*c)/(9*b^3*(a + b*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(b^2d\,x^{12}-4abd\,x^6+3b^2c\,x^6-8a^2d+6abc)}{9b^3\,(b\,x^6+a)}$$

input
```
int(x^11*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*( - 8*a**2*d + 6*a*b*c - 4*a*b*d*x**6 + 3*b**2*c*x**6 +
b**2*d*x**12))/(9*b**3*(a + b*x**6))
```

3.4 $\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{-bc+ad}{3b^2\sqrt{a+bx^6}} + \frac{d\sqrt{a+bx^6}}{3b^2}$$

output
```
1/3*(a*d-b*c)/b^2/(b*x^6+a)^(1/2)+1/3*d*(b*x^6+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{-bc+2ad+bdx^6}{3b^2\sqrt{a+bx^6}}$$

input
```
Integrate[(x^5*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(-(b*c) + 2*a*d + b*d*x^6)/(3*b^2*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \ 946$$

$$\frac{1}{6}\int \frac{dx^6+c}{(bx^6+a)^{3/2}}dx^6$$

$$\Big\downarrow \ 53$$

$$\frac{1}{6}\int \left(\frac{d}{b\sqrt{bx^6+a}}+\frac{bc-ad}{b\,(bx^6+a)^{3/2}}\right)dx^6$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}\left(\frac{2d\sqrt{a+bx^6}}{b^2}-\frac{2(bc-ad)}{b^2\sqrt{a+bx^6}}\right)$$

input
```
Int[(x^5*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
((-2*(b*c - a*d))/(b^2*Sqrt[a + b*x^6]) + (2*d*Sqrt[a + b*x^6])/b^2)/6
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x^{6} b d +2 a d -c b}{3 \sqrt{b \,x^{6}+a}\, b^{2}}$	30
trager	$\frac{x^{6} b d +2 a d -c b}{3 \sqrt{b \,x^{6}+a}\, b^{2}}$	30
orering	$\frac{x^{6} b d +2 a d -c b}{3 \sqrt{b \,x^{6}+a}\, b^{2}}$	30
pseudoelliptic	$\frac{\left(d \,x^{6}-c \right) b +2 a d}{3 \sqrt{b \,x^{6}+a}\, b^{2}}$	31
risch	$\frac{a d -c b}{3 b^{2} \sqrt{b \,x^{6}+a}}+\frac{d \sqrt{b \,x^{6}+a}}{3 b^{2}}$	39

input
```
int(x^5*(d*x^6+c)/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*d*x^6+2*a*d-b*c)/(b*x^6+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\frac{(bdx^6-bc+2\,ad)\sqrt{bx^6+a}}{3\,(b^3x^6+ab^2)}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b*d*x^6 - b*c + 2*a*d)*sqrt(b*x^6 + a)/(b^3*x^6 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \begin{cases} \frac{2ad}{3b^2\sqrt{a+bx^6}} - \frac{c}{3b\sqrt{a+bx^6}} + \frac{dx^6}{3b\sqrt{a+bx^6}} & \text{for } b \neq 0 \\ \frac{\frac{cx^6}{6}+\frac{dx^{12}}{12}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
Piecewise((2*a*d/(3*b**2*sqrt(a + b*x**6)) - c/(3*b*sqrt(a + b*x**6)) + d*
x**6/(3*b*sqrt(a + b*x**6)), Ne(b, 0)), ((c*x**6/6 + d*x**12/12)/a**(3/2),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{1}{3}\,d\left(\frac{\sqrt{bx^6+a}}{b^2} + \frac{a}{\sqrt{bx^6+a}\,b^2}\right) - \frac{c}{3\sqrt{bx^6+a}\,b}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*d*(sqrt(b*x^6 + a)/b^2 + a/(sqrt(b*x^6 + a)*b^2)) - 1/3*c/(sqrt(b*x^6
+ a)*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{bx^6+a}d}{3\,b^2} - \frac{bc-ad}{3\,\sqrt{bx^6+a}b^2}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^6 + a)*d/b^2 - 1/3*(b*c - a*d)/(sqrt(b*x^6 + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{d\,(b\,x^6+a)+a\,d-b\,c}{3\,b^2\,\sqrt{b\,x^6+a}}$$

input
```
int((x^5*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
(d*(a + b*x^6) + a*d - b*c)/(3*b^2*(a + b*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(bd\,x^6+2ad-bc)}{3b^2\,(b\,x^6+a)}$$

input
```
int(x^5*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*(2*a*d - b*c + b*d*x**6))/(3*b**2*(a + b*x**6))
```

3.5 $\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx = \frac{bc-ad}{3ab\sqrt{a+bx^6}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
1/3*(-a*d+b*c)/a/b/(b*x^6+a)^(1/2)-1/3*c*arctanh((b*x^6+a)^(1/2)/a^(1/2))/
a^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx = \frac{bc-ad}{3ab\sqrt{a+bx^6}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x*(a + b*x^6)^(3/2)),x]
```

output
```
(b*c - a*d)/(3*a*b*Sqrt[a + b*x^6]) - (c*ArcTanh[Sqrt[a + b*x^6]/Sqrt[a]])
/(3*a^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{dx^6+c}{x^6\left(bx^6+a\right)^{3/2}}dx^6$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6}\left(\frac{c\int \frac{1}{x^6\sqrt{bx^6+a}}dx^6}{a}+\frac{2(bc-ad)}{ab\sqrt{a+bx^6}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{2c\int \frac{1}{\frac{x^{12}}{b}-\frac{a}{b}}d\sqrt{bx^6+a}}{ab}+\frac{2(bc-ad)}{ab\sqrt{a+bx^6}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{2(bc-ad)}{ab\sqrt{a+bx^6}}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[(c + d*x^6)/(x*(a + b*x^6)^(3/2)),x]
```

output
```
((2*(b*c - a*d))/(a*b*Sqrt[a + b*x^6]) - (2*c*ArcTanh[Sqrt[a + b*x^6]/Sqrt
[a]])/a^(3/2))/6
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{-\frac{ad-cb}{a\sqrt{b\,x^{6}+a}}-\frac{bc\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{3b}$	50

input
```
int((d*x^6+c)/x/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-(a*d-b*c)/a/(b*x^6+a)^(1/2)-b/a^(3/2)*c*arctanh((b*x^6+a)^(1/2)/a^(1
/2)))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.86

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx = \left[\frac{(b^2cx^6+abc)\sqrt{a}\log\left(\frac{bx^6-2\sqrt{bx^6+a}\sqrt{a}+2a}{x^6}\right)+2\sqrt{bx^6+a}(abc-a^2d)}{6\left(a^2b^2x^6+a^3b\right)}, \frac{(b^2cx^6+abc)\sqrt{}}{}\right.$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*((b^2*c*x^6 + a*b*c)*sqrt(a)*log((b*x^6 - 2*sqrt(b*x^6 + a)*sqrt(a) +
 2*a)/x^6) + 2*sqrt(b*x^6 + a)*(a*b*c - a^2*d))/(a^2*b^2*x^6 + a^3*b), 1/3
*((b^2*c*x^6 + a*b*c)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^6 + a)) + sqrt(b*x
^6 + a)*(a*b*c - a^2*d))/(a^2*b^2*x^6 + a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 11.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{bc\,\mathrm{atan}\left(\frac{\sqrt{a+bx^6}}{\sqrt{-a}}\right)}{6a\sqrt{-a}}-\frac{ad-bc}{6a\sqrt{a+bx^6}}\right)}{b} & \text{for } b\neq 0 \\ \frac{c\log\left(dx^6\right)+dx^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**6+c)/x/(b*x**6+a)**(3/2),x)
```

output
```
Piecewise((2*(b*c*atan(sqrt(a + b*x**6)/sqrt(-a))/(6*a*sqrt(-a)) - (a*d -
b*c)/(6*a*sqrt(a + b*x**6)))/b, Ne(b, 0)), ((c*log(d*x**6) + d*x**6)/(6*a*
*(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx=\frac{1}{6}\,c\left(\frac{\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^6+a}a}\right)-\frac{d}{3\sqrt{bx^6+a}b}$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*c*(log((sqrt(b*x^6 + a) - sqrt(a))/(sqrt(b*x^6 + a) + sqrt(a)))/a^(3/2
) + 2/(sqrt(b*x^6 + a)*a)) - 1/3*d/(sqrt(b*x^6 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx=\frac{c\arctan\left(\frac{\sqrt{bx^6+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a}+\frac{bc-ad}{3\sqrt{bx^6+a}ab}$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*c*arctan(sqrt(b*x^6 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/3*(b*c - a*d)/(sqr
t(b*x^6 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx=\frac{c}{3\,a\,\sqrt{b\,x^6+a}}-\frac{d}{3\,b\,\sqrt{b\,x^6+a}}-\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^6+a}}{\sqrt{a}}\right)}{3\,a^{3/2}}$$

input
```
int((c + d*x^6)/(x*(a + b*x^6)^(3/2)),x)
```

output
```
c/(3*a*(a + b*x^6)^(1/2)) - d/(3*b*(a + b*x^6)^(1/2)) - (c*atanh((a + b*x^
6)^(1/2)/a^(1/2)))/(3*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.26

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^6+a}\,a^2d+2\sqrt{b\,x^6+a}\,abc+\sqrt{a}\log\left(\sqrt{b\,x^6+a}-\sqrt{a}\right)abc+\sqrt{a}\log\left(\sqrt{b\,x^6\cdots\right.}{6a^2b\left(b\,x\cdots\right.}$$

input
```
int((d*x^6+c)/x/(b*x^6+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**6)*a**2*d + 2*sqrt(a + b*x**6)*a*b*c + sqrt(a)*log(sqr
t(a + b*x**6) - sqrt(a))*a*b*c + sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*b
**2*c*x**6 - sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a*b*c - sqrt(a)*log(s
qrt(a + b*x**6) + sqrt(a))*b**2*c*x**6)/(6*a**2*b*(a + b*x**6))
```

3.6 $\int \frac{c+dx^6}{x^7(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{3/2}}\,dx=-\frac{bc-ad}{3a^2\sqrt{a+bx^6}}-\frac{c\sqrt{a+bx^6}}{6a^2x^6}+\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{6a^{5/2}}$$

output
```
-1/3*(-a*d+b*c)/a^2/(b*x^6+a)^(1/2)-1/6*c*(b*x^6+a)^(1/2)/a^2/x^6+1/6*(-2*
a*d+3*b*c)*arctanh((b*x^6+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{3/2}}\,dx=\frac{-ac-3bcx^6+2adx^6}{6a^2x^6\sqrt{a+bx^6}}+\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{6a^{5/2}}$$

input
```
Integrate[(c + d*x^6)/(x^7*(a + b*x^6)^(3/2)),x]
```

output
```
(-(a*c) - 3*b*c*x^6 + 2*a*d*x^6)/(6*a^2*x^6*Sqrt[a + b*x^6]) + ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b*x^6]/Sqrt[a]])/(6*a^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{6}\int \frac{dx^6+c}{x^{12} (bx^6+a)^{3/2}}dx^6$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{6}\left(-\frac{(3bc-2ad)\int \frac{1}{x^6 (bx^6+a)^{3/2}}dx^6}{2a}-\frac{c}{ax^6\sqrt{a+bx^6}}\right)$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{6}\left(-\frac{(3bc-2ad)\left(\frac{\int \frac{1}{x^6\sqrt{bx^6+a}}dx^6}{a}+\frac{2}{a\sqrt{a+bx^6}}\right)}{2a}-\frac{c}{ax^6\sqrt{a+bx^6}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{6}\left(-\frac{(3bc-2ad)\left(\frac{2\int \frac{1}{\frac{x^{12}}{b}-\frac{a}{b}}d\sqrt{bx^6+a}}{ab}+\frac{2}{a\sqrt{a+bx^6}}\right)}{2a}-\frac{c}{ax^6\sqrt{a+bx^6}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{6}\left(-\frac{\left(\frac{2}{a\sqrt{a+bx^6}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{a^{3/2}}\right)(3bc-2ad)}{2a}-\frac{c}{ax^6\sqrt{a+bx^6}}\right)$$

input
```
Int[(c + d*x^6)/(x^7*(a + b*x^6)^(3/2)),x]
```

output
```
(-(c/(a*x^6*Sqrt[a + b*x^6])) - ((3*b*c - 2*a*d)*(2/(a*Sqrt[a + b*x^6]) -
(2*ArcTanh[Sqrt[a + b*x^6]/Sqrt[a]])/a^(3/2)))/(2*a))/6
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$-\frac{2\sqrt{b\,x^{6}+a}\,x^{6}\left(ad-\frac{3cb}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{\sqrt{a}}\right)+\left(\left(-2d\,x^{6}+c\right)a+3bc\,x^{6}\right)\sqrt{a}}{6\sqrt{b\,x^{6}+a}\,a^{\frac{5}{2}}\,x^{6}}$	77

input
```
int((d*x^6+c)/x^7/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(b*x^6+a)^(1/2)*(2*(b*x^6+a)^(1/2)*x^6*(a*d-3/2*c*b)*arctanh((b*x^6+a
)^(1/2)/a^(1/2))+((-2*d*x^6+c)*a+3*b*c*x^6)*a^(1/2))/a^(5/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.68

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{3/2}}\,dx = \left[-\frac{((3\,b^2c-2\,abd)x^{12}+(3\,abc-2\,a^2d)x^6)\sqrt{a}\log\left(\frac{bx^6-2\,\sqrt{bx^6+a}\sqrt{a}+2\,a}{x^6}\right)+2\,((3\,abc-}{12\,(a^3bx^{12}+a^4x^6)}\right.$$
$$\left.-\frac{((3\,b^2c-2\,abd)x^{12}+(3\,abc-2\,a^2d)x^6)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^6+a}}\right)+((3\,abc-2\,a^2d)x^6+a^2c)\sqrt{bx^6+a}}{6\,(a^3bx^{12}+a^4x^6)}\right]$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((3*b^2*c - 2*a*b*d)*x^12 + (3*a*b*c - 2*a^2*d)*x^6)*sqrt(a)*log((
b*x^6 - 2*sqrt(b*x^6 + a)*sqrt(a) + 2*a)/x^6) + 2*((3*a*b*c - 2*a^2*d)*x^6
 + a^2*c)*sqrt(b*x^6 + a))/(a^3*b*x^12 + a^4*x^6), -1/6*(((3*b^2*c - 2*a*b
*d)*x^12 + (3*a*b*c - 2*a^2*d)*x^6)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^6 +
a)) + ((3*a*b*c - 2*a^2*d)*x^6 + a^2*c)*sqrt(b*x^6 + a))/(a^3*b*x^12 + a^4
*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(75) = 150$.

Time = 62.68 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.08

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{3/2}}\,dx = c\left(-\frac{1}{6a\sqrt{b}x^9\sqrt{\frac{a}{bx^6}+1}}-\frac{\sqrt{b}}{2a^2x^3\sqrt{\frac{a}{bx^6}+1}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^3}\right)}{2a^{\frac{5}{2}}}\right)+d\left(\frac{2a^3\sqrt{1+\frac{bx^6}{a}}}{6a^{\frac{9}{2}}+6a^{\frac{7}{2}}bx^6}+\frac{a^3\log\left(\frac{bx^6}{a}\right)}{6a^{\frac{9}{2}}+6a^{\frac{7}{2}}bx^6}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^6}{a}}+1\right)}{6a^{\frac{9}{2}}+6a^{\frac{7}{2}}bx^6}+\frac{a^2bx^6\log\left(\frac{bx^6}{a}\right)}{6a^{\frac{9}{2}}+6a^{\frac{7}{2}}bx^6}-\frac{2a^2bx^6\log\left(\sqrt{1+\frac{bx^6}{a}}+1\right)}{6a^{\frac{9}{2}}+6a^{\frac{7}{2}}bx^6}\right)$$

input
```
integrate((d*x**6+c)/x**7/(b*x**6+a)**(3/2),x)
```

output
```
c*(-1/(6*a*sqrt(b)*x**9*sqrt(a/(b*x**6) + 1)) - sqrt(b)/(2*a**2*x**3*sqrt(
a/(b*x**6) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**3))/(2*a**(5/2))) + d*(2*a*
*3*sqrt(1 + b*x**6/a)/(6*a**(9/2) + 6*a**(7/2)*b*x**6) + a**3*log(b*x**6/a
)/(6*a**(9/2) + 6*a**(7/2)*b*x**6) - 2*a**3*log(sqrt(1 + b*x**6/a) + 1)/(6
*a**(9/2) + 6*a**(7/2)*b*x**6) + a**2*b*x**6*log(b*x**6/a)/(6*a**(9/2) + 6
*a**(7/2)*b*x**6) - 2*a**2*b*x**6*log(sqrt(1 + b*x**6/a) + 1)/(6*a**(9/2)
+ 6*a**(7/2)*b*x**6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(69) = 138$.

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.69

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{3/2}}\,dx = -\frac{1}{12}\,c\left(\frac{2\,(3\,(bx^6+a)b-2\,ab)}{(bx^6+a)^{\frac{3}{2}}a^2-\sqrt{bx^6+a}a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right) + \frac{1}{6}\,d\left(\frac{\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^6+a}a}\right)$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/12*c*(2*(3*(b*x^6 + a)*b - 2*a*b)/((b*x^6 + a)^(3/2)*a^2 - sqrt(b*x^6 +
 a)*a^3) + 3*b*log((sqrt(b*x^6 + a) - sqrt(a))/(sqrt(b*x^6 + a) + sqrt(a))
)/a^(5/2)) + 1/6*d*(log((sqrt(b*x^6 + a) - sqrt(a))/(sqrt(b*x^6 + a) + sqr
t(a)))/a^(3/2) + 2/(sqrt(b*x^6 + a)*a))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{3/2}}\,dx = -\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{bx^6+a}}{\sqrt{-a}}\right)}{6\,\sqrt{-a}a^2} - \frac{3\,(bx^6+a)bc-2\,abc-2\,(bx^6+a)ad+2\,a^2d}{6\left((bx^6+a)^{\frac{3}{2}}-\sqrt{bx^6+a}a\right)a^2}$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
-1/6*(3*b*c - 2*a*d)*arctan(sqrt(b*x^6 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/6
*(3*(b*x^6 + a)*b*c - 2*a*b*c - 2*(b*x^6 + a)*a*d + 2*a^2*d)/(((b*x^6 + a)
^(3/2) - sqrt(b*x^6 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{3/2}}\,dx = \frac{d}{3\,a\,\sqrt{b\,x^6+a}} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^6+a}}{\sqrt{a}}\right)}{3\,a^{3/2}} - \frac{c}{6\,a\,x^6\,\sqrt{b\,x^6+a}} + \frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^6+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{b\,c}{2\,a^2\,\sqrt{b\,x^6+a}}$$

input
```
int((c + d*x^6)/(x^7*(a + b*x^6)^(3/2)),x)
```

output
```
d/(3*a*(a + b*x^6)^(1/2)) - (d*atanh((a + b*x^6)^(1/2)/a^(1/2)))/(3*a^(3/2
)) - c/(6*a*x^6*(a + b*x^6)^(1/2)) + (b*c*atanh((a + b*x^6)^(1/2)/a^(1/2))
)/(2*a^(5/2)) - (b*c)/(2*a^2*(a + b*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.98

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^6+a}\,a^2c+4\sqrt{b\,x^6+a}\,a^2d\,x^6-6\sqrt{b\,x^6+a}\,abc\,x^6+2\sqrt{a}\log\left(\sqrt{b\,x^6+a}-\right.}{}$$

input
```
int((d*x^6+c)/x^7/(b*x^6+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**6)*a**2*c + 4*sqrt(a + b*x**6)*a**2*d*x**6 - 6*sqrt(a
+ b*x**6)*a*b*c*x**6 + 2*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a**2*d*x*
*6 - 3*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a*b*c*x**6 + 2*sqrt(a)*log(
sqrt(a + b*x**6) - sqrt(a))*a*b*d*x**12 - 3*sqrt(a)*log(sqrt(a + b*x**6) -
 sqrt(a))*b**2*c*x**12 - 2*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a**2*d*
x**6 + 3*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a*b*c*x**6 - 2*sqrt(a)*lo
g(sqrt(a + b*x**6) + sqrt(a))*a*b*d*x**12 + 3*sqrt(a)*log(sqrt(a + b*x**6)
 + sqrt(a))*b**2*c*x**12)/(12*a**3*x**6*(a + b*x**6))
```

3.7 $\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{(bc-ad)x^3}{3b^2\sqrt{a+bx^6}} + \frac{dx^3\sqrt{a+bx^6}}{6b^2} + \frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{6b^{5/2}}$$

output
```
-1/3*(-a*d+b*c)*x^3/b^2/(b*x^6+a)^(1/2)+1/6*d*x^3*(b*x^6+a)^(1/2)/b^2+1/6*
(-3*a*d+2*b*c)*arctanh(b^(1/2)*x^3/(b*x^6+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^3(-2bc+3ad+bdx^6)}{6b^2\sqrt{a+bx^6}} + \frac{(2bc-3ad)\log\left(\sqrt{b}x^3+\sqrt{a+bx^6}\right)}{6b^{5/2}}$$

input
```
Integrate[(x^8*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^3*(-2*b*c + 3*a*d + b*d*x^6))/(6*b^2*Sqrt[a + b*x^6]) + ((2*b*c - 3*a*d
)*Log[Sqrt[b]*x^3 + Sqrt[a + b*x^6]])/(6*b^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {959, 807, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2bc-3ad)\int \frac{x^8}{(bx^6+a)^{3/2}}dx}{2b}+\frac{dx^9}{6b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2bc-3ad)\int \frac{x^6}{(bx^6+a)^{3/2}}dx^3}{6b}+\frac{dx^9}{6b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(2bc-3ad)\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^3}{b}-\frac{x^3}{b\sqrt{a+bx^6}}\right)}{6b}+\frac{dx^9}{6b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2bc-3ad)\left(\frac{\int \frac{1}{1-bx^6}d\frac{x^3}{\sqrt{bx^6+a}}}{b}-\frac{x^3}{b\sqrt{a+bx^6}}\right)}{6b}+\frac{dx^9}{6b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{b^{3/2}}-\frac{x^3}{b\sqrt{a+bx^6}}\right)(2bc-3ad)}{6b}+\frac{dx^9}{6b\sqrt{a+bx^6}}$$

input
```
Int[(x^8*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^9)/(6*b*Sqrt[a + b*x^6]) + ((2*b*c - 3*a*d)*(-(x^3/(b*Sqrt[a + b*x^6]
)) + ArcTanh[(Sqrt[b]*x^3)/Sqrt[a + b*x^6]]/b^(3/2)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 5.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{\sqrt{b\,x^{6}+a}\,\left(-c b+\frac{3 a d}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{x^{3} \sqrt{b}}\right)+\left(\left(-\frac{d\,x^{6}}{2}+c\right) b-\frac{3 a d}{2}\right) x^{3} \sqrt{b}}{3 \sqrt{b\,x^{6}+a}\, b^{\frac{5}{2}}}$	74

input
```
int(x^8*(d*x^6+c)/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((b*x^6+a)^(1/2)*(-c*b+3/2*a*d)*arctanh((b*x^6+a)^(1/2)/x^3/b^(1/2))+
((-1/2*d*x^6+c)*b-3/2*a*d)*x^3*b^(1/2))/(b*x^6+a)^(1/2)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.46

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \left[-\frac{((2\,b^2c-3\,abd)x^6+2\,abc-3\,a^2d)\sqrt{b}\log\left(-2\,bx^6+2\,\sqrt{bx^6+a}\sqrt{b}x^3-a\right)-2\,(b^2}{12\,(b^4x^6+ab^3)}\right.$$
$$\left.-\frac{((2\,b^2c-3\,abd)x^6+2\,abc-3\,a^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3}{\sqrt{bx^6+a}}\right)-(b^2dx^9-(2\,b^2c-3\,abd)x^3)\sqrt{bx^6+a}}{6\,(b^4x^6+ab^3)}\right]$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((2*b^2*c - 3*a*b*d)*x^6 + 2*a*b*c - 3*a^2*d)*sqrt(b)*log(-2*b*x^6
 + 2*sqrt(b*x^6 + a)*sqrt(b)*x^3 - a) - 2*(b^2*d*x^9 - (2*b^2*c - 3*a*b*d)
*x^3)*sqrt(b*x^6 + a))/(b^4*x^6 + a*b^3), -1/6*(((2*b^2*c - 3*a*b*d)*x^6 +
 2*a*b*c - 3*a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x^3/sqrt(b*x^6 + a)) - (b^2*d
*x^9 - (2*b^2*c - 3*a*b*d)*x^3)*sqrt(b*x^6 + a))/(b^4*x^6 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 43.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = c\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^3}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{x^3}{3\sqrt{a}b\sqrt{1+\frac{bx^6}{a}}}\right) + d\left(\frac{\sqrt{a}x^3}{2b^2\sqrt{1+\frac{bx^6}{a}}} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x^3}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^9}{6\sqrt{a}b\sqrt{1+\frac{bx^6}{a}}}\right)$$

input
```
integrate(x**8*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*(asinh(sqrt(b)*x**3/sqrt(a))/(3*b**(3/2)) - x**3/(3*sqrt(a)*b*sqrt(1 + b
*x**6/a))) + d*(sqrt(a)*x**3/(2*b**2*sqrt(1 + b*x**6/a)) - a*asinh(sqrt(b)
*x**3/sqrt(a))/(2*b**(5/2)) + x**9/(6*sqrt(a)*b*sqrt(1 + b*x**6/a)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{1}{6}\left(\frac{2\,x^3}{\sqrt{bx^6+a}b} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{3}{2}}}\right)c + \frac{1}{12}\,d\left(\frac{2\left(2\,ab-\frac{3\,(bx^6+a)a}{x^6}\right)}{\frac{\sqrt{bx^6+a}b^3}{x^3}-\frac{(bx^6+a)^{\frac{3}{2}}b^2}{x^9}} + \frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{5}{2}}}\right)$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*(2*x^3/(sqrt(b*x^6 + a)*b) + log(-(sqrt(b) - sqrt(b*x^6 + a)/x^3)/(sq
rt(b) + sqrt(b*x^6 + a)/x^3))/b^(3/2))*c + 1/12*d*(2*(2*a*b - 3*(b*x^6 + a
)*a/x^6)/(sqrt(b*x^6 + a)*b^3/x^3 - (b*x^6 + a)^(3/2)*b^2/x^9) + 3*a*log(-
(sqrt(b) - sqrt(b*x^6 + a)/x^3)/(sqrt(b) + sqrt(b*x^6 + a)/x^3))/b^(5/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb/t_nostep^6) ignored2*(486*sageVARb^4
*sageVARd*1/5832/sageVARb^5*sageVARx*sageVARx*sageVARx*sageVARx*sageVARx*s
ageVARx-(972*sageVA
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{x^8\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^8*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^8*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.96

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{6\sqrt{b\,x^6+a}\,abd\,x^3 - 4\sqrt{b\,x^6+a}\,b^2c\,x^3 + 2\sqrt{b\,x^6+a}\,b^2d\,x^9 + 3\sqrt{b}\log\Big(\sqrt{b\,x^6+a} - \sqrt{}}{}$$

input
```
int(x^8*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(6*sqrt(a + b*x**6)*a*b*d*x**3 - 4*sqrt(a + b*x**6)*b**2*c*x**3 + 2*sqrt(a
 + b*x**6)*b**2*d*x**9 + 3*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a*
*2*d - 2*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a*b*c + 3*sqrt(b)*lo
g(sqrt(a + b*x**6) - sqrt(b)*x**3)*a*b*d*x**6 - 2*sqrt(b)*log(sqrt(a + b*x
**6) - sqrt(b)*x**3)*b**2*c*x**6 - 3*sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b
)*x**3)*a**2*d + 2*sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a*b*c - 3*
sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a*b*d*x**6 + 2*sqrt(b)*log(sq
rt(a + b*x**6) + sqrt(b)*x**3)*b**2*c*x**6)/(12*b**3*(a + b*x**6))
```

3.8 $\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}} dx = \frac{(bc-ad)x^3}{3ab\sqrt{a+bx^6}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{3b^{3/2}}$$

output
```
1/3*(-a*d+b*c)*x^3/a/b/(b*x^6+a)^(1/2)+1/3*d*arctanh(b^(1/2)*x^3/(b*x^6+a)
^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}} dx = -\frac{(-bc+ad)x^3}{3ab\sqrt{a+bx^6}} + \frac{d\log\left(\sqrt{b}x^3+\sqrt{a+bx^6}\right)}{3b^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
-1/3*((-(b*c) + a*d)*x^3)/(a*b*Sqrt[a + b*x^6]) + (d*Log[Sqrt[b]*x^3 + Sqr
t[a + b*x^6]])/(3*b^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {954, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^6)}{(a+bx^6)^{3/2}} \, dx$$

$$\Big\downarrow \; 954$$

$$\frac{d \int \frac{x^2}{\sqrt{bx^6+a}}dx}{b} + \frac{x^3(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 807$$

$$\frac{d \int \frac{1}{\sqrt{bx^6+a}}dx^3}{3b} + \frac{x^3(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 224$$

$$\frac{d \int \frac{1}{1-bx^6}d\frac{x^3}{\sqrt{bx^6+a}}}{3b} + \frac{x^3(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 219$$

$$\frac{d \operatorname{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{3b^{3/2}} + \frac{x^3(bc-ad)}{3ab\sqrt{a+bx^6}}$$

input
```
Int[(x^2*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
((b*c - a*d)*x^3)/(3*a*b*Sqrt[a + b*x^6]) + (d*ArcTanh[(Sqrt[b]*x^3)/Sqrt[
a + b*x^6]])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\frac{ad\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{x^{3}\sqrt{b}}\right)}{b^{\frac{3}{2}}}-\frac{(ad-cb)x^{3}}{b\sqrt{b\,x^{6}+a}}}{3a}$	55

input
```
int(x^2*(d*x^6+c)/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a/b^(3/2)*d*arctanh((b*x^6+a)^(1/2)/x^3/b^(1/2))-(a*d-b*c)*x^3/b/(b*x
^6+a)^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.75

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \left[\frac{2\sqrt{bx^6+a}(b^2c-abd)x^3+(abdx^6+a^2d)\sqrt{b}\log\left(-2\,bx^6-2\sqrt{bx^6+a}\sqrt{b}x^3-a\right)}{6\,(ab^3x^6+a^2b^2)},\right.$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(2*sqrt(b*x^6 + a)*(b^2*c - a*b*d)*x^3 + (a*b*d*x^6 + a^2*d)*sqrt(b)*
log(-2*b*x^6 - 2*sqrt(b*x^6 + a)*sqrt(b)*x^3 - a))/(a*b^3*x^6 + a^2*b^2),
1/3*(sqrt(b*x^6 + a)*(b^2*c - a*b*d)*x^3 - (a*b*d*x^6 + a^2*d)*sqrt(-b)*ar
ctan(sqrt(-b)*x^3/sqrt(b*x^6 + a)))/(a*b^3*x^6 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 10.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^3}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{x^3}{3\sqrt{a}b\sqrt{1+\frac{bx^6}{a}}}\right)+\frac{cx^3}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^6}{a}}}$$

input
```
integrate(x**2*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
d*(asinh(sqrt(b)*x**3/sqrt(a))/(3*b**(3/2)) - x**3/(3*sqrt(a)*b*sqrt(1 + b
*x**6/a))) + c*x**3/(3*a**(3/2)*sqrt(1 + b*x**6/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^3}{3\sqrt{bx^6+a}a} - \frac{1}{6}\left(\frac{2\,x^3}{\sqrt{bx^6+a}b} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{3}{2}}}\right)d$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*c*x^3/(sqrt(b*x^6 + a)*a) - 1/6*(2*x^3/(sqrt(b*x^6 + a)*b) + log(-(sqr
t(b) - sqrt(b*x^6 + a)/x^3)/(sqrt(b) + sqrt(b*x^6 + a)/x^3))/b^(3/2))*d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{(bc-ad)x^3}{3\sqrt{bx^6+a}ab} - \frac{d\log\left(\left|-\sqrt{b}x^3+\sqrt{bx^6+a}\right|\right)}{3\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(b*c - a*d)*x^3/(sqrt(b*x^6 + a)*a*b) - 1/3*d*log(abs(-sqrt(b)*x^3 + s
qrt(b*x^6 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{x^2\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^2*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.36

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^6+a}\,abd\,x^3+2\sqrt{b\,x^6+a}\,b^2c\,x^3-\sqrt{b}\log\left(\sqrt{b\,x^6+a}-\sqrt{b}\,x^3\right)a^2d-\sqrt{b}\log\left(\right.}{6}$$

input
```
int(x^2*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**6)*a*b*d*x**3 + 2*sqrt(a + b*x**6)*b**2*c*x**3 - sqrt(
b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a**2*d - sqrt(b)*log(sqrt(a + b*x*
*6) - sqrt(b)*x**3)*a*b*d*x**6 + sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x*
*3)*a**2*d + sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a*b*d*x**6)/(6*a
*b**2*(a + b*x**6))
```

3.9 $\int \frac{c+dx^6}{x^4(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx=-\frac{c}{3ax^3\sqrt{a+bx^6}}-\frac{(2bc-ad)x^3}{3a^2\sqrt{a+bx^6}}$$

output
```
-1/3*c/a/x^3/(b*x^6+a)^(1/2)-1/3*(-a*d+2*b*c)*x^3/a^2/(b*x^6+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx=\frac{-ac-2bcx^6+adx^6}{3a^2x^3\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^4*(a + b*x^6)^(3/2)),x]
```

output
```
(-(a*c) - 2*b*c*x^6 + a*d*x^6)/(3*a^2*x^3*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^4 (a+bx^6)^{3/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-ad)\int \frac{x^2}{(bx^6+a)^{3/2}}dx}{a}-\frac{c}{3ax^3\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^3(2bc-ad)}{3a^2\sqrt{a+bx^6}}-\frac{c}{3ax^3\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^4*(a + b*x^6)^(3/2)),x]
```

output
```
-1/3*c/(a*x^3*Sqrt[a + b*x^6]) - ((2*b*c - a*d)*x^3)/(3*a^2*Sqrt[a + b*x^6
])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{-a d\,x^{6}+2 b c\,x^{6}+a c}{3 \sqrt{b\,x^{6}+a}\,x^{3} a^{2}}$	36
trager	$-\frac{-a d\,x^{6}+2 b c\,x^{6}+a c}{3 \sqrt{b\,x^{6}+a}\,x^{3} a^{2}}$	36
pseudoelliptic	$-\frac{\left(-d\,x^{6}+c \right) a+2 b c\,x^{6}}{3 \sqrt{b\,x^{6}+a}\,x^{3} a^{2}}$	36
orering	$-\frac{-a d\,x^{6}+2 b c\,x^{6}+a c}{3 \sqrt{b\,x^{6}+a}\,x^{3} a^{2}}$	36
risch	$-\frac{c \sqrt{b\,x^{6}+a}}{3 a^{2} x^{3}}+\frac{x^{3} \left(a d -c b \right)}{3 \sqrt{b\,x^{6}+a}\,a^{2}}$	45

input
```
int((d*x^6+c)/x^4/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*d*x^6+2*b*c*x^6+a*c)/(b*x^6+a)^(1/2)/x^3/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx=-\frac{\left((2\,bc-ad)x^6+ac\right)\sqrt{bx^6+a}}{3\left(a^2bx^9+a^3x^3\right)}$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((2*b*c - a*d)*x^6 + a*c)*sqrt(b*x^6 + a)/(a^2*b*x^9 + a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 24.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx=c\left(-\frac{1}{3a\sqrt{b}x^6\sqrt{\frac{a}{bx^6}+1}}-\frac{2\sqrt{b}}{3a^2\sqrt{\frac{a}{bx^6}+1}}\right)+\frac{dx^3}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^6}{a}}}$$

input
```
integrate((d*x**6+c)/x**4/(b*x**6+a)**(3/2),x)
```

output
```
c*(-1/(3*a*sqrt(b)*x**6*sqrt(a/(b*x**6) + 1)) - 2*sqrt(b)/(3*a**2*sqrt(a/(
b*x**6) + 1))) + d*x**3/(3*a**(3/2)*sqrt(1 + b*x**6/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx=\frac{dx^3}{3\sqrt{bx^6+a}a}-\frac{1}{3}\left(\frac{bx^3}{\sqrt{bx^6+a}a^2}+\frac{\sqrt{bx^6+a}}{a^2x^3}\right)c$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*d*x^3/(sqrt(b*x^6 + a)*a) - 1/3*(b*x^3/(sqrt(b*x^6 + a)*a^2) + sqrt(b*
x^6 + a)/(a^2*x^3))*c
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{c+dx^6}{x^4(a+bx^6)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{(bc-ad)x^3}{3\sqrt{bx^6+a}a^2} + \frac{c\left(\frac{\sqrt{b}\,\mathrm{sgn}(x)}{a} - \frac{\sqrt{b+\frac{a}{x^6}}}{a\,\mathrm{sgn}(x)}\right)}{3a} - \frac{bignored}{t_{nostep}^6}$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= -1/3*(b*c - a*d)*x^3/(sqrt(b*x^6 + a)*a^2) + 1/3
*c*(sqrt(b)*sgn(x)/a - sqrt(b + a/x^6)/(a*sgn(x)))/a - b*ignored/t_nostep^
6
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^6}{x^4(a+bx^6)^{3/2}}\,dx = \frac{a\,c-2\,c\,(b\,x^6+a)+a\,d\,x^6}{3\,a^2\,x^3\,\sqrt{b\,x^6+a}}$$

input
```
int((c + d*x^6)/(x^4*(a + b*x^6)^(3/2)),x)
```

output
```
(a*c - 2*c*(a + b*x^6) + a*d*x^6)/(3*a^2*x^3*(a + b*x^6)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{3/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,\left(ad\,x^6-2bc\,x^6-ac\right)}{3a^2x^3\left(b\,x^6+a\right)}$$

input
```
int((d*x^6+c)/x^4/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*( - a*c + a*d*x**6 - 2*b*c*x**6))/(3*a**2*x**3*(a + b*x*
*6))
```

3.10 $\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx=-\frac{c}{9ax^9\sqrt{a+bx^6}}-\frac{4bc-3ad}{9a^2x^3\sqrt{a+bx^6}}+\frac{2(4bc-3ad)\sqrt{a+bx^6}}{9a^3x^3}$$

output
```
-1/9*c/a/x^9/(b*x^6+a)^(1/2)-1/9*(-3*a*d+4*b*c)/a^2/x^3/(b*x^6+a)^(1/2)+2/
9*(-3*a*d+4*b*c)*(b*x^6+a)^(1/2)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx=\frac{-a^2c+4abcx^6-3a^2dx^6+8b^2cx^{12}-6abdx^{12}}{9a^3x^9\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^10*(a + b*x^6)^(3/2)),x]
```

output
```
(-(a^2*c) + 4*a*b*c*x^6 - 3*a^2*d*x^6 + 8*b^2*c*x^12 - 6*a*b*d*x^12)/(9*a^
3*x^9*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4bc-3ad)\int \frac{1}{x^4(bx^6+a)^{3/2}}dx}{3a}-\frac{c}{9ax^9\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-3ad)\left(-\frac{2b\int \frac{x^2}{\left(bx^6+a\right)^{3/2}}dx}{a}-\frac{1}{3ax^3\sqrt{a+bx^6}}\right)}{3a}-\frac{c}{9ax^9\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{2bx^3}{3a^2\sqrt{a+bx^6}}-\frac{1}{3ax^3\sqrt{a+bx^6}}\right)(4bc-3ad)}{3a}-\frac{c}{9ax^9\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^10*(a + b*x^6)^(3/2)),x]
```

output
```
-1/9*c/(a*x^9*Sqrt[a + b*x^6]) - ((4*b*c - 3*a*d)*(-1/3*1/(a*x^3*Sqrt[a +
b*x^6]) - (2*b*x^3)/(3*a^2*Sqrt[a + b*x^6])))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

method	result	size
pseudoelliptic	$-\frac{\left(3 d \,x^{6}+c \right) a^{2}-4 b \,x^{6} \left(-\frac{3 d \,x^{6}}{2}+c \right) a -8 b^{2} c \,x^{12}}{9 \sqrt{b \,x^{6}+a}\, x^{9} a^{3}}$	55
gosper	$-\frac{6 a b d \,x^{12}-8 b^{2} c \,x^{12}+3 a^{2} d \,x^{6}-4 a b c \,x^{6}+a^{2} c}{9 \sqrt{b \,x^{6}+a}\, x^{9} a^{3}}$	58
trager	$-\frac{6 a b d \,x^{12}-8 b^{2} c \,x^{12}+3 a^{2} d \,x^{6}-4 a b c \,x^{6}+a^{2} c}{9 \sqrt{b \,x^{6}+a}\, x^{9} a^{3}}$	58
orering	$-\frac{6 a b d \,x^{12}-8 b^{2} c \,x^{12}+3 a^{2} d \,x^{6}-4 a b c \,x^{6}+a^{2} c}{9 \sqrt{b \,x^{6}+a}\, x^{9} a^{3}}$	58
risch	$-\frac{\sqrt{b \,x^{6}+a}\, \left(3 a d \,x^{6}-5 b c \,x^{6}+a c \right)}{9 a^{3} x^{9}}-\frac{x^{3} \left(a d -c b \right) b}{3 \sqrt{b \,x^{6}+a}\, a^{3}}$	63

input
```
int((d*x^6+c)/x^10/(b*x^6+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((3*d*x^6+c)*a^2-4*b*x^6*(-3/2*d*x^6+c)*a-8*b^2*c*x^12)/(b*x^6+a)^(1/
2)/x^9/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = \frac{\left(2\left(4\,b^2c-3\,abd\right)x^{12}+\left(4\,abc-3\,a^2d\right)x^6-a^2c\right)\sqrt{bx^6+a}}{9\left(a^3bx^{15}+a^4x^9\right)}$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/9*(2*(4*b^2*c - 3*a*b*d)*x^12 + (4*a*b*c - 3*a^2*d)*x^6 - a^2*c)*sqrt(b*
x^6 + a)/(a^3*b*x^15 + a^4*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(76) = 152.

Time = 92.75 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.46

$$\begin{aligned}\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = c\Bigg(&-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^6}+1}}{9a^5b^4x^6+18a^4b^5x^{12}+9a^3b^6x^{18}}\\&+\frac{3a^2b^{\frac{11}{2}}x^6\sqrt{\frac{a}{bx^6}+1}}{9a^5b^4x^6+18a^4b^5x^{12}+9a^3b^6x^{18}}+\frac{12ab^{\frac{13}{2}}x^{12}\sqrt{\frac{a}{bx^6}+1}}{9a^5b^4x^6+18a^4b^5x^{12}+9a^3b^6x^{18}}\\&+\frac{8b^{\frac{15}{2}}x^{18}\sqrt{\frac{a}{bx^6}+1}}{9a^5b^4x^6+18a^4b^5x^{12}+9a^3b^6x^{18}}\Bigg)+d\left(-\frac{1}{3a\sqrt{b}x^6\sqrt{\frac{a}{bx^6}+1}}-\frac{2\sqrt{b}}{3a^2\sqrt{\frac{a}{bx^6}+1}}\right)\end{aligned}$$

input
```
integrate((d*x**6+c)/x**10/(b*x**6+a)**(3/2),x)
```

output
```
c*(-a**3*b**(9/2)*sqrt(a/(b*x**6) + 1)/(9*a**5*b**4*x**6 + 18*a**4*b**5*x*
*12 + 9*a**3*b**6*x**18) + 3*a**2*b**(11/2)*x**6*sqrt(a/(b*x**6) + 1)/(9*a
**5*b**4*x**6 + 18*a**4*b**5*x**12 + 9*a**3*b**6*x**18) + 12*a*b**(13/2)*x
**12*sqrt(a/(b*x**6) + 1)/(9*a**5*b**4*x**6 + 18*a**4*b**5*x**12 + 9*a**3*
b**6*x**18) + 8*b**(15/2)*x**18*sqrt(a/(b*x**6) + 1)/(9*a**5*b**4*x**6 + 1
8*a**4*b**5*x**12 + 9*a**3*b**6*x**18)) + d*(-1/(3*a*sqrt(b)*x**6*sqrt(a/(
b*x**6) + 1)) - 2*sqrt(b)/(3*a**2*sqrt(a/(b*x**6) + 1)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = \frac{1}{9}\left(\frac{3\,b^2x^3}{\sqrt{bx^6+a}a^3}+\frac{\frac{6\,\sqrt{bx^6+a}b}{x^3}-\frac{\left(bx^6+a\right)^{\frac{3}{2}}}{x^9}}{a^3}\right)c - \frac{1}{3}\left(\frac{bx^3}{\sqrt{bx^6+a}a^2}+\frac{\sqrt{bx^6+a}}{a^2x^3}\right)d$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(3*b^2*x^3/(sqrt(b*x^6 + a)*a^3) + (6*sqrt(b*x^6 + a)*b/x^3 - (b*x^6 +
 a)^(3/2)/x^9)/a^3)*c - 1/3*(b*x^3/(sqrt(b*x^6 + a)*a^2) + sqrt(b*x^6 + a)
/(a^2*x^3))*d
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = Recursiveaassumption \geq \frac{(b^2c-abd)x^3}{3\,\sqrt{bx^6+a}a^3} - \frac{\frac{\left(5\,b^{\frac{3}{2}}c-3\,a\sqrt{b}d\right)\mathrm{sgn}(x)}{a}+\frac{\left(b+\frac{a}{x^6}\right)^{\frac{3}{2}}c}{a\mathrm{sgn}(x)}-\frac{3\,(2\,bc-ad)\sqrt{b+\frac{a}{x^6}}}{a\mathrm{sgn}(x)}}{9\,a^2} - \frac{bignored}{t_{nostep}^6}$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= 1/3*(b^2*c - a*b*d)*x^3/(sqrt(b*x^6 + a)*a^3) -
1/9*((5*b^(3/2)*c - 3*a*sqrt(b)*d)*sgn(x)/a + (b + a/x^6)^(3/2)*c/(a*sgn(x
)) - 3*(2*b*c - a*d)*sqrt(b + a/x^6)/(a*sgn(x)))/a^2 - b*ignored/t_nostep^
6
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = -\frac{8\,c\,{\left(b\,x^6+a\right)}^2+3\,a^2\,c+3\,a^2\,d\,x^6-12\,a\,c\,\left(b\,x^6+a\right)-6\,a\,d\,x^6\,\left(b\,x^6+a\right)}{\left(\frac{9\,a^4\,x^3}{b}-\frac{9\,a^3\,x^3\,\left(b\,x^6+a\right)}{b}\right)\,\sqrt{b\,x^6+a}}$$

input
```
int((c + d*x^6)/(x^10*(a + b*x^6)^(3/2)),x)
```

output
```
-(8*c*(a + b*x^6)^2 + 3*a^2*c + 3*a^2*d*x^6 - 12*a*c*(a + b*x^6) - 6*a*d*x
^6*(a + b*x^6))/(((9*a^4*x^3)/b - (9*a^3*x^3*(a + b*x^6))/b)*(a + b*x^6)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{3/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(-6abd\,x^{12}+8b^2c\,x^{12}-3a^2d\,x^6+4abc\,x^6-a^2c)}{9a^3x^9\,(b\,x^6+a)}$$

input
```
int((d*x^6+c)/x^10/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*( - a**2*c - 3*a**2*d*x**6 + 4*a*b*c*x**6 - 6*a*b*d*x**1
2 + 8*b**2*c*x**12))/(9*a**3*x**9*(a + b*x**6))
```

3.11 $\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 284

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{(bc-ad)x^2}{3b^2\sqrt{a+bx^6}} + \frac{dx^2\sqrt{a+bx^6}}{5b^2}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(5bc-8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right), -7}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/3*(-a*d+b*c)*x^2/b^2/(b*x^6+a)^(1/2)+1/5*d*x^2*(b*x^6+a)^(1/2)/b^2+2/45
*(1/2*6^(1/2)+1/2*2^(1/2))*(-8*a*d+5*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2
)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x^2),I*3^(1/2)+2*I)*3^(3/4)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^2\left(-5bc+8ad+3bdx^6+(5bc-8ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a}\right)\right)}{15b^2\sqrt{a+bx^6}}$$

input
```
Integrate[(x^7*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^2*(-5*b*c + 8*a*d + 3*b*d*x^6 + (5*b*c - 8*a*d)*Sqrt[1 + (b*x^6)/a]*Hyp
ergeometric2F1[1/3, 1/2, 4/3, -((b*x^6)/a)]))/(15*b^2*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 807, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(5bc-8ad)\int \frac{x^7}{(bx^6+a)^{3/2}}dx}{5b}+\frac{dx^8}{5b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(5bc-8ad)\int \frac{x^6}{(bx^6+a)^{3/2}}dx^2}{10b}+\frac{dx^8}{5b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5bc-8ad)\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{10b}+\frac{dx^8}{5b\sqrt{a+bx^6}}$$

$$\Big\downarrow 759$$

$$\frac{(5bc-8ad)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{10b}$$

$$\frac{dx^8}{5b\sqrt{a+bx^6}}$$

input
```
Int[(x^7*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^8)/(5*b*Sqrt[a + b*x^6]) + ((5*b*c - 8*a*d)*((-2*x^2)/(3*b*Sqrt[a + b
*x^6]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^
6])))/(10*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{7}(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{3}{2}}}dx$$

input
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.35

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{2\left((5\,b^2c-8\,abd)x^6+5\,abc-8\,a^2d\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right)+\left(3\,b^2dx^8-\right.}{15\,(b^4x^6+ab^3)}$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(2*((5*b^2*c - 8*a*b*d)*x^6 + 5*a*b*c - 8*a^2*d)*sqrt(b)*weierstrassP
Inverse(0, -4*a/b, x^2) + (3*b^2*d*x^8 - (5*b^2*c - 8*a*b*d)*x^2)*sqrt(b*x
^6 + a))/(b^4*x^6 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 13.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{dx^{14}\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**7*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**8*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(3/2)*gamma(7/3)) + d*x**14*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**6
*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^7}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^7/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^7}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^7/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{x^7\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^7*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^7*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{8\sqrt{b\,x^6+a}\,ad\,x^2-5\sqrt{b\,x^6+a}\,bc\,x^2+\sqrt{b\,x^6+a}\,bd\,x^8-16\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^3d\,\cdots}{\cdots}$$

input
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(8*sqrt(a + b*x**6)*a*d*x**2 - 5*sqrt(a + b*x**6)*b*c*x**2 + sqrt(a + b*x*
*6)*b*d*x**8 - 16*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**2*x**12
),x)*a**3*d + 10*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**2*x**12)
,x)*a**2*b*c - 16*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**2*x**12
),x)*a**2*b*d*x**6 + 10*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**2
*x**12),x)*a*b**2*c*x**6)/(5*b**2*(a + b*x**6))
```

3.12 $\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 267

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{(bc-ad)x^2}{3ab\sqrt{a+bx^6}} + \frac{\sqrt{2+\sqrt{3}}(bc+2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/3*(-a*d+b*c)*x^2/a/b/(b*x^6+a)^(1/2)+1/9*(1/2*6^(1/2)+1/2*2^(1/2))*(2*a*
d+b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((
1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^
(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a/b^(4
/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1
/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.28

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^2\left(2bc-2ad+(bc+2ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a}\right)\right)}{6ab\sqrt{a+bx^6}}$$

input
```
Integrate[(x*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^2*(2*b*c - 2*a*d + (b*c + 2*a*d)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[
1/3, 1/2, 4/3, -((b*x^6)/a)]))/(6*a*b*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {957, 807, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(2ad+bc)\int \frac{x}{\sqrt{bx^6+a}}dx}{3ab}+\frac{x^2(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 807$$

$$\frac{(2ad+bc)\int \frac{1}{\sqrt{bx^6+a}}dx^2}{6ab}+\frac{x^2(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 759$$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}(2ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$\frac{x^2(bc-ad)}{3ab\sqrt{a+bx^6}}$$

input
```
Int[(x*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
((b*c - a*d)*x^2)/(3*a*b*Sqrt[a + b*x^6]) + (Sqrt[2 + Sqrt[3]]*(b*c + 2*a*
d)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x
^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{3}{2}}}dx$$

input
```
int(x*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.32

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{bx^6+a}(b^2c-abd)x^2+((b^2c+2\,abd)x^6+abc+2\,a^2d)\sqrt{b}\text{weierstrassPInverse}(0,-}{3\,(ab^3x^6+a^2b^2)}$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(b*x^6 + a)*(b^2*c - a*b*d)*x^2 + ((b^2*c + 2*a*b*d)*x^6 + a*b*c
+ 2*a^2*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x^2))/(a*b^3*x^6 + a^2*b
^2)
```

Sympy [A] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.30

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**2*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(3/2)*gamma(4/3)) + d*x**8*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**6*e
xp_polar(I*pi)/a)/(6*a**(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{x\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d\,x^2+2\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^2d+\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^2x^{12}+2ab\,x^6+a^2}dx\right)abc+2\left(\int \frac{\sqrt{b}}{b^2x^{12}}\right.}{b\,(b\,x^6+a)}$$

input
```
int(x*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x**2 + 2*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**
6 + b**2*x**12),x)*a**2*d + int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 +
b**2*x**12),x)*a*b*c + 2*int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**
2*x**12),x)*a*b*d*x**6 + int((sqrt(a + b*x**6)*x)/(a**2 + 2*a*b*x**6 + b**
2*x**12),x)*b**2*c*x**6)/(b*(a + b*x**6))
```

3.13 $\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{3/2}}\,dx = -\frac{c}{4ax^4\sqrt{a+bx^6}} - \frac{(7bc-4ad)x^2}{12a^2\sqrt{a+bx^6}} - \frac{\sqrt{2+\sqrt{3}}(7bc-4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7\right.}{12\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/4*c/a/x^4/(b*x^6+a)^(1/2)-1/12*(-4*a*d+7*b*c)*x^2/a^2/(b*x^6+a)^(1/2)-1
/36*(1/2*6^(1/2)+1/2*2^(1/2))*(-4*a*d+7*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/
3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(
1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a^2/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)
/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.30

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{3/2}}\,dx = \frac{-6ac-14bcx^6+8adx^6+(-7bc+4ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\right.}{24a^2x^4\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^5*(a + b*x^6)^(3/2)),x]
```

output
```
(-6*a*c - 14*b*c*x^6 + 8*a*d*x^6 + (-7*b*c + 4*a*d)*x^6*Sqrt[1 + (b*x^6)/a
]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^6)/a)])/(24*a^2*x^4*Sqrt[a + b*x
^6])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 807, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(7bc-4ad)\int \frac{x}{(bx^6+a)^{3/2}}dx}{4a}-\frac{c}{4ax^4\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(7bc-4ad)\int \frac{1}{(bx^6+a)^{3/2}}dx^2}{8a}-\frac{c}{4ax^4\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 749}$$

$$-\frac{(7bc-4ad)\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3a}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{8a}-\frac{c}{4ax^4\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 759$$

$$-\frac{(7bc-4ad)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{8a}$$

$$\frac{c}{4ax^4\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^5*(a + b*x^6)^(3/2)),x]
```

output
```
-1/4*c/(a*x^4*Sqrt[a + b*x^6]) - ((7*b*c - 4*a*d)*((2*x^2)/(3*a*Sqrt[a + b
*x^6]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6])))/(8*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{5} \left(b\,x^{6}+a \right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.36

$$\int \frac{c+dx^6}{x^5 \left(a+bx^6\right)^{3/2}}\,dx = -\frac{\left((7\,b^2c-4\,abd)x^{10}+(7\,abc-4\,a^2d)x^4\right)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right)+\left((7\,b^2c-4\,abd)x^6+3\,abc\right)}{12\,\left(a^2b^2x^{10}+a^3bx^4\right)}$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/12*(((7*b^2*c - 4*a*b*d)*x^10 + (7*a*b*c - 4*a^2*d)*x^4)*sqrt(b)*weiers
trassPInverse(0, -4*a/b, x^2) + ((7*b^2*c - 4*a*b*d)*x^6 + 3*a*b*c)*sqrt(b
*x^6 + a))/(a^2*b^2*x^10 + a^3*b*x^4)
```

Sympy [A] (verification not implemented)

Time = 28.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.29

$$\int \frac{c+dx^6}{x^5 \left(a+bx^6\right)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{1}{3} \end{matrix}\middle| {\frac{bx^6e^{i\pi}}{a}} \right)}{6a^{\frac{3}{2}}x^4\Gamma\left(\frac{1}{3}\right)} + \frac{dx^2\Gamma\left(\frac{1}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{4}{3} \end{matrix}\middle| {\frac{bx^6e^{i\pi}}{a}} \right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((d*x**6+c)/x**5/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
3/2)*x**4*gamma(1/3)) + d*x**2*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**6
*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}} x^5}\,dx$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^5), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}} x^5}\,dx$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{3/2}}\,dx = \int \frac{d\,x^6+c}{x^5\,(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^5*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^5*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c + dx^6}{x^5 (a + bx^6)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6 + a}\,d - 4\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{17}+2ab\,x^{11}+a^2x^5}dx\right) a^2 d\,x^4 + 7\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{17}+2ab\,x^{11}+a^2x^5}dx\right) abc\,x^4}{7b\,x^4 \left(b\,x^6 + a\right.}$$

input
```
int((d*x^6+c)/x^5/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 4*int(sqrt(a + b*x**6)/(a**2*x**5 + 2*a*b*x**11 +
 b**2*x**17),x)*a**2*d*x**4 + 7*int(sqrt(a + b*x**6)/(a**2*x**5 + 2*a*b*x*
*11 + b**2*x**17),x)*a*b*c*x**4 - 4*int(sqrt(a + b*x**6)/(a**2*x**5 + 2*a*
b*x**11 + b**2*x**17),x)*a*b*d*x**10 + 7*int(sqrt(a + b*x**6)/(a**2*x**5 +
 2*a*b*x**11 + b**2*x**17),x)*b**2*c*x**10)/(7*b*x**4*(a + b*x**6))
```

3.14 $\int \frac{c+dx^6}{x^{11}(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\int \frac{c+dx^6}{x^{11}(a+bx^6)^{3/2}}\,dx = -\frac{c}{10ax^{10}\sqrt{a+bx^6}}-\frac{13bc-10ad}{30a^2x^4\sqrt{a+bx^6}}+\frac{7(13bc-10ad)\sqrt{a+bx^6}}{120a^3x^4} + \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}(13bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)\right.}{120\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/10*c/a/x^10/(b*x^6+a)^(1/2)-1/30*(-10*a*d+13*b*c)/a^2/x^4/(b*x^6+a)^(1/
2)+7/120*(-10*a*d+13*b*c)*(b*x^6+a)^(1/2)/a^3/x^4+7/360*(1/2*6^(1/2)+1/2*2
^(1/2))*b^(2/3)*(-10*a*d+13*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b
^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*Ellipti
cF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3
^(1/2)+2*I)*3^(3/4)/a^3/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.23

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\frac{-4ac+(13bc-10ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{3}{2},\frac{1}{3},-\frac{bx^6}{a}\right)}{40a^2x^{10}\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^11*(a + b*x^6)^(3/2)),x]
```

output
```
(-4*a*c + (13*b*c - 10*a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[-2/3
, 3/2, 1/3, -((b*x^6)/a)])/(40*a^2*x^10*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 807, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13bc-10ad)\int \frac{1}{x^5\left(bx^6+a\right)^{3/2}}dx}{10a}-\frac{c}{10ax^{10}\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(13bc-10ad)\int \frac{1}{x^6\left(bx^6+a\right)^{3/2}}dx^2}{20a}-\frac{c}{10ax^{10}\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(13bc-10ad)\left(\frac{7\int \frac{1}{x^6\sqrt{bx^6+a}}dx^2}{3a}+\frac{2}{3ax^4\sqrt{a+bx^6}}\right)}{20a}-\frac{c}{10ax^{10}\sqrt{a+bx^6}}$$

$\Big\downarrow$ 847

$$-\frac{(13bc-10ad)\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^6+a}}dx^2}{4a}-\frac{\sqrt{a+bx^6}}{2ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^6}}\right)}{20a}-\frac{c}{10ax^{10}\sqrt{a+bx^6}}$$

$\Big\downarrow$ 759

$$-\frac{(13bc-10ad)\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{\sqrt{a+bx^6}}{2ax^4}\right)}{3a}\right.}{20a}$$

$$\frac{c}{10ax^{10}\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^11*(a + b*x^6)^(3/2)),x]
```

output
```
-1/10*c/(a*x^10*Sqrt[a + b*x^6]) - ((13*b*c - 10*a*d)*(2/(3*a*x^4*Sqrt[a +
 b*x^6]) + (7*(-1/2*Sqrt[a + b*x^6]/(a*x^4) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(
a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[
3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*a)))/(20*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{11}\left(b\,x^{6}+a\right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.38

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\frac{7\left(\left(13\,b^2c-10\,abd\right)x^{16}+\left(13\,abc-10\,a^2d\right)x^{10}\right)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right)}{120\left(a^3bx^{16}+a^4\right.}$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/120*(7*((13*b^2*c - 10*a*b*d)*x^16 + (13*a*b*c - 10*a^2*d)*x^10)*sqrt(b)
*weierstrassPInverse(0, -4*a/b, x^2) + (7*(13*b^2*c - 10*a*b*d)*x^12 + 3*(
13*a*b*c - 10*a^2*d)*x^6 - 12*a^2*c)*sqrt(b*x^6 + a))/(a^3*b*x^16 + a^4*x^
10)
```

Sympy [A] (verification not implemented)

Time = 128.87 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.28

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\frac{c\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^{10}\Gamma\left(-\frac{2}{3}\right)}+\frac{d\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^{4}\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((d*x**6+c)/x**11/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**6*exp_polar(I*pi)/a)/(6*a**
(3/2)*x**10*gamma(-2/3)) + d*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**6
*exp_polar(I*pi)/a)/(6*a**(3/2)*x**4*gamma(1/3))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^{11}}\,dx$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^11), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^{11}}\,dx$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{d\,x^6+c}{x^{11}\,\left(b\,x^6+a\right)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^11*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^11*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-10\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{23}+2ab\,x^{17}+a^2x^{11}}dx\right)a^2d\,x^{10}+13\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{23}+2ab\,x^{17}+a^2x^{11}}dx\right)a}{13b\,x^{10}\,(b}$$

input
```
int((d*x^6+c)/x^11/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 10*int(sqrt(a + b*x**6)/(a**2*x**11 + 2*a*b*x**17
 + b**2*x**23),x)*a**2*d*x**10 + 13*int(sqrt(a + b*x**6)/(a**2*x**11 + 2*a
*b*x**17 + b**2*x**23),x)*a*b*c*x**10 - 10*int(sqrt(a + b*x**6)/(a**2*x**1
1 + 2*a*b*x**17 + b**2*x**23),x)*a*b*d*x**16 + 13*int(sqrt(a + b*x**6)/(a*
*2*x**11 + 2*a*b*x**17 + b**2*x**23),x)*b**2*c*x**16)/(13*b*x**10*(a + b*x
**6))
```

3.15 $\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{(bc-ad)x}{3b^2\sqrt{a+bx^6}} + \frac{dx\sqrt{a+bx^6}}{4b^2} + \frac{(4bc-7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{24\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/3*(-a*d+b*c)*x/b^2/(b*x^6+a)^(1/2)+1/4*d*x*(b*x^6+a)^(1/2)/b^2+1/72*(-7
*a*d+4*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*
x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^
(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1
/2)+1/4*2^(1/2))*3^(3/4)/a^(1/3)/b^2/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.28

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x\left(-4bc+7ad+3bdx^6+(4bc-7ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a}\right)\right)}{12b^2\sqrt{a+bx^6}}$$

input
```
Integrate[(x^6*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x*(-4*b*c + 7*a*d + 3*b*d*x^6 + (4*b*c - 7*a*d)*Sqrt[1 + (b*x^6)/a]*Hyper
geometric2F1[1/6, 1/2, 7/6, -((b*x^6)/a)]))/(12*b^2*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 817, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4bc-7ad)\int \frac{x^6}{(bx^6+a)^{3/2}}dx}{4b}+\frac{dx^7}{4b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(4bc-7ad)\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx}{3b}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{4b}+\frac{dx^7}{4b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(4bc-7ad)\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}\sqrt[3]{a}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{4b}+\frac{dx^7}{4b\sqrt{a+bx^6}}$$

input
```
Int[(x^6*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^7)/(4*b*Sqrt[a + b*x^6]) + ((4*b*c - 7*a*d)*(-1/3*x/(b*Sqrt[a + b*x^6
]) + (x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2
/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3
) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2
+ Sqrt[3])/4])/(6*3^(1/4)*a^(1/3)*b*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x
^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(4*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{6} \left(d\,x^{6}+c\right)}{\left(b\,x^{6}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((d*x^12 + c*x^6)*sqrt(b*x^6 + a)/(b^2*x^12 + 2*a*b*x^6 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.86 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\frac{cx^7\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{13}{6}\right)}+\frac{dx^{13}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate(x**6*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**7*gamma(7/6)*hyper((7/6, 3/2), (13/6,), b*x**6*exp_polar(I*pi)/a)/(6*
a**(3/2)*gamma(13/6)) + d*x**13*gamma(13/6)*hyper((3/2, 13/6), (19/6,), b*
x**6*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(19/6))
```

Maxima [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^6/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^6/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{x^6\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^6*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^6*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\frac{7\sqrt{b\,x^6+a}\,adx-4\sqrt{b\,x^6+a}\,bcx+2\sqrt{b\,x^6+a}\,bd\,x^7-7\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^3d+4}{8b^2}$$

input
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(7*sqrt(a + b*x**6)*a*d*x - 4*sqrt(a + b*x**6)*b*c*x + 2*sqrt(a + b*x**6)*
b*d*x**7 - 7*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*a**3
*d + 4*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*a**2*b*c -
 7*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*a**2*b*d*x**6
+ 4*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*a*b**2*c*x**6
)/(8*b**2*(a + b*x**6))
```

3.16 $\int \frac{c+dx^6}{\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 256

$$\int \frac{c+dx^6}{\left(a+bx^6\right)^{3/2}}\,dx=\frac{(bc-ad)x}{3ab\sqrt{a+bx^6}}+\frac{(2bc+ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}a^{4/3}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/3*(-a*d+b*c)*x/a/b/(b*x^6+a)^(1/2)+1/18*(a*d+2*b*c)*x*(a^(1/3)+b^(1/3)*x
^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)
/b/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)
^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.28

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx = \frac{x\left(bc-ad+(2bc+ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a}\right)\right)}{3ab\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(a + b*x^6)^(3/2),x]
```

output
```
(x*(b*c - a*d + (2*b*c + a*d)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/6, 1
/2, 7/6, -((b*x^6)/a)]))/(3*a*b*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {910, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(ad+2bc)\int \frac{1}{\sqrt{bx^6+a}}dx}{3ab}+\frac{x(bc-ad)}{3ab\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 766$$

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}(ad+2bc)\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{6\sqrt[4]{3}a^{4/3}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{x(bc-ad)}{3ab\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(a + b*x^6)^(3/2),x]
```

output
```
((b*c - a*d)*x)/(3*a*b*Sqrt[a + b*x^6]) + ((2*b*c + a*d)*x*(a^(1/3) + b^(1
/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/
3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(6*3^(1/
4)*a^(4/3)*b*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{\left(b\,x^{6}+a\right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx=\int \frac{dx^{6}+c}{\left(bx^{6}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^2*x^12 + 2*a*b*x^6 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx=\frac{cx\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{6},\frac{3}{2}\\\frac{7}{6}\end{matrix}\right|\frac{bx^{6}e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{dx^{7}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{6},\frac{3}{2}\\\frac{13}{6}\end{matrix}\right|\frac{bx^{6}e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
3/2)*gamma(7/6)) + d*x**7*gamma(7/6)*hyper((7/6, 3/2), (13/6,), b*x**6*exp
_polar(I*pi)/a)/(6*a**(3/2)*gamma(13/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{(a+bx^6)^{3/2}}\,dx = \int \frac{d\,x^6+c}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(a + b*x^6)^(3/2),x)
```

output
```
int((c + d*x^6)/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{\left(a+bx^6\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,dx+\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^2d+2\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{12}+2ab\,x^6+a^2}dx\right)abc+\left(\int \frac{\sqrt{b\,x^6}}{b^2x^{12}+2a}\right.}{2b\left(b\,x^6+a\right)}$$

input
```
int((d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x + int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*
x**12),x)*a**2*d + 2*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12)
,x)*a*b*c + int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*a*b*d
*x**6 + 2*int(sqrt(a + b*x**6)/(a**2 + 2*a*b*x**6 + b**2*x**12),x)*b**2*c*
x**6)/(2*b*(a + b*x**6))
```

3.17 $\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 274

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx=-\frac{c}{5ax^5\sqrt{a+bx^6}}-\frac{(8bc-5ad)x}{15a^2\sqrt{a+bx^6}}-\frac{(8bc-5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/5*c/a/x^5/(b*x^6+a)^(1/2)-1/15*(-5*a*d+8*b*c)*x/a^2/(b*x^6+a)^(1/2)-1/4
5*(-5*a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(
2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arcco
s((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4
*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.32

$$\int \frac{c+dx^6}{x^6(a+bx^6)^{3/2}}\,dx = \frac{-3ac-8bcx^6+5adx^6+2(-8bc+5ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\right.}{15a^2x^5\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^6*(a + b*x^6)^(3/2)),x]
```

output
```
(-3*a*c - 8*b*c*x^6 + 5*a*d*x^6 + 2*(-8*b*c + 5*a*d)*x^6*Sqrt[1 + (b*x^6)/
a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^6)/a)])/(15*a^2*x^5*Sqrt[a + b*
x^6])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 749, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^6(a+bx^6)^{3/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(8bc-5ad)\int \frac{1}{(bx^6+a)^{3/2}}dx}{5a}-\frac{c}{5ax^5\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(8bc-5ad)\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx}{3a}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 766$$

$$-\frac{(8bc-5ad)\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{5a}-\frac{c}{5ax^5\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^6*(a + b*x^6)^(3/2)),x]
```

output
```
-1/5*c/(a*x^5*Sqrt[a + b*x^6]) - ((8*b*c - 5*a*d)*(x/(3*a*Sqrt[a + b*x^6])
 + (x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3
)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 +
Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(5*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{6}\left(b\,x^{6}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^2*x^18 + 2*a*b*x^12 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.30

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^5\Gamma\left(\frac{1}{6}\right)}+\frac{dx\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((d*x**6+c)/x**6/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
3/2)*x**5*gamma(1/6)) + d*x*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**6*ex
p_polar(I*pi)/a)/(6*a**(3/2)*gamma(7/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{d\,x^6+c}{x^6\,{\left(b\,x^6+a\right)}^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^6*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^6*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{3/2}}\,dx=\frac{-\sqrt{b\,x^6+a}\,d-5\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{18}+2ab\,x^{12}+a^2x^6}dx\right)a^2d\,x^5+8\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{18}+2ab\,x^{12}+a^2x^6}dx\right)abc\,x^5}{8b\,x^5\left(b\,x^6+a\right.}$$

input
```
int((d*x^6+c)/x^6/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 5*int(sqrt(a + b*x**6)/(a**2*x**6 + 2*a*b*x**12 +
 b**2*x**18),x)*a**2*d*x**5 + 8*int(sqrt(a + b*x**6)/(a**2*x**6 + 2*a*b*x*
*12 + b**2*x**18),x)*a*b*c*x**5 - 5*int(sqrt(a + b*x**6)/(a**2*x**6 + 2*a*
b*x**12 + b**2*x**18),x)*a*b*d*x**11 + 8*int(sqrt(a + b*x**6)/(a**2*x**6 +
 2*a*b*x**12 + b**2*x**18),x)*b**2*c*x**11)/(8*b*x**5*(a + b*x**6))
```

3.18 $\int \frac{c+dx^6}{x^{12}(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 307

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx=-\frac{c}{11ax^{11}\sqrt{a+bx^6}}-\frac{14bc-11ad}{33a^2x^5\sqrt{a+bx^6}}+\frac{8(14bc-11ad)\sqrt{a+bx^6}}{165a^3x^5}$$
$$+\frac{8b(14bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{\ldots}\right.\right.}{165\sqrt[4]{3}a^{10/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/11*c/a/x^11/(b*x^6+a)^(1/2)-1/33*(-11*a*d+14*b*c)/a^2/x^5/(b*x^6+a)^(1/
2)+8/165*(-11*a*d+14*b*c)*(b*x^6+a)^(1/2)/a^3/x^5+8/495*b*(-11*a*d+14*b*c)
*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^
(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(
1/2))*3^(3/4)/a^(10/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1
/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.23

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx=\frac{-5ac+(14bc-11ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{3}{2},\frac{1}{6},-\frac{bx^6}{a}\right)}{55a^2x^{11}\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^12*(a + b*x^6)^(3/2)),x]
```

output
```
(-5*a*c + (14*b*c - 11*a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[-5/6
, 3/2, 1/6, -((b*x^6)/a)])/(55*a^2*x^11*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 819, 847, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(14bc-11ad)\int \frac{1}{x^6\left(bx^6+a\right)^{3/2}}dx}{11a}-\frac{c}{11ax^{11}\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(14bc-11ad)\left(\frac{8\int \frac{1}{x^6\sqrt{bx^6+a}}dx}{3a}+\frac{1}{3ax^5\sqrt{a+bx^6}}\right)}{11a}-\frac{c}{11ax^{11}\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(14bc-11ad)\left(\frac{8\left(-\frac{2b\int\frac{1}{\sqrt{bx^6+a}}dx}{5a}-\frac{\sqrt{a+bx^6}}{5ax^5}\right)}{3a}+\frac{1}{3ax^5\sqrt{a+bx^6}}\right)}{11a}-\frac{c}{11ax^{11}\sqrt{a+bx^6}}$$

$\downarrow$ 766

$$-\frac{(14bc-11ad)\left(\frac{8\left(-\frac{bx\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{\sqrt{a+bx^6}}{5ax^5}\right)}{3a}+\frac{1}{3a\ldots}\right)}{11a}-\frac{c}{11ax^{11}\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^12*(a + b*x^6)^(3/2)),x]
```

output
```
-1/11*c/(a*x^11*Sqrt[a + b*x^6]) - ((14*b*c - 11*a*d)*(1/(3*a*x^5*Sqrt[a +
 b*x^6]) + (8*(-1/5*Sqrt[a + b*x^6]/(a*x^5) - (b*x*(a^(1/3) + b^(1/3)*x^2)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/
3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*a)))/(11*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{12} \left(b\,x^{6}+a \right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^2*x^24 + 2*a*b*x^18 + a^2*x^12), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 143.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.29

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},\frac{3}{2}\\-\frac{5}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^{11}\Gamma\left(-\frac{5}{6}\right)} + \frac{d\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\\frac{1}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^5\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate((d*x**6+c)/x**12/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-11/6)*hyper((-11/6, 3/2), (-5/6,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(3/2)*x**11*gamma(-5/6)) + d*gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x*
*6*exp_polar(I*pi)/a)/(6*a**(3/2)*x**5*gamma(1/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^12), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \int \frac{d\,x^6+c}{x^{12}\,\left(b\,x^6+a\right)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^12*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^12*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-11\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{24}+2ab\,x^{18}+a^2x^{12}}dx\right)a^2d\,x^{11}+14\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{24}+2ab\,x^{18}+a^2x^{12}}dx\right)a}{14b\,x^{11}\,(b}$$

input
```
int((d*x^6+c)/x^12/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 11*int(sqrt(a + b*x**6)/(a**2*x**12 + 2*a*b*x**18
 + b**2*x**24),x)*a**2*d*x**11 + 14*int(sqrt(a + b*x**6)/(a**2*x**12 + 2*a
*b*x**18 + b**2*x**24),x)*a*b*c*x**11 - 11*int(sqrt(a + b*x**6)/(a**2*x**1
2 + 2*a*b*x**18 + b**2*x**24),x)*a*b*d*x**17 + 14*int(sqrt(a + b*x**6)/(a*
*2*x**12 + 2*a*b*x**18 + b**2*x**24),x)*b**2*c*x**17)/(14*b*x**11*(a + b*x
**6))
```

3.19 $\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{(bc-ad)x^5}{3b^2\sqrt{a+bx^6}}+\frac{dx^5\sqrt{a+bx^6}}{8b^2}+\frac{5\left(1+\sqrt{3}\right)(8bc-11ad)x\sqrt{a+bx^6}}{48b^{8/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$

$$-\frac{5\sqrt[3]{a}(8bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}(8bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right.\right.}{96\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/3*(-a*d+b*c)*x^5/b^2/(b*x^6+a)^(1/2)+1/8*d*x^5*(b*x^6+a)^(1/2)/b^2+5/48
*(1+3^(1/2))*(-11*a*d+8*b*c)*x*(b*x^6+a)^(1/2)/b^(8/3)/(a^(1/3)+(1+3^(1/2)
)*b^(1/3)*x^2)-5/48*a^(1/3)*(-11*a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^
(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)+(1+3^(1/2)
)*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/b^(8/3)/(b^(1/3)*
x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^
6+a)^(1/2)-5/288*(1-3^(1/2))*a^(1/3)*(-11*a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^
2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a
^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(8/3)/
(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1
/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^5\left(8bc-11ad+2bdx^6+(-8bc+11ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^6}{a}\right)\right)}{16b^2\sqrt{a+bx^6}}$$

input
```
Integrate[(x^10*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^5*(8*b*c - 11*a*d + 2*b*d*x^6 + (-8*b*c + 11*a*d)*Sqrt[1 + (b*x^6)/a]*H
ypergeometric2F1[5/6, 3/2, 11/6, -((b*x^6)/a)]))/(16*b^2*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 817, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8bc-11ad)\int \frac{x^{10}}{(bx^6+a)^{3/2}}dx}{8b}+\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(8bc-11ad)\left(\frac{5\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{8b}+\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(8bc-11ad)\left(\frac{5\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{8b}+\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(8bc-11ad)\left(\frac{5\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{8b}+\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(8bc-11ad)\left(\frac{5\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{3b}\right.}{8b}$$

$$\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

$\downarrow$ 2420

$$\frac{(8bc-11ad)\left(\frac{\frac{5\left(\frac{(1+\sqrt{3})x\sqrt{a+bx^6}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x^2+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x^2+\sqrt[3]{a}}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{2b^{2/3}}}{3b}\right.}{8b}$$

$$\frac{dx^{11}}{8b\sqrt{a+bx^6}}$$

input
```
Int[(x^10*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^11)/(8*b*Sqrt[a + b*x^6]) + ((8*b*c - 11*a*d)*(-1/3*x^5/(b*Sqrt[a + b
*x^6]) + (5*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a
^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]
*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1
/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^
(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*
x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt
[a + b*x^6])))/(3*b)))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{x^{10}(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{3}{2}}}dx$$

input
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((d*x^16 + c*x^10)*sqrt(b*x^6 + a)/(b^2*x^12 + 2*a*b*x^6 + a^2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 79.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^{11}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)} + \frac{dx^{17}\Gamma\left(\frac{17}{6}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix}\middle| \frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{23}{6}\right)}$$

input

```
integrate(x**10*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output

```
c*x**11*gamma(11/6)*hyper((3/2, 11/6), (17/6,), b*x**6*exp_polar(I*pi)/a)/
(6*a**(3/2)*gamma(17/6)) + d*x**17*gamma(17/6)*hyper((3/2, 17/6), (23/6,),
 b*x**6*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(23/6))
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output

```
integrate((d*x^6 + c)*x^10/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input

```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^10/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{x^{10}\,\left(d\,x^6+c\right)}{{\left(b\,x^6+a\right)}^{3/2}}\,dx$$

input
```
int((x^10*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^10*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\frac{-11\sqrt{b\,x^{6}+a}\,a d\,x^{5}+8\sqrt{b\,x^{6}+a}\,b c\,x^{5}+2\sqrt{b\,x^{6}+a}\,b d\,x^{11}+55\left(\int \frac{\sqrt{b\,x^{6}+a}\,x^{4}}{b^{2}x^{12}+2ab\,x^{6}+a^{2}}d x\right)}{}$$

input
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
( - 11*sqrt(a + b*x**6)*a*d*x**5 + 8*sqrt(a + b*x**6)*b*c*x**5 + 2*sqrt(a
+ b*x**6)*b*d*x**11 + 55*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x**6 +
b**2*x**12),x)*a**3*d - 40*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x**6
+ b**2*x**12),x)*a**2*b*c + 55*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x
**6 + b**2*x**12),x)*a**2*b*d*x**6 - 40*int((sqrt(a + b*x**6)*x**4)/(a**2
+ 2*a*b*x**6 + b**2*x**12),x)*a*b**2*c*x**6)/(16*b**2*(a + b*x**6))
```

3.20 $\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 562

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{(bc-ad)x^5}{3ab\sqrt{a+bx^6}} - \frac{\left(1+\sqrt{3}\right)(2bc-5ad)x\sqrt{a+bx^6}}{6ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$
$$+ \frac{(2bc-5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\,3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$+ \frac{\left(1-\sqrt{3}\right)(2bc-5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\Bigg(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}(2}{12\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/3*(-a*d+b*c)*x^5/a/b/(b*x^6+a)^(1/2)-1/6*(1+3^(1/2))*(-5*a*d+2*b*c)*x*(b
*x^6+a)^(1/2)/a/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)+1/6*(-5*a*d+2*b*
c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b
^(1/3)*x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*3^(1/4)/a^(2/3)/b^(5/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3
)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+1/36*(1-3^(1/2))*(-5*a
*d+2*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^
4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1
/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2
)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/b^(5/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.13

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^5\left(5ad+(2bc-5ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^6}{a}\right)\right)}{10ab\sqrt{a+bx^6}}$$

input
```
Integrate[(x^4*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^5*(5*a*d + (2*b*c - 5*a*d)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[5/6, 3
/2, 11/6, -((b*x^6)/a)]))/(10*a*b*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 819, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^6\right)}{\left(a+bx^6\right)^{3/2}}dx$$

$$\Big\downarrow \; 959$$

$$\frac{(2bc-5ad)\int \frac{x^4}{\left(bx^6+a\right)^{3/2}}dx}{2b}+\frac{dx^5}{2b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 819$$

$$\frac{(2bc-5ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3a}\right)}{2b}+\frac{dx^5}{2b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 837$$

$$\frac{(2bc-5ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{2b}+\frac{dx^5}{2b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 25$$

$$\frac{(2bc-5ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{2b}+\frac{dx^5}{2b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 766$$

$$\frac{(2bc-5ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{}\right.}{\left(1+\sqrt{}\right.}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{3a}\right)}{2b}$$

$$\frac{dx^5}{2b\sqrt{a+bx^6}}$$

$$\downarrow 2420$$

$$\frac{(2bc-5ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{2b^{2/3}}\right.}{2b}$$

$$\frac{dx^5}{2b\sqrt{a+bx^6}}$$

input
```
Int[(x^4*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^5)/(2*b*Sqrt[a + b*x^6]) + ((2*b*c - 5*a*d)*(x^5/(3*a*Sqrt[a + b*x^6]
) - (2*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Elli
pticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x
^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)
) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(
a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a +
b*x^6]))))/(3*a)))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{x^{4}\left(d\,x^{6}+c\right)}{\left(b\,x^{6}+a\right)^{\frac{3}{2}}}d x$$

input
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^4)*sqrt(b*x^6 + a)/(b^2*x^12 + 2*a*b*x^6 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)} + \frac{dx^{11}\Gamma\left(\frac{11}{6}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate(x**4*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**5*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**6*exp_polar(I*pi)/a)/(6*
a**(3/2)*gamma(11/6)) + d*x**11*gamma(11/6)*hyper((3/2, 11/6), (17/6,), b*
x**6*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(17/6))
```

Maxima [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^4/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^4/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{x^4\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^4*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^4*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\frac{\sqrt{b\,x^6+a}\,d\,x^5-5\left(\int \frac{\sqrt{b\,x^6+a}\,x^4}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^2d+2\left(\int \frac{\sqrt{b\,x^6+a}\,x^4}{b^2x^{12}+2ab\,x^6+a^2}dx\right)abc-5\left(\int \frac{\sqrt{b\,x}}{b^2x^{12}+}\right.}{2b\,(b\,x^6+a)}$$

input
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*d*x**5 - 5*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x**
6 + b**2*x**12),x)*a**2*d + 2*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x*
*6 + b**2*x**12),x)*a*b*c - 5*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a*b*x*
*6 + b**2*x**12),x)*a*b*d*x**6 + 2*int((sqrt(a + b*x**6)*x**4)/(a**2 + 2*a
*b*x**6 + b**2*x**12),x)*b**2*c*x**6)/(2*b*(a + b*x**6))
```

3.21 $\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 578

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{3/2}}\,dx = -\frac{c}{ax\sqrt{a+bx^6}}-\frac{(4bc-ad)x^5}{3a^2\sqrt{a+bx^6}}+\frac{\left(1+\sqrt{3}\right)(4bc-ad)x\sqrt{a+bx^6}}{3a^2b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$
$$-\frac{(4bc-ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$-\frac{\left(1-\sqrt{3}\right)(4bc-ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}(2}{6\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-c/a/x/(b*x^6+a)^(1/2)-1/3*(-a*d+4*b*c)*x^5/a^2/(b*x^6+a)^(1/2)+1/3*(1+3^(
1/2))*(-a*d+4*b*c)*x*(b*x^6+a)^(1/2)/a^2/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1
/3)*x^2)-1/3*(-a*d+4*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3
)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1
-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(5/3)/b^(2/3)/(b^(1/3)*x^2*(a^(1/
3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
-1/18*(1-3^(1/2))*(-a*d+4*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b
^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*Inverse
JacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/b^(2/3)/(b^(1/3)*x^2*(
a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.12

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{3/2}}\,dx=\frac{-5ac+(-4bc+ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^6}{a}\right)}{5a^2x\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^2*(a + b*x^6)^(3/2)),x]
```

output
```
(-5*a*c + (-4*b*c + a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[5/6, 3/
2, 11/6, -((b*x^6)/a)])/(5*a^2*x*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 819, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{3/2}}dx$$

$\Big\downarrow$ 955

$$-\frac{(4bc-ad)\int \frac{x^4}{\left(bx^6+a\right)^{3/2}}dx}{a}-\frac{c}{ax\sqrt{a+bx^6}}$$

$\Big\downarrow$ 819

$$-\frac{(4bc-ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^6}}$$

$\Big\downarrow$ 837

$$-\frac{(4bc-ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^6}}$$

$\Big\downarrow$ 25

$$-\frac{(4bc-ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{a}-\frac{c}{ax\sqrt{a+bx^6}}$$

$\Big\downarrow$ 766

$$-\frac{(4bc-ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\cdots\right.}{\left(1+\cdots\right.}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{3a}\right.}{a}$$

$$\frac{c}{ax\sqrt{a+bx^6}}$$

$\downarrow$ 2420

$$-\frac{(4bc-ad)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{2b^{2/3}}\right.}{\cdots}\right.}{a}$$

$$\frac{c}{ax\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^2*(a + b*x^6)^(3/2)),x]
```

output
```
-(c/(a*x*Sqrt[a + b*x^6])) - ((4*b*c - a*d)*(x^5/(3*a*Sqrt[a + b*x^6]) - (
2*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2
) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticE
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)) - (
(1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^
(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Elliptic
F[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^(1/
3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6
])))/(3*a)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{2} \left(b\,x^{6}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^2*x^14 + 2*a*b*x^8 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.14

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx=\frac{c\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x\Gamma\left(\frac{5}{6}\right)}+\frac{dx^5\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((d*x**6+c)/x**2/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
3/2)*x*gamma(5/6)) + d*x**5*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**6*e
xp_polar(I*pi)/a)/(6*a**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx = \int \frac{d\,x^6+c}{x^2\,(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^2*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^2*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{14}+2ab\,x^8+a^2x^2}dx\right)a^2dx+4\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{14}+2ab\,x^8+a^2x^2}dx\right)abcx-\left(\int\right.}{4bx\,(b\,x^6+a)}$$

input
```
int((d*x^6+c)/x^2/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - int(sqrt(a + b*x**6)/(a**2*x**2 + 2*a*b*x**8 + b*
*2*x**14),x)*a**2*d*x + 4*int(sqrt(a + b*x**6)/(a**2*x**2 + 2*a*b*x**8 + b
**2*x**14),x)*a*b*c*x - int(sqrt(a + b*x**6)/(a**2*x**2 + 2*a*b*x**8 + b**
2*x**14),x)*a*b*d*x**7 + 4*int(sqrt(a + b*x**6)/(a**2*x**2 + 2*a*b*x**8 +
b**2*x**14),x)*b**2*c*x**7)/(4*b*x*(a + b*x**6))
```

3.22 $\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 612

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx = -\frac{c}{7ax^7\sqrt{a+bx^6}}-\frac{10bc-7ad}{21a^2x\sqrt{a+bx^6}}$$
$$+\frac{4(10bc-7ad)\sqrt{a+bx^6}}{21a^3x}-\frac{4\left(1+\sqrt{3}\right)\sqrt[3]{b}(10bc-7ad)x\sqrt{a+bx^6}}{21a^3\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$
$$+\frac{4\sqrt[3]{b}(10bc-7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\ 3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$+\frac{2\left(1-\sqrt{3}\right)\sqrt[3]{b}(10bc-7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)\right.}{21\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/7*c/a/x^7/(b*x^6+a)^(1/2)-1/21*(-7*a*d+10*b*c)/a^2/x/(b*x^6+a)^(1/2)+4/
21*(-7*a*d+10*b*c)*(b*x^6+a)^(1/2)/a^3/x-4/21*(1+3^(1/2))*b^(1/3)*(-7*a*d+
10*b*c)*x*(b*x^6+a)^(1/2)/a^3/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)+4/21*b^(1/
3)*(-7*a*d+10*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b
^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1-(a^(1/
3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2),1
/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(8/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+2/63*(1-3^(1/2
))*b^(1/3)*(-7*a*d+10*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/
3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJaco
biAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(8/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx = \frac{-ac+(10bc-7ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{2},\frac{5}{6},-\frac{bx^6}{a}\right)}{7a^2x^7\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^8*(a + b*x^6)^(3/2)),x]
```

output
```
(-(a*c) + (10*b*c - 7*a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[-1/6,
 3/2, 5/6, -((b*x^6)/a)])/(7*a^2*x^7*Sqrt[a + b*x^6])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 589, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 847, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx$$

$$\Big\downarrow\ 955$$

$$-\frac{(10bc-7ad)\int \frac{1}{x^2\left(bx^6+a\right)^{3/2}}dx}{7a}-\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$$\Big\downarrow\ 819$$

$$-\frac{(10bc-7ad)\left(\frac{4\int \frac{1}{x^2\sqrt{bx^6+a}}dx}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$$\Big\downarrow\ 847$$

$$-\frac{(10bc-7ad)\left(\frac{4\left(\frac{2b\int \frac{x^4}{\sqrt{bx^6+a}}dx}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$$\Big\downarrow\ 837$$

$$-\frac{(10bc-7ad)\left(\frac{4\left(\frac{2b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$$\Big\downarrow\ 25$$

$$-\frac{(10bc-7ad)\left(\frac{4\left(\frac{2b\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int\frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{7a}-\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$\downarrow$ 766

$$-\frac{(10bc-7ad)\left(\frac{4\left(\frac{2b\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{a}\right.}{3a}\right.}{7a}$$

$$\frac{c}{7ax^7\sqrt{a+bx^6}}$$

$\downarrow$ 2420

$$-\frac{(10bc-7ad)\left(4\left(2b\,\frac{\left(\frac{(1+\sqrt{3})x\sqrt{a+bx^6}}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\,E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}x^2+\sqrt[3]{a}}{(1+\sqrt{3})\sqrt[3]{b}x^2+\sqrt[3]{a}}\right)|\frac{1}{4}(2-\cdots}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\cdots\right)}{2b^{2/3}}\cdots\right)\cdots\right)}{\cdots}$$

$$\frac{c}{7ax^7\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^8*(a + b*x^6)^(3/2)),x]
```

output
```
-1/7*c/(a*x^7*Sqrt[a + b*x^6]) - ((10*b*c - 7*a*d)*(1/(3*a*x*Sqrt[a + b*x^
6]) + (4*(-(Sqrt[a + b*x^6]/(a*x)) + (2*b*((((1 + Sqrt[3])*x*Sqrt[a + b*x^
6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) +
b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqr
t[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) +
 b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*
b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*
3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/a))/(3*a)))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{8}\left(b\,x^{6}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^8 (a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^2*x^20 + 2*a*b*x^14 + a^2*x^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 67.80 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{c+dx^6}{x^8 (a+bx^6)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{6}, \frac{3}{2} \\ -\frac{1}{6} \end{matrix}\middle| \frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^7\Gamma\left(-\frac{1}{6}\right)} + \frac{d\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{3}{2} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate((d*x**6+c)/x**8/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-7/6)*hyper((-7/6, 3/2), (-1/6,), b*x**6*exp_polar(I*pi)/a)/(6*a**
(3/2)*x**7*gamma(-1/6)) + d*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**6*
exp_polar(I*pi)/a)/(6*a**(3/2)*x*gamma(5/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx=\int \frac{d\,x^6+c}{x^8\,\left(b\,x^6+a\right)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^8*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^8*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-7\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{20}+2ab\,x^{14}+a^2x^8}dx\right)a^2d\,x^7+10\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{20}+2ab\,x^{14}+a^2x^8}dx\right)abc\,x^7}{10b\,x^7\left(b\,x^6+\right.}$$

input
```
int((d*x^6+c)/x^8/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 7*int(sqrt(a + b*x**6)/(a**2*x**8 + 2*a*b*x**14 +
 b**2*x**20),x)*a**2*d*x**7 + 10*int(sqrt(a + b*x**6)/(a**2*x**8 + 2*a*b*x
**14 + b**2*x**20),x)*a*b*c*x**7 - 7*int(sqrt(a + b*x**6)/(a**2*x**8 + 2*a
*b*x**14 + b**2*x**20),x)*a*b*d*x**13 + 10*int(sqrt(a + b*x**6)/(a**2*x**8
 + 2*a*b*x**14 + b**2*x**20),x)*b**2*c*x**13)/(10*b*x**7*(a + b*x**6))
```

3.23 $\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 576

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{(bc-ad)x^4}{3b^2\sqrt{a+bx^6}} + \frac{dx^4\sqrt{a+bx^6}}{7b^2} + \frac{4(7bc-10ad)\sqrt{a+bx^6}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$+\frac{4\sqrt{2}\sqrt[3]{a}(7bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/3*(-a*d+b*c)*x^4/b^2/(b*x^6+a)^(1/2)+1/7*d*x^4*(b*x^6+a)^(1/2)/b^2+4/21
*(-10*a*d+7*b*c)*(b*x^6+a)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)
-2/21*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(-10*a*d+7*b*c)*(a^(1/3)+b^(1/3)*x
^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+4/63*2^
(1/2)*a^(1/3)*(-10*a*d+7*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(
((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1
/2)+2*I)*3^(3/4)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^4\left(7bc-10ad+bdx^6+(-7bc+10ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^6}{a}\right)\right)}{7b^2\sqrt{a+bx^6}}$$

input
```
Integrate[(x^9*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^4*(7*b*c - 10*a*d + b*d*x^6 + (-7*b*c + 10*a*d)*Sqrt[1 + (b*x^6)/a]*Hyp
ergeometric2F1[2/3, 3/2, 5/3, -((b*x^6)/a)]))/(7*b^2*Sqrt[a + b*x^6])
```

Rubi [A] (warning: unable to verify)

Time = 0.98 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 807, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9 \left(c+dx^6\right)}{\left(a+bx^6\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(7bc-10ad)\int \frac{x^9}{\left(bx^6+a\right)^{3/2}}dx}{7b}+\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 807$$

$$\frac{(7bc-10ad)\int \frac{x^8}{\left(bx^6+a\right)^{3/2}}dx^2}{14b}+\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 817$$

$$\frac{(7bc-10ad)\left(\frac{4\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{14b}+\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 832$$

$$\frac{(7bc-10ad)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{14b}+\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 759$$

$$\frac{(7bc-10ad)\Bigg(\frac{4\Bigg(\frac{\int \frac{\sqrt[3]{b}x^2+(1-\sqrt{3})\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}} - \frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{b}x^2)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x^2)^2}}\,\text{EllipticF}\Big(\arcsin\Big(\frac{\sqrt[3]{b}x^2+(1-\cdots}{\sqrt[3]{b}x^2+(1+\cdots}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}(\sqrt[3]{a}+\sqrt[3]{b}x^2)}{((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x^2)^2}}\sqrt{a+bx^6}}}{3b}}{14b}$$

$$\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

$$\downarrow 2416$$

$$(7bc-10ad)\left(\frac{4\left(\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{\sqrt[3]{b}}\right.$$

$$\frac{dx^{10}}{7b\sqrt{a+bx^6}}$$

input
```
Int[(x^9*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(d*x^10)/(7*b*Sqrt[a + b*x^6]) + ((7*b*c - 10*a*d)*((-2*x^4)/(3*b*Sqrt[a +
 b*x^6]) + (4*(((2*Sqrt[a + b*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a
 + b*x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3)
+ b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^
(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*b)))/(14*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^9(d\,x^6+c)}{(b\,x^6+a)^{\frac{3}{2}}}dx$$

input
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.19

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = -\frac{4\left((7\,b^2c-10\,abd)x^6+7\,abc-10\,a^2d\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right)\right)-(3}{21\,(b^4x^6+ab^3)}$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/21*(4*((7*b^2*c - 10*a*b*d)*x^6 + 7*a*b*c - 10*a^2*d)*sqrt(b)*weierstra
ssZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x^2)) - (3*b^2*d*x^10 - (
7*b^2*c - 10*a*b*d)*x^4)*sqrt(b*x^6 + a))/(b^4*x^6 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^{10}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}+\frac{dx^{16}\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**9*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**10*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**6*exp_polar(I*pi)/a)/(6*
a**(3/2)*gamma(8/3)) + d*x**16*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**
6*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{(dx^6+c)x^9}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^9/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{(dx^6+c)x^9}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^9/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx=\int \frac{x^9\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^9*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^9*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{-10\sqrt{b\,x^6+a}\,ad\,x^4+7\sqrt{b\,x^6+a}\,bc\,x^4+\sqrt{b\,x^6+a}\,bd\,x^{10}+40\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^2x^{12}+2ab\,x^6+a^2}dx\right)}{}$$

input
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
( - 10*sqrt(a + b*x**6)*a*d*x**4 + 7*sqrt(a + b*x**6)*b*c*x**4 + sqrt(a +
b*x**6)*b*d*x**10 + 40*int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**6 + b*
*2*x**12),x)*a**3*d - 28*int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**6 +
b**2*x**12),x)*a**2*b*c + 40*int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**
6 + b**2*x**12),x)*a**2*b*d*x**6 - 28*int((sqrt(a + b*x**6)*x**3)/(a**2 +
2*a*b*x**6 + b**2*x**12),x)*a*b**2*c*x**6)/(7*b**2*(a + b*x**6))
```

3.24 $\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 557

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{(bc-ad)x^4}{3ab\sqrt{a+bx^6}} - \frac{(bc-4ad)\sqrt{a+bx^6}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)} + \frac{\sqrt{2-\sqrt{3}}(bc-4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4\sqrt{3}\right)}{2\ 3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}} - \frac{\sqrt{2}(bc-4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/3*(-a*d+b*c)*x^4/a/b/(b*x^6+a)^(1/2)-1/3*(-4*a*d+b*c)*(b*x^6+a)^(1/2)/a/
b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)+1/6*(1/2*6^(1/2)-1/2*2^(1/2))*(-
4*a*d+b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3
)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)/a^
(2/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x^2)^2)^(1/2)/(b*x^6+a)^(1/2)-1/9*2^(1/2)*(-4*a*d+b*c)*(a^(1/3)+b^(1/3)*x^
2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/b^(5/3)/(a^(1/3)*(a^(1/3
)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.13

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{x^4\left(4ad+(bc-4ad)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^6}{a}\right)\right)}{4ab\sqrt{a+bx^6}}$$

input
```
Integrate[(x^3*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
(x^4*(4*a*d + (b*c - 4*a*d)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[2/3, 3/2
, 5/3, -((b*x^6)/a)]))/(4*a*b*Sqrt[a + b*x^6])
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {957, 807, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^6\right)}{\left(a+bx^6\right)^{3/2}}dx$$

$\big\downarrow$ 957

$$\frac{x^4(bc-ad)}{3ab\sqrt{a+bx^6}}-\frac{(bc-4ad)\int \frac{x^3}{\sqrt{bx^6+a}}dx}{3ab}$$

$\big\downarrow$ 807

$$\frac{x^4(bc-ad)}{3ab\sqrt{a+bx^6}}-\frac{(bc-4ad)\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{6ab}$$

$\big\downarrow$ 832

$$\frac{x^4(bc-ad)}{3ab\sqrt{a+bx^6}}-\frac{(bc-4ad)\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{6ab}$$

$\big\downarrow$ 759

$$\frac{x^4(bc-ad)}{3ab\sqrt{a+bx^6}}-$$

$$\frac{(bc-4ad)\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+(\cdots}{\sqrt[3]{b}x^2+(\cdots}\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}\cdots}\right)}{6ab}$$

$\big\downarrow$ 2416

$$\frac{x^4(bc-ad)}{3ab\sqrt{a+bx^6}}-\frac{(bc-4ad)\left(\frac{\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{\sqrt[3]{b}}\right.}{6ab}$$

input
```
Int[(x^3*(c + d*x^6))/(a + b*x^6)^(3/2),x]
```

output
```
((b*c - a*d)*x^4)/(3*a*b*Sqrt[a + b*x^6]) - ((b*c - 4*a*d)*(((2*Sqrt[a + b
*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 -
 Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 -
 Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a +
b*x^6])))/(6*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^3(d\,x^6+c)}{(b\,x^6+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.17

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{bx^6+a}(b^2c-abd)x^4+((b^2c-4\,abd)x^6+abc-4\,a^2d)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\right.}{3\,(ab^3x^6+a^2b^2)}$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(b*x^6 + a)*(b^2*c - a*b*d)*x^4 + ((b^2*c - 4*a*b*d)*x^6 + a*b*c
- 4*a^2*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x^2)))/(a*b^3*x^6 + a^2*b^2)
```

Sympy [A] (verification not implemented)

Time = 8.60 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{cx^4\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{3}{2}\\\frac{5}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{dx^{10}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**3*(d*x**6+c)/(b*x**6+a)**(3/2),x)
```

output
```
c*x**4*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(3/2)*gamma(5/3)) + d*x**10*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**6*
exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^3}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^3/(b*x^6 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{(dx^6+c)x^3}{(bx^6+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^3/(b*x^6 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \int \frac{x^3\,(d\,x^6+c)}{(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x^6))/(a + b*x^6)^(3/2),x)
```

output
```
int((x^3*(c + d*x^6))/(a + b*x^6)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{3/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,d\,x^4-4\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^2x^{12}+2ab\,x^6+a^2}dx\right)a^2d+\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^2x^{12}+2ab\,x^6+a^2}dx\right)abc-4\left(\int \frac{\sqrt{b\,x^6}}{b^2x^{12}+2}\right.}{b\,(b\,x^6+a)}$$

input
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**6)*d*x**4 - 4*int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**
6 + b**2*x**12),x)*a**2*d + int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**6
 + b**2*x**12),x)*a*b*c - 4*int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x**6
 + b**2*x**12),x)*a*b*d*x**6 + int((sqrt(a + b*x**6)*x**3)/(a**2 + 2*a*b*x
**6 + b**2*x**12),x)*b**2*c*x**6)/(b*(a + b*x**6))
```

3.25 $\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 580

$$\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{3/2}}\,dx = -\frac{c}{2ax^2\sqrt{a+bx^6}} - \frac{(5bc-2ad)x^4}{6a^2\sqrt{a+bx^6}} + \frac{(5bc-2ad)\sqrt{a+bx^6}}{6a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$
$$-\frac{\sqrt{2-\sqrt{3}}(5bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4\sqrt{3}\right)}{4\,3^{3/4}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$+\frac{(5bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/2*c/a/x^2/(b*x^6+a)^(1/2)-1/6*(-2*a*d+5*b*c)*x^4/a^2/(b*x^6+a)^(1/2)+1/
6*(-2*a*d+5*b*c)*(b*x^6+a)^(1/2)/a^2/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x^2)-1/12*(1/2*6^(1/2)-1/2*2^(1/2))*(-2*a*d+5*b*c)*(a^(1/3)+b^(1/3)*x^2)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2
)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)/a^(5/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+1/18
*(-2*a*d+5*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3
)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a
^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*2^(1/
2)*3^(3/4)/a^(5/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{c+dx^6}{x^3(a+bx^6)^{3/2}}\,dx = \frac{-4ac+(-5bc+2ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^6}{a}\right)}{8a^2x^2\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^3*(a + b*x^6)^(3/2)),x]
```

output
```
(-4*a*c + (-5*b*c + 2*a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[2/3,
3/2, 5/3, -((b*x^6)/a)])/(8*a^2*x^2*Sqrt[a + b*x^6])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 807, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(5bc-2ad)\int \frac{x^3}{\left(bx^6+a\right)^{3/2}}dx}{2a}-\frac{c}{2ax^2\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 807$$

$$-\frac{(5bc-2ad)\int \frac{x^2}{\left(bx^6+a\right)^{3/2}}dx^2}{4a}-\frac{c}{2ax^2\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(5bc-2ad)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3a}\right)}{4a}-\frac{c}{2ax^2\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 832$$

$$-\frac{(5bc-2ad)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}}{3a}\right)}{4a}-\frac{c}{2ax^2\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 759$$

$$-\frac{(5bc-2ad)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{3a}\right.}{4a}$$

$$\frac{c}{2ax^2\sqrt{a+bx^6}}$$

$$\Big\downarrow \; 2416$$

$$-\frac{(5bc-2ad)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\cdots}{\sqrt[3]{b}x^2+\cdots}\right)\cdots\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{\sqrt[3]{b}}\cdots}{\cdots}\right)}{\cdots}-\frac{c}{2ax^2\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^3*(a + b*x^6)^(3/2)),x]
```

output
```
-1/2*c/(a*x^2*Sqrt[a + b*x^6]) - ((5*b*c - 2*a*d)*((2*x^4)/(3*a*Sqrt[a + b
*x^6]) - (((2*Sqrt[a + b*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(
1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(3*a)))/(4*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{3} \left(b\,x^{6}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.19

$$\int \frac{c+dx^6}{x^3 \left(a+bx^6\right)^{3/2}}\,dx = -\frac{((5\,b^2c-2\,abd)x^8+(5\,abc-2\,a^2d)x^2)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right)\right)+((}{6\left(a^2b^2x^8+a^3bx^2\right)}$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(((5*b^2*c - 2*a*b*d)*x^8 + (5*a*b*c - 2*a^2*d)*x^2)*sqrt(b)*weierstr
assZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x^2)) + ((5*b^2*c - 2*a*
b*d)*x^6 + 3*a*b*c)*sqrt(b*x^6 + a))/(a^2*b^2*x^8 + a^3*b*x^2)
```

Sympy [A] (verification not implemented)

Time = 21.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^6 e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right)} + \frac{dx^4\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^6 e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x**6+c)/x**3/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
3/2)*x**2*gamma(2/3)) + d*x**4*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**6
*exp_polar(I*pi)/a)/(6*a**(3/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{3/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{3/2}}\,dx = \int \frac{d\,x^6+c}{x^3\,(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^3*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^3*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d - 2\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{15}+2ab\,x^9+a^2x^3}dx\right)a^2d\,x^2 + 5\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{15}+2ab\,x^9+a^2x^3}dx\right)abc\,x^2 -}{5b\,x^2\,(b\,x^6+a)}$$

input
```
int((d*x^6+c)/x^3/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 2*int(sqrt(a + b*x**6)/(a**2*x**3 + 2*a*b*x**9 +
b**2*x**15),x)*a**2*d*x**2 + 5*int(sqrt(a + b*x**6)/(a**2*x**3 + 2*a*b*x**
9 + b**2*x**15),x)*a*b*c*x**2 - 2*int(sqrt(a + b*x**6)/(a**2*x**3 + 2*a*b*
x**9 + b**2*x**15),x)*a*b*d*x**8 + 5*int(sqrt(a + b*x**6)/(a**2*x**3 + 2*a
*b*x**9 + b**2*x**15),x)*b**2*c*x**8)/(5*b*x**2*(a + b*x**6))
```

3.26 $\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 610

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{3/2}}\,dx = -\frac{c}{8ax^8\sqrt{a+bx^6}}-\frac{11bc-8ad}{24a^2x^2\sqrt{a+bx^6}}$$
$$+\frac{5(11bc-8ad)\sqrt{a+bx^6}}{48a^3x^2}-\frac{5\sqrt[3]{b}(11bc-8ad)\sqrt{a+bx^6}}{48a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$
$$+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}(11bc-8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4}{32\,3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$-\frac{5\sqrt[3]{b}(11bc-8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{}\right.}{24\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/8*c/a/x^8/(b*x^6+a)^(1/2)-1/24*(-8*a*d+11*b*c)/a^2/x^2/(b*x^6+a)^(1/2)+
5/48*(-8*a*d+11*b*c)*(b*x^6+a)^(1/2)/a^3/x^2-5/48*b^(1/3)*(-8*a*d+11*b*c)*
(b*x^6+a)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)+5/96*(1/2*6^(1/2)-1/
2*2^(1/2))*b^(1/3)*(-8*a*d+11*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*Ellip
ticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I
*3^(1/2)+2*I)*3^(1/4)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)-5/144*b^(1/3)*(-8*a*d+11*b*c
)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)
*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(
8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(
1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{3/2}}\,dx=\frac{-2ac+(11bc-8ad)x^6\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},-\frac{bx^6}{a}\right)}{16a^2x^8\sqrt{a+bx^6}}$$

input
```
Integrate[(c + d*x^6)/(x^9*(a + b*x^6)^(3/2)),x]
```

output
```
(-2*a*c + (11*b*c - 8*a*d)*x^6*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[-1/3,
 3/2, 2/3, -((b*x^6)/a)])/(16*a^2*x^8*Sqrt[a + b*x^6])
```

Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {955, 807, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(11bc-8ad)\int \frac{1}{x^3(bx^6+a)^{3/2}}dx}{8a}-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(11bc-8ad)\int \frac{1}{x^4(bx^6+a)^{3/2}}dx^2}{16a}-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(11bc-8ad)\left(\frac{5\int \frac{1}{x^4\sqrt{bx^6+a}}dx^2}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{16a}-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{(11bc-8ad)\left(\frac{5\left(\frac{b\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{2a}-\frac{\sqrt{a+bx^6}}{ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{16a}-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{(11bc-8ad)\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^6}}{ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{16a}-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(11bc-8ad)\left(\frac{5\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2\cdots}{\sqrt[3]{b}x^2\cdots}\right)\cdots\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{2a}\cdots\right)}{3a}\cdots\right)}{16a}$$

$$\frac{c}{8ax^8\sqrt{a+bx^6}}$$

$\downarrow$ 2416

$$(11bc-8ad)\left(5\left(b\left(\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.\right.\right.$$

$$\frac{}{\sqrt[3]{b}}$$

$$-\frac{c}{8ax^8\sqrt{a+bx^6}}$$

input
```
Int[(c + d*x^6)/(x^9*(a + b*x^6)^(3/2)),x]
```

output
```
-1/8*c/(a*x^8*Sqrt[a + b*x^6]) - ((11*b*c - 8*a*d)*(2/(3*a*x^2*Sqrt[a + b*
x^6]) + (5*(-(Sqrt[a + b*x^6]/(a*x^2)) + (b*(((2*Sqrt[a + b*x^6])/(b^(1/3)
*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x
^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*S
qrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[
2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(2*a)
))/(3*a)))/(16*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{9} \left(b\,x^{6}+a \right)^{\frac{3}{2}}}d x$$

input
```
int((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x)
```

output
```
int((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.21

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{3/2}}\,dx = \frac{5\left((11\,b^2c-8\,abd)x^{14}+(11\,abc-8\,a^2d)x^8\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassI}\right.}{48\,(a^3b}$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x, algorithm="fricas")
```

output
```
1/48*(5*((11*b^2*c - 8*a*b*d)*x^14 + (11*a*b*c - 8*a^2*d)*x^8)*sqrt(b)*wei
erstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x^2)) + (5*(11*b^2*
c - 8*a*b*d)*x^12 + 3*(11*a*b*c - 8*a^2*d)*x^6 - 6*a^2*c)*sqrt(b*x^6 + a))
/(a^3*b*x^14 + a^4*x^8)
```

Sympy [A] (verification not implemented)

Time = 76.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.15

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{3/2}}\,dx = \frac{c\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^8\Gamma\left(-\frac{1}{3}\right)}+\frac{d\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\\frac{2}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((d*x**6+c)/x**9/(b*x**6+a)**(3/2),x)
```

output
```
c*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**6*exp_polar(I*pi)/a)/(6*a**
(3/2)*x**8*gamma(-1/3)) + d*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**6*
exp_polar(I*pi)/a)/(6*a**(3/2)*x**2*gamma(2/3))
```

Maxima [F]

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{3/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^9}\,dx$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^9), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{3/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{3}{2}}x^9}\,dx$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(3/2)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{3/2}}\,dx=\int \frac{d\,x^6+c}{x^9\,(b\,x^6+a)^{3/2}}\,dx$$

input
```
int((c + d*x^6)/(x^9*(a + b*x^6)^(3/2)),x)
```

output
```
int((c + d*x^6)/(x^9*(a + b*x^6)^(3/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-8\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{21}+2ab\,x^{15}+a^2x^9}dx\right)a^2d\,x^8+11\left(\int \frac{\sqrt{b\,x^6+a}}{b^2x^{21}+2ab\,x^{15}+a^2x^9}dx\right)abc\,x^8}{11b\,x^8\left(b\,x^6+\right.}$$

input
```
int((d*x^6+c)/x^9/(b*x^6+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 8*int(sqrt(a + b*x**6)/(a**2*x**9 + 2*a*b*x**15 +
 b**2*x**21),x)*a**2*d*x**8 + 11*int(sqrt(a + b*x**6)/(a**2*x**9 + 2*a*b*x
**15 + b**2*x**21),x)*a*b*c*x**8 - 8*int(sqrt(a + b*x**6)/(a**2*x**9 + 2*a
*b*x**15 + b**2*x**21),x)*a*b*d*x**14 + 11*int(sqrt(a + b*x**6)/(a**2*x**9
 + 2*a*b*x**15 + b**2*x**21),x)*b**2*c*x**14)/(11*b*x**8*(a + b*x**6))
```

3.27 $\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}} \, dx = -\frac{a^2(bc-ad)}{9b^4(a+bx^6)^{3/2}} + \frac{a(2bc-3ad)}{3b^4\sqrt{a+bx^6}} + \frac{(bc-3ad)\sqrt{a+bx^6}}{3b^4} + \frac{d(a+bx^6)^{3/2}}{9b^4}$$

output
```
-1/9*a^2*(-a*d+b*c)/b^4/(b*x^6+a)^(3/2)+1/3*a*(-3*a*d+2*b*c)/b^4/(b*x^6+a)
^(1/2)+1/3*(-3*a*d+b*c)*(b*x^6+a)^(1/2)/b^4+1/9*d*(b*x^6+a)^(3/2)/b^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}} \, dx = \frac{-16a^3d + 8a^2b(c-3dx^6) - 6ab^2x^6(-2c+dx^6) + b^3x^{12}(3c+dx^6)}{9b^4(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^17*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(-16*a^3*d + 8*a^2*b*(c - 3*d*x^6) - 6*a*b^2*x^6*(-2*c + d*x^6) + b^3*x^12
*(3*c + d*x^6))/(9*b^4*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{6}\int \frac{x^{12}\left(dx^6+c\right)}{\left(bx^6+a\right)^{5/2}}dx^6$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{6}\int \left(-\frac{(ad-bc)a^2}{b^3\left(bx^6+a\right)^{5/2}}+\frac{(3ad-2bc)a}{b^3\left(bx^6+a\right)^{3/2}}+\frac{d\sqrt{bx^6+a}}{b^3}+\frac{bc-3ad}{b^3\sqrt{bx^6+a}}\right)dx^6$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}\left(-\frac{2a^2(bc-ad)}{3b^4\left(a+bx^6\right)^{3/2}}+\frac{2a(2bc-3ad)}{b^4\sqrt{a+bx^6}}+\frac{2\sqrt{a+bx^6}(bc-3ad)}{b^4}+\frac{2d\left(a+bx^6\right)^{3/2}}{3b^4}\right)$$

input
```
Int[(x^17*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((-2*a^2*(b*c - a*d))/(3*b^4*(a + b*x^6)^(3/2)) + (2*a*(2*b*c - 3*a*d))/(b
^4*Sqrt[a + b*x^6]) + (2*(b*c - 3*a*d)*Sqrt[a + b*x^6])/b^4 + (2*d*(a + b*
x^6)^(3/2))/(3*b^4))/6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{(d\,x^{18}+3c\,x^{12})b^{3}+12\left(-\frac{d\,x^{6}}{2}+c\right)x^{6}a\,b^{2}+8a^{2}\left(-3d\,x^{6}+c\right)b-16a^{3}d}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{4}}$	69
gosper	$-\frac{-d\,x^{18}b^{3}+6a\,b^{2}d\,x^{12}-3b^{3}c\,x^{12}+24a^{2}bd\,x^{6}-12a\,b^{2}c\,x^{6}+16a^{3}d-8a^{2}bc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{4}}$	77
trager	$-\frac{-d\,x^{18}b^{3}+6a\,b^{2}d\,x^{12}-3b^{3}c\,x^{12}+24a^{2}bd\,x^{6}-12a\,b^{2}c\,x^{6}+16a^{3}d-8a^{2}bc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{4}}$	77
orering	$-\frac{-d\,x^{18}b^{3}+6a\,b^{2}d\,x^{12}-3b^{3}c\,x^{12}+24a^{2}bd\,x^{6}-12a\,b^{2}c\,x^{6}+16a^{3}d-8a^{2}bc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{4}}$	77
risch	$-\frac{\left(-x^{6}bd+8ad-3cb\right)\sqrt{b\,x^{6}+a}}{9b^{4}}-\frac{\sqrt{b\,x^{6}+a}\,\left(9abd\,x^{6}-6b^{2}c\,x^{6}+8a^{2}d-5abc\right)a}{9b^{4}\left(x^{12}b^{2}+2a\,x^{6}b+a^{2}\right)}$	96

input
```
int(x^17*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*((d*x^18+3*c*x^12)*b^3+12*(-1/2*d*x^6+c)*x^6*a*b^2+8*a^2*(-3*d*x^6+c)*
b-16*a^3*d)/(b*x^6+a)^(3/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{(b^3dx^{18}+3\,(b^3c-2\,ab^2d)x^{12}+12\,(ab^2c-2\,a^2bd)x^6+8\,a^2bc-16\,a^3d)\sqrt{bx^6+a}}{9\,(b^6x^{12}+2\,ab^5x^6+a^2b^4)}$$

input
```
integrate(x^17*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(b^3*d*x^18 + 3*(b^3*c - 2*a*b^2*d)*x^12 + 12*(a*b^2*c - 2*a^2*b*d)*x^
6 + 8*a^2*b*c - 16*a^3*d)*sqrt(b*x^6 + a)/(b^6*x^12 + 2*a*b^5*x^6 + a^2*b^
4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(90) = 180.

Time = 2.27 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.27

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \begin{cases} -\frac{16a^3d}{9ab^4\sqrt{a+bx^6}+9b^5x^6\sqrt{a+bx^6}} + \frac{8a^2bc}{9ab^4\sqrt{a+bx^6}+9b^5x^6\sqrt{a+bx^6}} - \frac{24a^2bdx^6}{9ab^4\sqrt{a+bx^6}+9b^5x^6\sqrt{a+bx^6}} + \frac{}{9ab^4\sqrt{a+} \\ \frac{\frac{cx^{18}}{18}+\frac{dx^{24}}{24}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**17*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Piecewise((-16*a**3*d/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6*sqrt(a + b*
x**6)) + 8*a**2*b*c/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6*sqrt(a + b*x*
*6)) - 24*a**2*b*d*x**6/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6*sqrt(a +
b*x**6)) + 12*a*b**2*c*x**6/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6*sqrt(
a + b*x**6)) - 6*a*b**2*d*x**12/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6*s
qrt(a + b*x**6)) + 3*b**3*c*x**12/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6
*sqrt(a + b*x**6)) + b**3*d*x**18/(9*a*b**4*sqrt(a + b*x**6) + 9*b**5*x**6
*sqrt(a + b*x**6)), Ne(b, 0)), ((c*x**18/18 + d*x**24/24)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{1}{9}\,d\left(\frac{(bx^6+a)^{\frac{3}{2}}}{b^4} - \frac{9\,\sqrt{bx^6+a}a}{b^4} - \frac{9\,a^2}{\sqrt{bx^6+a}b^4} + \frac{a^3}{(bx^6+a)^{\frac{3}{2}}b^4}\right) + \frac{1}{9}\,c\left(\frac{3\,\sqrt{bx^6+a}}{b^3} + \frac{6\,a}{\sqrt{bx^6+a}b^3} - \frac{a^2}{(bx^6+a)^{\frac{3}{2}}b^3}\right)$$

input
```
integrate(x^17*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*d*((b*x^6 + a)^(3/2)/b^4 - 9*sqrt(b*x^6 + a)*a/b^4 - 9*a^2/(sqrt(b*x^6
 + a)*b^4) + a^3/((b*x^6 + a)^(3/2)*b^4)) + 1/9*c*(3*sqrt(b*x^6 + a)/b^3 +
 6*a/(sqrt(b*x^6 + a)*b^3) - a^2/((b*x^6 + a)^(3/2)*b^3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{6\,(bx^6+a)abc - a^2bc - 9\,(bx^6+a)a^2d + a^3d}{9\,(bx^6+a)^{\frac{3}{2}}b^4} + \frac{3\,\sqrt{bx^6+a}b^9c + (bx^6+a)^{\frac{3}{2}}b^8d - 9\,\sqrt{bx^6+a}ab^8d}{9\,b^{12}}$$

input
```
integrate(x^17*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
1/9*(6*(b*x^6 + a)*a*b*c - a^2*b*c - 9*(b*x^6 + a)*a^2*d + a^3*d)/((b*x^6
+ a)^(3/2)*b^4) + 1/9*(3*sqrt(b*x^6 + a)*b^9*c + (b*x^6 + a)^(3/2)*b^8*d -
 9*sqrt(b*x^6 + a)*a*b^8*d)/b^12
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{d\,(b\,x^6+a)^3+a^3\,d-9\,a\,d\,(b\,x^6+a)^2+3\,b\,c\,(b\,x^6+a)^2-9\,a^2\,d\,(b\,x^6+a)-a^2\,b\,c+}{9\,b^4\,(b\,x^6+a)^{3/2}}$$

input
```
int((x^17*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
(d*(a + b*x^6)^3 + a^3*d - 9*a*d*(a + b*x^6)^2 + 3*b*c*(a + b*x^6)^2 - 9*a
^2*d*(a + b*x^6) - a^2*b*c + 6*a*b*c*(a + b*x^6))/(9*b^4*(a + b*x^6)^(3/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\int \frac{x^{17}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(b^3d\,x^{18}-6a\,b^2d\,x^{12}+3b^3c\,x^{12}-24a^2bd\,x^6+12a\,b^2c\,x^6-16a^3d+8a^2bc)}{9b^4\,(b^2x^{12}+2ab\,x^6+a^2)}$$

input
```
int(x^17*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*( - 16*a**3*d + 8*a**2*b*c - 24*a**2*b*d*x**6 + 12*a*b**
2*c*x**6 - 6*a*b**2*d*x**12 + 3*b**3*c*x**12 + b**3*d*x**18))/(9*b**4*(a**
2 + 2*a*b*x**6 + b**2*x**12))
```

3.28 $\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{a(bc-ad)}{9b^3(a+bx^6)^{3/2}} - \frac{bc-2ad}{3b^3\sqrt{a+bx^6}} + \frac{d\sqrt{a+bx^6}}{3b^3}$$

output
```
1/9*a*(-a*d+b*c)/b^3/(b*x^6+a)^(3/2)-1/3*(-2*a*d+b*c)/b^3/(b*x^6+a)^(1/2)+
1/3*d*(b*x^6+a)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-2abc+8a^2d-3b^2cx^6+12abdx^6+3b^2dx^{12}}{9b^3(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^11*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(-2*a*b*c + 8*a^2*d - 3*b^2*c*x^6 + 12*a*b*d*x^6 + 3*b^2*d*x^12)/(9*b^3*(a
 + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^6(dx^6+c)}{(bx^6+a)^{5/2}}dx^6$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{6}\int \left(\frac{d}{b^2\sqrt{bx^6+a}}+\frac{bc-2ad}{b^2(bx^6+a)^{3/2}}+\frac{a(ad-bc)}{b^2(bx^6+a)^{5/2}}\right)dx^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(-\frac{2(bc-2ad)}{b^3\sqrt{a+bx^6}}+\frac{2a(bc-ad)}{3b^3(a+bx^6)^{3/2}}+\frac{2d\sqrt{a+bx^6}}{b^3}\right)$$

input
```
Int[(x^11*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((2*a*(b*c - a*d))/(3*b^3*(a + b*x^6)^(3/2)) - (2*(b*c - 2*a*d))/(b^3*Sqrt
[a + b*x^6]) + (2*d*Sqrt[a + b*x^6])/b^3)/6
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$\frac{-3x^{6}\left(-d\,x^{6}+c\right)b^{2}-2a\left(-6d\,x^{6}+c\right)b+8a^{2}d}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{3}}$	50
gosper	$\frac{3d\,x^{12}b^{2}+12abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-2abc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{3}}$	53
trager	$\frac{3d\,x^{12}b^{2}+12abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-2abc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{3}}$	53
orering	$\frac{3d\,x^{12}b^{2}+12abd\,x^{6}-3b^{2}c\,x^{6}+8a^{2}d-2abc}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{3}}$	53
risch	$\frac{d\sqrt{b\,x^{6}+a}}{3b^{3}}+\frac{\sqrt{b\,x^{6}+a}\,\left(6abd\,x^{6}-3b^{2}c\,x^{6}+5a^{2}d-2abc\right)}{9b^{3}\left(x^{12}b^{2}+2a\,x^{6}b+a^{2}\right)}$	80

input
```
int(x^11*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-3*x^6*(-d*x^6+c)*b^2-2*a*(-6*d*x^6+c)*b+8*a^2*d)/(b*x^6+a)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{(3\,b^2dx^{12}-3\,(b^2c-4\,abd)x^6-2\,abc+8\,a^2d)\sqrt{bx^6+a}}{9\,(b^5x^{12}+2\,ab^4x^6+a^2b^3)}$$

input
```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(3*b^2*d*x^12 - 3*(b^2*c - 4*a*b*d)*x^6 - 2*a*b*c + 8*a^2*d)*sqrt(b*x^
6 + a)/(b^5*x^12 + 2*a*b^4*x^6 + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(65) = 130.

Time = 1.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.29

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \begin{cases} \frac{8a^2d}{9ab^3\sqrt{a+bx^6}+9b^4x^6\sqrt{a+bx^6}} - \frac{2abc}{9ab^3\sqrt{a+bx^6}+9b^4x^6\sqrt{a+bx^6}} + \frac{12abdx^6}{9ab^3\sqrt{a+bx^6}+9b^4x^6\sqrt{a+bx^6}} - \frac{3}{9ab^3\sqrt{a+bx}} \\ \frac{\frac{cx^{12}}{12}+\frac{dx^{18}}{18}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**11*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Piecewise((8*a**2*d/(9*a*b**3*sqrt(a + b*x**6) + 9*b**4*x**6*sqrt(a + b*x*
*6)) - 2*a*b*c/(9*a*b**3*sqrt(a + b*x**6) + 9*b**4*x**6*sqrt(a + b*x**6))
+ 12*a*b*d*x**6/(9*a*b**3*sqrt(a + b*x**6) + 9*b**4*x**6*sqrt(a + b*x**6))
 - 3*b**2*c*x**6/(9*a*b**3*sqrt(a + b*x**6) + 9*b**4*x**6*sqrt(a + b*x**6)
) + 3*b**2*d*x**12/(9*a*b**3*sqrt(a + b*x**6) + 9*b**4*x**6*sqrt(a + b*x**
6)), Ne(b, 0)), ((c*x**12/12 + d*x**18/18)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{1}{9}\,d\left(\frac{3\sqrt{bx^6+a}}{b^3}+\frac{6\,a}{\sqrt{bx^6+a}b^3}-\frac{a^2}{(bx^6+a)^{\frac{3}{2}}b^3}\right) - \frac{1}{9}\,c\left(\frac{3}{\sqrt{bx^6+a}b^2}-\frac{a}{(bx^6+a)^{\frac{3}{2}}b^2}\right)$$

input
```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*d*(3*sqrt(b*x^6 + a)/b^3 + 6*a/(sqrt(b*x^6 + a)*b^3) - a^2/((b*x^6 + a
)^(3/2)*b^3)) - 1/9*c*(3/(sqrt(b*x^6 + a)*b^2) - a/((b*x^6 + a)^(3/2)*b^2)
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\sqrt{bx^6+a}d}{3\,b^3} - \frac{3\,(bx^6+a)bc-abc-6\,(bx^6+a)ad+a^2d}{9\,(bx^6+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^11*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*sqrt(b*x^6 + a)*d/b^3 - 1/9*(3*(b*x^6 + a)*b*c - a*b*c - 6*(b*x^6 + a)
*a*d + a^2*d)/((b*x^6 + a)^(3/2)*b^3)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{3\,d\,(b\,x^6+a)^2-a^2\,d+6\,a\,d\,(b\,x^6+a)-3\,b\,c\,(b\,x^6+a)+a\,b\,c}{9\,b^3\,(b\,x^6+a)^{3/2}}$$

input
```
int((x^11*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
(3*d*(a + b*x^6)^2 - a^2*d + 6*a*d*(a + b*x^6) - 3*b*c*(a + b*x^6) + a*b*c
)/(9*b^3*(a + b*x^6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(3b^2d\,x^{12}+12abd\,x^6-3b^2c\,x^6+8a^2d-2abc)}{9b^3\,(b^2x^{12}+2ab\,x^6+a^2)}$$

input
```
int(x^11*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*(8*a**2*d - 2*a*b*c + 12*a*b*d*x**6 - 3*b**2*c*x**6 + 3*
b**2*d*x**12))/(9*b**3*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.29 $\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-bc+ad}{9b^2(a+bx^6)^{3/2}} - \frac{d}{3b^2\sqrt{a+bx^6}}$$

output
```
1/9*(a*d-b*c)/b^2/(b*x^6+a)^(3/2)-1/3*d/b^2/(b*x^6+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-bc-2ad-3bdx^6}{9b^2(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^5*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(-(b*c) - 2*a*d - 3*b*d*x^6)/(9*b^2*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c+d x^6)}{(a+b x^6)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 946}$$

$$\frac{1}{6} \int \frac{d x^6+c}{(b x^6+a)^{5/2}} dx^6$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{6} \int \left(\frac{d}{b (b x^6+a)^{3/2}}+\frac{b c-a d}{b (b x^6+a)^{5/2}}\right) dx^6$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6} \left(-\frac{2 (b c-a d)}{3 b^2 (a+b x^6)^{3/2}}-\frac{2 d}{b^2 \sqrt{a+b x^6}}\right)$$

input
```
Int[(x^5*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((-2*(b*c - a*d))/(3*b^2*(a + b*x^6)^(3/2)) - (2*d)/(b^2*Sqrt[a + b*x^6]))
/6
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{3x^{6}bd+2ad+cb}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}} b^{2}}$	30
trager	$-\frac{3x^{6}bd+2ad+cb}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}} b^{2}}$	30
pseudoelliptic	$-\frac{2\left(\frac{\left(3d\,x^{6}+c\right)b}{2}+ad\right)}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}} b^{2}}$	30
orering	$-\frac{3x^{6}bd+2ad+cb}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}} b^{2}}$	30

input
```
int(x^5*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3*b*d*x^6+2*a*d+b*c)/(b*x^6+a)^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(3\,bdx^6+bc+2\,ad)\sqrt{bx^6+a}}{9\,(b^4x^{12}+2\,ab^3x^6+a^2b^2)}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(3*b*d*x^6 + b*c + 2*a*d)*sqrt(b*x^6 + a)/(b^4*x^12 + 2*a*b^3*x^6 + a
^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(39) = 78.

Time = 0.78 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.11

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \begin{cases} -\frac{2ad}{9ab^2\sqrt{a+bx^6}+9b^3x^6\sqrt{a+bx^6}} - \frac{bc}{9ab^2\sqrt{a+bx^6}+9b^3x^6\sqrt{a+bx^6}} - \frac{3bdx^6}{9ab^2\sqrt{a+bx^6}+9b^3x^6\sqrt{a+bx^6}} & \text{for } b \neq (\\ \frac{\frac{cx^6}{6}+\frac{dx^{12}}{12}}{a^{\frac{5}{2}}} & \text{otherwi} \end{cases}$$

input
```
integrate(x**5*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Piecewise((-2*a*d/(9*a*b**2*sqrt(a + b*x**6) + 9*b**3*x**6*sqrt(a + b*x**6
)) - b*c/(9*a*b**2*sqrt(a + b*x**6) + 9*b**3*x**6*sqrt(a + b*x**6)) - 3*b*
d*x**6/(9*a*b**2*sqrt(a + b*x**6) + 9*b**3*x**6*sqrt(a + b*x**6)), Ne(b, 0
)), ((c*x**6/6 + d*x**12/12)/a**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{1}{9}\,d\left(\frac{3}{\sqrt{bx^6+a}b^2} - \frac{a}{(bx^6+a)^{\frac{3}{2}}b^2}\right) - \frac{c}{9\,(bx^6+a)^{\frac{3}{2}}b}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*d*(3/(sqrt(b*x^6 + a)*b^2) - a/((b*x^6 + a)^(3/2)*b^2)) - 1/9*c/((b*x
^6 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{bc+3\,(bx^6+a)d-ad}{9\,(bx^6+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^5*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
-1/9*(b*c + 3*(b*x^6 + a)*d - a*d)/((b*x^6 + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{3\,d\,(b\,x^6+a)-a\,d+b\,c}{9\,b^2\,(b\,x^6+a)^{3/2}}$$

input
```
int((x^5*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
-(3*d*(a + b*x^6) - a*d + b*c)/(9*b^2*(a + b*x^6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(-3bd\,x^6-2ad-bc)}{9b^2\,(b^2x^{12}+2ab\,x^6+a^2)}$$

input
```
int(x^5*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*( - 2*a*d - b*c - 3*b*d*x**6))/(9*b**2*(a**2 + 2*a*b*x**
6 + b**2*x**12))
```

3.30 $\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx=\frac{bc-ad}{9ab\left(a+bx^6\right)^{3/2}}+\frac{c}{3a^2\sqrt{a+bx^6}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{3a^{5/2}}$$

output
```
1/9*(-a*d+b*c)/a/b/(b*x^6+a)^(3/2)+1/3*c/a^2/(b*x^6+a)^(1/2)-1/3*c*arctanh
((b*x^6+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx=\frac{4abc-a^2d+3b^2cx^6}{9a^2b\left(a+bx^6\right)^{3/2}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
Integrate[(c + d*x^6)/(x*(a + b*x^6)^(5/2)),x]
```

output
```
(4*a*b*c - a^2*d + 3*b^2*c*x^6)/(9*a^2*b*(a + b*x^6)^(3/2)) - (c*ArcTanh[S
qrt[a + b*x^6]/Sqrt[a]])/(3*a^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{dx^6+c}{x^6\left(bx^6+a\right)^{5/2}}dx^6$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6}\left(\frac{c\int \frac{1}{x^6\left(bx^6+a\right)^{3/2}}dx^6}{a}+\frac{2(bc-ad)}{3ab\left(a+bx^6\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{6}\left(\frac{c\left(\frac{\int \frac{1}{x^6\sqrt{bx^6+a}}dx^6}{a}+\frac{2}{a\sqrt{a+bx^6}}\right)}{a}+\frac{2(bc-ad)}{3ab\left(a+bx^6\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{c\left(\frac{2\int \frac{1}{\frac{x^{12}}{b}-\frac{a}{b}}d\sqrt{bx^6+a}}{ab}+\frac{2}{a\sqrt{a+bx^6}}\right)}{a}+\frac{2(bc-ad)}{3ab\left(a+bx^6\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{c\left(\frac{2}{a\sqrt{a+bx^6}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(bc-ad)}{3ab\left(a+bx^6\right)^{3/2}}\right)$$

input
```
Int[(c + d*x^6)/(x*(a + b*x^6)^(5/2)),x]
```

output
```
((2*(b*c - a*d))/(3*a*b*(a + b*x^6)^(3/2)) + (c*(2/(a*Sqrt[a + b*x^6]) - (
2*ArcTanh[Sqrt[a + b*x^6]/Sqrt[a]])/a^(3/2)))/a)/6
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$-\frac{3bc\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{\sqrt{a}}\right)a^{2}\left(b\,x^{6}+a\right)^{\frac{3}{2}}+a^{\frac{5}{2}}\left(-3b^{2}c\,x^{6}+a^{2}d-4abc\right)}{9\left(b\,x^{6}+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}b}$	73

input
```
int((d*x^6+c)/x/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3*b*c*arctanh((b*x^6+a)^(1/2)/a^(1/2))*a^2*(b*x^6+a)^(3/2)+a^(5/2)*(
-3*b^2*c*x^6+a^2*d-4*a*b*c))/(b*x^6+a)^(3/2)/a^(9/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.12

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx = \left[\frac{3\left(b^3cx^{12}+2\,ab^2cx^6+a^2bc\right)\sqrt{a}\log\left(\frac{bx^6-2\,\sqrt{bx^6+a}\sqrt{a}+2\,a}{x^6}\right)+2\left(3\,ab^2cx^6+4\,a^2bc-a^3d\right)\sqrt{bx^6+a}}{18\left(a^3b^3x^{12}+2\,a^4b^2x^6+a^5b\right)}\right.$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/18*(3*(b^3*c*x^12 + 2*a*b^2*c*x^6 + a^2*b*c)*sqrt(a)*log((b*x^6 - 2*sqr
t(b*x^6 + a)*sqrt(a) + 2*a)/x^6) + 2*(3*a*b^2*c*x^6 + 4*a^2*b*c - a^3*d)*s
qrt(b*x^6 + a))/(a^3*b^3*x^12 + 2*a^4*b^2*x^6 + a^5*b), 1/9*(3*(b^3*c*x^12
 + 2*a*b^2*c*x^6 + a^2*b*c)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^6 + a)) + (3
*a*b^2*c*x^6 + 4*a^2*b*c - a^3*d)*sqrt(b*x^6 + a))/(a^3*b^3*x^12 + 2*a^4*b
^2*x^6 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{ad-bc}{18a\left(a+bx^6\right)^{\frac{3}{2}}}+\frac{bc}{6a^2\sqrt{a+bx^6}}+\frac{bc\,\mathrm{atan}\left(\frac{\sqrt{a+bx^6}}{\sqrt{-a}}\right)}{6a^2\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{c\log\left(dx^6\right)+dx^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**6+c)/x/(b*x**6+a)**(5/2),x)
```

output
```
Piecewise((2*(-(a*d - b*c)/(18*a*(a + b*x**6)**(3/2)) + b*c/(6*a**2*sqrt(a
 + b*x**6)) + b*c*atan(sqrt(a + b*x**6)/sqrt(-a))/(6*a**2*sqrt(-a)))/b, Ne
(b, 0)), ((c*log(d*x**6) + d*x**6)/(6*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^6}{x\left(a+bx^6\right)^{5/2}}\,dx = \frac{1}{18}\,c\left(\frac{3\,\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx^6+4\,a)}{\left(bx^6+a\right)^{\frac{3}{2}}a^2}\right)-\frac{d}{9\left(bx^6+a\right)^{\frac{3}{2}}b}$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
1/18*c*(3*log((sqrt(b*x^6 + a) - sqrt(a))/(sqrt(b*x^6 + a) + sqrt(a)))/a^(
5/2) + 2*(3*b*x^6 + 4*a)/((b*x^6 + a)^(3/2)*a^2)) - 1/9*d/((b*x^6 + a)^(3/
2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{c+dx^6}{x(a+bx^6)^{5/2}}\,dx = \frac{c\arctan\left(\frac{\sqrt{bx^6+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}a^2} + \frac{3(bx^6+a)bc+abc-a^2d}{9(bx^6+a)^{\frac{3}{2}}a^2b}$$

input
```
integrate((d*x^6+c)/x/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*c*arctan(sqrt(b*x^6 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/9*(3*(b*x^6 + a)
*b*c + a*b*c - a^2*d)/((b*x^6 + a)^(3/2)*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^6}{x(a+bx^6)^{5/2}}\,dx = \frac{\frac{c}{3a}+\frac{c(bx^6+a)}{a^2}}{3(bx^6+a)^{3/2}} - \frac{d}{9b(bx^6+a)^{3/2}} - \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{bx^6+a}}{\sqrt{a}}\right)}{3a^{5/2}}$$

input
```
int((c + d*x^6)/(x*(a + b*x^6)^(5/2)),x)
```

output
```
(c/(3*a) + (c*(a + b*x^6))/a^2)/(3*(a + b*x^6)^(3/2)) - d/(9*b*(a + b*x^6)
^(3/2)) - (c*atanh((a + b*x^6)^(1/2)/a^(1/2)))/(3*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.83

$$\int \frac{c+dx^6}{x(a+bx^6)^{5/2}}\,dx = \frac{-2\sqrt{bx^6+a}\,a^3d+8\sqrt{bx^6+a}\,a^2bc+6\sqrt{bx^6+a}\,ab^2cx^6+3\sqrt{a}\log\left(\sqrt{bx^6+a}-\sqrt{\right.}}{}$$

input
```
int((d*x^6+c)/x/(b*x^6+a)^(5/2),x)
```

output
```
( - 2*sqrt(a + b*x**6)*a**3*d + 8*sqrt(a + b*x**6)*a**2*b*c + 6*sqrt(a + b
*x**6)*a*b**2*c*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a**2*b*c
+ 6*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a*b**2*c*x**6 + 3*sqrt(a)*log(
sqrt(a + b*x**6) - sqrt(a))*b**3*c*x**12 - 3*sqrt(a)*log(sqrt(a + b*x**6)
+ sqrt(a))*a**2*b*c - 6*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a*b**2*c*x
**6 - 3*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*b**3*c*x**12)/(18*a**3*b*(
a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.31 $\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{5/2}}\,dx = -\frac{bc-ad}{9a^2\left(a+bx^6\right)^{3/2}}-\frac{2bc-ad}{3a^3\sqrt{a+bx^6}}-\frac{c\sqrt{a+bx^6}}{6a^3x^6}+\frac{(5bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{6a^{7/2}}$$

output
```
-1/9*(-a*d+b*c)/a^2/(b*x^6+a)^(3/2)-1/3*(-a*d+2*b*c)/a^3/(b*x^6+a)^(1/2)-1
/6*c*(b*x^6+a)^(1/2)/a^3/x^6+1/6*(-2*a*d+5*b*c)*arctanh((b*x^6+a)^(1/2)/a^
(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{5/2}}\,dx = \frac{-3a^2c-20abcx^6+8a^2dx^6-15b^2cx^{12}+6abdx^{12}}{18a^3x^6\left(a+bx^6\right)^{3/2}}+\frac{(5bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{6a^{7/2}}$$

input
```
Integrate[(c + d*x^6)/(x^7*(a + b*x^6)^(5/2)),x]
```

output
```
(-3*a^2*c - 20*a*b*c*x^6 + 8*a^2*d*x^6 - 15*b^2*c*x^12 + 6*a*b*d*x^12)/(18
*a^3*x^6*(a + b*x^6)^(3/2)) + ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^6]/Sqr
t[a]])/(6*a^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{5/2}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6} \int \frac{dx^6+c}{x^{12} (bx^6+a)^{5/2}} dx^6$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6} \left(-\frac{(5bc-2ad) \int \frac{1}{x^6 (bx^6+a)^{5/2}} dx^6}{2a} - \frac{c}{ax^6 (a+bx^6)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{6} \left(-\frac{(5bc-2ad) \left(\frac{\int \frac{1}{x^6 \left(bx^6+a\right)^{3/2}} dx^6}{a} + \frac{2}{3a(a+bx^6)^{3/2}}\right)}{2a} - \frac{c}{ax^6 (a+bx^6)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{6}\left(-\frac{(5bc-2ad)\left(\frac{\frac{\int \frac{1}{x^6\sqrt{bx^6+a}}dx^6}{a}+\frac{2}{a\sqrt{a+bx^6}}}{a}+\frac{2}{3a(a+bx^6)^{3/2}}\right)}{2a}-\frac{c}{ax^6\left(a+bx^6\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{6}\left(-\frac{(5bc-2ad)\left(\frac{\frac{2\int \frac{1}{\frac{x^{12}}{b}-\frac{a}{b}}d\sqrt{bx^6+a}}{ab}+\frac{2}{a\sqrt{a+bx^6}}}{a}+\frac{2}{3a(a+bx^6)^{3/2}}\right)}{2a}-\frac{c}{ax^6\left(a+bx^6\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(-\frac{\left(\frac{\frac{2}{a\sqrt{a+bx^6}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^6}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx^6)^{3/2}}\right)(5bc-2ad)}{2a}-\frac{c}{ax^6\left(a+bx^6\right)^{3/2}}\right)$$

input
```
Int[(c + d*x^6)/(x^7*(a + b*x^6)^(5/2)),x]
```

output
```
(-(c/(a*x^6*(a + b*x^6)^(3/2))) - ((5*b*c - 2*a*d)*(2/(3*a*(a + b*x^6)^(3/
2)) + (2/(a*Sqrt[a + b*x^6]) - (2*ArcTanh[Sqrt[a + b*x^6]/Sqrt[a]])/a^(3/2
))/a))/(2*a))/6
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{(b x^6+a)^{\frac{3}{2}} x^6 \left(ad-\frac{5cb}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^6+a}}{\sqrt{a}}\right)+\frac{10\left(-\frac{3d x^6}{10}+c\right) b x^6 a^{\frac{3}{2}}}{3}+\left(-\frac{4d x^6}{3}+\frac{c}{2}\right) a^{\frac{5}{2}}+\frac{5\sqrt{a}\, b^2 c x^{12}}{2}}{3a^{\frac{7}{2}} (b x^6+a)^{\frac{3}{2}} x^6}$	97

input
```
int((d*x^6+c)/x^7/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^(7/2)/(b*x^6+a)^(3/2)*((b*x^6+a)^(3/2)*x^6*(a*d-5/2*c*b)*arctanh((b
*x^6+a)^(1/2)/a^(1/2))+10/3*(-3/10*d*x^6+c)*b*x^6*a^(3/2)+(-4/3*d*x^6+1/2*
c)*a^(5/2)+5/2*a^(1/2)*b^2*c*x^12)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.11

$$\int \frac{c+dx^6}{x^7\left(a+bx^6\right)^{5/2}}\,dx = \left[-\frac{3\left((5\,b^3c-2\,ab^2d)x^{18}+2\,(5\,ab^2c-2\,a^2bd)x^{12}+(5\,a^2bc-2\,a^3d)x^6\right)\sqrt{a}\log\left(\frac{bx^6\cdots}{\cdots}\right)\cdots}{36\,(a^4b^2x^{18}+\cdots}\right.$$

$$-\frac{3\left((5\,b^3c-2\,ab^2d)x^{18}+2\,(5\,ab^2c-2\,a^2bd)x^{12}+(5\,a^2bc-2\,a^3d)x^6\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^6+a}}\right)+\left(3\,(5\,ab^2c-\cdots\right.}{18\,(a^4b^2x^{18}+2\,a^5bx^{12}+a^6x^6)}$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/36*(3*((5*b^3*c - 2*a*b^2*d)*x^18 + 2*(5*a*b^2*c - 2*a^2*b*d)*x^12 + (
5*a^2*b*c - 2*a^3*d)*x^6)*sqrt(a)*log((b*x^6 - 2*sqrt(b*x^6 + a)*sqrt(a) +
 2*a)/x^6) + 2*(3*(5*a*b^2*c - 2*a^2*b*d)*x^12 + 4*(5*a^2*b*c - 2*a^3*d)*x
^6 + 3*a^3*c)*sqrt(b*x^6 + a))/(a^4*b^2*x^18 + 2*a^5*b*x^12 + a^6*x^6), -1
/18*(3*((5*b^3*c - 2*a*b^2*d)*x^18 + 2*(5*a*b^2*c - 2*a^2*b*d)*x^12 + (5*a
^2*b*c - 2*a^3*d)*x^6)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^6 + a)) + (3*(5*a
*b^2*c - 2*a^2*b*d)*x^12 + 4*(5*a^2*b*c - 2*a^3*d)*x^6 + 3*a^3*c)*sqrt(b*x
^6 + a))/(a^4*b^2*x^18 + 2*a^5*b*x^12 + a^6*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**7/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{5/2}}\,dx = -\frac{1}{36}\,c\left(\frac{2\left(15\,(bx^6+a)^2b-10\,(bx^6+a)ab-2\,a^2b\right)}{(bx^6+a)^{\frac{5}{2}}a^3-(bx^6+a)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right) + \frac{1}{18}\,d\left(\frac{3\,\log\left(\frac{\sqrt{bx^6+a}-\sqrt{a}}{\sqrt{bx^6+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx^6+4\,a)}{(bx^6+a)^{\frac{3}{2}}a^2}\right)$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/36*c*(2*(15*(b*x^6 + a)^2*b - 10*(b*x^6 + a)*a*b - 2*a^2*b)/((b*x^6 + a
)^(5/2)*a^3 - (b*x^6 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^6 + a) - sqrt(a)
)/(sqrt(b*x^6 + a) + sqrt(a)))/a^(7/2)) + 1/18*d*(3*log((sqrt(b*x^6 + a) -
 sqrt(a))/(sqrt(b*x^6 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^6 + 4*a)/((b*x^6
 + a)^(3/2)*a^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^6}{x^7(a+bx^6)^{5/2}}\,dx = -\frac{(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{bx^6+a}}{\sqrt{-a}}\right)}{6\,\sqrt{-a}a^3} - \frac{6\,(bx^6+a)bc+abc-3\,(bx^6+a)ad-a^2d}{9\,(bx^6+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^6+a}c}{6\,a^3x^6}$$

input
```
integrate((d*x^6+c)/x^7/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
-1/6*(5*b*c - 2*a*d)*arctan(sqrt(b*x^6 + a)/sqrt(-a))/(sqrt(-a)*a^3) - 1/9
*(6*(b*x^6 + a)*b*c + a*b*c - 3*(b*x^6 + a)*a*d - a^2*d)/((b*x^6 + a)^(3/2
)*a^3) - 1/6*sqrt(b*x^6 + a)*c/(a^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^6}{x^7(a+bx^6)^{5/2}}\,dx = \frac{\frac{d}{3\,a}+\frac{d\,(b\,x^6+a)}{a^2}}{3\,(b\,x^6+a)^{3/2}} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^6+a}}{\sqrt{a}}\right)}{3\,a^{5/2}} - \frac{c}{6\,a\,x^6\,(b\,x^6+a)^{3/2}} + \frac{5\,b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^6+a}}{\sqrt{a}}\right)}{6\,a^{7/2}} - \frac{10\,b\,c}{9\,a^2\,(b\,x^6+a)^{3/2}} - \frac{5\,b^2\,c\,x^6}{6\,a^3\,(b\,x^6+a)^{3/2}}$$

input
```
int((c + d*x^6)/(x^7*(a + b*x^6)^(5/2)),x)
```

output
```
(d/(3*a) + (d*(a + b*x^6))/a^2)/(3*(a + b*x^6)^(3/2)) - (d*atanh((a + b*x^
6)^(1/2)/a^(1/2)))/(3*a^(5/2)) - c/(6*a*x^6*(a + b*x^6)^(3/2)) + (5*b*c*at
anh((a + b*x^6)^(1/2)/a^(1/2)))/(6*a^(7/2)) - (10*b*c)/(9*a^2*(a + b*x^6)^
(3/2)) - (5*b^2*c*x^6)/(6*a^3*(a + b*x^6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.66

$$\int \frac{c+dx^6}{x^7 (a+bx^6)^{5/2}}\,dx = \frac{-6\sqrt{b\,x^6+a}\,a^3c+16\sqrt{b\,x^6+a}\,a^3d\,x^6-40\sqrt{b\,x^6+a}\,a^2bc\,x^6+12\sqrt{b\,x^6+a}\,a^2bd\,x}{}$$

input
```
int((d*x^6+c)/x^7/(b*x^6+a)^(5/2),x)
```

output
```
( - 6*sqrt(a + b*x**6)*a**3*c + 16*sqrt(a + b*x**6)*a**3*d*x**6 - 40*sqrt(
a + b*x**6)*a**2*b*c*x**6 + 12*sqrt(a + b*x**6)*a**2*b*d*x**12 - 30*sqrt(a
 + b*x**6)*a*b**2*c*x**12 + 6*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a**3
*d*x**6 - 15*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a**2*b*c*x**6 + 12*sq
rt(a)*log(sqrt(a + b*x**6) - sqrt(a))*a**2*b*d*x**12 - 30*sqrt(a)*log(sqrt
(a + b*x**6) - sqrt(a))*a*b**2*c*x**12 + 6*sqrt(a)*log(sqrt(a + b*x**6) -
sqrt(a))*a*b**2*d*x**18 - 15*sqrt(a)*log(sqrt(a + b*x**6) - sqrt(a))*b**3*
c*x**18 - 6*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a**3*d*x**6 + 15*sqrt(
a)*log(sqrt(a + b*x**6) + sqrt(a))*a**2*b*c*x**6 - 12*sqrt(a)*log(sqrt(a +
 b*x**6) + sqrt(a))*a**2*b*d*x**12 + 30*sqrt(a)*log(sqrt(a + b*x**6) + sqr
t(a))*a*b**2*c*x**12 - 6*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*a*b**2*d*
x**18 + 15*sqrt(a)*log(sqrt(a + b*x**6) + sqrt(a))*b**3*c*x**18)/(36*a**4*
x**6*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.32 $\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{a(bc-ad)x^3}{9b^3\left(a+bx^6\right)^{3/2}}-\frac{(4bc-7ad)x^3}{9b^3\sqrt{a+bx^6}}+\frac{dx^3\sqrt{a+bx^6}}{6b^3}+\frac{(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{6b^{7/2}}$$

output
```
1/9*a*(-a*d+b*c)*x^3/b^3/(b*x^6+a)^(3/2)-1/9*(-7*a*d+4*b*c)*x^3/b^3/(b*x^6
+a)^(1/2)+1/6*d*x^3*(b*x^6+a)^(1/2)/b^3+1/6*(-5*a*d+2*b*c)*arctanh(b^(1/2)
*x^3/(b*x^6+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{x^3\left(-6abc+15a^2d-8b^2cx^6+20abdx^6+3b^2dx^{12}\right)}{18b^3\left(a+bx^6\right)^{3/2}}+\frac{(2bc-5ad)\log\left(\sqrt{b}x^3+\sqrt{a+bx^6}\right)}{6b^{7/2}}$$

input
```
Integrate[(x^14*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^3*(-6*a*b*c + 15*a^2*d - 8*b^2*c*x^6 + 20*a*b*d*x^6 + 3*b^2*d*x^12))/(1
8*b^3*(a + b*x^6)^(3/2)) + ((2*b*c - 5*a*d)*Log[Sqrt[b]*x^3 + Sqrt[a + b*x
^6]])/(6*b^(7/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 807, 252, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2bc-5ad)\int \frac{x^{14}}{(bx^6+a)^{5/2}}dx}{2b}+\frac{dx^{15}}{6b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(2bc-5ad)\int \frac{x^{12}}{(bx^6+a)^{5/2}}dx^3}{6b}+\frac{dx^{15}}{6b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(2bc-5ad)\left(\frac{\int \frac{x^6}{(bx^6+a)^{3/2}}dx^3}{b}-\frac{x^9}{3b(a+bx^6)^{3/2}}\right)}{6b}+\frac{dx^{15}}{6b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(2bc-5ad)\left(\frac{\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^3}{b}-\frac{x^3}{b\sqrt{a+bx^6}}}{b}-\frac{x^9}{3b(a+bx^6)^{3/2}}\right)}{6b}+\frac{dx^{15}}{6b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2bc-5ad)\left(\frac{\frac{\int \frac{1}{1-bx^6}d\frac{x^3}{\sqrt{bx^6+a}}}{b}-\frac{x^3}{b\sqrt{a+bx^6}}}{b}-\frac{x^9}{3b(a+bx^6)^{3/2}}\right)}{6b}+\frac{dx^{15}}{6b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{b^{3/2}}-\frac{x^3}{b\sqrt{a+bx^6}}}{b}-\frac{x^9}{3b(a+bx^6)^{3/2}}\right)(2bc-5ad)}{6b}+\frac{dx^{15}}{6b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^14*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^15)/(6*b*(a + b*x^6)^(3/2)) + ((2*b*c - 5*a*d)*(-1/3*x^9/(b*(a + b*x^
6)^(3/2)) + (-(x^3/(b*Sqrt[a + b*x^6])) + ArcTanh[(Sqrt[b]*x^3)/Sqrt[a + b
*x^6]]/b^(3/2))/b))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 9.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{x^{3} a \left(-10 d \,x^{6}+3 c \right) b^{\frac{3}{2}}+\frac{x^{9} \left(-3 d \,x^{6}+8 c \right) b^{\frac{5}{2}}}{2}-\frac{15 \sqrt{b}\, a^{2} d \,x^{3}}{2}+\frac{3 \left(5 a d -2 c b \right) \left(b \,x^{6}+a \right)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{6}+a}}{x^{3} \sqrt{b}}\right)}{2}}{9 \left(b \,x^{6}+a \right)^{\frac{3}{2}} b^{\frac{7}{2}}}$	101

input
```
int(x^14*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(x^3*a*(-10*d*x^6+3*c)*b^(3/2)+1/2*x^9*(-3*d*x^6+8*c)*b^(5/2)-15/2*b^
(1/2)*a^2*d*x^3+3/2*(5*a*d-2*b*c)*(b*x^6+a)^(3/2)*arctanh((b*x^6+a)^(1/2)/
x^3/b^(1/2)))/(b*x^6+a)^(3/2)/b^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.80

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \left[-\frac{3\,((2\,b^3c-5\,ab^2d)x^{12}+2\,(2\,ab^2c-5\,a^2bd)x^6+2\,a^2bc-5\,a^3d)\sqrt{b}\log\left(-2\,bx^6+2\right.}{36\,(b^6x^{12}}\right.$$

$$-\frac{3\,((2\,b^3c-5\,ab^2d)x^{12}+2\,(2\,ab^2c-5\,a^2bd)x^6+2\,a^2bc-5\,a^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3}{\sqrt{bx^6+a}}\right)-(3\,b^3dx^{15}-4\,(2\,b}{18\,(b^6x^{12}+2\,ab^5x^6+a^2b^4)}$$

input
```
integrate(x^14*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/36*(3*((2*b^3*c - 5*a*b^2*d)*x^12 + 2*(2*a*b^2*c - 5*a^2*b*d)*x^6 + 2*
a^2*b*c - 5*a^3*d)*sqrt(b)*log(-2*b*x^6 + 2*sqrt(b*x^6 + a)*sqrt(b)*x^3 -
a) - 2*(3*b^3*d*x^15 - 4*(2*b^3*c - 5*a*b^2*d)*x^9 - 3*(2*a*b^2*c - 5*a^2*
b*d)*x^3)*sqrt(b*x^6 + a))/(b^6*x^12 + 2*a*b^5*x^6 + a^2*b^4), -1/18*(3*((
2*b^3*c - 5*a*b^2*d)*x^12 + 2*(2*a*b^2*c - 5*a^2*b*d)*x^6 + 2*a^2*b*c - 5*
a^3*d)*sqrt(-b)*arctan(sqrt(-b)*x^3/sqrt(b*x^6 + a)) - (3*b^3*d*x^15 - 4*(
2*b^3*c - 5*a*b^2*d)*x^9 - 3*(2*a*b^2*c - 5*a^2*b*d)*x^3)*sqrt(b*x^6 + a))
/(b^6*x^12 + 2*a*b^5*x^6 + a^2*b^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**14*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(102) = 204.

Time = 0.11 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.68

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{1}{18}\left(\frac{2\left(b+\frac{3\,(bx^6+a)}{x^6}\right)x^9}{(bx^6+a)^{\frac{3}{2}}b^2} + \frac{3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{5}{2}}}\right)c + \frac{1}{36}\,d\left(\frac{2\left(2\,ab^2+\frac{10\,(bx^6+a)ab}{x^6}-\frac{15\,(bx^6+a)^2a}{x^{12}}\right)}{\frac{(bx^6+a)^{\frac{3}{2}}b^4}{x^9}-\frac{(bx^6+a)^{\frac{5}{2}}b^3}{x^{15}}} + \frac{15\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{7}{2}}}\right)$$

input
```
integrate(x^14*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/18*(2*(b + 3*(b*x^6 + a)/x^6)*x^9/((b*x^6 + a)^(3/2)*b^2) + 3*log(-(sqr
t(b) - sqrt(b*x^6 + a)/x^3)/(sqrt(b) + sqrt(b*x^6 + a)/x^3))/b^(5/2))*c +
1/36*d*(2*(2*a*b^2 + 10*(b*x^6 + a)*a*b/x^6 - 15*(b*x^6 + a)^2*a/x^12)/((b
*x^6 + a)^(3/2)*b^4/x^9 - (b*x^6 + a)^(5/2)*b^3/x^15) + 15*a*log(-(sqrt(b)
 - sqrt(b*x^6 + a)/x^3)/(sqrt(b) + sqrt(b*x^6 + a)/x^3))/b^(7/2))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x^14*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb/t_nostep^6) ignored2*((86093442*sage
VARb^10*sageVARd*sageVARa^5*1/1033121304/sageVARb^11/sageVARa^5*sageVARx*s
ageVARx*sageVARx*sa
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^{14}\,\left(d\,x^6+c\right)}{{\left(b\,x^6+a\right)}^{5/2}}\,dx$$

input
```
int((x^14*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^14*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 440, normalized size of antiderivative = 3.61

$$\int \frac{x^{14}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{30\sqrt{b\,x^6+a}\,a^2bd\,x^3-12\sqrt{b\,x^6+a}\,a\,b^2c\,x^3+40\sqrt{b\,x^6+a}\,a\,b^2d\,x^9-16\sqrt{b\,x^6+a}\,b^3c}{}$$

input
```
int(x^14*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(30*sqrt(a + b*x**6)*a**2*b*d*x**3 - 12*sqrt(a + b*x**6)*a*b**2*c*x**3 + 4
0*sqrt(a + b*x**6)*a*b**2*d*x**9 - 16*sqrt(a + b*x**6)*b**3*c*x**9 + 6*sqr
t(a + b*x**6)*b**3*d*x**15 + 15*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**
3)*a**3*d - 6*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a**2*b*c + 30*s
qrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a**2*b*d*x**6 - 12*sqrt(b)*log
(sqrt(a + b*x**6) - sqrt(b)*x**3)*a*b**2*c*x**6 + 15*sqrt(b)*log(sqrt(a +
b*x**6) - sqrt(b)*x**3)*a*b**2*d*x**12 - 6*sqrt(b)*log(sqrt(a + b*x**6) -
sqrt(b)*x**3)*b**3*c*x**12 - 15*sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**
3)*a**3*d + 6*sqrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a**2*b*c - 30*s
qrt(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a**2*b*d*x**6 + 12*sqrt(b)*log
(sqrt(a + b*x**6) + sqrt(b)*x**3)*a*b**2*c*x**6 - 15*sqrt(b)*log(sqrt(a +
b*x**6) + sqrt(b)*x**3)*a*b**2*d*x**12 + 6*sqrt(b)*log(sqrt(a + b*x**6) +
sqrt(b)*x**3)*b**3*c*x**12)/(36*b**4*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.33 $\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^3}{9b^2(a+bx^6)^{3/2}} + \frac{(bc-4ad)x^3}{9ab^2\sqrt{a+bx^6}} + \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{3b^{5/2}}$$

output
```
-1/9*(-a*d+b*c)*x^3/b^2/(b*x^6+a)^(3/2)+1/9*(-4*a*d+b*c)*x^3/a/b^2/(b*x^6+
a)^(1/2)+1/3*d*arctanh(b^(1/2)*x^3/(b*x^6+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-3a^2dx^3+b^2cx^9-4abdx^9}{9ab^2(a+bx^6)^{3/2}} + \frac{d\log\left(\sqrt{b}x^3+\sqrt{a+bx^6}\right)}{3b^{5/2}}$$

input
```
Integrate[(x^8*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(-3*a^2*d*x^3 + b^2*c*x^9 - 4*a*b*d*x^9)/(9*a*b^2*(a + b*x^6)^(3/2)) + (d*
Log[Sqrt[b]*x^3 + Sqrt[a + b*x^6]])/(3*b^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {954, 807, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (c+dx^6)}{(a+bx^6)^{5/2}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int \frac{x^8}{(bx^6+a)^{3/2}}dx}{b}+\frac{x^9 (bc-ad)}{9ab (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{d\int \frac{x^6}{(bx^6+a)^{3/2}}dx^3}{3b}+\frac{x^9 (bc-ad)}{9ab (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^3}{b}-\frac{x^3}{b\sqrt{a+bx^6}}\right)}{3b}+\frac{x^9 (bc-ad)}{9ab (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{d\left(\frac{\int \frac{1}{1-bx^6}d\frac{x^3}{\sqrt{bx^6+a}}}{b}-\frac{x^3}{b\sqrt{a+bx^6}}\right)}{3b}+\frac{x^9 (bc-ad)}{9ab (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^3}{\sqrt{a+bx^6}}\right)}{b^{3/2}}-\frac{x^3}{b\sqrt{a+bx^6}}\right)}{3b}+\frac{x^9 (bc-ad)}{9ab (a+bx^6)^{3/2}}$$

input
```
Int[(x^8*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^9)/(9*a*b*(a + b*x^6)^(3/2)) + (d*(-(x^3/(b*Sqrt[a + b*x^6]
)) + ArcTanh[(Sqrt[b]*x^3)/Sqrt[a + b*x^6]]/b^(3/2)))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{da\ \mathrm{arctanh}\left(\frac{\sqrt{b\,x^{6}+a}}{x^{3}\sqrt{b}}\right)b^{2}\left(b\,x^{6}+a\right)^{\frac{3}{2}}+\frac{b^{\frac{5}{2}}\left(-4abd\,x^{6}+b^{2}c\,x^{6}-3a^{2}d\right)x^{3}}{3}}{3\left(b\,x^{6}+a\right)^{\frac{3}{2}}b^{\frac{9}{2}}a}$	82

input
```
int(x^8*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*a*arctanh((b*x^6+a)^(1/2)/x^3/b^(1/2))*b^2*(b*x^6+a)^(3/2)+1/3*b^(5
/2)*(-4*a*b*d*x^6+b^2*c*x^6-3*a^2*d)*x^3)/(b*x^6+a)^(3/2)/b^(9/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.71

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \left[\frac{3\,(ab^2dx^{12}+2\,a^2bdx^6+a^3d)\sqrt{b}\log\left(-2\,bx^6-2\,\sqrt{bx^6+a}\sqrt{b}x^3-a\right)+2\,((b^3c-4\,a}{18\,(ab^5x^{12}+2\,a^2b^4x^6+a^3b^3)}\right.$$

$$\left.-\frac{3\,(ab^2dx^{12}+2\,a^2bdx^6+a^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3}{\sqrt{bx^6+a}}\right)-((b^3c-4\,ab^2d)x^9-3\,a^2bdx^3)\sqrt{bx^6+a}}{9\,(ab^5x^{12}+2\,a^2b^4x^6+a^3b^3)}\right]$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/18*(3*(a*b^2*d*x^12 + 2*a^2*b*d*x^6 + a^3*d)*sqrt(b)*log(-2*b*x^6 - 2*s
qrt(b*x^6 + a)*sqrt(b)*x^3 - a) + 2*((b^3*c - 4*a*b^2*d)*x^9 - 3*a^2*b*d*x
^3)*sqrt(b*x^6 + a))/(a*b^5*x^12 + 2*a^2*b^4*x^6 + a^3*b^3), -1/9*(3*(a*b^
2*d*x^12 + 2*a^2*b*d*x^6 + a^3*d)*sqrt(-b)*arctan(sqrt(-b)*x^3/sqrt(b*x^6
+ a)) - ((b^3*c - 4*a*b^2*d)*x^9 - 3*a^2*b*d*x^3)*sqrt(b*x^6 + a))/(a*b^5*
x^12 + 2*a^2*b^4*x^6 + a^3*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{cx^9}{9\,(bx^6+a)^{\frac{3}{2}}a} - \frac{1}{18}\left(\frac{2\left(b+\frac{3\,(bx^6+a)}{x^6}\right)x^9}{(bx^6+a)^{\frac{3}{2}}b^2} + \frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^6+a}}{x^3}}{\sqrt{b}+\frac{\sqrt{bx^6+a}}{x^3}}\right)}{b^{\frac{5}{2}}}\right)d$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*c*x^9/((b*x^6 + a)^(3/2)*a) - 1/18*(2*(b + 3*(b*x^6 + a)/x^6)*x^9/((b*
x^6 + a)^(3/2)*b^2) + 3*log(-(sqrt(b) - sqrt(b*x^6 + a)/x^3)/(sqrt(b) + sq
rt(b*x^6 + a)/x^3))/b^(5/2))*d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x^8*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb/t_nostep^6) ignored2*(-(-9565938*sag
eVARb^7*sageVARa^4*sageVARc+38263752*sageVARb^6*sageVARd*sageVARa^5)*1/172
186884/sageVARb^7/s
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{x^8\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^8*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^8*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.63

$$\int \frac{x^8(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{-6\sqrt{b\,x^6+a}\,a^2bd\,x^3-8\sqrt{b\,x^6+a}\,a\,b^2d\,x^9+2\sqrt{b\,x^6+a}\,b^3c\,x^9-3\sqrt{b}\log\left(\sqrt{b\,x^6+a}\right.}{}$$

input
```
int(x^8*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - 6*sqrt(a + b*x**6)*a**2*b*d*x**3 - 8*sqrt(a + b*x**6)*a*b**2*d*x**9 +
2*sqrt(a + b*x**6)*b**3*c*x**9 - 3*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*
x**3)*a**3*d - 6*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a**2*b*d*x**
6 - 3*sqrt(b)*log(sqrt(a + b*x**6) - sqrt(b)*x**3)*a*b**2*d*x**12 + 3*sqrt
(b)*log(sqrt(a + b*x**6) + sqrt(b)*x**3)*a**3*d + 6*sqrt(b)*log(sqrt(a + b
*x**6) + sqrt(b)*x**3)*a**2*b*d*x**6 + 3*sqrt(b)*log(sqrt(a + b*x**6) + sq
rt(b)*x**3)*a*b**2*d*x**12)/(18*a*b**3*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.34 $\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{(bc-ad)x^3}{9ab\,(a+bx^6)^{3/2}}+\frac{(2bc+ad)x^3}{9a^2b\sqrt{a+bx^6}}$$

output
```
1/9*(-a*d+b*c)*x^3/a/b/(b*x^6+a)^(3/2)+1/9*(a*d+2*b*c)*x^3/a^2/b/(b*x^6+a)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{3acx^3+2bcx^9+adx^9}{9a^2\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(3*a*c*x^3 + 2*b*c*x^9 + a*d*x^9)/(9*a^2*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^6)}{(a+bx^6)^{5/2}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(ad+2bc)\int \frac{x^2}{(bx^6+a)^{3/2}}dx}{3ab}+\frac{x^3(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^3(ad+2bc)}{9a^2b\sqrt{a+bx^6}}+\frac{x^3(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

input
```
Int[(x^2*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^3)/(9*a*b*(a + b*x^6)^(3/2)) + ((2*b*c + a*d)*x^3)/(9*a^2*b
*Sqrt[a + b*x^6])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{x^{3}\left(a d \,x^{6}+2 b c \,x^{6}+3 a c \right)}{9 \left(b \,x^{6}+a \right)^{\frac{3}{2}} a^{2}}$	36
trager	$\frac{x^{3}\left(a d \,x^{6}+2 b c \,x^{6}+3 a c \right)}{9 \left(b \,x^{6}+a \right)^{\frac{3}{2}} a^{2}}$	36
pseudoelliptic	$\frac{x^{3}\left(a d \,x^{6}+2 b c \,x^{6}+3 a c \right)}{9 \left(b \,x^{6}+a \right)^{\frac{3}{2}} a^{2}}$	36
orering	$\frac{x^{3}\left(a d \,x^{6}+2 b c \,x^{6}+3 a c \right)}{9 \left(b \,x^{6}+a \right)^{\frac{3}{2}} a^{2}}$	36

input
```
int(x^2*(d*x^6+c)/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^3*(a*d*x^6+2*b*c*x^6+3*a*c)/(b*x^6+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{((2\,bc+ad)x^9+3\,acx^3)\sqrt{bx^6+a}}{9\,(a^2b^2x^{12}+2\,a^3bx^6+a^4)}$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/9*((2*b*c + a*d)*x^9 + 3*a*c*x^3)*sqrt(b*x^6 + a)/(a^2*b^2*x^12 + 2*a^3*
b*x^6 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(56) = 112.

Time = 164.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = c\left(\frac{3ax^3}{9a^{\frac{7}{2}}\sqrt{1+\frac{bx^6}{a}}+9a^{\frac{5}{2}}bx^6\sqrt{1+\frac{bx^6}{a}}} + \frac{2bx^9}{9a^{\frac{7}{2}}\sqrt{1+\frac{bx^6}{a}}+9a^{\frac{5}{2}}bx^6\sqrt{1+\frac{bx^6}{a}}}\right) + \frac{dx^9}{9a^{\frac{5}{2}}\sqrt{1+\frac{bx^6}{a}}+9a^{\frac{3}{2}}bx^6\sqrt{1+\frac{bx^6}{a}}}$$

input
```
integrate(x**2*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
c*(3*a*x**3/(9*a**(7/2)*sqrt(1 + b*x**6/a) + 9*a**(5/2)*b*x**6*sqrt(1 + b*
x**6/a)) + 2*b*x**9/(9*a**(7/2)*sqrt(1 + b*x**6/a) + 9*a**(5/2)*b*x**6*sqr
t(1 + b*x**6/a))) + d*x**9/(9*a**(5/2)*sqrt(1 + b*x**6/a) + 9*a**(3/2)*b*x
**6*sqrt(1 + b*x**6/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{\left(b-\frac{3\,(bx^6+a)}{x^6}\right)cx^9}{9\,(bx^6+a)^{\frac{3}{2}}a^2}+\frac{dx^9}{9\,(bx^6+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*(b - 3*(b*x^6 + a)/x^6)*c*x^9/((b*x^6 + a)^(3/2)*a^2) + 1/9*d*x^9/((b
*x^6 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\left(\frac{(2\,b^2c+abd)x^6}{a^2b}+\frac{3\,c}{a}\right)x^3}{9\,(bx^6+a)^{\frac{3}{2}}}$$

input
```
integrate(x^2*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
1/9*((2*b^2*c + a*b*d)*x^6/(a^2*b) + 3*c/a)*x^3/(b*x^6 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.54

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{3\,a\,c\,x^3+a\,d\,x^9+2\,b\,c\,x^9}{9\,a^2\,(b\,x^6+a)^{3/2}}$$

input
```
int((x^2*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
(3*a*c*x^3 + a*d*x^9 + 2*b*c*x^9)/(9*a^2*(a + b*x^6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^2(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,x^3(ad\,x^6+2bc\,x^6+3ac)}{9a^2\,(b^2x^{12}+2ab\,x^6+a^2)}$$

input
```
int(x^2*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*x**3*(3*a*c + a*d*x**6 + 2*b*c*x**6))/(9*a**2*(a**2 + 2*
a*b*x**6 + b**2*x**12))
```

3.35 $\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{5/2}}\,dx=-\frac{c}{3ax^3\left(a+bx^6\right)^{3/2}}-\frac{(4bc-ad)x^3}{9a^2\left(a+bx^6\right)^{3/2}}-\frac{2(4bc-ad)x^3}{9a^3\sqrt{a+bx^6}}$$

output
```
-1/3*c/a/x^3/(b*x^6+a)^(3/2)-1/9*(-a*d+4*b*c)*x^3/a^2/(b*x^6+a)^(3/2)-2/9*
(-a*d+4*b*c)*x^3/a^3/(b*x^6+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{5/2}}\,dx=\frac{-3a^2c-12abcx^6+3a^2dx^6-8b^2cx^{12}+2abdx^{12}}{9a^3x^3\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^4*(a + b*x^6)^(5/2)),x]
```

output
```
(-3*a^2*c - 12*a*b*c*x^6 + 3*a^2*d*x^6 - 8*b^2*c*x^12 + 2*a*b*d*x^12)/(9*a
^3*x^3*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^4 \left(a+bx^6\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(4bc-ad)\int \frac{x^2}{\left(bx^6+a\right)^{5/2}}dx}{a}-\frac{c}{3ax^3 \left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(4bc-ad)\left(\frac{2b\int \frac{x^8}{\left(bx^6+a\right)^{5/2}}dx}{a}+\frac{x^3}{3a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{c}{3ax^3 \left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(\frac{2bx^9}{9a^2\left(a+bx^6\right)^{3/2}}+\frac{x^3}{3a\left(a+bx^6\right)^{3/2}}\right)(4bc-ad)}{a}-\frac{c}{3ax^3 \left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^4*(a + b*x^6)^(5/2)),x]
```

output
```
-1/3*c/(a*x^3*(a + b*x^6)^(3/2)) - ((4*b*c - a*d)*(x^3/(3*a*(a + b*x^6)^(3
/2)) + (2*b*x^9)/(9*a^2*(a + b*x^6)^(3/2))))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{(3d\,x^{6}-3c)a^{2}-12b\,x^{6}\left(-\frac{d\,x^{6}}{6}+c\right)a-8b^{2}c\,x^{12}}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{3}a^{3}}$	57
gosper	$-\frac{-2abd\,x^{12}+8b^{2}c\,x^{12}-3a^{2}d\,x^{6}+12abc\,x^{6}+3a^{2}c}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{3}a^{3}}$	59
trager	$-\frac{-2abd\,x^{12}+8b^{2}c\,x^{12}-3a^{2}d\,x^{6}+12abc\,x^{6}+3a^{2}c}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{3}a^{3}}$	59
orering	$-\frac{-2abd\,x^{12}+8b^{2}c\,x^{12}-3a^{2}d\,x^{6}+12abc\,x^{6}+3a^{2}c}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{3}a^{3}}$	59
risch	$-\frac{c\sqrt{b\,x^{6}+a}}{3a^{3}x^{3}}+\frac{\sqrt{b\,x^{6}+a}\,x^{3}\left(2abd\,x^{6}-5b^{2}c\,x^{6}+3a^{2}d-6abc\right)}{9a^{3}\left(x^{12}b^{2}+2a\,x^{6}b+a^{2}\right)}$	86

input
```
int((d*x^6+c)/x^4/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*((3*d*x^6-3*c)*a^2-12*b*x^6*(-1/6*d*x^6+c)*a-8*b^2*c*x^12)/(b*x^6+a)^(
3/2)/x^3/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^6}{x^4(a+bx^6)^{5/2}}\,dx = -\frac{(2\,(4\,b^2c-abd)x^{12}+3\,(4\,abc-a^2d)x^6+3\,a^2c)\sqrt{bx^6+a}}{9\,(a^3b^2x^{15}+2\,a^4bx^9+a^5x^3)}$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/9*(2*(4*b^2*c - a*b*d)*x^12 + 3*(4*a*b*c - a^2*d)*x^6 + 3*a^2*c)*sqrt(b
*x^6 + a)/(a^3*b^2*x^15 + 2*a^4*b*x^9 + a^5*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^4(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**4/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{5/2}}\,dx=-\frac{\left(b-\frac{3\,(bx^6+a)}{x^6}\right)dx^9}{9\,(bx^6+a)^{\frac{3}{2}}a^2}+\frac{1}{9}\left(\frac{\left(b^2-\frac{6\,(bx^6+a)b}{x^6}\right)x^9}{(bx^6+a)^{\frac{3}{2}}a^3}-\frac{3\,\sqrt{bx^6+a}}{a^3x^3}\right)c$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*(b - 3*(b*x^6 + a)/x^6)*d*x^9/((b*x^6 + a)^(3/2)*a^2) + 1/9*((b^2 - 6
*(b*x^6 + a)*b/x^6)*x^9/((b*x^6 + a)^(3/2)*a^3) - 3*sqrt(b*x^6 + a)/(a^3*x
^3))*c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+dx^6}{x^4\left(a+bx^6\right)^{5/2}}\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate((d*x^6+c)/x^4/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb/t_nostep^6) ignored2*(-(47829690*sag
eVARb^7*sageVARa^6*sageVARc-19131876*sageVARb^6*sageVARa^7*sageVARd)*1/172
186884/sageVARb^5/s
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^6}{x^4 (a+bx^6)^{5/2}} \, dx = \frac{a^2 c - 8 c (b x^6 + a)^2 + a^2 d x^6 + 4 a c (b x^6 + a) + 2 a d x^6 (b x^6 + a)}{9 a^3 x^3 (b x^6 + a)^{3/2}}$$

input
```
int((c + d*x^6)/(x^4*(a + b*x^6)^(5/2)),x)
```

output
```
(a^2*c - 8*c*(a + b*x^6)^2 + a^2*d*x^6 + 4*a*c*(a + b*x^6) + 2*a*d*x^6*(a
+ b*x^6))/(9*a^3*x^3*(a + b*x^6)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^6}{x^4 (a+bx^6)^{5/2}} \, dx = \frac{\sqrt{b x^6 + a} \, (2abd \, x^{12} - 8b^2 c \, x^{12} + 3a^2 d \, x^6 - 12abc \, x^6 - 3a^2 c)}{9a^3 x^3 (b^2 x^{12} + 2ab \, x^6 + a^2)}$$

input
```
int((d*x^6+c)/x^4/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*( - 3*a**2*c + 3*a**2*d*x**6 - 12*a*b*c*x**6 + 2*a*b*d*x
**12 - 8*b**2*c*x**12))/(9*a**3*x**3*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.36 $\int \frac{c+dx^6}{x^{10}(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = -\frac{c}{9ax^9\left(a+bx^6\right)^{3/2}} - \frac{2bc-ad}{9a^2x^3\left(a+bx^6\right)^{3/2}} - \frac{4(2bc-ad)}{9a^3x^3\sqrt{a+bx^6}} + \frac{8(2bc-ad)\sqrt{a+bx^6}}{9a^4x^3}$$

output
```
-1/9*c/a/x^9/(b*x^6+a)^(3/2)-1/9*(-a*d+2*b*c)/a^2/x^3/(b*x^6+a)^(3/2)-4/9*
(-a*d+2*b*c)/a^3/x^3/(b*x^6+a)^(1/2)+8/9*(-a*d+2*b*c)*(b*x^6+a)^(1/2)/a^4/
x^3
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \frac{16b^3cx^{18}+6a^2bx^6(c-2dx^6)-8ab^2x^{12}(-3c+dx^6)-a^3(c+3dx^6)}{9a^4x^9\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^10*(a + b*x^6)^(5/2)),x]
```

output
```
(16*b^3*c*x^18 + 6*a^2*b*x^6*(c - 2*d*x^6) - 8*a*b^2*x^12*(-3*c + d*x^6) -
 a^3*(c + 3*d*x^6))/(9*a^4*x^9*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(2bc-ad)\int \frac{1}{x^4(bx^6+a)^{5/2}}dx}{a}-\frac{c}{9ax^9\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(2bc-ad)\left(-\frac{4b\int \frac{x^2}{\left(bx^6+a\right)^{5/2}}dx}{a}-\frac{1}{3ax^3(a+bx^6)^{3/2}}\right)}{a}-\frac{c}{9ax^9\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{(2bc-ad)\left(-\frac{4b\left(\frac{2b\int \frac{x^8}{\left(bx^6+a\right)^{5/2}}dx}{a}+\frac{x^3}{3a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^6)^{3/2}}\right)}{a}-\frac{c}{9ax^9\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(-\frac{4b\left(\frac{2bx^9}{9a^2\left(a+bx^6\right)^{3/2}}+\frac{x^3}{3a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^6)^{3/2}}\right)(2bc-ad)}{a}-\frac{c}{9ax^9\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^10*(a + b*x^6)^(5/2)),x]
```

output
```
-1/9*c/(a*x^9*(a + b*x^6)^(3/2)) - ((2*b*c - a*d)*(-1/3*1/(a*x^3*(a + b*x^
6)^(3/2)) - (4*b*(x^3/(3*a*(a + b*x^6)^(3/2)) + (2*b*x^9)/(9*a^2*(a + b*x^
6)^(3/2))))/a))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{(3d\,x^{6}+c)a^{3}-6b\,x^{6}(-2d\,x^{6}+c)a^{2}-24\left(-\frac{d\,x^{6}}{3}+c\right)b^{2}x^{12}a-16b^{3}c\,x^{18}}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{9}a^{4}}$	74
gosper	$-\frac{8a\,b^{2}d\,x^{18}-16b^{3}c\,x^{18}+12a^{2}bd\,x^{12}-24a\,b^{2}c\,x^{12}+3a^{3}d\,x^{6}-6a^{2}bc\,x^{6}+c\,a^{3}}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{9}a^{4}}$	82
trager	$-\frac{8a\,b^{2}d\,x^{18}-16b^{3}c\,x^{18}+12a^{2}bd\,x^{12}-24a\,b^{2}c\,x^{12}+3a^{3}d\,x^{6}-6a^{2}bc\,x^{6}+c\,a^{3}}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{9}a^{4}}$	82
orering	$-\frac{8a\,b^{2}d\,x^{18}-16b^{3}c\,x^{18}+12a^{2}bd\,x^{12}-24a\,b^{2}c\,x^{12}+3a^{3}d\,x^{6}-6a^{2}bc\,x^{6}+c\,a^{3}}{9(b\,x^{6}+a)^{\frac{3}{2}}x^{9}a^{4}}$	82
risch	$-\frac{\sqrt{b\,x^{6}+a}\,(3ad\,x^{6}-8bc\,x^{6}+ac)}{9a^{4}x^{9}}-\frac{\sqrt{b\,x^{6}+a}\,x^{3}(5abd\,x^{6}-8b^{2}c\,x^{6}+6a^{2}d-9abc)b}{9a^{4}(x^{12}b^{2}+2a\,x^{6}b+a^{2})}$	104

input
```
int((d*x^6+c)/x^10/(b*x^6+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9/(b*x^6+a)^(3/2)*((3*d*x^6+c)*a^3-6*b*x^6*(-2*d*x^6+c)*a^2-24*(-1/3*d*
x^6+c)*b^2*x^12*a-16*b^3*c*x^18)/x^9/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \frac{(8\,(2\,b^3c-ab^2d)x^{18}+12\,(2\,ab^2c-a^2bd)x^{12}+3\,(2\,a^2bc-a^3d)x^6-a^3c)\sqrt{bx^6+a}}{9\,(a^4b^2x^{21}+2\,a^5bx^{15}+a^6x^9)}$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(8*(2*b^3*c - a*b^2*d)*x^18 + 12*(2*a*b^2*c - a^2*b*d)*x^12 + 3*(2*a^2
*b*c - a^3*d)*x^6 - a^3*c)*sqrt(b*x^6 + a)/(a^4*b^2*x^21 + 2*a^5*b*x^15 +
a^6*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**10/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = -\frac{1}{9}\left(\frac{\left(b^3-\frac{9\,(bx^6+a)b^2}{x^6}\right)x^9}{\left(bx^6+a\right)^{\frac{3}{2}}a^4}-\frac{\frac{9\,\sqrt{bx^6+a}b}{x^3}-\frac{(bx^6+a)^{\frac{3}{2}}}{x^9}}{a^4}\right)c + \frac{1}{9}\left(\frac{\left(b^2-\frac{6\,(bx^6+a)b}{x^6}\right)x^9}{\left(bx^6+a\right)^{\frac{3}{2}}a^3}-\frac{3\,\sqrt{bx^6+a}}{a^3x^3}\right)d$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/9*((b^3 - 9*(b*x^6 + a)*b^2/x^6)*x^9/((b*x^6 + a)^(3/2)*a^4) - (9*sqrt(
b*x^6 + a)*b/x^3 - (b*x^6 + a)^(3/2)/x^9)/a^4)*c + 1/9*((b^2 - 6*(b*x^6 +
a)*b/x^6)*x^9/((b*x^6 + a)^(3/2)*a^3) - 3*sqrt(b*x^6 + a)/(a^3*x^3))*d
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((d*x^6+c)/x^10/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb/t_nostep^6) ignored2*(-(-76527504*sa
geVARb^8*sageVARa^7*sageVARc+47829690*sageVARb^7*sageVARa^8*sageVARd)*1/17
2186884/sageVARb^5/
```

Mupad [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \frac{x^3\left(\frac{b\,(2\,a\,d-3\,b\,c)}{2\,a^3}-\frac{a\left(\frac{2\,b^3\,c-a\,b^2\,d}{9\,a^4}+\frac{11\,b^2\,(2\,a\,d-3\,b\,c)}{18\,a^4}\right)}{b}\right)}{\left(b\,x^6+a\right)^{3/2}} - \frac{c\,\sqrt{b\,x^6+a}}{9\,a^3\,x^9} + \frac{x^3\,\left(8\,b^2\,c-5\,a\,b\,d\right)}{9\,a^4\,\sqrt{b\,x^6+a}} - \frac{\sqrt{b\,x^6+a}\,\left(3\,a^3\,d-8\,a^2\,b\,c\right)}{9\,a^6\,x^3}$$

input
```
int((c + d*x^6)/(x^10*(a + b*x^6)^(5/2)),x)
```

output
```
(x^3*((b*(2*a*d - 3*b*c))/(2*a^3) - (a*((2*b^3*c - a*b^2*d)/(9*a^4) + (11*
b^2*(2*a*d - 3*b*c))/(18*a^4)))/b))/(a + b*x^6)^(3/2) - (c*(a + b*x^6)^(1/
2))/(9*a^3*x^9) + (x^3*(8*b^2*c - 5*a*b*d))/(9*a^4*(a + b*x^6)^(1/2)) - ((
a + b*x^6)^(1/2)*(3*a^3*d - 8*a^2*b*c))/(9*a^6*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^6}{x^{10}\left(a+bx^6\right)^{5/2}}\,dx = \frac{\sqrt{b\,x^6+a}\,(-8a\,b^2d\,x^{18}+16b^3c\,x^{18}-12a^2bd\,x^{12}+24a\,b^2c\,x^{12}-3a^3d\,x^6+6a^2bc\,x}{9a^4x^9\left(b^2x^{12}+2ab\,x^6+a^2\right)}$$

input
```
int((d*x^6+c)/x^10/(b*x^6+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**6)*( - a**3*c - 3*a**3*d*x**6 + 6*a**2*b*c*x**6 - 12*a**2*b
*d*x**12 + 24*a*b**2*c*x**12 - 8*a*b**2*d*x**18 + 16*b**3*c*x**18))/(9*a**
4*x**9*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.37 $\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 315

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}} \, dx = \frac{a(bc-ad)x^2}{9b^3(a+bx^6)^{3/2}} - \frac{(11bc-20ad)x^2}{27b^3\sqrt{a+bx^6}} + \frac{dx^2\sqrt{a+bx^6}}{5b^3} + \frac{16\sqrt{2+\sqrt{3}}(5bc-14ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-\right.}{135\sqrt[4]{3}b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*a*(-a*d+b*c)*x^2/b^3/(b*x^6+a)^(3/2)-1/27*(-20*a*d+11*b*c)*x^2/b^3/(b*
x^6+a)^(1/2)+1/5*d*x^2*(b*x^6+a)^(1/2)/b^3+16/405*(1/2*6^(1/2)+1/2*2^(1/2)
)*(-14*a*d+5*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2
/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))
*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(
3/4)/b^(10/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.35

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^2\left(112a^2d + b^2x^6(-55c+27dx^6) + ab(-40c+154dx^6) + 8(5bc-14ad)\,(a+bx^6)\right.}{135b^3\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^13*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^2*(112*a^2*d + b^2*x^6*(-55*c + 27*d*x^6) + a*b*(-40*c + 154*d*x^6) + 8
*(5*b*c - 14*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^6)/a)]))/(135*b^3*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 807, 817, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(5bc-14ad)\int \frac{x^{13}}{(bx^6+a)^{5/2}}dx}{5b} + \frac{dx^{14}}{5b\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{(5bc-14ad)\int \frac{x^{12}}{(bx^6+a)^{5/2}}dx^2}{10b} + \frac{dx^{14}}{5b\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(5bc-14ad)\left(\frac{8\int \frac{x^6}{\left(bx^6+a\right)^{3/2}}dx^2}{9b}-\frac{2x^8}{9b(a+bx^6)^{3/2}}\right)}{10b}+\frac{dx^{14}}{5b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(5bc-14ad)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{2x^8}{9b(a+bx^6)^{3/2}}\right)}{10b}+\frac{dx^{14}}{5b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(5bc-14ad)\left(\frac{8\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{}{9}\right)}{10b}$$

$$\frac{dx^{14}}{5b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^13*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^14)/(5*b*(a + b*x^6)^(3/2)) + ((5*b*c - 14*a*d)*((-2*x^8)/(9*b*(a + b
*x^6)^(3/2)) + (8*((-2*x^2)/(3*b*Sqrt[a + b*x^6]) + (4*Sqrt[2 + Sqrt[3]]*(
a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[
3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(9*b)))/(10*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{13}(d\,x^6+c)}{(b\,x^6+a)^{\frac{5}{2}}}dx$$

input
```
int(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.50

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{16\left((5\,b^3c-14\,ab^2d)x^{12}+2\,(5\,ab^2c-14\,a^2bd)x^6+5\,a^2bc-14\,a^3d\right)\sqrt{b}\text{weierstrassPIn}}{135\,(b^6x^{12}}$$

input
```
integrate(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/135*(16*((5*b^3*c - 14*a*b^2*d)*x^12 + 2*(5*a*b^2*c - 14*a^2*b*d)*x^6 +
5*a^2*b*c - 14*a^3*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x^2) + (27*b^
3*d*x^14 - 11*(5*b^3*c - 14*a*b^2*d)*x^8 - 8*(5*a*b^2*c - 14*a^2*b*d)*x^2)
*sqrt(b*x^6 + a))/(b^6*x^12 + 2*a*b^5*x^6 + a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**13*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{13}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^13/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{13}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^13/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^{13}\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^13*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^13*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{13}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{112\sqrt{b\,x^6+a}\,a^2d\,x^2-40\sqrt{b\,x^6+a}\,abc\,x^2+98\sqrt{b\,x^6+a}\,abd\,x^8-35\sqrt{b\,x^6+a}\,b^2c\,x^8}{}$$

input
```
int(x^13*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(112*sqrt(a + b*x**6)*a**2*d*x**2 - 40*sqrt(a + b*x**6)*a*b*c*x**2 + 98*sq
rt(a + b*x**6)*a*b*d*x**8 - 35*sqrt(a + b*x**6)*b**2*c*x**8 + 7*sqrt(a + b
*x**6)*b**2*d*x**14 - 224*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 +
 3*a*b**2*x**12 + b**3*x**18),x)*a**5*d + 80*int((sqrt(a + b*x**6)*x)/(a**
3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**4*b*c - 448*int((sq
rt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*
a**4*b*d*x**6 + 160*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b
**2*x**12 + b**3*x**18),x)*a**3*b**2*c*x**6 - 224*int((sqrt(a + b*x**6)*x)
/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b**2*d*x**12
 + 80*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b*
*3*x**18),x)*a**2*b**3*c*x**12)/(35*b**3*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.38 $\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^2}{9b^2(a+bx^6)^{3/2}} + \frac{(2bc-11ad)x^2}{27ab^2\sqrt{a+bx^6}}$$
$$+ \frac{2\sqrt{2+\sqrt{3}}(bc+8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-\right.}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output

```
-1/9*(-a*d+b*c)*x^2/b^2/(b*x^6+a)^(3/2)+1/27*(-11*a*d+2*b*c)*x^2/a/b^2/(b*
x^6+a)^(1/2)+2/81*(1/2*6^(1/2)+1/2*2^(1/2))*(8*a*d+b*c)*(a^(1/3)+b^(1/3)*x
^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.34

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^2\left(-8a^2d+2b^2cx^6-ab(c+11dx^6)+(bc+8ad)\,(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometri}\right.}{27ab^2\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^7*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^2*(-8*a^2*d + 2*b^2*c*x^6 - a*b*(c + 11*d*x^6) + (b*c + 8*a*d)*(a + b*x
^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^6)/a)]))/(
27*a*b^2*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {957, 807, 817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(8ad+bc)\int \frac{x^7}{(bx^6+a)^{3/2}}dx}{9ab}+\frac{x^8(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{(8ad+bc)\int \frac{x^6}{(bx^6+a)^{3/2}}dx^2}{18ab}+\frac{x^8(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(8ad+bc)\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{18ab}+\frac{x^8(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{(8ad+bc)\left(\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{2x^2}{3b\sqrt{a+bx^6}}\right)}{18ab}$$
$$\frac{x^8(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^7*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^8)/(9*a*b*(a + b*x^6)^(3/2)) + ((b*c + 8*a*d)*((-2*x^2)/(3*
b*Sqrt[a + b*x^6]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*S
qrt[a + b*x^6])))/(18*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{7}(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{5}{2}}}dx$$

input
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.49

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{2\left((b^3c+8\,ab^2d)x^{12}+2\,(ab^2c+8\,a^2bd)x^6+a^2bc+8\,a^3d\right)\sqrt{b}\text{weierstrassPInverse}(0,-}{27\,(ab^5x^{12}+2\,a^2b^4x^6+a^3b}$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(2*((b^3*c + 8*a*b^2*d)*x^12 + 2*(a*b^2*c + 8*a^2*b*d)*x^6 + a^2*b*c
+ 8*a^3*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x^2) + ((2*b^3*c - 11*a*
b^2*d)*x^8 - (a*b^2*c + 8*a^2*b*d)*x^2)*sqrt(b*x^6 + a))/(a*b^5*x^12 + 2*a
^2*b^4*x^6 + a^3*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**7*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^7}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^7/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^7}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^7/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^7\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^7*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^7*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^7(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-8\sqrt{b\,x^6+a}\,ad\,x^2-\sqrt{b\,x^6+a}\,bc\,x^2-7\sqrt{b\,x^6+a}\,bd\,x^8+16\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a}\right.}{}$$

input
```
int(x^7*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - 8*sqrt(a + b*x**6)*a*d*x**2 - sqrt(a + b*x**6)*b*c*x**2 - 7*sqrt(a + b
*x**6)*b*d*x**8 + 16*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*
b**2*x**12 + b**3*x**18),x)*a**4*d + 2*int((sqrt(a + b*x**6)*x)/(a**3 + 3*
a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*c + 32*int((sqrt(a +
b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*
d*x**6 + 4*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12
 + b**3*x**18),x)*a**2*b**2*c*x**6 + 16*int((sqrt(a + b*x**6)*x)/(a**3 + 3
*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**2*d*x**12 + 2*int((
sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x
)*a*b**3*c*x**12)/(7*b**2*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.39 $\int \frac{x\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 301

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{(bc-ad)x^2}{9ab\,(a+bx^6)^{3/2}} + \frac{(7bc+2ad)x^2}{27a^2b\sqrt{a+bx^6}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}(7bc+2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-\cdots}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*(-a*d+b*c)*x^2/a/b/(b*x^6+a)^(3/2)+1/27*(2*a*d+7*b*c)*x^2/a^2/b/(b*x^6
+a)^(1/2)+1/81*(1/2*6^(1/2)+1/2*2^(1/2))*(2*a*d+7*b*c)*(a^(1/3)+b^(1/3)*x^
2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a^2/b^(4/3)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.34

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^2\left(-2a^2d+14b^2cx^6+4ab(5c+dx^6)+(7bc+2ad)\,(a+bx^6)\,\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeome}\right.}{54a^2b\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^2*(-2*a^2*d + 14*b^2*c*x^6 + 4*a*b*(5*c + d*x^6) + (7*b*c + 2*a*d)*(a +
 b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^6)/a)]
))/(54*a^2*b*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {957, 807, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(2ad+7bc)\int \frac{x}{(bx^6+a)^{3/2}}dx}{9ab}+\frac{x^2(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{(2ad+7bc)\int \frac{1}{(bx^6+a)^{3/2}}dx^2}{18ab}+\frac{x^2(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 749$$

$$\frac{(2ad+7bc)\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3a}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{18ab}+\frac{x^2(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(2ad+7bc)\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{18ab}$$

$$\frac{x^2(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^2)/(9*a*b*(a + b*x^6)^(3/2)) + ((7*b*c + 2*a*d)*((2*x^2)/(3
*a*Sqrt[a + b*x^6]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2
]*Sqrt[a + b*x^6])))/(18*a*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{5}{2}}}dx$$

input
```
int(x*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.50

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{((7\,b^3c+2\,ab^2d)x^{12}+2\,(7\,ab^2c+2\,a^2bd)x^6+7\,a^2bc+2\,a^3d)\sqrt{b}\text{weierstrassPInverse}(\ldots}{27\,(a^2b^4x^{12}+2\,a^3b^3x^6+\ldots}$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(((7*b^3*c + 2*a*b^2*d)*x^12 + 2*(7*a*b^2*c + 2*a^2*b*d)*x^6 + 7*a^2*
b*c + 2*a^3*d)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x^2) + ((7*b^3*c + 2
*a*b^2*d)*x^8 + (10*a*b^2*c - a^2*b*d)*x^2)*sqrt(b*x^6 + a))/(a^2*b^4*x^12
 + 2*a^3*b^3*x^6 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 101.90 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{cx^2\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^8\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\right|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
c*x**2*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(5/2)*gamma(4/3)) + d*x**8*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**6*e
xp_polar(I*pi)/a)/(6*a**(5/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d\,x^2+2\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right)a^3d+7\left(\int \frac{\sqrt{b\,x^6+a}\,x}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right.}{}$$

input
```
int(x*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x**2 + 2*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*
x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*d + 7*int((sqrt(a + b*x**6)*x)
/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b*c + 4*int(
(sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),
x)*a**2*b*d*x**6 + 14*int((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a
*b**2*x**12 + b**3*x**18),x)*a*b**2*c*x**6 + 2*int((sqrt(a + b*x**6)*x)/(a
**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**2*d*x**12 + 7*i
nt((sqrt(a + b*x**6)*x)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**1
8),x)*b**3*c*x**12)/(7*b*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.40 $\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 318

$$\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{5/2}}\,dx=-\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}-\frac{(13bc-4ad)x^2}{36a^2\left(a+bx^6\right)^{3/2}}-\frac{7(13bc-4ad)x^2}{108a^3\sqrt{a+bx^6}}$$
$$-\frac{7\sqrt{2+\sqrt{3}}(13bc-4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-\right.}{108\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/4*c/a/x^4/(b*x^6+a)^(3/2)-1/36*(-4*a*d+13*b*c)*x^2/a^2/(b*x^6+a)^(3/2)-
7/108*(-4*a*d+13*b*c)*x^2/a^3/(b*x^6+a)^(1/2)-7/324*(1/2*6^(1/2)+1/2*2^(1/
2))*(-4*a*d+13*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^
(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3
^(3/4)/a^3/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.36

$$\int \frac{c+dx^6}{x^5 \left(a+bx^6\right)^{5/2}}\,dx = \frac{-182b^2cx^{12}+4abx^6(-65c+14dx^6)+a^2(-54c+80dx^6)+7(-13bc+4ad)x^6(a+}{216a^3x^4\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^5*(a + b*x^6)^(5/2)),x]
```

output
```
(-182*b^2*c*x^12 + 4*a*b*x^6*(-65*c + 14*d*x^6) + a^2*(-54*c + 80*d*x^6) +
 7*(-13*b*c + 4*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((b*x^6)/a)])/(216*a^3*x^4*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 807, 749, 749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^5 \left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(13bc-4ad)\int \frac{x}{\left(bx^6+a\right)^{5/2}}dx}{4a}-\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(13bc-4ad)\int \frac{1}{\left(bx^6+a\right)^{5/2}}dx^2}{8a}-\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(13bc-4ad)\left(\frac{7\int \frac{1}{\left(bx^6+a\right)^{3/2}}dx^2}{9a}+\frac{2x^2}{9a(a+bx^6)^{3/2}}\right)}{8a}-\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}$$

$\Big\downarrow$ 749

$$-\frac{(13bc-4ad)\left(\frac{7\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx^2}{3a}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{9a}+\frac{2x^2}{9a(a+bx^6)^{3/2}}\right)}{8a}-\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}$$

$\Big\downarrow$ 759

$$-\frac{(13bc-4ad)\left(\frac{7\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{2x^2}{3a\sqrt{a+bx^6}}\right)}{9a}+\cdots\right)}{8a}$$

$$\frac{c}{4ax^4\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^5*(a + b*x^6)^(5/2)),x]
```

output
```
-1/4*c/(a*x^4*(a + b*x^6)^(3/2)) - ((13*b*c - 4*a*d)*((2*x^2)/(9*a*(a + b*
x^6)^(3/2)) + (7*((2*x^2)/(3*a*Sqrt[a + b*x^6]) + (2*Sqrt[2 + Sqrt[3]]*(a^
(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]
])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(9*a)))/(8*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{5} \left(b\,x^{6}+a \right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.52

$$\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{5/2}}\,dx =$$
$$-\frac{7\left((13\,b^3c-4\,ab^2d)x^{16}+2\,(13\,ab^2c-4\,a^2bd)x^{10}+(13\,a^2bc-4\,a^3d)x^4\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x^2\right.}{108\left(a^3b^3x^{16}+2\,a^4b^2x^{10}+a^5bx^4\right.}$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/108*(7*((13*b^3*c - 4*a*b^2*d)*x^16 + 2*(13*a*b^2*c - 4*a^2*b*d)*x^10 +
 (13*a^2*b*c - 4*a^3*d)*x^4)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x^2) +
 (7*(13*b^3*c - 4*a*b^2*d)*x^12 + 10*(13*a*b^2*c - 4*a^2*b*d)*x^6 + 27*a^2
*b*c)*sqrt(b*x^6 + a))/(a^3*b^3*x^16 + 2*a^4*b^2*x^10 + a^5*b*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**5/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^5\left(a+bx^6\right)^{5/2}}\,dx = \int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^5), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^5}\,dx$$

input
```
integrate((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{5/2}}\,dx = \int \frac{d\,x^6+c}{x^5\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^5*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^5*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^5 (a+bx^6)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-4\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{23}+3a\,b^2x^{17}+3a^2b\,x^{11}+a^3x^5}dx\right)a^3d\,x^4+13\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{23}+3a\,b^2x^{17}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^6+c)/x^5/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 4*int(sqrt(a + b*x**6)/(a**3*x**5 + 3*a**2*b*x**1
1 + 3*a*b**2*x**17 + b**3*x**23),x)*a**3*d*x**4 + 13*int(sqrt(a + b*x**6)/
(a**3*x**5 + 3*a**2*b*x**11 + 3*a*b**2*x**17 + b**3*x**23),x)*a**2*b*c*x**
4 - 8*int(sqrt(a + b*x**6)/(a**3*x**5 + 3*a**2*b*x**11 + 3*a*b**2*x**17 +
b**3*x**23),x)*a**2*b*d*x**10 + 26*int(sqrt(a + b*x**6)/(a**3*x**5 + 3*a**
2*b*x**11 + 3*a*b**2*x**17 + b**3*x**23),x)*a*b**2*c*x**10 - 4*int(sqrt(a
+ b*x**6)/(a**3*x**5 + 3*a**2*b*x**11 + 3*a*b**2*x**17 + b**3*x**23),x)*a*
b**2*d*x**16 + 13*int(sqrt(a + b*x**6)/(a**3*x**5 + 3*a**2*b*x**11 + 3*a*b
**2*x**17 + b**3*x**23),x)*b**3*c*x**16)/(13*b*x**4*(a**2 + 2*a*b*x**6 + b
**2*x**12))
```

3.41 $\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 348

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{5/2}}\,dx = -\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}-\frac{19bc-10ad}{90a^2x^4\left(a+bx^6\right)^{3/2}}-\frac{13(19bc-10ad)}{270a^3x^4\sqrt{a+bx^6}}+\frac{91(19bc-10ad)\sqrt{a+bx^6}}{1080a^4x^4}+\frac{91\sqrt{2+\sqrt{3}}b^{2/3}(19bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)\right.}{1080\sqrt[4]{3}a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output

```
-1/10*c/a/x^10/(b*x^6+a)^(3/2)-1/90*(-10*a*d+19*b*c)/a^2/x^4/(b*x^6+a)^(3/
2)-13/270*(-10*a*d+19*b*c)/a^3/x^4/(b*x^6+a)^(1/2)+91/1080*(-10*a*d+19*b*c
)*(b*x^6+a)^(1/2)/a^4/x^4+91/3240*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(-10*a
*d+19*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3
)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/a^
4/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2
)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.23

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{5/2}}\,dx = \frac{-4a^2c+(19bc-10ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{2},\frac{1}{3},-\frac{bx^6}{a}\right)}{40a^3x^{10}\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^11*(a + b*x^6)^(5/2)),x]
```

output
```
(-4*a^2*c + (19*b*c - 10*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeo
metric2F1[-2/3, 5/2, 1/3, -((b*x^6)/a)])/(40*a^3*x^10*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {955, 807, 819, 819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(19bc-10ad)\int \frac{1}{x^5\left(bx^6+a\right)^{5/2}}dx}{10a}-\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(19bc-10ad)\int \frac{1}{x^6\left(bx^6+a\right)^{5/2}}dx^2}{20a}-\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(19bc-10ad)\left(\frac{13\int \frac{1}{x^6\left(bx^6+a\right)^{3/2}}dx^2}{9a}+\frac{2}{9ax^4(a+bx^6)^{3/2}}\right)}{20a}-\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 819

$$-\frac{(19bc-10ad)\left(\frac{13\left(\frac{7\int \frac{1}{x^6\sqrt{bx^6+a}}dx^2}{3a}+\frac{2}{3ax^4\sqrt{a+bx^6}}\right)}{9a}+\frac{2}{9ax^4(a+bx^6)^{3/2}}\right)}{20a}-\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 847

$$-\frac{(19bc-10ad)\left(\frac{13\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^6+a}}dx^2}{4a}-\frac{\sqrt{a+bx^6}}{2ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^6}}\right)}{9a}+\frac{2}{9ax^4(a+bx^6)^{3/2}}\right)}{20a}-\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 759

$$-\frac{(19bc-10ad)\left(\frac{13\left(\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{\sqrt{a+}}{2ax}}{3a}}{9a}}{20a}$$

$$\frac{c}{10ax^{10}\left(a+bx^6\right)^{3/2}}$$

input

```
Int[(c + d*x^6)/(x^11*(a + b*x^6)^(5/2)),x]
```

output

```
-1/10*c/(a*x^10*(a + b*x^6)^(3/2)) - ((19*b*c - 10*a*d)*(2/(9*a*x^4*(a + b
*x^6)^(3/2)) + (13*(2/(3*a*x^4*Sqrt[a + b*x^6]) + (7*(-1/2*Sqrt[a + b*x^6]
/(a*x^4) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^
2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6])))/(3*a)))/(9*a)))/(20*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{11} \left(b\,x^{6}+a \right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

$$\int \frac{c+dx^6}{x^{11} (a+bx^6)^{5/2}}\, dx = \frac{91\,((19\,b^3c-10\,ab^2d)x^{22}+2\,(19\,ab^2c-10\,a^2bd)x^{16}+(19\,a^2bc-10\,a^3d)x^{10})\sqrt{b}\text{we}}{}$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/1080*(91*((19*b^3*c - 10*a*b^2*d)*x^22 + 2*(19*a*b^2*c - 10*a^2*b*d)*x^1
6 + (19*a^2*b*c - 10*a^3*d)*x^10)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x
^2) + (91*(19*b^3*c - 10*a*b^2*d)*x^18 + 130*(19*a*b^2*c - 10*a^2*b*d)*x^1
2 + 27*(19*a^2*b*c - 10*a^3*d)*x^6 - 108*a^3*c)*sqrt(b*x^6 + a))/(a^4*b^2*
x^22 + 2*a^5*b*x^16 + a^6*x^10)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{11} (a+bx^6)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**11/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^{11}(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^{11}}\,dx$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^11), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^{11}(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^{11}}\,dx$$

input
```
integrate((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{11}(a+bx^6)^{5/2}}\,dx = \int \frac{d\,x^6+c}{x^{11}\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^11*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^11*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^{11}\left(a+bx^6\right)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-10\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{29}+3a\,b^2x^{23}+3a^2b\,x^{17}+a^3x^{11}}dx\right)a^3d\,x^{10}+19\left(\int \frac{\sqrt{b\,x^6\cdots}}{b^3x^{29}+3a\,b^2x^{23}+3\cdots}\right.}{}$$

input
```
int((d*x^6+c)/x^11/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 10*int(sqrt(a + b*x**6)/(a**3*x**11 + 3*a**2*b*x*
*17 + 3*a*b**2*x**23 + b**3*x**29),x)*a**3*d*x**10 + 19*int(sqrt(a + b*x**
6)/(a**3*x**11 + 3*a**2*b*x**17 + 3*a*b**2*x**23 + b**3*x**29),x)*a**2*b*c
*x**10 - 20*int(sqrt(a + b*x**6)/(a**3*x**11 + 3*a**2*b*x**17 + 3*a*b**2*x
**23 + b**3*x**29),x)*a**2*b*d*x**16 + 38*int(sqrt(a + b*x**6)/(a**3*x**11
 + 3*a**2*b*x**17 + 3*a*b**2*x**23 + b**3*x**29),x)*a*b**2*c*x**16 - 10*in
t(sqrt(a + b*x**6)/(a**3*x**11 + 3*a**2*b*x**17 + 3*a*b**2*x**23 + b**3*x*
*29),x)*a*b**2*d*x**22 + 19*int(sqrt(a + b*x**6)/(a**3*x**11 + 3*a**2*b*x*
*17 + 3*a*b**2*x**23 + b**3*x**29),x)*b**3*c*x**22)/(19*b*x**10*(a**2 + 2*
a*b*x**6 + b**2*x**12))
```

3.42 $\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 303

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{a(bc-ad)x}{9b^3(a+bx^6)^{3/2}} - \frac{(10bc-19ad)x}{27b^3\sqrt{a+bx^6}} + \frac{dx\sqrt{a+bx^6}}{4b^3}$$
$$+ \frac{7(4bc-13ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{216\sqrt[4]{3}\sqrt[3]{a}b^3\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*a*(-a*d+b*c)*x/b^3/(b*x^6+a)^(3/2)-1/27*(-19*a*d+10*b*c)*x/b^3/(b*x^6+
a)^(1/2)+1/4*d*x*(b*x^6+a)^(1/2)/b^3+7/648*(-13*a*d+4*b*c)*x*(a^(1/3)+b^(1
/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b
^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x
^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^
(1/3)/b^3/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.35

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x\left(91a^2d+b^2x^6(-40c+27dx^6)+ab(-28c+130dx^6)+7(4bc-13ad)\left(a+bx^6\right)\sqrt{1\right.}{108b^3\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(x^12*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x*(91*a^2*d + b^2*x^6*(-40*c + 27*d*x^6) + a*b*(-28*c + 130*d*x^6) + 7*(4
*b*c - 13*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/6, 1/2,
 7/6, -((b*x^6)/a)]))/(108*b^3*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 817, 817, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(4bc-13ad)\int \frac{x^{12}}{(bx^6+a)^{5/2}}dx}{4b}+\frac{dx^{13}}{4b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(4bc-13ad)\left(\frac{7\int \frac{x^6}{(bx^6+a)^{3/2}}dx}{9b}-\frac{x^7}{9b(a+bx^6)^{3/2}}\right)}{4b}+\frac{dx^{13}}{4b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 817$$

$$\frac{(4bc-13ad)\left(\frac{7\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx}{3b}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{x^7}{9b(a+bx^6)^{3/2}}\right)}{4b}+\frac{dx^{13}}{4b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 766$$

$$\frac{(4bc-13ad)\left(\frac{7\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}\sqrt[3]{a}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{x^7}{9b(a+\cdots}\right)}{4b}$$

$$\frac{dx^{13}}{4b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^12*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^13)/(4*b*(a + b*x^6)^(3/2)) + ((4*b*c - 13*a*d)*(-1/9*x^7/(b*(a + b*x
^6)^(3/2)) + (7*(-1/3*x/(b*Sqrt[a + b*x^6]) + (x*(a^(1/3) + b^(1/3)*x^2)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(1/3)
*b*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(9*b)))/(4*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{12}(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{5}{2}}}dx$$

input
```
int(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{12}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^18 + c*x^12)*sqrt(b*x^6 + a)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^
2*b*x^6 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**12*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{12}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^12/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{12}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^12/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^{12}\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^12*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^12*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{12}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{91\sqrt{b\,x^6+a}\,a^2dx - 28\sqrt{b\,x^6+a}\,abcx + 104\sqrt{b\,x^6+a}\,abd\,x^7 - 32\sqrt{b\,x^6+a}\,b^2c\,x^7 +}{}$$

input
```
int(x^12*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(91*sqrt(a + b*x**6)*a**2*d*x - 28*sqrt(a + b*x**6)*a*b*c*x + 104*sqrt(a +
 b*x**6)*a*b*d*x**7 - 32*sqrt(a + b*x**6)*b**2*c*x**7 + 16*sqrt(a + b*x**6
)*b**2*d*x**13 - 91*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*
x**12 + b**3*x**18),x)*a**5*d + 28*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x
**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**4*b*c - 182*int(sqrt(a + b*x**6)/
(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**4*b*d*x**6 + 56
*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18)
,x)*a**3*b**2*c*x**6 - 91*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a
*b**2*x**12 + b**3*x**18),x)*a**3*b**2*d*x**12 + 28*int(sqrt(a + b*x**6)/(
a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**3*c*x**12)/
(64*b**3*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.43 $\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 284

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x}{9b^2(a+bx^6)^{3/2}}+\frac{(bc-10ad)x}{27ab^2\sqrt{a+bx^6}} + \frac{(2bc+7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{54\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/9*(-a*d+b*c)*x/b^2/(b*x^6+a)^(3/2)+1/27*(-10*a*d+b*c)*x/a/b^2/(b*x^6+a)
^(1/2)+1/162*(7*a*d+2*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/
3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJaco
biAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/b^2/(b^(1/3)*x^2*(a^(1/3)+
b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.36

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-7a^2dx+b^2cx^7-2abx(c+5dx^6)+(2bc+7ad)x(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometri}}{27ab^2\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^6*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(-7*a^2*d*x + b^2*c*x^7 - 2*a*b*x*(c + 5*d*x^6) + (2*b*c + 7*a*d)*x*(a + b
*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^6)/a)])/
(27*a*b^2*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {957, 817, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\,(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(7ad+2bc)\int \frac{x^6}{(bx^6+a)^{3/2}}dx}{9ab}+\frac{x^7(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7ad+2bc)\left(\frac{\int \frac{1}{\sqrt{bx^6+a}}dx}{3b}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(7ad+2bc)\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}\sqrt[3]{a}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{x}{3b\sqrt{a+bx^6}}\right)}{9ab}+\frac{x^7(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^6*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^7)/(9*a*b*(a + b*x^6)^(3/2)) + ((2*b*c + 7*a*d)*(-1/3*x/(b*
Sqrt[a + b*x^6]) + (x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x^2)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(1/3)*b*Sqrt[(b^(1/3)*x^2*(a^(1/
3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6
])))/(9*a*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{6} \left(d\,x^{6}+c\right)}{\left(b\,x^{6}+a\right)^{\frac{5}{2}}}d x$$

input
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^12 + c*x^6)*sqrt(b*x^6 + a)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^2
*b*x^6 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^6/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^6}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^6/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^6\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^6*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^6*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^6(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-7\sqrt{b\,x^6+a}\,adx - 2\sqrt{b\,x^6+a}\,bcx - 8\sqrt{b\,x^6+a}\,bd\,x^7 + 7\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}d\right.}{}$$

input
```
int(x^6*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - 7*sqrt(a + b*x**6)*a*d*x - 2*sqrt(a + b*x**6)*b*c*x - 8*sqrt(a + b*x**
6)*b*d*x**7 + 7*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**1
2 + b**3*x**18),x)*a**4*d + 2*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 +
 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*c + 14*int(sqrt(a + b*x**6)/(a**3
+ 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*d*x**6 + 4*int(sq
rt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**
2*b**2*c*x**6 + 7*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x*
*12 + b**3*x**18),x)*a**2*b**2*d*x**12 + 2*int(sqrt(a + b*x**6)/(a**3 + 3*
a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**3*c*x**12)/(16*b**2*(a*
*2 + 2*a*b*x**6 + b**2*x**12))
```

3.44 $\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 286

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx = \frac{(bc-ad)x}{9ab(a+bx^6)^{3/2}} + \frac{(8bc+ad)x}{27a^2b\sqrt{a+bx^6}} + \frac{(8bc+ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}b\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*(-a*d+b*c)*x/a/b/(b*x^6+a)^(3/2)+1/27*(a*d+8*b*c)*x/a^2/b/(b*x^6+a)^(1
/2)+1/81*(a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2
+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(a
rccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2))
,1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/b/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*
x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.36

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx=\frac{-2a^2dx+8b^2cx^7+abx(11c+dx^6)+2(8bc+ad)x(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\text{ Hypergeometr}}{27a^2b\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(a + b*x^6)^(5/2),x]
```

output
```
(-2*a^2*d*x + 8*b^2*c*x^7 + a*b*x*(11*c + d*x^6) + 2*(8*b*c + a*d)*x*(a +
b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^6)/a)])
/(27*a^2*b*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 749, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(ad+8bc)\int \frac{1}{(bx^6+a)^{3/2}}dx}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{ 749}$$

$$\frac{(ad+8bc)\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx}{3a}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{9ab}+\frac{x(bc-ad)}{9ab\,(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(ad+8bc)\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{9ab}+\frac{x(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x)/(9*a*b*(a + b*x^6)^(3/2)) + ((8*b*c + a*d)*(x/(3*a*Sqrt[a
+ b*x^6]) + (x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2
 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1
/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(9*a
*b)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{\left(b\,x^{6}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx=\int \frac{dx^{6}+c}{(bx^{6}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^2*b*x^
6 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 119.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.27

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx = \frac{cx\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{5}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{dx^7\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{5}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
c*x*gamma(1/6)*hyper((1/6, 5/2), (7/6,), b*x**6*exp_polar(I*pi)/a)/(6*a**(
5/2)*gamma(7/6)) + d*x**7*gamma(7/6)*hyper((7/6, 5/2), (13/6,), b*x**6*exp
_polar(I*pi)/a)/(6*a**(5/2)*gamma(13/6))
```

Maxima [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{c+dx^6}{(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{\left(a+bx^6\right)^{5/2}}\,dx=\int \frac{d\,x^6+c}{{\left(b\,x^6+a\right)}^{5/2}} \,d x$$

input
```
int((c + d*x^6)/(a + b*x^6)^(5/2),x)
```

output
```
int((c + d*x^6)/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{\left(a+bx^6\right)^{5/2}}\,dx=\frac{-\sqrt{b\,x^6+a}\,dx+\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right)a^3d+8\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right) \cdots}{\cdots}$$

input
```
int((d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x + int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*
a*b**2*x**12 + b**3*x**18),x)*a**3*d + 8*int(sqrt(a + b*x**6)/(a**3 + 3*a*
*2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b*c + 2*int(sqrt(a + b*x*
*6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b*d*x**6
+ 16*int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x*
*18),x)*a*b**2*c*x**6 + int(sqrt(a + b*x**6)/(a**3 + 3*a**2*b*x**6 + 3*a*b
**2*x**12 + b**3*x**18),x)*a*b**2*d*x**12 + 8*int(sqrt(a + b*x**6)/(a**3 +
 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*b**3*c*x**12)/(8*b*(a**2
+ 2*a*b*x**6 + b**2*x**12))
```

3.45 $\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 302

$$\int \frac{c+dx^6}{x^6\left(a+bx^6\right)^{5/2}}\,dx = -\frac{c}{5ax^5\left(a+bx^6\right)^{3/2}} - \frac{(14bc-5ad)x}{45a^2\left(a+bx^6\right)^{3/2}} - \frac{8(14bc-5ad)x}{135a^3\sqrt{a+bx^6}}$$
$$-\frac{8(14bc-5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\left(2+\sqrt{3}\right.}{135\sqrt[4]{3}a^{10/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/5*c/a/x^5/(b*x^6+a)^(3/2)-1/45*(-5*a*d+14*b*c)*x/a^2/(b*x^6+a)^(3/2)-8/
135*(-5*a*d+14*b*c)*x/a^3/(b*x^6+a)^(1/2)-8/405*(-5*a*d+14*b*c)*x*(a^(1/3)
+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/
4)/a^(10/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.38

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \frac{-112b^2cx^{12}+2abx^6(-77c+20dx^6)+a^2(-27c+55dx^6)+16(-14bc+5ad)x^6(a}{135a^3x^5 (a+bx^6)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^6*(a + b*x^6)^(5/2)),x]
```

output
```
(-112*b^2*c*x^12 + 2*a*b*x^6*(-77*c + 20*d*x^6) + a^2*(-27*c + 55*d*x^6) +
 16*(-14*b*c + 5*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F
1[1/6, 1/2, 7/6, -((b*x^6)/a)])/(135*a^3*x^5*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {955, 749, 749, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(14bc-5ad)\int \frac{1}{(bx^6+a)^{5/2}}dx}{5a}-\frac{c}{5ax^5 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{749}$$

$$-\frac{(14bc-5ad)\left(\frac{8\int \frac{1}{(bx^6+a)^{3/2}}dx}{9a}+\frac{x}{9a(a+bx^6)^{3/2}}\right)}{5a}-\frac{c}{5ax^5 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \; 749$$

$$-\frac{(14bc-5ad)\left(\frac{8\left(\frac{2\int \frac{1}{\sqrt{bx^6+a}}dx}{3a}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{9a}+\frac{x}{9a(a+bx^6)^{3/2}}\right)}{5a}-\frac{c}{5ax^5\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 766$$

$$-\frac{(14bc-5ad)\left(\frac{8\left(\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}+\frac{x}{3a\sqrt{a+bx^6}}\right)}{9a}+\frac{x}{9a(\ldots}\right)}{5a}$$

$$\frac{c}{5ax^5\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^6*(a + b*x^6)^(5/2)),x]
```

output
```
-1/5*c/(a*x^5*(a + b*x^6)^(3/2)) - ((14*b*c - 5*a*d)*(x/(9*a*(a + b*x^6)^(
3/2)) + (8*(x/(3*a*Sqrt[a + b*x^6]) + (x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(4/3)*Sqrt[(b
^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^
2]*Sqrt[a + b*x^6])))/(9*a)))/(5*a)
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{6}\left(b\,x^{6}+a\right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^3*x^24 + 3*a*b^2*x^18 + 3*a^2*b*x^
12 + a^3*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**6/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^6), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^6}\,dx$$

input
```
integrate((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \int \frac{d\,x^6+c}{x^6\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^6*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^6*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^6 (a+bx^6)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-5\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{24}+3a\,b^2x^{18}+3a^2b\,x^{12}+a^3x^6}dx\right)a^3d\,x^5+14\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{24}+3a\,b^2x^{18}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^6+c)/x^6/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 5*int(sqrt(a + b*x**6)/(a**3*x**6 + 3*a**2*b*x**1
2 + 3*a*b**2*x**18 + b**3*x**24),x)*a**3*d*x**5 + 14*int(sqrt(a + b*x**6)/
(a**3*x**6 + 3*a**2*b*x**12 + 3*a*b**2*x**18 + b**3*x**24),x)*a**2*b*c*x**
5 - 10*int(sqrt(a + b*x**6)/(a**3*x**6 + 3*a**2*b*x**12 + 3*a*b**2*x**18 +
 b**3*x**24),x)*a**2*b*d*x**11 + 28*int(sqrt(a + b*x**6)/(a**3*x**6 + 3*a*
*2*b*x**12 + 3*a*b**2*x**18 + b**3*x**24),x)*a*b**2*c*x**11 - 5*int(sqrt(a
 + b*x**6)/(a**3*x**6 + 3*a**2*b*x**12 + 3*a*b**2*x**18 + b**3*x**24),x)*a
*b**2*d*x**17 + 14*int(sqrt(a + b*x**6)/(a**3*x**6 + 3*a**2*b*x**12 + 3*a*
b**2*x**18 + b**3*x**24),x)*b**3*c*x**17)/(14*b*x**5*(a**2 + 2*a*b*x**6 +
b**2*x**12))
```

3.46 $\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 337

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{5/2}}\,dx = -\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}} - \frac{20bc-11ad}{99a^2x^5\left(a+bx^6\right)^{3/2}} - \frac{14(20bc-11ad)}{297a^3x^5\sqrt{a+bx^6}} + \frac{112(20bc-11ad)\sqrt{a+bx^6}}{1485a^4x^5} + \frac{112b(20bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}(2+}{1485\sqrt[4]{3}a^{13/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/11*c/a/x^11/(b*x^6+a)^(3/2)-1/99*(-11*a*d+20*b*c)/a^2/x^5/(b*x^6+a)^(3/
2)-14/297*(-11*a*d+20*b*c)/a^3/x^5/(b*x^6+a)^(1/2)+112/1485*(-11*a*d+20*b*
c)*(b*x^6+a)^(1/2)/a^4/x^5+112/4455*b*(-11*a*d+20*b*c)*x*(a^(1/3)+b^(1/3)*
x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(13/
3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)
^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.24

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{5/2}}\,dx = \frac{-5a^2c+(20bc-11ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{2},\frac{1}{6},-\frac{bx^6}{a}\right)}{55a^3x^{11}\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^12*(a + b*x^6)^(5/2)),x]
```

output
```
(-5*a^2*c + (20*b*c - 11*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeo
metric2F1[-5/6, 5/2, 1/6, -((b*x^6)/a)])/(55*a^3*x^11*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 819, 819, 847, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^{12}\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \ 955$$

$$-\frac{(20bc-11ad)\int \frac{1}{x^6\left(bx^6+a\right)^{5/2}}dx}{11a}-\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(20bc-11ad)\left(\frac{14\int \frac{1}{x^6\left(bx^6+a\right)^{3/2}}dx}{9a}+\frac{1}{9ax^5\left(a+bx^6\right)^{3/2}}\right)}{11a}-\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \ 819$$

$$-\frac{(20bc-11ad)\left(\frac{14\left(\frac{8\int \frac{1}{x^6\sqrt{bx^6+a}}dx}{3a}+\frac{1}{3ax^5\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax^5(a+bx^6)^{3/2}}\right)}{11a}-\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}}$$

$\big\downarrow$ 847

$$-\frac{(20bc-11ad)\left(\frac{14\left(\frac{8\left(-\frac{2b\int \frac{1}{\sqrt{bx^6+a}}dx}{5a}-\frac{\sqrt{a+bx^6}}{5ax^5}\right)}{3a}+\frac{1}{3ax^5\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax^5(a+bx^6)^{3/2}}\right)}{11a}-\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}}$$

$\big\downarrow$ 766

$$-\frac{(20bc-11ad)\left(\frac{14\left(\frac{8\left(-\frac{bx\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}-\frac{\sqrt{a+bx^6}}{5ax^5}\right)}{3a}+\cdots\right)}{9a}}{11a}$$

$$\frac{c}{11ax^{11}\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^12*(a + b*x^6)^(5/2)),x]
```

output
```
-1/11*c/(a*x^11*(a + b*x^6)^(3/2)) - ((20*b*c - 11*a*d)*(1/(9*a*x^5*(a + b
*x^6)^(3/2)) + (14*(1/(3*a*x^5*Sqrt[a + b*x^6]) + (8*(-1/5*Sqrt[a + b*x^6]
/(a*x^5) - (b*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^
2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos
[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^
2)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(
1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*
a))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{12}\left(b\,x^{6}+a\right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^{12}(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^3*x^30 + 3*a*b^2*x^24 + 3*a^2*b*x^
18 + a^3*x^12), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{12}(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**12/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^{12}(a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^12), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^{12} (a+bx^6)^{5/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^{12}}\,dx$$

input
```
integrate((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^{12} (a+bx^6)^{5/2}}\,dx=\int \frac{d\,x^6+c}{x^{12}\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^12*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^12*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^{12} (a+bx^6)^{5/2}}\,dx=\frac{-\sqrt{b\,x^6+a}\,d-11\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{30}+3a\,b^2x^{24}+3a^2b\,x^{18}+a^3x^{12}}dx\right)a^3d\,x^{11}+20\left(\int \frac{\sqrt{b\,x^6\text{-}}}{b^3x^{30}+3a\,b^2x^{24}+3}\right.}{}$$

input
```
int((d*x^6+c)/x^12/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 11*int(sqrt(a + b*x**6)/(a**3*x**12 + 3*a**2*b*x*
*18 + 3*a*b**2*x**24 + b**3*x**30),x)*a**3*d*x**11 + 20*int(sqrt(a + b*x**
6)/(a**3*x**12 + 3*a**2*b*x**18 + 3*a*b**2*x**24 + b**3*x**30),x)*a**2*b*c
*x**11 - 22*int(sqrt(a + b*x**6)/(a**3*x**12 + 3*a**2*b*x**18 + 3*a*b**2*x
**24 + b**3*x**30),x)*a**2*b*d*x**17 + 40*int(sqrt(a + b*x**6)/(a**3*x**12
 + 3*a**2*b*x**18 + 3*a*b**2*x**24 + b**3*x**30),x)*a*b**2*c*x**17 - 11*in
t(sqrt(a + b*x**6)/(a**3*x**12 + 3*a**2*b*x**18 + 3*a*b**2*x**24 + b**3*x*
*30),x)*a*b**2*d*x**23 + 20*int(sqrt(a + b*x**6)/(a**3*x**12 + 3*a**2*b*x*
*18 + 3*a*b**2*x**24 + b**3*x**30),x)*b**3*c*x**23)/(20*b*x**11*(a**2 + 2*
a*b*x**6 + b**2*x**12))
```

3.47 $\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^{11}}{9b^2(a+bx^6)^{3/2}} - \frac{(11bc-20ad)x^5}{27b^3\sqrt{a+bx^6}} + \frac{dx^5\sqrt{a+bx^6}}{8b^3} + \frac{55(1+\sqrt{3})(8bc-17ad)x\sqrt{a+bx^6}}{432b^{11/3}\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)}$$

$$-\frac{55\sqrt[3]{a}(8bc-17ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x^2}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{144\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$-\frac{55(1-\sqrt{3})\sqrt[3]{a}(8bc-17ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x^2}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}\right.\right.}{864\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/9*(-a*d+b*c)*x^11/b^2/(b*x^6+a)^(3/2)-1/27*(-20*a*d+11*b*c)*x^5/b^3/(b*
x^6+a)^(1/2)+1/8*d*x^5*(b*x^6+a)^(1/2)/b^3+55/432*(1+3^(1/2))*(-17*a*d+8*b
*c)*x*(b*x^6+a)^(1/2)/b^(11/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)-55/432*a^
(1/3)*(-17*a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^
2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1-(a^
(1/3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/b^(11/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*
x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)-55/2592*(1
-3^(1/2))*a^(1/3)*(-17*a*d+8*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3
)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*Inve
rseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(11/3)/(b^(1/3)*x^2*(a^(1
/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.18

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^5\left(-187a^2d+ab(88c-68dx^6)+8b^2x^6(4c+dx^6)+11(-8bc+17ad)\,(a+bx^6)\,\sqrt{1\right.}{64b^3\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^16*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^5*(-187*a^2*d + a*b*(88*c - 68*d*x^6) + 8*b^2*x^6*(4*c + d*x^6) + 11*(-
8*b*c + 17*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[5/6, 5/2
, 11/6, -((b*x^6)/a)]))/(64*b^3*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {959, 817, 817, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(8bc-17ad)\int \frac{x^{16}}{(bx^6+a)^{5/2}}dx}{8b}+\frac{dx^{17}}{8b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(8bc-17ad)\left(\frac{11\int \frac{x^{10}}{(bx^6+a)^{3/2}}dx}{9b}-\frac{x^{11}}{9b(a+bx^6)^{3/2}}\right)}{8b}+\frac{dx^{17}}{8b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(8bc-17ad)\left(\frac{11\left(\frac{5\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^6)^{3/2}}\right)}{8b}+\frac{dx^{17}}{8b(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(8bc-17ad)\left(\frac{11\left(\frac{5\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^6)^{3/2}}\right)}{8b}+\frac{dx^{17}}{8b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(8bc-17ad)\left(\frac{11\left(\frac{5\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3b}-\frac{x^5}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{x^{11}}{9b(a+bx^6)^{3/2}}\right)}{8b}+\frac{dx^{17}}{8b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{(8bc-17ad)\left(\frac{11\left(\frac{5\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{3b}\right.}{9b}\right.}{8b}$$

$$\frac{dx^{17}}{8b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 2420$$

$$(8bc-17ad)\frac{\left(11\frac{\left(5\frac{\left(\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{\cdots}\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\cdots\right.}{2b^{2/3}}\cdots\right.}{3b\cdots}\cdots\right.}{\cdots}\cdots$$

$$\frac{dx^{17}}{8b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^16*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^17)/(8*b*(a + b*x^6)^(3/2)) + ((8*b*c - 17*a*d)*(-1/9*x^11/(b*(a + b*
x^6)^(3/2)) + (11*(-1/3*x^5/(b*Sqrt[a + b*x^6]) + (5*((((1 + Sqrt[3])*x*Sq
rt[a + b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*
(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticE[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3
])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x
*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[
3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*b)))/(9*b)))/(8
*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{x^{16}(d\,x^6+c)}{(b\,x^6+a)^{\frac{5}{2}}}dx$$

input
```
int(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{16}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^22 + c*x^16)*sqrt(b*x^6 + a)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^
2*b*x^6 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**16*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{16}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^16/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^{16}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^16/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^{16}\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^16*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^16*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{16}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-187\sqrt{b\,x^6+a}\,a^2d\,x^5+88\sqrt{b\,x^6+a}\,abc\,x^5-68\sqrt{b\,x^6+a}\,abd\,x^{11}+32\sqrt{b\,x^6+a}\,b^2c}{}$$

input
```
int(x^16*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - 187*sqrt(a + b*x**6)*a**2*d*x**5 + 88*sqrt(a + b*x**6)*a*b*c*x**5 - 68
*sqrt(a + b*x**6)*a*b*d*x**11 + 32*sqrt(a + b*x**6)*b**2*c*x**11 + 8*sqrt(
a + b*x**6)*b**2*d*x**17 + 935*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*
b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**5*d - 440*int((sqrt(a + b*x**6
)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**4*b*c +
 1870*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 +
 b**3*x**18),x)*a**4*b*d*x**6 - 880*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*
a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b**2*c*x**6 + 935*int((
sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18
),x)*a**3*b**2*d*x**12 - 440*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*
x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**3*c*x**12)/(64*b**3*(a**2 +
 2*a*b*x**6 + b**2*x**12))
```

3.48 $\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 592

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^5}{9b^2(a+bx^6)^{3/2}}$$

$$+\frac{(5bc-14ad)x^5}{27ab^2\sqrt{a+bx^6}}-\frac{5\left(1+\sqrt{3}\right)(2bc-11ad)x\sqrt{a+bx^6}}{54ab^{8/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$

$$+\frac{5(2bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{18\ 3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$+\frac{5\left(1-\sqrt{3}\right)(2bc-11ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}\right.}{108\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output

```
-1/9*(-a*d+b*c)*x^5/b^2/(b*x^6+a)^(3/2)+1/27*(-14*a*d+5*b*c)*x^5/a/b^2/(b*
x^6+a)^(1/2)-5/54*(1+3^(1/2))*(-11*a*d+2*b*c)*x*(b*x^6+a)^(1/2)/a/b^(8/3)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)+5/54*(-11*a*d+2*b*c)*x*(a^(1/3)+b^(1/3)*
x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x^2)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(2/3
)/b^(8/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+5/324*(1-3^(1/2))*(-11*a*d+2*b*c)*x*(a^(1/3)
+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/
4)/a^(2/3)/b^(8/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))
*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.16

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^5\left(a(-2bc+11ad+4bdx^6)+(2bc-11ad)\left(a+bx^6\right)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\right.}{8ab^2\left(a+bx^6\right)^{3/2}}$$

input

```
Integrate[(x^10*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output

```
(x^5*(a*(-2*b*c + 11*a*d + 4*b*d*x^6) + (2*b*c - 11*a*d)*(a + b*x^6)*Sqrt[
1 + (b*x^6)/a]*Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^6)/a)]))/(8*a*b^2*
(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {959, 817, 819, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(2bc-11ad)\int \frac{x^{10}}{(bx^6+a)^{5/2}}dx}{2b}+\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{(2bc-11ad)\left(\frac{5\int \frac{x^4}{\left(bx^6+a\right)^{3/2}}dx}{9b}-\frac{x^5}{9b(a+bx^6)^{3/2}}\right)}{2b}+\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{(2bc-11ad)\left(\frac{5\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3a}\right)}{9b}-\frac{x^5}{9b(a+bx^6)^{3/2}}\right)}{2b}+\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{(2bc-11ad)\left(\frac{5\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int\frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9b}-\frac{x^5}{9b(a+bx^6)^{3/2}}\right)}{2b}+\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{(2bc-11ad)\left(\frac{5\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int\frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9b}-\frac{x^5}{9b(a+bx^6)^{3/2}}\right)}{2b}+\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 766$$

$$(2bc-11ad)\left(\frac{5\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\cdots\right.}{\left(1+\cdots\right.}\right.\right.\cdots}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{3a}\right)}{9b}\cdots\right.}{2b}$$

$$\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 2420

$$\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$(2bc-11ad)\frac{\left(5\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{\left(2\,\frac{\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\cdots}{2b^{2/3}}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{dx^{11}}{2b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^10*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^11)/(2*b*(a + b*x^6)^(3/2)) + ((2*b*c - 11*a*d)*(-1/9*x^5/(b*(a + b*x
^6)^(3/2)) + (5*(x^5/(3*a*Sqrt[a + b*x^6]) - (2*((((1 + Sqrt[3])*x*Sqrt[a
+ b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1
/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4]
)/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(
1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4
])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*a)))/(9*b)))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{x^{10}(d\,x^{6}+c)}{(b\,x^{6}+a)^{\frac{5}{2}}}dx$$

input
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^16 + c*x^10)*sqrt(b*x^6 + a)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^
2*b*x^6 + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**10*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^10/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{10}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^10/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{x^{10}\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^10*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^10*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{10}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{11\sqrt{b\,x^6+a}\,ad\,x^5-2\sqrt{b\,x^6+a}\,bc\,x^5+4\sqrt{b\,x^6+a}\,bd\,x^{11}-55\left(\int \frac{\sqrt{b\,x^6+a}\,x^4}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6}\right.}{}$$

input
```
int(x^10*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output

```
(11*sqrt(a + b*x**6)*a*d*x**5 - 2*sqrt(a + b*x**6)*b*c*x**5 + 4*sqrt(a + b
*x**6)*b*d*x**11 - 55*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 +
3*a*b**2*x**12 + b**3*x**18),x)*a**4*d + 10*int((sqrt(a + b*x**6)*x**4)/(a
**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*c - 110*int((
sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18
),x)*a**3*b*d*x**6 + 20*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6
+ 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**2*c*x**6 - 55*int((sqrt(a + b*x*
*6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**
2*d*x**12 + 10*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**
2*x**12 + b**3*x**18),x)*a*b**3*c*x**12)/(8*b**2*(a**2 + 2*a*b*x**6 + b**2
*x**12))
```

3.49 $\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 595

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{(bc-ad)x^5}{9ab\,(a+bx^6)^{3/2}}$$
$$+\frac{(4bc+5ad)x^5}{27a^2b\sqrt{a+bx^6}}-\frac{\left(1+\sqrt{3}\right)(4bc+5ad)x\sqrt{a+bx^6}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$
$$+\frac{(4bc+5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{9\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$+\frac{\left(1-\sqrt{3}\right)(4bc+5ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right),\frac{1}{4}(2\right.}{54\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*(-a*d+b*c)*x^5/a/b/(b*x^6+a)^(3/2)+1/27*(5*a*d+4*b*c)*x^5/a^2/b/(b*x^6
+a)^(1/2)-1/27*(1+3^(1/2))*(5*a*d+4*b*c)*x*(b*x^6+a)^(1/2)/a^2/b^(5/3)/(a^
(1/3)+(1+3^(1/2))*b^(1/3)*x^2)+1/27*(5*a*d+4*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*
((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^
2)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2)^2/(a^(1/3)+(1+3
^(1/2))*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(5/3)/b^(
5/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^
2)^(1/2)/(b*x^6+a)^(1/2)+1/162*(1-3^(1/2))*(5*a*d+4*b*c)*x*(a^(1/3)+b^(1/3
)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x^2
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5
/3)/b^(5/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^5\left(-5a^2d+(4bc+5ad)\left(a+bx^6\right)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\frac{11}{6},-\frac{bx^6}{a}\right)\right)}{20a^2b\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(x^4*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^5*(-5*a^2*d + (4*b*c + 5*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeom
etric2F1[5/6, 5/2, 11/6, -((b*x^6)/a)]))/(20*a^2*b*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 576, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 819, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5ad+4bc)\int \frac{x^4}{(bx^6+a)^{3/2}}dx}{9ab}+\frac{x^5(bc-ad)}{9ab(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$\frac{(5ad+4bc)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3a}\right)}{9ab}+\frac{x^5(bc-ad)}{9ab(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{(5ad+4bc)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9ab}+\frac{x^5(bc-ad)}{9ab(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(5ad+4bc)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9ab}+\frac{x^5(bc-ad)}{9ab(a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{(5ad+4bc)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{\cdots}\right.}{\left(1+\sqrt{\cdots}\right.}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\cdots\right)}{3a}\cdots\right)}{9ab}$$

$$\frac{x^5(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 2420}$$

$$\frac{(5ad+4bc)\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{2b^{2/3}}\right.}{9ab}$$

$$\frac{x^5(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input

```
Int[(x^4*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^5)/(9*a*b*(a + b*x^6)^(3/2)) + ((4*b*c + 5*a*d)*(x^5/(3*a*S
qrt[a + b*x^6]) - (2*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
)*x^2)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/
3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6
]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[
(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2
)^2]*Sqrt[a + b*x^6])))/(3*a)))/(9*a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{x^{4} \left(d \,x^{6}+c \right)}{\left(b \,x^{6}+a \right)^{\frac{5}{2}}}d x$$

input
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^10 + c*x^4)*sqrt(b*x^6 + a)/(b^3*x^18 + 3*a*b^2*x^12 + 3*a^2
*b*x^6 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 137.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{cx^5\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{5}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{dx^{11}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{11}{6},\frac{5}{2}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate(x**4*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
c*x**5*gamma(5/6)*hyper((5/6, 5/2), (11/6,), b*x**6*exp_polar(I*pi)/a)/(6*
a**(5/2)*gamma(11/6)) + d*x**11*gamma(11/6)*hyper((11/6, 5/2), (17/6,), b*
x**6*exp_polar(I*pi)/a)/(6*a**(5/2)*gamma(17/6))
```

Maxima [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^4/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^4}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^4/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^4\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^4*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^4*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^4(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d\,x^5+5\left(\int \frac{\sqrt{b\,x^6+a}\,x^4}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right)a^3d+4\left(\int \frac{\sqrt{b\,x^6+a}\,x^4}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right.}{}$$

input
```
int(x^4*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x**5 + 5*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2
*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*d + 4*int((sqrt(a + b*x**6)
*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b*c +
10*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b*
*3*x**18),x)*a**2*b*d*x**6 + 8*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*
b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**2*c*x**6 + 5*int((sqrt(a + b
*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**
2*d*x**12 + 4*int((sqrt(a + b*x**6)*x**4)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2
*x**12 + b**3*x**18),x)*b**3*c*x**12)/(4*b*(a**2 + 2*a*b*x**6 + b**2*x**12
))
```

3.50 $\int \frac{c+dx^6}{x^2(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 610

$$\int \frac{c+dx^6}{x^2(a+bx^6)^{5/2}}\,dx = -\frac{c}{ax(a+bx^6)^{3/2}} - \frac{(10bc-ad)x^5}{9a^2(a+bx^6)^{3/2}} - \frac{4(10bc-ad)x^5}{27a^3\sqrt{a+bx^6}} + \frac{4(1+\sqrt{3})(10bc-ad)x\sqrt{a+bx^6}}{27a^3b^{2/3}\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)}$$
$$-\frac{4(10bc-ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x^2}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{9\,3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$-\frac{2(1-\sqrt{3})(10bc-ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x^2}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2}\right),\frac{1}{4}\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-c/a/x/(b*x^6+a)^(3/2)-1/9*(-a*d+10*b*c)*x^5/a^2/(b*x^6+a)^(3/2)-4/27*(-a*
d+10*b*c)*x^5/a^3/(b*x^6+a)^(1/2)+4/27*(1+3^(1/2))*(-a*d+10*b*c)*x*(b*x^6+
a)^(1/2)/a^3/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)-4/27*(-a*d+10*b*c)*
x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3
)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1
/3)*x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1
/2))*3^(1/4)/a^(8/3)/b^(2/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)-2/81*(1-3^(1/2))*(-a*d+10
*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+
(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1/
4*2^(1/2))*3^(3/4)/a^(8/3)/b^(2/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{c+dx^6}{x^2 (a+bx^6)^{5/2}}\,dx = \frac{-5a^2c+(-10bc+ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{5}{2},\frac{11}{6},-\frac{bx^6}{a}\right)}{5a^3x\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^2*(a + b*x^6)^(5/2)),x]
```

output
```
(-5*a^2*c + (-10*b*c + a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeome
tric2F1[5/6, 5/2, 11/6, -((b*x^6)/a)])/(5*a^3*x*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 819, 819, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 955$$

$$-\frac{(10bc-ad)\int \frac{x^4}{\left(bx^6+a\right)^{5/2}}dx}{a}-\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(10bc-ad)\left(\frac{4\int \frac{x^4}{\left(bx^6+a\right)^{3/2}}dx}{9a}+\frac{x^5}{9a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$-\frac{(10bc-ad)\left(\frac{4\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\int \frac{x^4}{\sqrt{bx^6+a}}dx}{3a}\right)}{9a}+\frac{x^5}{9a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 837$$

$$-\frac{(10bc-ad)\left(\frac{4\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9a}+\frac{x^5}{9a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{(10bc-ad)\left(\frac{4\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int\frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{3a}\right)}{9a}+\frac{x^5}{9a\left(a+bx^6\right)^{3/2}}\right)}{a}-\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 766

$$-\frac{(10bc-ad)\left(\frac{4\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\frac{\int\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\cdots\right.}{\left(1-\cdots\right.}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}\cdots}\right)}{3a}\cdots\right)}{9a}\cdots\right)}{a}$$

$$\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

↓ 2420

$$\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{\cdots}}\cdots\right.\right.}{\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}\cdots}$$

$$2\left(\cdots\right)\Big/\,2b^{2/3}$$

$$4\left(\frac{x^5}{3a\sqrt{a+bx^6}}-\frac{2\left(\cdots\right)}{\cdots}\right.$$

$$-\frac{(10bc-ad)\left(\frac{4\left(\cdots\right)}{\cdots}\right.}{\cdots}$$

$$\frac{c}{ax\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^2*(a + b*x^6)^(5/2)),x]
```

output
```
-(c/(a*x*(a + b*x^6)^(3/2))) - ((10*b*c - a*d)*(x^5/(9*a*(a + b*x^6)^(3/2)
) + (4*(x^5/(3*a*Sqrt[a + b*x^6]) - (2*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(
1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1
 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1
/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(
b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)
^2]*Sqrt[a + b*x^6]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^
(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x^2)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(
1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(
1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*a)))/(9*a)))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{2}\left(b\,x^{6}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{5/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^3*x^20 + 3*a*b^2*x^14 + 3*a^2*b*x^
8 + a^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**6+c)/x**2/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^2\left(a+bx^6\right)^{5/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^2\,(a+bx^6)^{5/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^2\,(a+bx^6)^{5/2}}\,dx=\int \frac{d\,x^6+c}{x^2\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^2*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^2*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^2\,(a+bx^6)^{5/2}}\,dx=\frac{-\sqrt{b\,x^6+a}\,d-\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{20}+3a\,b^2x^{14}+3a^2b\,x^8+a^3x^2}dx\right)a^3dx+10\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{20}+3a\,b^2x^{14}+3a^2b\,x^8+a}\right.}{}$$

input
```
int((d*x^6+c)/x^2/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - int(sqrt(a + b*x**6)/(a**3*x**2 + 3*a**2*b*x**8 +
 3*a*b**2*x**14 + b**3*x**20),x)*a**3*d*x + 10*int(sqrt(a + b*x**6)/(a**3*
x**2 + 3*a**2*b*x**8 + 3*a*b**2*x**14 + b**3*x**20),x)*a**2*b*c*x - 2*int(
sqrt(a + b*x**6)/(a**3*x**2 + 3*a**2*b*x**8 + 3*a*b**2*x**14 + b**3*x**20)
,x)*a**2*b*d*x**7 + 20*int(sqrt(a + b*x**6)/(a**3*x**2 + 3*a**2*b*x**8 + 3
*a*b**2*x**14 + b**3*x**20),x)*a*b**2*c*x**7 - int(sqrt(a + b*x**6)/(a**3*
x**2 + 3*a**2*b*x**8 + 3*a*b**2*x**14 + b**3*x**20),x)*a*b**2*d*x**13 + 10
*int(sqrt(a + b*x**6)/(a**3*x**2 + 3*a**2*b*x**8 + 3*a*b**2*x**14 + b**3*x
**20),x)*b**3*c*x**13)/(10*b*x*(a**2 + 2*a*b*x**6 + b**2*x**12))
```

3.51 $\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 642

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx = -\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}-\frac{16bc-7ad}{63a^2x\left(a+bx^6\right)^{3/2}}-\frac{10(16bc-7ad)}{189a^3x\sqrt{a+bx^6}}$$

$$+\frac{40(16bc-7ad)\sqrt{a+bx^6}}{189a^4x}-\frac{40\left(1+\sqrt{3}\right)\sqrt[3]{b}(16bc-7ad)x\sqrt{a+bx^6}}{189a^4\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)}$$

$$+\frac{40\sqrt[3]{b}(16bc-7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{63\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$+\frac{20\left(1-\sqrt{3}\right)\sqrt[3]{b}(16bc-7ad)x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}\right.\right.}{189\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/7*c/a/x^7/(b*x^6+a)^(3/2)-1/63*(-7*a*d+16*b*c)/a^2/x/(b*x^6+a)^(3/2)-10
/189*(-7*a*d+16*b*c)/a^3/x/(b*x^6+a)^(1/2)+40/189*(-7*a*d+16*b*c)*(b*x^6+a
)^(1/2)/a^4/x-40/189*(1+3^(1/2))*b^(1/3)*(-7*a*d+16*b*c)*x*(b*x^6+a)^(1/2)
/a^4/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)+40/189*b^(1/3)*(-7*a*d+16*b*c)*x*(a
^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*
x^2)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*3^(1/4)/a^(11/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+20/567*(1-3^(1/2))*b^(1/3)*(-7*a*d+1
6*b*c)*x*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)
+(1-3^(1/2))*b^(1/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x^2)),1/4*6^(1/2)+1
/4*2^(1/2))*3^(3/4)/a^(11/3)/(b^(1/3)*x^2*(a^(1/3)+b^(1/3)*x^2)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.13

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx = \frac{-a^2c+(16bc-7ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{2},\frac{5}{6},-\frac{bx^6}{a}\right)}{7a^3x^7\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^8*(a + b*x^6)^(5/2)),x]
```

output
```
(-(a^2*c) + (16*b*c - 7*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeom
etric2F1[-1/6, 5/2, 5/6, -((b*x^6)/a)])/(7*a^3*x^7*(a + b*x^6)^(3/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 618, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {955, 819, 819, 847, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^8 \left(a+bx^6\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(16bc-7ad)\int \frac{1}{x^2(bx^6+a)^{5/2}}dx}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(16bc-7ad)\left(\frac{10\int \frac{1}{x^2\left(bx^6+a\right)^{3/2}}dx}{9a}+\frac{1}{9ax(a+bx^6)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(16bc-7ad)\left(\frac{10\left(\frac{4\int \frac{1}{x^2\sqrt{bx^6+a}}dx}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax(a+bx^6)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(16bc-7ad)\left(\frac{10\left(\frac{4\left(\frac{2b\int \frac{x^4}{\sqrt{bx^6+a}}dx}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax(a+bx^6)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$-\frac{(16bc-7ad)\left(\frac{10\left(\frac{4\left(\frac{2b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax(a+bx^6)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{(16bc-7ad)\left(\frac{10\left(\frac{4\left(\frac{2b\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{bx^6+a}}dx}{2b^{2/3}}\right)}{a}-\frac{\sqrt{a+bx^6}}{ax}\right)}{3a}+\frac{1}{3ax\sqrt{a+bx^6}}\right)}{9a}+\frac{1}{9ax(a+bx^6)^{3/2}}\right)}{7a}-\frac{c}{7ax^7\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{766}$$

$$(16bc-7ad)\left(\frac{10\left(\frac{4\left(\frac{2b\left(\frac{\int \frac{2b^{2/3}x^4+\left(1-\sqrt{3}\right)a^{2/3}}{\sqrt{bx^6+a}}\,dx}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{a}\right.}{3a}\right.}{9a}\right.$$

↓ 2420

$$(16bc-7ad)\left(10\left(4\left(2b\left(\frac{\left(1+\sqrt{3}\right)x\sqrt{a+bx^6}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2}-\frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\,E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2+\sqrt[3]{a}}\right)\right|}{2b^{2/3}\sqrt{\frac{\sqrt[3]{b}x^2\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.\right.\right.\right.$$

input
```
Int[(c + d*x^6)/(x^8*(a + b*x^6)^(5/2)),x]
```

output
```
-1/7*c/(a*x^7*(a + b*x^6)^(3/2)) - ((16*b*c - 7*a*d)*(1/(9*a*x*(a + b*x^6)
^(3/2)) + (10*(1/(3*a*x*Sqrt[a + b*x^6]) + (4*(-(Sqrt[a + b*x^6]/(a*x)) +
(2*b*((((1 + Sqrt[3])*x*Sqrt[a + b*x^6])/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x^2) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*x^2*(a^(1/3) + b^(1/3)*x^2
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(2*b^(2/3))
- ((1 - Sqrt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x^2 + b^(2/3)*x^4)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x^2)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x^2*(a^
(1/3) + b^(1/3)*x^2))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6])))/a))/(3*a)))/(9*a)))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{8}\left(b\,x^{6}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{5}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^6 + a)*(d*x^6 + c)/(b^3*x^26 + 3*a*b^2*x^20 + 3*a^2*b*x^
14 + a^3*x^8), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**6+c)/x**8/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^8\left(a+bx^6\right)^{5/2}}\,dx=\int \frac{dx^6+c}{\left(bx^6+a\right)^{\frac{5}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^8), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^8 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^8}\,dx$$

input
```
integrate((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^8 (a+bx^6)^{5/2}}\,dx = \int \frac{d\,x^6+c}{x^8\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^8*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^8*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^8 (a+bx^6)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-7\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{26}+3a\,b^2x^{20}+3a^2b\,x^{14}+a^3x^8}dx\right)a^3d\,x^7+16\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{26}+3a\,b^2x^{20}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^6+c)/x^8/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 7*int(sqrt(a + b*x**6)/(a**3*x**8 + 3*a**2*b*x**1
4 + 3*a*b**2*x**20 + b**3*x**26),x)*a**3*d*x**7 + 16*int(sqrt(a + b*x**6)/
(a**3*x**8 + 3*a**2*b*x**14 + 3*a*b**2*x**20 + b**3*x**26),x)*a**2*b*c*x**
7 - 14*int(sqrt(a + b*x**6)/(a**3*x**8 + 3*a**2*b*x**14 + 3*a*b**2*x**20 +
 b**3*x**26),x)*a**2*b*d*x**13 + 32*int(sqrt(a + b*x**6)/(a**3*x**8 + 3*a*
*2*b*x**14 + 3*a*b**2*x**20 + b**3*x**26),x)*a*b**2*c*x**13 - 7*int(sqrt(a
 + b*x**6)/(a**3*x**8 + 3*a**2*b*x**14 + 3*a*b**2*x**20 + b**3*x**26),x)*a
*b**2*d*x**19 + 16*int(sqrt(a + b*x**6)/(a**3*x**8 + 3*a**2*b*x**14 + 3*a*
b**2*x**20 + b**3*x**26),x)*b**3*c*x**19)/(16*b*x**7*(a**2 + 2*a*b*x**6 +
b**2*x**12))
```

3.52 $\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 606

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^{10}}{9b^2(a+bx^6)^{3/2}} - \frac{(10bc-19ad)x^4}{27b^3\sqrt{a+bx^6}} + \frac{dx^4\sqrt{a+bx^6}}{7b^3} + \frac{40(7bc-16ad)\sqrt{a+bx^6}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$

$$-\frac{20\sqrt{2-\sqrt{3}}\sqrt[3]{a}(7bc-16ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)\middle|-7-\right.}{63\,3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$+\frac{40\sqrt{2}\sqrt[3]{a}(7bc-16ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-\right.}{189\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/9*(-a*d+b*c)*x^10/b^2/(b*x^6+a)^(3/2)-1/27*(-19*a*d+10*b*c)*x^4/b^3/(b*
x^6+a)^(1/2)+1/7*d*x^4*(b*x^6+a)^(1/2)/b^3+40/189*(-16*a*d+7*b*c)*(b*x^6+a
)^(1/2)/b^(11/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)-20/189*(1/2*6^(1/2)-1/2
*2^(1/2))*a^(1/3)*(-16*a*d+7*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*Ellipt
icE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*
3^(1/2)+2*I)*3^(1/4)/b^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+40/567*2^(1/2)*a^(1/3)*(-16*
a*d+7*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3
)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(3/4)/b^
(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)
^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.18

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^4\Big(-32a^2d+2ab(7c-8dx^6)+b^2x^6(7c+dx^6)+2(-7bc+16ad)\,(a+bx^6)\,\sqrt{1+\frac{bx}{a}}}{7b^3\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^15*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^4*(-32*a^2*d + 2*a*b*(7*c - 8*d*x^6) + b^2*x^6*(7*c + d*x^6) + 2*(-7*b*
c + 16*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeometric2F1[2/3, 5/2, 5/
3, -((b*x^6)/a)]))/(7*b^3*(a + b*x^6)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {959, 807, 817, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(7bc-16ad)\int \frac{x^{15}}{\left(bx^6+a\right)^{5/2}}dx}{7b}+\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(7bc-16ad)\int \frac{x^{14}}{\left(bx^6+a\right)^{5/2}}dx^2}{14b}+\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-16ad)\left(\frac{10\int \frac{x^8}{\left(bx^6+a\right)^{3/2}}dx^2}{9b}-\frac{2x^{10}}{9b\left(a+bx^6\right)^{3/2}}\right)}{14b}+\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{817}$$

$$\frac{(7bc-16ad)\left(\frac{10\left(\frac{4\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{2x^{10}}{9b\left(a+bx^6\right)^{3/2}}\right)}{14b}+\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{832}$$

$$\frac{(7bc-16ad)\left(\frac{10\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{9b}-\frac{2x^{10}}{9b(a+bx^6)^{3/2}}\right)}{14b}+$$

$$\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 759

$$\frac{(7bc-16ad)\left(\frac{10\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+}{\sqrt[3]{b}x^2+}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{3b}\right.}{9b}\right.}{14b}$$

$$\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

$\Big\downarrow$ 2416

$$(7bc-16ad)\left(\frac{10\left(\frac{4\left(\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{\sqrt[3]{b}}\right.}{}\right.}{}\right)$$

$$\frac{dx^{16}}{7b\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(x^15*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(d*x^16)/(7*b*(a + b*x^6)^(3/2)) + ((7*b*c - 16*a*d)*((-2*x^10)/(9*b*(a +
b*x^6)^(3/2)) + (10*((-2*x^4)/(3*b*Sqrt[a + b*x^6]) + (4*(((2*Sqrt[a + b*x
^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 - S
qrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^
2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcSin
[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^
2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 - S
qrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6])))/(3*b)))/(9*b)))/(14*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^{15}(d\,x^6+c)}{(b\,x^6+a)^{\frac{5}{2}}}dx$$

input
```
int(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.27

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{40\left((7\,b^3c-16\,ab^2d)x^{12}+2\,(7\,ab^2c-16\,a^2bd)x^6+7\,a^2bc-16\,a^3d\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstras}\right.}{189\,(b^6x^{12}+2\,ab}$$

input
```
integrate(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/189*(40*((7*b^3*c - 16*a*b^2*d)*x^12 + 2*(7*a*b^2*c - 16*a^2*b*d)*x^6 +
 7*a^2*b*c - 16*a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInve
rse(0, -4*a/b, x^2)) - (27*b^3*d*x^16 - 13*(7*b^3*c - 16*a*b^2*d)*x^10 - 1
0*(7*a*b^2*c - 16*a^2*b*d)*x^4)*sqrt(b*x^6 + a))/(b^6*x^12 + 2*a*b^5*x^6 +
 a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**15*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{15}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^15/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{(dx^6+c)x^{15}}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^15/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{x^{15}\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^15*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^15*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^{15}(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{-32\sqrt{b\,x^6+a}\,a^2d\,x^4+14\sqrt{b\,x^6+a}\,abc\,x^4-16\sqrt{b\,x^6+a}\,abd\,x^{10}+7\sqrt{b\,x^6+a}\,b^2c\,x^{1}}{}$$

input
```
int(x^15*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - 32*sqrt(a + b*x**6)*a**2*d*x**4 + 14*sqrt(a + b*x**6)*a*b*c*x**4 - 16*
sqrt(a + b*x**6)*a*b*d*x**10 + 7*sqrt(a + b*x**6)*b**2*c*x**10 + sqrt(a +
b*x**6)*b**2*d*x**16 + 128*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x*
*6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**5*d - 56*int((sqrt(a + b*x**6)*x**
3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**4*b*c + 256*
int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*
x**18),x)*a**4*b*d*x**6 - 112*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b
*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b**2*c*x**6 + 128*int((sqrt(a
 + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a
**3*b**2*d*x**12 - 56*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 +
3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**3*c*x**12)/(7*b**3*(a**2 + 2*a*b*x
**6 + b**2*x**12))
```

3.53 $\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 587

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = -\frac{(bc-ad)x^4}{9b^2\left(a+bx^6\right)^{3/2}}$$

$$+\frac{(4bc-13ad)x^4}{27ab^2\sqrt{a+bx^6}} - \frac{4(bc-10ad)\sqrt{a+bx^6}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$

$$+\frac{2\sqrt{2-\sqrt{3}}(bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4\sqrt{3}\right)}{9\ 3^{3/4}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$-\frac{4\sqrt{2}(bc-10ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{\ }\right.}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/9*(-a*d+b*c)*x^4/b^2/(b*x^6+a)^(3/2)+1/27*(-13*a*d+4*b*c)*x^4/a/b^2/(b*
x^6+a)^(1/2)-4/27*(-10*a*d+b*c)*(b*x^6+a)^(1/2)/a/b^(8/3)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x^2)+2/27*(1/2*6^(1/2)-1/2*2^(1/2))*(-10*a*d+b*c)*(a^(1/3)+b^
(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(8/3)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a
)^(1/2)-4/81*2^(1/2)*(-10*a*d+b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*Ellip
ticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I
*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.16

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^4\left(a(-bc+10ad+5bdx^6)+(bc-10ad)\,(a+bx^6)\,\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3}\right.\right.}{5ab^2\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^9*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^4*(a*(-(b*c) + 10*a*d + 5*b*d*x^6) + (b*c - 10*a*d)*(a + b*x^6)*Sqrt[1
+ (b*x^6)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^6)/a)]))/(5*a*b^2*(a
+ b*x^6)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {957, 807, 817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\int \frac{x^9}{\left(bx^6+a\right)^{3/2}}dx}{9ab}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\int \frac{x^8}{\left(bx^6+a\right)^{3/2}}dx^2}{18ab}$$

$$\Big\downarrow \text{ 817}$$

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\left(\frac{4\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{18ab}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\left(\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^6}}\right)}{18ab}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\left(\frac{4\left(\frac{\int\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{b}x^2+\left(1+\sqrt{\cdots}\right.}\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{3b}\cdots\right)}{18ab}$$

$\downarrow$ 2416

$$\frac{x^{10}(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}-\frac{(bc-10ad)\left(\frac{4\left(\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x^2+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right)}{\sqrt[3]{b}}\cdots\right)}{1\cdots}$$

input
```
Int[(x^9*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
((b*c - a*d)*x^10)/(9*a*b*(a + b*x^6)^(3/2)) - ((b*c - 10*a*d)*((-2*x^4)/(
3*b*Sqrt[a + b*x^6]) + (4*(((2*Sqrt[a + b*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^
2)^2]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/
3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x
^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3
])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*S
qrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(3*b)))/(18*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^9(d\,x^6+c)}{(b\,x^6+a)^{\frac{5}{2}}}dx$$

input
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.26

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{4\left((b^3c-10\,ab^2d)x^{12}+2\,(ab^2c-10\,a^2bd)x^6+a^2bc-10\,a^3d\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\right.}{27\,(ab^5x^{12}}$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(4*((b^3*c - 10*a*b^2*d)*x^12 + 2*(a*b^2*c - 10*a^2*b*d)*x^6 + a^2*b*
c - 10*a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x^2)) + ((4*b^3*c - 13*a*b^2*d)*x^10 + (a*b^2*c - 10*a^2*b*d)*x^4)*s
qrt(b*x^6 + a))/(a*b^5*x^12 + 2*a^2*b^4*x^6 + a^3*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**9*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^9}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^9/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^9}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^9/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{x^9\,(d\,x^6+c)}{(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((x^9*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^9*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^9(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{10\sqrt{b\,x^6+a}\,ad\,x^4 - \sqrt{b\,x^6+a}\,bc\,x^4 + 5\sqrt{b\,x^6+a}\,bd\,x^{10} - 40\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a}\right.}{}$$

input
```
int(x^9*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
(10*sqrt(a + b*x**6)*a*d*x**4 - sqrt(a + b*x**6)*b*c*x**4 + 5*sqrt(a + b*x
**6)*b*d*x**10 - 40*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*
a*b**2*x**12 + b**3*x**18),x)*a**4*d + 4*int((sqrt(a + b*x**6)*x**3)/(a**3
 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*b*c - 80*int((sqrt
(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)
*a**3*b*d*x**6 + 8*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a
*b**2*x**12 + b**3*x**18),x)*a**2*b**2*c*x**6 - 40*int((sqrt(a + b*x**6)*x
**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b**2*d*x
**12 + 4*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**1
2 + b**3*x**18),x)*a*b**3*c*x**12)/(5*b**2*(a**2 + 2*a*b*x**6 + b**2*x**12
))
```

3.54 $\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 593

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{(bc-ad)x^4}{9ab\,(a+bx^6)^{3/2}} + \frac{(5bc+4ad)x^4}{27a^2b\sqrt{a+bx^6}} - \frac{(5bc+4ad)\sqrt{a+bx^6}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$
$$+ \frac{\sqrt{2-\sqrt{3}}(5bc+4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4\sqrt{3}\right)}{18\ 3^{3/4}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$
$$- \frac{\sqrt{2}(5bc+4ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{3}\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
1/9*(-a*d+b*c)*x^4/a/b/(b*x^6+a)^(3/2)+1/27*(4*a*d+5*b*c)*x^4/a^2/b/(b*x^6
+a)^(1/2)-1/27*(4*a*d+5*b*c)*(b*x^6+a)^(1/2)/a^2/b^(5/3)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x^2)+1/54*(1/2*6^(1/2)-1/2*2^(1/2))*(4*a*d+5*b*c)*(a^(1/3)+b^(
1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)/a^(5/3)/b^(5/3)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)
^(1/2)-1/81*2^(1/2)*(4*a*d+5*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*Ellipt
icF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*
3^(1/2)+2*I)*3^(3/4)/a^(5/3)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.14

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{x^4\left(-4a^2d+(5bc+4ad)\,(a+bx^6)\,\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^6}{a}\right)\right)}{20a^2b\,(a+bx^6)^{3/2}}$$

input
```
Integrate[(x^3*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output
```
(x^4*(-4*a^2*d + (5*b*c + 4*a*d)*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeom
etric2F1[2/3, 5/2, 5/3, -((b*x^6)/a)]))/(20*a^2*b*(a + b*x^6)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {957, 807, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^6\right)}{\left(a+bx^6\right)^{5/2}}dx$$

$$\Big\downarrow \; 957$$

$$\frac{(4ad+5bc)\int \frac{x^3}{\left(bx^6+a\right)^{3/2}}dx}{9ab}+\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 807$$

$$\frac{(4ad+5bc)\int \frac{x^2}{\left(bx^6+a\right)^{3/2}}dx^2}{18ab}+\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 819$$

$$\frac{(4ad+5bc)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3a}\right)}{18ab}+\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 832$$

$$\frac{(4ad+5bc)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}}{3a}\right)}{18ab}+\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \; 759$$

$$\frac{(4ad+5bc)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{3a}\right)}{18ab}$$

$$\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

$\Big\downarrow$ 2416

$$\frac{(4ad+5bc)\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+(1}{\sqrt[3]{b}x^2+(1}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{\sqrt[3]{b}}}{18a}$$

$$\frac{x^4(bc-ad)}{9ab\left(a+bx^6\right)^{3/2}}$$

input

```
Int[(x^3*(c + d*x^6))/(a + b*x^6)^(5/2),x]
```

output

```
((b*c - a*d)*x^4)/(9*a*b*(a + b*x^6)^(3/2)) + ((5*b*c + 4*a*d)*((2*x^4)/(3
*a*Sqrt[a + b*x^6]) - (((2*Sqrt[a + b*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*
x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt
[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2
]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[
3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/(3*a)))/(18*a*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^3(d\,x^6+c)}{(b\,x^6+a)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.26

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{((5\,b^3c+4\,ab^2d)x^{12}+2\,(5\,ab^2c+4\,a^2bd)x^6+5\,a^2bc+4\,a^3d)\sqrt{b}\text{weierstrassZeta}\left(0,-\right.}{27\,(a^2b^4x^{12}+}$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(((5*b^3*c + 4*a*b^2*d)*x^12 + 2*(5*a*b^2*c + 4*a^2*b*d)*x^6 + 5*a^2*
b*c + 4*a^3*d)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -
4*a/b, x^2)) + ((5*b^3*c + 4*a*b^2*d)*x^10 + (8*a*b^2*c + a^2*b*d)*x^4)*sq
rt(b*x^6 + a))/(a^2*b^4*x^12 + 2*a^3*b^3*x^6 + a^4*b^2)
```

Sympy [A] (verification not implemented)

Time = 100.83 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.13

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \frac{cx^4\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{dx^{10}\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^6e^{i\pi}}{a}\right)}{6a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**3*(d*x**6+c)/(b*x**6+a)**(5/2),x)
```

output
```
c*x**4*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**6*exp_polar(I*pi)/a)/(6*a
**(5/2)*gamma(5/3)) + d*x**10*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**6*
exp_polar(I*pi)/a)/(6*a**(5/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^3}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)*x^3/(b*x^6 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx = \int \frac{(dx^6+c)x^3}{(bx^6+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)*x^3/(b*x^6 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\int \frac{x^3\,(d\,x^6+c)}{{(b\,x^6+a)}^{5/2}}\,dx$$

input
```
int((x^3*(c + d*x^6))/(a + b*x^6)^(5/2),x)
```

output
```
int((x^3*(c + d*x^6))/(a + b*x^6)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^3(c+dx^6)}{(a+bx^6)^{5/2}}\,dx=\frac{-\sqrt{b\,x^6+a}\,d\,x^4+4\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right)a^3d+5\left(\int \frac{\sqrt{b\,x^6+a}\,x^3}{b^3x^{18}+3a\,b^2x^{12}+3a^2b\,x^6+a^3}dx\right.}{}$$

input
```
int(x^3*(d*x^6+c)/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d*x**4 + 4*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2
*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**3*d + 5*int((sqrt(a + b*x**6)
*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a**2*b*c +
8*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**
3*x**18),x)*a**2*b*d*x**6 + 10*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*
b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**2*c*x**6 + 4*int((sqrt(a + b
*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2*x**12 + b**3*x**18),x)*a*b**
2*d*x**12 + 5*int((sqrt(a + b*x**6)*x**3)/(a**3 + 3*a**2*b*x**6 + 3*a*b**2
*x**12 + b**3*x**18),x)*b**3*c*x**12)/(5*b*(a**2 + 2*a*b*x**6 + b**2*x**12
))
```

3.55 $\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 610

$$
\begin{aligned}
\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{5/2}}\,dx &= -\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}-\frac{(11bc-2ad)x^4}{18a^2\left(a+bx^6\right)^{3/2}} \\
&-\frac{5(11bc-2ad)x^4}{54a^3\sqrt{a+bx^6}}+\frac{5(11bc-2ad)\sqrt{a+bx^6}}{54a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)} \\
&-\frac{5\sqrt{2-\sqrt{3}}(11bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4\sqrt{3}\right)}{36\ 3^{3/4}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}} \\
&+\frac{5(11bc-2ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4\sqrt{3}\right)}{27\sqrt{2}\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}
\end{aligned}
$$

output
```
-1/2*c/a/x^2/(b*x^6+a)^(3/2)-1/18*(-2*a*d+11*b*c)*x^4/a^2/(b*x^6+a)^(3/2)-
5/54*(-2*a*d+11*b*c)*x^4/a^3/(b*x^6+a)^(1/2)+5/54*(-2*a*d+11*b*c)*(b*x^6+a
)^(1/2)/a^3/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)-5/108*(1/2*6^(1/2)-1
/2*2^(1/2))*(-2*a*d+11*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3
)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticE(((
1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2
)+2*I)*3^(1/4)/a^(8/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1/2)+5/162*(-2*a*d+11*b*c)*(a^(1
/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a
^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/b^
(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^
(1/2)/(b*x^6+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.13

$$\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{5/2}}\,dx = \frac{-4a^2c+(-11bc+2ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{2},\frac{5}{3},-\frac{bx^6}{a}\right)}{8a^3x^2\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^3*(a + b*x^6)^(5/2)),x]
```

output
```
(-4*a^2*c + (-11*b*c + 2*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeo
metric2F1[2/3, 5/2, 5/3, -((b*x^6)/a)])/(8*a^3*x^2*(a + b*x^6)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {955, 807, 819, 819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^3 \left(a+bx^6\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 955}$$

$$-\frac{(11bc-2ad)\int \frac{x^3}{\left(bx^6+a\right)^{5/2}}dx}{2a}-\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{(11bc-2ad)\int \frac{x^2}{\left(bx^6+a\right)^{5/2}}dx^2}{4a}-\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(11bc-2ad)\left(\frac{5\int \frac{x^2}{\left(bx^6+a\right)^{3/2}}dx^2}{9a}+\frac{2x^4}{9a\left(a+bx^6\right)^{3/2}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 819}$$

$$-\frac{(11bc-2ad)\left(\frac{5\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{3a}\right)}{9a}+\frac{2x^4}{9a\left(a+bx^6\right)^{3/2}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 832}$$

$$-\frac{(11bc-2ad)\left(\frac{5\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}}{3a}\right)}{9a}+\frac{2x^4}{9a\left(a+bx^6\right)^{3/2}}\right)}{4a}-\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{(11bc-2ad)\left(\frac{5\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{\int\frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\text{arcsi}\cdots\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^{\cdots}}}}{3a}\cdots\right)}{9a}\cdots\right)}{4a}\cdots$$

$$\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 2416

$$-\frac{(11bc-2ad)\left(5\left(\frac{2x^4}{3a\sqrt{a+bx^6}}-\frac{\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{b}x}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}}{\sqrt[3]{b}}}{}\right)\right.}{}}{}$$

$$\frac{c}{2ax^2\left(a+bx^6\right)^{3/2}}$$

input
```
Int[(c + d*x^6)/(x^3*(a + b*x^6)^(5/2)),x]
```

output
```
-1/2*c/(a*x^2*(a + b*x^6)^(3/2)) - ((11*b*c - 2*a*d)*((2*x^4)/(9*a*(a + b*
x^6)^(3/2)) + (5*((2*x^4)/(3*a*Sqrt[a + b*x^6]) - (((2*Sqrt[a + b*x^6])/(b
^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]
*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(
2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7
 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/b^(1/3) - (2*(1 - Sqrt[3])
*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
 b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*x^6]))/
(3*a)))/(9*a)))/(4*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{3} \left(b\,x^{6}+a \right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.28

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{5/2}}\,dx = -\frac{5\left((11\,b^3c-2\,ab^2d)x^{14}+2\,(11\,ab^2c-2\,a^2bd)x^8+(11\,a^2bc-2\,a^3d)x^2\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierst}\right.}{54\,(a^3b^3x^{14}+2\,a^4b^2}$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/54*(5*((11*b^3*c - 2*a*b^2*d)*x^14 + 2*(11*a*b^2*c - 2*a^2*b*d)*x^8 + (
11*a^2*b*c - 2*a^3*d)*x^2)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassP
Inverse(0, -4*a/b, x^2)) + (5*(11*b^3*c - 2*a*b^2*d)*x^12 + 8*(11*a*b^2*c
- 2*a^2*b*d)*x^6 + 27*a^2*b*c)*sqrt(b*x^6 + a))/(a^3*b^3*x^14 + 2*a^4*b^2*
x^8 + a^5*b*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**3/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{5/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{5/2}}\,dx=\int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^3 (a+bx^6)^{5/2}}\,dx=\int \frac{d\,x^6+c}{x^3\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^3*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^3*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^3\left(a+bx^6\right)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-2\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{21}+3a\,b^2x^{15}+3a^2b\,x^9+a^3x^3}dx\right)a^3d\,x^2+11\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{21}+3a\,b^2x^{15}+3a^2b\,x^9}\right.}{}$$

input
```
int((d*x^6+c)/x^3/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 2*int(sqrt(a + b*x**6)/(a**3*x**3 + 3*a**2*b*x**9
 + 3*a*b**2*x**15 + b**3*x**21),x)*a**3*d*x**2 + 11*int(sqrt(a + b*x**6)/(
a**3*x**3 + 3*a**2*b*x**9 + 3*a*b**2*x**15 + b**3*x**21),x)*a**2*b*c*x**2
- 4*int(sqrt(a + b*x**6)/(a**3*x**3 + 3*a**2*b*x**9 + 3*a*b**2*x**15 + b**
3*x**21),x)*a**2*b*d*x**8 + 22*int(sqrt(a + b*x**6)/(a**3*x**3 + 3*a**2*b*
x**9 + 3*a*b**2*x**15 + b**3*x**21),x)*a*b**2*c*x**8 - 2*int(sqrt(a + b*x*
*6)/(a**3*x**3 + 3*a**2*b*x**9 + 3*a*b**2*x**15 + b**3*x**21),x)*a*b**2*d*
x**14 + 11*int(sqrt(a + b*x**6)/(a**3*x**3 + 3*a**2*b*x**9 + 3*a*b**2*x**1
5 + b**3*x**21),x)*b**3*c*x**14)/(11*b*x**2*(a**2 + 2*a*b*x**6 + b**2*x**1
2))
```

3.56 $\int \frac{c+dx^6}{x^9(a+bx^6)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 640

$$\int \frac{c+dx^6}{x^9(a+bx^6)^{5/2}}\,dx = -\frac{c}{8ax^8(a+bx^6)^{3/2}} - \frac{17bc-8ad}{72a^2x^2(a+bx^6)^{3/2}}$$

$$-\frac{11(17bc-8ad)}{216a^3x^2\sqrt{a+bx^6}} + \frac{55(17bc-8ad)\sqrt{a+bx^6}}{432a^4x^2} - \frac{55\sqrt[3]{b}(17bc-8ad)\sqrt{a+bx^6}}{432a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}$$

$$+\frac{55\sqrt{2-\sqrt{3}}\sqrt[3]{b}(17bc-8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right)|-7-4}{288\ 3^{3/4}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

$$-\frac{55\sqrt[3]{b}(17bc-8ad)\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2}\right),-7-4}{216\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}$$

output
```
-1/8*c/a/x^8/(b*x^6+a)^(3/2)-1/72*(-8*a*d+17*b*c)/a^2/x^2/(b*x^6+a)^(3/2)-
11/216*(-8*a*d+17*b*c)/a^3/x^2/(b*x^6+a)^(1/2)+55/432*(-8*a*d+17*b*c)*(b*x
^6+a)^(1/2)/a^4/x^2-55/432*b^(1/3)*(-8*a*d+17*b*c)*(b*x^6+a)^(1/2)/a^4/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x^2)+55/864*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(-
8*a*d+17*b*c)*(a^(1/3)+b^(1/3)*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*
x^4)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(
1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*3^(1/4)
/a^(11/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)
^2)^(1/2)/(b*x^6+a)^(1/2)-55/1296*b^(1/3)*(-8*a*d+17*b*c)*(a^(1/3)+b^(1/3)
*x^2)*((a^(2/3)-a^(1/3)*b^(1/3)*x^2+b^(2/3)*x^4)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x^2)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x^2),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(11/3)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x^2)^2)^(1/2)/(b*x^6+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.13

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{5/2}}\,dx=\frac{-2a^2c+(17bc-8ad)x^6(a+bx^6)\sqrt{1+\frac{bx^6}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{2},\frac{2}{3},-\frac{bx^6}{a}\right)}{16a^3x^8\left(a+bx^6\right)^{3/2}}$$

input
```
Integrate[(c + d*x^6)/(x^9*(a + b*x^6)^(5/2)),x]
```

output
```
(-2*a^2*c + (17*b*c - 8*a*d)*x^6*(a + b*x^6)*Sqrt[1 + (b*x^6)/a]*Hypergeom
etric2F1[-1/3, 5/2, 2/3, -((b*x^6)/a)])/(16*a^3*x^8*(a + b*x^6)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 1.06 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {955, 807, 819, 819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^6}{x^9 (a+bx^6)^{5/2}}dx$$

$$\Big\downarrow \text{955}$$

$$-\frac{(17bc-8ad)\int \frac{1}{x^3(bx^6+a)^{5/2}}dx}{8a}-\frac{c}{8ax^8 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(17bc-8ad)\int \frac{1}{x^4(bx^6+a)^{5/2}}dx^2}{16a}-\frac{c}{8ax^8 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(17bc-8ad)\left(\frac{11\int \frac{1}{x^4\left(bx^6+a\right)^{3/2}}dx^2}{9a}+\frac{2}{9ax^2(a+bx^6)^{3/2}}\right)}{16a}-\frac{c}{8ax^8 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{819}$$

$$-\frac{(17bc-8ad)\left(\frac{11\left(\frac{5\int \frac{1}{x^4\sqrt{bx^6+a}}dx^2}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{9a}+\frac{2}{9ax^2(a+bx^6)^{3/2}}\right)}{16a}-\frac{c}{8ax^8 (a+bx^6)^{3/2}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{(17bc-8ad)\left(\frac{11\left(\frac{5\left(\frac{b\int \frac{x^2}{\sqrt{bx^6+a}}dx^2}{2a}-\frac{\sqrt{a+bx^6}}{ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{9a}+\frac{2}{9ax^2(a+bx^6)^{3/2}}\right)}{16a}-\frac{c}{8ax^8\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 832

$$-\frac{(17bc-8ad)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^6}}{ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^6}}\right)}{9a}+\frac{2}{9ax^2(a+bx^6)^{3/2}}\right)}{16a}-\frac{c}{8ax^8\left(a+bx^6\right)^{3/2}}$$

$\downarrow$ 759

$$(17bc-8ad)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x^2+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^6+a}}dx^2}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\cdots\right.}{2a}\cdots\right.}{3a}\cdots\right.}{9a}\cdots\right.$$

↓ 2416

$$(17bc-8ad)\left(\frac{11\left(\frac{5\left(\frac{b\left(\frac{2\sqrt{a+bx^6}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x^2+b^{2/3}x^4}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x^2+(1-}{\sqrt[3]{b}x^2+(1-}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x^2\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x^2\right)^2}}\sqrt{a+bx^6}}\right.}{\sqrt[3]{b}}\right.}{}\right.}{}\right.$$

input
```
Int[(c + d*x^6)/(x^9*(a + b*x^6)^(5/2)),x]
```

output
```
-1/8*c/(a*x^8*(a + b*x^6)^(3/2)) - ((17*b*c - 8*a*d)*(2/(9*a*x^2*(a + b*x^
6)^(3/2)) + (11*(2/(3*a*x^2*Sqrt[a + b*x^6]) + (5*(-(Sqrt[a + b*x^6]/(a*x^
2)) + (b*(((2*Sqrt[a + b*x^6])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
^2)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x^2)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x^2)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)^2]*Sqrt[a + b*
x^6]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(
1/3)*x^2)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x^2 + b^(2/3)*x^4)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x^2)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x^2)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x^2))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x^2)^2]*Sqrt[a + b*x^6])))/(2*a)))/(3*a)))/(9*a)))/(16*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{d\,x^{6}+c}{x^{9} \left(b\,x^{6}+a \right)^{\frac{5}{2}}}d x$$

input
```
int((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x)
```

output
```
int((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.29

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{5/2}}\,dx = \frac{55\left(\left(17\,b^3c-8\,ab^2d\right)x^{20}+2\left(17\,ab^2c-8\,a^2bd\right)x^{14}+\left(17\,a^2bc-8\,a^3d\right)x^8\right)\sqrt{b}\mathrm{weierstr}}{}$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x, algorithm="fricas")
```

output
```
1/432*(55*((17*b^3*c - 8*a*b^2*d)*x^20 + 2*(17*a*b^2*c - 8*a^2*b*d)*x^14 +
 (17*a^2*b*c - 8*a^3*d)*x^8)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x^2)) + (55*(17*b^3*c - 8*a*b^2*d)*x^18 + 88*(17*a*b^
2*c - 8*a^2*b*d)*x^12 + 27*(17*a^2*b*c - 8*a^3*d)*x^6 - 54*a^3*c)*sqrt(b*x
^6 + a))/(a^4*b^2*x^20 + 2*a^5*b*x^14 + a^6*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**6+c)/x**9/(b*x**6+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx^6}{x^9 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^9}\,dx$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^9), x)
```

Giac [F]

$$\int \frac{c+dx^6}{x^9 (a+bx^6)^{5/2}}\,dx = \int \frac{dx^6+c}{(bx^6+a)^{\frac{5}{2}}x^9}\,dx$$

input
```
integrate((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x^6 + c)/((b*x^6 + a)^(5/2)*x^9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^6}{x^9 (a+bx^6)^{5/2}}\,dx = \int \frac{d\,x^6+c}{x^9\,(b\,x^6+a)^{5/2}}\,dx$$

input
```
int((c + d*x^6)/(x^9*(a + b*x^6)^(5/2)),x)
```

output
```
int((c + d*x^6)/(x^9*(a + b*x^6)^(5/2)), x)
```

Reduce [F]

$$\int \frac{c+dx^6}{x^9\left(a+bx^6\right)^{5/2}}\,dx = \frac{-\sqrt{b\,x^6+a}\,d-8\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{27}+3a\,b^2x^{21}+3a^2b\,x^{15}+a^3x^9}dx\right)a^3d\,x^8+17\left(\int \frac{\sqrt{b\,x^6+a}}{b^3x^{27}+3a\,b^2x^{21}+3a^2b\,x}\right.}{}$$

input
```
int((d*x^6+c)/x^9/(b*x^6+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**6)*d - 8*int(sqrt(a + b*x**6)/(a**3*x**9 + 3*a**2*b*x**1
5 + 3*a*b**2*x**21 + b**3*x**27),x)*a**3*d*x**8 + 17*int(sqrt(a + b*x**6)/
(a**3*x**9 + 3*a**2*b*x**15 + 3*a*b**2*x**21 + b**3*x**27),x)*a**2*b*c*x**
8 - 16*int(sqrt(a + b*x**6)/(a**3*x**9 + 3*a**2*b*x**15 + 3*a*b**2*x**21 +
 b**3*x**27),x)*a**2*b*d*x**14 + 34*int(sqrt(a + b*x**6)/(a**3*x**9 + 3*a*
*2*b*x**15 + 3*a*b**2*x**21 + b**3*x**27),x)*a*b**2*c*x**14 - 8*int(sqrt(a
 + b*x**6)/(a**3*x**9 + 3*a**2*b*x**15 + 3*a*b**2*x**21 + b**3*x**27),x)*a
*b**2*d*x**20 + 17*int(sqrt(a + b*x**6)/(a**3*x**9 + 3*a**2*b*x**15 + 3*a*
b**2*x**21 + b**3*x**27),x)*b**3*c*x**20)/(17*b*x**8*(a**2 + 2*a*b*x**6 +
b**2*x**12))
```

3.57 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=-\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2}+\frac{(c+dx^6)^{3/2}}{9bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

output
```
-1/3*(a*d+b*c)*(d*x^6+c)^(1/2)/b^2/d^2+1/9*(d*x^6+c)^(3/2)/b/d^2-1/3*a^2*a
rctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{\sqrt{c+dx^6}(-2bc-3ad+bdx^6)}{9b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(3*b^(5/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^{12}}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{6}\int \left(\frac{a^2}{b^2(bx^6+a)\sqrt{dx^6+c}}+\frac{\sqrt{dx^6+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^6+c}}\right)dx^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^6}(ad+bc)}{b^2d^2}+\frac{2\left(c+dx^6\right)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^6])/(b^2*d^2) + (2*(c + d*x^6)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/6
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{-a^{2} \arctan \left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{\left(a d -c b \right) b}}\right) d^{2}+\sqrt{d\,x^{6}+c}\,\sqrt{\left(a d -c b \right) b}\,\left(\frac{\left(-d\,x^{6}+2 c \right) b}{3}+a d \right)}{3 \sqrt{\left(a d -c b \right) b}\,b^{2} d^{2}}$	93

input
```
int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a^2*arctan((d*x^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))*d^2+(d*x^6+c)^(1/
2)*((a*d-b*c)*b)^(1/2)*(1/3*(-d*x^6+2*c)*b+a*d))/((a*d-b*c)*b)^(1/2)/b^2/d
^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left((b^3cd-ab^2d^2)x^6-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{}}{18\left(b^4cd^2-ab^3d^3\right)}\right.$$

input
```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d
*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6
 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 -
a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\frac{3\,a^2d^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^6+c)^{\frac{3}{2}}b^2-3\sqrt{dx^6+c}b^2c-3\sqrt{dx^6+c}abd}{b^3}}{9\,d^2}$$

input
```
integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/9*(3*a^2*d^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b^2) + ((d*x^6 + c)^(3/2)*b^2 - 3*sqrt(d*x^6 + c)*b^2*c - 3*sqrt
(d*x^6 + c)*a*b*d)/b^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{(d\,x^6+c)^{3/2}}{9\,b\,d^2} - \left(\frac{2\,c}{3\,b\,d^2} + \frac{3\,a\,d^3-3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\sqrt{d\,x^6+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*
b^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d -
 b*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{-2\sqrt{d\,x^6+c}\,c+\sqrt{d\,x^6+c}\,d\,x^6-9\left(\int \frac{\sqrt{d\,x^6+c}\,x^{11}}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx\right)a\,d^2}{9b\,d^2}$$

input
```
int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**6)*c + sqrt(c + d*x**6)*d*x**6 - 9*int((sqrt(c + d*x**
6)*x**11)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)*a*d**2)/(9*b*d**2)
```

3.58 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

output

```
1/3*(d*x^6+c)^(1/2)/b/d+1/3*a*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(
1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\left(\frac{\sqrt{c+dx^6}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input

```
Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output

```
(Sqrt[c + d*x^6]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{1}{6}\int \frac{x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \ 90$$

$$\frac{1}{6}\left(\frac{2\sqrt{c+dx^6}}{bd}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{b}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{6}\left(\frac{2\sqrt{c+dx^6}}{bd}-\frac{2a\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bd}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{6}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^6}}{bd}\right)$$

input
```
Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((2*Sqrt[c + d*x^6])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/6
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{6}+c}}{d}-\frac{a \arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{(a d-c b)b}}\right)}{\sqrt{(a d-c b)b}}}{3b}$	59

input
```
int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*(1/d*(d*x^6+c)^(1/2)-a/((a*d-b*c)*b)^(1/2)*arctan((d*x^6+c)^(1/2)*b/
((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{6\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-\sqrt{dx^6+c}(b^2c-abd)}{3\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}-\frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

input
```
integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^6 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c
)^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^{11}}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**11)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.59 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/3*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{3\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]]/(3*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{6}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{3d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
- a*d])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{\left(a d -c b \right) b}}\right)}{3 \sqrt{\left(a d -c b \right) b}}$	39

input
```
int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((a*d-b*c)*b)^(1/2)*arctan((d*x^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\,(b^2c-abd)}\right]$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(
b*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 12.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^6}{6a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^6 & \text{for } \sqrt{c}=0 \\ \frac{\log\left(6a\sqrt{c}+6b\sqrt{c}x^6\right)}{6b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**6/(6*a*sqrt(c)), Eq(b, 0)), (zoo*x**6, Eq
(sqrt(c), 0)), (log(6*a*sqrt(c) + 6*b*sqrt(c)*x**6)/(6*b*sqrt(c)), True)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}}$$

input
```
integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\mathrm{atan}\left(\frac{b\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2)
)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^5}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**5)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.60 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

output
```
-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/3*b^(1/2)*arctanh(b^(1/2
)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

input
```
Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/3*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c])/a
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{1}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{6}\left(\frac{\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{2\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/6
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b \arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{(a d-c b) b}}\right)}{\sqrt{(a d-c b) b}}}{3a}$	65

input
```
int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*(-arctanh((d*x^6+c)^(1/2)/c^(1/2))/c^(1/2)-b/((a*d-b*c)*b)^(1/2)*arc
tan((d*x^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.53

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)}{6\,ac},\right.$$

$$-\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^6+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{c}\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)}{6\,ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{}}{b}\right.}{}$$

$$\left.-\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^6+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^6+c}}\right)}{3\,ac}\right]$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), -1/6*(2*c*sqrt(-b/(b*c - a*d))*a
rctan(sqrt(d*x^6 + c)*sqrt(-b/(b*c - a*d))) - sqrt(c)*log((d*x^6 - 2*sqrt(
d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(b/(b*c - a*d))*log((b*d
*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^6 + c)))/(a*c), -1/3*(c*s
qrt(-b/(b*c - a*d))*arctan(sqrt(d*x^6 + c)*sqrt(-b/(b*c - a*d))) - sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^6 + c)))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 10.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \begin{cases} \frac{2\left(-\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{6a\sqrt{\frac{ad-bc}{b}}}+\frac{d\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{6a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+x^6\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{3b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(6*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**6)/sqrt(-c))/(6*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**6)/sqrt(-a**2/b**2))/(3*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x}\,dx$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a} + \frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = -\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)}{3\,a\,\sqrt{c}} - \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6\cdots}}{27}\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6\cdots}}{27}\cdots\right.}{\cdots}}\right)\cdots}{3\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
- atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*
b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c
- a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)))*1i)/(a^2*d - a
*b*c) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c
 - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(
c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d -
 a*b*c)))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*
x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d
^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d
 - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2
)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d
^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b
*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c)))*
(b^2*c - a*b*d)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c}\log\left(\sqrt{d\,x^6+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^6+c}+\sqrt{c}\right)-6\left(\int \frac{\sqrt{d\,x^6+c}\,x^5}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx\right)bc}{6ac}$$

input
```
int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**6) +
sqrt(c)) - 6*int((sqrt(c + d*x**6)*x**5)/(a*c + a*d*x**6 + b*c*x**6 + b*d*
x**12),x)*b*c)/(6*a*c)
```

3.61 $\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

output
```
-1/6*(d*x^6+c)^(1/2)/a/c/x^6+1/6*(a*d+2*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/
2))/a^2/c^(3/2)-1/3*b^(3/2)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/
2))/a^2/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^7(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{-\frac{a\sqrt{c+dx^6}}{cx^6}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^2}$$

input
```
Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-((a*Sqrt[c + d*x^6])/(c*x^6)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
6])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6} \int \frac{1}{x^{12} (bx^6+a) \sqrt{dx^6+c}} dx^6$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{6} \left(-\frac{\int \frac{bdx^6+2bc+ad}{2x^6 (bx^6+a)\sqrt{dx^6+c}} dx^6}{ac} - \frac{\sqrt{c+dx^6}}{acx^6} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6} \left(-\frac{\int \frac{bdx^6+2bc+ad}{x^6 (bx^6+a)\sqrt{dx^6+c}} dx^6}{2ac} - \frac{\sqrt{c+dx^6}}{acx^6} \right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{6} \left(-\frac{\frac{(ad+2bc) \int \frac{1}{x^6 \sqrt{dx^6+c}} dx^6}{a} - \frac{2b^2 c \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}} dx^6}{a}}{2ac} - \frac{\sqrt{c+dx^6}}{acx^6} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-\frac{\frac{2(ad+2bc)\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{4b^2c\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(-\frac{\frac{4b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^6)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{6}+c}}{c\,x^{6}}+\frac{(ad+2cb)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2b^{2}\arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{6a^{2}}$	92

input
```
int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(-a/c*(d*x^6+c)^(1/2)/x^6+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^6+c)^(1
/2)/c^(1/2))+2*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x^6+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.43

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}} \, dx$$
$$= \left[\frac{2\,bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+(2\,bc+ad)\sqrt{c}x^6\log\left(\frac{dx^6+2\,\sqrt{dx^6+c}\sqrt{c}+2\,c}{x^6}\right)-2\,\sqrt{}}{12\,a^2c^2x^6} \right.$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt
(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*
sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6), 1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan
(sqrt(d*x^6 + c)*sqrt(-b/(b*c - a*d))) + (2*b*c + a*d)*sqrt(c)*x^6*log((d*
x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6 + c)*a*c)/(a^2*
c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d -
2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) - (2*b*c +
 a*d)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^6 + c)) - sqrt(d*x^6 + c)*a*c)
/(a^2*c^2*x^6), 1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^6 +
c)*sqrt(-b/(b*c - a*d))) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt
(d*x^6 + c)) - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^7 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

input
```
integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\frac{b^2\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}c}-\frac{\sqrt{dx^6+c}}{6\,acx^6}$$

input
```
integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^7 (a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{\ln\left(\sqrt{d\,x^6+c}\,(b^4\,c-a\,b^3\,d)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}$$
$$-\frac{\ln\left(\sqrt{d\,x^6+c}\,(b^4\,c-a\,b^3\,d)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,(a^3\,d-a^2\,b\,c)}$$
$$-\frac{\sqrt{d\,x^6+c}}{6\,a\,c\,x^6}$$
$$-\frac{\operatorname{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{18\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,c}+\frac{5\,a\,b^3\,d^5}{108\,c^2}+\frac{a^2\,b^2\,d^6}{108\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^6+c}\,1\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{108\,a}+\frac{b^2\,d^6}{108\,c}+\frac{b^4\,c\,d^4}{18\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^6+c}\,5\mathrm{i}}{108\,\sqrt{c^3}\,\left(\frac{b^4\,d^4}{18\,a}+\frac{5\,b^3\,d^5}{108\,c}+\frac{a\,b^2\,d^6}{108\,c^2}\right)}\right)\,(a\,d+2\,b\,c}{6\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
(log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x
^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x
^6) - (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*
c) + (5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d
*x^6)^(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c)
+ (b^4*c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)
*((b^4*d^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d +
 2*b*c)*1i)/(6*a^2*(c^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^7 \left(a+bx^6\right) \sqrt{c+dx^6}}\, dx = \frac{-2\sqrt{d\,x^6+c}\,a^2cd+4\sqrt{d\,x^6+c}\,b^2c^2x^6-\sqrt{c}\log\left(\sqrt{d\,x^6+c}-\sqrt{c}\right)a^2d^2x^6-2\sqrt{c}\log\left(\sqrt{d\,x^6+c}-\sqrt{c}\right)a\ldots}{}$$

input
```
int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**6)*a**2*c*d + 4*sqrt(c + d*x**6)*b**2*c**2*x**6 - sqrt
(c)*log(sqrt(c + d*x**6) - sqrt(c))*a**2*d**2*x**6 - 2*sqrt(c)*log(sqrt(c
+ d*x**6) - sqrt(c))*a*b*c*d*x**6 + sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c)
)*a**2*d**2*x**6 + 2*sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*a*b*c*d*x**6
- 12*int((sqrt(c + d*x**6)*x**11)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),
x)*b**3*c**2*d*x**6)/(12*a**3*c**2*d*x**6)
```

3.62 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

output
```
1/6*x^3*(d*x^6+c)^(1/2)/b/d+1/3*a^(3/2)*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2
)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(1/2)-1/6*(2*a*d+b*c)*arctanh(d^(1/2)*x^
3/(d*x^6+c)^(1/2))/b^2/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\frac{bx^3\sqrt{c+dx^6}}{d} + \frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{(bc+2ad)\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{d^{3/2}}}{6b^2}$$

input
```
Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]
*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{3}\int \frac{x^{12}}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{ 381}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\int \frac{(bc+2ad)x^6+ac}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2bd}\right)$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}-\frac{2a^2d\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{2a^2d\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}}{2bd}\right)$$

$\Big\downarrow$ 291

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}}{2bd}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{3}\left(\frac{x^3\sqrt{c+dx^6}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input
```
Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
((x^3*Sqrt[c + d*x^6])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]])/(b*Sqrt[d]))/(2*b*d))/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{-\frac{2a^2 \operatorname{arctanh}\left(\frac{a\sqrt{d\,x^6+c}}{x^3\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\frac{\sqrt{d\,x^6+c}\,b\,x^3}{d}+\frac{(2ad+cb)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{x^3\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{6b^2}$	98

input
```
int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^2*(-2*a^2/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d
-b*c))^(1/2))-(d*x^6+c)^(1/2)*b/d*x^3+(2*a*d+b*c)/d^(3/2)*arctanh((d*x^6+c
)^(1/2)/x^3/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 739, normalized size of antiderivative = 6.01

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{2\sqrt{dx^6+c}bdx^3 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(ab\right.}{b^2x^{12}+2\,abx^6+a^2}}{12\,b^2d^2} \right.$$

input
```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2
+ 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt
(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2)
, 1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2
 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqr
t(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c
+ 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2
*sqrt(d*x^6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)
) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c
))/(b^2*d^2), 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arc
tan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^9 + a*c*x^3)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6
+ c)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.24 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.93

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = Recursiveassumptionc \geq \frac{\sqrt{dx^6+c}x^3}{6\,bd} - \frac{a^2\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}b^2\mathrm{sgn}(x)} + \frac{\left(2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - \sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) - 2\sqrt{abc-a^2d}ad\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x}{6\sqrt{abc-a^2d}b^2\sqrt{-d}d} + \frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{6\,b^2\sqrt{-d}d\mathrm{sgn}(x)} - \frac{dignored}{t_{nostep}^6}$$

input
```
integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Recursive*assumption*c >= 1/6*sqrt(d*x^6 + c)*x^3/(b*d) - 1/3*a^2*arctan(a
*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b^2*sgn(x)) + 1
/6*(2*a^2*sqrt(-d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c -
a^2*d)*b*c*arctan(sqrt(d)/sqrt(-d)) - 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqr
t(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^2*sqrt(-d)*d) + 1/6*(b*c + 2
*a*d)*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b^2*sqrt(-d)*d*sgn(x)) - d*ignored
/t_nostep^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{2\sqrt{d\,x^6+c}\,bd\,x^3+2\sqrt{d}\log\left(\sqrt{d\,x^6+c}-\sqrt{d}\,x^3\right)ad+\sqrt{d}\log\left(\sqrt{d\,x^6+c}-\sqrt{d}\,x^3\right)bc-2\sqrt{d}\log\left(\sqrt{d\,x^6}\right.}{12b^2d^2}$$

input
```
int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x**6)*b*d*x**3 + 2*sqrt(d)*log(sqrt(c + d*x**6) - sqrt(d)*x*
*3)*a*d + sqrt(d)*log(sqrt(c + d*x**6) - sqrt(d)*x**3)*b*c - 2*sqrt(d)*log
(sqrt(c + d*x**6) + sqrt(d)*x**3)*a*d - sqrt(d)*log(sqrt(c + d*x**6) + sqr
t(d)*x**3)*b*c + 12*int((sqrt(c + d*x**6)*x**2)/(a*c + a*d*x**6 + b*c*x**6
 + b*d*x**12),x)*a**2*d**2)/(12*b**2*d**2)
```

3.63 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}$$

output
```
-1/3*a^(1/2)*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/b/(-a*d+
b*c)^(1/2)+1/3*arctanh(d^(1/2)*x^3/(d*x^6+c)^(1/2))/b/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{3b}$$

input
```
Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^3 + Sqrt[c + d*x
^6]]/Sqrt[d])/(3*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{385}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}\right)$$

input
```
Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(b*Sqrt[d]))/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\frac{a\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)}{\sqrt{d}}}{3b}$	70

input
```
int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*(a/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^
(1/2))-1/d^(1/2)*arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.95

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-\cdots}}}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,bd}\right.$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d))
)/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 +
c)*sqrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^
6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*ar
ctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d
*x^9 + a*c*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 -
c))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c
)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*ar
ctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}} + \frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b\,\mathrm{sgn}(x)} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\,\mathrm{sgn}(x)}$$

input
```
integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*
d)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3
*a*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sg
n(x)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{-\sqrt{d}\log\left(\sqrt{d\,x^6+c}-\sqrt{d}\,x^3\right)+\sqrt{d}\log\left(\sqrt{d\,x^6+c}+\sqrt{d}\,x^3\right)-6\left(\int \frac{\sqrt{d\,x^6+c}\,x^2}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx\right)ad}{6bd}$$

input
```
int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
( - sqrt(d)*log(sqrt(c + d*x**6) - sqrt(d)*x**3) + sqrt(d)*log(sqrt(c + d*
x**6) + sqrt(d)*x**3) - 6*int((sqrt(c + d*x**6)*x**2)/(a*c + a*d*x**6 + b*
c*x**6 + b*d*x**12),x)*a*d)/(6*b*d)
```

3.64 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/3*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(3*Sqrt[a]*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \ 965$$

$$\frac{1}{3}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \ 291$$

$$\frac{1}{3}\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*
c - a*d])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{3\sqrt{a(ad-cb)}}$	42

input
```
int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.11 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2-4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\,(abc-a^2d)}, \operatorname{arcta}\right.$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*
sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c
 - a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a
*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*
b*c - a^2*d)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{3\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^2}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**2)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.65 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/3*(d*x^6+c)^(1/2)/a/c/x^3-1/3*b*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*
x^6+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{c+dx^6}}{3acx^3}-\frac{b\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/3*Sqrt[c + d*x^6]/(a*c*x^3) - (b*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3
 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(3*a^(3/2)*Sqrt[b*c - a*d
])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^6+a) \sqrt{dx^6+c}} dx^3$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{3} \left(\frac{\int -\frac{bc}{(bx^6+a)\sqrt{dx^6+c}} dx^3}{ac} - \frac{\sqrt{c+dx^6}}{acx^3} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(-\frac{\int \frac{bc}{(bx^6+a)\sqrt{dx^6+c}} dx^3}{ac} - \frac{\sqrt{c+dx^6}}{acx^3} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3} \left(-\frac{b \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}} dx^3}{a} - \frac{\sqrt{c+dx^6}}{acx^3} \right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3} \left(-\frac{b \int \frac{1}{a-(ad-bc)x^6} d\frac{x^3}{\sqrt{dx^6+c}}}{a} - \frac{\sqrt{c+dx^6}}{acx^3} \right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(-\frac{b\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{acx^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^3)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*S
qrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}}{x^{3}}-\frac{bc\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{3ac}$	67

input
```
int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a*(-(d*x^6+c)^(1/2)/x^3-b*c/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1
/2)/x^3/(a*(a*d-b*c))^(1/2)))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.

Time = 0.13 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^4 (a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \left[-\frac{\sqrt{-abc+a^2d}bcx^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4\,((bc-2\,ad)x^9-acx^3)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+}{12\,(a^2bc^2-a^3cd)x^3}\right.$$
$$\left.-\frac{\sqrt{abc-a^2d}bcx^3\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)+2\,\sqrt{dx^6+c}(abc-a^2d)}{6\,(a^2bc^2-a^3cd)x^3}\right]$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2
)) + 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*
(sqrt(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x
^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)
*x^3)) + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx$$

input
```
integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}}\, dx = \int \frac{1}{x^4\,(b\,x^6 + a)\,\sqrt{d\,x^6 + c}}\, dx$$

input
```
int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^6) \sqrt{c + dx^6}}\, dx = \int \frac{\sqrt{d\,x^6 + c}}{bd\,x^{16} + ad\,x^{10} + bc\,x^{10} + ac\,x^4} dx$$

input
```
int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a*c*x**4 + a*d*x**10 + b*c*x**10 + b*d*x**16),x)
```

3.66 $\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

output
```
-1/9*(d*x^6+c)^(1/2)/a/c/x^9+1/9*(2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c^2/x^3
+1/3*b^2*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*
d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}\left(-ac+3bcx^6+2adx^6\right)}{9a^2c^2x^9} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{3a^{5/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]*(-(a*c) + 3*b*c*x^6 + 2*a*d*x^6))/(9*a^2*c^2*x^9) + (b^2*
ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(3*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3} \int \frac{1}{x^{12} (bx^6+a) \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{3} \left(\frac{\int -\frac{2bdx^6+3bc+2ad}{x^6 (bx^6+a) \sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(-\frac{\int \frac{2bdx^6+3bc+2ad}{x^6 (bx^6+a) \sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3} \left(-\frac{-\frac{\int \frac{3b^2c^2}{(bx^6+a) \sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6} (2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(2ad+3bc)}{acx^3}}{3ac}-\frac{\sqrt{c+dx^6}}{3acx^9}\right)$$

input
```
Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-1/3*Sqrt[c + d*x^6]/(a*c*x^9) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(a*
c*x^3)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]
)/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 6.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,\left(-2ad\,x^{6}-3bc\,x^{6}+ac\right)}{3x^{9}}+\frac{b^{2}c^{2}\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a\left(ad-cb\right)}}\right)}{\sqrt{a\left(ad-cb\right)}}}{3a^{2}c^{2}}$	88

input
```
int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-1/3*(d*x^6+c)^(1/2)*(-2*a*d*x^6-3*b*c*x^6+a*c)/x^9+b^2*c^2/(a*(a
*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2)))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^9\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left(\left(bc-2\,ad\right)x^9-acx^3\right)\sqrt{dx^6+c}\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right.}{36\left(a^3bc^3-a^4c^2d\right)x^9}\right.$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*
x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6
 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*
c*d)*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c -
a^2*d)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sq
rt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2
*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*
x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \int \frac{1}{x^{10}\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{-\sqrt{d\,x^6+c}\,c+2\sqrt{d\,x^6+c}\,d\,x^6-9\left(\int \frac{\sqrt{d\,x^6+c}}{bd\,x^{16}+ad\,x^{10}+bc\,x^{10}+ac\,x^4}dx\right)b\,c^2x^9}{9a\,c^2x^9}$$

input
```
int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**6)*c + 2*sqrt(c + d*x**6)*d*x**6 - 9*int(sqrt(c + d*x**6
)/(a*c*x**4 + a*d*x**10 + b*c*x**10 + b*d*x**16),x)*b*c**2*x**9)/(9*a*c**2
*x**9)
```

3.67 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

output
```
1/5*x^5*(1+d*x^6/c)^(1/2)*AppellF1(5/6,1,1/2,11/6,-b*x^6/a,-d*x^6/c)/a/(d*
x^6+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx=\frac{x^5\sqrt{\frac{c+dx^6}{c}}\operatorname{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

input
```
Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^
6)/a)])/(5*a*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^4}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{5}{6},1,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

input
```
Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^
6)/c)])/(5*a*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}dx$$

input
```
int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^4}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**4)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.68 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

output
```
1/4*x^4*(1+d*x^6/c)^(1/2)*AppellF1(2/3,1,1/2,5/3,-b*x^6/a,-d*x^6/c)/a/(d*x
^6+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

input
```
Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6
)/a)])/(4*a*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^6+a)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^2}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},1,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

input
```
Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6
)/c)])/(4*a*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}dx$$

input
```
int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^3}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**3)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.69 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}} dx = \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

output
```
1/2*x^2*(1+d*x^6/c)^(1/2)*AppellF1(1/3,1,1/2,4/3,-b*x^6/a,-d*x^6/c)/a/(d*x
^6+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}} dx = \frac{x^2\sqrt{\frac{c+dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

input
```
Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6
)/a)])/(2*a*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {965, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},1,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

input
```
Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6
)/c)])/(2*a*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{(b\,x^{6}+a)\,\sqrt{d\,x^{6}+c}}dx$$

input
```
int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{x}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(x/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.70 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

output
```
x*(1+d*x^6/c)^(1/2)*AppellF1(1/6,1,1/2,7/6,-b*x^6/a,-d*x^6/c)/a/(d*x^6+c)^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx =$$

$$-\frac{7acx\operatorname{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{(a+bx^6)\sqrt{c+dx^6}\left(-7ac\operatorname{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3x^6\left(2bc\operatorname{AppellF1}\left(\frac{7}{6},\frac{1}{2},2,\frac{13}{6},-\frac{dx^6}{c},-\frac{b}{}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*
x^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b
*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^
6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \; 937$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \; 936$$

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},1,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

input
```
Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{6}+a)\,\sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^6)\,\sqrt{c+dx^6}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

Sympy [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{1}{(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}}{bd\,x^{12}+ad\,x^6+bc\,x^6+ac}dx$$

input
```
int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a*c + a*d*x**6 + b*c*x**6 + b*d*x**12),x)
```

3.71 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

output
```
-(1+d*x^6/c)^(1/2)*AppellF1(-1/6,1,1/2,5/6,-b*x^6/a,-d*x^6/c)/a/x/(d*x^6+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$=\frac{-55a(c+dx^6)-11(bc-2ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+10bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{Appel}}{55a^2cx\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6
, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)
/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*
Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1} \int \frac{1}{x^2 (bx^6+a) \sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1} \operatorname{AppellF1}\left(-\frac{1}{6},1,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

input
```
Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)
/c)])/(a*x*Sqrt[c + d*x^6]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{6}+a \right) \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx=\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\int \frac{\sqrt{d\,x^6+c}}{bd\,x^{14}+ad\,x^8+bc\,x^8+ac\,x^2}dx$$

input
```
int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a*c*x**2 + a*d*x**8 + b*c*x**8 + b*d*x**14),x)
```

3.72 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

output
```
-1/2*(1+d*x^6/c)^(1/2)*AppellF1(-1/3,1,1/2,2/3,-b*x^6/a,-d*x^6/c)/a/x^2/(d
*x^6+c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$$
$$= \frac{-20a(c+dx^6)+5(-2bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bdx^{12}\sqrt{1+\frac{dx^6}{c}}\,\text{Appell}}{40a^2cx^2\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sq
rt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^6+a) \sqrt{dx^6+c}} dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1} \int \frac{1}{x^4 (bx^6+a) \sqrt{\frac{dx^6}{c}+1}} dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1} \, \text{AppellF1}\left(-\frac{1}{3},1,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

input
```
Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a*x^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^3\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}dx$$

input
```
int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} \, dx$$

input
```
integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^6) \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+cx^3}} \, dx$$

input
```
integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^3 \left(b\,x^6+a\right) \sqrt{d\,x^6+c}} \, dx$$

input
```
int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^6) \sqrt{c+dx^6}} \, dx = \int \frac{\sqrt{d\,x^6+c}}{bd\,x^{15}+ad\,x^9+bc\,x^9+ac\,x^3} dx$$

input
```
int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a*c*x**3 + a*d*x**9 + b*c*x**9 + b*d*x**15),x)
```

3.73 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

output
```
-1/4*(1+d*x^6/c)^(1/2)*AppellF1(-2/3,1,1/2,1/3,-b*x^6/a,-d*x^6/c)/a/x^4/(d
*x^6+c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$
$$=\frac{-8a(c+dx^6)-4(4bc+ad)x^6\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-bdx^{12}\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\right.}{32a^2cx^4\sqrt{c+dx^6}}$$

input
```
Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
(-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3,
1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Ap
pellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c
 + d*x^6])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^6\left(bx^6+a\right)\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},1,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

input
```
Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

output
```
-1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a*x^4*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b \,x^{6}+a \right) \sqrt{d \,x^{6}+c}}d x$$

input
```
int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^6) \sqrt{c + dx^6}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a + bx^6) \sqrt{c + dx^6}}\, dx = \int \frac{1}{x^5 (a + bx^6) \sqrt{c + dx^6}}\, dx$$

input
```
integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^6) \sqrt{c + dx^6}}\, dx = \int \frac{1}{(bx^6 + a)\sqrt{dx^6 + c}x^5}\, dx$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^5\,(b\,x^6+a)\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^6) \sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}}{bd\,x^{17}+ad\,x^{11}+bc\,x^{11}+ac\,x^5}dx$$

input
```
int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a*c*x**5 + a*d*x**11 + b*c*x**11 + b*d*x**17),x)
```

3.74 $\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\,(a+bx^6)} + \frac{a(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/3*(d*x^6+c)^(1/2)/b^2/d-1/6*a^2*(d*x^6+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^6+a)
+1/6*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^6}(-3a^2d+2b^2cx^6+2ab(c-dx^6))}{d(bc-ad)(a+bx^6)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{5/2}}$$

input
```
Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^6]*(-3*a^2*d + 2*b^2*c*x^6 + 2*a*b*(c - d*x^6)))/(d
*(b*c - a*d)*(a + b*x^6)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(6*b^(5/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^{12}}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{6}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^6}{2(bx^6+a)\sqrt{dx^6+c}}dx^6}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{6}\left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{d}-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^6}}{b^2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(-\frac{a^2\sqrt{c+dx^6}}{b^2\,(a+bx^6)\,(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^6}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^6])/(b^2*(b*c - a*d)*(a + b*x^6))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^6])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-(b x^6+a) d a \left(a d-\frac{4 c b}{3}\right) \arctan\left(\frac{\sqrt{d x^6+c}\, b}{\sqrt{(a d-c b) b}}\right)+\sqrt{(a d-c b) b}\,\sqrt{d x^6+c}\left(-\frac{2 b^2 c x^6}{3}-\frac{2 a \left(-d x^6+c\right) b}{3}+a^2 d\right)}{2 \sqrt{(a d-c b) b}\, d b^2 (a d-c b) (b x^6+a)}$	133

input
```
int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*(-(b*x^6+a)*d*a*(a*d-4/3*c*b)*arctan((d*x^6+c)^(1/
2)*b/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(d*x^6+c)^(1/2)*(-2/3*b^2*c*
x^6-2/3*a*(-d*x^6+c)*b+a^2*d))/d/b^2/(a*d-b*c)/(b*x^6+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.12 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,(2\,(b^4c^2-\cdots}{12\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+\cdots}\right.$$
$$-\frac{((4\,ab^2cd-3\,a^2bd^2)x^6+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)-(2\,(b^4c^2-2\,ab^3cd\cdots}{6\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2\cdots}$$

input
```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^
2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a
*b*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a
*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*
a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^
6), -1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt
(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*
c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b
^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^
3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\frac{\sqrt{dx^6+c}a^2d^3}{(b^3c-ab^2d)((dx^6+c)b-bc+ad)} + \frac{(4\,abcd^2-3\,a^2d^3)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c-ab^2d)\sqrt{-b^2c+abd}} - \frac{2\sqrt{dx^6+c}d}{b^2}}{6\,d^2}$$

input
```
integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(sqrt(d*x^6 + c)*a^2*d^3/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*
d)) + (4*a*b*c*d^2 - 3*a^2*d^3)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b
*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) - 2*sqrt(d*x^6 + c)*d/b^2)/d
^2
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{d\,x^6+c}}{3\,b^2\,d} - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{6\,b^{5/2}\,(a\,d-b\,c)^{3/2}} + \frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,(3\,b^3\,(d\,x^6+c)-3\,b^3\,c+3\,a\,b^2\,d)}$$

input
```
int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*
(c + d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

Reduce [F]

$$\int \frac{x^{17}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{-2\sqrt{d\,x^6+c}\,ac+\sqrt{d\,x^6+c}\,ad\,x^6-2\sqrt{d\,x^6+c}\,bc\,x^6-9\left(\int \frac{\sqrt{d\,x^6+c}\,x^{11}}{a\,b^2d^2x^{18}-2b^3cd\,x^{18}+2a^2b\,d^2x^{12}-3a\,b^2cd\,x^{12}-2b^3c^2x^{12}+}\right.}{}$$

input
```
int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**6)*a*c + sqrt(c + d*x**6)*a*d*x**6 - 2*sqrt(c + d*x**6
)*b*c*x**6 - 9*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2
*a**2*b*c**2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**
12 + a*b**2*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**4*d**
3 + 30*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*
c**2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b
**2*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**3*b*c*d**2 -
9*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d
**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**3*b*d**3*x**6 - 24
*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2 +
 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d*
*2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**2*b**2*c**2*d + 30*
int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2 +
2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d**
2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**2*b**2*c*d**2*x**6 -
 24*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**
2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2
*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a*b**3*c**2*d*x**6)
/(3*b*d*(a**2*d - 2*a*b*c + a*b*d*x**6 - 2*b**2*c*x**6))
```

3.75 $\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\,(a+bx^6)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

output
```
1/6*a*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)-1/6*(-a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{6b^{3/2}}$$

input
```
Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(6*b^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{x^6}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{6}\left(\frac{(2bc-ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2b(bc-ad)}+\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{a\sqrt{c+dx^6}}{b(a+bx^6)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[c + d*x^6])/(b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/6
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,a}{b\,x^{6}+a}+\frac{(a d-2 c b) \arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{(a d-c b) b}}\right)}{\sqrt{(a d-c b) b}}}{6(a d-c b) b}$	83

input
```
int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(a*d-b*c)/b*(-(d*x^6+c)^(1/2)*a/(b*x^6+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan((d*x^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[\frac{((2\,b^2c-abd)x^6+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(ab^2c-a^2b}{12\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^6)}\right.$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)
) + 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^
3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a
*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)
)/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*
b^3*d^2)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{\sqrt{dx^6+c}ad^2}{(b^2c-abd)((dx^6+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{6\,d}$$

input
```
integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{6\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,(a\,d-b\,c)\,(3\,b\,(d\,x^6+c)+3\,a\,d-3\,b\,c)}$$

input
```
int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(atan(((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c
 + d*x^6) + 3*a*d - 3*b*c))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^{11}}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**11)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*
d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.76 $\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}(bc-ad)^{3/2}}$$

output
```
-1/6*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)+1/6*d*arctanh(b^(1/2)*(d*x^6+c)^
(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\left(-\frac{\sqrt{c+dx^6}}{(bc-ad)\,(a+bx^6)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{6}\int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\left(-\frac{d\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^6}{2(bc-ad)}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\left(-\frac{\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{bc-ad}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\left(\frac{d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{(a+bx^6)(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/((b*c - a*d)*(a + b*x^6))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/6
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{6}+c}}{b\,x^{6}+a}+\frac{d \arctan\left(\frac{\sqrt{d\,x^{6}+c}\,b}{\sqrt{(a d -c b) b}}\right)}{\sqrt{(a d -c b) b}}}{6 a d -6 c b}$	71

input
```
int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(a*d-b*c)*((d*x^6+c)^(1/2)/(b*x^6+a)+d/((a*d-b*c)*b)^(1/2)*arctan((d*x
^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(71) = 142$.

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \left[-\frac{(bdx^6+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}(b^2c-abd)}{12\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}, -\frac{(bdx^6+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^6+c}\sqrt{-b^2c+abd}}{bdx^6+bc}\right)+\sqrt{dx^6+c}(b^2c-abd)}{6\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2
*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^
2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a
^2*b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(
b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^6+c}d}{6\left((dx^6+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{d\sqrt{d\,x^6+c}}{2\,(a\,d-b\,c)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

input
```
int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) +
(d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d -
b*c)^(3/2))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^5}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**5)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d
*x**12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.77 $\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{b\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}$$

output
```
1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)-1/3*arctanh((d*x^6+c)^(1/2)/c
^(1/2))/a^2/c^(1/2)+1/6*b^(1/2)*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(
1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{-\frac{ab\sqrt{c+dx^6}}{(-bc+ad)(a+bx^6)} + \frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

input
```
Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^6])/((-(b*c) + a*d)*(a + b*x^6))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c])/(6*a^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{6}\int \frac{1}{x^6\left(bx^6+a\right)^2\sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \; 114$$

$$\frac{1}{6}\left(\frac{\int \frac{bdx^6+2bc-2ad}{2x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(\frac{\int \frac{bdx^6+2(bc-ad)}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{6}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{6}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{a\left(a+bx^6\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((b*Sqrt[c + d*x^6])/(a*(b*c - a*d)*(a + b*x^6)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result	size
pseudoelliptic	$\frac{(b\,x^6+a)\left(cb-\frac{3ad}{2}\right)b\sqrt{c}\,\arctan\left(\frac{\sqrt{d\,x^6+c}\,b}{\sqrt{(ad-cb)b}}\right)-\frac{\sqrt{(ad-cb)b}\left(2(ad-cb)\left(b\,x^6+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)+\sqrt{d\,x^6+c}\,\sqrt{c}\,ab\right)}{2}}{3\sqrt{c}\,\sqrt{(ad-cb)b}\,a^2(ad-cb)(b\,x^6+a)}$	146

input
```
int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/c^(1/2)/((a*d-b*c)*b)^(1/2)*((b*x^6+a)*(c*b-3/2*a*d)*b*c^(1/2)*arctan(
(d*x^6+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))-1/2*((a*d-b*c)*b)^(1/2)*(2*(a*d-b*c
)*(b*x^6+a)*arctanh((d*x^6+c)^(1/2)/c^(1/2))+(d*x^6+c)^(1/2)*c^(1/2)*a*b))
/a^2/(a*d-b*c)/(b*x^6+a)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.18

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2
- 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6
 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x
^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/
x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x
^6 + c)*a*b*c - ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt
(-b/(b*c - a*d))*arctan(sqrt(d*x^6 + c)*sqrt(-b/(b*c - a*d))) + ((b^2*c -
a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c)
 + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/12*
(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^6 + c)) + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*
c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d
*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))/((a^2*b^2*c^2 - a
^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c)*a*b*c - ((2*b^2
*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan
(sqrt(d*x^6 + c)*sqrt(-b/(b*c - a*d))) + 2*((b^2*c - a*b*d)*x^6 + a*b*c -
a^2*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^6 + c)))/((a^2*b^2*c^2 - a^3*b*c*
d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{\sqrt{dx^6+c}bd}{6\,(abc-a^2d)((dx^6+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/
6*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 3025, normalized size of antiderivative = 22.92

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2
*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c
*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7
*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))
/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3
*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(1
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^
2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d
^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a
^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^
3)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*
a^5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^
2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c
^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)
/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b...
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**6)*a*b*c + sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c))*a**
2*d - 2*sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c))*a*b*c + sqrt(c)*log(sqrt(c
 + d*x**6) - sqrt(c))*a*b*d*x**6 - 2*sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c
))*b**2*c*x**6 - sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*a**2*d + 2*sqrt(c
)*log(sqrt(c + d*x**6) + sqrt(c))*a*b*c - sqrt(c)*log(sqrt(c + d*x**6) + s
qrt(c))*a*b*d*x**6 + 2*sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*b**2*c*x**6
 - 18*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c
**2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b*
*2*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**3*b**2*c*d**2
+ 48*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c*
*2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**
2*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**2*b**3*c**2*d -
 18*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**
2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2
*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**2*b**3*c*d**2*x*
*6 - 24*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b
*c**2 + 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*
b**2*d**2*x**18 - 2*b**3*c**2*x**12 - 2*b**3*c*d*x**18),x)*a*b**4*c**3 + 4
8*int((sqrt(c + d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**...
```

3.78 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)(a+bx^6)} - \frac{\sqrt{c+dx^6}}{6acx^6(a+bx^6)} + \frac{(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}$$

output
```
-1/6*b*(-a*d+2*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^6+a)-1/6*(d*x^6+
c)^(1/2)/a/c/x^6/(b*x^6+a)+1/6*(a*d+4*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2)
)/a^3/c^(3/2)-1/6*b^(3/2)*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(
-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{\frac{a\sqrt{c+dx^6}(-a^2d+2b^2cx^6+ab(c-dx^6))}{c(-bc+ad)x^6(a+bx^6)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{c^{3/2}}}{6a^3}$$

input
```
Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*Sqrt[c + d*x^6]*(-(a^2*d) + 2*b^2*c*x^6 + a*b*(c - d*x^6)))/(c*(-(b*c)
 + a*d)*x^6*(a + b*x^6)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^6])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^6]/Sqrt[c]])/c^(3/2))/(6*a^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{6}\int \frac{1}{x^{12} (bx^6+a)^2 \sqrt{dx^6+c}}dx^6$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{6}\left(-\frac{\int \frac{3bdx^6+4bc+ad}{2x^6(bx^6+a)^2\sqrt{dx^6+c}}dx^6}{ac} - \frac{\sqrt{c+dx^6}}{acx^6(a+bx^6)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{6}\left(-\frac{\int \frac{3bdx^6+4bc+ad}{x^6(bx^6+a)^2\sqrt{dx^6+c}}dx^6}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \; 168$$

$$\frac{1}{6}\left(-\frac{\frac{\int \frac{bd(2bc-ad)x^6+(bc-ad)(4bc+ad)}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^6\sqrt{dx^6+c}}dx^6}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^6}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^{12}}{d}-\frac{c}{d}}d\sqrt{dx^6+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^{12}}{d}+a-\frac{bc}{d}}d\sqrt{dx^6+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{6}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^6}(2bc-ad)}{a(a+bx^6)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^6}}{acx^6\left(a+bx^6\right)}\right)$$

input
```
Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-(Sqrt[c + d*x^6]/(a*c*x^6*(a + b*x^6))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^6])/(a*(b*c - a*d)*(a + b*x^6)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^6]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{2\left(\left(b x^{6}+a\right)\left(c b-\frac{5 a d}{4}\right) c^{\frac{5}{2}} b^{2} x^{6} \arctan\left(\frac{\sqrt{d x^{6}+c}\, b}{\sqrt{(a d-c b) b}}\right)-\frac{\left(c x^{6}\left(b x^{6}+a\right)(a d+4 c b)(a d-c b) \operatorname{arctanh}\left(\frac{\sqrt{d x^{6}+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}} \sqrt{d x^{6}+c}\, a\left(2 b \cdots\right.\right.}{4}\right.}{3 \sqrt{(a d-c b) b}\, c^{\frac{5}{2}} x^{6} a^{3}(a d-c b)\left(b x^{6}+a\right)}$

input
```
int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((b*x^6+a)*(c*b-5/4*a*d)*c^(5/2)*b^2*x^6*arctan((d*x^6+c)^(1/2)*b/((a
*d-b*c)*b)^(1/2))-1/4*(c*x^6*(b*x^6+a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^
6+c)^(1/2)/c^(1/2))+c^(3/2)*(d*x^6+c)^(1/2)*a*(2*b^2*c*x^6+a*(-d*x^6+c)*b-
a^2*d))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/c^(5/2)/x^6/a^3/(a*d-b*c)
/(b*x^6+a)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.43

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x
^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*
c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a
^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d
*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d
)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)
*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), 1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(sqr
t(d*x^6 + c)*sqrt(-b/(b*c - a*d))) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2
)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6 + 2
*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 +
a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 +
(a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d
^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt
(-c)/sqrt(d*x^6 + c)) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 -
 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sq
rt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((2*a*b^2*
c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3
 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), 1/6*(((4*b^3*c^3 - 5*
a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a...
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^6+c}b}{\sqrt{-b^2c+abd}}\right)}{6\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^6+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^6+c}b^2c^2d-(dx^6+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^6+c}abcd^2-\sqrt{dx^6+c}a^2d^3}{6\,(a^2bc^2-a^3cd)\left((dx^6+c)^2b-2\,(dx^6+c)bc+bc^2+(dx^6+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6
+ c)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 +
 c)^2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b
*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 3860, normalized size of antiderivative = 20.86

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
(((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*
d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^
3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2
*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*
a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*
d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*
d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d
^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a
^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*
a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 +
a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c
)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^
8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*
(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5...
```

Reduce [F]

$$\int \frac{1}{x^7 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \text{Too large to display}$$

input
```
int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output

```
( - 2*sqrt(c + d*x**6)*a**3*c*d + 4*sqrt(c + d*x**6)*a**2*b*c**2 - 2*sqrt(
c + d*x**6)*a**2*b*c*d*x**6 + 16*sqrt(c + d*x**6)*a*b**2*c**2*x**6 - sqrt(
c)*log(sqrt(c + d*x**6) - sqrt(c))*a**3*d**2*x**6 - 2*sqrt(c)*log(sqrt(c +
 d*x**6) - sqrt(c))*a**2*b*c*d*x**6 - sqrt(c)*log(sqrt(c + d*x**6) - sqrt(
c))*a**2*b*d**2*x**12 + 8*sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c))*a*b**2*c
**2*x**6 - 2*sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c))*a*b**2*c*d*x**12 + 8*
sqrt(c)*log(sqrt(c + d*x**6) - sqrt(c))*b**3*c**2*x**12 + sqrt(c)*log(sqrt
(c + d*x**6) + sqrt(c))*a**3*d**2*x**6 + 2*sqrt(c)*log(sqrt(c + d*x**6) +
sqrt(c))*a**2*b*c*d*x**6 + sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*a**2*b*
d**2*x**12 - 8*sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*a*b**2*c**2*x**6 +
2*sqrt(c)*log(sqrt(c + d*x**6) + sqrt(c))*a*b**2*c*d*x**12 - 8*sqrt(c)*log
(sqrt(c + d*x**6) + sqrt(c))*b**3*c**2*x**12 + 60*int((sqrt(c + d*x**6)*x*
*11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x**12 - 4*
a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d**2*x**18 - 2*b**3*c**2*x*
*12 - 2*b**3*c*d*x**18),x)*a**3*b**3*c**2*d**2*x**6 - 168*int((sqrt(c + d*
x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2 + 2*a**2*b*d**2*x*
*12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d**2*x**18 - 2*b**3
*c**2*x**12 - 2*b**3*c*d*x**18),x)*a**2*b**4*c**3*d*x**6 + 60*int((sqrt(c
+ d*x**6)*x**11)/(a**3*c*d + a**3*d**2*x**6 - 2*a**2*b*c**2 + 2*a**2*b*d**
2*x**12 - 4*a*b**2*c**2*x**6 - 3*a*b**2*c*d*x**12 + a*b**2*d**2*x**18 -...
```

3.79 $\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\,(a+bx^6)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

output
```
1/6*a*x^3*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)-1/6*a^(1/2)*(-2*a*d+3*b*c
)*arctan((-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(3/2
)+1/3*arctanh(d^(1/2)*x^3/(d*x^6+c)^(1/2))/b^2/d^(1/2)
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\frac{abx^3\sqrt{c+dx^6}}{(bc-ad)(a+bx^6)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{d}}}{6b^2}$$

input
```
Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^3 + Sqrt[c + d*x^
6]])/Sqrt[d])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \; 965$$

$$\frac{1}{3}\int \frac{x^{12}}{(bx^6+a)^2\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \; 372$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b(a+bx^6)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^6}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 398$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b(a+bx^6)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^6+c}}dx^3}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b(a+bx^6)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{ax^3\sqrt{c+dx^6}}{2b\left(a+bx^6\right)(bc-ad)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((a*x^3*Sqrt[c + d*x^6])/(2*b*(b*c - a*d)*(a + b*x^6)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{(2ad-3cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{ad-cb}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{d}}\right)}{\sqrt{d}}}{6b^{2}}$	117

input
```
int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b^2*(-a/(a*d-b*c)*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(2*a*d-3*b*c)/(a*
(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2)))-2/d^(
1/2)*arctanh((d*x^6+c)^(1/2)/x^3/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1077, normalized size of antiderivative = 7.64

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^
2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sq
rt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c
*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*
b*d*x^3 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2
)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b
^2*d^2), 1/12*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6
 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^
6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^
2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)
*sqrt(d)*x^3 - c))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2),
1/12*(2*sqrt(d*x^6 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.43

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\left(3\,abc\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,a^2\sqrt{-d}d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,\sqrt{abc-a^2d}bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 2\,\sqrt{c}\cdots\right.}{6\,\left(\sqrt{abc-a^2d}b^3c\sqrt{-d} - \sqrt{abc-a^2d}ab^2\sqrt{-d}\cdots\right.} + \frac{ac\sqrt{d+\frac{c}{x^6}}}{6\,(b^2c\,\mathrm{sgn}\,(x) - abd\,\mathrm{sgn}\,(x))\left(bc + a\left(d+\frac{c}{x^6}\right) - ad\right)} + \frac{(3\,abc - 2\,a^2d)\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{6\,(b^3c\,\mathrm{sgn}\,(x) - ab^2d\,\mathrm{sgn}\,(x))\sqrt{abc-a^2d}} - \frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b^2\sqrt{-d}\,\mathrm{sgn}\,(x)}$$

input
```
integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(
-d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*ar
ctan(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)
) + sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^
3*c*sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/
x^6)/((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*
a*b*c - 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn
(x) - a*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sq
rt(-d))/(b^2*sqrt(-d)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^{14}}{{\left(b\,x^6+a\right)}^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Too large to display}$$

input
```
int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x**6)*a*b*d*x**3 - 2*sqrt(d)*log(sqrt(c + d*x**6) - sqrt(
d)*x**3)*a**2*d + sqrt(d)*log(sqrt(c + d*x**6) - sqrt(d)*x**3)*a*b*c - 2*s
qrt(d)*log(sqrt(c + d*x**6) - sqrt(d)*x**3)*a*b*d*x**6 + sqrt(d)*log(sqrt(
c + d*x**6) - sqrt(d)*x**3)*b**2*c*x**6 + 2*sqrt(d)*log(sqrt(c + d*x**6) +
 sqrt(d)*x**3)*a**2*d - sqrt(d)*log(sqrt(c + d*x**6) + sqrt(d)*x**3)*a*b*c
 + 2*sqrt(d)*log(sqrt(c + d*x**6) + sqrt(d)*x**3)*a*b*d*x**6 - sqrt(d)*log
(sqrt(c + d*x**6) + sqrt(d)*x**3)*b**2*c*x**6 - 24*int((sqrt(c + d*x**6)*x
**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*
a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x
**12 - b**3*c*d*x**18),x)*a**5*d**3 + 48*int((sqrt(c + d*x**6)*x**2)/(2*a*
*3*c*d + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**
2*x**12 - 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**
3*c*d*x**18),x)*a**4*b*c*d**2 - 24*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d
 + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**1
2 - 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*
x**18),x)*a**4*b*d**3*x**6 - 18*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d +
2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 -
 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**
18),x)*a**3*b**2*c**2*d + 48*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a
**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 -...
```

3.80 $\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\,(a+bx^6)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}(bc-ad)^{3/2}}$$

output
```
-1/6*x^3*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)+1/6*c*arctan((-a*d+b*c)^(1/2
)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\left(-\frac{x^3\sqrt{c+dx^6}}{(bc-ad)\,(a+bx^6)} + \frac{c\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input
```
Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-((x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^6+a)^2\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{3}\left(\frac{\int \frac{c}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{c\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{2(bc-ad)}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{c\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{2(a+bx^6)(bc-ad)}\right)$$

input
```
Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(-1/2*(x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{6}+c}\,x^{3}}{c\left(b\,x^{6}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a\left(a d-c b\right)}}\right)}{\sqrt{a\left(a d-c b\right)}}\right)}{6\left(a d-c b\right)}$	81

input
```
int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*c/(a*d-b*c)*(-(d*x^6+c)^(1/2)*x^3/c/(b*x^6+a)+1/(a*(a*d-b*c))^(1/2)*a
rctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.16 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \left[-\frac{4\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(3\,abc^2-4\,a^2cd)x^6+a^2c^2+4}{b^2x^{12}+2\,abx^6+a}\right.}{24\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right.$$
$$\left.-\frac{2\sqrt{dx^6+c}(abc-a^2d)x^3-(bcx^6+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^6-ac)\sqrt{dx^6+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^9+(abc^2-a^2cd)x^3)}\right)}{12\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right]$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*s
qrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^
2*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sq
rt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*ar
ctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a
*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2
*c*d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**8/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^8}{(b\,x^6+a)^2\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{\sqrt{d\,x^6+c}\,x^3-6\left(\int \frac{\sqrt{d\,x^6+c}\,x^2}{2a\,b^2d^2x^{18}-b^3cd\,x^{18}+4a^2b\,d^2x^{12}-b^3c^2x^{12}+2a^3d^2x^6+3a^2bcd\,x^6-2a\,b^2c^2x^6+2a^3cd-a^2b\,c^2}dx\right)a^3cd+3\left(\int\right.}{}$$

input
```
int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output

```
(sqrt(c + d*x**6)*x**3 - 6*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**
3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*
b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x
)*a**3*c*d + 3*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6
- a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x*
*6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a**2*b*c**
2 - 6*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**2*b*
c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 + 2*a*
b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a**2*b*c*d*x**6 + 3
*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**2*b*c**2
+ 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 + 2*a*b**2*
d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a*b**2*c**2*x**6)/(3*(2*
a**2*d - a*b*c + 2*a*b*d*x**6 - b**2*c*x**6))
```

3.81 $\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)(a+bx^6)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/6*b*x^3*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)+1/6*(-2*a*d+b*c)*arctan((
-a*d+b*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=-\frac{bx^3\sqrt{c+dx^6}}{6a(-bc+ad)(a+bx^6)}+\frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/6*(b*x^3*Sqrt[c + d*x^6])/(a*(-(b*c) + a*d)*(a + b*x^6)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(6*a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \ 965$$

$$\frac{1}{3}\int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \ 296$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{bx^3\sqrt{c+dx^6}}{2a(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \ 291$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{2a(bc-ad)}+\frac{bx^3\sqrt{c+dx^6}}{2a(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \ 218$$

$$\frac{1}{3}\left(\frac{(bc-2ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{2a(a+bx^6)(bc-ad)}\right)$$

input
```
Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((b*x^3*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/3
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}+\frac{(2ad-cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{6a(ad-cb)}$	90

input
```
int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6/a/(a*d-b*c)*(-(d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)+(2*a*d-b*c)/(a*(a*d-b*c)
)^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(88) = 176$.

Time = 0.17 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \left[\frac{4\sqrt{dx^6+c}(ab^2c-a^2bd)x^3-((b^2c-2\,abd)x^6+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\,(}{}\right.}{24\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd}\right.$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9
 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d
 + a^4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^
2*c - 2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d
^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2
+ (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^2}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{1}{6}d^{\frac{3}{2}}\left(\frac{(bc-2ad)\arctan\left(\frac{\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^2 bc}{\left(\left(\sqrt{d}x^3-\sqrt{dx^6+c}\right)^4 b-2\left(\sqrt{d}x^3-\sqrt{dx^6+\right.\right.}\right.$$

input
```
integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2
*a*d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqr
t(d*x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^2}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{\sqrt{d\,x^6+c}\,x^2}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**2)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d
*x**12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.82 $\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3}+\frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3(a+bx^6)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

output

```
-1/6*(-2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/6*b*(d*x^6+c)^(
1/2)/a/(-a*d+b*c)/x^3/(b*x^6+a)-1/6*b*(-4*a*d+3*b*c)*arctan((-a*d+b*c)^(1/
2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{c+dx^6}(2abc-2a^2d+3b^2cx^6-2abdx^6)}{6a^2c(-bc+ad)x^3(a+bx^6)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^6+bx^3\sqrt{c+dx^6}}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(Sqrt[c + d*x^6]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^6 - 2*a*b*d*x^6))/(6*a^2*c
*(-(b*c) + a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^6 + b*x^3*Sqrt[c + d*x^6])/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(5
/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^6+a)^2 \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{3}\left(\frac{b\sqrt{c+dx^6}}{2ax^3 (a+bx^6)(bc-ad)}-\frac{\int -\frac{2bdx^6+3bc-2ad}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{2bdx^6+3bc-2ad}{x^6(bx^6+a)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3 (a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3 (a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{27}$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{(bx^6+a)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{291}$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3(a+bx^6)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{218}$$

$$\frac{1}{3}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}(3bc-2ad)}{acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^3(a+bx^6)(bc-ad)}\right)$$

input
```
Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((b*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*x^3*(a + b*x^6)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^6])/(a*c*x^3)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 7.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}}{x^{3}}+\frac{b c \left(\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{\left(4 a d-3 c b\right) \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{6}+c}}{x^{3} \sqrt{a \left(a d-c b\right)}}\right)}{\sqrt{a \left(a d-c b\right)}}\right)}{2 a d-2 c b}}{3 a^{2} c}$	112

input
```
int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(-(d*x^6+c)^(1/2)/x^3+1/2*b*c/(a*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*
x^6+a)-(4*a*d-3*b*c)/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*
(a*d-b*c))^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.20 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^4 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx$$

$$= \left[-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^9 + (3\,ab^2c^2 - 4\,a^2bcd)x^3)\sqrt{-abc + a^2d}\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^{12} - 2\,(3\,abc^2 - 4\,a^2cd)x^6 +}{b^2x^{12} + 2} \right.}{24\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a} \right.$$

$$-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^9 + (3\,ab^2c^2 - 4\,a^2bcd)x^3)\sqrt{abc - a^2d}\arctan\left(\frac{((bc - 2\,ad)x^6 - ac)\sqrt{dx^6 + c}\sqrt{abc - a^2d}}{2\,((abcd - a^2d^2)x^9 + (abc^2 - a^2cd)x^3)}\right) + 2\,((}{12\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^9 + (a^4b^2c^3 - 2}$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*
x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3
*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x
^9 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 -
4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*ar
ctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a
*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^
2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sq
rt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b
^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^4 \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$
$$= \frac{-2\sqrt{d\,x^6+c}\,ad+\sqrt{d\,x^6+c}\,bc-2\sqrt{d\,x^6+c}\,bd\,x^6-24\left(\int \frac{\sqrt{d\,x^6+c}\,x^2}{2a\,b^2d^2x^{18}-b^3cd\,x^{18}+4a^2b\,d^2x^{12}-b^3c^2x^{12}+2a^3d^2x^6+3a^2bc}\right.}{}$$

input
```
int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**6)*a*d + sqrt(c + d*x**6)*b*c - 2*sqrt(c + d*x**6)*b*d
*x**6 - 24*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a*
*2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 +
 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a**3*b*c*d**2*
x**3 + 30*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**
2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 +
2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a**2*b**2*c**2*
d*x**3 - 24*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a
**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6
+ 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a**2*b**2*c*d
**2*x**9 - 9*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 -
a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6
 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a*b**3*c**3*
x**3 + 30*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**
2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 +
2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a*b**3*c**2*d*x
**9 - 9*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d**2*x**6 - a**2*
b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2*c**2*x**6 + 2*
a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*b**4*c**3*x**9)/(
3*a*c*x**3*(2*a**2*d - a*b*c + 2*a*b*d*x**6 - b**2*c*x**6))
```

3.83 $\int \frac{1}{x^{10}(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

output
```
-1/18*(-2*a*d+5*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/x^9+1/18*(-4*a^2*d^2
-8*a*b*c*d+15*b^2*c^2)*(d*x^6+c)^(1/2)/a^3/c^2/(-a*d+b*c)/x^3+1/6*b*(d*x^6
+c)^(1/2)/a/(-a*d+b*c)/x^9/(b*x^6+a)+1/6*b^2*(-6*a*d+5*b*c)*arctan((-a*d+b
*c)^(1/2)*x^3/a^(1/2)/(d*x^6+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = -\frac{\sqrt{c+dx^6}(15b^3c^2x^{12}+2ab^2cx^6(5c-4dx^6)+2a^3d(c-2dx^6)-2a^2b(c^2+3cdx^6+2d^2x^{12}))}{18a^3c^2(-bc+ad)x^9(a+bx^6)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^3\left(\sqrt{d}x^3+\sqrt{c+dx^6}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{6a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/18*(Sqrt[c + d*x^6]*(15*b^3*c^2*x^12 + 2*a*b^2*c*x^6*(5*c - 4*d*x^6) +
2*a^3*d*(c - 2*d*x^6) - 2*a^2*b*(c^2 + 3*c*d*x^6 + 2*d^2*x^12)))/(a^3*c^2*
(-(b*c) + a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] +
 b*x^3*(Sqrt[d]*x^3 + Sqrt[c + d*x^6]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(6*a^(
7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{3}\int \frac{1}{x^{12} (bx^6+a)^2 \sqrt{dx^6+c}}dx^3$$

$$\Big\downarrow \text{ 374}$$

$$\frac{1}{3}\left(\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}-\frac{\int -\frac{4bdx^6+5bc-2ad}{x^{12}\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{\int \frac{4bdx^6+5bc-2ad}{x^{12}\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(\frac{-\frac{\int \frac{2bd(5bc-2ad)x^6+15b^2c^2-4a^2d^2-8abcd}{x^6\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{ac}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}}dx^3}{a}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^6}d\frac{x^3}{\sqrt{dx^6+c}}}{a}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{3}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^3}}{3ac}-\frac{\sqrt{c+dx^6}(5bc-2ad)}{3acx^9}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^6}}{2ax^9\left(a+bx^6\right)(bc-ad)}\right)$$

input
```
Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
((b*Sqrt[c + d*x^6])/(2*a*(b*c - a*d)*x^9*(a + b*x^6)) + (-1/3*((5*b*c - 2
*a*d)*Sqrt[c + d*x^6])/(a*c*x^9) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)/c
)*Sqrt[c + d*x^6])/x^3) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]
*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*a
*(b*c - a*d)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 13.88 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{6}+c}\,\left(-2ad\,x^{6}-6bc\,x^{6}+ac\right)}{3x^{9}}-\frac{b^{2}c^{2}\left(\frac{\sqrt{d\,x^{6}+c}\,b\,x^{3}}{b\,x^{6}+a}-\frac{(6ad-5cb)\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{6}+c}}{x^{3}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{2(ad-cb)}}{3a^{3}c^{2}}$	134

input
```
int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a^3*(-1/3*(d*x^6+c)^(1/2)*(-2*a*d*x^6-6*b*c*x^6+a*c)/x^9-1/2*b^2*c^2/(
a*d-b*c)*((d*x^6+c)^(1/2)*b*x^3/(b*x^6+a)-(6*a*d-5*b*c)/(a*(a*d-b*c))^(1/2
)*arctanh(a*(d*x^6+c)^(1/2)/x^3/(a*(a*d-b*c))^(1/2))))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3
)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*
((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 -
2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2
*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*
a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c
^2*d^2)*x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6
*a^2*b^2*c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a
*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^
2 - a^2*c*d)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2
 + 4*a^4*b*d^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*
a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 +
 c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4
- 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^9)]
```

Sympy [F]

$$\int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} \, dx = \int \frac{1}{x^{10} (a+bx^6)^2 \sqrt{c+dx^6}} \, dx$$

input
```
integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**10*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^{10}\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{-2\sqrt{d\,x^6+c}\,a^2cd+4\sqrt{d\,x^6+c}\,a^2d^2x^6+\sqrt{d\,x^6+c}\,ab\,c^2+8\sqrt{d\,x^6+c}\,abcd\,x^6+4\sqrt{d\,x^6+c}\,ab\,d^2x^{12}-}{}$$

input
```
int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**6)*a**2*c*d + 4*sqrt(c + d*x**6)*a**2*d**2*x**6 + sqrt
(c + d*x**6)*a*b*c**2 + 8*sqrt(c + d*x**6)*a*b*c*d*x**6 + 4*sqrt(c + d*x**
6)*a*b*d**2*x**12 - 5*sqrt(c + d*x**6)*b**2*c**2*x**6 + 10*sqrt(c + d*x**6
)*b**2*c*d*x**12 + 108*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a**3*d*
*2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*a*b**2
*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18),x)*a*
*3*b**2*c**2*d**2*x**9 - 144*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d + 2*a
**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 - 2*
a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**18)
,x)*a**2*b**3*c**3*d*x**9 + 108*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*d +
2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**12 -
 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d*x**
18),x)*a**2*b**3*c**2*d**2*x**15 + 45*int((sqrt(c + d*x**6)*x**2)/(2*a**3*
c*d + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x
**12 - 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c
*d*x**18),x)*a*b**4*c**4*x**9 - 144*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*
d + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*x**
12 - 2*a*b**2*c**2*x**6 + 2*a*b**2*d**2*x**18 - b**3*c**2*x**12 - b**3*c*d
*x**18),x)*a*b**4*c**3*d*x**15 + 45*int((sqrt(c + d*x**6)*x**2)/(2*a**3*c*
d + 2*a**3*d**2*x**6 - a**2*b*c**2 + 3*a**2*b*c*d*x**6 + 4*a**2*b*d**2*...
```

3.84 $\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

output
```
1/5*x^5*(1+d*x^6/c)^(1/2)*AppellF1(5/6,2,1/2,11/6,-b*x^6/a,-d*x^6/c)/a^2/(
d*x^6+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 169 vs. $2(64) = 128$.

Time = 10.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.64

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{x^5\Big(55ab(c+dx^6)+11(bc-6ad)(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{5}{6},\frac{1}{2},1,\frac{11}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-10bdx^6(a+b}{330a^2(bc-ad)(a+bx^6)\sqrt{c+dx^6}}$$

input
```
Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c
]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a
+ b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -(
(b*x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^4}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\operatorname{AppellF1}\left(\frac{5}{6},2,\frac{1}{2},\frac{11}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

input
```
Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^
6)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}dx$$

input
```
int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b
*c + a^2*d)*x^6 + a^2*c), x)
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x^4}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x^4}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**4)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d
*x**12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.85 $\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x^4\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

output
```
1/4*x^4*(1+d*x^6/c)^(1/2)*AppellF1(2/3,2,1/2,5/3,-b*x^6/a,-d*x^6/c)/a^2/(d
*x^6+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.

Time = 10.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx =$$
$$-\frac{x^4\left(-10ab(c+dx^6)-5(bc-3ad)\,(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^6(a+b\right.}{60a^2(bc-ad)\,(a+bx^6)\sqrt{c+dx^6}}$$

input
```
Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/60*(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*
x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(
a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -(
(b*x^6)/a)]))/(a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^6+a)^2\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{x^2}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{2}{3},2,\frac{1}{2},\frac{5}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

input
```
Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6
)/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}dx$$

input
```
int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$

input
```
integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{x^3}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{x^3}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\int \frac{\sqrt{d\,x^6+c}\,x^3}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x**3)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d
*x**12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.86 $\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\frac{x^2\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

output
```
1/2*x^2*(1+d*x^6/c)^(1/2)*AppellF1(1/3,2,1/2,4/3,-b*x^6/a,-d*x^6/c)/a^2/(d
*x^6+c)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.

Time = 10.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{8abx^2(c+dx^6)+8(2bc-3ad)x^2(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+bdx^8(a+bx^6)\ldots}{48a^2(bc-ad)\,(a+bx^6)\sqrt{c+dx^6}}$$

input
```
Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6
)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a +
 b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*
x^6)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {965, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{ 937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 936}$$

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{3},2,\frac{1}{2},\frac{4}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

input
```
Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6
)/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{(b\,x^{6}+a)^{2}\sqrt{d\,x^{6}+c}}dx$$

input
```
int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx=\text{Timed out}$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{x}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}\,x}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**6)*x)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x*
*12 + b**2*c*x**12 + b**2*d*x**18),x)
```

3.87 $\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \frac{x\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

output
```
x*(1+d*x^6/c)^(1/2)*AppellF1(1/6,2,1/2,7/6,-b*x^6/a,-d*x^6/c)/a^2/(d*x^6+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(59) = 118.

Time = 10.22 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.58

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{x\left(-2bdx^6\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(\frac{7}{6},\frac{1}{2},1,\frac{13}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(7ac(6ad-b(6c+dx^6))\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3\cdots\right.}{(a+bx^6)\left(-7ac\,\text{AppellF1}\left(\frac{1}{6},\frac{1}{2},1,\frac{7}{6},-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+3\cdots\right.}}{42a^2(-bc+ad)\sqrt{c+dx^6}}$$

input
```
Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c
), -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2
, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF
1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2,
1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6,
 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2
, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -
((d*x^6)/c), -((b*x^6)/a)]))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(\frac{1}{6},2,\frac{1}{2},\frac{7}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

input
```
Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{6}+a)^{2}\,\sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)^2\,\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}}\,dx$$

input
```
integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx = \int \frac{\sqrt{d\,x^6+c}}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx$$

input
```
int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**6)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12
+ b**2*c*x**12 + b**2*d*x**18),x)
```

3.88 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx = -\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

output
```
-(1+d*x^6/c)^(1/2)*AppellF1(-1/6,2,1/2,5/6,-b*x^6/a,-d*x^6/c)/a^2/x/(d*x^6
+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(62) = 124.

Time = 10.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$$
$$= \frac{55a(c+dx^6)(6a^2d-7b^2cx^6-6ab(c-dx^6))-11(7b^2c^2-24abcd+12a^2d^2)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{App}}{330a^3c(bc-ad)x}$$

input
```
Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*
c^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF
1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*
x^12*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6
)/c), -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1} \int \frac{1}{x^2 (bx^6+a)^2 \sqrt{\frac{dx^6}{c}+1}}dx}{\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1} \operatorname{AppellF1}\left(-\frac{1}{6},2,\frac{1}{2},\frac{5}{6},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2 x \sqrt{c+dx^6}}$$

input
```
Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)
/c)])/(a^2*x*Sqrt[c + d*x^6]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{6}+a \right)^{2} \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c +
 a^2*d)*x^8 + a^2*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^6)^2 \sqrt{c + dx^6}}\,dx = \int \frac{1}{x^2 (b\,x^6 + a)^2 \sqrt{d\,x^6 + c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^6)^2 \sqrt{c + dx^6}}\,dx$$
$$= \frac{-\sqrt{d\,x^6+c} - 4\left(\int \frac{\sqrt{d\,x^6+c}\,x^{10}}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx\right) abdx - 4\left(\int \frac{\sqrt{d\,x^6+c}\,x^{10}}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x}\right.}{}$$

input
```
int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**6) - 4*int((sqrt(c + d*x**6)*x**10)/(a**2*c + a**2*d*x**
6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x
 - 4*int((sqrt(c + d*x**6)*x**10)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2
*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*d*x**7 + 2*int((sqrt(c
 + d*x**6)*x**4)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b*
*2*c*x**12 + b**2*d*x**18),x)*a**2*d*x - 7*int((sqrt(c + d*x**6)*x**4)/(a*
*2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*
x**18),x)*a*b*c*x + 2*int((sqrt(c + d*x**6)*x**4)/(a**2*c + a**2*d*x**6 +
2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x**7
- 7*int((sqrt(c + d*x**6)*x**4)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a
*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*c*x**7)/(a*c*x*(a + b*x*
*6))
```

3.89 $\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

output
```
-1/2*(1+d*x^6/c)^(1/2)*AppellF1(-1/3,2,1/2,2/3,-b*x^6/a,-d*x^6/c)/a^2/x^2/
(d*x^6+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.

Time = 10.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{20a(c+dx^6)\left(3a^2d-4b^2cx^6-3ab(c-dx^6)\right)-5\left(8b^2c^2-15abcd+3a^2d^2\right)x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{Appell}}{120a^3c(bc-ad)x^2\,(a}$$

input
```
Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c
^2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12
*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c),
-((b*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^6+a)^2 \sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^4(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{1}{3},2,\frac{1}{2},\frac{2}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

input
```
Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/2*(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a^2*x^2*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

input
```
integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^3\,(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$
$$= \frac{-\sqrt{d\,x^6+c} - 5\left(\int \frac{\sqrt{d\,x^6+c}\,x^9}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\,x^6+a^2c}dx\right)abd\,x^2 - 5\left(\int \frac{\sqrt{d\,x^6+c}\,x^9}{b^2d\,x^{18}+2abd\,x^{12}+b^2c\,x^{12}+a^2d\,x^6+2abc\right.}}{}$$

input
```
int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**6) - 5*int((sqrt(c + d*x**6)*x**9)/(a**2*c + a**2*d*x**6
 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x*
*2 - 5*int((sqrt(c + d*x**6)*x**9)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 +
2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*d*x**8 + int((sqrt(c
+ d*x**6)*x**3)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**
2*c*x**12 + b**2*d*x**18),x)*a**2*d*x**2 - 8*int((sqrt(c + d*x**6)*x**3)/(
a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*
d*x**18),x)*a*b*c*x**2 + int((sqrt(c + d*x**6)*x**3)/(a**2*c + a**2*d*x**6
 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x*
*8 - 8*int((sqrt(c + d*x**6)*x**3)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 +
2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*c*x**8)/(2*a*c*x**2*(
a + b*x**6))
```

3.90 $\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=-\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

output
```
-1/4*(1+d*x^6/c)^(1/2)*AppellF1(-2/3,2,1/2,1/3,-b*x^6/a,-d*x^6/c)/a^2/x^4/
(d*x^6+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 225 vs. 2(64) = 128.

Time = 10.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.52

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$
$$=\frac{8a(c+dx^6)\left(3a^2d-5b^2cx^6-3ab(c-dx^6)\right)+4(-20b^2c^2+21abcd+3a^2d^2)\,x^6(a+bx^6)\sqrt{1+\frac{dx^6}{c}}\,\text{Appe}}{96a^3c(bc-ad)x^4\,(}$$

input
```
Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
(8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*
c^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12
*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c),
-((b*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {965, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}} \, dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{1}{x^6 (bx^6+a)^2 \sqrt{dx^6+c}}dx^2$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^6}{c}+1}\int \frac{1}{x^6(bx^6+a)^2\sqrt{\frac{dx^6}{c}+1}}dx^2}{2\sqrt{c+dx^6}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,\text{AppellF1}\left(-\frac{2}{3},2,\frac{1}{2},\frac{1}{3},-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

input
```
Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

output
```
-1/4*(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x
^6)/c)])/(a^2*x^4*Sqrt[c + d*x^6])
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}dx$$

input
```
int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^5 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \int \frac{1}{x^5 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx$$

input
```
integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^6)^2 \sqrt{c + dx^6}} \, dx = \int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c} x^5} \, dx$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{(bx^6+a)^2\sqrt{dx^6+c}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx = \int \frac{1}{x^5\,(b\,x^6+a)^2\,\sqrt{d\,x^6+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^6)^2 \sqrt{c+dx^6}}\,dx$$
$$= \frac{-\sqrt{d\,x^6+c} - 7\left(\int \frac{\sqrt{d\,x^6+c}\,x^7}{b^2 d\,x^{18}+2abd\,x^{12}+b^2 c\,x^{12}+a^2 d\,x^6+2abc\,x^6+a^2 c}dx\right) abd\,x^4 - 7\left(\int \frac{\sqrt{d\,x^6+c}\,x^7}{b^2 d\,x^{18}+2abd\,x^{12}+b^2 c\,x^{12}+a^2 d\,x^6+2abc}\right.}{}$$

input
```
int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**6) - 7*int((sqrt(c + d*x**6)*x**7)/(a**2*c + a**2*d*x**6
 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x*
*4 - 7*int((sqrt(c + d*x**6)*x**7)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 +
2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*d*x**10 - int((sqrt(c
 + d*x**6)*x)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*
c*x**12 + b**2*d*x**18),x)*a**2*d*x**4 - 10*int((sqrt(c + d*x**6)*x)/(a**2
*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x*
*18),x)*a*b*c*x**4 - int((sqrt(c + d*x**6)*x)/(a**2*c + a**2*d*x**6 + 2*a*
b*c*x**6 + 2*a*b*d*x**12 + b**2*c*x**12 + b**2*d*x**18),x)*a*b*d*x**10 - 1
0*int((sqrt(c + d*x**6)*x)/(a**2*c + a**2*d*x**6 + 2*a*b*c*x**6 + 2*a*b*d*
x**12 + b**2*c*x**12 + b**2*d*x**18),x)*b**2*c*x**10)/(4*a*c*x**4*(a + b*x
**6))
```

3.91 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=-\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

output
```
-1/4*(a*d+b*c)*(d*x^8+c)^(1/2)/b^2/d^2+1/12*(d*x^8+c)^(3/2)/b/d^2-1/4*a^2*
arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{c+dx^8}(-2bc-3ad+bdx^8)}{12b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) + (a^2*ArcTan[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) + a*d]
)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{x^{16}}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{8}\int \left(\frac{a^2}{b^2(bx^8+a)\sqrt{dx^8+c}}+\frac{\sqrt{dx^8+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^8+c}}\right)dx^8$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^8}(ad+bc)}{b^2d^2}+\frac{2\left(c+dx^8\right)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^8])/(b^2*d^2) + (2*(c + d*x^8)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/8
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{-a^{2} \arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(a d-c b) b}}\right) d^{2}+\sqrt{d\,x^{8}+c}\,\left(\frac{\left(-d\,x^{8}+2 c\right) b}{3}+a d\right) \sqrt{(a d-c b) b}}{4 \sqrt{(a d-c b) b}\, b^{2} d^{2}}$	93

input
```
int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-a^2*arctan((d*x^8+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))*d^2+(d*x^8+c)^(1/
2)*(1/3*(-d*x^8+2*c)*b+a*d)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d
^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.77

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left((b^3cd-ab^2d^2)x^8-2\,b^3c^2-ab^2cd+3\,a^2bd^2\right)\sqrt{}}{24\,(b^4cd^2-ab^3d^3)} \right.$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d
*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8
 - 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^
3*d^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 -
 a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

Sympy [F]

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**23/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\frac{3\,a^2d^2\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^8+c)^{\frac{3}{2}}b^2-3\sqrt{dx^8+c}b^2c-3\sqrt{dx^8+c}abd}{b^3}}{12\,d^2}$$

input
```
integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*a^2*d^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b^2) + ((d*x^8 + c)^(3/2)*b^2 - 3*sqrt(d*x^8 + c)*b^2*c - 3*sqr
t(d*x^8 + c)*a*b*d)/b^3)/d^2
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\left(d\,x^8+c\right)^{3/2}}{12\,b\,d^2}-\left(\frac{c}{2\,b\,d^2}+\frac{4\,a\,d^3-4\,b\,c\,d^2}{16\,b^2\,d^4}\right)\sqrt{d\,x^8+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^
2*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b
*c)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-2\sqrt{d\,x^8+c}\,c+\sqrt{d\,x^8+c}\,d\,x^8-12\left(\int \frac{\sqrt{d\,x^8+c}\,x^{15}}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx\right)a\,d^2}{12b\,d^2}$$

input
```
int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**8)*c + sqrt(c + d*x**8)*d*x**8 - 12*int((sqrt(c + d*x*
*8)*x**15)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)*a*d**2)/(12*b*d**2)
```

3.92 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{\sqrt{c+dx^8}}{4bd}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

output
```
1/4*(d*x^8+c)^(1/2)/b/d+1/4*a*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(
1/2))/b^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{1}{4}\left(\frac{\sqrt{c+dx^8}}{bd}-\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}\right)$$

input
```
Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) +
 a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d]))/4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{8}\left(\frac{2\sqrt{c+dx^8}}{bd}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(\frac{2\sqrt{c+dx^8}}{bd}-\frac{2a\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^8}}{bd}\right)$$

input
```
Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((2*Sqrt[c + d*x^8])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/8
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{8}+c}}{d}-\frac{a \arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(a d-c b)b}}\right)}{\sqrt{(a d-c b)b}}}{4b}$	59

input
```
int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*(1/d*(d*x^8+c)^(1/2)-a/((a*d-b*c)*b)^(1/2)*arctan((d*x^8+c)^(1/2)*b/
((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.77

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{8\,(b^3cd-ab^2d^2)}, -\frac{\sqrt{-b^2c+abd}ad\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-\sqrt{dx^8+c}(b^2c-abd)}{4\,(b^3cd-ab^2d^2)}\right]$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/
(b^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 +
c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d)
)/(b^3*c*d - a*b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

input
```
integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*
b*d)*b) - sqrt(d*x^8 + c)/b)/d
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

Reduce [F]

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^{15}}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**15)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.93 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

output
```
-1/4*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{4\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]]/(4*Sqrt[b]*Sqrt[-(b*c
) + a*d])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {946, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{8}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{4d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c
- a*d])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{4\sqrt{(ad-cb)b}}$	39

input
```
int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/((a*d-b*c)*b)^(1/2)*arctan((d*x^8+c)^(1/2)*b/((a*d-b*c)*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,(b^2c-abd)}\right]$$

input
```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(
b*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^
8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

Sympy [A] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^8}{8a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^8 & \text{for } \sqrt{c}=0 \\ \frac{\log\left(8a\sqrt{c}+8b\sqrt{c}x^8\right)}{8b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c
)/b)), Ne(d, 0)), (Piecewise((x**8/(8*a*sqrt(c)), Eq(b, 0)), (zoo*x**8, Eq
(sqrt(c), 0)), (log(8*a*sqrt(c) + 8*b*sqrt(c)*x**8)/(8*b*sqrt(c)), True)),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}}$$

input
```
integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\operatorname{atan}\left(\frac{b\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2)
)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^7}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**7)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.94 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

output
```
-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/4*b^(1/2)*arctanh(b^(1/2
)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx=-\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

input
```
Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/4*((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/Sqrt[
-(b*c) + a*d] + ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c])/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{1}{x^8\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{ 97}$$

$$\frac{1}{8}\left(\frac{\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{b\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{8}\left(\frac{2\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{8}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/8
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(a d-c b) b}}\right)}{\sqrt{(a d-c b) b}}}{4 a}$	65

input
```
int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(-arctanh((d*x^8+c)^(1/2)/c^(1/2))/c^(1/2)-b/((a*d-b*c)*b)^(1/2)*arc
tan((d*x^8+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 385, normalized size of antiderivative = 4.53

$$\int \frac{1}{x\,(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+\sqrt{c}\log\left(\frac{dx^8-2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)}{8\,ac},\right.$$
$$-\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^8+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{c}\log\left(\frac{dx^8-2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)}{8\,ac}, \frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{}}{b}\right.}{}$$
$$\left.-\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\sqrt{dx^8+c}\sqrt{-\frac{b}{bc-ad}}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^8+c}}\right)}{4\,ac}\right]$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)
*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), -1/8*(2*c*sqrt(-b/(b*c - a*d))*a
rctan(sqrt(d*x^8 + c)*sqrt(-b/(b*c - a*d))) - sqrt(c)*log((d*x^8 - 2*sqrt(
d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(b/(b*c - a*d))*log((b*d
*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b
*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^8 + c)))/(a*c), -1/4*(c*s
qrt(-b/(b*c - a*d))*arctan(sqrt(d*x^8 + c)*sqrt(-b/(b*c - a*d))) - sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^8 + c)))/(a*c)]
```

Sympy [A] (verification not implemented)

Time = 12.95 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{8a\sqrt{\frac{ad-bc}{b}}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{8a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^8\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{4b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(8*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**8)/sqrt(-c))/(8*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**8)/sqrt(-a**2/b**2))/(4*b*sqrt(c)*sqrt(-a**2/b**
2)), True))
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}x}\,dx$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.67

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{4\,a\,\sqrt{c}} - \frac{\mathrm{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{8\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}+\cdots\right.}{\cdots}}\right)\cdots}{4\,\left(a^2\,d-a\,b\,c\right)}$$

input
```
int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
- atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)
^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^
3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^
(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c))
+ ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)
^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2
)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8
*(a^2*d - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4
- ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2
)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d
 - a*b*c))))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d
*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 1
6*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*
c))))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i
)/(4*(a^2*d - a*b*c))
```

Reduce [F]

$$\int \frac{1}{x\,(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c}\log\left(\sqrt{d\,x^8+c}-\sqrt{c}\right)-\sqrt{c}\log\left(\sqrt{d\,x^8+c}+\sqrt{c}\right)-8\left(\int\frac{\sqrt{d\,x^8+c}\,x^7}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx\right)bc}{8ac}$$

input
```
int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
(sqrt(c)*log(sqrt(c + d*x**8) - sqrt(c)) - sqrt(c)*log(sqrt(c + d*x**8) +
sqrt(c)) - 8*int((sqrt(c + d*x**8)*x**7)/(a*c + a*d*x**8 + b*c*x**8 + b*d*
x**16),x)*b*c)/(8*a*c)
```

3.95 $\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

output
```
-1/8*(d*x^8+c)^(1/2)/a/c/x^8+1/8*(a*d+2*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/
2))/a^2/c^(3/2)-1/4*b^(3/2)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/
2))/a^2/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^9(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-\frac{a\sqrt{c+dx^8}}{cx^8}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^2}$$

input
```
Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((a*Sqrt[c + d*x^8])/(c*x^8)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
8])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8} \int \frac{1}{x^{16} (bx^8+a) \sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{8} \left(-\frac{\int \frac{bdx^8+2bc+ad}{2x^8(bx^8+a)\sqrt{dx^8+c}}dx^8}{ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} \left(-\frac{\int \frac{bdx^8+2bc+ad}{x^8(bx^8+a)\sqrt{dx^8+c}}dx^8}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{8} \left(-\frac{\frac{(ad+2bc)\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{2b^2c\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{a}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(-\frac{\frac{2(ad+2bc)\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{4b^2c\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(-\frac{\frac{4b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^8)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{8}+c}}{c\,x^{8}}+\frac{(ad+2cb)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2b^{2}\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{8a^{2}}$	92

input
```
int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*(-a/c*(d*x^8+c)^(1/2)/x^8+(a*d+2*b*c)/c^(3/2)*arctanh((d*x^8+c)^(1
/2)/c^(1/2))+2*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x^8+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 518, normalized size of antiderivative = 4.43

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{2\,bc^2x^8\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2\,bc+ad)\sqrt{c}x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\sqrt{c}+2\,c}{x^8}\right)-2\,\sqrt{}}{16\,a^2c^2x^8}\right.$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt
(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*
sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8), 1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan
(sqrt(d*x^8 + c)*sqrt(-b/(b*c - a*d))) + (2*b*c + a*d)*sqrt(c)*x^8*log((d*
x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8 + c)*a*c)/(a^2*
c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d -
2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) - (2*b*c +
 a*d)*sqrt(-c)*x^8*arctan(sqrt(-c)/sqrt(d*x^8 + c)) - sqrt(d*x^8 + c)*a*c)
/(a^2*c^2*x^8), 1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(sqrt(d*x^8 +
c)*sqrt(-b/(b*c - a*d))) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(-c)/sqrt
(d*x^8 + c)) - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

Sympy [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^9} \, dx$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^9 (a + bx^8) \sqrt{c + dx^8}} \, dx = \frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c + abd}a^2} - \frac{(2\,bc + ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}c} - \frac{\sqrt{dx^8 + c}}{8\,acx^8}$$

input
```
integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)
*c) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)
```

Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.38

$$\int \frac{1}{x^9 (a + bx^8) \sqrt{c + dx^8}} \, dx$$

$$= \frac{\ln\left(\sqrt{d\,x^8 + c}\,(b^4\,c - a\,b^3\,d)^{3/2} + b^6\,c^2 + a^2\,b^4\,d^2 - 2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c - a\,b^3\,d}}{8\,a^3\,d - 8\,a^2\,b\,c}$$
$$- \frac{\ln\left(\sqrt{d\,x^8 + c}\,(b^4\,c - a\,b^3\,d)^{3/2} - b^6\,c^2 - a^2\,b^4\,d^2 + 2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c - a\,b^3\,d}}{8\,(a^3\,d - a^2\,b\,c)}$$
$$- \frac{\sqrt{d\,x^8 + c}}{8\,a\,c\,x^8}$$
$$- \frac{\operatorname{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^8+c}\,3\mathrm{i}}{128\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,c}+\frac{5\,a\,b^3\,d^5}{256\,c^2}+\frac{a^2\,b^2\,d^6}{256\,c^3}\right)} + \frac{b^2\,d^6\,\sqrt{d\,x^8+c}\,1\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{256\,a}+\frac{b^2\,d^6}{256\,c}+\frac{3\,b^4\,c\,d^4}{128\,a^2}\right)} + \frac{b^3\,d^5\,\sqrt{d\,x^8+c}\,5\mathrm{i}}{256\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{128\,a}+\frac{5\,b^3\,d^5}{256\,c}+\frac{a\,b^2\,d^6}{256\,c^2}\right)}\right)\,(a\,d + 2}{8\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
(log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x
^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x
^8) - (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(
128*c) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c
 + d*x^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256
*c) + (3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3
)^(1/2)*((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2)
)))*(a*d + 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^9 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^9 (b\,x^8+a) \sqrt{d\,x^8+c}} dx$$

input
```
int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

3.96 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^4\sqrt{c+dx^8}}{8bd}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

output
```
1/8*x^4*(d*x^8+c)^(1/2)/b/d+1/4*a^(3/2)*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2
)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(1/2)-1/8*(2*a*d+b*c)*arctanh(d^(1/2)*x^
4/(d*x^8+c)^(1/2))/b^2/d^(3/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\frac{bx^4\sqrt{c+dx^8}}{d}+\frac{2a^{3/2}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{(bc+2ad)\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{d^{3/2}}}{8b^2}$$

input
```
Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]
*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d] - ((b
*c + 2*a*d)*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^{16}}{(bx^8+a)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{381}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\int \frac{(bc+2ad)x^8+ac}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2bd}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}-\frac{2a^2d\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{2a^2d\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}}{2bd}\right)$$

$\Big\downarrow$ 291

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^2d\int\frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}}{2bd}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{4}\left(\frac{x^4\sqrt{c+dx^8}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}}{2bd}\right)$$

input
```
Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((x^4*Sqrt[c + d*x^8])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x^
4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTan
h[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]])/(b*Sqrt[d]))/(2*b*d))/4
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 10.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{-\frac{2a^{2}\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}-\frac{\sqrt{d\,x^{8}+c}\,b\,x^{4}}{d}+\frac{(2ad+cb)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{8b^{2}}$	98

input
```
int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^2*(-2*a^2/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d
-b*c))^(1/2))-(d*x^8+c)^(1/2)*b/d*x^4+(2*a*d+b*c)/d^(3/2)*arctanh((d*x^8+c
)^(1/2)/x^4/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 745, normalized size of antiderivative = 6.06

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{2\sqrt{dx^8+c}bdx^4 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(a}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,b^2d^2}\right.$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^
2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2
+ 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqr
t(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c +
2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2
), 1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^
2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*s
qrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*
c + 2*a*d)*sqrt(-d)*arctan(sqrt(d*x^8 + c)*sqrt(-d)/(d*x^4)))/(b^2*d^2), 1
/16*(2*sqrt(d*x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a
*c*x^4)) + (b*c + 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*
x^4 - c))/(b^2*d^2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*
d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*
d))/(a*d*x^12 + a*c*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(d*x^8 + c)*
sqrt(-d)/(d*x^4)))/(b^2*d^2)]
```

Sympy [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{2\sqrt{d\,x^8+c}\,bd\,x^4+2\sqrt{d}\log\left(\sqrt{d\,x^8+c}-\sqrt{d}\,x^4\right)ad+\sqrt{d}\log\left(\sqrt{d\,x^8+c}-\sqrt{d}\,x^4\right)bc-2\sqrt{d}\log\left(\sqrt{d\,x^8}\right.}{16b^2d^2}$$

input
```
int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x**8)*b*d*x**4 + 2*sqrt(d)*log(sqrt(c + d*x**8) - sqrt(d)*x*
*4)*a*d + sqrt(d)*log(sqrt(c + d*x**8) - sqrt(d)*x**4)*b*c - 2*sqrt(d)*log
(sqrt(c + d*x**8) + sqrt(d)*x**4)*a*d - sqrt(d)*log(sqrt(c + d*x**8) + sqr
t(d)*x**4)*b*c + 16*int((sqrt(c + d*x**8)*x**3)/(a*c + a*d*x**8 + b*c*x**8
 + b*d*x**16),x)*a**2*d**2)/(16*b**2*d**2)
```

3.97 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}$$

output

```
-1/4*a^(1/2)*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/b/(-a*d+
b*c)^(1/2)+1/4*arctanh(d^(1/2)*x^4/(d*x^8+c)^(1/2))/b/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{4b}$$

input

```
Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sq
rt[a]*Sqrt[b*c - a*d])])/Sqrt[b*c - a*d]) + Log[Sqrt[d]*x^4 + Sqrt[c + d*x
^8]]/Sqrt[d])/(4*b)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{385}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}\right)$$

input
```
Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sq
rt[b*c - a*d])) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(b*Sqrt[d]))/4
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 7.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{\frac{a\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a\left(a d-c b\right)}}\right)}{\sqrt{a\left(a d-c b\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)}{\sqrt{d}}}{4b}$	70

input
```
int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*(a/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^
(1/2))-1/d^(1/2)*arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 638, normalized size of antiderivative = 7.01

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\,bd}\right.$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)
))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 +
 c)*sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*
x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*
arctan(sqrt(d*x^8 + c)*sqrt(-d)/(d*x^4)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d)
)*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d)
)/(a*d*x^12 + a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)
*x^4 - c))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^
8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqr
t(-d)*arctan(sqrt(d*x^8 + c)*sqrt(-d)/(d*x^4)))/(b*d)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-\sqrt{d}\log\left(\sqrt{d\,x^8+c}-\sqrt{d}\,x^4\right)+\sqrt{d}\log\left(\sqrt{d\,x^8+c}+\sqrt{d}\,x^4\right)-8\left(\int \frac{\sqrt{d\,x^8+c}\,x^3}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx\right)ad}{8bd}$$

input
```
int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
( - sqrt(d)*log(sqrt(c + d*x**8) - sqrt(d)*x**4) + sqrt(d)*log(sqrt(c + d*
x**8) + sqrt(d)*x**4) - 8*int((sqrt(c + d*x**8)*x**3)/(a*c + a*d*x**8 + b*
c*x**8 + b*d*x**16),x)*a*d)/(8*b*d)
```

3.98 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

output
```
1/4*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b
*c - a*d])]/(4*Sqrt[a]*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {965, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*
c - a*d])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 5.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a(ad-cb)}}\right)}{4\sqrt{a(ad-cb)}}$	42

input
```
int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.54

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\,(abc-a^2d)}, \operatorname{arcta}\right.$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*
c - a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(
a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(
a*b*c - a^2*d)]
```

Sympy [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}}$$

input
```
integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^3}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^3}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**3)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.99 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

output
```
-1/4*(d*x^8+c)^(1/2)/a/c/x^4-1/4*b*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*
x^8+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/4*Sqrt[c + d*x^8]/(a*c*x^4) - (b*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4
 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(4*a^(3/2)*Sqrt[b*c - a*d
])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4} \int \frac{1}{x^8 (bx^8+a) \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{4} \left(\frac{\int -\frac{bc}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{ac} - \frac{\sqrt{c+dx^8}}{acx^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\frac{\int \frac{bc}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{ac} - \frac{\sqrt{c+dx^8}}{acx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(-\frac{b\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{a} - \frac{\sqrt{c+dx^8}}{acx^4}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4} \left(-\frac{b\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a} - \frac{\sqrt{c+dx^8}}{acx^4}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(-\frac{b\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{acx^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^4)) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*S
qrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}}{x^{4}}-\frac{bc\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{4ac}$	67

input
```
int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(-(d*x^8+c)^(1/2)/x^4-b*c/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1
/2)/x^4/(a*(a*d-b*c))^(1/2)))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(64) = 128.

Time = 0.14 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^5\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$
$$=\left[-\frac{\sqrt{-abc+a^2d}bcx^4\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)+}{16\,(a^2bc^2-a^3cd)x^4}\right.$$
$$\left.-\frac{\sqrt{abc-a^2d}bcx^4\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)+2\,\sqrt{dx^8+c}(abc-a^2d)}{8\,(a^2bc^2-a^3cd)x^4}\right]$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^
2)) + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8
*(sqrt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*
d)*x^4)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} dx = \int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} dx$$

input
```
integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^5} dx$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$$
$$= \frac{1}{4}d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}ad}+\frac{2}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2-c\right)ad}\right)$$

input
```
integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*
x^4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{\sqrt{d\,x^8 + c}}{bd\,x^{21} + ad\,x^{13} + bc\,x^{13} + ac\,x^5} dx$$

input
```
int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c*x**5 + a*d*x**13 + b*c*x**13 + b*d*x**21),x)
```

3.100 $\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

output

```
-1/12*(d*x^8+c)^(1/2)/a/c/x^12+1/12*(2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c^2/
x^4+1/4*b^2*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(
-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}\left(-ac+3bcx^8+2adx^8\right)}{12a^2c^2x^{12}} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{4a^{5/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]*(-(a*c) + 3*b*c*x^8 + 2*a*d*x^8))/(12*a^2*c^2*x^12) + (b^
2*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt
[b*c - a*d])])/(4*a^(5/2)*Sqrt[b*c - a*d])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13} (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4} \int \frac{1}{x^{16} (bx^8+a) \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{4} \left(\frac{\int -\frac{2bdx^8+3bc+2ad}{x^8 (bx^8+a) \sqrt{dx^8+c}}dx^4}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(-\frac{\int \frac{2bdx^8+3bc+2ad}{x^8 (bx^8+a) \sqrt{dx^8+c}}dx^4}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{4} \left(-\frac{-\frac{\int \frac{3b^2c^2}{\left(bx^8+a\right) \sqrt{dx^8+c}}dx^4}{ac}-\frac{\sqrt{c+dx^8} (2ad+3bc)}{acx^4}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{-\frac{3b^2c\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{a}-\frac{\sqrt{c+dx^8}(2ad+3bc)}{acx^4}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

$\downarrow$ 291

$$\frac{1}{4}\left(-\frac{-\frac{3b^2c\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a}-\frac{\sqrt{c+dx^8}(2ad+3bc)}{acx^4}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

$\downarrow$ 218

$$\frac{1}{4}\left(-\frac{-\frac{3b^2c\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(2ad+3bc)}{acx^4}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^{12}}\right)$$

input
```
Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-1/3*Sqrt[c + d*x^8]/(a*c*x^12) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(a
*c*x^4)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])
])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 20.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}\,\left(-2 a d\,x^{8}-3 b c\,x^{8}+a c\right)}{3 x^{12}}+\frac{b^{2} c^{2} \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{8}+c}}{x^{4} \sqrt{a \left(a d -c b \right)}}\right)}{\sqrt{a \left(a d -c b \right)}}}{4 a^{2} c^{2}}$	88

input
```
int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-1/3*(d*x^8+c)^(1/2)*(-2*a*d*x^8-3*b*c*x^8+a*c)/x^12+b^2*c^2/(a*(
a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2)))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.62

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = \left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^{12}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((bc-2\,ad)x^{12}-acx^4)\sqrt{dx^8+c}\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right.}{48\,(a^3bc^3-a^4c^2d)x^{12}}\right.$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)
*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x
^8 + a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^
3*c*d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c
 - a^2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c
)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)
) + 2*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sq
rt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{13}(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}\,x^{13}}\,dx$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(95) = 190$.

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=$$
$$-\frac{1}{12}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8}\right.\right.}\right.$$

input
```
integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
 + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(
3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*b*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{-\sqrt{d\,x^8+c}\,c+2\sqrt{d\,x^8+c}\,d\,x^8-12\left(\int \frac{\sqrt{d\,x^8+c}}{bd\,x^{21}+ad\,x^{13}+bc\,x^{13}+ac\,x^5}dx\right)b\,c^2x^{12}}{12a\,c^2x^{12}}$$

input
```
int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
( - sqrt(c + d*x**8)*c + 2*sqrt(c + d*x**8)*d*x**8 - 12*int(sqrt(c + d*x**
8)/(a*c*x**5 + a*d*x**13 + b*c*x**13 + b*d*x**21),x)*b*c**2*x**12)/(12*a*c
**2*x**12)
```

3.101 $\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 672

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^2\sqrt{c+dx^8}}{6bd} - \frac{(-a)^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{7/4}\sqrt{-bc+ad}} - \frac{(-a)^{5/4}\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{7/4}\sqrt{-bc+ad}}$$

$$- \frac{c^{3/4}(bc+4ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12bd^{5/4}(bc+ad)\sqrt{c+dx^8}}$$

$$+ \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}$$

$$+ \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}$$

output
```
1/6*x^2*(d*x^8+c)^(1/2)/b/d-1/8*(-a)^(5/4)*arctan((a*d-b*c)^(1/2)*x^2/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)-1/8*(-a)^(5/4)*arc
tanh((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(7/4)/(a*d-
b*c)^(1/2)-1/12*c^(3/4)*(4*a*d+b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1
/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/
2*2^(1/2))/b/d^(5/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*a*(b^(1/2)*c^(1/2)+(-a
)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)
^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)
-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^2
/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)+1/16
*a*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(
c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))
),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4
)/(d*x^8+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.21

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx =$$
$$-\frac{x^2\left(-5a(c+dx^8)+5ac\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+(bc+3ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF}\right.}{30abd\sqrt{c+dx^8}}$$

input
```
Integrate[x^17/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/30*(x^2*(-5*a*(c + d*x^8) + 5*a*c*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + (b*c + 3*a*d)*x^8*Sqrt[1 + (d*x^8)
/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*b*d*Sqrt[c
 + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.61 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {965, 979, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{16}}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{979}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\int \frac{(bc+3ad)x^8+ac}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{3bd}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}-\frac{3a^2d\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}}{3bd}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(3ad+bc)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}}{3bd}\right)$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d\left(\frac{\int\frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int\frac{\quad}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}\cdots\right.}}{\quad}\cdots\right.}{b}}{3bd}\right.$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(3ad+bc)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{c+dx^8}}{3bd}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(3ad+bc)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d\left(\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{c}\cdots}}{ad+bc}\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 2221

$$\frac{1}{2}\left(\frac{x^2\sqrt{dx^8+c}}{3bd}-\frac{\frac{(bc+3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{3a^2d\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{\cdots}{\left(\sqrt{\cdots}\right.}}}{2\sqrt[4]{c}(\cdots}\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 2223

$$\frac{1}{2}\left(\frac{x^2\sqrt{dx^8+c}}{3bd}-\frac{(bc+3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{3a^2d\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\cdots}}{2\sqrt[4]{c}(\cdots}\cdots\right.}{\cdots}\right.$$

input
```
Int[x^17/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((x^2*Sqrt[c + d*x^8])/(3*b*d) - (((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8]) - (3*a^2*d*(((a*((
Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^
(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*
Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqr
t[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/
(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqr
t[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^
(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sq
rt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4
)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)
^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sq
rt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^{17}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**17/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**17/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^17/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^17/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^17/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^17/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{17}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}\,x^2-6\left(\int \frac{\sqrt{d\,x^8+c}\,x^9}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx\right)ad-2\left(\int \frac{\sqrt{d\,x^8+c}\,x^9}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx\right)bc-2\left(\int \frac{\sqrt{d\,x^8+c}\,x}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}d\right.}{6bd}$$

input
```
int(x^17/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**8)*x**2 - 6*int((sqrt(c + d*x**8)*x**9)/(a*c + a*d*x**8 + b
*c*x**8 + b*d*x**16),x)*a*d - 2*int((sqrt(c + d*x**8)*x**9)/(a*c + a*d*x**
8 + b*c*x**8 + b*d*x**16),x)*b*c - 2*int((sqrt(c + d*x**8)*x)/(a*c + a*d*x
**8 + b*c*x**8 + b*d*x**16),x)*a*c)/(6*b*d)
```

3.102 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 635

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt[4]{-a}\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}}-\frac{\sqrt[4]{-a}\,\text{arctanh}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}}+\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16b\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}$$

output
```
-1/8*(-a)^(1/4)*arctan((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/b^(3/4)/(a*d-b*c)^(1/2)-1/8*(-a)^(1/4)*arctanh((a*d-b*c)^(1/2)*x^2/(-
a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/4*c^(3/4)*(c^(
1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(
1/2)-1/16*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x
^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c
^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))/b/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/
d^(1/4)/(d*x^8+c)^(1/2)-1/16*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)
+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*a
rctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a
)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b/c^(1/4)/(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^{10}\sqrt{\frac{c+dx^8}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

input
```
Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^
8)/a)])/(10*a*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.19 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.54, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {965, 983, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{ 983}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{ 925}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+}}\right.}{b}\right.$$

$$\Big\downarrow \text{ 1541}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\cdots\right.}{\cdots}}{2a}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int\frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-\cdots}\right.}{\cdots}}{2a}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 761

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{\cdots}}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2221

$$\frac{1}{2}\left(\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{a\left(\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 2223$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{a\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{Elli}\cdots}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}\cdots}\cdots\right.}{\cdots}\right.$$

input
```
Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c +
 d*x^8]) - (a*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c]
 + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
 + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sq
rt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4
)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*S
qrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a
]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1
/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sq
rt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c
^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]
*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[
b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c
 - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] +
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^9}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**9)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.103 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 635

$$
\begin{aligned}
&\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx \\
&= -\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}} - \frac{\sqrt[4]{b}\,\mathrm{arctanh}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}} \\
&\quad + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}
\end{aligned}
$$

output
```
-1/8*b^(1/4)*arctan((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(3/4)/(a*d-b*c)^(1/2)-1/8*b^(1/4)*arctanh((a*d-b*c)^(1/2)*x^2/(-a)^
(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/4*d^(3/4)*(c^(
1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiA
M(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(
1/2)+1/16*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x
^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c
^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))/a/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/
d^(1/4)/(d*x^8+c)^(1/2)+1/16*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)
+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*a
rctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a
)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/c^(1/4)/(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^2\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

input
```
Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8
)/a)])/(2*a*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 876, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {965, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)$$

$$\Big\downarrow \text{1541}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+ \cdots}{\cdots}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-\cdots}\right.}{\cdots}}{2a}\right.$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.$$

$\Big\downarrow$ 2221

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.$$

$\Big\downarrow$ 2223

$$\frac{1}{2}\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\cdots\right.}{2\sqrt[4]{b}\sqrt{bc\cdots}}\right.}{\cdots}}{2a}\right.$$

input
```
Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a
] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x
^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b
])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sq
rt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt
[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c +
a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-
a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2
)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4
)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}\,dx$$

input
```
int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.104 $\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 672

$$\begin{aligned}
&\int \frac{1}{x^7(a+bx^8)\sqrt{c+dx^8}}\,dx \\
&= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\arctan\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}} - \frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}} \\
&\quad - \frac{d^{3/4}(4bc+ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12ac^{5/4}(bc+ad)\sqrt{c+dx^8}} \\
&\quad - \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}} \\
&\quad - \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}
\end{aligned}$$

output
```
-1/6*(d*x^8+c)^(1/2)/a/c/x^6-1/8*b^(5/4)*arctan((a*d-b*c)^(1/2)*x^2/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/8*b^(5/4)*arcta
nh((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d
-b*c)^(1/2)-1/12*d^(3/4)*(a*d+4*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(
1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1
/2*2^(1/2))/a/c^(5/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*b*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2
)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^
2/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)-1/1
6*b*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/
(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d
^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/
4)/(d*x^8+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}}\,dx$$
$$= \frac{-5a(c+dx^8)-5(3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-bdx^{16}\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\right.}{30a^2cx^6\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Ap
pellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c
 + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {965, 980, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^8 (bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{980}$$

$$\frac{1}{2}\left(\frac{\int -\frac{bdx^8+3bc+ad}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{bdx^8+3bc+ad}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(-\frac{3bc\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2+d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(-\frac{3bc\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}-\frac{\sqrt{c+dx^8}}{3acx^6}\right)$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}\cdots}{3ac}\cdots\right.$$

$\big\downarrow$ 1541

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{3ac}\cdots\right.$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\cdots\right.}{\cdots}}{2a}\cdots\right.}{3ac}\cdots\right.$$

$\big\downarrow$ 761

$$\frac{1}{2}\left(-\frac{3bc\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}+\right.}{}\right.$$

↓ 2221

$$\frac{1}{2}\left(-\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\,a\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}\right.}{}\right.$$

↓ 2223

$$\frac{1}{2}\left(-\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\,a\right.}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}\right.}{}\right.$$

input
```
Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-1/3*Sqrt[c + d*x^8]/(a*c*x^6) - ((d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + 3*b*c*(((a*((Sqrt[b]*Sqrt[c])/S
qrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c
^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]
*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt
[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*
c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a
*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d
])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqr...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{1}{x^7 (b x^8 + a) \sqrt{d x^8 + c}} dx$$

input
```
int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^7 (a + bx^8) \sqrt{c + dx^8}} dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^7} dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b*d*x^23 + (b*c + a*d)*x^15 + a*c*x^7), x)
```

Sympy [F]

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^7 \left(b x^8+a\right) \sqrt{d x^8+c}} \, dx$$

input
```
int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{\sqrt{d x^8+c}}{bd\, x^{23}+ad\, x^{15}+bc\, x^{15}+ac\, x^7} dx$$

input
```
int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c*x**7 + a*d*x**15 + b*c*x**15 + b*d*x**23),x)
```

3.105 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1017

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

output
```
1/2*x^2*(d*x^8+c)^(1/2)/b/d^(1/2)/(c^(1/2)+d^(1/2)*x^4)+1/8*(-a)^(3/4)*arc
tan((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(5/4)/(-a*d
+b*c)^(1/2)-1/8*(-a)^(3/4)*arctanh((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/b^(5/4)/(-a*d+b*c)^(1/2)-1/2*c^(1/4)*(c^(1/2)+d^(1/2)*x^
4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticE(sin(2*arctan(d^(1/4
)*x^2/c^(1/4))),1/2*2^(1/2))/b/d^(3/4)/(d*x^8+c)^(1/2)+1/4*c^(1/4)*(c^(1/2
)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2
*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b/d^(3/4)/(d*x^8+c)^(1/2)+1/8*a*
d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*In
verseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^8+c)^(1/2)+1/8*a*d^(1/4)*(c^(1/2)
+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*
arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(b^(1/2)*c^(1/2)+
(-a)^(1/2)*d^(1/2))/(d*x^8+c)^(1/2)-1/16*a*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*Elli
pticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)
*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(3/2)/c^(1/4
)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)-1/16*((-a
)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/
2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.06

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^{14}\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

input
```
Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x
^8)/a)])/(14*a*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 1111, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {965, 983, 834, 27, 761, 993, 1510, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{983}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4}{\sqrt{dx^8+c}}dx^2}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{834}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{c}\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{993}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x^4}{\sqrt{dx^8+c}}dx^2}{\sqrt{d}}}{b}-\frac{a\left(\frac{\int\frac{1}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}-\frac{\int\frac{}{\left(\sqrt{-a}-\sqrt{\cdots}\right.}\cdots}{\cdots}\right.\cdots}{b}\cdots\right.$$

$\downarrow$ 1510

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+d\cdots}}{\sqrt{c}+\sqrt{d}\cdots}}{\sqrt{d}}\cdots}{b}\cdots\right.$$

$\downarrow$ 1541

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+d\cdots}}{\sqrt{c}+\sqrt{d}\cdots}}{\sqrt{d}}\cdots}{b}\cdots\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+d\cdots}}{\sqrt{c}+\sqrt{d}\cdots}}{\sqrt{d}}\cdots}{b}\cdots\right.$$

$\downarrow$ 761

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^8}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{x^2\sqrt{c+d}}{\sqrt{c}+\sqrt{d}}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{ 2221}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^8+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{x^2\sqrt{dx^8}}{\sqrt{d}x^4+}}{\sqrt{d}}}{b}\right.$$

$$\Big\downarrow \text{ 2223}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{dx^8+c}}-\frac{\frac{\sqrt[4]{c}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{x^2\sqrt{dx^8}}{\sqrt{d}x^4+}}{\sqrt{d}}}{b}\right.$$

input
```
Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
((-((-((x^2*Sqrt[c + d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + (c^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^8]))/Sqrt[d]) + (c
^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^
8]))/b - (a*(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c]
+ Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
+ (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*
x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2)
+ Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/
4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/
(c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-
a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]
*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^{13}}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{13}}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)*x^13/(b*d*x^16 + (b*c + a*d)*x^8 + a*c), x)
```

Sympy [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^{13}}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**13)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.106 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 788

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt{c+dx^8}}$$
$$- \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{8\sqrt{b}\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16\sqrt{b}\sqrt[4]{c}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}$$
$$+ \frac{\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{16a\sqrt{b}\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}\sqrt{c+dx^8}}$$

output
```
1/8*arctan((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(
1/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*arctanh((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/
b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*d^(1/4)*(
c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJaco
biAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^(1/2)/c^(1/4)/(b^(1/2)*c
^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^8+c)^(1/2)-1/8*d^(1/4)*(c^(1/2)+d^(1/2)*x^
4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1
/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*
d^(1/2))/(d*x^8+c)^(1/2)+1/16*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2
)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*
arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^(1/2)/c^(1/4)/((-a)^(1/2)
*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(d*x^8+c)^(1/2)+1/16*((-a)^(1/2)*b^(1/
2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^
4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
/a/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(d*x^8+c)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.08

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^6\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

input
```
Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8
)/a)])/(6*a*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 881, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {965, 993, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{993}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^4\right)\sqrt{dx^8+c}}dx^2}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1541}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\sqrt{c}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}-\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}}{2\sqrt{b}}-\frac{\frac{\sqrt{d}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\cdots}{\cdots}}{2\cdots}\right.$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(\sqrt{b}x^4+\sqrt{-a}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2\sqrt{b}}\right.$$

↓ 2221

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)}{4\sqrt{-a}\sqrt{b}\sqrt{d}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}\right.}{ad+bc}}{2\sqrt{b}}\right.$$

↓ 2223

$$\frac{1}{2}\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\arctan\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^8+c}}\right)}{2\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}+\frac{\left(\frac{\sqrt{c}}{\sqrt{-a}}+\frac{\sqrt{d}}{\sqrt{b}}\right)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{c}\left(\sqrt{b}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{c}}\right)}{4\sqrt{-a}\sqrt{b}\sqrt{d}}\right.}{4\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}\right.}{bc+ad}}{2\sqrt{b}}\right.$$

input
```
Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-1/2*(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])
*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(
-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - (((a*Sqrt[c])/(-a)^(3/2) + Sqrt[d]/Sq
rt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*S
qrt[c + d*x^8])))/(b*c + a*d))/Sqrt[b] + (-1/2*((Sqrt[b]*Sqrt[c] + Sqrt[-a
]*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*
(((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) +
((Sqrt[c]/Sqrt[-a] + Sqrt[d]/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[c]*(Sqrt[b] - (Sqrt[
-a]*Sqrt[d])/Sqrt[c])^2)/(Sqrt[-a]*Sqrt[b]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*Sq
rt[b]))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^5}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**5)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.107 $\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1053

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

output
```
-1/2*(d*x^8+c)^(1/2)/a/c/x^2+1/2*d^(1/2)*x^2*(d*x^8+c)^(1/2)/a/c/(c^(1/2)+
d^(1/2)*x^4)+1/8*b^(3/4)*arctan((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d
*x^8+c)^(1/2))/(-a)^(5/4)/(-a*d+b*c)^(1/2)-1/8*b^(3/4)*arctanh((-a*d+b*c)^
(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/(-a*d+b*c)^(1/2)-
1/2*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2
)*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))/a/c^(3/4)/(d*x
^8+c)^(1/2)+1/4*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*
x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a
/c^(3/4)/(d*x^8+c)^(1/2)+1/8*b^(1/2)*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8
+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^
(1/4)),1/2*2^(1/2))/a/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^8+
c)^(1/2)+1/8*b^(1/2)*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(
1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/
2))/a/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^8+c)^(1/2)-1/16*b^
(1/2)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c
)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/
4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))/a/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^
(1/4)/(d*x^8+c)^(1/2)-1/16*b^(1/2)*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*
(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*Ellipti...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx = \frac{-21a(c+dx^8)+7(-bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF}}{42a^2cx^2\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4
, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*S
qrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 1021, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {965, 980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \ 965$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^8+a) \sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \ 980$$

$$\frac{1}{2}\left(\frac{\int -\frac{x^4(-bdx^8+bc-ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{ac}-\frac{\sqrt{c+dx^8}}{acx^2}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(-\frac{\int \frac{x^4(-bdx^8+bc-ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{ac}-\frac{\sqrt{c+dx^8}}{acx^2}\right)$$

$$\Big\downarrow \ 1054$$

$$\frac{1}{2}\left(-\frac{\int \left(\frac{bcx^4}{(bx^8+a)\sqrt{dx^8+c}}-\frac{dx^4}{\sqrt{dx^8+c}}\right)dx^2}{ac}-\frac{\sqrt{c+dx^8}}{acx^2}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{-\frac{\sqrt{b}c^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{b^{3/4}c\arctan\left(\frac{\cdots}{\sqrt[4]{-\cdots}}\right.}{4\sqrt[4]{-a}\sqrt{\cdots}}}{\cdots}\right.$$

input
```
Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^2)) - (-((Sqrt[d]*x^2*Sqrt[c + d*x^8])/(Sqrt[c]
+ Sqrt[d]*x^4)) + (b^(3/4)*c*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1
/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) - (b^(3/4)*c*ArcTanh
[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4
)*Sqrt[b*c - a*d]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/Sqrt[c + d*x^8] - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
+ d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(2*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/
Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*(b*c + a*d)
*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x
^8]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^8]) + (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt
[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}}\, dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^3 \left(b\,x^8+a\right) \sqrt{d\,x^8+c}} \, dx$$

input
```
int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{\sqrt{d\,x^8+c}}{bd\,x^{19}+ad\,x^{11}+bc\,x^{11}+ac\,x^3} dx$$

input
```
int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c*x**3 + a*d*x**11 + b*c*x**11 + b*d*x**19),x)
```

3.108 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

output
```
1/5*x^5*(1+d*x^8/c)^(1/2)*AppellF1(5/8,1,1/2,13/8,-b*x^8/a,-d*x^8/c)/a/(d*
x^8+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

input
```
Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^
8)/a)])/(5*a*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^4}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},1,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

input
```
Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^
8)/c)])/(5*a*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^4}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^4}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**4)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.109 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^3\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

output
```
1/3*x^3*(1+d*x^8/c)^(1/2)*AppellF1(3/8,1,1/2,11/8,-b*x^8/a,-d*x^8/c)/a/(d*
x^8+c)^(1/2)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx=\frac{x^3\sqrt{\frac{c+dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

input
```
Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^
8)/a)])/(3*a*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^2}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{3}{8},1,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

input
```
Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^
8)/c)])/(3*a*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^2}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**2)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.110 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}} \, dx = \frac{x\sqrt{1+\frac{dx^8}{c}} \operatorname{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

output
```
x*(1+d*x^8/c)^(1/2)*AppellF1(1/8,1,1/2,9/8,-b*x^8/a,-d*x^8/c)/a/(d*x^8+c)^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 161 vs. 2(59) = 118.

Time = 10.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.73

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}} \, dx =$$
$$-\frac{9acx \operatorname{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\left(-9ac \operatorname{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x^8\left(2bc \operatorname{AppellF1}\left(\frac{9}{8},\frac{1}{2},2,\frac{17}{8},-\frac{dx^8}{c},-\frac{b}{}\right.\right.\right.}$$

input
```
Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*
x^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b
*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^
8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{936}$$

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\operatorname{AppellF1}\left(\frac{1}{8},1,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

input
```
Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^{8}+a)\,\sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}}{bd\,x^{16}+ad\,x^8+bc\,x^8+ac}dx$$

input
```
int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c + a*d*x**8 + b*c*x**8 + b*d*x**16),x)
```

3.111 $\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

output
```
-(1+d*x^8/c)^(1/2)*AppellF1(-1/8,1,1/2,7/8,-b*x^8/a,-d*x^8/c)/a/x/(d*x^8+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(62) = 124.

Time = 10.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.27

$$\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{-35a(c+dx^8)-5(bc-3ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{7}{8},\frac{1}{2},1,\frac{15}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF}}{35a^2cx\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sq
rt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \ 1013$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{x^2 (bx^8+a) \sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \ 1012$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1} \operatorname{AppellF1}\left(-\frac{1}{8},1,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

input
```
Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)
/c)])/(a*x*Sqrt[c + d*x^8]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{8}+a \right) \sqrt{d \,x^{8}+c}}d x$$

input
```
int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right) \sqrt{c+dx^8}} \, dx=\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} \, dx$$

input
```
integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^2 \left(b\,x^8+a\right) \sqrt{d\,x^8+c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^8) \sqrt{c+dx^8}} \, dx = \int \frac{\sqrt{d\,x^8+c}}{bd\,x^{18}+ad\,x^{10}+bc\,x^{10}+ac\,x^2} dx$$

input
```
int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c*x**2 + a*d*x**10 + b*c*x**10 + b*d*x**18),x)
```

3.112 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

output
```
-1/3*(1+d*x^8/c)^(1/2)*AppellF1(-3/8,1,1/2,5/8,-b*x^8/a,-d*x^8/c)/a/x^3/(d
*x^8+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.

Time = 10.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{-65a(c+dx^8)+13(-3bc+ad)x^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bdx^{16}\sqrt{1+\frac{dx^8}{c}}\,\text{Appe}}{195a^2cx^3\sqrt{c+dx^8}}$$

input
```
Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
(-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/
8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)
/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x
^3*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{x^4(bx^8+a)\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1} \text{AppellF1}\left(-\frac{3}{8},1,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

input
```
Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

output
```
-1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x
^8)/c)])/(a*x^3*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{8}+a \right) \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+b x^8\right) \sqrt{c+d x^8}} \, dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx$$

input
```
integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + bx^8) \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^4\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^8) \sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}}{bd\,x^{20}+ad\,x^{12}+bc\,x^{12}+ac\,x^4}dx$$

input
```
int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a*c*x**4 + a*d*x**12 + b*c*x**12 + b*d*x**20),x)
```

3.113 $\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}}{4b^2d} - \frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\,(a+bx^8)} + \frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}$$

output
```
1/4*(d*x^8+c)^(1/2)/b^2/d-1/8*a^2*(d*x^8+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^8+a)
+1/8*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^
(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{\sqrt{b}\sqrt{c+dx^8}(-3a^2d+2b^2cx^8+2ab(c-dx^8))}{d(bc-ad)(a+bx^8)} + \frac{a(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{5/2}}$$

input
```
Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x^8]*(-3*a^2*d + 2*b^2*c*x^8 + 2*a*b*(c - d*x^8)))/(d
*(b*c - a*d)*(a + b*x^8)) + (a*(4*b*c - 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*
x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {948, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8}\int \frac{x^{16}}{(bx^8+a)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{8}\left(\frac{\int -\frac{a(2bc-ad)-2b(bc-ad)x^8}{2(bx^8+a)\sqrt{dx^8+c}}dx^8}{b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-\frac{\int \frac{a(2bc-ad)-2b(bc-ad)x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{8}\left(-\frac{a(4bc-3ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(-\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{d}-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}-\frac{a^2\sqrt{c+dx^8}}{b^2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(-\frac{a^2\sqrt{c+dx^8}}{b^2\left(a+bx^8\right)(bc-ad)}-\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c+dx^8}(bc-ad)}{d}}{2b^2(bc-ad)}\right)$$

input
```
Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^8])/(b^2*(b*c - a*d)*(a + b*x^8))) - ((-4*(b*c - a*d)
*Sqrt[c + d*x^8])/d - (2*a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8
])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{-\frac{3d\left(b\,x^{8}+a\right)a\left(ad-\frac{4cb}{3}\right)\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{8}+\frac{3\sqrt{d\,x^{8}+c}\,\left(-\frac{2b^{2}c\,x^{8}}{3}-\frac{2a\left(-d\,x^{8}+c\right)b}{3}+a^{2}d\right)\sqrt{(ad-cb)b}}{8}}{b^{2}(ad-cb)d(b\,x^{8}+a)\sqrt{(ad-cb)b}}$	133

input
```
int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8/((a*d-b*c)*b)^(1/2)*(-d*(b*x^8+a)*a*(a*d-4/3*c*b)*arctan((d*x^8+c)^(1/
2)*b/((a*d-b*c)*b)^(1/2))+(d*x^8+c)^(1/2)*(-2/3*b^2*c*x^8-2/3*a*(-d*x^8+c)
*b+a^2*d)*((a*d-b*c)*b)^(1/2))/d/b^2/(a*d-b*c)/(b*x^8+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(103) = 206$.

Time = 0.13 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.86

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \left[\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,(2\,(b^4c^2-\cdots}{16\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+\cdots} -\frac{((4\,ab^2cd-3\,a^2bd^2)x^8+4\,a^2bcd-3\,a^3d^2)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)-(2\,(b^4c^2-2\,ab^3cd\cdots}{8\,(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+(b^6c^2d-2\,ab^5cd^2+a^2\cdots}\right.$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^
2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a
*b*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a
*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*
a^2*b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^
8), -1/8*((((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt
(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*
c)) - (2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b
^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^
3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\frac{\sqrt{dx^8+c}a^2d^3}{(b^3c-ab^2d)((dx^8+c)b-bc+ad)} + \frac{(4\,abcd^2-3\,a^2d^3)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c-ab^2d)\sqrt{-b^2c+abd}} - \frac{2\sqrt{dx^8+c}d}{b^2}}{8\,d^2}$$

input
```
integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(sqrt(d*x^8 + c)*a^2*d^3/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*
d)) + (4*a*b*c*d^2 - 3*a^2*d^3)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b
*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) - 2*sqrt(d*x^8 + c)*d/b^2)/d
^2
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{d\,x^8+c}}{4\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,(3\,a\,d-4\,b\,c)}{(3\,a^2\,d-4\,a\,b\,c)\,\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-4\,b\,c)}{8\,b^{5/2}\,{(a\,d-b\,c)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,(a\,d-b\,c)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

input
```
int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d
- 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(
5/2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*
(c + d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

Reduce [F]

$$\int \frac{x^{23}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{-2\sqrt{d\,x^8+c}\,ac+\sqrt{d\,x^8+c}\,ad\,x^8-2\sqrt{d\,x^8+c}\,bc\,x^8-12\left(\int \frac{\sqrt{d\,x^8+c}\,x^{15}}{a\,b^2d^2x^{24}-2b^3cd\,x^{24}+2a^2b\,d^2x^{16}-3a\,b^2cd\,x^{16}-2b^3c^2x^{16}}\right.}{}$$

input
```
int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**8)*a*c + sqrt(c + d*x**8)*a*d*x**8 - 2*sqrt(c + d*x**8
)*b*c*x**8 - 12*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 -
2*a**2*b*c**2 + 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x*
*16 + a*b**2*d**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a**4*d*
*3 + 40*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 - 2*a**2*b
*c**2 + 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x**16 + a*
b**2*d**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a**3*b*c*d**2 -
 12*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 - 2*a**2*b*c**
2 + 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x**16 + a*b**2
*d**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a**3*b*d**3*x**8 -
32*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 - 2*a**2*b*c**2
 + 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x**16 + a*b**2*
d**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a**2*b**2*c**2*d + 4
0*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 - 2*a**2*b*c**2
+ 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x**16 + a*b**2*d
**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a**2*b**2*c*d**2*x**8
 - 32*int((sqrt(c + d*x**8)*x**15)/(a**3*c*d + a**3*d**2*x**8 - 2*a**2*b*c
**2 + 2*a**2*b*d**2*x**16 - 4*a*b**2*c**2*x**8 - 3*a*b**2*c*d*x**16 + a*b*
*2*d**2*x**24 - 2*b**3*c**2*x**16 - 2*b**3*c*d*x**24),x)*a*b**3*c**2*d*x**
8)/(4*b*d*(a**2*d - 2*a*b*c + a*b*d*x**8 - 2*b**2*c*x**8))
```

3.114 $\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\,(a+bx^8)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

output
```
1/8*a*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)-1/8*(-a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{8b^{3/2}}$$

input
```
Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{1}{8}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \; 87$$

$$\frac{1}{8}\left(\frac{(2bc-ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2b(bc-ad)}+\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{8}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{8}\left(\frac{a\sqrt{c+dx^8}}{b(a+bx^8)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[c + d*x^8])/(b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/8
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^{8}+c}}{b\,x^{8}+a}+\frac{(ad-2cb)\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{8(ad-cb)b}$	83

input
```
int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a*d-b*c)/b*(-a*(d*x^8+c)^(1/2)/(b*x^8+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan((d*x^8+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 3.52

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \left[\frac{((2\,b^2c-abd)x^8+2\,abc-a^2d)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(ab^2c-a^2bd)}{16\,((b^5c^2-2\,ab^4cd+a^2b^3d^2)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2)}\right.$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((
b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)
) + 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b
^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a
*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*s
qrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)
)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d +
 a^3*b^2*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{\sqrt{dx^8+c}ad^2}{(b^2c-abd)((dx^8+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{8\,d}$$

input
```
integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,(a\,d-2\,b\,c)}{8\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,(a\,d-b\,c)\,(4\,b\,(d\,x^8+c)+4\,a\,d-4\,b\,c)}$$

input
```
int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c
 + d*x^8) + 4*a*d - 4*b*c))
```

Reduce [F]

$$\int \frac{x^{15}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^{15}}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**15)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*
d*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.115 $\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=-\frac{\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)}+\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}(bc-ad)^{3/2}}$$

output
```
-1/8*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*d*arctanh(b^(1/2)*(d*x^8+c)^
(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\left(-\frac{\sqrt{c+dx^8}}{(bc-ad)\,(a+bx^8)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {946, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{946}$$

$$\frac{1}{8}\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{8}\left(-\frac{d\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^8}{2(bc-ad)}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(-\frac{\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{bc-ad}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}}{(a+bx^8)(bc-ad)}\right)$$

input
```
Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/((b*c - a*d)*(a + b*x^8))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/8
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^{8}+c}}{b\,x^{8}+a}+\frac{d \arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)}{\sqrt{(ad-cb)b}}}{8ad-8cb}$	71

input
```
int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/(a*d-b*c)*((d*x^8+c)^(1/2)/(b*x^8+a)+d/((a*d-b*c)*b)^(1/2)*arctan((d*x
^8+c)^(1/2)*b/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.47

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{(bdx^8+ad)\sqrt{b^2c-abd}\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}(b^2c-abd)}{16\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)},\right.$$
$$\left.-\frac{(bdx^8+ad)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{dx^8+c}\sqrt{-b^2c+abd}}{bdx^8+bc}\right)+\sqrt{dx^8+c}(b^2c-abd)}{8\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2)}\right]$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2
*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^
2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a
^2*b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(
b^2*c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^8+c}d}{8\left((dx^8+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{d\sqrt{d\,x^8+c}}{2\,(a\,d-b\,c)\,(4\,b\,(d\,x^8+c)+4\,a\,d-4\,b\,c)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) +
(d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d -
b*c)^(3/2))
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}\,x^7}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**7)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.116 $\int \frac{1}{x(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}$$

output
```
1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/4*arctanh((d*x^8+c)^(1/2)/c
^(1/2))/a^2/c^(1/2)+1/8*b^(1/2)*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(
1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{-\frac{ab\sqrt{c+dx^8}}{(-bc+ad)(a+bx^8)}+\frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

input
```
Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^8])/((-(b*c) + a*d)*(a + b*x^8))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c])/(8*a^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{8}\int \frac{1}{x^8\left(bx^8+a\right)^2\sqrt{dx^8+c}}dx^8$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{8}\left(\frac{\int \frac{bdx^8+2bc-2ad}{2x^8(bx^8+a)\sqrt{dx^8+c}}dx^8}{a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{\int \frac{bdx^8+2(bc-ad)}{x^8(bx^8+a)\sqrt{dx^8+c}}dx^8}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{8}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{8}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{a\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(a*(b*c - a*d)*(a + b*x^8)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{-2\left(b\,x^{8}+a\right)\sqrt{c}\,b\left(cb-\frac{3ad}{2}\right)\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{(ad-cb)b}}\right)+\left(2(ad-cb)\left(b\,x^{8}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)+\sqrt{d\,x^{8}+c}\,\sqrt{c}\,ab\right)\sqrt{(ad-cb}}{8\sqrt{c}\,\sqrt{(ad-cb)b}\,a^{2}(ad-cb)\left(b\,x^{8}+a\right)}$

input
```
int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^(1/2)*(-2*(b*x^8+a)*c^(1/2)*b*(c*b-3/2*a*d)*arctan((d*x^8+c)^(1/2)*
b/((a*d-b*c)*b)^(1/2))+(2*(a*d-b*c)*(b*x^8+a)*arctanh((d*x^8+c)^(1/2)/c^(1
/2))+(d*x^8+c)^(1/2)*c^(1/2)*a*b)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)
/a^2/(a*d-b*c)/(b*x^8+a)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 816, normalized size of antiderivative = 6.18

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2
- 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8
 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x
^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/
x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x
^8 + c)*a*b*c - ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt
(-b/(b*c - a*d))*arctan(sqrt(d*x^8 + c)*sqrt(-b/(b*c - a*d))) + ((b^2*c -
a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c)
 + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/16*
(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^8 + c)) + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*
c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d
*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))/((a^2*b^2*c^2 - a
^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c)*a*b*c - ((2*b^2
*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c - a*d))*arctan
(sqrt(d*x^8 + c)*sqrt(-b/(b*c - a*d))) + 2*((b^2*c - a*b*d)*x^8 + a*b*c -
a^2*d)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^8 + c)))/((a^2*b^2*c^2 - a^3*b*c*
d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

Sympy [F]

$$\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx=\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx=\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}x}\,dx$$

input
```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx=\frac{\sqrt{dx^8+c}bd}{8\,(abc-a^2d)((dx^8+c)b-bc+ad)}-\frac{(2\,b^2c-3\,abd)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(a^2bc-a^3d)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/
8*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))
/(a^2*sqrt(-c))
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 3017, normalized size of antiderivative = 22.86

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*
b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
- ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2
*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(
512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b
^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c
)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*
d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^
2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2
- 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*
b^4*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^
3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))
*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/
(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^...
```

Reduce [F]

$$\int \frac{1}{x\,(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \int \frac{1}{x\,(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.117 $\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}$$

output

```
-1/8*b*(-a*d+2*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^8+a)-1/8*(d*x^8+
c)^(1/2)/a/c/x^8/(b*x^8+a)+1/8*(a*d+4*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2)
)/a^3/c^(3/2)-1/8*b^(3/2)*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(
-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \frac{\frac{a\sqrt{c+dx^8}(-a^2d+2b^2cx^8+ab(c-dx^8))}{c(-bc+ad)x^8(a+bx^8)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{c^{3/2}}}{8a^3}$$

input
```
Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*Sqrt[c + d*x^8]*(-(a^2*d) + 2*b^2*c*x^8 + a*b*(c - d*x^8)))/(c*(-(b*c)
 + a*d)*x^8*(a + b*x^8)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^8])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^8]/Sqrt[c]])/c^(3/2))/(8*a^3)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {948, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8} \int \frac{1}{x^{16} (bx^8+a)^2 \sqrt{dx^8+c}} dx^8$$

$$\Big\downarrow \text{ 114}$$

$$\frac{1}{8} \left(-\frac{\int \frac{3bdx^8+4bc+ad}{2x^8(bx^8+a)^2\sqrt{dx^8+c}} dx^8}{ac} - \frac{\sqrt{c+dx^8}}{acx^8 (a+bx^8)} \right)$$

$$\Big\downarrow 27$$

$$\frac{1}{8}\left(-\frac{\int \frac{3bdx^8+4bc+ad}{x^8(bx^8+a)^2\sqrt{dx^8+c}}dx^8}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow 168$$

$$\frac{1}{8}\left(-\frac{\frac{\int \frac{bd(2bc-ad)x^8+(bc-ad)(4bc+ad)}{x^8\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^8\sqrt{dx^8+c}}dx^8}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^8}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^{16}}{d}-\frac{c}{d}}d\sqrt{dx^8+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^{16}}{d}+a-\frac{bc}{d}}d\sqrt{dx^8+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{8}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^8}(2bc-ad)}{a(a+bx^8)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^8}}{acx^8\left(a+bx^8\right)}\right)$$

input
```
Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-(Sqrt[c + d*x^8]/(a*c*x^8*(a + b*x^8))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^8])/(a*(b*c - a*d)*(a + b*x^8)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^8]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-4\left(b\,x^{8}+a\right)\left(cb-\frac{5ad}{4}\right)c^{\frac{5}{2}}b^{2}x^{8}\arctan\left(\frac{\sqrt{d\,x^{8}+c}\,b}{\sqrt{\left(ad-cb\right)b}}\right)+\sqrt{\left(ad-cb\right)b}\,\left(c\,x^{8}\left(b\,x^{8}+a\right)\left(ad+4cb\right)\left(ad-cb\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{\sqrt{c}}\right)+c^{\frac{3}{2}}\right.}{8\sqrt{\left(ad-cb\right)b}\,c^{\frac{5}{2}}x^{8}a^{3}\left(ad-cb\right)\left(b\,x^{8}+a\right)}$

input
```
int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-4*(b*x^8+a)*(c*b-5/4*a*d)*c^(5/2)*b^2*x^8*arctan((d*x^8+c)^(1/2)*b/(
(a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(c*x^8*(b*x^8+a)*(a*d+4*b*c)*(a*d-
b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))+c^(3/2)*(d*x^8+c)^(1/2)*a*(2*b^2*c*x
^8+a*(-d*x^8+c)*b-a^2*d)))/((a*d-b*c)*b)^(1/2)/c^(5/2)/x^8/a^3/(a*d-b*c)/(
b*x^8+a)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1189, normalized size of antiderivative = 6.43

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x
^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*
c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a
^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d
*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d
)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)
*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), 1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)
*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(sqr
t(d*x^8 + c)*sqrt(-b/(b*c - a*d))) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2
)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8 + 2
*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 +
a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 +
(a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d
^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt
(-c)/sqrt(d*x^8 + c)) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 -
 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sq
rt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((2*a*b^2*
c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3
 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), 1/8*(((4*b^3*c^3 - 5*
a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a...
```

Sympy [F]

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**9*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^9 (a + bx^8)^2 \sqrt{c + dx^8}}\, dx = \int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c}x^9}\, dx$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^9 (a + bx^8)^2 \sqrt{c + dx^8}}\, dx = \frac{(4\,b^3c - 5\,ab^2d)\arctan\left(\frac{\sqrt{dx^8+c}b}{\sqrt{-b^2c+abd}}\right)}{8\,(a^3bc - a^4d)\sqrt{-b^2c + abd}} - \frac{2\,(dx^8 + c)^{\frac{3}{2}}b^2cd - 2\,\sqrt{dx^8 + c}b^2c^2d - (dx^8 + c)^{\frac{3}{2}}abd^2 + 2\,\sqrt{dx^8 + c}abcd^2 - \sqrt{dx^8 + c}a^2d^3}{8\,(a^2bc^2 - a^3cd)\left((dx^8 + c)^2b - 2\,(dx^8 + c)bc + bc^2 + (dx^8 + c)ad - acd\right)} - \frac{(4\,bc + ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8
+ c)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 +
 c)^2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b
*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 3832, normalized size of antiderivative = 20.71

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
(((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) +
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b
^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*
d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 -
2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^
7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)
*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*
a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b
^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3
 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*
d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^
2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b
^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(
a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(
c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^...
```

Reduce [F]

$$\int \frac{1}{x^9 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^9 (b\,x^8+a)^2 \sqrt{d\,x^8+c}}dx$$

input
```
int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.118 $\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} - \frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

output
```
1/8*a*x^4*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)-1/8*a^(1/2)*(-2*a*d+3*b*c
)*arctan((-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(3/2
)+1/4*arctanh(d^(1/2)*x^4/(d*x^8+c)^(1/2))/b^2/d^(1/2)
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\frac{abx^4\sqrt{c+dx^8}}{(bc-ad)(a+bx^8)} + \frac{\sqrt{a}(-3bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{d}}}{8b^2}$$

input
```
Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c +
2*a*d)*ArcTan[(a*Sqrt[d] + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]
*Sqrt[b*c - a*d])])/(b*c - a*d)^(3/2) + (2*Log[Sqrt[d]*x^4 + Sqrt[c + d*x^
8]])/Sqrt[d])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^{16}}{(bx^8+a)^2\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b(a+bx^8)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^8}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b(a+bx^8)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^8+c}}dx^4}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b(a+bx^8)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\int \frac{1}{1-dx^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int\frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{ax^4\sqrt{c+dx^8}}{2b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{b\sqrt{d}}}{2b(bc-ad)}\right)$$

input
```
Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*x^4*Sqrt[c + d*x^8])/(2*b*(b*c - a*d)*(a + b*x^8)) - ((Sqrt[a]*(3*b*c
- 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*Sqrt[
b*c - a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]])/(b*Sq
rt[d]))/(2*b*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 20.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{-\frac{a\left(-\frac{\sqrt{d\,x^{8}+c}\,b\,x^{4}}{b\,x^{8}+a}-\frac{(2ad-3cb)\,\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}\right)}{ad-cb}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{d}}\right)}{\sqrt{d}}}{8b^{2}}$	117

input
```
int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^2*(-a/(a*d-b*c)*(-(d*x^8+c)^(1/2)*b*x^4/(b*x^8+a)-(2*a*d-3*b*c)/(a*
(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2)))-2/d^(
1/2)*arctanh((d*x^8+c)^(1/2)/x^4/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1083, normalized size of antiderivative = 7.68

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*
d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d
 - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^
2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*s
qrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*
c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a
*b*d*x^4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(d*
x^8 + c)*sqrt(-d)/(d*x^4)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*
a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^
2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)
))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d -
 a^2*b^2*d^2), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2
)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) +
 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x
^8 + c)*sqrt(d)*x^4 - c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2
*d^2), 1/16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 +
3*a*b*c*d - 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x...
```

Sympy [F]

$$\int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**19/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(117) = 234.

Time = 0.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.11

$$\int \frac{x^{19}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = -\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 abc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4 b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 ad+bc^2\right)(b^3c-ab^2c} - \frac{\log\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2\right)}{8\,b^2\sqrt{d}}$$

input
```
integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*s
qrt(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqr
t(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/
(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)
) - 1/8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{19}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{19}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

input
```
int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output

```
( - 4*sqrt(c + d*x**8)*a*b*d*x**4 - 2*sqrt(d)*log(sqrt(c + d*x**8) - sqrt(
d)*x**4)*a**2*d + sqrt(d)*log(sqrt(c + d*x**8) - sqrt(d)*x**4)*a*b*c - 2*s
qrt(d)*log(sqrt(c + d*x**8) - sqrt(d)*x**4)*a*b*d*x**8 + sqrt(d)*log(sqrt(
c + d*x**8) - sqrt(d)*x**4)*b**2*c*x**8 + 2*sqrt(d)*log(sqrt(c + d*x**8) +
 sqrt(d)*x**4)*a**2*d - sqrt(d)*log(sqrt(c + d*x**8) + sqrt(d)*x**4)*a*b*c
 + 2*sqrt(d)*log(sqrt(c + d*x**8) + sqrt(d)*x**4)*a*b*d*x**8 - sqrt(d)*log
(sqrt(c + d*x**8) + sqrt(d)*x**4)*b**2*c*x**8 - 32*int((sqrt(c + d*x**8)*x
**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*
a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x
**16 - b**3*c*d*x**24),x)*a**5*d**3 + 64*int((sqrt(c + d*x**8)*x**3)/(2*a*
*3*c*d + 2*a**3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**
2*x**16 - 2*a*b**2*c**2*x**8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**
3*c*d*x**24),x)*a**4*b*c*d**2 - 32*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d
 + 2*a**3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**1
6 - 2*a*b**2*c**2*x**8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*
x**24),x)*a**4*b*d**3*x**8 - 24*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d +
2*a**3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 -
 2*a*b**2*c**2*x**8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**
24),x)*a**3*b**2*c**2*d + 64*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a
**3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 -...
```

3.119 $\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\,(a+bx^8)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}(bc-ad)^{3/2}}$$

output
```
-1/8*x^4*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*c*arctan((-a*d+b*c)^(1/2
)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\left(-\frac{x^4\sqrt{c+dx^8}}{(bc-ad)\,(a+bx^8)} + \frac{c\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}\right)$$

input
```
Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-((x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8))) + (c*ArcTan[(a*Sqrt[d]
 + b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqr
t[a]*(b*c - a*d)^(3/2)))/8
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{4}\left(\frac{\int \frac{c}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{c\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{2(bc-ad)}-\frac{x^4\sqrt{c+dx^8}}{2(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{c\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{2(a+bx^8)(bc-ad)}\right)$$

input
```
Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-1/2*(x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*
c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(2*Sqrt[a]*(b*c - a*d)^(3/2)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 15.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{8}+c}\,x^{4}}{c\left(b\,x^{8}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a\left(a d-c b\right)}}\right)}{\sqrt{a\left(a d-c b\right)}}\right)}{8\left(a d-c b\right)}$	81

input
```
int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*c/(a*d-b*c)*(-(d*x^8+c)^(1/2)*x^4/c/(b*x^8+a)+1/(a*(a*d-b*c))^(1/2)*a
rctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(77) = 154$.

Time = 0.20 (sec) , antiderivative size = 426, normalized size of antiderivative = 4.58

$$\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[-\frac{4\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2+4}{b^2x^{16}+2\,abx^8+a}\right.}{32\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right.$$
$$\left.-\frac{2\sqrt{dx^8+c}(abc-a^2d)x^4-(bcx^8+ac)\sqrt{abc-a^2d}\arctan\left(\frac{((bc-2\,ad)x^8-ac)\sqrt{dx^8+c}\sqrt{abc-a^2d}}{2\,((abcd-a^2d^2)x^{12}+(abc^2-a^2cd)x^4)}\right)}{16\,((ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2)}\right]$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*
c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*
a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*
sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*s
qrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*a
rctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((
a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

Sympy [F]

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**11/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{11}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(77) = 154.

Time = 0.36 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{8\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{4\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2ad+bc^2\right)(b^2c-abd}$$

input
```
integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a
*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*(
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b
 - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d + b*c^2)*(b^2*c - a*b*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{11}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{\sqrt{d\,x^8+c}\,x^4 - 8\left(\int \frac{\sqrt{d\,x^8+c}\,x^3}{2a\,b^2d^2x^{24}-b^3cd\,x^{24}+4a^2b\,d^2x^{16}-b^3c^2x^{16}+2a^3d^2x^8+3a^2bcd\,x^8-2a\,b^2c^2x^8+2a^3cd-a^2b\,c^2}dx\right)a^3cd + 4\left(\int\right.}{}$$

input
```
int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
(sqrt(c + d*x**8)*x**4 - 8*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**
3*d**2*x**8 - a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*
b**2*c**2*x**8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x
)*a**3*c*d + 4*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8
- a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x*
*8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a**2*b*c**
2 - 8*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a**2*b*
c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8 + 2*a*
b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a**2*b*c*d*x**8 + 4
*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a**2*b*c**2
+ 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8 + 2*a*b**2*
d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a*b**2*c**2*x**8)/(4*(2*
a**2*d - a*b*c + 2*a*b*d*x**8 - b**2*c*x**8))
```

3.120 $\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/8*b*x^4*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)+1/8*(-2*a*d+b*c)*arctan((
-a*d+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=-\frac{bx^4\sqrt{c+dx^8}}{8a(-bc+ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/8*(b*x^4*Sqrt[c + d*x^8])/(a*(-(b*c) + a*d)*(a + b*x^8)) + ((b*c - 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqr
t[b*c - a*d])])/(8*a^(3/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {965, 296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{296}$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{bx^4\sqrt{c+dx^8}}{2a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{2a(bc-ad)}+\frac{bx^4\sqrt{c+dx^8}}{2a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{4}\left(\frac{(bc-2ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{2a(a+bx^8)(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^4*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*Ar
cTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(2*a^(3/2)*(b*c - a
*d)^(3/2)))/4
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 15.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}\,b\,x^{4}}{b\,x^{8}+a}+\frac{(2ad-cb)\operatorname{arctanh}\left(\frac{a\sqrt{d\,x^{8}+c}}{x^{4}\sqrt{a(ad-cb)}}\right)}{\sqrt{a(ad-cb)}}}{8a(ad-cb)}$	90

input
```
int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a/(a*d-b*c)*(-(d*x^8+c)^(1/2)*b*x^4/(b*x^8+a)+(2*a*d-b*c)/(a*(a*d-b*c)
)^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(88) = 176.

Time = 0.22 (sec) , antiderivative size = 467, normalized size of antiderivative = 4.49

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \left[\frac{4\sqrt{dx^8+c}(ab^2c-a^2bd)x^4-((b^2c-2\,abd)x^8+abc-2\,a^2d)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(}{}\right.}{32\,((a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x^8+a^3b^2c^2-2\,a^4bcd+}\right.$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8
+ a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^1
2 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 +
 a^2)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a
^4*b*c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b
^2*c - 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*
c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*
d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4
*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

Sympy [F]

$$\int \frac{x^3}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^3}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^3}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^3}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(88) = 176$.

Time = 0.18 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.28

$$\int \frac{x^3}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = -\frac{1}{8}\,d^{\frac{3}{2}} \left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2 bc}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4 b - 2\left(\sqrt{d}x^4-\sqrt{dx^8+}\right.\right.}\right.$$

input
```
integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2
*a*d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqr
t(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*
b*c*d - a^2*d^2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^3}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^3}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**3)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.121 $\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4(a+bx^8)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/8*(-2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^4+1/8*b*(d*x^8+c)^(
1/2)/a/(-a*d+b*c)/x^4/(b*x^8+a)-1/8*b*(-4*a*d+3*b*c)*arctan((-a*d+b*c)^(1/
2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{\sqrt{c+dx^8}(2abc-2a^2d+3b^2cx^8-2abdx^8)}{8a^2c(-bc+ad)x^4(a+bx^8)} - \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^8+bx^4\sqrt{c+dx^8}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(Sqrt[c + d*x^8]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^8 - 2*a*b*d*x^8))/(8*a^2*c
*(-(b*c) + a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] +
b*Sqrt[d]*x^8 + b*x^4*Sqrt[c + d*x^8])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5
/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {965, 374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{4}\int \frac{1}{x^8 (bx^8+a)^2 \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{374}$$

$$\frac{1}{4}\left(\frac{b\sqrt{c+dx^8}}{2ax^4 (a+bx^8)(bc-ad)}-\frac{\int -\frac{2bdx^8+3bc-2ad}{x^8(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{\int \frac{2bdx^8+3bc-2ad}{x^8(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4 (a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{4}\left(\frac{-\frac{\int \frac{bc(3bc-4ad)}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{ac}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4 (a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^4}{a}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 291$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow 218$$

$$\frac{1}{4}\left(\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}(3bc-2ad)}{acx^4}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^4\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*x^4*(a + b*x^8)) + (-(((3*b*c - 2*a*
d)*Sqrt[c + d*x^8])/(a*c*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c -
a*d)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 24.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}}{x^{4}}+\frac{b c \left(\frac{\sqrt{d\,x^{8}+c}\,b\,x^{4}}{b\,x^{8}+a}-\frac{(4 a d -3 c b) \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{8}+c}}{x^{4} \sqrt{a (a d -c b)}}\right)}{\sqrt{a (a d -c b)}}\right)}{2 a d -2 c b}}{4 a^{2} c}$	112

input
```
int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*(-(d*x^8+c)^(1/2)/x^4+1/2*b*c/(a*d-b*c)*((d*x^8+c)^(1/2)*b*x^4/(b*
x^8+a)-(4*a*d-3*b*c)/(a*(a*d-b*c))^(1/2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*
(a*d-b*c))^(1/2))))/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(129) = 258$.

Time = 0.21 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^5 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx$$
$$= \left[-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^{12} + (3\,ab^2c^2 - 4\,a^2bcd)x^4)\sqrt{-abc + a^2d}\log\left(\frac{(b^2c^2 - 8\,abcd + 8\,a^2d^2)x^{16} - 2\,(3\,abc^2 - 4\,a^2cd)x^8}{b^2x^{16} +}\right.}{32\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a} \right.$$
$$-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^{12} + (3\,ab^2c^2 - 4\,a^2bcd)x^4)\sqrt{abc - a^2d}\arctan\left(\frac{((bc - 2\,ad)x^8 - ac)\sqrt{dx^8 + c}\sqrt{abc - a^2d}}{2\,((abcd - a^2d^2)x^{12} + (abc^2 - a^2cd)x^4)}\right) + 2\,(}{16\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^{12} + (a^4b^2c^3 - }$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(
d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b
^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d +
2*a^4*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)
*x^12 + (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2
 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d
)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)
/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 -
 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^
2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 +
(a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

Sympy [F]

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**5*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}\,x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(129) = 258$.

Time = 0.43 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$
$$= \frac{1}{8}d^{\frac{5}{2}}\left(\frac{(3\,b^2c-4\,abd)\arctan\left(\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^2bcd^2-a^3d^3)\sqrt{abcd-a^2d^2}} + \frac{2\left(3\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b^2c-4\left(\sqrt{d}x^4-\sqrt{\right.\right.}{\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^6b-3\left(\sqrt{d}x^4-\sqrt{\right.\right.}\right.$$

input
```
integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c
))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sq
rt(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 -
 sqrt(d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(
d)*x^4 - sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^
2 - 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*
d^3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^5\,(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$
$$= \frac{-2\sqrt{d\,x^8+c}\,ad+\sqrt{d\,x^8+c}\,bc-2\sqrt{d\,x^8+c}\,bd\,x^8-32\left(\int \frac{\sqrt{d\,x^8+c}\,x^3}{2a\,b^2d^2x^{24}-b^3cd\,x^{24}+4a^2b\,d^2x^{16}-b^3c^2x^{16}+2a^3d^2x^8+3a^2bc}\right.}{}$$

input
```
int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x**8)*a*d + sqrt(c + d*x**8)*b*c - 2*sqrt(c + d*x**8)*b*d
*x**8 - 32*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a*
*2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8 +
 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a**3*b*c*d**2*
x**4 + 40*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a**
2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8 +
2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a**2*b**2*c**2*
d*x**4 - 32*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a
**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8
+ 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a**2*b**2*c*d
**2*x**12 - 12*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8
- a**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x*
*8 + 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a*b**3*c**
3*x**4 + 40*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a
**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8
+ 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*a*b**3*c**2*d
*x**12 - 12*int((sqrt(c + d*x**8)*x**3)/(2*a**3*c*d + 2*a**3*d**2*x**8 - a
**2*b*c**2 + 3*a**2*b*c*d*x**8 + 4*a**2*b*d**2*x**16 - 2*a*b**2*c**2*x**8
+ 2*a*b**2*d**2*x**24 - b**3*c**2*x**16 - b**3*c*d*x**24),x)*b**4*c**3*x**
12)/(4*a*c*x**4*(2*a**2*d - a*b*c + 2*a*b*d*x**8 - b**2*c*x**8))
```

3.122 $\int \frac{1}{x^{13}(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = & -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} \\ & +\frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} \\ & +\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} \\ & +\frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}\end{aligned}$$

output
```
-1/24*(-2*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^12+1/24*(-4*a^2*d^
2-8*a*b*c*d+15*b^2*c^2)*(d*x^8+c)^(1/2)/a^3/c^2/(-a*d+b*c)/x^4+1/8*b*(d*x^
8+c)^(1/2)/a/(-a*d+b*c)/x^12/(b*x^8+a)+1/8*b^2*(-6*a*d+5*b*c)*arctan((-a*d
+b*c)^(1/2)*x^4/a^(1/2)/(d*x^8+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} dx = -\frac{\sqrt{c+dx^8}(15b^3c^2x^{16}+2ab^2cx^8(5c-4dx^8)+2a^3d(c-2dx^8)-2a^2b(c^2+3cdx^8+2d^2x^{16}))}{24a^3c^2(-bc+ad)x^{12}(a+bx^8)} + \frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx^4\left(\sqrt{d}x^4+\sqrt{c+dx^8}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/24*(Sqrt[c + d*x^8]*(15*b^3*c^2*x^16 + 2*a*b^2*c*x^8*(5*c - 4*d*x^8) +
2*a^3*d*(c - 2*d*x^8) - 2*a^2*b*(c^2 + 3*c*d*x^8 + 2*d^2*x^16)))/(a^3*c^2*
(-(b*c) + a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d]
+ b*x^4*(Sqrt[d]*x^4 + Sqrt[c + d*x^8]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^
(7/2)*(b*c - a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {965, 374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{4}\int \frac{1}{x^{16} (bx^8+a)^2 \sqrt{dx^8+c}}dx^4$$

$$\Big\downarrow \text{ 374}$$

$$\frac{1}{4}\left(\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}-\frac{\int -\frac{4bdx^8+5bc-2ad}{x^{16}(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{4}\left(\frac{\int \frac{4bdx^8+5bc-2ad}{x^{16}(bx^8+a)\sqrt{dx^8+c}}dx^4}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{1}{4}\left(\frac{-\frac{\int \frac{2bd(5bc-2ad)x^8+15b^2c^2-4a^2d^2-8abcd}{x^8\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 445}$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{ac}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^4}{a}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 291}$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-(ad-bc)x^8}d\frac{x^4}{\sqrt{dx^8+c}}}{a}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{4}\left(\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x^4}}{3ac}-\frac{\sqrt{c+dx^8}(5bc-2ad)}{3acx^{12}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{2ax^{12}\left(a+bx^8\right)(bc-ad)}\right)$$

input
```
Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(2*a*(b*c - a*d)*x^12*(a + b*x^8)) + (-1/3*((5*b*c -
2*a*d)*Sqrt[c + d*x^8])/(a*c*x^12) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)
/c)*Sqrt[c + d*x^8])/x^4) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*
d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2
*a*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 45.94 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{8}+c}\,\left(-2 a d\,x^{8}-6 b c\,x^{8}+a c \right)}{3 x^{12}}-\frac{b^{2} c^{2} \left(\frac{\sqrt{d\,x^{8}+c}\,b\,x^{4}}{b\,x^{8}+a}-\frac{\left(6 a d -5 c b \right) \operatorname{arctanh}\left(\frac{a \sqrt{d\,x^{8}+c}}{x^{4} \sqrt{a \left(a d -c b \right)}}\right)}{\sqrt{a \left(a d -c b \right)}}\right)}{2 \left(a d -c b \right)}}{4 a^{3} c^{2}}$	134

input
```
int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3*(-1/3*(d*x^8+c)^(1/2)*(-2*a*d*x^8-6*b*c*x^8+a*c)/x^12-1/2*b^2*c^2/
(a*d-b*c)*((d*x^8+c)^(1/2)*b*x^4/(b*x^8+a)-(6*a*d-5*b*c)/(a*(a*d-b*c))^(1/
2)*arctanh(a*(d*x^8+c)^(1/2)/x^4/(a*(a*d-b*c))^(1/2))))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 760, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16
 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x
^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) -
4*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16
+ 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^
3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 -
2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7
*c^2*d^2)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3
- 6*a^2*b^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8
 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a
*b*c^2 - a^2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*
c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d
^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*
x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2
*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^12)]
```

Sympy [F]

$$\int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^{13} (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

input
```
integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**13*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^{13}}\,dx$$

input
```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(184) = 368.

Time = 0.54 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{24}d^{\frac{7}{2}}\left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}x^4-\sqrt{dx^8+\dots\right.\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^4b\dots\right.}\right.$$

input
```
integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d
^4)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c
- 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a
^4*d^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8
 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*
c - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*
x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^{13}\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}} \,dx$$

input
```
int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^{13}\,{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.123 $\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 788

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{(4bc-7ad)x^2\sqrt{c+dx^8}}{24b^2d(bc-ad)} + \frac{ax^{10}\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{(-a)^{5/4}(9bc-7ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32b^{11/4}(-bc+ad)^{3/2}} + \frac{(-a)^{5/4}(9bc-7ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32b^{11/4}(-bc+ad)^{3/2}} - \frac{c^{3/4}(bc+7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12b^2d^{5/4}(bc+ad)\sqrt{c+dx^8}} + \frac{a\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(9bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\right.}{64b^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}} + \frac{a\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(9bc-7ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64b^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

output
```
1/24*(-7*a*d+4*b*c)*x^2*(d*x^8+c)^(1/2)/b^2/d/(-a*d+b*c)+1/8*a*x^10*(d*x^8
+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)+1/32*(-a)^(5/4)*(-7*a*d+9*b*c)*arctan((a*
d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(11/4)/(a*d-b*c)^(3
/2)+1/32*(-a)^(5/4)*(-7*a*d+9*b*c)*arctanh((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/
b^(1/4)/(d*x^8+c)^(1/2))/b^(11/4)/(a*d-b*c)^(3/2)-1/12*c^(3/4)*(7*a*d+b*c)
*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJa
cobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^2/d^(5/4)/(a*d+b*c)/(d
*x^8+c)^(1/2)+1/64*a*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-7*a*d+9*b*c)*(
c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(
sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^3/c^(1/4)/(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*a*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-7*a*d+9*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*
x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))/b^3/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2)
)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.29

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^2\left(-5a(c+dx^8)\left(7a^2d-4b^2cx^8-4ab(c-dx^8)\right)+5ac(-4bc+7ad)\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\right.\right.}{120ab^2d(bc-ad)}$$

input
```
Integrate[x^25/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*(-5*a*(c + d*x^8)*(7*a^2*d - 4*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*a*c
*(-4*b*c + 7*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/
4, -((d*x^8)/c), -((b*x^8)/a)] - (4*b^2*c^2 + 20*a*b*c*d - 21*a^2*d^2)*x^8
*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c),
-((b*x^8)/a)]))/(120*a*b^2*d*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.72 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {965, 970, 1052, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{24}}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{970}$$

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b(a+bx^8)(bc-ad)}-\frac{\int \frac{x^8(5ac-(4bc-7ad)x^8)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1052}$$

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b(a+bx^8)(bc-ad)}-\frac{-\frac{\int -\frac{(4b^2c^2+20abdc-21a^2d^2)x^8+ac(4bc-7ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{3bd}-\frac{x^2\sqrt{c+dx^8}(4bc-7ad)}{3bd}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b(a+bx^8)(bc-ad)}-\frac{\frac{\int \frac{(4b^2c^2+20abdc-21a^2d^2)x^8+ac(4bc-7ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{3bd}-\frac{x^2\sqrt{c+dx^8}(4bc-7ad)}{3bd}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b(a+bx^8)(bc-ad)}-\frac{\frac{\frac{(-21a^2d^2+20abcd+4b^2c^2)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}-\frac{3a^2d(9bc-7ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}}{3bd}-\frac{x^2\sqrt{c+dx^8}(4bc-7ad)}{3bd}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d(9bc-7ad)\int\frac{}{\left(bx^8-\right.}}{b}}{3bd}}{4b(bc-ad)}\right.$$

$\downarrow$ 925

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d(9bc-7ad)\left(\int\frac{}{\left(1-\right.}\right.}{}}{3bd}}{4b(bc-ad)}\right.$$

$\downarrow$ 1541

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d(9bc-7ad)\left(\frac{\sqrt{d}\left(\sqrt{\right.}}{}\right.}{}}{}\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d(9bc-7ad)\left(\frac{\sqrt{d}\left(\sqrt{\right.}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 761

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(-21a^2d^2+20abcd+4b^2c^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}-\frac{3a^2d(9bc-7ad)\left(\frac{\sqrt{b}\left(\sqrt{\right.}\right.}{}\right.}{}}{}\right.$$

$\downarrow$ 2221

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{dx^8+c}}{4b(bc-ad)\left(bx^8+a\right)}-\frac{\frac{\left(4b^2c^2+20abdc-21a^2d^2\right)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-3a^2d(9bc-7ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{-c}}\right.}{}\right.}{}\right.$$

$\downarrow$ 2223

$$\frac{1}{2}\left(\frac{ax^{10}\sqrt{dx^8+c}}{4b(bc-ad)\left(bx^8+a\right)}-\frac{\frac{\left(4b^2c^2+20abdc-21a^2d^2\right)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{dx^8+c}}-\frac{3a^2d(9bc-7ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{-c}}\right.}{}\right.}{}}{}\right.$$

input
```
Int[x^25/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*x^10*Sqrt[c + d*x^8])/(4*b*(b*c - a*d)*(a + b*x^8)) - (-1/3*((4*b*c -
7*a*d)*x^2*Sqrt[c + d*x^8])/(b*d) + (((4*b^2*c^2 + 20*a*b*c*d - 21*a^2*d^2
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c +
 d*x^8]) - (3*a^2*d*(9*b*c - 7*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((
-a)^(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x
^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + (
(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-
a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) +
(((Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])
])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^{25}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x**25/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{25}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^25/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{25}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^25/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{25}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^25/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^25/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{25}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{25}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^25/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.124 $\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 735

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{ax^2\sqrt{c+dx^8}}{8b(bc-ad)(a+bx^8)} + \frac{\sqrt[4]{-a}(5bc-3ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32b^{7/4}(-bc+ad)^{3/2}}$$
$$+\frac{\sqrt[4]{-a}(5bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32b^{7/4}(-bc+ad)^{3/2}}$$
$$+\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4b\sqrt[4]{d}(bc+ad)\sqrt{c+dx^8}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$
$$-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(5bc-3ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64b^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

output
```
1/8*a*x^2*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)+1/32*(-a)^(1/4)*(-3*a*d+5
*b*c)*arctan((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(7/
4)/(a*d-b*c)^(3/2)+1/32*(-a)^(1/4)*(-3*a*d+5*b*c)*arctanh((a*d-b*c)^(1/2)*
x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(7/4)/(a*d-b*c)^(3/2)+1/4*c^(3/4
)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJ
acobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b/d^(1/4)/(a*d+b*c)/(d*
x^8+c)^(1/2)-1/64*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(c^(
1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin
(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/64*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*(-3*a*d+5*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c
)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/
4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)
/d^(1/2),1/2*2^(1/2))/b^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(
1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.24

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^2\left(5a^2(c+dx^8)-5ac(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+(4bc-3ad)x^8(a+bx^8)\right.}{40ab(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

input
```
Integrate[x^17/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*(5*a^2*(c + d*x^8) - 5*a*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + (4*b*c - 3*a*d)*x^8*(a + b*
x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)]))/(40*a*b*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.35 (sec) , antiderivative size = 1052, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {965, 970, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{16}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{970}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\int \frac{ac-(4bc-3ad)x^8}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}-\frac{(4bc-3ad)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(5bc-3ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(4bc-3ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{2b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(5bc-3ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)}{b}-\frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(4bc-\cdots}{2b\sqrt[4]{c}\cdots}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \; 1541$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{a\sqrt{d}\left(\cdots\right.}{\cdots}\right.}{b}\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{\cdots}}{\sqrt{-a}\cdots}\right.}{\cdots}\right.}{b}\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+dx^8}}{4b\left(a+bx^8\right)(bc-ad)}-\frac{a(5bc-3ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad-}}{2a}\right.}{}\right.$$

$\big\downarrow$ 2221

$$\frac{1}{2}\left(\frac{ax^2\sqrt{dx^8+c}}{4b(bc-ad)\left(bx^8+a\right)}-\frac{a(5bc-3ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-}\right.}{}}{}\right.}{}\right.$$

$\big\downarrow$ 2223

$$\frac{1}{2}\left(\frac{ax^2\sqrt{dx^8+c}}{4b(bc-ad)\,(bx^8+a)}-\frac{a(5bc-3ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-\right.}}{}\right.}{}\right.$$

input
```
Int[x^17/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((a*x^2*Sqrt[c + d*x^8])/(4*b*(b*c - a*d)*(a + b*x^8)) - (-1/2*((4*b*c - 3
*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(b*c^(1/4)*d^(1/4)*Sqrt[c
 + d*x^8]) + (a*(5*b*c - 3*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d]
)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)
^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d
)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^
(3/4)*((Sqrt[b]*Sqrt[c])/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqr
t[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8
)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*S
qrt[d])^2/(Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(4*c^(1/4)*d^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((S
qrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^
(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*
Sqrt[c] - Sqrt[-a]*Sqrt[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[
d])*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt
[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^{17}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**17/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**17/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^17/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^{17}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^17/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{17}}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^17/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^17/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{17}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^{17}}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^17/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**17)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*
d*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.125 $\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 627

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}$$

$$+\frac{(bc+ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(-bc+ad)^{3/2}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(bc+ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\ldots\right.\right.}{64ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

$$+\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc+ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\ldots\right.\right.}{64ab\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}$$

output
```
-1/8*x^2*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/32*(a*d+b*c)*arctan((a*d-b
*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-
b*c)^(3/2)+1/32*(a*d+b*c)*arctanh((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(
d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/2)+1/64*(b^(1/2)*c^(1/2)+(
-a)^(1/2)*d^(1/2))*(a*d+b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(
1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))/a/b/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*
c)/(d*x^8+c)^(1/2)+1/64*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(a*d+b*c)*(c^
(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(si
n(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a/b/c^(1/4)/(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.25

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x^2\left(5a(c+dx^8)-5c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{A}\right.}{40a(bc-ad)\left(a+bx^8\right)\sqrt{c+dx^8}}$$

input
```
Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/40*(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1
+ (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*(
b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 1033, normalized size of antiderivative = 1.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {965, 971, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{ 965}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{ 971}$$

$$\frac{1}{2}\left(\frac{\int \frac{c-dx^8}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1021}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}-\frac{d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{b}}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^8}}}{4(bc-ad)}-\frac{x^2\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 925}$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)}{b}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^8}}}{4(bc-ad)} - \ldots\right)$$

$$\Big\downarrow \; 1541$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\ldots\right)}{\ldots}}{2\ldots}\right)}{b}}{4(bc-ad)}\ldots\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\frac{(ad+bc)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{-a}\sqrt{d}+\ldots\right)}{\ldots}}{2a}\right)}{b}}{4(bc-ad)}\ldots\right)$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{(ad+bc)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}+\cdots\right)}{b}\cdots\right)$$

$\downarrow$ 2221

$$\frac{1}{2}\left(\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\cdots\right)}{2\sqrt[4]{b}\sqrt{bc-\cdots}}\cdots\right)}{\cdots}}{2a}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 2223

$$\frac{1}{2}\left(\frac{(bc+ad)\left(\frac{\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}(bc+ad)\sqrt{dx^8+c}}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\left(\frac{(-a)^{3/4}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}-\sqrt{d}\right)\arctan\left(\cdots\right.}{2\sqrt[4]{b}\sqrt{bc-\cdots}}\right.}{\cdots}}{2a}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-1/4*(x^2*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (-1/2*(d^(3/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF
[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(b*c^(1/4)*Sqrt[c + d*x^8]) + ((b*
c + a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (S
qrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c]
)/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d
])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^
4)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqr
t[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d
^(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
 + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] +
Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4
)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
 a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*
d]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt
[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + S...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x^9}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**9/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^9}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^9}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^9}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^9}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**9)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.126 $\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 735

$$
\begin{aligned}
&\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx \\
&= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt[4]{b}(3bc-5ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(-bc+ad)^{3/2}} \\
&\quad - \frac{\sqrt[4]{b}(3bc-5ad)\text{arctanh}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(-bc+ad)^{3/2}} \\
&\quad + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{4a\sqrt[4]{c}(bc+ad)\sqrt{c+dx^8}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}} \\
&\quad + \frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(3bc-5ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^2\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output
```
1/8*b*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/32*b^(1/4)*(-5*a*d+3*b*
c)*arctan((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/
4)/(a*d-b*c)^(3/2)-1/32*b^(1/4)*(-5*a*d+3*b*c)*arctanh((a*d-b*c)^(1/2)*x^2
/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(3/2)+1/4*d^(3/4
)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJ
acobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a/c^(1/4)/(a*d+b*c)/(d*
x^8+c)^(1/2)+1/64*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-5*a*d+3*b*c)*(c^(
1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin
(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*(-5*a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c
)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/
4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)
/d^(1/2),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/d^(
1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.23

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^2\left(5ab(c+dx^8)+5(3bc-4ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+bdx^8(a+bx^8)\right.}{40a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

input
```
Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c
]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*
x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.44 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {965, 931, 25, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{931}$$

$$\frac{1}{2}\left(\frac{bx^2\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}-\frac{\int -\frac{bdx^8+3bc-4ad}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{bdx^8+3bc-4ad}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2+d\int \frac{1}{\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\int \frac{1}{(bx^8+a)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{4a(bc-ad)}+\frac{bx^2\sqrt{c+dx^8}}{4a(a+bx^8)(bc-\right.$$

$$\Big\downarrow \text{925}$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}}\cdots}{4a(bc-ad)}\right.$$

$$\Big\downarrow \; 1541$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}\cdots}{\cdots}\right.\cdots}{4a(bc-ad\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\cdots}{\cdots}}{\cdots}\right.\cdots}{4a(bc-ad)\cdots}\right.$$

$$\Big\downarrow \; 761$$

$$\frac{1}{2}\left(\frac{(3bc-5ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.}{}\right.$$

$$\Big\downarrow \; 2221$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}x^2}{4a(bc-ad)\left(bx^8+a\right)}+\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{\cdots}}{\cdots}\right.}{}\right.$$

$$\Big\downarrow \; 2223$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}\,x^2}{4a(bc-ad)\left(bx^8+a\right)}+\frac{\frac{d^{3/4}\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+(3bc-5ad)\left(\frac{a\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{\ }}{}\right.}{}\right.$$

input
```
Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^2*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*(a + b*x^8)) + ((d^(3/4)*(Sqrt[c
] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*A
rcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + (3*b*c -
 5*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sq
rt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])
/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt
[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^
(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
+ a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[
(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}dx$$

input
```
int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \int \frac{x}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\, dx$$

input
```
int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^8)^2 \sqrt{c+dx^8}}\, dx = \int \frac{\sqrt{d\,x^8+c}\,x}{b^2 d\,x^{24}+2abd\,x^{16}+b^2 c\,x^{16}+a^2 d\,x^8+2abc\,x^8+a^2 c} dx$$

input
```
int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d*x*
*16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.127 $\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 788

$$
\begin{aligned}
\int \frac{1}{x^7(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} \\
&+ \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6(a+bx^8)} - \frac{b^{5/4}(7bc-9ad)\arctan\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(-bc+ad)^{3/2}} \\
&- \frac{b^{5/4}(7bc-9ad)\operatorname{arctanh}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{32(-a)^{11/4}(-bc+ad)^{3/2}} \\
&- \frac{d^{3/4}(7bc+ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{12a^2c^{5/4}(bc+ad)\sqrt{c+dx^8}} \\
&- \frac{b\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}} \\
&- \frac{b\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(7bc-9ad)\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\right.\right.}{64a^3\sqrt[4]{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\sqrt[4]{d}(bc-ad)\sqrt{c+dx^8}}
\end{aligned}
$$

output
```
-1/24*(-4*a*d+7*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6+1/8*b*(d*x^8+c)^
(1/2)/a/(-a*d+b*c)/x^6/(b*x^8+a)-1/32*b^(5/4)*(-9*a*d+7*b*c)*arctan((a*d-b
*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4)/(a*d-b*c)^(3
/2)-1/32*b^(5/4)*(-9*a*d+7*b*c)*arctanh((a*d-b*c)^(1/2)*x^2/(-a)^(1/4)/b^(
1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4)/(a*d-b*c)^(3/2)-1/12*d^(3/4)*(a*d+7*b*c)
*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJa
cobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a^2/c^(5/4)/(a*d+b*c)/(d
*x^8+c)^(1/2)-1/64*b*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(-9*a*d+7*b*c)*(
c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(
sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))/a^3/c^(1/4)/(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/64*b*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*(-9*a*d+7*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*
x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))/a^3/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2)
)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^7 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \frac{5a(c+dx^8)\left(4a^2d-7b^2cx^8-4ab(c-dx^8)\right)+5(-21b^2c^2+20abcd+4a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appe}}{120a^3c(bc-ad)x^6\,(}$$

input
```
Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*
c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16
*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c),
-((b*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (warning: unable to verify)

Time = 2.68 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {965, 972, 25, 1053, 1021, 761, 925, 1541, 27, 761, 2221, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+b x^8\right)^2 \sqrt{c+d x^8}} \, dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(b x^8+a\right)^2 \sqrt{d x^8+c}}dx^2$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2} \left(\frac{b \sqrt{c+d x^8}}{4 a x^6 \left(a+b x^8\right) (b c-a d)}-\frac{\int -\frac{5 b d x^8+7 b c-4 a d}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^2}{4 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(\frac{\int \frac{5 b d x^8+7 b c-4 a d}{x^8 \left(b x^8+a\right) \sqrt{d x^8+c}}dx^2}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{4 a x^6 \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2} \left(\frac{-\frac{\int \frac{b d (7 b c-4 a d) x^8+21 b^2 c^2-4 a^2 d^2-20 a b c d}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^2}{3 a c}-\frac{\sqrt{c+d x^8} (7 b c-4 a d)}{3 a c x^6}}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{4 a x^6 \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{1021}$$

$$\frac{1}{2} \left(\frac{-\frac{d (7 b c-4 a d) \int \frac{1}{\sqrt{d x^8+c}}dx^2+3 b c (7 b c-9 a d) \int \frac{1}{\left(b x^8+a\right) \sqrt{d x^8+c}}dx^2}{3 a c}-\frac{\sqrt{c+d x^8} (7 b c-4 a d)}{3 a c x^6}}{4 a (b c-a d)}+\frac{b \sqrt{c+d x^8}}{4 a x^6 \left(a+b x^8\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\frac{-\frac{3bc(7bc-9ad)\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}-\frac{\sqrt{c+dx^8}(7bc-4ad)}{3acx^6}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 925

$$\frac{1}{2}\left(\frac{-\frac{3bc(7bc-9ad)\left(\frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{2a}+\frac{\int \frac{1}{\left(\frac{\sqrt{b}x^4}{\sqrt{-a}}+1\right)\sqrt{dx^8+c}}dx^2}{2a}\right)+\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}(7bc-4ad)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x\cdots}{\sqrt[4]{c}\cdots}\cdots\right.\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}}}{3ac}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 1541

$$\frac{1}{2}\left(\frac{-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\sqrt{c}\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\cdots}{\cdots}\right.}{3ac}\cdots}{4a\cdots}\cdots\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(\frac{-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{d}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int \frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}}{2a}+\frac{\frac{a\sqrt{d}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\int \frac{1}{\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt{b}\left(\sqrt{\cdots}\right.}{\cdots}}{2\cdots}\right.}{3ac}\cdots}{4a(bc\cdots}\cdots\right.$$

$\big\downarrow$ 761

$$\frac{1}{2}\left(-\frac{3bc(7bc-9ad)\left(\frac{\frac{\sqrt{b}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\int\frac{\sqrt{d}x^4+\sqrt{c}}{\left(1-\frac{\sqrt{b}x^4}{\sqrt{-a}}\right)\sqrt{dx^8+c}}dx^2}{ad+bc}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{-a}\sqrt{b}\sqrt{c}+a\sqrt{d}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{2\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}}{2a}\right.}{}\right.$$

$\big\downarrow$ 2221

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}b}{4a(bc-ad)x^6\left(bx^8+a\right)}+\frac{-\frac{\sqrt{dx^8+c}(7bc-4ad)}{3acx^6}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc(}{}}{}\right.$$

$\big\downarrow$ 2223

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}\,b}{4a(bc-ad)x^6(bx^8+a)}+\frac{-\frac{\sqrt{dx^8+c}(7bc-4ad)}{3acx^6}-\frac{\frac{d^{3/4}(7bc-4ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{dx^8+c}}+3bc(}{}}{}\right.$$

input
```
Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*x^6*(a + b*x^8)) + (-1/3*((7*b*c - 4
*a*d)*Sqrt[c + d*x^8])/(a*c*x^6) - ((d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sq
rt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[
(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*Sqrt[c + d*x^8]) + 3*b*c*(7*b*c -
 9*a*d)*(((a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sq
rt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*(((-a)^(3/4)*((Sqrt[b]*Sqrt[c])
/Sqrt[-a] - Sqrt[d])*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt
[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticPi[-1/4*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(Sqrt[-a]*Sqrt
[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*d^
(1/4)*Sqrt[c + d*x^8])))/(b*c + a*d))/(2*a) + (((Sqrt[-a]*Sqrt[b]*Sqrt[c]
+ a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*c^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])*(((-a)^(1/4)*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*ArcTanh[(Sqrt[b*c -
a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(2*b^(1/4)*Sqrt[b*c - a*d
]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*(Sqrt[c] + Sqrt[d]*x^4)*Sq...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(b \,x^{8}+a \right)^{2} \sqrt{d \,x^{8}+c}}d x$$

input
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \text{Timed out}$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^7 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^7 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx$$

input
```
integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**7*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^7 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^7} \, dx$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}\, x^7}\, dx$$

input
```
integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^7 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\, dx$$

input
```
int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{x^7 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\, dx$$

input
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.128 $\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1177

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

output
```
1/8*d^(1/2)*x^2*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^4)-1/8*x^6
*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/32*(-a*d+3*b*c)*arctan((-a*d+b*c)^
(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c
)^(3/2)-1/32*(-a*d+3*b*c)*arctanh((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)-1/8*c^(1/4)*d^(1/4)*(
c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*EllipticE(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))/b/(-a*d+b*c)/(d*x^8+c)^(1/2
)+1/16*c^(1/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x
^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b/
(-a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^4)
*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4
)*x^2/c^(1/4)),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^
(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(c^(1/2)+d^(1/
2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan
(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/b^(3/2)/c^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(
1/2)*d^(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(b^(1/2)*c^(1/2)+(-a)^(1/2)*
d^(1/2))*(-a*d+3*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^
4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2)
)/b^(3/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(1/4)/(-a*d+...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.14

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$= \frac{x^6\left(-7a(c+dx^8)+7c(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+dx^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\mathrm{A}\right.}{56a(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*
x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b
*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 1107, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {965, 971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4(dx^8+3c)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4(bc-ad)}-\frac{x^6\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{dx^4}{b\sqrt{dx^8+c}}+\frac{(3bc-ad)x^4}{b(bx^8+a)\sqrt{dx^8+c}}\right)dx^2}{4(bc-ad)}-\frac{x^6\sqrt{c+dx^8}}{4(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(3bc-ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8\sqrt{-a}b^{3/2}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}+\frac{(3bc-ad)\arctan\Big(\ldots}{4\sqrt[4]{-a}b^{5\ldots}}\right.$$

input
```
Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(-1/4*(x^6*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((Sqrt[d]*x^2*Sqrt
[c + d*x^8])/(b*(Sqrt[c] + Sqrt[d]*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(5/4)*
Sqrt[b*c - a*d]) - ((3*b*c - a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(5/4)*Sqrt[b*c - a*d]) - (c^(
1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^8]
) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + S
qrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*Sqrt[
c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*
d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqr
t[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d...
```

Defintions of rubi rules used

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^{13}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**13/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{x^{13}}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^{13}}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{13}}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^{13}}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**13)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*
d*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.129 $\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1187

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

output
```
-1/8*d^(1/2)*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^4)+1/8*b*
x^6*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/32*(-3*a*d+b*c)*arctan((-a*d+
b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*
d+b*c)^(3/2)+1/32*(-3*a*d+b*c)*arctanh((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(
1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)+1/8*c^(1/4)*d^(1
/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*Ellipt
icE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))/a/(-a*d+b*c)/(d*x^8+c)
^(1/2)-1/16*c^(1/4)*d^(1/4)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1
/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2
))/a/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(c^(1/2)+d^(1/2)
*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*arctan(d
^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(
1/2)*d^(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(c^(1/2
)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2
*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a/b^(1/2)/c^(1/4)/(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))*(-3*a*d+b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d
^(1/2)*x^4)^2)^(1/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b
^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/
2*2^(1/2))/a/b^(1/2)/c^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/d^(...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.14

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$
$$= \frac{x^6\left(21ab(c+dx^8)+7(bc-4ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-3bdx^8(a+bx^8)\right.}{168a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b
*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x
^8)/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 1093, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {965, 972, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \ 965$$

$$\frac{1}{2}\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \ 972$$

$$\frac{1}{2}\left(\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}-\frac{\int -\frac{x^4(-bdx^8+bc-4ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{\int \frac{x^4(-bdx^8+bc-4ad)}{(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \ 1054$$

$$\frac{1}{2}\left(\frac{\int \left(\frac{(bc-3ad)x^4}{(bx^8+a)\sqrt{dx^8+c}}-\frac{dx^4}{\sqrt{dx^8+c}}\right)dx^2}{4a(bc-ad)}+\frac{bx^6\sqrt{c+dx^8}}{4a(a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{b\sqrt{dx^8+c}x^6}{4a(bc-ad)(bx^8+a)}+\frac{-\frac{(bc-3ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\arctan\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right),\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-\right.}{8\sqrt{-a}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}}{}\right.$$

input
```
Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*x^6*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*(a + b*x^8)) + (-((Sqrt[d]*x^2*S
qrt[c + d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + ((b*c - 3*a*d)*ArcTan[(Sqrt[b*c
 - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(1/4)*
Sqrt[b*c - a*d]) - ((b*c - 3*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*b^(1/4)*Sqrt[b*c - a*d]) + (c^(
1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x
^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/Sqrt[c + d*x^8] -
(c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[
d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*Sqrt[c + d*
x^8]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqr
t[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x
^8]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt
[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*
Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^5}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^5}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^5}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**5)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.130 $\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1266

$$\int \frac{1}{x^3(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Too large to display}$$

output
```
-1/8*(-4*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1/8*d^(1/2)*(-4*a
*d+5*b*c)*x^2*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+d^(1/2)*x^4)+1/8*b
*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^8+a)-1/32*b^(3/4)*(-7*a*d+5*b*c)*ar
ctan((-a*d+b*c)^(1/2)*x^2/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(
-a*d+b*c)^(3/2)+1/32*b^(3/4)*(-7*a*d+5*b*c)*arctanh((-a*d+b*c)^(1/2)*x^2/(
-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/8*d^(1/4)
*(-4*a*d+5*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^
(1/2)*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))/a^2/c^(3/4
)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/32*b^(1/2)*d^(1/4)*(-7*a*d+5*b*c)*(c^(1/2)+
d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*InverseJacobiAM(2*a
rctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/32*b^(1/2)*d^(1/4)*(-7*a*d+5*b
*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*Invers
eJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a^2/c^(1/4)/(b^(1/2)*
c^(1/2)+(-a)^(1/2)*d^(1/2))/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/16*d^(1/4)*(-4*a*
d+5*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1/2)*I
nverseJacobiAM(2*arctan(d^(1/4)*x^2/c^(1/4)),1/2*2^(1/2))/a^2/c^(3/4)/(-a*
d+b*c)/(d*x^8+c)^(1/2)-1/64*b^(1/2)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*(
-7*a*d+5*b*c)*(c^(1/2)+d^(1/2)*x^4)*((d*x^8+c)/(c^(1/2)+d^(1/2)*x^4)^2)^(1
/2)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)...
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.23 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$
$$= \frac{21a(c+dx^8)\left(4a^2d-5b^2cx^8-4ab(c-dx^8)\right)-7(5b^2c^2-12abcd+4a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appell}}{168a^3c(bc-ad)x^2\,(a}$$

input
```
Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c
^2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[
3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16
*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c),
 -((b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 1170, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {965, 972, 25, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{965}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^8+a)^2 \sqrt{dx^8+c}}dx^2$$

$$\Big\downarrow \text{972}$$

$$\frac{1}{2}\left(\frac{b\sqrt{c+dx^8}}{4ax^2 (a+bx^8)(bc-ad)}-\frac{\int -\frac{3bdx^8+5bc-4ad}{x^4(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{3bdx^8+5bc-4ad}{x^4(bx^8+a)\sqrt{dx^8+c}}dx^2}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2 (a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$\frac{1}{2}\left(\frac{-\frac{\int \frac{x^4\left((bc-2ad)(5bc-2ad)-bd(5bc-4ad)x^8\right)}{\left(bx^8+a\right)\sqrt{dx^8+c}}dx^2}{ac}-\frac{\sqrt{c+dx^8}(5bc-4ad)}{acx^2}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2 (a+bx^8)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$\frac{1}{2}\left(\frac{-\frac{\int \left(\frac{\left(5b^2c^2-7abcd\right)x^4}{\left(bx^8+a\right)\sqrt{dx^8+c}}-\frac{d(5bc-4ad)x^4}{\sqrt{dx^8+c}}\right)dx^2}{ac}-\frac{\sqrt{c+dx^8}(5bc-4ad)}{acx^2}}{4a(bc-ad)}+\frac{b\sqrt{c+dx^8}}{4ax^2\left(a+bx^8\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\sqrt{dx^8+c}\,b}{4a(bc-ad)x^2\left(bx^8+a\right)}+\frac{-\frac{\sqrt{dx^8+c}(5bc-4ad)}{acx^2}-\frac{-\frac{\sqrt{b}c^{3/4}(5bc-7ad)\left(\sqrt{d}x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}x^4+\sqrt{c}\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}},2\operatorname{arc}\cdots\right.}{8\sqrt{-a}\sqrt[4]{d}(bc+ad)\sqrt{dx^8+c}}\cdots}{\cdots}\right.$$

input
```
Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
((b*Sqrt[c + d*x^8])/(4*a*(b*c - a*d)*x^2*(a + b*x^8)) + (-(((5*b*c - 4*a*
d)*Sqrt[c + d*x^8])/(a*c*x^2)) - (-((Sqrt[d]*(5*b*c - 4*a*d)*x^2*Sqrt[c +
d*x^8])/(Sqrt[c] + Sqrt[d]*x^4)) + (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt
[b*c - a*d]) - (b^(3/4)*c*(5*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x^2)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(4*(-a)^(1/4)*Sqrt[b*c - a*d]) + (c^(
1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/Sq
rt[c + d*x^8] - (c^(1/4)*d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2
)/c^(1/4)], 1/2])/(2*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] - (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)
], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*c^(3/4)*(Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(4*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*c^(3/4)*(Sqrt[b]*Sq
rt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-
a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 965
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k -
 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; Free
Q[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^{3} \left(a + b x^{8}\right)^{2} \sqrt{c + d x^{8}}}\, dx$$

input
```
integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}\, x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{(bx^8+a)^2 \sqrt{dx^8+c}\, x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^3 (b\,x^8+a)^2 \sqrt{d\,x^8+c}} \, dx$$

input
```
int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^3 (b\,x^8+a)^2 \sqrt{d\,x^8+c}}dx$$

input
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.131 $\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

output
```
1/5*x^5*(1+d*x^8/c)^(1/2)*AppellF1(5/8,2,1/2,13/8,-b*x^8/a,-d*x^8/c)/a^2/(
d*x^8+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.

Time = 10.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^5\left(65ab(c+dx^8)+13(3bc-8ad)(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{5}{8},\frac{1}{2},1,\frac{13}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5bdx^8(a+b\ldots\right.}{520a^2(bc-ad)(a+bx^8)\sqrt{c+dx^8}}$$

input
```
Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)
/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -
((b*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^4}{(bx^8+a)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(\frac{5}{8},2,\frac{1}{2},\frac{13}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

input
```
Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^
8)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^4}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^4}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**4)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.132 $\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

output
```
1/3*x^3*(1+d*x^8/c)^(1/2)*AppellF1(3/8,2,1/2,11/8,-b*x^8/a,-d*x^8/c)/a^2/(
d*x^8+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 170 vs. 2(64) = 128.

Time = 10.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{x^3\left(33ab(c+dx^8)+11(5bc-8ad)\,(a+bx^8)\,\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{3}{8},\frac{1}{2},1,\frac{11}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^8(a+b\right.}{264a^2(bc-ad)\,(a+bx^8)\,\sqrt{c+dx^8}}$$

input
```
Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)
/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -
((b*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1013, 1012\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}dx$$
$$\Big\downarrow \text{ 1013}$$
$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{x^2}{(bx^8+a)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$
$$\Big\downarrow \text{ 1012}$$
$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\operatorname{AppellF1}\left(\frac{3}{8},2,\frac{1}{2},\frac{11}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

input
```
Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^
8)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}dx$$

input
```
int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2\,\sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{x^2}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{x^2}{{\left(b\,x^8+a\right)}^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\int \frac{\sqrt{d\,x^8+c}\,x^2}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int((sqrt(c + d*x**8)*x**2)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d
*x**16 + b**2*c*x**16 + b**2*d*x**24),x)
```

3.133 $\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx=\frac{x\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

output
```
x*(1+d*x^8/c)^(1/2)*AppellF1(1/8,2,1/2,9/8,-b*x^8/a,-d*x^8/c)/a^2/(d*x^8+c
)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 328 vs. $2(59) = 118$.

Time = 10.22 (sec) , antiderivative size = 328, normalized size of antiderivative = 5.56

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{x\left(bdx^8\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(\frac{9}{8},\frac{1}{2},1,\frac{17}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(9ac\left(8ad-b\left(8c+dx^8\right)\right)\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4b\right.}{(a+bx^8)\left(-9ac\,\text{AppellF1}\left(\frac{1}{8},\frac{1}{2},1,\frac{9}{8},-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+4x\right.}\right.}{24a^2(-bc+ad)\sqrt{c+dx^8}}$$

input
```
Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/24*(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8
)/c), -((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8,
1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*Appe
llF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/
2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1
/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8,
1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8
, -((d*x^8)/c), -((b*x^8)/a)])))))/(a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \ 937$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{(bx^8+a)^2 \sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \ 936$$

$$\frac{x\sqrt{\frac{dx^8}{c}+1} \operatorname{AppellF1}\left(\frac{1}{8},2,\frac{1}{2},\frac{9}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

input
```
Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

input
```
integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{1}{(b\,x^8+a)^2\,\sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \int \frac{\sqrt{d\,x^8+c}}{b^2d\,x^{24}+2abd\,x^{16}+b^2c\,x^{16}+a^2d\,x^8+2abc\,x^8+a^2c}dx$$

input
```
int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(sqrt(c + d*x**8)/(a**2*c + a**2*d*x**8 + 2*a*b*c*x**8 + 2*a*b*d*x**16
+ b**2*c*x**16 + b**2*d*x**24),x)
```

3.134 $\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}$$

output
```
-(1+d*x^8/c)^(1/2)*AppellF1(-1/8,2,1/2,7/8,-b*x^8/a,-d*x^8/c)/a^2/x/(d*x^8
+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(62) = 124.

Time = 10.41 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.65

$$\int \frac{1}{x^2(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{35a(c+dx^8)\left(8a^2d-9b^2cx^8-8ab(c-dx^8)\right)-5\left(9b^2c^2-40abcd+24a^2d^2\right)x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Appel}}{280a^3c(bc-ad)x\,(}$$

input
```
Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c
^2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[7/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/
c), -((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1} \int \frac{1}{x^2 (bx^8+a)^2 \sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1} \operatorname{AppellF1}\left(-\frac{1}{8},2,\frac{1}{2},\frac{7}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2 x \sqrt{c+dx^8}}$$

input
```
Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)
/c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\, x^8+a\right)^2 \sqrt{d\, x^8+c}}dx$$

input
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx = \int \frac{1}{\left(bx^8+a\right)^2 \sqrt{dx^8+c}x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c +
 a^2*d)*x^10 + a^2*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \int \frac{1}{x^2 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx$$

input
```
integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx = \int \frac{1}{(bx^8 + a)^2 \sqrt{dx^8 + c} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^2 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^8)^2 \sqrt{c+dx^8}}\,dx = \int \frac{1}{x^2 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}}dx$$

input
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.135 $\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx = -\frac{\sqrt{1+\frac{dx^8}{c}}\,\text{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

output
```
-1/3*(1+d*x^8/c)^(1/2)*AppellF1(-3/8,2,1/2,5/8,-b*x^8/a,-d*x^8/c)/a^2/x^3/
(d*x^8+c)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 226 vs. 2(64) = 128.

Time = 10.23 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{1}{x^4(a+bx^8)^2\sqrt{c+dx^8}}\,dx = \frac{65a(c+dx^8)\left(8a^2d-11b^2cx^8-8ab(c-dx^8)\right)-13(33b^2c^2-56abcd+8a^2d^2)\,x^8(a+bx^8)\sqrt{1+\frac{dx^8}{c}}\,\text{Ap}}{1560a^3c(bc-ad)}$$

input
```
Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
(65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^
2*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*Appell
F1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)
*x^16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^
8)/c), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x
^8])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{dx^8}{c}+1}\int \frac{1}{x^4(bx^8+a)^2\sqrt{\frac{dx^8}{c}+1}}dx}{\sqrt{c+dx^8}}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,\text{AppellF1}\left(-\frac{3}{8},2,\frac{1}{2},\frac{5}{8},-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

input
```
Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

output
```
-1/3*(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x
^8)/c)])/(a^2*x^3*Sqrt[c + d*x^8])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}}dx$$

input
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx=\text{Timed out}$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

input
```
integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

Maxima [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^4 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}} \, dx$$

input
```
int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^8)^2 \sqrt{c+dx^8}} \, dx = \int \frac{1}{x^4 \left(b\,x^8+a\right)^2 \sqrt{d\,x^8+c}} dx$$

input
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

output
```
int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

3.136 $\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \frac{dx^9}{7b\sqrt[4]{a+bx^8}} + \frac{(7bc-9ad)x^9\sqrt[4]{1+\frac{bx^8}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{4},\frac{17}{8},-\frac{bx^8}{a}\right)}{63ab\sqrt[4]{a+bx^8}}$$

output
```
1/7*d*x^9/b/(b*x^8+a)^(1/4)+1/63*(-9*a*d+7*b*c)*x^9*(1+b*x^8/a)^(1/4)*hype
rgeom([9/8, 5/4],[17/8],-b*x^8/a)/a/b/(b*x^8+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \frac{x^9\sqrt[4]{1+\frac{bx^8}{a}}\left(17c\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{4},\frac{17}{8},-\frac{bx^8}{a}\right)+9dx^8\,\text{Hypergeometric2F1}\left(\right.\right.}{153a\sqrt[4]{a+bx^8}}$$

input
```
Integrate[(x^8*(c + d*x^8))/(a + b*x^8)^(5/4),x]
```

output
```
(x^9*(1 + (b*x^8)/a)^(1/4)*(17*c*Hypergeometric2F1[9/8, 5/4, 17/8, -((b*x^
8)/a)] + 9*d*x^8*Hypergeometric2F1[5/4, 17/8, 25/8, -((b*x^8)/a)]))/(153*a
*(a + b*x^8)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx$$

$$\Big\downarrow \ 959$$

$$\frac{(7bc-9ad)\int \frac{x^8}{(bx^8+a)^{5/4}}dx}{7b}+\frac{dx^9}{7b\sqrt[4]{a+bx^8}}$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt[4]{\frac{bx^8}{a}+1}(7bc-9ad)\int \frac{x^8}{\left(\frac{bx^8}{a}+1\right)^{5/4}}dx}{7ab\sqrt[4]{a+bx^8}}+\frac{dx^9}{7b\sqrt[4]{a+bx^8}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^9\sqrt[4]{\frac{bx^8}{a}+1}(7bc-9ad)\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{4},\frac{17}{8},-\frac{bx^8}{a}\right)}{63ab\sqrt[4]{a+bx^8}}+\frac{dx^9}{7b\sqrt[4]{a+bx^8}}$$

input
```
Int[(x^8*(c + d*x^8))/(a + b*x^8)^(5/4),x]
```

output
```
(d*x^9)/(7*b*(a + b*x^8)^(1/4)) + ((7*b*c - 9*a*d)*x^9*(1 + (b*x^8)/a)^(1/
4)*Hypergeometric2F1[9/8, 5/4, 17/8, -((b*x^8)/a)])/(63*a*b*(a + b*x^8)^(1
/4))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^8(d\,x^8+c)}{(b\,x^8+a)^{\frac{5}{4}}}dx$$

input
```
int(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x)
```

output
```
int(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \int \frac{(dx^8+c)x^8}{(bx^8+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*x^16 + c*x^8)*(b*x^8 + a)^(3/4)/(b^2*x^16 + 2*a*b*x^8 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 51.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \frac{cx^9\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{5}{4}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8a^{\frac{5}{4}}\Gamma\left(\frac{17}{8}\right)} + \frac{dx^{17}\Gamma\left(\frac{17}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{17}{8}\\ \frac{25}{8}\end{matrix}\middle|\frac{bx^8e^{i\pi}}{a}\right)}{8a^{\frac{5}{4}}\Gamma\left(\frac{25}{8}\right)}$$

input
```
integrate(x**8*(d*x**8+c)/(b*x**8+a)**(5/4),x)
```

output
```
c*x**9*gamma(9/8)*hyper((9/8, 5/4), (17/8,), b*x**8*exp_polar(I*pi)/a)/(8*
a**(5/4)*gamma(17/8)) + d*x**17*gamma(17/8)*hyper((5/4, 17/8), (25/8,), b*
x**8*exp_polar(I*pi)/a)/(8*a**(5/4)*gamma(25/8))
```

Maxima [F]

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \int \frac{(dx^8+c)x^8}{(bx^8+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^8 + c)*x^8/(b*x^8 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \int \frac{(dx^8+c)x^8}{(bx^8+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^8 + c)*x^8/(b*x^8 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \int \frac{x^8\,(d\,x^8+c)}{(b\,x^8+a)^{5/4}}\,dx$$

input
```
int((x^8*(c + d*x^8))/(a + b*x^8)^(5/4),x)
```

output
```
int((x^8*(c + d*x^8))/(a + b*x^8)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^8(c+dx^8)}{(a+bx^8)^{5/4}}\,dx = \left(\int \frac{x^{16}}{(b\,x^8+a)^{\frac{1}{4}}\,a+(b\,x^8+a)^{\frac{1}{4}}\,b\,x^8}dx\right)d + \left(\int \frac{x^8}{(b\,x^8+a)^{\frac{1}{4}}\,a+(b\,x^8+a)^{\frac{1}{4}}\,b\,x^8}dx\right)c$$

input
```
int(x^8*(d*x^8+c)/(b*x^8+a)^(5/4),x)
```

output
```
int(x**16/((a + b*x**8)**(1/4)*a + (a + b*x**8)**(1/4)*b*x**8),x)*d + int(
x**8/((a + b*x**8)**(1/4)*a + (a + b*x**8)**(1/4)*b*x**8),x)*c
```

3.137 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx = \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c^2} + \frac{(6bc+ad)\sqrt{c+\frac{d}{x^2}}x^4}{24c} + \frac{1}{6}a\sqrt{c+\frac{d}{x^2}}x^6 - \frac{d^2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}$$

output
```
1/16*d*(-a*d+2*b*c)*(c+d/x^2)^(1/2)*x^2/c^2+1/24*(a*d+6*b*c)*(c+d/x^2)^(1/
2)*x^4/c+1/6*a*(c+d/x^2)^(1/2)*x^6-1/16*d^2*(-a*d+2*b*c)*arctanh((c+d/x^2)
^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^5 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x(6bc(d+2cx^2)+a(-3d^2+2cdx^2+8c^2x^4)) + \frac{6d^2(-2bc+ad)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{48c^{5/2}}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*(6*b*c*(d + 2*c*x^2) + a*(-3*d^2 + 2*c*d*x^2
 + 8*c^2*x^4)) + (6*d^2*(-2*b*c + a*d)*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqr
t[d + c*x^2])])/Sqrt[d + c*x^2]))/(48*c^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^8d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(\frac{a x^6 \left(c+\frac{d}{x^2}\right)^{3/2}}{3 c}-\frac{(2 b c-a d) \int \sqrt{c+\frac{d}{x^2}}x^6d\frac{1}{x^2}}{2 c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(\frac{a x^6 \left(c+\frac{d}{x^2}\right)^{3/2}}{3 c}-\frac{(2 b c-a d) \left(\frac{1}{4} d \int \frac{x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-\frac{1}{2} x^4 \sqrt{c+\frac{d}{x^2}}\right)}{2 c}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(-\frac{d\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(-\frac{\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-\frac{(2bc-ad)\left(\frac{1}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)-\frac{1}{2}x^4\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^6)/(3*c) - ((2*b*c - a*d)*(-1/2*(Sqrt[c + d/x^2]*x
^4) + (d*(-((Sqrt[c + d/x^2]*x^2)/c) + (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
)/c^(3/2)))/4))/(2*c))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x^{2}\left(8a\,c^{2}x^{4}+2ad\,x^{2}c+12b\,c^{2}x^{2}-3a\,d^{2}+6dbc\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48c^{2}}+\frac{d^{2}\left(ad-2cb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16c^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(8c^{\frac{3}{2}}\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,x^{3}-6\sqrt{c}\left(c\,x^{2}+d\right)^{\frac{3}{2}}adx+12c^{\frac{3}{2}}\left(c\,x^{2}+d\right)^{\frac{3}{2}}bx+3\sqrt{c}\sqrt{c\,x^{2}+d}\,a\,d^{2}x-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,bdx+3\ln\left(\sqrt{c}\,x+\sqrt{c\right.}\right.}{48\sqrt{c\,x^{2}+d}\,c^{\frac{5}{2}}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*x^2*(8*a*c^2*x^4+2*a*c*d*x^2+12*b*c^2*x^2-3*a*d^2+6*b*c*d)/c^2*((c*x^
2+d)/x^2)^(1/2)+1/16*d^2*(a*d-2*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.03

$$\int\left(a+\frac{b}{x^{2}}\right)\sqrt{c+\frac{d}{x^{2}}}x^{5}\,dx$$
$$=\left[-\frac{3\left(2\,bcd^{2}-ad^{3}\right)\sqrt{c}\log\left(-2\,cx^{2}-2\,\sqrt{c}x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(8\,ac^{3}x^{6}+2\left(6\,bc^{3}+ac^{2}d\right)x^{4}+3\left(2\,bc^{2}d-\right.\right.}{96\,c^{3}}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^5,x, algorithm="fricas")
```

output
```
[-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c
*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*
c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d
^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a
*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((
c*x^2 + d)/x^2))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(105) = 210$.

Time = 35.87 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.90

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx = \frac{acx^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**5,x)
```

output
```
a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2
/d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c*
*2*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b
*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sq
rt(d))/(8*c**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(99) = 198$.

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx =$$

$$-\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c^2-3\left(c+\frac{d}{x^2}\right)^2c^3+3\left(c+\frac{d}{x^2}\right)c^4-c^5}\right)a$$

$$+\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^5,x, algorithm="maxima")
```

output
```
-1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(
c + d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^
2)*c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x
^2) + sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2
)/((c + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$

$$=\frac{1}{48}\left(2\left(4\,ax^2\mathrm{sgn}(x)+\frac{6\,bc^4\mathrm{sgn}(x)+ac^3d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\left(2\,bc^3d\mathrm{sgn}(x)-ac^2d^2\mathrm{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+d}x$$
$$+\frac{\left(2\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{5}{2}}}$$
$$-\frac{\left(2\,bcd^2\log\left(|d|\right)-ad^3\log\left(|d|\right)\right)\mathrm{sgn}(x)}{32\,c^{\frac{5}{2}}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^5,x, algorithm="giac")
```

output
```
1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(
2*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*
d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2)
- 1/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx=\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{16\,c^{5/2}}$$

input
```
int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d
/x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^
2)^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5
/2)) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^5\,dx$$
$$=\frac{8\sqrt{c\,x^2+d}\,a\,c^3x^5+2\sqrt{c\,x^2+d}\,a\,c^2d\,x^3-3\sqrt{c\,x^2+d}\,ac\,d^2x+12\sqrt{c\,x^2+d}\,b\,c^3x^3+6\sqrt{c\,x^2+d}\,b\,c^2dx+}{48c^3}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^5,x)
```

output
```
(8*sqrt(c*x**2 + d)*a*c**3*x**5 + 2*sqrt(c*x**2 + d)*a*c**2*d*x**3 - 3*sqr
t(c*x**2 + d)*a*c*d**2*x + 12*sqrt(c*x**2 + d)*b*c**3*x**3 + 6*sqrt(c*x**2
 + d)*b*c**2*d*x + 3*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a
*d**3 - 6*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d**2)/(4
8*c**3)
```

3.138 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3 \, dx=\frac{(4bc+ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c}+\frac{1}{4}a\sqrt{c+\frac{d}{x^2}}x^4+\frac{d(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

output
```
1/8*(a*d+4*b*c)*(c+d/x^2)^(1/2)*x^2/c+1/4*a*(c+d/x^2)^(1/2)*x^4+1/8*d*(-a*
d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^3 \, dx =\frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(4bc+a(d+2cx^2))+d(-4bc+ad)\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)\right)}{8c^{3/2}\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(4*b*c + a*(d + 2*c*x^2)) +
d*(-4*b*c + a*d)*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]]))/(8*c^(3/2)*Sqrt[d +
 c*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{3/2}}{2 c}-\frac{(4 b c-a d) \int \sqrt{c+\frac{d}{x^2}} x^4 d\frac{1}{x^2}}{4 c}\right)$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{3/2}}{2 c}-\frac{(4 b c-a d) \left(\frac{1}{2} d \int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}} d\frac{1}{x^2}-x^2 \sqrt{c+\frac{d}{x^2}}\right)}{4 c}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2} \left(\frac{a x^4 \left(c+\frac{d}{x^2}\right)^{3/2}}{2 c}-\frac{(4 b c-a d) \left(\int \frac{1}{\frac{1}{d x^4}-\frac{c}{d}} d\sqrt{c+\frac{d}{x^2}}-x^2 \sqrt{c+\frac{d}{x^2}}\right)}{4 c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}-\frac{(4bc-ad)\left(x^2\left(-\sqrt{c+\frac{d}{x^2}}\right)-\frac{d\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{4c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^4)/(2*c) - ((4*b*c - a*d)*(-(Sqrt[c + d/x^2]*x^2)
- (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]))/(4*c))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{x^{2}\left(2ac\,x^{2}+ad+4cb\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{8c}-\frac{d\left(ad-4cb\right)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{8c^{\frac{3}{2}}\sqrt{c\,x^{2}+d}}$	91
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(2\sqrt{c}\,\left(c\,x^{2}+d\right)^{\frac{3}{2}}ax-\sqrt{c}\,\sqrt{c\,x^{2}+d}\,adx+4c^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,bx-\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)a\,d^{2}+4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)bcd\right)}{8\sqrt{c\,x^{2}+d}\,c^{\frac{3}{2}}}$	122

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2*(2*a*c*x^2+a*d+4*b*c)/c*((c*x^2+d)/x^2)^(1/2)-1/8*d*(a*d-4*b*c)/c^
(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.22

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \left[-\frac{(4\,bcd-ad^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},\ -\frac{(4\,bcd-ad^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4+(4\,bc^2+acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c^2}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^3,x, algorithm="fricas")
```

output
```
[-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)
/x^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x
^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*
x^2 + d)/x^2))/c^2]
```

Sympy [A] (verification not implemented)

Time = 33.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.67

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3a\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**3,x)
```

output
```
a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/
d + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/s
qrt(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c
)*x/sqrt(d))/(2*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a + \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^3,x, algorithm="maxima")
```

output
```
1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(
3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c
- 2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{1}{8}\left(2\,ax^2\mathrm{sgn}(x)+\frac{4\,bc^2\mathrm{sgn}(x)+acd\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{(4\,bcd\mathrm{sgn}(x)-ad^2\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}} + \frac{(4\,bcd\log(|d|)-ad^2\log(|d|))\mathrm{sgn}(x)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^3,x, algorithm="giac")
```

output
```
1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)
*x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
 d)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx = \frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c} + \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}} - \frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

input
```
int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/
x^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (
a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^3\,dx$$
$$=\frac{2\sqrt{c\,x^2+d}\,a\,c^2x^3+\sqrt{c\,x^2+d}\,acdx+4\sqrt{c\,x^2+d}\,b\,c^2x-\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)a\,d^2+4\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{}}{\sqrt{d}}\right.}{8c^2}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^3,x)
```

output
```
(2*sqrt(c*x**2 + d)*a*c**2*x**3 + sqrt(c*x**2 + d)*a*c*d*x + 4*sqrt(c*x**2
 + d)*b*c**2*x - sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*d**
2 + 4*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d)/(8*c**2)
```

3.139 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx=-b\sqrt{c+\frac{d}{x^2}}+\frac{1}{2}a\sqrt{c+\frac{d}{x^2}}x^2+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

output
```
-b*(c+d/x^2)^(1/2)+1/2*a*(c+d/x^2)^(1/2)*x^2+1/2*(a*d+2*b*c)*arctanh((c+d/
x^2)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x \, dx=\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(-2b+ax^2+\frac{2(2bc+ad)x\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

output
```
(Sqrt[c + d/x^2]*(-2*b + a*x^2 + (2*(2*b*c + a*d)*x*ArcTanh[(Sqrt[c]*x)/(-
Sqrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt[d + c*x^2])))/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(ad+2bc)\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

output
```
((a*(c + d/x^2)^(3/2)*x^2)/c - ((2*b*c + a*d)*(2*Sqrt[c + d/x^2] - 2*Sqrt[
c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]))/(2*c))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

method	result	si
risch	$\frac{(a x^2-2b)\sqrt{\frac{c x^2+d}{x^2}}}{2}+\frac{\left(\frac{ad}{2}+cb\right)\ln\left(\sqrt{c}\,x+\sqrt{c x^2+d}\right)\sqrt{\frac{c x^2+d}{x^2}}\,x}{\sqrt{c}\,\sqrt{c x^2+d}}$	77
default	$\frac{\sqrt{\frac{c x^2+d}{x^2}}\left(2c^{\frac{3}{2}}\sqrt{c x^2+d}\,b x^2+\sqrt{c}\,\sqrt{c x^2+d}\,a d x^2-2\sqrt{c}\,(c x^2+d)^{\frac{3}{2}} b+\ln\left(\sqrt{c}\,x+\sqrt{c x^2+d}\right) a d^2 x+2\ln\left(\sqrt{c}\,x+\sqrt{c x^2+d}\right) b c d x\right)}{2\sqrt{c x^2+d}\,d\sqrt{c}}$	12

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2-2*b)*((c*x^2+d)/x^2)^(1/2)+(1/2*a*d+c*b)*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \left[\frac{(2\,bc+ad)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c},\ -\frac{(2\,bc+ad)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(acx^2-2\,bc)\sqrt{\frac{cx^2+d}{x^2}}}{2\,c}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x,x, algorithm="fricas")
```

output
```
[1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/
x^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a
*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c
*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] (verification not implemented)

Time = 33.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.55

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2\sqrt{c}} + b\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x,x)
```

output
```
a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c)
) + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.57

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d
/x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c)
)/(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx=\frac{1}{2}\sqrt{cx^2+d}ax\mathrm{sgn}(x)+\frac{2b\sqrt{c}d\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d}-\frac{(2bc\mathrm{sgn}(x)+ad\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{4\sqrt{c}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c
*x^2 + d))^2 - d) - 1/4*(2*b*c*sgn(x) + a*d*sgn(x))*log((sqrt(c)*x - sqrt(
c*x^2 + d))^2)/sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 4.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}}+b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

input
```
int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d
/x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x\,dx = \frac{4\sqrt{c\,x^2+d}\,ac\,x^2-8\sqrt{c\,x^2+d}\,bc+4\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)adx+8\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)bcx-\sqrt{c}\,adx-}{8cx}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x,x)
```

output
```
(4*sqrt(c*x**2 + d)*a*c*x**2 - 8*sqrt(c*x**2 + d)*b*c + 4*sqrt(c)*log((sqr
t(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*d*x + 8*sqrt(c)*log((sqrt(c*x**2 + d
) + sqrt(c)*x)/sqrt(d))*b*c*x - sqrt(c)*a*d*x - 8*sqrt(c)*b*c*x)/(8*c*x)
```

3.140 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=-a\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}+a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-a*(c+d/x^2)^(1/2)-1/3*b*(c+d/x^2)^(3/2)/d+a*c^(1/2)*arctanh((c+d/x^2)^(1/
2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b(d+cx^2)+\frac{3a\sqrt{c}dx^3\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{3dx^2}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

output
```
-1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d + c*x^2) + (3*a*Sqrt[c]*d*x^3*Log[
-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(d*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-a\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-a\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

output
```
((-2*b*(c + d/x^2)^(3/2))/(3*d) - a*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh
[Sqrt[c + d/x^2]/Sqrt[c]]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

method	result	size
risch	$-\frac{\left(3 a d \,x^{2}+b c \,x^{2}+b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{3 x^{2} d}+\frac{a \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{\sqrt{c \,x^{2}+d}}$	84
default	$\frac{\sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, \left(3 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}\, a \,x^{4}-3 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{3}{2}} a \,x^{2}+3 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) a c d \,x^{3}-\sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{3}{2}} b \right)}{3 x^{2} \sqrt{c \,x^{2}+d}\, d \sqrt{c}}$	110

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*a*d*x^2+b*c*x^2+b*d)/x^2/d*((c*x^2+d)/x^2)^(1/2)+a*c^(1/2)*ln(c^(1
/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = \left[\frac{3\,a\sqrt{c}dx^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,((bc+3\,ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,dx^2},\right.$$
$$\left.-\frac{3\,a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+((bc+3\,ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{3\,dx^2}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
 - d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(
3*a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ ((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

Sympy [A] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.44

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=-\frac{a\left(\begin{cases}\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2\sqrt{c+\frac{d}{x^2}} & \text{for}\ d\neq 0\\ -\sqrt{c}\log\left(x^2\right) & \text{otherwise}\end{cases}\right)}{2}+\frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

output
```
-a*Piecewise((2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) + 2*sqrt(c + d/
x**2), Ne(d, 0)), (-sqrt(c)*log(x**2), True))/2 + b*Piecewise((-sqrt(c)/x*
*2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx=-\frac{1}{2}\left(\sqrt{c}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)a-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
 + 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(47) = 94$.

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = -\frac{1}{2}a\sqrt{c}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\operatorname{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d\operatorname{sgn}(x)-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d^2}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*
sgn(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt
(c*x^2 + d))^2 - d)^3
```

Mupad [B] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx = a\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - a\sqrt{c+\frac{d}{x^2}} - \frac{b\sqrt{c+\frac{d}{x^2}}\left(c\,x^2+d\right)}{3\,d\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
```

output
```
a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
 d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$$
$$= \frac{-3\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\,x^2+d}\,bc\,x^2-\sqrt{c\,x^2+d}\,bd+3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)ad\,x^3+\sqrt{c}\,ad\,x^3-\sqrt{c}\,bc\,x}{3d\,x^3}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x)
```

output
```
( - 3*sqrt(c*x**2 + d)*a*d*x**2 - sqrt(c*x**2 + d)*b*c*x**2 - sqrt(c*x**2
+ d)*b*d + 3*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*d*x**3
+ sqrt(c)*a*d*x**3 - sqrt(c)*b*c*x**3)/(3*d*x**3)
```

3.141 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

output
```
1/3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^2-1/5*b*(c+d/x^2)^(5/2)/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-3bd+2bcx^2-5adx^2\right)}{15d^2x^4}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-3*b*d + 2*b*c*x^2 - 5*a*d*x^2))/(15*d^2*x^4
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}}{x^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2} \int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d}+\frac{(ad-bc)\sqrt{c+\frac{d}{x^2}}}{d}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2\left(c+\frac{d}{x^2}\right)^{3/2} (bc-ad)}{3d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

output
```
((2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (2*b*(c + d/x^2)^(5/2))/(5*d^
2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2bc\,x^{2}+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5ad\,x^{2}-2bc\,x^{2}+3bd\right)\left(c\,x^{2}+d\right)}{15d^{2}x^{4}}$	48
orering	$-\frac{\left(5ad\,x^{2}-2bc\,x^{2}+3bd\right)\left(c\,x^{2}+d\right)\left(a+\frac{b}{x^{2}}\right)\sqrt{c+\frac{d}{x^{2}}}}{15d^{2}\left(a\,x^{2}+b\right)x^{2}}$	60
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(5acd\,x^{4}-2b\,c^{2}x^{4}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)}{15x^{4}d^{2}}$	62
trager	$-\frac{\left(5acd\,x^{4}-2b\,c^{2}x^{4}+5a\,d^{2}x^{2}+bcd\,x^{2}+3b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15x^{4}d^{2}}$	66

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\left((2\,bc^2-5\,acd)x^4-3\,bd^2-(bcd+5\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(d^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

output
```
-a*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, True)
)/2 - b*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d*
*2, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx=-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
 d/x^2)^(3/2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(38) = 76.

Time = 0.66 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$$
$$=\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}\mathrm{sgn}(x)-30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d\mathrm{sg}\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x
- sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*a*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x
) + 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*
x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 +
 d))^2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn
(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(b\,c^2+a\,d\,c)}{5\,d^2} - \frac{b\sqrt{c+\frac{d}{x^2}}}{5\,x^4} - \frac{\sqrt{c+\frac{d}{x^2}}\,(5\,a\,d^2+b\,c\,d)}{15\,d^2\,x^2} - \frac{c\sqrt{c+\frac{d}{x^2}}\,(8\,a\,d+b\,c)}{15\,d^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

output
```
((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4
) - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1
/2)*(8*a*d + b*c))/(15*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx = \frac{-5\sqrt{c\,x^2+d}\,acd\,x^4-5\sqrt{c\,x^2+d}\,a\,d^2x^2+2\sqrt{c\,x^2+d}\,b\,c^2x^4-\sqrt{c\,x^2+d}\,bcd\,x^2-3\sqrt{c\,x^2+d}\,b\,d^2-\sqrt{c}}{15d^2x^5}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x)
```

output
```
( - 5*sqrt(c*x**2 + d)*a*c*d*x**4 - 5*sqrt(c*x**2 + d)*a*d**2*x**2 + 2*sqr
t(c*x**2 + d)*b*c**2*x**4 - sqrt(c*x**2 + d)*b*c*d*x**2 - 3*sqrt(c*x**2 +
d)*b*d**2 - sqrt(c)*a*c*d*x**5 - 2*sqrt(c)*b*c**2*x**5)/(15*d**2*x**5)
```

3.142 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

output
```
-1/3*c*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^3+1/5*(-a*d+2*b*c)*(c+d/x^2)^(5/2)/d^3
-1/7*b*(c+d/x^2)^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-15bd^2+12bcdx^2-21ad^2x^2-8bc^2x^4+14acdx^4\right)}{105d^3x^6}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-15*b*d^2 + 12*b*c*d*x^2 - 21*a*d^2*x^2 - 8*
b*c^2*x^4 + 14*a*c*d*x^4))/(105*d^3*x^6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^2}+\frac{(ad-2bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}+\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

output
```
((-2*c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + (2*(2*b*c - a*d)*(c + d/x^
2)^(5/2))/(5*d^3) - (2*b*(c + d/x^2)^(7/2))/(7*d^3))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a c d\,x^{4}-8 b\,c^{2} x^{4}-21 a\,d^{2} x^{2}+12 b c d\,x^{2}-15 b\,d^{2}\right) \left(c\,x^{2}+d \right)}{105 d^{3} x^{6}}$	70
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a c d\,x^{4}-8 b\,c^{2} x^{4}-21 a\,d^{2} x^{2}+12 b c d\,x^{2}-15 b\,d^{2}\right) \left(c\,x^{2}+d \right)}{105 d^{3} x^{6}}$	70
orering	$\frac{\left(14 a c d\,x^{4}-8 b\,c^{2} x^{4}-21 a\,d^{2} x^{2}+12 b c d\,x^{2}-15 b\,d^{2}\right) \left(c\,x^{2}+d \right) \left(a+\frac{b}{x^{2}}\right) \sqrt{c+\frac{d}{x^{2}}}}{105 d^{3} \left(a\,x^{2}+b \right) x^{4}}$	82
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(14 a\,c^{2} d\,x^{6}-8 b\,c^{3} x^{6}-7 a c\,d^{2} x^{4}+4 b\,c^{2} d\,x^{4}-21 a\,d^{3} x^{2}-3 b c\,d^{2} x^{2}-15 b\,d^{3}\right)}{105 x^{6} d^{3}}$	87
trager	$\frac{\left(14 a\,c^{2} d\,x^{6}-8 b\,c^{3} x^{6}-7 a c\,d^{2} x^{4}+4 b\,c^{2} d\,x^{4}-21 a\,d^{3} x^{2}-3 b c\,d^{2} x^{2}-15 b\,d^{3}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105 x^{6} d^{3}}$	91

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*
d*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{\left(2\,(4\,bc^3-7\,ac^2d)x^6-(4\,bc^2d-7\,acd^2)x^4+15\,bd^3+3\,(bcd^2+7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d
^3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

Sympy [A] (verification not implemented)

Time = 2.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

output
```
-a*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, N
e(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*Piecewise((2*(c**2*(c + d/x**2)*
*(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d,
0)), (sqrt(c)/(3*x**6), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = -\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right) - \frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c +
d/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3
/2)*c/d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(62) = 124.

Time = 0.96 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.19

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{4\left(105\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{8}ac\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output

```
4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c
)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d
*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(7/2)*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x
) + 4*b*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d)^7
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6} - \frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4} - \frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3} - \frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2} - \frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4} + \frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

input

```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

output

```
(2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*
(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*
(c + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*
c^2*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx = \frac{14\sqrt{c\,x^2+d}\,a\,c^2d\,x^6 - 7\sqrt{c\,x^2+d}\,ac\,d^2x^4 - 21\sqrt{c\,x^2+d}\,a\,d^3x^2 - 8\sqrt{c\,x^2+d}\,b\,c^3x^6 + 4\sqrt{c\,x^2+d}\,b\,c^2d\,x}{105d^3x^7}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x)
```

output
```
(14*sqrt(c*x**2 + d)*a*c**2*d*x**6 - 7*sqrt(c*x**2 + d)*a*c*d**2*x**4 - 21
*sqrt(c*x**2 + d)*a*d**3*x**2 - 8*sqrt(c*x**2 + d)*b*c**3*x**6 + 4*sqrt(c*
x**2 + d)*b*c**2*d*x**4 - 3*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 15*sqrt(c*x**
2 + d)*b*d**3 - 14*sqrt(c)*a*c**2*d*x**7 + 8*sqrt(c)*b*c**3*x**7)/(105*d**
3*x**7)
```

3.143 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

output
```
1/3*c^2*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^4-1/5*c*(-2*a*d+3*b*c)*(c+d/x^2)^(5/2
)/d^4+1/7*(-a*d+3*b*c)*(c+d/x^2)^(7/2)/d^4-1/9*b*(c+d/x^2)^(9/2)/d^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(-35bd^3+30bcd^2x^2-45ad^3x^2-24bc^2dx^4+36acd^2x^4+16bc^3x^6-24ac^2dx^6\right)}{315d^4x^8}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(-35*b*d^3 + 30*b*c*d^2*x^2 - 45*a*d^3*x^2 -
24*b*c^2*d*x^4 + 36*a*c*d^2*x^4 + 16*b*c^3*x^6 - 24*a*c^2*d*x^6))/(315*d^4
*x^8)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}+\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}-\frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}\right)$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

output
```
((2*c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (2*c*(3*b*c - 2*a*d)*(c +
 d/x^2)^(5/2))/(5*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (2*
b*(c + d/x^2)^(9/2))/(9*d^4))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{2}d\,x^{6}-16b\,c^{3}x^{6}-36ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+45a\,d^{3}x^{2}-30bc\,d^{2}x^{2}+35b\,d^{3}\right)\left(c\,x^{2}+d\right)}{315d^{4}x^{8}}$	94
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{2}d\,x^{6}-16b\,c^{3}x^{6}-36ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+45a\,d^{3}x^{2}-30bc\,d^{2}x^{2}+35b\,d^{3}\right)\left(c\,x^{2}+d\right)}{315d^{4}x^{8}}$	94
orering	$-\frac{\left(24a\,c^{2}d\,x^{6}-16b\,c^{3}x^{6}-36ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+45a\,d^{3}x^{2}-30bc\,d^{2}x^{2}+35b\,d^{3}\right)\left(c\,x^{2}+d\right)\left(a+\frac{b}{x^{2}}\right)\sqrt{c+\frac{d}{x^{2}}}}{315d^{4}\left(a\,x^{2}+b\right)x^{6}}$	106
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(24a\,c^{3}d\,x^{8}-16b\,c^{4}x^{8}-12a\,c^{2}d^{2}x^{6}+8b\,c^{3}d\,x^{6}+9ac\,d^{3}x^{4}-6b\,c^{2}d^{2}x^{4}+45a\,d^{4}x^{2}+5bc\,d^{3}x^{2}+35b\,d^{4}\right)}{315x^{8}d^{4}}$	111
trager	$-\frac{\left(24a\,c^{3}d\,x^{8}-16b\,c^{4}x^{8}-12a\,c^{2}d^{2}x^{6}+8b\,c^{3}d\,x^{6}+9ac\,d^{3}x^{4}-6b\,c^{2}d^{2}x^{4}+45a\,d^{4}x^{2}+5bc\,d^{3}x^{2}+35b\,d^{4}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315x^{8}d^{4}}$	115

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+2
4*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{\left(8\left(2\,bc^4-3\,ac^3d\right)x^8-4\left(2\,bc^3d-3\,ac^2d^2\right)x^6-35\,bd^4+3\left(2\,bc^2d^2-3\,acd^3\right)x^4-5\left(bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx}{}}}{315\,d^4x^8}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*
b*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^4*x^8)
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\frac{a\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

output
```
-a*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 +
(c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - b*Pi
ecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3
*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c
)/(4*x**8), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = -\frac{1}{315}\,b\left(\frac{35\,\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right) - \frac{1}{105}\,a\left(\frac{15\,\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\,\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\,\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c
+ d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c +
 d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/
d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(88) = 176.

Time = 0.95 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.56

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2
+ d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2
)*d*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168
*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4
*b*c^(9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*
sgn(x) + 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqr
t(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x)
+ 3*a*c^(7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx = \frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3} - \frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6} - \frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4} - \frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6} + \frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2} + \frac{2\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4} - \frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

output
```
(16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (
8*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*
c*(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*
a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*
d^2*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$$
$$= \frac{-24\sqrt{c\,x^2+d}\,a\,c^3d\,x^8+12\sqrt{c\,x^2+d}\,a\,c^2d^2x^6-9\sqrt{c\,x^2+d}\,ac\,d^3x^4-45\sqrt{c\,x^2+d}\,a\,d^4x^2+16\sqrt{c\,x^2+d}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x)
```

output
```
( - 24*sqrt(c*x**2 + d)*a*c**3*d*x**8 + 12*sqrt(c*x**2 + d)*a*c**2*d**2*x*
*6 - 9*sqrt(c*x**2 + d)*a*c*d**3*x**4 - 45*sqrt(c*x**2 + d)*a*d**4*x**2 +
16*sqrt(c*x**2 + d)*b*c**4*x**8 - 8*sqrt(c*x**2 + d)*b*c**3*d*x**6 + 6*sqr
t(c*x**2 + d)*b*c**2*d**2*x**4 - 5*sqrt(c*x**2 + d)*b*c*d**3*x**2 - 35*sqr
t(c*x**2 + d)*b*d**4 + 24*sqrt(c)*a*c**3*d*x**9 - 16*sqrt(c)*b*c**4*x**9)/
(315*d**4*x**9)
```

3.144 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx=-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

output
```
-1/3*c^3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^5+1/5*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(
5/2)/d^5-3/7*c*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^5+1/9*(-a*d+4*b*c)*(c+d/x^2)
^(9/2)/d^5-1/11*b*(c+d/x^2)^(11/2)/d^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)\left(11adx^2(-35d^3+30cd^2x^2-24c^2dx^4+16c^3x^6)+b(-315d^4+280cd^3x^2-240c^2d^2x^4\right.}{3465d^5x^{10}}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)*(11*a*d*x^2*(-35*d^3 + 30*c*d^2*x^2 - 24*c^2*
d*x^4 + 16*c^3*x^6) + b*(-315*d^4 + 280*c*d^3*x^2 - 240*c^2*d^2*x^4 + 192*
c^3*d*x^6 - 128*c^4*x^8)))/(3465*d^5*x^10)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$
$$\Big\downarrow \text{948}$$
$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^6}d\frac{1}{x^2}$$
$$\Big\downarrow \text{86}$$
$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^4}+\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}-\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^4}+\frac{c^3(bc\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{2c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{6c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-a$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

output
```
((-2*c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (2*c^2*(4*b*c - 3*a*d)*(
c + d/x^2)^(5/2))/(5*d^5) - (6*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5)
+ (2*(4*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (2*b*(c + d/x^2)^(11/2))/(
11*d^5))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4)\,(c\,x^2+d}{3465d^5x^{10}}$
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4)\,(c\,x^2+d}{3465d^5x^{10}}$
orering	$\frac{(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d^3x^2-315b\,d^4)\,(c\,x^2+d)\left(a+\frac{b}{x^2}\right)}{3465d^5(a\,x^2+b)x^8}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,(176a\,c^4d\,x^{10}-128b\,c^5x^{10}-88a\,c^3d^2x^8+64b\,c^4d\,x^8+66a\,c^2d^3x^6-48b\,c^3d^2x^6-55ac\,d^4x^4+40b\,c^2d^3x^4-385a\,d^5x^2-35bc\,d}{3465x^{10}d^5}$
trager	$\frac{(176a\,c^4d\,x^{10}-128b\,c^5x^{10}-88a\,c^3d^2x^8+64b\,c^4d\,x^8+66a\,c^2d^3x^6-48b\,c^3d^2x^6-55ac\,d^4x^4+40b\,c^2d^3x^4-385a\,d^5x^2-35bc\,d^4x^2-315b}{3465x^{10}d^5}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*
x^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*
c*d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx =$$

$$-\frac{(16\,(8\,bc^5-11\,ac^4d)x^{10}-8\,(8\,bc^4d-11\,ac^3d^2)x^8+6\,(8\,bc^3d^2-11\,ac^2d^3)x^6+315\,bd^5-5\,(8\,bc^2d^3-1}{3465\,d^5x^{10}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
-1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
 + 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*
c*d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

Sympy [A] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.32

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$
$$= -\frac{a\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{b\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

input

```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
```

output

```
-a*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/
5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (s
qrt(c)/(4*x**8), True))/2 - b*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4
*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x*
*2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10
), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx =$$

$$-\frac{1}{3465}\,b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^5}\right.$$

$$\left.-\frac{1}{315}\,a\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 297
0*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d
/x^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)
^(7/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d
^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(114) = 228.

Time = 1.84 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$

$$= \frac{32\left(3465\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\operatorname{sgn}(x)+11088\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\operatorname{sgn}(x)-4851\left(\sqrt{c}x-\sqrt{cx^2+\cdots}\right.\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

output
```
32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(s
qrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(
c*x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*
b*c^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*s
gn(x) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(
sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d)
)^6*a*c^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*
d^4*sgn(x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88
*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c
^(9/2)*d^7*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx = \frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4} - \frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}} - \frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5} - \frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6} - \frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8} + \frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4} - \frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3\,x^2} + \frac{8\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{693\,d^2\,x^6} - \frac{16\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^4} + \frac{64\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^4\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)
```

output
```
(16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) -
 (a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5)
- (a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8)
+ (2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/
(315*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c +
 d/x^2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2
)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.73

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$$
$$= \frac{176\sqrt{c\,x^2+d}\,a\,c^4 d\,x^{10}-88\sqrt{c\,x^2+d}\,a\,c^3 d^2 x^8+66\sqrt{c\,x^2+d}\,a\,c^2 d^3 x^6-55\sqrt{c\,x^2+d}\,ac\,d^4 x^4-385\sqrt{c\,x^2}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x)
```

output
```
(176*sqrt(c*x**2 + d)*a*c**4*d*x**10 - 88*sqrt(c*x**2 + d)*a*c**3*d**2*x**
8 + 66*sqrt(c*x**2 + d)*a*c**2*d**3*x**6 - 55*sqrt(c*x**2 + d)*a*c*d**4*x*
*4 - 385*sqrt(c*x**2 + d)*a*d**5*x**2 - 128*sqrt(c*x**2 + d)*b*c**5*x**10
+ 64*sqrt(c*x**2 + d)*b*c**4*d*x**8 - 48*sqrt(c*x**2 + d)*b*c**3*d**2*x**6
 + 40*sqrt(c*x**2 + d)*b*c**2*d**3*x**4 - 35*sqrt(c*x**2 + d)*b*c*d**4*x**
2 - 315*sqrt(c*x**2 + d)*b*d**5 - 176*sqrt(c)*a*c**4*d*x**11 + 128*sqrt(c)
*b*c**5*x**11)/(3465*d**5*x**11)
```

3.145 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^{10} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^{10} \, dx = -\frac{16 d^3 (11 b c-8 a d) \left(c+\frac{d}{x^2}\right)^{3/2} x^3}{3465 c^5} + \frac{8 d^2 (11 b c-8 a d) \left(c+\frac{d}{x^2}\right)^{3/2} x^5}{1155 c^4} - \frac{2 d (11 b c-8 a d) \left(c+\frac{d}{x^2}\right)^{3/2} x^7}{231 c^3} + \frac{(11 b c-8 a d) \left(c+\frac{d}{x^2}\right)^{3/2} x^9}{99 c^2} + \frac{a \left(c+\frac{d}{x^2}\right)^{3/2} x^{11}}{11 c}$$

output

```
-16/3465*d^3*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^3/c^5+8/1155*d^2*(-8*a*d+11
*b*c)*(c+d/x^2)^(3/2)*x^5/c^4-2/231*d*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^7/
c^3+1/99*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^9/c^2+1/11*a*(c+d/x^2)^(3/2)*x^
11/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(11bc(-16d^3+24cd^2x^2-30c^2dx^4+35c^3x^6)+a(128d^4-192cd^3x^2+240c^2d^2x^4-2\right.}{3465c^5}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*
x^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3
*d*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$
$$\Big\downarrow \text{955}$$
$$\frac{(11bc-8ad)\int \sqrt{c+\frac{d}{x^2}}x^8dx}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$
$$\Big\downarrow \text{803}$$
$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\int \sqrt{c+\frac{d}{x^2}}x^6dx}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$
$$\Big\downarrow \text{803}$$

$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\int\sqrt{c+\frac{d}{x^2}}x^4dx}{7c}\right)}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

$\downarrow$ 803

$$\frac{(11bc-8ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int\sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}\right)}{3c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

$\downarrow$ 796

$$\frac{\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}-\frac{2d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)}{7c}\right)}{3c}\right)(11bc-8ad)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{3/2}}{11c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^11)/(11*c) + ((11*b*c - 8*a*d)*(((c + d/x^2)^(3/2)*
x^9)/(9*c) - (2*d*(((c + d/x^2)^(3/2)*x^7)/(7*c) - (4*d*((-2*d*(c + d/x^2)
^(3/2)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)*x^5)/(5*c)))/(7*c)))/(3*c)))/(11
*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
gosper	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(315a\,x^8c^4-280a\,c^3d\,x^6+385b\,c^4x^6+240a\,c^2d^2x^4-330b\,c^3d\,x^4-192ac\,d^3x^2+264b\,c^2d^2x^2+128a\,d^4-176bc\,d^3\right)\left(c\,x^2+d\right)}{3465c^5}$
default	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(315a\,x^8c^4-280a\,c^3d\,x^6+385b\,c^4x^6+240a\,c^2d^2x^4-330b\,c^3d\,x^4-192ac\,d^3x^2+264b\,c^2d^2x^2+128a\,d^4-176bc\,d^3\right)\left(c\,x^2+d\right)}{3465c^5}$
orering	$\frac{\left(315a\,x^8c^4-280a\,c^3d\,x^6+385b\,c^4x^6+240a\,c^2d^2x^4-330b\,c^3d\,x^4-192ac\,d^3x^2+264b\,c^2d^2x^2+128a\,d^4-176bc\,d^3\right)\left(c\,x^2+d\right)x^3\left(a+\frac{b}{x^2}\right)}{3465c^5\left(a\,x^2+b\right)}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,x\left(315a\,c^5x^{10}+35a\,c^4d\,x^8+385b\,c^5x^8-40a\,c^3d^2x^6+55b\,c^4d\,x^6+48a\,c^2d^3x^4-66b\,c^3d^2x^4-64ac\,d^4x^2+88b\,c^2d^3x^2+128a\,d^5\right.}{3465c^5}$
trager	$\frac{\left(315a\,c^5x^{10}+35a\,c^4d\,x^8+385b\,c^5x^8-40a\,c^3d^2x^6+55b\,c^4d\,x^6+48a\,c^2d^3x^4-66b\,c^3d^2x^4-64ac\,d^4x^2+88b\,c^2d^3x^2+128a\,d^5-176bc\,d^4\right)}{3465c^5}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^10,x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^
6+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*
a*d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx$$
$$=\frac{\left(315\,ac^5x^{11}+35\,(11\,bc^5+ac^4d)x^9+5\,(11\,bc^4d-8\,ac^3d^2)x^7-6\,(11\,bc^3d^2-8\,ac^2d^3)x^5+8\,(11\,bc^2d^3-8\right.}{3465\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^10,x, algorithm="fricas")
```

output
```
1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a
*c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a
*c*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(146) = 292$.

Time = 4.79 (sec) , antiderivative size = 1386, normalized size of antiderivative = 9.24

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**10,x)
```

output
```
315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/
(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13
860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(
c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d
**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c
**4*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d
**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) + 280*a*c**3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d
**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**
19*x**2 + 3465*c**5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(34
65*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4} + \frac{\left(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5+1155\left(c+\frac{d}{x^2}\right)\right.}{3465\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^10,x, algorithm="maxima")
```

output
```
1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d
/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(
c + d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(
7/2)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4
*x^3)*a/c^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx = \frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5} + \frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1540\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-1485\left(cx^2+d\right)^{\frac{7}{2}}bcd\mathrm{sg}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^10,x, algorithm="giac")
```

output
```
16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 +
d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(
9/2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(
7/2)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 +
d)^(5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^
2 + d)^(3/2)*a*d^4*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^{11}}{11}+\frac{x\,(128\,a\,d^5-176\,b\,c\,d^4)}{3465\,c^5}+\frac{x^9\,(385\,b\,c^5+35\,a\,d\,c^4)}{3465\,c^5}-\frac{d\,x^7\,(8\,a\,d-11\,b\,c)}{693\,c^2}+\frac{2\,d^2\,x^5\,(8\,a\,d-11\,b\,c)}{1155\,c^3}-\frac{8\,d^3\,x^3\,(8\,a\,d-11\,b\,c)}{3465\,c^4}\right)$$

input
```
int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5)
+ (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(69
3*c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*
b*c))/(3465*c^4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^{10}\,dx=\frac{\sqrt{c\,x^2+d}\,(315a\,c^5x^{10}+35a\,c^4d\,x^8+385b\,c^5x^8-40a\,c^3d^2x^6+55b\,c^4d\,x^6+48a\,c^2d^3x^4-66b\,c^3d^2x^4-64}{3465c^5}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^10,x)
```

output
```
(sqrt(c*x**2 + d)*(315*a*c**5*x**10 + 35*a*c**4*d*x**8 - 40*a*c**3*d**2*x*
*6 + 48*a*c**2*d**3*x**4 - 64*a*c*d**4*x**2 + 128*a*d**5 + 385*b*c**5*x**8
 + 55*b*c**4*d*x**6 - 66*b*c**3*d**2*x**4 + 88*b*c**2*d**3*x**2 - 176*b*c*
d**4))/(3465*c**5)
```

3.146 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^8 \, dx = \frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{315c^4} - \frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{105c^3} + \frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{21c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^9}{9c}$$

output
```
8/315*d^2*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^3/c^4-4/105*d*(-2*a*d+3*b*c)*(c
+d/x^2)^(3/2)*x^5/c^3+1/21*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^7/c^2+1/9*a*(c
+d/x^2)^(3/2)*x^9/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)\left(24bcd^2-16ad^3-36bc^2dx^2+24acd^2x^2+45bc^3x^4-30ac^2dx^4+35ac^3x^6\right)}{315c^4}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(24*b*c*d^2 - 16*a*d^3 - 36*b*c^2*d*x^2 + 2
4*a*c*d^2*x^2 + 45*b*c^3*x^4 - 30*a*c^2*d*x^4 + 35*a*c^3*x^6))/(315*c^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$
$$\Big\downarrow \text{955}$$
$$\frac{(3bc-2ad)\int \sqrt{c+\frac{d}{x^2}}x^6dx}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$
$$\Big\downarrow \text{803}$$
$$\frac{(3bc-2ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\int \sqrt{c+\frac{d}{x^2}}x^4dx}{7c}\right)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$
$$\Big\downarrow \text{803}$$

$$\frac{(3bc-2ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int\sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}\right)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}-\frac{4d\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)}{7c}\right)(3bc-2ad)}{3c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^9)/(9*c) + ((3*b*c - 2*a*d)*(((c + d/x^2)^(3/2)*x^7
)/(7*c) - (4*d*((-2*d*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)
*x^5)/(5*c)))/(7*c)))/(3*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35a\,x^{6}c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)}{315c^{4}}$	89
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35a\,x^{6}c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)}{315c^{4}}$	89
orering	$\frac{\left(35a\,x^{6}c^{3}-30a\,c^{2}d\,x^{4}+45b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-36b\,c^{2}d\,x^{2}-16a\,d^{3}+24bc\,d^{2}\right)\left(c\,x^{2}+d\right)x^{3}\left(a+\frac{b}{x^{2}}\right)\sqrt{c+\frac{d}{x^{2}}}}{315c^{4}\left(a\,x^{2}+b\right)}$	103
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(35a\,x^{8}c^{4}+5a\,c^{3}d\,x^{6}+45b\,c^{4}x^{6}-6a\,c^{2}d^{2}x^{4}+9b\,c^{3}d\,x^{4}+8ac\,d^{3}x^{2}-12b\,c^{2}d^{2}x^{2}-16a\,d^{4}+24bc\,d^{3}\right)}{315c^{4}}$	106
trager	$\frac{\left(35a\,x^{8}c^{4}+5a\,c^{3}d\,x^{6}+45b\,c^{4}x^{6}-6a\,c^{2}d^{2}x^{4}+9b\,c^{3}d\,x^{4}+8ac\,d^{3}x^{2}-12b\,c^{2}d^{2}x^{2}-16a\,d^{4}+24bc\,d^{3}\right)x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315c^{4}}$	110

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24
*a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int\left(a+\frac{b}{x^{2}}\right)\sqrt{c+\frac{d}{x^{2}}}x^{8}\,dx$$
$$=\frac{(35\,ac^{4}x^{9}+5\,(9\,bc^{4}+ac^{3}d)x^{7}+3\,(3\,bc^{3}d-2\,ac^{2}d^{2})x^{5}-4\,(3\,bc^{2}d^{2}-2\,acd^{3})x^{3}+8\,(3\,bcd^{3}-2\,ad^{4})x)\sqrt{}}{315\,c^{4}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^8,x, algorithm="fricas")
```

output
```
1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d
^2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqr
t((c*x^2 + d)/x^2)/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(112) = 224$.

Time = 3.71 (sec) , antiderivative size = 910, normalized size of antiderivative = 7.78

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**8,x)
```

output
```
35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**
6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2
)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5
*a*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d
**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x*
*4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5
*d**11*x**2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(
315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*
d**12) - 16*a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x*
*10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c
**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*
x**2 + 105*c**3*d**6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3}$$
$$+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)a}{315\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^8,x, algorithm="maxima")
```

output
```
1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x
^2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^
2)^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3
*x^3)*a/c^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx=-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,ad^{\frac{9}{2}}\right)\operatorname{sgn}(x)}{315\,c^4}$$
$$+\frac{35\left(cx^2+d\right)^{\frac{9}{2}}a\operatorname{sgn}(x)+45\left(cx^2+d\right)^{\frac{7}{2}}bc\operatorname{sgn}(x)-135\left(cx^2+d\right)^{\frac{7}{2}}ad\operatorname{sgn}(x)-126\left(cx^2+d\right)^{\frac{5}{2}}bcd\operatorname{sgn}(x)-}{315\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^8,x, algorithm="giac")
```

output
```
-8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9
/2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d
*sgn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2
*sgn(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d
^3*sgn(x))/c^4
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^9}{9}-\frac{x\,(16\,a\,d^4-24\,b\,c\,d^3)}{315\,c^4}+\frac{x^7\,(45\,b\,c^4+5\,a\,d\,c^3)}{315\,c^4}-\frac{d\,x^5\,(2\,a\,d-3\,b\,c)}{105\,c^2}+\frac{4\,d^2\,x^3\,(2\,a\,d-3\,b\,c)}{315\,c^3}\right)$$

input
```
int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^
7*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) +
(4*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^8\,dx$$
$$=\frac{\sqrt{c\,x^2+d}\,(35a\,c^4x^8+5a\,c^3d\,x^6+45b\,c^4x^6-6a\,c^2d^2x^4+9b\,c^3d\,x^4+8ac\,d^3x^2-12b\,c^2d^2x^2-16a\,d^4+24}{315c^4}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^8,x)
```

output
```
(sqrt(c*x**2 + d)*(35*a*c**4*x**8 + 5*a*c**3*d*x**6 - 6*a*c**2*d**2*x**4 +
 8*a*c*d**3*x**2 - 16*a*d**4 + 45*b*c**4*x**6 + 9*b*c**3*d*x**4 - 12*b*c**
2*d**2*x**2 + 24*b*c*d**3))/(315*c**4)
```

3.147 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{105c^3} + \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{35c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^7}{7c}$$

output
```
-2/105*d*(-4*a*d+7*b*c)*(c+d/x^2)^(3/2)*x^3/c^3+1/35*(-4*a*d+7*b*c)*(c+d/x
^2)^(3/2)*x^5/c^2+1/7*a*(c+d/x^2)^(3/2)*x^7/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^6 \, dx = \frac{\sqrt{c+\frac{d}{x^2}} x(d+cx^2)\left(-14bcd+8ad^2+21bc^2x^2-12acdx^2+15ac^2x^4\right)}{105c^3}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(-14*b*c*d + 8*a*d^2 + 21*b*c^2*x^2 - 12*a*
c*d*x^2 + 15*a*c^2*x^4))/(105*c^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(7bc-4ad)\int \sqrt{c+\frac{d}{x^2}}x^4dx}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(7bc-4ad)\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2d\int \sqrt{c+\frac{d}{x^2}}x^2dx}{5c}\right)}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}}{15c^2}\right)(7bc-4ad)}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^7)/(7*c) + ((7*b*c - 4*a*d)*((-2*d*(c + d/x^2)^(3/2
)*x^3)/(15*c^2) + ((c + d/x^2)^(3/2)*x^5)/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(15 a \,c^{2} x^{4}-12 a d \,x^{2} c +21 b \,c^{2} x^{2}+8 a \,d^{2}-14 d b c \right) \left(c \,x^{2}+d \right)}{105 c^{3}}$	65
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(15 a \,c^{2} x^{4}-12 a d \,x^{2} c +21 b \,c^{2} x^{2}+8 a \,d^{2}-14 d b c \right) \left(c \,x^{2}+d \right)}{105 c^{3}}$	65
orering	$\frac{\left(15 a \,c^{2} x^{4}-12 a d \,x^{2} c +21 b \,c^{2} x^{2}+8 a \,d^{2}-14 d b c \right) \left(c \,x^{2}+d \right) x^{3} \left(a +\frac{b}{x^{2}}\right) \sqrt{c +\frac{d}{x^{2}}}}{105 c^{3} \left(a \,x^{2}+b \right)}$	79
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(15 a \,x^{6} c^{3}+3 a \,c^{2} d \,x^{4}+21 b \,c^{3} x^{4}-4 a c \,d^{2} x^{2}+7 b \,c^{2} d \,x^{2}+8 a \,d^{3}-14 b c \,d^{2}\right)}{105 c^{3}}$	82
trager	$\frac{\left(15 a \,x^{6} c^{3}+3 a \,c^{2} d \,x^{4}+21 b \,c^{3} x^{4}-4 a c \,d^{2} x^{2}+7 b \,c^{2} d \,x^{2}+8 a \,d^{3}-14 b c \,d^{2}\right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{105 c^{3}}$	86

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*
d^2-14*b*c*d)*(c*x^2+d)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{(15\,ac^3x^7+3\,(7\,bc^3+ac^2d)x^5+(7\,bc^2d-4\,acd^2)x^3-2\,(7\,bcd^2-4\,ad^3)x)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^6,x, algorithm="fricas")
```

output
```
1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*
x^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(78) = 156$.

Time = 2.69 (sec) , antiderivative size = 422, normalized size of antiderivative = 5.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3ac^2d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{12acd^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{8ad^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{b\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**6,x)
```

output
```
15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/
(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(
13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 1
05*c**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*
x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*
a*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
105*c**3*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqr
t(c*x**2/d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2} + \frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^6,x, algorithm="maxima")
```

output
```
1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(
15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/
2)*d^2*x^3)*a/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx = \frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3} + \frac{15\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\left(cx^2+d\right)^{\frac{3}{2}}bcd\mathrm{sgn}(x)+3}{105\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^6,x, algorithm="giac")
```

output
```
2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/
2)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*s
gn(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn
(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^7}{7}+\frac{x\,(8\,a\,d^3-14\,b\,c\,d^2)}{105\,c^3}+\frac{x^5\,(21\,b\,c^3+3\,a\,d\,c^2)}{105\,c^3}-\frac{d\,x^3\,(4\,a\,d-7\,b\,c)}{105\,c^2}\right)$$

input
```
int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5
*(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^6\,dx=\frac{\sqrt{c\,x^2+d}\,(15a\,c^3x^6+3a\,c^2d\,x^4+21b\,c^3x^4-4ac\,d^2x^2+7b\,c^2d\,x^2+8a\,d^3-14bc\,d^2)}{105c^3}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^6,x)
```

output
```
(sqrt(c*x**2 + d)*(15*a*c**3*x**6 + 3*a*c**2*d*x**4 - 4*a*c*d**2*x**2 + 8*
a*d**3 + 21*b*c**3*x**4 + 7*b*c**2*d*x**2 - 14*b*c*d**2))/(105*c**3)
```

3.148 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx = \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^5}{5c}$$

output
```
1/15*(-2*a*d+5*b*c)*(c+d/x^2)^(3/2)*x^3/c^2+1/5*a*(c+d/x^2)^(3/2)*x^5/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} x^4 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\, x(d+cx^2)\left(5bc-2ad+3acx^2\right)}{15c^2}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}} dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-2ad) \int \sqrt{c+\frac{d}{x^2}} x^2 dx}{5c}+\frac{a x^5 \left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^3 \left(c+\frac{d}{x^2}\right)^{3/2} (5bc-2ad)}{15c^2}+\frac{a x^5 \left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

output
```
((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^
5)/(5*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(3 a c\,x^{2}-2 a d+5 c b \right) \left(c\,x^{2}+d \right)}{15 c^{2}}$	43
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(3 a c\,x^{2}-2 a d+5 c b \right) \left(c\,x^{2}+d \right)}{15 c^{2}}$	43
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(3 a\,c^{2} x^{4}+a d\,x^{2} c+5 b\,c^{2} x^{2}-2 a\,d^{2}+5 d b c \right)}{15 c^{2}}$	57
orering	$\frac{\left(3 a c\,x^{2}-2 a d+5 c b \right) \left(c\,x^{2}+d \right) x^{3} \left(a+\frac{b}{x^{2}}\right) \sqrt{c+\frac{d}{x^{2}}}}{15 c^{2} \left(a\,x^{2}+b \right)}$	57
trager	$\frac{\left(3 a\,c^{2} x^{4}+a d\,x^{2} c+5 b\,c^{2} x^{2}-2 a\,d^{2}+5 d b c \right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15 c^{2}}$	61

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{(3\,ac^2x^5+(5\,bc^2+acd)x^3+(5\,bcd-2\,ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^4,x, algorithm="fricas")
```

output
```
1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(46) = 92.

Time = 2.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx = \frac{a\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**4,x)
```

output
```
a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(
15*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*
x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)a}{15\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^4,x, algorithm="maxima")
```

output
```
1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x
^2)^(3/2)*d*x^3)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}a\mathrm{sgn}(x)+5\left(cx^2+d\right)^{\frac{3}{2}}bc\mathrm{sgn}(x)-5\left(cx^2+d\right)^{\frac{3}{2}}ad\mathrm{sgn}(x)}{15\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^4,x, algorithm="giac")
```

output
```
-1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)
*a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x
))/c^2
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^5}{5}-\frac{x\,(2\,a\,d^2-5\,b\,c\,d)}{15\,c^2}+\frac{x^3\,(5\,b\,c^2+a\,d\,c)}{15\,c^2}\right)$$

input
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
(c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*
b*c^2 + a*c*d))/(15*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^4\,dx=\frac{\sqrt{c\,x^2+d}\,(3a\,c^2x^4+acd\,x^2+5b\,c^2x^2-2a\,d^2+5bcd)}{15c^2}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^4,x)
```

output
```
(sqrt(c*x**2 + d)*(3*a*c**2*x**4 + a*c*d*x**2 - 2*a*d**2 + 5*b*c**2*x**2 +
 5*b*c*d))/(15*c**2)
```

3.149 $\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^2 \, dx=b\sqrt{c+\frac{d}{x^2}}x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2} x^3}{3c}-b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)$$

output
```
b*(c+d/x^2)^(1/2)*x+1/3*a*(c+d/x^2)^(3/2)*x^3/c-b*d^(1/2)*arctanh(d^(1/2)/
(c+d/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^2 \, dx=\frac{1}{3}\sqrt{c+\frac{d}{x^2}}x\left(3b+a\left(\frac{d}{c}+x^2\right)-\frac{3b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

output
```
(Sqrt[c + d/x^2]*x*(3*b + a*(d/c + x^2) - (3*b*Sqrt[d]*ArcTanh[Sqrt[d + c*
x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {953, 773, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$

$$\Big\downarrow \text{953}$$

$$b\int \sqrt{c+\frac{d}{x^2}}dx+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(d\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-x\sqrt{c+\frac{d}{x^2}}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(d\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}-x\sqrt{c+\frac{d}{x^2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}-b\left(\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)-x\sqrt{c+\frac{d}{x^2}}\right)$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

output
```
(a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*(-(Sqrt[c + d/x^2]*x) + Sqrt[d]*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x\left(3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bc-a\left(c\,x^{2}+d\right)^{\frac{3}{2}}-3\sqrt{c\,x^{2}+d}\,bc\right)}{3\sqrt{c\,x^{2}+d}\,c}$	83

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x
)*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.27

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$$
$$=\left[\frac{3\,bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(acx^3+(3\,bc+ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c},\frac{3\,bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)\ldots}{3\ldots}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^2,x, algorithm="fricas")
```

output
```
[1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)
/x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c
*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + (a*c*x^3 + (3*b*c +
 a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^2 \, dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c} + \frac{b\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}} - b\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{bd}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)*x**2,x)
```

output
```
a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
+ b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))
+ b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right) \sqrt{c+\frac{d}{x^2}}x^2 \, dx = \frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c} + \frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x + \sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x - \sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x + \sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^2,x, algorithm="maxima")
```

output
```
1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sq
rt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.76

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = \frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}} - \frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\sqrt{d}+a\sqrt{-d}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{3\,c\sqrt{-d}} + \frac{(cx^2+d)^{\frac{3}{2}}ac^2\operatorname{sgn}(x)+3\,\sqrt{cx^2+d}bc^3\operatorname{sgn}(x)}{3\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)*x^2,x, algorithm="giac")
```

output
```
b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan
(sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(
c*sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^
3*sgn(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx = b\,x\,\sqrt{c+\frac{d}{x^2}} + \frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,(c\,x^2+d)}{3\,c} + \frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

input
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(
1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x
^2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.50

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2\,dx$$
$$=\frac{\sqrt{c\,x^2+d}\,ac\,x^2+\sqrt{c\,x^2+d}\,ad+3\sqrt{c\,x^2+d}\,bc+3\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)bc-3\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x+}{\sqrt{d}}\right.}{3c}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)*x^2,x)
```

output
```
(sqrt(c*x**2 + d)*a*c*x**2 + sqrt(c*x**2 + d)*a*d + 3*sqrt(c*x**2 + d)*b*c
 + 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c - 3
*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c)/(3*c)
```

3.150 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=-\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c}-\frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

output
```
-1/2*(2*a*d+b*c)*(c+d/x^2)^(1/2)/c/x+a*(c+d/x^2)^(3/2)*x/c-1/2*(2*a*d+b*c)
*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(-b+2ax^2-\frac{(bc+2ad)x^2\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{2x}$$

input
```
Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2
]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {899, 359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}dx$$
$$\Big\downarrow \text{899}$$
$$-\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{359}$$
$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}}{c}$$
$$\Big\downarrow \text{211}$$
$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$
$$\Big\downarrow \text{224}$$
$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$
$$\Big\downarrow \text{219}$$
$$\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}-\frac{(2ad+bc)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{c}$$

input
```
Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

output
```
(a*(c + d/x^2)^(3/2)*x)/c - ((b*c + 2*a*d)*(Sqrt[c + d/x^2]/(2*x) + (c*Arc
Tanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])))/c
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{b\sqrt{\frac{c\,x^2+d}{x^2}}}{2x}+\frac{\left(-\frac{(2ad+cb)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{2\sqrt{d}}+a\sqrt{c\,x^2+d}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{\sqrt{c\,x^2+d}}$
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)a\,x^2+\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)bc\,x^2-2\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\,x^2+d}\,bc\,x^2+\left(c\,x^2+d\right)^{\frac{3}{2}}b\right)}{2x\sqrt{c\,x^2+d}\,d}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/x*((c*x^2+d)/x^2)^(1/2)+(-1/2*(2*a*d+b*c)/d^(1/2)*ln((2*d+2*d^(1/2)
*(c*x^2+d)^(1/2))/x)+a*(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.86

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = \left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,adx^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx}, \frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\cdots}{\cdots}\right)\cdots}{2\,\cdots}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((
b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + (2*a*
d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

Sympy [A] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{a\sqrt{c}x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
a*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x
) - b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt
(c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2
 - d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/sqrt(d))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$$
$$=\frac{1}{2}\left(\frac{2\sqrt{cx^2+d}a\text{sgn}(x)}{c}+\frac{(bc\text{sgn}(x)+2\,ad\text{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{c\sqrt{-d}}-\frac{\sqrt{cx^2+d}b\text{sgn}(x)}{cx^2}\right)c$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(c*x^2 + d)*a*sgn(x)/c + (b*c*sgn(x) + 2*a*d*sgn(x))*arctan(sqr
t(c*x^2 + d)/sqrt(-d))/(c*sqrt(-d)) - sqrt(c*x^2 + d)*b*sgn(x)/(c*x^2))*c
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.14

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx=a\,x\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\,\sqrt{d}}+\frac{a\,\sqrt{d}\,\text{asin}\left(\frac{\sqrt{d}\,1\text{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1\text{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

input
```
int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

output
```
a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)
^(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x
))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$$
$$=\frac{2\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\,x^2+d}\,bd+2\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)ad\,x^2+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)bc\,x^2-2}{2d\,x^2}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2),x)
```

output
```
(2*sqrt(c*x**2 + d)*a*d*x**2 - sqrt(c*x**2 + d)*b*d + 2*sqrt(d)*log((sqrt(
c*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*a*d*x**2 + sqrt(d)*log((sqrt(c
*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c*x**2 - 2*sqrt(d)*log((sqrt(
c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*a*d*x**2 - sqrt(d)*log((sqrt(c
*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c*x**2)/(2*d*x**2)
```

3.151 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx} + \frac{c(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}$$

output
```
1/8*(-4*a*d+b*c)*(c+d/x^2)^(1/2)/d/x-1/4*b*(c+d/x^2)^(3/2)/d/x+1/8*c*(-4*a
*d+b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(2bd+bcx^2+4adx^2)+\frac{c(bc-4ad)x^4\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d+cx^2}}\right)}{8d^{3/2}x^3}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(-(Sqrt[d]*(2*b*d + b*c*x^2 + 4*a*d*x^2)) + (c*(b*c - 4*a
*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/(8*d^(3/2)*x^3
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 858, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(bc-4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}dx}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{858}$$

$$\frac{(bc-4ad)\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{211}$$

$$\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-4ad)\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-4ad)\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)}{4d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

output
```
-1/4*(b*(c + d/x^2)^(3/2))/(d*x) + ((b*c - 4*a*d)*(Sqrt[c + d/x^2]/(2*x) +
 (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])))/(4*d)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{(4ad\,x^2+bc\,x^2+2bd)\sqrt{\frac{c\,x^2+d}{x^2}}}{8x^3d}-\frac{c(4ad-cb)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{\frac{c\,x^2+d}{x^2}}\,x}{8d^{\frac{3}{2}}\sqrt{c\,x^2+d}}$
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(4d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)ac\,x^4-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)b\,c^2x^4-4\sqrt{c\,x^2+d}\,acd\,x^4+\sqrt{c\,x^2+d}\,b\,c^2x^4+4(c\,x^2+d)^{\frac{3}{2}}c}{8x^3\sqrt{c\,x^2+d}\,d^2}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*a*d*x^2+b*c*x^2+2*b*d)/x^3/d*((c*x^2+d)/x^2)^(1/2)-1/8*c*(4*a*d-b*
c)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(
c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.07

$$\int\frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$
$$=\left[-\frac{(bc^2-4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2+(bcd+4\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3},\right.$$
$$\left.-\frac{(bc^2-4\,acd)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)+\left(2\,bd^2+(bcd+4\,ad^2)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{8\,d^2x^3}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2
+ d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)
*x*sqrt((c*x^2 + d)/x^2)/d) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2))/(d^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2\sqrt{d}} - \frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} - \frac{bd}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

output
```
-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*
sqrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*s
qrt(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*
d/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(75) = 150$.

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}}\right)a - \frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)
*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((s
qrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c
 + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 -
2*(c + d/x^2)*d^2*x^2 + d^3))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = -\frac{\frac{\left(bc^3\operatorname{sgn}(x)-4\,ac^2d\operatorname{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\operatorname{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\operatorname{sgn}(x)+\sqrt{cx^2+d}bc^3d\operatorname{sgn}(x)-4\sqrt{cx^2+d}ac^2d^2\operatorname{sgn}(}{c^2dx^4}}{8\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(
sqrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*
d*sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sg
n(x))/(c^2*d*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx = \frac{-4\sqrt{c\,x^2+d}\,a\,d^2x^2-\sqrt{c\,x^2+d}\,bcd\,x^2-2\sqrt{c\,x^2+d}\,b\,d^2+4\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)acd\,x^4-\sqrt{d}\log\left(\right.}{8d^2x^4}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x)
```

output
```
( - 4*sqrt(c*x**2 + d)*a*d**2*x**2 - sqrt(c*x**2 + d)*b*c*d*x**2 - 2*sqrt(
c*x**2 + d)*b*d**2 + 4*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d)
)/sqrt(d))*a*c*d*x**4 - sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d
))/sqrt(d))*b*c**2*x**4 - 4*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sq
rt(d))/sqrt(d))*a*c*d*x**4 + sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + s
qrt(d))/sqrt(d))*b*c**2*x**4)/(8*d**2*x**4)
```

3.152 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=\frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{c^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{16d^{5/2}}$$

output
```
1/8*(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d/x^3-1/6*b*(c+d/x^2)^(3/2)/d/x^3+1/16*c*
(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d^2/x-1/16*c^2*(-2*a*d+b*c)*arctanh(d^(1/2)/(
c+d/x^2)^(1/2)/x)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}\left(-8bd^2-2bcdx^2-12ad^2x^2+3bc^2x^4-6acdx^4\right)}{48d^2x^5}-\frac{c^2(bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x\,\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{16d^{5/2}\sqrt{d+cx^2}}$$

input
```
Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

output
```
(Sqrt[c + d/x^2]*(-8*b*d^2 - 2*b*c*d*x^2 - 12*a*d^2*x^2 + 3*b*c^2*x^4 - 6*
a*c*d*x^4))/(48*d^2*x^5) - (c^2*(b*c - 2*a*d)*Sqrt[c + d/x^2]*x*ArcTanh[Sq
rt[d + c*x^2]/Sqrt[d]])/(16*d^(5/2)*Sqrt[d + c*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 858, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(bc-2ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}dx}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{(bc-2ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x}}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-2ad)\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)}{2d}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

input
```
Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

output
```
-1/6*(b*(c + d/x^2)^(3/2))/(d*x^3) + ((b*c - 2*a*d)*(Sqrt[c + d/x^2]/(4*x^
3) + (c*(Sqrt[c + d/x^2]/(2*d*x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
)/(2*d^(3/2))))/4))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(6acd\,x^{4}-3b\,c^{2}x^{4}+12a\,d^{2}x^{2}+2bcd\,x^{2}+8b\,d^{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48x^{5}d^{2}}+\frac{c^{2}\left(2ad-cb\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(6d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{2}x^{6}-3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{3}x^{6}-6\sqrt{c\,x^{2}+d}\,a\,c^{2}d\,x^{6}+3\sqrt{c\,x^{2}+d}\,b\,c^{3}x^{6}+6\left(c\,x^{2}+d\right)}{48x^{5}\sqrt{c\,x^{2}+d}\,d^{3}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(6*a*c*d*x^4-3*b*c^2*x^4+12*a*d^2*x^2+2*b*c*d*x^2+8*b*d^2)/x^5/d^2*(
(c*x^2+d)/x^2)^(1/2)+1/16*c^2*(2*a*d-b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2
+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$= \left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}x^5\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3\,(bc^2d-2\,acd^2)x^4-8\,bd^3-2\,(bcd^2+6\,ad^3)x}{96\,d^3x^5}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((
c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2
*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3
 - 2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + (3
*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*
x^2 + d)/x^2))/(d^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(107) = 214$.

Time = 13.01 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{3}{2}}} -\frac{ad}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}} -\frac{5b\sqrt{c}}{24x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{5}{2}}}-\frac{bd}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

output
```
-a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/
(c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(
c)*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))
) + b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqr
t(1 + d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d
/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$= -\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a$$
$$+\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5-8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3d^2x^6-3\left(c+\frac{d}{x^2}\right)^2d^3x^4+3\left(c+\frac{d}{x^2}\right)d^4x^2-d^5}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c +
d/x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c
 + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c +
d/x^2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c
^3*d^2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)
*d^4*x^2 - d^5))*b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$=\frac{1}{48}c^3\left(\frac{3\,(bc\text{sgn}(x)-2\,ad\text{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{c\sqrt{-d}d^2}+\frac{3\,(cx^2+d)^{\frac{5}{2}}bc\text{sgn}(x)-6\,(cx^2+d)^{\frac{5}{2}}ad\text{sgn}(x)-8\,(\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/48*c^3*(3*(b*c*sgn(x) - 2*a*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(
c*sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d^2*sg
n(x) + 6*sqrt(c*x^2 + d)*a*d^3*sgn(x))/(c^4*d^2*x^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$
$$= \frac{-6\sqrt{c\,x^2+d}\,ac\,d^2x^4-12\sqrt{c\,x^2+d}\,a\,d^3x^2+3\sqrt{c\,x^2+d}\,b\,c^2d\,x^4-2\sqrt{c\,x^2+d}\,bc\,d^2x^2-8\sqrt{c\,x^2+d}\,b\,d^3}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x)
```

output
```
( - 6*sqrt(c*x**2 + d)*a*c*d**2*x**4 - 12*sqrt(c*x**2 + d)*a*d**3*x**2 + 3
*sqrt(c*x**2 + d)*b*c**2*d*x**4 - 2*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 8*sqr
t(c*x**2 + d)*b*d**3 - 6*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(
d))/sqrt(d))*a*c**2*d*x**6 + 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x -
 sqrt(d))/sqrt(d))*b*c**3*x**6 + 6*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)
*x + sqrt(d))/sqrt(d))*a*c**2*d*x**6 - 3*sqrt(d)*log((sqrt(c*x**2 + d) + s
qrt(c)*x + sqrt(d))/sqrt(d))*b*c**3*x**6)/(48*d**3*x**6)
```

3.153 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{d(10bc+ad)\sqrt{c+\frac{d}{x^2}}x^2}{16c} + \frac{1}{8}(2bc+ad)\sqrt{c+\frac{d}{x^2}}x^4 + \frac{1}{6}a\left(c+\frac{d}{x^2}\right)^{3/2} x^6 + \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}$$

output
```
1/16*d*(a*d+10*b*c)*(c+d/x^2)^(1/2)*x^2/c+1/8*(a*d+2*b*c)*(c+d/x^2)^(1/2)*
x^4+1/6*a*(c+d/x^2)^(3/2)*x^6+1/16*d^2*(-a*d+6*b*c)*arctanh((c+d/x^2)^(1/2
)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x\left(\sqrt{c}x\sqrt{d+cx^2}(6bc(5d+2cx^2)+a(3d^2+14cdx^2+8c^2x^4))+3d^2(-6bc+ad)\text{l}\right.}{48c^{3/2}\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

output
```
(Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[d + c*x^2]*(6*b*c*(5*d + 2*c*x^2) + a*(
3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) + 3*d^2*(-6*b*c + a*d)*Log[-(Sqrt[c]*x) +
 Sqrt[d + c*x^2]]))/(48*c^(3/2)*Sqrt[d + c*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6d\frac{1}{x^2}}{6c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\int \sqrt{c+\frac{d}{x^2}}x^4d\frac{1}{x^2}-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-x^2\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}-x^2\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(6bc-ad)\left(\frac{3}{4}d\left(x^2\left(-\sqrt{c+\frac{d}{x^2}}\right)-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{1}{2}x^4\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{6c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^6)/(3*c) - ((6*b*c - a*d)*(-1/2*((c + d/x^2)^(3/2)
*x^4) + (3*d*(-(Sqrt[c + d/x^2]*x^2) - (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
)/Sqrt[c]))/4))/(6*c))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

method	result
risch	$\frac{x^{2}\left(8 a \,c^{2} x^{4}+14 a d \,x^{2} c +12 b \,c^{2} x^{2}+3 a \,d^{2}+30 d b c \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{48 c}-\frac{d^{2}\left(a d -6 c b \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{16 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}}$
default	$\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(8 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{5}{2}} a x -2 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{3}{2}} a d x +12 c^{\frac{3}{2}} \left(c \,x^{2}+d \right)^{\frac{3}{2}} b x -3 \sqrt{c}\, \sqrt{c \,x^{2}+d}\, a \,d^{2} x +18 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}\, b d x -3 \ln \left(\sqrt{c}\, x +\right.}{48 \left(c \,x^{2}+d \right)^{\frac{3}{2}} c^{\frac{3}{2}}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48/c*x^2*(8*a*c^2*x^4+14*a*c*d*x^2+12*b*c^2*x^2+3*a*d^2+30*b*c*d)*((c*x^
2+d)/x^2)^(1/2)-1/16*d^2*(a*d-6*b*c)/c^(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.11

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \left[-\frac{3\,(6\,bcd^2-ad^3)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{96\,c^2},\ -\frac{3\,(6\,bcd^2-ad^3)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(8\,ac^3x^6+2\,(6\,bc^3+7\,ac^2d)x^4+3\,(10\,bc^2d+acd^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{48\,c^2}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c
*x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10
*b*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 -
 a*d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) -
(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.

Time = 68.92 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.20

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}x^5}{24\sqrt{\frac{cx^2}{d}+1}}+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3bc\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{bd^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3bd^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

output
```
a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(
c*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x
/(16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2))
 + b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt
(c*x**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt
(c*x**2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(95) = 190$.

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.09

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a - \frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

output
```
1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
 + d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*
c^3 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^
2) + sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*
d^2)/((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{1}{48}\left(2\left(4\,acx^2\mathrm{sgn}(x)+\frac{6\,bc^5\mathrm{sgn}(x)+7\,ac^4d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\,(10\,bc^4d\mathrm{sgn}(x)+ac^3d^2\mathrm{sgn}(x)}{c^4}\right. - \frac{(6\,bcd^2\mathrm{sgn}(x)-ad^3\mathrm{sgn}(x))\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{16\,c^{\frac{3}{2}}} + \frac{(6\,bcd^2\log(|d|)-ad^3\log(|d|))\mathrm{sgn}(x)}{32\,c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

output
```
1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
 3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6
*b*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(
3/2) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{3/2}}{6} + \frac{5\,b\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8} + \frac{a\,x^6\,\left(c+\frac{d}{x^2}\right)^{5/2}}{16\,c} + \frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}} - \frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16} - \frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8} + \frac{a\,d^3\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{16\,c^{3/2}}$$

input
```
int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c +
d/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16
*c^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*
x^6*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^5\,dx = \frac{8\sqrt{c\,x^2+d}\,a\,c^3x^5+14\sqrt{c\,x^2+d}\,a\,c^2d\,x^3+3\sqrt{c\,x^2+d}\,ac\,d^2x+12\sqrt{c\,x^2+d}\,b\,c^3x^3+30\sqrt{}}{48c^2}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x)
```

output
```
(8*sqrt(c*x**2 + d)*a*c**3*x**5 + 14*sqrt(c*x**2 + d)*a*c**2*d*x**3 + 3*sq
rt(c*x**2 + d)*a*c*d**2*x + 12*sqrt(c*x**2 + d)*b*c**3*x**3 + 30*sqrt(c*x*
*2 + d)*b*c**2*d*x - 3*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))
*a*d**3 + 18*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d**2)
/(48*c**2)
```

3.154 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = -bd\sqrt{c+\frac{d}{x^2}} + \frac{1}{8}(4bc+3ad)\sqrt{c+\frac{d}{x^2}}x^2 + \frac{1}{4}a\left(c+\frac{d}{x^2}\right)^{3/2} x^4 + \frac{3d(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

output
```
-b*d*(c+d/x^2)^(1/2)+1/8*(3*a*d+4*b*c)*(c+d/x^2)^(1/2)*x^2+1/4*a*(c+d/x^2)
^(3/2)*x^4+3/8*d*(a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^3 \, dx = \frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(-8bd+4bcx^2 + 5adx^2+2acx^4+\frac{6d(4bc+ad)x\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{c}\sqrt{d+cx^2}}\right)$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

output
```
(Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (6*d*(4*b*c
 + a*d)*x*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/(Sqrt[c]*Sqrt
[d + c*x^2])))/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\left(\frac{2c\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}-\frac{(ad+4bc)\left(\frac{3}{2}d\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)-x^2\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{4c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^4)/(2*c) - ((4*b*c + a*d)*(-((c + d/x^2)^(3/2)*x^2
) + (3*d*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]))
/2))/(4*c))/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\left(2 a c \,x^{4}+5 a d \,x^{2}+4 b c \,x^{2}-8 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{8}+\frac{3 d \left(a d +4 c b \right) \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{8 \sqrt{c}\, \sqrt{c \,x^{2}+d}}$
default	$\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{2} \left(8 c^{\frac{3}{2}} \left(c \,x^{2}+d \right)^{\frac{3}{2}} b \,x^{2}+12 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}\, b d \,x^{2}+2 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{3}{2}} a d \,x^{2}-8 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{5}{2}} b +3 \sqrt{c}\, \sqrt{c \,x^{2}+d}\, a \,d^{2} x^{2}+3 \ln \left(\sqrt{c}\, x \cdots\right.\right.}{8 \left(c \,x^{2}+d \right)^{\frac{3}{2}} d \sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a*c*x^4+5*a*d*x^2+4*b*c*x^2-8*b*d)*((c*x^2+d)/x^2)^(1/2)+3/8*d*(a*d
+4*b*c)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/c^(1/2)*((c*x^2+d)/x^2)^(1/2)*x/(c*x
^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.07

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \left[\frac{3\,(4\,bcd+ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,a}{16\,c}, -\frac{3\,(4\,bcd+ad^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(2\,ac^2x^4-8\,bcd+(4\,bc^2+5\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{8\,c}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

output
```
[1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqr
t((c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*
x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2
 + 5*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(90) = 180$.

Time = 85.70 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.20

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx=\frac{ac^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3ac\sqrt{d}x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8\sqrt{c}}+\frac{3b\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2}+\frac{bc\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{bc\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{bd^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

output
```
a*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*c*sqrt(d)*x**3/(8*sqrt(c*
x**2/d + 1)) + a*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + a*d**(3/2)*x/(8*sqrt(c*
x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c)) + 3*b*sqrt(c)
*d*asinh(sqrt(c)*x/sqrt(d))/2 + b*c*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 - b*c*s
qrt(d)*x/sqrt(c*x**2/d + 1) - b*d**(3/2)/(x*sqrt(c*x**2/d + 1))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(80) = 160.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx=-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-3\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2-2\left(c+\frac{d}{x^2}\right)c+c^2}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2-3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}d\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

output
```
-1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^
2)^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c
)*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(
c + d/x^2)*d)*b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{2\,b\sqrt{c}d^2\mathrm{sgn}(x)}{\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d} + \frac{1}{8}\left(2\,acx^2\mathrm{sgn}(x)+\frac{4\,bc^3\mathrm{sgn}(x)+5\,ac^2d\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}x - \frac{3\,(4\,bcd\mathrm{sgn}(x)+ad^2\mathrm{sgn}(x))\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{16\,\sqrt{c}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

output
```
2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*
x^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x -
3/16*(4*b*c*d*sgn(x) + a*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/
sqrt(c)
```

Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^3\,dx = \frac{5\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8} - b\,d\,\sqrt{c+\frac{d}{x^2}} + \frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2} + \frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}} - \frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8} + \frac{b\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

input
```
int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*ata
nh((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2
)))/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(
1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.47

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^3\,dx = \frac{2\sqrt{c\,x^2+d}\,a\,c^2x^4+5\sqrt{c\,x^2+d}\,acd\,x^2+4\sqrt{c\,x^2+d}\,b\,c^2x^2-8\sqrt{c\,x^2+d}\,bcd+3\sqrt{c}\log\Big(\sqrt{}}{8cx}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x)
```

output
```
(2*sqrt(c*x**2 + d)*a*c**2*x**4 + 5*sqrt(c*x**2 + d)*a*c*d*x**2 + 4*sqrt(c
*x**2 + d)*b*c**2*x**2 - 8*sqrt(c*x**2 + d)*b*c*d + 3*sqrt(c)*log((sqrt(c*
x**2 + d) + sqrt(c)*x)/sqrt(d))*a*d**2*x + 12*sqrt(c)*log((sqrt(c*x**2 + d
) + sqrt(c)*x)/sqrt(d))*b*c*d*x - sqrt(c)*a*d**2*x - 9*sqrt(c)*b*c*d*x)/(8
*c*x)
```

3.155 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x\,dx$

Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = -\left((bc+ad)\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} +\frac{1}{2}ac\sqrt{c+\frac{d}{x^2}}x^2+\frac{1}{2}\sqrt{c}(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-(a*d+b*c)*(c+d/x^2)^(1/2)-1/3*b*(c+d/x^2)^(3/2)+1/2*a*c*(c+d/x^2)^(1/2)*x
^2+1/2*c^(1/2)*(3*a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int \left(a+\frac{b}{x^2}\right) \left(c +\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-6adx^2+3acx^4-2b(d+4cx^2)+\frac{6\sqrt{c}(2bc+3ad)x^3\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{\sqrt{d+cx^2}}\right)}{6x^2}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

output
```
(Sqrt[c + d/x^2]*(-6*a*d*x^2 + 3*a*c*x^4 - 2*b*(d + 4*c*x^2) + (6*Sqrt[c]*
(2*b*c + 3*a*d)*x^3*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/Sqr
t[d + c*x^2]))/(6*x^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {948, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(3ad+2bc)\left(c\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{2c}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

output
```
((a*(c + d/x^2)^(5/2)*x^2)/c - ((2*b*c + 3*a*d)*((2*(c + d/x^2)^(3/2))/3 +
 c*(2*Sqrt[c + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])))/(2*c
))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\left(3 a c \,x^{4}-6 a d \,x^{2}-8 b c \,x^{2}-2 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{6 x^{2}}+\frac{\left(3 a d +2 c b \right) \sqrt{c}\, \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{2 \sqrt{c \,x^{2}+d}}$
default	$\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(4 c^{\frac{5}{2}} \left(c \,x^{2}+d \right)^{\frac{3}{2}} b \,x^{4}+6 c^{\frac{5}{2}} \sqrt{c \,x^{2}+d}\, b d \,x^{4}+6 c^{\frac{3}{2}} \left(c \,x^{2}+d \right)^{\frac{3}{2}} a d \,x^{4}-4 c^{\frac{3}{2}} \left(c \,x^{2}+d \right)^{\frac{5}{2}} b \,x^{2}+9 c^{\frac{3}{2}} \sqrt{c \,x^{2}+d}\, a \,d^{2} x^{4}-6 \sqrt{c}\, \left(c \,x^{2}+\right.\right.}{6 \left(c \,x^{2}+d \right)^{\frac{3}{2}} d^{2} \sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*a*c*x^4-6*a*d*x^2-8*b*c*x^2-2*b*d)/x^2*((c*x^2+d)/x^2)^(1/2)+1/2*(3
*a*d+2*b*c)*c^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2)*x/
(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}x^2\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\,(3\,acx^4-2\,(4\,bc+3\,ad)x^2-\right.$$

$$\left.-\frac{3\,(2\,bc+3\,ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd)\sqrt{\frac{cx^2+d}{x^2}}}{6\,x^2}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c
*x^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*
x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^
2 - 2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

Sympy [A] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.01

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{3a\sqrt{c}d\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2} + \frac{ac\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2} - \frac{ac\sqrt{d}x}{\sqrt{\frac{cx^2}{d}+1}} - \frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}} + bc^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) - \frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}} + bd\left(\begin{cases} -\frac{\sqrt{c}}{2x^2} & \text{for } d=0 \\ -\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

output
```
3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1
)/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1))
 + b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d +
 1)) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**
2), Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}cx^2 - 3\sqrt{c}d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right) - 4\sqrt{c+\frac{d}{x^2}}d\right)a - \frac{1}{6}\left(3c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right) + 2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} + 6\sqrt{c+\frac{d}{x^2}}c\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*lo
g((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)
^(3/2) + 6*sqrt(c + d/x^2)*c)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(75) = 150$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.42

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{1}{2}\sqrt{cx^2+d}acx\,\mathrm{sgn}(x) - \frac{1}{4}\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x) + 3\,a\sqrt{c}d\,\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right) + \frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4 bc^{\frac{3}{2}}d\,\mathrm{sgn}(x) + 3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4 a\sqrt{c}d^2\mathrm{sgn}(x) - 6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2 bc^{\frac{3}{2}}d^2\mathrm{s}}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right.}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d
*sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c
*x^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt
(c)*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) -
6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*s
gn(x) + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right) - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3} - a\,d\sqrt{c+\frac{d}{x^2}} - b\,c\sqrt{c+\frac{d}{x^2}} + \frac{3\,a\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2} + \frac{a\,c\,x^2\sqrt{c+\frac{d}{x^2}}}{2}$$

input
```
int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d
*(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x
^2)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = \frac{6\sqrt{c\,x^2+d}\,ac\,x^4-12\sqrt{c\,x^2+d}\,ad\,x^2-16\sqrt{c\,x^2+d}\,bc\,x^2-4\sqrt{c\,x^2+d}\,bd+18\sqrt{c}\log\left(\frac{\sqrt{c}\right.}{12x^3}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x)
```

output
```
(6*sqrt(c*x**2 + d)*a*c*x**4 - 12*sqrt(c*x**2 + d)*a*d*x**2 - 16*sqrt(c*x*
*2 + d)*b*c*x**2 - 4*sqrt(c*x**2 + d)*b*d + 18*sqrt(c)*log((sqrt(c*x**2 +
d) + sqrt(c)*x)/sqrt(d))*a*d*x**3 + 12*sqrt(c)*log((sqrt(c*x**2 + d) + sqr
t(c)*x)/sqrt(d))*b*c*x**3 + 5*sqrt(c)*a*d*x**3)/(12*x**3)
```

3.156 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$

output
```
-a*c*(c+d/x^2)^(1/2)-1/3*a*(c+d/x^2)^(3/2)-1/5*b*(c+d/x^2)^(5/2)/d+a*c^(3/
2)*arctanh((c+d/x^2)^(1/2)/c^(1/2))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{3b(d+cx^2)^2}{d} - 5ax^2(d+4cx^2) - \frac{15ac^{3/2}x^5\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{d+cx^2}}\right)}{15x^4}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

output
```
(Sqrt[c + d/x^2]*((-3*b*(d + c*x^2)^2)/d - 5*a*x^2*(d + 4*c*x^2) - (15*a*c
^(3/2)*x^5*Log[-(Sqrt[c]*x) + Sqrt[d + c*x^2]])/Sqrt[d + c*x^2]))/(15*x^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-a\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x^2}+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(-a\left(c\left(c\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-a\left(c\left(\frac{2c\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}+2\sqrt{c+\frac{d}{x^2}}\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(-a\left(c\left(2\sqrt{c+\frac{d}{x^2}}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+\frac{d}{x^2}\right)^{3/2}\right)-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

output
```
((-2*b*(c + d/x^2)^(5/2))/(5*d) - a*((2*(c + d/x^2)^(3/2))/3 + c*(2*Sqrt[c
 + d/x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{\left(20 a c d \,x^{4}+3 b \,c^{2} x^{4}+5 a \,d^{2} x^{2}+6 b c d \,x^{2}+3 b \,d^{2}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{15 x^{4} d}+\frac{a \,c^{\frac{3}{2}} \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{\sqrt{c \,x^{2}+d}}$
default	$\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(10 c^{\frac{5}{2}} \left(c \,x^{2}+d \right)^{\frac{3}{2}} a \,x^{6}+15 c^{\frac{5}{2}} \sqrt{c \,x^{2}+d}\, a d \,x^{6}-10 c^{\frac{3}{2}} \left(c \,x^{2}+d \right)^{\frac{5}{2}} a \,x^{4}+15 \ln \left(\sqrt{c}\, x +\sqrt{c \,x^{2}+d}\right) a \,c^{2} d^{2} x^{5}-5 \sqrt{c}\, \left(c \,x^{2}+d \right)^{\frac{5}{2}} a c}{15 x^{2} \left(c \,x^{2}+d \right)^{\frac{3}{2}} d^{2} \sqrt{c}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(20*a*c*d*x^4+3*b*c^2*x^4+5*a*d^2*x^2+6*b*c*d*x^2+3*b*d^2)/x^4/d*((c
*x^2+d)/x^2)^(1/2)+a*c^(3/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)
^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.80

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,((3\,bc^2+20\,acd)x^4+3\,bc}{30\,dx^4}\right.$$
$$\left.-\frac{15\,a\sqrt{-c}cdx^4\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+((3\,bc^2+20\,acd)x^4+3\,bd^2+(6\,bcd+5\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{15\,dx^4}\right]$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^
2) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(
-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3
*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

Sympy [A] (verification not implemented)

Time = 15.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx=\frac{\begin{cases} -\frac{2ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2ac\sqrt{c+\frac{d}{x^2}}-\frac{2a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d} & \text{for}\ d\neq 0\\ -ac^{\frac{3}{2}}\log\left(-\frac{bc^{\frac{3}{2}}}{x^2}\right)-\frac{bc^{\frac{3}{2}}}{x^2} & \text{otherwise}\end{cases}}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

output
```
Piecewise((-2*a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*a*c*sqrt
(c + d/x**2) - 2*a*(c + d/x**2)**(3/2)/3 - 2*b*(c + d/x**2)**(5/2)/(5*d),
Ne(d, 0)), (-a*c**(3/2)*log(-b*c**(3/2)/x**2) - b*c**(3/2)/x**2, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx=-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}c\right)a$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c)
)/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c
)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(60) = 120$.

Time = 0.77 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\mathrm{sgn}(x)$$
$$+\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\mathrm{sgn}(x)+30\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d\mathrm{sgn}(x)-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}d^2\mathrm{s}\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
-1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqr
t(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2
+ d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*
d^2*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110
*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sq
rt(c*x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3
/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = a\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)$$
$$-\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{3}-a\,c\,\sqrt{c+\frac{d}{x^2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)^2}{5\,d\,x^4}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
```

output
```
a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c
*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx = \frac{-20\sqrt{c\,x^2+d}\,acd\,x^4-5\sqrt{c\,x^2+d}\,a\,d^2x^2-3\sqrt{c\,x^2+d}\,b\,c^2x^4-6\sqrt{c\,x^2+d}\,b}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x)
```

output
```
( - 20*sqrt(c*x**2 + d)*a*c*d*x**4 - 5*sqrt(c*x**2 + d)*a*d**2*x**2 - 3*sq
rt(c*x**2 + d)*b*c**2*x**4 - 6*sqrt(c*x**2 + d)*b*c*d*x**2 - 3*sqrt(c*x**2
 + d)*b*d**2 + 15*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*c*
d*x**5 + 8*sqrt(c)*a*c*d*x**5 - 3*sqrt(c)*b*c**2*x**5)/(15*d*x**5)
```

3.157 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=\frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

output
```
1/5*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^2-1/7*b*(c+d/x^2)^(7/2)/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}\,(d+cx^2)^2\,(5bd-2bcx^2+7adx^2)}{35d^2x^6}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

output
```
-1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2
*x^6)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d}+\frac{(ad-bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

output
```
((2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (2*b*(c + d/x^2)^(7/2))/(7*d^
2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

method	result	size
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(7 a d\,x^{2}-2 b c\,x^{2}+5 b d\right) \left(c\,x^{2}+d\right)}{35 d^{2} x^{4}}$	48
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(7 a d\,x^{2}-2 b c\,x^{2}+5 b d\right) \left(c\,x^{2}+d\right)}{35 d^{2} x^{4}}$	48
orering	$-\frac{\left(7 a d\,x^{2}-2 b c\,x^{2}+5 b d\right) \left(c\,x^{2}+d\right) \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{35 d^{2} \left(a\,x^{2}+b\right) x^{2}}$	60
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(7 a\,c^{2} d\,x^{6}-2 b\,c^{3} x^{6}+14 a c\,d^{2} x^{4}+b\,c^{2} d\,x^{4}+7 a\,d^{3} x^{2}+8 b c\,d^{2} x^{2}+5 b\,d^{3}\right)}{35 x^{6} d^{2}}$	86
trager	$-\frac{\left(7 a\,c^{2} d\,x^{6}-2 b\,c^{3} x^{6}+14 a c\,d^{2} x^{4}+b\,c^{2} d\,x^{4}+7 a\,d^{3} x^{2}+8 b c\,d^{2} x^{2}+5 b\,d^{3}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35 x^{6} d^{2}}$	90

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{\left((2\,bc^3-7\,ac^2d)x^6-(bc^2d+14\,acd^2)x^4-5\,bd^3-(8\,bcd^2+7\,ad^3)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,d^2x^6}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (
8*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.11

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{x^2} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

output
```
-a*c*Piecewise((2*(c + d/x**2)**(3/2)/(3*d), Ne(d, 0)), (sqrt(c)/x**2, Tru
e))/2 - a*d*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5
)/d**2, Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - b*c*Piecewise((2*(-c*(c +
 d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2, Ne(d, 0)), (sqrt(c)/(2*x*
*4), True))/2 - b*d*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/
x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6)
, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = -\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d} - \frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2} - \frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)
^(5/2)*c/d^2)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(38) = 76.

Time = 1.21 (sec) , antiderivative size = 370, normalized size of antiderivative = 8.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}\mathrm{sgn}(x)-7\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
 - sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))
^10*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sg
n(x) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqr
t(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c
*x^2 + d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c
^(7/2)*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x
) + 14*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*
x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a
*c^(5/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

output
```
(2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (
2*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*
c*(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(
c + d/x^2)^(1/2))/(35*d*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx = \frac{-7\sqrt{c\,x^2+d}\,a\,c^2 d\,x^6-14\sqrt{c\,x^2+d}\,ac\,d^2x^4-7\sqrt{c\,x^2+d}\,a\,d^3x^2+2\sqrt{c\,x^2+}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x)
```

output
```
( - 7*sqrt(c*x**2 + d)*a*c**2*d*x**6 - 14*sqrt(c*x**2 + d)*a*c*d**2*x**4 -
 7*sqrt(c*x**2 + d)*a*d**3*x**2 + 2*sqrt(c*x**2 + d)*b*c**3*x**6 - sqrt(c*
x**2 + d)*b*c**2*d*x**4 - 8*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 5*sqrt(c*x**2
 + d)*b*d**3 - 3*sqrt(c)*a*c**2*d*x**7 - 2*sqrt(c)*b*c**3*x**7)/(35*d**2*x
**7)
```

3.158 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx=-\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

output
```
-1/5*c*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^3+1/7*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^3
-1/9*b*(c+d/x^2)^(9/2)/d^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(9adx^2(-5d+2cx^2)+b(-35d^2+20cdx^2-8c^2x^4)\right)}{315d^3x^8}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 +
20*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \ 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{d^2}+\frac{(ad-2bc)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^2}+\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

output
```
((-2*c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + (2*(2*b*c - a*d)*(c + d/x^
2)^(7/2))/(7*d^3) - (2*b*(c + d/x^2)^(9/2))/(9*d^3))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(18acd\,x^{4}-8b\,c^{2} x^{4}-45a\,d^{2} x^{2}+20bcd\,x^{2}-35b\,d^{2}\right) \left(c\,x^{2}+d\right)}{315 d^{3} x^{6}}$	70
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(18acd\,x^{4}-8b\,c^{2} x^{4}-45a\,d^{2} x^{2}+20bcd\,x^{2}-35b\,d^{2}\right) \left(c\,x^{2}+d\right)}{315 d^{3} x^{6}}$	70
orering	$\frac{\left(18acd\,x^{4}-8b\,c^{2} x^{4}-45a\,d^{2} x^{2}+20bcd\,x^{2}-35b\,d^{2}\right) \left(c\,x^{2}+d\right) \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{315 d^{3} \left(a\,x^{2}+b\right) x^{4}}$	82
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(18a\,c^{3} d\,x^{8}-8b\,c^{4} x^{8}-9a\,c^{2} d^{2} x^{6}+4b\,c^{3} d\,x^{6}-72ac\,d^{3} x^{4}-3b\,c^{2} d^{2} x^{4}-45a\,d^{4} x^{2}-50bc\,d^{3} x^{2}-35b\,d^{4}\right)}{315 x^{8} d^{3}}$	111
trager	$\frac{\left(18a\,c^{3} d\,x^{8}-8b\,c^{4} x^{8}-9a\,c^{2} d^{2} x^{6}+4b\,c^{3} d\,x^{6}-72ac\,d^{3} x^{4}-3b\,c^{2} d^{2} x^{4}-45a\,d^{4} x^{2}-50bc\,d^{3} x^{2}-35b\,d^{4}\right) \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315 x^{8} d^{3}}$	115

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*
d*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b
*d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt
((c*x^2 + d)/x^2)/(d^3*x^8)
```

Sympy [A] (verification not implemented)

Time = 4.70 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.49

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{2x^4} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

output
```
-a*c*Piecewise((2*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2,
 Ne(d, 0)), (sqrt(c)/(2*x**4), True))/2 - a*d*Piecewise((2*(c**2*(c + d/x*
*2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne
(d, 0)), (sqrt(c)/(3*x**6), True))/2 - b*c*Piecewise((2*(c**2*(c + d/x**2)
**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3, Ne(d,
 0)), (sqrt(c)/(3*x**6), True))/2 - b*d*Piecewise((2*(-c**3*(c + d/x**2)**
(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c +
d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = -\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a - \frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*
(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*
c^2/d^3)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(62) = 124.

Time = 1.75 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c
)*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
 d))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/
2)*d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) +
1764*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x
 - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 +
 d))^6*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2
)*d^4*sgn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) -
9*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - s
qrt(c*x^2 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(7/2)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/
((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2} - \frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3} - \frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4} - \frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6} - \frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6} - \frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2} - \frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4} + \frac{4\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

output
```
(2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^
3) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6)
- (10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) -
(a*c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^
4) + (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.59

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx = \frac{18\sqrt{c\,x^2+d}\,a\,c^3d\,x^8-9\sqrt{c\,x^2+d}\,a\,c^2d^2x^6-72\sqrt{c\,x^2+d}\,ac\,d^3x^4-45\sqrt{c\,x^2}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x)
```

output
```
(18*sqrt(c*x**2 + d)*a*c**3*d*x**8 - 9*sqrt(c*x**2 + d)*a*c**2*d**2*x**6 -
 72*sqrt(c*x**2 + d)*a*c*d**3*x**4 - 45*sqrt(c*x**2 + d)*a*d**4*x**2 - 8*s
qrt(c*x**2 + d)*b*c**4*x**8 + 4*sqrt(c*x**2 + d)*b*c**3*d*x**6 - 3*sqrt(c*
x**2 + d)*b*c**2*d**2*x**4 - 50*sqrt(c*x**2 + d)*b*c*d**3*x**2 - 35*sqrt(c
*x**2 + d)*b*d**4 - 18*sqrt(c)*a*c**3*d*x**9 + 8*sqrt(c)*b*c**4*x**9)/(315
*d**3*x**9)
```

3.159 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx=\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

output
```
1/5*c^2*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^4-1/7*c*(-2*a*d+3*b*c)*(c+d/x^2)^(7/2
)/d^4+1/9*(-a*d+3*b*c)*(c+d/x^2)^(9/2)/d^4-1/11*b*(c+d/x^2)^(11/2)/d^4
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(-11adx^2(35d^2-20cdx^2+8c^2x^4)-3b(105d^3-70cd^2x^2\right.}{3465d^4x^{10}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x
^4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4
*x^10)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^3}+\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}-\frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}\right)$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

output
```
((2*c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (2*c*(3*b*c - 2*a*d)*(c +
 d/x^2)^(7/2))/(7*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (2*
b*(c + d/x^2)^(11/2))/(11*d^4))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
gosper	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-220 a c\,d^{2} x^{4}+120 b\,c^{2} d\,x^{4}+385 a\,d^{3} x^{2}-210 b c\,d^{2} x^{2}+315 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465 d^{4} x^{8}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(88 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-220 a c\,d^{2} x^{4}+120 b\,c^{2} d\,x^{4}+385 a\,d^{3} x^{2}-210 b c\,d^{2} x^{2}+315 b\,d^{3}\right) \left(c\,x^{2}+d\right)}{3465 d^{4} x^{8}}$
orering	$-\frac{\left(88 a\,c^{2} d\,x^{6}-48 b\,c^{3} x^{6}-220 a c\,d^{2} x^{4}+120 b\,c^{2} d\,x^{4}+385 a\,d^{3} x^{2}-210 b c\,d^{2} x^{2}+315 b\,d^{3}\right) \left(c\,x^{2}+d\right) \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{3465 d^{4} \left(a\,x^{2}+b\right) x^{6}}$
risch	$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, \left(88 a\,c^{4} d\,x^{10}-48 b\,c^{5} x^{10}-44 a\,c^{3} d^{2} x^{8}+24 b\,c^{4} d\,x^{8}+33 a\,c^{2} d^{3} x^{6}-18 b\,c^{3} d^{2} x^{6}+550 a c\,d^{4} x^{4}+15 b\,c^{2} d^{3} x^{4}+385 a\,d^{5} x^{2}+420 b}{3465 x^{10} d^{4}}$
trager	$-\frac{\left(88 a\,c^{4} d\,x^{10}-48 b\,c^{5} x^{10}-44 a\,c^{3} d^{2} x^{8}+24 b\,c^{4} d\,x^{8}+33 a\,c^{2} d^{3} x^{6}-18 b\,c^{3} d^{2} x^{6}+550 a c\,d^{4} x^{4}+15 b\,c^{2} d^{3} x^{4}+385 a\,d^{5} x^{2}+420 b c\,d^{4} x^{2}+31}{3465 x^{10} d^{4}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4
+120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^
8
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{(8\,(6\,bc^5-11\,ac^4d)x^{10}-4\,(6\,bc^4d-11\,ac^3d^2)x^8+3\,(6\,bc^3d^2-11\,ac^2d^3)x^6-}{3465\,d^4x^1}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
 3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c
*d^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^1
0)
```

Sympy [A] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{3x^6} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for } d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

output
```
-a*c*Piecewise((2*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5
+ (c + d/x**2)**(7/2)/7)/d**3, Ne(d, 0)), (sqrt(c)/(3*x**6), True))/2 - a*
d*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5
 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)), (sq
rt(c)/(4*x**8), True))/2 - b*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 +
 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**
(9/2)/9)/d**4, Ne(d, 0)), (sqrt(c)/(4*x**8), True))/2 - b*d*Piecewise((2*(
c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/
x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5
, Ne(d, 0)), (sqrt(c)/(5*x**10), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = -\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3} - \frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3} + \frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a - \frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4} - \frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4} + \frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4} - \frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/
x^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^
2)^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3
/d^4)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(88) = 176$.

Time = 1.44 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\left(2310\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\mathrm{sgn}(x)+6930\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\mathrm{sgn}\right.}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sq
rt(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c
*x^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*
b*c^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*s
gn(x) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 485
1*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x
- sqrt(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2
+ d))^8*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11
/2)*d^4*sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x)
- 330*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)
*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2
+ d))^2*b*c^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9
/2)*d^7*sgn(x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt
(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4} - \frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3} - \frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6} - \frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8} - \frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8} - \frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}} - \frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4} + \frac{4\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{315\,d^2\,x^2} - \frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6} + \frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4} - \frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

output
```
(16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315
*d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x
^8) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^1
0) - (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(
315*d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2
)^(1/2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7}\,dx = \frac{-88\sqrt{c\,x^2+d}\,a\,c^4d\,x^{10}+44\sqrt{c\,x^2+d}\,a\,c^3d^2x^8-33\sqrt{c\,x^2+d}\,a\,c^2d^3x^6-550$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x)
```

output
```
( - 88*sqrt(c*x**2 + d)*a*c**4*d*x**10 + 44*sqrt(c*x**2 + d)*a*c**3*d**2*x
**8 - 33*sqrt(c*x**2 + d)*a*c**2*d**3*x**6 - 550*sqrt(c*x**2 + d)*a*c*d**4
*x**4 - 385*sqrt(c*x**2 + d)*a*d**5*x**2 + 48*sqrt(c*x**2 + d)*b*c**5*x**1
0 - 24*sqrt(c*x**2 + d)*b*c**4*d*x**8 + 18*sqrt(c*x**2 + d)*b*c**3*d**2*x*
*6 - 15*sqrt(c*x**2 + d)*b*c**2*d**3*x**4 - 420*sqrt(c*x**2 + d)*b*c*d**4*
x**2 - 315*sqrt(c*x**2 + d)*b*d**5 + 88*sqrt(c)*a*c**4*d*x**11 - 48*sqrt(c
)*b*c**5*x**11)/(3465*d**4*x**11)
```

3.160 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=-\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

output
```
-1/5*c^3*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^5+1/7*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(
7/2)/d^5-1/3*c*(-a*d+2*b*c)*(c+d/x^2)^(9/2)/d^5+1/11*(-a*d+4*b*c)*(c+d/x^2
)^(11/2)/d^5-1/13*b*(c+d/x^2)^(13/2)/d^5
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(d+cx^2)^2\left(13adx^2(-105d^3+70cd^2x^2-40c^2dx^4+16c^3x^6)+b(-11\right.}{15015d^5x^{12}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

output
```
(Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c
^2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 +
320*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \ 86$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{9/2}}{d^4}+\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{d^4}-\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}+\frac{c^3(bc$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{2c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{2c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{2c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-a}{3d^5}\right.$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

output
```
((-2*c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (2*c^2*(4*b*c - 3*a*d)*(
c + d/x^2)^(7/2))/(7*d^5) - (2*c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5)
+ (2*(4*b*c - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (2*b*(c + d/x^2)^(13/2))
/(13*d^5))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

method	result
gosper	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} (208a\,c^3 d\,x^8-128b\,c^4 x^8-520a\,c^2 d^2 x^6+320b\,c^3 d\,x^6+910ac\,d^3 x^4-560b\,c^2 d^2 x^4-1365a\,d^4 x^2+840bc\,d^3 x^2-1155b\,d^4) (c\,x \cdots}{15015d^5 x^{10}}$
default	$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} (208a\,c^3 d\,x^8-128b\,c^4 x^8-520a\,c^2 d^2 x^6+320b\,c^3 d\,x^6+910ac\,d^3 x^4-560b\,c^2 d^2 x^4-1365a\,d^4 x^2+840bc\,d^3 x^2-1155b\,d^4) (c\,x \cdots}{15015d^5 x^{10}}$
orering	$\frac{(208a\,c^3 d\,x^8-128b\,c^4 x^8-520a\,c^2 d^2 x^6+320b\,c^3 d\,x^6+910ac\,d^3 x^4-560b\,c^2 d^2 x^4-1365a\,d^4 x^2+840bc\,d^3 x^2-1155b\,d^4) (c\,x^2+d) (a+\frac{b}{x \cdots}}{15015d^5 (a\,x^2+b) x^8}$
risch	$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\, (208a\,c^5 d\,x^{12}-128b\,c^6 x^{12}-104a\,c^4 d^2 x^{10}+64b\,c^5 d\,x^{10}+78a\,c^3 d^3 x^8-48b\,c^4 d^2 x^8-65a\,c^2 d^4 x^6+40b\,c^3 d^3 x^6-1820ac\,d^5 x^4- \cdots}{15015x^{12} d^5}$
trager	$\frac{(208a\,c^5 d\,x^{12}-128b\,c^6 x^{12}-104a\,c^4 d^2 x^{10}+64b\,c^5 d\,x^{10}+78a\,c^3 d^3 x^8-48b\,c^4 d^2 x^8-65a\,c^2 d^4 x^6+40b\,c^3 d^3 x^6-1820ac\,d^5 x^4-35b\,c^2 d^4 x \cdots}{15015x^{12} d^5}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2
*x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*
b*c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{(16\,(8\,bc^6-13\,ac^5d)x^{12}-8\,(8\,bc^5d-13\,ac^4d^2)x^{10}+6\,(8\,bc^4d^2-13\,ac^3d^3)x^8+1155\,bd^6-5\,(8\,bc^3d^3-1}{15015\,d^5x^{12}}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^
10 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13
*a*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a
*d^6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

Sympy [A] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = -\frac{ac\left(\begin{cases}\frac{2\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{4x^8} & \text{otherwise}\end{cases}\right)}{2} - \frac{ad\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2} - \frac{bc\left(\begin{cases}\frac{2\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right)}{d^5} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{5x^{10}} & \text{otherwise}\end{cases}\right)}{2} - \frac{bd\left(\begin{cases}\frac{2\left(-\frac{c^5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+c^4\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}-\frac{10c^3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{10c^2\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}-\frac{5c\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{13}\right)}{d^6} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}}{6x^{12}} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

output
```
-a*c*Piecewise((2*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2
)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4, Ne(d, 0)),
(sqrt(c)/(4*x**8), True))/2 - a*d*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3
 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c +
d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x
**10), True))/2 - b*c*Piecewise((2*(c**4*(c + d/x**2)**(3/2)/3 - 4*c**3*(c
 + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)**(9/
2)/9 + (c + d/x**2)**(11/2)/11)/d**5, Ne(d, 0)), (sqrt(c)/(5*x**10), True)
)/2 - b*d*Piecewise((2*(-c**5*(c + d/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(
5/2) - 10*c**3*(c + d/x**2)**(7/2)/7 + 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c
*(c + d/x**2)**(11/2)/11 + (c + d/x**2)**(13/2)/13)/d**6, Ne(d, 0)), (sqrt
(c)/(6*x**12), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a$$
$$-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-\frac{5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}c}{d^5}+\frac{10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c^2}{d^5}-\frac{8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^3}{d^5}+\frac{3003\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^5}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
 + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155
*(c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2
)^(9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*
c^4/d^5)*b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(114) = 228$.

Time = 2.35 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048
*(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sq
rt(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d)
)^14*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2
)*d^2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(
x) - 28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*
(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2
 + d))^8*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^
(11/2)*d^5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sg
n(x) + 3718*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(s
qrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sq
rt(c*x^2 + d))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))
^2*b*c^(13/2)*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*
d^8*sgn(x) + 8*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)
*x - sqrt(c*x^2 + d))^2 - d)^13
```

Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=\frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{385\,d^2\,x^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^3\,x^2}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{429\,d\,x^8}+\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{3003\,d^2\,x^6}-\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{5005\,d^3\,x^4}+\frac{64\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^4\,x^2}$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)
```

output
```
(16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(1
5015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(
11*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))
/(13*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^
(1/2))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2
*(c + d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^
6) - (16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(
1/2))/(15015*d^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx=\frac{208\sqrt{c\,x^2+d}\,a\,c^5d\,x^{12}-104\sqrt{c\,x^2+d}\,a\,c^4d^2x^{10}+78\sqrt{c\,x^2+d}\,a\,c^3d^3x^8-65}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x)
```

output
```
(208*sqrt(c*x**2 + d)*a*c**5*d*x**12 - 104*sqrt(c*x**2 + d)*a*c**4*d**2*x*
*10 + 78*sqrt(c*x**2 + d)*a*c**3*d**3*x**8 - 65*sqrt(c*x**2 + d)*a*c**2*d*
*4*x**6 - 1820*sqrt(c*x**2 + d)*a*c*d**5*x**4 - 1365*sqrt(c*x**2 + d)*a*d*
*6*x**2 - 128*sqrt(c*x**2 + d)*b*c**6*x**12 + 64*sqrt(c*x**2 + d)*b*c**5*d
*x**10 - 48*sqrt(c*x**2 + d)*b*c**4*d**2*x**8 + 40*sqrt(c*x**2 + d)*b*c**3
*d**3*x**6 - 35*sqrt(c*x**2 + d)*b*c**2*d**4*x**4 - 1470*sqrt(c*x**2 + d)*
b*c*d**5*x**2 - 1155*sqrt(c*x**2 + d)*b*d**6 - 208*sqrt(c)*a*c**5*d*x**13
+ 128*sqrt(c)*b*c**6*x**13)/(15015*d**5*x**13)
```

3.161 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 150

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^5}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^7}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^9}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2}x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^{13}}{13c}$$

output
```
-16/15015*d^3*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^5/c^5+8/3003*d^2*(-8*a*d+1
3*b*c)*(c+d/x^2)^(5/2)*x^7/c^4-2/429*d*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^9
/c^3+1/143*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^11/c^2+1/13*a*(c+d/x^2)^(5/2)
*x^13/c
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2\left(13bc(-16d^3+40cd^2x^2-70c^2dx^4+105c^3x^6)+a(128d^4-320cd^3x\right.}{15015c^5}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*
d*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*
c^3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^{10}dx}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^8dx}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}\right)}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(13bc-8ad)\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}-\frac{6d\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)}{9c}\right)}{11c}\right)(13bc-8ad)}{13c}+\frac{ax^{13}\left(c+\frac{d}{x^2}\right)^{5/2}}{13c}$$

input

```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^13)/(13*c) + ((13*b*c - 8*a*d)*(((c + d/x^2)^(5/2)*
x^11)/(11*c) - (6*d*(((c + d/x^2)^(5/2)*x^9)/(9*c) - (4*d*((-2*d*(c + d/x^
2)^(5/2)*x^5)/(35*c^2) + ((c + d/x^2)^(5/2)*x^7)/(7*c)))/(9*c)))/(11*c)))/
(13*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

method	result
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(1155 a\,x^{8} c^{4}-840 a\,c^{3} d\,x^{6}+1365 b\,c^{4} x^{6}+560 a\,c^{2} d^{2} x^{4}-910 b\,c^{3} d\,x^{4}-320 a c\,d^{3} x^{2}+520 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-208 b c\,d^{3}\right) \left(c\,x\right.}{15015 c^{5}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(1155 a\,x^{8} c^{4}-840 a\,c^{3} d\,x^{6}+1365 b\,c^{4} x^{6}+560 a\,c^{2} d^{2} x^{4}-910 b\,c^{3} d\,x^{4}-320 a c\,d^{3} x^{2}+520 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-208 b c\,d^{3}\right) \left(c\,x\right.}{15015 c^{5}}$
orering	$\frac{\left(1155 a\,x^{8} c^{4}-840 a\,c^{3} d\,x^{6}+1365 b\,c^{4} x^{6}+560 a\,c^{2} d^{2} x^{4}-910 b\,c^{3} d\,x^{4}-320 a c\,d^{3} x^{2}+520 b\,c^{2} d^{2} x^{2}+128 a\,d^{4}-208 b c\,d^{3}\right) \left(c\,x^{2}+d\right) x^{5} \left(a+\frac{b}{x}\right.}{15015 c^{5} \left(a\,x^{2}+b\right)}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(1155 a\,c^{6} x^{12}+1470 a\,c^{5} d\,x^{10}+1365 b\,c^{6} x^{10}+35 a\,c^{4} d^{2} x^{8}+1820 b\,c^{5} d\,x^{8}-40 a\,c^{3} d^{3} x^{6}+65 b\,c^{4} d^{2} x^{6}+48 a\,c^{2} d^{4} x^{4}-78 b\,c^{3} d^{3} x\right.}{15015 c^{5}}$
trager	$\frac{\left(1155 a\,c^{6} x^{12}+1470 a\,c^{5} d\,x^{10}+1365 b\,c^{6} x^{10}+35 a\,c^{4} d^{2} x^{8}+1820 b\,c^{5} d\,x^{8}-40 a\,c^{3} d^{3} x^{6}+65 b\,c^{4} d^{2} x^{6}+48 a\,c^{2} d^{4} x^{4}-78 b\,c^{3} d^{3} x^{4}-64 a c\,d^{5}\right.}{15015 c^{5}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x,method=_RETURNVERBOSE)
```

output
```
1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c
^4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2
+128*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{(1155\,ac^6x^{13}+105\,(13\,bc^6+14\,ac^5d)x^{11}+35\,(52\,bc^5d+ac^4d^2)x^9+5\,(13\,bc^4d^2-8\,ac^3d^3\cdots}{1501\cdots}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

output
```
1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5
*d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3
 - 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 -
8*a*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3351 vs. $2(146) = 292$.

Time = 7.23 (sec) , antiderivative size = 3351, normalized size of antiderivative = 22.34

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

output
```
693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 4
5045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45
045*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*
d**30) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28
*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x
**16*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 +
 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9
009*c**6*d**30) + 315*a*c**9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9
*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d
**19*x**2 + 3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d +
 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x*
*6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 12
95*a*c**8*d**(37/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 756*a*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**2
5*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**2
8*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx = \frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4} + \frac{\left(1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-5460\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}dx^{11}+10010\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}d^2x^9-8580\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^3x^7+3003\left(c+\frac{d}{x^2}\right)\right.}{15015\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

output
```
1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
 + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(
1155*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c +
 d/x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^
(5/2)*d^4*x^5)*a/c^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.17

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{12}\,dx = \frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5} + \frac{1155\left(cx^2+d\right)^{\frac{13}{2}}a\mathrm{sgn}(x)+1365\left(cx^2+d\right)^{\frac{11}{2}}bc\mathrm{sgn}(x)-5460\left(cx^2+d\right)^{\frac{11}{2}}ad\mathrm{sgn}(x)-5005\left(cx^2+d\right)^{\frac{9}{2}}bcd\mathrm{sg}}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

output
```
16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^
2 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2
+ d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^
2 + d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(
c*x^2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 30
03*(c*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{x\,(128\,a\,d^6-208\,b\,c\,d^5)}{15015\,c^5}+\frac{x^{11}\,(1365\,b\,c^6+1470\,a\,d\,c^5)}{15015\,c^5}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^9\,(a\,d+52\,b\,c)}{429\,c}-\frac{d^2\,x^7\,(8\,a\,d-13\,b\,c)}{3003\,c^2}+\frac{2\,d^3\,x^5\,(8\,a\,d-13\,b\,c)}{5005\,c^3}-\frac{8\,d^4\,x^3\,(8\,a\,d-13\,b\,c)}{15015\,c^4}\right)$$

input
```
int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365
*b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c
))/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 1
3*b*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx=\frac{\sqrt{c\,x^2+d}\,(1155a\,c^6x^{12}+1470a\,c^5d\,x^{10}+1365b\,c^6x^{10}+35a\,c^4d^2x^8+1820b\,c^5d\,x^8-40a}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x)
```

output
```
(sqrt(c*x**2 + d)*(1155*a*c**6*x**12 + 1470*a*c**5*d*x**10 + 35*a*c**4*d**
2*x**8 - 40*a*c**3*d**3*x**6 + 48*a*c**2*d**4*x**4 - 64*a*c*d**5*x**2 + 12
8*a*d**6 + 1365*b*c**6*x**10 + 1820*b*c**5*d*x**8 + 65*b*c**4*d**2*x**6 -
78*b*c**3*d**3*x**4 + 104*b*c**2*d**4*x**2 - 208*b*c*d**5))/(15015*c**5)
```

3.162 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx = \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c}$$

output
```
8/3465*d^2*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^5/c^4-4/693*d*(-6*a*d+11*b*c)
*(c+d/x^2)^(5/2)*x^7/c^3+1/99*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^9/c^2+1/11
*a*(c+d/x^2)^(5/2)*x^11/c
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^{10} \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 \left(11bc(8d^2-20cdx^2+35c^2x^4)+3a(-16d^3+40cd^2x^2-70c^2dx^4+\right.}{3465c^4}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
 + 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(11bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^8dx}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(11bc-6ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(11bc-6ad)\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}\right)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}-\frac{4d\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)}{9c}\right)(11bc-6ad)}{11c}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^11)/(11*c) + ((11*b*c - 6*a*d)*(((c + d/x^2)^(5/2)*
x^9)/(9*c) - (4*d*((-2*d*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + ((c + d/x^2)^(5
/2)*x^7)/(7*c)))/(9*c)))/(11*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

method	result
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(315 a\,x^{6} c^{3}-210 a\,c^{2} d\,x^{4}+385 b\,c^{3} x^{4}+120 a c\,d^{2} x^{2}-220 b\,c^{2} d\,x^{2}-48 a\,d^{3}+88 b c\,d^{2}\right) \left(c\,x^{2}+d\right)}{3465 c^{4}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(315 a\,x^{6} c^{3}-210 a\,c^{2} d\,x^{4}+385 b\,c^{3} x^{4}+120 a c\,d^{2} x^{2}-220 b\,c^{2} d\,x^{2}-48 a\,d^{3}+88 b c\,d^{2}\right) \left(c\,x^{2}+d\right)}{3465 c^{4}}$
orering	$\frac{\left(315 a\,x^{6} c^{3}-210 a\,c^{2} d\,x^{4}+385 b\,c^{3} x^{4}+120 a c\,d^{2} x^{2}-220 b\,c^{2} d\,x^{2}-48 a\,d^{3}+88 b c\,d^{2}\right) \left(c\,x^{2}+d\right) x^{5} \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{3465 c^{4} \left(a\,x^{2}+b\right)}$
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(315 a\,c^{5} x^{10}+420 a\,c^{4} d\,x^{8}+385 b\,c^{5} x^{8}+15 a\,c^{3} d^{2} x^{6}+550 b\,c^{4} d\,x^{6}-18 a\,c^{2} d^{3} x^{4}+33 b\,c^{3} d^{2} x^{4}+24 a c\,d^{4} x^{2}-44 b\,c^{2} d^{3} x^{2}-48 a\,d\right.}{3465 c^{4}}$
trager	$\frac{\left(315 a\,c^{5} x^{10}+420 a\,c^{4} d\,x^{8}+385 b\,c^{5} x^{8}+15 a\,c^{3} d^{2} x^{6}+550 b\,c^{4} d\,x^{6}-18 a\,c^{2} d^{3} x^{4}+33 b\,c^{3} d^{2} x^{4}+24 a c\,d^{4} x^{2}-44 b\,c^{2} d^{3} x^{2}-48 a\,d^{5}+88 b c\,d^{4}\right)}{3465 c^{4}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x,method=_RETURNVERBOSE)
```

output
```
1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*
x^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx=\frac{(315\,ac^5x^{11}+35\,(11\,bc^5+12\,ac^4d)x^9+5\,(110\,bc^4d+3\,ac^3d^2)x^7+3\,(11\,bc^3d^2-6\,ac^2d^3)}{3465\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

output
```
1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
 3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 -
 6*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2304 vs. $2(112) = 224$.

Time = 5.27 (sec) , antiderivative size = 2304, normalized size of antiderivative = 19.69

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^{10}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

output
```
315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*
c**5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**1
9*x**2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)
/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1
3860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt
(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*
d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2
)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**
2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*
x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**
12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5
*d**11*x**2 + 315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1
)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 +
13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*
x**2 + 315*c**4*d**12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 138...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3} + \frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)a}{1155\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

output
```
1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x
^2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c +
d/x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)
*d^3*x^5)*a/c^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = -\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4} + \frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1155\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-990\left(cx^2+d\right)^{\frac{7}{2}}bcd\mathrm{sgn}(x)}{3465\,c^4}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

output
```
-8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 +
d)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(
9/2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7
/2)*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^
(5/2)*a*d^3*sgn(x))/c^4
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{x^9\,(385\,b\,c^5+420\,a\,d\,c^4)}{3465\,c^4}-\frac{x\,(48\,a\,d^5-88\,b\,c\,d^4)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,(3\,a\,d+110\,b\,c)}{693\,c}-\frac{d^2\,x^5\,(6\,a\,d-11\,b\,c)}{1155\,c^2}+\frac{4\,d^3\,x^3\,(6\,a\,d-11\,b\,c)}{3465\,c^3}\right)$$

input
```
int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d
^5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(
693*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*
c))/(3465*c^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx=\frac{\sqrt{c\,x^2+d}\,(315a\,c^5x^{10}+420a\,c^4d\,x^8+385b\,c^5x^8+15a\,c^3d^2x^6+550b\,c^4d\,x^6-18a\,c^2d^3x^4}{3465c^4}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x)
```

output
```
(sqrt(c*x**2 + d)*(315*a*c**5*x**10 + 420*a*c**4*d*x**8 + 15*a*c**3*d**2*x
**6 - 18*a*c**2*d**3*x**4 + 24*a*c*d**4*x**2 - 48*a*d**5 + 385*b*c**5*x**8
 + 550*b*c**4*d*x**6 + 33*b*c**3*d**2*x**4 - 44*b*c**2*d**3*x**2 + 88*b*c*
d**4))/(3465*c**4)
```

3.163 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = -\frac{2d(9bc-4ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc-4ad) \left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c}$$

output
```
-2/315*d*(-4*a*d+9*b*c)*(c+d/x^2)^(5/2)*x^5/c^3+1/63*(-4*a*d+9*b*c)*(c+d/x
^2)^(5/2)*x^7/c^2+1/9*a*(c+d/x^2)^(5/2)*x^9/c
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^8 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2 \left(9bc(-2d+5cx^2)+a(8d^2-20cdx^2+35c^2x^4)\right)}{315c^3}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c
*d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(9bc-4ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^6dx}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(9bc-4ad)\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2d\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}\right)}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

$$\Big\downarrow \text{796}$$

$$\frac{\left(\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}}{35c^2}\right)(9bc-4ad)}{9c}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^9)/(9*c) + ((9*b*c - 4*a*d)*((-2*d*(c + d/x^2)^(5/2
)*x^5)/(35*c^2) + ((c + d/x^2)^(5/2)*x^7)/(7*c)))/(9*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(35 a\,c^{2} x^{4}-20 a d\,x^{2} c+45 b\,c^{2} x^{2}+8 a\,d^{2}-18 d b c \right) \left(c\,x^{2}+d \right)}{315 c^{3}}$	67
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(35 a\,c^{2} x^{4}-20 a d\,x^{2} c+45 b\,c^{2} x^{2}+8 a\,d^{2}-18 d b c \right) \left(c\,x^{2}+d \right)}{315 c^{3}}$	67
orering	$\frac{\left(35 a\,c^{2} x^{4}-20 a d\,x^{2} c+45 b\,c^{2} x^{2}+8 a\,d^{2}-18 d b c \right) \left(c\,x^{2}+d \right) x^{5} \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{315 c^{3} \left(a\,x^{2}+b \right)}$	79
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(35 a\,x^{8} c^{4}+50 a\,c^{3} d\,x^{6}+45 b\,c^{4} x^{6}+3 a\,c^{2} d^{2} x^{4}+72 b\,c^{3} d\,x^{4}-4 a c\,d^{3} x^{2}+9 b\,c^{2} d^{2} x^{2}+8 a\,d^{4}-18 b c\,d^{3}\right)}{315 c^{3}}$	106
trager	$\frac{\left(35 a\,x^{8} c^{4}+50 a\,c^{3} d\,x^{6}+45 b\,c^{4} x^{6}+3 a\,c^{2} d^{2} x^{4}+72 b\,c^{3} d\,x^{4}-4 a c\,d^{3} x^{2}+9 b\,c^{2} d^{2} x^{2}+8 a\,d^{4}-18 b c\,d^{3}\right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{315 c^{3}}$	110

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*
a*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\frac{(35\,ac^4x^9+5\,(9\,bc^4+10\,ac^3d)x^7+3\,(24\,bc^3d+ac^2d^2)x^5+(9\,bc^2d^2-4\,acd^3)x^3-2\,(9\,bcd$$
$$315\,c^3$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

output
```
1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2
*d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqr
t((c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1340 vs. 2(78) = 156.

Time = 4.18 (sec) , antiderivative size = 1340, normalized size of antiderivative = 15.95

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\text{Too large to display}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**8,x)
```

output
```
35*a*c**8*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**
6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2
)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945
*c**5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x**10*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 40*a*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 1
5*a*c**5*d**(11/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(
315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*
d**12) + 33*a*c**4*d**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
 210*c**4*d**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
 315*c**4*d**12) + 17*a*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d*
*11*x**2 + 315*c**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)
*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d
**11*x**2 + 315*c**4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

output
```
1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(
35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/
2)*d^2*x^5)*a/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3}$$
$$+\frac{35\,(cx^2+d)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\,(cx^2+d)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\,(cx^2+d)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\,(cx^2+d)^{\frac{5}{2}}bcd\mathrm{sgn}(x)+63\,(}{315\,c^3}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

output
```
2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/
2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*s
gn(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn
(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{x\,(8\,a\,d^4-18\,b\,c\,d^3)}{315\,c^3}\right.$$
$$\left.+\frac{x^7\,(45\,b\,c^4+50\,a\,d\,c^3)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,(a\,d+24\,b\,c)}{105\,c}-\frac{d^2\,x^3\,(4\,a\,d-9\,b\,c)}{315\,c^2}\right)$$

input
```
int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
 50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (
d^2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{\sqrt{c\,x^2+d}\,(35a\,c^4x^8+50a\,c^3d\,x^6+45b\,c^4x^6+3a\,c^2d^2x^4+72b\,c^3d\,x^4-4ac\,d^3x^2+9b\,c^2d^2}{315c^3}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x)
```

output
```
(sqrt(c*x**2 + d)*(35*a*c**4*x**8 + 50*a*c**3*d*x**6 + 3*a*c**2*d**2*x**4
- 4*a*c*d**3*x**2 + 8*a*d**4 + 45*b*c**4*x**6 + 72*b*c**3*d*x**4 + 9*b*c**
2*d**2*x**2 - 18*b*c*d**3))/(315*c**3)
```

3.164 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx = \frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}$$

output
```
1/35*(-2*a*d+7*b*c)*(c+d/x^2)^(5/2)*x^5/c^2+1/7*a*(c+d/x^2)^(5/2)*x^7/c
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^6 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}x(d+cx^2)^2\,(7bc-2ad+5acx^2)}{35c^2}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

output
```
(Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(7bc-2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^4dx}{7c}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2}(7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

output
```
((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^
7)/(7*c)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(5 a c\,x^{2}-2 a d+7 c b\right) \left(c\,x^{2}+d\right)}{35 c^{2}}$	45
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(5 a c\,x^{2}-2 a d+7 c b\right) \left(c\,x^{2}+d\right)}{35 c^{2}}$	45
orering	$\frac{\left(5 a c\,x^{2}-2 a d+7 c b\right) \left(c\,x^{2}+d\right) x^{5} \left(a+\frac{b}{x^{2}}\right) \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{35 c^{2} \left(a\,x^{2}+b\right)}$	57
risch	$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x \left(5 a\,x^{6} c^{3}+8 a\,c^{2} d\,x^{4}+7 b\,c^{3} x^{4}+a c\,d^{2} x^{2}+14 b\,c^{2} d\,x^{2}-2 a\,d^{3}+7 b c\,d^{2}\right)}{35 c^{2}}$	81
trager	$\frac{\left(5 a\,x^{6} c^{3}+8 a\,c^{2} d\,x^{4}+7 b\,c^{3} x^{4}+a c\,d^{2} x^{2}+14 b\,c^{2} d\,x^{2}-2 a\,d^{3}+7 b c\,d^{2}\right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35 c^{2}}$	85

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx = \frac{\left(5\,ac^3x^7+\left(7\,bc^3+8\,ac^2d\right)x^5+\left(14\,bc^2d+acd^2\right)x^3+\left(7\,bcd^2-2\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

output
```
1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
 + (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(46) = 92.

Time = 3.18 (sec) , antiderivative size = 498, normalized size of antiderivative = 9.40

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx = \frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{3ac^3d^{\frac{15}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{12ac^2d^{\frac{17}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{8acd^{\frac{19}{2}}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6} + \frac{ad^{\frac{3}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{ad^{\frac{5}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c} - \frac{2ad^{\frac{7}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2} + \frac{bc\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{2bd^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{bd^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

output
```
15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4
*d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/
(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(
13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 1
05*c**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt
(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) +
 8*a*c*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x*
*2*sqrt(c*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) +
 b*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d +
 1)/5 + b*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)a}{35\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

output
```
1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x
^2)^(5/2)*d*x^5)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{35\,c^2}+\frac{5\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+7\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-7\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)}{35\,c^2}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

output
```
-1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)
*a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x
))/c^2
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\sqrt{c+\frac{d}{x^2}}\left(\frac{x^5\,(7\,b\,c^3+8\,a\,d\,c^2)}{35\,c^2}-\frac{x\,(2\,a\,d^3-7\,b\,c\,d^2)}{35\,c^2}+\frac{a\,c\,x^7}{7}+\frac{d\,x^3\,(a\,d+14\,b\,c)}{35\,c}\right)$$

input
```
int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*
b*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^6\,dx=\frac{\sqrt{c\,x^2+d}\,(5a\,c^3x^6+8a\,c^2d\,x^4+7b\,c^3x^4+ac\,d^2x^2+14b\,c^2d\,x^2-2a\,d^3+7bc\,d^2)}{35c^2}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x)
```

output
```
(sqrt(c*x**2 + d)*(5*a*c**3*x**6 + 8*a*c**2*d*x**4 + a*c*d**2*x**2 - 2*a*d
**3 + 7*b*c**3*x**4 + 14*b*c**2*d*x**2 + 7*b*c*d**2))/(35*c**2)
```

3.165 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = \frac{4}{3} b d \sqrt{c+\frac{d}{x^2}} x + \frac{1}{3} b c \sqrt{c+\frac{d}{x^2}} x^3 + \frac{a \left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5 c} - b d^{3/2} \text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}} x}\right)$$

output
```
4/3*b*d*(c+d/x^2)^(1/2)*x+1/3*b*c*(c+d/x^2)^(1/2)*x^3+1/5*a*(c+d/x^2)^(5/2
)*x^5/c-b*d^(3/2)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^4 \, dx = \frac{1}{15} \sqrt{c+\frac{d}{x^2}} x \left(\frac{3 a (d+c x^2)^2}{c} + 5 b (4 d + c x^2) - \frac{15 b d^{3/2} \text{arctanh}\left(\frac{\sqrt{d+c x^2}}{\sqrt{d}}\right)}{\sqrt{d+c x^2}}\right)$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

output
```
(Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3
/2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {953, 858, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{953}$$

$$b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^2dx+\frac{ax^5 \left(c+\frac{d}{x^2}\right)^{5/2}}{5c}$$

$$\Big\downarrow \text{858}$$

$$\frac{ax^5 \left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^4d\frac{1}{x}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^5 \left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\int \sqrt{c+\frac{d}{x^2}}x^2d\frac{1}{x}-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^5 \left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(d\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(d\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-b\left(d\left(\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)-x\sqrt{c+\frac{d}{x^2}}\right)-\frac{1}{3}x^3\left(c+\frac{d}{x^2}\right)^{3/2}\right)$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^5)/(5*c) - b*(-1/3*((c + d/x^2)^(3/2)*x^3) + d*(-(S
qrt[c + d/x^2]*x) + Sqrt[d]*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 953
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[d/e^n   Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] &&
(IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && G
tQ[m + n, -1]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} \left(-3 a \left(c\,x^{2}+d\right)^{\frac{5}{2}}+15 d^{\frac{3}{2}} \ln\left(\frac{2 d+2 \sqrt{d}\, \sqrt{c\,x^{2}+d}}{x}\right) b c-5 \left(c\,x^{2}+d\right)^{\frac{3}{2}} b c-15 \sqrt{c\,x^{2}+d}\, b c d\right)}{15 \left(c\,x^{2}+d\right)^{\frac{3}{2}} c}$	99

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(-3*a*(c*x^2+d)^(5/2)+15*d^(3/2)*ln(2*(d^(
1/2)*(c*x^2+d)^(1/2)+d)/x)*b*c-5*(c*x^2+d)^(3/2)*b*c-15*(c*x^2+d)^(1/2)*b*
c*d)/(c*x^2+d)^(3/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.19

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx = \left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,ac^2x^5+(5\,bc^2+6\,acd)x^3+(20\,bcd+3\,ad^2)x}{30\,c}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

output
```
[1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*
d)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*
x)*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqr
t((c*x^2 + d)/x^2)/d) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d
 + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(80) = 160$.

Time = 3.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.04

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx = \frac{ac\sqrt{d}x^4\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5} + \frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c} + \frac{b\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}} + \frac{bc\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3} + \frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3} - bd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right) + \frac{bd^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

output
```
a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d +
1)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x
**2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d +
 1)/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 +
 d/(c*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

output
```
1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c +
d/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*
x + sqrt(d))))*b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^4\,dx=\frac{bd^2\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\operatorname{sgn}(x)}{\sqrt{-d}}-\frac{\left(15\,bcd^2\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+20\,bc\sqrt{-d}d^{\frac{3}{2}}+3\,a\sqrt{-d}d^{\frac{5}{2}}\right)\operatorname{sgn}(x)}{15\,c\sqrt{-d}}+\frac{3\,(cx^2+d)^{\frac{5}{2}}ac^4\operatorname{sgn}(x)+5\,(cx^2+d)^{\frac{3}{2}}bc^5\operatorname{sgn}(x)+15\,\sqrt{cx^2+d}bc^5d\operatorname{sgn}(x)}{15\,c^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

output
```
b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*
arctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))
*sgn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 +
 d)^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx=\int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$$

input
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.57

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx=\frac{3\sqrt{c\,x^2+d}\,a\,c^2x^4+6\sqrt{c\,x^2+d}\,acd\,x^2+3\sqrt{c\,x^2+d}\,a\,d^2+5\sqrt{c\,x^2+d}\,b\,c^2x^2+20\sqrt{c\,x^2+}{15c}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x)
```

output
```
(3*sqrt(c*x**2 + d)*a*c**2*x**4 + 6*sqrt(c*x**2 + d)*a*c*d*x**2 + 3*sqrt(c
*x**2 + d)*a*d**2 + 5*sqrt(c*x**2 + d)*b*c**2*x**2 + 20*sqrt(c*x**2 + d)*b
*c*d + 15*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*
c*d - 15*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c
*d)/(15*c)
```

3.166 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{6cx} + \frac{1}{3}(3bc+2ad)\sqrt{c+\frac{d}{x^2}}\,x + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^3}{3c} - \frac{1}{2}\sqrt{d}(3bc+2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

output
```
-1/6*d*(2*a*d+3*b*c)*(c+d/x^2)^(1/2)/c/x+1/3*(2*a*d+3*b*c)*(c+d/x^2)^(1/2)
*x+1/3*a*(c+d/x^2)^(5/2)*x^3/c-1/2*d^(1/2)*(2*a*d+3*b*c)*arctanh(d^(1/2)/(
c+d/x^2)^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2 \, dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d+cx^2}(-3bd+6bcx^2+8adx^2+2acx^4)-3\sqrt{d}(3bc+2ad)x^2\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)\right)}{6x\sqrt{d+cx^2}}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*
x^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6
*x*Sqrt[d + c*x^2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {955, 773, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(2ad+3bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}dx}{3c}+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x}}{3c}$$

$$\Big\downarrow \text{247}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{211}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{1}{2}c\int\frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}-\frac{(2ad+3bc)\left(3d\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)-x\left(c+\frac{d}{x^2}\right)^{3/2}\right)}{3c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

output
```
(a*(c + d/x^2)^(5/2)*x^3)/(3*c) - ((3*b*c + 2*a*d)*(-((c + d/x^2)^(3/2)*x)
 + 3*d*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(
2*Sqrt[d]))))/(3*c)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{db\sqrt{\frac{c x^2+d}{x^2}}}{2x}+\frac{\left(-\frac{\sqrt{d}\,(2ad+3cb)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c x^2+d}}{x}\right)}{2}+\sqrt{c x^2+d}\,bc+a c^2\left(\frac{x^2\sqrt{c x^2+d}}{3c}-\frac{2d\sqrt{c x^2+d}}{3c^2}\right)+2ad\sqrt{c x^2+d}\right)\sqrt{\frac{c x^2+d}{x^2}}\, x}{\sqrt{c x^2+d}}$
default	$-\frac{\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} x\left(6d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c x^2+d}}{x}\right)a x^2+9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c x^2+d}}{x}\right)bc x^2-2(c x^2+d)^{\frac{3}{2}}ad x^2-3(c x^2+d)^{\frac{3}{2}}bc x^2+3(c x^2+d\right)}{6(c x^2+d)^{\frac{3}{2}}d}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*b/x*((c*x^2+d)/x^2)^(1/2)+(-1/2*d^(1/2)*(2*a*d+3*b*c)*ln((2*d+2*d^(
1/2)*(c*x^2+d)^(1/2))/x)+(c*x^2+d)^(1/2)*b*c+a*c^2*(1/3*x^2/c*(c*x^2+d)^(1
/2)-2/3*d/c^2*(c*x^2+d)^(1/2))+2*a*d*(c*x^2+d)^(1/2))*((c*x^2+d)/x^2)^(1/2
)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.56

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd)\sqrt{}}{12\,x}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
 d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt(
(c*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*s
qrt((c*x^2 + d)/x^2)/d) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt
((c*x^2 + d)/x^2))/x]
```

Sympy [A] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.71

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{a\sqrt{c}dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)+\frac{ad^2}{\sqrt{c}x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{b\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

output
```
a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
 + a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x))
 + a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x*
*2)) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/
(c*x**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.38

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a+\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx-\frac{2\sqrt{c+\frac{d}{x^2}}cdx}{(c+\frac{d}{x^2})x^2-d}+3\,c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

output
```
1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt
(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c
 + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d
)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2\,dx=\frac{1}{6}\,c\left(\frac{3\,(3\,bcd\mathrm{sgn}(x)+2\,ad^2\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{c\sqrt{-d}}-\frac{3\sqrt{cx^2+d}bd\mathrm{sgn}(x)}{cx^2}+\frac{2\left((cx^2+\right.}{}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

output
```
1/6*c*(3*(3*b*c*d*sgn(x) + 2*a*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d)
)/(c*sqrt(-d)) - 3*sqrt(c*x^2 + d)*b*d*sgn(x)/(c*x^2) + 2*((c*x^2 + d)^(3/
2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*sgn(x) + 3*sqrt(c*x^2 + d)*a*c^2
*d*sgn(x))/c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx=\int x^2\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

input
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.58

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx=\frac{2\sqrt{c\,x^2+d}\,ac\,x^4+8\sqrt{c\,x^2+d}\,ad\,x^2+6\sqrt{c\,x^2+d}\,bc\,x^2-3\sqrt{c\,x^2+d}\,bd+6\sqrt{d}\log\left(\frac{\sqrt{c\,x^2\cdots}}{\cdots}\right.}{\cdots}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x)
```

output

```
(2*sqrt(c*x**2 + d)*a*c*x**4 + 8*sqrt(c*x**2 + d)*a*d*x**2 + 6*sqrt(c*x**2
 + d)*b*c*x**2 - 3*sqrt(c*x**2 + d)*b*d + 6*sqrt(d)*log((sqrt(c*x**2 + d)
+ sqrt(c)*x - sqrt(d))/sqrt(d))*a*d*x**2 + 9*sqrt(d)*log((sqrt(c*x**2 + d)
 + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c*x**2 - 6*sqrt(d)*log((sqrt(c*x**2 + d
) + sqrt(c)*x + sqrt(d))/sqrt(d))*a*d*x**2 - 9*sqrt(d)*log((sqrt(c*x**2 +
d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c*x**2)/(6*x**2)
```

3.167 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 112

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx = -\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x} - \frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c} - \frac{3c(bc+4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8\sqrt{d}}$$

output
```
-3/8*(4*a*d+b*c)*(c+d/x^2)^(1/2)/x-1/4*(4*a*d+b*c)*(c+d/x^2)^(3/2)/c/x+a*(
c+d/x^2)^(5/2)*x/c-3/8*c*(4*a*d+b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^
(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-2bd-5bcx^2-4adx^2+8acx^4-\frac{3c(bc+4ad)x^4\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{d+cx^2}}\right)}{8x^3}$$

input
```
Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

output
```
(Sqrt[c + d/x^2]*(-2*b*d - 5*b*c*x^2 - 4*a*d*x^2 + 8*a*c*x^4 - (3*c*(b*c +
 4*a*d)*x^4*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/
(8*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {899, 359, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^2d\frac{1}{x}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\int \left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}-\frac{(4ad+bc)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{c}$$

input
```
Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

output
```
(a*(c + d/x^2)^(5/2)*x)/c - ((b*c + 4*a*d)*((c + d/x^2)^(3/2)/(4*x) + (3*c
*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt
[d])))/4))/c
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(4 a d \,x^{2}+5 b c \,x^{2}+2 b d \right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}{8 x^{3}}+\frac{c \left(-\frac{\left(12 a d +3 c b \right) \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right)}{\sqrt{d}}+8 a \sqrt{c \,x^{2}+d}\right) \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, x}{8 \sqrt{c \,x^{2}+d}}$
default	$-\frac{\left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} \left(12 d^{\frac{5}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) a c \,x^{4}+3 d^{\frac{3}{2}} \ln \left(\frac{2 d +2 \sqrt{d}\, \sqrt{c \,x^{2}+d}}{x}\right) b \,c^{2} x^{4}-4 \left(c \,x^{2}+d \right)^{\frac{3}{2}} a c d \,x^{4}-\left(c \,x^{2}+d \right)^{\frac{3}{2}} b \,c^{2} x^{4}+4 (c \,x^{2}}{8 x \left(c \,x^{2}+d \right)^{\frac{3}{2}} d^{2}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*a*d*x^2+5*b*c*x^2+2*b*d)/x^3*((c*x^2+d)/x^2)^(1/2)+1/8*c*(-(12*a*d
+3*b*c)/d^(1/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)+8*a*(c*x^2+d)^(1/2))
*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.88

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx = \left[\frac{3\,(bc^2+4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(8\,acdx^4-2\,bd^2-(5\,bcd+4\,ad^2)x^2)\cdot}{16\,dx^3}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*
x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3
*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + (8*a*c*d*x^4 - 2*b*d^2 - (5*
b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(100) = 200.

Time = 6.83 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.93

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx = \frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}} - \frac{a\sqrt{c}d\sqrt{1+\frac{d}{cx^2}}}{2x} + \frac{a\sqrt{c}d}{x\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2} - \frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3b\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{bd^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

output
```
a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
 + a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqr
t(c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(
1 + d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*
asinh(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d
/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(94) = 188$.

Time = 0.11 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.85

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx = \frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}cx - \frac{2\sqrt{c+\frac{d}{x^2}}cdx}{\left(c+\frac{d}{x^2}\right)x^2-d} + 3c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)\right)a + \frac{1}{16}\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4 - 2\left(c+\frac{d}{x^2}\right)dx^2 + d^2}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
 + 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt
(d))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*
x + sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)
*c^2*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{8\sqrt{cx^2+d}ac^2\mathrm{sgn}(x)+\frac{3\left(bc^3\mathrm{sgn}(x)+4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{5\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2ds}{}}{8\,c}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*
arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn
(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x
) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx=\frac{a\,x\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{{\left(\frac{d}{c}+x^2\right)}^{3/2}}-\frac{b\,{\left(c\,x^2+d\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\,{\left(\frac{d}{c}+x^2\right)}^{3/2}}$$

input
```
int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

output
```
(a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^
2)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(
x*(d/c + x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.80

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} dx = \frac{8\sqrt{c x^2+d}\, acd\, x^4 - 4\sqrt{c x^2+d}\, a\, d^2 x^2 - 5\sqrt{c x^2+d}\, bcd\, x^2 - 2\sqrt{c x^2+d}\, b\, d^2 + 12\sqrt{d}\log\left(\sqrt{\right.}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2),x)
```

output
```
(8*sqrt(c*x**2 + d)*a*c*d*x**4 - 4*sqrt(c*x**2 + d)*a*d**2*x**2 - 5*sqrt(c
*x**2 + d)*b*c*d*x**2 - 2*sqrt(c*x**2 + d)*b*d**2 + 12*sqrt(d)*log((sqrt(c
*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*a*c*d*x**4 + 3*sqrt(d)*log((sqr
t(c*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c**2*x**4 - 12*sqrt(d)*log
((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*a*c*d*x**4 - 3*sqrt(d)*
log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**2*x**4)/(8*d*x*
*4)
```

3.168 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx}+\frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}+\frac{c^2(bc-6ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{16d^{3/2}}$$

output
```
1/16*c*(-6*a*d+b*c)*(c+d/x^2)^(1/2)/d/x+1/24*(-6*a*d+b*c)*(c+d/x^2)^(3/2)/
d/x-1/6*b*(c+d/x^2)^(5/2)/d/x+1/16*c^2*(-6*a*d+b*c)*arctanh(d^(1/2)/(c+d/x
^2)^(1/2)/x)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\left(-\sqrt{d}(6adx^2(2d+5cx^2)+b(8d^2+14cdx^2+3c^2x^4))+\frac{3c^2(bc-6ad)x^6}{\cdots}\right.}{48d^{3/2}x^5}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[c + d/x^2]*(-(Sqrt[d]*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 + 14*c*d
*x^2 + 3*c^2*x^4))) + (3*c^2*(b*c - 6*a*d)*x^6*ArcTanh[Sqrt[d + c*x^2]/Sqr
t[d]])/Sqrt[d + c*x^2]))/(48*d^(3/2)*x^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {959, 858, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(bc-6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}dx}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{858}$$

$$\frac{(bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}d\frac{1}{x}}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{211}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\int \sqrt{c+\frac{d}{x^2}}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{211}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-6ad)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt{c+\frac{d}{x^2}}}{2x}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{4x}\right)}{6d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

output
```
-1/6*(b*(c + d/x^2)^(5/2))/(d*x) + ((b*c - 6*a*d)*((c + d/x^2)^(3/2)/(4*x)
 + (3*c*(Sqrt[c + d/x^2]/(2*x) + (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/
(2*Sqrt[d])))/4))/(6*d)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\left(30acd\,x^{4}+3b\,c^{2}x^{4}+12a\,d^{2}x^{2}+14bcd\,x^{2}+8b\,d^{2}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{48x^{5}d}-\frac{c^{2}\left(6ad-cb\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}{16d^{\frac{3}{2}}\sqrt{c\,x^{2}+d}}$
default	$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(18d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{2}x^{6}-3d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{3}x^{6}-6\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,c^{2}d\,x^{6}+\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\,c^{3}x^{6}+6(c\right.}{48x^{3}}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(30*a*c*d*x^4+3*b*c^2*x^4+12*a*d^2*x^2+14*b*c*d*x^2+8*b*d^2)/x^5/d*(
(c*x^2+d)/x^2)^(1/2)-1/16*c^2*(6*a*d-b*c)/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2
+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.95

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\left[-\frac{3\,(bc^3-6\,ac^2d)\sqrt{d}x^5\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(3\,(bc^2d+10\,acd^2)x^{\cdots}}{96\,d^2x^5}\right.$$

$$-\frac{3\,(bc^3-6\,ac^2d)\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)+(3\,(bc^2d+10\,acd^2)x^4+8\,bd^3+2\,(7\,bcd^2+6\,ad^3)x^2)\sqrt{\frac{c\cdots}{}}}{48\,d^2x^5}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((
c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 +
2*(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b
*c^3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d)
+ (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(102) = 204$.

Time = 14.05 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.06

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x} - \frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}} - \frac{3a\sqrt{c}d}{8x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8\sqrt{d}} - \frac{ad^2}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11b\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{bd^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

output
```
-a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**
2))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d
)/(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
- b*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1
 + d/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*a
sinh(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 +
d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.24

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = \frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3 - 3\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4 - 2\left(c+\frac{d}{x^2}\right)dx^2 + d^2}\right)a - \frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5 + 8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3 - 3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6 - 3\left(c+\frac{d}{x^2}\right)^2d^2x^4 + 3\left(c+\frac{d}{x^2}\right)d^3x^2 - d^4}\right)b$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/(
(c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c +
 d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*
c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*
d^3*x^2 - d^4))*b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{1}{48}c^3\left(\frac{3\left(bc\mathrm{sgn}(x) - 6\,ad\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{c\sqrt{-d}d} + \frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x) + 30\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x) + 8$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
-1/48*c^3*(3*(b*c*sgn(x) - 6*a*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(c*sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*d^2*s
gn(x) - 3*sqrt(c*x^2 + d)*b*c*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*d^3*sgn(x)
)/(c^4*d*x^6))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx=\frac{-30\sqrt{c\,x^2+d}\,ac\,d^2x^4-12\sqrt{c\,x^2+d}\,a\,d^3x^2-3\sqrt{c\,x^2+d}\,b\,c^2d\,x^4-14\sqrt{c\,x^2}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x)
```

output
```
( - 30*sqrt(c*x**2 + d)*a*c*d**2*x**4 - 12*sqrt(c*x**2 + d)*a*d**3*x**2 -
3*sqrt(c*x**2 + d)*b*c**2*d*x**4 - 14*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 8*s
qrt(c*x**2 + d)*b*d**3 + 18*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sq
rt(d))/sqrt(d))*a*c**2*d*x**6 - 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*
x - sqrt(d))/sqrt(d))*b*c**3*x**6 - 18*sqrt(d)*log((sqrt(c*x**2 + d) + sqr
t(c)*x + sqrt(d))/sqrt(d))*a*c**2*d*x**6 + 3*sqrt(d)*log((sqrt(c*x**2 + d)
 + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**3*x**6)/(48*d**2*x**6)
```

3.169 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{128d^2x}-\frac{c^3(3bc-8ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{128d^{5/2}}$$

output
```
1/64*c*(-8*a*d+3*b*c)*(c+d/x^2)^(1/2)/d/x^3+1/48*(-8*a*d+3*b*c)*(c+d/x^2)^
(3/2)/d/x^3-1/8*b*(c+d/x^2)^(5/2)/d/x^3+1/128*c^2*(-8*a*d+3*b*c)*(c+d/x^2)
^(1/2)/d^2/x-1/128*c^3*(-8*a*d+3*b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d
^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{d}\sqrt{d+cx^2}(8adx^2(8d^2+14cdx^2+3c^2x^4)+b(48d^3+72cd^2x^2+6c^2dx^4-9c^3x^6))+3c^3(3bc\right.}{384d^{5/2}x^7\sqrt{d+cx^2}}$$

input
```
Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

output
```
-1/384*(Sqrt[c + d/x^2]*(Sqrt[d]*Sqrt[d + c*x^2]*(8*a*d*x^2*(8*d^2 + 14*c*
d*x^2 + 3*c^2*x^4) + b*(48*d^3 + 72*c*d^2*x^2 + 6*c^2*d*x^4 - 9*c^3*x^6))
+ 3*c^3*(3*b*c - 8*a*d)*x^8*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(d^(5/2)*x^
7*Sqrt[d + c*x^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {959, 858, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{ 959}$$

$$-\frac{(3bc-8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}dx}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{(3bc-8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}d\frac{1}{x}}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\int\frac{\sqrt{c+\frac{d}{x^2}}}{x^2}d\frac{1}{x}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\int\frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int\frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int\frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3bc-8ad)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)+\frac{\sqrt{c+\frac{d}{x^2}}}{4x^3}\right)+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{6x^3}\right)}{8d}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}$$

input
```
Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

output
```
-1/8*(b*(c + d/x^2)^(5/2))/(d*x^3) + ((3*b*c - 8*a*d)*((c + d/x^2)^(3/2)/(
6*x^3) + (c*(Sqrt[c + d/x^2]/(4*x^3) + (c*(Sqrt[c + d/x^2]/(2*d*x) - (c*Ar
cTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(3/2))))/4))/2))/(8*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\left(24a\,c^{2}d\,x^{6}-9b\,c^{3}x^{6}+112ac\,d^{2}x^{4}+6b\,c^{2}d\,x^{4}+64a\,d^{3}x^{2}+72bc\,d^{2}x^{2}+48b\,d^{3}\right)\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}{384x^{7}d^{2}}+\frac{c^{3}\left(8ad-3cb\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{}}{128d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}}$
default	$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(24d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)a\,c^{3}x^{8}-9d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{4}x^{8}-8\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,c^{3}d\,x^{8}+3\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\,c^{4}x^{8}+8\left(c\,x\right.\right.}{}$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/384*(24*a*c^2*d*x^6-9*b*c^3*x^6+112*a*c*d^2*x^4+6*b*c^2*d*x^4+64*a*d^3*
x^2+72*b*c*d^2*x^2+48*b*d^3)/x^7/d^2*((c*x^2+d)/x^2)^(1/2)+1/128*c^3*(8*a*
d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)*((c*x^2+d)/x^2)^(1/
2)*x/(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.84

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx=\left[-\frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{d}x^7\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(3\,bc^3d-8\,ac^2d^2\right.\right.}{768\,d^3}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqr
t((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b
*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7
*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + (3*(3*b*c^3*d - 8*a*c^2*d^2)
*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^
4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(141) = 282$.

Time = 43.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.81

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}} - \frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}} - \frac{11a\sqrt{c}d}{24x^5\sqrt{1+\frac{d}{cx^2}}} + \frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{16d^{\frac{3}{2}}} - \frac{ad^2}{6\sqrt{c}x^7\sqrt{1+\frac{d}{cx^2}}} + \frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{bc^{\frac{5}{2}}}{128dx^3\sqrt{1+\frac{d}{cx^2}}} - \frac{13bc^{\frac{3}{2}}}{64x^5\sqrt{1+\frac{d}{cx^2}}} - \frac{5b\sqrt{c}d}{16x^7\sqrt{1+\frac{d}{cx^2}}} - \frac{3bc^4\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{128d^{\frac{5}{2}}} - \frac{bd^2}{8\sqrt{c}x^9\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

output
```
-a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1
+ d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*as
inh(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d
/(c*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/
(128*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x
**2))) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqr
t(d)/(sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x*
*2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(135) = 270$.

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)a + \frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^4x^7-11\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^4dx^5-11\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4d^2x^3+3\sqrt{c+}\right.}{\left(c+\frac{d}{x^2}\right)^4d^2x^8-4\left(c+\frac{d}{x^2}\right)^3d^3x^6+6\left(c+\frac{d}{x^2}\right)^2d^4x^4-4\left(c+\frac{d}{x^2}\right)d^5x^2}\right.$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
)))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x
^3 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d
^2*x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^
2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^
(7/2)*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*
d^2*x^3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x
^2)^3*d^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = \frac{3\left(3\,bc^5\mathrm{sgn}(x)-8\,ac^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2} + \frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\mathrm{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\mathrm{sgn}(x)-33}{}$$

input
```
integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-
d))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7
/2)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^
(5/2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2
 + d)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqr
t(c*x^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx=\frac{-24\sqrt{c\,x^2+d}\,a\,c^2d^2x^6-112\sqrt{c\,x^2+d}\,ac\,d^3x^4-64\sqrt{c\,x^2+d}\,a\,d^4x^2+9\sqrt{c\,x}}{}$$

input
```
int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x)
```

output
```
( - 24*sqrt(c*x**2 + d)*a*c**2*d**2*x**6 - 112*sqrt(c*x**2 + d)*a*c*d**3*x
**4 - 64*sqrt(c*x**2 + d)*a*d**4*x**2 + 9*sqrt(c*x**2 + d)*b*c**3*d*x**6 -
 6*sqrt(c*x**2 + d)*b*c**2*d**2*x**4 - 72*sqrt(c*x**2 + d)*b*c*d**3*x**2 -
 48*sqrt(c*x**2 + d)*b*d**4 - 24*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x
 - sqrt(d))/sqrt(d))*a*c**3*d*x**8 + 9*sqrt(d)*log((sqrt(c*x**2 + d) + sqr
t(c)*x - sqrt(d))/sqrt(d))*b*c**4*x**8 + 24*sqrt(d)*log((sqrt(c*x**2 + d)
+ sqrt(c)*x + sqrt(d))/sqrt(d))*a*c**3*d*x**8 - 9*sqrt(d)*log((sqrt(c*x**2
 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**4*x**8)/(384*d**3*x**8)
```

3.170 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c}-\frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
1/8*(-3*a*d+4*b*c)*(c+d/x^2)^(1/2)*x^2/c^2+1/4*a*(c+d/x^2)^(1/2)*x^4/c-1/8
*d*(-3*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c}(d+cx^2)\left(4bc-3ad+2acx^2\right)+\frac{2d(-4bc+3ad)\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{d}+\sqrt{d+cx^2}}\right)}{x}}{8c^{5/2}\sqrt{c+\frac{d}{x^2}}}$$

input
```
Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c]*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + (2*d*(-4*b*c + 3*a*d)*S
qrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])])/x)/(8*c^
(5/2)*Sqrt[c + d/x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}\,dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^6}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\int\frac{x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \textbf{52}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(-\frac{d\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(-\frac{\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{c}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{2c}-\frac{(4bc-3ad)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{x^2\sqrt{c+\frac{d}{x^2}}}{c}\right)}{4c}\right)$$

input
```
Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

output
```
((a*Sqrt[c + d/x^2]*x^4)/(2*c) - ((4*b*c - 3*a*d)*(-((Sqrt[c + d/x^2]*x^2)
/c) + (d*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^(3/2)))/(4*c))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

method	result	siz
risch	$\frac{(2ac\,x^2-3ad+4cb)(c\,x^2+d)}{8c^2\sqrt{\frac{c\,x^2+d}{x^2}}}+\frac{d(3ad-4cb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}}{8c^{\frac{5}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	99
default	$\frac{\sqrt{c\,x^2+d}\left(2c^{\frac{5}{2}}\sqrt{c\,x^2+d}\,a\,x^3-3c^{\frac{3}{2}}\sqrt{c\,x^2+d}\,adx+4c^{\frac{5}{2}}\sqrt{c\,x^2+d}\,bx+3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)ac\,d^2-4\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)b\,c^2d\right)}{8\sqrt{\frac{c\,x^2+d}{x^2}}\,x\,c^{\frac{7}{2}}}$	12

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*a*c*x^2-3*a*d+4*b*c)*(c*x^2+d)/c^2/((c*x^2+d)/x^2)^(1/2)+1/8*d*(3*a
*d-4*b*c)/c^(5/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c
*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$$
$$=\left[-\frac{(4\,bcd-3\,ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+(4\,bc^2-3\,acd)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3},\ \frac{(4\,bc\cdots}{\cdots}\right.$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*s
qrt((c*x^2 + d)/x^2))/c^3]
```

Sympy [A] (verification not implemented)

Time = 24.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} - \frac{a\sqrt{d}x^3}{8c\sqrt{\frac{cx^2}{d}+1}} - \frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}} + \frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}} + \frac{b\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c} - \frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

output
```
a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d
+ 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c
)*x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asi
nh(sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(74) = 148.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{4}b\left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2} + \frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{1}{16}a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}} + \frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2 - 5\sqrt{c+\frac{d}{x^2}}cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2 - 2\left(c+\frac{d}{x^2}\right)c^3 + c^4}\right)$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2)
- sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt
(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/
x^2)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^
2)*c^3 + c^4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{1}{8}\sqrt{cx^2+d}x\left(\frac{2\,ax^2}{c\mathrm{sgn}\left(x\right)}+\frac{4\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right) - \frac{\left(4\,bcd\log\left(|d|\right)-3\,ad^2\log\left(|d|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{5}{2}}} + \frac{\left(4\,bcd-3\,ad^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(c*x^2 + d)*x*(2*a*x^2/(c*sgn(x)) + (4*b*c^2*sgn(x) - 3*a*c*d*sgn(
x))/c^3) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))*sgn(x)/c^(5/2)
 + 1/8*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)
*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{5\,a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8\,c} - \frac{3\,a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c^2} + \frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c} - \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}} + \frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

input
```
int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) +
(b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(
2*c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$$
$$= \frac{2\sqrt{c\,x^2+d}\,a\,c^2x^3-3\sqrt{c\,x^2+d}\,acdx+4\sqrt{c\,x^2+d}\,b\,c^2x+3\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)a\,d^2-4\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}}{\sqrt{c}}\right)}{8c^3}$$

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x)
```

output
```
(2*sqrt(c*x**2 + d)*a*c**2*x**3 - 3*sqrt(c*x**2 + d)*a*c*d*x + 4*sqrt(c*x*
*2 + d)*b*c**2*x + 3*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a
*d**2 - 4*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d)/(8*c*
*3)
```

3.171 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}x^2}{2c}+\frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

output
```
1/2*a*(c+d/x^2)^(1/2)*x^2/c+1/2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/
2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c}x(d+cx^2)+(-2bc+ad)\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{2c^{3/2}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c]*x*(d + c*x^2) + (-2*b*c + a*d)*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x)
 + Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}-\frac{(2bc-ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}-\frac{(2bc-ad)\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{c}\right)$$

input
```
Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

output
```
((a*Sqrt[c + d/x^2]*x^2)/c + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c
]])/c^(3/2))/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{a(c\,x^2+d)}{2c\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{(ad-2cb)\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)\sqrt{c\,x^2+d}}{2c^{\frac{3}{2}}\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	82
default	$-\frac{\sqrt{c\,x^2+d}\left(-c^{\frac{3}{2}}\sqrt{c\,x^2+d}\,ax+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)acd-2b\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)c^2\right)}{2\sqrt{\frac{c\,x^2+d}{x^2}}\,x\,c^{\frac{5}{2}}}$	90

input
```
int((a+b/x^2)*x/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/c*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)-1/2*(a*d-2*b*c)/c^(3/2)*ln(c^(1/2)
*x+(c*x^2+d)^(1/2))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.47

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2}, \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\,\mathrm{arc}}{2\,c^2}\right.$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
 + 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2
 + d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)))/c^2]
```

Sympy [A] (verification not implemented)

Time = 29.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{d}x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{c}}$$

input
```
integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**
(3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(47) = 94.

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{1}{4}a\left(\frac{2\sqrt{c+\frac{d}{x^2}}d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2)
- sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x
^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{cx^2+d}ax}{2\,c\text{sgn}\,(x)} + \frac{(2\,bc\log\left(|d|\right)-ad\log\left(|d|\right))\text{sgn}(x)}{4\,c^{\frac{3}{2}}} - \frac{(2\,bc-ad)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{3}{2}}\text{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(c*x^2 + d)*a*x/(c*sgn(x)) + 1/4*(2*b*c*log(abs(d)) - a*d*log(abs(
d)))*sgn(x)/c^(3/2) - 1/2*(2*b*c - a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 +
d)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{b\,\text{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2\,c} - \frac{a\,d\,\text{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

input
```
int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(
2*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c\,x^2+d}\,acx-\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)ad+2\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)bc}{2c^2}$$

input
```
int((a+b/x^2)*x/(c+d/x^2)^(1/2),x)
```

output
```
(sqrt(c*x**2 + d)*a*c*x - sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(
d))*a*d + 2*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c)/(2*c*
*2)
```

3.172 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x} dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x} dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{d}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-b*(c+d/x^2)^(1/2)/d+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x} dx = \frac{-b\sqrt{c}(d+cx^2)-adx\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{\sqrt{c}d\sqrt{c+\frac{d}{x^2}}x^2}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

output
```
(-(b*Sqrt[c]*(d + c*x^2)) - a*d*x*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-a\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{d}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

output
```
((-2*b*Sqrt[c + d/x^2])/d + (2*a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]
)/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)d x-b\sqrt{c\,x^{2}+d}\,\sqrt{c}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{2}\sqrt{c}\,d}$	69
risch	$-\frac{b\left(c\,x^{2}+d\right)}{d\,x^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{a\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}}{\sqrt{c}\,\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	77

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+d)^(1/2)*(a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x-b*(c*x^2+d)^(1/2)*c^(
1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.02

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = \left[\frac{a\sqrt{c}d\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd},\ -\frac{a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d)
- 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*s
qrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

Sympy [A] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx = -\frac{a\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}} & \text{for}\ d\neq 0\\ -\frac{\log\left(x^2\right)}{\sqrt{c}} & \text{otherwise}\end{cases}\right)}{2}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}x^2} & \text{for}\ d=0\\ -\frac{2\sqrt{c+\frac{d}{x^2}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x,x)
```

output
```
-a*Piecewise((2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c), Ne(d, 0)), (-log
(x**2)/sqrt(c), True))/2 + b*Piecewise((-1/(sqrt(c)*x**2), Eq(d, 0)), (-2*
sqrt(c + d/x**2)/d, True))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx=-\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{2\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c
) - b*sqrt(c + d/x^2)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x}\,dx=-\frac{a\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)}{2\sqrt{c}\operatorname{sgn}(x)}+\frac{2b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*a*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)/(sqrt(c)*sgn(x)) + 2*b*sqrt(c)
/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx=\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

output
```
(a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx=\frac{-\sqrt{c\,x^2+d}\,bc+\sqrt{c}\,\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)adx-\sqrt{c}\,bcx}{cdx}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x,x)
```

output
```
( - sqrt(c*x**2 + d)*b*c + sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt
(d))*a*d*x - sqrt(c)*b*c*x)/(c*d*x)
```

3.173 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx = \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

output
```
(-a*d+b*c)*(c+d/x^2)^(1/2)/d^2-1/3*b*(c+d/x^2)^(3/2)/d^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3} dx = -\frac{\sqrt{c+\frac{d}{x^2}}(3adx^2+b(d-2cx^2))}{3d^2x^2}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

output
```
-1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^3\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{c+\frac{d}{x^2}}b}{d}+\frac{ad-bc}{d\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^2}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

output
```
((2*(b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (2*b*(c + d/x^2)^(3/2))/(3*d^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
trager	$-\frac{\left(3 a d \,x^{2}-2 b c \,x^{2}+b d \right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{3 x^{2} d^{2}}$	44
gosper	$-\frac{\left(3 a d \,x^{2}-2 b c \,x^{2}+b d \right) \left(c \,x^{2}+d \right)}{3 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{2} x^{4}}$	47
default	$-\frac{\left(3 a d \,x^{2}-2 b c \,x^{2}+b d \right) \left(c \,x^{2}+d \right)}{3 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{2} x^{4}}$	47
risch	$-\frac{\left(3 a d \,x^{2}-2 b c \,x^{2}+b d \right) \left(c \,x^{2}+d \right)}{3 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{2} x^{4}}$	47
orering	$-\frac{\left(3 a d \,x^{2}-2 b c \,x^{2}+b d \right) \left(c \,x^{2}+d \right) \left(a +\frac{b}{x^{2}}\right)}{3 d^{2} \left(a \,x^{2}+b \right) x^{2} \sqrt{c +\frac{d}{x^{2}}}}$	59

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^2*(3*a*d*x^2-2*b*c*x^2+b*d)/d^2*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx=\frac{((2\,bc-3\,ad)x^2-bd)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx=\frac{\begin{cases}\frac{-2a\sqrt{c+\frac{d}{x^2}}-\frac{2b\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for}\ d\neq 0\\ \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{otherwise}\end{cases}}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x**3,x)
```

output
```
Piecewise(((-2*a*sqrt(c + d/x**2) - 2*b*(-c*sqrt(c + d/x**2) + (c + d/x**2
)**(3/2)/3)/d)/d, Ne(d, 0)), ((-a/x**2 - b/(2*x**4))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx=-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)-\frac{a\sqrt{c+\frac{d}{x^2}}}{d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^
2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.88

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx$$

$$=\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}+6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}-6\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2a\sqrt{c}d-2\,bc^{\frac{3}{2}}d+3\,a\sqrt{}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c) + 6*(sqrt(c)*x - sqrt(c*x
^2 + d))^2*b*c^(3/2) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d - 2*b
*c^(3/2)*d + 3*a*sqrt(c)*d^2)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3*sgn
(x))
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,(b\,d+3\,a\,d\,x^2-2\,b\,c\,x^2)}{3\,d^2\,x^2}$$

input
```
int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^3}\,dx = \frac{-3\sqrt{c\,x^2+d}\,ad\,x^2+2\sqrt{c\,x^2+d}\,bc\,x^2-\sqrt{c\,x^2+d}\,bd+\sqrt{c}\,ad\,x^3-2\sqrt{c}\,bc\,x^3}{3d^2x^3}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^3,x)
```

output
```
( - 3*sqrt(c*x**2 + d)*a*d*x**2 + 2*sqrt(c*x**2 + d)*b*c*x**2 - sqrt(c*x**
2 + d)*b*d + sqrt(c)*a*d*x**3 - 2*sqrt(c)*b*c*x**3)/(3*d**2*x**3)
```

3.174 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=-\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

output
```
-c*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^3+1/3*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^3-1/5
*b*(c+d/x^2)^(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}(-5adx^2(d-2cx^2)+b(-3d^2+4cdx^2-8c^2x^4))}{15d^3x^4}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

output
```
(Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2
*x^4)))/(15*d^3*x^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^5\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d^2}+\frac{(ad-2bc)\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c(bc-ad)}{d^2\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{2c\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^3}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

output
```
((-2*c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3 + (2*(2*b*c - a*d)*(c + d/x^2)^(3/
2))/(3*d^3) - (2*b*(c + d/x^2)^(5/2))/(5*d^3))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{\left(10 a c d \,x^{4}-8 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+4 b c d \,x^{2}-3 b \,d^{2}\right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 x^{4} d^{3}}$	67
gosper	$\frac{\left(10 a c d \,x^{4}-8 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+4 b c d \,x^{2}-3 b \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{3} x^{6}}$	70
default	$\frac{\left(10 a c d \,x^{4}-8 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+4 b c d \,x^{2}-3 b \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{3} x^{6}}$	70
risch	$\frac{\left(10 a c d \,x^{4}-8 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+4 b c d \,x^{2}-3 b \,d^{2}\right) \left(c \,x^{2}+d \right)}{15 \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, d^{3} x^{6}}$	70
orering	$\frac{\left(10 a c d \,x^{4}-8 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+4 b c d \,x^{2}-3 b \,d^{2}\right) \left(c \,x^{2}+d \right) \left(a +\frac{b}{x^{2}}\right)}{15 d^{3} \left(a \,x^{2}+b \right) x^{4} \sqrt{c +\frac{d}{x^{2}}}}$	82

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/15/x^4*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)/d^3*(-
(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{(2\,(4\,bc^2-5\,acd)x^4+3\,bd^2-(4\,bcd-5\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2)/(d^3*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = \frac{\begin{cases} \frac{-\frac{2a\left(-c\sqrt{c+\frac{d}{x^2}}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}-\frac{2b\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x**5,x)
```

output
```
Piecewise(((-2*a*(-c*sqrt(c + d/x**2) + (c + d/x**2)**(3/2)/3)/d - 2*b*(c*
*2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d
**2)/d, Ne(d, 0)), ((-a/(2*x**4) - b/(3*x**6))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right) - \frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}c}{d^2}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c
+ d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(62) = 124.

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.50

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = \frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{3}{2}}+40\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{5}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4ac^{\frac{3}{2}}d-20\left(\sqrt{c}x-\sqrt{\cdots}\right.\right.}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^5\operatorname{sgn}\left(x\cdots\right.}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2) + 40*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*b*c^(5/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d -
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d + 25*(sqrt(c)*x - sqrt(c*x
^2 + d))^2*a*c^(3/2)*d^2 + 4*b*c^(5/2)*d^2 - 5*a*c^(3/2)*d^3)/(((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^5*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2)}{15\,d^3\,x^4}$$

input
```
int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 -
4*b*c*d*x^2))/(15*d^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^5}\,dx$$
$$= \frac{10\sqrt{c\,x^2+d}\,acd\,x^4-5\sqrt{c\,x^2+d}\,a\,d^2x^2-8\sqrt{c\,x^2+d}\,b\,c^2x^4+4\sqrt{c\,x^2+d}\,bcd\,x^2-3\sqrt{c\,x^2+d}\,b\,d^2-10}{15d^3x^5}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^5,x)
```

output
```
(10*sqrt(c*x**2 + d)*a*c*d*x**4 - 5*sqrt(c*x**2 + d)*a*d**2*x**2 - 8*sqrt(
c*x**2 + d)*b*c**2*x**4 + 4*sqrt(c*x**2 + d)*b*c*d*x**2 - 3*sqrt(c*x**2 +
d)*b*d**2 - 10*sqrt(c)*a*c*d*x**5 + 8*sqrt(c)*b*c**2*x**5)/(15*d**3*x**5)
```

3.175 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx = \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4} - \frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

output
```
c^2*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^4-1/3*c*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)/d^
4+1/5*(-a*d+3*b*c)*(c+d/x^2)^(5/2)/d^4-1/7*b*(c+d/x^2)^(7/2)/d^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7} dx = \frac{(d+cx^2)\left(-15bd^3+18bcd^2x^2-21ad^3x^2-24bc^2dx^4+28acd^2x^4+48bc^3x^6-56ac^2dx^6\right)}{105d^4\sqrt{c+\frac{d}{x^2}}x^8}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

output
```
((d + c*x^2)*(-15*b*d^3 + 18*b*c*d^2*x^2 - 21*a*d^3*x^2 - 24*b*c^2*d*x^4 +
 28*a*c*d^2*x^4 + 48*b*c^3*x^6 - 56*a*c^2*d*x^6))/(105*d^4*Sqrt[c + d/x^2]
*x^8)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^7\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^3}+\frac{(ad-3bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}+\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{c^2(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2c^2\sqrt{c+\frac{d}{x^2}}(bc-ad)}{d^4}+\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}\right)$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

output
```
((2*c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (2*c*(3*b*c - 2*a*d)*(c + d/x^2
)^(3/2))/(3*d^4) + (2*(3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (2*b*(c +
 d/x^2)^(7/2))/(7*d^4))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

method	result	size
trager	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{105x^{6}d^{4}}$	91
gosper	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94
default	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94
risch	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)}{105\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{4}x^{8}}$	94
orering	$-\frac{\left(56a\,c^{2}d\,x^{6}-48b\,c^{3}x^{6}-28ac\,d^{2}x^{4}+24b\,c^{2}d\,x^{4}+21a\,d^{3}x^{2}-18bc\,d^{2}x^{2}+15b\,d^{3}\right)\left(c\,x^{2}+d\right)\left(a+\frac{b}{x^{2}}\right)}{105d^{4}\left(a\,x^{2}+b\right)x^{6}\sqrt{c+\frac{d}{x^{2}}}}$	106

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/105/x^6*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a
*d^3*x^2-18*b*c*d^2*x^2+15*b*d^3)/d^4*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{(8\,(6\,bc^3-7\,ac^2d)x^6-4\,(6\,bc^2d-7\,acd^2)x^4-15\,bd^3+3\,(6\,bcd^2-7\,ad^3)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*
d^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

Sympy [A] (verification not implemented)

Time = 3.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.32

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{\begin{cases} \frac{-\frac{2a\left(c^2\sqrt{c+\frac{d}{x^2}}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{2b\left(-c^3\sqrt{c+\frac{d}{x^2}}+c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for}\ d\neq 0 \\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x**7,x)
```

output
```
Piecewise(((-2*a*(c**2*sqrt(c + d/x**2) - 2*c*(c + d/x**2)**(3/2)/3 + (c +
 d/x**2)**(5/2)/5)/d**2 - 2*b*(-c**3*sqrt(c + d/x**2) + c**2*(c + d/x**2)*
*(3/2) - 3*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3)/d, Ne(d,
 0)), ((-a/(3*x**6) - b/(4*x**8))/sqrt(c), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$
$$= -\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}c^3}{d^4}\right)$$
$$-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^3}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/
x^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(
5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(87) = 174.

Time = 0.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx$$
$$= \frac{16\left(70\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{5}{2}}+210\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{7}{2}}-175\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6ac^{\frac{5}{2}}d-126\left(\sqrt{c}\right.\right.}{105}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2) + 210*(sqrt(c)*x - sq
rt(c*x^2 + d))^6*b*c^(7/2) - 175*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)
*d - 126*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d + 147*(sqrt(c)*x - sq
rt(c*x^2 + d))^4*a*c^(5/2)*d^2 + 42*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7
/2)*d^2 - 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^3 - 6*b*c^(7/2)*d
^3 + 7*a*c^(5/2)*d^4)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}\,(48\,b\,c^3-56\,a\,c^2\,d)}{105\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6} - \frac{\sqrt{c+\frac{d}{x^2}}\,(24\,b\,c^2-28\,a\,c\,d)}{105\,d^3\,x^2}-\frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d-6\,b\,c)}{35\,d^2\,x^4}$$

input
```
int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)
```

output
```
((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/
2))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) -
((c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^7}\,dx = \frac{-56\sqrt{c\,x^2+d}\,a\,c^2d\,x^6+28\sqrt{c\,x^2+d}\,ac\,d^2x^4-21\sqrt{c\,x^2+d}\,a\,d^3x^2+48\sqrt{c\,x^2+d}\,b\,c^3x^6-24\sqrt{c\,x^2+d}\,b}{105d^4x^7}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^7,x)
```

output
```
( - 56*sqrt(c*x**2 + d)*a*c**2*d*x**6 + 28*sqrt(c*x**2 + d)*a*c*d**2*x**4
- 21*sqrt(c*x**2 + d)*a*d**3*x**2 + 48*sqrt(c*x**2 + d)*b*c**3*x**6 - 24*s
qrt(c*x**2 + d)*b*c**2*d*x**4 + 18*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 15*sqr
t(c*x**2 + d)*b*d**3 + 56*sqrt(c)*a*c**2*d*x**7 - 48*sqrt(c)*b*c**3*x**7)/
(105*d**4*x**7)
```

3.176 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x}{15c^3}+\frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}x^3}{15c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^5}{5c}$$

output
```
-2/15*d*(-4*a*d+5*b*c)*(c+d/x^2)^(1/2)*x/c^3+1/15*(-4*a*d+5*b*c)*(c+d/x^2)
^(1/2)*x^3/c^2+1/5*a*(c+d/x^2)^(1/2)*x^5/c
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c+\frac{d}{x^2}}x(5bc(-2d+cx^2)+a(8d^2-4cdx^2+3c^2x^4))}{15c^3}$$

input
```
Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^
4)))/(15*c^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {955, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}dx}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

$$\Big\downarrow \text{803}$$

$$\frac{(5bc-4ad)\left(\frac{x^3\sqrt{c+\frac{d}{x^2}}}{3c}-\frac{2d\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}dx}{3c}\right)}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

$$\Big\downarrow \text{746}$$

$$\frac{\left(\frac{x^3\sqrt{c+\frac{d}{x^2}}}{3c}-\frac{2dx\sqrt{c+\frac{d}{x^2}}}{3c^2}\right)(5bc-4ad)}{5c}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

input
```
Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c + d/x^2]*x^5)/(5*c) + ((5*b*c - 4*a*d)*((-2*d*Sqrt[c + d/x^2]*x)
/(3*c^2) + (Sqrt[c + d/x^2]*x^3)/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{\left(3 a \,c^{2} x^{4}-4 a d \,x^{2} c +5 b \,c^{2} x^{2}+8 a \,d^{2}-10 d b c \right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 c^{3}}$	62
gosper	$\frac{\left(3 a \,c^{2} x^{4}-4 a d \,x^{2} c +5 b \,c^{2} x^{2}+8 a \,d^{2}-10 d b c \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67
default	$\frac{\left(3 a \,c^{2} x^{4}-4 a d \,x^{2} c +5 b \,c^{2} x^{2}+8 a \,d^{2}-10 d b c \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67
risch	$\frac{\left(3 a \,c^{2} x^{4}-4 a d \,x^{2} c +5 b \,c^{2} x^{2}+8 a \,d^{2}-10 d b c \right) \left(c \,x^{2}+d \right)}{15 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{3}}$	67
orering	$\frac{\left(3 a \,c^{2} x^{4}-4 a d \,x^{2} c +5 b \,c^{2} x^{2}+8 a \,d^{2}-10 d b c \right) \left(c \,x^{2}+d \right) x \left(a +\frac{b}{x^{2}}\right)}{15 c^{3} \left(a \,x^{2}+b \right) \sqrt{c +\frac{d}{x^{2}}}}$	77

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*x/c^3*(-(-c*x^
2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(3\,ac^2x^5+\left(5\,bc^2-4\,acd\right)x^3-2\,\left(5\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqr
t((c*x^2 + d)/x^2)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(76) = 152$.

Time = 2.79 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.12

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{12acd^{\frac{15}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{8ad^{\frac{17}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6} + \frac{b\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}$$

input
```
integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

output
```
3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**
5*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c*
*5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4
*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6)
 + 12*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d
**5*x**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*
x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d +
1)/(3*c) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c+\frac{d}{x^2}}dx\right)b}{3\,c^2} + \frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5 - 10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3 + 15\sqrt{c+\frac{d}{x^2}}d^2x\right)a}{15\,c^3}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d
/x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a
/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{2\left(5\,bcd^{\frac{3}{2}} - 4\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^3} - \frac{(bcd-ad^2)\sqrt{cx^2+d}}{c^3\mathrm{sgn}\,(x)} + \frac{3\,(cx^2+d)^{\frac{5}{2}}a + 5\,(cx^2+d)^{\frac{3}{2}}bc - 10\,(cx^2+d)^{\frac{3}{2}}ad}{15\,c^3\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*b*c*d^(3/2) - 4*a*d^(5/2))*sgn(x)/c^3 - (b*c*d - a*d^2)*sqrt(c*x^2
 + d)/(c^3*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a + 5*(c*x^2 + d)^(3/2)*b*c
 - 10*(c*x^2 + d)^(3/2)*a*d)/(c^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{x\sqrt{c+\frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4+5\,b\,c^2\,x^2-4\,a\,c\,d\,x^2-10\,b\,c\,d+8\,a\,d^2\right)}{15\,c^3}$$

input
```
int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a
*c*d*x^2))/(15*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{c\,x^2+d}\,(3a\,c^2x^4-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd)}{15c^3}$$

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x)
```

output
```
(sqrt(c*x**2 + d)*(3*a*c**2*x**4 - 4*a*c*d*x**2 + 8*a*d**2 + 5*b*c**2*x**2
 - 10*b*c*d))/(15*c**3)
```

3.177 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}x^3}{3c}$$

output
```
1/3*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)*x/c^2+1/3*a*(c+d/x^2)^(1/2)*x^3/c
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{\sqrt{c+\frac{d}{x^2}}x(3bc-2ad+acx^2)}{3c^2}$$

input
```
Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

output
```
(Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {955, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x^2}\right)}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}dx}{3c}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

$$\Big\downarrow \text{746}$$

$$\frac{x\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

input
```
Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

output
```
((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c
)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result	size
trager	$\frac{\left(a c \,x^{2}-2 a d +3 c b \right) x \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{3 c^{2}}$	39
gosper	$\frac{\left(a c \,x^{2}-2 a d +3 c b \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44
default	$\frac{\left(a c \,x^{2}-2 a d +3 c b \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44
risch	$\frac{\left(a c \,x^{2}-2 a d +3 c b \right) \left(c \,x^{2}+d \right)}{3 x \sqrt{\frac{c \,x^{2}+d}{x^{2}}}\, c^{2}}$	44
orering	$\frac{\left(a c \,x^{2}-2 a d +3 c b \right) \left(c \,x^{2}+d \right) x \left(a +\frac{b}{x^{2}}\right)}{3 c^{2} \left(a \,x^{2}+b \right) \sqrt{c +\frac{d}{x^{2}}}}$	54

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*c*x^2-2*a*d+3*b*c)*x/c^2*(-(-c*x^2-d)/x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\left(acx^3+(3\,bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}x^2\sqrt{\frac{cx^2}{d}+1}}{3c} - \frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2} + \frac{b\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c}$$

input
```
integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/
(3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{b\sqrt{c+\frac{d}{x^2}}x}{c} + \frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{c+\frac{d}{x^2}}dx\right)a}{3\,c^2}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output
```
b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x
)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,c^2}+\frac{(cx^2+d)^{\frac{3}{2}}a}{3\,c^2\mathrm{sgn}\,(x)}+\frac{\sqrt{cx^2+d}(bc-ad)}{c^2\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(3*b*c*sqrt(d) - 2*a*d^(3/2))*sgn(x)/c^2 + 1/3*(c*x^2 + d)^(3/2)*a/(c
^2*sgn(x)) + sqrt(c*x^2 + d)*(b*c - a*d)/(c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\,x^3\,\sqrt{c+\frac{d}{x^2}}\,\left(c-\frac{2\,d}{x^2}\right)}{3\,c^2}+\frac{b\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}$$

input
```
int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

output
```
(a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(
1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.55

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c\,x^2+d}\,(ac\,x^2-2ad+3bc)}{3c^2}$$

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x)
```

output
```
(sqrt(c*x**2 + d)*(a*c*x**2 - 2*a*d + 3*b*c))/(3*c**2)
```

3.178 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{c+\frac{d}{x^2}}x}{c}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{\sqrt{d}}$$

output
```
a*(c+d/x^2)^(1/2)*x/c-b*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx=\frac{a\sqrt{d}(d+cx^2)-bc\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{c\sqrt{d}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[
d]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {899, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{358}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}$$

$$\Big\downarrow \text{224}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-b\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}$$

$$\Big\downarrow \text{219}$$

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

input
```
Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

output
```
(a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{\sqrt{c\,x^{2}+d}\,\left(a\sqrt{c\,x^{2}+d}\,\sqrt{d}-b\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)c\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x c\sqrt{d}}$	73

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)
+d)/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.66

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)}{cd}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*
x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) +
 b*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d))/(c*d)]
```

Sympy [A] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}{c} - \frac{b\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

output
```
a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} + \frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

input

```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

output

```
a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c +
d/x^2)*x + sqrt(d)))/sqrt(d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\frac{\left(bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+a\sqrt{-d}\sqrt{d}\right)\mathrm{sgn}(x)}{c\sqrt{-d}} + \frac{\frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\sqrt{cx^2+d}a}{c}}{\mathrm{sgn}\,(x)}$$

input

```
integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

output

```
-(b*c*arctan(sqrt(d)/sqrt(-d)) + a*sqrt(-d)*sqrt(d))*sgn(x)/(c*sqrt(-d)) +
 (b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) + sqrt(c*x^2 + d)*a/c)/sgn(x
)
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)} - \frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

input
```
int((a + b/x^2)/(c + d/x^2)^(1/2),x)
```

output
```
(a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1)
) - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{\sqrt{c\,x^2+d}\,ad+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)bc-\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x+\sqrt{d}}{\sqrt{d}}\right)bc}{cd}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2),x)
```

output
```
(sqrt(c*x**2 + d)*a*d + sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d
))/sqrt(d))*b*c - sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqr
t(d))*b*c)/(c*d)
```

3.179 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx=-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}+\frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{2d^{3/2}}$$

output
```
-1/2*b*(c+d/x^2)^(1/2)/d/x+1/2*(-2*a*d+b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2
)/x)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.31

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx=\frac{-b\sqrt{d}(d+cx^2)+(bc-2ad)x^2\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{c+\frac{d}{x^2}}x^3}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

output
```
(-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt
[d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {959, 858, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^2\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \text{ 959}$$

$$-\frac{(bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}dx}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{(bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(bc-2ad)\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

output
```
-1/2*(b*Sqrt[c + d/x^2])/(d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c +
d/x^2]*x)])/(2*d^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{b\left(c\,x^{2}+d\right)}{2d\,x^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(2ad-cb\right)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}}{2d^{\frac{3}{2}}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	93
default	$-\frac{\sqrt{c\,x^{2}+d}\,\left(2a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)d^{2}x^{2}-\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)bcd\,x^{2}+d^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,b\right)}{2\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{3}d^{\frac{5}{2}}}$	105

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)-1/2*(2*a*d-b*c)/d^(3/2)*ln((2
*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \left[-\frac{(bc-2\,ad)\sqrt{d}x\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,bd\sqrt{\frac{cx^2+d}{x^2}}}{4\,d^2x}, -\frac{(bc-2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2\,d^2x}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/
x^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*
d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d) + b*d*sqrt((c*x^2
 + d)/x^2))/(d^2*x)]
```

Sympy [A] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{\sqrt{d}} - \frac{b\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx} + \frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x**2,x)
```

output
```
-a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*
d*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.98

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)b+\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{2\sqrt{d}}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/
x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((s
qrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = -\frac{c\left(\frac{(bc-2\,ad)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{c\sqrt{-d}d}+\frac{\sqrt{cx^2+d}b}{cdx^2}\right)}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*c*((b*c - 2*a*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(c*sqrt(-d)*d) + sq
rt(c*x^2 + d)*b/(c*d*x^2))/sgn(x)
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.54

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx = \begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if } d=0 \\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if } d\neq 0 \end{cases}$$

input
```
int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

output
```
piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/
x^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*lo
g(2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.36

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^2}\,dx$$

$$= \frac{-\sqrt{c\,x^2+d}\,bd+2\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)ad\,x^2-\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)bc\,x^2-2\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{}}{\sqrt{d}}\right.}{2d^2x^2}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^2,x)
```

output
```
( - sqrt(c*x**2 + d)*b*d + 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - s
qrt(d))/sqrt(d))*a*d*x**2 - sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sq
rt(d))/sqrt(d))*b*c*x**2 - 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + s
qrt(d))/sqrt(d))*a*d*x**2 + sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sq
rt(d))/sqrt(d))*b*c*x**2)/(2*d**2*x**2)
```

3.180 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} \, dx=-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{8d^{5/2}}$$

output
```
-1/4*b*(c+d/x^2)^(1/2)/d/x^3+1/8*(-4*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^2/x-1/8*
c*(-4*a*d+3*b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4} \, dx =\frac{-\sqrt{d}(d+cx^2)\left(2bd-3bcx^2+4adx^2\right)-c(3bc-4ad)x^4\sqrt{d+cx^2}\text{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

input
```
Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

output
```
(-(Sqrt[d]*(d + c*x^2)*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2)) - c*(3*b*c - 4*a*d
)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(8*d^(5/2)*Sqrt[c
+ d/x^2]*x^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 858, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^4\sqrt{c+\frac{d}{x^2}}}dx$$

$$\Big\downarrow \ 959$$

$$-\frac{(3bc-4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^4}dx}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \ 858$$

$$\frac{(3bc-4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \ 262$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\Big\downarrow \ 224$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

$$\downarrow 219$$

$$\frac{(3bc-4ad)\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)}{4d}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

input
```
Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

output
```
-1/4*(b*Sqrt[c + d/x^2])/(d*x^3) + ((3*b*c - 4*a*d)*(Sqrt[c + d/x^2]/(2*d*
x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(3/2))))/(4*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(c\,x^{2}+d)\left(4ad\,x^{2}-3bc\,x^{2}+2bd\right)}{8d^{2}x^{5}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{c(4ad-3cb)\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}}{8d^{\frac{5}{2}}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$
default	$-\frac{\sqrt{c\,x^{2}+d}\,\left(-4\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)ac\,d^{2}x^{4}+3\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)b\,c^{2}d\,x^{4}+4d^{\frac{5}{2}}\sqrt{c\,x^{2}+d}\,a\,x^{2}-3d^{\frac{3}{2}}\sqrt{c\,x^{2}+d}\,bc\,x^{2}+2d^{\frac{5}{2}}\sqrt{c\,x}}{8\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x^{5}d^{\frac{7}{2}}}$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+d)*(4*a*d*x^2-3*b*c*x^2+2*b*d)/d^2/x^5/((c*x^2+d)/x^2)^(1/2)+1
/8*c*(4*a*d-3*b*c)/d^(5/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)/((c*x^2+d
)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.10

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = \left[-\frac{(3\,bc^2-4\,acd)\sqrt{d}x^3\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(2\,bd^2-(3\,bcd-4\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3}, \frac{(3\,bc^2-4\,ac}{}\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*
x^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*
x^2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqr
t(-d)*x*sqrt((c*x^2 + d)/x^2)/d) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^3)]
```

Sympy [A] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx = -\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}x^5\sqrt{1+\frac{d}{cx^2}}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(1/2)/x**4,x)
```

output
```
-a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(
2*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*
d*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(
5/2)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(77) = 154$.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.15

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$$
$$= -\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a$$
$$+\frac{1}{16}b\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-5\sqrt{c+\frac{d}{x^2}}c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2d^2x^4-2\left(c+\frac{d}{x^2}\right)d^3x^2+d^4}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/
x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^
2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2)
 + 2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2
)^2*d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$$
$$= \frac{\frac{(3\,bc^3-4\,ac^2d)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d^2}+\frac{3\,(cx^2+d)^{\frac{3}{2}}bc^3-4\,(cx^2+d)^{\frac{3}{2}}ac^2d-5\,\sqrt{cx^2+d}bc^3d+4\,\sqrt{cx^2+d}ac^2d^2}{c^2d^2x^4}}{8\,c\,\mathrm{csgn}\,(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/8*((3*b*c^3 - 4*a*c^2*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2)
 + (3*(c*x^2 + d)^(3/2)*b*c^3 - 4*(c*x^2 + d)^(3/2)*a*c^2*d - 5*sqrt(c*x^2
 + d)*b*c^3*d + 4*sqrt(c*x^2 + d)*a*c^2*d^2)/(c^2*d^2*x^4))/(c*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx=\int \frac{a+\frac{b}{x^2}}{x^4\,\sqrt{c+\frac{d}{x^2}}}\,dx$$

input
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
```

output
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.00

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}x^4}\,dx$$
$$=\frac{-4\sqrt{c\,x^2+d}\,a\,d^2x^2+3\sqrt{c\,x^2+d}\,bcd\,x^2-2\sqrt{c\,x^2+d}\,b\,d^2-4\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)acd\,x^4+3\sqrt{d}\log}{8d^3x^4}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(c*x**2 + d)*a*d**2*x**2 + 3*sqrt(c*x**2 + d)*b*c*d*x**2 - 2*sqr
t(c*x**2 + d)*b*d**2 - 4*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(
d))/sqrt(d))*a*c*d*x**4 + 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sq
rt(d))/sqrt(d))*b*c**2*x**4 + 4*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x
+ sqrt(d))/sqrt(d))*a*c*d*x**4 - 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)
*x + sqrt(d))/sqrt(d))*b*c**2*x**4)/(8*d**3*x**4)
```

3.181 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{d(bc-ad)}{c^3\sqrt{c+\frac{d}{x^2}}} + \frac{(4bc-7ad)\sqrt{c+\frac{d}{x^2}}x^2}{8c^3} + \frac{a\sqrt{c+\frac{d}{x^2}}x^4}{4c^2} - \frac{3d(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

output
```
d*(-a*d+b*c)/c^3/(c+d/x^2)^(1/2)+1/8*(-7*a*d+4*b*c)*(c+d/x^2)^(1/2)*x^2/c^
3+1/4*a*(c+d/x^2)^(1/2)*x^4/c^2-3/8*d*(-5*a*d+4*b*c)*arctanh((c+d/x^2)^(1/
2)/c^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{c}x(4bc(3d+cx^2)+a(-15d^2-5cdx^2+2c^2x^4))+24bcd\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx}}\right.}{8c^{7/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

output
```
(Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 2
4*b*c*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] +
 30*a*d^2*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])
])/(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^6}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\int \frac{x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{4c}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\int\frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{\int\frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{2\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^4}{2c\sqrt{c+\frac{d}{x^2}}}-\frac{(4bc-5ad)\left(-\frac{3d\left(\frac{2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{4c}\right)$$

input
```
Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

output
```
((a*x^4)/(2*c*Sqrt[c + d/x^2]) - ((4*b*c - 5*a*d)*(-(x^2/(c*Sqrt[c + d/x^2
])) - (3*d*(2/(c*Sqrt[c + d/x^2]) - (2*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c
^(3/2)))/(2*c)))/(4*c))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(c\,x^{2}+d\right)\left(2c^{\frac{7}{2}}a\,x^{5}-5c^{\frac{5}{2}}ad\,x^{3}+4c^{\frac{7}{2}}b\,x^{3}-15c^{\frac{3}{2}}a\,d^{2}x+12c^{\frac{5}{2}}bdx+15\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}\,ac\,d^{2}-12\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c}\right.}{8\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c^{\frac{9}{2}}}$
risch	$\frac{\left(2ac\,x^{2}-7ad+4cb\right)\left(c\,x^{2}+d\right)}{8c^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{d\left(3c(5ad-4cb)\left(-\frac{x}{c\sqrt{c\,x^{2}+d}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)}{c^{\frac{3}{2}}}\right)+\frac{7adx}{\sqrt{c\,x^{2}+d}}-\frac{4bcx}{\sqrt{c\,x^{2}+d}}\right)\sqrt{c\,x^{2}+d}}{8c^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*x^2+d)*(2*c^(7/2)*a*x^5-5*c^(5/2)*a*d*x^3+4*c^(7/2)*b*x^3-15*c^(3/2
)*a*d^2*x+12*c^(5/2)*b*d*x+15*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2
)*a*c*d^2-12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*b*c^2*d)/((c*x^
2+d)/x^2)^(3/2)/x^3/c^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \left[-\frac{3\left(4\,bcd^2-5\,ad^3+(4\,bc^2d-5\,acd^2)x^2\right)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2}{16\left(c^5x^2+c^4d\right)}\right.$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(
-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*
b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d
^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) +
 (2*a*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)
*sqrt((c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

Sympy [A] (verification not implemented)

Time = 66.64 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{5\sqrt{d}x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}}+\frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right)+b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$

input
```
integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

output
```
a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x
**2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sq
rt(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1))
+ 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(
2*c**(5/2)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(96) = 192$.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.89

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx =$$

$$-\frac{1}{16}a\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2d^2-25\left(c+\frac{d}{x^2}\right)cd^2+8c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4+\sqrt{c+\frac{d}{x^2}}c^5}+\frac{15\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)$$
$$+\frac{1}{4}b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
 d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^
2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) +
1/4*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2
)*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(x^2\left(\frac{2\,ax^2}{c\mathrm{sgn}(x)}+\frac{4\,bc^4\mathrm{sgn}(x)-5\,ac^3d\mathrm{sgn}(x)}{c^5}\right)+\frac{3\left(4\,bc^3d\mathrm{sgn}(x)-5\,ac^2d^2\mathrm{sgn}(x)\right)}{c^5}\right)x}{8\sqrt{cx^2+d}} - \frac{3\left(4\,bcd\log\left(|d|\right)-5\,ad^2\log\left(|d|\right)\right)\mathrm{sgn}(x)}{16\,c^{\frac{7}{2}}} + \frac{3\left(4\,bcd-5\,ad^2\right)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(x^2*(2*a*x^2/(c*sgn(x)) + (4*b*c^4*sgn(x) - 5*a*c^3*d*sgn(x))/c^5) +
3*(4*b*c^3*d*sgn(x) - 5*a*c^2*d^2*sgn(x))/c^5)*x/sqrt(c*x^2 + d) - 3/16*(4
*b*c*d*log(abs(d)) - 5*a*d^2*log(abs(d)))*sgn(x)/c^(7/2) + 3/8*(4*b*c*d -
5*a*d^2)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(7/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}} - \frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}} + \frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}} - \frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}} + \frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}} + \frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}} - \frac{5\,a\,d\,x^2}{8\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (
b*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/
(2*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3
*b*d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{2\sqrt{c\,x^2+d}\,a\,c^3x^5-5\sqrt{c\,x^2+d}\,a\,c^2d\,x^3-15\sqrt{c\,x^2+d}\,ac\,d^2x+4\sqrt{c\,x^2+d}\,b\,c^3x^3+12}{}$$

input
```
int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x)
```

output
```
(2*sqrt(c*x**2 + d)*a*c**3*x**5 - 5*sqrt(c*x**2 + d)*a*c**2*d*x**3 - 15*sq
rt(c*x**2 + d)*a*c*d**2*x + 4*sqrt(c*x**2 + d)*b*c**3*x**3 + 12*sqrt(c*x**
2 + d)*b*c**2*d*x + 15*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))
*a*c*d**2*x**2 + 15*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*
d**3 - 12*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c**2*d*x**
2 - 12*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d**2 - 10*s
qrt(c)*a*c*d**2*x**2 - 10*sqrt(c)*a*d**3 + 9*sqrt(c)*b*c**2*d*x**2 + 9*sqr
t(c)*b*c*d**2)/(8*c**4*(c*x**2 + d))
```

3.182 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 83

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=-\frac{bc-ad}{c^2\sqrt{c+\frac{d}{x^2}}}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c^2}+\frac{(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

output
```
-(-a*d+b*c)/c^2/(c+d/x^2)^(1/2)+1/2*a*(c+d/x^2)^(1/2)*x^2/c^2+1/2*(-3*a*d+
2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{6ad\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d}-\sqrt{d+cx^2}}\right)+\sqrt{c}\left(-2bcx+3adx+acx^3+4b\sqrt{c}\sqrt{d+cx^2}\text{arcta}\right.}{2c^{5/2}\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

output
```
(6*a*d*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/(Sqrt[d] - Sqrt[d + c*x^2])] +
Sqrt[c]*(-2*b*c*x + 3*a*d*x + a*c*x^3 + 4*b*Sqrt[c]*Sqrt[d + c*x^2]*ArcTan
h[(Sqrt[c]*x)/(-Sqrt[d] + Sqrt[d + c*x^2])]))/(2*c^(5/2)*Sqrt[c + d/x^2]*x
)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 948$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}}{2c}\right)$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{2\int\frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}+\frac{2}{c\sqrt{c+\frac{d}{x^2}}}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{ax^2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(2bc-3ad)\left(\frac{2}{c\sqrt{c+\frac{d}{x^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}\right)$$

input
```
Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

output
```
((a*x^2)/(c*Sqrt[c + d/x^2]) - ((2*b*c - 3*a*d)*(2/(c*Sqrt[c + d/x^2]) - (
2*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^(3/2)))/(2*c))/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37

method	result	size
default	$\frac{(c\,x^{2}+d)\left(c^{\frac{5}{2}}\,a\,x^{3}+3c^{\frac{3}{2}}\,adx-2c^{\frac{5}{2}}\,bx-3\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}\,acd+2\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}\,b\,c^{2}\right)}{2\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c^{\frac{7}{2}}}$	114
risch	$\frac{a(c\,x^{2}+d)}{2c^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(\frac{adx}{\sqrt{c\,x^{2}+d}}+c(3ad-2cb)\left(-\frac{x}{c\sqrt{c\,x^{2}+d}}+\frac{\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)}{c^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^{2}+d}}{2c^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	119

input
```
int((a+b/x^2)*x/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^2+d)*(c^(5/2)*a*x^3+3*c^(3/2)*a*d*x-2*c^(5/2)*b*x-3*ln(c^(1/2)*x+
(c*x^2+d)^(1/2))*(c*x^2+d)^(1/2)*a*c*d+2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*
x^2+d)^(1/2)*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \left[-\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{c}\log\left(-2\,cx^2+2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\,(ac^2x\cdots}{4\,(c^4x^2+c^3d)}\right.$$

$$\left.-\frac{(2\,bcd-3\,ad^2+(2\,bc^2-3\,acd)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)-(ac^2x^4-(2\,bc^2-3\,acd)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(c^4x^2+c^3d)}\right]$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a
*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*
d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 +
d)/x^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
 d)/x^2))/(c^4*x^2 + c^3*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(71) = 142.

Time = 28.97 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)$$
$$+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right.$$
$$\left.-\frac{c^2d\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^2d\log\left(\sqrt{1+\frac{d}{cx^2}}+1\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}\right)$$

input
```
integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

output
```
a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2
/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sq
rt(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x
**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**
2)) + 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(
9/2)*x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(
9/2)*x**2 + 2*c**(7/2)*d))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(69) = 138$.

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.73

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{4}a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}c^3}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right) - \frac{1}{2}b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}c}\right)$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2
)*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/
c^(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(
c)))/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{x\left(\frac{ax^2}{c\mathrm{sgn}(x)} - \frac{2\,bc^2\mathrm{sgn}(x)-3\,acd\mathrm{sgn}(x)}{c^3}\right)}{2\,\sqrt{cx^2+d}} + \frac{(2\,bc\log(|d|)-3\,ad\log(|d|))\mathrm{sgn}(x)}{4\,c^{\frac{5}{2}}} - \frac{(2\,bc-3\,ad)\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{2\,c^{\frac{5}{2}}\,\mathrm{sgn}(x)}$$

input
```
integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(a*x^2/(c*sgn(x)) - (2*b*c^2*sgn(x) - 3*a*c*d*sgn(x))/c^3)/sqrt(c*x^
2 + d) + 1/4*(2*b*c*log(abs(d)) - 3*a*d*log(abs(d)))*sgn(x)/c^(5/2) - 1/2*
(2*b*c - 3*a*d)*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(5/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{b}{c\,\sqrt{c+\frac{d}{x^2}}} + \frac{a\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}} - \frac{3\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}} + \frac{3\,a\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (
a*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/
(2*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.52

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{4\sqrt{c\,x^2+d}\,a\,c^2x^3+12\sqrt{c\,x^2+d}\,acdx-8\sqrt{c\,x^2+d}\,b\,c^2x-12\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)ac}{}$$

input
```
int((a+b/x^2)*x/(c+d/x^2)^(3/2),x)
```

output
```
(4*sqrt(c*x**2 + d)*a*c**2*x**3 + 12*sqrt(c*x**2 + d)*a*c*d*x - 8*sqrt(c*x
**2 + d)*b*c**2*x - 12*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))
*a*c*d*x**2 - 12*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*d**
2 + 8*sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c**2*x**2 + 8*
sqrt(c)*log((sqrt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*b*c*d + 9*sqrt(c)*a*c*
d*x**2 + 9*sqrt(c)*a*d**2 - 8*sqrt(c)*b*c**2*x**2 - 8*sqrt(c)*b*c*d)/(8*c*
*3*(c*x**2 + d))
```

3.183 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx = \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

output
```
(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx = \frac{\sqrt{c}(bc-ad)x-ad\sqrt{d+cx^2}\log\left(-\sqrt{c}x+\sqrt{d+cx^2}\right)}{c^{3/2}d\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

output
```
(Sqrt[c]*(b*c - a*d)*x - a*d*Sqrt[d + c*x^2]*Log[-(Sqrt[c]*x) + Sqrt[d + c
*x^2]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}-\frac{a\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x^2}}{c}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}-\frac{2a\int \frac{1}{\frac{1}{dx^4}-\frac{c}{d}}d\sqrt{c+\frac{d}{x^2}}}{cd}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(bc-ad)}{cd\sqrt{c+\frac{d}{x^2}}}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

output
```
((2*(b*c - a*d))/(c*d*Sqrt[c + d/x^2]) + (2*a*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/c^(3/2))/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{\left(c\,x^{2}+d\right)\left(c^{\frac{5}{2}} b x-c^{\frac{3}{2}} a d x+\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)\sqrt{c\,x^{2}+d}\,a c d\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} d\,c^{\frac{5}{2}}}$	75

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+d)*(c^(5/2)*b*x-c^(3/2)*a*d*x+ln(c^(1/2)*x+(c*x^2+d)^(1/2))*(c*x^2+
d)^(1/2)*a*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/d/c^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = \left[\frac{2\,(bc^2-acd)x^2\sqrt{\frac{cx^2+d}{x^2}}+(acdx^2+ad^2)\sqrt{c}\log\left(-2\,cx^2-2\,\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\,(c^3dx^2+c^2d^2)}, \frac{(bc$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sq
rt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2
+ c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^
2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d
*x^2 + c^2*d^2)]
```

Sympy [A] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = \begin{cases} \frac{2\left(-\frac{ad\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{2c\sqrt{-c}}-\frac{ad-bc}{2c\sqrt{c+\frac{d}{x^2}}}\right)}{d} & \text{for } d\neq 0 \\ \frac{-a\log\left(-\frac{b}{x^2}\right)-\frac{b}{x^2}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

output
```
Piecewise((2*(-a*d*atan(sqrt(c + d/x**2)/sqrt(-c))/(2*c*sqrt(-c)) - (a*d -
 b*c)/(2*c*sqrt(c + d/x**2)))/d, Ne(d, 0)), ((-a*log(-b/x**2) - b/x**2)/(2
*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx = -\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}c}\right)+\frac{b}{\sqrt{c+\frac{d}{x^2}}d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/
2) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx = \frac{a\log\left(|d|\right)\operatorname{sgn}(x)}{2\,c^{\frac{3}{2}}} + \frac{(bc\operatorname{sgn}(x)-ad\operatorname{sgn}(x))x}{\sqrt{cx^2+d}cd} - \frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2+d}\right|\right)}{c^{\frac{3}{2}}\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*a*log(abs(d))*sgn(x)/c^(3/2) + (b*c*sgn(x) - a*d*sgn(x))*x/(sqrt(c*x^2
 + d)*c*d) - a*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/(c^(3/2)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}} - \frac{a}{c\sqrt{c+\frac{d}{x^2}}} + \frac{b\sqrt{x^2}}{d\sqrt{c\,x^2+d}}$$

input
```
int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
```

output
```
(a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (
b*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.56

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x}\,dx = \frac{-\sqrt{c\,x^2+d}\,acdx+\sqrt{c\,x^2+d}\,b\,c^2x+\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}}\right)acd\,x^2+\sqrt{c}\log\left(\frac{\sqrt{c\,x^2+d}+}{\sqrt{d}}\right.}{c^2d\,(c\,x^2+d)}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x)
```

output
```
( - sqrt(c*x**2 + d)*a*c*d*x + sqrt(c*x**2 + d)*b*c**2*x + sqrt(c)*log((sq
rt(c*x**2 + d) + sqrt(c)*x)/sqrt(d))*a*c*d*x**2 + sqrt(c)*log((sqrt(c*x**2
 + d) + sqrt(c)*x)/sqrt(d))*a*d**2 - sqrt(c)*a*c*d*x**2 - sqrt(c)*a*d**2 +
 sqrt(c)*b*c**2*x**2 + sqrt(c)*b*c*d)/(c**2*d*(c*x**2 + d))
```

3.184 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

output
```
-(-a*d+b*c)/d^2/(c+d/x^2)^(1/2)-b*(c+d/x^2)^(1/2)/d^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx=\frac{adx^2-b(d+2cx^2)}{d^2\sqrt{c+\frac{d}{x^2}}x^2}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

output
```
(a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^3\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{b}{d\sqrt{c+\frac{d}{x^2}}}+\frac{ad-bc}{d\left(c+\frac{d}{x^2}\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2(bc-ad)}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{2b\sqrt{c+\frac{d}{x^2}}}{d^2}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

output
```
((-2*(b*c - a*d))/(d^2*Sqrt[c + d/x^2]) - (2*b*Sqrt[c + d/x^2])/d^2)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

method	result	size
gosper	$\frac{(a d\,x^{2}-2 b c\,x^{2}-b d)\,(c\,x^{2}+d)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{2} x^{4}}$	46
default	$\frac{(a d\,x^{2}-2 b c\,x^{2}-b d)\,(c\,x^{2}+d)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{2} x^{4}}$	46
trager	$\frac{(a d\,x^{2}-2 b c\,x^{2}-b d)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{d^{2}(c\,x^{2}+d)}$	49
risch	$-\frac{b(c\,x^{2}+d)}{d^{2} x^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{a d-c b}{d^{2}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	56
orering	$\frac{(a d\,x^{2}-2 b c\,x^{2}-b d)\,(c\,x^{2}+d)\left(a+\frac{b}{x^{2}}\right)}{d^{2}(a\,x^{2}+b)x^{2}\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	58

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = -\frac{((2\,bc-ad)x^2+bd)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2+d^3}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
-((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}} - \frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d \neq 0 \\ \frac{-\frac{a}{2x^2}-\frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
```

output
```
Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x
**2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = -b\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)+\frac{a}{\sqrt{c+\frac{d}{x^2}}\,d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{2\,b\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)d\mathrm{sgn}\,(x)} - \frac{(bc-ad)x}{\sqrt{cx^2+d}d^2\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*b*sqrt(c)/(((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)*d*sgn(x)) - (b*c - a*d)
*x/(sqrt(c*x^2 + d)*d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^3}\,dx = \frac{x\,\sqrt{c+\frac{d}{x^2}}\,\left(x^2\,\left(\frac{a}{d}-\frac{2\,b\,c}{d^2}\right)-\frac{b}{d}\right)}{c\,x^3+d\,x}$$

input
```
int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
```

output
```
(x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.40

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3}\,dx = \frac{\sqrt{c\,x^2+d}\,acd\,x^2-2\sqrt{c\,x^2+d}\,b\,c^2x^2-\sqrt{c\,x^2+d}\,bcd+\sqrt{c}\,acd\,x^3+\sqrt{c}\,a\,d^2x-2\sqrt{}}{c\,d^2x\,(c\,x^2+d)}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x)
```

output
```
(sqrt(c*x**2 + d)*a*c*d*x**2 - 2*sqrt(c*x**2 + d)*b*c**2*x**2 - sqrt(c*x**
2 + d)*b*c*d + sqrt(c)*a*c*d*x**3 + sqrt(c)*a*d**2*x - 2*sqrt(c)*b*c**2*x*
*3 - 2*sqrt(c)*b*c*d*x)/(c*d**2*x*(c*x**2 + d))
```

3.185 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx=\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

output
```
c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)+(-a*d+2*b*c)*(c+d/x^2)^(1/2)/d^3-1/3*b*(c
+d/x^2)^(3/2)/d^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx=\frac{-3adx^2(d+2cx^2)+b(-d^2+4cdx^2+8c^2x^4)}{3d^3\sqrt{c+\frac{d}{x^2}}x^4}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

output
```
(-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[
c + d/x^2]*x^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^5\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{\sqrt{c+\frac{d}{x^2}}b}{d^2}+\frac{ad-2bc}{d^2\sqrt{c+\frac{d}{x^2}}}+\frac{c(bc-ad)}{d^2\left(c+\frac{d}{x^2}\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+\frac{d}{x^2}}(2bc-ad)}{d^3}+\frac{2c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{2b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

output
```
((2*c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + (2*(2*b*c - a*d)*Sqrt[c + d/x^2
])/d^3 - (2*b*(c + d/x^2)^(3/2))/(3*d^3))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

method	result	size
gosper	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\left(c\,x^{2}+d\right)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
default	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\left(c\,x^{2}+d\right)}{3\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{3}x^{6}}$	69
trager	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3x^{2}d^{3}\left(c\,x^{2}+d\right)}$	75
risch	$-\frac{\left(c\,x^{2}+d\right)\left(3ad\,x^{2}-5bc\,x^{2}+bd\right)}{3d^{3}x^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{\left(ad-cb\right)c}{d^{3}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	75
orering	$-\frac{\left(6acd\,x^{4}-8b\,c^{2}x^{4}+3a\,d^{2}x^{2}-4bcd\,x^{2}+b\,d^{2}\right)\left(c\,x^{2}+d\right)\left(a+\frac{b}{x^{2}}\right)}{3d^{3}\left(a\,x^{2}+b\right)x^{4}\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	81

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c
*x^2+d)/x^2)^(3/2)/d^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx=\frac{\left(2\left(4\,bc^2-3\,acd\right)x^4-bd^2+\left(4\,bcd-3\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(cd^3x^4+d^4x^2\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx=\begin{cases}\frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{6d^2}-\frac{c(ad-bc)}{2d^2\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{2d^2}\right)}{d} & \text{for } d\neq 0\\ \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{2c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(3/2)/(6*d**2) - c*(a*d - b*c)/(2*d**2*sqrt
(c + d/x**2)) - sqrt(c + d/x**2)*(a*d - 2*b*c)/(2*d**2))/d, Ne(d, 0)), ((-
a/(2*x**4) - b/(3*x**6))/(2*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = -\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3} - \frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3} - \frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right) - a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c +
d/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(60) = 120$.

Time = 0.40 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.76

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^5}\,dx = \frac{(bc^2-acd)x}{\sqrt{cx^2+d}\,d^3\operatorname{sgn}(x)} - \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}} - 3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}d - 12\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2bc^{\frac{3}{2}}d + 6\left(\sqrt{c}x-\sqrt{c\right.}\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2-d\right)^3d^2\operatorname{sgn}(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
(b*c^2 - a*c*d)*x/(sqrt(c*x^2 + d)*d^3*sgn(x)) - 2/3*(3*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*b*c^(3/2) - 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)*d -
12*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d + 6*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*sqrt(c)*d^2 + 5*b*c^(3/2)*d^2 - 3*a*sqrt(c)*d^3)/(((sqrt(c)*x -
 sqrt(c*x^2 + d))^2 - d)^3*d^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4+6\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+3\,a\,d^2\,x^2+b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2+d\right)}$$

input
```
int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b
*c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.06

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5}\,dx = \frac{-6\sqrt{c\,x^2+d}\,acd\,x^4-3\sqrt{c\,x^2+d}\,a\,d^2x^2+8\sqrt{c\,x^2+d}\,b\,c^2x^4+4\sqrt{c\,x^2+d}\,bcd\,x^2-}{3d^3x^3\left(c\,x^2+d\right)}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x)
```

output
```
( - 6*sqrt(c*x**2 + d)*a*c*d*x**4 - 3*sqrt(c*x**2 + d)*a*d**2*x**2 + 8*sqr
t(c*x**2 + d)*b*c**2*x**4 + 4*sqrt(c*x**2 + d)*b*c*d*x**2 - sqrt(c*x**2 +
d)*b*d**2 + 6*sqrt(c)*a*c*d*x**5 + 6*sqrt(c)*a*d**2*x**3 - 8*sqrt(c)*b*c**
2*x**5 - 8*sqrt(c)*b*c*d*x**3)/(3*d**3*x**3*(c*x**2 + d))
```

3.186 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx = -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}} - \frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4} + \frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

output
```
-c^2*(-a*d+b*c)/d^4/(c+d/x^2)^(1/2)-c*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^4+1
/3*(-a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4-1/5*b*(c+d/x^2)^(5/2)/d^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx = \frac{-5adx^2(d^2-4cdx^2-8c^2x^4)-3b(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)}{15d^4\sqrt{c+\frac{d}{x^2}}x^6}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

output
```
(-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2
*d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^7\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \; 86$$

$$-\frac{1}{2}\int \left(-\frac{(bc-ad)c^2}{d^3\left(c+\frac{d}{x^2}\right)^{3/2}}+\frac{(3bc-2ad)c}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{d^3}+\frac{(ad-3bc)\sqrt{c+\frac{d}{x^2}}}{d^3}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{2\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{2c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{2b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

output
```
((-2*c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2]) - (2*c*(3*b*c - 2*a*d)*Sqrt[c
+ d/x^2])/d^4 + (2*(3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (2*b*(c + d/
x^2)^(5/2))/(5*d^4))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{4} x^{8}}$	94
default	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \left(c \,x^{2}+d \right)}{15 \left(\frac{c \,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} d^{4} x^{8}}$	94
risch	$\frac{\left(c \,x^{2}+d \right) \left(25 a c d \,x^{4}-33 b \,c^{2} x^{4}-5 a \,d^{2} x^{2}+9 b c d \,x^{2}-3 b \,d^{2}\right)}{15 d^{4} x^{6} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}+\frac{c^{2} \left(a d -c b \right)}{d^{4} \sqrt{\frac{c \,x^{2}+d}{x^{2}}}}$	99
trager	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \sqrt{-\frac{-c \,x^{2}-d}{x^{2}}}}{15 x^{4} d^{4} \left(c \,x^{2}+d \right)}$	100
orering	$\frac{\left(40 a \,c^{2} d \,x^{6}-48 b \,c^{3} x^{6}+20 a c \,d^{2} x^{4}-24 b \,c^{2} d \,x^{4}-5 a \,d^{3} x^{2}+6 b c \,d^{2} x^{2}-3 b \,d^{3}\right) \left(c \,x^{2}+d \right) \left(a +\frac{b}{x^{2}}\right)}{15 d^{4} \left(a \,x^{2}+b \right) x^{6} \left(c +\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	106

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^
2+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = -\frac{\left(8\left(6\,bc^3-5\,ac^2d\right)x^6+4\left(6\,bc^2d-5\,acd^2\right)x^4+3\,bd^3-\left(6\,bcd^2-5\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(cd^4x^6+d^5x^4\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d
^3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4
)
```

Sympy [A] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{10d^3}+\frac{c^2(ad-bc)}{2d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}(ad-3bc)}{6d^3}-\frac{\sqrt{c+\frac{d}{x^2}}\left(-2acd+3bc^2\right)}{2d^3}\right)}{d} & \text{for } d\neq 0 \\ \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(5/2)/(10*d**3) + c**2*(a*d - b*c)/(2*d**3*
sqrt(c + d/x**2)) - (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(6*d**3) - sqrt(c +
d/x**2)*(-2*a*c*d + 3*b*c**2)/(2*d**3))/d, Ne(d, 0)), ((-a/(3*x**6) - b/(4
*x**8))/(2*c**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{1}{5}b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right) - \frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}c}{d^3}-\frac{3c^2}{\sqrt{c+\frac{d}{x^2}}d^3}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/
x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3
 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(88) = 176.

Time = 0.86 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.03

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^7}\,dx = -\frac{(bc^3-ac^2d)x}{\sqrt{cx^2+d}d^4\mathrm{sgn}\left(x\right)} + \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}-15\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}d-90\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^6bc^{\frac{5}{2}}d+90\left(\sqrt{c}x-\right.\right.}{}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-(b*c^3 - a*c^2*d)*x/(sqrt(c*x^2 + d)*d^4*sgn(x)) + 2/15*(15*(sqrt(c)*x -
sqrt(c*x^2 + d))^8*b*c^(5/2) - 15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2
)*d - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(5/2)*d + 90*(sqrt(c)*x - sqr
t(c*x^2 + d))^6*a*c^(3/2)*d^2 + 240*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5
/2)*d^2 - 160*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3 - 150*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(5/2)*d^3 + 110*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^4 + 33*b*c^(5/2)*d^4 - 25*a*c^(3/2)*d^5)/(((sqrt(c)*x - sqrt
(c*x^2 + d))^2 - d)^5*d^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = -\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3\right)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

input
```
int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

output
```
-((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
 - 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^
2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7}\,dx = \frac{40\sqrt{c\,x^2+d}\,a\,c^2 d\,x^6+20\sqrt{c\,x^2+d}\,ac\,d^2x^4-5\sqrt{c\,x^2+d}\,a\,d^3x^2-48\sqrt{c\,x^2+d}\,b\,c^3x}{}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x)
```

output

```
(40*sqrt(c*x**2 + d)*a*c**2*d*x**6 + 20*sqrt(c*x**2 + d)*a*c*d**2*x**4 - 5
*sqrt(c*x**2 + d)*a*d**3*x**2 - 48*sqrt(c*x**2 + d)*b*c**3*x**6 - 24*sqrt(
c*x**2 + d)*b*c**2*d*x**4 + 6*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 3*sqrt(c*x*
*2 + d)*b*d**3 - 40*sqrt(c)*a*c**2*d*x**7 - 40*sqrt(c)*a*c*d**2*x**5 + 48*
sqrt(c)*b*c**3*x**7 + 48*sqrt(c)*b*c**2*d*x**5)/(15*d**4*x**5*(c*x**2 + d)
)
```

3.187 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5}+\frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

output
```
c^3*(-a*d+b*c)/d^5/(c+d/x^2)^(1/2)+c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(1/2)/d^5-
c*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^5+1/5*(-a*d+4*b*c)*(c+d/x^2)^(5/2)/d^5-1/
7*b*(c+d/x^2)^(7/2)/d^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{-7adx^2(d^3-2cd^2x^2+8c^2dx^4+16c^3x^6)+b(-5d^4+8cd^3x^2-16c^2d^2x^4+64c^3dx^6}{35d^5\sqrt{c+\frac{d}{x^2}}\,x^8}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

output
```
(-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8
*c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c
+ d/x^2]*x^8)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^9\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2}\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{1}{2}\int \left(\frac{(bc-ad)c^3}{d^4\left(c+\frac{d}{x^2}\right)^{3/2}}-\frac{(4bc-3ad)c^2}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{3(2bc-ad)\sqrt{c+\frac{d}{x^2}}c}{d^4}+\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{d^4}+\frac{(ad-4bc)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^4}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{2c^2\sqrt{c+\frac{d}{x^2}}(4bc-3ad)}{d^5}+\frac{2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{2c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{2b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}\right)$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

output
```
((2*c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (2*c^2*(4*b*c - 3*a*d)*Sqrt[c
 + d/x^2])/d^5 - (2*c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + (2*(4*b*c - a
*d)*(c + d/x^2)^(5/2))/(5*d^5) - (2*b*(c + d/x^2)^(7/2))/(7*d^5))/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{\left(112a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}+56a\,c^{2}d^{2}x^{6}-64b\,c^{3}d\,x^{6}-14ac\,d^{3}x^{4}+16b\,c^{2}d^{2}x^{4}+7a\,d^{4}x^{2}-8bc\,d^{3}x^{2}+5b\,d^{4}\right)\left(c\,x^{2}+d\right)}{35\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{5}x^{10}}$	118
default	$-\frac{\left(112a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}+56a\,c^{2}d^{2}x^{6}-64b\,c^{3}d\,x^{6}-14ac\,d^{3}x^{4}+16b\,c^{2}d^{2}x^{4}+7a\,d^{4}x^{2}-8bc\,d^{3}x^{2}+5b\,d^{4}\right)\left(c\,x^{2}+d\right)}{35\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}d^{5}x^{10}}$	118
trager	$-\frac{\left(112a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}+56a\,c^{2}d^{2}x^{6}-64b\,c^{3}d\,x^{6}-14ac\,d^{3}x^{4}+16b\,c^{2}d^{2}x^{4}+7a\,d^{4}x^{2}-8bc\,d^{3}x^{2}+5b\,d^{4}\right)\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{35x^{6}d^{5}\left(c\,x^{2}+d\right)}$	124
risch	$-\frac{\left(c\,x^{2}+d\right)\left(77a\,c^{2}d\,x^{6}-93b\,c^{3}x^{6}-21ac\,d^{2}x^{4}+29b\,c^{2}d\,x^{4}+7a\,d^{3}x^{2}-13bc\,d^{2}x^{2}+5b\,d^{3}\right)}{35d^{5}x^{8}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}-\frac{c^{3}\left(ad-cb\right)}{d^{5}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}}$	124
orering	$-\frac{\left(112a\,c^{3}d\,x^{8}-128b\,c^{4}x^{8}+56a\,c^{2}d^{2}x^{6}-64b\,c^{3}d\,x^{6}-14ac\,d^{3}x^{4}+16b\,c^{2}d^{2}x^{4}+7a\,d^{4}x^{2}-8bc\,d^{3}x^{2}+5b\,d^{4}\right)\left(c\,x^{2}+d\right)\left(a+\frac{b}{x^{2}}\right)}{35d^{5}\left(a\,x^{2}+b\right)x^{8}\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	130

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*
c*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((
c*x^2+d)/x^2)^(3/2)/d^5/x^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \frac{(16\,(8\,bc^4-7\,ac^3d)x^8+8\,(8\,bc^3d-7\,ac^2d^2)x^6-5\,bd^4-2\,(8\,bc^2d^2-7\,acd^3)x^4+(8}{35\,(cd^5x^8+d^6x^6)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b
*d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

Sympy [A] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^9}\,dx = \begin{cases} \frac{2\left(-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{14d^4}-\frac{c^3(ad-bc)}{2d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{10d^4}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{6d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\cdot\left(3ac^2d-4bc^3\right)}{2d^4}\right)}{d} & \text{for } c \\ \frac{-\frac{a}{4x^8}-\frac{b}{5x^{10}}}{2c^{\frac{3}{2}}} & \text{othe} \end{cases}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

output
```
Piecewise((2*(-b*(c + d/x**2)**(7/2)/(14*d**4) - c**3*(a*d - b*c)/(2*d**4*
sqrt(c + d/x**2)) - (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(10*d**4) - (c + d/x
**2)**(3/2)*(-3*a*c*d + 6*b*c**2)/(6*d**4) - sqrt(c + d/x**2)*(3*a*c**2*d
- 4*b*c**3)/(2*d**4))/d, Ne(d, 0)), ((-a/(4*x**8) - b/(5*x**10))/(2*c**(3/
2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = -\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}d^5}\right) - \frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}d^4}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/
x^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)
*d^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqr
t(c + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(112) = 224.

Time = 1.49 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.29

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = \frac{(bc^4-ac^3d)x}{\sqrt{cx^2+d}\,d^5\,\mathrm{sgn}\,(x)} - \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{7}{2}}-35\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{5}{2}}d-280\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{7}{2}}d+280\left(\sqrt{c}\right.\right.}{}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
(b*c^4 - a*c^3*d)*x/(sqrt(c*x^2 + d)*d^5*sgn(x)) - 2/35*(35*(sqrt(c)*x - s
qrt(c*x^2 + d))^12*b*c^(7/2) - 35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/
2)*d - 280*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(7/2)*d + 280*(sqrt(c)*x -
 sqrt(c*x^2 + d))^10*a*c^(5/2)*d^2 + 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*
b*c^(7/2)*d^2 - 1015*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^3 - 2240*
(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^3 + 1680*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*a*c^(5/2)*d^4 + 1673*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*
d^4 - 1337*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^5 - 616*(sqrt(c)*x
- sqrt(c*x^2 + d))^2*b*c^(7/2)*d^5 + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a
*c^(5/2)*d^6 + 93*b*c^(7/2)*d^6 - 77*a*c^(5/2)*d^7)/(((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d)^7*d^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.22

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{c\sqrt{c+\frac{d}{x^2}}\,(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2} - \frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6} - \frac{\sqrt{c+\frac{d}{x^2}}\,(7\,a\,d^2-13\,b\,c\,d)}{35\,d^4\,x^4} - \frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-93\,b\,c)}{35\,d^5}\right)+\frac{c^2\,(77\,a\,d-93\,b\,c)}{35\,d^4}\right)}{c\,x^2+d}$$

input
```
int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

output
```
(c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2
))/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - (
(c + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d
- 93*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.80

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx = \frac{-112\sqrt{c\,x^2+d}\,a\,c^3 d\,x^8 - 56\sqrt{c\,x^2+d}\,a\,c^2 d^2 x^6 + 14\sqrt{c\,x^2+d}\,ac\,d^3 x^4 - 7\sqrt{c\,x^2+a}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x)
```

output
```
( - 112*sqrt(c*x**2 + d)*a*c**3*d*x**8 - 56*sqrt(c*x**2 + d)*a*c**2*d**2*x
**6 + 14*sqrt(c*x**2 + d)*a*c*d**3*x**4 - 7*sqrt(c*x**2 + d)*a*d**4*x**2 +
 128*sqrt(c*x**2 + d)*b*c**4*x**8 + 64*sqrt(c*x**2 + d)*b*c**3*d*x**6 - 16
*sqrt(c*x**2 + d)*b*c**2*d**2*x**4 + 8*sqrt(c*x**2 + d)*b*c*d**3*x**2 - 5*
sqrt(c*x**2 + d)*b*d**4 + 112*sqrt(c)*a*c**3*d*x**9 + 112*sqrt(c)*a*c**2*d
**2*x**7 - 128*sqrt(c)*b*c**4*x**9 - 128*sqrt(c)*b*c**3*d*x**7)/(35*d**5*x
**7*(c*x**2 + d))
```

3.188 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4} - \frac{(5bc-6ad)x^3}{5c^2\sqrt{c+\frac{d}{x^2}}} + \frac{4(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^3} + \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

output
```
-8/15*d*(-6*a*d+5*b*c)*(c+d/x^2)^(1/2)*x/c^4-1/5*(-6*a*d+5*b*c)*x^3/c^2/(c
+d/x^2)^(1/2)+4/15*(-6*a*d+5*b*c)*(c+d/x^2)^(1/2)*x^3/c^3+1/5*a*x^5/c/(c+d
/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{5bc(-8d^2-4cdx^2+c^2x^4)+3a(16d^3+8cd^2x^2-2c^2dx^4+c^3x^6)}{15c^4\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

output
```
(5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*
d*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {955, 803, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{803}$$

$$\frac{(5bc-6ad)\left(\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{4d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{3c}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{773}$$

$$\frac{(5bc-6ad)\left(\frac{4d\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{245}$$

$$\frac{(5bc-6ad)\left(\frac{4d\left(-\frac{2d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4d\left(-\frac{2d}{c^2x\sqrt{c+\frac{d}{x^2}}}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}+\frac{x^3}{3c\sqrt{c+\frac{d}{x^2}}}\right)(5bc-6ad)}{5c}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

output
```
(a*x^5)/(5*c*Sqrt[c + d/x^2]) + ((5*b*c - 6*a*d)*(x^3/(3*c*Sqrt[c + d/x^2]
) + (4*d*((-2*d)/(c^2*Sqrt[c + d/x^2]*x) - x/(c*Sqrt[c + d/x^2])))/(3*c)))
/(5*c)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{(3a\,x^{6}c^{3}-6a\,c^{2}d\,x^{4}+5b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-20b\,c^{2}d\,x^{2}+48a\,d^{3}-40bc\,d^{2})\,(c\,x^{2}+d)}{15\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c^{4}}$	91
default	$\frac{(3a\,x^{6}c^{3}-6a\,c^{2}d\,x^{4}+5b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-20b\,c^{2}d\,x^{2}+48a\,d^{3}-40bc\,d^{2})\,(c\,x^{2}+d)}{15\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c^{4}}$	91
trager	$\frac{(3a\,x^{6}c^{3}-6a\,c^{2}d\,x^{4}+5b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-20b\,c^{2}d\,x^{2}+48a\,d^{3}-40bc\,d^{2})\,x\sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{15(c\,x^{2}+d)c^{4}}$	95
risch	$\frac{(3a\,c^{2}x^{4}-9ad\,x^{2}c+5b\,c^{2}x^{2}+33a\,d^{2}-25dbc)\,(c\,x^{2}+d)}{15c^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}+\frac{(ad-cb)d^{2}}{c^{4}\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,x}$	99
orering	$\frac{(3a\,x^{6}c^{3}-6a\,c^{2}d\,x^{4}+5b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-20b\,c^{2}d\,x^{2}+48a\,d^{3}-40bc\,d^{2})\,(c\,x^{2}+d)\left(a+\frac{b}{x^{2}}\right)}{15c^{4}(a\,x^{2}+b)x\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	103

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+
48*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{(3\,ac^3x^7+(5\,bc^3-6\,ac^2d)x^5-4\,(5\,bc^2d-6\,acd^2)x^3-8\,(5\,bcd^2-6\,ad^3)x)\sqrt{\frac{cx^2+d}{x^2}}}{15\,(c^5x^2+c^4d)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*
x^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(105) = 210$.

Time = 4.64 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{c^5d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{5c^3d^{\frac{23}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{30c^2d^{\frac{25}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{40cd^{\frac{27}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}} + \frac{16d^{\frac{29}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c^7d^9x^6+15c^6d^{10}x^4+15c^5d^{11}x^2+5c^4d^{12}}\right) + b\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6} - \frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)$$

input
```
integrate((a+b/x**2)*x***4/(c+d/x**2)**(3/2),x)
```

output
```
a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**
10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(
c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2
+ 5*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x
**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(2
7/2)*x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c
**5*d**11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d
**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c
**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2
+ 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**
4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\,\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3\,d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)+\frac{1}{5}\,a\left(\frac{5\,d^3}{\sqrt{c+\frac{d}{x^2}}c^4x}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\,\sqrt{c+\frac{d}{x^2}}d^2x}{c^4}\right)$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
 + d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5
/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.30

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{8\,(5\,bcd^2-6\,ad^3)\mathrm{sgn}(x)}{15\,c^4\sqrt{d}} - \frac{bcd^2-ad^3}{\sqrt{cx^2+d}c^4\mathrm{sgn}\left(x\right)} + \frac{3\,(cx^2+d)^{\frac{5}{2}}ac^{16}+5\,(cx^2+d)^{\frac{3}{2}}bc^{17}-15\,(cx^2+d)^{\frac{3}{2}}ac^{16}d-30\,\sqrt{cx^2+d}bc^{17}d+45\,\sqrt{cx^2+d}ac^{16}d^2}{15\,c^{20}\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
8/15*(5*b*c*d^2 - 6*a*d^3)*sgn(x)/(c^4*sqrt(d)) - (b*c*d^2 - a*d^3)/(sqrt(
c*x^2 + d)*c^4*sgn(x)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^16 + 5*(c*x^2 + d)^
(3/2)*b*c^17 - 15*(c*x^2 + d)^(3/2)*a*c^16*d - 30*sqrt(c*x^2 + d)*b*c^17*d
 + 45*sqrt(c*x^2 + d)*a*c^16*d^2)/(c^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{3\,a\,c^3\,x^6+5\,b\,c^3\,x^4-6\,a\,c^2\,d\,x^4-20\,b\,c^2\,d\,x^2+24\,a\,c\,d^2\,x^2-40\,b\,c\,d^2+48\,a\,d^3}{15\,c^4\,x\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*
c^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c\,x^2+d}\,(3a\,c^3x^6-6a\,c^2d\,x^4+5b\,c^3x^4+24ac\,d^2x^2-20b\,c^2d\,x^2+48a\,d^3-40bc\,d^2)}{15c^4\,(c\,x^2+d)}$$

input
```
int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x)
```

output
```
(sqrt(c*x**2 + d)*(3*a*c**3*x**6 - 6*a*c**2*d*x**4 + 24*a*c*d**2*x**2 + 48
*a*d**3 + 5*b*c**3*x**4 - 20*b*c**2*d*x**2 - 40*b*c*d**2))/(15*c**4*(c*x**
2 + d))
```

3.189 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

output
```
-1/3*(-4*a*d+3*b*c)*x/c^2/(c+d/x^2)^(1/2)+2/3*(-4*a*d+3*b*c)*(c+d/x^2)^(1/
2)*x/c^3+1/3*a*x^3/c/(c+d/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{3bc(2d+cx^2)+a(-8d^2-4cdx^2+c^2x^4)}{3c^3\sqrt{c+\frac{d}{x^2}}\,x}$$

input
```
Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d
/x^2]*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {955, 773, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+\frac{b}{x^2}\right)}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{955}$$

$$\frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx}{3c}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{773}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{(3bc-4ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{3c}$$

$$\Big\downarrow \text{245}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{(3bc-4ad)\left(-\frac{2d\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)}{3c}$$

$$\Big\downarrow \text{208}$$

$$\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}-\frac{\left(-\frac{2d}{c^2x\sqrt{c+\frac{d}{x^2}}}-\frac{x}{c\sqrt{c+\frac{d}{x^2}}}\right)(3bc-4ad)}{3c}$$

input
```
Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(a*x^3)/(3*c*Sqrt[c + d/x^2]) - ((3*b*c - 4*a*d)*((-2*d)/(c^2*Sqrt[c + d/x
^2]*x) - x/(c*Sqrt[c + d/x^2])))/(3*c)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 955
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1))   Int[(e
*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*
c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) ||
(LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{\left(a\,c^{2} x^{4}-4 a d\,x^{2} c+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 d b c\right) \left(c\,x^{2}+d\right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{3}}$	66
default	$\frac{\left(a\,c^{2} x^{4}-4 a d\,x^{2} c+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 d b c\right) \left(c\,x^{2}+d\right)}{3 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{3} c^{3}}$	66
trager	$\frac{\left(a\,c^{2} x^{4}-4 a d\,x^{2} c+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 d b c\right) x \sqrt{-\frac{-c\,x^{2}-d}{x^{2}}}}{3 \left(c\,x^{2}+d\right) c^{3}}$	70
risch	$\frac{\left(a c\,x^{2}-5 a d+3 c b\right) \left(c\,x^{2}+d\right)}{3 c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}-\frac{\left(a d-c b\right) d}{c^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	75
orering	$\frac{\left(a\,c^{2} x^{4}-4 a d\,x^{2} c+3 b\,c^{2} x^{2}-8 a\,d^{2}+6 d b c\right) \left(c\,x^{2}+d\right) \left(a+\frac{b}{x^{2}}\right)}{3 c^{3} \left(a\,x^{2}+b\right) x \left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}$	78

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+
d)/x^2)^(3/2)/x^3/c^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\left(a+\frac{b}{x^2}\right) x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\, dx = \frac{(ac^2x^5 + (3\,bc^2 - 4\,acd)x^3 + 2\,(3\,bcd - 4\,ad^2)x)\sqrt{\frac{cx^2+d}{x^2}}}{3\,(c^4x^2 + c^3d)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((
c*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(70) = 140$.

Time = 4.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.38

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}\right)+b\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)$$

input
```
integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

output
```
a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x
**2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4
*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/
d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*s
qrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b
*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = b\left(\frac{\sqrt{c+\frac{d}{x^2}}x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}c^2x}\right)+\frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\sqrt{c+\frac{d}{x^2}}dx}{c^3}-\frac{3d^2}{\sqrt{c+\frac{d}{x^2}}c^3x}\right)$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2
)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\frac{2\,(3\,bcd-4\,ad^2)\mathrm{sgn}(x)}{3\,c^3\sqrt{d}} + \frac{bcd-ad^2}{\sqrt{cx^2+d}c^3\mathrm{sgn}\,(x)} + \frac{(cx^2+d)^{\frac{3}{2}}ac^6+3\,\sqrt{cx^2+d}bc^7-6\,\sqrt{cx^2+d}ac^6d}{3\,c^9\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
-2/3*(3*b*c*d - 4*a*d^2)*sgn(x)/(c^3*sqrt(d)) + (b*c*d - a*d^2)/(sqrt(c*x^
2 + d)*c^3*sgn(x)) + 1/3*((c*x^2 + d)^(3/2)*a*c^6 + 3*sqrt(c*x^2 + d)*b*c^
7 - 6*sqrt(c*x^2 + d)*a*c^6*d)/(c^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{b\,c^2\,x^4+3\,b\,c\,d\,x^2+2\,b\,d^2}{c^2\,x^3\,\left(c+\frac{d}{x^2}\right)^{3/2}} - \frac{-a\,c^2\,x^4+4\,a\,c\,d\,x^2+8\,a\,d^2}{3\,c^3\,x\,\sqrt{c+\frac{d}{x^2}}}$$

input
```
int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

output
```
(2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2
 - a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+d}\,(a\,c^2x^4-4acd\,x^2+3b\,c^2x^2-8a\,d^2+6bcd)}{3c^3\,(c\,x^2+d)}$$

input
```
int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x)
```

output
```
(sqrt(c*x**2 + d)*(a*c**2*x**4 - 4*a*c*d*x**2 - 8*a*d**2 + 3*b*c**2*x**2 +
 6*b*c*d))/(3*c**3*(c*x**2 + d))
```

3.190 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx = -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}x} + \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}$$

output
```
-(-2*a*d+b*c)/c^2/(c+d/x^2)^(1/2)/x+a*x/c/(c+d/x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}} dx = \frac{-bc+2ad+acx^2}{c^2\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

output
```
(-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {899, 359, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{899}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}$$

$$\Big\downarrow \text{359}$$

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{(bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}d\frac{1}{x}}{c}$$

$$\Big\downarrow \text{208}$$

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

output
```
-((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(a c\,x^2+2ad-cb)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 c^2}$	43
default	$\frac{(a c\,x^2+2ad-cb)(c\,x^2+d)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} x^3 c^2}$	43
trager	$\frac{x(a c\,x^2+2ad-cb)\sqrt{-\frac{-c\,x^2-d}{x^2}}}{(c\,x^2+d)c^2}$	47
orering	$\frac{(a c\,x^2+2ad-cb)(c\,x^2+d)\left(a+\frac{b}{x^2}\right)}{c^2(a\,x^2+b)x\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}$	55
risch	$\frac{a(c\,x^2+d)}{c^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}+\frac{ad-cb}{c^2\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$	58

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{(acx^3-(bc-2\,ad)x)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

output
```
(a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

Sympy [A] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

output
```
a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1
))) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = a\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

output
```
a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)
*c*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{(bc-2\,ad)\mathrm{sgn}(x)}{c^2\sqrt{d}} + \frac{\sqrt{cx^2+d}a}{c^2\mathrm{sgn}\,(x)} - \frac{bc-ad}{\sqrt{cx^2+d}c^2\mathrm{sgn}\,(x)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

output
```
(b*c - 2*a*d)*sgn(x)/(c^2*sqrt(d)) + sqrt(c*x^2 + d)*a/(c^2*sgn(x)) - (b*c
 - a*d)/(sqrt(c*x^2 + d)*c^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{\left(c\,x^2+d\right)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,\left(c+\frac{d}{x^2}\right)^{3/2}}$$

input
```
int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

output
```
((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{\sqrt{c\,x^2+d}\,(ac\,x^2+2ad-bc)}{c^2\,(c\,x^2+d)}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2),x)
```

output
```
(sqrt(c*x**2 + d)*(a*c*x**2 + 2*a*d - b*c))/(c**2*(c*x**2 + d))
```

3.191 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx = \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x}\right)}{d^{3/2}}$$

output
```
(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)/x-b*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(3
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx = \frac{\sqrt{d}(bc-ad) - bc\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{cd^{3/2}\sqrt{c+\frac{d}{x^2}}x}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

output
```
(Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]
])/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {954, 858, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^2\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{954}$$

$$\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}dx}{d}+\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{d}$$

$$\Big\downarrow \text{224}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

output
```
(b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*
x)])/d^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\left(c\,x^{2}+d\right)\left(a\,d^{\frac{5}{2}}-d^{\frac{3}{2}}bc+\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}\,bcd\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}x^{3}c\,d^{\frac{5}{2}}}$	79

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+d)*(a*d^(5/2)-d^(3/2)*b*c+ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)*(c*x
^2+d)^(1/2)*b*c*d)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \left[\frac{2\,(bcd-ad^2)x\sqrt{\frac{cx^2+d}{x^2}}+(bc^2x^2+bcd)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,(c^2d^2x^2+cd^3)}, \frac{(bcd-ad^2)}{}\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt
(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*
x^2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*
d)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/d))/(c^2*d^2*x^2 + c*d
^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(46) = 92$.

Time = 6.01 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.49

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = -\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}} + b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

output
```
-a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d
**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2
+ 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d*
*3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}dx}\right)-\frac{a}{\sqrt{c+\frac{d}{x^2}}cx}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^
(3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(51) = 102.

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.83

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}\,dx = \frac{b\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}d\mathrm{sgn}\left(x\right)} - \frac{\left(bc\sqrt{d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+bc\sqrt{-d}-a\sqrt{-d}d\right)\mathrm{sgn}(x)}{c\sqrt{-d}d^{\frac{3}{2}}}+\frac{bc-ad}{\sqrt{cx^2+d}cd\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
b*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d*sgn(x)) - (b*c*sqrt(d)*arct
an(sqrt(d)/sqrt(-d)) + b*c*sqrt(-d) - a*sqrt(-d)*d)*sgn(x)/(c*sqrt(-d)*d^(
3/2)) + (b*c - a*d)/(sqrt(c*x^2 + d)*c*d*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2}\,dx = \frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}} - \frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}} - \frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

input
```
int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

output
```
b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)
^(1/2) + d^(1/2)/x))/d^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.80

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2}\,dx = \frac{-\sqrt{c\,x^2+d}\,a\,d^2+\sqrt{c\,x^2+d}\,bcd+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right) b\,c^2x^2+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+}}{}\right.}{c\,d^2\,(c\,x^2}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x)
```

output
```
( - sqrt(c*x**2 + d)*a*d**2 + sqrt(c*x**2 + d)*b*c*d + sqrt(d)*log((sqrt(c
*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c**2*x**2 + sqrt(d)*log((sqrt
(c*x**2 + d) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c*d - sqrt(d)*log((sqrt(c*x
**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**2*x**2 - sqrt(d)*log((sqrt(c
*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c*d)/(c*d**2*(c*x**2 + d))
```

3.192 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}\,x}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2d^2x}+\frac{(3bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}$$

output
```
-(-a*d+b*c)/d^2/(c+d/x^2)^(1/2)/x-1/2*b*(c+d/x^2)^(1/2)/d^2/x+1/2*(-2*a*d+
3*b*c)*arctanh(d^(1/2)/(c+d/x^2)^(1/2)/x)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=\frac{\sqrt{d}(2adx^2-b(d+3cx^2))+(3bc-2ad)x^2\sqrt{d+cx^2}\operatorname{arctanh}\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{c+\frac{d}{x^2}}\,x^3}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

output
```
(Sqrt[d]*(2*a*d*x^2 - b*(d + 3*c*x^2)) + (3*b*c - 2*a*d)*x^2*Sqrt[d + c*x^
2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/(2*d^(5/2)*Sqrt[c + d/x^2]*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {959, 858, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(3bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}dx}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(3bc-2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^2}d\frac{1}{x}}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{d}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3bc-2ad)\left(\frac{\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{d}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3bc-2ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}-\frac{1}{dx\sqrt{c+\frac{d}{x^2}}}\right)}{2d}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

output
```
-1/2*b/(d*Sqrt[c + d/x^2]*x^3) + ((3*b*c - 2*a*d)*(-(1/(d*Sqrt[c + d/x^2]*
x)) + ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]/d^(3/2)))/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{b\left(c\,x^{2}+d\right)}{2 d^{2} x^{3} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}}+\frac{\left(\frac{b c}{\sqrt{c\,x^{2}+d}}+d\left(2 a d -3 c b \right)\left(\frac{1}{d \sqrt{c\,x^{2}+d}}-\frac{\ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^{2}+d}}{2 d^{2} \sqrt{\frac{c\,x^{2}+d}{x^{2}}}\, x}$	127
default	$\frac{\left(c\,x^{2}+d \right)\left(2 d^{\frac{5}{2}} a\,x^{2}-3 d^{\frac{3}{2}} b c\,x^{2}-2 \ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}\, a\,d^{2} x^{2}+3 \ln\left(\frac{2 d +2 \sqrt{d}\,\sqrt{c\,x^{2}+d}}{x}\right)\sqrt{c\,x^{2}+d}\, b c d\,x^{2}-d^{\frac{5}{2}} b \right)}{2 \left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}} x^{5} d^{\frac{7}{2}}}$	132

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^2*b*(c*x^2+d)/x^3/((c*x^2+d)/x^2)^(1/2)+1/2/d^2*(b*c/(c*x^2+d)^(1/2
)+d*(2*a*d-3*b*c)*(1/d/(c*x^2+d)^(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+
d)^(1/2))/x)))/((c*x^2+d)/x^2)^(1/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.72

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4}\,dx = \left[-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{d}\log\left(-\frac{cx^2-2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,(bd^2+}{4\,(cd^3x^3+d^4x)}\right.$$

$$\left.-\frac{((3\,bc^2-2\,acd)x^3+(3\,bcd-2\,ad^2)x)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x\sqrt{\frac{cx^2+d}{x^2}}}{d}\right)+(bd^2+(3\,bcd-2\,ad^2)x^2)\sqrt{\frac{cx^2+d}{x^2}}}{2\,(cd^3x^3+d^4x)}\right]$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x
^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d -
 2*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2
 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((
c*x^2 + d)/x^2)/d) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^
2))/(c*d^3*x^3 + d^4*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(73) = 146$.

Time = 9.80 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.94

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} + \frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}} - \frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right) + b\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}} + \frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}} - \frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right)$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

output
```
a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x
**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*
sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(
2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*
d**(7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2)
)) + 3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqr
t(1 + d/(c*x**2))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right) + \frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}dx}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
-1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(
c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)
*x + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c
 + d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx = -\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{2\,\sqrt{-d}d^2\mathrm{sgn}\left(x\right)} - \frac{3\,(cx^2+d)bc-2\,(cx^2+d)ad-2\,bcd+2\,ad^2}{2\left((cx^2+d)^{\frac{3}{2}}-\sqrt{cx^2+d}d\right)d^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*(3*b*c - 2*a*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^2*sgn(x)
) - 1/2*(3*(c*x^2 + d)*b*c - 2*(c*x^2 + d)*a*d - 2*b*c*d + 2*a*d^2)/(((c*x
^2 + d)^(3/2) - sqrt(c*x^2 + d)*d)*d^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
```

output
```
int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.65

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx=\frac{2\sqrt{c\,x^2+d}\,a\,d^2x^2-3\sqrt{c\,x^2+d}\,bcd\,x^2-\sqrt{c\,x^2+d}\,b\,d^2+2\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}}{\sqrt{d}}\right)}{}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x)
```

output
```
(2*sqrt(c*x**2 + d)*a*d**2*x**2 - 3*sqrt(c*x**2 + d)*b*c*d*x**2 - sqrt(c*x
**2 + d)*b*d**2 + 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d))/s
qrt(d))*a*c*d*x**4 + 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqrt(d)
)/sqrt(d))*a*d**2*x**2 - 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqr
t(d))/sqrt(d))*b*c**2*x**4 - 3*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x -
 sqrt(d))/sqrt(d))*b*c*d*x**2 - 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*
x + sqrt(d))/sqrt(d))*a*c*d*x**4 - 2*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(
c)*x + sqrt(d))/sqrt(d))*a*d**2*x**2 + 3*sqrt(d)*log((sqrt(c*x**2 + d) + s
qrt(c)*x + sqrt(d))/sqrt(d))*b*c**2*x**4 + 3*sqrt(d)*log((sqrt(c*x**2 + d)
 + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c*d*x**2)/(2*d**3*x**2*(c*x**2 + d))
```

3.193 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{4d^2x^3} + \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}\,x} + \frac{(7bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{7/2}}$$

output
```
-1/4*b*(c+d/x^2)^(1/2)/d^2/x^3+c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)/x+1/8*(-4*
a*d+7*b*c)*(c+d/x^2)^(1/2)/d^3/x-3/8*c*(-4*a*d+5*b*c)*arctanh(d^(1/2)/(c+d
/x^2)^(1/2)/x)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{\sqrt{d}(-4adx^2(d+3cx^2)+b(-2d^2+5cdx^2+15c^2x^4))-3c(5bc-4ad)x^4\sqrt{d+cx^2}\mathrm{ar}}{8d^{7/2}\sqrt{c+\frac{d}{x^2}}x^5}$$

input
```
Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

output
```
(Sqrt[d]*(-4*a*d*x^2*(d + 3*c*x^2) + b*(-2*d^2 + 5*c*d*x^2 + 15*c^2*x^4))
- 3*c*(5*b*c - 4*a*d)*x^4*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(8*d^(7/2)*Sqrt[c + d/x^2]*x^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {959, 858, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{(5bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}dx}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(5bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}d\frac{1}{x}}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(5bc-4ad)\left(\frac{3\int \frac{1}{\sqrt{c+\frac{d}{x^2}}x^2}d\frac{1}{x}}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}d\frac{1}{x}}{2d}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\int \frac{1}{1-\frac{d}{x^2}}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{2d}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(5bc-4ad)\left(\frac{3\left(\frac{\sqrt{c+\frac{d}{x^2}}}{2dx}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}\right)}{d}-\frac{1}{dx^3\sqrt{c+\frac{d}{x^2}}}\right)}{4d}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

input
```
Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

output
```
-1/4*b/(d*Sqrt[c + d/x^2]*x^5) + ((5*b*c - 4*a*d)*(-(1/(d*Sqrt[c + d/x^2]*
x^3)) + (3*(Sqrt[c + d/x^2]/(2*d*x) - (c*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*
x)])/(2*d^(3/2))))/d))/(4*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{(c\,x^2+d)\,(4ad\,x^2-7bc\,x^2+2bd)}{8d^3x^5\sqrt{\frac{c\,x^2+d}{x^2}}}-\frac{c\left(-\frac{4ad-7cb}{\sqrt{c\,x^2+d}}+3d(4ad-5cb)\left(\frac{1}{d\sqrt{c\,x^2+d}}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}\right)\right)\sqrt{c\,x^2+d}}{8d^3\sqrt{\frac{c\,x^2+d}{x^2}}\,x}$
default	$-\frac{(c\,x^2+d)\left(12d^{\frac{5}{2}}\,ac\,x^4-15d^{\frac{3}{2}}\,b\,c^2x^4-12\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{c\,x^2+d}\,ac\,d^2x^4+15\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{c\,x^2+d}}{x}\right)\sqrt{c\,x^2+d}\,b\,c^2d\,x^4+4d^{\frac{7}{2}}\right.}{8\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}x^7d^{\frac{9}{2}}}$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(c*x^2+d)*(4*a*d*x^2-7*b*c*x^2+2*b*d)/d^3/x^5/((c*x^2+d)/x^2)^(1/2)-1
/8*c/d^3*(-(4*a*d-7*b*c)/(c*x^2+d)^(1/2)+3*d*(4*a*d-5*b*c)*(1/d/(c*x^2+d)^
(1/2)-1/d^(3/2)*ln((2*d+2*d^(1/2)*(c*x^2+d)^(1/2))/x)))/((c*x^2+d)/x^2)^(1
/2)/x*(c*x^2+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.57

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=\left[-\frac{3\,((5\,bc^3-4\,ac^2d)x^5+(5\,bc^2d-4\,acd^2)x^3)\sqrt{d}\log\left(-\frac{cx^2+2\,\sqrt{d}x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\,(3}{16\,(cd^4x^5+d^5x^3)}\right.$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d
)*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*
c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2
+ d)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b
*c^2*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/
d) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)
*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]
```

Sympy [A] (verification not implemented)

Time = 18.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.50

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}dx^3\sqrt{1+\frac{d}{cx^2}}}\right) + b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}x}\right)}{8d^{\frac{7}{2}}}-\frac{1}{4\sqrt{c}dx^5\sqrt{1+\frac{d}{cx^2}}}\right)$$

input
```
integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

output
```
a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)
*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**
(3/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/
(c*x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c
)*d*x**5*sqrt(1 + d/(c*x**2))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(102) = 204$.

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.02

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}\,d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right) - \frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\,d^3x}+\frac{3\,c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2
)/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^
2)*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
sqrt(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(
3/2)*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(
d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.23

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx = \frac{3\left(5\,bc^2-4\,acd\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{8\sqrt{-d}\,d^3\mathrm{sgn}\left(x\right)}+\frac{bc^2-acd}{\sqrt{cx^2+d}\,d^3\mathrm{sgn}\left(x\right)} + \frac{7\left(cx^2+d\right)^{\frac{3}{2}}bc^2-4\left(cx^2+d\right)^{\frac{3}{2}}acd-9\sqrt{cx^2+d}\,bc^2d+4\sqrt{cx^2+d}\,acd^2}{8\,c^2d^3x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
3/8*(5*b*c^2 - 4*a*c*d)*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(sqrt(-d)*d^3*sgn
(x)) + (b*c^2 - a*c*d)/(sqrt(c*x^2 + d)*d^3*sgn(x)) + 1/8*(7*(c*x^2 + d)^(
3/2)*b*c^2 - 4*(c*x^2 + d)^(3/2)*a*c*d - 9*sqrt(c*x^2 + d)*b*c^2*d + 4*sqr
t(c*x^2 + d)*a*c*d^2)/(c^2*d^3*x^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

input
```
int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
```

output
```
int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.11

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^6}\,dx=\frac{-12\sqrt{c\,x^2+d}\,ac\,d^2x^4-4\sqrt{c\,x^2+d}\,a\,d^3x^2+15\sqrt{c\,x^2+d}\,b\,c^2d\,x^4+5\sqrt{c\,x^2+d}\,bc\,d}{}$$

input
```
int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x)
```

output
```
( - 12*sqrt(c*x**2 + d)*a*c*d**2*x**4 - 4*sqrt(c*x**2 + d)*a*d**3*x**2 + 1
5*sqrt(c*x**2 + d)*b*c**2*d*x**4 + 5*sqrt(c*x**2 + d)*b*c*d**2*x**2 - 2*sq
rt(c*x**2 + d)*b*d**3 - 12*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x - sqr
t(d))/sqrt(d))*a*c**2*d*x**6 - 12*sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*
x - sqrt(d))/sqrt(d))*a*c*d**2*x**4 + 15*sqrt(d)*log((sqrt(c*x**2 + d) + s
qrt(c)*x - sqrt(d))/sqrt(d))*b*c**3*x**6 + 15*sqrt(d)*log((sqrt(c*x**2 + d
) + sqrt(c)*x - sqrt(d))/sqrt(d))*b*c**2*d*x**4 + 12*sqrt(d)*log((sqrt(c*x
**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*a*c**2*d*x**6 + 12*sqrt(d)*log((s
qrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*a*c*d**2*x**4 - 15*sqrt(d)
*log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**3*x**6 - 15*sq
rt(d)*log((sqrt(c*x**2 + d) + sqrt(c)*x + sqrt(d))/sqrt(d))*b*c**2*d*x**4)
/(8*d**4*x**4*(c*x**2 + d))
```

3.194 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5\,\text{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
1/5*(a+b/x^2)^p*(c+d/x^2)^q*x^5*AppellF1(-5/2,-p,-q,-3/2,-b/a/x^2,-d/c/x^2
)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^5 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{5}{2}-p-q,-p,-q,\frac{7}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-5+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 +
(c*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{997}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^6 d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{5}x^5 \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{5}{2},-p,-q,-\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2))
, -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4 dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
```

output
```
integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int(x^4*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x^4*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx=\text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

output
```
(3*(c*x**2 + d)**q*(a*x**2 + b)**p*a**2*c**2*x**5 + 2*(c*x**2 + d)**q*(a*x
**2 + b)**p*a**2*c*d*q*x**3 + 4*(c*x**2 + d)**q*(a*x**2 + b)**p*a**2*d**2*
q**2*x - 6*(c*x**2 + d)**q*(a*x**2 + b)**p*a**2*d**2*q*x + 2*(c*x**2 + d)*
*q*(a*x**2 + b)**p*a*b*c**2*p*x**3 + 8*(c*x**2 + d)**q*(a*x**2 + b)**p*a*b
*c*d*p*q*x + 4*(c*x**2 + d)**q*(a*x**2 + b)**p*b**2*c**2*p**2*x - 6*(c*x**
2 + d)**q*(a*x**2 + b)**p*b**2*c**2*p*x + 8*x**(2*p + 2*q)*int(((c*x**2 +
d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*
x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a**3*d**3*q**3 - 1
6*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q
)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p +
 2*q)*b*d),x)*a**3*d**3*q**2 + 6*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x*
*2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(
2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a**3*d**3*q + 24*x**(2*p + 2*
q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x
**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*
a**2*b*c*d**2*p*q**2 - 16*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)
**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2
*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a**2*b*c*d**2*p*q + 24*x**(2*p + 2*q
)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x*
*(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x...
```

3.195 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx = \frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 \operatorname{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
1/3*(a+b/x^2)^p*(c+d/x^2)^q*x^3*AppellF1(-3/2,-p,-q,-1/2,-b/a/x^2,-d/c/x^2
)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{2}-p-q,-p,-q,\frac{5}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-3+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 +
(c*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

$$\Big\downarrow \text{997}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int \left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{395}$$

$$-\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int \left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2))
, -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

output
```
integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \int x^2 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int(x^2*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x^2*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = \text{Too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

output
```
((c*x**2 + d)**q*(a*x**2 + b)**p*a*c*x**3 + 2*(c*x**2 + d)**q*(a*x**2 + b)
**p*a*d*q*x + 2*(c*x**2 + d)**q*(a*x**2 + b)**p*b*c*p*x + 4*x**(2*p + 2*q)
*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**
(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a*
*2*d**2*q**2 - 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)
/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x
**2 + x**(2*p + 2*q)*b*d),x)*a**2*d**2*q + 8*x**(2*p + 2*q)*int(((c*x**2 +
 d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d
*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a*b*c*d*p*q + 4*x
**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a
*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*
q)*b*d),x)*b**2*c**2*p**2 - 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2
+ b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p
 + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b**2*c**2*p + 4*x**(2*p + 2*q)*i
nt(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p +
2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a*b*d**2*
p*q + 4*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*
q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p
+ 2*q)*b*d),x)*a*b*d**2*q**2 - 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x*
*2 + b)**p)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*...
```

3.196 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x \,\text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b
/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.32

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{-1+2p+2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {899, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{ 899}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q x^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 394}$$

$$x\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -
(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

input
```
int((a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = \frac{(c\,x^2+d)^q\,(a\,x^2+b)^p\,x+2x^{2p+2q}\left(\int \frac{(c\,x^2+d)^q\,(a\,x^2+b)^p\,x^2}{x^{2p+2q}acx^4+x^{2p+2q}adx^2+x^{2p+2q}bcx^2+x^{2p+2q}bd}dx\right)adq+2x^{2p+2q}\left(\int \frac{}{x^{2p+2q}ac}\right.}{}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

output
```
((c*x**2 + d)**q*(a*x**2 + b)**p*x + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q
*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2
+ x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a*d*q + 2*x**(2*p + 2*q
)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4 + x*
*(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b
*c*p + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2
*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p
 + 2*q)*b*d),x)*b*d*p + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)
**p)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b
*c*x**2 + x**(2*p + 2*q)*b*d),x)*b*d*q)/x**(2*p + 2*q)
```

3.197 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

output
```
-(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/2,-p,-q,3/2,-b/a/x^2,-d/c/x^2)/((1+b/a
/x^2)^p)/((1+d/c/x^2)^q)/x
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(-\frac{1}{2}-p-q,-p,-q,\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+2p+2q)x}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {997, 334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} \, dx$$

$$\Big\downarrow \text{ 997}$$

$$-\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{ 334}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{ 334}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q d\frac{1}{x}$$

$$\Big\downarrow \text{ 333}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx=\text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

output
```
( - (c*x**2 + d)**q*(a*x**2 + b)**p*a*d - (c*x**2 + d)**q*(a*x**2 + b)**p*
b*c + 4*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(2*x**(2
*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*c*d*x**4 + 2*x**(2*p + 2*q
)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a**2*d**2*x**2 + 2*x**(2*p + 2*q)*a*b*
c**2*p*x**4 + x**(2*p + 2*q)*a*b*c**2*x**4 + 2*x**(2*p + 2*q)*a*b*c*d*p*x*
*2 + 2*x**(2*p + 2*q)*a*b*c*d*q*x**2 + 2*x**(2*p + 2*q)*a*b*c*d*x**2 + 2*x
**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*q)*a*b*d**2 + 2*x**(2*p + 2*q)*b**2
*c**2*p*x**2 + x**(2*p + 2*q)*b**2*c**2*x**2 + 2*x**(2*p + 2*q)*b**2*c*d*p
 + x**(2*p + 2*q)*b**2*c*d),x)*a**3*c*d**2*q**2*x + 2*x**(2*p + 2*q)*int((
(c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(2*x**(2*p + 2*q)*a**2*c*d*q*x**4 +
x**(2*p + 2*q)*a**2*c*d*x**4 + 2*x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p
 + 2*q)*a**2*d**2*x**2 + 2*x**(2*p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)
*a*b*c**2*x**4 + 2*x**(2*p + 2*q)*a*b*c*d*p*x**2 + 2*x**(2*p + 2*q)*a*b*c*
d*q*x**2 + 2*x**(2*p + 2*q)*a*b*c*d*x**2 + 2*x**(2*p + 2*q)*a*b*d**2*q + x
**(2*p + 2*q)*a*b*d**2 + 2*x**(2*p + 2*q)*b**2*c**2*p*x**2 + x**(2*p + 2*q
)*b**2*c**2*x**2 + 2*x**(2*p + 2*q)*b**2*c*d*p + x**(2*p + 2*q)*b**2*c*d),
x)*a**3*c*d**2*q*x + 8*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p
*x**2)/(2*x**(2*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*c*d*x**4 +
2*x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a**2*d**2*x**2 + 2*x**(
2*p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)*a*b*c**2*x**4 + 2*x**(2*p +...
```

3.198 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

output
```
-1/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/2,-p,-q,5/2,-b/a/x^2,-d/c/x^2)/((1
+b/a/x^2)^p)/((1+d/c/x^2)^q)/x^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{2}-p-q,-p,-q,-\frac{1}{2}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+2p+2q)x^3}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

output
```
-(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 +
 (c*x^2)/d)^q))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {997, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$$

$$\Big\downarrow \text{ 997}$$

$$-\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \int \frac{\left(\frac{b}{a x^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{ 395}$$

$$-\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{c x^2}+1\right)^{-q} \int \frac{\left(\frac{b}{a x^2}+1\right)^p \left(\frac{d}{c x^2}+1\right)^q}{x^2} d\frac{1}{x}$$

$$\Big\downarrow \text{ 394}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{a x^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{c x^2}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{b}{a x^2},-\frac{d}{c x^2}\right)}{3 x^3}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

output
```
-1/3*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^4} \, dx = \text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

output

```
( - (c*x**2 + d)**q*(a*x**2 + b)**p + 8*x**(2*p + 2*q)*int(((c*x**2 + d)**
q*(a*x**2 + b)**p)/(4*x**(2*p + 2*q)*a**2*c*d*p*q*x**6 + 6*x**(2*p + 2*q)*
a**2*c*d*p*x**6 + 4*x**(2*p + 2*q)*a**2*c*d*q**2*x**6 + 12*x**(2*p + 2*q)*
a**2*c*d*q*x**6 + 9*x**(2*p + 2*q)*a**2*c*d*x**6 + 4*x**(2*p + 2*q)*a**2*d
**2*p*q*x**4 + 6*x**(2*p + 2*q)*a**2*d**2*p*x**4 + 4*x**(2*p + 2*q)*a**2*d
**2*q**2*x**4 + 12*x**(2*p + 2*q)*a**2*d**2*q*x**4 + 9*x**(2*p + 2*q)*a**2
*d**2*x**4 + 4*x**(2*p + 2*q)*a*b*c**2*p**2*x**6 + 4*x**(2*p + 2*q)*a*b*c*
*2*p*q*x**6 + 12*x**(2*p + 2*q)*a*b*c**2*p*x**6 + 6*x**(2*p + 2*q)*a*b*c**
2*q*x**6 + 9*x**(2*p + 2*q)*a*b*c**2*x**6 + 4*x**(2*p + 2*q)*a*b*c*d*p**2*
x**4 + 8*x**(2*p + 2*q)*a*b*c*d*p*q*x**4 + 18*x**(2*p + 2*q)*a*b*c*d*p*x**
4 + 4*x**(2*p + 2*q)*a*b*c*d*q**2*x**4 + 18*x**(2*p + 2*q)*a*b*c*d*q*x**4
+ 18*x**(2*p + 2*q)*a*b*c*d*x**4 + 4*x**(2*p + 2*q)*a*b*d**2*p*q*x**2 + 6*
x**(2*p + 2*q)*a*b*d**2*p*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q**2*x**2 + 12*
x**(2*p + 2*q)*a*b*d**2*q*x**2 + 9*x**(2*p + 2*q)*a*b*d**2*x**2 + 4*x**(2*
p + 2*q)*b**2*c**2*p**2*x**4 + 4*x**(2*p + 2*q)*b**2*c**2*p*q*x**4 + 12*x*
*(2*p + 2*q)*b**2*c**2*p*x**4 + 6*x**(2*p + 2*q)*b**2*c**2*q*x**4 + 9*x**(
2*p + 2*q)*b**2*c**2*x**4 + 4*x**(2*p + 2*q)*b**2*c*d*p**2*x**2 + 4*x**(2*
p + 2*q)*b**2*c*d*p*q*x**2 + 12*x**(2*p + 2*q)*b**2*c*d*p*x**2 + 6*x**(2*p
 + 2*q)*b**2*c*d*q*x**2 + 9*x**(2*p + 2*q)*b**2*c*d*x**2),x)*a**2*d**2*p**
2*q*x**3 + 12*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(4*x...
```

3.199 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,3,-q,2+p,1+\frac{b}{ax^2},-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2a^3(1+p)}$$

output
```
1/2*b^2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,-q,3,2+p,-d*(a+b/x^2)/(-a
*d+b*c),1+b/a/x^2)/a^3/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(2-p-q,-p,-q,3-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-2+p+q)}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^
2)/d)^q)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{154}$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^6 d\frac{1}{x^2}$$

$$\Big\downarrow \text{153}$$

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,3,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

output
```
(b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*
(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))
/(b*c - a*d))^q)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3 dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
```

output
```
integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\int x^3\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int(x^3*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x^3*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx=\text{Too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

output
```
((c*x**2 + d)**q*(a*x**2 + b)**p*a*c*x**4 + (c*x**2 + d)**q*(a*x**2 + b)**
p*a*d*q*x**2 + (c*x**2 + d)**q*(a*x**2 + b)**p*b*c*p*x**2 - (c*x**2 + d)**
q*(a*x**2 + b)**p*b*d*p - (c*x**2 + d)**q*(a*x**2 + b)**p*b*d*q + (c*x**2
+ d)**q*(a*x**2 + b)**p*b*d + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**
2 + b)**p*x**3)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2
*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*a**2*d**2*q**2 - 2*x**(2*p + 2
*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**3)/(x**(2*p + 2*q)*a*c*x**4 +
x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)
*a**2*d**2*q + 4*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**3)
/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x
**2 + x**(2*p + 2*q)*b*d),x)*a*b*c*d*p*q + 2*x**(2*p + 2*q)*int(((c*x**2 +
 d)**q*(a*x**2 + b)**p*x**3)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d
*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b**2*c**2*p**2 -
2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**3)/(x**(2*p + 2*q
)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p +
 2*q)*b*d),x)*b**2*c**2*p - 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2
+ b)**p)/(x**(2*p + 2*q)*a*c*x**5 + x**(2*p + 2*q)*a*d*x**3 + x**(2*p + 2*
q)*b*c*x**3 + x**(2*p + 2*q)*b*d*x),x)*b**2*d**2*p**2 - 4*x**(2*p + 2*q)*i
nt(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a*c*x**5 + x**(2*p +
2*q)*a*d*x**3 + x**(2*p + 2*q)*b*c*x**3 + x**(2*p + 2*q)*b*d*x),x)*b**2...
```

3.200 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,2,-q,2+p,1+\frac{b}{ax^2},-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2a^2(1+p)}$$

output
```
-1/2*b*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,-q,2,2+p,-d*(a+b/x^2)/(-a*
d+b*c),1+b/a/x^2)/a^2/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(-1+p+q)}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^
2)/d)^q)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \; 948$$

$$-\frac{1}{2}\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^4 d\frac{1}{x^2}$$

$$\Big\downarrow \; 154$$

$$-\frac{1}{2}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p\left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^4 d\frac{1}{x^2}$$

$$\Big\downarrow \; 153$$

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

output
```
-1/2*(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -(
(d*(a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(a^2*(1 + p)*((b*(c + d/x^2)
)/(b*c - a*d))^q)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !G
tQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
```

output
```
integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx=\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx=\int x\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \,dx$$

input
```
int(x*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int(x*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx=\text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

output
```
((c*x**2 + d)**q*(a*x**2 + b)**p*a*d*q*x**2 + (c*x**2 + d)**q*(a*x**2 + b)
**p*b*c*p*x**2 - (c*x**2 + d)**q*(a*x**2 + b)**p*b*d*p - (c*x**2 + d)**q*(
a*x**2 + b)**p*b*d*q + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)*
*p*x**3)/(x**(2*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*d**2*q*x**2
 + x**(2*p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)*a*b*c*d*p*x**2 + x**(2*
p + 2*q)*a*b*c*d*q*x**2 + x**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*q)*b**2*
c**2*p*x**2 + x**(2*p + 2*q)*b**2*c*d*p),x)*a**3*d**3*q**3 + 6*x**(2*p + 2
*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**3)/(x**(2*p + 2*q)*a**2*c*d*q*
x**4 + x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a*b*c**2*p*x**4 +
x**(2*p + 2*q)*a*b*c*d*p*x**2 + x**(2*p + 2*q)*a*b*c*d*q*x**2 + x**(2*p +
2*q)*a*b*d**2*q + x**(2*p + 2*q)*b**2*c**2*p*x**2 + x**(2*p + 2*q)*b**2*c*
d*p),x)*a**2*b*c*d**2*p*q**2 + 6*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x*
*2 + b)**p*x**3)/(x**(2*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*d**
2*q*x**2 + x**(2*p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)*a*b*c*d*p*x**2
+ x**(2*p + 2*q)*a*b*c*d*q*x**2 + x**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*
q)*b**2*c**2*p*x**2 + x**(2*p + 2*q)*b**2*c*d*p),x)*a*b**2*c**2*d*p**2*q +
 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**3)/(x**(2*p + 2*
q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a*b*
c**2*p*x**4 + x**(2*p + 2*q)*a*b*c*d*p*x**2 + x**(2*p + 2*q)*a*b*c*d*q*x**
2 + x**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*q)*b**2*c**2*p*x**2 + x**(2...
```

3.201 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx = \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,1,-q,2+p,1+\frac{b}{ax^2},-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2a(1+p)}$$

output
```
1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^q*AppellF1(p+1,-q,1,2+p,-d*(a+b/x^2)/(-a*d+b
*c),1+b/a/x^2)/a/(p+1)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx = -\frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a
*x^2)/b), -((c*x^2)/d)])/((p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 154}$$

$$-\frac{1}{2} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q x^2 d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 153}$$

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

output
```
((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
 b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c
- a*d))^q)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx=\int \frac{\left(c\,x^2+d\right)^q\left(a\,x^2+b\right)^p}{x^{2p+2q}x}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

output
```
int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*x),x)
```

3.202 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^{1+q} \text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2(bc-ad)(1+p)}$$

output
```
-1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b/x^
2)/(-a*d+b*c))/(-a*d+b*c)/(p+1)
```

Mathematica [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx = -\frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{ax^2}{b}\right)^{-p} (d+cx^2) \left(1+\frac{cx^2}{d}\right)^p \text{Hypergeometric2F1}\left(-p,-1-p-q,-p-q,\right.}{2d(1+p+q)x^2}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

output
```
-1/2*((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeome
tric2F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(d*(1
+ p + q)*x^2*(1 + (a*x^2)/b)^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {946, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2} \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x^2}$$

$$\Big\downarrow \text{80}$$

$$-\frac{1}{2} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q d\frac{1}{x^2}$$

$$\Big\downarrow \text{79}$$

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

output
```
-1/2*((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p
, -((d*(a + b/x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d)
)^q)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx=\text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

output
```
( - (c*x**2 + d)**q*(a*x**2 + b)**p*a*c*p*x**2 - (c*x**2 + d)**q*(a*x**2 +
 b)**p*a*c*q*x**2 - (c*x**2 + d)**q*(a*x**2 + b)**p*a*d*q - (c*x**2 + d)**
q*(a*x**2 + b)**p*b*c*p + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 +
b)**p)/(x**(2*p + 2*q)*a**2*c*d*p*q*x**5 + x**(2*p + 2*q)*a**2*c*d*q**2*x*
*5 + x**(2*p + 2*q)*a**2*c*d*q*x**5 + x**(2*p + 2*q)*a**2*d**2*p*q*x**3 +
x**(2*p + 2*q)*a**2*d**2*q**2*x**3 + x**(2*p + 2*q)*a**2*d**2*q*x**3 + x**
(2*p + 2*q)*a*b*c**2*p**2*x**5 + x**(2*p + 2*q)*a*b*c**2*p*q*x**5 + x**(2*
p + 2*q)*a*b*c**2*p*x**5 + x**(2*p + 2*q)*a*b*c*d*p**2*x**3 + 2*x**(2*p +
2*q)*a*b*c*d*p*q*x**3 + x**(2*p + 2*q)*a*b*c*d*p*x**3 + x**(2*p + 2*q)*a*b
*c*d*q**2*x**3 + x**(2*p + 2*q)*a*b*c*d*q*x**3 + x**(2*p + 2*q)*a*b*d**2*p
*q*x + x**(2*p + 2*q)*a*b*d**2*q**2*x + x**(2*p + 2*q)*a*b*d**2*q*x + x**(
2*p + 2*q)*b**2*c**2*p**2*x**3 + x**(2*p + 2*q)*b**2*c**2*p*q*x**3 + x**(2
*p + 2*q)*b**2*c**2*p*x**3 + x**(2*p + 2*q)*b**2*c*d*p**2*x + x**(2*p + 2*
q)*b**2*c*d*p*q*x + x**(2*p + 2*q)*b**2*c*d*p*x),x)*a**3*d**3*p**2*q**2*x*
*2 + 2*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q
)*a**2*c*d*p*q*x**5 + x**(2*p + 2*q)*a**2*c*d*q**2*x**5 + x**(2*p + 2*q)*a
**2*c*d*q*x**5 + x**(2*p + 2*q)*a**2*d**2*p*q*x**3 + x**(2*p + 2*q)*a**2*d
**2*q**2*x**3 + x**(2*p + 2*q)*a**2*d**2*q*x**3 + x**(2*p + 2*q)*a*b*c**2*
p**2*x**5 + x**(2*p + 2*q)*a*b*c**2*p*q*x**5 + x**(2*p + 2*q)*a*b*c**2*p*x
**5 + x**(2*p + 2*q)*a*b*c*d*p**2*x**3 + 2*x**(2*p + 2*q)*a*b*c*d*p*q*x...
```

3.203 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5} dx = -\frac{\left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^{1+q}}{2bd(2+p+q)} + \frac{(bc(1+p)+ad(1+q)) \left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^{1+q} \text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2bd(bc-ad)(1+p)(2+p+q)}$$

output
```
-1/2*(a+b/x^2)^(p+1)*(c+d/x^2)^(1+q)/b/d/(2+p+q)+1/2*(b*c*(p+1)+a*d*(1+q))
*(a+b/x^2)^(p+1)*(c+d/x^2)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b/x^2)/(
-a*d+b*c))/b/d/(-a*d+b*c)/(p+1)/(2+p+q)
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5} dx = \frac{\left(a+\frac{b}{x^2}\right)^{1+p} \left(c+\frac{d}{x^2}\right)^q \left(-b\left(c+\frac{d}{x^2}\right) + \frac{(bc(1+p)+ad(1+q))\left(\frac{b(d+cx^2)}{(bc-ad)x^2}\right)^{-q} \text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{d(b+ax^2)}{(bc-ad)x^2}\right)}{1+p}\right)}{2b^2d(2+p+q)}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^5,x]
```

output
```
((a + b/x^2)^(1 + p)*(c + d/x^2)^q*(-(b*(c + d/x^2)) + ((b*c*(1 + p) + a*d
*(1 + q))*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(b + a*x^2))/((b*c - a*
d)*x^2))])/((1 + p)*((b*(d + c*x^2))/((b*c - a*d)*x^2))^q)))/(2*b^2*d*(2 +
 p + q))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {948, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5} \, dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2} \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^2} d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2} \left(\frac{(ad(q+1)+bc(p+1)) \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x^2}}{bd(p+q+2)}-\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^{q+1}}{bd(p+q+2)}\right)$$

$$\Big\downarrow \text{80}$$

$$\frac{1}{2} \left(\frac{\left(c+\frac{d}{x^2}\right)^q (ad(q+1)+bc(p+1)) \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} \int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q d\frac{1}{x^2}}{bd(p+q+2)}-\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right.}{bd(p+q+2)}\right.$$

$$\Big\downarrow \text{79}$$

$$\frac{1}{2}\left(\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q(ad(q+1)+bc(p+1))\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right.}{b^2d(p+1)(p+q+2)}\right.$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^5,x]
```

output
```
(-(((a + b/x^2)^(1 + p)*(c + d/x^2)^(1 + q))/(b*d*(2 + p + q))) + ((b*c*(1
 + p) + a*d*(1 + q))*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1
 + p, -q, 2 + p, -((d*(a + b/x^2))/(b*c - a*d))])/(b^2*d*(1 + p)*(2 + p +
q)*((b*(c + d/x^2))/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^5}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^5,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^5,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^5,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^5, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^5,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^5, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^5,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^5,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^5, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^5}\,dx = \text{too large to display}$$

input

```
int((a+b/x^2)^p*(c+d/x^2)^q/x^5,x)
```

output

```
((c*x**2 + d)**q*(a*x**2 + b)**p*a**2*c*d*p*x**4 - (c*x**2 + d)**q*(a*x**2
 + b)**p*a**2*d**2*p*q*x**2 + (c*x**2 + d)**q*(a*x**2 + b)**p*a*b*c**2*q*x
**4 - (c*x**2 + d)**q*(a*x**2 + b)**p*a*b*c*d*p**2*x**2 - (c*x**2 + d)**q*
(a*x**2 + b)**p*a*b*c*d*q**2*x**2 - (c*x**2 + d)**q*(a*x**2 + b)**p*a*b*d*
*2*p*q - (c*x**2 + d)**q*(a*x**2 + b)**p*a*b*d**2*q**2 - (c*x**2 + d)**q*(
a*x**2 + b)**p*a*b*d**2*q - (c*x**2 + d)**q*(a*x**2 + b)**p*b**2*c**2*p*q*
x**2 - (c*x**2 + d)**q*(a*x**2 + b)**p*b**2*c*d*p**2 - (c*x**2 + d)**q*(a*
x**2 + b)**p*b**2*c*d*p*q - (c*x**2 + d)**q*(a*x**2 + b)**p*b**2*c*d*p - 2
*x**(2*p + 2*q)*int(((c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a**2
*c*d*p**2*q*x**5 + 2*x**(2*p + 2*q)*a**2*c*d*p*q**2*x**5 + 3*x**(2*p + 2*q
)*a**2*c*d*p*q*x**5 + x**(2*p + 2*q)*a**2*c*d*q**3*x**5 + 3*x**(2*p + 2*q)
*a**2*c*d*q**2*x**5 + 2*x**(2*p + 2*q)*a**2*c*d*q*x**5 + x**(2*p + 2*q)*a*
*2*d**2*p**2*q*x**3 + 2*x**(2*p + 2*q)*a**2*d**2*p*q**2*x**3 + 3*x**(2*p +
 2*q)*a**2*d**2*p*q*x**3 + x**(2*p + 2*q)*a**2*d**2*q**3*x**3 + 3*x**(2*p
+ 2*q)*a**2*d**2*q**2*x**3 + 2*x**(2*p + 2*q)*a**2*d**2*q*x**3 + x**(2*p +
 2*q)*a*b*c**2*p**3*x**5 + 2*x**(2*p + 2*q)*a*b*c**2*p**2*q*x**5 + 3*x**(2
*p + 2*q)*a*b*c**2*p**2*x**5 + x**(2*p + 2*q)*a*b*c**2*p*q**2*x**5 + 3*x**
(2*p + 2*q)*a*b*c**2*p*q*x**5 + 2*x**(2*p + 2*q)*a*b*c**2*p*x**5 + x**(2*p
 + 2*q)*a*b*c*d*p**3*x**3 + 3*x**(2*p + 2*q)*a*b*c*d*p**2*q*x**3 + 3*x**(2
*p + 2*q)*a*b*c*d*p**2*x**3 + 3*x**(2*p + 2*q)*a*b*c*d*p*q**2*x**3 + 6*...
```

3.204 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 230

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx = \frac{(bc(2+p)+ad(4+p+2q))\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} - \frac{\left(a+\frac{b}{x^2}\right)^{2+p}\left(c+\frac{d}{x^2}\right)^{1+q}}{2b^2d(3+p+q)} - \frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2))\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^{1+q}\text{Hypergeometr}}{2b^2d^2(bc-ad)(1+p)(2+p+q)(3+p+q)}$$

output
```
1/2*(b*c*(2+p)+a*d*(4+p+2*q))*(a+b/x^2)^(p+1)*(c+d/x^2)^(1+q)/b^2/d^2/(2+p
+q)/(3+p+q)-1/2*(a+b/x^2)^(2+p)*(c+d/x^2)^(1+q)/b^2/d/(3+p+q)-1/2*(b^2*c^2
*(p^2+3*p+2)+2*a*b*c*d*(p+1)*(1+q)+a^2*d^2*(q^2+3*q+2))*(a+b/x^2)^(p+1)*(c
+d/x^2)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b/x^2)/(-a*d+b*c))/b^2/d^2/
(-a*d+b*c)/(p+1)/(2+p+q)/(3+p+q)
```

Mathematica [A] (warning: unable to verify)

Time = 5.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx$$

$$= \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(-\frac{d+cx^2}{x^4}+\frac{(bc(2+p)+ad(2+q))(d+cx^2)}{bd(2+p+q)x^2}-\frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2))\left(\frac{b(d\cdots}{(bc-\cdots}\right.}{b^2d(1+p)(\cdots}\right.}{2bd(3+p+q)}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^7,x]
```

output
```
((a + b/x^2)^(1 + p)*(c + d/x^2)^q*(-((d + c*x^2)/x^4) + ((b*c*(2 + p) + a
*d*(2 + q))*(d + c*x^2))/(b*d*(2 + p + q)*x^2) - ((b^2*c^2*(2 + 3*p + p^2)
 + 2*a*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*Hypergeometric2F1[
1 + p, -q, 2 + p, -((d*(b + a*x^2))/((b*c - a*d)*x^2))])/(b^2*d*(1 + p)*(2
 + p + q)*((b*(d + c*x^2))/((b*c - a*d)*x^2))^q)))/(2*b*d*(3 + p + q))
```

Rubi [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 101, 25, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$-\frac{1}{2}\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 101}$$

$$\frac{1}{2}\left(-\frac{\int -\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(ac+\frac{bc(p+2)+ad(q+2)}{x^2}\right)d\frac{1}{x^2}}{bd(p+q+3)}-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^{q+1}}{bdx^2(p+q+3)}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(\frac{\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(ac+\frac{bc(p+2)+ad(q+2)}{x^2}\right)d\frac{1}{x^2}}{bd(p+q+3)}-\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^{q+1}}{bdx^2(p+q+3)}\right)$$

$\Big\downarrow$ 90

$$\frac{1}{2}\left(\frac{\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{x^2}+\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^{q+1}(ad(q+2)+bc(p+2))}{bd(p+q+2)}}{bd(p+q+3)}-\right.$$

$\Big\downarrow$ 80

$$\frac{1}{2}\left(\frac{\left(c+\frac{d}{x^2}\right)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\int \left(a+\frac{b}{x^2}\right)^p \left(\frac{bc}{bc-ad}+\frac{bd}{(bc-ad)x^2}\right)^q d\frac{1}{x^2}+\frac{\left(a+\right.}{}}{bd(p+q+3)}\right.$$

$\Big\downarrow$ 79

$$\frac{1}{2}\left(\frac{\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{b(p+1)}+\frac{\left(a\right.}{}}{bd(p+q+3)}\right.$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^7,x]
```

output
```
(-(((a + b/x^2)^(1 + p)*(c + d/x^2)^(1 + q))/(b*d*(3 + p + q)*x^2)) + (((b
*c*(2 + p) + a*d*(2 + q))*(a + b/x^2)^(1 + p)*(c + d/x^2)^(1 + q))/(b*d*(2
 + p + q)) + ((a*c - ((b*c*(1 + p) + a*d*(1 + q))*(b*c*(2 + p) + a*d*(2 +
q)))/(b*d*(2 + p + q)))*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F
1[1 + p, -q, 2 + p, -((d*(a + b/x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d
/x^2))/(b*c - a*d))^q))/(b*d*(3 + p + q)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^7,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^7,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7}\, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7}\, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^7,x, algorithm="fricas")
```

output
```
integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^7, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7} \, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/x**7,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7} \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^7,x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^7, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7} \, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^7} \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/x^7,x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^7,x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/x^7, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^7}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^7,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/x^7,x)
```

3.205 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},\right.}{5e}$$

output
```
2/5*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2)*AppellF1(-5/4,-p,-q,-1/4,-b/a/x^2,
-d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=$$
$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x(ex)^{3/2}\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{5}{4}-p-q,-p,-q,\frac{9}{4}-p-q,-\frac{ax^2}{b},-\right.}{-5+4p+4q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q
, 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e^3\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int e^3\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int e^3\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q x^3 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(-\frac{5}{4},-p,-q,-\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

output
```
(2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(
b/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{\frac{3}{2}}\,dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*e*x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\int (e\,x)^{3/2}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx=\text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

output

```
(2*sqrt(e)*e*(4*sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*a*d*q*x**2 - sqrt(
x)*(c*x**2 + d)**q*(a*x**2 + b)**p*a*d*x**2 + 4*sqrt(x)*(c*x**2 + d)**q*(a
*x**2 + b)**p*b*c*p*x**2 - sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*b*c*x**
2 - 4*sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*b*d*p - 4*sqrt(x)*(c*x**2 +
d)**q*(a*x**2 + b)**p*b*d*q + 32*x**(2*p + 2*q)*int((sqrt(x)*(c*x**2 + d)*
*q*(a*x**2 + b)**p*x**3)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 - x**(2*p + 2*q
)*a**2*c*d*x**4 + 4*x**(2*p + 2*q)*a**2*d**2*q*x**2 - x**(2*p + 2*q)*a**2*
d**2*x**2 + 4*x**(2*p + 2*q)*a*b*c**2*p*x**4 - x**(2*p + 2*q)*a*b*c**2*x**
4 + 4*x**(2*p + 2*q)*a*b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 - 2*
x**(2*p + 2*q)*a*b*c*d*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q - x**(2*p + 2*q)
*a*b*d**2 + 4*x**(2*p + 2*q)*b**2*c**2*p*x**2 - x**(2*p + 2*q)*b**2*c**2*x
**2 + 4*x**(2*p + 2*q)*b**2*c*d*p - x**(2*p + 2*q)*b**2*c*d),x)*a**3*d**3*
q**3 - 16*x**(2*p + 2*q)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x**3
)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 - x**(2*p + 2*q)*a**2*c*d*x**4 + 4*x**
(2*p + 2*q)*a**2*d**2*q*x**2 - x**(2*p + 2*q)*a**2*d**2*x**2 + 4*x**(2*p +
 2*q)*a*b*c**2*p*x**4 - x**(2*p + 2*q)*a*b*c**2*x**4 + 4*x**(2*p + 2*q)*a*
b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 - 2*x**(2*p + 2*q)*a*b*c*d*
x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q - x**(2*p + 2*q)*a*b*d**2 + 4*x**(2*p +
 2*q)*b**2*c**2*p*x**2 - x**(2*p + 2*q)*b**2*c**2*x**2 + 4*x**(2*p + 2*q)*
b**2*c*d*p - x**(2*p + 2*q)*b**2*c*d),x)*a**3*d**3*q**2 + 2*x**(2*p + 2...
```

3.206 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{3/2}\,\text{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

output
```
2/3*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2)*AppellF1(-3/4,-p,-q,1/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\sqrt{ex}\left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \text{AppellF1}\left(\frac{3}{4}-p-q,-p,-q,\frac{7}{4}-p-q,-\frac{ax^2}{b},-\right.}{-3+4p+4q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q,
7/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e^2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int e^2\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int e^2\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q x^2 d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

output
```
(2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = \int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx=\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx=\int \sqrt{e\,x}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$$
$$=\frac{2\sqrt{e}\left(\sqrt{x}\,(c\,x^2+d)^q\,(a\,x^2+b)^p\,x+2x^{2p+2q}\left(\int \frac{\sqrt{x}\,(c\,x^2+d)^q\,(a\,x^2+b)^p\,x^2}{x^{2p+2q}ac\,x^4+x^{2p+2q}ad\,x^2+x^{2p+2q}bc\,x^2+x^{2p+2q}bd}dx\right)adq+2x^{2p+2q}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

output
```
(2*sqrt(e)*(sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x + 2*x**(2*p + 2*q)*i
nt((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*x**4
 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d)
,x)*a*d*q + 2*x**(2*p + 2*q)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*
x**2)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*
b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b*c*p + 2*x**(2*p + 2*q)*int((sqrt(x)*(c
*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a
*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d),x)*b*d*p + 2*x**(2
*p + 2*q)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a*
c*x**4 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q
)*b*d),x)*b*d*q))/(3*x**(2*p + 2*q))
```

3.207 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\sqrt{ex}\,\text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

output
```
2*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2)*AppellF1(-1/4,-p,-q,3/4,-b/a/x^2,-d/
c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{4}-p-q,-p,-q,\frac{5}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(-1+4p+4q)\sqrt{ex}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int e\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\int e\left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\int e\left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q xd\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

output
```
(2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(
a*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2),x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = \text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

output

```
(2*sqrt(e)*( - sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*a*d - sqrt(x)*(c*x*
*2 + d)**q*(a*x**2 + b)**p*b*c + 8*x**(2*p + 2*q)*int((sqrt(x)*(c*x**2 + d
)**q*(a*x**2 + b)**p*x**3)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 - x**(2*p + 2
*q)*a**2*c*d*x**4 + 4*x**(2*p + 2*q)*a**2*d**2*q*x**2 - x**(2*p + 2*q)*a**
2*d**2*x**2 + 4*x**(2*p + 2*q)*a*b*c**2*p*x**4 - x**(2*p + 2*q)*a*b*c**2*x
**4 + 4*x**(2*p + 2*q)*a*b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 -
2*x**(2*p + 2*q)*a*b*c*d*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q - x**(2*p + 2*
q)*a*b*d**2 + 4*x**(2*p + 2*q)*b**2*c**2*p*x**2 - x**(2*p + 2*q)*b**2*c**2
*x**2 + 4*x**(2*p + 2*q)*b**2*c*d*p - x**(2*p + 2*q)*b**2*c*d),x)*a**3*c*d
**2*q**2 - 2*x**(2*p + 2*q)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x
**3)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 - x**(2*p + 2*q)*a**2*c*d*x**4 + 4*
x**(2*p + 2*q)*a**2*d**2*q*x**2 - x**(2*p + 2*q)*a**2*d**2*x**2 + 4*x**(2*
p + 2*q)*a*b*c**2*p*x**4 - x**(2*p + 2*q)*a*b*c**2*x**4 + 4*x**(2*p + 2*q)
*a*b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 - 2*x**(2*p + 2*q)*a*b*c
*d*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q - x**(2*p + 2*q)*a*b*d**2 + 4*x**(2*
p + 2*q)*b**2*c**2*p*x**2 - x**(2*p + 2*q)*b**2*c**2*x**2 + 4*x**(2*p + 2*
q)*b**2*c*d*p - x**(2*p + 2*q)*b**2*c*d),x)*a**3*c*d**2*q + 16*x**(2*p + 2
*q)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x**3)/(4*x**(2*p + 2*q)*a
**2*c*d*q*x**4 - x**(2*p + 2*q)*a**2*c*d*x**4 + 4*x**(2*p + 2*q)*a**2*d**2
*q*x**2 - x**(2*p + 2*q)*a**2*d**2*x**2 + 4*x**(2*p + 2*q)*a*b*c**2*p*x...
```

3.208 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

output
```
-2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/4,-p,-q,5/4,-b/a/x^2,-d/c/x^2)/e/((1
+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.25

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{4}-p-q,-p,-q,\frac{3}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(1+4p+4q)(ex)^{3/2}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {998, 937, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}} \, dx$$

$$\Big\downarrow \ 998$$

$$-\frac{2\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \ 937$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \ 937$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \ 936$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -
(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x\right)^{\frac{3}{2}}}d x$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^2*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\, dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

output
```
(2*sqrt(e)*( - (c*x**2 + d)**q*(a*x**2 + b)**p*a*d - (c*x**2 + d)**q*(a*x*
*2 + b)**p*b*c + 8*x**((4*p + 4*q + 1)/2)*int((sqrt(x)*(c*x**2 + d)**q*(a*
x**2 + b)**p*x**2)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2
*c*d*x**4 + 4*x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a**2*d**2*x
**2 + 4*x**(2*p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)*a*b*c**2*x**4 + 4*
x**(2*p + 2*q)*a*b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 + 2*x**(2*
p + 2*q)*a*b*c*d*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*q)*a*b*d
**2 + 4*x**(2*p + 2*q)*b**2*c**2*p*x**2 + x**(2*p + 2*q)*b**2*c**2*x**2 +
4*x**(2*p + 2*q)*b**2*c*d*p + x**(2*p + 2*q)*b**2*c*d),x)*a**3*c*d**2*q**2
 + 2*x**((4*p + 4*q + 1)/2)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x
**2)/(4*x**(2*p + 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*c*d*x**4 + 4*
x**(2*p + 2*q)*a**2*d**2*q*x**2 + x**(2*p + 2*q)*a**2*d**2*x**2 + 4*x**(2*
p + 2*q)*a*b*c**2*p*x**4 + x**(2*p + 2*q)*a*b*c**2*x**4 + 4*x**(2*p + 2*q)
*a*b*c*d*p*x**2 + 4*x**(2*p + 2*q)*a*b*c*d*q*x**2 + 2*x**(2*p + 2*q)*a*b*c
*d*x**2 + 4*x**(2*p + 2*q)*a*b*d**2*q + x**(2*p + 2*q)*a*b*d**2 + 4*x**(2*
p + 2*q)*b**2*c**2*p*x**2 + x**(2*p + 2*q)*b**2*c**2*x**2 + 4*x**(2*p + 2*
q)*b**2*c*d*p + x**(2*p + 2*q)*b**2*c*d),x)*a**3*c*d**2*q + 16*x**((4*p +
4*q + 1)/2)*int((sqrt(x)*(c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(4*x**(2*p
+ 2*q)*a**2*c*d*q*x**4 + x**(2*p + 2*q)*a**2*c*d*x**4 + 4*x**(2*p + 2*q)*a
**2*d**2*q*x**2 + x**(2*p + 2*q)*a**2*d**2*x**2 + 4*x**(2*p + 2*q)*a*b*...
```

3.209 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

output
```
-2/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/4,-p,-q,7/4,-b/a/x^2,-d/c/x^2)/e/(
(1+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\left(1+\frac{ax^2}{b}\right)^{-p}\left(1+\frac{cx^2}{d}\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4}-p-q,-p,-q,\frac{1}{4}-p-q,-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(3+4p+4q)(ex)^{5/2}}$$

input
```
Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p -
 q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)
/b)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {998, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{998}$$

$$-\frac{2\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\int \frac{\left(\frac{b}{ax^2}+1\right)^p\left(c+\frac{d}{x^2}\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1013}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\int \frac{\left(\frac{b}{ax^2}+1\right)^p\left(\frac{d}{cx^2}+1\right)^q}{ex}d\frac{1}{\sqrt{ex}}}{e}$$

$$\Big\downarrow \text{1012}$$

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

input
```
Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -
(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Defintions of rubi rules used

rule 998
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, Simp[-g/e   Subst[Int[(a +
b/(e^n*x^(g*n)))^p*((c + d/(e^n*x^(g*n)))^q/x^(g*(m + 1) + 1)), x], x, 1/(e
*x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fractio
nQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x^{2}}\right)^{p} \left(c+\frac{d}{x^{2}}\right)^{q}}{\left(e x \right)^{\frac{5}{2}}}d x$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{5/2}}\,dx$$

input
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x)
```

output
```
int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

output
```
(2*sqrt(e)*( - (c*x**2 + d)**q*(a*x**2 + b)**p + 24*x**((4*p + 4*q + 1)/2)
*int(((c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(16*x**((4*p + 4*q + 1)/2)*a**
2*c*d*p*q*x**4 + 12*x**((4*p + 4*q + 1)/2)*a**2*c*d*p*x**4 + 16*x**((4*p +
 4*q + 1)/2)*a**2*c*d*q**2*x**4 + 24*x**((4*p + 4*q + 1)/2)*a**2*c*d*q*x**
4 + 9*x**((4*p + 4*q + 1)/2)*a**2*c*d*x**4 + 16*x**((4*p + 4*q + 1)/2)*a**
2*d**2*p*q*x**2 + 12*x**((4*p + 4*q + 1)/2)*a**2*d**2*p*x**2 + 16*x**((4*p
 + 4*q + 1)/2)*a**2*d**2*q**2*x**2 + 24*x**((4*p + 4*q + 1)/2)*a**2*d**2*q
*x**2 + 9*x**((4*p + 4*q + 1)/2)*a**2*d**2*x**2 + 16*x**((4*p + 4*q + 1)/2
)*a*b*c**2*p**2*x**4 + 16*x**((4*p + 4*q + 1)/2)*a*b*c**2*p*q*x**4 + 24*x*
*((4*p + 4*q + 1)/2)*a*b*c**2*p*x**4 + 12*x**((4*p + 4*q + 1)/2)*a*b*c**2*
q*x**4 + 9*x**((4*p + 4*q + 1)/2)*a*b*c**2*x**4 + 16*x**((4*p + 4*q + 1)/2
)*a*b*c*d*p**2*x**2 + 32*x**((4*p + 4*q + 1)/2)*a*b*c*d*p*q*x**2 + 36*x**(
(4*p + 4*q + 1)/2)*a*b*c*d*p*x**2 + 16*x**((4*p + 4*q + 1)/2)*a*b*c*d*q**2
*x**2 + 36*x**((4*p + 4*q + 1)/2)*a*b*c*d*q*x**2 + 18*x**((4*p + 4*q + 1)/
2)*a*b*c*d*x**2 + 16*x**((4*p + 4*q + 1)/2)*a*b*d**2*p*q + 12*x**((4*p + 4
*q + 1)/2)*a*b*d**2*p + 16*x**((4*p + 4*q + 1)/2)*a*b*d**2*q**2 + 24*x**((
4*p + 4*q + 1)/2)*a*b*d**2*q + 9*x**((4*p + 4*q + 1)/2)*a*b*d**2 + 16*x**(
(4*p + 4*q + 1)/2)*b**2*c**2*p**2*x**2 + 16*x**((4*p + 4*q + 1)/2)*b**2*c*
*2*p*q*x**2 + 24*x**((4*p + 4*q + 1)/2)*b**2*c**2*p*x**2 + 12*x**((4*p + 4
*q + 1)/2)*b**2*c**2*q*x**2 + 9*x**((4*p + 4*q + 1)/2)*b**2*c**2*x**2 +...
```

3.210 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} (ex)^{1+m} \operatorname{AppellF1}\left(\frac{1}{2}(-1-m),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e(1+m)}$$

output
```
(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1+m)*AppellF1(-1/2-1/2*m,-p,-q,1/2-1/2*m,-b
/a/x^2,-d/c/x^2)/e/(1+m)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x(ex)^m \left(1+\frac{ax^2}{b}\right)^{-p} \left(1+\frac{cx^2}{d}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}(1+m-2p-2q),-p,-q,\frac{1}{2}(3+m-\right.}{1+m-2p-2q}$$

input
```
Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
 -q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2
*q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {999, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

$$\Big\downarrow \ 999$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}$$

$$\Big\downarrow \ 395$$

$$\left(\frac{1}{x}\right)^m (ex)^m \left(-\left(a+\frac{b}{x^2}\right)^p\right) \left(\frac{b}{ax^2}+1\right)^{-p} \int \left(\frac{b}{ax^2}+1\right)^p \left(c+\frac{d}{x^2}\right)^q \left(\frac{1}{x}\right)^{-m-2} d\frac{1}{x}$$

$$\Big\downarrow \ 395$$

$$\left(\frac{1}{x}\right)^m (ex)^m \left(-\left(a+\frac{b}{x^2}\right)^p\right) \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \int \left(\frac{b}{ax^2}+1\right)^p \left(\frac{d}{cx^2}+1\right)^q \left(\frac{1}{x}\right)^{-m-2} d$$

$$\Big\downarrow \ 394$$

$$\frac{x(ex)^m \left(a+\frac{b}{x^2}\right)^p \left(\frac{b}{ax^2}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{d}{cx^2}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2}(-m-1),-p,-q,\frac{1-m}{2},-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{m+1}$$

input
```
Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

output
```
((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m
)/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2
))^q)
```

Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Maple [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

output
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
```

output
```
integral((e*x)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \text{Timed out}$$

input
```
integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = \int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \int (e\,x)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$$

input
```
int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

output
```
int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx = \text{Too large to display}$$

input
```
int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

output

```
(e**m*(x**m*(c*x**2 + d)**q*(a*x**2 + b)**p*x + 2*x**(2*p + 2*q)*int((x**m
*(c*x**2 + d)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*m*x**4 + x**(2*
p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*m*x**2 + x**(2*p + 2*q)*a*d*x**2 +
x**(2*p + 2*q)*b*c*m*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d*m
 + x**(2*p + 2*q)*b*d),x)*a*d*m*q + 2*x**(2*p + 2*q)*int((x**m*(c*x**2 + d
)**q*(a*x**2 + b)**p*x**2)/(x**(2*p + 2*q)*a*c*m*x**4 + x**(2*p + 2*q)*a*c
*x**4 + x**(2*p + 2*q)*a*d*m*x**2 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*
q)*b*c*m*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d*m + x**(2*p +
 2*q)*b*d),x)*a*d*q + 2*x**(2*p + 2*q)*int((x**m*(c*x**2 + d)**q*(a*x**2 +
 b)**p*x**2)/(x**(2*p + 2*q)*a*c*m*x**4 + x**(2*p + 2*q)*a*c*x**4 + x**(2*
p + 2*q)*a*d*m*x**2 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*m*x**2
+ x**(2*p + 2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d*m + x**(2*p + 2*q)*b*d),x)*
b*c*m*p + 2*x**(2*p + 2*q)*int((x**m*(c*x**2 + d)**q*(a*x**2 + b)**p*x**2)
/(x**(2*p + 2*q)*a*c*m*x**4 + x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d
*m*x**2 + x**(2*p + 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*m*x**2 + x**(2*p +
2*q)*b*c*x**2 + x**(2*p + 2*q)*b*d*m + x**(2*p + 2*q)*b*d),x)*b*c*p + 2*x*
*(2*p + 2*q)*int((x**m*(c*x**2 + d)**q*(a*x**2 + b)**p)/(x**(2*p + 2*q)*a*
c*m*x**4 + x**(2*p + 2*q)*a*c*x**4 + x**(2*p + 2*q)*a*d*m*x**2 + x**(2*p +
 2*q)*a*d*x**2 + x**(2*p + 2*q)*b*c*m*x**2 + x**(2*p + 2*q)*b*c*x**2 + x**
(2*p + 2*q)*b*d*m + x**(2*p + 2*q)*b*d),x)*b*d*m*p + 2*x**(2*p + 2*q)*i...
```

3.211 $\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 533

$$\begin{aligned}
&\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx \\
&= \frac{\sqrt{5-\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}-\frac{2\sqrt[10]{2}c^{2/5}}{\sqrt{5-\sqrt{5}}\sqrt[5]{d}x\sqrt[5]{c+dx^5}}\right)}{20\sqrt[10]{2}ac^{2/5}d^{4/5}} \\
&\quad-\frac{\sqrt{5+\sqrt{5}}\arctan\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}+\frac{2\sqrt[10]{2}c^{2/5}}{\sqrt{5+\sqrt{5}}\sqrt[5]{d}x\sqrt[5]{c+dx^5}}\right)}{20\sqrt[10]{2}ac^{2/5}d^{4/5}} \\
&\quad-\frac{\log\left(2^{2/5}\sqrt[5]{d}+\frac{c^{2/5}}{x\sqrt[5]{c+dx^5}}\right)}{10\ 2^{3/5}ac^{2/5}d^{4/5}} \\
&\quad+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{4/5}-2^{2/5}c^{2/5}\sqrt[5]{d}x\sqrt[5]{c+dx^5}-2^{2/5}\sqrt{5}c^{2/5}\sqrt[5]{d}x\sqrt[5]{c+dx^5}+2\ 2^{4/5}d^{2/5}x^2\left(c+dx^5\right)^{2/5}}{x^2\left(c+dx^5\right)^{2/5}}\right)}{40\ 2^{3/5}ac^{2/5}d^{4/5}} \\
&\quad+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{4/5}-2^{2/5}c^{2/5}\sqrt[5]{d}x\sqrt[5]{c+dx^5}+2^{2/5}\sqrt{5}c^{2/5}\sqrt[5]{d}x\sqrt[5]{c+dx^5}+2\ 2^{4/5}d^{2/5}x^2\left(c+dx^5\right)^{2/5}}{x^2\left(c+dx^5\right)^{2/5}}\right)}{40\ 2^{3/5}ac^{2/5}d^{4/5}}
\end{aligned}$$

output
```
-1/40*(5-5^(1/2))^(1/2)*arctan(-1/5*(25+10*5^(1/2))^(1/2)+2*2^(1/10)*c^(2/
5)/(5-5^(1/2))^(1/2)/d^(1/5)/x/(d*x^5+c)^(1/5))*2^(9/10)/a/c^(2/5)/d^(4/5)
-1/40*(5+5^(1/2))^(1/2)*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*2^(1/10)*c^(2/5
)/(5+5^(1/2))^(1/2)/d^(1/5)/x/(d*x^5+c)^(1/5))*2^(9/10)/a/c^(2/5)/d^(4/5)-
1/20*ln(2^(2/5)*d^(1/5)+c^(2/5)/x/(d*x^5+c)^(1/5))*2^(2/5)/a/c^(2/5)/d^(4/
5)+1/80*(5^(1/2)+1)*ln((2*c^(4/5)-2^(2/5)*c^(2/5)*d^(1/5)*x*(d*x^5+c)^(1/5
)-2^(2/5)*5^(1/2)*c^(2/5)*d^(1/5)*x*(d*x^5+c)^(1/5)+2*2^(4/5)*d^(2/5)*x^2*
(d*x^5+c)^(2/5))/x^2/(d*x^5+c)^(2/5))*2^(2/5)/a/c^(2/5)/d^(4/5)+1/80*(-5^(
1/2)+1)*ln((2*c^(4/5)-2^(2/5)*c^(2/5)*d^(1/5)*x*(d*x^5+c)^(1/5)+2^(2/5)*5^
(1/2)*c^(2/5)*d^(1/5)*x*(d*x^5+c)^(1/5)+2*2^(4/5)*d^(2/5)*x^2*(d*x^5+c)^(2
/5))/x^2/(d*x^5+c)^(2/5))*2^(2/5)/a/c^(2/5)/d^(4/5)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \frac{x^4\sqrt[5]{\frac{c+dx^5}{c}}\,\text{AppellF1}\left(\frac{4}{5},\frac{1}{5},1,\frac{9}{5},-\frac{dx^5}{c},-\frac{2dx^5}{c}\right)}{4ac\sqrt[5]{c+dx^5}}$$

input
```
Integrate[x^3/((c + d*x^5)^(1/5)*(a*c + 2*a*d*x^5)),x]
```

output
```
(x^4*((c + d*x^5)/c)^(1/5)*AppellF1[4/5, 1/5, 1, 9/5, -((d*x^5)/c), (-2*d*
x^5)/c])/(4*a*c*(c + d*x^5)^(1/5))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx$$

$\Big\downarrow$ 1013

$$\frac{\sqrt[5]{\frac{dx^5}{c}+1}\int \frac{x^3}{a(2dx^5+c)\sqrt[5]{\frac{dx^5}{c}+1}}dx}{\sqrt[5]{c+dx^5}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt[5]{\frac{dx^5}{c}+1}\int \frac{x^3}{(2dx^5+c)\sqrt[5]{\frac{dx^5}{c}+1}}dx}{a\sqrt[5]{c+dx^5}}$$

$\Big\downarrow$ 1012

$$\frac{x^4\sqrt[5]{\frac{dx^5}{c}+1}\,\text{AppellF1}\left(\frac{4}{5},1,\frac{1}{5},\frac{9}{5},-\frac{2dx^5}{c},-\frac{dx^5}{c}\right)}{4ac\sqrt[5]{c+dx^5}}$$

input
```
Int[x^3/((c + d*x^5)^(1/5)*(a*c + 2*a*d*x^5)),x]
```

output
```
(x^4*(1 + (d*x^5)/c)^(1/5)*AppellF1[4/5, 1, 1/5, 9/5, (-2*d*x^5)/c, -((d*x
^5)/c)])/(4*a*c*(c + d*x^5)^(1/5))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{(d\,x^5+c)^{\frac{1}{5}}\,(2x^5ad+ac)}dx$$

input
```
int(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x)
```

output
```
int(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (trace 0)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \frac{\int \frac{x^3}{c\sqrt[5]{c+dx^5}+2dx^5\sqrt[5]{c+dx^5}}\,dx}{a}$$

input
```
integrate(x**3/(d*x**5+c)**(1/5)/(2*a*d*x**5+a*c),x)
```

output
```
Integral(x**3/(c*(c + d*x**5)**(1/5) + 2*d*x**5*(c + d*x**5)**(1/5)), x)/a
```

Maxima [F]

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \int \frac{x^3}{(2\,adx^5+ac)(dx^5+c)^{\frac{1}{5}}}\,dx$$

input
```
integrate(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x, algorithm="maxima")
```

output
```
integrate(x^3/((2*a*d*x^5 + a*c)*(d*x^5 + c)^(1/5)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \int \frac{x^3}{(2\,adx^5+ac)(dx^5+c)^{\frac{1}{5}}}\,dx$$

input
```
integrate(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x, algorithm="giac")
```

output
```
integrate(x^3/((2*a*d*x^5 + a*c)*(d*x^5 + c)^(1/5)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \int \frac{x^3}{{\left(d\,x^5+c\right)}^{1/5}\,\left(2\,a\,d\,x^5+a\,c\right)}\,dx$$

input
```
int(x^3/((c + d*x^5)^(1/5)*(a*c + 2*a*d*x^5)),x)
```

output
```
int(x^3/((c + d*x^5)^(1/5)*(a*c + 2*a*d*x^5)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[5]{c+dx^5}\,(ac+2adx^5)}\,dx = \frac{\int \frac{x^3}{(d\,x^5+c)^{\frac{1}{5}}\,c+2(d\,x^5+c)^{\frac{1}{5}}\,d\,x^5}dx}{a}$$

input
```
int(x^3/(d*x^5+c)^(1/5)/(2*a*d*x^5+a*c),x)
```

output
```
int(x**3/((c + d*x**5)**(1/5)*c + 2*(c + d*x**5)**(1/5)*d*x**5),x)/a
```

3.212 $\int \frac{x^2}{\sqrt[4]{c+dx^4}(ac+2adx^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 162

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{c}}{\sqrt[4]{d}x\sqrt[4]{c+dx^4}}\right)}{8a\sqrt{c}d^{3/4}} - \frac{\arctan\left(1+\frac{\sqrt{c}}{\sqrt[4]{d}x\sqrt[4]{c+dx^4}}\right)}{8a\sqrt{c}d^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{2\sqrt{c}\sqrt[4]{d}}{x\sqrt[4]{c+dx^4}\left(2\sqrt{d}+\frac{c}{x^2\sqrt{c+dx^4}}\right)}\right)}{8a\sqrt{c}d^{3/4}}$$

output
```
1/8*arctan(1-c^(1/2)/d^(1/4)/x/(d*x^4+c)^(1/4))/a/c^(1/2)/d^(3/4)-1/8*arct
an(1+c^(1/2)/d^(1/4)/x/(d*x^4+c)^(1/4))/a/c^(1/2)/d^(3/4)-1/8*arctanh(2*c^
(1/2)*d^(1/4)/x/(d*x^4+c)^(1/4)/(2*d^(1/2)+c/x^2/(d*x^4+c)^(1/2)))/a/c^(1/
2)/d^(3/4)
```

Mathematica [A] (verified)

Time = 8.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \frac{\arctan\left(\frac{-\frac{\sqrt{c}}{2\sqrt[4]{d}}+\frac{\sqrt[4]{d}x^2\sqrt{c+dx^4}}{\sqrt{c}}}{x\sqrt[4]{c+dx^4}}\right)-\operatorname{arctanh}\left(\frac{2\sqrt{c}\sqrt[4]{d}x\sqrt[4]{c+dx^4}}{c+2\sqrt{d}x^2\sqrt{c+dx^4}}\right)}{8a\sqrt{c}d^{3/4}}$$

input
```
Integrate[x^2/((c + d*x^4)^(1/4)*(a*c + 2*a*d*x^4)),x]
```

output
```
(ArcTan[(-1/2*Sqrt[c]/d^(1/4) + (d^(1/4)*x^2*Sqrt[c + d*x^4])/Sqrt[c])/(x*
(c + d*x^4)^(1/4))] - ArcTanh[(2*Sqrt[c]*d^(1/4)*x*(c + d*x^4)^(1/4))/(c +
 2*Sqrt[d]*x^2*Sqrt[c + d*x^4])])/(8*a*Sqrt[c]*d^(3/4))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt[4]{\frac{dx^4}{c}+1}\int \frac{x^2}{a(2dx^4+c)\sqrt[4]{\frac{dx^4}{c}+1}}dx}{\sqrt[4]{c+dx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt[4]{\frac{dx^4}{c}+1}\int \frac{x^2}{(2dx^4+c)\sqrt[4]{\frac{dx^4}{c}+1}}dx}{a\sqrt[4]{c+dx^4}}$$

$$\Big\downarrow \; 1012$$

$$\frac{x^3\sqrt[4]{\frac{dx^4}{c}+1}\,\text{AppellF1}\left(\frac{3}{4},1,\frac{1}{4},\frac{7}{4},-\frac{2dx^4}{c},-\frac{dx^4}{c}\right)}{3ac\sqrt[4]{c+dx^4}}$$

input
```
Int[x^2/((c + d*x^4)^(1/4)*(a*c + 2*a*d*x^4)),x]
```

output
```
(x^3*(1 + (d*x^4)/c)^(1/4)*AppellF1[3/4, 1, 1/4, 7/4, (-2*d*x^4)/c, -((d*x
^4)/c)])/(3*a*c*(c + d*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(d\,x^4+c)^{\frac{1}{4}}\,(2ad\,x^4+ac)}dx$$

input
```
int(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x)
```

output
```
int(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \frac{\int \frac{x^2}{c\sqrt[4]{c+dx^4}+2dx^4\sqrt[4]{c+dx^4}}\,dx}{a}$$

input
```
integrate(x**2/(d*x**4+c)**(1/4)/(2*a*d*x**4+a*c),x)
```

output
```
Integral(x**2/(c*(c + d*x**4)**(1/4) + 2*d*x**4*(c + d*x**4)**(1/4)), x)/a
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \int \frac{x^2}{(2\,adx^4+ac)(dx^4+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x, algorithm="maxima")
```

output
```
integrate(x^2/((2*a*d*x^4 + a*c)*(d*x^4 + c)^(1/4)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \int \frac{x^2}{(2\,adx^4+ac)(dx^4+c)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x, algorithm="giac")
```

output
```
integrate(x^2/((2*a*d*x^4 + a*c)*(d*x^4 + c)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \int \frac{x^2}{(d\,x^4+c)^{1/4}\,(2\,a\,d\,x^4+a\,c)}\,dx$$

input
```
int(x^2/((c + d*x^4)^(1/4)*(a*c + 2*a*d*x^4)),x)
```

output
```
int(x^2/((c + d*x^4)^(1/4)*(a*c + 2*a*d*x^4)), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{c+dx^4}\,(ac+2adx^4)}\,dx = \frac{\int \frac{x^2}{(d\,x^4+c)^{\frac{1}{4}}c+2(d\,x^4+c)^{\frac{1}{4}}d\,x^4}dx}{a}$$

input
```
int(x^2/(d*x^4+c)^(1/4)/(2*a*d*x^4+a*c),x)
```

output
```
int(x**2/((c + d*x**4)**(1/4)*c + 2*(c + d*x**4)**(1/4)*d*x**4),x)/a
```

3.213 $\int \frac{x}{\sqrt[3]{c+dx^3}(ac+2adx^3)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{d}-\frac{\sqrt[3]{2}c^{2/3}}{x\sqrt[3]{c+dx^3}}}{\sqrt{3}\sqrt[3]{d}}\right)}{2\sqrt[3]{2}\sqrt{3}ac^{2/3}d^{2/3}}-\frac{\log\left(2^{2/3}\sqrt[3]{d}+\frac{c^{2/3}}{x\sqrt[3]{c+dx^3}}\right)}{6\sqrt[3]{2}ac^{2/3}d^{2/3}}+\frac{\log\left(2\sqrt[3]{2}d^{2/3}+\frac{c^{4/3}}{x^2(c+dx^3)^{2/3}}-\frac{2^{2/3}c^{2/3}\sqrt[3]{d}}{x\sqrt[3]{c+dx^3}}\right)}{12\sqrt[3]{2}ac^{2/3}d^{2/3}}$$

output
```
1/12*arctan(1/3*(d^(1/3)-2^(1/3)*c^(2/3)/x/(d*x^3+c)^(1/3))*3^(1/2)/d^(1/3
))*2^(2/3)*3^(1/2)/a/c^(2/3)/d^(2/3)-1/12*ln(2^(2/3)*d^(1/3)+c^(2/3)/x/(d*
x^3+c)^(1/3))*2^(2/3)/a/c^(2/3)/d^(2/3)+1/24*ln(2*2^(1/3)*d^(2/3)+c^(4/3)/
x^2/(d*x^3+c)^(2/3)-2^(2/3)*c^(2/3)*d^(1/3)/x/(d*x^3+c)^(1/3))*2^(2/3)/a/c
^(2/3)/d^(2/3)
```

Mathematica [A] (verified)

Time = 5.77 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{d}x\sqrt[3]{c+dx^3}}{\sqrt[3]{2}c^{2/3}-\sqrt[3]{d}x\sqrt[3]{c+dx^3}}\right)+2\log\left(x\sqrt[3]{c+dx^3}\right)-\log\left(x^2(c+dx^3)^{2/3}\right)-2\log\left(\sqrt[3]{2}c^{2/3}+2\right.}{12\sqrt[3]{2}ac^{2/3}d^{2/3}}$$

input
```
Integrate[x/((c + d*x^3)^(1/3)*(a*c + 2*a*d*x^3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(Sqrt[3]*d^(1/3)*x*(c + d*x^3)^(1/3))/(2^(1/3)*c^(2/3) -
 d^(1/3)*x*(c + d*x^3)^(1/3))] + 2*Log[x*(c + d*x^3)^(1/3)] - Log[x^2*(c +
 d*x^3)^(2/3)] - 2*Log[2^(1/3)*c^(2/3) + 2*d^(1/3)*x*(c + d*x^3)^(1/3)] +
Log[2^(2/3)*c^(4/3) - 2*2^(1/3)*c^(2/3)*d^(1/3)*x*(c + d*x^3)^(1/3) + 4*d^
(2/3)*x^2*(c + d*x^3)^(2/3)])/(12*2^(1/3)*a*c^(2/3)*d^(2/3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.31, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx$$
$$\Big\downarrow \text{ 1013}$$
$$\frac{\sqrt[3]{\frac{dx^3}{c}+1}\int \frac{x}{a(2dx^3+c)\sqrt[3]{\frac{dx^3}{c}+1}}dx}{\sqrt[3]{c+dx^3}}$$

$$\downarrow 27$$

$$\frac{\sqrt[3]{\frac{dx^3}{c}+1}\int \frac{x}{(2dx^3+c)\sqrt[3]{\frac{dx^3}{c}+1}}dx}{a\sqrt[3]{c+dx^3}}$$

$$\downarrow 1012$$

$$\frac{x^2\sqrt[3]{\frac{dx^3}{c}+1}\operatorname{AppellF1}\left(\frac{2}{3},1,\frac{1}{3},\frac{5}{3},-\frac{2dx^3}{c},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{c+dx^3}}$$

input
```
Int[x/((c + d*x^3)^(1/3)*(a*c + 2*a*d*x^3)),x]
```

output
```
(x^2*(1 + (d*x^3)/c)^(1/3)*AppellF1[2/3, 1, 1/3, 5/3, (-2*d*x^3)/c, -((d*x
^3)/c)])/(2*a*c*(c + d*x^3)^(1/3))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(d\,x^{3}+c\right)^{\frac{1}{3}} \left(2 a d\,x^{3}+a c\right)}d x$$

input
```
int(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x)
```

output
```
int(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \text{Timed out}$$

input
```
integrate(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \frac{\int \frac{x}{c\sqrt[3]{c+dx^3}+2dx^3\sqrt[3]{c+dx^3}}\,dx}{a}$$

input
```
integrate(x/(d*x**3+c)**(1/3)/(2*a*d*x**3+a*c),x)
```

output
```
Integral(x/(c*(c + d*x**3)**(1/3) + 2*d*x**3*(c + d*x**3)**(1/3)), x)/a
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \int \frac{x}{(2\,adx^3+ac)(dx^3+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x, algorithm="maxima")
```

output
```
integrate(x/((2*a*d*x^3 + a*c)*(d*x^3 + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \int \frac{x}{(2\,adx^3+ac)(dx^3+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x, algorithm="giac")
```

output
```
integrate(x/((2*a*d*x^3 + a*c)*(d*x^3 + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \int \frac{x}{(d\,x^3+c)^{1/3}\,(2\,a\,d\,x^3+a\,c)}\,dx$$

input
```
int(x/((c + d*x^3)^(1/3)*(a*c + 2*a*d*x^3)),x)
```

output
```
int(x/((c + d*x^3)^(1/3)*(a*c + 2*a*d*x^3)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[3]{c+dx^3}\,(ac+2adx^3)}\,dx = \frac{\int \frac{x}{(dx^3+c)^{\frac{1}{3}}c+2(dx^3+c)^{\frac{1}{3}}dx^3}dx}{a}$$

input
```
int(x/(d*x^3+c)^(1/3)/(2*a*d*x^3+a*c),x)
```

output
```
int(x/((c + d*x**3)**(1/3)*c + 2*(c + d*x**3)**(1/3)*d*x**3),x)/a
```

3.214 $\int \frac{1}{\sqrt{c+dx^2}(ac+2adx^2)} dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int \frac{1}{\sqrt{c+dx^2}\left(ac+2adx^2\right)} dx = \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{ac\sqrt{d}}$$

output
```
arctan(d^(1/2)*x/(d*x^2+c)^(1/2))/a/c/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{c+dx^2}\left(ac+2adx^2\right)} dx = -\frac{\arctan\left(1+\frac{2dx^2}{c}-\frac{2\sqrt{d}x\sqrt{c+dx^2}}{c}\right)}{ac\sqrt{d}}$$

input
```
Integrate[1/(Sqrt[c + d*x^2]*(a*c + 2*a*d*x^2)),x]
```

output
```
-(ArcTan[1 + (2*d*x^2)/c - (2*Sqrt[d]*x*Sqrt[c + d*x^2])/c]/(a*c*Sqrt[d]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx$$

$$\Big\downarrow \text{ 291}$$

$$\int \frac{1}{\frac{acdx^2}{c+dx^2}+ac}d\frac{x}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{ac\sqrt{d}}$$

input
```
Int[1/(Sqrt[c + d*x^2]*(a*c + 2*a*d*x^2)),x]
```

output
```
ArcTan[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(a*c*Sqrt[d])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{ac\sqrt{d}}$
default	$\frac{-\frac{\sqrt{2}\,\ln\left(\frac{c+\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{2d}\right)+\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{4d\left(x-\frac{\sqrt{-2cd}}{2d}\right)^2+4\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{2d}\right)+2c}}{2}}{x-\frac{\sqrt{-2cd}}{2d}}\right)}{2\sqrt{-2cd}\,\sqrt{c}}+\frac{\sqrt{2}\,\ln\left(\frac{c-\sqrt{-2cd}\left(x+\frac{\sqrt{-2cd}}{2d}\right)+\frac{\sqrt{2}\,\sqrt{}}{}}{}\right.}{}}{a}$

input
```
int(1/(d*x^2+c)^(1/2)/(2*a*d*x^2+a*c),x,method=_RETURNVERBOSE)
```

output
```
-1/a/c/d^(1/2)*arctan((d*x^2+c)^(1/2)/x/d^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = \left[-\frac{\sqrt{-d}\log\left(-\frac{4\,d^2x^4+4\,cdx^2-4\,\sqrt{dx^2+c}c\sqrt{-d}x-c^2}{4\,d^2x^4+4\,cdx^2+c^2}\right)}{4\,acd}, -\frac{\arctan\left(\frac{\sqrt{dx^2+c}c\sqrt{d}}{2\,(d^2x^3+cdx)}\right)}{2\,ac\sqrt{d}}\right]$$

input
```
integrate(1/(d*x^2+c)^(1/2)/(2*a*d*x^2+a*c),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-d)*log(-(4*d^2*x^4 + 4*c*d*x^2 - 4*sqrt(d*x^2 + c)*c*sqrt(-d)*
x - c^2)/(4*d^2*x^4 + 4*c*d*x^2 + c^2))/(a*c*d), -1/2*arctan(1/2*sqrt(d*x^
2 + c)*c*sqrt(d)/(d^2*x^3 + c*d*x))/(a*c*sqrt(d))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = \frac{\int \frac{1}{c\sqrt{c+dx^2}+2dx^2\sqrt{c+dx^2}}\,dx}{a}$$

input
```
integrate(1/(d*x**2+c)**(1/2)/(2*a*d*x**2+a*c),x)
```

output
```
Integral(1/(c*sqrt(c + d*x**2) + 2*d*x**2*sqrt(c + d*x**2)), x)/a
```

Maxima [F]

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = \int \frac{1}{(2\,adx^2+ac)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(d*x^2+c)^(1/2)/(2*a*d*x^2+a*c),x, algorithm="maxima")
```

output
```
integrate(1/((2*a*d*x^2 + a*c)*sqrt(d*x^2 + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = -\frac{\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2}{c}\right)}{ac\sqrt{d}}$$

input
```
integrate(1/(d*x^2+c)^(1/2)/(2*a*d*x^2+a*c),x, algorithm="giac")
```

output
```
-arctan((sqrt(d)*x - sqrt(d*x^2 + c))^2/c)/(a*c*sqrt(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = \begin{cases} \frac{x}{a\,c^{3/2}} & \text{if } d=0 \\ -\frac{1}{4\,a\,d^{3/2}\,x\,\sqrt{x^2}} & \text{if } c=0\wedge d\neq 0 \\ \frac{\operatorname{atan}\left(\frac{x\,\sqrt{a\,c\,d}}{\sqrt{a\,c}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{a^2\,c^2\,d}} & \text{if } 0<a\,c\,d \\ \frac{\ln\left(-\frac{x\,\sqrt{-a\,c\,d}+\sqrt{a\,c\,(d\,x^2+c)}}{x\,\sqrt{-a\,c\,d}-\sqrt{a\,c\,(d\,x^2+c)}}\right)}{2\,\sqrt{-a^2\,c^2\,d}} & \text{if } a\,c\,d<0 \\ \int \frac{1}{\sqrt{d\,x^2+c}\,(2\,a\,d\,x^2+a\,c)}\,dx & \text{if } a\,c\,d\notin\mathbb{R} \end{cases}$$

input
```
int(1/((c + d*x^2)^(1/2)*(a*c + 2*a*d*x^2)),x)
```

output
```
piecewise(d == 0, x/(a*c^(3/2)), c == 0 & d ~= 0, -1/(4*a*d^(3/2)*x*(x^2)^
(1/2)), 0 < a*c*d, atan((x*(a*c*d)^(1/2))/((a*c)^(1/2)*(c + d*x^2)^(1/2)))
/(a^2*c^2*d)^(1/2), a*c*d < 0, log(-(x*(-a*c*d)^(1/2) + (a*c*(c + d*x^2))^
(1/2))/(x*(-a*c*d)^(1/2) - (a*c*(c + d*x^2))^(1/2)))/(2*(-a^2*c^2*d)^(1/2)
), ~in(a*c*d, 'real'), int(1/((c + d*x^2)^(1/2)*(a*c + 2*a*d*x^2)), x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{c+dx^2}\,(ac+2adx^2)}\,dx = \frac{\sqrt{d}\left(atan\left(\frac{2\sqrt{d\,x^2+c}-\sqrt{c}\,\sqrt{2}+2\sqrt{d}\,x}{\sqrt{c}\,\sqrt{2}}\right)-atan\left(\frac{2\sqrt{d\,x^2+c}+\sqrt{c}\,\sqrt{2}+2\sqrt{d}\,x}{\sqrt{c}\,\sqrt{2}}\right)\right)}{acd}$$

input
```
int(1/(d*x^2+c)^(1/2)/(2*a*d*x^2+a*c),x)
```

output
```
(sqrt(d)*(atan((2*sqrt(c + d*x**2) - sqrt(c)*sqrt(2) + 2*sqrt(d)*x)/(sqrt(
c)*sqrt(2))) - atan((2*sqrt(c + d*x**2) + sqrt(c)*sqrt(2) + 2*sqrt(d)*x)/(
sqrt(c)*sqrt(2)))))/(a*c*d)
```

3.215 $\int \frac{1}{x(c+dx)(ac+2adx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 38

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx=\frac{\log(x)}{ac^2}+\frac{\log(c+dx)}{ac^2}-\frac{2\log(c+2dx)}{ac^2}$$

output
```
ln(x)/a/c^2+ln(d*x+c)/a/c^2-2*ln(2*d*x+c)/a/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx=\frac{\log(x)}{ac^2}+\frac{\log(c+dx)}{ac^2}-\frac{2\log(c+2dx)}{ac^2}$$

input
```
Integrate[1/(x*(c + d*x)*(a*c + 2*a*d*x)),x]
```

output
```
Log[x]/(a*c^2) + Log[c + d*x]/(a*c^2) - (2*Log[c + 2*d*x])/(a*c^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(c+dx)(ac+2adx)} \, dx$$
$$\Big\downarrow \ 93$$
$$\int \left(\frac{d}{ac^2(c+dx)}-\frac{4d}{ac^2(c+2dx)}+\frac{1}{ac^2x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{\log(c+dx)}{ac^2}-\frac{2\log(c+2dx)}{ac^2}+\frac{\log(x)}{ac^2}$$

input
```
Int[1/(x*(c + d*x)*(a*c + 2*a*d*x)),x]
```

output
```
Log[x]/(a*c^2) + Log[c + d*x]/(a*c^2) - (2*Log[c + 2*d*x])/(a*c^2)
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$\frac{\ln(x)-2\ln\left(dx+\frac{c}{2}\right)+\ln(dx+c)}{a\,c^{2}}$	27
default	$\frac{\frac{\ln(x)}{c^{2}}-\frac{2\ln(2dx+c)}{c^{2}}+\frac{\ln(dx+c)}{c^{2}}}{a}$	34
risch	$-\frac{2\ln(-2dx-c)}{a\,c^{2}}+\frac{\ln(-d\,x^{2}-cx)}{a\,c^{2}}$	38
norman	$\frac{\ln(x)}{a\,c^{2}}+\frac{\ln(dx+c)}{a\,c^{2}}-\frac{2\ln(2dx+c)}{a\,c^{2}}$	39

input
```
int(1/x/(d*x+c)/(2*a*d*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-2*ln(d*x+1/2*c)+ln(d*x+c))/a/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx=\frac{\log\left(dx^{2}+cx\right)-2\,\log\left(2\,dx+c\right)}{ac^{2}}$$

input
```
integrate(1/x/(d*x+c)/(2*a*d*x+a*c),x, algorithm="fricas")
```

output
```
(log(d*x^2 + c*x) - 2*log(2*d*x + c))/(a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx=-\frac{2\log\left(\frac{c}{2d}+x\right)}{ac^{2}}+\frac{\log\left(\frac{cx}{d}+x^{2}\right)}{ac^{2}}$$

input
```
integrate(1/x/(d*x+c)/(2*a*d*x+a*c),x)
```

output
```
-2*log(c/(2*d) + x)/(a*c**2) + log(c*x/d + x**2)/(a*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx = -\frac{2\,\log\left(2\,dx+c\right)}{ac^2}+\frac{\log\left(dx+c\right)}{ac^2}+\frac{\log\left(x\right)}{ac^2}$$

input
```
integrate(1/x/(d*x+c)/(2*a*d*x+a*c),x, algorithm="maxima")
```

output
```
-2*log(2*d*x + c)/(a*c^2) + log(d*x + c)/(a*c^2) + log(x)/(a*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx = -\frac{2\,\log\left(|2\,dx+c|\right)}{ac^2}+\frac{\log\left(|dx+c|\right)}{ac^2}+\frac{\log\left(|x|\right)}{ac^2}$$

input
```
integrate(1/x/(d*x+c)/(2*a*d*x+a*c),x, algorithm="giac")
```

output
```
-2*log(abs(2*d*x + c))/(a*c^2) + log(abs(d*x + c))/(a*c^2) + log(abs(x))/(
a*c^2)
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx = \frac{\ln\left(x\,\left(c+d\,x\right)\right)}{a\,c^2}-\frac{2\,\ln\left(-c-2\,d\,x\right)}{a\,c^2}$$

input
```
int(1/(x*(a*c + 2*a*d*x)*(c + d*x)),x)
```

output
```
log(x*(c + d*x))/(a*c^2) - (2*log(- c - 2*d*x))/(a*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{1}{x(c+dx)(ac+2adx)}\,dx = \frac{\log(dx+c)-2\log(2dx+c)+\log(x)}{a\,c^2}$$

input
```
int(1/x/(d*x+c)/(2*a*d*x+a*c),x)
```

output
```
(log(c + d*x) - 2*log(c + 2*d*x) + log(x))/(a*c**2)
```

3.216 $\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx=-\frac{1}{4ax^2}-\frac{c}{4adx}+\frac{c^2\log\left(c+\frac{2d}{x}\right)}{8ad^2}$$

output
```
-1/4/a/x^2-1/4*c/a/d/x+1/8*c^2*ln(c+2*d/x)/a/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx=-\frac{1}{4ax^2}-\frac{c}{4adx}-\frac{c^2\log(x)}{8ad^2}+\frac{c^2\log(2d+cx)}{8ad^2}$$

input
```
Integrate[(c + d/x)/((a*c + (2*a*d)/x)*x^3),x]
```

output
```
-1/4*1/(a*x^2) - c/(4*a*d*x) - (c^2*Log[x])/(8*a*d^2) + (c^2*Log[2*d + c*x
])/(8*a*d^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {947, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+\frac{d}{x}}{x^3\left(ac+\frac{2ad}{x}\right)}dx$$
$$\Big\downarrow \; 947$$
$$\int \frac{cx+d}{x^3(acx+2ad)}dx$$
$$\Big\downarrow \; 86$$
$$\int \left(\frac{c^3}{8ad^2(cx+2d)}-\frac{c^2}{8ad^2x}+\frac{c}{4adx^2}+\frac{1}{2ax^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{c^2\log(x)}{8ad^2}+\frac{c^2\log(cx+2d)}{8ad^2}-\frac{c}{4adx}-\frac{1}{4ax^2}$$

input
```
Int[(c + d/x)/((a*c + (2*a*d)/x)*x^3),x]
```

output
```
-1/4*1/(a*x^2) - c/(4*a*d*x) - (c^2*Log[x])/(8*a*d^2) + (c^2*Log[2*d + c*x
])/(8*a*d^2)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{-\frac{1}{4x^2}-\frac{c}{4dx}-\frac{c^2\ln(x)}{8d^2}+\frac{c^2\ln(cx+2d)}{8d^2}}{a}$	46
parallelrisch	$-\frac{c^2\ln(x)x^2-c^2\ln(cx+2d)x^2+2cdx+2d^2}{8d^2a\,x^2}$	48
risch	$\frac{-\frac{cx}{4d}-\frac{1}{4}}{a\,x^2}+\frac{c^2\ln(-cx-2d)}{8a\,d^2}-\frac{c^2\ln(x)}{8a\,d^2}$	51
norman	$\frac{-\frac{x}{4a}-\frac{c\,x^2}{4ad}}{x^3}-\frac{c^2\ln(x)}{8a\,d^2}+\frac{c^2\ln(cx+2d)}{8a\,d^2}$	57

input
```
int((c+d/x)/(a*c+2*a*d/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/4/x^2-1/4*c/d/x-1/8*c^2/d^2*ln(x)+1/8*c^2/d^2*ln(c*x+2*d))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx=\frac{c^2x^2\log\left(cx+2\,d\right)-c^2x^2\log\left(x\right)-2\,cdx-2\,d^2}{8\,ad^2x^2}$$

input
```
integrate((c+d/x)/(a*c+2*a*d/x)/x^3,x, algorithm="fricas")
```

output
```
1/8*(c^2*x^2*log(c*x + 2*d) - c^2*x^2*log(x) - 2*c*d*x - 2*d^2)/(a*d^2*x^2
)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx=\frac{c^2\left(-\frac{\log\left(x\right)}{8}+\frac{\log\left(x+\frac{2d}{c}\right)}{8}\right)}{ad^2}+\frac{-cx-d}{4adx^2}$$

input
```
integrate((c+d/x)/(a*c+2*a*d/x)/x**3,x)
```

output
```
c**2*(-log(x)/8 + log(x + 2*d/c)/8)/(a*d**2) + (-c*x - d)/(4*a*d*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx=\frac{c^2\log\left(cx+2\,d\right)}{8\,ad^2}-\frac{c^2\log\left(x\right)}{8\,ad^2}-\frac{cx+d}{4\,adx^2}$$

input
```
integrate((c+d/x)/(a*c+2*a*d/x)/x^3,x, algorithm="maxima")
```

output
```
1/8*c^2*log(c*x + 2*d)/(a*d^2) - 1/8*c^2*log(x)/(a*d^2) - 1/4*(c*x + d)/(a
*d*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx = \frac{c^2\log\left(|cx+2\,d|\right)}{8\,ad^2} - \frac{c^2\log\left(|x|\right)}{8\,ad^2} - \frac{cdx+d^2}{4\,ad^2x^2}$$

input
```
integrate((c+d/x)/(a*c+2*a*d/x)/x^3,x, algorithm="giac")
```

output
```
1/8*c^2*log(abs(c*x + 2*d))/(a*d^2) - 1/8*c^2*log(abs(x))/(a*d^2) - 1/4*(c
*d*x + d^2)/(a*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx = \frac{c^2\,\mathrm{atanh}\left(\frac{c\,x}{d}+1\right)}{4\,a\,d^2} - \frac{\frac{c\,x}{4\,d}+\frac{1}{4}}{a\,x^2}$$

input
```
int((c + d/x)/(x^3*(a*c + (2*a*d)/x)),x)
```

output
```
(c^2*atanh((c*x)/d + 1))/(4*a*d^2) - ((c*x)/(4*d) + 1/4)/(a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{c+\frac{d}{x}}{\left(ac+\frac{2ad}{x}\right)x^3}\,dx = \frac{\log(cx+2d)\,c^2x^2-\log(x)\,c^2x^2-2cdx-2d^2}{8a\,d^2x^2}$$

input
```
int((c+d/x)/(a*c+2*a*d/x)/x^3,x)
```

output
```
(log(c*x + 2*d)*c**2*x**2 - log(x)*c**2*x**2 - 2*c*d*x - 2*d**2)/(8*a*d**2
*x**2)
```

3.217 $\int \frac{d+cx}{x^3(2ad+acx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx=-\frac{1}{4ax^2}-\frac{c}{4adx}-\frac{c^2\log(x)}{8ad^2}+\frac{c^2\log(2d+cx)}{8ad^2}$$

output
```
-1/4/a/x^2-1/4*c/a/d/x-1/8*c^2*ln(x)/a/d^2+1/8*c^2*ln(c*x+2*d)/a/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx=-\frac{1}{4ax^2}-\frac{c}{4adx}-\frac{c^2\log(x)}{8ad^2}+\frac{c^2\log(2d+cx)}{8ad^2}$$

input
```
Integrate[(d + c*x)/(x^3*(2*a*d + a*c*x)),x]
```

output
```
-1/4*1/(a*x^2) - c/(4*a*d*x) - (c^2*Log[x])/(8*a*d^2) + (c^2*Log[2*d + c*x
])/(8*a*d^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx+d}{x^3(acx+2ad)} \, dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{c^3}{8ad^2(cx+2d)}-\frac{c^2}{8ad^2x}+\frac{c}{4adx^2}+\frac{1}{2ax^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\log(x)}{8ad^2}+\frac{c^2\log(cx+2d)}{8ad^2}-\frac{c}{4adx}-\frac{1}{4ax^2}$$

input
```
Int[(d + c*x)/(x^3*(2*a*d + a*c*x)),x]
```

output
```
-1/4*1/(a*x^2) - c/(4*a*d*x) - (c^2*Log[x])/(8*a*d^2) + (c^2*Log[2*d + c*x
])/(8*a*d^2)
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{-\frac{1}{4x^{2}}-\frac{c}{4dx}-\frac{c^{2}\ln(x)}{8d^{2}}+\frac{c^{2}\ln(cx+2d)}{8d^{2}}}{a}$	46
parallelrisch	$-\frac{c^{2}\ln(x)x^{2}-c^{2}\ln(cx+2d)x^{2}+2cdx+2d^{2}}{8d^{2}a\,x^{2}}$	48
risch	$\frac{-\frac{cx}{4d}-\frac{1}{4}}{a\,x^{2}}+\frac{c^{2}\ln(-cx-2d)}{8a\,d^{2}}-\frac{c^{2}\ln(x)}{8a\,d^{2}}$	51
norman	$\frac{-\frac{1}{4a}-\frac{cx}{4ad}}{x^{2}}-\frac{c^{2}\ln(x)}{8a\,d^{2}}+\frac{c^{2}\ln(cx+2d)}{8a\,d^{2}}$	54

input
```
int((c*x+d)/x^3/(a*c*x+2*a*d),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-1/4/x^2-1/4*c/d/x-1/8*c^2/d^2*ln(x)+1/8*c^2/d^2*ln(c*x+2*d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{c^2x^2\log(cx+2\,d)-c^2x^2\log(x)-2\,cdx-2\,d^2}{8\,ad^2x^2}$$

input
```
integrate((c*x+d)/x^3/(a*c*x+2*a*d),x, algorithm="fricas")
```

output
```
1/8*(c^2*x^2*log(c*x + 2*d) - c^2*x^2*log(x) - 2*c*d*x - 2*d^2)/(a*d^2*x^2
)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{c^2\left(-\frac{\log(x)}{8}+\frac{\log\left(x+\frac{2d}{c}\right)}{8}\right)}{ad^2}+\frac{-cx-d}{4adx^2}$$

input
```
integrate((c*x+d)/x**3/(a*c*x+2*a*d),x)
```

output
```
c**2*(-log(x)/8 + log(x + 2*d/c)/8)/(a*d**2) + (-c*x - d)/(4*a*d*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{c^2\log(cx+2\,d)}{8\,ad^2}-\frac{c^2\log(x)}{8\,ad^2}-\frac{cx+d}{4\,adx^2}$$

input
```
integrate((c*x+d)/x^3/(a*c*x+2*a*d),x, algorithm="maxima")
```

output
```
1/8*c^2*log(c*x + 2*d)/(a*d^2) - 1/8*c^2*log(x)/(a*d^2) - 1/4*(c*x + d)/(a
*d*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{c^2\log(|cx+2\,d|)}{8\,ad^2}-\frac{c^2\log(|x|)}{8\,ad^2}-\frac{cdx+d^2}{4\,ad^2x^2}$$

input
```
integrate((c*x+d)/x^3/(a*c*x+2*a*d),x, algorithm="giac")
```

output
```
1/8*c^2*log(abs(c*x + 2*d))/(a*d^2) - 1/8*c^2*log(abs(x))/(a*d^2) - 1/4*(c
*d*x + d^2)/(a*d^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.62

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{c^2\,\mathrm{atanh}\left(\frac{c\,x}{d}+1\right)}{4\,a\,d^2}-\frac{\frac{c\,x}{4\,d}+\frac{1}{4}}{a\,x^2}$$

input
```
int((d + c*x)/(x^3*(2*a*d + a*c*x)),x)
```

output
```
(c^2*atanh((c*x)/d + 1))/(4*a*d^2) - ((c*x)/(4*d) + 1/4)/(a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{d+cx}{x^3(2ad+acx)}\,dx = \frac{\log(cx+2d)\,c^2x^2-\log(x)\,c^2x^2-2cdx-2d^2}{8a\,d^2x^2}$$

input
```
int((c*x+d)/x^3/(a*c*x+2*a*d),x)
```

output
```
(log(c*x + 2*d)*c**2*x**2 - log(x)*c**2*x**2 - 2*c*d*x - 2*d**2)/(8*a*d**2
*x**2)
```

3.218 $\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 59

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}+\frac{c\arctan\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{4ad^{3/2}}$$

output
```
-1/4*(c+d/x^2)^(1/2)/a/d/x+1/4*c*arctan(d^(1/2)/(c+d/x^2)^(1/2)/x)/a/d^(3/
2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx=-\frac{\frac{\sqrt{c+\frac{d}{x^2}}}{d}+\frac{c\sqrt{c+\frac{d}{x^2}}\,x^2\arctan\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{d+cx^2}}}{4ax}$$

input
```
Integrate[Sqrt[c + d/x^2]/((a*c + (2*a*d)/x^2)*x^4),x]
```

output
```
-1/4*(Sqrt[c + d/x^2]/d + (c*Sqrt[c + d/x^2]*x^2*ArcTan[Sqrt[d + c*x^2]/Sq
rt[d]])/(d^(3/2)*Sqrt[d + c*x^2]))/(a*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {997, 380, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4\left(ac+\frac{2ad}{x^2}\right)}dx$$

$$\Big\downarrow \text{997}$$

$$-\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{380}$$

$$\frac{\int \frac{c^2}{\sqrt{c+\frac{d}{x^2}}\left(c+\frac{2d}{x^2}\right)}d\frac{1}{x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\left(c+\frac{2d}{x^2}\right)}d\frac{1}{x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{291}$$

$$\frac{c^2\int \frac{1}{\frac{dc}{x^2}+c}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{218}$$

$$\frac{c\arctan\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{4ad^{3/2}}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

input
```
Int[Sqrt[c + d/x^2]/((a*c + (2*a*d)/x^2)*x^4),x]
```

output
```
-1/4*Sqrt[c + d/x^2]/(a*d*x) + (c*ArcTan[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(4*
a*d^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(47) = 94$.

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-2\sqrt{c\,x^2+d}\,\sqrt{-d}\,c\,x^2-\ln\left(-\frac{2c\left(\sqrt{-2cd}\,x+\sqrt{-d}\,\sqrt{c\,x^2+d}+d\right)}{-cx+\sqrt{-2cd}}\right)cd\,x^2-\ln\left(-\frac{2c\left(\sqrt{-2cd}\,x-\sqrt{-d}\,\sqrt{c\,x^2+d}-d\right)}{cx+\sqrt{-2cd}}\right)cd\,x^2+2(c\cdots\right.}{8x\sqrt{c\,x^2+d}\,a\,d^2\sqrt{-d}}$
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}}{4dxa}+\frac{\left(\frac{c\ln\left(\frac{-2d+2\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{c}\right)+2\sqrt{-d}\sqrt{c\left(x-\frac{\sqrt{-2cd}}{c}\right)^2+2\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{c}\right)-d}}{x-\frac{\sqrt{-2cd}}{c}}\right)}{8d\sqrt{-d}}+\frac{c\ln\left(\frac{-2d-2\sqrt{-2cd}\left(x+\frac{\sqrt{-2cd}}{c}\cdots\right.\right.}{\cdots}\right.}{a\sqrt{c\,x^2+d}\cdots}$

input
```
int((c+d/x^2)^(1/2)/(a*c+2*a*d/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*((c*x^2+d)/x^2)^(1/2)/x*(-2*(c*x^2+d)^(1/2)*(-d)^(1/2)*c*x^2-ln(-2*c*
((-2*c*d)^(1/2)*x+(-d)^(1/2)*(c*x^2+d)^(1/2)+d)/(-c*x+(-2*c*d)^(1/2)))*c*d
*x^2-ln(-2*c*((-2*c*d)^(1/2)*x-(-d)^(1/2)*(c*x^2+d)^(1/2)-d)/(c*x+(-2*c*d)
^(1/2)))*c*d*x^2+2*(c*x^2+d)^(3/2)*(-d)^(1/2))/(c*x^2+d)^(1/2)/a/d^2/(-d)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx = \left[-\frac{c\sqrt{-d}x\log\left(\frac{c^2x^4+4\,c\sqrt{-d}x^3\sqrt{\frac{cx^2+d}{x^2}}-4\,cdx^2-4\,d^2}{c^2x^4+4\,cdx^2+4\,d^2}\right)+4\,d\sqrt{\frac{cx^2+d}{x^2}}}{16\,ad^2x},\right.$$
$$\left.-\frac{c\sqrt{d}x\arctan\left(\frac{c\sqrt{d}x^3\sqrt{\frac{cx^2+d}{x^2}}}{2\,(cdx^2+d^2)}\right)+2\,d\sqrt{\frac{cx^2+d}{x^2}}}{8\,ad^2x}\right]$$

input
```
integrate((c+d/x^2)^(1/2)/(a*c+2*a*d/x^2)/x^4,x, algorithm="fricas")
```

output
```
[-1/16*(c*sqrt(-d)*x*log((c^2*x^4 + 4*c*sqrt(-d)*x^3*sqrt((c*x^2 + d)/x^2)
 - 4*c*d*x^2 - 4*d^2)/(c^2*x^4 + 4*c*d*x^2 + 4*d^2)) + 4*d*sqrt((c*x^2 + d
)/x^2))/(a*d^2*x), -1/8*(c*sqrt(d)*x*arctan(1/2*c*sqrt(d)*x^3*sqrt((c*x^2
+ d)/x^2)/(c*d*x^2 + d^2)) + 2*d*sqrt((c*x^2 + d)/x^2))/(a*d^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx = \frac{\int \frac{\sqrt{c+\frac{d}{x^2}}}{cx^4+2dx^2}\,dx}{a}$$

input
```
integrate((c+d/x**2)**(1/2)/(a*c+2*a*d/x**2)/x**4,x)
```

output
```
Integral(sqrt(c + d/x**2)/(c*x**4 + 2*d*x**2), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx = \int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2\,ad}{x^2}\right)x^4}\,dx$$

input
```
integrate((c+d/x^2)^(1/2)/(a*c+2*a*d/x^2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c + d/x^2)/((a*c + 2*a*d/x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx = -\frac{c\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\operatorname{sgn}(x)}{4\,ad^{\frac{3}{2}}} - \frac{\sqrt{cx^2+d}\operatorname{sgn}(x)}{4\,adx^2}$$

input
```
integrate((c+d/x^2)^(1/2)/(a*c+2*a*d/x^2)/x^4,x, algorithm="giac")
```

output
```
-1/4*c*arctan(sqrt(c*x^2 + d)/sqrt(d))*sgn(x)/(a*d^(3/2)) - 1/4*sqrt(c*x^2
 + d)*sgn(x)/(a*d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx = \int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4\,\left(a\,c+\frac{2\,a\,d}{x^2}\right)}\,dx$$

input
```
int((c + d/x^2)^(1/2)/(x^4*(a*c + (2*a*d)/x^2)),x)
```

output
```
int((c + d/x^2)^(1/2)/(x^4*(a*c + (2*a*d)/x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^4}\,dx$$
$$= \frac{-2\sqrt{d}\,atan\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}\,\sqrt{2}+\sqrt{d}}\right)c\,x^2-2\sqrt{c\,x^2+d}\,d-\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i}{\sqrt{d}}\right)c\,i\,x^2+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}}{}\right.}{8a\,d^2x^2}$$

input
```
int((c+d/x^2)^(1/2)/(a*c+2*a*d/x^2)/x^4,x)
```

output
```
( - 2*sqrt(d)*atan((sqrt(c*x**2 + d) + sqrt(c)*x)/(sqrt(d)*sqrt(2) + sqrt(
d)))*c*x**2 - 2*sqrt(c*x**2 + d)*d - sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(
c)*x - sqrt(d)*sqrt(2)*i + sqrt(d)*i)/sqrt(d))*c*i*x**2 + sqrt(d)*log((sqr
t(c*x**2 + d) + sqrt(c)*x + sqrt(d)*sqrt(2)*i - sqrt(d)*i)/sqrt(d))*c*i*x*
*2)/(8*a*d**2*x**2)
```

3.219 $\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2(2ad+acx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 59

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\,(2ad+acx^2)}\,dx=-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}+\frac{c\arctan\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{4ad^{3/2}}$$

output
```
-1/4*(c+d/x^2)^(1/2)/a/d/x+1/4*c*arctan(d^(1/2)/(c+d/x^2)^(1/2)/x)/a/d^(3/
2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\,(2ad+acx^2)}\,dx=-\frac{\frac{\sqrt{c+\frac{d}{x^2}}}{d}+\frac{c\sqrt{c+\frac{d}{x^2}}\,x^2\arctan\left(\frac{\sqrt{d+cx^2}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{d+cx^2}}}{4ax}$$

input
```
Integrate[Sqrt[c + d/x^2]/(x^2*(2*a*d + a*c*x^2)),x]
```

output
```
-1/4*(Sqrt[c + d/x^2]/d + (c*Sqrt[c + d/x^2]*x^2*ArcTan[Sqrt[d + c*x^2]/Sq
rt[d]])/(d^(3/2)*Sqrt[d + c*x^2]))/(a*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1016, 997, 380, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\left(acx^2+2ad\right)}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4\left(ac+\frac{2ad}{x^2}\right)}dx$$

$$\Big\downarrow \text{997}$$

$$-\int \frac{\sqrt{c+\frac{d}{x^2}}}{\left(ac+\frac{2ad}{x^2}\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{380}$$

$$\frac{\int \frac{c^2}{\sqrt{c+\frac{d}{x^2}}\left(c+\frac{2d}{x^2}\right)}d\frac{1}{x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\left(c+\frac{2d}{x^2}\right)}d\frac{1}{x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{291}$$

$$\frac{c^2\int \frac{1}{\frac{dc}{x^2}+c}d\frac{1}{\sqrt{c+\frac{d}{x^2}}x}}{4ad}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

$$\Big\downarrow \text{218}$$

$$\frac{c\arctan\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{4ad^{3/2}}-\frac{\sqrt{c+\frac{d}{x^2}}}{4adx}$$

input
```
Int[Sqrt[c + d/x^2]/(x^2*(2*a*d + a*c*x^2)),x]
```

output
```
-1/4*Sqrt[c + d/x^2]/(a*d*x) + (c*ArcTan[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(4*
a*d^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.98

method	result
default	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-2\sqrt{c\,x^2+d}\,\sqrt{-d}\,c\,x^2-\ln\left(-\frac{2c\left(\sqrt{-2cd}\,x+\sqrt{-d}\,\sqrt{c\,x^2+d}+d\right)}{-cx+\sqrt{-2cd}}\right)cd\,x^2-\ln\left(-\frac{2c\left(\sqrt{-2cd}\,x-\sqrt{-d}\,\sqrt{c\,x^2+d}-d\right)}{cx+\sqrt{-2cd}}\right)cd\,x^2+2(c\right.}{8x\sqrt{c\,x^2+d}\,a\,d^2\sqrt{-d}}$
risch	$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}}{4dxa}+\frac{\left(\frac{c\ln\left(\frac{-2d+2\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{c}\right)+2\sqrt{-d}\sqrt{c\left(x-\frac{\sqrt{-2cd}}{c}\right)^2+2\sqrt{-2cd}\left(x-\frac{\sqrt{-2cd}}{c}\right)-d}}{x-\frac{\sqrt{-2cd}}{c}}\right)}{8d\sqrt{-d}}+\frac{c\ln\left(\frac{-2d-2\sqrt{-2cd}\left(x+\frac{\sqrt{-2cd}}{c}\right.}{}\right.}{}\right.}{a\sqrt{c\,x^2+d}}$

input
```
int((c+d/x^2)^(1/2)/x^2/(a*c*x^2+2*a*d),x,method=_RETURNVERBOSE)
```

output
```
-1/8*((c*x^2+d)/x^2)^(1/2)/x*(-2*(c*x^2+d)^(1/2)*(-d)^(1/2)*c*x^2-ln(-2*c*
((-2*c*d)^(1/2)*x+(-d)^(1/2)*(c*x^2+d)^(1/2)+d)/(-c*x+(-2*c*d)^(1/2)))*c*d
*x^2-ln(-2*c*((-2*c*d)^(1/2)*x-(-d)^(1/2)*(c*x^2+d)^(1/2)-d)/(c*x+(-2*c*d)
^(1/2)))*c*d*x^2+2*(c*x^2+d)^(3/2)*(-d)^(1/2))/(c*x^2+d)^(1/2)/a/d^2/(-d)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2(2ad+acx^2)}\,dx = \left[-\frac{c\sqrt{-d}x\log\left(\frac{c^2x^4+4\,c\sqrt{-d}x^3\sqrt{\frac{cx^2+d}{x^2}}-4\,cdx^2-4\,d^2}{c^2x^4+4\,cdx^2+4\,d^2}\right)+4\,d\sqrt{\frac{cx^2+d}{x^2}}}{16\,ad^2x}, -\frac{c\sqrt{d}x\arctan\left(\frac{c\sqrt{d}x^3\sqrt{\frac{cx^2+d}{x^2}}}{2\,(cdx^2+d^2)}\right)+2\,d\sqrt{\frac{cx^2+d}{x^2}}}{8\,ad^2x}\right]$$

input
```
integrate((c+d/x^2)^(1/2)/x^2/(a*c*x^2+2*a*d),x, algorithm="fricas")
```

output
```
[-1/16*(c*sqrt(-d)*x*log((c^2*x^4 + 4*c*sqrt(-d)*x^3*sqrt((c*x^2 + d)/x^2)
 - 4*c*d*x^2 - 4*d^2)/(c^2*x^4 + 4*c*d*x^2 + 4*d^2)) + 4*d*sqrt((c*x^2 + d
)/x^2))/(a*d^2*x), -1/8*(c*sqrt(d)*x*arctan(1/2*c*sqrt(d)*x^3*sqrt((c*x^2
+ d)/x^2)/(c*d*x^2 + d^2)) + 2*d*sqrt((c*x^2 + d)/x^2))/(a*d^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2(2ad+acx^2)}\,dx = \frac{\int \frac{\sqrt{c+\frac{d}{x^2}}}{cx^4+2dx^2}\,dx}{a}$$

input
```
integrate((c+d/x**2)**(1/2)/x**2/(a*c*x**2+2*a*d),x)
```

output
```
Integral(sqrt(c + d/x**2)/(c*x**4 + 2*d*x**2), x)/a
```

Maxima [F]

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\left(2ad+acx^2\right)}\,dx=\int \frac{\sqrt{c+\frac{d}{x^2}}}{(acx^2+2\,ad)x^2}\,dx$$

input
```
integrate((c+d/x^2)^(1/2)/x^2/(a*c*x^2+2*a*d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c + d/x^2)/((a*c*x^2 + 2*a*d)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\left(2ad+acx^2\right)}\,dx=-\frac{c\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\operatorname{sgn}(x)}{4\,ad^{\frac{3}{2}}}-\frac{\sqrt{cx^2+d}\operatorname{sgn}(x)}{4\,adx^2}$$

input
```
integrate((c+d/x^2)^(1/2)/x^2/(a*c*x^2+2*a*d),x, algorithm="giac")
```

output
```
-1/4*c*arctan(sqrt(c*x^2 + d)/sqrt(d))*sgn(x)/(a*d^(3/2)) - 1/4*sqrt(c*x^2
 + d)*sgn(x)/(a*d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\left(2ad+acx^2\right)}\,dx=\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\,\left(a\,c\,x^2+2\,a\,d\right)}\,dx$$

input
```
int((c + d/x^2)^(1/2)/(x^2*(2*a*d + a*c*x^2)),x)
```

output
```
int((c + d/x^2)^(1/2)/(x^2*(2*a*d + a*c*x^2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2\,(2ad+acx^2)}\,dx$$
$$= \frac{-2\sqrt{d}\,atan\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x}{\sqrt{d}\,\sqrt{2}+\sqrt{d}}\right)c\,x^2-2\sqrt{c\,x^2+d}\,d-\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+d}+\sqrt{c}\,x-\sqrt{d}\,\sqrt{2}\,i+\sqrt{d}\,i}{\sqrt{d}}\right)ci\,x^2+\sqrt{d}\log\left(\frac{\sqrt{c\,x^2+c}\cdots}{}\right.}{8a\,d^2x^2}$$

input
```
int((c+d/x^2)^(1/2)/x^2/(a*c*x^2+2*a*d),x)
```

output
```
( - 2*sqrt(d)*atan((sqrt(c*x**2 + d) + sqrt(c)*x)/(sqrt(d)*sqrt(2) + sqrt(
d)))*c*x**2 - 2*sqrt(c*x**2 + d)*d - sqrt(d)*log((sqrt(c*x**2 + d) + sqrt(
c)*x - sqrt(d)*sqrt(2)*i + sqrt(d)*i)/sqrt(d))*c*i*x**2 + sqrt(d)*log((sqr
t(c*x**2 + d) + sqrt(c)*x + sqrt(d)*sqrt(2)*i - sqrt(d)*i)/sqrt(d))*c*i*x*
*2)/(8*a*d**2*x**2)
```

3.220 $\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 270

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = -\frac{\sqrt[3]{c+\frac{d}{x^3}}}{4adx} - \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\arctan\left(\frac{c^{2/3}-2\ 2^{2/3}\sqrt[3]{d}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}}{\sqrt{3}c^{2/3}}\right)}{4\ 2^{2/3}\sqrt{3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x} + \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\log\left(c^{2/3}+2^{2/3}\sqrt[3]{d}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}\right)}{8\ 2^{2/3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x} - \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\log\left(c+\frac{2d}{x^3}\right)}{12\ 2^{2/3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x}$$

output
```
-1/4*(c+d/x^3)^(1/3)/a/d/x-1/24*c^(2/3)*(c+d/x^3)^(1/3)*arctan(1/3*(c^(2/3
)-2*2^(2/3)*d^(1/3)*((c+d/x^3)/x^3)^(1/3))*3^(1/2)/c^(2/3))*2^(1/3)*3^(1/2
)/a/d^(4/3)/((c+d/x^3)/x^3)^(1/3)/x+1/16*c^(2/3)*(c+d/x^3)^(1/3)*ln(c^(2/3
)+2^(2/3)*d^(1/3)*((c+d/x^3)/x^3)^(1/3))*2^(1/3)/a/d^(4/3)/((c+d/x^3)/x^3)
^(1/3)/x-1/24*c^(2/3)*(c+d/x^3)^(1/3)*ln(c+2*d/x^3)*2^(1/3)/a/d^(4/3)/((c+
d/x^3)/x^3)^(1/3)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = -\frac{\sqrt[3]{c+\frac{d}{x^3}}\left(8d(d+cx^3)+c^2x^6\left(1+\frac{cx^3}{d}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{cx^3}{d},-\frac{cx^3}{2d}\right)\right)}{32ad^2x\,(d+cx^3)}$$

input
```
Integrate[(c + d/x^3)^(1/3)/((a*c + (2*a*d)/x^3)*x^5),x]
```

output
```
-1/32*((c + d/x^3)^(1/3)*(8*d*(d + c*x^3) + c^2*x^6*(1 + (c*x^3)/d)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((c*x^3)/d), -1/2*(c*x^3)/d]))/(a*d^2*x*(d + c
*x^3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {997, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^5\left(ac+\frac{2ad}{x^3}\right)}dx$$

$$\Big\downarrow \text{ 997}$$

$$-\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 1013}$$

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\int \frac{\sqrt[3]{\frac{d}{cx^3}+1}}{a\left(c+\frac{2d}{x^3}\right)x^3}d\frac{1}{x}}{\sqrt[3]{\frac{d}{cx^3}+1}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\int \frac{\sqrt[3]{\frac{d}{cx^3}+1}}{\left(c+\frac{2d}{x^3}\right)x^3}d\frac{1}{x}}{a\sqrt[3]{\frac{d}{cx^3}+1}}$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{3},\frac{7}{3},-\frac{2d}{cx^3},-\frac{d}{cx^3}\right)}{4acx^4\sqrt[3]{\frac{d}{cx^3}+1}}$$

input
```
Int[(c + d/x^3)^(1/3)/((a*c + (2*a*d)/x^3)*x^5),x]
```

output
```
-1/4*((c + d/x^3)^(1/3)*AppellF1[4/3, 1, -1/3, 7/3, (-2*d)/(c*x^3), -(d/(c
*x^3))])/(a*c*(1 + d/(c*x^3))^(1/3)*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}dx$$

input
```
int((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x)
```

output
```
int((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \text{Timed out}$$

input
```
integrate((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \frac{\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{cx^5+2dx^2}\,dx}{a}$$

input
```
integrate((c+d/x**3)**(1/3)/(a*c+2*a*d/x**3)/x**5,x)
```

output
```
Integral((c + d/x**3)**(1/3)/(c*x**5 + 2*d*x**2), x)/a
```

Maxima [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{3}}}{\left(ac+\frac{2\,ad}{x^3}\right)x^5}\,dx$$

input
```
integrate((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x, algorithm="maxima")
```

output
```
integrate((c + d/x^3)^(1/3)/((a*c + 2*a*d/x^3)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{3}}}{\left(ac+\frac{2\,ad}{x^3}\right)x^5}\,dx$$

input
```
integrate((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x, algorithm="giac")
```

output
```
integrate((c + d/x^3)^(1/3)/((a*c + 2*a*d/x^3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \int \frac{\left(c+\frac{d}{x^3}\right)^{1/3}}{x^5\,\left(a\,c+\frac{2\,a\,d}{x^3}\right)}\,dx$$

input
```
int((c + d/x^3)^(1/3)/(x^5*(a*c + (2*a*d)/x^3)),x)
```

output
```
int((c + d/x^3)^(1/3)/(x^5*(a*c + (2*a*d)/x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^5}\,dx = \frac{-(c\,x^3+d)^{\frac{1}{3}}-\left(\int \frac{(c\,x^3+d)^{\frac{1}{3}}x^3}{c^2x^6+3cd\,x^3+2d^2}dx\right)c^2x^2}{4ad\,x^2}$$

input
```
int((c+d/x^3)^(1/3)/(a*c+2*a*d/x^3)/x^5,x)
```

output
```
( - ((c*x**3 + d)**(1/3) + int(((c*x**3 + d)**(1/3)*x**3)/(c**2*x**6 + 3*c
*d*x**3 + 2*d**2),x)*c**2*x**2))/(4*a*d*x**2)
```

3.221 $\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2(2ad+acx^3)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 270

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\left(2ad+acx^3\right)}\,dx = -\frac{\sqrt[3]{c+\frac{d}{x^3}}}{4adx} - \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\arctan\left(\frac{c^{2/3}-2\ 2^{2/3}\sqrt[3]{d}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}}{\sqrt{3}c^{2/3}}\right)}{4\ 2^{2/3}\sqrt{3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x} + \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\log\left(c^{2/3}+2^{2/3}\sqrt[3]{d}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}\right)}{8\ 2^{2/3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x} - \frac{c^{2/3}\sqrt[3]{c+\frac{d}{x^3}}\log\left(c+\frac{2d}{x^3}\right)}{12\ 2^{2/3}ad^{4/3}\sqrt[3]{\frac{c+\frac{d}{x^3}}{x^3}}x}$$

output
```
-1/4*(c+d/x^3)^(1/3)/a/d/x-1/24*c^(2/3)*(c+d/x^3)^(1/3)*arctan(1/3*(c^(2/3
)-2*2^(2/3)*d^(1/3)*((c+d/x^3)/x^3)^(1/3))*3^(1/2)/c^(2/3))*2^(1/3)*3^(1/2
)/a/d^(4/3)/((c+d/x^3)/x^3)^(1/3)/x+1/16*c^(2/3)*(c+d/x^3)^(1/3)*ln(c^(2/3
)+2^(2/3)*d^(1/3)*((c+d/x^3)/x^3)^(1/3))*2^(1/3)/a/d^(4/3)/((c+d/x^3)/x^3)
^(1/3)/x-1/24*c^(2/3)*(c+d/x^3)^(1/3)*ln(c+2*d/x^3)*2^(1/3)/a/d^(4/3)/((c+
d/x^3)/x^3)^(1/3)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 11.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\left(2ad+acx^3\right)}\,dx = -\frac{\sqrt[3]{c+\frac{d}{x^3}}\left(8d(d+cx^3)+c^2x^6\left(1+\frac{cx^3}{d}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3},-\frac{cx^3}{d},-\frac{cx^3}{2d}\right)\right)}{32ad^2x\left(d+cx^3\right)}$$

input
```
Integrate[(c + d/x^3)^(1/3)/(x^2*(2*a*d + a*c*x^3)),x]
```

output
```
-1/32*((c + d/x^3)^(1/3)*(8*d*(d + c*x^3) + c^2*x^6*(1 + (c*x^3)/d)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((c*x^3)/d), -1/2*(c*x^3)/d]))/(a*d^2*x*(d + c
*x^3))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1016, 997, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2 (acx^3+2ad)}dx$$

$\Big\downarrow$ 1016

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^5 \left(ac+\frac{2ad}{x^3}\right)}dx$$

$\Big\downarrow$ 997

$$-\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{\left(ac+\frac{2ad}{x^3}\right)x^3}d\frac{1}{x}$$

$\Big\downarrow$ 1013

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\int \frac{\sqrt[3]{\frac{d}{cx^3}+1}}{a\left(c+\frac{2d}{x^3}\right)x^3}d\frac{1}{x}}{\sqrt[3]{\frac{d}{cx^3}+1}}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\int \frac{\sqrt[3]{\frac{d}{cx^3}+1}}{\left(c+\frac{2d}{x^3}\right)x^3}d\frac{1}{x}}{a\sqrt[3]{\frac{d}{cx^3}+1}}$$

$\Big\downarrow$ 1012

$$-\frac{\sqrt[3]{c+\frac{d}{x^3}}\,\text{AppellF1}\left(\frac{4}{3},1,-\frac{1}{3},\frac{7}{3},-\frac{2d}{cx^3},-\frac{d}{cx^3}\right)}{4acx^4\sqrt[3]{\frac{d}{cx^3}+1}}$$

input
```
Int[(c + d/x^3)^(1/3)/(x^2*(2*a*d + a*c*x^3)),x]
```

output
```
-1/4*((c + d/x^3)^(1/3)*AppellF1[4/3, 1, -1/3, 7/3, (-2*d)/(c*x^3), -(d/(c
*x^3))])/(a*c*(1 + d/(c*x^3))^(1/3)*x^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 997
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^(m + 2)), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [F]

$$\int \frac{\left(c+\frac{d}{x^{3}}\right)^{\frac{1}{3}}}{x^{2} \left(a c \,x^{3}+2 a d \right)}d x$$

input
```
int((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x)
```

output
```
int((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\left(2ad+acx^3\right)}\,dx = \text{Timed out}$$

input
```
integrate((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\left(2ad+acx^3\right)}\,dx = \frac{\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{cx^5+2dx^2}\,dx}{a}$$

input
```
integrate((c+d/x**3)**(1/3)/x**2/(a*c*x**3+2*a*d),x)
```

output
```
Integral((c + d/x**3)**(1/3)/(c*x**5 + 2*d*x**2), x)/a
```

Maxima [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\left(2ad+acx^3\right)}\,dx = \int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{3}}}{(acx^3+2\,ad)x^2}\,dx$$

input
```
integrate((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x, algorithm="maxima")
```

output
```
integrate((c + d/x^3)^(1/3)/((a*c*x^3 + 2*a*d)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\,(2ad+acx^3)}\,dx=\int \frac{\left(c+\frac{d}{x^3}\right)^{\frac{1}{3}}}{(acx^3+2\,ad)x^2}\,dx$$

input
```
integrate((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x, algorithm="giac")
```

output
```
integrate((c + d/x^3)^(1/3)/((a*c*x^3 + 2*a*d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\,(2ad+acx^3)}\,dx=\int \frac{\left(c+\frac{d}{x^3}\right)^{1/3}}{x^2\,\left(a\,c\,x^3+2\,a\,d\right)}\,dx$$

input
```
int((c + d/x^3)^(1/3)/(x^2*(2*a*d + a*c*x^3)),x)
```

output
```
int((c + d/x^3)^(1/3)/(x^2*(2*a*d + a*c*x^3)), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{c+\frac{d}{x^3}}}{x^2\,(2ad+acx^3)}\,dx=\frac{-(c\,x^3+d)^{\frac{1}{3}}-\left(\int \frac{(c\,x^3+d)^{\frac{1}{3}}x^3}{c^2x^6+3cd\,x^3+2d^2}dx\right)c^2x^2}{4ad\,x^2}$$

input
```
int((c+d/x^3)^(1/3)/x^2/(a*c*x^3+2*a*d),x)
```

output

```
( - ((c*x**3 + d)**(1/3) + int(((c*x**3 + d)**(1/3)*x**3)/(c**2*x**6 + 3*c
*d*x**3 + 2*d**2),x)*c**2*x**2))/(4*a*d*x**2)
```

3.222 $\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{4a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{9b^4} - \frac{2x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{9b^2}$$

output
```
-4/9*a^2*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^4-2/9*x^3*(a-b*x^(3/2))
^(1/2)*(a+b*x^(3/2))^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}(2a^2+b^2x^3)}{9b^4}$$

input
```
Integrate[x^5/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]*(2*a^2 + b^2*x^3))/(9*b^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {799, 111, 27, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{799}$$

$$\frac{2}{3}\int \frac{x^{9/2}}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}$$

$$\Big\downarrow \text{111}$$

$$\frac{2}{3}\left(-\frac{\int -\frac{2a^2x^{3/2}}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}}{3b^2}-\frac{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\left(\frac{2a^2\int \frac{x^{3/2}}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}}{3b^2}-\frac{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}\right)$$

$$\Big\downarrow \text{83}$$

$$\frac{2}{3}\left(-\frac{2a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^4}-\frac{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}\right)$$

input
```
Int[x^5/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(2*((-2*a^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(3*b^4) - (x^3*Sqrt[a
 - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(3*b^2)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 799
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1
*x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
 && EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

method	result	size
orering	$-\frac{2\left(b^{2} x^{3}+2 a^{2}\right)\left(-b^{2} x^{3}+a^{2}\right)}{9 b^{4} \sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}$	50

input
```
int(x^5/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b^2*x^3+2*a^2)/b^4*(-b^2*x^3+a^2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\left(b^2x^3+2\,a^2\right)\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{9\,b^4}$$

input
```
integrate(x^5/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-2/9*(b^2*x^3 + 2*a^2)*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)/b^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 35.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{ia^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^4} - \frac{a^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^4}$$

input
```
integrate(x**5/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
-I*a**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4,
 -1/2, 0), ()), a**2/(b**2*x**3))/(6*pi**(3/2)*b**4) - a**3*meijerg(((-2,
-7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), a**2*e
xp_polar(-2*I*pi)/(b**2*x**3))/(6*pi**(3/2)*b**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{-b^2x^3+a^2}x^3}{9\,b^2} - \frac{4\sqrt{-b^2x^3+a^2}a^2}{9\,b^4}$$

input
```
integrate(x^5/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
-2/9*sqrt(-b^2*x^3 + a^2)*x^3/b^2 - 4/9*sqrt(-b^2*x^3 + a^2)*a^2/b^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}\left(\left(bx^{\frac{3}{2}}+a\right)\left(\frac{bx^{\frac{3}{2}}+a}{b^3}-\frac{2\,a}{b^3}\right)+\frac{3\,a^2}{b^3}\right)}{9\,b}$$

input
```
integrate(x^5/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
-2/9*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*((b*x^(3/2) + a)*((b*x^(3/2)
 + a)/b^3 - 2*a/b^3) + 3*a^2/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{a-b\,x^{3/2}}\left(\frac{4\,a^3}{9\,b^4}+\frac{2\,x^{9/2}}{9\,b}+\frac{2\,a\,x^3}{9\,b^2}+\frac{4\,a^2\,x^{3/2}}{9\,b^3}\right)}{\sqrt{a+b\,x^{3/2}}}$$

input
```
int(x^5/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
-((a - b*x^(3/2))^(1/2)*((4*a^3)/(9*b^4) + (2*x^(9/2))/(9*b) + (2*a*x^3)/(
9*b^2) + (4*a^2*x^(3/2))/(9*b^3)))/(a + b*x^(3/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,(-b^2x^3-2a^2)}{9b^4}$$

input
```
int(x^5/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
(2*sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*( - 2*a**2 - b**2*x**3))
/(9*b**4)
```

3.223 $\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 34

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}} \, dx = -\frac{2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}$$

output
```
-2/3*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}} \, dx = -\frac{2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}$$

input
```
Integrate[x^2/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {794}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{794}$$

$$-\frac{2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3b^2}$$

input
```
Int[x^2/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(3*b^2)
```

Defintions of rubi rules used

rule 794
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_.))^(p_)*((a2_) + (b2_.)*(x_)^(n_.))
^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1)/(2*b1
*b2*n*(p + 1))), x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a
1*b2, 0] && EqQ[m, 2*n - 1] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}{3b^{2}}$	25
default	$-\frac{2\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}{3b^{2}}$	25
orering	$-\frac{2\left(-b^{2}x^{3}+a^{2}\right)}{3b^{2}\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}$	37

input
```
int(x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\,\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{3\,b^2}$$

input
```
integrate(x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-2/3*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.00

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{iaG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^2} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^2}$$

input
```
integrate(x**2/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
-I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
 ()), a**2/(b**2*x**3))/(6*pi**(3/2)*b**2) - a*meijerg(((-1, -3/4, -1/2, -
1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), a**2*exp_polar(-2*I*
pi)/(b**2*x**3))/(6*pi**(3/2)*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{-b^2x^3+a^2}}{3\,b^2}$$

input
```
integrate(x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
-2/3*sqrt(-b^2*x^3 + a^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{3\,b^2}$$

input
```
integrate(x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{a-b\,x^{3/2}}\left(\frac{2\,a}{3\,b^2}+\frac{2\,x^{3/2}}{3\,b}\right)}{\sqrt{a+b\,x^{3/2}}}$$

input
```
int(x^2/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
-((a - b*x^(3/2))^(1/2)*((2*a)/(3*b^2) + (2*x^(3/2))/(3*b)))/(a + b*x^(3/2
))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{3b^2}$$

input
```
int(x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
( - 2*sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(3*b**2)
```

3.224 $\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 39

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a}\right)}{3a}$$

output
```
-2/3*arctanh((a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a)/a
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.03

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{2\log\left(-\sqrt{a-bx^{3/2}}+\sqrt{a+bx^{3/2}}\right)}{3a}-\frac{2\log\left(a\sqrt{a-bx^{3/2}}+a\sqrt{a+bx^{3/2}}\right)}{3a}$$

input
```
Integrate[1/(x*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(2*Log[-Sqrt[a - b*x^(3/2)] + Sqrt[a + b*x^(3/2)]])/(3*a) - (2*Log[a*Sqrt[
a - b*x^(3/2)] + a*Sqrt[a + b*x^(3/2)]])/(3*a)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {799, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{799}$$

$$\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}$$

$$\Big\downarrow \text{103}$$

$$-\frac{2}{3}b\int \frac{1}{a^2b-bx^3}d\left(\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}\right)$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a}\right)}{3a}$$

input
```
Int[1/(x*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*ArcTanh[(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/a])/(3*a)
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 799
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1
*x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
 && EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.72

method	result	size
derivativedivides	$-\frac{2\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}\,\ln\left(\frac{2a\left(\operatorname{csgn}(a)\sqrt{-b^{2}x^{3}+a^{2}}+a\right)}{x^{\frac{3}{2}}}\right)\operatorname{csgn}(a)}{3a\sqrt{-b^{2}x^{3}+a^{2}}}$	67
default	$-\frac{2\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}\,\ln\left(\frac{2a\left(\operatorname{csgn}(a)\sqrt{-b^{2}x^{3}+a^{2}}+a\right)}{x^{\frac{3}{2}}}\right)\operatorname{csgn}(a)}{3a\sqrt{-b^{2}x^{3}+a^{2}}}$	67

input
```
int(1/x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a*ln(2*a*(csgn(a)*(-b^2*x^3+a
^2)^(1/2)+a)/x^(3/2))*csgn(a)/(-b^2*x^3+a^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\log\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}+a\right)-\log\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}-a\right)}{3\,a}$$

input
```
integrate(1/x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-1/3*(log(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a) + a) - log(sqrt(b*x^(3/
2) + a)*sqrt(-b*x^(3/2) + a) - a))/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}a} - \frac{G_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}a}$$

input
```
integrate(1/x/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), a
**2/(b**2*x**3))/(6*pi**(3/2)*a) - meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
 ((1/4, 3/4), (0, 1/2, 1/2, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))/(6*p
i**(3/2)*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\,\log\left(\frac{2\,a^2}{x^{\frac{3}{2}}}+\frac{2\,\sqrt{-b^2x^3+a^2}a}{x^{\frac{3}{2}}}\right)}{3\,a}$$

input
```
integrate(1/x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
-2/3*log(2*a^2/x^(3/2) + 2*sqrt(-b^2*x^3 + a^2)*a/x^(3/2))/a
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{2\left(\ln\left(\frac{\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^2}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^2}-1\right)-\ln\left(\frac{\sqrt{a+b\,x^{3/2}}-\sqrt{a}}{\sqrt{a-b\,x^{3/2}}-\sqrt{a}}\right)\right)}{3\,a}$$

input
```
int(1/(x*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
(2*(log(((a + b*x^(3/2))^(1/2) - a^(1/2))^2/((a - b*x^(3/2))^(1/2) - a^(1/
2))^2 - 1) - log(((a + b*x^(3/2))^(1/2) - a^(1/2))/((a - b*x^(3/2))^(1/2)
- a^(1/2)))))/(3*a)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx$$

input
```
int(1/x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),
x)
```

3.225 $\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 80

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{3a^2x^3}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a}\right)}{3a^3}$$

output
```
-1/3*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a^2/x^3-1/3*b^2*arctanh((a-b*
x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a)/a^3
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=-\frac{a\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}+2b^2x^3\text{arctanh}\left(\frac{\sqrt{a-bx^{3/2}}}{\sqrt{a+bx^{3/2}}}\right)}{3a^3x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-1/3*(a*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)] + 2*b^2*x^3*ArcTanh[Sqrt[a
 - b*x^(3/2)]/Sqrt[a + b*x^(3/2)]])/(a^3*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {799, 114, 25, 27, 103, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{799}$$

$$\frac{2}{3}\int \frac{1}{x^{9/2}\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}$$

$$\Big\downarrow \text{114}$$

$$\frac{2}{3}\left(-\frac{\int -\frac{b^2}{x^{3/2}\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}}{2a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}\left(\frac{\int \frac{b^2}{x^{3/2}\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}}{2a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^3}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\left(\frac{b^2\int \frac{1}{x^{3/2}\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx^{3/2}}{2a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^3}\right)$$

$$\Big\downarrow \text{103}$$

$$\frac{2}{3}\left(-\frac{b^3\int \frac{1}{a^2b-bx^3}d\left(\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}\right)}{2a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2}{3}\left(-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a}\right)}{2a^3}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(2*(-1/2*(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(a^2*x^3) - (b^2*ArcTan
h[(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/a])/(2*a^3)))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 799
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1
*x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
 && EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Maple [F]

$$\int \frac{1}{x^4 \sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{b^2x^3\log\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}+a\right)-b^2x^3\log\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}-a\right)+2\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}\ldots}{6\,a^3x^3}$$

input
```
integrate(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fric
as")
```

output
```
-1/6*(b^2*x^3*log(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a) + a) - b^2*x^3*
log(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a) - a) + 2*sqrt(b*x^(3/2) + a)*
sqrt(-b*x^(3/2) + a)*a)/(a^3*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{b^2\log\left(\frac{2\,a^2}{x^{\frac{3}{2}}}+\frac{2\,\sqrt{-b^2x^3+a^2}a}{x^{\frac{3}{2}}}\right)}{3\,a^3}-\frac{\sqrt{-b^2x^3+a^2}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/3*b^2*log(2*a^2/x^(3/2) + 2*sqrt(-b^2*x^3 + a^2)*a/x^(3/2))/a^3 - 1/3*s
qrt(-b^2*x^3 + a^2)/(a^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(60) = 120.

Time = 0.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.62

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx =$$

$$-\frac{\frac{b^3\log\left(\left|-\frac{\sqrt{2}\sqrt{a}-\sqrt{-bx^{\frac{3}{2}}+a}}{\sqrt{bx^{\frac{3}{2}}+a}}+\frac{\sqrt{bx^{\frac{3}{2}}+a}}{\sqrt{2}\sqrt{a}-\sqrt{-bx^{\frac{3}{2}}+a}}+2\right|\right)}{a^3}-\frac{b^3\log\left(\left|-\frac{\sqrt{2}\sqrt{a}-\sqrt{-bx^{\frac{3}{2}}+a}}{\sqrt{bx^{\frac{3}{2}}+a}}+\frac{\sqrt{bx^{\frac{3}{2}}+a}}{\sqrt{2}\sqrt{a}-\sqrt{-bx^{\frac{3}{2}}+a}}-2\right|\right)}{a^3}-\frac{4\left(b^3\left(\frac{\sqrt{2}\sqrt{a}-\sqrt{-bx^{\frac{3}{2}}+a}}{\sqrt{bx^{\frac{3}{2}}+a}}-\cdots\right.\right.}{\left(\left(\cdots\right.\right.}}{3\,b}$$

input
```
integrate(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac
")
```

output
```
-1/3*(b^3*log(abs(-(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a))/sqrt(b*x^(3/2)
 + a) + sqrt(b*x^(3/2) + a)/(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a)) + 2))
/a^3 - b^3*log(abs(-(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a))/sqrt(b*x^(3/2
) + a) + sqrt(b*x^(3/2) + a)/(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a)) - 2)
)/a^3 - 4*(b^3*((sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a))/sqrt(b*x^(3/2) +
a) - sqrt(b*x^(3/2) + a)/(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a)))^3 + 4*b
^3*((sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a))/sqrt(b*x^(3/2) + a) - sqrt(b*
x^(3/2) + a)/(sqrt(2)*sqrt(a) - sqrt(-b*x^(3/2) + a))))/((((sqrt(2)*sqrt(a
) - sqrt(-b*x^(3/2) + a))/sqrt(b*x^(3/2) + a) - sqrt(b*x^(3/2) + a)/(sqrt(
2)*sqrt(a) - sqrt(-b*x^(3/2) + a)))^2 - 4)^2*a^3))/b
```

Mupad [B] (verification not implemented)

Time = 8.07 (sec) , antiderivative size = 344, normalized size of antiderivative = 4.30

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{b^2\,\ln\left(\frac{\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^2}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^2}-1\right)}{3\,a^3} - \frac{\frac{b^2\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^2}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^2}-\frac{b^2}{2}+\frac{15\,b^2\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^4}{2\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^4}}{\frac{24\,a^3\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^2}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^2}-\frac{48\,a^3\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^4}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^4}+\frac{24\,a^3\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^6}{\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^6}} - \frac{b^2\,\ln\left(\frac{\sqrt{a+b\,x^{3/2}}-\sqrt{a}}{\sqrt{a-b\,x^{3/2}}-\sqrt{a}}\right)}{3\,a^3} + \frac{b^2\left(\sqrt{a+b\,x^{3/2}}-\sqrt{a}\right)^2}{48\,a^3\left(\sqrt{a-b\,x^{3/2}}-\sqrt{a}\right)^2}$$

input
```
int(1/(x^4*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
(b^2*log(((a + b*x^(3/2))^(1/2) - a^(1/2))^2/((a - b*x^(3/2))^(1/2) - a^(1
/2))^2 - 1))/(3*a^3) - ((b^2*((a + b*x^(3/2))^(1/2) - a^(1/2))^2)/((a - b*
x^(3/2))^(1/2) - a^(1/2))^2 - b^2/2 + (15*b^2*((a + b*x^(3/2))^(1/2) - a^(
1/2))^4)/(2*((a - b*x^(3/2))^(1/2) - a^(1/2))^4))/((24*a^3*((a + b*x^(3/2)
)^(1/2) - a^(1/2))^2)/((a - b*x^(3/2))^(1/2) - a^(1/2))^2 - (48*a^3*((a +
b*x^(3/2))^(1/2) - a^(1/2))^4)/((a - b*x^(3/2))^(1/2) - a^(1/2))^4 + (24*a
^3*((a + b*x^(3/2))^(1/2) - a^(1/2))^6)/((a - b*x^(3/2))^(1/2) - a^(1/2))^
6) - (b^2*log(((a + b*x^(3/2))^(1/2) - a^(1/2))/((a - b*x^(3/2))^(1/2) - a
^(1/2))))/(3*a^3) + (b^2*((a + b*x^(3/2))^(1/2) - a^(1/2))^2)/(48*a^3*((a
- b*x^(3/2))^(1/2) - a^(1/2))^2)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{-2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}+3\left(\int\frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^4+a^2x}dx\right)b^2x^3}{6a^2x^3}$$

input
```
int(1/x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
( - 2*sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a) + 3*int((sqrt(sqrt(x)
*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2*x - b**2*x**4),x)*b**2*x**3)/(6*
a**2*x**3)
```

3.226 $\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 309

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{16a^2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{55b^4}-\frac{2x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{11b^2} - \frac{32\sqrt{2+\sqrt{3}}a^4\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}b^{14/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
-16/55*a^2*x*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^4-2/11*x^4*(a-b*x^(
3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^2-32/165*(1/2*6^(1/2)+1/2*2^(1/2))*a^4*(
a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2))*a
^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/((1+3
^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(14/3)/(a^(2/3)*(a^(2/
3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)
/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{x^7\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{7}{3};\frac{10}{3};\frac{b^2x^3}{a^2}\right)}{7\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[x^6/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(x^7*Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{1/2, 7/3}, {10/3}, (b^2*x^
3)/a^2])/(7*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {845, 845, 785, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$$

$$\Big\downarrow \text{845}$$

$$\frac{8a^2\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{11b^2}-\frac{2x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{11b^2}$$

$$\Big\downarrow \text{845}$$

$$\frac{8a^2\left(\frac{2a^2\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{5b^2}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}\right)}{11b^2}-\frac{2x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{11b^2}$$

$$\Big\downarrow \text{785}$$

$$\frac{8a^2\left(\frac{2a^2\sqrt{a^2-b^2x^3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{5b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}\right)}{11b^2}-\frac{2x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{11b^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{8a^2\left(-\frac{4\sqrt{2+\sqrt{3}}a^2\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{8/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}\right.}{11b^2}$$

$$\frac{2x^4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{11b^2}$$

input
```
Int[x^6/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*x^4*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(11*b^2) + (8*a^2*((-2*x*
Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(5*b^2) - (4*Sqrt[2 + Sqrt[3]]*a^
2*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/(
(1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(2
/3) - b^(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(5
*3^(1/4)*b^(8/3)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/
3) - b^(2/3)*x)^2]*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])))/(11*b^2)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Maple [F]

$$\int \frac{x^{6}}{\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\left(16\,\sqrt{-b^2}a^4\text{weierstrassPInverse}\left(0,\frac{4\,a^2}{b^2},x\right)+\left(5\,b^4x^4+8\,a^2b^2x\right)\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}\right)}{55\,b^6}$$

input
```
integrate(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-2/55*(16*sqrt(-b^2)*a^4*weierstrassPInverse(0, 4*a^2/b^2, x) + (5*b^4*x^4
 + 8*a^2*b^2*x)*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 103.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.46

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{ia^{\frac{11}{3}} G_{6,6}^{5,3}\left(\begin{matrix} -\frac{19}{12}, -\frac{13}{12}, 1 & -\frac{4}{3}, -\frac{4}{3}, -\frac{5}{6} \\ -\frac{11}{6}, -\frac{19}{12}, -\frac{4}{3}, -\frac{13}{12}, -\frac{5}{6} & 0 \end{matrix}\middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^{\frac{14}{3}}} + \frac{a^{\frac{11}{3}} G_{6,6}^{2,6}\left(\begin{matrix} -\frac{7}{3}, -\frac{25}{12}, -\frac{11}{6}, -\frac{19}{12}, -\frac{4}{3}, 1 & \\ -\frac{25}{12}, -\frac{19}{12} & -\frac{7}{3}, -\frac{11}{6}, -\frac{11}{6}, 0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}b^{\frac{14}{3}}}$$

input
```
integrate(x**6/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*a**(11/3)*meijerg(((-19/12, -13/12, 1), (-4/3, -4/3, -5/6)), ((-11/6, -1
9/12, -4/3, -13/12, -5/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*b**(14/3)
) + a**(11/3)*meijerg(((-7/3, -25/12, -11/6, -19/12, -4/3, 1), ()), ((-25/
12, -19/12), (-7/3, -11/6, -11/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3)
)*exp(I*pi/3)/(6*pi**(3/2)*b**(14/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x^6}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^6/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^6}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^6}{\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(x^6/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(x^6/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{-\frac{16\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,a^2x}{55}-\frac{2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,b^2x^4}{11}+\frac{16\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^3+a^2}\right)}{55}}{b^4}$$

input
```
int(x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
(2*( - 8*sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*a**2*x - 5*sqrt(sq
rt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*b**2*x**4 + 8*int((sqrt(sqrt(x)*b*
x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2 - b**2*x**3),x)*a**4))/(55*b**4)
```

3.227 $\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 269

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2} - \frac{4\sqrt{2+\sqrt{3}}a^2\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{8/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output

```
-2/5*x*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^2-4/15*(1/2*6^(1/2)+1/2*2
^(1/2))*a^2*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/(
(1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3)-b^(
2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(8/3)/(a^
(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^
(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.27

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{x^4\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\frac{b^2x^3}{a^2}\right)}{4\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[x^3/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(x^4*Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{1/2, 4/3}, {7/3}, (b^2*x^3
)/a^2])/(4*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {845, 785, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{2a^2\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{5b^2}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}$$

$$\Big\downarrow \text{785}$$

$$\frac{2a^2\sqrt{a^2-b^2x^3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{5b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}$$

$$\Big\downarrow \text{759}$$

$$-\frac{4\sqrt{2+\sqrt{3}}a^2\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{8/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5b^2}$$

input
```
Int[x^3/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*x*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(5*b^2) - (4*Sqrt[2 + Sqrt[
3]]*a^2*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*
x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*a^(2/3) - b^(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3
]])/(5*3^(1/4)*b^(8/3)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])
*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Maple [F]

$$\int \frac{x^{3}}{\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}b^{2}x+2\sqrt{-b^{2}}a^{2}\text{weierstrassPInverse}\left(0,\frac{4\,a^{2}}{b^{2}},x\right)\right)}{5\,b^{4}}$$

input
```
integrate(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-2/5*(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*b^2*x + 2*sqrt(-b^2)*a^2*we
ierstrassPInverse(0, 4*a^2/b^2, x))/b^4
```

Sympy [A] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.51

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{ia^{\frac{5}{3}}G_{6,6}^{5,3}\left(\begin{matrix} -\frac{7}{12},-\frac{1}{12},1 & -\frac{1}{3},-\frac{1}{3},\frac{1}{6} \\ -\frac{5}{6},-\frac{7}{12},-\frac{1}{3},-\frac{1}{12},\frac{1}{6} & 0 \end{matrix}\middle|\,\frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^{\frac{8}{3}}} + \frac{a^{\frac{5}{3}}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{4}{3},-\frac{13}{12},-\frac{5}{6},-\frac{7}{12},-\frac{1}{3},1 & \\ -\frac{13}{12},-\frac{7}{12} & -\frac{4}{3},-\frac{5}{6},-\frac{5}{6},0 \end{matrix}\middle|\,\frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}b^{\frac{8}{3}}}$$

input
```
integrate(x**3/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*a**(5/3)*meijerg(((-7/12, -1/12, 1), (-1/3, -1/3, 1/6)), ((-5/6, -7/12,
-1/3, -1/12, 1/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*b**(8/3)) + a**(5
/3)*meijerg(((-4/3, -13/12, -5/6, -7/12, -1/3, 1), ()), ((-13/12, -7/12),
(-4/3, -5/6, -5/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(I*pi/3)/(
6*pi**(3/2)*b**(8/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x^3}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^3/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^3}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^3}{\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(x^3/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(x^3/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{-\frac{2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,x}{5}+\frac{2\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^3+a^2}dx\right)a^2}{5}}{b^2}$$

input
```
int(x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
(2*( - sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*x + int((sqrt(sqrt(x
)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2 - b**2*x**3),x)*a**2))/(5*b**2)
```

3.228 $\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 228

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/
3)*x+b^(4/3)*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3
^(1/2))*a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*
3^(3/4)/b^(2/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*
x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{x\sqrt{1-\frac{b^2x^3}{a^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\frac{b^2x^3}{a^2}\right)}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[1/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(x*Sqrt[1 - (b^2*x^3)/a^2]*Hypergeometric2F1[1/3, 1/2, 4/3, (b^2*x^3)/a^2]
)/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {785, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{785}$$

$$\frac{\sqrt{a^2-b^2x^3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

$$\Big\downarrow \text{759}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Int[1/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3
)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin
[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)],
 -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((
1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2
)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\sqrt{-b^2}\text{weierstrassPInverse}\left(0, \frac{4a^2}{b^2}, x\right)}{b^2}$$

input
```
integrate(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b^2)*weierstrassPInverse(0, 4*a^2/b^2, x)/b^2
```

Sympy [A] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{12}, \frac{11}{12}, 1 & \frac{2}{3}, \frac{2}{3}, \frac{7}{6} \\ \frac{1}{6}, \frac{5}{12}, \frac{2}{3}, \frac{11}{12}, \frac{7}{6} & 0 \end{matrix} \middle| {\frac{a^2}{b^2x^3}} \right)}{6\pi^{\frac{3}{2}}\sqrt[3]{a}b^{\frac{2}{3}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{12}, \frac{1}{6}, \frac{5}{12}, \frac{2}{3}, 1 & \\ -\frac{1}{12}, \frac{5}{12} & -\frac{1}{3}, \frac{1}{6}, \frac{1}{6}, 0 \end{matrix} \middle| {\frac{a^2e^{-2i\pi}}{b^2x^3}} \right)e^{\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}\sqrt[3]{a}b^{\frac{2}{3}}}$$

input
```
integrate(1/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*meijerg(((5/12, 11/12, 1), (2/3, 2/3, 7/6)), ((1/6, 5/12, 2/3, 11/12, 7/
6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*a**(1/3)*b**(2/3)) + meijerg(((-
1/3, -1/12, 1/6, 5/12, 2/3, 1), ()), ((-1/12, 5/12), (-1/3, 1/6, 1/6, 0)),
 a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(I*pi/3)/(6*pi**(3/2)*a**(1/3)*b*
*(2/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - bx^{3/2}}\sqrt{a + bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}} + a}\sqrt{-bx^{\frac{3}{2}} + a}}\,dx$$

input
```
integrate(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a - bx^{3/2}}\sqrt{a + bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}} + a}\sqrt{-bx^{\frac{3}{2}} + a}}\,dx$$

input
```
integrate(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - bx^{3/2}}\sqrt{a + bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{a + b\,x^{3/2}}\,\sqrt{a - b\,x^{3/2}}}\,dx$$

input
```
int(1/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(1/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^3+a^2}dx$$

input
```
int(1/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2 - b**2*x**3),x)
```

3.229 $\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 271

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2} - \frac{\sqrt{2+\sqrt{3}}b^{4/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a^2\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
-1/2*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a^2/x^2-1/6*(1/2*6^(1/2)+1/2*
2^(1/2))*b^(4/3)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x
^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3
)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^2/(a
^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x
^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\frac{b^2x^3}{a^2}\right)}{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[1/(x^3*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-1/2*(Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{-2/3, 1/2}, {1/3}, (b^2*x
^3)/a^2])/(x^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {849, 785, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{ 849}$$

$$\frac{b^2\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{4a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}$$

$$\Big\downarrow \text{ 785}$$

$$\frac{b^2\sqrt{a^2-b^2x^3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{4a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}-\frac{\sqrt{2+\sqrt{3}}b^{4/3}(a^{2/3}-b^{2/3}x)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a^2\sqrt{\frac{a^{2/3}(a^{2/3}-b^{2/3}x)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Int[1/(x^3*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-1/2*(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(a^2*x^2) - (Sqrt[2 + Sqrt[
3]]*b^(4/3)*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4
/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(2/3) - b^(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sq
rt[3]])/(2*3^(1/4)*a^2*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])
*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

rule 849
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1)))   Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^
p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && I
GtQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{3} \sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{-b^2}x^2\text{weierstrassPInverse}\left(0,\frac{4\,a^2}{b^2},x\right)+\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fric
as")
```

output
```
-1/2*(sqrt(-b^2)*x^2*weierstrassPInverse(0, 4*a^2/b^2, x) + sqrt(b*x^(3/2)
 + a)*sqrt(-b*x^(3/2) + a))/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 15.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{ib^{\frac{4}{3}}G_{6,6}^{5,3}\left(\begin{matrix} \frac{17}{12}, \frac{23}{12}, 1 & \frac{5}{3}, \frac{5}{3}, \frac{13}{6} \\ \frac{7}{6}, \frac{17}{12}, \frac{5}{3}, \frac{23}{12}, \frac{13}{6} & 0 \end{matrix} \middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}a^{\frac{7}{3}}} + \frac{b^{\frac{4}{3}}G_{6,6}^{2,6}\left(\begin{matrix} \frac{2}{3}, \frac{11}{12}, \frac{7}{6}, \frac{17}{12}, \frac{5}{3}, 1 & \\ \frac{11}{12}, \frac{17}{12} & \frac{2}{3}, \frac{7}{6}, \frac{7}{6}, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}a^{\frac{7}{3}}}$$

input
```
integrate(1/x**3/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*b**(4/3)*meijerg(((17/12, 23/12, 1), (5/3, 5/3, 13/6)), ((7/6, 17/12, 5/
3, 23/12, 13/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*a**(7/3)) + b**(4/3
)*meijerg(((2/3, 11/12, 7/6, 17/12, 5/3, 1), ()), ((11/12, 17/12), (2/3, 7
/6, 7/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(I*pi/3)/(6*pi**(3/2
)*a**(7/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^3}\,dx$$

input
```
integrate(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^3}\,dx$$

input
```
integrate(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(1/(x^3*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^6+a^2x^3}\,dx$$

input
```
int(1/x^3/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2*x**3 - b**2*x**
6),x)
```

3.230 $\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 311

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5a^2x^5}-\frac{7b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{20a^4x^2}-\frac{7\sqrt{2+\sqrt{3}}\,b^{10/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{20\sqrt[4]{3}a^4\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
-1/5*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a^2/x^5-7/20*b^2*(a-b*x^(3/2)
)^(1/2)*(a+b*x^(3/2))^(1/2)/a^4/x^2-7/60*(1/2*6^(1/2)+1/2*2^(1/2))*b^(10/3
)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2)
)*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/((
1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^4/(a^(2/3)*(a^(2/3)
-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)/(
a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\frac{b^2x^3}{a^2}\right)}{5x^5\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[1/(x^6*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-1/5*(Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{-5/3, 1/2}, {-2/3}, (b^2*
x^3)/a^2])/(x^5*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {849, 849, 785, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{849}$$

$$\frac{7b^2\int \frac{1}{x^3\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{10a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5a^2x^5}$$

$$\Big\downarrow \text{849}$$

$$\frac{7b^2\left(\frac{b^2\int \frac{1}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{4a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}\right)}{10a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5a^2x^5}$$

$$\Big\downarrow \text{785}$$

$$\frac{7b^2\left(\frac{b^2\sqrt{a^2-b^2x^3}\int\frac{1}{\sqrt{a^2-b^2x^3}}dx}{4a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}\right)}{10a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5a^2x^5}$$

$$\Big\downarrow \text{759}$$

$$\frac{7b^2\left(-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{2a^2x^2}-\frac{\sqrt{2+\sqrt{3}}b^{4/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a^2\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\right)}{10a^2}$$
$$\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{5a^2x^5}$$

input
```
Int[1/(x^6*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-1/5*(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(a^2*x^5) + (7*b^2*(-1/2*(S
qrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(a^2*x^2) - (Sqrt[2 + Sqrt[3]]*b^(
4/3)*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2
)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a
^(2/3) - b^(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])
/(2*3^(1/4)*a^2*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3
) - b^(2/3)*x)^2]*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])))/(10*a^2)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

rule 849
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1)))   Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^
p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && I
GtQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{7\,\sqrt{-b^2}b^2x^5\text{weierstrassPInverse}\left(0,\frac{4\,a^2}{b^2},x\right)+\left(7\,b^2x^3+4\,a^2\right)\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{20\,a^4x^5}$$

input
```
integrate(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fric
as")
```

output
```
-1/20*(7*sqrt(-b^2)*b^2*x^5*weierstrassPInverse(0, 4*a^2/b^2, x) + (7*b^2*
x^3 + 4*a^2)*sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a))/(a^4*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**6/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^6}\,dx$$

input
```
integrate(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^6}\,dx$$

input
```
integrate(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{1}{x^6\,\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(1/(x^6*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(1/(x^6*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^9+a^2x^6}dx$$

input
```
int(1/x^6/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2*x**6 - b**2*x**
9),x)
```

3.231 $\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 579

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$
$$+\frac{8a^2(a^2-b^2x^3)}{7b^{10/3}\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{8/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right)|-7-4\sqrt{3}\right)}{7b^{10/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$
$$+\frac{8\sqrt{2}a^{8/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{10/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
-2/7*x^2*(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/b^2+8/7*a^2*(-b^2*x^3+a^2
)/b^(10/3)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2
))^(1/2)-4/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(8/3)*(a^(2/3)-b^(2/3)*x)
*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^
2)^(1/2)*EllipticE(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^
(2/3)*x),I*3^(1/2)+2*I)/b^(10/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))
*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)+8/21*
2^(1/2)*a^(8/3)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^
2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3)
-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(10/3
)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a
-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{x^5\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\frac{b^2x^3}{a^2}\right)}{5\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[x^4/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(x^5*Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{1/2, 5/3}, {8/3}, (b^2*x^3
)/a^2])/(5*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {845, 890, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$\Big\downarrow$ 845

$$\frac{4a^2\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{7b^2}-\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$

$\Big\downarrow$ 890

$$\frac{4a^2\sqrt{a^2-b^2x^3}\int \frac{x}{\sqrt{a^2-b^2x^3}}dx}{7b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$

$\Big\downarrow$ 832

$$\frac{4a^2\sqrt{a^2-b^2x^3}\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}\right)}{7b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$

$\Big\downarrow$ 759

$$\frac{4a^2\sqrt{a^2-b^2x^3}\left(-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}\right)}{7b^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$

$\Big\downarrow$ 2416

$$\frac{4a^2\sqrt{a^2-b^2x^3}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}-\right.}{7b^2\sqrt{a-bx^{3/2}}\sqrt{a}}$$

$$\frac{2x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{7b^2}$$

input
```
Int[x^4/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(-2*x^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(7*b^2) + (4*a^2*Sqrt[a^2
 - b^2*x^3]*(-(((-2*Sqrt[a^2 - b^2*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(2/3) -
 b^(2/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(2/3)*(a^(2/3) - b^(2/3)*x)*Sq
rt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^
(2/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/((1 + Sqr
t[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(2/3)*(a^(2
/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a^2 - b^2*x^
3]))/b^(2/3)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(2/3)*(a^(2/3) - b^(2
/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(
2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/
((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sq
rt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*
Sqrt[a^2 - b^2*x^3])))/(7*b^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{2\left(\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}b^2x^2-4\sqrt{-b^2}a^2\mathrm{weierstrassZeta}\left(0,\frac{4a^2}{b^2},\mathrm{weierstrassPInverse}\left(0,\frac{4a^2}{b^2},x\right)\right)\right)}{7b^4}$$

input
```
integrate(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas
")
```

output
```
-2/7*(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*b^2*x^2 - 4*sqrt(-b^2)*a^2*
weierstrassZeta(0, 4*a^2/b^2, weierstrassPInverse(0, 4*a^2/b^2, x)))/b^4
```

Sympy [A] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.24

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{ia^{\frac{7}{3}}G_{6,6}^{5,3}\left(\begin{matrix}-\frac{11}{12},-\frac{5}{12},1 & -\frac{2}{3},-\frac{2}{3},-\frac{1}{6}\\ -\frac{7}{6},-\frac{11}{12},-\frac{2}{3},-\frac{5}{12},-\frac{1}{6} & 0\end{matrix}\middle|\frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^{\frac{10}{3}}} + \frac{a^{\frac{7}{3}}G_{6,6}^{2,6}\left(\begin{matrix}-\frac{5}{3},-\frac{17}{12},-\frac{7}{6},-\frac{11}{12},-\frac{2}{3},1 & \\ -\frac{17}{12},-\frac{11}{12} & -\frac{5}{3},-\frac{7}{6},-\frac{7}{6},0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{-\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}b^{\frac{10}{3}}}$$

input
```
integrate(x**4/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*a**(7/3)*meijerg(((-11/12, -5/12, 1), (-2/3, -2/3, -1/6)), ((-7/6, -11/1
2, -2/3, -5/12, -1/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*b**(10/3)) +
a**(7/3)*meijerg(((-5/3, -17/12, -7/6, -11/12, -2/3, 1), ()), ((-17/12, -1
1/12), (-5/3, -7/6, -7/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(-I
*pi/3)/(6*pi**(3/2)*b**(10/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^4}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(x^4/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^4}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x^4}{\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(x^4/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(x^4/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{-\frac{2\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,x^2}{7}+\frac{4\left(\int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,x}{-b^2x^3+a^2}dx\right)a^2}{7}}{b^2}$$

input
```
int(x^4/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
(2*( - sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*x**2 + 2*int((sqrt(s
qrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),x)*a**2))/(
7*b**2)
```

3.232 $\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 533

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{2(a^2-b^2x^3)}{b^{4/3}\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$
$$-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right)|-7-4\sqrt{3}\right)}{b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$
$$+\frac{2\sqrt{2}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

output
```
2*(-b^2*x^3+a^2)/b^(4/3)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)/(a-b*x^(3/2))^(1/
2)/(a+b*x^(3/2))^(1/2)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(2/3)*(a^(2/3)-
b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2))*a^(2/3)-b
^(2/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*
a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)/b^(4/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1
+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1
/2)+2/3*2^(1/2)*a^(2/3)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^
(4/3)*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))
*a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)
/b^(4/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(
1/2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\frac{x^2\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\frac{b^2x^3}{a^2}\right)}{2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[x/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
(x^2*Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{1/2, 2/3}, {5/3}, (b^2*x^3
)/a^2])/(2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Rubi [A] (warning: unable to verify)

Time = 0.93 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {890, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$$\Big\downarrow \text{890}$$

$$\frac{\sqrt{a^2-b^2x^3}\int \frac{x}{\sqrt{a^2-b^2x^3}}dx}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

$$\Big\downarrow \text{832}$$

$$\frac{\sqrt{a^2-b^2x^3}\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}\right)}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{a^2-b^2x^3}\left(-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}\ldots}{\left(1+\sqrt{3}\right)a^{2/3}\ldots}\right.\right.}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}\right.}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\sqrt{a^2-b^2x^3}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}-\frac{\sqrt[4]{3}\ldots}{\ldots}\right.}{\sqrt{a-bx^{3/2}}\sqrt{a+b\ldots}}$$

input
```
Int[x/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output

```
(Sqrt[a^2 - b^2*x^3]*(-(((-2*Sqrt[a^2 - b^2*x^3])/(b^(2/3)*((1 + Sqrt[3])*
a^(2/3) - b^(2/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(2/3)*(a^(2/3) - b^(2
/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(
2/3) - b^(2/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/
((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(2
/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a^2
 - b^2*x^3]))/b^(2/3)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(2/3)*(a^(2/
3) - b^(2/3)*x)*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqr
t[3])*a^(2/3) - b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^
(2/3)*x)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b
^(4/3)*Sqrt[(a^(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/
3)*x)^2]*Sqrt[a^2 - b^2*x^3])))/(Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 890

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] && !IntegerQ[p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{a-b\,x^{\frac{3}{2}}}\,\sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.06

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{2\sqrt{-b^2}\text{weierstrassZeta}\left(0,\frac{4\,a^2}{b^2},\text{weierstrassPInverse}\left(0,\frac{4\,a^2}{b^2},x\right)\right)}{b^2}$$

input
```
integrate(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(-b^2)*weierstrassZeta(0, 4*a^2/b^2, weierstrassPInverse(0, 4*a^2/b^
2, x))/b^2
```

Sympy [A] (verification not implemented)

Time = 2.96 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.23

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{i\sqrt[3]{a}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{12}, \frac{7}{12}, 1 & \frac{1}{3}, \frac{1}{3}, \frac{5}{6} \\ -\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, \frac{7}{12}, \frac{5}{6} & 0 \end{matrix} \middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}b^{\frac{4}{3}}} + \frac{\sqrt[3]{a}G_{6,6}^{2,6}\left(\begin{matrix} -\frac{2}{3}, -\frac{5}{12}, -\frac{1}{6}, \frac{1}{12}, \frac{1}{3}, 1 & \\ -\frac{5}{12}, \frac{1}{12} & -\frac{2}{3}, -\frac{1}{6}, -\frac{1}{6}, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{-\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}b^{\frac{4}{3}}}$$

input
```
integrate(x/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*a**(1/3)*meijerg(((1/12, 7/12, 1), (1/3, 1/3, 5/6)), ((-1/6, 1/12, 1/3,
7/12, 5/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*b**(4/3)) + a**(1/3)*mei
jerg(((-2/3, -5/12, -1/6, 1/12, 1/3, 1), ()), ((-5/12, 1/12), (-2/3, -1/6,
 -1/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(-I*pi/3)/(6*pi**(3/2)
*b**(4/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{x}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}\,dx$$

input
```
integrate(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{x}{\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(x/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(x/((a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=\int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}\,x}{-b^{2}x^{3}+a^{2}}dx$$

input
```
int(x/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a)*x)/(a**2 - b**2*x**3),
x)
```

3.233 $\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 573

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx &= -\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x} \\ &- \frac{b^{2/3}(a^2-b^2x^3)}{a^2\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}} \\ &+ \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}b^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right)|-7-4\sqrt{3}\right)}{2a^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}} \\ &- \frac{\sqrt{2}b^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{4/3}+a^{2/3}b^{2/3}x+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\end{aligned}$$

output
```
-(a-b*x^(3/2))^(1/2)*(a+b*x^(3/2))^(1/2)/a^2/x-b^(2/3)*(-b^2*x^3+a^2)/a^2/
((1+3^(1/2))*a^(2/3)-b^(2/3)*x)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)+1/
2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(2/3)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+
a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*
3^(1/2)+2*I)/a^(4/3)/(a^(2/3)*(a^(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(
2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2)-1/3*2^(1/2)*b^(2/
3)*(a^(2/3)-b^(2/3)*x)*((a^(4/3)+a^(2/3)*b^(2/3)*x+b^(4/3)*x^2)/((1+3^(1/2
))*a^(2/3)-b^(2/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(2/3)-b^(2/3)*x)/(
(1+3^(1/2))*a^(2/3)-b^(2/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(4/3)/(a^(2/3)*(a^
(2/3)-b^(2/3)*x)/((1+3^(1/2))*a^(2/3)-b^(2/3)*x)^2)^(1/2)/(a-b*x^(3/2))^(1
/2)/(a+b*x^(3/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.99 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx=-\frac{\sqrt{1-\frac{b^2x^3}{a^2}}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\frac{b^2x^3}{a^2}\right)}{x\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

input
```
Integrate[1/(x^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-((Sqrt[1 - (b^2*x^3)/a^2]*HypergeometricPFQ[{-1/3, 1/2}, {2/3}, (b^2*x^3)
/a^2])/(x*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]))
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.161$, Rules used = {849, 890, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}dx$$

$\big\downarrow$ 849

$$-\frac{b^2\int \frac{x}{\sqrt{a-bx^{3/2}}\sqrt{bx^{3/2}+a}}dx}{2a^2}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x}$$

$\big\downarrow$ 890

$$-\frac{b^2\sqrt{a^2-b^2x^3}\int \frac{x}{\sqrt{a^2-b^2x^3}}dx}{2a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x}$$

$\big\downarrow$ 832

$$-\frac{b^2\sqrt{a^2-b^2x^3}\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}\int \frac{1}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}\right)}{2a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x}$$

$\big\downarrow$ 759

$$-\frac{b^2\sqrt{a^2-b^2x^3}\left(-\frac{\int \frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\sqrt{a^2-b^2x^3}}dx}{b^{2/3}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}\right.}{2a^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}$$

$$\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x}$$

$\big\downarrow$ 2416

$$-\frac{\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}{a^2x}-$$

$$\frac{b^2\sqrt{a^2-b^2x^3}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}a^{2/3}\left(a^{2/3}-b^{2/3}x\right)\sqrt{\frac{a^{2/3}b^{2/3}x+a^{4/3}+b^{4/3}x^2}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)a^{2/3}-b^{2/3}x}{\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{4/3}\sqrt{\frac{a^{2/3}\left(a^{2/3}-b^{2/3}x\right)}{\left(\left(1+\sqrt{3}\right)a^{2/3}-b^{2/3}x\right)^2}}\sqrt{a^2-b^2x^3}}-\right.}{2a^2\sqrt{a-bx^{3/2}}\sqrt{a}}$$

input
```
Int[1/(x^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)]),x]
```

output
```
-((Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])/(a^2*x)) - (b^2*Sqrt[a^2 - b^2
*x^3]*(-(((-2*Sqrt[a^2 - b^2*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(2/3) - b^(2/
3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(2/3)*(a^(2/3) - b^(2/3)*x)*Sqrt[(a^
(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/((1 + Sqrt[3])*
a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(2/3)*(a^(2/3) -
b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a^2 - b^2*x^3]))/b
^(2/3)) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(2/3)*(a^(2/3) - b^(2/3)*x)
*Sqrt[(a^(4/3) + a^(2/3)*b^(2/3)*x + b^(4/3)*x^2)/((1 + Sqrt[3])*a^(2/3) -
 b^(2/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(2/3) - b^(2/3)*x)/((1 +
Sqrt[3])*a^(2/3) - b^(2/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(4/3)*Sqrt[(a^
(2/3)*(a^(2/3) - b^(2/3)*x))/((1 + Sqrt[3])*a^(2/3) - b^(2/3)*x)^2]*Sqrt[a
^2 - b^2*x^3])))/(2*a^2*Sqrt[a - b*x^(3/2)]*Sqrt[a + b*x^(3/2)])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 849
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1)))   Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^
p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && I
GtQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^{2} \sqrt{a-b\,x^{\frac{3}{2}}}\, \sqrt{a+b\,x^{\frac{3}{2}}}}dx$$

input
```
int(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = -\frac{\sqrt{-b^2}x\mathrm{weierstrassZeta}\left(0, \frac{4\,a^2}{b^2}, \mathrm{weierstrassPInverse}\left(0, \frac{4\,a^2}{b^2}, x\right)\right) + \sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}}{a^2x}$$

input
```
integrate(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="fric
as")
```

output
```
-(sqrt(-b^2)*x*weierstrassZeta(0, 4*a^2/b^2, weierstrassPInverse(0, 4*a^2/
b^2, x)) + sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a))/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \frac{ib^{\frac{2}{3}}G_{6,6}^{5,3}\left(\begin{matrix} \frac{13}{12},\frac{19}{12},1 & \frac{4}{3},\frac{4}{3},\frac{11}{6} \\ \frac{5}{6},\frac{13}{12},\frac{4}{3},\frac{19}{12},\frac{11}{6} & 0 \end{matrix}\middle| \frac{a^2}{b^2x^3}\right)}{6\pi^{\frac{3}{2}}a^{\frac{5}{3}}} + \frac{b^{\frac{2}{3}}G_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{3},\frac{7}{12},\frac{5}{6},\frac{13}{12},\frac{4}{3},1 & \\ \frac{7}{12},\frac{13}{12} & \frac{1}{3},\frac{5}{6},\frac{5}{6},0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^3}\right)e^{-\frac{i\pi}{3}}}{6\pi^{\frac{3}{2}}a^{\frac{5}{3}}}$$

input
```
integrate(1/x**2/(a-b*x**(3/2))**(1/2)/(a+b*x**(3/2))**(1/2),x)
```

output
```
I*b**(2/3)*meijerg(((13/12, 19/12, 1), (4/3, 4/3, 11/6)), ((5/6, 13/12, 4/
3, 19/12, 11/6), (0,)), a**2/(b**2*x**3))/(6*pi**(3/2)*a**(5/3)) + b**(2/3
)*meijerg(((1/3, 7/12, 5/6, 13/12, 4/3, 1), ()), ((7/12, 13/12), (1/3, 5/6
, 5/6, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**3))*exp(-I*pi/3)/(6*pi**(3/2)
*a**(5/3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{\sqrt{bx^{\frac{3}{2}}+a}\sqrt{-bx^{\frac{3}{2}}+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/(sqrt(b*x^(3/2) + a)*sqrt(-b*x^(3/2) + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,x^{3/2}}\,\sqrt{a-b\,x^{3/2}}}\,dx$$

input
```
int(1/(x^2*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^(3/2))^(1/2)*(a - b*x^(3/2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a-bx^{3/2}}\sqrt{a+bx^{3/2}}}\,dx = \int \frac{\sqrt{\sqrt{x}\,bx+a}\,\sqrt{-\sqrt{x}\,bx+a}}{-b^2x^5+a^2x^2}dx$$

input
```
int(1/x^2/(a-b*x^(3/2))^(1/2)/(a+b*x^(3/2))^(1/2),x)
```

output
```
int((sqrt(sqrt(x)*b*x + a)*sqrt( - sqrt(x)*b*x + a))/(a**2*x**2 - b**2*x**
5),x)
```

3.234 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = -\frac{5}{64}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2} - \frac{5\operatorname{arccosh}\left(\sqrt{x}\right)}{64}$$

output
```
-5/64*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-5/96*(-1+x^(1/2))^(1/2)
*(1+x^(1/2))^(1/2)*x^(3/2)-1/24*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/
2)+1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(7/2)-5/64*arccosh(x^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{192}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\left(-15-15\sqrt{x}-10x-10x^{3/2}-8x^2-8x^{5/2}+48x^3+4\right.\right.$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-15 - 15*Sqrt[x] - 10*x - 10*
x^(3/2) - 8*x^2 - 8*x^(5/2) + 48*x^3 + 48*x^(7/2)) - 30*ArcTanh[Sqrt[(-1 +
 Sqrt[x])/(1 + Sqrt[x])]])/192
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {812, 845, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\,dx$$

$$\Big\downarrow \text{812}$$

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}-\frac{1}{8}\int \frac{x^{5/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)+\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-\right.$$
$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\right.$$
$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

$$\Big\downarrow 43$$

$$\frac{1}{8}\left(-\frac{5}{6}\left(\frac{3}{4}\left(\text{arccosh}(\sqrt{x})+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)-\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}\right)$$
$$\frac{1}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}$$

input
```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 + (-1/3*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*x^(5/2)) - (5*((Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*
x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sq
rt[x]]))/4))/6)/8
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{192\sqrt{x-1}}$	75
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{192\sqrt{x-1}}$	75

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-48*(x-1)^(1/2)*x^(7/2)+8*x^(
5/2)*(x-1)^(1/2)+10*x^(3/2)*(x-1)^(1/2)+15*x^(1/2)*(x-1)^(1/2)+15*ln(x^(1/
2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{192}\left(48\,x^3-8\,x^2-10\,x-15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{5}{128}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2),x, algorithm="frica
s")
```

output
```
1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x)
- 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)*x**(5/2),x)
```

output
```
Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.42

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{4}\,(x-1)^{\frac{3}{2}}x^{\frac{5}{2}} + \frac{5}{24}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}} + \frac{5}{32}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{5}{64}\,\sqrt{x-1}\sqrt{x} - \frac{5}{64}\,\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2),x, algorithm="maxim
a")
```

output
```
1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2
)*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}\right.\right.\right.\right.\right.$$
$$+\frac{1}{840}\left(\left(2\left(\left(4\left(5\left(6\sqrt{x}-37\right)\left(\sqrt{x}+1\right)+661\right)\left(\sqrt{x}+1\right)-4551\right)\left(\sqrt{x}+1\right)+4781\right)\left(\sqrt{x}+1\right)-6335\right)\left(\sqrt{x}\right.\right.$$
$$+\frac{5}{32}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2),x, algorithm="giac"
)
```

output
```
1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1)
- 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 594
65)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2
*((4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(
x) + 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 49.47 (sec) , antiderivative size = 831, normalized size of antiderivative = 6.16

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output
```
((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (2
35*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283
*((x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*
((x^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*
((x^(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (109941
81*((x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (174
57599*((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (
17457599*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17)
+ (10994181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^1
9) + (4181067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)
^21) + (848801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1
)^23) + (72283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1
)^25) + (1723*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)
^27) - (235*((x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^2
9) + (5*((x^(1/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) +
 (5*((x^(1/2) - 1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^
(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) -
1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/(
(x^(1/2) + 1)^(1/2) - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}\,dx = \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^3}{4} - \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{24} - \frac{5\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{96} - \frac{5\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{64} - \frac{5\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)}{32}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2),x)
```

output

```
(48*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**3 - 8*sqrt(x)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1)*x**2 - 10*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x
) - 1)*x - 15*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 30*log((sqrt(s
qrt(x) - 1) + sqrt(sqrt(x) + 1))/sqrt(2)))/192
```

3.235 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} - \frac{\operatorname{arccosh}(\sqrt{x})}{8}$$

output
```
-1/8*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-1/12*(-1+x^(1/2))^(1/2)*
(1+x^(1/2))^(1/2)*x^(3/2)+1/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2)
-1/8*arccosh(x^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{24}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}(-3-3\sqrt{x}-2x-2x^{3/2}+8x^2+8x^{5/2})-6\operatorname{arctanh}\left(\sqrt{\right.\right.$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(-3 - 3*Sqrt[x] - 2*x - 2*x^(3
/2) + 8*x^2 + 8*x^(5/2)) - 6*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/
24
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {812, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\,dx$$

$$\Big\downarrow \text{812}$$

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}-\frac{1}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{6}\left(-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{6}\left(-\frac{3}{4}\left(\operatorname{arccosh}(\sqrt{x})+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)-\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

input
```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 + (-1/2*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*x^(3/2)) - (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*S
qrt[x] + ArcCosh[Sqrt[x]]))/4)/6
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-8x^{\frac{5}{2}}\sqrt{x-1}+2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{24\sqrt{x-1}}$	65
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-8x^{\frac{5}{2}}\sqrt{x-1}+2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{24\sqrt{x-1}}$	65

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-8*x^(5/2)*(x-1)^(1/2)+2*x^(3/
2)*(x-1)^(1/2)+3*x^(1/2)*(x-1)^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx=\frac{1}{24}\left(8\,x^{2}-2\,x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{16}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(3/2),x, algorithm="frica
s")
```

output
```
1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*
log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \int x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)*x**(3/2),x)
```

output
```
Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}} + \frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x} + \frac{1}{8}\,\sqrt{x-1}\sqrt{x} - \frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(3/2),x, algorithm="maxim
a")
```

output
```
1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sq
rt(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.22

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+7\right.\right.$$
$$+\frac{1}{60}\left(\left(2\left(3\left(4\sqrt{x}-17\right)\left(\sqrt{x}+1\right)+133\right)\left(\sqrt{x}+1\right)-295\right)\left(\sqrt{x}+1\right)+195\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}$$
$$+\frac{1}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(3/2),x, algorithm="giac"
)
```

output
```
1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(
sqrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
1) + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295
)*(sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(
sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 27.17 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output
```
- atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x
^(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2
) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^
(1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/
2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) -
1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^1
5)/(3*((x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)
/(2*((x^(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2
*((x^(1/2) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x
^(1/2) + 1)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) +
1)^(1/2) - 1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*(
(x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2)
- 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2
) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^1
0)/((x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^
(1/2) + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) +
 1)^(1/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/
2) - 1)^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) -...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}\,dx = \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{3} - \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{12} - \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{8} - \frac{\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)}{4}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(3/2),x)
```

output
```
(8*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**2 - 2*sqrt(x)*sqrt(sqrt(
x) + 1)*sqrt(sqrt(x) - 1)*x - 3*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1
) - 6*log((sqrt(sqrt(x) - 1) + sqrt(sqrt(x) + 1))/sqrt(2)))/24
```

3.236 $\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} - \frac{\operatorname{arccosh}(\sqrt{x})}{4}$$

output
```
-1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)+1/2*(-1+x^(1/2))^(1/2)*(
1+x^(1/2))^(1/2)*x^(3/2)-1/4*arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 404 vs. 2(73) = 146.

Time = 1.41 (sec) , antiderivative size = 404, normalized size of antiderivative = 5.53

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{-4\sqrt{1+\sqrt{x}}\left(-18816+28224\sqrt{x}+55360x+17296x^{3/2}+7240x^2-1096x^{5/2}-4752x^3-1136x^{7/2}\right)-}{-12416} + \operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
(-4*Sqrt[1 + Sqrt[x]]*(-18816 + 28224*Sqrt[x] + 55360*x + 17296*x^(3/2) +
7240*x^2 - 1096*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*(32592 + 74488*Sqrt[x] + 38632*x + 6992*x^(3/2) - 104*x^
2 - 6079*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[x]
]*(-18816 - 52416*Sqrt[x] - 41472*x - 10928*x^(3/2) - 1192*x^2 + 3832*x^(5
/2) + 3408*x^3 + 656*x^(7/2)) - 4*(10864 - 10872*Sqrt[x] - 41440*x - 23268
*x^(3/2) - 6678*x^2 - 1148*x^(5/2) + 3416*x^3 + 1800*x^(7/2) + 112*x^4)))/
(-12416 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqr
t[1 + Sqrt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 +
 Sqrt[x]]*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1
 + Sqrt[x]]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2))) + ArcTanh[(
-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {812, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\,dx$$

$$\Big\downarrow \text{ 812}$$

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}-\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 845}$$

$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{ 852}$$

$$\frac{1}{4}\left(-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(-\text{arccosh}\left(\sqrt{x}\right)-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

input
```
Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (-(Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]]*Sqrt[x]) - ArcCosh[Sqrt[x]])/4
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-2x^{\frac{3}{2}}\sqrt{x-1}+\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	52
default	$-\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(-2x^{\frac{3}{2}}\sqrt{x-1}+\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	52

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(-2*x^(3/2)*(x-1)^(1/2)+x^(1/2)*
(x-1)^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{4}\,(2\,x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="frica
s")
```

output
```
1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt
(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)*x**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{2}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{4}\,\sqrt{x-1}\sqrt{x}-\frac{1}{4}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1
) + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx$$
$$= \frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}$$
$$+\frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}$$
$$+\frac{1}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2),x, algorithm="giac"
)
```

output
```
1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqr
t(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt
(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x)
 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \int \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

input
```
int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

output
```
int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{2} - \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{4} - \frac{\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)}{2}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x - sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1) - 2*log((sqrt(sqrt(x) - 1) + sqrt(sqrt(x) + 1))/sqrt(
2)))/4
```

3.237 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx=\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}-\text{arccosh}\left(\sqrt{x}\right)$$

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)-arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 264 vs. 2(37) = 74.

Time = 1.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 7.14

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$
$$=4\left(\frac{4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\left(\right.}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\right.}\right.$$
$$\left.\left.+\text{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)\right)\right)$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

output
```
4*((4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(2
8 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8
*Sqrt[x] + 5*x + 3*x^(3/2))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*Sq
rt[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt[
x]) + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) + ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]]
)/(Sqrt[3] - Sqrt[1 + Sqrt[x]])])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {812, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 812}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx$$

$$\Big\downarrow \text{ 852}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}$$

$$\Big\downarrow \text{ 43}$$

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\text{arccosh}\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

output
```
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.95

method	result	size
derivativedivides	$\sqrt{-1+\sqrt{x}}\,\left(1+\sqrt{x}\right)^{\frac{3}{2}}-\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}-\frac{\sqrt{\left(1+\sqrt{x}\right)\left(-1+\sqrt{x}\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{1+\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$	72
default	$\sqrt{-1+\sqrt{x}}\,\left(1+\sqrt{x}\right)^{\frac{3}{2}}-\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}-\frac{\sqrt{\left(1+\sqrt{x}\right)\left(-1+\sqrt{x}\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{1+\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$	72

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(3/2)-(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-
((1+x^(1/2))*(-1+x^(1/2)))^(1/2)/(1+x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)*ln(x
^(1/2)+(x-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="frica
s")
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(
x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = 4\sqrt{\sqrt{x}-1}\left(\frac{\left(\sqrt{x}+1\right)^{\frac{3}{2}}}{4}-\frac{\sqrt{\sqrt{x}+1}}{4}\right) - 2\log\left(2\sqrt{\sqrt{x}-1}+2\sqrt{\sqrt{x}+1}\right)$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

output
```
4*sqrt(sqrt(x) - 1)*((sqrt(x) + 1)**(3/2)/4 - sqrt(sqrt(x) + 1)/4) - 2*log
(2*sqrt(sqrt(x) - 1) + 2*sqrt(sqrt(x) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x-1}\sqrt{x} - \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxim
a")
```

output
```
sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}(\sqrt{x}-2) + 2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + 2\log\left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac"
)
```

output
```
sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sq
rt(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\ln\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}+\sqrt{x}\right)$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

output
```
x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*
(x^(1/2) + 1)^(1/2) + x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x)
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log((sqrt(sqrt(x) - 1) + s
qrt(sqrt(x) + 1))/sqrt(2))
```

3.238 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 38

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}} + 2\operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
-2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)+2*arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 184 vs. 2(38) = 76.

Time = 0.96 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.84

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}\right.}{\left(-3-2\sqrt{-1+\sqrt{x}}+2\sqrt{3}\sqrt{1+\sqrt{x}}+\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}-2\sqrt{x}\right.} - 8\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((-3 - 2*Sqrt[-1 + Sqrt[x]
] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt
[x]] - 2*Sqrt[x])*Sqrt[x]) - 8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3]
- Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {849, 812, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{3/2}}dx$$

$$\Big\downarrow \text{849}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{812}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx\right)$$

$$\Big\downarrow \text{852}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{43}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}-\operatorname{arccosh}\left(\sqrt{x}\right)\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*(Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]])
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 812
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^p*((a2 + b2*x^n)
^p/(c*(m + 2*n*p + 1))), x] + Simp[2*a1*a2*n*(p/(m + 2*n*p + 1))   Int[(c*x
)^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a
2, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && N
eQ[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 849
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2
*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a
1*a2*c^(2*n)*(m + 1)))   Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^
p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && I
GtQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)\sqrt{x}-\sqrt{x-1}\right)}{\sqrt{x}\,\sqrt{x-1}}$	47
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)\sqrt{x}-\sqrt{x-1}\right)}{\sqrt{x}\,\sqrt{x-1}}$	47

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(ln(x^(1/2)+(x-1)^(1/2))*x^(1/2)-(x
-1)^(1/2))/x^(1/2)/(x-1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="frica
s")
```

output
```
-(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = -\frac{2\sqrt{x-1}}{\sqrt{x}} + 2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxim
a")
```

output
```
-2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac"
)
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.39

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = 8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) - \frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}} - \frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
```

output
```
8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1
/2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/(((x^(1/2)
- 1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i
)/((x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1
)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{-2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+4\sqrt{x}\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)-2\sqrt{x}}{\sqrt{x}}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x)
```

output
```
(2*( - sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*sqrt(x)*log((sqrt(sqrt(x) -
 1) + sqrt(sqrt(x) + 1))/sqrt(2)) - sqrt(x)))/sqrt(x)
```

3.239 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

output
```
2/3*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 421 vs. 2(31) = 62.

Time = 1.94 (sec) , antiderivative size = 421, normalized size of antiderivative = 13.58

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}\right.}{}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(7 + 12*Sqrt[-1 + Sqrt[x
]] - 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] - 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + S
qrt[x]]) + 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 3
*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-23 - 144*Sqrt
[-1 + Sqrt[x]] + 32*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 77*Sqrt[3]*Sqrt[-1 + Sqrt[
x]]*Sqrt[1 + Sqrt[x]])*x + 2*(-140 - 106*Sqrt[-1 + Sqrt[x]] + 62*Sqrt[3]*S
qrt[1 + Sqrt[x]] + 21*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2
) - 73*x^2))/(12*(-3 - 2*Sqrt[-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]]
+ Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{797\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{797}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)}{3x^{\frac{3}{2}}}$	23
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)}{3x^{\frac{3}{2}}}$	23
orering	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)}{3x^{\frac{3}{2}}}$	23

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x-1)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="frica
s")
```

output
```
2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxim
a")
```

output
```
2/3*(x - 1)^(3/2)/x^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac"
)
```

output
```
16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2)
)/3))/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx = \frac{\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{3}-\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{3}+\frac{2\sqrt{x}\,x}{3}}{\sqrt{x}\,x}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x)
```

output
```
(2*(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x - sqrt(sqrt(x) + 1)*sqrt(sqrt(x)
 - 1) + sqrt(x)*x))/(3*sqrt(x)*x)
```

3.240 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}$$

output
```
2/5*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)+4/15*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(3/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 693 vs. 2(63) = 126.

Time = 8.07 (sec) , antiderivative size = 693, normalized size of antiderivative = 11.00

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx=\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\sqrt{x}}\right.}{}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(-384*(97 - 168*Sqrt[-1 + S
qrt[x]] - 56*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 97*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]) - 192*(-499 - 1112*Sqrt[-1 + Sqrt[x]] + 344*Sqrt[3]*Sqrt[1
 + Sqrt[x]] + 545*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] +
32*(9925 + 1656*Sqrt[-1 + Sqrt[x]] - 4616*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 535*
Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x + 16*(2243 - 20096*Sqrt[-1
 + Sqrt[x]] + 2720*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 10385*Sqrt[3]*Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) + 8*(-33645 - 39152*Sqrt[-1 + Sqrt[x]] +
15056*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 13135*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1
+ Sqrt[x]])*x^2 + 8*(-21349 - 18772*Sqrt[-1 + Sqrt[x]] + 6180*Sqrt[3]*Sqrt
[1 + Sqrt[x]] + 6379*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(5/2)
 + 4*(-22053 - 18788*Sqrt[-1 + Sqrt[x]] + 8788*Sqrt[3]*Sqrt[1 + Sqrt[x]] +
 5745*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^3 + 2*(-19920 - 7252
*Sqrt[-1 + Sqrt[x]] + 4188*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 715*Sqrt[3]*Sqrt[-1
 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(7/2) - 2477*x^4))/(240*(-3 - 2*Sqrt[-1 +
 Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[
1 + Sqrt[x]] - 2*Sqrt[x])^5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x
])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$	28
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$	28
orering	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)(2x+3)}{15x^{\frac{5}{2}}}$	28

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x-1)*(2*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{2\left(2\,x^{3}+\left(2\,x^{2}+x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^{3}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="frica
s")
```

output
```
2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))
/x^3
```

Sympy [F]

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
```

output
```
Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{4\,(x-1)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxim
a")
```

output
```
4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\right.}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac"
)
```

output
```
128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
 1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4
+ 64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{15}-\frac{2\,\sqrt{\sqrt{x}+1}}{5}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2
))/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx = \frac{\frac{4\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{15}+\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{15}-\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{5}-\frac{4\sqrt{x}\,x^2}{15}}{\sqrt{x}\,x^2}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x)
```

output
```
(2*(2*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**2 + sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1)*x - 3*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*sqrt(x)*x**2))/(1
5*sqrt(x)*x**2)
```

3.241 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}} + \frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}} + \frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}$$

output
```
2/7*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(7/2)+8/35*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(5/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/
2)
```

Mathematica [A] (verified)

Time = 10.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(15+12x+8x^2)}{105x^{7/2}}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7
/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 804}$$

$$\frac{4}{7}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{ 804}$$

$$\frac{4}{7}\left(\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

$$\Big\downarrow \text{ 797}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\left(\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

output
```
(4*((2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sq
rt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))))/7 + (2*(-1 + Sqrt[x])^(3/
2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2))
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33
orering	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(8x^{2}+12x+15\right)}{105x^{\frac{7}{2}}}$	33

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x-1)*(8*x^2+12*x+15)/x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{2\left(8\,x^4+(8\,x^3+4\,x^2+3\,x-15)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{105\,x^4}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="frica
s")
```

output
```
2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(s
qrt(x) - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}}+\frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxim
a")
```

output
```
16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3
/2)/x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+16\right.}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\right.\right.}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac"
)
```

output
```
4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
 + 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1)
)^8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35}-\frac{2\,\sqrt{\sqrt{x}+1}}{7}+\frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2
))/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105)
)/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx = \frac{\frac{16\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^3}{105}+\frac{8\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{105}+\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{35}-\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{7}-\frac{1}{}}{\sqrt{x}\,x^3}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x)
```

output
```
(2*(8*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**3 + 4*sqrt(sqrt(x) + 1)*sqrt(
sqrt(x) - 1)*x**2 + 3*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x - 15*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1) - 8*sqrt(x)*x**3))/(105*sqrt(x)*x**3)
```

3.242 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}} + \frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}} + \frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{5/2}} + \frac{32\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{315x^{3/2}}$$

output
```
2/9*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(9/2)+4/21*(-1+x^(1/2))^(3/2)*(
1+x^(1/2))^(3/2)/x^(7/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/
2)+32/315*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Mathematica [A] (verified)

Time = 10.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}(35+30x+24x^2+16x^3)}{315x^{9/2}}$$

input
```
Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

output
```
(2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))
/(315*x^(9/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {804, 804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{11/2}} \, dx$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{9/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\left(\frac{4}{7}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{7/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\left(\frac{4}{7}\left(\frac{2}{5}\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{5/2}}dx+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}\right)+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\left(\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\left(\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}\right)\right)$$

input
```
Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

output
```
(2*((4*((2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1
+ Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))))/7 + (2*(-1 + Sqrt[x])
^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2))))/3 + (2*(-1 + Sqrt[x])^(3/2)*(1 +
 Sqrt[x])^(3/2))/(9*x^(9/2))
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$	38
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$	38
orering	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(x-1)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$	38

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x-1)*(16*x^3+24*x^2+30*x+35)/x
^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{2\left(16\,x^5+(16\,x^4+8\,x^3+6\,x^2+5\,x-35)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{315\,x^5}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="fric
as")
```

output
```
2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1))/x^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}} + \frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}} + \frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}} + \frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxi
ma")
```

output
```
32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)
^(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20} - 756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16} + \cdots}{3\cdots}$$

input
```
integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac
")
```

output
```
16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x)
- 1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqr
t(x) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x
) - 1))^4 + 4)^9
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{63}-\frac{2\,\sqrt{\sqrt{x}+1}}{9}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{315}+\frac{32\,x^4\,\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

input
```
int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2
))/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315
+ (32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx = \frac{\frac{32\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^4}{315}+\frac{16\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^3}{315}+\frac{4\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{105}+\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{63}}{\sqrt{x}\,x^4}$$

input
```
int((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x)
```

output
```
(2*(16*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**4 + 8*sqrt(sqrt(x) + 1)*sqrt
(sqrt(x) - 1)*x**3 + 6*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**2 + 5*sqrt(s
qrt(x) + 1)*sqrt(sqrt(x) - 1)*x - 35*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) -
 16*sqrt(x)*x**4))/(315*sqrt(x)*x**4)
```

3.243 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2} + \frac{5\operatorname{arccosh}\left(\sqrt{x}\right)}{8}$$

output
```
5/8*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)+5/12*(-1+x^(1/2))^(1/2)*(
1+x^(1/2))^(1/2)*x^(3/2)+1/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(5/2)+
5/8*arccosh(x^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 1.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\sqrt{x}\big(15+15\sqrt{x} + 10x+10x^{3/2}+8x^2+8x^{5/2}\big) + \frac{5}{4}\operatorname{arctanh}\left(\sqrt{\frac{-1+\sqrt{x}}{1+\sqrt{x}}}\right)$$

input
```
Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]*Sqrt[x]*(15 + 15*Sqrt[x] + 10*x + 10*x
^(3/2) + 8*x^2 + 8*x^(5/2)))/24 + (5*ArcTanh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt
[x])]])/4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {845, 845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

$$\Big\downarrow \; 43$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\operatorname{arccosh}(\sqrt{x})+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}\right)+\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}$$

input
```
Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))/3 + (5*((Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*S
qrt[x] + ArcCosh[Sqrt[x]]))/4))/6
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{24\sqrt{x-1}}$	65
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(8x^{\frac{5}{2}}\sqrt{x-1}+10x^{\frac{3}{2}}\sqrt{x-1}+15\sqrt{x}\,\sqrt{x-1}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{24\sqrt{x-1}}$	65

input
```
int(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^(5/2)*(x-1)^(1/2)+10*x^(3/2
)*(x-1)^(1/2)+15*x^(1/2)*(x-1)^(1/2)+15*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.55

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\left(8\,x^{2}+10\,x+15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{5}{16}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/1
6*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.45

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{x-1}x^{\frac{5}{2}} + \frac{5}{12}\sqrt{x-1}x^{\frac{3}{2}} + \frac{5}{8}\sqrt{x-1}\sqrt{x} + \frac{5}{8}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(
x) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{24}\left(\left(2\left(\left(4\left(\sqrt{x}+1\right)\left(\sqrt{x}-4\right)+45\right)\left(\sqrt{x}+1\right)-55\right)\left(\sqrt{x}+1\right)+85\right)\left(\sqrt{x}+\right.$$
$$- \frac{5}{4}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x
) + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4
*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 23.79 (sec) , antiderivative size = 632, normalized size of antiderivative = 6.08

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
(5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*
((x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(
1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2)
 - 1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) -
1)^(1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^
(1/2) - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i
)^15)/(3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^
17)/(2*((x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)
/(2*((x^(1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6
*((x^(1/2) + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^
(1/2) + 1)^(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) +
1)^(1/2) - 1)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) -
1)^4 - (12*((x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (22
0*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/
2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(
1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) -
1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)
/((x^(1/2) + 1)^(1/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1
/2) + 1)^(1/2) - 1)^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{3} + \frac{5\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{12} + \frac{5\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{8} + \frac{5\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)}{4}$$

input
```
int(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

output
```
(8*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**2 + 10*sqrt(x)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1)*x + 15*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 30*log((sqrt(sqrt(x) - 1) + sqrt(sqrt(x) + 1))/sqrt(2)))/24
```

3.244 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 73

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2} + \frac{3\text{arccosh}\left(\sqrt{x}\right)}{4}$$

output
```
3/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)+1/2*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)*x^(3/2)+3/4*arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 406 vs. 2(73) = 146.

Time = 1.52 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.56

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{-4\sqrt{1+\sqrt{x}}\left(-29568+50496\sqrt{x}+98112x+21840x^{3/2}-2264x^2-3368x\right.}{}$$
$$-3\text{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(-4*Sqrt[1 + Sqrt[x]]*(-29568 + 50496*Sqrt[x] + 98112*x + 21840*x^(3/2) -
2264*x^2 - 3368*x^(5/2) - 4752*x^3 - 1136*x^(7/2)) - 4*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*(51216 + 120600*Sqrt[x] + 56904*x - 4016*x^(3/2) - 6344*
x^2 - 6467*x^(5/2) - 3120*x^3 - 194*x^(7/2)) + Sqrt[3]*(-4*Sqrt[-1 + Sqrt[
x]]*(-29568 - 84416*Sqrt[x] - 64000*x - 7152*x^(3/2) + 5624*x^2 + 5144*x^(
5/2) + 3408*x^3 + 656*x^(7/2)) - 4*(17072 - 20632*Sqrt[x] - 73312*x - 3624
4*x^(3/2) - 510*x^2 + 2452*x^(5/2) + 3640*x^3 + 1800*x^(7/2) + 112*x^4)))/
(-12416 + 13312*Sqrt[x] + 49408*x + 24960*x^(3/2) + 1552*x^2 + Sqrt[3]*Sqr
t[1 + Sqrt[x]]*(7168 - 11264*Sqrt[x] - 22016*x - 5248*x^(3/2)) + Sqrt[-1 +
 Sqrt[x]]*(21504 + 60416*Sqrt[x] + 47104*x + 9088*x^(3/2) + Sqrt[3]*Sqrt[1
 + Sqrt[x]]*(-12416 - 28672*Sqrt[x] - 14400*x - 896*x^(3/2)))) - 3*ArcTanh
[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {845, 845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{845}$$

$$\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{845}$$

$$\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \text{852}$$

$$\frac{3}{4}\left(\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

$$\Big\downarrow \; 43$$

$$\frac{3}{4}\left(\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\right)+\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}$$

input
```
Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2))/2 + (3*(Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]))/4
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	55
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(2x^{\frac{3}{2}}\sqrt{x-1}+3\sqrt{x}\,\sqrt{x-1}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{4\sqrt{x-1}}$	55

input
```
int(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(2*x^(3/2)*(x-1)^(1/2)+3*x^(1/2)*
(x-1)^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\,(2\,x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{8}\,\log\left(2\,\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt
(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{2}\sqrt{x-1}x^{\frac{3}{2}} + \frac{3}{4}\sqrt{x-1}\sqrt{x} + \frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1)
+ 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{4}\left(\left(2\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
 1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 429, normalized size of antiderivative = 5.88

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = 3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right) + \frac{\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{13}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{13}}-}{1+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{70\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\frac{8\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{14}}+}$$

input
```
int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(
1/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)
^(1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) -
1i)^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((
x^(1/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2)
+ 1)^(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/
2) - 1)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^1
5 - (3*((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/
2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/
2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6
)/((x^(1/2) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2
) + 1)^(1/2) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1
/2) - 1)^10 + (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)
^12 - (8*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x
^(1/2) - 1)^(1/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{2} + \frac{3\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{4} + \frac{3\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)}{2}$$

input
```
int(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x + 3*sqrt(x)*sqrt(sqrt(x)
+ 1)*sqrt(sqrt(x) - 1) + 6*log((sqrt(sqrt(x) - 1) + sqrt(sqrt(x) + 1))/sqr
t(2)))/4
```

3.245 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 35

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}+\operatorname{arccosh}(\sqrt{x})$$

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*x^(1/2)+arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 265 vs. 2(35) = 70.

Time = 1.41 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.57

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{4\left(4\sqrt{1+\sqrt{x}}\left(-12-24\sqrt{x}+x+5x^{3/2}\right)+\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\left(-84-10\sqrt{x}+28x+7x^{3/2}\right)+\sqrt{3}\left(2\right.\right.}{56-16\sqrt{3}\sqrt{1+\sqrt{x}}\left(2+3\sqrt{x}\right)+\sqrt{-1+\sqrt{x}}\left(96-8\sqrt{3}\sqrt{1+\right.} - 4\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
(4*(4*Sqrt[1 + Sqrt[x]]*(-12 - 24*Sqrt[x] + x + 5*x^(3/2)) + Sqrt[-1 + Sqr
t[x]]*Sqrt[1 + Sqrt[x]]*(-84 - 10*Sqrt[x] + 28*x + 7*x^(3/2)) + Sqrt[3]*(2
8 + 70*Sqrt[x] + 18*x - 14*x^(3/2) - 4*x^2 - 4*Sqrt[-1 + Sqrt[x]]*(-12 - 8
*Sqrt[x] + 5*x + 3*x^(3/2)))))/(56 - 16*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(2 + 3*S
qrt[x]) + Sqrt[-1 + Sqrt[x]]*(96 - 8*Sqrt[3]*Sqrt[1 + Sqrt[x]]*(7 + 2*Sqrt
[x]) + 80*Sqrt[x]) + 112*Sqrt[x] + 28*x) - 4*ArcTanh[(-1 + Sqrt[-1 + Sqrt[
x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {845, 852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}dx$$

$$\Big\downarrow \text{ 845}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

$$\Big\downarrow \text{ 852}$$

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

$$\Big\downarrow \text{ 43}$$

$$\text{arccosh}\left(\sqrt{x}\right)+\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}$$

input
```
Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

output
```
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 845
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n)
^(p + 1)*((a2 + b2*x^n)^(p + 1)/(b1*b2*(m + 2*n*p + 1))), x] - Simp[a1*a2*c
^(2*n)*((m - 2*n + 1)/(b1*b2*(m + 2*n*p + 1)))   Int[(c*x)^(m - 2*n)*(a1 +
b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && Eq
Q[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1
, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{\sqrt{x-1}}$	41
default	$\frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\left(\sqrt{x}\,\sqrt{x-1}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)\right)}{\sqrt{x-1}}$	41

input
```
int(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(x^(1/2)*(x-1)^(1/2)+ln(x^(1/2)+(x-1)
^(1/2)))/(x-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - \frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x + 1\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(
x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(x)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x-1}\sqrt{x} + \log\left(2\sqrt{x-1} + 2\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2\log\left(\sqrt{\sqrt{x}+1} - \sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sq
rt(sqrt(x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

input
```
int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+2\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)$$

input
```
int(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x)
```

output
```
sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*log((sqrt(sqrt(x) - 1) + s
qrt(sqrt(x) + 1))/sqrt(2))
```

3.246 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 8

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\operatorname{arccosh}\left(\sqrt{x}\right)$$

output
```
2*arccosh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(8) = 16.

Time = 0.86 (sec) , antiderivative size = 38, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -8\operatorname{arctanh}\left(\frac{-1+\sqrt{-1+\sqrt{x}}}{\sqrt{3}-\sqrt{1+\sqrt{x}}}\right)$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

output
```
-8*ArcTanh[(-1 + Sqrt[-1 + Sqrt[x]])/(Sqrt[3] - Sqrt[1 + Sqrt[x]])]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {852, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}}dx$$

$$\Big\downarrow \text{852}$$

$$2\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}d\sqrt{x}$$

$$\Big\downarrow \text{43}$$

$$2\text{arccosh}\left(\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

output
```
2*ArcCosh[Sqrt[x]]
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 852
```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/c   Subst[Int[x^
(k*(m + 1) - 1)*(a1 + b1*(x^(k*n)/c^n))^p*(a2 + b2*(x^(k*n)/c^n))^p, x], x,
 (c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2
, 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n,
m, p, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(6) = 12$.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

method	result	size
derivativedivides	$\frac{2\sqrt{(1+\sqrt{x})(-1+\sqrt{x})}\,\ln(\sqrt{x}+\sqrt{x-1})}{\sqrt{1+\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$	40
default	$\frac{2\sqrt{(1+\sqrt{x})(-1+\sqrt{x})}\,\ln(\sqrt{x}+\sqrt{x-1})}{\sqrt{1+\sqrt{x}}\,\sqrt{-1+\sqrt{x}}}$	40

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
2*((1+x^(1/2))*(-1+x^(1/2)))^(1/2)/(1+x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)*ln
(x^(1/2)+(x-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fri
cas")
```

output
```
-log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2)/x**(1/2),x)
```

output
```
Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="max
ima")
```

output
```
2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(6) = 12.

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = -4\,\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="gia
c")
```

output
```
-4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
2*acosh(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx = 4\log\left(\frac{\sqrt{\sqrt{x}-1}+\sqrt{\sqrt{x}+1}}{\sqrt{2}}\right)$$

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(1/2),x)
```

output
```
4*log((sqrt(sqrt(x) - 1) + sqrt(sqrt(x) + 1))/sqrt(2))
```

3.247 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 29

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

output
```
2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.

Time = 0.91 (sec) , antiderivative size = 146, normalized size of antiderivative = 5.03

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx=\frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\right.}{\left(3+2\sqrt{-1+\sqrt{x}}-2\sqrt{3}\sqrt{1+\sqrt{x}}-\sqrt{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}+2\right.}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x]))/((3 + 2*Sqrt[-1 + Sqrt[x]]
 - 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[
x]] + 2*Sqrt[x])*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{797\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx$$
$$\Big\downarrow \text{ 797}$$
$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

output
```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}$	20
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}$	20
orering	$\frac{2x-2}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}$	23

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fri
cas")
```

output
```
2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2)/x**(3/2),x)
```

output
```
Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{x-1}}{\sqrt{x}}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="max
ima")
```

output
```
2*sqrt(x - 1)/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="gia
c")
```

output
```
16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
(2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{3/2}}\,dx = \frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+2\sqrt{x}}{\sqrt{x}}$$

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(3/2),x)
```

output
```
(2*(sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + sqrt(x)))/sqrt(x)
```

3.248 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 63

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3x^{3/2}} + \frac{4\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{3\sqrt{x}}$$

output
```
2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+4/3*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(63) = 126.

Time = 1.98 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.46

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\left(-1+\sqrt{-1+\sqrt{x}}\right)\left(\sqrt{3}-\sqrt{1+\sqrt{x}}\right)\left(-2+\sqrt{-1+\sqrt{x}}+\sqrt{3}\sqrt{1+\right.}{}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

output
```
((-1 + Sqrt[-1 + Sqrt[x]])*(Sqrt[3] - Sqrt[1 + Sqrt[x]])*(-2 + Sqrt[-1 + S
qrt[x]] + Sqrt[3]*Sqrt[1 + Sqrt[x]] - Sqrt[x])*(8*(-7 - 12*Sqrt[-1 + Sqrt[
x]] + 4*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 +
Sqrt[x]]) - 4*(49 + 8*Sqrt[-1 + Sqrt[x]] - 24*Sqrt[3]*Sqrt[1 + Sqrt[x]] +
3*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 2*(-61 + 16*Sqrt
[3]*Sqrt[1 + Sqrt[x]] + 7*Sqrt[3]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x
+ (-56 - 28*Sqrt[-1 + Sqrt[x]] + 20*Sqrt[3]*Sqrt[1 + Sqrt[x]] + 6*Sqrt[3]*
Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])*x^(3/2) - 11*x^2))/(12*(-3 - 2*Sqrt[
-1 + Sqrt[x]] + 2*Sqrt[3]*Sqrt[1 + Sqrt[x]] + Sqrt[3]*Sqrt[-1 + Sqrt[x]]*S
qrt[1 + Sqrt[x]] - 2*Sqrt[x])^3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}\,dx$$

$$\Big\downarrow \text{804}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

output
```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x
]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.40

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(1+2x)}{3x^{\frac{3}{2}}}$	25
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(1+2x)}{3x^{\frac{3}{2}}}$	25
orering	$\frac{2(1+2x)(x-1)}{3x^{\frac{3}{2}}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}$	28

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(1+2*x)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{2\left((2\,x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\,x^2\right)}{3\,x^2}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fri
cas")
```

output
```
2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2)/x**(5/2),x)
```

output
```
Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{4\sqrt{x-1}}{3\sqrt{x}} + \frac{2\sqrt{x-1}}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="max
ima")
```

output
```
4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="gia
c")
```

output
```
128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^3
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{4x}{3}+\frac{2\sqrt{x}}{3}+\frac{4x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\sqrt{\sqrt{x}+1}}$$

input
```
int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(
3/2)*(x^(1/2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{5/2}}\,dx = \frac{\frac{4\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{3}+\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{3}-\frac{4\sqrt{x}\,x}{3}}{\sqrt{x}\,x}$$

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(5/2),x)
```

output
```
(2*(2*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x + sqrt(sqrt(x) + 1)*sqrt(sqrt(
x) - 1) - 2*sqrt(x)*x))/(3*sqrt(x)*x)
```

3.249 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{5x^{5/2}} + \frac{8\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15x^{3/2}} + \frac{16\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{15\sqrt{x}}$$

output
```
2/5*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2)+8/15*(-1+x^(1/2))^(1/2)*(
1+x^(1/2))^(1/2)/x^(3/2)+16/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2
)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}(3+4x+8x^2)}{15x^{5/2}}$$

input
```
Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

output
```
(2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {804, 804, 797}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{7/2}}dx$$

$$\Big\downarrow \text{804}$$

$$\frac{4}{5}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{5/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{804}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}x^{3/2}}dx+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}\right)+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

$$\Big\downarrow \text{797}$$

$$\frac{4}{5}\left(\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{3\sqrt{x}}\right)+\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{5x^{5/2}}$$

input
```
Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

output
```
(4*((2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sq
rt[x]]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])))/5 + (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 +
 Sqrt[x]])/(5*x^(5/2))
```

Defintions of rubi rules used

rule 797
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b
2*x^n)^(p + 1)/(a1*a2*c*(m + 1))), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

rule 804
```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[x^(m + 1)*(a1 + b1*x^n)^(p + 1)*((a2 + b2*x^n)^(p + 1
)/(a1*a2*(m + 1))), x] - Simp[b1*b2*((m + 2*n*(p + 1) + 1)/(a1*a2*(m + 1)))
   Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1
, a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2
*n) + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.32

method	result	size
derivativedivides	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30
default	$\frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,(8x^2+4x+3)}{15x^{\frac{5}{2}}}$	30
orering	$\frac{2(8x^2+4x+3)(x-1)}{15x^{\frac{5}{2}}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}$	33

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
2/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{2\left(8\,x^{3} + (8\,x^{2} + 4\,x + 3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^{3}}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fri
cas")
```

output
```
2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1
))/x^3
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \int \frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

input
```
integrate(1/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2)/x**(7/2),x)
```

output
```
Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{16\sqrt{x-1}}{15\sqrt{x}} + \frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}} + \frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="max
ima")
```

output
```
16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(
5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+8\right.}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

input
```
integrate(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="gia
c")
```

output
```
4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4
+ 4)^5
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{\sqrt{\sqrt{x}-1}\left(\frac{8\,x}{15}+\frac{16\,x^2}{15}+\frac{2\,\sqrt{x}}{5}+\frac{8\,x^{3/2}}{15}+\frac{16\,x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\,\sqrt{\sqrt{x}+1}}$$

input
```
int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

output
```
((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))
/15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}x^{7/2}}\,dx = \frac{\frac{16\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x^2}{15}+\frac{8\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\,x}{15}+\frac{2\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}{5}-\frac{16\sqrt{x}\,x^2}{15}}{\sqrt{x}\,x^2}$$

input
```
int(1/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2)/x^(7/2),x)
```

output
```
(2*(8*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*x**2 + 4*sqrt(sqrt(x) + 1)*sqrt(
sqrt(x) - 1)*x + 3*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 8*sqrt(x)*x**2))/
(15*sqrt(x)*x**2)
```

3.250 $\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m\,dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m\,dx = \frac{x^{1+m}\operatorname{AppellF1}\left(2(1+m),-p,-q,3+2m,-b\sqrt{x},-d\sqrt{x}\right)}{1+m}$$

output
```
x^(1+m)*AppellF1(2+2*m,-p,-q,3+2*m,-b*x^(1/2),-d*x^(1/2))/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m\,dx = \frac{x^{1+m}\operatorname{AppellF1}\left(2(1+m),-p,-q,1+2(1+m),-b\sqrt{x},-d\sqrt{x}\right)}{1+m}$$

input
```
Integrate[(1 + b*Sqrt[x])^p*(1 + d*Sqrt[x])^q*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[2*(1 + m), -p, -q, 1 + 2*(1 + m), -(b*Sqrt[x]), -(d*Sq
rt[x])])/(1 + m)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1000, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(b\sqrt{x}+1\right)^p \left(d\sqrt{x}+1\right)^q \, dx$$

$$\Big\downarrow \ 1000$$

$$2\int \left(\sqrt{x}b+1\right)^p \left(\sqrt{x}d+1\right)^q x^{\frac{1}{2}(2m+1)} d\sqrt{x}$$

$$\Big\downarrow \ 150$$

$$\frac{x^{m+1} \text{AppellF1}\left(2(m+1),-p,-q,2m+3,-b\sqrt{x},-d\sqrt{x}\right)}{m+1}$$

input
```
Int[(1 + b*Sqrt[x])^p*(1 + d*Sqrt[x])^q*x^m,x]
```

output
```
(x^(1 + m)*AppellF1[2*(1 + m), -p, -q, 3 + 2*m, -(b*Sqrt[x]), -(d*Sqrt[x])
])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1000
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g*(m + 1) -
1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b
, c, d, m, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Maple [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m dx$$

input
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x)
```

output
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-1)]

Timed out.

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \text{Timed out}$$

input
```
integrate((1+b*x**(1/2))**p*(1+d*x**(1/2))**q*x**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \int (b\sqrt{x}+1)^p (d\sqrt{x}+1)^q x^m \, dx$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(x) + 1)^p*(d*sqrt(x) + 1)^q*x^m, x)
```

Giac [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \int (b\sqrt{x}+1)^p (d\sqrt{x}+1)^q x^m \, dx$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x, algorithm="giac")
```

output
```
integrate((b*sqrt(x) + 1)^p*(d*sqrt(x) + 1)^q*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \int x^m \left(b\sqrt{x}+1\right)^p \left(d\sqrt{x}+1\right)^q \, dx$$

input
```
int(x^m*(b*x^(1/2) + 1)^p*(d*x^(1/2) + 1)^q,x)
```

output
```
int(x^m*(b*x^(1/2) + 1)^p*(d*x^(1/2) + 1)^q, x)
```

Reduce [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q x^m \, dx = \text{too large to display}$$

input

```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*x^m,x)
```

output

```
(4*x**((2*m + 1)/2)*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2*m*q + 2*x**
((2*m + 1)/2)*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2*p*q + 4*x**((2*m
+ 1)/2)*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d*m*p + 4*x**((2*m + 1)/2)
*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d*m*q + 2*x**((2*m + 1)/2)*(sqrt(
x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d*p**2 + 2*x**((2*m + 1)/2)*(sqrt(x)*d +
 1)**q*(sqrt(x)*b + 1)**p*b*d*q**2 + 4*x**((2*m + 1)/2)*(sqrt(x)*d + 1)**q
*(sqrt(x)*b + 1)**p*d**2*m*p + 2*x**((2*m + 1)/2)*(sqrt(x)*d + 1)**q*(sqrt
(x)*b + 1)**p*d**2*p*q + 8*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2
*d*m**2*x + 8*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2*d*m*p*x + 4*
x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2*d*m*q*x + 4*x**m*(sqrt(x)*
d + 1)**q*(sqrt(x)*b + 1)**p*b**2*d*m*x + 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(
x)*b + 1)**p*b**2*d*p**2*x + 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*
b**2*d*p*q*x + 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b**2*d*p*x + 8
*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d**2*m**2*x + 4*x**m*(sqrt(x
)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d**2*m*p*x + 8*x**m*(sqrt(x)*d + 1)**q*(s
qrt(x)*b + 1)**p*b*d**2*m*q*x + 4*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)*
*p*b*d**2*m*x + 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d**2*p*q*x
+ 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d**2*q**2*x + 2*x**m*(sqr
t(x)*d + 1)**q*(sqrt(x)*b + 1)**p*b*d**2*q*x - 4*x**m*(sqrt(x)*d + 1)**q*(
sqrt(x)*b + 1)**p*b*m*q - 2*x**m*(sqrt(x)*d + 1)**q*(sqrt(x)*b + 1)**p*...
```

3.251 $\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx$

Optimal result

Integrand size = 28, antiderivative size = 49

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx = \frac{(ex)^{1+m}\operatorname{AppellF1}\left(2(1+m),-p,-q,3+2m,-b\sqrt{x},-d\sqrt{x}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*AppellF1(2+2*m,-p,-q,3+2*m,-b*x^(1/2),-d*x^(1/2))/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx = \frac{x(ex)^m\operatorname{AppellF1}\left(2(1+m),-p,-q,1+2(1+m),-b\sqrt{x},-d\sqrt{x}\right)}{1+m}$$

input
```
Integrate[(1 + b*Sqrt[x])^p*(1 + d*Sqrt[x])^q*(e*x)^m,x]
```

output
```
(x*(e*x)^m*AppellF1[2*(1 + m), -p, -q, 1 + 2*(1 + m), -(b*Sqrt[x]), -(d*Sq
rt[x])])/(1 + m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1001, 1000, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b\sqrt{x}+1\right)^p \left(d\sqrt{x}+1\right)^q (ex)^m\,dx$$

$$\Big\downarrow \text{1001}$$

$$x^{-m}(ex)^m \int \left(\sqrt{x}b+1\right)^p \left(\sqrt{x}d+1\right)^q x^m dx$$

$$\Big\downarrow \text{1000}$$

$$2x^{-m}(ex)^m \int \left(\sqrt{x}b+1\right)^p \left(\sqrt{x}d+1\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(ex)^m \operatorname{AppellF1}\left(2(m+1),-p,-q,2m+3,-b\sqrt{x},-d\sqrt{x}\right)}{m+1}$$

input
```
Int[(1 + b*Sqrt[x])^p*(1 + d*Sqrt[x])^q*(e*x)^m,x]
```

output
```
(x*(e*x)^m*AppellF1[2*(1 + m), -p, -q, 3 + 2*m, -(b*Sqrt[x]), -(d*Sqrt[x])
])/(1 + m)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1000
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g*(m + 1) -
1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b
, c, d, m, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1001
```
Int[((e_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
^(q_), x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   I
nt[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, p, q
}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Maple [F]

$$\int \left(1+b\sqrt{x}\right)^{p} \left(1+d\sqrt{x}\right)^{q} (ex)^{m}\, dx$$

input
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x)
```

output
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(1+b\sqrt{x}\right)^{p} \left(1+d\sqrt{x}\right)^{q} (ex)^{m}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-1)]

Timed out.

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx = \text{Timed out}$$

input
```
integrate((1+b*x**(1/2))**p*(1+d*x**(1/2))**q*(e*x)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx = \int (ex)^m \left(b\sqrt{x}+1\right)^p \left(d\sqrt{x}+1\right)^q\,dx$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*sqrt(x) + 1)^p*(d*sqrt(x) + 1)^q, x)
```

Giac [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m\,dx = \int (ex)^m \left(b\sqrt{x}+1\right)^p \left(d\sqrt{x}+1\right)^q\,dx$$

input
```
integrate((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*sqrt(x) + 1)^p*(d*sqrt(x) + 1)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m \, dx = \int (e\,x)^m \left(b\,\sqrt{x}+1\right)^p \left(d\,\sqrt{x}+1\right)^q \, dx$$

input
```
int((e*x)^m*(b*x^(1/2) + 1)^p*(d*x^(1/2) + 1)^q,x)
```

output
```
int((e*x)^m*(b*x^(1/2) + 1)^p*(d*x^(1/2) + 1)^q, x)
```

Reduce [F]

$$\int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m \, dx = \int \left(1+b\sqrt{x}\right)^p \left(1+d\sqrt{x}\right)^q (ex)^m \, dx$$

input
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x)
```

output
```
int((1+b*x^(1/2))^p*(1+d*x^(1/2))^q*(e*x)^m,x)
```

3.252 $\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m \, dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m \, dx = \frac{\left(a+b\sqrt{x}\right)^p \left(1+\frac{b\sqrt{x}}{a}\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(1+\frac{d\sqrt{x}}{c}\right)^{-q} x^{1+m} \operatorname{AppellF1}\left(2(1+m),-p,-q,3+2m,-\frac{b\sqrt{x}}{a},-\right.}{1+m}$$

output
```
(a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^(1+m)*AppellF1(2+2*m,-p,-q,3+2*m,-b*x^(1
/2)/a,-d*x^(1/2)/c)/(1+m)/((1+b*x^(1/2)/a)^p)/((1+d*x^(1/2)/c)^q)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m \, dx = \frac{\left(a+b\sqrt{x}\right)^p \left(1+\frac{b\sqrt{x}}{a}\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(1+\frac{d\sqrt{x}}{c}\right)^{-q} x^{1+m} \operatorname{AppellF1}\left(2+2m,-p,-q,3+2m,-\frac{b\sqrt{x}}{a},-\frac{d\sqrt{}}{c}\right.}{1+m}$$

input
```
Integrate[(a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x^m,x]
```

output
```
((a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x^(1 + m)*AppellF1[2 + 2*m, -p, -q, 3
 + 2*m, -((b*Sqrt[x])/a), -((d*Sqrt[x])/c)])/((1 + m)*(1 + (b*Sqrt[x])/a)^
p*(1 + (d*Sqrt[x])/c)^q)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1000, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q \, dx$$

$$\Big\downarrow \text{1000}$$

$$2\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{152}$$

$$2\left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \int \left(\frac{\sqrt{x}b}{a}+1\right)^p \left(c+d\sqrt{x}\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{152}$$

$$2\left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(\frac{d\sqrt{x}}{c}+1\right)^{-q} \int \left(\frac{\sqrt{x}b}{a}+1\right)^p \left(\frac{\sqrt{x}d}{c}+1\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x^{m+1}\left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(\frac{d\sqrt{x}}{c}+1\right)^{-q} \text{AppellF1}\left(2(m+1),-p,-q,2m+3,-\frac{b\sqrt{x}}{a},-\frac{d\sqrt{x}}{c}\right)}{m+1}$$

input
```
Int[(a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x^m,x]
```

output
```
((a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x^(1 + m)*AppellF1[2*(1 + m), -p, -q,
 3 + 2*m, -((b*Sqrt[x])/a), -((d*Sqrt[x])/c)])/((1 + m)*(1 + (b*Sqrt[x])/a
)^p*(1 + (d*Sqrt[x])/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 1000
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g*(m + 1) -
1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b
, c, d, m, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^{p}\left(c+d\sqrt{x}\right)^{q}x^{m}dx$$

input
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x)
```

output
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-2)]

Exception generated.

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**(1/2))**p*(c+d*x**(1/2))**q*x**m,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx = \int \left(b\sqrt{x}+a\right)^p \left(d\sqrt{x}+c\right)^q x^m\,dx$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x, algorithm="maxima")
```

output
```
integrate((b*sqrt(x) + a)^p*(d*sqrt(x) + c)^q*x^m, x)
```

Giac [F]

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx=\int \left(b\sqrt{x}+a\right)^p\left(d\sqrt{x}+c\right)^q x^m\,dx$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x, algorithm="giac")
```

output
```
integrate((b*sqrt(x) + a)^p*(d*sqrt(x) + c)^q*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx=\int x^m\,\left(a+b\,\sqrt{x}\right)^p\,\left(c+d\,\sqrt{x}\right)^q\,dx$$

input
```
int(x^m*(a + b*x^(1/2))^p*(c + d*x^(1/2))^q,x)
```

output
```
int(x^m*(a + b*x^(1/2))^p*(c + d*x^(1/2))^q, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m\,dx=\text{too large to display}$$

input
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*x^m,x)
```

output
```
(4*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*d**2*m*p +
2*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*d**2*p*q + 4
*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*m*p + 4*x*
*((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*m*q + 2*x**((
2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*p**2 + 2*x**((2*
m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*q**2 + 4*x**((2*m
+ 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c**2*m*q + 2*x**((2*m +
 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c**2*p*q - 4*x**m*(sqrt(
x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*c*d*m*p - 2*x**m*(sqrt(x)*d + c)**q*(
sqrt(x)*b + a)**p*a**2*c*d*p - 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**
p*a*b*c**2*m*q - 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c**2*q +
 8*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*m**2*x + 4*x**m*(sq
rt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*m*p*x + 8*x**m*(sqrt(x)*d + c)
**q*(sqrt(x)*b + a)**p*a*b*d**2*m*q*x + 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)
*b + a)**p*a*b*d**2*m*x + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b
*d**2*p*q*x + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*q**2*x
 + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*q*x + 8*x**m*(sqr
t(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c*d*m**2*x + 8*x**m*(sqrt(x)*d + c)
**q*(sqrt(x)*b + a)**p*b**2*c*d*m*p*x + 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)
*b + a)**p*b**2*c*d*m*q*x + 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**...
```

3.253 $\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx$$
$$= \frac{\left(a+b\sqrt{x}\right)^p \left(1+\frac{b\sqrt{x}}{a}\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(1+\frac{d\sqrt{x}}{c}\right)^{-q} (ex)^{1+m}\,\text{AppellF1}\left(2(1+m),-p,-q,3+2m,-\frac{b\sqrt{x}}{a},\right.}{e(1+m)}$$

output
```
(a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^(1+m)*AppellF1(2+2*m,-p,-q,3+2*m,-b*
x^(1/2)/a,-d*x^(1/2)/c)/e/(1+m)/((1+b*x^(1/2)/a)^p)/((1+d*x^(1/2)/c)^q)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx$$
$$= \frac{\left(a+b\sqrt{x}\right)^p \left(1+\frac{b\sqrt{x}}{a}\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(1+\frac{d\sqrt{x}}{c}\right)^{-q} x(ex)^m\,\text{AppellF1}\left(2+2m,-p,-q,3+2m,-\frac{b\sqrt{x}}{a},-\right.}{1+m}$$

input
```
Integrate[(a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*(e*x)^m,x]
```

output
```
((a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x*(e*x)^m*AppellF1[2 + 2*m, -p, -q, 3
 + 2*m, -((b*Sqrt[x])/a), -((d*Sqrt[x])/c)])/((1 + m)*(1 + (b*Sqrt[x])/a)^
p*(1 + (d*Sqrt[x])/c)^q)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1001, 1000, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q \, dx$$

$$\Big\downarrow \text{1001}$$

$$x^{-m}(ex)^m \int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^m dx$$

$$\Big\downarrow \text{1000}$$

$$2x^{-m}(ex)^m \int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{152}$$

$$2x^{-m}(ex)^m \left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \int \left(\frac{\sqrt{x}b}{a}+1\right)^p \left(c+d\sqrt{x}\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{152}$$

$$2x^{-m}(ex)^m \left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(\frac{d\sqrt{x}}{c}+1\right)^{-q} \int \left(\frac{\sqrt{x}b}{a}+1\right)^p \left(\frac{\sqrt{x}d}{c}+1\right)^q x^{\frac{1}{2}(2m+1)}d\sqrt{x}$$

$$\Big\downarrow \text{150}$$

$$\frac{x(ex)^m \left(a+b\sqrt{x}\right)^p \left(\frac{b\sqrt{x}}{a}+1\right)^{-p} \left(c+d\sqrt{x}\right)^q \left(\frac{d\sqrt{x}}{c}+1\right)^{-q} \text{AppellF1}\left(2(m+1),-p,-q,2m+3,-\frac{b\sqrt{x}}{a},-\frac{d\sqrt{x}}{c}\right)}{m+1}$$

input
```
Int[(a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*(e*x)^m,x]
```

output
```
((a + b*Sqrt[x])^p*(c + d*Sqrt[x])^q*x*(e*x)^m*AppellF1[2*(1 + m), -p, -q,
 3 + 2*m, -((b*Sqrt[x])/a), -((d*Sqrt[x])/c)])/((1 + m)*(1 + (b*Sqrt[x])/a
)^p*(1 + (d*Sqrt[x])/c)^q)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 1000
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g*(m + 1) -
1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b
, c, d, m, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1001
```
Int[((e_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))
^(q_), x_Symbol] :> Simp[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m])   I
nt[x^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, p, q
}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Maple [F]

$$\int \left(a+b\sqrt{x}\right)^{p} \left(c+d\sqrt{x}\right)^{q} \left(ex\right)^{m} dx$$

input
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x)
```

output
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\sqrt{x})^p (c+d\sqrt{x})^q (ex)^m \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [F(-2)]

Exception generated.

$$\int (a+b\sqrt{x})^p (c+d\sqrt{x})^q (ex)^m \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+b*x**(1/2))**p*(c+d*x**(1/2))**q*(e*x)**m,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+b\sqrt{x})^p (c+d\sqrt{x})^q (ex)^m \, dx = \int (ex)^m (b\sqrt{x}+a)^p (d\sqrt{x}+c)^q \, dx$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*(b*sqrt(x) + a)^p*(d*sqrt(x) + c)^q, x)
```

Giac [F]

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx=\int (ex)^m \left(b\sqrt{x}+a\right)^p \left(d\sqrt{x}+c\right)^q\,dx$$

input
```
integrate((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x, algorithm="giac")
```

output
```
integrate((e*x)^m*(b*sqrt(x) + a)^p*(d*sqrt(x) + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx=\int (e\,x)^m \left(a+b\,\sqrt{x}\right)^p \left(c+d\,\sqrt{x}\right)^q\,dx$$

input
```
int((e*x)^m*(a + b*x^(1/2))^p*(c + d*x^(1/2))^q,x)
```

output
```
int((e*x)^m*(a + b*x^(1/2))^p*(c + d*x^(1/2))^q, x)
```

Reduce [F]

$$\int \left(a+b\sqrt{x}\right)^p \left(c+d\sqrt{x}\right)^q (ex)^m\,dx=\text{too large to display}$$

input
```
int((a+b*x^(1/2))^p*(c+d*x^(1/2))^q*(e*x)^m,x)
```

output

```
(e**m*(4*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*d**2*
m*p + 2*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*d**2*p
*q + 4*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*m*p
+ 4*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*m*q + 2
*x**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*p**2 + 2*x
**((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*d*q**2 + 4*x**
((2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c**2*m*q + 2*x**(
(2*m + 1)/2)*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c**2*p*q - 4*x**m*
(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a**2*c*d*m*p - 2*x**m*(sqrt(x)*d + c
)**q*(sqrt(x)*b + a)**p*a**2*c*d*p - 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b
+ a)**p*a*b*c**2*m*q - 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*c*
*2*q + 8*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*m**2*x + 4*x*
*m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*m*p*x + 8*x**m*(sqrt(x)*
d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*m*q*x + 4*x**m*(sqrt(x)*d + c)**q*(s
qrt(x)*b + a)**p*a*b*d**2*m*x + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)*
*p*a*b*d**2*p*q*x + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*
q**2*x + 2*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*a*b*d**2*q*x + 8*x**
m*(sqrt(x)*d + c)**q*(sqrt(x)*b + a)**p*b**2*c*d*m**2*x + 8*x**m*(sqrt(x)*
d + c)**q*(sqrt(x)*b + a)**p*b**2*c*d*m*p*x + 4*x**m*(sqrt(x)*d + c)**q*(s
qrt(x)*b + a)**p*b**2*c*d*m*q*x + 4*x**m*(sqrt(x)*d + c)**q*(sqrt(x)*b ...
```

3.254 $\int x^2(a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 45

$$\int x^2(a+bx^n)(A+Bx^n)\,dx=\frac{1}{3}aAx^3+\frac{(Ab+aB)x^{3+n}}{3+n}+\frac{bBx^{3+2n}}{3+2n}$$

output
```
1/3*a*A*x^3+(A*b+B*a)*x^(3+n)/(3+n)+b*B*x^(3+2*n)/(3+2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)(A+Bx^n)\,dx=\frac{1}{3}aAx^3+\frac{(Ab+aB)x^{3+n}}{3+n}+\frac{bBx^{3+2n}}{3+2n}$$

input
```
Integrate[x^2*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^(3 + n))/(3 + n) + (b*B*x^(3 + 2*n))/(3 + 2*n
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)\left(A+Bx^n\right)dx$$

$$\Big\downarrow \; 950$$

$$\int \left(x^{n+2}(aB+Ab)+aAx^2+bBx^{2(n+1)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{n+3}(aB+Ab)}{n+3}+\frac{1}{3}aAx^3+\frac{bBx^{2n+3}}{2n+3}$$

input
```
Int[x^2*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^(3 + n))/(3 + n) + (b*B*x^(3 + 2*n))/(3 + 2*n
)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(Ab+Ba)x^3x^n}{3+n}+\frac{Bb\,x^3x^{2n}}{3+2n}+\frac{aA\,x^3}{3}$
norman	$\frac{(Ab+Ba)x^3\mathrm{e}^{n\ln(x)}}{3+n}+\frac{Bb\,x^3\mathrm{e}^{2n\ln(x)}}{3+2n}+\frac{aA\,x^3}{3}$
parallelrisch	$\frac{3B\,x^3x^{2n}bn+6A\,x^3x^nbn+2A\,x^3a\,n^2+9Bb\,x^3x^{2n}+6B\,x^3x^nan+9A\,x^3x^nb+9A\,x^3an+9B\,x^3x^na+9aA\,x^3}{3(3+n)(3+2n)}$
orering	$\frac{x^3(2n^2+15n+19)(a+b\,x^n)(A+B\,x^n)}{3(3+n)(3+2n)}-\frac{x^2(2+n)(2x(a+b\,x^n)(A+B\,x^n)+xb\,x^nn(A+B\,x^n)+x(a+b\,x^n)B\,x^nn)}{(3+n)(3+2n)}+\frac{x^3(2(}{}$

input
```
int(x^2*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(3+n)*x^3*x^n+B*b/(3+2*n)*x^3*(x^n)^2+1/3*a*A*x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int x^2(a+bx^n)\,(A+Bx^n)\;dx$$
$$=\frac{3\,(Bbn+3\,Bb)x^3x^{2\,n}+3\,(3\,Ba+3\,Ab+2\,(Ba+Ab)n)x^3x^n+(2\,Aan^2+9\,Aan+9\,Aa)x^3}{3\,(2\,n^2+9\,n+9)}$$

input
```
integrate(x^2*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
1/3*(3*(B*b*n + 3*B*b)*x^3*x^(2*n) + 3*(3*B*a + 3*A*b + 2*(B*a + A*b)*n)*x
^3*x^n + (2*A*a*n^2 + 9*A*a*n + 9*A*a)*x^3)/(2*n^2 + 9*n + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(37) = 74$.

Time = 0.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 6.09

$$\int x^2(a+bx^n)(A+Bx^n)\,dx$$
$$= \begin{cases} \frac{Aax^3}{3} + Ab\log(x) + Ba\log(x) - \frac{Bb}{3x^3} \\ \frac{Aax^3}{3} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + Bb\log(x) \\ \frac{2Aan^2x^3}{6n^2+27n+27} + \frac{9Aanx^3}{6n^2+27n+27} + \frac{9Aax^3}{6n^2+27n+27} + \frac{6Abnx^3x^n}{6n^2+27n+27} + \frac{9Abx^3x^n}{6n^2+27n+27} + \frac{6Banx^3x^n}{6n^2+27n+27} + \frac{9Bax^3x^n}{6n^2+27n+27} + \frac{3Bbnx^3x^{2n}}{6n^2+27n+2} \end{cases}$$

input
```
integrate(x**2*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Piecewise((A*a*x**3/3 + A*b*log(x) + B*a*log(x) - B*b/(3*x**3), Eq(n, -3))
, (A*a*x**3/3 + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + B*b*log(x), Eq(n, -3
/2)), (2*A*a*n**2*x**3/(6*n**2 + 27*n + 27) + 9*A*a*n*x**3/(6*n**2 + 27*n
+ 27) + 9*A*a*x**3/(6*n**2 + 27*n + 27) + 6*A*b*n*x**3*x**n/(6*n**2 + 27*n
 + 27) + 9*A*b*x**3*x**n/(6*n**2 + 27*n + 27) + 6*B*a*n*x**3*x**n/(6*n**2
+ 27*n + 27) + 9*B*a*x**3*x**n/(6*n**2 + 27*n + 27) + 3*B*b*n*x**3*x**(2*n
)/(6*n**2 + 27*n + 27) + 9*B*b*x**3*x**(2*n)/(6*n**2 + 27*n + 27), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int x^2(a+bx^n)(A+Bx^n)\,dx = \frac{1}{3}Aax^3 + \frac{Bbx^{2n+3}}{2n+3} + \frac{Bax^{n+3}}{n+3} + \frac{Abx^{n+3}}{n+3}$$

input
```
integrate(x^2*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
1/3*A*a*x^3 + B*b*x^(2*n + 3)/(2*n + 3) + B*a*x^(n + 3)/(n + 3) + A*b*x^(n
 + 3)/(n + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.38

$$\int x^2(a+bx^n)\,(A+Bx^n)\;dx$$
$$=\frac{3\,Bbnx^3x^{2\,n}+6\,Banx^3x^n+6\,Abnx^3x^n+2\,Aan^2x^3+9\,Bbx^3x^{2\,n}+9\,Bax^3x^n+9\,Abx^3x^n+9\,Aanx^3+9}{3\,(2\,n^2+9\,n+9)}$$

input
```
integrate(x^2*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
1/3*(3*B*b*n*x^3*x^(2*n) + 6*B*a*n*x^3*x^n + 6*A*b*n*x^3*x^n + 2*A*a*n^2*x
^3 + 9*B*b*x^3*x^(2*n) + 9*B*a*x^3*x^n + 9*A*b*x^3*x^n + 9*A*a*n*x^3 + 9*A
*a*x^3)/(2*n^2 + 9*n + 9)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)\,(A+Bx^n)\;dx=\frac{A\,a\,x^3}{3}+\frac{x^n\,x^3\,(A\,b+B\,a)}{n+3}+\frac{B\,b\,x^{2\,n}\,x^3}{2\,n+3}$$

input
```
int(x^2*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(A*a*x^3)/3 + (x^n*x^3*(A*b + B*a))/(n + 3) + (B*b*x^(2*n)*x^3)/(2*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int x^2(a+bx^n)(A+Bx^n)\,dx$$
$$=\frac{x^3(3x^{2n}b^2n+9x^{2n}b^2+12x^nabn+18x^nab+2a^2n^2+9a^2n+9a^2)}{6n^2+27n+27}$$

input
```
int(x^2*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(x**3*(3*x**(2*n)*b**2*n + 9*x**(2*n)*b**2 + 12*x**n*a*b*n + 18*x**n*a*b +
 2*a**2*n**2 + 9*a**2*n + 9*a**2))/(3*(2*n**2 + 9*n + 9))
```

3.255 $\int x(a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int x(a+bx^n)(A+Bx^n)\,dx=\frac{1}{2}aAx^2+\frac{bBx^{2(1+n)}}{2(1+n)}+\frac{(Ab+aB)x^{2+n}}{2+n}$$

output
```
1/2*a*A*x^2+b*B*x^(2+2*n)/(2+2*n)+(A*b+B*a)*x^(2+n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int x(a+bx^n)(A+Bx^n)\,dx=\frac{1}{2}x^2\left(aA+\frac{2(Ab+aB)x^n}{2+n}+\frac{bBx^{2n}}{1+n}\right)$$

input
```
Integrate[x*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(x^2*(a*A + (2*(A*b + a*B)*x^n)/(2 + n) + (b*B*x^(2*n))/(1 + n)))/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)(A+Bx^n)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n+1}(aB+Ab)+aAx+bBx^{2n+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{n+2}(aB+Ab)}{n+2}+\frac{1}{2}aAx^2+\frac{bBx^{2(n+1)}}{2(n+1)}$$

input
```
Int[x*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(a*A*x^2)/2 + (b*B*x^(2*(1 + n)))/(2*(1 + n)) + ((A*b + a*B)*x^(2 + n))/(2
 + n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result
risch	$\frac{(Ab+Ba)x^2x^n}{2+n}+\frac{aA\,x^2}{2}+\frac{Bb\,x^2x^{2n}}{2+2n}$
norman	$\frac{(Ab+Ba)x^2\mathrm{e}^{n\ln(x)}}{2+n}+\frac{aA\,x^2}{2}+\frac{Bb\,x^2\mathrm{e}^{2n\ln(x)}}{2+2n}$
parallelrisch	$\frac{B\,x^2x^{2n}bn+2A\,x^2x^n bn+A\,x^2a\,n^2+2Bb\,x^2x^{2n}+2B\,x^2x^n an+2A\,x^2x^n b+3A\,x^2an+2B\,x^2x^n a+2aA\,x^2}{2(2+n)(1+n)}$
orering	$\frac{x^2(7+2n)(a+b\,x^n)(A+B\,x^n)}{8+4n}-\frac{3x^2((a+b\,x^n)(A+B\,x^n)+b\,x^n n(A+B\,x^n)+(a+b\,x^n)B\,x^n n)}{4(2+n)}+\frac{x^3\left(\frac{b\,x^n n(A+B\,x^n)}{x}+(a+\right.}{}$

input
```
int(x*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(2+n)*x^2*x^n+1/2*a*A*x^2+1/2*B*b/(1+n)*x^2*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int x(a+bx^n)\left(A+Bx^n\right)\,dx$$
$$=\frac{(Bbn+2\,Bb)x^2x^{2\,n}+2\,(Ba+Ab+(Ba+Ab)n)x^2x^n+(Aan^2+3\,Aan+2\,Aa)x^2}{2\,(n^2+3\,n+2)}$$

input
```
integrate(x*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
1/2*((B*b*n + 2*B*b)*x^2*x^(2*n) + 2*(B*a + A*b + (B*a + A*b)*n)*x^2*x^n +
 (A*a*n^2 + 3*A*a*n + 2*A*a)*x^2)/(n^2 + 3*n + 2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(37) = 74$.

Time = 0.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 5.54

$$\int x(a+bx^n)\,(A+Bx^n)\;dx = \begin{cases} \frac{Aax^2}{2} + Ab\log{(x)} + Ba\log{(x)} - \frac{Bb}{2x^2} \\ \frac{Aax^2}{2} + Abx + Bax + Bb\log{(x)} \\ \frac{Aan^2x^2}{2n^2+6n+4} + \frac{3Aanx^2}{2n^2+6n+4} + \frac{2Aax^2}{2n^2+6n+4} + \frac{2Abnx^2x^n}{2n^2+6n+4} + \frac{2Abx^2x^n}{2n^2+6n+4} + \frac{2Banx^2x^n}{2n^2+6n+4} + \frac{2Bax^2x^n}{2n^2+6n+4} + \frac{Bbnx^2x^{2n}}{2n^2+6n+4} + \frac{2Bbx^2x^{2n}}{2n^2+6n+4} \end{cases}$$

input
```
integrate(x*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Piecewise((A*a*x**2/2 + A*b*log(x) + B*a*log(x) - B*b/(2*x**2), Eq(n, -2))
, (A*a*x**2/2 + A*b*x + B*a*x + B*b*log(x), Eq(n, -1)), (A*a*n**2*x**2/(2*
n**2 + 6*n + 4) + 3*A*a*n*x**2/(2*n**2 + 6*n + 4) + 2*A*a*x**2/(2*n**2 + 6
*n + 4) + 2*A*b*n*x**2*x**n/(2*n**2 + 6*n + 4) + 2*A*b*x**2*x**n/(2*n**2 +
 6*n + 4) + 2*B*a*n*x**2*x**n/(2*n**2 + 6*n + 4) + 2*B*a*x**2*x**n/(2*n**2
 + 6*n + 4) + B*b*n*x**2*x**(2*n)/(2*n**2 + 6*n + 4) + 2*B*b*x**2*x**(2*n)
/(2*n**2 + 6*n + 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x(a+bx^n)\,(A+Bx^n)\;dx = \frac{1}{2}\,Aax^2 + \frac{Bbx^{2\,n+2}}{2\,(n+1)} + \frac{Bax^{n+2}}{n+2} + \frac{Abx^{n+2}}{n+2}$$

input
```
integrate(x*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
1/2*A*a*x^2 + 1/2*B*b*x^(2*n + 2)/(n + 1) + B*a*x^(n + 2)/(n + 2) + A*b*x^
(n + 2)/(n + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(42) = 84$.

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

$$\int x(a + bx^n)(A + Bx^n)\, dx$$
$$= \frac{Bbnx^2x^{2n} + 2Banx^2x^n + 2Abnx^2x^n + Aan^2x^2 + 2Bbx^2x^{2n} + 2Bax^2x^n + 2Abx^2x^n + 3Aanx^2 + 2A\ldots}{2(n^2 + 3n + 2)}$$

input
```
integrate(x*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
1/2*(B*b*n*x^2*x^(2*n) + 2*B*a*n*x^2*x^n + 2*A*b*n*x^2*x^n + A*a*n^2*x^2 +
 2*B*b*x^2*x^(2*n) + 2*B*a*x^2*x^n + 2*A*b*x^2*x^n + 3*A*a*n*x^2 + 2*A*a*x
^2)/(n^2 + 3*n + 2)
```

Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int x(a + bx^n)(A + Bx^n)\, dx = \frac{A\, a\, x^2}{2} + \frac{x^n\, x^2\, (A\, b + B\, a)}{n + 2} + \frac{B\, b\, x^{2n}\, x^2}{2\, n + 2}$$

input
```
int(x*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(A*a*x^2)/2 + (x^n*x^2*(A*b + B*a))/(n + 2) + (B*b*x^(2*n)*x^2)/(2*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int x(a+bx^n)\left(A+Bx^n\right)\,dx = \frac{x^2(x^{2n}b^2n+2x^{2n}b^2+4x^nabn+4x^nab+a^2n^2+3a^2n+2a^2)}{2n^2+6n+4}$$

input
```
int(x*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(x**2*(x**(2*n)*b**2*n + 2*x**(2*n)*b**2 + 4*x**n*a*b*n + 4*x**n*a*b + a**
2*n**2 + 3*a**2*n + 2*a**2))/(2*(n**2 + 3*n + 2))
```

3.256 $\int (a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int (a+bx^n)(A+Bx^n)\,dx = aAx+\frac{(Ab+aB)x^{1+n}}{1+n}+\frac{bBx^{1+2n}}{1+2n}$$

output
```
a*A*x+(A*b+B*a)*x^(1+n)/(1+n)+b*B*x^(1+2*n)/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (a+bx^n)(A+Bx^n)\,dx = x\left(aA+\frac{(Ab+aB)x^n}{1+n}+\frac{bBx^{2n}}{1+2n}\right)$$

input
```
Integrate[(a + b*x^n)*(A + B*x^n),x]
```

output
```
x*(a*A + ((A*b + a*B)*x^n)/(1 + n) + (b*B*x^(2*n))/(1 + 2*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)(A+Bx^n)\,dx$$

$$\Big\downarrow \text{ 897}$$

$$\int \left(x^n(aB+Ab)+aA+bBx^{2n}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{n+1}(aB+Ab)}{n+1}+aAx+\frac{bBx^{2n+1}}{2n+1}$$

input
```
Int[(a + b*x^n)*(A + B*x^n),x]
```

output
```
a*A*x + ((A*b + a*B)*x^(1 + n))/(1 + n) + (b*B*x^(1 + 2*n))/(1 + 2*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result
risch	$aAx+\frac{(Ab+Ba)x\,x^{n}}{1+n}+\frac{Bbx\,x^{2n}}{1+2n}$
norman	$aAx+\frac{(Ab+Ba)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{Bbx\,\mathrm{e}^{2n\ln(x)}}{1+2n}$
parallelrisch	$\frac{Bx\,x^{2n}bn+2Ax\,x^{n}bn+2Axa\,n^{2}+Bbx\,x^{2n}+2Bx\,x^{n}an+Ax\,x^{n}b+3Axan+Bx\,x^{n}a+aAx}{(1+n)(1+2n)}$
orering	$x(a+b\,x^{n})\,(A+B\,x^{n})-\frac{3nx^{2}\left(\frac{b\,x^{n}n(A+B\,x^{n})}{x}+\frac{(a+b\,x^{n})B\,x^{n}n}{x}\right)}{2n^{2}+3n+1}+\frac{x^{3}\left(\frac{b\,x^{n}n^{2}(A+B\,x^{n})}{x^{2}}-\frac{b\,x^{n}n(A+B\,x^{n})}{x^{2}}+\frac{2x^{2}}{}\right.}{2n^{2}+}$

input
```
int((a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+(A*b+B*a)/(1+n)*x*x^n+B*b/(1+2*n)*x*(x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int(a+bx^{n})\,(A+Bx^{n})\;dx=\frac{(Bbn+Bb)xx^{2\,n}+(Ba+Ab+2\,(Ba+Ab)n)xx^{n}+(2\,Aan^{2}+3\,Aan+Aa)x}{2\,n^{2}+3\,n+1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
((B*b*n + B*b)*x*x^(2*n) + (B*a + A*b + 2*(B*a + A*b)*n)*x*x^n + (2*A*a*n^
2 + 3*A*a*n + A*a)*x)/(2*n^2 + 3*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(34) = 68$.

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 5.90

$$\int (a+bx^n)(A+Bx^n)\,dx$$
$$= \begin{cases} Aax + Ab\log(x) + Ba\log(x) - \frac{Bb}{x} \\ Aax + 2Ab\sqrt{x} + 2Ba\sqrt{x} + Bb\log(x) \\ \frac{2Aan^2x}{2n^2+3n+1} + \frac{3Aanx}{2n^2+3n+1} + \frac{Aax}{2n^2+3n+1} + \frac{2Abnxx^n}{2n^2+3n+1} + \frac{Abxx^n}{2n^2+3n+1} + \frac{2Banxx^n}{2n^2+3n+1} + \frac{Baxx^n}{2n^2+3n+1} + \frac{Bbnxx^{2n}}{2n^2+3n+1} + \frac{Bbxx^{2n}}{2n^2+3n+1} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n),x)
```

output
```
Piecewise((A*a*x + A*b*log(x) + B*a*log(x) - B*b/x, Eq(n, -1)), (A*a*x + 2
*A*b*sqrt(x) + 2*B*a*sqrt(x) + B*b*log(x), Eq(n, -1/2)), (2*A*a*n**2*x/(2*
n**2 + 3*n + 1) + 3*A*a*n*x/(2*n**2 + 3*n + 1) + A*a*x/(2*n**2 + 3*n + 1)
+ 2*A*b*n*x*x**n/(2*n**2 + 3*n + 1) + A*b*x*x**n/(2*n**2 + 3*n + 1) + 2*B*
a*n*x*x**n/(2*n**2 + 3*n + 1) + B*a*x*x**n/(2*n**2 + 3*n + 1) + B*b*n*x*x*
*(2*n)/(2*n**2 + 3*n + 1) + B*b*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int (a+bx^n)(A+Bx^n)\,dx = Aax + \frac{Bbx^{2n+1}}{2n+1} + \frac{Bax^{n+1}}{n+1} + \frac{Abx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
A*a*x + B*b*x^(2*n + 1)/(2*n + 1) + B*a*x^(n + 1)/(n + 1) + A*b*x^(n + 1)/
(n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int (a+bx^n)(A+Bx^n)\,dx = \frac{2\,Aan^2x + Bbnxx^{2\,n} + 2\,Banxx^n + 2\,Abnxx^n + 3\,Aanx + Bbxx^{2\,n} + Baxx^n + Abxx^n + Aax}{2\,n^2 + 3\,n + 1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
(2*A*a*n^2*x + B*b*n*x*x^(2*n) + 2*B*a*n*x*x^n + 2*A*b*n*x*x^n + 3*A*a*n*x
 + B*b*x*x^(2*n) + B*a*x*x^n + A*b*x*x^n + A*a*x)/(2*n^2 + 3*n + 1)
```

Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (a+bx^n)(A+Bx^n)\,dx = A\,a\,x + \frac{x\,x^n\,(A\,b+B\,a)}{n+1} + \frac{B\,b\,x\,x^{2\,n}}{2\,n+1}$$

input
```
int((A + B*x^n)*(a + b*x^n),x)
```

output
```
A*a*x + (x*x^n*(A*b + B*a))/(n + 1) + (B*b*x*x^(2*n))/(2*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int (a+bx^n)(A+Bx^n)\,dx = \frac{x(x^{2n}b^2n + x^{2n}b^2 + 4x^nabn + 2x^nab + 2a^2n^2 + 3a^2n + a^2)}{2n^2 + 3n + 1}$$

input
```
int((a+b*x^n)*(A+B*x^n),x)
```

output
```
(x*(x**(2*n)*b**2*n + x**(2*n)*b**2 + 4*x**n*a*b*n + 2*x**n*a*b + 2*a**2*n
**2 + 3*a**2*n + a**2))/(2*n**2 + 3*n + 1)
```

3.257 $\int \frac{(a+bx^n)(A+Bx^n)}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 34

$$\int \frac{(a+bx^n)(A+Bx^n)}{x} dx = \frac{(Ab+aB)x^n}{n} + \frac{bBx^{2n}}{2n} + aA\log(x)$$

output
```
(A*b+B*a)*x^n/n+1/2*b*B*x^(2*n)/n+a*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)(A+Bx^n)}{x} dx = \frac{x^n(2Ab+2aB+bBx^n)}{2n} + \frac{aA\log(x^n)}{n}$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x,x]
```

output
```
(x^n*(2*A*b + 2*a*B + b*B*x^n))/(2*n) + (a*A*Log[x^n])/n
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x} \, dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int x^{-n}(bx^n+a)(Bx^n+A)\,dx^n}{n}$$

$$\Big\downarrow \; 85$$

$$\frac{\int (aAx^{-n}+bBx^n+Ab+aB)\,dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^n(aB+Ab)+aA\log(x^n)+\frac{1}{2}bBx^{2n}}{n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x,x]
```

output
```
((A*b + a*B)*x^n + (b*B*x^(2*n))/2 + a*A*Log[x^n])/n
```

Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{B\,x^{2n} b}{2}+A\,x^{n} b+B\,x^{n} a+A a \ln\left(x^{n}\right)}{n}$	34
default	$\frac{\frac{B\,x^{2n} b}{2}+A\,x^{n} b+B\,x^{n} a+A a \ln\left(x^{n}\right)}{n}$	34
parallelrisch	$\frac{2 a A \ln\left(x\right) n+B\,x^{2n} b+2 A\,x^{n} b+2 B\,x^{n} a}{2 n}$	36
norman	$a A \ln\left(x\right)+\frac{\left(A b+B a\right) {\mathrm e}^{n \ln\left(x\right)}}{n}+\frac{B b\,{\mathrm e}^{2 n \ln\left(x\right)}}{2 n}$	37
risch	$a A \ln\left(x\right)+\frac{x^{n} A b}{n}+\frac{x^{n} B a}{n}+\frac{b B\,x^{2 n}}{2 n}$	37

input
```
int((a+b*x^n)*(A+B*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*B*(x^n)^2*b+A*x^n*b+B*x^n*a+A*a*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x}\,dx = \frac{2\,Aan\log\left(x\right)+Bbx^{2\,n}+2\,(Ba+Ab)x^n}{2\,n}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x,x, algorithm="fricas")
```

output
```
1/2*(2*A*a*n*log(x) + B*b*x^(2*n) + 2*(B*a + A*b)*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^n)(A+Bx^n)}{x}\,dx = \begin{cases} Aa\log(x) + \frac{Abx^n}{n} + \frac{Bax^n}{n} + \frac{Bbx^{2n}}{2n} & \text{for } n \neq 0 \\ (A+B)(a+b)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x,x)
```

output
```
Piecewise((A*a*log(x) + A*b*x**n/n + B*a*x**n/n + B*b*x**(2*n)/(2*n), Ne(n
, 0)), ((A + B)*(a + b)*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)(A+Bx^n)}{x}\,dx = Aa\log(x) + \frac{Bbx^{2n}}{2n} + \frac{Bax^n}{n} + \frac{Abx^n}{n}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x,x, algorithm="maxima")
```

output
```
A*a*log(x) + 1/2*B*b*x^(2*n)/n + B*a*x^n/n + A*b*x^n/n
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x}\,dx=\int \frac{(Bx^n+A)(bx^n+a)}{x}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)(A+Bx^n)}{x}\,dx=\frac{x^n\,(A\,b+B\,a)}{n}+A\,a\,\ln(x)+\frac{B\,b\,x^{2\,n}}{2\,n}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x,x)
```

output
```
(x^n*(A*b + B*a))/n + A*a*log(x) + (B*b*x^(2*n))/(2*n)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^n)(A+Bx^n)}{x}\,dx=\frac{x^{2n}b^2+4x^nab+2\log(x)\,a^2n}{2n}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x,x)
```

output
```
(x**(2*n)*b**2 + 4*x**n*a*b + 2*log(x)*a**2*n)/(2*n)
```

3.258 $\int \frac{(a+bx^n)(A+Bx^n)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2} dx = -\frac{aA}{x} - \frac{(Ab+aB)x^{-1+n}}{1-n} - \frac{bBx^{-1+2n}}{1-2n}$$

output
```
-a*A/x-(A*b+B*a)*x^(-1+n)/(1-n)-b*B*x^(-1+2*n)/(1-2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2} dx = \frac{-aA + \frac{(Ab+aB)x^n}{-1+n} + \frac{bBx^{2n}}{-1+2n}}{x}$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^2,x]
```

output
```
(-(a*A) + ((A*b + a*B)*x^n)/(-1 + n) + (b*B*x^(2*n))/(-1 + 2*n))/x
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n-2}(aB+Ab)+\frac{aA}{x^2}+bBx^{2(n-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^{n-1}(aB+Ab)}{1-n}-\frac{aA}{x}-\frac{bBx^{2n-1}}{1-2n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^2,x]
```

output
```
-((a*A)/x) - ((A*b + a*B)*x^(-1 + n))/(1 - n) - (b*B*x^(-1 + 2*n))/(1 - 2*
n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{(Ab+Ba)e^{n\ln(x)}}{-1+n}+\frac{Bb\,e^{2n\ln(x)}}{-1+2n}-Aa}{x}$
risch	$-\frac{aA}{x}+\frac{Bb\,x^{2n}}{(-1+2n)x}+\frac{(Ab+Ba)x^{n}}{(-1+n)x}$
parallelrisch	$\frac{bnB\,x^{2n}+2A\,x^{n}bn-2Aa\,n^{2}-B\,x^{2n}b+2B\,x^{n}an-A\,x^{n}b+3Aan-B\,x^{n}a-Aa}{x(-1+2n)(-1+n)}$
orering	$-\frac{(2n-7)(a+b\,x^{n})(A+B\,x^{n})}{x(-1+2n)}+\frac{3x^{2}(-2+n)\left(\frac{b\,x^{n}n(A+B\,x^{n})}{x^{3}}+\frac{(a+b\,x^{n})B\,x^{n}n}{x^{3}}-\frac{2(a+b\,x^{n})(A+B\,x^{n})}{x^{3}}\right)}{2n^{2}-3n+1}-\frac{x^{3}\left(\frac{b\,x^{n}n^{2}(A+B}{x^{4}}\right.}{}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
((A*b+B*a)/(-1+n)*exp(n*ln(x))+B*b/(-1+2*n)*exp(n*ln(x))^2-A*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2}\,dx = -\frac{2\,Aan^2-3\,Aan+Aa-(Bbn-Bb)x^{2\,n}+(Ba+Ab-2\,(Ba+Ab)n)x^n}{(2\,n^2-3\,n+1)x}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^2,x, algorithm="fricas")
```

output
```
-(2*A*a*n^2 - 3*A*a*n + A*a - (B*b*n - B*b)*x^(2*n) + (B*a + A*b - 2*(B*a
+ A*b)*n)*x^n)/((2*n^2 - 3*n + 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(36) = 72$.

Time = 0.35 (sec) , antiderivative size = 248, normalized size of antiderivative = 5.28

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2}\,dx$$
$$= \begin{cases} -\frac{Aa}{x} - \frac{2Ab}{\sqrt{x}} - \frac{2Ba}{\sqrt{x}} + Bb\log(x) \\ -\frac{Aa}{x} + Ab\log(x) + Ba\log(x) + Bbx \\ -\frac{2Aan^2}{2n^2x-3nx+x} + \frac{3Aan}{2n^2x-3nx+x} - \frac{Aa}{2n^2x-3nx+x} + \frac{2Abnx^n}{2n^2x-3nx+x} - \frac{Abx^n}{2n^2x-3nx+x} + \frac{2Banx^n}{2n^2x-3nx+x} - \frac{Bax^n}{2n^2x-3nx+x} + \frac{Bbr}{2n^2x-} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**2,x)
```

output
```
Piecewise((-A*a/x - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + B*b*log(x), Eq(n, 1/2)
), (-A*a/x + A*b*log(x) + B*a*log(x) + B*b*x, Eq(n, 1)), (-2*A*a*n**2/(2*n
**2*x - 3*n*x + x) + 3*A*a*n/(2*n**2*x - 3*n*x + x) - A*a/(2*n**2*x - 3*n*
x + x) + 2*A*b*n*x**n/(2*n**2*x - 3*n*x + x) - A*b*x**n/(2*n**2*x - 3*n*x
+ x) + 2*B*a*n*x**n/(2*n**2*x - 3*n*x + x) - B*a*x**n/(2*n**2*x - 3*n*x +
x) + B*b*n*x**(2*n)/(2*n**2*x - 3*n*x + x) - B*b*x**(2*n)/(2*n**2*x - 3*n*
x + x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^2}\,dx=\int \frac{(Bx^n+A)(bx^n+a)}{x^2}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^2}\,dx=\frac{x^n\,(A\,b+B\,a)}{x\,(n-1)}-\frac{A\,a}{x}+\frac{B\,b\,x^{2\,n}}{x\,(2\,n-1)}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^2,x)
```

output
```
(x^n*(A*b + B*a))/(x*(n - 1)) - (A*a)/x + (B*b*x^(2*n))/(x*(2*n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^2}\,dx=\frac{x^{2n}b^2n-x^{2n}b^2+4x^nabn-2x^nab-2a^2n^2+3a^2n-a^2}{x\,(2n^2-3n+1)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^2,x)
```

output
```
(x**(2*n)*b**2*n - x**(2*n)*b**2 + 4*x**n*a*b*n - 2*x**n*a*b - 2*a**2*n**2
 + 3*a**2*n - a**2)/(x*(2*n**2 - 3*n + 1))
```

3.259 $\int \frac{(a+bx^n)(A+Bx^n)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3} dx = -\frac{aA}{2x^2} - \frac{bBx^{-2(1-n)}}{2(1-n)} - \frac{(Ab+aB)x^{-2+n}}{2-n}$$

output
```
-1/2*a*A/x^2-1/2*b*B/(1-n)/(x^(2-2*n))-(A*b+B*a)*x^(-2+n)/(2-n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3} dx = \frac{-aA + \frac{2(Ab+aB)x^n}{-2+n} + \frac{bBx^{2n}}{-1+n}}{2x^2}$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^3,x]
```

output
```
(-(a*A) + (2*(A*b + a*B)*x^n)/(-2 + n) + (b*B*x^(2*n))/(-1 + n))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(x^{n-3}(aB+Ab)+\frac{aA}{x^3}+bBx^{2n-3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^{n-2}(aB+Ab)}{2-n}-\frac{aA}{2x^2}-\frac{bBx^{-2(1-n)}}{2(1-n)}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (b*B)/(2*(1 - n)*x^(2*(1 - n))) - ((A*b + a*B)*x^(-2 + n)
)/(2 - n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result
norman	$\frac{\frac{(Ab+Ba)\mathrm{e}^{n\ln(x)}}{-2+n}-\frac{Aa}{2}+\frac{Bb\,\mathrm{e}^{2n\ln(x)}}{-2+2n}}{x^2}$
risch	$-\frac{aA}{2x^2}+\frac{Bb\,x^{2n}}{2(-1+n)x^2}+\frac{(Ab+Ba)x^n}{(-2+n)x^2}$
parallelrisch	$\frac{bnB\,x^{2n}+2A\,x^n bn-Aa\,n^2-2B\,x^{2n}b+2B\,x^n an-2A\,x^n b+3Aan-2B\,x^n a-2Aa}{2x^2(-1+n)(-2+n)}$
orering	$-\frac{(2n^2-15n+19)(a+b\,x^n)(A+B\,x^n)}{4x^2(-1+n)(-2+n)}+\frac{3x^2(-3+n)\left(\frac{b\,x^n n(A+B\,x^n)}{x^4}+\frac{(a+b\,x^n)B\,x^n n}{x^4}-\frac{3(a+b\,x^n)(A+B\,x^n)}{x^4}\right)}{4(-1+n)(-2+n)}-\frac{x^3\left(\frac{b\,x^n}{}\right.}{}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
((A*b+B*a)/(-2+n)*exp(n*ln(x))-1/2*A*a+1/2*B*b/(-1+n)*exp(n*ln(x))^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = -\frac{Aan^2-3\,Aan+2\,Aa-(Bbn-2\,Bb)x^{2\,n}+2\,(Ba+Ab-(Ba+Ab)n)x^n}{2\,(n^2-3\,n+2)x^2}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(A*a*n^2 - 3*A*a*n + 2*A*a - (B*b*n - 2*B*b)*x^(2*n) + 2*(B*a + A*b -
 (B*a + A*b)*n)*x^n)/((n^2 - 3*n + 2)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(39) = 78$.

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.91

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = \begin{cases} -\frac{Aa}{2x^2} - \frac{Ab}{x} - \frac{Ba}{x} + Bb\log(x) \\ -\frac{Aa}{2x^2} + Ab\log(x) + Ba\log(x) + \frac{Bbx^2}{2} \\ -\frac{Aan^2}{2n^2x^2-6nx^2+4x^2} + \frac{3Aan}{2n^2x^2-6nx^2+4x^2} - \frac{2Aa}{2n^2x^2-6nx^2+4x^2} + \frac{2Abnx^n}{2n^2x^2-6nx^2+4x^2} - \frac{2Abx^n}{2n^2x^2-6nx^2+4x^2} + \frac{2Banx^n}{2n^2x^2-6nx^2+4x^2} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**3,x)
```

output
```
Piecewise((-A*a/(2*x**2) - A*b/x - B*a/x + B*b*log(x), Eq(n, 1)), (-A*a/(2
*x**2) + A*b*log(x) + B*a*log(x) + B*b*x**2/2, Eq(n, 2)), (-A*a*n**2/(2*n*
*2*x**2 - 6*n*x**2 + 4*x**2) + 3*A*a*n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) -
 2*A*a/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) + 2*A*b*n*x**n/(2*n**2*x**2 - 6*n
*x**2 + 4*x**2) - 2*A*b*x**n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) + 2*B*a*n*x
**n/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) - 2*B*a*x**n/(2*n**2*x**2 - 6*n*x**2
 + 4*x**2) + B*b*n*x**(2*n)/(2*n**2*x**2 - 6*n*x**2 + 4*x**2) - 2*B*b*x**(
2*n)/(2*n**2*x**2 - 6*n*x**2 + 4*x**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)}{x^3}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = \frac{x^n\,(A\,b+B\,a)}{x^2\,(n-2)} - \frac{A\,a}{2\,x^2} + \frac{B\,b\,x^{2\,n}}{x^2\,(2\,n-2)}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^3,x)
```

output
```
(x^n*(A*b + B*a))/(x^2*(n - 2)) - (A*a)/(2*x^2) + (B*b*x^(2*n))/(x^2*(2*n
- 2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^3}\,dx = \frac{x^{2n}b^2n - 2x^{2n}b^2 + 4x^nabn - 4x^nab - a^2n^2 + 3a^2n - 2a^2}{2x^2\,(n^2-3n+2)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^3,x)
```

output
```
(x**(2*n)*b**2*n - 2*x**(2*n)*b**2 + 4*x**n*a*b*n - 4*x**n*a*b - a**2*n**2
 + 3*a**2*n - 2*a**2)/(2*x**2*(n**2 - 3*n + 2))
```

3.260 $\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = -\frac{aA}{3x^3} - \frac{(Ab+aB)x^{-3+n}}{3-n} - \frac{bBx^{-3+2n}}{3-2n}$$

output
```
-1/3*a*A/x^3-(A*b+B*a)*x^(-3+n)/(3-n)-b*B*x^(-3+2*n)/(3-2*n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = \frac{-aA + \frac{3(Ab+aB)x^n}{-3+n} + \frac{3bBx^{2n}}{-3+2n}}{3x^3}$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^4,x]
```

output
```
(-(a*A) + (3*(A*b + a*B)*x^n)/(-3 + n) + (3*b*B*x^(2*n))/(-3 + 2*n))/(3*x^
3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n-4}(aB+Ab)+\frac{aA}{x^4}+bBx^{2(n-2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x^{n-3}(aB+Ab)}{3-n}-\frac{aA}{3x^3}-\frac{bBx^{2n-3}}{3-2n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - ((A*b + a*B)*x^(-3 + n))/(3 - n) - (b*B*x^(-3 + 2*n))/(3
- 2*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{(Ab+Ba)\mathrm{e}^{n\ln(x)}}{-3+n}+\frac{Bb\,\mathrm{e}^{2n\ln(x)}}{-3+2n}-\frac{Aa}{3}}{x^3}$
risch	$-\frac{aA}{3x^3}+\frac{Bb\,x^{2n}}{(-3+2n)x^3}+\frac{(Ab+Ba)x^n}{(-3+n)x^3}$
parallelrisch	$\frac{3bnB\,x^{2n}+6A\,x^n bn-2Aa\,n^2-9B\,x^{2n}b+6B\,x^n an-9A\,x^n b+9Aan-9B\,x^n a-9Aa}{3x^3(-3+2n)(-3+n)}$
orering	$-\frac{(2n^2-21n+37)(a+b\,x^n)(A+B\,x^n)}{3x^3(-3+2n)(-3+n)}+\frac{x^2(-4+n)\left(\frac{b\,x^n n(A+B\,x^n)}{x^5}+\frac{(a+b\,x^n)B\,x^n n}{x^5}-\frac{4(a+b\,x^n)(A+B\,x^n)}{x^5}\right)}{(-3+2n)(-3+n)}-\frac{x^3\left(\frac{b\,x^n n}{}\right.}{}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^4,x,method=_RETURNVERBOSE)
```

output
```
((A*b+B*a)/(-3+n)*exp(n*ln(x))+B*b/(-3+2*n)*exp(n*ln(x))^2-1/3*A*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = -\frac{2\,Aan^2-9\,Aan+9\,Aa-3\,(Bbn-3\,Bb)x^{2\,n}+3\,(3\,Ba+3\,Ab-2\,(Ba+Ab)n)x^n}{3\,(2\,n^2-9\,n+9)x^3}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*A*a*n^2 - 9*A*a*n + 9*A*a - 3*(B*b*n - 3*B*b)*x^(2*n) + 3*(3*B*a +
 3*A*b - 2*(B*a + A*b)*n)*x^n)/((2*n^2 - 9*n + 9)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(39) = 78$.

Time = 0.55 (sec) , antiderivative size = 332, normalized size of antiderivative = 6.78

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = \begin{cases} -\frac{Aa}{3x^3} - \frac{2Ab}{3x^{\frac{3}{2}}} - \frac{2Ba}{3x^{\frac{3}{2}}} + Bb\log(x) \\ -\frac{Aa}{3x^3} + Ab\log(x) + Ba\log(x) + \frac{Bbx^3}{3} \\ -\frac{2Aan^2}{6n^2x^3-27nx^3+27x^3} + \frac{9Aan}{6n^2x^3-27nx^3+27x^3} - \frac{9Aa}{6n^2x^3-27nx^3+27x^3} + \frac{6Abnx^n}{6n^2x^3-27nx^3+27x^3} - \frac{9Abx^n}{6n^2x^3-27nx^3+27x^3} + \frac{6B}{6n^2x^3-2} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**4,x)
```

output
```
Piecewise((-A*a/(3*x**3) - 2*A*b/(3*x**(3/2)) - 2*B*a/(3*x**(3/2)) + B*b*l
og(x), Eq(n, 3/2)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3,
 Eq(n, 3)), (-2*A*a*n**2/(6*n**2*x**3 - 27*n*x**3 + 27*x**3) + 9*A*a*n/(6*
n**2*x**3 - 27*n*x**3 + 27*x**3) - 9*A*a/(6*n**2*x**3 - 27*n*x**3 + 27*x**
3) + 6*A*b*n*x**n/(6*n**2*x**3 - 27*n*x**3 + 27*x**3) - 9*A*b*x**n/(6*n**2
*x**3 - 27*n*x**3 + 27*x**3) + 6*B*a*n*x**n/(6*n**2*x**3 - 27*n*x**3 + 27*
x**3) - 9*B*a*x**n/(6*n**2*x**3 - 27*n*x**3 + 27*x**3) + 3*B*b*n*x**(2*n)/
(6*n**2*x**3 - 27*n*x**3 + 27*x**3) - 9*B*b*x**(2*n)/(6*n**2*x**3 - 27*n*x
**3 + 27*x**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-4>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)(bx^n+a)}{x^4}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^4}\,dx = \frac{x^n\,(A\,b+B\,a)}{x^3\,(n-3)} - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^{2\,n}}{x^3\,(2\,n-3)}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^4,x)
```

output
```
(x^n*(A*b + B*a))/(x^3*(n - 3)) - (A*a)/(3*x^3) + (B*b*x^(2*n))/(x^3*(2*n
- 3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{(a + bx^n)(A + Bx^n)}{x^4} \, dx$$
$$= \frac{3x^{2n}b^2n - 9x^{2n}b^2 + 12x^n abn - 18x^n ab - 2a^2n^2 + 9a^2n - 9a^2}{3x^3 (2n^2 - 9n + 9)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^4,x)
```

output
```
(3*x**(2*n)*b**2*n - 9*x**(2*n)*b**2 + 12*x**n*a*b*n - 18*x**n*a*b - 2*a**
2*n**2 + 9*a**2*n - 9*a**2)/(3*x**3*(2*n**2 - 9*n + 9))
```

3.261 $\int x^2(a+bx^n)^2 (A+Bx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int x^2(a+bx^n)^2 (A+Bx^n) \, dx = \frac{1}{3}a^2Ax^3 + \frac{b^2Bx^{3(1+n)}}{3(1+n)} + \frac{a(2Ab+aB)x^{3+n}}{3+n} + \frac{b(Ab+2aB)x^{3+2n}}{3+2n}$$

output
```
1/3*a^2*A*x^3+b^2*B*x^(3+3*n)/(3+3*n)+a*(2*A*b+B*a)*x^(3+n)/(3+n)+b*(A*b+2
*B*a)*x^(3+2*n)/(3+2*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^n)^2 (A+Bx^n) \, dx = \frac{1}{3}x^3\left(a^2A + \frac{3a(2Ab+aB)x^n}{3+n} + \frac{3b(Ab+2aB)x^{2n}}{3+2n} + \frac{b^2Bx^{3n}}{1+n}\right)$$

input
```
Integrate[x^2*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(x^3*(a^2*A + (3*a*(2*A*b + a*B)*x^n)/(3 + n) + (3*b*(A*b + 2*a*B)*x^(2*n)
)/(3 + 2*n) + (b^2*B*x^(3*n))/(1 + n)))/3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)^2(A+Bx^n)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^2+bx^{2(n+1)}(2aB+Ab)+ax^{n+2}(aB+2Ab)+b^2Bx^{3n+2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{ax^{n+3}(aB+2Ab)}{n+3}+\frac{bx^{2n+3}(2aB+Ab)}{2n+3}+\frac{b^2Bx^{3(n+1)}}{3(n+1)}$$

input
```
Int[x^2*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(a^2*A*x^3)/3 + (b^2*B*x^(3*(1 + n)))/(3*(1 + n)) + (a*(2*A*b + a*B)*x^(3
+ n))/(3 + n) + (b*(A*b + 2*a*B)*x^(3 + 2*n))/(3 + 2*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
risch	$\frac{a(2Ab+Ba)x^3x^n}{3+n}+\frac{b(Ab+2Ba)x^3x^{2n}}{3+2n}+\frac{a^2A\,x^3}{3}+\frac{b^2B\,x^3x^{3n}}{3+3n}$
norman	$\frac{a(2Ab+Ba)x^3\mathrm{e}^{n\ln(x)}}{3+n}+\frac{b(Ab+2Ba)x^3\mathrm{e}^{2n\ln(x)}}{3+2n}+\frac{a^2A\,x^3}{3}+\frac{b^2B\,x^3\mathrm{e}^{3n\ln(x)}}{3+3n}$
parallelrisch	$2B\,x^3x^{3n}b^2n^2+3A\,x^3x^{2n}b^2n^2+9B\,x^3x^{3n}b^2n+6B\,x^3x^{2n}ab\,n^2+12A\,x^3x^{2n}b^2n+12A\,x^3x^n ab\,n^2+2A\,x^3a^2n^3+9b^2B\,x^3x^{3n}+$
orering	$\frac{x^3(6n^2+49n+65)(a+b\,x^n)^2(A+B\,x^n)}{18n^2+81n+81}-\frac{x^2(11n+25)\left(2x(a+b\,x^n)^2(A+B\,x^n)+2x(a+b\,x^n)(A+B\,x^n)b\,x^n n+x(a+b\,x^n)^2\right.}{9(2n^2+9n+9)}$

input
```
int(x^2*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*(2*A*b+B*a)/(3+n)*x^3*x^n+b*(A*b+2*B*a)/(3+2*n)*x^3*(x^n)^2+1/3*a^2*A*x^
3+1/3*b^2*B/(1+n)*x^3*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.49

$$\int x^2(a+bx^n)^2\left(A+Bx^n\right)\,dx$$
$$=\frac{(2\,Bb^2n^2+9\,Bb^2n+9\,Bb^2)x^3x^{3\,n}+3\,(6\,Bab+3\,Ab^2+(2\,Bab+Ab^2)n^2+4\,(2\,Bab+Ab^2)n)x^3x^{2\,n}+3}{3\,(2\,n^3+1}$$

input
```
integrate(x^2*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
1/3*((2*B*b^2*n^2 + 9*B*b^2*n + 9*B*b^2)*x^3*x^(3*n) + 3*(6*B*a*b + 3*A*b^
2 + (2*B*a*b + A*b^2)*n^2 + 4*(2*B*a*b + A*b^2)*n)*x^3*x^(2*n) + 3*(3*B*a^
2 + 6*A*a*b + 2*(B*a^2 + 2*A*a*b)*n^2 + 5*(B*a^2 + 2*A*a*b)*n)*x^3*x^n + (
2*A*a^2*n^3 + 11*A*a^2*n^2 + 18*A*a^2*n + 9*A*a^2)*x^3)/(2*n^3 + 11*n^2 +
18*n + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(66) = 132$.

Time = 0.69 (sec) , antiderivative size = 770, normalized size of antiderivative = 10.13

$$\int x^2(a+bx^n)^2\,(A+Bx^n)\;dx$$
$$= \begin{cases} \frac{Aa^2x^3}{3} + 2Aab\log(x) - \frac{Ab^2}{3x^3} + Ba^2\log(x) - \frac{2Bab}{3x^3} - \frac{Bb^2}{6x^6} \\ \frac{Aa^2x^3}{3} + \frac{4Aabx^{\frac{3}{2}}}{3} + Ab^2\log(x) + \frac{2Ba^2x^{\frac{3}{2}}}{3} + 2Bab\log(x) - \frac{2Bb^2}{3x^{\frac{3}{2}}} \\ \frac{Aa^2x^3}{3} + Aabx^2 + Ab^2x + \frac{Ba^2x^2}{2} + 2Babx + Bb^2\log(x) \\ \frac{2Aa^2n^3x^3}{6n^3+33n^2+54n+27} + \frac{11Aa^2n^2x^3}{6n^3+33n^2+54n+27} + \frac{18Aa^2nx^3}{6n^3+33n^2+54n+27} + \frac{9Aa^2x^3}{6n^3+33n^2+54n+27} + \frac{12Aabn^2x^3x^n}{6n^3+33n^2+54n+27} + \frac{30Aabnx^3x}{6n^3+33n^2+54} \end{cases}$$

input
```
integrate(x**2*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Piecewise((A*a**2*x**3/3 + 2*A*a*b*log(x) - A*b**2/(3*x**3) + B*a**2*log(x
) - 2*B*a*b/(3*x**3) - B*b**2/(6*x**6), Eq(n, -3)), (A*a**2*x**3/3 + 4*A*a
*b*x**(3/2)/3 + A*b**2*log(x) + 2*B*a**2*x**(3/2)/3 + 2*B*a*b*log(x) - 2*B
*b**2/(3*x**(3/2)), Eq(n, -3/2)), (A*a**2*x**3/3 + A*a*b*x**2 + A*b**2*x +
 B*a**2*x**2/2 + 2*B*a*b*x + B*b**2*log(x), Eq(n, -1)), (2*A*a**2*n**3*x**
3/(6*n**3 + 33*n**2 + 54*n + 27) + 11*A*a**2*n**2*x**3/(6*n**3 + 33*n**2 +
 54*n + 27) + 18*A*a**2*n*x**3/(6*n**3 + 33*n**2 + 54*n + 27) + 9*A*a**2*x
**3/(6*n**3 + 33*n**2 + 54*n + 27) + 12*A*a*b*n**2*x**3*x**n/(6*n**3 + 33*
n**2 + 54*n + 27) + 30*A*a*b*n*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) +
18*A*a*b*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) + 3*A*b**2*n**2*x**3*x**
(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 12*A*b**2*n*x**3*x**(2*n)/(6*n**3 +
 33*n**2 + 54*n + 27) + 9*A*b**2*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n +
27) + 6*B*a**2*n**2*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) + 15*B*a**2*n
*x**3*x**n/(6*n**3 + 33*n**2 + 54*n + 27) + 9*B*a**2*x**3*x**n/(6*n**3 + 3
3*n**2 + 54*n + 27) + 6*B*a*b*n**2*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n
+ 27) + 24*B*a*b*n*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 18*B*a*b
*x**3*x**(2*n)/(6*n**3 + 33*n**2 + 54*n + 27) + 2*B*b**2*n**2*x**3*x**(3*n
)/(6*n**3 + 33*n**2 + 54*n + 27) + 9*B*b**2*n*x**3*x**(3*n)/(6*n**3 + 33*n
**2 + 54*n + 27) + 9*B*b**2*x**3*x**(3*n)/(6*n**3 + 33*n**2 + 54*n + 27),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.26

$$\int x^2(a+bx^n)^2\,(A+Bx^n)\;dx=\frac{1}{3}\,Aa^2x^3+\frac{Bb^2x^{3\,n+3}}{3\,(n+1)}+\frac{2\,Babx^{2\,n+3}}{2\,n+3}+\frac{Ab^2x^{2\,n+3}}{2\,n+3}+\frac{Ba^2x^{n+3}}{n+3}+\frac{2\,Aabx^{n+3}}{n+3}$$

input
```
integrate(x^2*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
1/3*A*a^2*x^3 + 1/3*B*b^2*x^(3*n + 3)/(n + 1) + 2*B*a*b*x^(2*n + 3)/(2*n +
 3) + A*b^2*x^(2*n + 3)/(2*n + 3) + B*a^2*x^(n + 3)/(n + 3) + 2*A*a*b*x^(n
 + 3)/(n + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(72) = 144$.

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int x^2(a+bx^n)^2(A+Bx^n)\,dx$$
$$= \frac{2\,Bb^2n^2x^3x^{3\,n}+6\,Babn^2x^3x^{2\,n}+3\,Ab^2n^2x^3x^{2\,n}+6\,Ba^2n^2x^3x^n+12\,Aabn^2x^3x^n+2\,Aa^2n^3x^3+9\,Bb^2nx}{}$$

input
```
integrate(x^2*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
1/3*(2*B*b^2*n^2*x^3*x^(3*n) + 6*B*a*b*n^2*x^3*x^(2*n) + 3*A*b^2*n^2*x^3*x
^(2*n) + 6*B*a^2*n^2*x^3*x^n + 12*A*a*b*n^2*x^3*x^n + 2*A*a^2*n^3*x^3 + 9*
B*b^2*n*x^3*x^(3*n) + 24*B*a*b*n*x^3*x^(2*n) + 12*A*b^2*n*x^3*x^(2*n) + 15
*B*a^2*n*x^3*x^n + 30*A*a*b*n*x^3*x^n + 11*A*a^2*n^2*x^3 + 9*B*b^2*x^3*x^(
3*n) + 18*B*a*b*x^3*x^(2*n) + 9*A*b^2*x^3*x^(2*n) + 9*B*a^2*x^3*x^n + 18*A
*a*b*x^3*x^n + 18*A*a^2*n*x^3 + 9*A*a^2*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int x^2(a+bx^n)^2(A+Bx^n)\,dx = \frac{A\,a^2\,x^3}{3} + \frac{x^{2\,n}\,x^3\,(A\,b^2+2\,B\,a\,b)}{2\,n+3} + \frac{x^n\,x^3\,(B\,a^2+2\,A\,b\,a)}{n+3} + \frac{B\,b^2\,x^{3\,n}\,x^3}{3\,n+3}$$

input
```
int(x^2*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(A*a^2*x^3)/3 + (x^(2*n)*x^3*(A*b^2 + 2*B*a*b))/(2*n + 3) + (x^n*x^3*(B*a^
2 + 2*A*a*b))/(n + 3) + (B*b^2*x^(3*n)*x^3)/(3*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int x^2(a+bx^n)^2\,(A+Bx^n)\;dx$$
$$=\frac{x^3(2x^{3n}b^3n^2+9x^{3n}b^3n+9x^{3n}b^3+9x^{2n}a\,b^2n^2+36x^{2n}a\,b^2n+27x^{2n}a\,b^2+18x^na^2b\,n^2+45x^na^2bn+27x}{6n^3+33n^2+54n+27}$$

input
```
int(x^2*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(x**3*(2*x**(3*n)*b**3*n**2 + 9*x**(3*n)*b**3*n + 9*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 36*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 45*x**n*a**2*b*n + 27*x**n*a**2*b + 2*a**3*n**3 + 11*a**3*n**2
+ 18*a**3*n + 9*a**3))/(3*(2*n**3 + 11*n**2 + 18*n + 9))
```

3.262 $\int x(a+bx^n)^2\,(A+Bx^n)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 76

$$\int x(a+bx^n)^2\,(A+Bx^n)\,dx = \frac{1}{2}a^2Ax^2+\frac{b(Ab+2aB)x^{2(1+n)}}{2(1+n)}+\frac{a(2Ab+aB)x^{2+n}}{2+n}+\frac{b^2Bx^{2+3n}}{2+3n}$$

output
```
1/2*a^2*A*x^2+b*(A*b+2*B*a)*x^(2+2*n)/(2+2*n)+a*(2*A*b+B*a)*x^(2+n)/(2+n)+
b^2*B*x^(2+3*n)/(2+3*n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int x(a+bx^n)^2\,(A+Bx^n)\,dx = \frac{1}{2}x^2\left(a^2A+\frac{2a(2Ab+aB)x^n}{2+n}+\frac{b(Ab+2aB)x^{2n}}{1+n}+\frac{2b^2Bx^{3n}}{2+3n}\right)$$

input
```
Integrate[x*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(x^2*(a^2*A + (2*a*(2*A*b + a*B)*x^n)/(2 + n) + (b*(A*b + 2*a*B)*x^(2*n))/
(1 + n) + (2*b^2*B*x^(3*n))/(2 + 3*n)))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^2(A+Bx^n)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax+ax^{n+1}(aB+2Ab)+bx^{2n+1}(2aB+Ab)+b^2Bx^{3n+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^2Ax^2+\frac{bx^{2(n+1)}(2aB+Ab)}{2(n+1)}+\frac{ax^{n+2}(aB+2Ab)}{n+2}+\frac{b^2Bx^{3n+2}}{3n+2}$$

input
```
Int[x*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(a^2*A*x^2)/2 + (b*(A*b + 2*a*B)*x^(2*(1 + n)))/(2*(1 + n)) + (a*(2*A*b +
a*B)*x^(2 + n))/(2 + n) + (b^2*B*x^(2 + 3*n))/(2 + 3*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
risch	$\frac{a(2Ab+Ba)x^2x^n}{2+n}+\frac{b^2B\,x^2x^{3n}}{2+3n}+\frac{a^2A\,x^2}{2}+\frac{b(Ab+2Ba)x^2x^{2n}}{2+2n}$
norman	$\frac{a(2Ab+Ba)x^2\mathrm{e}^{n\ln(x)}}{2+n}+\frac{b^2B\,x^2\mathrm{e}^{3n\ln(x)}}{2+3n}+\frac{a^2A\,x^2}{2}+\frac{b(Ab+2Ba)x^2\mathrm{e}^{2n\ln(x)}}{2+2n}$
parallelrisch	$2B\,x^2x^{3n}b^2n^2+3A\,x^2x^{2n}b^2n^2+6B\,x^2x^{3n}b^2n+6B\,x^2x^{2n}ab\,n^2+8A\,x^2x^{2n}b^2n+12A\,x^2x^{n}ab\,n^2+3A\,x^2a^2n^3+4b^2B\,x^2x^{3n}+1$
orering	$\frac{3x^2(2n^2+9n+5)(a+b\,x^n)^2(A+B\,x^n)}{4(3n^2+8n+4)}-\frac{(11n+7)x^2\left((a+b\,x^n)^2(A+B\,x^n)+2(a+b\,x^n)(A+B\,x^n)b\,x^nn+(a+b\,x^n)^2B\,x^nn\right)}{4(3n^2+8n+4)}$

input
```
int(x*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
a*(2*A*b+B*a)/(2+n)*x^2*x^n+b^2*B/(2+3*n)*x^2*(x^n)^3+1/2*a^2*A*x^2+1/2*b*
(A*b+2*B*a)/(1+n)*x^2*(x^n)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(72) = 144.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.49

$$\int x(a+bx^n)^2\left(A+Bx^n\right)\,dx$$
$$=\frac{2\left(Bb^2n^2+3\,Bb^2n+2\,Bb^2\right)x^2x^{3\,n}+\left(8\,Bab+4\,Ab^2+3\left(2\,Bab+Ab^2\right)n^2+8\left(2\,Bab+Ab^2\right)n\right)x^2x^{2\,n}+2}{2\left(3\,n^3+1\right.}$$

input
```
integrate(x*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*(B*b^2*n^2 + 3*B*b^2*n + 2*B*b^2)*x^2*x^(3*n) + (8*B*a*b + 4*A*b^2
+ 3*(2*B*a*b + A*b^2)*n^2 + 8*(2*B*a*b + A*b^2)*n)*x^2*x^(2*n) + 2*(2*B*a^
2 + 4*A*a*b + 3*(B*a^2 + 2*A*a*b)*n^2 + 5*(B*a^2 + 2*A*a*b)*n)*x^2*x^n + (
3*A*a^2*n^3 + 11*A*a^2*n^2 + 12*A*a^2*n + 4*A*a^2)*x^2)/(3*n^3 + 11*n^2 +
12*n + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(66) = 132$.

Time = 0.53 (sec) , antiderivative size = 767, normalized size of antiderivative = 10.09

$$\int x(a+bx^n)^2\,(A+Bx^n)\,dx$$
$$= \begin{cases} \frac{Aa^2x^2}{2} + 2Aab\log(x) - \frac{Ab^2}{2x^2} + Ba^2\log(x) - \frac{Bab}{x^2} - \frac{Bb^2}{4x^4} \\ \frac{Aa^2x^2}{2} + 2Aabx + Ab^2\log(x) + Ba^2x + 2Bab\log(x) - \frac{Bb^2}{x} \\ \frac{Aa^2x^2}{2} + \frac{3Aabx^{\frac{4}{3}}}{2} + \frac{3Ab^2x^{\frac{2}{3}}}{2} + \frac{3Ba^2x^{\frac{4}{3}}}{4} + 3Babx^{\frac{2}{3}} + Bb^2\log(x) \\ \frac{3Aa^2n^3x^2}{6n^3+22n^2+24n+8} + \frac{11Aa^2n^2x^2}{6n^3+22n^2+24n+8} + \frac{12Aa^2nx^2}{6n^3+22n^2+24n+8} + \frac{4Aa^2x^2}{6n^3+22n^2+24n+8} + \frac{12Aabn^2x^2x^n}{6n^3+22n^2+24n+8} + \frac{20Aabnx^2x^n}{6n^3+22n^2+24n+8} \end{cases}$$

input
```
integrate(x*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Piecewise((A*a**2*x**2/2 + 2*A*a*b*log(x) - A*b**2/(2*x**2) + B*a**2*log(x
) - B*a*b/x**2 - B*b**2/(4*x**4), Eq(n, -2)), (A*a**2*x**2/2 + 2*A*a*b*x +
 A*b**2*log(x) + B*a**2*x + 2*B*a*b*log(x) - B*b**2/x, Eq(n, -1)), (A*a**2
*x**2/2 + 3*A*a*b*x**(4/3)/2 + 3*A*b**2*x**(2/3)/2 + 3*B*a**2*x**(4/3)/4 +
 3*B*a*b*x**(2/3) + B*b**2*log(x), Eq(n, -2/3)), (3*A*a**2*n**3*x**2/(6*n*
*3 + 22*n**2 + 24*n + 8) + 11*A*a**2*n**2*x**2/(6*n**3 + 22*n**2 + 24*n +
8) + 12*A*a**2*n*x**2/(6*n**3 + 22*n**2 + 24*n + 8) + 4*A*a**2*x**2/(6*n**
3 + 22*n**2 + 24*n + 8) + 12*A*a*b*n**2*x**2*x**n/(6*n**3 + 22*n**2 + 24*n
 + 8) + 20*A*a*b*n*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 8*A*a*b*x**2*
x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 3*A*b**2*n**2*x**2*x**(2*n)/(6*n**3 +
 22*n**2 + 24*n + 8) + 8*A*b**2*n*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n +
 8) + 4*A*b**2*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 6*B*a**2*n**2
*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) + 10*B*a**2*n*x**2*x**n/(6*n**3 +
 22*n**2 + 24*n + 8) + 4*B*a**2*x**2*x**n/(6*n**3 + 22*n**2 + 24*n + 8) +
6*B*a*b*n**2*x**2*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 16*B*a*b*n*x**2
*x**(2*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 8*B*a*b*x**2*x**(2*n)/(6*n**3 +
22*n**2 + 24*n + 8) + 2*B*b**2*n**2*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n
 + 8) + 6*B*b**2*n*x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8) + 4*B*b**2*
x**2*x**(3*n)/(6*n**3 + 22*n**2 + 24*n + 8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int x(a+bx^n)^2(A+Bx^n)\,dx = \frac{1}{2}Aa^2x^2 + \frac{Bb^2x^{3n+2}}{3n+2} + \frac{Babx^{2n+2}}{n+1} + \frac{Ab^2x^{2n+2}}{2(n+1)} + \frac{Ba^2x^{n+2}}{n+2} + \frac{2Aabx^{n+2}}{n+2}$$

input
```
integrate(x*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
1/2*A*a^2*x^2 + B*b^2*x^(3*n + 2)/(3*n + 2) + B*a*b*x^(2*n + 2)/(n + 1) +
1/2*A*b^2*x^(2*n + 2)/(n + 1) + B*a^2*x^(n + 2)/(n + 2) + 2*A*a*b*x^(n + 2
)/(n + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(72) = 144$.

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.62

$$\int x(a+bx^n)^2 (A+Bx^n)\, dx$$
$$= \frac{2\, Bb^2n^2x^2x^{3\,n} + 6\, Babn^2x^2x^{2\,n} + 3\, Ab^2n^2x^2x^{2\,n} + 6\, Ba^2n^2x^2x^n + 12\, Aabn^2x^2x^n + 3\, Aa^2n^3x^2 + 6\, Bb^2nx}{}$$

input
```
integrate(x*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
1/2*(2*B*b^2*n^2*x^2*x^(3*n) + 6*B*a*b*n^2*x^2*x^(2*n) + 3*A*b^2*n^2*x^2*x
^(2*n) + 6*B*a^2*n^2*x^2*x^n + 12*A*a*b*n^2*x^2*x^n + 3*A*a^2*n^3*x^2 + 6*
B*b^2*n*x^2*x^(3*n) + 16*B*a*b*n*x^2*x^(2*n) + 8*A*b^2*n*x^2*x^(2*n) + 10*
B*a^2*n*x^2*x^n + 20*A*a*b*n*x^2*x^n + 11*A*a^2*n^2*x^2 + 4*B*b^2*x^2*x^(3
*n) + 8*B*a*b*x^2*x^(2*n) + 4*A*b^2*x^2*x^(2*n) + 4*B*a^2*x^2*x^n + 8*A*a*
b*x^2*x^n + 12*A*a^2*n*x^2 + 4*A*a^2*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int x(a+bx^n)^2 (A+Bx^n)\, dx = \frac{A\,a^2\,x^2}{2} + \frac{x^{2\,n}\,x^2\,\left(A\,b^2+2\,B\,a\,b\right)}{2\,n+2} + \frac{x^n\,x^2\,\left(B\,a^2+2\,A\,b\,a\right)}{n+2} + \frac{B\,b^2\,x^{3\,n}\,x^2}{3\,n+2}$$

input
```
int(x*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(A*a^2*x^2)/2 + (x^(2*n)*x^2*(A*b^2 + 2*B*a*b))/(2*n + 2) + (x^n*x^2*(B*a^
2 + 2*A*a*b))/(n + 2) + (B*b^2*x^(3*n)*x^2)/(3*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.99

$$\int x(a+bx^n)^2(A+Bx^n)\,dx$$
$$=\frac{x^2(2x^{3n}b^3n^2+6x^{3n}b^3n+4x^{3n}b^3+9x^{2n}a\,b^2n^2+24x^{2n}a\,b^2n+12x^{2n}a\,b^2+18x^na^2b\,n^2+30x^na^2bn+12x}{6n^3+22n^2+24n+8}$$

input
```
int(x*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(x**2*(2*x**(3*n)*b**3*n**2 + 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(
2*n)*a*b**2*n**2 + 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**
2*b*n**2 + 30*x**n*a**2*b*n + 12*x**n*a**2*b + 3*a**3*n**3 + 11*a**3*n**2
+ 12*a**3*n + 4*a**3))/(2*(3*n**3 + 11*n**2 + 12*n + 4))
```

3.263 $\int (a+bx^n)^2 (A+Bx^n)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^n)^2 (A+Bx^n)\, dx = a^2Ax+\frac{a(2Ab+aB)x^{1+n}}{1+n}+\frac{b(Ab+2aB)x^{1+2n}}{1+2n}+\frac{b^2Bx^{1+3n}}{1+3n}$$

output
```
a^2*A*x+a*(2*A*b+B*a)*x^(1+n)/(1+n)+b*(A*b+2*B*a)*x^(1+2*n)/(1+2*n)+b^2*B*
x^(1+3*n)/(1+3*n)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int (a+bx^n)^2 (A+Bx^n)\, dx = \frac{Bx(a+bx^n)^3-(aB-A(b+3bn))x\left(a^2+\frac{2abx^n}{1+n}+\frac{b^2x^{2n}}{1+2n}\right)}{b+3bn}$$

input
```
Integrate[(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(B*x*(a + b*x^n)^3 - (a*B - A*(b + 3*b*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) +
(b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {897, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^2 (A+Bx^n) \, dx$$

$$\Big\downarrow \text{897}$$

$$\int \left(a^2A+bx^{2n}(2aB+Ab)+ax^n(aB+2Ab)+b^2Bx^{3n}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{ax^{n+1}(aB+2Ab)}{n+1}+\frac{bx^{2n+1}(2aB+Ab)}{2n+1}+\frac{b^2Bx^{3n+1}}{3n+1}$$

input
```
Int[(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^(1 + n))/(1 + n) + (b*(A*b + 2*a*B)*x^(1 + 2*
n))/(1 + 2*n) + (b^2*B*x^(1 + 3*n))/(1 + 3*n)
```

Defintions of rubi rules used

rule 897
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
risch	$a^2Ax+\frac{a(2Ab+Ba)x\,x^n}{1+n}+\frac{b(Ab+2Ba)x\,x^{2n}}{1+2n}+\frac{b^2Bx\,x^{3n}}{1+3n}$
norman	$a^2Ax+\frac{a(2Ab+Ba)x\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{b(Ab+2Ba)x\,\mathrm{e}^{2n\ln(x)}}{1+2n}+\frac{b^2Bx\,\mathrm{e}^{3n\ln(x)}}{1+3n}$
parallelrisch	$\frac{2Bx\,x^{3n}b^2n^2+3Ax\,x^{2n}b^2n^2+3Bx\,x^{3n}b^2n+6Bx\,x^{2n}ab\,n^2+4Ax\,x^{2n}b^2n+12Ax\,x^nab\,n^2+6Ax\,a^2n^3+b^2Bx\,x^{3n}+8Bx\,x^{2n}abn}{(1+n)(1+2n}$
orering	$x(a+b\,x^n)^2\,(A+B\,x^n)-\frac{x^2(11n^2+1)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x}+\frac{(a+b\,x^n)^2B\,x^n n}{x}\right)}{(2n^2+3n+1)(1+3n)}+\frac{2x^3(-1+3n)\left(\frac{2b^2x^{2n}n^2}{}\right.}{}$

input
```
int((a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2*A*x+a*(2*A*b+B*a)/(1+n)*x*x^n+b*(A*b+2*B*a)/(1+2*n)*x*(x^n)^2+b^2*B/(1
+3*n)*x*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.50

$$\int (a+bx^n)^2\,(A+Bx^n)\,dx$$
$$=\frac{(2\,Bb^2n^2+3\,Bb^2n+Bb^2)xx^{3\,n}+(2\,Bab+Ab^2+3\,(2\,Bab+Ab^2)n^2+4\,(2\,Bab+Ab^2)n)xx^{2\,n}+(Ba^2}{6\,n^3+11\,n^2+}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
((2*B*b^2*n^2 + 3*B*b^2*n + B*b^2)*x*x^(3*n) + (2*B*a*b + A*b^2 + 3*(2*B*a
*b + A*b^2)*n^2 + 4*(2*B*a*b + A*b^2)*n)*x*x^(2*n) + (B*a^2 + 2*A*a*b + 6*
(B*a^2 + 2*A*a*b)*n^2 + 5*(B*a^2 + 2*A*a*b)*n)*x*x^n + (6*A*a^2*n^3 + 11*A
*a^2*n^2 + 6*A*a^2*n + A*a^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(63) = 126$.

Time = 0.55 (sec) , antiderivative size = 726, normalized size of antiderivative = 10.37

$$\int (a+bx^n)^2 (A+Bx^n)\,dx$$
$$= \begin{cases} Aa^2x + 2Aab\log(x) - \frac{Ab^2}{x} + Ba^2\log(x) - \frac{2Bab}{x} - \frac{Bb^2}{2x^2} \\ Aa^2x + 4Aab\sqrt{x} + Ab^2\log(x) + 2Ba^2\sqrt{x} + 2Bab\log(x) - \frac{2Bb^2}{\sqrt{x}} \\ Aa^2x + 3Aabx^{\frac{2}{3}} + 3Ab^2\sqrt[3]{x} + \frac{3Ba^2x^{\frac{2}{3}}}{2} + 6Bab\sqrt[3]{x} + Bb^2\log(x) \\ \frac{6Aa^2n^3x}{6n^3+11n^2+6n+1} + \frac{11Aa^2n^2x}{6n^3+11n^2+6n+1} + \frac{6Aa^2nx}{6n^3+11n^2+6n+1} + \frac{Aa^2x}{6n^3+11n^2+6n+1} + \frac{12Aabn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10Aabnxx^n}{6n^3+11n^2+6n+1} + \frac{}{6n^3} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Piecewise((A*a**2*x + 2*A*a*b*log(x) - A*b**2/x + B*a**2*log(x) - 2*B*a*b/
x - B*b**2/(2*x**2), Eq(n, -1)), (A*a**2*x + 4*A*a*b*sqrt(x) + A*b**2*log(
x) + 2*B*a**2*sqrt(x) + 2*B*a*b*log(x) - 2*B*b**2/sqrt(x), Eq(n, -1/2)), (
A*a**2*x + 3*A*a*b*x**(2/3) + 3*A*b**2*x**(1/3) + 3*B*a**2*x**(2/3)/2 + 6*
B*a*b*x**(1/3) + B*b**2*log(x), Eq(n, -1/3)), (6*A*a**2*n**3*x/(6*n**3 + 1
1*n**2 + 6*n + 1) + 11*A*a**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*A*a*
*2*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + A*a**2*x/(6*n**3 + 11*n**2 + 6*n + 1
) + 12*A*a*b*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*A*a*b*n*x*x**n/
(6*n**3 + 11*n**2 + 6*n + 1) + 2*A*a*b*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1)
 + 3*A*b**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*A*b**2*n*x*x*
*(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + A*b**2*x*x**(2*n)/(6*n**3 + 11*n**2
+ 6*n + 1) + 6*B*a**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*B*a**2*
n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + B*a**2*x*x**n/(6*n**3 + 11*n**2 +
6*n + 1) + 6*B*a*b*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*B*a*b*
n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*B*a*b*x*x**(2*n)/(6*n**3 + 1
1*n**2 + 6*n + 1) + 2*B*b**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 3*B*b**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + B*b**2*x*x**(3*n)/(
6*n**3 + 11*n**2 + 6*n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int (a+bx^n)^2 (A+Bx^n)\, dx = Aa^2x + \frac{Bb^2x^{3n+1}}{3n+1} + \frac{2\, Babx^{2n+1}}{2n+1} + \frac{Ab^2x^{2n+1}}{2n+1} + \frac{Ba^2x^{n+1}}{n+1} + \frac{2\, Aabx^{n+1}}{n+1}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
A*a^2*x + B*b^2*x^(3*n + 1)/(3*n + 1) + 2*B*a*b*x^(2*n + 1)/(2*n + 1) + A*
b^2*x^(2*n + 1)/(2*n + 1) + B*a^2*x^(n + 1)/(n + 1) + 2*A*a*b*x^(n + 1)/(n
 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(70) = 140.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.31

$$\int (a+bx^n)^2 (A+Bx^n)\, dx = \frac{6\, Aa^2n^3x + 2\, Bb^2n^2xx^{3n} + 6\, Babn^2xx^{2n} + 3\, Ab^2n^2xx^{2n} + 6\, Ba^2n^2xx^n + 12\, Aabn^2xx^n + 11\, Aa^2n^2x + }{}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
(6*A*a^2*n^3*x + 2*B*b^2*n^2*x*x^(3*n) + 6*B*a*b*n^2*x*x^(2*n) + 3*A*b^2*n
^2*x*x^(2*n) + 6*B*a^2*n^2*x*x^n + 12*A*a*b*n^2*x*x^n + 11*A*a^2*n^2*x + 3
*B*b^2*n*x*x^(3*n) + 8*B*a*b*n*x*x^(2*n) + 4*A*b^2*n*x*x^(2*n) + 5*B*a^2*n
*x*x^n + 10*A*a*b*n*x*x^n + 6*A*a^2*n*x + B*b^2*x*x^(3*n) + 2*B*a*b*x*x^(2
*n) + A*b^2*x*x^(2*n) + B*a^2*x*x^n + 2*A*a*b*x*x^n + A*a^2*x)/(6*n^3 + 11
*n^2 + 6*n + 1)
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int (a+bx^n)^2 (A+Bx^n)\,dx = A\,a^2\,x + \frac{x\,x^{2\,n}\,(A\,b^2+2\,B\,a\,b)}{2\,n+1} + \frac{x\,x^n\,(B\,a^2+2\,A\,b\,a)}{n+1} + \frac{B\,b^2\,x\,x^{3\,n}}{3\,n+1}$$

input
```
int((A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
A*a^2*x + (x*x^(2*n)*(A*b^2 + 2*B*a*b))/(2*n + 1) + (x*x^n*(B*a^2 + 2*A*a*
b))/(n + 1) + (B*b^2*x*x^(3*n))/(3*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.09

$$\int (a+bx^n)^2 (A+Bx^n)\,dx = \frac{x(2x^{3n}b^3n^2+3x^{3n}b^3n+x^{3n}b^3+9x^{2n}a\,b^2n^2+12x^{2n}a\,b^2n+3x^{2n}a\,b^2+18x^na^2b\,n^2+15x^na^2bn+3x^na^2b}{6n^3+11n^2+6n+1}$$

input
```
int((a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(x*(2*x**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*
a*b**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 + 15*x**n*a**2*b*n + 3*x**n*a**2*b + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**
3*n + a**3))/(6*n**3 + 11*n**2 + 6*n + 1)
```

3.264 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x} \, dx = \frac{2aAbx^n}{n} + \frac{Ab^2x^{2n}}{2n} + \frac{B(a+bx^n)^3}{3bn} + a^2A\log(x)$$

output
```
2*a*A*b*x^n/n+1/2*A*b^2*x^(2*n)/n+1/3*B*(a+b*x^n)^3/b/n+a^2*A*ln(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x} \, dx = \frac{x^n(6a^2B+6ab(2A+Bx^n)+b^2x^n(3A+2Bx^n))+6a^2A\log(x^n)}{6n}$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x,x]
```

output
```
(x^n*(6*a^2*B + 6*a*b*(2*A + B*x^n) + b^2*x^n*(3*A + 2*B*x^n)) + 6*a^2*A*L
og[x^n])/(6*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{-n}(bx^n+a)^2 (Bx^n+A) \, dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{A\int x^{-n}(bx^n+a)^2 \, dx^n+\frac{B(a+bx^n)^3}{3b}}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{A\int \left(a^2x^{-n}+b^2x^n+2ab\right) dx^n+\frac{B(a+bx^n)^3}{3b}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{A\left(a^2\log(x^n)+2abx^n+\frac{1}{2}b^2x^{2n}\right)+\frac{B(a+bx^n)^3}{3b}}{n}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x,x]
```

output
```
((B*(a + b*x^n)^3)/(3*b) + A*(2*a*b*x^n + (b^2*x^(2*n))/2 + a^2*Log[x^n]))
/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{\frac{b^{2} B\, x^{3 n}}{3}+\frac{A\, b^{2} x^{2 n}}{2}+B a b\, x^{2 n}+2 a b A\, x^{n}+a^{2} B\, x^{n}+a^{2} A \ln\left(x^{n}\right)}{n}$	62
default	$\frac{\frac{b^{2} B\, x^{3 n}}{3}+\frac{A\, b^{2} x^{2 n}}{2}+B a b\, x^{2 n}+2 a b A\, x^{n}+a^{2} B\, x^{n}+a^{2} A \ln\left(x^{n}\right)}{n}$	62
norman	$a^{2} A \ln\left(x\right)+\frac{a \left(2 A b+B a\right) {\mathrm e}^{n \ln\left(x\right)}}{n}+\frac{b \left(A b+2 B a\right) {\mathrm e}^{2 n \ln\left(x\right)}}{2 n}+\frac{b^{2} B\, {\mathrm e}^{3 n \ln\left(x\right)}}{3 n}$	64
parallelrisch	$\frac{2 b^{2} B\, x^{3 n}+3 A\, b^{2} x^{2 n}+6 a^{2} A \ln\left(x\right) n+6 B a b\, x^{2 n}+12 a b A\, x^{n}+6 a^{2} B\, x^{n}}{6 n}$	65
risch	$a^{2} A \ln\left(x\right)+\frac{2 a A b\, x^{n}}{n}+\frac{a^{2} x^{n} B}{n}+\frac{A\, b^{2} x^{2 n}}{2 n}+\frac{b\, x^{2 n} B a}{n}+\frac{b^{2} B\, x^{3 n}}{3 n}$	71

input
```
int((a+b*x^n)^2*(A+B*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/3*b^2*B*(x^n)^3+1/2*A*b^2*(x^n)^2+B*a*b*(x^n)^2+2*a*b*A*x^n+a^2*B*x
^n+a^2*A*ln(x^n))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} \, dx = \frac{6 \, Aa^2 n \log(x) + 2 \, Bb^2 x^{3 \, n} + 3 \, (2 \, Bab + Ab^2) x^{2 \, n} + 6 \, (Ba^2 + 2 \, Aab) x^n}{6 \, n}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x,x, algorithm="fricas")
```

output
```
1/6*(6*A*a^2*n*log(x) + 2*B*b^2*x^(3*n) + 3*(2*B*a*b + A*b^2)*x^(2*n) + 6*
(B*a^2 + 2*A*a*b)*x^n)/n
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} \, dx = \begin{cases} Aa^2 \log(x) + \frac{2Aabx^n}{n} + \frac{Ab^2x^{2n}}{2n} + \frac{Ba^2x^n}{n} + \frac{Babx^{2n}}{n} + \frac{Bb^2x^{3n}}{3n} & \text{for } n \neq 0 \\ (A+B)(a+b)^2 \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x,x)
```

output
```
Piecewise((A*a**2*log(x) + 2*A*a*b*x**n/n + A*b**2*x**(2*n)/(2*n) + B*a**2
*x**n/n + B*a*b*x**(2*n)/n + B*b**2*x**(3*n)/(3*n), Ne(n, 0)), ((A + B)*(a
 + b)**2*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} dx = Aa^2 \log(x) + \frac{Bb^2x^{3n}}{3n} + \frac{Babx^{2n}}{n} + \frac{Ab^2x^{2n}}{2n} + \frac{Ba^2x^n}{n} + \frac{2Aabx^n}{n}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x,x, algorithm="maxima")
```

output
```
A*a^2*log(x) + 1/3*B*b^2*x^(3*n)/n + B*a*b*x^(2*n)/n + 1/2*A*b^2*x^(2*n)/n
 + B*a^2*x^n/n + 2*A*a*b*x^n/n
```

Giac [F]

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x} dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} dx = A a^2 \ln(x) + \frac{x^n (B a^2 + 2 A b a)}{n} + \frac{x^{2n} (A b^2 + 2 B a b)}{2n} + \frac{B b^2 x^{3n}}{3n}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x,x)
```

output
```
A*a^2*log(x) + (x^n*(B*a^2 + 2*A*a*b))/n + (x^(2*n)*(A*b^2 + 2*B*a*b))/(2*
n) + (B*b^2*x^(3*n))/(3*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x} \, dx = \frac{2x^{3n}b^3 + 9x^{2n}a\,b^2 + 18x^n a^2 b + 6\log(x)\,a^3 n}{6n}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x,x)
```

output
```
(2*x**(3*n)*b**3 + 9*x**(2*n)*a*b**2 + 18*x**n*a**2*b + 6*log(x)*a**3*n)/(
6*n)
```

3.265 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 78

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2}\,dx = -\frac{a^2A}{x} - \frac{a(2Ab+aB)x^{-1+n}}{1-n} - \frac{b(Ab+2aB)x^{-1+2n}}{1-2n} - \frac{b^2Bx^{-1+3n}}{1-3n}$$

output
```
-a^2*A/x-a*(2*A*b+B*a)*x^(-1+n)/(1-n)-b*(A*b+2*B*a)*x^(-1+2*n)/(1-2*n)-b^2
*B*x^(-1+3*n)/(1-3*n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2}\,dx = \frac{-a^2A + \frac{a(2Ab+aB)x^n}{-1+n} + \frac{b(Ab+2aB)x^{2n}}{-1+2n} + \frac{b^2Bx^{3n}}{-1+3n}}{x}$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^2,x]
```

output
```
(-(a^2*A) + (a*(2*A*b + a*B)*x^n)/(-1 + n) + (b*(A*b + 2*a*B)*x^(2*n))/(-1
 + 2*n) + (b^2*B*x^(3*n))/(-1 + 3*n))/x
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2} \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^2A}{x^2}+ax^{n-2}(aB+2Ab)+bx^{2(n-1)}(2aB+Ab)+b^2Bx^{3n-2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{x}-\frac{ax^{n-1}(aB+2Ab)}{1-n}-\frac{bx^{2n-1}(2aB+Ab)}{1-2n}-\frac{b^2Bx^{3n-1}}{1-3n}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^2,x]
```

output
```
-((a^2*A)/x) - (a*(2*A*b + a*B)*x^(-1 + n))/(1 - n) - (b*(A*b + 2*a*B)*x^(
-1 + 2*n))/(1 - 2*n) - (b^2*B*x^(-1 + 3*n))/(1 - 3*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{a(2Ab+Ba)e^{n\ln(x)}}{-1+n}+\frac{b(Ab+2Ba)e^{2n\ln(x)}}{-1+2n}+\frac{b^2Be^{3n\ln(x)}}{-1+3n}-a^2A}{x}$
risch	$-\frac{a^2A}{x}+\frac{b^2B\,x^{3n}}{(-1+3n)x}+\frac{b(Ab+2Ba)x^{2n}}{(-1+2n)x}+\frac{a(2Ab+Ba)x^n}{(-1+n)x}$
parallelrisch	$\frac{2B\,x^{3n}b^2n^2+3A\,x^{2n}b^2n^2-3B\,x^{3n}b^2n+6B\,x^{2n}ab\,n^2-4A\,x^{2n}b^2n+12A\,x^n ab\,n^2-6A\,a^2n^3+b^2B\,x^{3n}-8B\,x^{2n}abn+6B\,x^n a^2n}{x(-1+3n)(-1+2n)(-1+n)}$
orering	$-\frac{3(2n^2-9n+5)(a+b\,x^n)^2(A+B\,x^n)}{x(-1+3n)(-1+2n)}+\frac{(11n-25)x^2\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^3}+\frac{(a+b\,x^n)^2B\,x^n n}{x^3}-\frac{2(a+b\,x^n)^2(A+B\,x^n)}{x^3}\right)}{6n^2-5n+1}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(a*(2*A*b+B*a)/(-1+n)*exp(n*ln(x))+b*(A*b+2*B*a)/(-1+2*n)*exp(n*ln(x))^2+b
^2*B/(-1+3*n)*exp(n*ln(x))^3-a^2*A)/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(73) = 146.

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.27

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{x^2}\,dx = -\frac{6\,Aa^2n^3-11\,Aa^2n^2+6\,Aa^2n-Aa^2-(2\,Bb^2n^2-3\,Bb^2n+Bb^2)x^{3\,n}-(2\,Bab+Ab^2+3\,(2\,Bab+A}{(6\,n^3-11\,n^2+6}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^2,x, algorithm="fricas")
```

output
```
-(6*A*a^2*n^3 - 11*A*a^2*n^2 + 6*A*a^2*n - A*a^2 - (2*B*b^2*n^2 - 3*B*b^2*
n + B*b^2)*x^(3*n) - (2*B*a*b + A*b^2 + 3*(2*B*a*b + A*b^2)*n^2 - 4*(2*B*a
*b + A*b^2)*n)*x^(2*n) - (B*a^2 + 2*A*a*b + 6*(B*a^2 + 2*A*a*b)*n^2 - 5*(B
*a^2 + 2*A*a*b)*n)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(65) = 130$.

Time = 0.48 (sec) , antiderivative size = 785, normalized size of antiderivative = 10.06

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2} \, dx$$
$$= \begin{cases} -\frac{Aa^2}{x} - \frac{3Aab}{x^{\frac{2}{3}}} - \frac{3Ab^2}{\sqrt[3]{x}} - \frac{3Ba^2}{2x^{\frac{2}{3}}} - \frac{6Bab}{\sqrt[3]{x}} + Bb^2 \log(x) \\ -\frac{Aa^2}{x} - \frac{4Aab}{\sqrt{x}} + Ab^2 \log(x) - \frac{2Ba^2}{\sqrt{x}} + 2Bab \log(x) + 2Bb^2\sqrt{x} \\ -\frac{Aa^2}{x} + 2Aab \log(x) + Ab^2x + Ba^2 \log(x) + 2Babx + \frac{Bb^2x^2}{2} \\ -\frac{6Aa^2n^3}{6n^3x-11n^2x+6nx-x} + \frac{11Aa^2n^2}{6n^3x-11n^2x+6nx-x} - \frac{6Aa^2n}{6n^3x-11n^2x+6nx-x} + \frac{Aa^2}{6n^3x-11n^2x+6nx-x} + \frac{12Aabn^2x^n}{6n^3x-11n^2x+6nx-x} - \frac{}{6n^3} \end{cases}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**2,x)
```

output
```
Piecewise((-A*a**2/x - 3*A*a*b/x**(2/3) - 3*A*b**2/x**(1/3) - 3*B*a**2/(2*
x**(2/3)) - 6*B*a*b/x**(1/3) + B*b**2*log(x), Eq(n, 1/3)), (-A*a**2/x - 4*
A*a*b/sqrt(x) + A*b**2*log(x) - 2*B*a**2/sqrt(x) + 2*B*a*b*log(x) + 2*B*b*
*2*sqrt(x), Eq(n, 1/2)), (-A*a**2/x + 2*A*a*b*log(x) + A*b**2*x + B*a**2*l
og(x) + 2*B*a*b*x + B*b**2*x**2/2, Eq(n, 1)), (-6*A*a**2*n**3/(6*n**3*x -
11*n**2*x + 6*n*x - x) + 11*A*a**2*n**2/(6*n**3*x - 11*n**2*x + 6*n*x - x)
 - 6*A*a**2*n/(6*n**3*x - 11*n**2*x + 6*n*x - x) + A*a**2/(6*n**3*x - 11*n
**2*x + 6*n*x - x) + 12*A*a*b*n**2*x**n/(6*n**3*x - 11*n**2*x + 6*n*x - x)
 - 10*A*a*b*n*x**n/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 2*A*a*b*x**n/(6*n*
*3*x - 11*n**2*x + 6*n*x - x) + 3*A*b**2*n**2*x**(2*n)/(6*n**3*x - 11*n**2
*x + 6*n*x - x) - 4*A*b**2*n*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) +
 A*b**2*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 6*B*a**2*n**2*x**n/(
6*n**3*x - 11*n**2*x + 6*n*x - x) - 5*B*a**2*n*x**n/(6*n**3*x - 11*n**2*x
+ 6*n*x - x) + B*a**2*x**n/(6*n**3*x - 11*n**2*x + 6*n*x - x) + 6*B*a*b*n*
*2*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 8*B*a*b*n*x**(2*n)/(6*n**
3*x - 11*n**2*x + 6*n*x - x) + 2*B*a*b*x**(2*n)/(6*n**3*x - 11*n**2*x + 6*
n*x - x) + 2*B*b**2*n**2*x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) - 3*B
*b**2*n*x**(3*n)/(6*n**3*x - 11*n**2*x + 6*n*x - x) + B*b**2*x**(3*n)/(6*n
**3*x - 11*n**2*x + 6*n*x - x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x^2} \, dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^2} \, dx = \frac{x^{2n} (A b^2 + 2 B a b)}{x (2n-1)} - \frac{A a^2}{x} + \frac{x^n (B a^2 + 2 A b a)}{x (n-1)} + \frac{B b^2 x^{3n}}{x (3n-1)}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^2,x)
```

output
```
(x^(2*n)*(A*b^2 + 2*B*a*b))/(x*(2*n - 1)) - (A*a^2)/x + (x^n*(B*a^2 + 2*A*
a*b))/(x*(n - 1)) + (B*b^2*x^(3*n))/(x*(3*n - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^2}\,dx$$
$$= \frac{2x^{3n}b^3n^2 - 3x^{3n}b^3n + x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 12x^{2n}a\,b^2n + 3x^{2n}a\,b^2 + 18x^n a^2b\,n^2 - 15x^n a^2bn + 3x^n a^2b -}{x\,(6n^3 - 11n^2 + 6n - 1)}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^2,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 9*x**(2*n)*a*b
**2*n**2 - 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n**2
- 15*x**n*a**2*b*n + 3*x**n*a**2*b - 6*a**3*n**3 + 11*a**3*n**2 - 6*a**3*n
 + a**3)/(x*(6*n**3 - 11*n**2 + 6*n - 1))
```

3.266 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} dx = -\frac{a^2 A}{2x^2} - \frac{b(Ab+2aB)x^{-2(1-n)}}{2(1-n)} - \frac{a(2Ab+aB)x^{-2+n}}{2-n} - \frac{b^2 B x^{-2+3n}}{2-3n}$$

output
```
-1/2*a^2*A/x^2-1/2*b*(A*b+2*B*a)/(1-n)/(x^(2-2*n))-a*(2*A*b+B*a)*x^(-2+n)/
(2-n)-b^2*B*x^(-2+3*n)/(2-3*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} dx = \frac{-a^2 A + \frac{2a(2Ab+aB)x^n}{-2+n} + \frac{b(Ab+2aB)x^{2n}}{-1+n} + \frac{2b^2 B x^{3n}}{-2+3n}}{2x^2}$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^3,x]
```

output
```
(-(a^2*A) + (2*a*(2*A*b + a*B)*x^n)/(-2 + n) + (b*(A*b + 2*a*B)*x^(2*n))/(
-1 + n) + (2*b^2*B*x^(3*n))/(-2 + 3*n))/(2*x^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^3}+ax^{n-3}(aB+2Ab)+bx^{2n-3}(2aB+Ab)+b^2Bx^{3(n-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{2x^2}-\frac{bx^{-2(1-n)}(2aB+Ab)}{2(1-n)}-\frac{ax^{n-2}(aB+2Ab)}{2-n}-\frac{b^2Bx^{3n-2}}{2-3n}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (b*(A*b + 2*a*B))/(2*(1 - n)*x^(2*(1 - n))) - (a*(2*A*b
 + a*B)*x^(-2 + n))/(2 - n) - (b^2*B*x^(-2 + 3*n))/(2 - 3*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{a^2 A}{2x^2}+\frac{b^2 B\,x^{3n}}{(-2+3n)x^2}+\frac{b(Ab+2Ba)x^{2n}}{2(-1+n)x^2}+\frac{a(2Ab+Ba)x^n}{(-2+n)x^2}$
parallelrisch	$\frac{2B\,x^{3n}b^2n^2+3A\,x^{2n}b^2n^2-6B\,x^{3n}b^2n+6B\,x^{2n}ab\,n^2-8A\,x^{2n}b^2n+12A\,x^n ab\,n^2-3A\,a^2n^3+4b^2B\,x^{3n}-16B\,x^{2n}abn+6B\,x^n a \cdots}{2x^2(-2+3n)(-1+n)(-2 \cdots}$
orering	$-\frac{(6n^2-49n+65)(a+b\,x^n)^2(A+B\,x^n)}{4x^2(3n^2-8n+4)}+\frac{x^2(11n^2-54n+55)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^4}+\frac{(a+b\,x^n)^2B\,x^n n}{x^4}-\frac{3(a+b\,x^n)^2(A+ \cdots}{x^4} \cdots\right.}{12n^3-44n^2+48n-16}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*A/x^2+1/(-2+3*n)*b^2*B/x^2*(x^n)^3+1/2/(-1+n)*b*(A*b+2*B*a)/x^2*(
x^n)^2+1/(-2+n)*a*(2*A*b+B*a)/x^2*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(74) = 148.

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{x^3}\,dx = -\frac{3\,Aa^2n^3-11\,Aa^2n^2+12\,Aa^2n-4\,Aa^2-2\,(Bb^2n^2-3\,Bb^2n+2\,Bb^2)x^{3\,n}-(8\,Bab+4\,Ab^2+3\,(2\,Ba \cdots}{2\,(3\,n^3-11\,n^2 \cdots}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(3*A*a^2*n^3 - 11*A*a^2*n^2 + 12*A*a^2*n - 4*A*a^2 - 2*(B*b^2*n^2 - 3
*B*b^2*n + 2*B*b^2)*x^(3*n) - (8*B*a*b + 4*A*b^2 + 3*(2*B*a*b + A*b^2)*n^2
 - 8*(2*B*a*b + A*b^2)*n)*x^(2*n) - 2*(2*B*a^2 + 4*A*a*b + 3*(B*a^2 + 2*A*
a*b)*n^2 - 5*(B*a^2 + 2*A*a*b)*n)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(68) = 136$.

Time = 0.60 (sec) , antiderivative size = 955, normalized size of antiderivative = 11.37

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**3,x)
```

output
```
Piecewise((-A*a**2/(2*x**2) - 3*A*a*b/(2*x**(4/3)) - 3*A*b**2/(2*x**(2/3))
 - 3*B*a**2/(4*x**(4/3)) - 3*B*a*b/x**(2/3) + B*b**2*log(x), Eq(n, 2/3)),
(-A*a**2/(2*x**2) - 2*A*a*b/x + A*b**2*log(x) - B*a**2/x + 2*B*a*b*log(x)
+ B*b**2*x, Eq(n, 1)), (-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2
+ B*a**2*log(x) + B*a*b*x**2 + B*b**2*x**4/4, Eq(n, 2)), (-3*A*a**2*n**3/(
6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 11*A*a**2*n**2/(6*n**3*
x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 12*A*a**2*n/(6*n**3*x**2 - 22*
n**2*x**2 + 24*n*x**2 - 8*x**2) + 4*A*a**2/(6*n**3*x**2 - 22*n**2*x**2 + 2
4*n*x**2 - 8*x**2) + 12*A*a*b*n**2*x**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n
*x**2 - 8*x**2) - 20*A*a*b*n*x**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2
- 8*x**2) + 8*A*a*b*x**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2)
 + 3*A*b**2*n**2*x**(2*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2
) - 8*A*b**2*n*x**(2*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2)
+ 4*A*b**2*x**(2*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 6*
B*a**2*n**2*x**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 10*B*
a**2*n*x**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 4*B*a**2*x
**n/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 6*B*a*b*n**2*x**(2
*n)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) - 16*B*a*b*n*x**(2*n
)/(6*n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 8*B*a*b*x**(2*n)/(6*
n**3*x**2 - 22*n**2*x**2 + 24*n*x**2 - 8*x**2) + 2*B*b**2*n**2*x**(3*n)...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x^3} \, dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^3} \, dx = \frac{x^{2n} (A b^2 + 2 B a b)}{x^2 (2n-2)} - \frac{A a^2}{2 x^2} + \frac{x^n (B a^2 + 2 A b a)}{x^2 (n-2)} + \frac{B b^2 x^{3n}}{x^2 (3n-2)}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^3,x)
```

output
```
(x^(2*n)*(A*b^2 + 2*B*a*b))/(x^2*(2*n - 2)) - (A*a^2)/(2*x^2) + (x^n*(B*a^
2 + 2*A*a*b))/(x^2*(n - 2)) + (B*b^2*x^(3*n))/(x^2*(3*n - 2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^3}\,dx$$
$$= \frac{2x^{3n}b^3n^2 - 6x^{3n}b^3n + 4x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 24x^{2n}a\,b^2n + 12x^{2n}a\,b^2 + 18x^n a^2b\,n^2 - 30x^n a^2bn + 12x^n a^2}{2x^2\left(3n^3 - 11n^2 + 12n - 4\right)}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^3,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 6*x**(3*n)*b**3*n + 4*x**(3*n)*b**3 + 9*x**(2*n)*a
*b**2*n**2 - 24*x**(2*n)*a*b**2*n + 12*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 - 30*x**n*a**2*b*n + 12*x**n*a**2*b - 3*a**3*n**3 + 11*a**3*n**2 - 12*a
**3*n + 4*a**3)/(2*x**2*(3*n**3 - 11*n**2 + 12*n - 4))
```

3.267 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^4} dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^4} dx = -\frac{a^2A}{3x^3} - \frac{b^2Bx^{-3(1-n)}}{3(1-n)} - \frac{a(2Ab+aB)x^{-3+n}}{3-n} - \frac{b(Ab+2aB)x^{-3+2n}}{3-2n}$$

output
```
-1/3*a^2*A/x^3-1/3*b^2*B/(1-n)/(x^(3-3*n))-a*(2*A*b+B*a)*x^(-3+n)/(3-n)-b*
(A*b+2*B*a)*x^(-3+2*n)/(3-2*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^4} dx = \frac{-a^2A + \frac{3a(2Ab+aB)x^n}{-3+n} + \frac{3b(Ab+2aB)x^{2n}}{-3+2n} + \frac{b^2Bx^{3n}}{-1+n}}{3x^3}$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^4,x]
```

output
```
(-(a^2*A) + (3*a*(2*A*b + a*B)*x^n)/(-3 + n) + (3*b*(A*b + 2*a*B)*x^(2*n))
/(-3 + 2*n) + (b^2*B*x^(3*n))/(-1 + n))/(3*x^3)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{x^4}+a x^{n-4}(aB+2Ab)+b x^{2(n-2)}(2aB+Ab)+b^2 B x^{3n-4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{3x^3}-\frac{a x^{n-3}(aB+2Ab)}{3-n}-\frac{b x^{2n-3}(2aB+Ab)}{3-2n}-\frac{b^2 B x^{-3(1-n)}}{3(1-n)}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (b^2*B)/(3*(1 - n)*x^(3*(1 - n))) - (a*(2*A*b + a*B)*x^
(-3 + n))/(3 - n) - (b*(A*b + 2*a*B)*x^(-3 + 2*n))/(3 - 2*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{a^{2} A}{3 x^{3}}+\frac{b^{2} B \,x^{3 n}}{3 \left(-1+n \right) x^{3}}+\frac{b \left(A b +2 B a \right) x^{2 n}}{\left(-3+2 n \right) x^{3}}+\frac{a \left(2 A b +B a \right) x^{n}}{\left(-3+n \right) x^{3}}$
parallelrisch	$\frac{2 B \,x^{3 n} b^{2} n^{2}+3 A \,x^{2 n} b^{2} n^{2}-9 B \,x^{3 n} b^{2} n +6 B \,x^{2 n} a b \,n^{2}-12 A \,x^{2 n} b^{2} n +12 A \,x^{n} a b \,n^{2}-2 A \,a^{2} n^{3}+9 b^{2} B \,x^{3 n}-24 B \,x^{2 n} a b n +6 B \,x^{n}}{3 x^{3} \left(-1+n \right) \left(-3+2 n \right) \left(-\right.}$
orering	$-\frac{\left(6 n^{3}-77 n^{2}+222 n -175\right) \left(a +b \,x^{n}\right)^{2} \left(A +B \,x^{n}\right)}{9 x^{3} \left(2 n^{3}-11 n^{2}+18 n -9\right)}+\frac{\left(11 n^{2}-72 n +97\right) x^{2} \left(\frac{2 \left(a +b \,x^{n}\right) \left(A +B \,x^{n}\right) b \,x^{n} n}{x^{5}}+\frac{\left(a +b \,x^{n}\right)^{2} B \,x^{n} n}{x^{5}}-\frac{4 (a+b}{}\right.}{18 n^{3}-99 n^{2}+162 n -81}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2*A/x^3+1/3/(-1+n)*b^2*B/x^3*(x^n)^3+1/(-3+2*n)*b*(A*b+2*B*a)/x^3*(
x^n)^2+1/(-3+n)*a*(2*A*b+B*a)/x^3*x^n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(74) = 148$.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^4}\,dx = -\frac{2\,Aa^2n^3-11\,Aa^2n^2+18\,Aa^2n-9\,Aa^2-(2\,Bb^2n^2-9\,Bb^2n+9\,Bb^2)x^{3\,n}-3\,(6\,Bab+3\,Ab^2+(2\,Ba}{3\,(2\,n^3-11\,n^2}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*A*a^2*n^3 - 11*A*a^2*n^2 + 18*A*a^2*n - 9*A*a^2 - (2*B*b^2*n^2 - 9
*B*b^2*n + 9*B*b^2)*x^(3*n) - 3*(6*B*a*b + 3*A*b^2 + (2*B*a*b + A*b^2)*n^2
 - 4*(2*B*a*b + A*b^2)*n)*x^(2*n) - 3*(3*B*a^2 + 6*A*a*b + 2*(B*a^2 + 2*A*
a*b)*n^2 - 5*(B*a^2 + 2*A*a*b)*n)*x^n)/((2*n^3 - 11*n^2 + 18*n - 9)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(68) = 136$.

Time = 0.62 (sec) , antiderivative size = 959, normalized size of antiderivative = 11.42

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**4,x)
```

output
```
Piecewise((-A*a**2/(3*x**3) - A*a*b/x**2 - A*b**2/x - B*a**2/(2*x**2) - 2*
B*a*b/x + B*b**2*log(x), Eq(n, 1)), (-A*a**2/(3*x**3) - 4*A*a*b/(3*x**(3/2
)) + A*b**2*log(x) - 2*B*a**2/(3*x**(3/2)) + 2*B*a*b*log(x) + 2*B*b**2*x**
(3/2)/3, Eq(n, 3/2)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
 B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(n, 3)), (-2*A*a**2*n**
3/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) + 11*A*a**2*n**2/(6*n
**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) - 18*A*a**2*n/(6*n**3*x**3
- 33*n**2*x**3 + 54*n*x**3 - 27*x**3) + 9*A*a**2/(6*n**3*x**3 - 33*n**2*x*
*3 + 54*n*x**3 - 27*x**3) + 12*A*a*b*n**2*x**n/(6*n**3*x**3 - 33*n**2*x**3
 + 54*n*x**3 - 27*x**3) - 30*A*a*b*n*x**n/(6*n**3*x**3 - 33*n**2*x**3 + 54
*n*x**3 - 27*x**3) + 18*A*a*b*x**n/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3
 - 27*x**3) + 3*A*b**2*n**2*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x*
*3 - 27*x**3) - 12*A*b**2*n*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x*
*3 - 27*x**3) + 9*A*b**2*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3
- 27*x**3) + 6*B*a**2*n**2*x**n/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 -
27*x**3) - 15*B*a**2*n*x**n/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x
**3) + 9*B*a**2*x**n/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) +
6*B*a*b*n**2*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) -
 24*B*a*b*n*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) +
18*B*a*b*x**(2*n)/(6*n**3*x**3 - 33*n**2*x**3 + 54*n*x**3 - 27*x**3) + ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-4>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x^4} \, dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx = \frac{x^{2n} (A b^2 + 2 B a b)}{x^3 (2n-3)} - \frac{A a^2}{3 x^3} + \frac{x^n (B a^2 + 2 A b a)}{x^3 (n-3)} + \frac{B b^2 x^{3n}}{x^3 (3n-3)}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^4,x)
```

output
```
(x^(2*n)*(A*b^2 + 2*B*a*b))/(x^3*(2*n - 3)) - (A*a^2)/(3*x^3) + (x^n*(B*a^
2 + 2*A*a*b))/(x^3*(n - 3)) + (B*b^2*x^(3*n))/(x^3*(3*n - 3))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^4} \, dx$$
$$= \frac{2x^{3n}b^3n^2 - 9x^{3n}b^3n + 9x^{3n}b^3 + 9x^{2n}a\,b^2n^2 - 36x^{2n}a\,b^2n + 27x^{2n}a\,b^2 + 18x^na^2b\,n^2 - 45x^na^2bn + 27x^na^2}{3x^3\,(2n^3 - 11n^2 + 18n - 9)}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^4,x)
```

output
```
(2*x**(3*n)*b**3*n**2 - 9*x**(3*n)*b**3*n + 9*x**(3*n)*b**3 + 9*x**(2*n)*a
*b**2*n**2 - 36*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 18*x**n*a**2*b*n*
*2 - 45*x**n*a**2*b*n + 27*x**n*a**2*b - 2*a**3*n**3 + 11*a**3*n**2 - 18*a
**3*n + 9*a**3)/(3*x**3*(2*n**3 - 11*n**2 + 18*n - 9))
```

3.268 $\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx = \frac{Bx^3}{3b} + \frac{(Ab-aB)x^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3ab}$$

output
```
1/3*B*x^3/b+1/3*(A*b-B*a)*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx = \frac{Ax^3 \text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a} + \frac{Bx^{3+n} \text{Hypergeometric2F1}\left(1,\frac{3+n}{n},2+\frac{3}{n},-\frac{bx^n}{a}\right)}{a(3+n)}$$

input
```
Integrate[(x^2*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(A*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a) + (B*x^(3
 + n)*Hypergeometric2F1[1, (3 + n)/n, 2 + 3/n, -((b*x^n)/a)])/(a*(3 + n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^2}{bx^n+a}dx}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3ab}+\frac{Bx^3}{3b}$$

input
```
Int[(x^2*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(B*x^3)/(3*b) + ((A*b - a*B)*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b
*x^n)/a)])/(3*a*b)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(A + B\,x^n)}{a + b\,x^n}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n),x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{x^2(A + Bx^n)}{a + bx^n}\,dx = \int \frac{(Bx^n + A)x^2}{bx^n + a}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^2*x^n + A*x^2)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.89

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx = \frac{3Aa^{\frac{3}{n}}a^{-1-\frac{3}{n}}x^3\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{3}{n}\right)\Gamma\left(\frac{3}{n}\right)}{n^2\Gamma\left(1+\frac{3}{n}\right)} + \frac{Ba^{-2-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{3Ba^{-2-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{n}\right)\Gamma\left(1+\frac{3}{n}\right)}{n^2\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n),x)
```

output
```
3*A*a**(3/n)*a**(-1 - 3/n)*x**3*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/n)
*gamma(3/n)/(n**2*gamma(1 + 3/n)) + B*a**(-2 - 3/n)*a**(1 + 3/n)*x**(n + 3
)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/n)*gamma(1 + 3/n)/(n*gamma(2
 + 3/n)) + 3*B*a**(-2 - 3/n)*a**(1 + 3/n)*x**(n + 3)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, 1 + 3/n)*gamma(1 + 3/n)/(n**2*gamma(2 + 3/n))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx = \int \frac{(Bx^n+A)x^2}{bx^n+a}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
1/3*B*x^3/b - (B*a - A*b)*integrate(x^2/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx=\int \frac{(Bx^n+A)x^2}{bx^n+a}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx=\int \frac{x^2\,(A+B\,x^n)}{a+b\,x^n}\,dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n),x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{x^2(A+Bx^n)}{a+bx^n}\,dx=\frac{x^3}{3}$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n),x)
```

output
```
x**3/3
```

3.269 $\int \frac{x(A+Bx^n)}{a+bx^n} dx$

Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x(A+Bx^n)}{a+bx^n} dx = \frac{Bx^2}{2b} + \frac{(Ab-aB)x^2 \operatorname{Hypergeometric2F1}\left(1, \frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2ab}$$

output
```
1/2*B*x^2/b+1/2*(A*b-B*a)*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int \frac{x(A+Bx^n)}{a+bx^n} dx = \frac{Ax^2 \operatorname{Hypergeometric2F1}\left(1, \frac{2}{n}, \frac{2+n}{n}, -\frac{bx^n}{a}\right)}{2a} + \frac{Bx^{2+n} \operatorname{Hypergeometric2F1}\left(1, \frac{2+n}{n}, 2\left(1+\frac{1}{n}\right), -\frac{bx^n}{a}\right)}{a(2+n)}$$

input
```
Integrate[(x*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(A*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a) + (B*x^(2
 + n)*Hypergeometric2F1[1, (2 + n)/n, 2*(1 + n^(-1)), -((b*x^n)/a)])/(a*(2
 + n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{a+bx^n}\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x}{bx^n+a}dx}{b}+\frac{Bx^2}{2b}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2ab}+\frac{Bx^2}{2b}$$

input
```
Int[(x*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(B*x^2)/(2*b) + ((A*b - a*B)*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b
*x^n)/a)])/(2*a*b)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(A+B\,x^n)}{a+b\,x^n}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n),x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{x(A+Bx^n)}{a+bx^n}\,dx=\int \frac{(Bx^n+A)x}{bx^n+a}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.89

$$\int \frac{x(A+Bx^n)}{a+bx^n}\,dx=\frac{2Aa^{\frac{2}{n}}a^{-1-\frac{2}{n}}x^2\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{2}{n}\right)\Gamma\left(\frac{2}{n}\right)}{n^2\Gamma\left(1+\frac{2}{n}\right)}+\frac{Ba^{-2-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}+\frac{2Ba^{-2-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{2}{n}\right)\Gamma\left(1+\frac{2}{n}\right)}{n^2\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n),x)
```

output
```
2*A*a**(2/n)*a**(-1 - 2/n)*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)
*gamma(2/n)/(n**2*gamma(1 + 2/n)) + B*a**(-2 - 2/n)*a**(1 + 2/n)*x**(n + 2
)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 2/n)*gamma(1 + 2/n)/(n*gamma(2
 + 2/n)) + 2*B*a**(-2 - 2/n)*a**(1 + 2/n)*x**(n + 2)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, 1 + 2/n)*gamma(1 + 2/n)/(n**2*gamma(2 + 2/n))
```

Maxima [F]

$$\int \frac{x(A + Bx^n)}{a + bx^n} \, dx = \int \frac{(Bx^n + A)x}{bx^n + a} \, dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*B*x^2/b - (B*a - A*b)*integrate(x/(b^2*x^n + a*b), x)
```

Giac [F]

$$\int \frac{x(A + Bx^n)}{a + bx^n} \, dx = \int \frac{(Bx^n + A)x}{bx^n + a} \, dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{a+bx^n} \, dx = \int \frac{x\,(A+B\,x^n)}{a+b\,x^n} \, dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n),x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{x(A+Bx^n)}{a+bx^n} \, dx = \frac{x^2}{2}$$

input
```
int(x*(A+B*x^n)/(a+b*x^n),x)
```

output
```
x**2/2
```

3.270 $\int \frac{A+Bx^n}{a+bx^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{A+Bx^n}{a+bx^n}\,dx=\frac{Bx}{b}+\frac{(Ab-aB)x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}$$

output
```
B*x/b+(A*b-B*a)*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^n}{a+bx^n}\,dx=\frac{x\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{ab}$$

input
```
Integrate[(A + B*x^n)/(a + b*x^n),x]
```

output
```
(x*(a*B + (A*b - a*B)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a
)]))/(a*b)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {913, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{a+bx^n} \, dx$$
$$\Big\downarrow \text{913}$$
$$\frac{(Ab-aB)\int \frac{1}{bx^n+a}dx}{b}+\frac{Bx}{b}$$
$$\Big\downarrow \text{778}$$
$$\frac{x(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab}+\frac{Bx}{b}$$

input
```
Int[(A + B*x^n)/(a + b*x^n),x]
```

output
```
(B*x)/b + ((A*b - a*B)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a*b)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{a+b\,x^{n}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{a+bx^n}\,dx = \int \frac{Bx^n+A}{bx^n+a}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.62

$$\int \frac{A+Bx^n}{a+bx^n}\,dx = \frac{Aa^{\frac{1}{n}}a^{-1-\frac{1}{n}}x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{n^2\Gamma\left(1+\frac{1}{n}\right)} - \frac{Ba^{-\frac{1}{n}}a^{1+\frac{1}{n}}b^{\frac{1}{n}}b^{-1-\frac{1}{n}}x\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/(a+b*x**n),x)
```

output
```
A*a**(1/n)*a**(-1 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamm
a(1/n)/(n**2*gamma(1 + 1/n)) - B*a**(1 + 1/n)*b**(1/n)*b**(-1 - 1/n)*x*ler
chphi(a*exp_polar(I*pi)/(b*x**n), 1, exp_polar(I*pi)/n)*gamma(1/n)/(a*a**(
1/n)*n**2*gamma(1 + 1/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{a+bx^n}\,dx=\int \frac{Bx^n+A}{bx^n+a}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*integrate(1/(b^2*x^n + a*b), x) + B*x/b
```

Giac [F]

$$\int \frac{A+Bx^n}{a+bx^n}\,dx=\int \frac{Bx^n+A}{bx^n+a}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{a+bx^n}\,dx=\int \frac{A+B\,x^n}{a+b\,x^n}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n),x)
```

output
```
int((A + B*x^n)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.02

$$\int \frac{A + Bx^n}{a + bx^n} \, dx = x$$

input
```
int((A+B*x^n)/(a+b*x^n),x)
```

output
```
x
```

3.271 $\int \frac{A+Bx^n}{x(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{A+Bx^n}{x(a+bx^n)}\,dx=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log(a+bx^n)}{abn}$$

output
```
A*ln(x)/a-(A*b-B*a)*ln(a+b*x^n)/a/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^n}{x(a+bx^n)}\,dx=\frac{Ab\log(x^n)+(-Ab+aB)\log(an(a+bx^n))}{abn}$$

input
```
Integrate[(A + B*x^n)/(x*(a + b*x^n)),x]
```

output
```
(A*b*Log[x^n] + (-(A*b) + a*B)*Log[a*n*(a + b*x^n)])/(a*b*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x(a+bx^n)}dx$$

$$\Big\downarrow \ 948$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{bx^n+a}dx^n}{n}$$

$$\Big\downarrow \ 86$$

$$\frac{\int \left(\frac{Ax^{-n}}{a}+\frac{aB-Ab}{a(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{A\log(x^n)}{a}-\frac{(Ab-aB)\log(a+bx^n)}{ab}}{n}$$

input
```
Int[(A + B*x^n)/(x*(a + b*x^n)),x]
```

output
```
((A*Log[x^n])/a - ((A*b - a*B)*Log[a + b*x^n])/(a*b))/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\frac{A\ln(x^n)}{a}+\frac{(-Ab+Ba)\ln(a+b\,x^n)}{ab}}{n}$	38
default	$\frac{\frac{A\ln(x^n)}{a}+\frac{(-Ab+Ba)\ln(a+b\,x^n)}{ab}}{n}$	38
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln\left(a+b\,e^{n\ln(x)}\right)}{abn}$	38
parallelrisch	$\frac{A\ln(x)bn-A\ln(a+b\,x^n)b+B\ln(a+b\,x^n)a}{abn}$	41
risch	$\frac{A\ln(x)}{a}-\frac{\ln\left(x^n+\frac{a}{b}\right)A}{an}+\frac{\ln\left(x^n+\frac{a}{b}\right)B}{bn}$	46

input
```
int((A+B*x^n)/x/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*(A/a*ln(x^n)+(-A*b+B*a)/a/b*ln(a+b*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^n}{x\,(a+bx^n)}\,dx=\frac{Abn\log{(x)}+(Ba-Ab)\log{(bx^n+a)}}{abn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n),x, algorithm="fricas")
```

output
```
(A*b*n*log(x) + (B*a - A*b)*log(b*x^n + a))/(a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(26) = 52$.

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \frac{A + Bx^n}{x\,(a + bx^n)}\,dx = \begin{cases} \tilde{\infty}(A + B)\log{(x)} & \text{for}\ a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{-\frac{Ax^{-n}}{n} - \frac{B\log{\left(x^{-n}\right)}}{n}}{b} & \text{for}\ a = 0 \\ \frac{A\log{(x)} + \frac{Bx^n}{n}}{a} & \text{for}\ b = 0 \\ \frac{(A+B)\log{(x)}}{a+b} & \text{for}\ n = 0 \\ \frac{A\log{(x)}}{a} - \frac{A\log{\left(\frac{a}{b} + x^n\right)}}{an} + \frac{B\log{\left(\frac{a}{b} + x^n\right)}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n),x)
```

output
```
Piecewise((zoo*(A + B)*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), ((-A/(n*x*
*n) - B*log(x**(-n))/n)/b, Eq(a, 0)), ((A*log(x) + B*x**n/n)/a, Eq(b, 0)),
 ((A + B)*log(x)/(a + b), Eq(n, 0)), (A*log(x)/a - A*log(a/b + x**n)/(a*n)
 + B*log(a/b + x**n)/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{A + Bx^n}{x\,(a + bx^n)}\,dx = A\left(\frac{\log{(x)}}{a} - \frac{\log{\left(\frac{bx^n+a}{b}\right)}}{an}\right) + \frac{B\log{\left(\frac{bx^n+a}{b}\right)}}{bn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n),x, algorithm="maxima")
```

output
```
A*(log(x)/a - log((b*x^n + a)/b)/(a*n)) + B*log((b*x^n + a)/b)/(b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x\,(a+bx^n)}\,dx = \int \frac{Bx^n+A}{(bx^n+a)x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^n}{x\,(a+bx^n)}\,dx = \frac{A\,\ln{(x)}}{a} - \frac{\ln{(a+b\,x^n)}\,\left(A\,b-B\,a\right)}{a\,b\,n}$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)),x)
```

output
```
(A*log(x))/a - (log(a + b*x^n)*(A*b - B*a))/(a*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.06

$$\int \frac{A+Bx^n}{x\,(a+bx^n)}\,dx = \log(x)$$

input
```
int((A+B*x^n)/x/(a+b*x^n),x)
```

output
```
log(x)
```

3.272 $\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx=-\frac{B}{bx}-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{abx}$$

output
```
-B/b/x-(A*b-B*a)*hypergeom([1, -1/n],[-(1-n)/n],-b*x^n/a)/a/b/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx =\frac{(A-An)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(1,\frac{-1+n}{n},2-\frac{1}{n},-\frac{bx^n}{a}\right)}{a(-1+n)x}$$

input
```
Integrate[(A + B*x^n)/(x^2*(a + b*x^n)),x]
```

output
```
((A - A*n)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] + B*x^n
*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), -((b*x^n)/a)])/(a*(-1 + n)*x
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{1}{x^2(bx^n+a)}dx}{b}-\frac{B}{bx}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{abx}-\frac{B}{bx}$$

input
```
Int[(A + B*x^n)/(x^2*(a + b*x^n)),x]
```

output
```
-(B/(b*x)) - ((A*b - a*B)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b
*x^n)/a)])/(a*b*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2}\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b*x^2*x^n + a*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx = \int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**2*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx = \int \frac{Bx^n+A}{(bx^n+a)x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*integrate(1/(b^2*x^2*x^n + a*b*x^2), x) - B/(b*x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)}\,dx = \int \frac{Bx^n+A}{(bx^n+a)x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)}\,dx=\int \frac{A+B\,x^n}{x^2\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.09

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)}\,dx=-\frac{1}{x}$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n),x)
```

output
```
( - 1)/x
```

3.273 $\int \frac{A+Bx^n}{x^3(a+bx^n)} dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=-\frac{B}{2bx^2}-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2abx^2}$$

output
```
-1/2*B/b/x^2-1/2*(A*b-B*a)*hypergeom([1, -2/n],[-(2-n)/n],-b*x^n/a)/a/b/x^
2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx = \frac{-A(-2+n)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(1,\frac{-2+n}{n},2-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a(-2+n)x^2}$$

input
```
Integrate[(A + B*x^n)/(x^3*(a + b*x^n)),x]
```

output
```
(-(A*(-2 + n)*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)]) + 2*B*
x^n*Hypergeometric2F1[1, (-2 + n)/n, 2 - 2/n, -((b*x^n)/a)])/(2*a*(-2 + n)
*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{1}{x^3(bx^n+a)}dx}{b}-\frac{B}{2bx^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2abx^2}-\frac{B}{2bx^2}$$

input
```
Int[(A + B*x^n)/(x^3*(a + b*x^n)),x]
```

output
```
-1/2*B/(b*x^2) - ((A*b - a*B)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((
b*x^n)/a)])/(2*a*b*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{3}\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b*x^3*x^n + a*x^3), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**3*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*integrate(1/(b^2*x^3*x^n + a*b*x^3), x) - 1/2*B/(b*x^2)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=\int \frac{A+B\,x^n}{x^3\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.08

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)}\,dx=-\frac{1}{2x^2}$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n),x)
```

output
```
( - 1)/(2*x**2)
```

3.274 $\int \frac{A+Bx^n}{x^4(a+bx^n)} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)} \, dx = -\frac{B}{3bx^3} - \frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3abx^3}$$

output
```
-1/3*B/b/x^3-1/3*(A*b-B*a)*hypergeom([1, -3/n],[-(3-n)/n],-b*x^n/a)/a/b/x^
3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)} \, dx = \frac{-A(-3+n)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(1,\frac{-3+n}{n},2-\frac{3}{n},-\frac{bx^n}{a}\right)}{3a(-3+n)x^3}$$

input
```
Integrate[(A + B*x^n)/(x^4*(a + b*x^n)),x]
```

output
```
(-(A*(-3 + n)*Hypergeometric2F1[1, -3/n, (-3 + n)/n, -((b*x^n)/a)]) + 3*B*
x^n*Hypergeometric2F1[1, (-3 + n)/n, 2 - 3/n, -((b*x^n)/a)])/(3*a*(-3 + n)
*x^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^4(a+bx^n)}dx$$
$$\Big\downarrow \text{959}$$
$$\frac{(Ab-aB)\int \frac{1}{x^4(bx^n+a)}dx}{b}-\frac{B}{3bx^3}$$
$$\Big\downarrow \text{888}$$
$$-\frac{(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3abx^3}-\frac{B}{3bx^3}$$

input
```
Int[(A + B*x^n)/(x^4*(a + b*x^n)),x]
```

output
```
-1/3*B/(b*x^3) - ((A*b - a*B)*Hypergeometric2F1[1, -3/n, -((3 - n)/n), -((
b*x^n)/a)])/(3*a*b*x^3)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{x^{4}\,(a + b\,x^{n})}dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A + Bx^n}{x^4\,(a + bx^n)}\,dx = \int \frac{Bx^n + A}{(bx^n + a)x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b*x^4*x^n + a*x^4), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)}\,dx = \int \frac{A+Bx^n}{x^4\left(a+bx^n\right)}\,dx$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**4*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)}\,dx = \int \frac{Bx^n+A}{(bx^n+a)x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n),x, algorithm="maxima")
```

output
```
-(B*a - A*b)*integrate(1/(b^2*x^4*x^n + a*b*x^4), x) - 1/3*B/(b*x^3)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)}\,dx = \int \frac{Bx^n+A}{(bx^n+a)x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4(a+bx^n)}\,dx=\int \frac{A+B\,x^n}{x^4\,(a+b\,x^n)}\,dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.08

$$\int \frac{A+Bx^n}{x^4(a+bx^n)}\,dx=-\frac{1}{3x^3}$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n),x)
```

output
```
( - 1)/(3*x**3)
```

3.275 $\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{(Ab-aB)x^3}{abn\,(a+bx^n)} + \frac{(3aB-Ab(3-n))x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2bn}$$

output
```
(A*b-B*a)*x^3/a/b/n/(a+b*x^n)+1/3*(3*B*a-A*b*(3-n))*x^3*hypergeom([1, 3/n]
,[(3+n)/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{Ax^3\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2} + \frac{Bx^{3+n}\,\text{Hypergeometric2F1}\left(2,\frac{3+n}{n},2+\frac{3}{n},-\frac{bx^n}{a}\right)}{a^2(3+n)}$$

input
```
Integrate[(x^2*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
(A*x^3*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2) + (B*x^
(3 + n)*Hypergeometric2F1[2, (3 + n)/n, 2 + 3/n, -((b*x^n)/a)])/(a^2*(3 +
n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3aB-Ab(3-n))\int \frac{x^2}{bx^n+a}dx}{abn}+\frac{x^3(Ab-aB)}{abn(a+bx^n)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(3aB-Ab(3-n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^2bn}+\frac{x^3(Ab-aB)}{abn(a+bx^n)}$$

input
```
Int[(x^2*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*x^3)/(a*b*n*(a + b*x^n)) + ((3*a*B - A*b*(3 - n))*x^3*Hyperge
ometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^2(A + B\,x^n)}{(a + b\,x^n)^2}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^2(A + Bx^n)}{(a + bx^n)^2}\,dx = \int \frac{(Bx^n + A)x^2}{(bx^n + a)^2}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2*x^n + A*x^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.19 (sec) , antiderivative size = 767, normalized size of antiderivative = 9.13

$$\int \frac{x^2(A + Bx^n)}{(a + bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n)**2,x)
```

output
```
A*(3*a*a**(3/n)*a**(-2 - 3/n)*n*x**3*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 3/n)*gamma(3/n)/(a*n**3*gamma(1 + 3/n) + b*n**3*x**n*gamma(1 + 3/n)) + 3*
a*a**(3/n)*a**(-2 - 3/n)*n*x**3*gamma(3/n)/(a*n**3*gamma(1 + 3/n) + b*n**3
*x**n*gamma(1 + 3/n)) - 9*a*a**(3/n)*a**(-2 - 3/n)*x**3*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, 3/n)*gamma(3/n)/(a*n**3*gamma(1 + 3/n) + b*n**3*x**n*g
amma(1 + 3/n)) + 3*a**(3/n)*a**(-2 - 3/n)*b*n*x**3*x**n*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, 3/n)*gamma(3/n)/(a*n**3*gamma(1 + 3/n) + b*n**3*x**n*g
amma(1 + 3/n)) - 9*a**(3/n)*a**(-2 - 3/n)*b*x**3*x**n*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, 3/n)*gamma(3/n)/(a*n**3*gamma(1 + 3/n) + b*n**3*x**n*gam
ma(1 + 3/n))) + B*(a*a**(-3 - 3/n)*a**(1 + 3/n)*n**2*x**(n + 3)*gamma(1 +
3/n)/(a*n**3*gamma(2 + 3/n) + b*n**3*x**n*gamma(2 + 3/n)) - 3*a*a**(-3 - 3
/n)*a**(1 + 3/n)*n*x**(n + 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/
n)*gamma(1 + 3/n)/(a*n**3*gamma(2 + 3/n) + b*n**3*x**n*gamma(2 + 3/n)) + 3
*a*a**(-3 - 3/n)*a**(1 + 3/n)*n*x**(n + 3)*gamma(1 + 3/n)/(a*n**3*gamma(2
+ 3/n) + b*n**3*x**n*gamma(2 + 3/n)) - 9*a*a**(-3 - 3/n)*a**(1 + 3/n)*x**(
n + 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/n)*gamma(1 + 3/n)/(a*n*
*3*gamma(2 + 3/n) + b*n**3*x**n*gamma(2 + 3/n)) - 3*a**(-3 - 3/n)*a**(1 +
3/n)*b*n*x**n*x**(n + 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/n)*ga
mma(1 + 3/n)/(a*n**3*gamma(2 + 3/n) + b*n**3*x**n*gamma(2 + 3/n)) - 9*a**(
-3 - 3/n)*a**(1 + 3/n)*b*x**n*x**(n + 3)*lerchphi(b*x**n*exp_polar(I*pi...
```

Maxima [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(B*a - A*b)*x^3/(a*b^2*n*x^n + a^2*b*n) + (A*b*(n - 3) + 3*B*a)*integrate
(x^2/(a*b^2*n*x^n + a^2*b*n), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{x^2\,(A+B\,x^n)}{(a+b\,x^n)^2}\,dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n)^2,x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{x^2}{x^n b+a}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x**2/(x**n*b + a),x)
```

3.276 $\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{(Ab-aB)x^2}{abn\,(a+bx^n)} + \frac{(2aB-Ab(2-n))x^2\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2bn}$$

output
```
(A*b-B*a)*x^2/a/b/n/(a+b*x^n)+1/2*(2*B*a-A*b*(2-n))*x^2*hypergeom([1, 2/n]
,[(2+n)/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{Ax^2\,\text{Hypergeometric2F1}\left(2,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2} + \frac{Bx^{2+n}\,\text{Hypergeometric2F1}\left(2,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^2(2+n)}$$

input
```
Integrate[(x*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
(A*x^2*Hypergeometric2F1[2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2) + (B*x^
(2 + n)*Hypergeometric2F1[2, (2 + n)/n, 2*(1 + n^(-1)), -((b*x^n)/a)])/(a^
2*(2 + n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB-Ab(2-n))\int \frac{x}{bx^n+a}dx}{abn}+\frac{x^2(Ab-aB)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(2aB-Ab(2-n))\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^2bn}+\frac{x^2(Ab-aB)}{abn\,(a+bx^n)}$$

input
```
Int[(x*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*x^2)/(a*b*n*(a + b*x^n)) + ((2*a*B - A*b*(2 - n))*x^2*Hyperge
ometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(A+B\,x^{n})}{(a+b\,x^{n})^{2}}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx=\int \frac{(Bx^n+A)x}{(bx^n+a)^2}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.27 (sec) , antiderivative size = 767, normalized size of antiderivative = 9.13

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n)**2,x)
```

output
```
A*(2*a*a**(2/n)*a**(-2 - 2/n)*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gamma(1 + 2/n)) + 2*
a*a**(2/n)*a**(-2 - 2/n)*n*x**2*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3
*x**n*gamma(1 + 2/n)) - 4*a*a**(2/n)*a**(-2 - 2/n)*x**2*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*g
amma(1 + 2/n)) + 2*a**(2/n)*a**(-2 - 2/n)*b*n*x**2*x**n*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*g
amma(1 + 2/n)) - 4*a**(2/n)*a**(-2 - 2/n)*b*x**2*x**n*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a*n**3*gamma(1 + 2/n) + b*n**3*x**n*gam
ma(1 + 2/n))) + B*(a*a**(-3 - 2/n)*a**(1 + 2/n)*n**2*x**(n + 2)*gamma(1 +
2/n)/(a*n**3*gamma(2 + 2/n) + b*n**3*x**n*gamma(2 + 2/n)) - 2*a*a**(-3 - 2
/n)*a**(1 + 2/n)*n*x**(n + 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 2/
n)*gamma(1 + 2/n)/(a*n**3*gamma(2 + 2/n) + b*n**3*x**n*gamma(2 + 2/n)) + 2
*a*a**(-3 - 2/n)*a**(1 + 2/n)*n*x**(n + 2)*gamma(1 + 2/n)/(a*n**3*gamma(2
+ 2/n) + b*n**3*x**n*gamma(2 + 2/n)) - 4*a*a**(-3 - 2/n)*a**(1 + 2/n)*x**(
n + 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 2/n)*gamma(1 + 2/n)/(a*n*
*3*gamma(2 + 2/n) + b*n**3*x**n*gamma(2 + 2/n)) - 2*a**(-3 - 2/n)*a**(1 +
2/n)*b*n*x**n*x**(n + 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 2/n)*ga
mma(1 + 2/n)/(a*n**3*gamma(2 + 2/n) + b*n**3*x**n*gamma(2 + 2/n)) - 4*a**(
-3 - 2/n)*a**(1 + 2/n)*b*x**n*x**(n + 2)*lerchphi(b*x**n*exp_polar(I*pi...
```

Maxima [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^2}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(B*a - A*b)*x^2/(a*b^2*n*x^n + a^2*b*n) + (A*b*(n - 2) + 2*B*a)*integrate
(x/(a*b^2*n*x^n + a^2*b*n), x)
```

Giac [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^2}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{x\,(A+B\,x^n)}{(a+b\,x^n)^2}\,dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n)^2,x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x(A + Bx^n)}{(a + bx^n)^2} dx = \int \frac{x}{x^n b + a} dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x/(x**n*b + a),x)
```

3.277 $\int \frac{A+Bx^n}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{A+Bx^n}{(a+bx^n)^2} dx = \frac{Bx}{b(1-n)(a+bx^n)} - \frac{(aB-Ab(1-n))x \,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2b(1-n)}$$

output
```
B*x/b/(1-n)/(a+b*x^n)-(B*a-A*b*(1-n))*x*hypergeom([2, 1/n],[1+1/n],-b*x^n/
a)/a^2/b/(1-n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^n}{(a+bx^n)^2} dx = \frac{x\left(\frac{B}{a+bx^n} - \frac{(aB+Ab(-1+n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

input
```
Integrate[(A + B*x^n)/(a + b*x^n)^2,x]
```

output
```
(x*(B/(a + b*x^n) - ((a*B + A*b*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
 n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(aB-Ab(1-n))\int \frac{1}{bx^n+a}dx}{abn}+\frac{x(Ab-aB)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x(aB-Ab(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(Ab-aB)}{abn\,(a+bx^n)}$$

input
```
Int[(A + B*x^n)/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*x)/(a*b*n*(a + b*x^n)) + ((a*B - A*b*(1 - n))*x*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^2}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.84 (sec) , antiderivative size = 741, normalized size of antiderivative = 10.44

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/(a+b*x**n)**2,x)
```

output
```
A*(a*a**(1/n)*a**(-2 - 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)
*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) + a*a**(1
/n)*a**(-2 - 1/n)*n*x*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamm
a(1 + 1/n)) - a*a**(1/n)*a**(-2 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, 1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))
+ a**(1/n)*a**(-2 - 1/n)*b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
1/n)*gamma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n)) - a**
(1/n)*a**(-2 - 1/n)*b*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*ga
mma(1/n)/(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + B*(a*a**(
-3 - 1/n)*a**(1 + 1/n)*n**2*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*n*x**(n +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*g
amma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n)) + a*a**(-3 - 1/n)*a**(1 + 1/n)
*n*x**(n + 1)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2
+ 1/n)) - a*a**(-3 - 1/n)*a**(1 + 1/n)*x**(n + 1)*lerchphi(b*x**n*exp_pola
r(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/n) + b*n**3*x**n
*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*n*x**n*x**(n + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*n**3*gamma(2 + 1/
n) + b*n**3*x**n*gamma(2 + 1/n)) - a**(-3 - 1/n)*a**(1 + 1/n)*b*x**n*x**(n
 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*...
```

Maxima [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^2}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(A*b*(n - 1) + B*a)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) - (B*a - A*b)*
x/(a*b^2*n*x^n + a^2*b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^2}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx = \int \frac{A+B\,x^n}{(a+b\,x^n)^2}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n)^2,x)
```

output
```
int((A + B*x^n)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^2}\,dx=\int \frac{1}{x^nb+a}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(1/(x**n*b + a),x)
```

3.278 $\int \frac{A+Bx^n}{x(a+bx^n)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^2} dx = \frac{Ab-aB}{abn\left(a+bx^n\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^n\right)}{a^2n}$$

output
```
(A*b-B*a)/a/b/n/(a+b*x^n)+A*ln(x)/a^2-A*ln(a+b*x^n)/a^2/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^2} dx = \frac{\frac{a(Ab-aB)}{b(a+bx^n)} + A\log\left(x^n\right) - A\log\left(a+bx^n\right)}{a^2n}$$

input
```
Integrate[(A + B*x^n)/(x*(a + b*x^n)^2),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x^n)) + A*Log[x^n] - A*Log[a + b*x^n])/(a^2*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x (a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{(bx^n+a)^2}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{Ax^{-n}}{a^2}-\frac{Ab}{a^2(bx^n+a)}+\frac{aB-Ab}{a(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{A\log(a+bx^n)}{a^2}+\frac{A\log(x^n)}{a^2}+\frac{Ab-aB}{ab(a+bx^n)}}{n}$$

input
```
Int[(A + B*x^n)/(x*(a + b*x^n)^2),x]
```

output
```
((A*b - a*B)/(a*b*(a + b*x^n)) + (A*Log[x^n])/a^2 - (A*Log[a + b*x^n])/a^2
)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{A\ln(x^n)}{a^2}-\frac{-Ab+Ba}{ab(a+b\,x^n)}-\frac{A\ln(a+b\,x^n)}{a^2}}{n}$	54
default	$\frac{\frac{A\ln(x^n)}{a^2}-\frac{-Ab+Ba}{ab(a+b\,x^n)}-\frac{A\ln(a+b\,x^n)}{a^2}}{n}$	54
risch	$\frac{A\ln(x)}{a^2}+\frac{A}{an(a+b\,x^n)}-\frac{B}{bn(a+b\,x^n)}-\frac{A\ln\left(x^n+\frac{a}{b}\right)}{a^2n}$	63
norman	$\frac{\frac{A\ln(x)}{a}+\frac{bA\ln(x)\mathrm{e}^{n\ln(x)}}{a^2}-\frac{(Ab-Ba)\mathrm{e}^{n\ln(x)}}{a^2n}}{a+b\,\mathrm{e}^{n\ln(x)}}-\frac{A\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{a^2n}$	75
parallelrisch	$\frac{A\,x^n\ln(x)b^2n+A\ln(x)abn-A\ln(a+b\,x^n)x^nb^2-A\ln(a+b\,x^n)ab+abA-a^2B}{a^2bn(a+b\,x^n)}$	79

input
```
int((A+B*x^n)/x/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*(A/a^2*ln(x^n)-(-A*b+B*a)/a/b/(a+b*x^n)-A/a^2*ln(a+b*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int\frac{A+Bx^n}{x\left(a+bx^n\right)^2}\,dx$$
$$=\frac{Ab^2nx^n\log\left(x\right)+Aabn\log\left(x\right)-Ba^2+Aab-\left(Ab^2x^n+Aab\right)\log\left(bx^n+a\right)}{a^2b^2nx^n+a^3bn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
(A*b^2*n*x^n*log(x) + A*a*b*n*log(x) - B*a^2 + A*a*b - (A*b^2*x^n + A*a*b)
*log(b*x^n + a))/(a^2*b^2*n*x^n + a^3*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(41) = 82$.

Time = 1.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.77

$$\int \frac{A + Bx^n}{x\,(a + bx^n)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}(A+B)\log(x) & \text{for } a = 0 \wedge \\ \frac{A\log(x)+\frac{Bx^n}{n}}{a^2} & \text{for } b = 0 \\ \frac{-\frac{Ax^{-2n}}{2n}-\frac{Bx^{-n}}{n}}{b^2} & \text{for } a = 0 \\ \tilde{\infty}\left(A\log(x)+\frac{Bx^n}{n}\right) & \text{for } b = -ax \\ \frac{(A+B)\log(x)}{(a+b)^2} & \text{for } n = 0 \\ \frac{Aabn\log(x)}{a^3bn+a^2b^2nx^n} - \frac{Aab\log\left(\frac{a}{b}+x^n\right)}{a^3bn+a^2b^2nx^n} + \frac{Aab}{a^3bn+a^2b^2nx^n} + \frac{Ab^2nx^n\log(x)}{a^3bn+a^2b^2nx^n} - \frac{Ab^2x^n\log\left(\frac{a}{b}+x^n\right)}{a^3bn+a^2b^2nx^n} - \frac{Ba^2}{a^3bn+a^2b^2nx^n} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n)**2,x)
```

output
```
Piecewise((zoo*(A + B)*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), ((A*log(x)
 + B*x**n/n)/a**2, Eq(b, 0)), ((-A/(2*n*x**(2*n)) - B/(n*x**n))/b**2, Eq(a
, 0)), (zoo*(A*log(x) + B*x**n/n), Eq(b, -a/x**n)), ((A + B)*log(x)/(a + b
)**2, Eq(n, 0)), (A*a*b*n*log(x)/(a**3*b*n + a**2*b**2*n*x**n) - A*a*b*log
(a/b + x**n)/(a**3*b*n + a**2*b**2*n*x**n) + A*a*b/(a**3*b*n + a**2*b**2*n
*x**n) + A*b**2*n*x**n*log(x)/(a**3*b*n + a**2*b**2*n*x**n) - A*b**2*x**n*
log(a/b + x**n)/(a**3*b*n + a**2*b**2*n*x**n) - B*a**2/(a**3*b*n + a**2*b*
*2*n*x**n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^2}\,dx = A\left(\frac{1}{abnx^n+a^2n}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^2n}\right)-\frac{B}{b^2nx^n+abn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
A*(1/(a*b*n*x^n + a^2*n) + log(x)/a^2 - log((b*x^n + a)/b)/(a^2*n)) - B/(b
^2*n*x^n + a*b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^2}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^2x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^2}\,dx = \frac{A\ln\left(x\right)}{a^2}-\frac{A\ln\left(a+b\,x^n\right)}{a^2\,n}+\frac{A\,b-B\,a}{a\,b\,n\,\left(a+b\,x^n\right)}$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)^2),x)
```

output
```
(A*log(x))/a^2 - (A*log(a + b*x^n))/(a^2*n) + (A*b - B*a)/(a*b*n*(a + b*x^
n))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.42

$$\int \frac{A + Bx^n}{x\,(a + bx^n)^2}\,dx = \frac{-\log(x^n b + a) + \log(x)\,n}{an}$$

input
```
int((A+B*x^n)/x/(a+b*x^n)^2,x)
```

output
```
( - log(x**n*b + a) + log(x)*n)/(a*n)
```

3.279 $\int \frac{A+Bx^n}{x^2(a+bx^n)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^2} dx = \frac{Ab-aB}{abnx(a+bx^n)} + \frac{(aB-Ab(1+n))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2bnx}$$

output
```
(A*b-B*a)/a/b/n/x/(a+b*x^n)+(B*a-A*b*(1+n))*hypergeom([1, -1/n],[-(1-n)/n]
,-b*x^n/a)/a^2/b/n/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^2} dx = \frac{(A-An)\,\text{Hypergeometric2F1}\left(2,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(2,\frac{-1+n}{n},2-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(-1+n)x}$$

input
```
Integrate[(A + B*x^n)/(x^2*(a + b*x^n)^2),x]
```

output
```
((A - A*n)*Hypergeometric2F1[2, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] + B*x^n
*Hypergeometric2F1[2, (-1 + n)/n, 2 - n^(-1), -((b*x^n)/a)])/(a^2*(-1 + n)
*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{abnx(a+bx^n)}-\frac{(aB-Ab(n+1))\int \frac{1}{x^2(bx^n+a)}dx}{abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(aB-Ab(n+1))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2bnx}+\frac{Ab-aB}{abnx(a+bx^n)}$$

input
```
Int[(A + B*x^n)/(x^2*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*x*(a + b*x^n)) + ((a*B - A*b*(1 + n))*Hypergeometric2F1
[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*b*n*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2} \left(a+b\,x^{n}\right)^{2}}d x$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^2}\, dx=\int \frac{Bx^{n}+A}{{\left(bx^{n}+a\right)}^{2}x^{2}}\, dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x^n + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.45 (sec) , antiderivative size = 780, normalized size of antiderivative = 9.63

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n)**2,x)
```

output
```
A*(-a*a**(-2 + 1/n)*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi
)/n)*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**
n*gamma(1 - 1/n)) - a*a**(-2 + 1/n)*n*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma
(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*gamma(1 - 1/n)) - a*a**(-2 + 1/n)*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*gamma(-1/n)/(a*a**(1/n
)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*gamma(1 - 1/n)) - a**(-2
+ 1/n)*b*n*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, exp_polar(I*pi)/n)*g
amma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)*b*n**3*x*x**n*gamm
a(1 - 1/n)) - a**(-2 + 1/n)*b*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, e
xp_polar(I*pi)/n)*gamma(-1/n)/(a*a**(1/n)*n**3*x*gamma(1 - 1/n) + a**(1/n)
*b*n**3*x*x**n*gamma(1 - 1/n))) + B*(a*a**(-3 + 1/n)*a**(1 - 1/n)*n**2*x**
(n - 1)*gamma(1 - 1/n)/(a*n**3*gamma(2 - 1/n) + b*n**3*x**n*gamma(2 - 1/n)
) + a*a**(-3 + 1/n)*a**(1 - 1/n)*n*x**(n - 1)*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, 1 - 1/n)*gamma(1 - 1/n)/(a*n**3*gamma(2 - 1/n) + b*n**3*x**n*gam
ma(2 - 1/n)) - a*a**(-3 + 1/n)*a**(1 - 1/n)*n*x**(n - 1)*gamma(1 - 1/n)/(a
*n**3*gamma(2 - 1/n) + b*n**3*x**n*gamma(2 - 1/n)) - a*a**(-3 + 1/n)*a**(1
 - 1/n)*x**(n - 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 1/n)*gamma(1
- 1/n)/(a*n**3*gamma(2 - 1/n) + b*n**3*x**n*gamma(2 - 1/n)) + a**(-3 + 1/n
)*a**(1 - 1/n)*b*n*x**n*x**(n - 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 - 1/n)*gamma(1 - 1/n)/(a*n**3*gamma(2 - 1/n) + b*n**3*x**n*gamma(2 - 1...
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(A*b*(n + 1) - B*a)*integrate(1/(a*b^2*n*x^2*x^n + a^2*b*n*x^2), x) - (B*a
 - A*b)/(a*b^2*n*x*x^n + a^2*b*n*x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{A+B\,x^n}{x^2\,\left(a+b\,x^n\right)^2}\,dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^n b\,x^2+a\,x^2}dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^2,x)
```

output
```
int(1/(x**n*b*x**2 + a*x**2),x)
```

3.280 $\int \frac{A+Bx^n}{x^3(a+bx^n)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^2} \, dx=\frac{Ab-aB}{abnx^2\left(a+bx^n\right)}+\frac{(2aB-Ab(2+n)) \text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2bnx^2}$$

output
```
(A*b-B*a)/a/b/n/x^2/(a+b*x^n)+1/2*(2*B*a-A*b*(2+n))*hypergeom([1, -2/n],[-
(2-n)/n],-b*x^n/a)/a^2/b/n/x^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^2} \, dx = \frac{-A(-2+n) \text{Hypergeometric2F1}\left(2,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n \text{Hypergeometric2F1}\left(2,\frac{-2+n}{n},2-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^2(-2+n)x^2}$$

input
```
Integrate[(A + B*x^n)/(x^3*(a + b*x^n)^2),x]
```

output
```
(-(A*(-2 + n)*Hypergeometric2F1[2, -2/n, (-2 + n)/n, -((b*x^n)/a)]) + 2*B*
x^n*Hypergeometric2F1[2, (-2 + n)/n, 2 - 2/n, -((b*x^n)/a)])/(2*a^2*(-2 +
n)*x^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3 (a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{abnx^2 (a+bx^n)}-\frac{(2aB-Ab(n+2)) \int \frac{1}{x^3 (bx^n+a)}dx}{abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(2aB-Ab(n+2)) \text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2bnx^2}+\frac{Ab-aB}{abnx^2 (a+bx^n)}$$

input
```
Int[(A + B*x^n)/(x^3*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*x^2*(a + b*x^n)) + ((2*a*B - A*b*(2 + n))*Hypergeometri
c2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*b*n*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{x^{3} \left(a + b\,x^{n}\right)^{2}}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A + Bx^{n}}{x^{3} \left(a + bx^{n}\right)^{2}}\, dx = \int \frac{Bx^{n} + A}{\left(bx^{n} + a\right)^{2} x^{3}}\, dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x^n + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.41 (sec) , antiderivative size = 821, normalized size of antiderivative = 9.66

$$\int \frac{A+Bx^n}{x^3 (a+bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n)**2,x)
```

output
```
A*(-2*a*a**(-2 + 2/n)*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(
I*pi)/n)*gamma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**
3*x**2*x**n*gamma(1 - 2/n)) - 2*a*a**(-2 + 2/n)*n*gamma(-2/n)/(a*a**(2/n)*
n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2/n)) - 4*a
*a**(-2 + 2/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*
gamma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x*
*n*gamma(1 - 2/n)) - 2*a**(-2 + 2/n)*b*n*x**n*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a*a**(2/n)*n**3*x**2*gamma(1 -
 2/n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2/n)) - 4*a**(-2 + 2/n)*b*x**n
*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2*exp_polar(I*pi)/n)*gamma(-2/n)/(a
*a**(2/n)*n**3*x**2*gamma(1 - 2/n) + a**(2/n)*b*n**3*x**2*x**n*gamma(1 - 2
/n))) + B*(a*a**(-3 + 2/n)*a**(1 - 2/n)*n**2*x**(n - 2)*gamma(1 - 2/n)/(a*
n**3*gamma(2 - 2/n) + b*n**3*x**n*gamma(2 - 2/n)) + 2*a*a**(-3 + 2/n)*a**(
1 - 2/n)*n*x**(n - 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 2/n)*gamma
(1 - 2/n)/(a*n**3*gamma(2 - 2/n) + b*n**3*x**n*gamma(2 - 2/n)) - 2*a*a**(-
3 + 2/n)*a**(1 - 2/n)*n*x**(n - 2)*gamma(1 - 2/n)/(a*n**3*gamma(2 - 2/n) +
 b*n**3*x**n*gamma(2 - 2/n)) - 4*a*a**(-3 + 2/n)*a**(1 - 2/n)*x**(n - 2)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 2/n)*gamma(1 - 2/n)/(a*n**3*gamma
(2 - 2/n) + b*n**3*x**n*gamma(2 - 2/n)) + 2*a**(-3 + 2/n)*a**(1 - 2/n)*b*n
*x**n*x**(n - 2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 2/n)*gamma(1...
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(A*b*(n + 2) - 2*B*a)*integrate(1/(a*b^2*n*x^3*x^n + a^2*b*n*x^3), x) - (B
*a - A*b)/(a*b^2*n*x^2*x^n + a^2*b*n*x^2)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^2}\,dx=\int \frac{A+B\,x^n}{x^3\,\left(a+b\,x^n\right)^2}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{x^3 (a + bx^n)^2} dx = \int \frac{1}{x^n b x^3 + a x^3} dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^2,x)
```

output
```
int(1/(x**n*b*x**3 + a*x**3),x)
```

3.281 $\int \frac{A+Bx^n}{x^4(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^2}\,dx=\frac{Ab-aB}{abnx^3\left(a+bx^n\right)}+\frac{\left(3aB-Ab(3+n)\right)\text{Hypergeometric2F1}\left(1,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3a^2bnx^3}$$

output
```
(A*b-B*a)/a/b/n/x^3/(a+b*x^n)+1/3*(3*B*a-A*b*(3+n))*hypergeom([1, -3/n],[-
(3-n)/n],-b*x^n/a)/a^2/b/n/x^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^2}\,dx=\frac{-A(-3+n)\,\text{Hypergeometric2F1}\left(2,-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(2,\frac{-3+n}{n},2-\frac{3}{n},-\frac{bx^n}{a}\right.}{3a^2(-3+n)x^3}$$

input
```
Integrate[(A + B*x^n)/(x^4*(a + b*x^n)^2),x]
```

output
```
(-(A*(-3 + n)*Hypergeometric2F1[2, -3/n, (-3 + n)/n, -((b*x^n)/a)]) + 3*B*
x^n*Hypergeometric2F1[2, (-3 + n)/n, 2 - 3/n, -((b*x^n)/a)])/(3*a^2*(-3 +
n)*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{abnx^3 (a+bx^n)}-\frac{(3aB-Ab(n+3)) \int \frac{1}{x^4 (bx^n+a)}dx}{abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(3aB-Ab(n+3)) \text{Hypergeometric2F1}\left(1,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3a^2bnx^3}+\frac{Ab-aB}{abnx^3 (a+bx^n)}$$

input
```
Int[(A + B*x^n)/(x^4*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*x^3*(a + b*x^n)) + ((3*a*B - A*b*(3 + n))*Hypergeometri
c2F1[1, -3/n, -((3 - n)/n), -((b*x^n)/a)])/(3*a^2*b*n*x^3)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{4} \left(a+b\,x^{n}\right)^{2}}d x$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^2}\,dx=\int \frac{Bx^{n}+A}{{\left(bx^{n}+a\right)}^{2}x^{4}}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^2*x^4*x^(2*n) + 2*a*b*x^4*x^n + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.21 (sec) , antiderivative size = 821, normalized size of antiderivative = 9.66

$$\int \frac{A+Bx^n}{x^4(a+bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n)**2,x)
```

output
```
A*(-3*a*a**(-2 + 3/n)*n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3*exp_polar(
I*pi)/n)*gamma(-3/n)/(a*a**(3/n)*n**3*x**3*gamma(1 - 3/n) + a**(3/n)*b*n**
3*x**3*x**n*gamma(1 - 3/n)) - 3*a*a**(-2 + 3/n)*n*gamma(-3/n)/(a*a**(3/n)*
n**3*x**3*gamma(1 - 3/n) + a**(3/n)*b*n**3*x**3*x**n*gamma(1 - 3/n)) - 9*a
*a**(-2 + 3/n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3*exp_polar(I*pi)/n)*
gamma(-3/n)/(a*a**(3/n)*n**3*x**3*gamma(1 - 3/n) + a**(3/n)*b*n**3*x**3*x*
*n*gamma(1 - 3/n)) - 3*a**(-2 + 3/n)*b*n*x**n*lerchphi(b*x**n*exp_polar(I*
pi)/a, 1, 3*exp_polar(I*pi)/n)*gamma(-3/n)/(a*a**(3/n)*n**3*x**3*gamma(1 -
 3/n) + a**(3/n)*b*n**3*x**3*x**n*gamma(1 - 3/n)) - 9*a**(-2 + 3/n)*b*x**n
*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3*exp_polar(I*pi)/n)*gamma(-3/n)/(a
*a**(3/n)*n**3*x**3*gamma(1 - 3/n) + a**(3/n)*b*n**3*x**3*x**n*gamma(1 - 3
/n))) + B*(a*a**(-3 + 3/n)*a**(1 - 3/n)*n**2*x**(n - 3)*gamma(1 - 3/n)/(a*
n**3*gamma(2 - 3/n) + b*n**3*x**n*gamma(2 - 3/n)) + 3*a*a**(-3 + 3/n)*a**(
1 - 3/n)*n*x**(n - 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 3/n)*gamma
(1 - 3/n)/(a*n**3*gamma(2 - 3/n) + b*n**3*x**n*gamma(2 - 3/n)) - 3*a*a**(-
3 + 3/n)*a**(1 - 3/n)*n*x**(n - 3)*gamma(1 - 3/n)/(a*n**3*gamma(2 - 3/n) +
 b*n**3*x**n*gamma(2 - 3/n)) - 9*a*a**(-3 + 3/n)*a**(1 - 3/n)*x**(n - 3)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 3/n)*gamma(1 - 3/n)/(a*n**3*gamma
(2 - 3/n) + b*n**3*x**n*gamma(2 - 3/n)) + 3*a**(-3 + 3/n)*a**(1 - 3/n)*b*n
*x**n*x**(n - 3)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 - 3/n)*gamma(1...
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^2} dx = \int \frac{Bx^n+A}{(bx^n+a)^2 x^4} dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
(A*b*(n + 3) - 3*B*a)*integrate(1/(a*b^2*n*x^4*x^n + a^2*b*n*x^4), x) - (B
*a - A*b)/(a*b^2*n*x^3*x^n + a^2*b*n*x^3)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^2} dx = \int \frac{Bx^n+A}{(bx^n+a)^2 x^4} dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^2} dx = \int \frac{A+B\,x^n}{x^4\,(a+b\,x^n)^2} dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{x^4 (a + bx^n)^2} \, dx = \int \frac{1}{x^n b \, x^4 + a \, x^4} dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^2,x)
```

output
```
int(1/(x**n*b*x**4 + a*x**4),x)
```

3.282 $\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{(Ab-aB)x^3}{2abn\,(a+bx^n)^2} + \frac{(3aB-Ab(3-2n))x^3\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{6a^3bn}$$

output
```
1/2*(A*b-B*a)*x^3/a/b/n/(a+b*x^n)^2+1/6*(3*B*a-A*b*(3-2*n))*x^3*hypergeom(
[2, 3/n],[(3+n)/n],-b*x^n/a)/a^3/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{Ax^3\,\text{Hypergeometric2F1}\left(3,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^3} + \frac{Bx^{3+n}\,\text{Hypergeometric2F1}\left(3,\frac{3+n}{n},2+\frac{3}{n},-\frac{bx^n}{a}\right)}{a^3(3+n)}$$

input
```
Integrate[(x^2*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
(A*x^3*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3) + (B*x^
(3 + n)*Hypergeometric2F1[3, (3 + n)/n, 2 + 3/n, -((b*x^n)/a)])/(a^3*(3 +
n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3aB-Ab(3-2n))\int \frac{x^2}{(bx^n+a)^2}dx}{2abn}+\frac{x^3(Ab-aB)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(3aB-Ab(3-2n))\,\text{Hypergeometric2F1}\left(2,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{6a^3bn}+\frac{x^3(Ab-aB)}{2abn\,(a+bx^n)^2}$$

input
```
Int[(x^2*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*x^3)/(2*a*b*n*(a + b*x^n)^2) + ((3*a*B - A*b*(3 - 2*n))*x^3*H
ypergeometric2F1[2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(6*a^3*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{2} \left(A+B\,x^{n}\right)}{\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx=\int \frac{(Bx^n+A)x^2}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2*x^n + A*x^2)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
+ a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 68.19 (sec) , antiderivative size = 2360, normalized size of antiderivative = 27.13

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
A*(6*a**2*a**(3/n)*a**(-3 - 3/n)*n**2*x**3*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, 3/n)*gamma(3/n)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*gamma
(1 + 3/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) + 9*a**2*a**(3/n)*a**(-3
- 3/n)*n**2*x**3*gamma(3/n)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*
gamma(1 + 3/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) - 27*a**2*a**(3/n)*a
**(-3 - 3/n)*n*x**3*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/n)*gamma(3/n)/
(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*gamma(1 + 3/n) + 2*b**2*n**4
*x**(2*n)*gamma(1 + 3/n)) - 9*a**2*a**(3/n)*a**(-3 - 3/n)*n*x**3*gamma(3/n
)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*gamma(1 + 3/n) + 2*b**2*n*
*4*x**(2*n)*gamma(1 + 3/n)) + 27*a**2*a**(3/n)*a**(-3 - 3/n)*x**3*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, 3/n)*gamma(3/n)/(2*a**2*n**4*gamma(1 + 3/n)
+ 4*a*b*n**4*x**n*gamma(1 + 3/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) +
12*a*a**(3/n)*a**(-3 - 3/n)*b*n**2*x**3*x**n*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 3/n)*gamma(3/n)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*gam
ma(1 + 3/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) + 6*a*a**(3/n)*a**(-3 -
 3/n)*b*n**2*x**3*x**n*gamma(3/n)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4
*x**n*gamma(1 + 3/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) - 54*a*a**(3/n
)*a**(-3 - 3/n)*b*n*x**3*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/n)*g
amma(3/n)/(2*a**2*n**4*gamma(1 + 3/n) + 4*a*b*n**4*x**n*gamma(1 + 3/n) + 2
*b**2*n**4*x**(2*n)*gamma(1 + 3/n)) - 9*a*a**(3/n)*a**(-3 - 3/n)*b*n*x**...
```

Maxima [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 9*n + 9)*A*b + 3*B*a*(n - 3))*integrate(1/2*x^2/(a^2*b^2*n^2*x^n
 + a^3*b*n^2), x) + 1/2*((A*b^2*(2*n - 3) + 3*B*a*b)*x^3*x^n + (3*A*a*b*(n
 - 1) - B*a^2*(n - 3))*x^3)/(a^2*b^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4
*b*n^2)
```

Giac [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{x^2\,(A+B\,x^n)}{(a+b\,x^n)^3}\,dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n)^3,x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{x^2}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x**2/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.283 $\int \frac{x(A+Bx^n)}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 86

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3} dx = \frac{(Ab-aB)x^2}{2abn(a+bx^n)^2} + \frac{(aB-Ab(1-n))x^2 \text{Hypergeometric2F1}\left(2,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^3bn}$$

output
```
1/2*(A*b-B*a)*x^2/a/b/n/(a+b*x^n)^2+1/2*(B*a-A*b*(1-n))*x^2*hypergeom([2,
2/n],[(2+n)/n],-b*x^n/a)/a^3/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3} dx = \frac{Ax^2 \text{Hypergeometric2F1}\left(3,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^3} + \frac{Bx^{2+n} \text{Hypergeometric2F1}\left(3,\frac{2+n}{n},2\left(1+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^3(2+n)}$$

input
```
Integrate[(x*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
(A*x^2*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3) + (B*x^
(2 + n)*Hypergeometric2F1[3, (2 + n)/n, 2*(1 + n^(-1)), -((b*x^n)/a)])/(a^
3*(2 + n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(aB-Ab(1-n))\int \frac{x}{(bx^n+a)^2}dx}{abn}+\frac{x^2(Ab-aB)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \; 888$$

$$\frac{x^2(aB-Ab(1-n))\,\text{Hypergeometric2F1}\left(2,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^3bn}+\frac{x^2(Ab-aB)}{2abn\,(a+bx^n)^2}$$

input
```
Int[(x*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*x^2)/(2*a*b*n*(a + b*x^n)^2) + ((a*B - A*b*(1 - n))*x^2*Hyper
geometric2F1[2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x(A+B\,x^{n})}{(a+b\,x^{n})^{3}}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x(A+Bx^{n})}{(a+bx^{n})^{3}}\,dx=\int \frac{(Bx^{n}+A)x}{(bx^{n}+a)^{3}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^
3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.54 (sec) , antiderivative size = 2315, normalized size of antiderivative = 26.92

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
A*(2*a**2*a**(2/n)*a**(-3 - 2/n)*n**2*x**2*lerchphi(b*x**n*exp_polar(I*pi)
/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1
 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) + 3*a**2*a**(2/n)*a**(-3 - 2/
n)*n**2*x**2*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(
1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) - 6*a**2*a**(2/n)*a**(-3 - 2
/n)*n*x**2*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*n**
4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gam
ma(1 + 2/n)) - 2*a**2*a**(2/n)*a**(-3 - 2/n)*n*x**2*gamma(2/n)/(a**2*n**4*
gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma
(1 + 2/n)) + 4*a**2*a**(2/n)*a**(-3 - 2/n)*x**2*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*ga
mma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n)) + 4*a*a**(2/n)*a**(-3 -
2/n)*b*n**2*x**2*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n
)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x
**(2*n)*gamma(1 + 2/n)) + 2*a*a**(2/n)*a**(-3 - 2/n)*b*n**2*x**2*x**n*gamm
a(2/n)/(a**2*n**4*gamma(1 + 2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n
**4*x**(2*n)*gamma(1 + 2/n)) - 12*a*a**(2/n)*a**(-3 - 2/n)*b*n*x**2*x**n*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 2/n)*gamma(2/n)/(a**2*n**4*gamma(1 +
2/n) + 2*a*b*n**4*x**n*gamma(1 + 2/n) + b**2*n**4*x**(2*n)*gamma(1 + 2/n))
 - 2*a*a**(2/n)*a**(-3 - 2/n)*b*n*x**2*x**n*gamma(2/n)/(a**2*n**4*gamma...
```

Maxima [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^3}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((n^2 - 3*n + 2)*A*b + B*a*(n - 2))*integrate(x/(a^2*b^2*n^2*x^n + a^3*b*n
^2), x) + 1/2*(2*(A*b^2*(n - 1) + B*a*b)*x^2*x^n + (A*a*b*(3*n - 2) - B*a^
2*(n - 2))*x^2)/(a^2*b^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

Giac [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^3}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{x\,(A+B\,x^n)}{(a+b\,x^n)^3}\,dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n)^3,x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x(A + Bx^n)}{(a + bx^n)^3} \, dx = \int \frac{x}{x^{2n}b^2 + 2x^n ab + a^2} dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.284 $\int \frac{A+Bx^n}{(a+bx^n)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{A+Bx^n}{(a+bx^n)^3} dx = \frac{Bx}{b(1-2n)(a+bx^n)^2} + \frac{\left(A-\frac{aB}{b-2bn}\right) x \,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}$$

output
```
B*x/b/(1-2*n)/(a+b*x^n)^2+(A-a*B/(-2*b*n+b))*x*hypergeom([3, 1/n],[1+1/n],
-b*x^n/a)/a^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^n}{(a+bx^n)^3} dx = \frac{x\left(\frac{B}{(a+bx^n)^2} - \frac{(aB+Ab(-1+2n))\,\text{Hypergeometric2F1}\left(3,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3}\right)}{b-2bn}$$

input
```
Integrate[(A + B*x^n)/(a + b*x^n)^3,x]
```

output
```
(x*(B/(a + b*x^n)^2 - ((a*B + A*b*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/a^3))/(b - 2*b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {910, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{ 910}$$

$$\frac{(aB-Ab(1-2n))\int \frac{1}{(bx^n+a)^2}dx}{2abn}+\frac{x(Ab-aB)}{2abn(a+bx^n)^2}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x(aB-Ab(1-2n))\text{Hypergeometric2F1}\left(2,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^3bn}+\frac{x(Ab-aB)}{2abn(a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*x)/(2*a*b*n*(a + b*x^n)^2) + ((a*B - A*b*(1 - 2*n))*x*Hyperge
ometric2F1[2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(2*a^3*b*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^3}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^3}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.66 (sec) , antiderivative size = 2319, normalized size of antiderivative = 38.02

$$\int \frac{A+Bx^n}{(a+bx^n)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
A*(2*a**2*a**(1/n)*a**(-3 - 1/n)*n**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1
+ 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 3*a**2*a**(1/n)*a**(-3 - 1
/n)*n**2*x*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(
1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) - 3*a**2*a**(1/n)*a**(-3 -
 1/n)*n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n*
*4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*
gamma(1 + 1/n)) - a**2*a**(1/n)*a**(-3 - 1/n)*n*x*gamma(1/n)/(2*a**2*n**4*
gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gam
ma(1 + 1/n)) + a**2*a**(1/n)*a**(-3 - 1/n)*x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gam
ma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/n)) + 4*a*a**(1/n)*a**(-3 -
 1/n)*b*n**2*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/
(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4
*x**(2*n)*gamma(1 + 1/n)) + 2*a*a**(1/n)*a**(-3 - 1/n)*b*n**2*x*x**n*gamma
(1/n)/(2*a**2*n**4*gamma(1 + 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**
2*n**4*x**(2*n)*gamma(1 + 1/n)) - 6*a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*le
rchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(2*a**2*n**4*gamma(1 +
 1/n) + 4*a*b*n**4*x**n*gamma(1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(1 + 1/
n)) - a*a**(1/n)*a**(-3 - 1/n)*b*n*x*x**n*gamma(1/n)/(2*a**2*n**4*gamma...
```

Maxima [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^3}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^3}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 - 3*n + 1)*A*b + B*a*(n - 1))*integrate(1/2/(a^2*b^2*n^2*x^n + a^3
*b*n^2), x) + 1/2*((A*b^2*(2*n - 1) + B*a*b)*x*x^n + (A*a*b*(3*n - 1) - B*
a^2*(n - 1))*x)/(a^2*b^3*n^2*x^(2*n) + 2*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

Giac [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^3}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^3}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(a+bx^n)^3}\,dx=\int \frac{A+B\,x^n}{(a+b\,x^n)^3}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n)^3,x)
```

output
```
int((A + B*x^n)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{(a + bx^n)^3} \, dx = \int \frac{1}{x^{2n}b^2 + 2x^nab + a^2} dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.285 $\int \frac{A+Bx^n}{x(a+bx^n)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{A+Bx^n}{x(a+bx^n)^3} dx = \frac{Ab-aB}{2abn(a+bx^n)^2} + \frac{A}{a^2n(a+bx^n)} + \frac{A\log(x)}{a^3} - \frac{A\log(a+bx^n)}{a^3n}$$

output
```
1/2*(A*b-B*a)/a/b/n/(a+b*x^n)^2+A/a^2/n/(a+b*x^n)+A*ln(x)/a^3-A*ln(a+b*x^n
)/a^3/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^n}{x(a+bx^n)^3} dx = \frac{\frac{a(3aAb-a^2B+2Ab^2x^n)}{b(a+bx^n)^2} + 2A\log(x^n) - 2A\log(a+bx^n)}{2a^3n}$$

input
```
Integrate[(A + B*x^n)/(x*(a + b*x^n)^3),x]
```

output
```
((a*(3*a*A*b - a^2*B + 2*A*b^2*x^n))/(b*(a + b*x^n)^2) + 2*A*Log[x^n] - 2*
A*Log[a + b*x^n])/(2*a^3*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x(a+bx^n)^3}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{(bx^n+a)^3}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{Ax^{-n}}{a^3}-\frac{Ab}{a^3(bx^n+a)}-\frac{Ab}{a^2(bx^n+a)^2}+\frac{aB-Ab}{a(bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{A\log(a+bx^n)}{a^3}+\frac{A\log(x^n)}{a^3}+\frac{A}{a^2(a+bx^n)}+\frac{Ab-aB}{2ab(a+bx^n)^2}}{n}$$

input
```
Int[(A + B*x^n)/(x*(a + b*x^n)^3),x]
```

output
```
((A*b - a*B)/(2*a*b*(a + b*x^n)^2) + A/(a^2*(a + b*x^n)) + (A*Log[x^n])/a^
3 - (A*Log[a + b*x^n])/a^3)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{A\ln\left(x^{n}\right)}{a^{3}}-\frac{-Ab+Ba}{2ab\left(a+b\,x^{n}\right)^{2}}-\frac{A\ln\left(a+b\,x^{n}\right)}{a^{3}}+\frac{A}{a^{2}\left(a+b\,x^{n}\right)}}{n}$
default	$\frac{\frac{A\ln\left(x^{n}\right)}{a^{3}}-\frac{-Ab+Ba}{2ab\left(a+b\,x^{n}\right)^{2}}-\frac{A\ln\left(a+b\,x^{n}\right)}{a^{3}}+\frac{A}{a^{2}\left(a+b\,x^{n}\right)}}{n}$
risch	$\frac{A\ln\left(x\right)}{a^{3}}+\frac{2A\,x^{n}b^{2}+3abA-a^{2}B}{2a^{2}bn\left(a+b\,x^{n}\right)^{2}}-\frac{A\ln\left(x^{n}+\frac{a}{b}\right)}{a^{3}n}$
norman	$\frac{\frac{A\ln\left(x\right)}{a}+\frac{b^{2}A\ln\left(x\right){\mathrm e}^{2n\ln\left(x\right)}}{a^{3}}-\frac{\left(2Ab-Ba\right){\mathrm e}^{n\ln\left(x\right)}}{a^{2}n}+\frac{2bA\ln\left(x\right){\mathrm e}^{n\ln\left(x\right)}}{a^{2}}-\frac{b\left(3Ab-Ba\right){\mathrm e}^{2n\ln\left(x\right)}}{2a^{3}n}}{\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)^{2}}-\frac{A\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{a^{3}n}$
parallelrisch	$\frac{2A\ln\left(x\right)x^{2n}b^{2}n+4A\ln\left(x\right)x^{n}abn-2A\ln\left(a+b\,x^{n}\right)x^{2n}b^{2}+2a^{2}A\ln\left(x\right)n-4A\ln\left(a+b\,x^{n}\right)x^{n}ab-3A\,b^{2}x^{2n}+Bab\,x^{2n}-2A\,l}{2a^{3}n\left(a+b\,x^{n}\right)^{2}}$

input
```
int((A+B*x^n)/x/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/n*(A/a^3*ln(x^n)-1/2*(-A*b+B*a)/a/b/(a+b*x^n)^2-A/a^3*ln(a+b*x^n)+A/a^2/
(a+b*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.85

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx$$
$$= \frac{2\,Ab^3nx^{2\,n}\log\left(x\right)+2\,Aa^2bn\log\left(x\right)-Ba^3+3\,Aa^2b+2\,(2\,Aab^2n\log\left(x\right)+Aab^2)x^n-2\,(Ab^3x^{2\,n}+2\,Aa}{2\,(a^3b^3nx^{2\,n}+2\,a^4b^2nx^n+a^5bn)}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*A*b^3*n*x^(2*n)*log(x) + 2*A*a^2*b*n*log(x) - B*a^3 + 3*A*a^2*b + 2
*(2*A*a*b^2*n*log(x) + A*a*b^2)*x^n - 2*(A*b^3*x^(2*n) + 2*A*a*b^2*x^n + A
*a^2*b)*log(b*x^n + a))/(a^3*b^3*n*x^(2*n) + 2*a^4*b^2*n*x^n + a^5*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(60) = 120$.

Time = 2.04 (sec) , antiderivative size = 542, normalized size of antiderivative = 7.53

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx$$
$$= \begin{cases} \tilde{\infty}(A+B)\log\left(x\right) \\ \frac{A\log\left(x\right)+\frac{Bx^n}{n}}{a^3} \\ \frac{-\frac{Ax^{-3n}}{3n}-\frac{Bx^{-2n}}{2n}}{b^3} \\ \tilde{\infty}\left(A\log\left(x\right)+\frac{Bx^n}{n}\right) \\ \frac{(A+B)\log\left(x\right)}{(a+b)^3} \\ \frac{2Aa^2bn\log\left(x\right)}{2a^5bn+4a^4b^2nx^n+2a^3b^3nx^{2n}}-\frac{2Aa^2b\log\left(\frac{a}{b}+x^n\right)}{2a^5bn+4a^4b^2nx^n+2a^3b^3nx^{2n}}+\frac{3Aa^2b}{2a^5bn+4a^4b^2nx^n+2a^3b^3nx^{2n}}+\frac{4Aab^2nx^n\log\left(x\right)}{2a^5bn+4a^4b^2nx^n+2a^3b^3nx^{2n}}- \end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n)**3,x)
```

output
```
Piecewise((zoo*(A + B)*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), ((A*log(x)
 + B*x**n/n)/a**3, Eq(b, 0)), ((-A/(3*n*x**(3*n)) - B/(2*n*x**(2*n)))/b**3
, Eq(a, 0)), (zoo*(A*log(x) + B*x**n/n), Eq(b, -a/x**n)), ((A + B)*log(x)/
(a + b)**3, Eq(n, 0)), (2*A*a**2*b*n*log(x)/(2*a**5*b*n + 4*a**4*b**2*n*x*
*n + 2*a**3*b**3*n*x**(2*n)) - 2*A*a**2*b*log(a/b + x**n)/(2*a**5*b*n + 4*
a**4*b**2*n*x**n + 2*a**3*b**3*n*x**(2*n)) + 3*A*a**2*b/(2*a**5*b*n + 4*a*
*4*b**2*n*x**n + 2*a**3*b**3*n*x**(2*n)) + 4*A*a*b**2*n*x**n*log(x)/(2*a**
5*b*n + 4*a**4*b**2*n*x**n + 2*a**3*b**3*n*x**(2*n)) - 4*A*a*b**2*x**n*log
(a/b + x**n)/(2*a**5*b*n + 4*a**4*b**2*n*x**n + 2*a**3*b**3*n*x**(2*n)) +
2*A*a*b**2*x**n/(2*a**5*b*n + 4*a**4*b**2*n*x**n + 2*a**3*b**3*n*x**(2*n))
 + 2*A*b**3*n*x**(2*n)*log(x)/(2*a**5*b*n + 4*a**4*b**2*n*x**n + 2*a**3*b*
*3*n*x**(2*n)) - 2*A*b**3*x**(2*n)*log(a/b + x**n)/(2*a**5*b*n + 4*a**4*b*
*2*n*x**n + 2*a**3*b**3*n*x**(2*n)) - B*a**3/(2*a**5*b*n + 4*a**4*b**2*n*x
**n + 2*a**3*b**3*n*x**(2*n)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx = \frac{1}{2}\,A\left(\frac{2\,bx^n+3\,a}{a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n}+\frac{2\,\log\left(x\right)}{a^3}-\frac{2\,\log\left(\frac{bx^n+a}{b}\right)}{a^3n}\right) - \frac{B}{2\left(b^3nx^{2\,n}+2\,ab^2nx^n+a^2bn\right)}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
1/2*A*((2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + 2*log
(x)/a^3 - 2*log((b*x^n + a)/b)/(a^3*n)) - 1/2*B/(b^3*n*x^(2*n) + 2*a*b^2*n
*x^n + a^2*b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx=\frac{A\,\ln\left(x\right)}{a^3}+\frac{A}{a^2\,n\,\left(a+b\,x^n\right)}-\frac{A\,\ln\left(a+b\,x^n\right)}{a^3\,n}+\frac{A\,b-B\,a}{2\,a\,b\,n\,\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)^3),x)
```

output
```
(A*log(x))/a^3 + A/(a^2*n*(a + b*x^n)) - (A*log(a + b*x^n))/(a^3*n) + (A*b
 - B*a)/(2*a*b*n*(a^2 + b^2*x^(2*n) + 2*a*b*x^n))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^3}\,dx=\frac{-x^n\log\left(x^nb+a\right)b+x^n\log\left(x\right)bn-x^nb-\log\left(x^nb+a\right)a+\log\left(x\right)an}{a^2n\left(x^nb+a\right)}$$

input
```
int((A+B*x^n)/x/(a+b*x^n)^3,x)
```

output
```
( - x**n*log(x**n*b + a)*b + x**n*log(x)*b*n - x**n*b - log(x**n*b + a)*a
+ log(x)*a*n)/(a**2*n*(x**n*b + a))
```

3.286 $\int \frac{A+Bx^n}{x^2(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^3}\,dx=\frac{Ab-aB}{2abnx\left(a+bx^n\right)^2}+\frac{(aB-A(b+2bn))\,\text{Hypergeometric2F1}\left(2,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{2a^3bnx}$$

output
```
1/2*(A*b-B*a)/a/b/n/x/(a+b*x^n)^2+1/2*(B*a-A*(2*b*n+b))*hypergeom([2, -1/n
],[-(1-n)/n],-b*x^n/a)/a^3/b/n/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^3}\,dx=\frac{(A-An)\,\text{Hypergeometric2F1}\left(3,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(3,\frac{-1+n}{n},2-\frac{1}{n},-\frac{bx^n}{a}\right)}{a^3(-1+n)x}$$

input
```
Integrate[(A + B*x^n)/(x^2*(a + b*x^n)^3),x]
```

output
```
((A - A*n)*Hypergeometric2F1[3, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] + B*x^n
*Hypergeometric2F1[3, (-1 + n)/n, 2 - n^(-1), -((b*x^n)/a)])/(a^3*(-1 + n)
*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{2abnx(a+bx^n)^2}-\frac{(aB-A(2bn+b))\int \frac{1}{x^2(bx^n+a)^2}dx}{2abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(aB-A(2bn+b))\,\text{Hypergeometric2F1}\left(2,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{2a^3bnx}+\frac{Ab-aB}{2abnx(a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(x^2*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*x*(a + b*x^n)^2) + ((a*B - A*(b + 2*b*n))*Hypergeomet
ric2F1[2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(2*a^3*b*n*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2} \left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^3}\,dx = \int \frac{Bx^{n} + A}{{\left(bx^{n} + a\right)}^{3} x^{2}}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^3*x^2*x^(3*n) + 3*a*b^2*x^2*x^(2*n) + 3*a^2*b*x^2*
x^n + a^3*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^3} \, dx = \text{Timed out}$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^3} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^3 x^2} \, dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 + 3*n + 1)*A*b - B*a*(n + 1))*integrate(1/2/(a^2*b^2*n^2*x^2*x^n +
 a^3*b*n^2*x^2), x) + 1/2*(A*a*b*(3*n + 1) - B*a^2*(n + 1) + (A*b^2*(2*n +
 1) - B*a*b)*x^n)/(a^2*b^3*n^2*x*x^(2*n) + 2*a^3*b^2*n^2*x*x^n + a^4*b*n^2
*x)
```

Giac [F]

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^3} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^3 x^2} \, dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^3} \, dx = \int \frac{A+B\,x^n}{x^2\,(a+b\,x^n)^3} \, dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^3} \, dx = \int \frac{1}{x^{2n}b^2x^2+2x^n ab\,x^2+a^2x^2} dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(2*n)*b**2*x**2 + 2*x**n*a*b*x**2 + a**2*x**2),x)
```

3.287 $\int \frac{A+Bx^n}{x^3(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^3}\,dx = \frac{Ab-aB}{2abnx^2(a+bx^n)^2} + \frac{(aB-Ab(1+n))\,\text{Hypergeometric2F1}\left(2,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3bnx^2}$$

output
```
1/2*(A*b-B*a)/a/b/n/x^2/(a+b*x^n)^2+1/2*(B*a-A*b*(1+n))*hypergeom([2, -2/n
],[-(2-n)/n],-b*x^n/a)/a^3/b/n/x^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^3}\,dx = \frac{-A(-2+n)\,\text{Hypergeometric2F1}\left(3,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(3,\frac{-2+n}{n},2-\frac{2}{n},-\frac{bx^n}{a}\right)}{2a^3(-2+n)x^2}$$

input
```
Integrate[(A + B*x^n)/(x^3*(a + b*x^n)^3),x]
```

output
```
(-(A*(-2 + n)*Hypergeometric2F1[3, -2/n, (-2 + n)/n, -((b*x^n)/a)]) + 2*B*
x^n*Hypergeometric2F1[3, (-2 + n)/n, 2 - 2/n, -((b*x^n)/a)])/(2*a^3*(-2 +
n)*x^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^3}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{Ab-aB}{2abnx^2(a+bx^n)^2}-\frac{(aB-Ab(n+1))\int \frac{1}{x^3(bx^n+a)^2}dx}{abn}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(aB-Ab(n+1))\,\text{Hypergeometric2F1}\left(2,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^3bnx^2}+\frac{Ab-aB}{2abnx^2(a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(x^3*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*x^2*(a + b*x^n)^2) + ((a*B - A*b*(1 + n))*Hypergeomet
ric2F1[2, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^3*b*n*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{3} \left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^3 (a+bx^n)^3} \, dx=\int \frac{Bx^{n} + A}{{\left(bx^{n} + a\right)}^{3} x^{3}} \,d x$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^3*x^3*x^(3*n) + 3*a*b^2*x^3*x^(2*n) + 3*a^2*b*x^3*
x^n + a^3*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*A*b - B*a*(n + 2))*integrate(1/(a^2*b^2*n^2*x^3*x^n + a^3
*b*n^2*x^3), x) + 1/2*(A*a*b*(3*n + 2) - B*a^2*(n + 2) + 2*(A*b^2*(n + 1)
- B*a*b)*x^n)/(a^2*b^3*n^2*x^2*x^(2*n) + 2*a^3*b^2*n^2*x^2*x^n + a^4*b*n^2
*x^2)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^3}\,dx=\int \frac{A+B\,x^n}{x^3\,\left(a+b\,x^n\right)^3}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^3}\,dx=\int \frac{1}{x^{2n}b^2x^3+2x^nab\,x^3+a^2x^3}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**3 + a**2*x**3),x)
```

3.288 $\int \frac{A+Bx^n}{x^4(a+bx^n)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3} dx = \frac{Ab-aB}{2abnx^3\left(a+bx^n\right)^2} + \frac{(3aB-Ab(3+2n))\,\text{Hypergeometric2F1}\left(2,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{6a^3bnx^3}$$

output
```
1/2*(A*b-B*a)/a/b/n/x^3/(a+b*x^n)^2+1/6*(3*B*a-A*b*(3+2*n))*hypergeom([2,
-3/n],[-(3-n)/n],-b*x^n/a)/a^3/b/n/x^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3} dx = \frac{-A(-3+n)\,\text{Hypergeometric2F1}\left(3,-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(3,\frac{-3+n}{n},2-\frac{3}{n},-\frac{bx^n}{a}\right.}{3a^3(-3+n)x^3}$$

input
```
Integrate[(A + B*x^n)/(x^4*(a + b*x^n)^3),x]
```

output
```
(-(A*(-3 + n)*Hypergeometric2F1[3, -3/n, (-3 + n)/n, -((b*x^n)/a)]) + 3*B*
x^n*Hypergeometric2F1[3, (-3 + n)/n, 2 - 3/n, -((b*x^n)/a)])/(3*a^3*(-3 +
n)*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^3} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{2abnx^3 (a+bx^n)^2}-\frac{(3aB-Ab(2n+3)) \int \frac{1}{x^4 (bx^n+a)^2}dx}{2abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(3aB-Ab(2n+3)) \text{Hypergeometric2F1}\left(2,-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{6a^3bnx^3}+\frac{Ab-aB}{2abnx^3 (a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(x^4*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*x^3*(a + b*x^n)^2) + ((3*a*B - A*b*(3 + 2*n))*Hyperge
ometric2F1[2, -3/n, -((3 - n)/n), -((b*x^n)/a)])/(6*a^3*b*n*x^3)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{4}\left(a+b\,x^{n}\right)^{3}}dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)/(b^3*x^4*x^(3*n) + 3*a*b^2*x^4*x^(2*n) + 3*a^2*b*x^4*
x^n + a^3*x^4), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
((2*n^2 + 9*n + 9)*A*b - 3*B*a*(n + 3))*integrate(1/2/(a^2*b^2*n^2*x^4*x^n
 + a^3*b*n^2*x^4), x) - 1/2*(B*a^2*(n + 3) - 3*A*a*b*(n + 1) - (A*b^2*(2*n
 + 3) - 3*B*a*b)*x^n)/(a^2*b^3*n^2*x^3*x^(2*n) + 2*a^3*b^2*n^2*x^3*x^n + a
^4*b*n^2*x^3)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx=\int \frac{A+B\,x^n}{x^4\,\left(a+b\,x^n\right)^3}\,dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^3}\,dx=\int \frac{1}{x^{2n}b^2x^4+2x^nab\,x^4+a^2x^4}dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^3,x)
```

output
```
int(1/(x**(2*n)*b**2*x***4 + 2*x***n*a*b*x***4 + a**2*x***4),x)
```

3.289 $\int x^{7/2}(a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^{7/2}(a+bx^n)(A+Bx^n)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2(Ab+aB)x^{\frac{9}{2}+n}}{9+2n}+\frac{2bBx^{\frac{9}{2}+2n}}{9+4n}$$

output
```
2/9*a*A*x^(9/2)+2*(A*b+B*a)*x^(9/2+n)/(9+2*n)+2*b*B*x^(9/2+2*n)/(9+4*n)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{7/2}(a+bx^n)(A+Bx^n)\,dx=2\left(\frac{1}{9}aAx^{9/2}+\frac{(Ab+aB)x^{\frac{9}{2}+n}}{9+2n}+\frac{bBx^{\frac{9}{2}+2n}}{9+4n}\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
2*((a*A*x^(9/2))/9 + ((A*b + a*B)*x^(9/2 + n))/(9 + 2*n) + (b*B*x^(9/2 + 2
*n))/(9 + 4*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx^n)\,(A+Bx^n)\;dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n+\frac{7}{2}}(aB+Ab)+aAx^{7/2}+bBx^{2n+\frac{7}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{n+\frac{9}{2}}(aB+Ab)}{2n+9}+\frac{2}{9}aAx^{9/2}+\frac{2bBx^{2n+\frac{9}{2}}}{4n+9}$$

input
```
Int[x^(7/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(9/2 + n))/(9 + 2*n) + (2*b*B*x^(9/2
+ 2*n))/(9 + 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.42

method	result
orering	$\frac{2x^{\frac{9}{2}}\left(8n^{2}+96n+193\right)\left(a+b\,x^{n}\right)\left(A+B\,x^{n}\right)}{9(9+4n)(9+2n)}-\frac{4x^{2}(7+2n)\left(\frac{7x^{\frac{5}{2}}\left(a+b\,x^{n}\right)\left(A+B\,x^{n}\right)}{2}+x^{\frac{5}{2}}b\,x^{n}n\left(A+B\,x^{n}\right)+x^{\frac{5}{2}}\left(a+b\,x^{n}\right)B\,x^{n}n\right)}{3(9+4n)(9+2n)}+$

input
```
int(x^(7/2)*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^(9/2)*(8*n^2+96*n+193)/(9+4*n)/(9+2*n)*(a+b*x^n)*(A+B*x^n)-4/3*x^2*(
7+2*n)/(9+4*n)/(9+2*n)*(7/2*x^(5/2)*(a+b*x^n)*(A+B*x^n)+x^(5/2)*b*x^n*n*(A
+B*x^n)+x^(5/2)*(a+b*x^n)*B*x^n*n)+8/9/(8*n^2+54*n+81)*x^3*(35/4*x^(3/2)*(
a+b*x^n)*(A+B*x^n)+6*x^(3/2)*b*x^n*n*(A+B*x^n)+6*x^(3/2)*(a+b*x^n)*B*x^n*n
+x^(3/2)*b*x^n*n^2*(A+B*x^n)+2*x^(3/2)*(x^n)^2*n^2*B*b+x^(3/2)*(a+b*x^n)*B
*x^n*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int x^{7/2}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2\left(9\left(2\,Bbn+9\,Bb\right)x^{\frac{9}{2}}x^{2\,n}+9\left(9\,Ba+9\,Ab+4\left(Ba+Ab\right)n\right)x^{\frac{9}{2}}x^{n}+\left(8\,Aan^{2}+54\,Aan+81\right.\right.}{9\left(8\,n^{2}+54\,n+81\right)}$$

input
```
integrate(x^(7/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/9*(9*(2*B*b*n + 9*B*b)*x^(9/2)*x^(2*n) + 9*(9*B*a + 9*A*b + 4*(B*a + A*b
)*n)*x^(9/2)*x^n + (8*A*a*n^2 + 54*A*a*n + 81*A*a)*x^(9/2))/(8*n^2 + 54*n
+ 81)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(a+bx^n)\left(A+Bx^n\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int x^{7/2}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2}{9}\,Aax^{\frac{9}{2}}+\frac{2\,Bbx^{2\,n+\frac{9}{2}}}{4\,n+9}+\frac{2\,Bax^{n+\frac{9}{2}}}{2\,n+9}+\frac{2\,Abx^{n+\frac{9}{2}}}{2\,n+9}$$

input
```
integrate(x^(7/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/9*A*a*x^(9/2) + 2*B*b*x^(2*n + 9/2)/(4*n + 9) + 2*B*a*x^(n + 9/2)/(2*n +
 9) + 2*A*b*x^(n + 9/2)/(2*n + 9)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int x^{7/2}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2}{9}\,Aax^{\frac{9}{2}}+\frac{2\,Bbx^{\frac{9}{2}}\sqrt{x}^{4\,n}}{4\,n+9}+\frac{2\,Bax^{\frac{9}{2}}\sqrt{x}^{2\,n}}{2\,n+9}+\frac{2\,Abx^{\frac{9}{2}}\sqrt{x}^{2\,n}}{2\,n+9}$$

input
```
integrate(x^(7/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
2/9*A*a*x^(9/2) + 2*B*b*x^(9/2)*sqrt(x)^(4*n)/(4*n + 9) + 2*B*a*x^(9/2)*sq
rt(x)^(2*n)/(2*n + 9) + 2*A*b*x^(9/2)*sqrt(x)^(2*n)/(2*n + 9)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2\,A\,a\,x^{9/2}}{9}+\frac{x^n\,x^{9/2}\,(2\,A\,b+2\,B\,a)}{2\,n+9}+\frac{2\,B\,b\,x^{2\,n}\,x^{9/2}}{4\,n+9}$$

input
```
int(x^(7/2)*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(2*A*a*x^(9/2))/9 + (x^n*x^(9/2)*(2*A*b + 2*B*a))/(2*n + 9) + (2*B*b*x^(2*
n)*x^(9/2))/(4*n + 9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int x^{7/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2\sqrt{x}\,x^4(18x^{2n}b^2n+81x^{2n}b^2+72x^nabn+162x^nab+8a^2n^2+54a^2n+81a^2)}{72n^2+486n+729}$$

input
```
int(x^(7/2)*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**4*(18*x**(2*n)*b**2*n + 81*x**(2*n)*b**2 + 72*x**n*a*b*n + 1
62*x**n*a*b + 8*a**2*n**2 + 54*a**2*n + 81*a**2))/(9*(8*n**2 + 54*n + 81))
```

3.290 $\int x^{5/2}(a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^{5/2}(a+bx^n)(A+Bx^n)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2(Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{2bBx^{\frac{7}{2}+2n}}{7+4n}$$

output
```
2/7*a*A*x^(7/2)+2*(A*b+B*a)*x^(7/2+n)/(7+2*n)+2*b*B*x^(7/2+2*n)/(7+4*n)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{5/2}(a+bx^n)(A+Bx^n)\,dx=2\left(\frac{1}{7}aAx^{7/2}+\frac{(Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{bBx^{\frac{7}{2}+2n}}{7+4n}\right)$$

input
```
Integrate[x^(5/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
2*((a*A*x^(7/2))/7 + ((A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (b*B*x^(7/2 + 2
*n))/(7 + 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx^n)\left(A+Bx^n\right)dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(x^{n+\frac{5}{2}}(aB+Ab)+aAx^{5/2}+bBx^{2n+\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x^{n+\frac{7}{2}}(aB+Ab)}{2n+7}+\frac{2}{7}aAx^{7/2}+\frac{2bBx^{2n+\frac{7}{2}}}{4n+7}$$

input
```
Int[x^(5/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (2*b*B*x^(7/2
+ 2*n))/(7 + 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.42

method	result
orering	$\frac{2x^{\frac{7}{2}}(8n^2+72n+109)(a+b\,x^n)(A+B\,x^n)}{7(7+4n)(7+2n)} - \frac{12x^2(5+2n)\left(\frac{5x^{\frac{3}{2}}(a+b\,x^n)(A+B\,x^n)}{2}+x^{\frac{3}{2}}b\,x^n n(A+B\,x^n)+x^{\frac{3}{2}}(a+b\,x^n)B\,x^n n\right)}{7(7+4n)(7+2n)} +$

input
```
int(x^(5/2)*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^(7/2)*(8*n^2+72*n+109)/(7+4*n)/(7+2*n)*(a+b*x^n)*(A+B*x^n)-12/7*x^2*
(5+2*n)/(7+4*n)/(7+2*n)*(5/2*x^(3/2)*(a+b*x^n)*(A+B*x^n)+x^(3/2)*b*x^n*n*(
A+B*x^n)+x^(3/2)*(a+b*x^n)*B*x^n*n)+8/7/(8*n^2+42*n+49)*x^3*(15/4*x^(1/2)*
(a+b*x^n)*(A+B*x^n)+4*x^(1/2)*b*x^n*n*(A+B*x^n)+4*x^(1/2)*(a+b*x^n)*B*x^n*
n+x^(1/2)*b*x^n*n^2*(A+B*x^n)+2*x^(1/2)*b*(x^n)^2*n^2*B+x^(1/2)*(a+b*x^n)*
B*x^n*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int x^{5/2}(a+bx^n)\left(A+Bx^n\right)\,dx = \frac{2\left(7\,(2\,Bbn+7\,Bb)x^{\frac{7}{2}}x^{2\,n}+7\,(7\,Ba+7\,Ab+4\,(Ba+Ab)n)x^{\frac{7}{2}}x^n+(8\,Aan^2+42\,Aan+49\,A\right.}{7\,(8\,n^2+42\,n+49)}$$

input
```
integrate(x^(5/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/7*(7*(2*B*b*n + 7*B*b)*x^(7/2)*x^(2*n) + 7*(7*B*a + 7*A*b + 4*(B*a + A*b
)*n)*x^(7/2)*x^n + (8*A*a*n^2 + 42*A*a*n + 49*A*a)*x^(7/2))/(8*n^2 + 42*n
+ 49)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx^n)\,(A+Bx^n)\;dx=\text{Timed out}$$

input
```
integrate(x**(5/2)*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int x^{5/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2}{7}\,Aax^{\frac{7}{2}}+\frac{2\,Bbx^{2\,n+\frac{7}{2}}}{4\,n+7}+\frac{2\,Bax^{n+\frac{7}{2}}}{2\,n+7}+\frac{2\,Abx^{n+\frac{7}{2}}}{2\,n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/7*A*a*x^(7/2) + 2*B*b*x^(2*n + 7/2)/(4*n + 7) + 2*B*a*x^(n + 7/2)/(2*n +
 7) + 2*A*b*x^(n + 7/2)/(2*n + 7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int x^{5/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2}{7}\,Aax^{\frac{7}{2}}+\frac{2\,Bbx^{\frac{7}{2}}\sqrt{x}^{4\,n}}{4\,n+7}+\frac{2\,Bax^{\frac{7}{2}}\sqrt{x}^{2\,n}}{2\,n+7}+\frac{2\,Abx^{\frac{7}{2}}\sqrt{x}^{2\,n}}{2\,n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
2/7*A*a*x^(7/2) + 2*B*b*x^(7/2)*sqrt(x)^(4*n)/(4*n + 7) + 2*B*a*x^(7/2)*sq
rt(x)^(2*n)/(2*n + 7) + 2*A*b*x^(7/2)*sqrt(x)^(2*n)/(2*n + 7)
```

Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2\,A\,a\,x^{7/2}}{7}+\frac{x^n\,x^{7/2}\,(2\,A\,b+2\,B\,a)}{2\,n+7}+\frac{2\,B\,b\,x^{2\,n}\,x^{7/2}}{4\,n+7}$$

input
```
int(x^(5/2)*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(2*A*a*x^(7/2))/7 + (x^n*x^(7/2)*(2*A*b + 2*B*a))/(2*n + 7) + (2*B*b*x^(2*
n)*x^(7/2))/(4*n + 7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int x^{5/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2\sqrt{x}\,x^3(14x^{2n}b^2n+49x^{2n}b^2+56x^nabn+98x^nab+8a^2n^2+42a^2n+49a^2)}{56n^2+294n+343}$$

input
```
int(x^(5/2)*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**3*(14*x**(2*n)*b**2*n + 49*x**(2*n)*b**2 + 56*x**n*a*b*n + 9
8*x**n*a*b + 8*a**2*n**2 + 42*a**2*n + 49*a**2))/(7*(8*n**2 + 42*n + 49))
```

3.291 $\int x^{3/2}(a+bx^n)\left(A+Bx^n\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^{3/2}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2(Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{2bBx^{\frac{5}{2}+2n}}{5+4n}$$

output
```
2/5*a*A*x^(5/2)+2*(A*b+B*a)*x^(5/2+n)/(5+2*n)+2*b*B*x^(5/2+2*n)/(5+4*n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{3/2}(a+bx^n)\left(A+Bx^n\right)\,dx=2\left(\frac{1}{5}aAx^{5/2}+\frac{(Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{bBx^{\frac{5}{2}+2n}}{5+4n}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
2*((a*A*x^(5/2))/5 + ((A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (b*B*x^(5/2 + 2
*n))/(5 + 4*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx^n)\left(A+Bx^n\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n+\frac{3}{2}}(aB+Ab)+aAx^{3/2}+bBx^{2n+\frac{3}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{n+\frac{5}{2}}(aB+Ab)}{2n+5}+\frac{2}{5}aAx^{5/2}+\frac{2bBx^{2n+\frac{5}{2}}}{4n+5}$$

input
```
Int[x^(3/2)*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (2*b*B*x^(5/2
+ 2*n))/(5 + 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.42

method	result
orering	$\frac{2x^{\frac{5}{2}}(8n^2+48n+49)(a+b\,x^n)(A+B\,x^n)}{5(5+4n)(5+2n)}-\frac{12x^2(3+2n)\left(\frac{3\sqrt{x}\,(a+b\,x^n)(A+B\,x^n)}{2}+\sqrt{x}\,b\,x^n n(A+B\,x^n)+\sqrt{x}\,(a+b\,x^n)B\,x^n n\right)}{5(5+4n)(5+2n)}+$

input
```
int(x^(3/2)*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*(8*n^2+48*n+49)/(5+4*n)/(5+2*n)*(a+b*x^n)*(A+B*x^n)-12/5*x^2*(
3+2*n)/(5+4*n)/(5+2*n)*(3/2*x^(1/2)*(a+b*x^n)*(A+B*x^n)+x^(1/2)*b*x^n*n*(A
+B*x^n)+x^(1/2)*(a+b*x^n)*B*x^n*n)+8/5/(8*n^2+30*n+25)*x^3*(3/4*(a+b*x^n)*
(A+B*x^n)/x^(1/2)+2/x^(1/2)*b*x^n*n*(A+B*x^n)+2/x^(1/2)*(a+b*x^n)*B*x^n*n+
1/x^(1/2)*b*x^n*n^2*(A+B*x^n)+2/x^(1/2)*b*(x^n)^2*n^2*B+1/x^(1/2)*(a+b*x^n
)*B*x^n*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int x^{3/2}(a+bx^n)\,(A+Bx^n)\;dx=\frac{2\left(5\,(2\,Bbn+5\,Bb)x^{\frac{5}{2}}x^{2\,n}+5\,(5\,Ba+5\,Ab+4\,(Ba+Ab)n)x^{\frac{5}{2}}x^{n}+(8\,Aan^{2}+30\,Aan+25\,A\right.}{5\,(8\,n^{2}+30\,n+25)}$$

input
```
integrate(x^(3/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/5*(5*(2*B*b*n + 5*B*b)*x^(5/2)*x^(2*n) + 5*(5*B*a + 5*A*b + 4*(B*a + A*b
)*n)*x^(5/2)*x^n + (8*A*a*n^2 + 30*A*a*n + 25*A*a)*x^(5/2))/(8*n^2 + 30*n
+ 25)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(51) = 102$.

Time = 16.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.47

$$\int x^{3/2}(a+bx^n)(A+Bx^n)\,dx = \begin{cases} \frac{2Aax^{\frac{5}{2}}}{5} + Ab\log(x) + Ba\log(x) - \frac{2Bb}{5x^{\frac{5}{2}}} \\ \frac{2Aax^{\frac{5}{2}}}{5} + \frac{4Abx^{\frac{5}{4}}}{5} + \frac{4Bax^{\frac{5}{4}}}{5} + Bb\log(x) \\ \frac{16Aan^2x^{\frac{5}{2}}}{40n^2+150n+125} + \frac{60Aanx^{\frac{5}{2}}}{40n^2+150n+125} + \frac{50Aax^{\frac{5}{2}}}{40n^2+150n+125} + \frac{40Abnx^{\frac{5}{2}}x^n}{40n^2+150n+125} + \frac{50Abx^{\frac{5}{2}}x^n}{40n^2+150n+125} + \frac{40Banx^{\frac{5}{2}}x^n}{40n^2+150n+125} \end{cases}$$

input
```
integrate(x**(3/2)*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Piecewise((2*A*a*x**(5/2)/5 + A*b*log(x) + B*a*log(x) - 2*B*b/(5*x**(5/2))
, Eq(n, -5/2)), (2*A*a*x**(5/2)/5 + 4*A*b*x**(5/4)/5 + 4*B*a*x**(5/4)/5 +
B*b*log(x), Eq(n, -5/4)), (16*A*a*n**2*x**(5/2)/(40*n**2 + 150*n + 125) +
60*A*a*n*x**(5/2)/(40*n**2 + 150*n + 125) + 50*A*a*x**(5/2)/(40*n**2 + 150
*n + 125) + 40*A*b*n*x**(5/2)*x**n/(40*n**2 + 150*n + 125) + 50*A*b*x**(5/
2)*x**n/(40*n**2 + 150*n + 125) + 40*B*a*n*x**(5/2)*x**n/(40*n**2 + 150*n
+ 125) + 50*B*a*x**(5/2)*x**n/(40*n**2 + 150*n + 125) + 20*B*b*n*x**(5/2)*
x**(2*n)/(40*n**2 + 150*n + 125) + 50*B*b*x**(5/2)*x**(2*n)/(40*n**2 + 150
*n + 125), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int x^{3/2}(a+bx^n)(A+Bx^n)\,dx = \frac{2}{5}Aax^{\frac{5}{2}} + \frac{2\,Bbx^{2\,n+\frac{5}{2}}}{4\,n+5} + \frac{2\,Bax^{n+\frac{5}{2}}}{2\,n+5} + \frac{2\,Abx^{n+\frac{5}{2}}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/5*A*a*x^(5/2) + 2*B*b*x^(2*n + 5/2)/(4*n + 5) + 2*B*a*x^(n + 5/2)/(2*n +
 5) + 2*A*b*x^(n + 5/2)/(2*n + 5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int x^{3/2}(a+bx^n)(A+Bx^n)\,dx = \frac{2}{5}\,Aax^{\frac{5}{2}} + \frac{2\,Bbx^{\frac{5}{2}}\sqrt{x}^{4\,n}}{4\,n+5} + \frac{2\,Bax^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5} + \frac{2\,Abx^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
2/5*A*a*x^(5/2) + 2*B*b*x^(5/2)*sqrt(x)^(4*n)/(4*n + 5) + 2*B*a*x^(5/2)*sq
rt(x)^(2*n)/(2*n + 5) + 2*A*b*x^(5/2)*sqrt(x)^(2*n)/(2*n + 5)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int x^{3/2}(a+bx^n)(A+Bx^n)\,dx = \frac{2\,A\,a\,x^{5/2}}{5} + \frac{x^n\,x^{5/2}\,(2\,A\,b+2\,B\,a)}{2\,n+5} + \frac{2\,B\,b\,x^{2\,n}\,x^{5/2}}{4\,n+5}$$

input
```
int(x^(3/2)*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(2*A*a*x^(5/2))/5 + (x^n*x^(5/2)*(2*A*b + 2*B*a))/(2*n + 5) + (2*B*b*x^(2*
n)*x^(5/2))/(4*n + 5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int x^{3/2}(a+bx^n)(A+Bx^n)\,dx = \frac{2\sqrt{x}\,x^2(10x^{2n}b^2n+25x^{2n}b^2+40x^nabn+50x^nab+8a^2n^2+30a^2n+25a^2)}{40n^2+150n+125}$$

input
```
int(x^(3/2)*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**2*(10*x**(2*n)*b**2*n + 25*x**(2*n)*b**2 + 40*x**n*a*b*n + 5
0*x**n*a*b + 8*a**2*n**2 + 30*a**2*n + 25*a**2))/(5*(8*n**2 + 30*n + 25))
```

3.292 $\int \sqrt{x}(a+bx^n)(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \sqrt{x}(a+bx^n)(A+Bx^n)\,dx = \frac{2}{3}aAx^{3/2} + \frac{2(Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{2bBx^{\frac{3}{2}+2n}}{3+4n}$$

output
```
2/3*a*A*x^(3/2)+2*(A*b+B*a)*x^(3/2+n)/(3+2*n)+2*b*B*x^(3/2+2*n)/(3+4*n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \sqrt{x}(a+bx^n)(A+Bx^n)\,dx = 2\left(\frac{1}{3}aAx^{3/2} + \frac{(Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{bBx^{\frac{3}{2}+2n}}{3+4n}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^n)*(A + B*x^n),x]
```

output
```
2*((a*A*x^(3/2))/3 + ((A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (b*B*x^(3/2 + 2
*n))/(3 + 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^n)\left(A+Bx^n\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n+\frac{1}{2}}(aB+Ab)+aA\sqrt{x}+bBx^{2n+\frac{1}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{n+\frac{3}{2}}(aB+Ab)}{2n+3}+\frac{2}{3}aAx^{3/2}+\frac{2bBx^{2n+\frac{3}{2}}}{4n+3}$$

input
```
Int[Sqrt[x]*(a + b*x^n)*(A + B*x^n),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (2*b*B*x^(3/2
+ 2*n))/(3 + 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.80

method	result
orering	$\frac{2x^{\frac{3}{2}}(8n^2+24n+13)(a+b\,x^n)(A+B\,x^n)}{3(3+4n)(3+2n)} - \frac{4x^2(1+2n)\left(\frac{(a+b\,x^n)(A+B\,x^n)}{2\sqrt{x}}+\frac{b\,x^n n(A+B\,x^n)}{\sqrt{x}}+\frac{(a+b\,x^n)B\,x^n n}{\sqrt{x}}\right)}{(3+4n)(3+2n)} + \frac{8x^3\left(-\frac{(a+b\,x^n}{}\right.}{}$

input
```
int(x^(1/2)*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*(8*n^2+24*n+13)/(3+4*n)/(3+2*n)*(a+b*x^n)*(A+B*x^n)-4*x^2*(1+2
*n)/(3+4*n)/(3+2*n)*(1/2*(a+b*x^n)*(A+B*x^n)/x^(1/2)+1/x^(1/2)*b*x^n*n*(A+
B*x^n)+1/x^(1/2)*(a+b*x^n)*B*x^n*n)+8/3/(8*n^2+18*n+9)*x^3*(-1/4*(a+b*x^n)
*(A+B*x^n)/x^(3/2)+1/x^(3/2)*b*x^n*n^2*(A+B*x^n)+2/x^(3/2)*b*(x^n)^2*n^2*B
+1/x^(3/2)*(a+b*x^n)*B*x^n*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \sqrt{x}(a+bx^n)\left(A+Bx^n\right)\,dx$$
$$= \frac{2\left(3\left(2\,Bbn+3\,Bb\right)x^{\frac{3}{2}}x^{2\,n}+3\left(3\,Ba+3\,Ab+4\left(Ba+Ab\right)n\right)x^{\frac{3}{2}}x^{n}+\left(8\,Aan^2+18\,Aan+9\,Aa\right)x^{\frac{3}{2}}\right)}{3\left(8\,n^2+18\,n+9\right)}$$

input
```
integrate(x^(1/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/3*(3*(2*B*b*n + 3*B*b)*x^(3/2)*x^(2*n) + 3*(3*B*a + 3*A*b + 4*(B*a + A*b
)*n)*x^(3/2)*x^n + (8*A*a*n^2 + 18*A*a*n + 9*A*a)*x^(3/2))/(8*n^2 + 18*n +
 9)
```

Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int \sqrt{x}(a+bx^n)(A+Bx^n)\,dx = \frac{2Aax^{\frac{3}{2}}}{3} + 2Ab\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for } n \neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + 2Ba\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for } n \neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right) + 2Bb\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{2n}}{4n+3} & \text{for } n \neq -\frac{3}{4}\\ x^{\frac{3}{2}}x^{2n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)*(a+b*x**n)*(A+B*x**n),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*A*b*Piecewise((x**(3/2)*x**n/(2*n + 3), Ne(n, -3/2)),
 (x**(3/2)*x**n*log(sqrt(x)), True)) + 2*B*a*Piecewise((x**(3/2)*x**n/(2*n
 + 3), Ne(n, -3/2)), (x**(3/2)*x**n*log(sqrt(x)), True)) + 2*B*b*Piecewise
((x**(3/2)*x**(2*n)/(4*n + 3), Ne(n, -3/4)), (x**(3/2)*x**(2*n)*log(sqrt(x
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(a+bx^n)(A+Bx^n)\,dx = \frac{2}{3}Aax^{\frac{3}{2}} + \frac{2\,Bbx^{2\,n+\frac{3}{2}}}{4\,n+3} + \frac{2\,Bax^{n+\frac{3}{2}}}{2\,n+3} + \frac{2\,Abx^{n+\frac{3}{2}}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/3*A*a*x^(3/2) + 2*B*b*x^(2*n + 3/2)/(4*n + 3) + 2*B*a*x^(n + 3/2)/(2*n +
 3) + 2*A*b*x^(n + 3/2)/(2*n + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \sqrt{x}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2}{3}\,Aax^{\frac{3}{2}}+\frac{2\,Bbx^{\frac{3}{2}}\sqrt{x}^{4\,n}}{4\,n+3}+\frac{2\,Bax^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3}+\frac{2\,Abx^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
2/3*A*a*x^(3/2) + 2*B*b*x^(3/2)*sqrt(x)^(4*n)/(4*n + 3) + 2*B*a*x^(3/2)*sq
rt(x)^(2*n)/(2*n + 3) + 2*A*b*x^(3/2)*sqrt(x)^(2*n)/(2*n + 3)
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^n)\left(A+Bx^n\right)\,dx=\frac{2\,A\,a\,x^{3/2}}{3}+\frac{x^n\,x^{3/2}\,\left(2\,A\,b+2\,B\,a\right)}{2\,n+3}+\frac{2\,B\,b\,x^{2\,n}\,x^{3/2}}{4\,n+3}$$

input
```
int(x^(1/2)*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(2*A*a*x^(3/2))/3 + (x^n*x^(3/2)*(2*A*b + 2*B*a))/(2*n + 3) + (2*B*b*x^(2*
n)*x^(3/2))/(4*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \sqrt{x}(a+bx^n)\left(A+Bx^n\right)\,dx$$
$$=\frac{2\sqrt{x}\,x(6x^{2n}b^2n+9x^{2n}b^2+24x^nabn+18x^nab+8a^2n^2+18a^2n+9a^2)}{24n^2+54n+27}$$

input
```
int(x^(1/2)*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x*(6*x**(2*n)*b**2*n + 9*x**(2*n)*b**2 + 24*x**n*a*b*n + 18*x**
n*a*b + 8*a**2*n**2 + 18*a**2*n + 9*a**2))/(3*(8*n**2 + 18*n + 9))
```

3.293 $\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{\sqrt{x}}\,dx = 2aA\sqrt{x}+\frac{2(Ab+aB)x^{\frac{1}{2}+n}}{1+2n}+\frac{2bBx^{\frac{1}{2}+2n}}{1+4n}$$

output
```
2*a*A*x^(1/2)+2*(A*b+B*a)*x^(1/2+n)/(1+2*n)+2*b*B*x^(1/2+2*n)/(1+4*n)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{\sqrt{x}}\,dx = 2\left(aA\sqrt{x}+\frac{(Ab+aB)x^{\frac{1}{2}+n}}{1+2n}+\frac{bBx^{\frac{1}{2}+2n}}{1+4n}\right)$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*(a*A*Sqrt[x] + ((A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (b*B*x^(1/2 + 2*n))
/(1 + 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx$$
$$\Big\downarrow \; 950$$
$$\int \left(x^{n-\frac{1}{2}}(aB+Ab)+\frac{aA}{\sqrt{x}}+bBx^{2n-\frac{1}{2}}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2x^{n+\frac{1}{2}}(aB+Ab)}{2n+1}+2aA\sqrt{x}+\frac{2bBx^{2n+\frac{1}{2}}}{4n+1}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (2*b*B*x^(1/2 + 2*
n))/(1 + 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.53

method	result
orering	$\frac{2\sqrt{x}\,(8n^2+1)(a+b\,x^n)(A+B\,x^n)}{(1+4n)(1+2n)} - \frac{12x^2(-1+2n)\left(\frac{b\,x^n n(A+B\,x^n)}{x^{\frac{3}{2}}} + \frac{(a+b\,x^n)B\,x^n n}{x^{\frac{3}{2}}} - \frac{(a+b\,x^n)(A+B\,x^n)}{2x^{\frac{3}{2}}}\right)}{(1+4n)(1+2n)} + \frac{8x^3\left(\frac{b\,x^n n^2(A+B}{x^{\frac{5}{2}}}\right.}{}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*(8*n^2+1)/(1+4*n)/(1+2*n)*(a+b*x^n)*(A+B*x^n)-12*x^2*(-1+2*n)/(1
+4*n)/(1+2*n)*(b*x^n*n/x^(3/2)*(A+B*x^n)+(a+b*x^n)*B*x^n*n/x^(3/2)-1/2*(a+
b*x^n)*(A+B*x^n)/x^(3/2))+8/(8*n^2+6*n+1)*x^3*(1/x^(5/2)*b*x^n*n^2*(A+B*x^
n)-2*b*x^n*n/x^(5/2)*(A+B*x^n)+2/x^(5/2)*b*(x^n)^2*n^2*B+1/x^(5/2)*(a+b*x^
n)*B*x^n*n^2-2*(a+b*x^n)*B*x^n*n/x^(5/2)+3/4*(a+b*x^n)*(A+B*x^n)/x^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = \frac{2\left((2\,Bbn+Bb)\sqrt{x}x^{2\,n} + (Ba+Ab+4\,(Ba+Ab)n)\sqrt{x}x^n + (8\,Aan^2+6\,Aan+Aa)\sqrt{x}\right)}{8\,n^2+6\,n+1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(1/2),x, algorithm="fricas")
```

output
```
2*((2*B*b*n + B*b)*sqrt(x)*x^(2*n) + (B*a + A*b + 4*(B*a + A*b)*n)*sqrt(x)
*x^n + (8*A*a*n^2 + 6*A*a*n + A*a)*sqrt(x))/(8*n^2 + 6*n + 1)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.53

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = 2Aa\sqrt{x} - 2Ab\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2} \\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Ba\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2} \\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Bb\left(\begin{cases}\frac{\sqrt{x}x^{2n}}{-4n-1} & \text{for } n \neq -\frac{1}{4} \\ \sqrt{x}x^{2n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) - 2*A*b*Piecewise((sqrt(x)*x**n/(-2*n - 1), Ne(n, -1/2)), (s
qrt(x)*x**n*log(1/sqrt(x)), True)) - 2*B*a*Piecewise((sqrt(x)*x**n/(-2*n -
 1), Ne(n, -1/2)), (sqrt(x)*x**n*log(1/sqrt(x)), True)) - 2*B*b*Piecewise(
(sqrt(x)*x**(2*n)/(-4*n - 1), Ne(n, -1/4)), (sqrt(x)*x**(2*n)*log(1/sqrt(x
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = 2\,Aa\sqrt{x} + \frac{2\,Bbx^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Bax^{n+\frac{1}{2}}}{2\,n+1} + \frac{2\,Abx^{n+\frac{1}{2}}}{2\,n+1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(1/2),x, algorithm="maxima")
```

output
```
2*A*a*sqrt(x) + 2*B*b*x^(2*n + 1/2)/(4*n + 1) + 2*B*a*x^(n + 1/2)/(2*n + 1
) + 2*A*b*x^(n + 1/2)/(2*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = 2\,Aa\sqrt{x} + \frac{2\,Bbx^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Bax^{n+\frac{1}{2}}}{2\,n+1} + \frac{2\,Abx^{n+\frac{1}{2}}}{2\,n+1}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(1/2),x, algorithm="giac")
```

output
```
2*A*a*sqrt(x) + 2*B*b*x^(2*n + 1/2)/(4*n + 1) + 2*B*a*x^(n + 1/2)/(2*n + 1
) + 2*A*b*x^(n + 1/2)/(2*n + 1)
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = 2\,A\,a\,\sqrt{x} + \frac{x^n\,\sqrt{x}\,(2\,A\,b+2\,B\,a)}{2\,n+1} + \frac{2\,B\,b\,x^{2\,n}\,\sqrt{x}}{4\,n+1}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^(1/2),x)
```

output
```
2*A*a*x^(1/2) + (x^n*x^(1/2)*(2*A*b + 2*B*a))/(2*n + 1) + (2*B*b*x^(2*n)*x
^(1/2))/(4*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^n)(A+Bx^n)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(2x^{2n}b^2n + x^{2n}b^2 + 8x^nabn + 2x^nab + 8a^2n^2 + 6a^2n + a^2)}{8n^2+6n+1}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(2*x**(2*n)*b**2*n + x**(2*n)*b**2 + 8*x**n*a*b*n + 2*x**n*a*b
+ 8*a**2*n**2 + 6*a**2*n + a**2))/(8*n**2 + 6*n + 1)
```

3.294 $\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = -\frac{2aA}{\sqrt{x}} - \frac{2(Ab+aB)x^{-\frac{1}{2}+n}}{1-2n} - \frac{2bBx^{-\frac{1}{2}+2n}}{1-4n}$$

output
```
-2*a*A/x^(1/2)-2*(A*b+B*a)*x^(-1/2+n)/(1-2*n)-2*b*B*x^(-1/2+2*n)/(1-4*n)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = 2\left(-\frac{aA}{\sqrt{x}} - \frac{(Ab+aB)x^{-\frac{1}{2}+n}}{1-2n} - \frac{bBx^{-\frac{1}{2}+2n}}{1-4n}\right)$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^(3/2),x]
```

output
```
2*(-((a*A)/Sqrt[x]) - ((A*b + a*B)*x^(-1/2 + n))/(1 - 2*n) - (b*B*x^(-1/2
+ 2*n))/(1 - 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n-\frac{3}{2}}(aB+Ab)+\frac{aA}{x^{3/2}}+bBx^{2n-\frac{3}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^{n-\frac{1}{2}}(aB+Ab)}{1-2n}-\frac{2aA}{\sqrt{x}}-\frac{2bBx^{2n-\frac{1}{2}}}{1-4n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] - (2*(A*b + a*B)*x^(-1/2 + n))/(1 - 2*n) - (2*b*B*x^(-1/2
 + 2*n))/(1 - 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.58

method	result
orering	$-\frac{2(8n^2-24n+13)(a+b\,x^n)(A+B\,x^n)}{\sqrt{x}\,(-1+4n)(-1+2n)}+\frac{12x^2(-3+2n)\left(\frac{b\,x^n n(A+B\,x^n)}{x^{\frac{5}{2}}}+\frac{(a+b\,x^n)B\,x^n n}{x^{\frac{5}{2}}}-\frac{3(a+b\,x^n)(A+B\,x^n)}{2x^{\frac{5}{2}}}\right)}{(-1+4n)(-1+2n)}-\frac{8x^3\left(\frac{b\,x^n n}{\cdots}\right.}{\cdots}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)*(8*n^2-24*n+13)/(-1+4*n)/(-1+2*n)*(a+b*x^n)*(A+B*x^n)+12*x^2*(-
3+2*n)/(-1+4*n)/(-1+2*n)*(b*x^n*n/x^(5/2)*(A+B*x^n)+(a+b*x^n)*B*x^n*n/x^(5
/2)-3/2*(a+b*x^n)*(A+B*x^n)/x^(5/2))-8/(8*n^2-6*n+1)*x^3*(1/x^(7/2)*b*x^n*
n^2*(A+B*x^n)-4*b*x^n*n/x^(7/2)*(A+B*x^n)+2*b*(x^n)^2*n^2/x^(7/2)*B+1/x^(7
/2)*(a+b*x^n)*B*x^n*n^2-4*(a+b*x^n)*B*x^n*n/x^(7/2)+15/4*(a+b*x^n)*(A+B*x^
n)/x^(7/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^{3/2}}\,dx = \frac{2\left((2\,Bbn-Bb)\sqrt{x}x^{2\,n}-(Ba+Ab-4\,(Ba+Ab)n)\sqrt{x}x^n-(8\,Aan^2-6\,A\cdots\right.}{(8\,n^2-6\,n+1)x}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(3/2),x, algorithm="fricas")
```

output
```
2*((2*B*b*n - B*b)*sqrt(x)*x^(2*n) - (B*a + A*b - 4*(B*a + A*b)*n)*sqrt(x)
*x^n - (8*A*a*n^2 - 6*A*a*n + A*a)*sqrt(x))/((8*n^2 - 6*n + 1)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(51) = 102$.

Time = 1.29 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.91

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = \begin{cases} -\frac{2Aa}{\sqrt{x}} - \frac{4Ab}{\sqrt[4]{x}} - \frac{4Ba}{\sqrt[4]{x}} + Bb\log(x) \\ -\frac{2Aa}{\sqrt{x}} + Ab\log(x) + Ba\log(x) + 2Bb\sqrt{x} \\ -\frac{16Aan^2}{8n^2\sqrt{x}-6n\sqrt{x}+\sqrt{x}} + \frac{12Aan}{8n^2\sqrt{x}-6n\sqrt{x}+\sqrt{x}} - \frac{2Aa}{8n^2\sqrt{x}-6n\sqrt{x}+\sqrt{x}} + \frac{8Abnx^n}{8n^2\sqrt{x}-6n\sqrt{x}+\sqrt{x}} - \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**(3/2),x)
```

output
```
Piecewise((-2*A*a/sqrt(x) - 4*A*b/x**(1/4) - 4*B*a/x**(1/4) + B*b*log(x),
Eq(n, 1/4)), (-2*A*a/sqrt(x) + A*b*log(x) + B*a*log(x) + 2*B*b*sqrt(x), Eq
(n, 1/2)), (-16*A*a*n**2/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x)) + 12*A*a
*n/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x)) - 2*A*a/(8*n**2*sqrt(x) - 6*n*
sqrt(x) + sqrt(x)) + 8*A*b*n*x**n/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x))
 - 2*A*b*x**n/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x)) + 8*B*a*n*x**n/(8*n
**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x)) - 2*B*a*x**n/(8*n**2*sqrt(x) - 6*n*sq
rt(x) + sqrt(x)) + 4*B*b*n*x**(2*n)/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x
)) - 2*B*b*x**(2*n)/(8*n**2*sqrt(x) - 6*n*sqrt(x) + sqrt(x)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = \frac{2\,B\,b\,x^{2\,n-\frac{1}{2}}}{4\,n-1} - \frac{2\,A\,a}{\sqrt{x}} + \frac{A\,b\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}} + \frac{B\,a\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^(3/2),x)
```

output
```
(2*B*b*x^(2*n - 1/2))/(4*n - 1) - (2*A*a)/x^(1/2) + (A*b*x^(n - 1/2))/(n -
 1/2) + (B*a*x^(n - 1/2))/(n - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{3/2}}\,dx = \frac{4x^{2n}b^2n - 2x^{2n}b^2 + 16x^n abn - 4x^n ab - 16a^2n^2 + 12a^2n - 2a^2}{\sqrt{x}\,(8n^2-6n+1)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(3/2),x)
```

output
```
(2*(2*x**(2*n)*b**2*n - x**(2*n)*b**2 + 8*x**n*a*b*n - 2*x**n*a*b - 8*a**2
*n**2 + 6*a**2*n - a**2))/(sqrt(x)*(8*n**2 - 6*n + 1))
```

3.295 $\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = -\frac{2aA}{3x^{3/2}} - \frac{2(Ab+aB)x^{-\frac{3}{2}+n}}{3-2n} - \frac{2bBx^{-\frac{3}{2}+2n}}{3-4n}$$

output
```
-2/3*a*A/x^(3/2)-2*(A*b+B*a)*x^(-3/2+n)/(3-2*n)-2*b*B*x^(-3/2+2*n)/(3-4*n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = 2\left(-\frac{aA}{3x^{3/2}} - \frac{(Ab+aB)x^{-\frac{3}{2}+n}}{3-2n} - \frac{bBx^{-\frac{3}{2}+2n}}{3-4n}\right)$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^(5/2),x]
```

output
```
2*(-1/3*(a*A)/x^(3/2) - ((A*b + a*B)*x^(-3/2 + n))/(3 - 2*n) - (b*B*x^(-3/
2 + 2*n))/(3 - 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n-\frac{5}{2}}(aB+Ab)+\frac{aA}{x^{5/2}}+bBx^{2n-\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^{n-\frac{3}{2}}(aB+Ab)}{3-2n}-\frac{2aA}{3x^{3/2}}-\frac{2bBx^{2n-\frac{3}{2}}}{3-4n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) - (2*(A*b + a*B)*x^(-3/2 + n))/(3 - 2*n) - (2*b*B*x^(
-3/2 + 2*n))/(3 - 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.42

method	result
orering	$-\frac{2(8n^2-48n+49)(a+b\,x^n)(A+B\,x^n)}{3x^{\frac{3}{2}}(-3+4n)(-3+2n)}+\frac{4x^2(-5+2n)\left(\frac{b\,x^n n(A+B\,x^n)}{x^{\frac{7}{2}}}+\frac{(a+b\,x^n)B\,x^n n}{x^{\frac{7}{2}}}-\frac{5(a+b\,x^n)(A+B\,x^n)}{2x^{\frac{7}{2}}}\right)}{(-3+4n)(-3+2n)}-\frac{8x^3\left(\frac{b\,x^n n^2}{\cdots}\right.}{\cdots}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)*(8*n^2-48*n+49)/(-3+4*n)/(-3+2*n)*(a+b*x^n)*(A+B*x^n)+4*x^2*(
-5+2*n)/(-3+4*n)/(-3+2*n)*(b*x^n*n/x^(7/2)*(A+B*x^n)+(a+b*x^n)*B*x^n*n/x^(
7/2)-5/2*(a+b*x^n)*(A+B*x^n)/x^(7/2))-8/3/(8*n^2-18*n+9)*x^3*(1/x^(9/2)*b*
x^n*n^2*(A+B*x^n)-6*b*x^n*n/x^(9/2)*(A+B*x^n)+2*b*(x^n)^2*n^2/x^(9/2)*B+1/
x^(9/2)*(a+b*x^n)*B*x^n*n^2-6*(a+b*x^n)*B*x^n*n/x^(9/2)+35/4*(a+b*x^n)*(A+
B*x^n)/x^(9/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \frac{2\left(3\left(2\,Bbn-3\,Bb\right)\sqrt{x}x^{2\,n}-3\left(3\,Ba+3\,Ab-4\,(Ba+Ab)n\right)\sqrt{x}x^{n}-(8\,Aan\cdots\right.}{3\left(8\,n^2-18\,n+9\right)x^2}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*(2*B*b*n - 3*B*b)*sqrt(x)*x^(2*n) - 3*(3*B*a + 3*A*b - 4*(B*a + A*b
)*n)*sqrt(x)*x^n - (8*A*a*n^2 - 18*A*a*n + 9*A*a)*sqrt(x))/((8*n^2 - 18*n
+ 9)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(53) = 106$.

Time = 6.35 (sec) , antiderivative size = 389, normalized size of antiderivative = 7.07

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \begin{cases} -\frac{2Aa}{3x^{\frac{3}{2}}} - \frac{4Ab}{3x^{\frac{3}{4}}} - \frac{4Ba}{3x^{\frac{3}{4}}} + Bb\log(x) \\ -\frac{2Aa}{3x^{\frac{3}{2}}} + Ab\log(x) + Ba\log(x) + \frac{2Bbx^{\frac{3}{2}}}{3} \\ -\frac{16Aan^2}{24n^2x^{\frac{3}{2}}-54nx^{\frac{3}{2}}+27x^{\frac{3}{2}}} + \frac{36Aan}{24n^2x^{\frac{3}{2}}-54nx^{\frac{3}{2}}+27x^{\frac{3}{2}}} - \frac{18Aa}{24n^2x^{\frac{3}{2}}-54nx^{\frac{3}{2}}+27x^{\frac{3}{2}}} + \frac{24Ab}{24n^2x^{\frac{3}{2}}-54n} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**(5/2),x)
```

output
```
Piecewise((-2*A*a/(3*x**(3/2)) - 4*A*b/(3*x**(3/4)) - 4*B*a/(3*x**(3/4)) +
 B*b*log(x), Eq(n, 3/4)), (-2*A*a/(3*x**(3/2)) + A*b*log(x) + B*a*log(x) +
 2*B*b*x**(3/2)/3, Eq(n, 3/2)), (-16*A*a*n**2/(24*n**2*x**(3/2) - 54*n*x**
(3/2) + 27*x**(3/2)) + 36*A*a*n/(24*n**2*x**(3/2) - 54*n*x**(3/2) + 27*x**
(3/2)) - 18*A*a/(24*n**2*x**(3/2) - 54*n*x**(3/2) + 27*x**(3/2)) + 24*A*b*
n*x**n/(24*n**2*x**(3/2) - 54*n*x**(3/2) + 27*x**(3/2)) - 18*A*b*x**n/(24*
n**2*x**(3/2) - 54*n*x**(3/2) + 27*x**(3/2)) + 24*B*a*n*x**n/(24*n**2*x**(
3/2) - 54*n*x**(3/2) + 27*x**(3/2)) - 18*B*a*x**n/(24*n**2*x**(3/2) - 54*n
*x**(3/2) + 27*x**(3/2)) + 12*B*b*n*x**(2*n)/(24*n**2*x**(3/2) - 54*n*x**(
3/2) + 27*x**(3/2)) - 18*B*b*x**(2*n)/(24*n**2*x**(3/2) - 54*n*x**(3/2) +
27*x**(3/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-5/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \frac{2\,B\,b\,x^{2\,n-\frac{3}{2}}}{4\,n-3} - \frac{2\,A\,a}{3\,x^{3/2}} + \frac{A\,b\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}} + \frac{B\,a\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^(5/2),x)
```

output
```
(2*B*b*x^(2*n - 3/2))/(4*n - 3) - (2*A*a)/(3*x^(3/2)) + (A*b*x^(n - 3/2))/
(n - 3/2) + (B*a*x^(n - 3/2))/(n - 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{5/2}}\,dx = \frac{4x^{2n}b^2n - 6x^{2n}b^2 + 16x^n abn - 12x^n ab - \frac{16a^2n^2}{3} + 12a^2n - 6a^2}{\sqrt{x}\,x\,(8n^2 - 18n + 9)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(5/2),x)
```

output
```
(2*(6*x**(2*n)*b**2*n - 9*x**(2*n)*b**2 + 24*x**n*a*b*n - 18*x**n*a*b - 8*
a**2*n**2 + 18*a**2*n - 9*a**2))/(3*sqrt(x)*x*(8*n**2 - 18*n + 9))
```

3.296 $\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^{7/2}}\,dx = -\frac{2aA}{5x^{5/2}} - \frac{2(Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{2bBx^{-\frac{5}{2}+2n}}{5-4n}$$

output
```
-2/5*a*A/x^(5/2)-2*(A*b+B*a)*x^(-5/2+n)/(5-2*n)-2*b*B*x^(-5/2+2*n)/(5-4*n)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^{7/2}}\,dx = 2\left(-\frac{aA}{5x^{5/2}} - \frac{(Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{bBx^{-\frac{5}{2}+2n}}{5-4n}\right)$$

input
```
Integrate[((a + b*x^n)*(A + B*x^n))/x^(7/2),x]
```

output
```
2*(-1/5*(a*A)/x^(5/2) - ((A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (b*B*x^(-5/
2 + 2*n))/(5 - 4*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^{n-\frac{7}{2}}(aB+Ab)+\frac{aA}{x^{7/2}}+bBx^{2n-\frac{7}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^{n-\frac{5}{2}}(aB+Ab)}{5-2n}-\frac{2aA}{5x^{5/2}}-\frac{2bBx^{2n-\frac{5}{2}}}{5-4n}$$

input
```
Int[((a + b*x^n)*(A + B*x^n))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (2*b*B*x^(
-5/2 + 2*n))/(5 - 4*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(47) = 94.

Time = 0.14 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.42

method	result
orering	$-\frac{2(8n^2-72n+109)(a+b\,x^n)(A+B\,x^n)}{5x^{\frac{5}{2}}(-5+4n)(-5+2n)}+\frac{12x^2(2n-7)\left(\frac{b\,x^n n(A+B\,x^n)}{x^{\frac{9}{2}}}+\frac{(a+b\,x^n)B\,x^n n}{x^{\frac{9}{2}}}-\frac{7(a+b\,x^n)(A+B\,x^n)}{2x^{\frac{9}{2}}}\right)}{5(-5+4n)(-5+2n)}-\frac{8x^3\left(\frac{b\,x^n n^2}{}\right.}{}$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^(5/2)*(8*n^2-72*n+109)/(-5+4*n)/(-5+2*n)*(a+b*x^n)*(A+B*x^n)+12/5*x
^2*(2*n-7)/(-5+4*n)/(-5+2*n)*(b*x^n*n/x^(9/2)*(A+B*x^n)+(a+b*x^n)*B*x^n*n/
x^(9/2)-7/2*(a+b*x^n)*(A+B*x^n)/x^(9/2))-8/5/(8*n^2-30*n+25)*x^3*(1/x^(11/
2)*b*x^n*n^2*(A+B*x^n)-8*b*x^n*n/x^(11/2)*(A+B*x^n)+2*b*(x^n)^2*n^2/x^(11/
2)*B+1/x^(11/2)*(a+b*x^n)*B*x^n*n^2-8*(a+b*x^n)*B*x^n*n/x^(11/2)+63/4*(a+b
*x^n)*(A+B*x^n)/x^(11/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^n)\,(A+Bx^n)}{x^{7/2}}\,dx=\frac{2\left(5\,(2\,Bbn-5\,Bb)\sqrt{x}x^{2\,n}-5\,(5\,Ba+5\,Ab-4\,(Ba+Ab)n)\sqrt{x}x^{n}-(8\,Aan\right.}{5\,(8\,n^2-30\,n+25)x^3}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(7/2),x, algorithm="fricas")
```

output
```
2/5*(5*(2*B*b*n - 5*B*b)*sqrt(x)*x^(2*n) - 5*(5*B*a + 5*A*b - 4*(B*a + A*b
)*n)*sqrt(x)*x^n - (8*A*a*n^2 - 30*A*a*n + 25*A*a)*sqrt(x))/((8*n^2 - 30*n
 + 25)*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(53) = 106$.

Time = 25.56 (sec) , antiderivative size = 389, normalized size of antiderivative = 7.07

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx = \begin{cases} -\frac{2Aa}{5x^{\frac{5}{2}}} - \frac{4Ab}{5x^{\frac{5}{4}}} - \frac{4Ba}{5x^{\frac{5}{4}}} + Bb\log(x) \\ -\frac{2Aa}{5x^{\frac{5}{2}}} + Ab\log(x) + Ba\log(x) + \frac{2Bbx^{\frac{5}{2}}}{5} \\ -\frac{16Aan^2}{40n^2x^{\frac{5}{2}} - 150nx^{\frac{5}{2}} + 125x^{\frac{5}{2}}} + \frac{60Aan}{40n^2x^{\frac{5}{2}} - 150nx^{\frac{5}{2}} + 125x^{\frac{5}{2}}} - \frac{50Aa}{40n^2x^{\frac{5}{2}} - 150nx^{\frac{5}{2}} + 125x^{\frac{5}{2}}} + \frac{\quad}{40n^2x} \end{cases}$$

input
```
integrate((a+b*x**n)*(A+B*x**n)/x**(7/2),x)
```

output
```
Piecewise((-2*A*a/(5*x**(5/2)) - 4*A*b/(5*x**(5/4)) - 4*B*a/(5*x**(5/4)) +
 B*b*log(x), Eq(n, 5/4)), (-2*A*a/(5*x**(5/2)) + A*b*log(x) + B*a*log(x) +
 2*B*b*x**(5/2)/5, Eq(n, 5/2)), (-16*A*a*n**2/(40*n**2*x**(5/2) - 150*n*x*
*(5/2) + 125*x**(5/2)) + 60*A*a*n/(40*n**2*x**(5/2) - 150*n*x**(5/2) + 125
*x**(5/2)) - 50*A*a/(40*n**2*x**(5/2) - 150*n*x**(5/2) + 125*x**(5/2)) + 4
0*A*b*n*x**n/(40*n**2*x**(5/2) - 150*n*x**(5/2) + 125*x**(5/2)) - 50*A*b*x
**n/(40*n**2*x**(5/2) - 150*n*x**(5/2) + 125*x**(5/2)) + 40*B*a*n*x**n/(40
*n**2*x**(5/2) - 150*n*x**(5/2) + 125*x**(5/2)) - 50*B*a*x**n/(40*n**2*x**
(5/2) - 150*n*x**(5/2) + 125*x**(5/2)) + 20*B*b*n*x**(2*n)/(40*n**2*x**(5/
2) - 150*n*x**(5/2) + 125*x**(5/2)) - 50*B*b*x**(2*n)/(40*n**2*x**(5/2) -
150*n*x**(5/2) + 125*x**(5/2)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-7/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*x^n)*(A+B*x^n)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)/x^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx = \frac{2\,B\,b\,x^{2\,n-\frac{5}{2}}}{4\,n-5} - \frac{2\,A\,a}{5\,x^{5/2}} + \frac{A\,b\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}} + \frac{B\,a\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n))/x^(7/2),x)
```

output
```
(2*B*b*x^(2*n - 5/2))/(4*n - 5) - (2*A*a)/(5*x^(5/2)) + (A*b*x^(n - 5/2))/
(n - 5/2) + (B*a*x^(n - 5/2))/(n - 5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^n)(A+Bx^n)}{x^{7/2}}\,dx = \frac{4x^{2n}b^2n - 10x^{2n}b^2 + 16x^nabn - 20x^nab - \frac{16a^2n^2}{5} + 12a^2n - 10a^2}{\sqrt{x}\,x^2(8n^2-30n+25)}$$

input
```
int((a+b*x^n)*(A+B*x^n)/x^(7/2),x)
```

output
```
(2*(10*x**(2*n)*b**2*n - 25*x**(2*n)*b**2 + 40*x**n*a*b*n - 50*x**n*a*b -
8*a**2*n**2 + 30*a**2*n - 25*a**2))/(5*sqrt(x)*x**2*(8*n**2 - 30*n + 25))
```

3.297 $\int x^{5/2}(a+bx^n)^2(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int x^{5/2}(a+bx^n)^2(A+Bx^n)\,dx=\frac{2}{7}a^2Ax^{7/2}+\frac{2a(2Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{2b(Ab+2aB)x^{\frac{7}{2}+2n}}{7+4n}+\frac{2b^2Bx^{\frac{7}{2}+3n}}{7+6n}$$

output
```
2/7*a^2*A*x^(7/2)+2*a*(2*A*b+B*a)*x^(7/2+n)/(7+2*n)+2*b*(A*b+2*B*a)*x^(7/2
+2*n)/(7+4*n)+2*b^2*B*x^(7/2+3*n)/(7+6*n)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int x^{5/2}(a+bx^n)^2(A+Bx^n)\,dx=2\left(\frac{1}{7}a^2Ax^{7/2}+\frac{a(2Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{b(Ab+2aB)x^{\frac{7}{2}+2n}}{7+4n}+\frac{b^2Bx^{\frac{7}{2}+3n}}{7+6n}\right)$$

input
```
Integrate[x^(5/2)*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
2*((a^2*A*x^(7/2))/7 + (a*(2*A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (b*(A*b +
 2*a*B)*x^(7/2 + 2*n))/(7 + 4*n) + (b^2*B*x^(7/2 + 3*n))/(7 + 6*n))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx^n)^2\left(A+Bx^n\right)dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2Ax^{5/2}+ax^{n+\frac{5}{2}}(aB+2Ab)+bx^{2n+\frac{5}{2}}(2aB+Ab)+b^2Bx^{3n+\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2ax^{n+\frac{7}{2}}(aB+2Ab)}{2n+7}+\frac{2bx^{2n+\frac{7}{2}}(2aB+Ab)}{4n+7}+\frac{2b^2Bx^{3n+\frac{7}{2}}}{6n+7}$$

input
```
Int[x^(5/2)*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (2*b*(A*
b + 2*a*B)*x^(7/2 + 2*n))/(7 + 4*n) + (2*b^2*B*x^(7/2 + 3*n))/(7 + 6*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(78) = 156.

Time = 0.45 (sec) , antiderivative size = 621, normalized size of antiderivative = 7.06

method	result
orering	$\frac{24x^{\frac{7}{2}}(4n^3+44n^2+109n+74)(a+bx^n)^2(A+Bx^n)}{7(48n^3+308n^2+588n+343)} - \frac{8x^2(22n^2+90n+77)\left(\frac{5x^{\frac{3}{2}}(a+bx^n)^2(A+Bx^n)}{2}+2x^{\frac{3}{2}}(a+bx^n)(A+Bx^n)bx^n\right.}{7(48n^3+308n^2+588n+343)}$

input
```
int(x^(5/2)*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
24/7*x^(7/2)*(4*n^3+44*n^2+109*n+74)/(48*n^3+308*n^2+588*n+343)*(a+b*x^n)^
2*(A+B*x^n)-8/7*x^2*(22*n^2+90*n+77)/(48*n^3+308*n^2+588*n+343)*(5/2*x^(3/
2)*(a+b*x^n)^2*(A+B*x^n)+2*x^(3/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+x^(3/2)*(a+
b*x^n)^2*B*x^n*n)+32/7*x^3*(4+3*n)/(48*n^3+308*n^2+588*n+343)*(15/4*x^(1/2
)*(a+b*x^n)^2*(A+B*x^n)+8*x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+4*x^(1/2)*(a
+b*x^n)^2*B*x^n*n+2*x^(1/2)*b^2*(x^n)^2*n^2*(A+B*x^n)+4*x^(1/2)*(a+b*x^n)*
B*(x^n)^2*n^2*b+2*x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n^2+x^(1/2)*(a+b*x^n)^
2*B*x^n*n^2)-16/7/(48*n^3+308*n^2+588*n+343)*x^4*(15/8*(a+b*x^n)^2*(A+B*x^
n)/x^(1/2)+23/2/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+23/4/x^(1/2)*(a+b*x^n)
^2*B*x^n*n+9/x^(1/2)*b^2*(x^n)^2*n^2*(A+B*x^n)+18/x^(1/2)*(a+b*x^n)*B*(x^n
)^2*n^2*b+9/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n^2+9/2/x^(1/2)*(a+b*x^n)^2*
B*x^n*n^2+6/x^(1/2)*b^2*(x^n)^2*n^3*(A+B*x^n)+6*(x^n)^3*b^2*n^3*B/x^(1/2)+
12/x^(1/2)*(a+b*x^n)*B*(x^n)^2*n^3*b+2/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n
^3+1/x^(1/2)*(a+b*x^n)^2*B*x^n*n^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.17

$$\int x^{5/2}(a+bx^n)^2 (A+Bx^n)\, dx = \frac{2\left(7\,(8\,Bb^2n^2+42\,Bb^2n+49\,Bb^2)x^{\frac{7}{2}}x^{3\,n}+7\,(98\,Bab+49\,Ab^2+12\,(2\,Bab+Ab^2)n^2+56\,(2\right.}{}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/7*(7*(8*B*b^2*n^2 + 42*B*b^2*n + 49*B*b^2)*x^(7/2)*x^(3*n) + 7*(98*B*a*b
 + 49*A*b^2 + 12*(2*B*a*b + A*b^2)*n^2 + 56*(2*B*a*b + A*b^2)*n)*x^(7/2)*x
^(2*n) + 7*(49*B*a^2 + 98*A*a*b + 24*(B*a^2 + 2*A*a*b)*n^2 + 70*(B*a^2 + 2
*A*a*b)*n)*x^(7/2)*x^n + (48*A*a^2*n^3 + 308*A*a^2*n^2 + 588*A*a^2*n + 343
*A*a^2)*x^(7/2))/(48*n^3 + 308*n^2 + 588*n + 343)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx^n)^2 (A+Bx^n)\, dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int x^{5/2}(a+bx^n)^2\,(A+Bx^n)\,dx = \frac{2}{7}\,Aa^2x^{\frac{7}{2}} + \frac{2\,Bb^2x^{3\,n+\frac{7}{2}}}{6\,n+7} + \frac{4\,Babx^{2\,n+\frac{7}{2}}}{4\,n+7} + \frac{2\,Ab^2x^{2\,n+\frac{7}{2}}}{4\,n+7} + \frac{2\,Ba^2x^{n+\frac{7}{2}}}{2\,n+7} + \frac{4\,Aabx^{n+\frac{7}{2}}}{2\,n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/7*A*a^2*x^(7/2) + 2*B*b^2*x^(3*n + 7/2)/(6*n + 7) + 4*B*a*b*x^(2*n + 7/2
)/(4*n + 7) + 2*A*b^2*x^(2*n + 7/2)/(4*n + 7) + 2*B*a^2*x^(n + 7/2)/(2*n +
 7) + 4*A*a*b*x^(n + 7/2)/(2*n + 7)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int x^{5/2}(a+bx^n)^2\,(A+Bx^n)\,dx = \frac{2}{7}\,Aa^2x^{\frac{7}{2}} + \frac{2\,Bb^2x^{\frac{7}{2}}\sqrt{x}^{6\,n}}{6\,n+7} + \frac{4\,Babx^{\frac{7}{2}}\sqrt{x}^{4\,n}}{4\,n+7} + \frac{2\,Ab^2x^{\frac{7}{2}}\sqrt{x}^{4\,n}}{4\,n+7} + \frac{2\,Ba^2x^{\frac{7}{2}}\sqrt{x}^{2\,n}}{2\,n+7} + \frac{4\,Aabx^{\frac{7}{2}}\sqrt{x}^{2\,n}}{2\,n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
2/7*A*a^2*x^(7/2) + 2*B*b^2*x^(7/2)*sqrt(x)^(6*n)/(6*n + 7) + 4*B*a*b*x^(7
/2)*sqrt(x)^(4*n)/(4*n + 7) + 2*A*b^2*x^(7/2)*sqrt(x)^(4*n)/(4*n + 7) + 2*
B*a^2*x^(7/2)*sqrt(x)^(2*n)/(2*n + 7) + 4*A*a*b*x^(7/2)*sqrt(x)^(2*n)/(2*n
 + 7)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int x^{5/2}(a+bx^n)^2\,(A+Bx^n)\;dx=\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{x^n\,x^{7/2}\,(2\,B\,a^2+4\,A\,b\,a)}{2\,n+7}+\frac{x^{2\,n}\,x^{7/2}\,(2\,A\,b^2+4\,B\,a\,b)}{4\,n+7}+\frac{2\,B\,b^2\,x^{3\,n}\,x^{7/2}}{6\,n+7}$$

input
```
int(x^(5/2)*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(2*A*a^2*x^(7/2))/7 + (x^n*x^(7/2)*(2*B*a^2 + 4*A*a*b))/(2*n + 7) + (x^(2*
n)*x^(7/2)*(2*A*b^2 + 4*B*a*b))/(4*n + 7) + (2*B*b^2*x^(3*n)*x^(7/2))/(6*n
 + 7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int x^{5/2}(a+bx^n)^2\,(A+Bx^n)\;dx=\frac{2\sqrt{x}\,x^3(56x^{3n}b^3n^2+294x^{3n}b^3n+343x^{3n}b^3+252x^{2n}a\,b^2n^2+1176x^{2n}a\,b^2n+1029x^{2n}a\,b^2+5}{336n^3+2156n^2+411}$$

input
```
int(x^(5/2)*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**3*(56*x**(3*n)*b**3*n**2 + 294*x**(3*n)*b**3*n + 343*x**(3*n
)*b**3 + 252*x**(2*n)*a*b**2*n**2 + 1176*x**(2*n)*a*b**2*n + 1029*x**(2*n)
*a*b**2 + 504*x**n*a**2*b*n**2 + 1470*x**n*a**2*b*n + 1029*x**n*a**2*b + 4
8*a**3*n**3 + 308*a**3*n**2 + 588*a**3*n + 343*a**3))/(7*(48*n**3 + 308*n*
*2 + 588*n + 343))
```

3.298 $\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2a(2Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{2b(Ab+2aB)x^{\frac{5}{2}+2n}}{5+4n}+\frac{2b^2Bx^{\frac{5}{2}+3n}}{5+6n}$$

output
```
2/5*a^2*A*x^(5/2)+2*a*(2*A*b+B*a)*x^(5/2+n)/(5+2*n)+2*b*(A*b+2*B*a)*x^(5/2
+2*n)/(5+4*n)+2*b^2*B*x^(5/2+3*n)/(5+6*n)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx=2\left(\frac{1}{5}a^2Ax^{5/2}+\frac{a(2Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{b(Ab+2aB)x^{\frac{5}{2}+2n}}{5+4n}+\frac{b^2Bx^{\frac{5}{2}+3n}}{5+6n}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
2*((a^2*A*x^(5/2))/5 + (a*(2*A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (b*(A*b +
 2*a*B)*x^(5/2 + 2*n))/(5 + 4*n) + (b^2*B*x^(5/2 + 3*n))/(5 + 6*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx$$

$$\Big\downarrow \ 950$$

$$\int \left(a^2Ax^{3/2}+ax^{n+\frac{3}{2}}(aB+2Ab)+bx^{2n+\frac{3}{2}}(2aB+Ab)+b^2Bx^{3n+\frac{3}{2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2ax^{n+\frac{5}{2}}(aB+2Ab)}{2n+5}+\frac{2bx^{2n+\frac{5}{2}}(2aB+Ab)}{4n+5}+\frac{2b^2Bx^{3n+\frac{5}{2}}}{6n+5}$$

input
```
Int[x^(3/2)*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (2*b*(A*
b + 2*a*B)*x^(5/2 + 2*n))/(5 + 4*n) + (2*b^2*B*x^(5/2 + 3*n))/(5 + 6*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 621, normalized size of antiderivative = 7.06

method	result
orering	$\frac{8(12n^3+88n^2+147n+68)x^{\frac{5}{2}}(a+b\,x^n)^2(A+B\,x^n)}{5(48n^3+220n^2+300n+125)} - \frac{8x^2(22n^2+54n+29)\left(\frac{3\sqrt{x}\,(a+b\,x^n)^2(A+B\,x^n)}{2}+2\sqrt{x}\,(a+b\,x^n)(A+B\,x^n)b\,x^{n}\right.}{5(48n^3+220n^2+300n+125)}$

input
```
int(x^(3/2)*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
8/5*(12*n^3+88*n^2+147*n+68)*x^(5/2)/(48*n^3+220*n^2+300*n+125)*(a+b*x^n)^
2*(A+B*x^n)-8/5*x^2*(22*n^2+54*n+29)/(48*n^3+220*n^2+300*n+125)*(3/2*x^(1/
2)*(a+b*x^n)^2*(A+B*x^n)+2*x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+x^(1/2)*(a+
b*x^n)^2*B*x^n*n)+32/5*x^3*(2+3*n)/(48*n^3+220*n^2+300*n+125)*(3/4*(a+b*x^
n)^2*(A+B*x^n)/x^(1/2)+4/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+2/x^(1/2)*(a+
b*x^n)^2*B*x^n*n+2/x^(1/2)*b^2*(x^n)^2*n^2*(A+B*x^n)+4/x^(1/2)*(a+b*x^n)*B
*(x^n)^2*n^2*b+2/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n^2+1/x^(1/2)*(a+b*x^n)
^2*B*x^n*n^2)-16/5/(48*n^3+220*n^2+300*n+125)*x^4*(-1/2*(a+b*x^n)*(A+B*x^n
)/x^(3/2)*b*x^n*n-1/4*(a+b*x^n)^2*B*x^n*n/x^(3/2)-3/8*(a+b*x^n)^2*(A+B*x^n
)/x^(3/2)+3/x^(3/2)*b^2*(x^n)^2*n^2*(A+B*x^n)+6/x^(3/2)*(a+b*x^n)*B*(x^n)^
2*n^2*b+3/x^(3/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n^2+3/2/x^(3/2)*(a+b*x^n)^2*B*
x^n*n^2+6/x^(3/2)*b^2*(x^n)^2*n^3*(A+B*x^n)+6/x^(3/2)*b^2*(x^n)^3*n^3*B+12
/x^(3/2)*(a+b*x^n)*B*(x^n)^2*n^3*b+2/x^(3/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n^3
+1/x^(3/2)*(a+b*x^n)^2*B*x^n*n^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.17

$$\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx = \frac{2\left(5\left(8\,Bb^2n^2+30\,Bb^2n+25\,Bb^2\right)x^{\frac{5}{2}}x^{3\,n}+5\left(50\,Bab+25\,Ab^2+12\left(2\,Bab+Ab^2\right)n^2+40\left(2\right.\right.\right.}{}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/5*(5*(8*B*b^2*n^2 + 30*B*b^2*n + 25*B*b^2)*x^(5/2)*x^(3*n) + 5*(50*B*a*b
 + 25*A*b^2 + 12*(2*B*a*b + A*b^2)*n^2 + 40*(2*B*a*b + A*b^2)*n)*x^(5/2)*x
^(2*n) + 5*(25*B*a^2 + 50*A*a*b + 24*(B*a^2 + 2*A*a*b)*n^2 + 50*(B*a^2 + 2
*A*a*b)*n)*x^(5/2)*x^n + (48*A*a^2*n^3 + 220*A*a^2*n^2 + 300*A*a^2*n + 125
*A*a^2)*x^(5/2))/(48*n^3 + 220*n^2 + 300*n + 125)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(82) = 164$.

Time = 177.03 (sec) , antiderivative size = 842, normalized size of antiderivative = 9.57

$$\int x^{3/2}(a+bx^n)^2(A+Bx^n)\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Piecewise((2*A*a**2*x**(5/2)/5 + 2*A*a*b*log(x) - 2*A*b**2/(5*x**(5/2)) +
B*a**2*log(x) - 4*B*a*b/(5*x**(5/2)) - B*b**2/(5*x**5), Eq(n, -5/2)), (2*A
*a**2*x**(5/2)/5 + 8*A*a*b*x**(5/4)/5 + A*b**2*log(x) + 4*B*a**2*x**(5/4)/
5 + 2*B*a*b*log(x) - 4*B*b**2/(5*x**(5/4)), Eq(n, -5/4)), (2*A*a**2*x**(5/
2)/5 + 6*A*a*b*x**(5/3)/5 + 6*A*b**2*x**(5/6)/5 + 3*B*a**2*x**(5/3)/5 + 12
*B*a*b*x**(5/6)/5 + B*b**2*log(x), Eq(n, -5/6)), (96*A*a**2*n**3*x**(5/2)/
(240*n**3 + 1100*n**2 + 1500*n + 625) + 440*A*a**2*n**2*x**(5/2)/(240*n**3
 + 1100*n**2 + 1500*n + 625) + 600*A*a**2*n*x**(5/2)/(240*n**3 + 1100*n**2
 + 1500*n + 625) + 250*A*a**2*x**(5/2)/(240*n**3 + 1100*n**2 + 1500*n + 62
5) + 480*A*a*b*n**2*x**(5/2)*x**n/(240*n**3 + 1100*n**2 + 1500*n + 625) +
1000*A*a*b*n*x**(5/2)*x**n/(240*n**3 + 1100*n**2 + 1500*n + 625) + 500*A*a
*b*x**(5/2)*x**n/(240*n**3 + 1100*n**2 + 1500*n + 625) + 120*A*b**2*n**2*x
**(5/2)*x**(2*n)/(240*n**3 + 1100*n**2 + 1500*n + 625) + 400*A*b**2*n*x**(
5/2)*x**(2*n)/(240*n**3 + 1100*n**2 + 1500*n + 625) + 250*A*b**2*x**(5/2)*
x**(2*n)/(240*n**3 + 1100*n**2 + 1500*n + 625) + 240*B*a**2*n**2*x**(5/2)*
x**n/(240*n**3 + 1100*n**2 + 1500*n + 625) + 500*B*a**2*n*x**(5/2)*x**n/(2
40*n**3 + 1100*n**2 + 1500*n + 625) + 250*B*a**2*x**(5/2)*x**n/(240*n**3 +
 1100*n**2 + 1500*n + 625) + 240*B*a*b*n**2*x**(5/2)*x**(2*n)/(240*n**3 +
1100*n**2 + 1500*n + 625) + 800*B*a*b*n*x**(5/2)*x**(2*n)/(240*n**3 + 1100
*n**2 + 1500*n + 625) + 500*B*a*b*x**(5/2)*x**(2*n)/(240*n**3 + 1100*n*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int x^{3/2}(a+bx^n)^2\,(A+Bx^n)\,dx = \frac{2}{5}\,Aa^2x^{\frac{5}{2}} + \frac{2\,Bb^2x^{3\,n+\frac{5}{2}}}{6\,n+5} + \frac{4\,Babx^{2\,n+\frac{5}{2}}}{4\,n+5} + \frac{2\,Ab^2x^{2\,n+\frac{5}{2}}}{4\,n+5} + \frac{2\,Ba^2x^{n+\frac{5}{2}}}{2\,n+5} + \frac{4\,Aabx^{n+\frac{5}{2}}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/5*A*a^2*x^(5/2) + 2*B*b^2*x^(3*n + 5/2)/(6*n + 5) + 4*B*a*b*x^(2*n + 5/2
)/(4*n + 5) + 2*A*b^2*x^(2*n + 5/2)/(4*n + 5) + 2*B*a^2*x^(n + 5/2)/(2*n +
 5) + 4*A*a*b*x^(n + 5/2)/(2*n + 5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int x^{3/2}(a+bx^n)^2 (A+Bx^n)\, dx = \frac{2}{5} Aa^2x^{\frac{5}{2}} + \frac{2\, Bb^2x^{\frac{5}{2}}\sqrt{x}^{6\,n}}{6\,n+5} + \frac{4\, Babx^{\frac{5}{2}}\sqrt{x}^{4\,n}}{4\,n+5} + \frac{2\, Ab^2x^{\frac{5}{2}}\sqrt{x}^{4\,n}}{4\,n+5} + \frac{2\, Ba^2x^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5} + \frac{4\, Aabx^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
2/5*A*a^2*x^(5/2) + 2*B*b^2*x^(5/2)*sqrt(x)^(6*n)/(6*n + 5) + 4*B*a*b*x^(5
/2)*sqrt(x)^(4*n)/(4*n + 5) + 2*A*b^2*x^(5/2)*sqrt(x)^(4*n)/(4*n + 5) + 2*
B*a^2*x^(5/2)*sqrt(x)^(2*n)/(2*n + 5) + 4*A*a*b*x^(5/2)*sqrt(x)^(2*n)/(2*n
 + 5)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int x^{3/2}(a+bx^n)^2 (A+Bx^n)\, dx = \frac{2\,A\,a^2\,x^{5/2}}{5} + \frac{x^n\,x^{5/2}\,\left(2\,B\,a^2+4\,A\,b\,a\right)}{2\,n+5} + \frac{x^{2\,n}\,x^{5/2}\,\left(2\,A\,b^2+4\,B\,a\,b\right)}{4\,n+5} + \frac{2\,B\,b^2\,x^{3\,n}\,x^{5/2}}{6\,n+5}$$

input
```
int(x^(3/2)*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(2*A*a^2*x^(5/2))/5 + (x^n*x^(5/2)*(2*B*a^2 + 4*A*a*b))/(2*n + 5) + (x^(2*
n)*x^(5/2)*(2*A*b^2 + 4*B*a*b))/(4*n + 5) + (2*B*b^2*x^(3*n)*x^(5/2))/(6*n
 + 5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.75

$$\int x^{3/2}(a+bx^n)^2\,(A+Bx^n)\;dx = \frac{2\sqrt{x}\,x^2(40x^{3n}b^3n^2+150x^{3n}b^3n+125x^{3n}b^3+180x^{2n}a\,b^2n^2+600x^{2n}a\,b^2n+375x^{2n}a\,b^2+360}{240n^3+1100n^2+1500n}$$

input
```
int(x^(3/2)*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**2*(40*x**(3*n)*b**3*n**2 + 150*x**(3*n)*b**3*n + 125*x**(3*n
)*b**3 + 180*x**(2*n)*a*b**2*n**2 + 600*x**(2*n)*a*b**2*n + 375*x**(2*n)*a
*b**2 + 360*x**n*a**2*b*n**2 + 750*x**n*a**2*b*n + 375*x**n*a**2*b + 48*a*
*3*n**3 + 220*a**3*n**2 + 300*a**3*n + 125*a**3))/(5*(48*n**3 + 220*n**2 +
 300*n + 125))
```

3.299 $\int \sqrt{x}(a+bx^n)^2 (A+Bx^n) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \sqrt{x}(a+bx^n)^2 (A+Bx^n) \, dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2a(2Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{2b(Ab+2aB)x^{\frac{3}{2}+2n}}{3+4n} + \frac{2b^2Bx^{\frac{3}{2}+3n}}{3(1+2n)}$$

output
```
2/3*a^2*A*x^(3/2)+2*a*(2*A*b+B*a)*x^(3/2+n)/(3+2*n)+2*b*(A*b+2*B*a)*x^(3/2
+2*n)/(3+4*n)+2*b^2*B*x^(3/2+3*n)/(3+6*n)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \sqrt{x}(a+bx^n)^2 (A+Bx^n) \, dx = 2\left(\frac{1}{3}a^2Ax^{3/2} + \frac{a(2Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{b(Ab+2aB)x^{\frac{3}{2}+2n}}{3+4n} + \frac{b^2Bx^{\frac{3}{2}+3n}}{3+6n}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
2*((a^2*A*x^(3/2))/3 + (a*(2*A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (b*(A*b +
 2*a*B)*x^(3/2 + 2*n))/(3 + 4*n) + (b^2*B*x^(3/2 + 3*n))/(3 + 6*n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^n)^2(A+Bx^n)\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(a^2A\sqrt{x}+ax^{n+\frac{1}{2}}(aB+2Ab)+bx^{2n+\frac{1}{2}}(2aB+Ab)+b^2Bx^{3n+\frac{1}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2ax^{n+\frac{3}{2}}(aB+2Ab)}{2n+3}+\frac{2bx^{2n+\frac{3}{2}}(2aB+Ab)}{4n+3}+\frac{2b^2Bx^{3n+\frac{3}{2}}}{3(2n+1)}$$

input
```
Int[Sqrt[x]*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (2*b*(A*
b + 2*a*B)*x^(3/2 + 2*n))/(3 + 4*n) + (2*b^2*B*x^(3/2 + 3*n))/(3*(1 + 2*n)
)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 564, normalized size of antiderivative = 6.27

method	result
orering	$\frac{8(6n^2+19n+10)x^{\frac{3}{2}}(a+b\,x^n)^2(A+B\,x^n)}{9(8n^2+18n+9)} - \frac{8x^2(22n^2+18n+5)\left(\frac{(a+b\,x^n)^2(A+B\,x^n)}{2\sqrt{x}} + \frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{\sqrt{x}} + \frac{(a+b\,x^n)^2 B\,x^n n}{\sqrt{x}}\right.}{9(16n^3+44n^2+36n+9)}$

input
```
int(x^(1/2)*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
8/9*(6*n^2+19*n+10)*x^(3/2)/(8*n^2+18*n+9)*(a+b*x^n)^2*(A+B*x^n)-8/9*x^2*(
22*n^2+18*n+5)/(16*n^3+44*n^2+36*n+9)*(1/2*(a+b*x^n)^2*(A+B*x^n)/x^(1/2)+2
/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b*x^n*n+1/x^(1/2)*(a+b*x^n)^2*B*x^n*n)+32/3*n
*x^3/(16*n^3+44*n^2+36*n+9)*(-1/4*(a+b*x^n)^2*(A+B*x^n)/x^(3/2)+2/x^(3/2)*
b^2*(x^n)^2*n^2*(A+B*x^n)+4/x^(3/2)*(a+b*x^n)*B*(x^n)^2*n^2*b+2/x^(3/2)*(a
+b*x^n)*(A+B*x^n)*b*x^n*n^2+1/x^(3/2)*(a+b*x^n)^2*B*x^n*n^2)-16/9/(16*n^3+
44*n^2+36*n+9)*x^4*(-1/2*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b*x^n*n-1/4*(a+b*x^n)
^2*B*x^n*n/x^(5/2)+3/8*(a+b*x^n)^2*(A+B*x^n)/x^(5/2)-3*b^2*(x^n)^2*n^2/x^(
5/2)*(A+B*x^n)+6/x^(5/2)*b^2*(x^n)^2*n^3*(A+B*x^n)+6*b^2*(x^n)^3*n^3*B/x^(
5/2)-6*(a+b*x^n)*B*(x^n)^2*n^2/x^(5/2)*b+12/x^(5/2)*(a+b*x^n)*B*(x^n)^2*n^
3*b-3*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b*x^n*n^2+2/x^(5/2)*(a+b*x^n)*(A+B*x^n)*
b*x^n*n^3-3/2*(a+b*x^n)^2*B*x^n*n^2/x^(5/2)+1/x^(5/2)*(a+b*x^n)^2*B*x^n*n^
3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(78) = 156$.

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.11

$$\int \sqrt{x}(a+bx^n)^2\,(A+Bx^n)\;dx$$
$$= \frac{2\left((8\,Bb^2n^2+18\,Bb^2n+9\,Bb^2)x^{\frac{3}{2}}x^{3\,n}+3\,(6\,Bab+3\,Ab^2+4\,(2\,Bab+Ab^2)n^2+8\,(2\,Bab+Ab^2)n)x^{\frac{3}{2}}x\right.}{3\,(16}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/3*((8*B*b^2*n^2 + 18*B*b^2*n + 9*B*b^2)*x^(3/2)*x^(3*n) + 3*(6*B*a*b + 3
*A*b^2 + 4*(2*B*a*b + A*b^2)*n^2 + 8*(2*B*a*b + A*b^2)*n)*x^(3/2)*x^(2*n)
+ 3*(3*B*a^2 + 6*A*a*b + 8*(B*a^2 + 2*A*a*b)*n^2 + 10*(B*a^2 + 2*A*a*b)*n)
*x^(3/2)*x^n + (16*A*a^2*n^3 + 44*A*a^2*n^2 + 36*A*a^2*n + 9*A*a^2)*x^(3/2
))/(16*n^3 + 44*n^2 + 36*n + 9)
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.42

$$\int \sqrt{x}(a+bx^n)^2\,(A+Bx^n)\;dx = \frac{2Aa^2x^{\frac{3}{2}}}{3} + 4Aab\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for}\; n \neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$
$$+ 2Ab^2\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{2n}}{4n+3} & \text{for}\; n \neq -\frac{3}{4}\\ x^{\frac{3}{2}}x^{2n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$
$$+ 2Ba^2\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for}\; n \neq -\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$
$$+ 4Bab\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{2n}}{4n+3} & \text{for}\; n \neq -\frac{3}{4}\\ x^{\frac{3}{2}}x^{2n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$
$$+ 2Bb^2\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{3n}}{6n+3} & \text{for}\; n \neq -\frac{1}{2}\\ x^{\frac{3}{2}}x^{3n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**(1/2)*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 4*A*a*b*Piecewise((x**(3/2)*x**n/(2*n + 3), Ne(n, -3
/2)), (x**(3/2)*x**n*log(sqrt(x)), True)) + 2*A*b**2*Piecewise((x**(3/2)*x
**(2*n)/(4*n + 3), Ne(n, -3/4)), (x**(3/2)*x**(2*n)*log(sqrt(x)), True)) +
 2*B*a**2*Piecewise((x**(3/2)*x**n/(2*n + 3), Ne(n, -3/2)), (x**(3/2)*x**n
*log(sqrt(x)), True)) + 4*B*a*b*Piecewise((x**(3/2)*x**(2*n)/(4*n + 3), Ne
(n, -3/4)), (x**(3/2)*x**(2*n)*log(sqrt(x)), True)) + 2*B*b**2*Piecewise((
x**(3/2)*x**(3*n)/(6*n + 3), Ne(n, -1/2)), (x**(3/2)*x**(3*n)*log(sqrt(x))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \sqrt{x}(a+bx^n)^2\,(A+Bx^n)\;dx = \frac{2}{3}\,Aa^2x^{\frac{3}{2}} + \frac{2\,Bb^2x^{3\,n+\frac{3}{2}}}{3\,(2\,n+1)} + \frac{4\,Babx^{2\,n+\frac{3}{2}}}{4\,n+3} + \frac{2\,Ab^2x^{2\,n+\frac{3}{2}}}{4\,n+3} + \frac{2\,Ba^2x^{n+\frac{3}{2}}}{2\,n+3} + \frac{4\,Aabx^{n+\frac{3}{2}}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/3*A*a^2*x^(3/2) + 2/3*B*b^2*x^(3*n + 3/2)/(2*n + 1) + 4*B*a*b*x^(2*n + 3
/2)/(4*n + 3) + 2*A*b^2*x^(2*n + 3/2)/(4*n + 3) + 2*B*a^2*x^(n + 3/2)/(2*n
 + 3) + 4*A*a*b*x^(n + 3/2)/(2*n + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37

$$\int \sqrt{x}(a+bx^n)^2\,(A+Bx^n)\;dx = \frac{2}{3}\,Aa^2x^{\frac{3}{2}} + \frac{2\,Bb^2x^{\frac{3}{2}}\sqrt{x}^{6\,n}}{3\,(2\,n+1)} + \frac{4\,Babx^{\frac{3}{2}}\sqrt{x}^{4\,n}}{4\,n+3} + \frac{2\,Ab^2x^{\frac{3}{2}}\sqrt{x}^{4\,n}}{4\,n+3} + \frac{2\,Ba^2x^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3} + \frac{4\,Aabx^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
2/3*A*a^2*x^(3/2) + 2/3*B*b^2*x^(3/2)*sqrt(x)^(6*n)/(2*n + 1) + 4*B*a*b*x^
(3/2)*sqrt(x)^(4*n)/(4*n + 3) + 2*A*b^2*x^(3/2)*sqrt(x)^(4*n)/(4*n + 3) +
2*B*a^2*x^(3/2)*sqrt(x)^(2*n)/(2*n + 3) + 4*A*a*b*x^(3/2)*sqrt(x)^(2*n)/(2
*n + 3)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^n)^2 (A+Bx^n)\, dx = \frac{2\,A\,a^2\,x^{3/2}}{3} + \frac{x^n\,x^{3/2}\,(2\,B\,a^2+4\,A\,b\,a)}{2\,n+3} + \frac{x^{2\,n}\,x^{3/2}\,(2\,A\,b^2+4\,B\,a\,b)}{4\,n+3} + \frac{2\,B\,b^2\,x^{3\,n}\,x^{3/2}}{6\,n+3}$$

input
```
int(x^(1/2)*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(2*A*a^2*x^(3/2))/3 + (x^n*x^(3/2)*(2*B*a^2 + 4*A*a*b))/(2*n + 3) + (x^(2*
n)*x^(3/2)*(2*A*b^2 + 4*B*a*b))/(4*n + 3) + (2*B*b^2*x^(3*n)*x^(3/2))/(6*n
 + 3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\int \sqrt{x}(a+bx^n)^2 (A+Bx^n)\, dx = \frac{2\sqrt{x}\,x(8x^{3n}b^3n^2+18x^{3n}b^3n+9x^{3n}b^3+36x^{2n}a\,b^2n^2+72x^{2n}a\,b^2n+27x^{2n}a\,b^2+72x^na^2b\,n^2+90x^na^2bn}{48n^3+132n^2+108n+27}$$

input
```
int(x^(1/2)*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x*(8*x**(3*n)*b**3*n**2 + 18*x**(3*n)*b**3*n + 9*x**(3*n)*b**3
+ 36*x**(2*n)*a*b**2*n**2 + 72*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 72
*x**n*a**2*b*n**2 + 90*x**n*a**2*b*n + 27*x**n*a**2*b + 16*a**3*n**3 + 44*
a**3*n**2 + 36*a**3*n + 9*a**3))/(3*(16*n**3 + 44*n**2 + 36*n + 9))
```

3.300 $\int \frac{(a+bx^n)^2(A+Bx^n)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x} + \frac{2a(2Ab+aB)x^{\frac{1}{2}+n}}{1+2n} + \frac{2b(Ab+2aB)x^{\frac{1}{2}+2n}}{1+4n} + \frac{2b^2Bx^{\frac{1}{2}+3n}}{1+6n}$$

output
```
2*a^2*A*x^(1/2)+2*a*(2*A*b+B*a)*x^(1/2+n)/(1+2*n)+2*b*(A*b+2*B*a)*x^(1/2+2
*n)/(1+4*n)+2*b^2*B*x^(1/2+3*n)/(1+6*n)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}}\,dx = 2\left(a^2A\sqrt{x} + \frac{a(2Ab+aB)x^{\frac{1}{2}+n}}{1+2n} + \frac{b(Ab+2aB)x^{\frac{1}{2}+2n}}{1+4n} + \frac{b^2Bx^{\frac{1}{2}+3n}}{1+6n}\right)$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*(a^2*A*Sqrt[x] + (a*(2*A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (b*(A*b + 2*a
*B)*x^(1/2 + 2*n))/(1 + 4*n) + (b^2*B*x^(1/2 + 3*n))/(1 + 6*n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2 A}{\sqrt{x}}+a x^{n-\frac{1}{2}} (aB+2Ab)+b x^{2n-\frac{1}{2}} (2aB+Ab)+b^2 B x^{3n-\frac{1}{2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2 A\sqrt{x}+\frac{2a x^{n+\frac{1}{2}} (aB+2Ab)}{2n+1}+\frac{2b x^{2n+\frac{1}{2}} (2aB+Ab)}{4n+1}+\frac{2b^2 B x^{3n+\frac{1}{2}}}{6n+1}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (2*b*(A*b +
2*a*B)*x^(1/2 + 2*n))/(1 + 4*n) + (2*b^2*B*x^(1/2 + 3*n))/(1 + 6*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.14

method	result
orering	$\frac{24\sqrt{x}\,n(4n^2+1)(a+b\,x^n)^2(A+B\,x^n)}{48n^3+44n^2+12n+1} - \frac{8x^2(22n^2-18n+5)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^{\frac{3}{2}}}+\frac{(a+b\,x^n)^2 B\,x^n n}{x^{\frac{3}{2}}}-\frac{(a+b\,x^n)^2(A+B\,x^n)}{2x^{\frac{3}{2}}}\right)}{48n^3+44n^2+12n+1}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
24*x^(1/2)*n*(4*n^2+1)/(48*n^3+44*n^2+12*n+1)*(a+b*x^n)^2*(A+B*x^n)-8*x^2*
(22*n^2-18*n+5)/(48*n^3+44*n^2+12*n+1)*(2*(a+b*x^n)*(A+B*x^n)/x^(3/2)*b*x^
n*n+(a+b*x^n)^2*B*x^n*n/x^(3/2)-1/2*(a+b*x^n)^2*(A+B*x^n)/x^(3/2))+32*x^3*
(-2+3*n)/(48*n^3+44*n^2+12*n+1)*(2*b^2*(x^n)^2*n^2/x^(5/2)*(A+B*x^n)+4*(a+
b*x^n)*B*(x^n)^2*n^2/x^(5/2)*b-4*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b*x^n*n+2*(a+
b*x^n)*(A+B*x^n)/x^(5/2)*b*x^n*n^2+(a+b*x^n)^2*B*x^n*n^2/x^(5/2)-2*(a+b*x^
n)^2*B*x^n*n/x^(5/2)+3/4*(a+b*x^n)^2*(A+B*x^n)/x^(5/2))-16/(48*n^3+44*n^2+
12*n+1)*x^4*(6*b^2*(x^n)^2*n^3/x^(7/2)*(A+B*x^n)-9*b^2*(x^n)^2*n^2/x^(7/2)
*(A+B*x^n)+6*b^2*(x^n)^3*n^3/x^(7/2)*B+12*(a+b*x^n)*B*(x^n)^2*n^3/x^(7/2)*
b-18*(a+b*x^n)*B*(x^n)^2*n^2/x^(7/2)*b+23/2*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b*
x^n*n-9*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b*x^n*n^2+2*(a+b*x^n)*(A+B*x^n)/x^(7/2
)*b*x^n*n^3+(a+b*x^n)^2*B*x^n*n^3/x^(7/2)-9/2*(a+b*x^n)^2*B*x^n*n^2/x^(7/2
)+23/4*(a+b*x^n)^2*B*x^n*n/x^(7/2)-15/8*(a+b*x^n)^2*(A+B*x^n)/x^(7/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(78) = 156$.

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}} \, dx$$
$$= \frac{2\left((8\,Bb^2n^2+6\,Bb^2n+Bb^2)\sqrt{x}x^{3\,n}+(2\,Bab+Ab^2+12\,(2\,Bab+Ab^2)n^2+8\,(2\,Bab+Ab^2)n)\sqrt{x}x^{2\,n}\right.}{48\,n^3+}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(1/2),x, algorithm="fricas")
```

output
```
2*((8*B*b^2*n^2 + 6*B*b^2*n + B*b^2)*sqrt(x)*x^(3*n) + (2*B*a*b + A*b^2 +
12*(2*B*a*b + A*b^2)*n^2 + 8*(2*B*a*b + A*b^2)*n)*sqrt(x)*x^(2*n) + (B*a^2
 + 2*A*a*b + 24*(B*a^2 + 2*A*a*b)*n^2 + 10*(B*a^2 + 2*A*a*b)*n)*sqrt(x)*x^
n + (48*A*a^2*n^3 + 44*A*a^2*n^2 + 12*A*a^2*n + A*a^2)*sqrt(x))/(48*n^3 +
44*n^2 + 12*n + 1)
```

Sympy [A] (verification not implemented)

Time = 3.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.71

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x} - 4Aab\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2}\\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Ab^2\left(\begin{cases}\frac{\sqrt{x}x^{2n}}{-4n-1} & \text{for } n \neq -\frac{1}{4}\\ \sqrt{x}x^{2n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Ba^2\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2}\\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 4Bab\left(\begin{cases}\frac{\sqrt{x}x^{2n}}{-4n-1} & \text{for } n \neq -\frac{1}{4}\\ \sqrt{x}x^{2n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Bb^2\left(\begin{cases}\frac{\sqrt{x}x^{3n}}{-6n-1} & \text{for } n \neq -\frac{1}{6}\\ \sqrt{x}x^{3n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) - 4*A*a*b*Piecewise((sqrt(x)*x**n/(-2*n - 1), Ne(n, -1/2)
), (sqrt(x)*x**n*log(1/sqrt(x)), True)) - 2*A*b**2*Piecewise((sqrt(x)*x**(
2*n)/(-4*n - 1), Ne(n, -1/4)), (sqrt(x)*x**(2*n)*log(1/sqrt(x)), True)) -
2*B*a**2*Piecewise((sqrt(x)*x**n/(-2*n - 1), Ne(n, -1/2)), (sqrt(x)*x**n*l
og(1/sqrt(x)), True)) - 4*B*a*b*Piecewise((sqrt(x)*x**(2*n)/(-4*n - 1), Ne
(n, -1/4)), (sqrt(x)*x**(2*n)*log(1/sqrt(x)), True)) - 2*B*b**2*Piecewise(
(sqrt(x)*x**(3*n)/(-6*n - 1), Ne(n, -1/6)), (sqrt(x)*x**(3*n)*log(1/sqrt(x
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}}\,dx = 2\,Aa^2\sqrt{x} + \frac{2\,Bb^2x^{3\,n+\frac{1}{2}}}{6\,n+1} + \frac{4\,Babx^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Ab^2x^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Ba^2x^{n+\frac{1}{2}}}{2\,n+1} + \frac{4\,Aabx^{n+\frac{1}{2}}}{2\,n+1}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(1/2),x, algorithm="maxima")
```

output
```
2*A*a^2*sqrt(x) + 2*B*b^2*x^(3*n + 1/2)/(6*n + 1) + 4*B*a*b*x^(2*n + 1/2)/
(4*n + 1) + 2*A*b^2*x^(2*n + 1/2)/(4*n + 1) + 2*B*a^2*x^(n + 1/2)/(2*n + 1
) + 4*A*a*b*x^(n + 1/2)/(2*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{\sqrt{x}}\,dx = 2\,Aa^2\sqrt{x} + \frac{2\,Bb^2x^{3\,n+\frac{1}{2}}}{6\,n+1} + \frac{4\,Babx^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Ab^2x^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Ba^2x^{n+\frac{1}{2}}}{2\,n+1} + \frac{4\,Aabx^{n+\frac{1}{2}}}{2\,n+1}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(1/2),x, algorithm="giac")
```

output
```
2*A*a^2*sqrt(x) + 2*B*b^2*x^(3*n + 1/2)/(6*n + 1) + 4*B*a*b*x^(2*n + 1/2)/
(4*n + 1) + 2*A*b^2*x^(2*n + 1/2)/(4*n + 1) + 2*B*a^2*x^(n + 1/2)/(2*n + 1
) + 4*A*a*b*x^(n + 1/2)/(2*n + 1)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{\sqrt{x}}\,dx = 2\,A\,a^2\,\sqrt{x}+\frac{x^n\,\sqrt{x}\,(2\,B\,a^2+4\,A\,b\,a)}{2\,n+1} + \frac{x^{2\,n}\,\sqrt{x}\,(2\,A\,b^2+4\,B\,a\,b)}{4\,n+1}+\frac{2\,B\,b^2\,x^{3\,n}\,\sqrt{x}}{6\,n+1}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^(1/2),x)
```

output
```
2*A*a^2*x^(1/2) + (x^n*x^(1/2)*(2*B*a^2 + 4*A*a*b))/(2*n + 1) + (x^(2*n)*x
^(1/2)*(2*A*b^2 + 4*B*a*b))/(4*n + 1) + (2*B*b^2*x^(3*n)*x^(1/2))/(6*n + 1
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(8x^{3n}b^3n^2+6x^{3n}b^3n+x^{3n}b^3+36x^{2n}a\,b^2n^2+24x^{2n}a\,b^2n+3x^{2n}a\,b^2+72x^na^2b\,n^2+30x^na^2bn+3x}{48n^3+44n^2+12n+1}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(8*x**(3*n)*b**3*n**2 + 6*x**(3*n)*b**3*n + x**(3*n)*b**3 + 36*
x**(2*n)*a*b**2*n**2 + 24*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 72*x**n*
a**2*b*n**2 + 30*x**n*a**2*b*n + 3*x**n*a**2*b + 48*a**3*n**3 + 44*a**3*n*
*2 + 12*a**3*n + a**3))/(48*n**3 + 44*n**2 + 12*n + 1)
```

3.301 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx = -\frac{2a^2A}{\sqrt{x}} - \frac{2a(2Ab+aB)x^{-\frac{1}{2}+n}}{1-2n} - \frac{2b(Ab+2aB)x^{-\frac{1}{2}+2n}}{1-4n} - \frac{2b^2Bx^{-\frac{1}{2}+3n}}{1-6n}$$

output
```
-2*a^2*A/x^(1/2)-2*a*(2*A*b+B*a)*x^(-1/2+n)/(1-2*n)-2*b*(A*b+2*B*a)*x^(-1/
2+2*n)/(1-4*n)-2*b^2*B*x^(-1/2+3*n)/(1-6*n)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx = 2\left(-\frac{a^2A}{\sqrt{x}} + \frac{a(2Ab+aB)x^{-\frac{1}{2}+n}}{-1+2n} + \frac{b(Ab+2aB)x^{-\frac{1}{2}+2n}}{-1+4n} + \frac{b^2Bx^{-\frac{1}{2}+3n}}{-1+6n}\right)$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^(3/2),x]
```

output
```
2*(-((a^2*A)/Sqrt[x]) + (a*(2*A*b + a*B)*x^(-1/2 + n))/(-1 + 2*n) + (b*(A*
b + 2*a*B)*x^(-1/2 + 2*n))/(-1 + 4*n) + (b^2*B*x^(-1/2 + 3*n))/(-1 + 6*n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^{3/2}}+ax^{n-\frac{3}{2}}(aB+2Ab)+bx^{2n-\frac{3}{2}}(2aB+Ab)+b^2Bx^{3n-\frac{3}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{\sqrt{x}}-\frac{2ax^{n-\frac{1}{2}}(aB+2Ab)}{1-2n}-\frac{2bx^{2n-\frac{1}{2}}(2aB+Ab)}{1-4n}-\frac{2b^2Bx^{3n-\frac{1}{2}}}{1-6n}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] - (2*a*(2*A*b + a*B)*x^(-1/2 + n))/(1 - 2*n) - (2*b*(A*
b + 2*a*B)*x^(-1/2 + 2*n))/(1 - 4*n) - (2*b^2*B*x^(-1/2 + 3*n))/(1 - 6*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 611, normalized size of antiderivative = 7.10

method	result
orering	$-\frac{8(6n^2-19n+10)(a+b\,x^n)^2(A+B\,x^n)}{\sqrt{x}\,(24n^2-10n+1)}+\frac{8x^2(22n^2-54n+29)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^{\frac{5}{2}}}+\frac{(a+b\,x^n)^2 B\,x^n n}{x^{\frac{5}{2}}}-\frac{3(a+b\,x^n)^2(A+B\,x^n}{2x^{\frac{5}{2}}}\right.}{48n^3-44n^2+12n-1}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(6*n^2-19*n+10)/x^(1/2)/(24*n^2-10*n+1)*(a+b*x^n)^2*(A+B*x^n)+8*x^2*(22
*n^2-54*n+29)/(48*n^3-44*n^2+12*n-1)*(2*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b*x^n*
n+(a+b*x^n)^2*B*x^n*n/x^(5/2)-3/2*(a+b*x^n)^2*(A+B*x^n)/x^(5/2))-32*x^3*(-
4+3*n)/(48*n^3-44*n^2+12*n-1)*(2*b^2*(x^n)^2*n^2/x^(7/2)*(A+B*x^n)+4*(a+b*
x^n)*B*(x^n)^2*n^2/x^(7/2)*b-8*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b*x^n*n+2*(a+b*
x^n)*(A+B*x^n)/x^(7/2)*b*x^n*n^2+(a+b*x^n)^2*B*x^n*n^2/x^(7/2)-4*(a+b*x^n)
^2*B*x^n*n/x^(7/2)+15/4*(a+b*x^n)^2*(A+B*x^n)/x^(7/2))+16/(48*n^3-44*n^2+1
2*n-1)*x^4*(6*b^2*(x^n)^2*n^3/x^(9/2)*(A+B*x^n)-15*b^2*(x^n)^2*n^2/x^(9/2)
*(A+B*x^n)+6*b^2*(x^n)^3*n^3/x^(9/2)*B+12*(a+b*x^n)*B*(x^n)^2*n^3/x^(9/2)*
b-30*(a+b*x^n)*B*(x^n)^2*n^2/x^(9/2)*b+71/2*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b*
x^n*n-15*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b*x^n*n^2+2*(a+b*x^n)*(A+B*x^n)/x^(9/
2)*b*x^n*n^3+(a+b*x^n)^2*B*x^n*n^3/x^(9/2)-15/2*(a+b*x^n)^2*B*x^n*n^2/x^(9
/2)+71/4*(a+b*x^n)^2*B*x^n*n/x^(9/2)-105/8*(a+b*x^n)^2*(A+B*x^n)/x^(9/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(78) = 156$.

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.20

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{3/2}}\,dx = \frac{2\left((8\,Bb^2n^2 - 6\,Bb^2n + Bb^2)\sqrt{x}x^{3\,n} + (2\,Bab + Ab^2 + 12\,(2\,Bab + Ab^2)n^2\right.}{}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(3/2),x, algorithm="fricas")
```

output
```
2*((8*B*b^2*n^2 - 6*B*b^2*n + B*b^2)*sqrt(x)*x^(3*n) + (2*B*a*b + A*b^2 +
12*(2*B*a*b + A*b^2)*n^2 - 8*(2*B*a*b + A*b^2)*n)*sqrt(x)*x^(2*n) + (B*a^2
 + 2*A*a*b + 24*(B*a^2 + 2*A*a*b)*n^2 - 10*(B*a^2 + 2*A*a*b)*n)*sqrt(x)*x^
n - (48*A*a^2*n^3 - 44*A*a^2*n^2 + 12*A*a^2*n - A*a^2)*sqrt(x))/((48*n^3 -
 44*n^2 + 12*n - 1)*x)
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.21

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}} + 2Aab\left(\begin{cases}\frac{x^n}{\sqrt{x}(n-\frac{1}{2})} & \text{for } n \neq \frac{1}{2}\\ \frac{x^n\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Ab^2\left(\begin{cases}\frac{x^{2n}}{\sqrt{x}(2n-\frac{1}{2})} & \text{for } n \neq \frac{1}{4}\\ \frac{x^{2n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Ba^2\left(\begin{cases}\frac{x^n}{\sqrt{x}(n-\frac{1}{2})} & \text{for } n \neq \frac{1}{2}\\ \frac{x^n\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + 2Bab\left(\begin{cases}\frac{x^{2n}}{\sqrt{x}(2n-\frac{1}{2})} & \text{for } n \neq \frac{1}{4}\\ \frac{x^{2n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}\frac{x^{3n}}{\sqrt{x}(3n-\frac{1}{2})} & \text{for } n \neq \frac{1}{6}\\ \frac{x^{3n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 2*A*a*b*Piecewise((x**n/(sqrt(x)*(n - 1/2)), Ne(n, 1/2
)), (x**n*log(x)/sqrt(x), True)) + A*b**2*Piecewise((x**(2*n)/(sqrt(x)*(2*
n - 1/2)), Ne(n, 1/4)), (x**(2*n)*log(x)/sqrt(x), True)) + B*a**2*Piecewis
e((x**n/(sqrt(x)*(n - 1/2)), Ne(n, 1/2)), (x**n*log(x)/sqrt(x), True)) + 2
*B*a*b*Piecewise((x**(2*n)/(sqrt(x)*(2*n - 1/2)), Ne(n, 1/4)), (x**(2*n)*l
og(x)/sqrt(x), True)) + B*b**2*Piecewise((x**(3*n)/(sqrt(x)*(3*n - 1/2)),
Ne(n, 1/6)), (x**(3*n)*log(x)/sqrt(x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{3/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{3/2}} \, dx = \frac{B\,a^2\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}} - \frac{2\,A\,a^2}{\sqrt{x}} + \frac{2\,A\,b^2\,x^{2\,n-\frac{1}{2}}}{4\,n-1} + \frac{2\,B\,b^2\,x^{3\,n-\frac{1}{2}}}{6\,n-1} + \frac{2\,A\,a\,b\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}} + \frac{4\,B\,a\,b\,x^{2\,n-\frac{1}{2}}}{4\,n-1}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^(3/2),x)
```

output
```
(B*a^2*x^(n - 1/2))/(n - 1/2) - (2*A*a^2)/x^(1/2) + (2*A*b^2*x^(2*n - 1/2)
)/(4*n - 1) + (2*B*b^2*x^(3*n - 1/2))/(6*n - 1) + (2*A*a*b*x^(n - 1/2))/(n
 - 1/2) + (4*B*a*b*x^(2*n - 1/2))/(4*n - 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{3/2}} \, dx = \frac{16x^{3n}b^3n^2 - 12x^{3n}b^3n + 2x^{3n}b^3 + 72x^{2n}a\,b^2n^2 - 48x^{2n}a\,b^2n + 6x^{2n}a\,b^2 + 14}{\sqrt{x}\,(48n^3 - 44n^2}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(3/2),x)
```

output
```
(2*(8*x**(3*n)*b**3*n**2 - 6*x**(3*n)*b**3*n + x**(3*n)*b**3 + 36*x**(2*n)
*a*b**2*n**2 - 24*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 72*x**n*a**2*b*n
**2 - 30*x**n*a**2*b*n + 3*x**n*a**2*b - 48*a**3*n**3 + 44*a**3*n**2 - 12*
a**3*n + a**3))/(sqrt(x)*(48*n**3 - 44*n**2 + 12*n - 1))
```

3.302 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}} - \frac{2a(2Ab+aB)x^{-\frac{3}{2}+n}}{3-2n} - \frac{2b(Ab+2aB)x^{-\frac{3}{2}+2n}}{3-4n} - \frac{2b^2Bx^{-\frac{3}{2}+3n}}{3(1-2n)}$$

output
```
-2/3*a^2*A/x^(3/2)-2*a*(2*A*b+B*a)*x^(-3/2+n)/(3-2*n)-2*b*(A*b+2*B*a)*x^(-
3/2+2*n)/(3-4*n)-2*b^2*B*x^(-3/2+3*n)/(3-6*n)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}} + \frac{2a(2Ab+aB)x^{-\frac{3}{2}+n}}{-3+2n} + \frac{2b(Ab+2aB)x^{-\frac{3}{2}+2n}}{-3+4n} - \frac{2b^2Bx^{-\frac{3}{2}+3n}}{3-6n}$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(-3/2 + n))/(-3 + 2*n) + (2*
b*(A*b + 2*a*B)*x^(-3/2 + 2*n))/(-3 + 4*n) - (2*b^2*B*x^(-3/2 + 3*n))/(3 -
 6*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^2A}{x^{5/2}}+ax^{n-\frac{5}{2}}(aB+2Ab)+bx^{2n-\frac{5}{2}}(2aB+Ab)+b^2Bx^{3n-\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2A}{3x^{3/2}}-\frac{2ax^{n-\frac{3}{2}}(aB+2Ab)}{3-2n}-\frac{2bx^{2n-\frac{3}{2}}(2aB+Ab)}{3-4n}-\frac{2b^2Bx^{3n-\frac{3}{2}}}{3(1-2n)}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) - (2*a*(2*A*b + a*B)*x^(-3/2 + n))/(3 - 2*n) - (2*b
*(A*b + 2*a*B)*x^(-3/2 + 2*n))/(3 - 4*n) - (2*b^2*B*x^(-3/2 + 3*n))/(3*(1
- 2*n))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 619, normalized size of antiderivative = 6.88

method	result
orering	$-\frac{8(12n^3-88n^2+147n-68)(a+b\,x^n)^2(A+B\,x^n)}{9x^{\frac{3}{2}}(16n^3-44n^2+36n-9)}+\frac{8x^2(22n^2-90n+77)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^{\frac{7}{2}}}+\frac{(a+b\,x^n)^2 B\,x^n n}{x^{\frac{7}{2}}}-\frac{5(a+b\,x^n)}{2\ldots}\right.}{9(16n^3-44n^2+36n-9)}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-8/9/x^(3/2)*(12*n^3-88*n^2+147*n-68)/(16*n^3-44*n^2+36*n-9)*(a+b*x^n)^2*(
A+B*x^n)+8/9*x^2*(22*n^2-90*n+77)/(16*n^3-44*n^2+36*n-9)*(2*(a+b*x^n)*(A+B
*x^n)/x^(7/2)*b*x^n*n+(a+b*x^n)^2*B*x^n*n/x^(7/2)-5/2*(a+b*x^n)^2*(A+B*x^n
)/x^(7/2))-32/3*x^3*(-2+n)/(16*n^3-44*n^2+36*n-9)*(2*b^2*(x^n)^2*n^2/x^(9/
2)*(A+B*x^n)+4*(a+b*x^n)*B*(x^n)^2*n^2/x^(9/2)*b-12*(a+b*x^n)*(A+B*x^n)/x^
(9/2)*b*x^n*n+2*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b*x^n*n^2+(a+b*x^n)^2*B*x^n*n^
2/x^(9/2)-6*(a+b*x^n)^2*B*x^n*n/x^(9/2)+35/4*(a+b*x^n)^2*(A+B*x^n)/x^(9/2)
)+16/9/(16*n^3-44*n^2+36*n-9)*x^4*(6*b^2*(x^n)^2*n^3/x^(11/2)*(A+B*x^n)-21
*b^2*(x^n)^2*n^2/x^(11/2)*(A+B*x^n)+6*b^2*(x^n)^3*n^3/x^(11/2)*B+12*(a+b*x
^n)*B*(x^n)^2*n^3/x^(11/2)*b-42*(a+b*x^n)*B*(x^n)^2*n^2/x^(11/2)*b+143/2*(
a+b*x^n)*(A+B*x^n)/x^(11/2)*b*x^n*n-21*(a+b*x^n)*(A+B*x^n)/x^(11/2)*b*x^n*
n^2+2*(a+b*x^n)*(A+B*x^n)/x^(11/2)*b*x^n*n^3+(a+b*x^n)^2*B*x^n*n^3/x^(11/2
)-21/2*(a+b*x^n)^2*B*x^n*n^2/x^(11/2)+143/4*(a+b*x^n)^2*B*x^n*n/x^(11/2)-3
15/8*(a+b*x^n)^2*(A+B*x^n)/x^(11/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}} dx = \frac{2 \left((8 Bb^2n^2 - 18 Bb^2n + 9 Bb^2)\sqrt{x}x^{3n} + 3 (6 Bab + 3 Ab^2 + 4 (2 Bab + Ab^2 \right.}{}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*((8*B*b^2*n^2 - 18*B*b^2*n + 9*B*b^2)*sqrt(x)*x^(3*n) + 3*(6*B*a*b + 3
*A*b^2 + 4*(2*B*a*b + A*b^2)*n^2 - 8*(2*B*a*b + A*b^2)*n)*sqrt(x)*x^(2*n)
+ 3*(3*B*a^2 + 6*A*a*b + 8*(B*a^2 + 2*A*a*b)*n^2 - 10*(B*a^2 + 2*A*a*b)*n)
*sqrt(x)*x^n - (16*A*a^2*n^3 - 44*A*a^2*n^2 + 36*A*a^2*n - 9*A*a^2)*sqrt(x
))/((16*n^3 - 44*n^2 + 36*n - 9)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. 2(83) = 166.

Time = 7.12 (sec) , antiderivative size = 1112, normalized size of antiderivative = 12.36

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}} dx = \text{Too large to display}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**(5/2),x)
```

output
```
Piecewise((-2*A*a**2/(3*x**(3/2)) - 2*A*a*b/x - 2*A*b**2/sqrt(x) - B*a**2/
x - 4*B*a*b/sqrt(x) + B*b**2*log(x), Eq(n, 1/2)), (-2*A*a**2/(3*x**(3/2))
- 8*A*a*b/(3*x**(3/4)) + A*b**2*log(x) - 4*B*a**2/(3*x**(3/4)) + 2*B*a*b*l
og(x) + 4*B*b**2*x**(3/4)/3, Eq(n, 3/4)), (-2*A*a**2/(3*x**(3/2)) + 2*A*a*
b*log(x) + 2*A*b**2*x**(3/2)/3 + B*a**2*log(x) + 4*B*a*b*x**(3/2)/3 + B*b*
*2*x**3/3, Eq(n, 3/2)), (-32*A*a**2*n**3/(48*n**3*x**(3/2) - 132*n**2*x**(
3/2) + 108*n*x**(3/2) - 27*x**(3/2)) + 88*A*a**2*n**2/(48*n**3*x**(3/2) -
132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) - 72*A*a**2*n/(48*n**3*x
**(3/2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) + 18*A*a**2/(4
8*n**3*x**(3/2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) + 96*A
*a*b*n**2*x**n/(48*n**3*x**(3/2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27
*x**(3/2)) - 120*A*a*b*n*x**n/(48*n**3*x**(3/2) - 132*n**2*x**(3/2) + 108*
n*x**(3/2) - 27*x**(3/2)) + 36*A*a*b*x**n/(48*n**3*x**(3/2) - 132*n**2*x**
(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) + 24*A*b**2*n**2*x**(2*n)/(48*n**3*x
**(3/2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) - 48*A*b**2*n*
x**(2*n)/(48*n**3*x**(3/2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3
/2)) + 18*A*b**2*x**(2*n)/(48*n**3*x**(3/2) - 132*n**2*x**(3/2) + 108*n*x*
*(3/2) - 27*x**(3/2)) + 48*B*a**2*n**2*x**n/(48*n**3*x**(3/2) - 132*n**2*x
**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) - 60*B*a**2*n*x**n/(48*n**3*x**(3/
2) - 132*n**2*x**(3/2) + 108*n*x**(3/2) - 27*x**(3/2)) + 18*B*a**2*x**n...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-5/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}}\,dx = \frac{B\,a^2\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}} - \frac{2\,A\,a^2}{3\,x^{3/2}} + \frac{2\,A\,b^2\,x^{2\,n-\frac{3}{2}}}{4\,n-3} + \frac{2\,B\,b^2\,x^{3\,n-\frac{3}{2}}}{3\,(2\,n-1)} + \frac{2\,A\,a\,b\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}} + \frac{4\,B\,a\,b\,x^{2\,n-\frac{3}{2}}}{4\,n-3}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^(5/2),x)
```

output
```
(B*a^2*x^(n - 3/2))/(n - 3/2) - (2*A*a^2)/(3*x^(3/2)) + (2*A*b^2*x^(2*n -
3/2))/(4*n - 3) + (2*B*b^2*x^(3*n - 3/2))/(3*(2*n - 1)) + (2*A*a*b*x^(n -
3/2))/(n - 3/2) + (4*B*a*b*x^(2*n - 3/2))/(4*n - 3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{5/2}}\,dx = \frac{\frac{16x^{3n}b^3n^2}{3} - 12x^{3n}b^3n + 6x^{3n}b^3 + 24x^{2n}a\,b^2n^2 - 48x^{2n}a\,b^2n + 18x^{2n}a\,b^2 + 48}{\sqrt{x}\,x\,(16n^3 - 44n^2 +}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(5/2),x)
```

output
```
(2*(8*x**(3*n)*b**3*n**2 - 18*x**(3*n)*b**3*n + 9*x**(3*n)*b**3 + 36*x**(2
*n)*a*b**2*n**2 - 72*x**(2*n)*a*b**2*n + 27*x**(2*n)*a*b**2 + 72*x**n*a**2
*b*n**2 - 90*x**n*a**2*b*n + 27*x**n*a**2*b - 16*a**3*n**3 + 44*a**3*n**2
- 36*a**3*n + 9*a**3))/(3*sqrt(x)*x*(16*n**3 - 44*n**2 + 36*n - 9))
```

3.303 $\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{7/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}} dx = -\frac{2a^2A}{5x^{5/2}} - \frac{2a(2Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{2b(Ab+2aB)x^{-\frac{5}{2}+2n}}{5-4n} - \frac{2b^2Bx^{-\frac{5}{2}+3n}}{5-6n}$$

output
```
-2/5*a^2*A/x^(5/2)-2*a*(2*A*b+B*a)*x^(-5/2+n)/(5-2*n)-2*b*(A*b+2*B*a)*x^(-
5/2+2*n)/(5-4*n)-2*b^2*B*x^(-5/2+3*n)/(5-6*n)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}} dx = 2\left(-\frac{a^2A}{5x^{5/2}} - \frac{a(2Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{b(Ab+2aB)x^{-\frac{5}{2}+2n}}{5-4n} - \frac{b^2Bx^{-\frac{5}{2}+3n}}{5-6n}\right)$$

input
```
Integrate[((a + b*x^n)^2*(A + B*x^n))/x^(7/2),x]
```

output
```
2*(-1/5*(a^2*A)/x^(5/2) - (a*(2*A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (b*(A
*b + 2*a*B)*x^(-5/2 + 2*n))/(5 - 4*n) - (b^2*B*x^(-5/2 + 3*n))/(5 - 6*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^2(A+Bx^n)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^2A}{x^{7/2}}+ax^{n-\frac{7}{2}}(aB+2Ab)+bx^{2n-\frac{7}{2}}(2aB+Ab)+b^2Bx^{3n-\frac{7}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2A}{5x^{5/2}}-\frac{2ax^{n-\frac{5}{2}}(aB+2Ab)}{5-2n}-\frac{2bx^{2n-\frac{5}{2}}(2aB+Ab)}{5-4n}-\frac{2b^2Bx^{3n-\frac{5}{2}}}{5-6n}$$

input
```
Int[((a + b*x^n)^2*(A + B*x^n))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (2*b
*(A*b + 2*a*B)*x^(-5/2 + 2*n))/(5 - 4*n) - (2*b^2*B*x^(-5/2 + 3*n))/(5 - 6
*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 621, normalized size of antiderivative = 7.06

method	result
orering	$-\frac{24(4n^3-44n^2+109n-74)(a+b\,x^n)^2(A+B\,x^n)}{5x^{\frac{5}{2}}(48n^3-220n^2+300n-125)}+\frac{8x^2(22n^2-126n+149)\left(\frac{2(a+b\,x^n)(A+B\,x^n)b\,x^n n}{x^{\frac{9}{2}}}+\frac{(a+b\,x^n)^2B\,x^n n}{x^{\frac{9}{2}}}-\frac{7(a+b\,x\right.}{}}{5(48n^3-220n^2+300n-125)}$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-24/5/x^(5/2)*(4*n^3-44*n^2+109*n-74)/(48*n^3-220*n^2+300*n-125)*(a+b*x^n)
^2*(A+B*x^n)+8/5*x^2*(22*n^2-126*n+149)/(48*n^3-220*n^2+300*n-125)*(2*(a+b
*x^n)*(A+B*x^n)/x^(9/2)*b*x^n*n+(a+b*x^n)^2*B*x^n*n/x^(9/2)-7/2*(a+b*x^n)^
2*(A+B*x^n)/x^(9/2))-32/5*x^3*(-8+3*n)/(48*n^3-220*n^2+300*n-125)*(2*b^2*(
x^n)^2*n^2/x^(11/2)*(A+B*x^n)+4*(a+b*x^n)*B*(x^n)^2*n^2/x^(11/2)*b-16*(a+b
*x^n)*(A+B*x^n)/x^(11/2)*b*x^n*n+2*(a+b*x^n)*(A+B*x^n)/x^(11/2)*b*x^n*n^2+
(a+b*x^n)^2*B*x^n*n^2/x^(11/2)-8*(a+b*x^n)^2*B*x^n*n/x^(11/2)+63/4*(a+b*x^
n)^2*(A+B*x^n)/x^(11/2))+16/5/(48*n^3-220*n^2+300*n-125)*x^4*(6*b^2*(x^n)^
2*n^3/x^(13/2)*(A+B*x^n)-27*b^2*(x^n)^2*n^2/x^(13/2)*(A+B*x^n)+6*b^2*(x^n)
^3*n^3/x^(13/2)*B+12*(a+b*x^n)*B*(x^n)^2*n^3/x^(13/2)*b-54*(a+b*x^n)*B*(x^
n)^2*n^2/x^(13/2)*b+239/2*(a+b*x^n)*(A+B*x^n)/x^(13/2)*b*x^n*n-27*(a+b*x^n
)*(A+B*x^n)/x^(13/2)*b*x^n*n^2+2*(a+b*x^n)*(A+B*x^n)/x^(13/2)*b*x^n*n^3+(a
+b*x^n)^2*B*x^n*n^3/x^(13/2)-27/2*(a+b*x^n)^2*B*x^n*n^2/x^(13/2)+239/4*(a+
b*x^n)^2*B*x^n*n/x^(13/2)-693/8*(a+b*x^n)^2*(A+B*x^n)/x^(13/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(78) = 156.

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}}\,dx = \frac{2\left(5\left(8\,Bb^2n^2 - 30\,Bb^2n + 25\,Bb^2\right)\sqrt{x}x^{3\,n} + 5\left(50\,Bab + 25\,Ab^2 + 12\left(2\,Bab\right.\right.\right.}{}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(7/2),x, algorithm="fricas")
```

output
```
2/5*(5*(8*B*b^2*n^2 - 30*B*b^2*n + 25*B*b^2)*sqrt(x)*x^(3*n) + 5*(50*B*a*b
 + 25*A*b^2 + 12*(2*B*a*b + A*b^2)*n^2 - 40*(2*B*a*b + A*b^2)*n)*sqrt(x)*x
^(2*n) + 5*(25*B*a^2 + 50*A*a*b + 24*(B*a^2 + 2*A*a*b)*n^2 - 50*(B*a^2 + 2
*A*a*b)*n)*sqrt(x)*x^n - (48*A*a^2*n^3 - 220*A*a^2*n^2 + 300*A*a^2*n - 125
*A*a^2)*sqrt(x))/((48*n^3 - 220*n^2 + 300*n - 125)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)**2*(A+B*x**n)/x**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-7/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{x^{7/2}}\,dx=\int \frac{(Bx^n+A)(bx^n+a)^2}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^2*(A+B*x^n)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2/x^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^n)^2\,(A+Bx^n)}{x^{7/2}}\,dx=\frac{B\,a^2\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}}-\frac{2\,A\,a^2}{5\,x^{5/2}}+\frac{2\,A\,b^2\,x^{2\,n-\frac{5}{2}}}{4\,n-5}+\frac{2\,B\,b^2\,x^{3\,n-\frac{5}{2}}}{6\,n-5}+\frac{2\,A\,a\,b\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}}+\frac{4\,B\,a\,b\,x^{2\,n-\frac{5}{2}}}{4\,n-5}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^2)/x^(7/2),x)
```

output
```
(B*a^2*x^(n - 5/2))/(n - 5/2) - (2*A*a^2)/(5*x^(5/2)) + (2*A*b^2*x^(2*n -
5/2))/(4*n - 5) + (2*B*b^2*x^(3*n - 5/2))/(6*n - 5) + (2*A*a*b*x^(n - 5/2)
)/(n - 5/2) + (4*B*a*b*x^(2*n - 5/2))/(4*n - 5)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^n)^2 (A+Bx^n)}{x^{7/2}}\,dx = \frac{16x^{3n}b^3n^2 - 60x^{3n}b^3n + 50x^{3n}b^3 + 72x^{2n}a\,b^2n^2 - 240x^{2n}a\,b^2n + 150x^{2n}a\,b^2}{\sqrt{x}\,x^2\,(48n^3 - 22}$$

input
```
int((a+b*x^n)^2*(A+B*x^n)/x^(7/2),x)
```

output
```
(2*(40*x**(3*n)*b**3*n**2 - 150*x**(3*n)*b**3*n + 125*x**(3*n)*b**3 + 180*
x**(2*n)*a*b**2*n**2 - 600*x**(2*n)*a*b**2*n + 375*x**(2*n)*a*b**2 + 360*x
**n*a**2*b*n**2 - 750*x**n*a**2*b*n + 375*x**n*a**2*b - 48*a**3*n**3 + 220
*a**3*n**2 - 300*a**3*n + 125*a**3))/(5*sqrt(x)*x**2*(48*n**3 - 220*n**2 +
 300*n - 125))
```

3.304 $\int x^{7/2}(a+bx^n)^3(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int x^{7/2}(a+bx^n)^3(A+Bx^n)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2a^2(3Ab+aB)x^{\frac{9}{2}+n}}{9+2n}+\frac{6ab(Ab+aB)x^{\frac{9}{2}+2n}}{9+4n}+\frac{2b^2(Ab+3aB)x^{\frac{9}{2}+3n}}{3(3+2n)}+\frac{2b^3Bx^{\frac{9}{2}+4n}}{9+8n}$$

output
```
2/9*a^3*A*x^(9/2)+2*a^2*(3*A*b+B*a)*x^(9/2+n)/(9+2*n)+6*a*b*(A*b+B*a)*x^(9
/2+2*n)/(9+4*n)+2*b^2*(A*b+3*B*a)*x^(9/2+3*n)/(9+6*n)+2*b^3*B*x^(9/2+4*n)/
(9+8*n)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int x^{7/2}(a+bx^n)^3(A+Bx^n)\,dx=2\left(\frac{1}{9}a^3Ax^{9/2}+\frac{a^2(3Ab+aB)x^{\frac{9}{2}+n}}{9+2n}+\frac{3ab(Ab+aB)x^{\frac{9}{2}+2n}}{9+4n}+\frac{b^2(Ab+3aB)x^{\frac{9}{2}+3n}}{9+6n}+\frac{b^3Bx^{\frac{9}{2}+4n}}{9+8n}\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
2*((a^3*A*x^(9/2))/9 + (a^2*(3*A*b + a*B)*x^(9/2 + n))/(9 + 2*n) + (3*a*b*
(A*b + a*B)*x^(9/2 + 2*n))/(9 + 4*n) + (b^2*(A*b + 3*a*B)*x^(9/2 + 3*n))/(
9 + 6*n) + (b^3*B*x^(9/2 + 4*n))/(9 + 8*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx^n)^3(A+Bx^n)\,dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(a^3Ax^{7/2}+a^2x^{n+\frac{7}{2}}(aB+3Ab)+b^2x^{3n+\frac{7}{2}}(3aB+Ab)+3abx^{2n+\frac{7}{2}}(aB+Ab)+b^3Bx^{4n+\frac{7}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2a^2x^{n+\frac{9}{2}}(aB+3Ab)}{2n+9}+\frac{2b^2x^{3n+\frac{9}{2}}(3aB+Ab)}{3(2n+3)}+\frac{6abx^{2n+\frac{9}{2}}(aB+Ab)}{4n+9}+\frac{2b^3Bx^{4n+\frac{9}{2}}}{8n+9}$$

input
```
Int[x^(7/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
(2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(9/2 + n))/(9 + 2*n) + (6*a*b
*(A*b + a*B)*x^(9/2 + 2*n))/(9 + 4*n) + (2*b^2*(A*b + 3*a*B)*x^(9/2 + 3*n)
)/(3*(3 + 2*n)) + (2*b^3*B*x^(9/2 + 4*n))/(9 + 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(107) = 214$.

Time = 1.03 (sec) , antiderivative size = 1249, normalized size of antiderivative = 10.32

method	result	size
orering	Expression too large to display	1249

input
```
int(x^(7/2)*(a+b*x^n)^3*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/27*x^(9/2)*(384*n^4+6400*n^3+27020*n^2+41600*n+21121)/(32*n^3+228*n^2+43
2*n+243)/(9+4*n)*(a+b*x^n)^3*(A+B*x^n)-40/27*x^2*(40*n^3+294*n^2+596*n+357
)/(32*n^3+228*n^2+432*n+243)/(9+4*n)*(7/2*x^(5/2)*(a+b*x^n)^3*(A+B*x^n)+3*
x^(5/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n+x^(5/2)*(a+b*x^n)^3*B*x^n*n)+80/27*x
^3*(14*n^2+48*n+37)/(32*n^3+228*n^2+432*n+243)/(9+4*n)*(35/4*x^(3/2)*(a+b*
x^n)^3*(A+B*x^n)+18*x^(3/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n+6*x^(3/2)*(a+b*x
^n)^3*B*x^n*n+6*x^(3/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^2+6*x^(3/2)*(a+b
*x^n)^2*B*(x^n)^2*n^2*b+3*x^(3/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^2+x^(3/2)*
(a+b*x^n)^3*B*x^n*n^2)-80/27*x^4*(5+4*n)/(128*n^4+1200*n^3+3780*n^2+4860*n
+2187)*(105/8*x^(1/2)*(a+b*x^n)^3*(A+B*x^n)+213/4*x^(1/2)*(a+b*x^n)^2*(A+B
*x^n)*b*x^n*n+71/4*x^(1/2)*(a+b*x^n)^3*B*x^n*n+45*x^(1/2)*(a+b*x^n)*(A+B*x
^n)*b^2*(x^n)^2*n^2+45*x^(1/2)*(a+b*x^n)^2*B*(x^n)^2*n^2*b+45/2*x^(1/2)*(a
+b*x^n)^2*(A+B*x^n)*b*x^n*n^2+15/2*x^(1/2)*(a+b*x^n)^3*B*x^n*n^2+6*x^(1/2)
*b^3*(x^n)^3*n^3*(A+B*x^n)+18*x^(1/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+18*x^(1/
2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^3+18*x^(1/2)*(a+b*x^n)^2*B*(x^n)^2*n^
3*b+3*x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^3+x^(1/2)*(a+b*x^n)^3*B*x^n*n^
3)+32/27/(128*n^4+1200*n^3+3780*n^2+4860*n+2187)*x^5*(105/16*(a+b*x^n)^3*(
A+B*x^n)/x^(1/2)+129/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^2+129/x^(1/
2)*(a+b*x^n)^2*B*(x^n)^2*n^2*b+48/x^(1/2)*b^3*(x^n)^3*n^3*(A+B*x^n)+144/x^
(1/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+144/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b^2*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(107) = 214$.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.80

$$\int x^{7/2}(a+bx^n)^3 (A+Bx^n)\,dx = \frac{2\left(9\,(16\,Bb^3n^3 + 132\,Bb^3n^2 + 324\,Bb^3n + 243\,Bb^3)x^{\frac{9}{2}}x^{4\,n} + 3\,(2187\,Bab^2 + 729\,Ab^3 + 64\,(3\,B\ldots\right.}{}$$

input
```
integrate(x^(7/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/9*(9*(16*B*b^3*n^3 + 132*B*b^3*n^2 + 324*B*b^3*n + 243*B*b^3)*x^(9/2)*x^
(4*n) + 3*(2187*B*a*b^2 + 729*A*b^3 + 64*(3*B*a*b^2 + A*b^3)*n^3 + 504*(3*
B*a*b^2 + A*b^3)*n^2 + 1134*(3*B*a*b^2 + A*b^3)*n)*x^(9/2)*x^(3*n) + 27*(2
43*B*a^2*b + 243*A*a*b^2 + 32*(B*a^2*b + A*a*b^2)*n^3 + 228*(B*a^2*b + A*a
*b^2)*n^2 + 432*(B*a^2*b + A*a*b^2)*n)*x^(9/2)*x^(2*n) + 9*(243*B*a^3 + 72
9*A*a^2*b + 64*(B*a^3 + 3*A*a^2*b)*n^3 + 312*(B*a^3 + 3*A*a^2*b)*n^2 + 486
*(B*a^3 + 3*A*a^2*b)*n)*x^(9/2)*x^n + (128*A*a^3*n^4 + 1200*A*a^3*n^3 + 37
80*A*a^3*n^2 + 4860*A*a^3*n + 2187*A*a^3)*x^(9/2))/(128*n^4 + 1200*n^3 + 3
780*n^2 + 4860*n + 2187)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\text{Timed out}$$

input
```
integrate(x**(7/2)*(a+b*x**n)**3*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int x^{7/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2\,Bb^3x^{4\,n+\frac{9}{2}}}{8\,n+9}+\frac{2\,Bab^2x^{3\,n+\frac{9}{2}}}{2\,n+3}+\frac{2\,Ab^3x^{3\,n+\frac{9}{2}}}{3\,(2\,n+3)}+\frac{6\,Ba^2bx^{2\,n+\frac{9}{2}}}{4\,n+9}+\frac{6\,Aab^2x^{2\,n+\frac{9}{2}}}{4\,n+9}+\frac{2\,Ba^3x^{n+\frac{9}{2}}}{2\,n+9}+\frac{6\,Aa^2bx^{n+\frac{9}{2}}}{2\,n+9}$$

input
```
integrate(x^(7/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/9*A*a^3*x^(9/2) + 2*B*b^3*x^(4*n + 9/2)/(8*n + 9) + 2*B*a*b^2*x^(3*n + 9
/2)/(2*n + 3) + 2/3*A*b^3*x^(3*n + 9/2)/(2*n + 3) + 6*B*a^2*b*x^(2*n + 9/2
)/(4*n + 9) + 6*A*a*b^2*x^(2*n + 9/2)/(4*n + 9) + 2*B*a^3*x^(n + 9/2)/(2*n
 + 9) + 6*A*a^2*b*x^(n + 9/2)/(2*n + 9)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int x^{7/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2\,Bb^3x^{\frac{9}{2}}\sqrt{x}^{8\,n}}{8\,n+9}+\frac{2\,Bab^2x^{\frac{9}{2}}\sqrt{x}^{6\,n}}{2\,n+3}+\frac{2\,Ab^3x^{\frac{9}{2}}\sqrt{x}^{6\,n}}{3\,(2\,n+3)}+\frac{6\,Ba^2bx^{\frac{9}{2}}\sqrt{x}^{4\,n}}{4\,n+9}+\frac{6\,Aab^2x^{\frac{9}{2}}\sqrt{x}^{4\,n}}{4\,n+9}+\frac{2\,Ba^3x^{\frac{9}{2}}\sqrt{x}^{2\,n}}{2\,n+9}+\frac{6\,Aa^2bx^{\frac{9}{2}}\sqrt{x}^{2\,n}}{2\,n+9}$$

input
```
integrate(x^(7/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="giac")
```

output
```
2/9*A*a^3*x^(9/2) + 2*B*b^3*x^(9/2)*sqrt(x)^(8*n)/(8*n + 9) + 2*B*a*b^2*x^
(9/2)*sqrt(x)^(6*n)/(2*n + 3) + 2/3*A*b^3*x^(9/2)*sqrt(x)^(6*n)/(2*n + 3)
+ 6*B*a^2*b*x^(9/2)*sqrt(x)^(4*n)/(4*n + 9) + 6*A*a*b^2*x^(9/2)*sqrt(x)^(4
*n)/(4*n + 9) + 2*B*a^3*x^(9/2)*sqrt(x)^(2*n)/(2*n + 9) + 6*A*a^2*b*x^(9/2
)*sqrt(x)^(2*n)/(2*n + 9)
```

Mupad [B] (verification not implemented)

Time = 4.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int x^{7/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{x^n\,x^{9/2}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)}{2\,n+9}+\frac{x^{3\,n}\,x^{9/2}\,\left(2\,A\,b^3+6\,B\,a\,b^2\right)}{6\,n+9}+\frac{2\,B\,b^3\,x^{4\,n}\,x^{9/2}}{8\,n+9}+\frac{6\,a\,b\,x^{2\,n}\,x^{9/2}\,\left(A\,b+B\,a\right)}{4\,n+9}$$

input
```
int(x^(7/2)*(A + B*x^n)*(a + b*x^n)^3,x)
```

output
```
(2*A*a^3*x^(9/2))/9 + (x^n*x^(9/2)*(2*B*a^3 + 6*A*a^2*b))/(2*n + 9) + (x^(
3*n)*x^(9/2)*(2*A*b^3 + 6*B*a*b^2))/(6*n + 9) + (2*B*b^3*x^(4*n)*x^(9/2))/
(8*n + 9) + (6*a*b*x^(2*n)*x^(9/2)*(A*b + B*a))/(4*n + 9)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.19

$$\int x^{7/2}(a+bx^n)^3\,(A+Bx^n)\,dx = \frac{2\sqrt{x}\,x^4(144x^{4n}b^4n^3+1188x^{4n}b^4n^2+2916x^{4n}b^4n+2187x^{4n}b^4+768x^{3n}a\,b^3n^3+6048x^{3n}a\,b^3n}{}$$

input
```
int(x^(7/2)*(a+b*x^n)^3*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**4*(144*x**(4*n)*b**4*n**3 + 1188*x**(4*n)*b**4*n**2 + 2916*x
**(4*n)*b**4*n + 2187*x**(4*n)*b**4 + 768*x**(3*n)*a*b**3*n**3 + 6048*x**(
3*n)*a*b**3*n**2 + 13608*x**(3*n)*a*b**3*n + 8748*x**(3*n)*a*b**3 + 1728*x
**(2*n)*a**2*b**2*n**3 + 12312*x**(2*n)*a**2*b**2*n**2 + 23328*x**(2*n)*a*
*2*b**2*n + 13122*x**(2*n)*a**2*b**2 + 2304*x**n*a**3*b*n**3 + 11232*x**n*
a**3*b*n**2 + 17496*x**n*a**3*b*n + 8748*x**n*a**3*b + 128*a**4*n**4 + 120
0*a**4*n**3 + 3780*a**4*n**2 + 4860*a**4*n + 2187*a**4))/(9*(128*n**4 + 12
00*n**3 + 3780*n**2 + 4860*n + 2187))
```

3.305 $\int x^{5/2}(a+bx^n)^3(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int x^{5/2}(a+bx^n)^3(A+Bx^n)\,dx = \frac{2}{7}a^3Ax^{7/2}+\frac{2a^2(3Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{6ab(Ab+aB)x^{\frac{7}{2}+2n}}{7+4n}+\frac{2b^2(Ab+3aB)x^{\frac{7}{2}+3n}}{7+6n}+\frac{2b^3Bx^{\frac{7}{2}+4n}}{7+8n}$$

output
```
2/7*a^3*A*x^(7/2)+2*a^2*(3*A*b+B*a)*x^(7/2+n)/(7+2*n)+6*a*b*(A*b+B*a)*x^(7
/2+2*n)/(7+4*n)+2*b^2*(A*b+3*B*a)*x^(7/2+3*n)/(7+6*n)+2*b^3*B*x^(7/2+4*n)/
(7+8*n)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int x^{5/2}(a+bx^n)^3(A+Bx^n)\,dx = 2\left(\frac{1}{7}a^3Ax^{7/2}+\frac{a^2(3Ab+aB)x^{\frac{7}{2}+n}}{7+2n}+\frac{3ab(Ab+aB)x^{\frac{7}{2}+2n}}{7+4n}+\frac{b^2(Ab+3aB)x^{\frac{7}{2}+3n}}{7+6n}+\frac{b^3Bx^{\frac{7}{2}+4n}}{7+8n}\right)$$

input
```
Integrate[x^(5/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
2*((a^3*A*x^(7/2))/7 + (a^2*(3*A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (3*a*b*
(A*b + a*B)*x^(7/2 + 2*n))/(7 + 4*n) + (b^2*(A*b + 3*a*B)*x^(7/2 + 3*n))/(
7 + 6*n) + (b^3*B*x^(7/2 + 4*n))/(7 + 8*n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(a+bx^n)^3\,(A+Bx^n)\;dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{n+\frac{5}{2}}(aB+3Ab)+b^2x^{3n+\frac{5}{2}}(3aB+Ab)+3abx^{2n+\frac{5}{2}}(aB+Ab)+b^3Bx^{4n+\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2a^2x^{n+\frac{7}{2}}(aB+3Ab)}{2n+7}+\frac{2b^2x^{3n+\frac{7}{2}}(3aB+Ab)}{6n+7}+\frac{6abx^{2n+\frac{7}{2}}(aB+Ab)}{4n+7}+\frac{2b^3Bx^{4n+\frac{7}{2}}}{8n+7}$$

input
```
Int[x^(5/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(7/2 + n))/(7 + 2*n) + (6*a*b
*(A*b + a*B)*x^(7/2 + 2*n))/(7 + 4*n) + (2*b^2*(A*b + 3*a*B)*x^(7/2 + 3*n)
)/(7 + 6*n) + (2*b^3*B*x^(7/2 + 4*n))/(7 + 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(107) = 214$.

Time = 1.04 (sec) , antiderivative size = 1249, normalized size of antiderivative = 10.50

method	result	size
orering	Expression too large to display	1249

input
```
int(x^(5/2)*(a+b*x^n)^3*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^(7/2)*(384*n^4+4800*n^3+15260*n^2+17760*n+6841)/(96*n^3+532*n^2+784*
n+343)/(7+4*n)*(a+b*x^n)^3*(A+B*x^n)-40/7*x^2*(40*n^3+210*n^2+308*n+135)/(
96*n^3+532*n^2+784*n+343)/(7+4*n)*(5/2*x^(3/2)*(a+b*x^n)^3*(A+B*x^n)+3*x^(
3/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n+x^(3/2)*(a+b*x^n)^3*B*x^n*n)+80/7*x^3*(
14*n^2+32*n+17)/(96*n^3+532*n^2+784*n+343)/(7+4*n)*(15/4*x^(1/2)*(a+b*x^n)
^3*(A+B*x^n)+12*x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n+4*x^(1/2)*(a+b*x^n)^
3*B*x^n*n+6*x^(1/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^2+6*x^(1/2)*(a+b*x^n
)^2*B*(x^n)^2*n^2*b+3*x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^2+x^(1/2)*(a+b
*x^n)^3*B*x^n*n^2)-80/7*x^4*(3+4*n)/(384*n^4+2800*n^3+6860*n^2+6860*n+2401
)*(15/8*(a+b*x^n)^3*(A+B*x^n)/x^(1/2)+69/4/x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b
*x^n*n+23/4/x^(1/2)*(a+b*x^n)^3*B*x^n*n+27/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b^2
*(x^n)^2*n^2+27/x^(1/2)*(a+b*x^n)^2*B*(x^n)^2*n^2*b+27/2/x^(1/2)*(a+b*x^n)
^2*(A+B*x^n)*b*x^n*n^2+9/2/x^(1/2)*(a+b*x^n)^3*B*x^n*n^2+6/x^(1/2)*b^3*(x^
n)^3*n^3*(A+B*x^n)+18/x^(1/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+18/x^(1/2)*(a+b*
x^n)*(A+B*x^n)*b^2*(x^n)^2*n^3+18/x^(1/2)*(a+b*x^n)^2*B*(x^n)^2*n^3*b+3/x^
(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^3+1/x^(1/2)*(a+b*x^n)^3*B*x^n*n^3)+32/
7/(384*n^4+2800*n^3+6860*n^2+6860*n+2401)*x^5*(24/x^(3/2)*b^3*(x^n)^3*n^3*
(A+B*x^n)+72/x^(3/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+72/x^(3/2)*(a+b*x^n)*(A+B
*x^n)*b^2*(x^n)^2*n^3+72/x^(3/2)*(a+b*x^n)^2*B*(x^n)^2*n^3*b+108/x^(3/2)*(
a+b*x^n)*B*(x^n)^3*n^4*b^2+42/x^(3/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(107) = 214$.

Time = 0.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.85

$$\int x^{5/2}(a+bx^n)^3 (A+Bx^n) \, dx = \frac{2 \left(7 \left(48 \, Bb^3n^3 + 308 \, Bb^3n^2 + 588 \, Bb^3n + 343 \, Bb^3\right)x^{\frac{7}{2}}x^{4\,n} + 7 \left(1029 \, Bab^2 + 343 \, Ab^3 + 64 \, (3 \, B \right.}{}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/7*(7*(48*B*b^3*n^3 + 308*B*b^3*n^2 + 588*B*b^3*n + 343*B*b^3)*x^(7/2)*x^
(4*n) + 7*(1029*B*a*b^2 + 343*A*b^3 + 64*(3*B*a*b^2 + A*b^3)*n^3 + 392*(3*
B*a*b^2 + A*b^3)*n^2 + 686*(3*B*a*b^2 + A*b^3)*n)*x^(7/2)*x^(3*n) + 21*(34
3*B*a^2*b + 343*A*a*b^2 + 96*(B*a^2*b + A*a*b^2)*n^3 + 532*(B*a^2*b + A*a*
b^2)*n^2 + 784*(B*a^2*b + A*a*b^2)*n)*x^(7/2)*x^(2*n) + 7*(343*B*a^3 + 102
9*A*a^2*b + 192*(B*a^3 + 3*A*a^2*b)*n^3 + 728*(B*a^3 + 3*A*a^2*b)*n^2 + 88
2*(B*a^3 + 3*A*a^2*b)*n)*x^(7/2)*x^n + (384*A*a^3*n^4 + 2800*A*a^3*n^3 + 6
860*A*a^3*n^2 + 6860*A*a^3*n + 2401*A*a^3)*x^(7/2))/(384*n^4 + 2800*n^3 +
6860*n^2 + 6860*n + 2401)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\text{Timed out}$$

input
```
integrate(x**(5/2)*(a+b*x**n)**3*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int x^{5/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2\,Bb^3x^{4\,n+\frac{7}{2}}}{8\,n+7}+\frac{6\,Bab^2x^{3\,n+\frac{7}{2}}}{6\,n+7}+\frac{2\,Ab^3x^{3\,n+\frac{7}{2}}}{6\,n+7}+\frac{6\,Ba^2bx^{2\,n+\frac{7}{2}}}{4\,n+7}+\frac{6\,Aab^2x^{2\,n+\frac{7}{2}}}{4\,n+7}+\frac{2\,Ba^3x^{n+\frac{7}{2}}}{2\,n+7}+\frac{6\,Aa^2bx^{n+\frac{7}{2}}}{2\,n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/7*A*a^3*x^(7/2) + 2*B*b^3*x^(4*n + 7/2)/(8*n + 7) + 6*B*a*b^2*x^(3*n + 7
/2)/(6*n + 7) + 2*A*b^3*x^(3*n + 7/2)/(6*n + 7) + 6*B*a^2*b*x^(2*n + 7/2)/
(4*n + 7) + 6*A*a*b^2*x^(2*n + 7/2)/(4*n + 7) + 2*B*a^3*x^(n + 7/2)/(2*n +
 7) + 6*A*a^2*b*x^(n + 7/2)/(2*n + 7)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.47

$$\int x^{5/2}(a+bx^n)^3 (A+Bx^n) \, dx = \frac{2}{7} Aa^3x^{\frac{7}{2}} + \frac{2 Bb^3x^{\frac{7}{2}}\sqrt{x}^{8n}}{8n+7} + \frac{6 Bab^2x^{\frac{7}{2}}\sqrt{x}^{6n}}{6n+7} + \frac{2 Ab^3x^{\frac{7}{2}}\sqrt{x}^{6n}}{6n+7} + \frac{6 Ba^2bx^{\frac{7}{2}}\sqrt{x}^{4n}}{4n+7} + \frac{6 Aab^2x^{\frac{7}{2}}\sqrt{x}^{4n}}{4n+7} + \frac{2 Ba^3x^{\frac{7}{2}}\sqrt{x}^{2n}}{2n+7} + \frac{6 Aa^2bx^{\frac{7}{2}}\sqrt{x}^{2n}}{2n+7}$$

input
```
integrate(x^(5/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="giac")
```

output
```
2/7*A*a^3*x^(7/2) + 2*B*b^3*x^(7/2)*sqrt(x)^(8*n)/(8*n + 7) + 6*B*a*b^2*x^
(7/2)*sqrt(x)^(6*n)/(6*n + 7) + 2*A*b^3*x^(7/2)*sqrt(x)^(6*n)/(6*n + 7) +
6*B*a^2*b*x^(7/2)*sqrt(x)^(4*n)/(4*n + 7) + 6*A*a*b^2*x^(7/2)*sqrt(x)^(4*n
)/(4*n + 7) + 2*B*a^3*x^(7/2)*sqrt(x)^(2*n)/(2*n + 7) + 6*A*a^2*b*x^(7/2)*
sqrt(x)^(2*n)/(2*n + 7)
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int x^{5/2}(a+bx^n)^3 (A+Bx^n) \, dx = \frac{2\,A\,a^3\,x^{7/2}}{7} + \frac{x^n\,x^{7/2}\,(2\,B\,a^3+6\,A\,b\,a^2)}{2\,n+7} + \frac{x^{3\,n}\,x^{7/2}\,(2\,A\,b^3+6\,B\,a\,b^2)}{6\,n+7} + \frac{2\,B\,b^3\,x^{4\,n}\,x^{7/2}}{8\,n+7} + \frac{6\,a\,b\,x^{2\,n}\,x^{7/2}\,(A\,b+B\,a)}{4\,n+7}$$

input
```
int(x^(5/2)*(A + B*x^n)*(a + b*x^n)^3,x)
```

output
```
(2*A*a^3*x^(7/2))/7 + (x^n*x^(7/2)*(2*B*a^3 + 6*A*a^2*b))/(2*n + 7) + (x^(
3*n)*x^(7/2)*(2*A*b^3 + 6*B*a*b^2))/(6*n + 7) + (2*B*b^3*x^(4*n)*x^(7/2))/
(8*n + 7) + (6*a*b*x^(2*n)*x^(7/2)*(A*b + B*a))/(4*n + 7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.23

$$\int x^{5/2}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2\sqrt{x}\,x^3(336x^{4n}b^4n^3+2156x^{4n}b^4n^2+4116x^{4n}b^4n+2401x^{4n}b^4+1792x^{3n}a\,b^3n^3+10976x^{3n}a\,b\cdots}{\cdots}$$

input
```
int(x^(5/2)*(a+b*x^n)^3*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**3*(336*x**(4*n)*b**4*n**3 + 2156*x**(4*n)*b**4*n**2 + 4116*x
**(4*n)*b**4*n + 2401*x**(4*n)*b**4 + 1792*x**(3*n)*a*b**3*n**3 + 10976*x*
*(3*n)*a*b**3*n**2 + 19208*x**(3*n)*a*b**3*n + 9604*x**(3*n)*a*b**3 + 4032
*x**(2*n)*a**2*b**2*n**3 + 22344*x**(2*n)*a**2*b**2*n**2 + 32928*x**(2*n)*
a**2*b**2*n + 14406*x**(2*n)*a**2*b**2 + 5376*x**n*a**3*b*n**3 + 20384*x**
n*a**3*b*n**2 + 24696*x**n*a**3*b*n + 9604*x**n*a**3*b + 384*a**4*n**4 + 2
800*a**4*n**3 + 6860*a**4*n**2 + 6860*a**4*n + 2401*a**4))/(7*(384*n**4 +
2800*n**3 + 6860*n**2 + 6860*n + 2401))
```

3.306 $\int x^{3/2}(a+bx^n)^3(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int x^{3/2}(a+bx^n)^3(A+Bx^n)\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2a^2(3Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{6ab(Ab+aB)x^{\frac{5}{2}+2n}}{5+4n}+\frac{2b^2(Ab+3aB)x^{\frac{5}{2}+3n}}{5+6n}+\frac{2b^3Bx^{\frac{5}{2}+4n}}{5+8n}$$

output
```
2/5*a^3*A*x^(5/2)+2*a^2*(3*A*b+B*a)*x^(5/2+n)/(5+2*n)+6*a*b*(A*b+B*a)*x^(5
/2+2*n)/(5+4*n)+2*b^2*(A*b+3*B*a)*x^(5/2+3*n)/(5+6*n)+2*b^3*B*x^(5/2+4*n)/
(5+8*n)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int x^{3/2}(a+bx^n)^3(A+Bx^n)\,dx=2\left(\frac{1}{5}a^3Ax^{5/2}+\frac{a^2(3Ab+aB)x^{\frac{5}{2}+n}}{5+2n}+\frac{3ab(Ab+aB)x^{\frac{5}{2}+2n}}{5+4n}+\frac{b^2(Ab+3aB)x^{\frac{5}{2}+3n}}{5+6n}+\frac{b^3Bx^{\frac{5}{2}+4n}}{5+8n}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
2*((a^3*A*x^(5/2))/5 + (a^2*(3*A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (3*a*b*
(A*b + a*B)*x^(5/2 + 2*n))/(5 + 4*n) + (b^2*(A*b + 3*a*B)*x^(5/2 + 3*n))/(
5 + 6*n) + (b^3*B*x^(5/2 + 4*n))/(5 + 8*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(a+bx^n)^3(A+Bx^n)\,dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3Ax^{3/2}+a^2x^{n+\frac{3}{2}}(aB+3Ab)+b^2x^{3n+\frac{3}{2}}(3aB+Ab)+3abx^{2n+\frac{3}{2}}(aB+Ab)+b^3Bx^{4n+\frac{3}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2a^2x^{n+\frac{5}{2}}(aB+3Ab)}{2n+5}+\frac{2b^2x^{3n+\frac{5}{2}}(3aB+Ab)}{6n+5}+\frac{6abx^{2n+\frac{5}{2}}(aB+Ab)}{4n+5}+\frac{2b^3Bx^{4n+\frac{5}{2}}}{8n+5}$$

input
```
Int[x^(3/2)*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(5/2 + n))/(5 + 2*n) + (6*a*b
*(A*b + a*B)*x^(5/2 + 2*n))/(5 + 4*n) + (2*b^2*(A*b + 3*a*B)*x^(5/2 + 3*n)
)/(5 + 6*n) + (2*b^3*B*x^(5/2 + 4*n))/(5 + 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. $2(107) = 214$.

Time = 0.55 (sec) , antiderivative size = 1054, normalized size of antiderivative = 8.86

method	result	size
orering	Expression too large to display	1054

input
```
int(x^(3/2)*(a+b*x^n)^3*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*(384*n^4+3200*n^3+6860*n^2+5440*n+1441)/(96*n^3+380*n^2+400*n+
125)/(5+4*n)*(a+b*x^n)^3*(A+B*x^n)-8*x^2*(40*n^3+126*n^2+116*n+33)/(96*n^3
+380*n^2+400*n+125)/(5+4*n)*(3/2*x^(1/2)*(a+b*x^n)^3*(A+B*x^n)+3*x^(1/2)*(
a+b*x^n)^2*(A+B*x^n)*b*x^n*n+x^(1/2)*(a+b*x^n)^3*B*x^n*n)+16*x^3*(14*n^2+1
6*n+5)/(96*n^3+380*n^2+400*n+125)/(5+4*n)*(3/4*(a+b*x^n)^3*(A+B*x^n)/x^(1/
2)+6/x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n+2/x^(1/2)*(a+b*x^n)^3*B*x^n*n+6
/x^(1/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^2+6/x^(1/2)*(a+b*x^n)^2*B*(x^n)
^2*n^2*b+3/x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^2+1/x^(1/2)*(a+b*x^n)^3*B
*x^n*n^2)-16*x^4*(1+4*n)/(384*n^4+2000*n^3+3500*n^2+2500*n+625)*(-3/4*(a+b
*x^n)^2*(A+B*x^n)/x^(3/2)*b*x^n*n-1/4*(a+b*x^n)^3*B*x^n*n/x^(3/2)-3/8*(a+b
*x^n)^3*(A+B*x^n)/x^(3/2)+9/x^(3/2)*(a+b*x^n)*(A+B*x^n)*b^2*(x^n)^2*n^2+9/
x^(3/2)*(a+b*x^n)^2*B*(x^n)^2*n^2*b+9/2/x^(3/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^
n*n^2+3/2/x^(3/2)*(a+b*x^n)^3*B*x^n*n^2+6/x^(3/2)*b^3*(x^n)^3*n^3*(A+B*x^n
)+18/x^(3/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+18/x^(3/2)*(a+b*x^n)*(A+B*x^n)*b^
2*(x^n)^2*n^3+18/x^(3/2)*(a+b*x^n)^2*B*(x^n)^2*n^3*b+3/x^(3/2)*(a+b*x^n)^2
*(A+B*x^n)*b*x^n*n^3+1/x^(3/2)*(a+b*x^n)^3*B*x^n*n^3)+32/5/(384*n^4+2000*n
^3+3500*n^2+2500*n+625)*x^5*(-15*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b^2*(x^n)^2*n
^2+108/x^(5/2)*(a+b*x^n)*B*(x^n)^3*n^4*b^2+42/x^(5/2)*(a+b*x^n)*(A+B*x^n)*
b^2*(x^n)^2*n^4+42/x^(5/2)*(a+b*x^n)^2*B*(x^n)^2*n^4*b-15*(a+b*x^n)^2*B*(x
^n)^2*n^2/x^(5/2)*b+9/16*(a+b*x^n)^3*(A+B*x^n)/x^(5/2)-5/2*(a+b*x^n)^3*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.85

$$\int x^{3/2}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2\left(5\,(48\,Bb^3n^3+220\,Bb^3n^2+300\,Bb^3n+125\,Bb^3)x^{\frac{5}{2}}x^{4\,n}+5\,(375\,Bab^2+125\,Ab^3+64\,(3\,B\ldots\right.}{}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/5*(5*(48*B*b^3*n^3 + 220*B*b^3*n^2 + 300*B*b^3*n + 125*B*b^3)*x^(5/2)*x^
(4*n) + 5*(375*B*a*b^2 + 125*A*b^3 + 64*(3*B*a*b^2 + A*b^3)*n^3 + 280*(3*B
*a*b^2 + A*b^3)*n^2 + 350*(3*B*a*b^2 + A*b^3)*n)*x^(5/2)*x^(3*n) + 15*(125
*B*a^2*b + 125*A*a*b^2 + 96*(B*a^2*b + A*a*b^2)*n^3 + 380*(B*a^2*b + A*a*b
^2)*n^2 + 400*(B*a^2*b + A*a*b^2)*n)*x^(5/2)*x^(2*n) + 5*(125*B*a^3 + 375*
A*a^2*b + 192*(B*a^3 + 3*A*a^2*b)*n^3 + 520*(B*a^3 + 3*A*a^2*b)*n^2 + 450*
(B*a^3 + 3*A*a^2*b)*n)*x^(5/2)*x^n + (384*A*a^3*n^4 + 2000*A*a^3*n^3 + 350
0*A*a^3*n^2 + 2500*A*a^3*n + 625*A*a^3)*x^(5/2))/(384*n^4 + 2000*n^3 + 350
0*n^2 + 2500*n + 625)
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\text{Timed out}$$

input
```
integrate(x**(3/2)*(a+b*x**n)**3*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int x^{3/2}(a+bx^n)^3\,(A+Bx^n)\;dx=\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2\,Bb^3x^{4\,n+\frac{5}{2}}}{8\,n+5}+\frac{6\,Bab^2x^{3\,n+\frac{5}{2}}}{6\,n+5}+\frac{2\,Ab^3x^{3\,n+\frac{5}{2}}}{6\,n+5}+\frac{6\,Ba^2bx^{2\,n+\frac{5}{2}}}{4\,n+5}+\frac{6\,Aab^2x^{2\,n+\frac{5}{2}}}{4\,n+5}+\frac{2\,Ba^3x^{n+\frac{5}{2}}}{2\,n+5}+\frac{6\,Aa^2bx^{n+\frac{5}{2}}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/5*A*a^3*x^(5/2) + 2*B*b^3*x^(4*n + 5/2)/(8*n + 5) + 6*B*a*b^2*x^(3*n + 5
/2)/(6*n + 5) + 2*A*b^3*x^(3*n + 5/2)/(6*n + 5) + 6*B*a^2*b*x^(2*n + 5/2)/
(4*n + 5) + 6*A*a*b^2*x^(2*n + 5/2)/(4*n + 5) + 2*B*a^3*x^(n + 5/2)/(2*n +
 5) + 6*A*a^2*b*x^(n + 5/2)/(2*n + 5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.47

$$\int x^{3/2}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2}{5}\,Aa^3x^{\frac{5}{2}} + \frac{2\,Bb^3x^{\frac{5}{2}}\sqrt{x}^{8\,n}}{8\,n+5} + \frac{6\,Bab^2x^{\frac{5}{2}}\sqrt{x}^{6\,n}}{6\,n+5} + \frac{2\,Ab^3x^{\frac{5}{2}}\sqrt{x}^{6\,n}}{6\,n+5} + \frac{6\,Ba^2bx^{\frac{5}{2}}\sqrt{x}^{4\,n}}{4\,n+5} + \frac{6\,Aab^2x^{\frac{5}{2}}\sqrt{x}^{4\,n}}{4\,n+5} + \frac{2\,Ba^3x^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5} + \frac{6\,Aa^2bx^{\frac{5}{2}}\sqrt{x}^{2\,n}}{2\,n+5}$$

input
```
integrate(x^(3/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="giac")
```

output
```
2/5*A*a^3*x^(5/2) + 2*B*b^3*x^(5/2)*sqrt(x)^(8*n)/(8*n + 5) + 6*B*a*b^2*x^
(5/2)*sqrt(x)^(6*n)/(6*n + 5) + 2*A*b^3*x^(5/2)*sqrt(x)^(6*n)/(6*n + 5) +
6*B*a^2*b*x^(5/2)*sqrt(x)^(4*n)/(4*n + 5) + 6*A*a*b^2*x^(5/2)*sqrt(x)^(4*n
)/(4*n + 5) + 2*B*a^3*x^(5/2)*sqrt(x)^(2*n)/(2*n + 5) + 6*A*a^2*b*x^(5/2)*
sqrt(x)^(2*n)/(2*n + 5)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int x^{3/2}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2\,A\,a^3\,x^{5/2}}{5} + \frac{x^n\,x^{5/2}\,(2\,B\,a^3+6\,A\,b\,a^2)}{2\,n+5} + \frac{x^{3\,n}\,x^{5/2}\,(2\,A\,b^3+6\,B\,a\,b^2)}{6\,n+5} + \frac{2\,B\,b^3\,x^{4\,n}\,x^{5/2}}{8\,n+5} + \frac{6\,a\,b\,x^{2\,n}\,x^{5/2}\,(A\,b+B\,a)}{4\,n+5}$$

input
```
int(x^(3/2)*(A + B*x^n)*(a + b*x^n)^3,x)
```

output
```
(2*A*a^3*x^(5/2))/5 + (x^n*x^(5/2)*(2*B*a^3 + 6*A*a^2*b))/(2*n + 5) + (x^(
3*n)*x^(5/2)*(2*A*b^3 + 6*B*a*b^2))/(6*n + 5) + (2*B*b^3*x^(4*n)*x^(5/2))/
(8*n + 5) + (6*a*b*x^(2*n)*x^(5/2)*(A*b + B*a))/(4*n + 5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.23

$$\int x^{3/2}(a+bx^n)^3(A+Bx^n)\,dx = \frac{2\sqrt{x}\,x^2(240x^{4n}b^4n^3+1100x^{4n}b^4n^2+1500x^{4n}b^4n+625x^{4n}b^4+1280x^{3n}a\,b^3n^3+5600x^{3n}a\,b^3n}{}$$

input
```
int(x^(3/2)*(a+b*x^n)^3*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x**2*(240*x**(4*n)*b**4*n**3 + 1100*x**(4*n)*b**4*n**2 + 1500*x
**(4*n)*b**4*n + 625*x**(4*n)*b**4 + 1280*x**(3*n)*a*b**3*n**3 + 5600*x**(
3*n)*a*b**3*n**2 + 7000*x**(3*n)*a*b**3*n + 2500*x**(3*n)*a*b**3 + 2880*x*
*(2*n)*a**2*b**2*n**3 + 11400*x**(2*n)*a**2*b**2*n**2 + 12000*x**(2*n)*a**
2*b**2*n + 3750*x**(2*n)*a**2*b**2 + 3840*x**n*a**3*b*n**3 + 10400*x**n*a*
*3*b*n**2 + 9000*x**n*a**3*b*n + 2500*x**n*a**3*b + 384*a**4*n**4 + 2000*a
**4*n**3 + 3500*a**4*n**2 + 2500*a**4*n + 625*a**4))/(5*(384*n**4 + 2000*n
**3 + 3500*n**2 + 2500*n + 625))
```

3.307 $\int \sqrt{x}(a+bx^n)^3 (A+Bx^n) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \sqrt{x}(a+bx^n)^3 (A+Bx^n) \, dx = \frac{2}{3}a^3Ax^{3/2} + \frac{2a^2(3Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{6ab(Ab+aB)x^{\frac{3}{2}+2n}}{3+4n} + \frac{2b^2(Ab+3aB)x^{\frac{3}{2}+3n}}{3(1+2n)} + \frac{2b^3Bx^{\frac{3}{2}+4n}}{3+8n}$$

output
```
2/3*a^3*A*x^(3/2)+2*a^2*(3*A*b+B*a)*x^(3/2+n)/(3+2*n)+6*a*b*(A*b+B*a)*x^(3
/2+2*n)/(3+4*n)+2*b^2*(A*b+3*B*a)*x^(3/2+3*n)/(3+6*n)+2*b^3*B*x^(3/2+4*n)/
(3+8*n)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \sqrt{x}(a+bx^n)^3 (A+Bx^n) \, dx = 2\left(\frac{1}{3}a^3Ax^{3/2} + \frac{a^2(3Ab+aB)x^{\frac{3}{2}+n}}{3+2n} + \frac{3ab(Ab+aB)x^{\frac{3}{2}+2n}}{3+4n} + \frac{b^2(Ab+3aB)x^{\frac{3}{2}+3n}}{3+6n} + \frac{b^3Bx^{\frac{3}{2}+4n}}{3+8n}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
2*((a^3*A*x^(3/2))/3 + (a^2*(3*A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (3*a*b*
(A*b + a*B)*x^(3/2 + 2*n))/(3 + 4*n) + (b^2*(A*b + 3*a*B)*x^(3/2 + 3*n))/(
3 + 6*n) + (b^3*B*x^(3/2 + 4*n))/(3 + 8*n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^n)^3 (A+Bx^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3A\sqrt{x}+a^2x^{n+\frac{1}{2}}(aB+3Ab)+b^2x^{3n+\frac{1}{2}}(3aB+Ab)+3abx^{2n+\frac{1}{2}}(aB+Ab)+b^3Bx^{4n+\frac{1}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2a^2x^{n+\frac{3}{2}}(aB+3Ab)}{2n+3}+\frac{2b^2x^{3n+\frac{3}{2}}(3aB+Ab)}{3(2n+1)}+\frac{6abx^{2n+\frac{3}{2}}(aB+Ab)}{4n+3}+\frac{2b^3Bx^{4n+\frac{3}{2}}}{8n+3}$$

input
```
Int[Sqrt[x]*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(3/2 + n))/(3 + 2*n) + (6*a*b
*(A*b + a*B)*x^(3/2 + 2*n))/(3 + 4*n) + (2*b^2*(A*b + 3*a*B)*x^(3/2 + 3*n)
)/(3*(1 + 2*n)) + (2*b^3*B*x^(3/2 + 4*n))/(3 + 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(107) = 214$.

Time = 0.56 (sec) , antiderivative size = 1190, normalized size of antiderivative = 9.83

method	result	size
orering	Expression too large to display	1190

input
```
int(x^(1/2)*(a+b*x^n)^3*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
2/9*(192*n^3+704*n^2+558*n+121)*x^(3/2)/(3+4*n)/(16*n^2+30*n+9)*(a+b*x^n)^
3*(A+B*x^n)-40/9*x^2*(40*n^3+42*n^2+20*n+3)/(32*n^3+76*n^2+48*n+9)/(3+4*n)
*(1/2*(a+b*x^n)^3*(A+B*x^n)/x^(1/2)+3/x^(1/2)*(a+b*x^n)^2*(A+B*x^n)*b*x^n*
n+1/x^(1/2)*(a+b*x^n)^3*B*x^n*n)+80/9*x^3*(14*n^2+1)/(32*n^3+76*n^2+48*n+9
)/(3+4*n)*(-1/4*(a+b*x^n)^3*(A+B*x^n)/x^(3/2)+6/x^(3/2)*(a+b*x^n)*(A+B*x^n
)*b^2*(x^n)^2*n^2+6/x^(3/2)*(a+b*x^n)^2*B*(x^n)^2*n^2*b+3/x^(3/2)*(a+b*x^n
)^2*(A+B*x^n)*b*x^n*n^2+1/x^(3/2)*(a+b*x^n)^3*B*x^n*n^2)-80/9*x^4*(-1+4*n)
/(128*n^4+400*n^3+420*n^2+180*n+27)*(-3/4*(a+b*x^n)^2*(A+B*x^n)/x^(5/2)*b*
x^n*n-1/4*(a+b*x^n)^3*B*x^n*n/x^(5/2)+3/8*(a+b*x^n)^3*(A+B*x^n)/x^(5/2)-9*
(a+b*x^n)*(A+B*x^n)/x^(5/2)*b^2*(x^n)^2*n^2+6/x^(5/2)*b^3*(x^n)^3*n^3*(A+B
*x^n)+18/x^(5/2)*(a+b*x^n)*B*(x^n)^3*n^3*b^2+18/x^(5/2)*(a+b*x^n)*(A+B*x^n
)*b^2*(x^n)^2*n^3-9*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(5/2)*b+18/x^(5/2)*(a+b*x^
n)^2*B*(x^n)^2*n^3*b-9/2*(a+b*x^n)^2*(A+B*x^n)/x^(5/2)*b*x^n*n^2+3/x^(5/2)
*(a+b*x^n)^2*(A+B*x^n)*b*x^n*n^3-3/2*(a+b*x^n)^3*B*x^n*n^2/x^(5/2)+1/x^(5/
2)*(a+b*x^n)^3*B*x^n*n^3)+32/9/(128*n^4+400*n^3+420*n^2+180*n+27)*x^5*(-72
*(a+b*x^n)^2*B*(x^n)^2*n^3/x^(7/2)*b+21*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b^2*(x
^n)^2*n^2+108/x^(7/2)*(a+b*x^n)*B*(x^n)^3*n^4*b^2+42/x^(7/2)*(a+b*x^n)*(A+
B*x^n)*b^2*(x^n)^2*n^4-24*b^3*(x^n)^3*n^3/x^(7/2)*(A+B*x^n)-72*(a+b*x^n)*B
*(x^n)^3*n^3/x^(7/2)*b^2-72*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b^2*(x^n)^2*n^3+42
/x^(7/2)*(a+b*x^n)^2*B*(x^n)^2*n^4*b+21*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(7/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(107) = 214$.

Time = 0.12 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.79

$$\int \sqrt{x}(a+bx^n)^3 (A+Bx^n)\,dx$$

$$= \frac{2\left(3\,(16\,Bb^3n^3+44\,Bb^3n^2+36\,Bb^3n+9\,Bb^3)x^{\frac{3}{2}}x^{4\,n}+(81\,Bab^2+27\,Ab^3+64\,(3\,Bab^2+Ab^3)n^3+168\right.}{}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="fricas")
```

output
```
2/3*(3*(16*B*b^3*n^3 + 44*B*b^3*n^2 + 36*B*b^3*n + 9*B*b^3)*x^(3/2)*x^(4*n
) + (81*B*a*b^2 + 27*A*b^3 + 64*(3*B*a*b^2 + A*b^3)*n^3 + 168*(3*B*a*b^2 +
 A*b^3)*n^2 + 126*(3*B*a*b^2 + A*b^3)*n)*x^(3/2)*x^(3*n) + 9*(9*B*a^2*b +
9*A*a*b^2 + 32*(B*a^2*b + A*a*b^2)*n^3 + 76*(B*a^2*b + A*a*b^2)*n^2 + 48*(
B*a^2*b + A*a*b^2)*n)*x^(3/2)*x^(2*n) + 3*(9*B*a^3 + 27*A*a^2*b + 64*(B*a^
3 + 3*A*a^2*b)*n^3 + 104*(B*a^3 + 3*A*a^2*b)*n^2 + 54*(B*a^3 + 3*A*a^2*b)*
n)*x^(3/2)*x^n + (128*A*a^3*n^4 + 400*A*a^3*n^3 + 420*A*a^3*n^2 + 180*A*a^
3*n + 27*A*a^3)*x^(3/2))/(128*n^4 + 400*n^3 + 420*n^2 + 180*n + 27)
```

Sympy [A] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.55

$$\begin{aligned}\int \sqrt{x}(a+bx^n)^3(A+Bx^n)\,dx = &\frac{2Aa^3x^{\frac{3}{2}}}{3} + 6Aa^2b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for}\ n\neq-\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+6Aab^2\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{2n}}{4n+3} & \text{for}\ n\neq-\frac{3}{4}\\ x^{\frac{3}{2}}x^{2n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+2Ab^3\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{3n}}{6n+3} & \text{for}\ n\neq-\frac{1}{2}\\ x^{\frac{3}{2}}x^{3n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+2Ba^3\left(\begin{cases}\frac{x^{\frac{3}{2}}x^n}{2n+3} & \text{for}\ n\neq-\frac{3}{2}\\ x^{\frac{3}{2}}x^n\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+6Ba^2b\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{2n}}{4n+3} & \text{for}\ n\neq-\frac{3}{4}\\ x^{\frac{3}{2}}x^{2n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+6Bab^2\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{3n}}{6n+3} & \text{for}\ n\neq-\frac{1}{2}\\ x^{\frac{3}{2}}x^{3n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\\ &+2Bb^3\left(\begin{cases}\frac{x^{\frac{3}{2}}x^{4n}}{8n+3} & \text{for}\ n\neq-\frac{3}{8}\\ x^{\frac{3}{2}}x^{4n}\log\left(\sqrt{x}\right) & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate(x**(1/2)*(a+b*x**n)**3*(A+B*x**n),x)
```

output
```
2*A*a**3*x**(3/2)/3 + 6*A*a**2*b*Piecewise((x**(3/2)*x**n/(2*n + 3), Ne(n,
 -3/2)), (x**(3/2)*x**n*log(sqrt(x)), True)) + 6*A*a*b**2*Piecewise((x**(3
/2)*x**(2*n)/(4*n + 3), Ne(n, -3/4)), (x**(3/2)*x**(2*n)*log(sqrt(x)), Tru
e)) + 2*A*b**3*Piecewise((x**(3/2)*x**(3*n)/(6*n + 3), Ne(n, -1/2)), (x**(
3/2)*x**(3*n)*log(sqrt(x)), True)) + 2*B*a**3*Piecewise((x**(3/2)*x**n/(2*
n + 3), Ne(n, -3/2)), (x**(3/2)*x**n*log(sqrt(x)), True)) + 6*B*a**2*b*Pie
cewise((x**(3/2)*x**(2*n)/(4*n + 3), Ne(n, -3/4)), (x**(3/2)*x**(2*n)*log(
sqrt(x)), True)) + 6*B*a*b**2*Piecewise((x**(3/2)*x**(3*n)/(6*n + 3), Ne(n
, -1/2)), (x**(3/2)*x**(3*n)*log(sqrt(x)), True)) + 2*B*b**3*Piecewise((x*
*(3/2)*x**(4*n)/(8*n + 3), Ne(n, -3/8)), (x**(3/2)*x**(4*n)*log(sqrt(x)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \sqrt{x}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2\,Bb^3x^{4\,n+\frac{3}{2}}}{8\,n+3} + \frac{2\,Bab^2x^{3\,n+\frac{3}{2}}}{2\,n+1} + \frac{2\,Ab^3x^{3\,n+\frac{3}{2}}}{3\,(2\,n+1)} + \frac{6\,Ba^2bx^{2\,n+\frac{3}{2}}}{4\,n+3} + \frac{6\,Aab^2x^{2\,n+\frac{3}{2}}}{4\,n+3} + \frac{2\,Ba^3x^{n+\frac{3}{2}}}{2\,n+3} + \frac{6\,Aa^2bx^{n+\frac{3}{2}}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="maxima")
```

output
```
2/3*A*a^3*x^(3/2) + 2*B*b^3*x^(4*n + 3/2)/(8*n + 3) + 2*B*a*b^2*x^(3*n + 3
/2)/(2*n + 1) + 2/3*A*b^3*x^(3*n + 3/2)/(2*n + 1) + 6*B*a^2*b*x^(2*n + 3/2
)/(4*n + 3) + 6*A*a*b^2*x^(2*n + 3/2)/(4*n + 3) + 2*B*a^3*x^(n + 3/2)/(2*n
 + 3) + 6*A*a^2*b*x^(n + 3/2)/(2*n + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.45

$$\int \sqrt{x}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2\,Bb^3x^{\frac{3}{2}}\sqrt{x}^{8\,n}}{8\,n+3} + \frac{2\,Bab^2x^{\frac{3}{2}}\sqrt{x}^{6\,n}}{2\,n+1} + \frac{2\,Ab^3x^{\frac{3}{2}}\sqrt{x}^{6\,n}}{3\,(2\,n+1)} + \frac{6\,Ba^2bx^{\frac{3}{2}}\sqrt{x}^{4\,n}}{4\,n+3} + \frac{6\,Aab^2x^{\frac{3}{2}}\sqrt{x}^{4\,n}}{4\,n+3} + \frac{2\,Ba^3x^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3} + \frac{6\,Aa^2bx^{\frac{3}{2}}\sqrt{x}^{2\,n}}{2\,n+3}$$

input
```
integrate(x^(1/2)*(a+b*x^n)^3*(A+B*x^n),x, algorithm="giac")
```

output
```
2/3*A*a^3*x^(3/2) + 2*B*b^3*x^(3/2)*sqrt(x)^(8*n)/(8*n + 3) + 2*B*a*b^2*x^
(3/2)*sqrt(x)^(6*n)/(2*n + 1) + 2/3*A*b^3*x^(3/2)*sqrt(x)^(6*n)/(2*n + 1)
+ 6*B*a^2*b*x^(3/2)*sqrt(x)^(4*n)/(4*n + 3) + 6*A*a*b^2*x^(3/2)*sqrt(x)^(4
*n)/(4*n + 3) + 2*B*a^3*x^(3/2)*sqrt(x)^(2*n)/(2*n + 3) + 6*A*a^2*b*x^(3/2
)*sqrt(x)^(2*n)/(2*n + 3)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(a+bx^n)^3\,(A+Bx^n)\;dx = \frac{2\,A\,a^3\,x^{3/2}}{3} + \frac{x^n\,x^{3/2}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)}{2\,n+3} + \frac{x^{3\,n}\,x^{3/2}\,\left(2\,A\,b^3+6\,B\,a\,b^2\right)}{6\,n+3} + \frac{2\,B\,b^3\,x^{4\,n}\,x^{3/2}}{8\,n+3} + \frac{6\,a\,b\,x^{2\,n}\,x^{3/2}\,\left(A\,b+B\,a\right)}{4\,n+3}$$

input
```
int(x^(1/2)*(A + B*x^n)*(a + b*x^n)^3,x)
```

output
```
(2*A*a^3*x^(3/2))/3 + (x^n*x^(3/2)*(2*B*a^3 + 6*A*a^2*b))/(2*n + 3) + (x^(
3*n)*x^(3/2)*(2*A*b^3 + 6*B*a*b^2))/(6*n + 3) + (2*B*b^3*x^(4*n)*x^(3/2))/
(8*n + 3) + (6*a*b*x^(2*n)*x^(3/2)*(A*b + B*a))/(4*n + 3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.17

$$\int \sqrt{x}(a+bx^n)^3\left(A+Bx^n\right)\,dx$$
$$= \frac{2\sqrt{x}\,x(48x^{4n}b^4n^3+132x^{4n}b^4n^2+108x^{4n}b^4n+27x^{4n}b^4+256x^{3n}a\,b^3n^3+672x^{3n}a\,b^3n^2+504x^{3n}a\,b^3n+}{}$$

input
```
int(x^(1/2)*(a+b*x^n)^3*(A+B*x^n),x)
```

output
```
(2*sqrt(x)*x*(48*x**(4*n)*b**4*n**3 + 132*x**(4*n)*b**4*n**2 + 108*x**(4*n
)*b**4*n + 27*x**(4*n)*b**4 + 256*x**(3*n)*a*b**3*n**3 + 672*x**(3*n)*a*b*
*3*n**2 + 504*x**(3*n)*a*b**3*n + 108*x**(3*n)*a*b**3 + 576*x**(2*n)*a**2*
b**2*n**3 + 1368*x**(2*n)*a**2*b**2*n**2 + 864*x**(2*n)*a**2*b**2*n + 162*
x**(2*n)*a**2*b**2 + 768*x**n*a**3*b*n**3 + 1248*x**n*a**3*b*n**2 + 648*x*
*n*a**3*b*n + 108*x**n*a**3*b + 128*a**4*n**4 + 400*a**4*n**3 + 420*a**4*n
**2 + 180*a**4*n + 27*a**4))/(3*(128*n**4 + 400*n**3 + 420*n**2 + 180*n +
27))
```

3.308 $\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx = 2a^3A\sqrt{x} + \frac{2a^2(3Ab+aB)x^{\frac{1}{2}+n}}{1+2n} + \frac{6ab(Ab+aB)x^{\frac{1}{2}+2n}}{1+4n} + \frac{2b^2(Ab+3aB)x^{\frac{1}{2}+3n}}{1+6n} + \frac{2b^3Bx^{\frac{1}{2}+4n}}{1+8n}$$

output
```
2*a^3*A*x^(1/2)+2*a^2*(3*A*b+B*a)*x^(1/2+n)/(1+2*n)+6*a*b*(A*b+B*a)*x^(1/2
+2*n)/(1+4*n)+2*b^2*(A*b+3*B*a)*x^(1/2+3*n)/(1+6*n)+2*b^3*B*x^(1/2+4*n)/(1
+8*n)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx = 2\left(a^3A\sqrt{x} + \frac{a^2(3Ab+aB)x^{\frac{1}{2}+n}}{1+2n} + \frac{3ab(Ab+aB)x^{\frac{1}{2}+2n}}{1+4n} + \frac{b^2(Ab+3aB)x^{\frac{1}{2}+3n}}{1+6n} + \frac{b^3Bx^{\frac{1}{2}+4n}}{1+8n}\right)$$

input
```
Integrate[((a + b*x^n)^3*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*(a^3*A*Sqrt[x] + (a^2*(3*A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (3*a*b*(A*b
 + a*B)*x^(1/2 + 2*n))/(1 + 4*n) + (b^2*(A*b + 3*a*B)*x^(1/2 + 3*n))/(1 +
6*n) + (b^3*B*x^(1/2 + 4*n))/(1 + 8*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \ 950$$

$$\int \left(\frac{a^3 A}{\sqrt{x}}+a^2 x^{n-\frac{1}{2}}(aB+3Ab)+b^2 x^{3n-\frac{1}{2}}(3aB+Ab)+3abx^{2n-\frac{1}{2}}(aB+Ab)+b^3 B x^{4n-\frac{1}{2}}\right) dx$$

$$\Big\downarrow \ 2009$$

$$2a^3 A\sqrt{x}+\frac{2a^2 x^{n+\frac{1}{2}}(aB+3Ab)}{2n+1}+\frac{2b^2 x^{3n+\frac{1}{2}}(3aB+Ab)}{6n+1}+\frac{6abx^{2n+\frac{1}{2}}(aB+Ab)}{4n+1}+\frac{2b^3 B x^{4n+\frac{1}{2}}}{8n+1}$$

input
```
Int[((a + b*x^n)^3*(A + B*x^n))/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(1/2 + n))/(1 + 2*n) + (6*a*b*(A*
b + a*B)*x^(1/2 + 2*n))/(1 + 4*n) + (2*b^2*(A*b + 3*a*B)*x^(1/2 + 3*n))/(1
 + 6*n) + (2*b^3*B*x^(1/2 + 4*n))/(1 + 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(107) = 214$.

Time = 0.52 (sec) , antiderivative size = 1241, normalized size of antiderivative = 10.61

method	result	size
orering	Expression too large to display	1241

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*(384*n^4+140*n^2+1)/(96*n^3+76*n^2+16*n+1)/(1+4*n)*(a+b*x^n)^3*(
A+B*x^n)-40*x^2*(40*n^3-42*n^2+20*n-3)/(96*n^3+76*n^2+16*n+1)/(1+4*n)*(3*(
a+b*x^n)^2*(A+B*x^n)/x^(3/2)*b*x^n*n+(a+b*x^n)^3*B*x^n*n/x^(3/2)-1/2*(a+b*
x^n)^3*(A+B*x^n)/x^(3/2))+80*x^3*(14*n^2-16*n+5)/(96*n^3+76*n^2+16*n+1)/(1
+4*n)*(6*(a+b*x^n)*(A+B*x^n)/x^(5/2)*b^2*(x^n)^2*n^2+6*(a+b*x^n)^2*B*(x^n)
^2*n^2/x^(5/2)*b-6*(a+b*x^n)^2*(A+B*x^n)/x^(5/2)*b*x^n*n+3*(a+b*x^n)^2*(A+
B*x^n)/x^(5/2)*b*x^n*n^2+(a+b*x^n)^3*B*x^n*n^2/x^(5/2)-2*(a+b*x^n)^3*B*x^n
*n/x^(5/2)+3/4*(a+b*x^n)^3*(A+B*x^n)/x^(5/2))-80*x^4*(-3+4*n)/(384*n^4+400
*n^3+140*n^2+20*n+1)*(6*b^3*(x^n)^3*n^3/x^(7/2)*(A+B*x^n)+18*(a+b*x^n)*B*(
x^n)^3*n^3/x^(7/2)*b^2-27*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b^2*(x^n)^2*n^2+18*(
a+b*x^n)*(A+B*x^n)/x^(7/2)*b^2*(x^n)^2*n^3+18*(a+b*x^n)^2*B*(x^n)^2*n^3/x^
(7/2)*b-27*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(7/2)*b+69/4*(a+b*x^n)^2*(A+B*x^n)/
x^(7/2)*b*x^n*n-27/2*(a+b*x^n)^2*(A+B*x^n)/x^(7/2)*b*x^n*n^2+3*(a+b*x^n)^2
*(A+B*x^n)/x^(7/2)*b*x^n*n^3+(a+b*x^n)^3*B*x^n*n^3/x^(7/2)-9/2*(a+b*x^n)^3
*B*x^n*n^2/x^(7/2)+23/4*(a+b*x^n)^3*B*x^n*n/x^(7/2)-15/8*(a+b*x^n)^3*(A+B*
x^n)/x^(7/2))+32/(384*n^4+400*n^3+140*n^2+20*n+1)*x^5*(105/16*(a+b*x^n)^3*
(A+B*x^n)/x^(9/2)+108*(a+b*x^n)*B*(x^n)^3*n^4/x^(9/2)*b^2+42*(a+b*x^n)^2*B
*(x^n)^2*n^4/x^(9/2)*b-48*b^3*(x^n)^3*n^3/x^(9/2)*(A+B*x^n)-144*(a+b*x^n)*
B*(x^n)^3*n^3/x^(9/2)*b^2-144*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b^2*(x^n)^2*n^3-
8*(a+b*x^n)^3*B*x^n*n^3/x^(9/2)+129*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(9/2)*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(107) = 214$.

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx = \frac{2\left((48\,Bb^3n^3+44\,Bb^3n^2+12\,Bb^3n+Bb^3)\sqrt{x}x^{4\,n}+(3\,Bab^2+Ab^3+64\,(3\,Bab^2+Ab^3)n^3+56\,(3\,Bab^2\right.}{}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(1/2),x, algorithm="fricas")
```

output
```
2*((48*B*b^3*n^3 + 44*B*b^3*n^2 + 12*B*b^3*n + B*b^3)*sqrt(x)*x^(4*n) + (3
*B*a*b^2 + A*b^3 + 64*(3*B*a*b^2 + A*b^3)*n^3 + 56*(3*B*a*b^2 + A*b^3)*n^2
 + 14*(3*B*a*b^2 + A*b^3)*n)*sqrt(x)*x^(3*n) + 3*(B*a^2*b + A*a*b^2 + 96*(
B*a^2*b + A*a*b^2)*n^3 + 76*(B*a^2*b + A*a*b^2)*n^2 + 16*(B*a^2*b + A*a*b^
2)*n)*sqrt(x)*x^(2*n) + (B*a^3 + 3*A*a^2*b + 192*(B*a^3 + 3*A*a^2*b)*n^3 +
 104*(B*a^3 + 3*A*a^2*b)*n^2 + 18*(B*a^3 + 3*A*a^2*b)*n)*sqrt(x)*x^n + (38
4*A*a^3*n^4 + 400*A*a^3*n^3 + 140*A*a^3*n^2 + 20*A*a^3*n + A*a^3)*sqrt(x))
/(384*n^4 + 400*n^3 + 140*n^2 + 20*n + 1)
```

Sympy [A] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x} - 6Aa^2b\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2}\\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 6Aab^2\left(\begin{cases}\frac{\sqrt{x}x^{2n}}{-4n-1} & \text{for } n \neq -\frac{1}{4}\\ \sqrt{x}x^{2n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Ab^3\left(\begin{cases}\frac{\sqrt{x}x^{3n}}{-6n-1} & \text{for } n \neq -\frac{1}{6}\\ \sqrt{x}x^{3n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Ba^3\left(\begin{cases}\frac{\sqrt{x}x^n}{-2n-1} & \text{for } n \neq -\frac{1}{2}\\ \sqrt{x}x^n\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 6Ba^2b\left(\begin{cases}\frac{\sqrt{x}x^{2n}}{-4n-1} & \text{for } n \neq -\frac{1}{4}\\ \sqrt{x}x^{2n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 6Bab^2\left(\begin{cases}\frac{\sqrt{x}x^{3n}}{-6n-1} & \text{for } n \neq -\frac{1}{6}\\ \sqrt{x}x^{3n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right) - 2Bb^3\left(\begin{cases}\frac{\sqrt{x}x^{4n}}{-8n-1} & \text{for } n \neq -\frac{1}{8}\\ \sqrt{x}x^{4n}\log\left(\frac{1}{\sqrt{x}}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)**3*(A+B*x**n)/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) - 6*A*a**2*b*Piecewise((sqrt(x)*x**n/(-2*n - 1), Ne(n, -1
/2)), (sqrt(x)*x**n*log(1/sqrt(x)), True)) - 6*A*a*b**2*Piecewise((sqrt(x)
*x**(2*n)/(-4*n - 1), Ne(n, -1/4)), (sqrt(x)*x**(2*n)*log(1/sqrt(x)), True
)) - 2*A*b**3*Piecewise((sqrt(x)*x**(3*n)/(-6*n - 1), Ne(n, -1/6)), (sqrt(
x)*x**(3*n)*log(1/sqrt(x)), True)) - 2*B*a**3*Piecewise((sqrt(x)*x**n/(-2*
n - 1), Ne(n, -1/2)), (sqrt(x)*x**n*log(1/sqrt(x)), True)) - 6*B*a**2*b*Pi
ecewise((sqrt(x)*x**(2*n)/(-4*n - 1), Ne(n, -1/4)), (sqrt(x)*x**(2*n)*log(
1/sqrt(x)), True)) - 6*B*a*b**2*Piecewise((sqrt(x)*x**(3*n)/(-6*n - 1), Ne
(n, -1/6)), (sqrt(x)*x**(3*n)*log(1/sqrt(x)), True)) - 2*B*b**3*Piecewise(
(sqrt(x)*x**(4*n)/(-8*n - 1), Ne(n, -1/8)), (sqrt(x)*x**(4*n)*log(1/sqrt(x
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{\sqrt{x}} \, dx = 2 \, Aa^3\sqrt{x} + \frac{2 \, Bb^3x^{4 \, n+\frac{1}{2}}}{8 \, n+1} + \frac{6 \, Bab^2x^{3 \, n+\frac{1}{2}}}{6 \, n+1} + \frac{2 \, Ab^3x^{3 \, n+\frac{1}{2}}}{6 \, n+1} + \frac{6 \, Ba^2bx^{2 \, n+\frac{1}{2}}}{4 \, n+1} + \frac{6 \, Aab^2x^{2 \, n+\frac{1}{2}}}{4 \, n+1} + \frac{2 \, Ba^3x^{n+\frac{1}{2}}}{2 \, n+1} + \frac{6 \, Aa^2bx^{n+\frac{1}{2}}}{2 \, n+1}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(1/2),x, algorithm="maxima")
```

output
```
2*A*a^3*sqrt(x) + 2*B*b^3*x^(4*n + 1/2)/(8*n + 1) + 6*B*a*b^2*x^(3*n + 1/2
)/(6*n + 1) + 2*A*b^3*x^(3*n + 1/2)/(6*n + 1) + 6*B*a^2*b*x^(2*n + 1/2)/(4
*n + 1) + 6*A*a*b^2*x^(2*n + 1/2)/(4*n + 1) + 2*B*a^3*x^(n + 1/2)/(2*n + 1
) + 6*A*a^2*b*x^(n + 1/2)/(2*n + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx = 2\,Aa^3\sqrt{x} + \frac{2\,Bb^3x^{4\,n+\frac{1}{2}}}{8\,n+1} + \frac{6\,Bab^2x^{3\,n+\frac{1}{2}}}{6\,n+1} + \frac{2\,Ab^3x^{3\,n+\frac{1}{2}}}{6\,n+1} + \frac{6\,Ba^2bx^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{6\,Aab^2x^{2\,n+\frac{1}{2}}}{4\,n+1} + \frac{2\,Ba^3x^{n+\frac{1}{2}}}{2\,n+1} + \frac{6\,Aa^2bx^{n+\frac{1}{2}}}{2\,n+1}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(1/2),x, algorithm="giac")
```

output
```
2*A*a^3*sqrt(x) + 2*B*b^3*x^(4*n + 1/2)/(8*n + 1) + 6*B*a*b^2*x^(3*n + 1/2
)/(6*n + 1) + 2*A*b^3*x^(3*n + 1/2)/(6*n + 1) + 6*B*a^2*b*x^(2*n + 1/2)/(4
*n + 1) + 6*A*a*b^2*x^(2*n + 1/2)/(4*n + 1) + 2*B*a^3*x^(n + 1/2)/(2*n + 1
) + 6*A*a^2*b*x^(n + 1/2)/(2*n + 1)
```

Mupad [B] (verification not implemented)

Time = 4.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{\sqrt{x}}\,dx = 2\,A\,a^3\,\sqrt{x} + \frac{x^n\,\sqrt{x}\,(2\,B\,a^3+6\,A\,b\,a^2)}{2\,n+1} + \frac{x^{3\,n}\,\sqrt{x}\,(2\,A\,b^3+6\,B\,a\,b^2)}{6\,n+1} + \frac{2\,B\,b^3\,x^{4\,n}\,\sqrt{x}}{8\,n+1} + \frac{6\,a\,b\,x^{2\,n}\,\sqrt{x}\,(A\,b+B\,a)}{4\,n+1}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^3)/x^(1/2),x)
```

output
```
2*A*a^3*x^(1/2) + (x^n*x^(1/2)*(2*B*a^3 + 6*A*a^2*b))/(2*n + 1) + (x^(3*n)
*x^(1/2)*(2*A*b^3 + 6*B*a*b^2))/(6*n + 1) + (2*B*b^3*x^(4*n)*x^(1/2))/(8*n
 + 1) + (6*a*b*x^(2*n)*x^(1/2)*(A*b + B*a))/(4*n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.21

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}\,(48x^{4n}b^4n^3+44x^{4n}b^4n^2+12x^{4n}b^4n+x^{4n}b^4+256x^{3n}a\,b^3n^3+224x^{3n}a\,b^3n^2+56x^{3n}a\,b^3n+4x^{3n}a\,b}{}$$

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(48*x**(4*n)*b**4*n**3 + 44*x**(4*n)*b**4*n**2 + 12*x**(4*n)*b*
*4*n + x**(4*n)*b**4 + 256*x**(3*n)*a*b**3*n**3 + 224*x**(3*n)*a*b**3*n**2
 + 56*x**(3*n)*a*b**3*n + 4*x**(3*n)*a*b**3 + 576*x**(2*n)*a**2*b**2*n**3
+ 456*x**(2*n)*a**2*b**2*n**2 + 96*x**(2*n)*a**2*b**2*n + 6*x**(2*n)*a**2*
b**2 + 768*x**n*a**3*b*n**3 + 416*x**n*a**3*b*n**2 + 72*x**n*a**3*b*n + 4*
x**n*a**3*b + 384*a**4*n**4 + 400*a**4*n**3 + 140*a**4*n**2 + 20*a**4*n +
a**4))/(384*n**4 + 400*n**3 + 140*n**2 + 20*n + 1)
```

3.309 $\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}}-\frac{2a^2(3Ab+aB)x^{-\frac{1}{2}+n}}{1-2n} -\frac{6ab(Ab+aB)x^{-\frac{1}{2}+2n}}{1-4n}-\frac{2b^2(Ab+3aB)x^{-\frac{1}{2}+3n}}{1-6n}-\frac{2b^3Bx^{-\frac{1}{2}+4n}}{1-8n}$$

output
```
-2*a^3*A/x^(1/2)-2*a^2*(3*A*b+B*a)*x^(-1/2+n)/(1-2*n)-6*a*b*(A*b+B*a)*x^(-
1/2+2*n)/(1-4*n)-2*b^2*(A*b+3*B*a)*x^(-1/2+3*n)/(1-6*n)-2*b^3*B*x^(-1/2+4*
n)/(1-8*n)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{3/2}}\,dx = 2\left(-\frac{a^3A}{\sqrt{x}}+\frac{a^2(3Ab+aB)x^{-\frac{1}{2}+n}}{-1+2n} +\frac{3ab(Ab+aB)x^{-\frac{1}{2}+2n}}{-1+4n}+\frac{b^2(Ab+3aB)x^{-\frac{1}{2}+3n}}{-1+6n}+\frac{b^3Bx^{-\frac{1}{2}+4n}}{-1+8n}\right)$$

input
```
Integrate[((a + b*x^n)^3*(A + B*x^n))/x^(3/2),x]
```

output
```
2*(-((a^3*A)/Sqrt[x]) + (a^2*(3*A*b + a*B)*x^(-1/2 + n))/(-1 + 2*n) + (3*a
*b*(A*b + a*B)*x^(-1/2 + 2*n))/(-1 + 4*n) + (b^2*(A*b + 3*a*B)*x^(-1/2 + 3
*n))/(-1 + 6*n) + (b^3*B*x^(-1/2 + 4*n))/(-1 + 8*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3 A}{x^{3/2}}+a^2 x^{n-\frac{3}{2}} (aB+3Ab)+b^2 x^{3n-\frac{3}{2}} (3aB+Ab)+3abx^{2n-\frac{3}{2}} (aB+Ab)+b^3 Bx^{4n-\frac{3}{2}}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3 A}{\sqrt{x}}-\frac{2a^2 x^{n-\frac{1}{2}} (aB+3Ab)}{1-2n}-\frac{2b^2 x^{3n-\frac{1}{2}} (3aB+Ab)}{1-6n}-\frac{6abx^{2n-\frac{1}{2}} (aB+Ab)}{1-4n}-\frac{2b^3 Bx^{4n-\frac{1}{2}}}{1-8n}$$

input
```
Int[((a + b*x^n)^3*(A + B*x^n))/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] - (2*a^2*(3*A*b + a*B)*x^(-1/2 + n))/(1 - 2*n) - (6*a*b
*(A*b + a*B)*x^(-1/2 + 2*n))/(1 - 4*n) - (2*b^2*(A*b + 3*a*B)*x^(-1/2 + 3*
n))/(1 - 6*n) - (2*b^3*B*x^(-1/2 + 4*n))/(1 - 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. $2(107) = 214$.

Time = 0.53 (sec) , antiderivative size = 1239, normalized size of antiderivative = 10.59

method	result	size
orering	Expression too large to display	1239

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(192*n^3-704*n^2+558*n-121)/x^(1/2)/(-1+4*n)/(48*n^2-14*n+1)*(a+b*x^n)^
3*(A+B*x^n)+40*x^2*(40*n^3-126*n^2+116*n-33)/(96*n^3-76*n^2+16*n-1)/(-1+4*
n)*(3*(a+b*x^n)^2*(A+B*x^n)/x^(5/2)*b*x^n*n+(a+b*x^n)^3*B*x^n*n/x^(5/2)-3/
2*(a+b*x^n)^3*(A+B*x^n)/x^(5/2))-80*x^3*(14*n^2-32*n+17)/(96*n^3-76*n^2+16
*n-1)/(-1+4*n)*(6*(a+b*x^n)*(A+B*x^n)/x^(7/2)*b^2*(x^n)^2*n^2+6*(a+b*x^n)^
2*B*(x^n)^2*n^2/x^(7/2)*b-12*(a+b*x^n)^2*(A+B*x^n)/x^(7/2)*b*x^n*n+3*(a+b*
x^n)^2*(A+B*x^n)/x^(7/2)*b*x^n*n^2+(a+b*x^n)^3*B*x^n*n^2/x^(7/2)-4*(a+b*x^
n)^3*B*x^n*n/x^(7/2)+15/4*(a+b*x^n)^3*(A+B*x^n)/x^(7/2))+80*x^4*(-5+4*n)/(
384*n^4-400*n^3+140*n^2-20*n+1)*(6*b^3*(x^n)^3*n^3/x^(9/2)*(A+B*x^n)+18*(a
+b*x^n)*B*(x^n)^3*n^3/x^(9/2)*b^2-45*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b^2*(x^n)
^2*n^2+18*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b^2*(x^n)^2*n^3+18*(a+b*x^n)^2*B*(x^
n)^2*n^3/x^(9/2)*b-45*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(9/2)*b+213/4*(a+b*x^n)^
2*(A+B*x^n)/x^(9/2)*b*x^n*n-45/2*(a+b*x^n)^2*(A+B*x^n)/x^(9/2)*b*x^n*n^2+3
*(a+b*x^n)^2*(A+B*x^n)/x^(9/2)*b*x^n*n^3+(a+b*x^n)^3*B*x^n*n^3/x^(9/2)-15/
2*(a+b*x^n)^3*B*x^n*n^2/x^(9/2)+71/4*(a+b*x^n)^3*B*x^n*n/x^(9/2)-105/8*(a+
b*x^n)^3*(A+B*x^n)/x^(9/2))-32/(384*n^4-400*n^3+140*n^2-20*n+1)*x^5*(42*(a
+b*x^n)*(A+B*x^n)/x^(11/2)*b^2*(x^n)^2*n^4-216*(a+b*x^n)^2*B*(x^n)^2*n^3/x
^(11/2)*b+309*(a+b*x^n)*(A+B*x^n)/x^(11/2)*b^2*(x^n)^2*n^2-279*(a+b*x^n)^2
*(A+B*x^n)/x^(11/2)*b*x^n*n+108*(a+b*x^n)*B*(x^n)^3*n^4/x^(11/2)*b^2+945/1
6*(a+b*x^n)^3*(A+B*x^n)/x^(11/2)-12*(a+b*x^n)^3*B*x^n*n^3/x^(11/2)+36*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(107) = 214$.

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}}\,dx = \frac{2\left((48\,Bb^3n^3 - 44\,Bb^3n^2 + 12\,Bb^3n - Bb^3)\sqrt{x}x^{4\,n} - (3\,Bab^2 + Ab^3 - 64\,(3\right.}{}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(3/2),x, algorithm="fricas")
```

output
```
2*((48*B*b^3*n^3 - 44*B*b^3*n^2 + 12*B*b^3*n - B*b^3)*sqrt(x)*x^(4*n) - (3
*B*a*b^2 + A*b^3 - 64*(3*B*a*b^2 + A*b^3)*n^3 + 56*(3*B*a*b^2 + A*b^3)*n^2
 - 14*(3*B*a*b^2 + A*b^3)*n)*sqrt(x)*x^(3*n) - 3*(B*a^2*b + A*a*b^2 - 96*(
B*a^2*b + A*a*b^2)*n^3 + 76*(B*a^2*b + A*a*b^2)*n^2 - 16*(B*a^2*b + A*a*b^
2)*n)*sqrt(x)*x^(2*n) - (B*a^3 + 3*A*a^2*b - 192*(B*a^3 + 3*A*a^2*b)*n^3 +
 104*(B*a^3 + 3*A*a^2*b)*n^2 - 18*(B*a^3 + 3*A*a^2*b)*n)*sqrt(x)*x^n - (38
4*A*a^3*n^4 - 400*A*a^3*n^3 + 140*A*a^3*n^2 - 20*A*a^3*n + A*a^3)*sqrt(x))
/((384*n^4 - 400*n^3 + 140*n^2 - 20*n + 1)*x)
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.35

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}} + 3Aa^2b\left(\begin{cases}\frac{x^n}{\sqrt{x}(n-\frac{1}{2})} & \text{for } n\neq\frac{1}{2}\\ \frac{x^n\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + 3Aab^2\left(\begin{cases}\frac{x^{2n}}{\sqrt{x}(2n-\frac{1}{2})} & \text{for } n\neq\frac{1}{4}\\ \frac{x^{2n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Ab^3\left(\begin{cases}\frac{x^{3n}}{\sqrt{x}(3n-\frac{1}{2})} & \text{for } n\neq\frac{1}{6}\\ \frac{x^{3n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Ba^3\left(\begin{cases}\frac{x^n}{\sqrt{x}(n-\frac{1}{2})} & \text{for } n\neq\frac{1}{2}\\ \frac{x^n\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + 3Ba^2b\left(\begin{cases}\frac{x^{2n}}{\sqrt{x}(2n-\frac{1}{2})} & \text{for } n\neq\frac{1}{4}\\ \frac{x^{2n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + 3Bab^2\left(\begin{cases}\frac{x^{3n}}{\sqrt{x}(3n-\frac{1}{2})} & \text{for } n\neq\frac{1}{6}\\ \frac{x^{3n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right) + Bb^3\left(\begin{cases}\frac{x^{4n}}{\sqrt{x}(4n-\frac{1}{2})} & \text{for } n\neq\frac{1}{8}\\ \frac{x^{4n}\log(x)}{\sqrt{x}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)**3*(A+B*x**n)/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 3*A*a**2*b*Piecewise((x**n/(sqrt(x)*(n - 1/2)), Ne(n,
1/2)), (x**n*log(x)/sqrt(x), True)) + 3*A*a*b**2*Piecewise((x**(2*n)/(sqrt
(x)*(2*n - 1/2)), Ne(n, 1/4)), (x**(2*n)*log(x)/sqrt(x), True)) + A*b**3*P
iecewise((x**(3*n)/(sqrt(x)*(3*n - 1/2)), Ne(n, 1/6)), (x**(3*n)*log(x)/sq
rt(x), True)) + B*a**3*Piecewise((x**n/(sqrt(x)*(n - 1/2)), Ne(n, 1/2)), (
x**n*log(x)/sqrt(x), True)) + 3*B*a**2*b*Piecewise((x**(2*n)/(sqrt(x)*(2*n
 - 1/2)), Ne(n, 1/4)), (x**(2*n)*log(x)/sqrt(x), True)) + 3*B*a*b**2*Piece
wise((x**(3*n)/(sqrt(x)*(3*n - 1/2)), Ne(n, 1/6)), (x**(3*n)*log(x)/sqrt(x
), True)) + B*b**3*Piecewise((x**(4*n)/(sqrt(x)*(4*n - 1/2)), Ne(n, 1/8)),
 (x**(4*n)*log(x)/sqrt(x), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-3/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^3}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^3/x^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}} \, dx = \frac{B\,a^3\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}} - \frac{2\,A\,a^3}{\sqrt{x}} + \frac{2\,A\,b^3\,x^{3\,n-\frac{1}{2}}}{6\,n-1} + \frac{2\,B\,b^3\,x^{4\,n-\frac{1}{2}}}{8\,n-1} + \frac{6\,A\,a\,b^2\,x^{2\,n-\frac{1}{2}}}{4\,n-1} + \frac{6\,B\,a^2\,b\,x^{2\,n-\frac{1}{2}}}{4\,n-1} + \frac{6\,B\,a\,b^2\,x^{3\,n-\frac{1}{2}}}{6\,n-1} + \frac{3\,A\,a^2\,b\,x^{n-\frac{1}{2}}}{n-\frac{1}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^3)/x^(3/2),x)
```

output
```
(B*a^3*x^(n - 1/2))/(n - 1/2) - (2*A*a^3)/x^(1/2) + (2*A*b^3*x^(3*n - 1/2)
)/(6*n - 1) + (2*B*b^3*x^(4*n - 1/2))/(8*n - 1) + (6*A*a*b^2*x^(2*n - 1/2)
)/(4*n - 1) + (6*B*a^2*b*x^(2*n - 1/2))/(4*n - 1) + (6*B*a*b^2*x^(3*n - 1/
2))/(6*n - 1) + (3*A*a^2*b*x^(n - 1/2))/(n - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{3/2}} \, dx = \frac{96x^{4n}b^4n^3 - 88x^{4n}b^4n^2 + 24x^{4n}b^4n - 2x^{4n}b^4 + 512x^{3n}a\,b^3n^3 - 448x^{3n}a\,b^3n^2}{}$$

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(3/2),x)
```

output
```
(2*(48*x**(4*n)*b**4*n**3 - 44*x**(4*n)*b**4*n**2 + 12*x**(4*n)*b**4*n - x
**(4*n)*b**4 + 256*x**(3*n)*a*b**3*n**3 - 224*x**(3*n)*a*b**3*n**2 + 56*x*
*(3*n)*a*b**3*n - 4*x**(3*n)*a*b**3 + 576*x**(2*n)*a**2*b**2*n**3 - 456*x*
*(2*n)*a**2*b**2*n**2 + 96*x**(2*n)*a**2*b**2*n - 6*x**(2*n)*a**2*b**2 + 7
68*x**n*a**3*b*n**3 - 416*x**n*a**3*b*n**2 + 72*x**n*a**3*b*n - 4*x**n*a**
3*b - 384*a**4*n**4 + 400*a**4*n**3 - 140*a**4*n**2 + 20*a**4*n - a**4))/(
sqrt(x)*(384*n**4 - 400*n**3 + 140*n**2 - 20*n + 1))
```

3.310 $\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx = -\frac{2a^3A}{3x^{3/2}} - \frac{2a^2(3Ab+aB)x^{-\frac{3}{2}+n}}{3-2n} - \frac{6ab(Ab+aB)x^{-\frac{3}{2}+2n}}{3-4n} - \frac{2b^2(Ab+3aB)x^{-\frac{3}{2}+3n}}{3(1-2n)} - \frac{2b^3Bx^{-\frac{3}{2}+4n}}{3-8n}$$

output
```
-2/3*a^3*A/x^(3/2)-2*a^2*(3*A*b+B*a)*x^(-3/2+n)/(3-2*n)-6*a*b*(A*b+B*a)*x^
(-3/2+2*n)/(3-4*n)-2*b^2*(A*b+3*B*a)*x^(-3/2+3*n)/(3-6*n)-2*b^3*B*x^(-3/2+
4*n)/(3-8*n)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx = \frac{2}{3}\left(-\frac{a^3A}{x^{3/2}} + \frac{3a^2(3Ab+aB)x^{-\frac{3}{2}+n}}{-3+2n} + \frac{9ab(Ab+aB)x^{-\frac{3}{2}+2n}}{-3+4n} + \frac{b^2(Ab+3aB)x^{-\frac{3}{2}+3n}}{-1+2n} + \frac{3b^3Bx^{-\frac{3}{2}+4n}}{-3+8n}\right)$$

input
```
Integrate[((a + b*x^n)^3*(A + B*x^n))/x^(5/2),x]
```

output
```
(2*(-((a^3*A)/x^(3/2)) + (3*a^2*(3*A*b + a*B)*x^(-3/2 + n))/(-3 + 2*n) + (
9*a*b*(A*b + a*B)*x^(-3/2 + 2*n))/(-3 + 4*n) + (b^2*(A*b + 3*a*B)*x^(-3/2
+ 3*n))/(-1 + 2*n) + (3*b^3*B*x^(-3/2 + 4*n))/(-3 + 8*n)))/3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(\frac{a^3 A}{x^{5/2}}+a^2 x^{n-\frac{5}{2}} (aB+3Ab)+b^2 x^{3n-\frac{5}{2}} (3aB+Ab)+3abx^{2n-\frac{5}{2}} (aB+Ab)+b^3 Bx^{4n-\frac{5}{2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3 A}{3x^{3/2}}-\frac{2a^2 x^{n-\frac{3}{2}} (aB+3Ab)}{3-2n}-\frac{2b^2 x^{3n-\frac{3}{2}} (3aB+Ab)}{3(1-2n)}-\frac{6abx^{2n-\frac{3}{2}} (aB+Ab)}{3-4n}-\frac{2b^3 Bx^{4n-\frac{3}{2}}}{3-8n}$$

input
```
Int[((a + b*x^n)^3*(A + B*x^n))/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) - (2*a^2*(3*A*b + a*B)*x^(-3/2 + n))/(3 - 2*n) - (6
*a*b*(A*b + a*B)*x^(-3/2 + 2*n))/(3 - 4*n) - (2*b^2*(A*b + 3*a*B)*x^(-3/2
+ 3*n))/(3*(1 - 2*n)) - (2*b^3*B*x^(-3/2 + 4*n))/(3 - 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(107) = 214$.

Time = 0.54 (sec) , antiderivative size = 1249, normalized size of antiderivative = 10.32

method	result	size
orering	Expression too large to display	1249

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9/x^(3/2)*(384*n^4-3200*n^3+6860*n^2-5440*n+1441)/(32*n^3-76*n^2+48*n-9
)/(-3+4*n)*(a+b*x^n)^3*(A+B*x^n)+40/9*x^2*(40*n^3-210*n^2+308*n-135)/(32*n
^3-76*n^2+48*n-9)/(-3+4*n)*(3*(a+b*x^n)^2*(A+B*x^n)/x^(7/2)*b*x^n*n+(a+b*x
^n)^3*B*x^n*n/x^(7/2)-5/2*(a+b*x^n)^3*(A+B*x^n)/x^(7/2))-80/9*x^3*(14*n^2-
48*n+37)/(32*n^3-76*n^2+48*n-9)/(-3+4*n)*(6*(a+b*x^n)*(A+B*x^n)/x^(9/2)*b^
2*(x^n)^2*n^2+6*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(9/2)*b-18*(a+b*x^n)^2*(A+B*x^
n)/x^(9/2)*b*x^n*n+3*(a+b*x^n)^2*(A+B*x^n)/x^(9/2)*b*x^n*n^2+(a+b*x^n)^3*B
*x^n*n^2/x^(9/2)-6*(a+b*x^n)^3*B*x^n*n/x^(9/2)+35/4*(a+b*x^n)^3*(A+B*x^n)/
x^(9/2))+80/9*x^4*(-7+4*n)/(128*n^4-400*n^3+420*n^2-180*n+27)*(6*b^3*(x^n)
^3*n^3/x^(11/2)*(A+B*x^n)+18*(a+b*x^n)*B*(x^n)^3*n^3/x^(11/2)*b^2-63*(a+b*
x^n)*(A+B*x^n)/x^(11/2)*b^2*(x^n)^2*n^2+18*(a+b*x^n)*(A+B*x^n)/x^(11/2)*b^
2*(x^n)^2*n^3+18*(a+b*x^n)^2*B*(x^n)^2*n^3/x^(11/2)*b-63*(a+b*x^n)^2*B*(x^
n)^2*n^2/x^(11/2)*b+429/4*(a+b*x^n)^2*(A+B*x^n)/x^(11/2)*b*x^n*n-63/2*(a+b
*x^n)^2*(A+B*x^n)/x^(11/2)*b*x^n*n^2+3*(a+b*x^n)^2*(A+B*x^n)/x^(11/2)*b*x^
n*n^3+(a+b*x^n)^3*B*x^n*n^3/x^(11/2)-21/2*(a+b*x^n)^3*B*x^n*n^2/x^(11/2)+1
43/4*(a+b*x^n)^3*B*x^n*n/x^(11/2)-315/8*(a+b*x^n)^3*(A+B*x^n)/x^(11/2))-32
/9/(128*n^4-400*n^3+420*n^2-180*n+27)*x^5*(42*(a+b*x^n)^2*B*(x^n)^2*n^4/x^
(13/2)*b-96*b^3*(x^n)^3*n^3/x^(13/2)*(A+B*x^n)-288*(a+b*x^n)*B*(x^n)^3*n^3
/x^(13/2)*b^2-288*(a+b*x^n)*(A+B*x^n)/x^(13/2)*b^2*(x^n)^2*n^3+561*(a+b*x^
n)^2*B*(x^n)^2*n^2/x^(13/2)*b+108*(a+b*x^n)*B*(x^n)^3*n^4/x^(13/2)*b^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}}\,dx = \frac{2\left(3\left(16\,Bb^3n^3 - 44\,Bb^3n^2 + 36\,Bb^3n - 9\,Bb^3\right)\sqrt{x}x^{4\,n} - \left(81\,Bab^2 + 27\,Ab^3\right.}{}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*(16*B*b^3*n^3 - 44*B*b^3*n^2 + 36*B*b^3*n - 9*B*b^3)*sqrt(x)*x^(4*n
) - (81*B*a*b^2 + 27*A*b^3 - 64*(3*B*a*b^2 + A*b^3)*n^3 + 168*(3*B*a*b^2 +
 A*b^3)*n^2 - 126*(3*B*a*b^2 + A*b^3)*n)*sqrt(x)*x^(3*n) - 9*(9*B*a^2*b +
9*A*a*b^2 - 32*(B*a^2*b + A*a*b^2)*n^3 + 76*(B*a^2*b + A*a*b^2)*n^2 - 48*(
B*a^2*b + A*a*b^2)*n)*sqrt(x)*x^(2*n) - 3*(9*B*a^3 + 27*A*a^2*b - 64*(B*a^
3 + 3*A*a^2*b)*n^3 + 104*(B*a^3 + 3*A*a^2*b)*n^2 - 54*(B*a^3 + 3*A*a^2*b)*
n)*sqrt(x)*x^n - (128*A*a^3*n^4 - 400*A*a^3*n^3 + 420*A*a^3*n^2 - 180*A*a^
3*n + 27*A*a^3)*sqrt(x))/((128*n^4 - 400*n^3 + 420*n^2 - 180*n + 27)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)**3*(A+B*x**n)/x**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-5/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^3}{x^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^3/x^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx = \frac{B\,a^3\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}} - \frac{2\,A\,a^3}{3\,x^{3/2}} + \frac{2\,A\,b^3\,x^{3\,n-\frac{3}{2}}}{3\,(2\,n-1)} + \frac{2\,B\,b^3\,x^{4\,n-\frac{3}{2}}}{8\,n-3} + \frac{6\,A\,a\,b^2\,x^{2\,n-\frac{3}{2}}}{4\,n-3} + \frac{2\,B\,a\,b^2\,x^{3\,n-\frac{3}{2}}}{2\,n-1} + \frac{6\,B\,a^2\,b\,x^{2\,n-\frac{3}{2}}}{4\,n-3} + \frac{3\,A\,a^2\,b\,x^{n-\frac{3}{2}}}{n-\frac{3}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^3)/x^(5/2),x)
```

output
```
(B*a^3*x^(n - 3/2))/(n - 3/2) - (2*A*a^3)/(3*x^(3/2)) + (2*A*b^3*x^(3*n -
3/2))/(3*(2*n - 1)) + (2*B*b^3*x^(4*n - 3/2))/(8*n - 3) + (6*A*a*b^2*x^(2*
n - 3/2))/(4*n - 3) + (2*B*a*b^2*x^(3*n - 3/2))/(2*n - 1) + (6*B*a^2*b*x^(
2*n - 3/2))/(4*n - 3) + (3*A*a^2*b*x^(n - 3/2))/(n - 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.21

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{5/2}} \, dx = \frac{32x^{4n}b^4n^3 - 88x^{4n}b^4n^2 + 72x^{4n}b^4n - 18x^{4n}b^4 + \frac{512x^{3n}ab^3n^3}{3} - 448x^{3n}ab^3n^2}{}$$

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(5/2),x)
```

output
```
(2*(48*x**(4*n)*b**4*n**3 - 132*x**(4*n)*b**4*n**2 + 108*x**(4*n)*b**4*n -
 27*x**(4*n)*b**4 + 256*x**(3*n)*a*b**3*n**3 - 672*x**(3*n)*a*b**3*n**2 +
504*x**(3*n)*a*b**3*n - 108*x**(3*n)*a*b**3 + 576*x**(2*n)*a**2*b**2*n**3
- 1368*x**(2*n)*a**2*b**2*n**2 + 864*x**(2*n)*a**2*b**2*n - 162*x**(2*n)*a
**2*b**2 + 768*x**n*a**3*b*n**3 - 1248*x**n*a**3*b*n**2 + 648*x**n*a**3*b*
n - 108*x**n*a**3*b - 128*a**4*n**4 + 400*a**4*n**3 - 420*a**4*n**2 + 180*
a**4*n - 27*a**4))/(3*sqrt(x)*x*(128*n**4 - 400*n**3 + 420*n**2 - 180*n +
27))
```

3.311 $\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{7/2}}\,dx = -\frac{2a^3A}{5x^{5/2}} - \frac{2a^2(3Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{6ab(Ab+aB)x^{-\frac{5}{2}+2n}}{5-4n} - \frac{2b^2(Ab+3aB)x^{-\frac{5}{2}+3n}}{5-6n} - \frac{2b^3Bx^{-\frac{5}{2}+4n}}{5-8n}$$

output
```
-2/5*a^3*A/x^(5/2)-2*a^2*(3*A*b+B*a)*x^(-5/2+n)/(5-2*n)-6*a*b*(A*b+B*a)*x^
(-5/2+2*n)/(5-4*n)-2*b^2*(A*b+3*B*a)*x^(-5/2+3*n)/(5-6*n)-2*b^3*B*x^(-5/2+
4*n)/(5-8*n)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^n)^3(A+Bx^n)}{x^{7/2}}\,dx = 2\left(-\frac{a^3A}{5x^{5/2}} - \frac{a^2(3Ab+aB)x^{-\frac{5}{2}+n}}{5-2n} - \frac{3ab(Ab+aB)x^{-\frac{5}{2}+2n}}{5-4n} - \frac{b^2(Ab+3aB)x^{-\frac{5}{2}+3n}}{5-6n} - \frac{b^3Bx^{-\frac{5}{2}+4n}}{5-8n}\right)$$

input
```
Integrate[((a + b*x^n)^3*(A + B*x^n))/x^(7/2),x]
```

output
```
2*(-1/5*(a^3*A)/x^(5/2) - (a^2*(3*A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (3*
a*b*(A*b + a*B)*x^(-5/2 + 2*n))/(5 - 4*n) - (b^2*(A*b + 3*a*B)*x^(-5/2 + 3
*n))/(5 - 6*n) - (b^3*B*x^(-5/2 + 4*n))/(5 - 8*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{7/2}} dx$$

$$\Big\downarrow \text{ 950}$$

$$\int \left(\frac{a^3A}{x^{7/2}}+a^2x^{n-\frac{7}{2}}(aB+3Ab)+b^2x^{3n-\frac{7}{2}}(3aB+Ab)+3abx^{2n-\frac{7}{2}}(aB+Ab)+b^3Bx^{4n-\frac{7}{2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2x^{n-\frac{5}{2}}(aB+3Ab)}{5-2n}-\frac{2b^2x^{3n-\frac{5}{2}}(3aB+Ab)}{5-6n}-\frac{6abx^{2n-\frac{5}{2}}(aB+Ab)}{5-4n}-\frac{2b^3Bx^{4n-\frac{5}{2}}}{5-8n}$$

input
```
Int[((a + b*x^n)^3*(A + B*x^n))/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B)*x^(-5/2 + n))/(5 - 2*n) - (6
*a*b*(A*b + a*B)*x^(-5/2 + 2*n))/(5 - 4*n) - (2*b^2*(A*b + 3*a*B)*x^(-5/2
+ 3*n))/(5 - 6*n) - (2*b^3*B*x^(-5/2 + 4*n))/(5 - 8*n)
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(107) = 214$.

Time = 0.54 (sec) , antiderivative size = 1249, normalized size of antiderivative = 10.50

method	result	size
orering	Expression too large to display	1249

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/x^(5/2)*(384*n^4-4800*n^3+15260*n^2-17760*n+6841)/(96*n^3-380*n^2+400
*n-125)/(-5+4*n)*(a+b*x^n)^3*(A+B*x^n)+8*x^2*(40*n^3-294*n^2+596*n-357)/(9
6*n^3-380*n^2+400*n-125)/(-5+4*n)*(3*(a+b*x^n)^2*(A+B*x^n)/x^(9/2)*b*x^n*n
+(a+b*x^n)^3*B*x^n*n/x^(9/2)-7/2*(a+b*x^n)^3*(A+B*x^n)/x^(9/2))-16*x^3*(14
*n^2-64*n+65)/(96*n^3-380*n^2+400*n-125)/(-5+4*n)*(6*(a+b*x^n)*(A+B*x^n)/x
^(11/2)*b^2*(x^n)^2*n^2+6*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(11/2)*b-24*(a+b*x^n
)^2*(A+B*x^n)/x^(11/2)*b*x^n*n+3*(a+b*x^n)^2*(A+B*x^n)/x^(11/2)*b*x^n*n^2+
(a+b*x^n)^3*B*x^n*n^2/x^(11/2)-8*(a+b*x^n)^3*B*x^n*n/x^(11/2)+63/4*(a+b*x^
n)^3*(A+B*x^n)/x^(11/2))+16*x^4*(-9+4*n)/(384*n^4-2000*n^3+3500*n^2-2500*n
+625)*(6*b^3*(x^n)^3*n^3/x^(13/2)*(A+B*x^n)+18*(a+b*x^n)*B*(x^n)^3*n^3/x^(
13/2)*b^2-81*(a+b*x^n)*(A+B*x^n)/x^(13/2)*b^2*(x^n)^2*n^2+18*(a+b*x^n)*(A+
B*x^n)/x^(13/2)*b^2*(x^n)^2*n^3+18*(a+b*x^n)^2*B*(x^n)^2*n^3/x^(13/2)*b-81
*(a+b*x^n)^2*B*(x^n)^2*n^2/x^(13/2)*b+717/4*(a+b*x^n)^2*(A+B*x^n)/x^(13/2)
*b*x^n*n-81/2*(a+b*x^n)^2*(A+B*x^n)/x^(13/2)*b*x^n*n^2+3*(a+b*x^n)^2*(A+B*
x^n)/x^(13/2)*b*x^n*n^3+(a+b*x^n)^3*B*x^n*n^3/x^(13/2)-27/2*(a+b*x^n)^3*B*
x^n*n^2/x^(13/2)+239/4*(a+b*x^n)^3*B*x^n*n/x^(13/2)-693/8*(a+b*x^n)^3*(A+B
*x^n)/x^(13/2))-32/5/(384*n^4-2000*n^3+3500*n^2-2500*n+625)*x^5*(885*(a+b*
x^n)*(A+B*x^n)/x^(15/2)*b^2*(x^n)^2*n^2+9009/16*(a+b*x^n)^3*(A+B*x^n)/x^(1
5/2)+36*b^3*(x^n)^3*n^4/x^(15/2)*(A+B*x^n)+24*b^3*(x^n)^4*n^4/x^(15/2)*B-6
0*(a+b*x^n)^2*(A+B*x^n)/x^(15/2)*b*x^n*n^3+3*(a+b*x^n)^2*(A+B*x^n)/x^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{7/2}}\,dx = \frac{2\left(5\left(48\,Bb^3n^3-220\,Bb^3n^2+300\,Bb^3n-125\,Bb^3\right)\sqrt{x}x^{4\,n}-5\left(375\,Bab^2+\right.\right.}{}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(7/2),x, algorithm="fricas")
```

output
```
2/5*(5*(48*B*b^3*n^3 - 220*B*b^3*n^2 + 300*B*b^3*n - 125*B*b^3)*sqrt(x)*x^
(4*n) - 5*(375*B*a*b^2 + 125*A*b^3 - 64*(3*B*a*b^2 + A*b^3)*n^3 + 280*(3*B
*a*b^2 + A*b^3)*n^2 - 350*(3*B*a*b^2 + A*b^3)*n)*sqrt(x)*x^(3*n) - 15*(125
*B*a^2*b + 125*A*a*b^2 - 96*(B*a^2*b + A*a*b^2)*n^3 + 380*(B*a^2*b + A*a*b
^2)*n^2 - 400*(B*a^2*b + A*a*b^2)*n)*sqrt(x)*x^(2*n) - 5*(125*B*a^3 + 375*
A*a^2*b - 192*(B*a^3 + 3*A*a^2*b)*n^3 + 520*(B*a^3 + 3*A*a^2*b)*n^2 - 450*
(B*a^3 + 3*A*a^2*b)*n)*sqrt(x)*x^n - (384*A*a^3*n^4 - 2000*A*a^3*n^3 + 350
0*A*a^3*n^2 - 2500*A*a^3*n + 625*A*a^3)*sqrt(x))/((384*n^4 - 2000*n^3 + 35
00*n^2 - 2500*n + 625)*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a + bx^n)^3 (A + Bx^n)}{x^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)**3*(A+B*x**n)/x**(7/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx^n)^3 (A + Bx^n)}{x^{7/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(n-7/2>0)', see `assume?` for mor
e details)
```

Giac [F]

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{7/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^3}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^3*(A+B*x^n)/x^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^3/x^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{7/2}}\,dx = \frac{B\,a^3\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}} - \frac{2\,A\,a^3}{5\,x^{5/2}} + \frac{2\,A\,b^3\,x^{3\,n-\frac{5}{2}}}{6\,n-5} + \frac{2\,B\,b^3\,x^{4\,n-\frac{5}{2}}}{8\,n-5} + \frac{6\,A\,a\,b^2\,x^{2\,n-\frac{5}{2}}}{4\,n-5} + \frac{6\,B\,a^2\,b\,x^{2\,n-\frac{5}{2}}}{4\,n-5} + \frac{6\,B\,a\,b^2\,x^{3\,n-\frac{5}{2}}}{6\,n-5} + \frac{3\,A\,a^2\,b\,x^{n-\frac{5}{2}}}{n-\frac{5}{2}}$$

input
```
int(((A + B*x^n)*(a + b*x^n)^3)/x^(7/2),x)
```

output
```
(B*a^3*x^(n - 5/2))/(n - 5/2) - (2*A*a^3)/(5*x^(5/2)) + (2*A*b^3*x^(3*n -
5/2))/(6*n - 5) + (2*B*b^3*x^(4*n - 5/2))/(8*n - 5) + (6*A*a*b^2*x^(2*n -
5/2))/(4*n - 5) + (6*B*a^2*b*x^(2*n - 5/2))/(4*n - 5) + (6*B*a*b^2*x^(3*n
- 5/2))/(6*n - 5) + (3*A*a^2*b*x^(n - 5/2))/(n - 5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^n)^3 (A+Bx^n)}{x^{7/2}}\,dx = \frac{96x^{4n}b^4n^3 - 440x^{4n}b^4n^2 + 600x^{4n}b^4n - 250x^{4n}b^4 + 512x^{3n}a\,b^3n^3 - 2240x^{3n}}{}$$

input
```
int((a+b*x^n)^3*(A+B*x^n)/x^(7/2),x)
```

output
```
(2*(240*x**(4*n)*b**4*n**3 - 1100*x**(4*n)*b**4*n**2 + 1500*x**(4*n)*b**4*
n - 625*x**(4*n)*b**4 + 1280*x**(3*n)*a*b**3*n**3 - 5600*x**(3*n)*a*b**3*n
**2 + 7000*x**(3*n)*a*b**3*n - 2500*x**(3*n)*a*b**3 + 2880*x**(2*n)*a**2*b
**2*n**3 - 11400*x**(2*n)*a**2*b**2*n**2 + 12000*x**(2*n)*a**2*b**2*n - 37
50*x**(2*n)*a**2*b**2 + 3840*x**n*a**3*b*n**3 - 10400*x**n*a**3*b*n**2 + 9
000*x**n*a**3*b*n - 2500*x**n*a**3*b - 384*a**4*n**4 + 2000*a**4*n**3 - 35
00*a**4*n**2 + 2500*a**4*n - 625*a**4))/(5*sqrt(x)*x**2*(384*n**4 - 2000*n
**3 + 3500*n**2 - 2500*n + 625))
```

3.312 $\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx=\frac{2Bx^{5/2}}{5b}+\frac{2(Ab-aB)x^{5/2}\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5ab}$$

output
```
2/5*B*x^(5/2)/b+2/5*(A*b-B*a)*x^(5/2)*hypergeom([1, 5/2/n],[1+5/2/n],-b*x^
n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx=\frac{2x^{5/2}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)\right)}{5ab}$$

input
```
Integrate[(x^(3/2)*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(2*x^(5/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), -
((b*x^n)/a)]))/(5*a*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^n+a}dx}{b}+\frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{5/2}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5ab}+\frac{2Bx^{5/2}}{5b}$$

input
```
Int[(x^(3/2)*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(2*B*x^(5/2))/(5*b) + (2*(A*b - a*B)*x^(5/2)*Hypergeometric2F1[1, 5/(2*n),
 1 + 5/(2*n), -((b*x^n)/a)])/(5*a*b)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}} (A + B\,x^{n})}{a + b\,x^{n}}dx$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n),x)
```

output
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{x^{3/2}(A + Bx^n)}{a + bx^n}\,dx = \int \frac{(Bx^n + A)x^{\frac{3}{2}}}{bx^n + a}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)*sqrt(x)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.06

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx = \frac{5Aa^{\frac{5}{2n}}a^{-1-\frac{5}{2n}}x^{\frac{5}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{5}{2n}\right)\Gamma\left(\frac{5}{2n}\right)}{2n^2\Gamma\left(1+\frac{5}{2n}\right)} + \frac{Ba^{-2-\frac{5}{2n}}a^{1+\frac{5}{2n}}x^{n+\frac{5}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{5}{2n}\right)\Gamma\left(1+\frac{5}{2n}\right)}{n\Gamma\left(2+\frac{5}{2n}\right)} + \frac{5Ba^{-2-\frac{5}{2n}}a^{1+\frac{5}{2n}}x^{n+\frac{5}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{5}{2n}\right)\Gamma\left(1+\frac{5}{2n}\right)}{2n^2\Gamma\left(2+\frac{5}{2n}\right)}$$

input
```
integrate(x**(3/2)*(A+B*x**n)/(a+b*x**n),x)
```

output
```
5*A*a**(5/(2*n))*a**(-1 - 5/(2*n))*x**(5/2)*lerchphi(b*x**n*exp_polar(I*pi
)/a, 1, 5/(2*n))*gamma(5/(2*n))/(2*n**2*gamma(1 + 5/(2*n))) + B*a**(-2 - 5
/(2*n))*a**(1 + 5/(2*n))*x**(n + 5/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1
, 1 + 5/(2*n))*gamma(1 + 5/(2*n))/(n*gamma(2 + 5/(2*n))) + 5*B*a**(-2 - 5/
(2*n))*a**(1 + 5/(2*n))*x**(n + 5/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1 + 5/(2*n))*gamma(1 + 5/(2*n))/(2*n**2*gamma(2 + 5/(2*n)))
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx = \int \frac{(Bx^n+A)x^{\frac{3}{2}}}{bx^n+a}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(x^(3/2)/(b^3*(2*n - 5)*x^(2*n) + 2*a*b^2*
(2*n - 5)*x^n + a^2*b*(2*n - 5)), x) + 2/5*(B*b*(2*n - 5)*x*x^n + (2*B*a*n
 - 5*A*b)*x)*x^(3/2)/(b^2*(2*n - 5)*x^n + a*b*(2*n - 5))
```

Giac [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx=\int \frac{(Bx^n+A)x^{\frac{3}{2}}}{bx^n+a}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^(3/2)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx=\int \frac{x^{3/2}\,(A+B\,x^n)}{a+b\,x^n}\,dx$$

input
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n),x)
```

output
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.11

$$\int \frac{x^{3/2}(A+Bx^n)}{a+bx^n}\,dx=\frac{2\sqrt{x}\,x^2}{5}$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n),x)
```

output
```
(2*sqrt(x)*x**2)/5
```

3.313 $\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx = \frac{2Bx^{3/2}}{3b}+\frac{2(Ab-aB)x^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3ab}$$

output
```
2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*x^(3/2)*hypergeom([1, 3/2/n],[1+3/2/n],-b*x^
n/a)/a/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx = \frac{2x^{3/2}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)\right)}{3ab}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(2*x^(3/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), -
((b*x^n)/a)]))/(3*a*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^n+a}dx}{b}+\frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{3/2}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3ab}+\frac{2Bx^{3/2}}{3b}$$

input
```
Int[(Sqrt[x]*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*x^(3/2)*Hypergeometric2F1[1, 3/(2*n),
 1 + 3/(2*n), -((b*x^n)/a)])/(3*a*b)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{x}\,(A+B\,x^{n})}{a+b\,x^{n}}dx$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n),x)
```

output
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx=\int \frac{(Bx^n+A)\sqrt{x}}{bx^n+a}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx = \frac{3Aa^{\frac{3}{2n}}a^{-1-\frac{3}{2n}}x^{\frac{3}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{3}{2n}\right)\Gamma\left(\frac{3}{2n}\right)}{2n^2\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Ba^{-2-\frac{3}{2n}}a^{1+\frac{3}{2n}}x^{n+\frac{3}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{2n}\right)\Gamma\left(1+\frac{3}{2n}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)} + \frac{3Ba^{-2-\frac{3}{2n}}a^{1+\frac{3}{2n}}x^{n+\frac{3}{2}}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,1+\frac{3}{2n}\right)\Gamma\left(1+\frac{3}{2n}\right)}{2n^2\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate(x**(1/2)*(A+B*x**n)/(a+b*x**n),x)
```

output
```
3*A*a**(3/(2*n))*a**(-1 - 3/(2*n))*x**(3/2)*lerchphi(b*x**n*exp_polar(I*pi
)/a, 1, 3/(2*n))*gamma(3/(2*n))/(2*n**2*gamma(1 + 3/(2*n))) + B*a**(-2 - 3
/(2*n))*a**(1 + 3/(2*n))*x**(n + 3/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1
, 1 + 3/(2*n))*gamma(1 + 3/(2*n))/(n*gamma(2 + 3/(2*n))) + 3*B*a**(-2 - 3/
(2*n))*a**(1 + 3/(2*n))*x**(n + 3/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1 + 3/(2*n))*gamma(1 + 3/(2*n))/(2*n**2*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx = \int \frac{(Bx^n+A)\sqrt{x}}{bx^n+a}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(sqrt(x)/(b^3*(2*n - 3)*x^(2*n) + 2*a*b^2*
(2*n - 3)*x^n + a^2*b*(2*n - 3)), x) + 2/3*(B*b*(2*n - 3)*x*x^n + (2*B*a*n
 - 3*A*b)*x)*sqrt(x)/(b^2*(2*n - 3)*x^n + a*b*(2*n - 3))
```

Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx=\int \frac{(Bx^n+A)\sqrt{x}}{bx^n+a}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(x)/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx=\int \frac{\sqrt{x}\,(A+B\,x^n)}{a+b\,x^n}\,dx$$

input
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n),x)
```

output
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{x}(A+Bx^n)}{a+bx^n}\,dx=\frac{2\sqrt{x}\,x}{3}$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n),x)
```

output
```
(2*sqrt(x)*x)/3
```

3.314 $\int \frac{A+Bx^n}{\sqrt{x}(a+bx^n)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)} \, dx = \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\sqrt{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ab}$$

output
```
2*B*x^(1/2)/b+2*(A*b-B*a)*x^(1/2)*hypergeom([1, 1/2/n],[1+1/2/n],-b*x^n/a)
/a/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)} \, dx = \frac{2\sqrt{x}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},-\frac{bx^n}{a}\right)\right)}{ab}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)),x]
```

output
```
(2*Sqrt[x]*(a*B + (A*b - a*B)*Hypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -
((b*x^n)/a)]))/(a*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(bx^n+a)}dx}{b}+\frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \; 888$$

$$\frac{2\sqrt{x}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ab}+\frac{2B\sqrt{x}}{b}$$

input
```
Int[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)),x]
```

output
```
(2*B*Sqrt[x])/b + (2*(A*b - a*B)*Sqrt[x]*Hypergeometric2F1[1, 1/(2*n), (2
+ n^(-1))/2, -((b*x^n)/a)])/(a*b)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\sqrt{x}\,\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b*x*x^n + a*x), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx$$

input
```
integrate((A+B*x**n)/x**(1/2)/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(sqrt(x)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(1/((b^3*(2*n - 1)*x^(2*n) + 2*a*b^2*(2*n
- 1)*x^n + a^2*b*(2*n - 1))*sqrt(x)), x) + 2*(B*b*(2*n - 1)*x*x^n + (2*B*a
*n - A*b)*x)/((b^2*(2*n - 1)*x^n + a*b*(2*n - 1))*sqrt(x))
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=\int \frac{A+B\,x^n}{\sqrt{x}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.07

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)}\,dx=2\sqrt{x}$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n),x)
```

output
```
2*sqrt(x)
```

3.315 $\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)}\,dx=-\frac{2B}{b\sqrt{x}}-\frac{2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{ab\sqrt{x}}$$

output
```
-2*B/b/x^(1/2)-2*(A*b-B*a)*hypergeom([1, -1/2/n],[1-1/2/n],-b*x^n/a)/a/b/x
^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)}\,dx=\frac{-2aB+(-2Ab+2aB)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{ab\sqrt{x}}$$

input
```
Integrate[(A + B*x^n)/(x^(3/2)*(a + b*x^n)),x]
```

output
```
(-2*a*B + (-2*A*b + 2*a*B)*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -((
b*x^n)/a)])/(a*b*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{1}{x^{3/2}(bx^n+a)}dx}{b}-\frac{2B}{b\sqrt{x}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{ab\sqrt{x}}-\frac{2B}{b\sqrt{x}}$$

input
```
Int[(A + B*x^n)/(x^(3/2)*(a + b*x^n)),x]
```

output
```
(-2*B)/(b*Sqrt[x]) - (2*(A*b - a*B)*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(
2*n), -((b*x^n)/a)])/(a*b*Sqrt[x])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{3}{2}}\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b*x^2*x^n + a*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)}\,dx=\int \frac{A+Bx^n}{x^{\frac{3}{2}}\left(a+bx^n\right)}\,dx$$

input
```
integrate((A+B*x**n)/x**(3/2)/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**(3/2)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^{3/2}\left(a + bx^n\right)}\, dx = \int \frac{Bx^n + A}{(bx^n + a)x^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(1/((b^3*(2*n + 1)*x^(2*n) + 2*a*b^2*(2*n
+ 1)*x^n + a^2*b*(2*n + 1))*x^(3/2)), x) - 2*(B*b*(2*n + 1)*x*x^n + (2*B*a
*n + A*b)*x)/((b^2*(2*n + 1)*x^n + a*b*(2*n + 1))*x^(3/2))
```

Giac [F]

$$\int \frac{A + Bx^n}{x^{3/2}\left(a + bx^n\right)}\, dx = \int \frac{Bx^n + A}{(bx^n + a)x^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^n}{x^{3/2}\left(a + bx^n\right)}\, dx = \int \frac{A + B\,x^n}{x^{3/2}\,\left(a + b\,x^n\right)}\, dx$$

input
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.10

$$\int \frac{A + Bx^n}{x^{3/2}(a + bx^n)} \, dx = -\frac{2}{\sqrt{x}}$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n),x)
```

output
```
( - 2)/sqrt(x)
```

3.316 $\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)}\,dx = -\frac{2B}{3bx^{3/2}} - \frac{2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3abx^{3/2}}$$

output
```
-2/3*B/b/x^(3/2)-2/3*(A*b-B*a)*hypergeom([1, -3/2/n],[1-3/2/n],-b*x^n/a)/a
/b/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)}\,dx = \frac{-2aB+(-2Ab+2aB)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3abx^{3/2}}$$

input
```
Integrate[(A + B*x^n)/(x^(5/2)*(a + b*x^n)),x]
```

output
```
(-2*a*B + (-2*A*b + 2*a*B)*Hypergeometric2F1[1, -3/(2*n), 1 - 3/(2*n), -((
b*x^n)/a)])/(3*a*b*x^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{1}{x^{5/2}(bx^n+a)}dx}{b}-\frac{2B}{3bx^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3abx^{3/2}}-\frac{2B}{3bx^{3/2}}$$

input
```
Int[(A + B*x^n)/(x^(5/2)*(a + b*x^n)),x]
```

output
```
(-2*B)/(3*b*x^(3/2)) - (2*(A*b - a*B)*Hypergeometric2F1[1, -3/(2*n), 1 - 3
/(2*n), -((b*x^n)/a)])/(3*a*b*x^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{5}{2}}\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b*x^3*x^n + a*x^3), x)
```

Sympy [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=\int \frac{A+Bx^n}{x^{\frac{5}{2}}\left(a+bx^n\right)}\,dx$$

input
```
integrate((A+B*x**n)/x**(5/2)/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**(5/2)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(1/((b^3*(2*n + 3)*x^(2*n) + 2*a*b^2*(2*n
+ 3)*x^n + a^2*b*(2*n + 3))*x^(5/2)), x) - 2/3*(B*b*(2*n + 3)*x*x^n + (2*B
*a*n + 3*A*b)*x)/((b^2*(2*n + 3)*x^n + a*b*(2*n + 3))*x^(5/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=\int \frac{A+B\,x^n}{x^{5/2}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)}\,dx=-\frac{2}{3\sqrt{x}\,x}$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n),x)
```

output
```
( - 2)/(3*sqrt(x)*x)
```

3.317 $\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)} dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)} dx = -\frac{2B}{5bx^{5/2}} - \frac{2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{5abx^{5/2}}$$

output
```
-2/5*B/b/x^(5/2)-2/5*(A*b-B*a)*hypergeom([1, -5/2/n],[1-5/2/n],-b*x^n/a)/a
/b/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)} dx = \frac{-2aB+(-2Ab+2aB)\,\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{5abx^{5/2}}$$

input
```
Integrate[(A + B*x^n)/(x^(7/2)*(a + b*x^n)),x]
```

output
```
(-2*a*B + (-2*A*b + 2*a*B)*Hypergeometric2F1[1, -5/(2*n), 1 - 5/(2*n), -((
b*x^n)/a)])/(5*a*b*x^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(Ab-aB)\int \frac{1}{x^{7/2}(bx^n+a)}dx}{b}-\frac{2B}{5bx^{5/2}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{2(Ab-aB)\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{5abx^{5/2}}-\frac{2B}{5bx^{5/2}}$$

input
```
Int[(A + B*x^n)/(x^(7/2)*(a + b*x^n)),x]
```

output
```
(-2*B)/(5*b*x^(5/2)) - (2*(A*b - a*B)*Hypergeometric2F1[1, -5/(2*n), 1 - 5
/(2*n), -((b*x^n)/a)])/(5*a*b*x^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{7}{2}}\left(a+b\,x^{n}\right)}dx$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n),x)
```

output
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)}\,dx=\int \frac{Bx^n+A}{(bx^n+a)x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b*x^4*x^n + a*x^4), x)
```

Sympy [F]

$$\int \frac{A + Bx^n}{x^{7/2} (a + bx^n)} dx = \int \frac{A + Bx^n}{x^{\frac{7}{2}} (a + bx^n)} dx$$

input
```
integrate((A+B*x**n)/x**(7/2)/(a+b*x**n),x)
```

output
```
Integral((A + B*x**n)/(x**(7/2)*(a + b*x**n)), x)
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^{7/2} (a + bx^n)} dx = \int \frac{Bx^n + A}{(bx^n + a)x^{\frac{7}{2}}} dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n),x, algorithm="maxima")
```

output
```
-2*(B*a^2*n - A*a*b*n)*integrate(1/((b^3*(2*n + 5)*x^(2*n) + 2*a*b^2*(2*n
+ 5)*x^n + a^2*b*(2*n + 5))*x^(7/2)), x) - 2/5*(B*b*(2*n + 5)*x*x^n + (2*B
*a*n + 5*A*b)*x)/((b^2*(2*n + 5)*x^n + a*b*(2*n + 5))*x^(7/2))
```

Giac [F]

$$\int \frac{A + Bx^n}{x^{7/2} (a + bx^n)} dx = \int \frac{Bx^n + A}{(bx^n + a)x^{\frac{7}{2}}} dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)}\,dx=\int \frac{A+B\,x^n}{x^{7/2}\,\left(a+b\,x^n\right)}\,dx$$

input
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)),x)
```

output
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.14

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)}\,dx=-\frac{2}{5\sqrt{x}\,x^2}$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n),x)
```

output
```
( - 2)/(5*sqrt(x)*x**2)
```

3.318 $\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{abn\,(a+bx^n)} + \frac{(5aB-Ab(5-2n))x^{5/2}\operatorname{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2bn}$$

output
```
(A*b-B*a)*x^(5/2)/a/b/n/(a+b*x^n)+1/5*(5*B*a-A*b*(5-2*n))*x^(5/2)*hypergeo
m([1, 5/2/n],[1+5/2/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{x^{5/2}\left(-\frac{5a(-Ab+aB)}{a+bx^n}+(5aB+Ab(-5+2n))\operatorname{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx}{a}\right.\right.}{5a^2bn}$$

input
```
Integrate[(x^(3/2)*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
(x^(5/2)*((-5*a*(-(A*b) + a*B))/(a + b*x^n) + (5*a*B + A*b*(-5 + 2*n))*Hyp
ergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), -((b*x^n)/a)]))/(5*a^2*b*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(5aB-Ab(5-2n))\int \frac{x^{3/2}}{bx^n+a}dx}{2abn}+\frac{x^{5/2}(Ab-aB)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \; 888$$

$$\frac{x^{5/2}(5aB-Ab(5-2n))\,\text{Hypergeometric2F1}\left(1,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2bn}+\frac{x^{5/2}(Ab-aB)}{abn\,(a+bx^n)}$$

input
```
Int[(x^(3/2)*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(a*b*n*(a + b*x^n)) + ((5*a*B - A*b*(5 - 2*n))*x^(5/
2)*Hypergeometric2F1[1, 5/(2*n), 1 + 5/(2*n), -((b*x^n)/a)])/(5*a^2*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}} \left(A +B \,x^{n}\right)}{\left(a +b \,x^{n}\right)^{2}}d x$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\, dx = \int \frac{(Bx^n+A)x^{\frac{3}{2}}}{(bx^n+a)^2}\, dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)*sqrt(x)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(A+B*x**n)/(a+b*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x^{\frac{3}{2}}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 - 5*n)*A*a*b + 5*B*a^2*n)*integrate(x^(3/2)/((8*n^2 - 30*n + 25)
*b^4*x^(3*n) + 3*(8*n^2 - 30*n + 25)*a*b^3*x^(2*n) + 3*(8*n^2 - 30*n + 25)
*a^2*b^2*x^n + (8*n^2 - 30*n + 25)*a^3*b), x) - 2*(B*b*(4*n - 5)*x*x^n + (
A*b*(2*n - 5) + 4*B*a*n)*x)*x^(3/2)/((8*n^2 - 30*n + 25)*b^3*x^(2*n) + 2*(
8*n^2 - 30*n + 25)*a*b^2*x^n + (8*n^2 - 30*n + 25)*a^2*b)
```

Giac [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)x^{\frac{3}{2}}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^(3/2)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx=\int \frac{x^{3/2}\,(A+B\,x^n)}{(a+b\,x^n)^2}\,dx$$

input
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n)^2,x)
```

output
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^2}\,dx=\int \frac{\sqrt{x}\,x}{x^n b+a}dx$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int((sqrt(x)*x)/(x**n*b + a),x)
```

3.319 $\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{abn(a+bx^n)} + \frac{(3aB-Ab(3-2n))x^{3/2}\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2bn}$$

output
```
(A*b-B*a)*x^(3/2)/a/b/n/(a+b*x^n)+1/3*(3*B*a-A*b*(3-2*n))*x^(3/2)*hypergeo
m([1, 3/2/n],[1+3/2/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{x^{3/2}\left(-\frac{3a(-Ab+aB)}{a+bx^n}+(3aB+Ab(-3+2n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)\right)}{3a^2bn}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
(x^(3/2)*((-3*a*(-(A*b) + a*B))/(a + b*x^n) + (3*a*B + A*b*(-3 + 2*n))*Hyp
ergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), -((b*x^n)/a)]))/(3*a^2*b*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(3aB-Ab(3-2n))\int \frac{\sqrt{x}}{bx^n+a}dx}{2abn}+\frac{x^{3/2}(Ab-aB)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{3/2}(3aB-Ab(3-2n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2bn}+\frac{x^{3/2}(Ab-aB)}{abn\,(a+bx^n)}$$

input
```
Int[(Sqrt[x]*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*x^(3/2))/(a*b*n*(a + b*x^n)) + ((3*a*B - A*b*(3 - 2*n))*x^(3/
2)*Hypergeometric2F1[1, 3/(2*n), 1 + 3/(2*n), -((b*x^n)/a)])/(3*a^2*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{\sqrt{x}\,(A+B\,x^{n})}{(a+b\,x^{n})^{2}}dx$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx=\int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.77 (sec) , antiderivative size = 932, normalized size of antiderivative = 10.13

$$\int \frac{\sqrt{x}(A + Bx^n)}{(a + bx^n)^2} \, dx = \text{Too large to display}$$

input

```
integrate(x**(1/2)*(A+B*x**n)/(a+b*x**n)**2,x)
```

output

```
A*(6*a*a**(3/(2*n))*a**(-2 - 3/(2*n))*n*x**(3/2)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 3/(2*n))*gamma(3/(2*n))/(4*a*n**3*gamma(1 + 3/(2*n)) + 4*b*n*
*3*x**n*gamma(1 + 3/(2*n))) + 6*a*a**(3/(2*n))*a**(-2 - 3/(2*n))*n*x**(3/2
)*gamma(3/(2*n))/(4*a*n**3*gamma(1 + 3/(2*n)) + 4*b*n**3*x**n*gamma(1 + 3/
(2*n))) - 9*a*a**(3/(2*n))*a**(-2 - 3/(2*n))*x**(3/2)*lerchphi(b*x**n*exp_
polar(I*pi)/a, 1, 3/(2*n))*gamma(3/(2*n))/(4*a*n**3*gamma(1 + 3/(2*n)) + 4
*b*n**3*x**n*gamma(1 + 3/(2*n))) + 6*a**(3/(2*n))*a**(-2 - 3/(2*n))*b*n*x*
*(3/2)*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3/(2*n))*gamma(3/(2*n))/
(4*a*n**3*gamma(1 + 3/(2*n)) + 4*b*n**3*x**n*gamma(1 + 3/(2*n))) - 9*a**(3
/(2*n))*a**(-2 - 3/(2*n))*b*x**(3/2)*x**n*lerchphi(b*x**n*exp_polar(I*pi)/
a, 1, 3/(2*n))*gamma(3/(2*n))/(4*a*n**3*gamma(1 + 3/(2*n)) + 4*b*n**3*x**n
*gamma(1 + 3/(2*n)))) + B*(4*a*a**(-3 - 3/(2*n))*a**(1 + 3/(2*n))*n**2*x**
(n + 3/2)*gamma(1 + 3/(2*n))/(4*a*n**3*gamma(2 + 3/(2*n)) + 4*b*n**3*x**n*
gamma(2 + 3/(2*n))) - 6*a*a**(-3 - 3/(2*n))*a**(1 + 3/(2*n))*n*x**(n + 3/2
)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 3/(2*n))*gamma(1 + 3/(2*n))/(4
*a*n**3*gamma(2 + 3/(2*n)) + 4*b*n**3*x**n*gamma(2 + 3/(2*n))) + 6*a*a**(-
3 - 3/(2*n))*a**(1 + 3/(2*n))*n*x**(n + 3/2)*gamma(1 + 3/(2*n))/(4*a*n**3*
gamma(2 + 3/(2*n)) + 4*b*n**3*x**n*gamma(2 + 3/(2*n))) - 9*a*a**(-3 - 3/(2
*n))*a**(1 + 3/(2*n))*x**(n + 3/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1
 + 3/(2*n))*gamma(1 + 3/(2*n))/(4*a*n**3*gamma(2 + 3/(2*n)) + 4*b*n**3*...
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 - 3*n)*A*a*b + 3*B*a^2*n)*integrate(sqrt(x)/((8*n^2 - 18*n + 9)*
b^4*x^(3*n) + 3*(8*n^2 - 18*n + 9)*a*b^3*x^(2*n) + 3*(8*n^2 - 18*n + 9)*a^
2*b^2*x^n + (8*n^2 - 18*n + 9)*a^3*b), x) - 2*(B*b*(4*n - 3)*x*x^n + (A*b*
(2*n - 3) + 4*B*a*n)*x)*sqrt(x)/((8*n^2 - 18*n + 9)*b^3*x^(2*n) + 2*(8*n^2
 - 18*n + 9)*a*b^2*x^n + (8*n^2 - 18*n + 9)*a^2*b)
```

Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^2}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(x)/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx = \int \frac{\sqrt{x}\,(A+B\,x^n)}{(a+b\,x^n)^2}\,dx$$

input
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n)^2,x)
```

output
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^2}\,dx=\int \frac{\sqrt{x}}{x^n b+a}dx$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int(sqrt(x)/(x**n*b + a),x)
```

3.320 $\int \frac{A+Bx^n}{\sqrt{x}(a+bx^n)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{abn\,(a+bx^n)}+\frac{(aB-Ab(1-2n))\sqrt{x}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn}$$

output
```
(A*b-B*a)*x^(1/2)/a/b/n/(a+b*x^n)+(B*a-A*b*(1-2*n))*x^(1/2)*hypergeom([1,
1/2/n],[1+1/2/n],-b*x^n/a)/a^2/b/n
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx = \frac{\sqrt{x}\left(a(Ab-aB)+(aB+Ab(-1+2n))\,(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},-\frac{bx^n}{a}\right)\right)}{a^2bn\,(a+bx^n)}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)^2),x]
```

output
```
(Sqrt[x]*(a*(A*b - a*B) + (a*B + A*b*(-1 + 2*n))*(a + b*x^n)*Hypergeometri
c2F1[1, 1/(2*n), 1 + 1/(2*n), -((b*x^n)/a)]))/(a^2*b*n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx$$

$$\Big\downarrow \; 957$$

$$\frac{(aB-Ab(1-2n))\int \frac{1}{\sqrt{x}(bx^n+a)}dx}{2abn}+\frac{\sqrt{x}(Ab-aB)}{abn\,(a+bx^n)}$$

$$\Big\downarrow \; 888$$

$$\frac{\sqrt{x}(aB-Ab(1-2n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^2bn}+\frac{\sqrt{x}(Ab-aB)}{abn\,(a+bx^n)}$$

input
```
Int[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)^2),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(a*b*n*(a + b*x^n)) + ((a*B - A*b*(1 - 2*n))*Sqrt[x]
*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))/2, -((b*x^n)/a)])/(a^2*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\sqrt{x}\,\left(a+b\,x^{n}\right)^{2}}d x$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^2\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^2*x*x^(2*n) + 2*a*b*x*x^n + a^2*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.37 (sec) , antiderivative size = 925, normalized size of antiderivative = 10.51

$$\int \frac{A + Bx^n}{\sqrt{x}\,(a + bx^n)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*x**n)/x**(1/2)/(a+b*x**n)**2,x)
```

output
```
A*(2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*sqrt(x)*lerchphi(b*x**n*exp_polar(
I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*n**3*gamma(1 + 1/(2*n)) + 4*b*n**
3*x**n*gamma(1 + 1/(2*n))) + 2*a*a**(1/(2*n))*a**(-2 - 1/(2*n))*n*sqrt(x)*
gamma(1/(2*n))/(4*a*n**3*gamma(1 + 1/(2*n)) + 4*b*n**3*x**n*gamma(1 + 1/(2
*n))) - a*a**(1/(2*n))*a**(-2 - 1/(2*n))*sqrt(x)*lerchphi(b*x**n*exp_polar
(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*n**3*gamma(1 + 1/(2*n)) + 4*b*n*
*3*x**n*gamma(1 + 1/(2*n))) + 2*a**(1/(2*n))*a**(-2 - 1/(2*n))*b*n*sqrt(x)
*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/(2*n))*gamma(1/(2*n))/(4*a*n
**3*gamma(1 + 1/(2*n)) + 4*b*n**3*x**n*gamma(1 + 1/(2*n))) - a**(1/(2*n))*
a**(-2 - 1/(2*n))*b*sqrt(x)*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/(
2*n))*gamma(1/(2*n))/(4*a*n**3*gamma(1 + 1/(2*n)) + 4*b*n**3*x**n*gamma(1
+ 1/(2*n)))) + B*(4*a*a**(-3 - 1/(2*n))*a**(1 + 1/(2*n))*n**2*x**(n + 1/2)
*gamma(1 + 1/(2*n))/(4*a*n**3*gamma(2 + 1/(2*n)) + 4*b*n**3*x**n*gamma(2 +
 1/(2*n))) - 2*a*a**(-3 - 1/(2*n))*a**(1 + 1/(2*n))*n*x**(n + 1/2)*lerchph
i(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/(2*n))*gamma(1 + 1/(2*n))/(4*a*n**3*g
amma(2 + 1/(2*n)) + 4*b*n**3*x**n*gamma(2 + 1/(2*n))) + 2*a*a**(-3 - 1/(2*
n))*a**(1 + 1/(2*n))*n*x**(n + 1/2)*gamma(1 + 1/(2*n))/(4*a*n**3*gamma(2 +
 1/(2*n)) + 4*b*n**3*x**n*gamma(2 + 1/(2*n))) - a*a**(-3 - 1/(2*n))*a**(1
+ 1/(2*n))*x**(n + 1/2)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/(2*n))
*gamma(1 + 1/(2*n))/(4*a*n**3*gamma(2 + 1/(2*n)) + 4*b*n**3*x**n*gamma(...
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^2\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 - n)*A*a*b + B*a^2*n)*integrate(1/((((8*n^2 - 6*n + 1)*b^4*x^(3*n
) + 3*(8*n^2 - 6*n + 1)*a*b^3*x^(2*n) + 3*(8*n^2 - 6*n + 1)*a^2*b^2*x^n +
(8*n^2 - 6*n + 1)*a^3*b)*sqrt(x)), x) - 2*(B*b*(4*n - 1)*x*x^n + (A*b*(2*n
 - 1) + 4*B*a*n)*x)/(((8*n^2 - 6*n + 1)*b^3*x^(2*n) + 2*(8*n^2 - 6*n + 1)*
a*b^2*x^n + (8*n^2 - 6*n + 1)*a^2*b)*sqrt(x))
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^2\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^2}\,dx = \int \frac{A+B\,x^n}{\sqrt{x}\,(a+b\,x^n)^2}\,dx$$

input
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{\sqrt{x}\,(a + bx^n)^2}\,dx = \int \frac{1}{x^{n+\frac{1}{2}}b + \sqrt{x}\,a}dx$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^2,x)
```

output
```
int(1/(x**((2*n + 1)/2)*b + sqrt(x)*a),x)
```

3.321 $\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}\,dx = \frac{Ab-aB}{abn\sqrt{x}\left(a+bx^n\right)} + \frac{(aB-A(b+2bn))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{a^2bn\sqrt{x}}$$

output
```
(A*b-B*a)/a/b/n/x^(1/2)/(a+b*x^n)+(B*a-A*(2*b*n+b))*hypergeom([1, -1/2/n],
[1-1/2/n],-b*x^n/a)/a^2/b/n/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}\,dx = \frac{a(Ab-aB)+(aB-A(b+2bn))\left(a+bx^n\right)\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\right.}{a^2bn\sqrt{x}\left(a+bx^n\right)}$$

input
```
Integrate[(A + B*x^n)/(x^(3/2)*(a + b*x^n)^2),x]
```

output
```
(a*(A*b - a*B) + (a*B - A*(b + 2*b*n))*(a + b*x^n)*Hypergeometric2F1[1, -1
/2*1/n, 1 - 1/(2*n), -((b*x^n)/a)])/(a^2*b*n*Sqrt[x]*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{Ab-aB}{abn\sqrt{x}\left(a+bx^n\right)}-\frac{(aB-A(2bn+b))\int \frac{1}{x^{3/2}(bx^n+a)}dx}{2abn}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(aB-A(2bn+b))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{a^2bn\sqrt{x}}+\frac{Ab-aB}{abn\sqrt{x}\left(a+bx^n\right)}$$

input
```
Int[(A + B*x^n)/(x^(3/2)*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*Sqrt[x]*(a + b*x^n)) + ((a*B - A*(b + 2*b*n))*Hypergeom
etric2F1[1, -1/2*1/n, 1 - 1/(2*n), -((b*x^n)/a)])/(a^2*b*n*Sqrt[x])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{3}{2}}\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x^n + a^2*x^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^n}{x^{3/2} (a + bx^n)^2} \, dx = \text{Timed out}$$

input
```
integrate((A+B*x**n)/x**(3/2)/(a+b*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^{3/2} (a + bx^n)^2} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^2 x^{\frac{3}{2}}} \, dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 + n)*A*a*b - B*a^2*n)*integrate(1/(((8*n^2 + 6*n + 1)*b^4*x^(3*n
) + 3*(8*n^2 + 6*n + 1)*a*b^3*x^(2*n) + 3*(8*n^2 + 6*n + 1)*a^2*b^2*x^n +
(8*n^2 + 6*n + 1)*a^3*b)*x^(3/2)), x) - 2*(B*b*(4*n + 1)*x*x^n + (A*b*(2*n
 + 1) + 4*B*a*n)*x)/(((8*n^2 + 6*n + 1)*b^3*x^(2*n) + 2*(8*n^2 + 6*n + 1)*
a*b^2*x^n + (8*n^2 + 6*n + 1)*a^2*b)*x^(3/2))
```

Giac [F]

$$\int \frac{A + Bx^n}{x^{3/2} (a + bx^n)^2} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^2 x^{\frac{3}{2}}} \, dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}\,dx=\int \frac{A+B\,x^n}{x^{3/2}\,{\left(a+b\,x^n\right)}^2}\,dx$$

input
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^{n+\frac{1}{2}}bx+\sqrt{x}\,ax}dx$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^2,x)
```

output
```
int(1/(x**((2*n + 1)/2)*b*x + sqrt(x)*a*x),x)
```

3.322 $\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx=\frac{Ab-aB}{abnx^{3/2}\left(a+bx^n\right)}+\frac{(3aB-Ab(3+2n))\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2bnx^{3/2}}$$

output
```
(A*b-B*a)/a/b/n/x^(3/2)/(a+b*x^n)+1/3*(3*B*a-A*b*(3+2*n))*hypergeom([1, -3
/2/n],[1-3/2/n],-b*x^n/a)/a^2/b/n/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx=\frac{3a(Ab-aB)+(3aB-Ab(3+2n))\left(a+bx^n\right)\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n}\right.}{3a^2bnx^{3/2}\left(a+bx^n\right)}$$

input
```
Integrate[(A + B*x^n)/(x^(5/2)*(a + b*x^n)^2),x]
```

output
```
(3*a*(A*b - a*B) + (3*a*B - A*b*(3 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1
, -3/(2*n), 1 - 3/(2*n), -((b*x^n)/a)])/(3*a^2*b*n*x^(3/2)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{abnx^{3/2}(a+bx^n)}-\frac{(3aB-Ab(2n+3))\int \frac{1}{x^{5/2}(bx^n+a)}dx}{2abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(3aB-Ab(2n+3))\,\text{Hypergeometric2F1}\left(1,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2bnx^{3/2}}+\frac{Ab-aB}{abnx^{3/2}(a+bx^n)}$$

input
```
Int[(A + B*x^n)/(x^(5/2)*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*x^(3/2)*(a + b*x^n)) + ((3*a*B - A*b*(3 + 2*n))*Hyperge
ometric2F1[1, -3/(2*n), 1 - 3/(2*n), -((b*x^n)/a)])/(3*a^2*b*n*x^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{x^{\frac{5}{2}}\left(a + b\,x^{n}\right)^{2}}dx$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A + Bx^n}{x^{5/2}\left(a + bx^n\right)^2}\,dx = \int \frac{Bx^n + A}{(bx^n + a)^2 x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x^n + a^2*x^3),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*x**n)/x**(5/2)/(a+b*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 + 3*n)*A*a*b - 3*B*a^2*n)*integrate(1/(((8*n^2 + 18*n + 9)*b^4*x
^(3*n) + 3*(8*n^2 + 18*n + 9)*a*b^3*x^(2*n) + 3*(8*n^2 + 18*n + 9)*a^2*b^2
*x^n + (8*n^2 + 18*n + 9)*a^3*b)*x^(5/2)), x) - 2*(B*b*(4*n + 3)*x*x^n + (
A*b*(2*n + 3) + 4*B*a*n)*x)/(((8*n^2 + 18*n + 9)*b^3*x^(2*n) + 2*(8*n^2 +
18*n + 9)*a*b^2*x^n + (8*n^2 + 18*n + 9)*a^2*b)*x^(5/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx=\int \frac{A+B\,x^n}{x^{5/2}\,\left(a+b\,x^n\right)^2}\,dx$$

input
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^{n+\frac{1}{2}}b\,x^2+\sqrt{x}\,a\,x^2}dx$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^2,x)
```

output
```
int(1/(x**((2*n + 1)/2)*b*x**2 + sqrt(x)*a*x**2),x)
```

3.323 $\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^2} \, dx = \frac{Ab-aB}{abnx^{5/2}(a+bx^n)} + \frac{(5aB-Ab(5+2n))\,\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2bnx^{5/2}}$$

output
```
(A*b-B*a)/a/b/n/x^(5/2)/(a+b*x^n)+1/5*(5*B*a-A*b*(5+2*n))*hypergeom([1, -5
/2/n],[1-5/2/n],-b*x^n/a)/a^2/b/n/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^2} \, dx = \frac{5a(Ab-aB)+(5aB-Ab(5+2n))\,(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n}\right.}{5a^2bnx^{5/2}(a+bx^n)}$$

input
```
Integrate[(A + B*x^n)/(x^(7/2)*(a + b*x^n)^2),x]
```

output
```
(5*a*(A*b - a*B) + (5*a*B - A*b*(5 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1
, -5/(2*n), 1 - 5/(2*n), -((b*x^n)/a)])/(5*a^2*b*n*x^(5/2)*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{abnx^{5/2}(a+bx^n)}-\frac{(5aB-Ab(2n+5))\int \frac{1}{x^{7/2}(bx^n+a)}dx}{2abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(5aB-Ab(2n+5))\,\text{Hypergeometric2F1}\left(1,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2bnx^{5/2}}+\frac{Ab-aB}{abnx^{5/2}(a+bx^n)}$$

input
```
Int[(A + B*x^n)/(x^(7/2)*(a + b*x^n)^2),x]
```

output
```
(A*b - a*B)/(a*b*n*x^(5/2)*(a + b*x^n)) + ((5*a*B - A*b*(5 + 2*n))*Hyperge
ometric2F1[1, -5/(2*n), 1 - 5/(2*n), -((b*x^n)/a)])/(5*a^2*b*n*x^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{7}{2}}\left(a+b\,x^{n}\right)^{2}}dx$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x)
```

output
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^2*x^4*x^(2*n) + 2*a*b*x^4*x^n + a^2*x^4),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**(7/2)/(a+b*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
4*((2*n^2 + 5*n)*A*a*b - 5*B*a^2*n)*integrate(1/(((8*n^2 + 30*n + 25)*b^4*
x^(3*n) + 3*(8*n^2 + 30*n + 25)*a*b^3*x^(2*n) + 3*(8*n^2 + 30*n + 25)*a^2*
b^2*x^n + (8*n^2 + 30*n + 25)*a^3*b)*x^(7/2)), x) - 2*(B*b*(4*n + 5)*x*x^n
 + (A*b*(2*n + 5) + 4*B*a*n)*x)/(((8*n^2 + 30*n + 25)*b^3*x^(2*n) + 2*(8*n
^2 + 30*n + 25)*a*b^2*x^n + (8*n^2 + 30*n + 25)*a^2*b)*x^(7/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^2x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^2*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx=\int \frac{A+B\,x^n}{x^{7/2}\,{\left(a+b\,x^n\right)}^2}\,dx$$

input
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)^2),x)
```

output
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^2}\,dx=\int \frac{1}{x^{n+\frac{1}{2}}b\,x^3+\sqrt{x}\,a\,x^3}dx$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^2,x)
```

output
```
int(1/(x**((2*n + 1)/2)*b*x**3 + sqrt(x)*a*x**3),x)
```

3.324 $\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{(Ab-aB)x^{5/2}}{2abn\,(a+bx^n)^2} + \frac{(5aB-Ab(5-4n))x^{5/2}\,\text{Hypergeometric2F1}\left(2,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{10a^3bn}$$

output
```
1/2*(A*b-B*a)*x^(5/2)/a/b/n/(a+b*x^n)^2+1/10*(5*B*a-A*b*(5-4*n))*x^(5/2)*h
ypergeom([2, 5/2/n],[1+5/2/n],-b*x^n/a)/a^3/b/n
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{x^{5/2}\left(-\frac{10a^2(-Ab+aB)n}{(a+bx^n)^2}+\frac{5a(5aB+Ab(-5+4n))}{a+bx^n}+(-5+2n)(5aB+Ab(-5+4n))\,\text{Hyper}\right.}{20a^3bn^2}$$

input
```
Integrate[(x^(3/2)*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
(x^(5/2)*((-10*a^2*(-(A*b) + a*B)*n)/(a + b*x^n)^2 + (5*a*(5*a*B + A*b*(-5
 + 4*n)))/(a + b*x^n) + (-5 + 2*n)*(5*a*B + A*b*(-5 + 4*n))*Hypergeometric
2F1[1, 5/(2*n), 1 + 5/(2*n), -((b*x^n)/a)]))/(20*a^3*b*n^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(5aB-Ab(5-4n))\int \frac{x^{3/2}}{(bx^n+a)^2}dx}{4abn}+\frac{x^{5/2}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{5/2}(5aB-Ab(5-4n))\,\text{Hypergeometric2F1}\left(2,\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{10a^3bn}+\frac{x^{5/2}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

input
```
Int[(x^(3/2)*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*x^(5/2))/(2*a*b*n*(a + b*x^n)^2) + ((5*a*B - A*b*(5 - 4*n))*x
^(5/2)*Hypergeometric2F1[2, 5/(2*n), 1 + 5/(2*n), -((b*x^n)/a)])/(10*a^3*b
*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}} \left(A+B\,x^{n}\right)}{\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx=\int \frac{(Bx^n+A)x^{\frac{3}{2}}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x*x^n + A*x)*sqrt(x)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*
x^n + a^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}(A + Bx^n)}{(a + bx^n)^3}\, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(3/2)*(A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{3/2}(A + Bx^n)}{(a + bx^n)^3}\, dx = \int \frac{(Bx^n + A)x^{\frac{3}{2}}}{(bx^n + a)^3}\, dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
6*((4*n^2 - 5*n)*A*a*b + 5*B*a^2*n)*integrate(x^(3/2)/((24*n^2 - 50*n + 25
)*b^5*x^(4*n) + 4*(24*n^2 - 50*n + 25)*a*b^4*x^(3*n) + 6*(24*n^2 - 50*n +
25)*a^2*b^3*x^(2*n) + 4*(24*n^2 - 50*n + 25)*a^3*b^2*x^n + (24*n^2 - 50*n
+ 25)*a^4*b), x) - 2*(B*b*(6*n - 5)*x*x^n + (A*b*(4*n - 5) + 6*B*a*n)*x)*x
^(3/2)/((24*n^2 - 50*n + 25)*b^4*x^(3*n) + 3*(24*n^2 - 50*n + 25)*a*b^3*x^
(2*n) + 3*(24*n^2 - 50*n + 25)*a^2*b^2*x^n + (24*n^2 - 50*n + 25)*a^3*b)
```

Giac [F]

$$\int \frac{x^{3/2}(A + Bx^n)}{(a + bx^n)^3}\, dx = \int \frac{(Bx^n + A)x^{\frac{3}{2}}}{(bx^n + a)^3}\, dx$$

input
```
integrate(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^(3/2)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx=\int \frac{x^{3/2}\,(A+B\,x^n)}{(a+b\,x^n)^3}\,dx$$

input
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n)^3,x)
```

output
```
int((x^(3/2)*(A + B*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{x^{3/2}(A+Bx^n)}{(a+bx^n)^3}\,dx=\int \frac{\sqrt{x}\,x}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(x^(3/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int((sqrt(x)*x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.325 $\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{(Ab-aB)x^{3/2}}{2abn\,(a+bx^n)^2}+\frac{(3aB-Ab(3-4n))x^{3/2}\,\text{Hypergeometric2F1}\left(2,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{6a^3bn}$$

output
```
1/2*(A*b-B*a)*x^(3/2)/a/b/n/(a+b*x^n)^2+1/6*(3*B*a-A*b*(3-4*n))*x^(3/2)*hy
pergeom([2, 3/2/n],[1+3/2/n],-b*x^n/a)/a^3/b/n
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{x^{3/2}\left(-\frac{6a^2(-Ab+aB)n}{(a+bx^n)^2}+\frac{3a(3aB+Ab(-3+4n))}{a+bx^n}+(-3+2n)(3aB+Ab(-3+4n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{2n},1\right.\right.}{12a^3bn^2}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
(x^(3/2)*((-6*a^2*(-(A*b) + a*B)*n)/(a + b*x^n)^2 + (3*a*(3*a*B + A*b*(-3
+ 4*n)))/(a + b*x^n) + (-3 + 2*n)*(3*a*B + A*b*(-3 + 4*n))*Hypergeometric2
F1[1, 3/(2*n), 1 + 3/(2*n), -((b*x^n)/a)]))/(12*a^3*b*n^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \ 957$$

$$\frac{(3aB-Ab(3-4n))\int \frac{\sqrt{x}}{(bx^n+a)^2}dx}{4abn}+\frac{x^{3/2}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \ 888$$

$$\frac{x^{3/2}(3aB-Ab(3-4n))\,\text{Hypergeometric2F1}\left(2,\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{6a^3bn}+\frac{x^{3/2}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

input
```
Int[(Sqrt[x]*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(2*a*b*n*(a + b*x^n)^2) + ((3*a*B - A*b*(3 - 4*n))*x
^(3/2)*Hypergeometric2F1[2, 3/(2*n), 1 + 3/(2*n), -((b*x^n)/a)])/(6*a^3*b*
n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{\sqrt{x}\,(A+B\,x^{n})}{(a+b\,x^{n})^{3}}dx$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx=\int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
+ a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)*(A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
2*((4*n^2 - 3*n)*A*a*b + 3*B*a^2*n)*integrate(sqrt(x)/((8*n^2 - 10*n + 3)*
b^5*x^(4*n) + 4*(8*n^2 - 10*n + 3)*a*b^4*x^(3*n) + 6*(8*n^2 - 10*n + 3)*a^
2*b^3*x^(2*n) + 4*(8*n^2 - 10*n + 3)*a^3*b^2*x^n + (8*n^2 - 10*n + 3)*a^4*
b), x) - 2/3*(3*B*b*(2*n - 1)*x*x^n + (A*b*(4*n - 3) + 6*B*a*n)*x)*sqrt(x)
/((8*n^2 - 10*n + 3)*b^4*x^(3*n) + 3*(8*n^2 - 10*n + 3)*a*b^3*x^(2*n) + 3*
(8*n^2 - 10*n + 3)*a^2*b^2*x^n + (8*n^2 - 10*n + 3)*a^3*b)
```

Giac [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)\sqrt{x}}{(bx^n+a)^3}\,dx$$

input
```
integrate(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(x)/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3} \, dx = \int \frac{\sqrt{x}\,(A+B\,x^n)}{(a+b\,x^n)^3} \, dx$$

input
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n)^3,x)
```

output
```
int((x^(1/2)*(A + B*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{x}(A+Bx^n)}{(a+bx^n)^3} \, dx = \int \frac{\sqrt{x}}{x^{2n}b^2+2x^nab+a^2} dx$$

input
```
int(x^(1/2)*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int(sqrt(x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.326 $\int \frac{A+Bx^n}{\sqrt{x}(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{2abn\,(a+bx^n)^2} + \frac{(aB-Ab(1-4n))\sqrt{x}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{2a^3bn}$$

output
```
1/2*(A*b-B*a)*x^(1/2)/a/b/n/(a+b*x^n)^2+1/2*(B*a-A*b*(1-4*n))*x^(1/2)*hype
rgeom([2, 1/2/n],[1+1/2/n],-b*x^n/a)/a^3/b/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx = \frac{\sqrt{x}\left(-\frac{2a^2(-Ab+aB)n}{(a+bx^n)^2}+\frac{a(aB+Ab(-1+4n))}{a+bx^n}+(-1+2n)(aB+Ab(-1+4n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\right.\right.}{4a^3bn^2}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)^3),x]
```

output
```
(Sqrt[x]*((-2*a^2*(-(A*b) + a*B)*n)/(a + b*x^n)^2 + (a*(a*B + A*b*(-1 + 4*
n)))/(a + b*x^n) + (-1 + 2*n)*(a*B + A*b*(-1 + 4*n))*Hypergeometric2F1[1,
1/(2*n), 1 + 1/(2*n), -((b*x^n)/a)]))/(4*a^3*b*n^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB-Ab(1-4n))\int \frac{1}{\sqrt{x}(bx^n+a)^2}dx}{4abn}+\frac{\sqrt{x}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt{x}(aB-Ab(1-4n))\,\text{Hypergeometric2F1}\left(2,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{2a^3bn}+\frac{\sqrt{x}(Ab-aB)}{2abn\,(a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(Sqrt[x]*(a + b*x^n)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(2*a*b*n*(a + b*x^n)^2) + ((a*B - A*b*(1 - 4*n))*Sqr
t[x]*Hypergeometric2F1[2, 1/(2*n), (2 + n^(-1))/2, -((b*x^n)/a)])/(2*a^3*b
*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\sqrt{x}\,\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^3\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^3*x*x^(3*n) + 3*a*b^2*x*x^(2*n) + 3*a^2*b*
x*x^n + a^3*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**(1/2)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^3\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
6*((4*n^2 - n)*A*a*b + B*a^2*n)*integrate(1/(((24*n^2 - 10*n + 1)*b^5*x^(4
*n) + 4*(24*n^2 - 10*n + 1)*a*b^4*x^(3*n) + 6*(24*n^2 - 10*n + 1)*a^2*b^3*
x^(2*n) + 4*(24*n^2 - 10*n + 1)*a^3*b^2*x^n + (24*n^2 - 10*n + 1)*a^4*b)*s
qrt(x)), x) - 2*(B*b*(6*n - 1)*x*x^n + (A*b*(4*n - 1) + 6*B*a*n)*x)/(((24*
n^2 - 10*n + 1)*b^4*x^(3*n) + 3*(24*n^2 - 10*n + 1)*a*b^3*x^(2*n) + 3*(24*
n^2 - 10*n + 1)*a^2*b^2*x^n + (24*n^2 - 10*n + 1)*a^3*b)*sqrt(x))
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^3\sqrt{x}}\,dx$$

input
```
integrate((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx=\int \frac{A+B\,x^n}{\sqrt{x}\,(a+b\,x^n)^3}\,dx$$

input
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^(1/2)*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{\sqrt{x}\,(a+bx^n)^3}\,dx=\int \frac{1}{x^{2n+\frac{1}{2}}b^2+2x^{n+\frac{1}{2}}ab+\sqrt{x}\,a^2}dx$$

input
```
int((A+B*x^n)/x^(1/2)/(a+b*x^n)^3,x)
```

output
```
int(1/(x**((4*n + 1)/2)*b**2 + 2*x**((2*n + 1)/2)*a*b + sqrt(x)*a**2),x)
```

3.327 $\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)^3}\,dx=\frac{Ab-aB}{2abn\sqrt{x}\,(a+bx^n)^2}+\frac{(aB-A(b+4bn))\,\text{Hypergeometric2F1}\left(2,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{2a^3bn\sqrt{x}}$$

output
```
1/2*(A*b-B*a)/a/b/n/x^(1/2)/(a+b*x^n)^2+1/2*(B*a-A*(4*b*n+b))*hypergeom([2
, -1/2/n],[1-1/2/n],-b*x^n/a)/a^3/b/n/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^n}{x^{3/2}(a+bx^n)^3}\,dx=\frac{\frac{2a^2(Ab-aB)n}{(a+bx^n)^2}+\frac{a(-aB+A(b+4bn))}{a+bx^n}+(1+2n)(aB-A(b+4bn))\,\text{Hypergeometric2F1}\left(1\right.}{4a^3bn^2\sqrt{x}}$$

input
```
Integrate[(A + B*x^n)/(x^(3/2)*(a + b*x^n)^3),x]
```

output
```
((2*a^2*(A*b - a*B)*n)/(a + b*x^n)^2 + (a*(-(a*B) + A*(b + 4*b*n)))/(a + b
*x^n) + (1 + 2*n)*(a*B - A*(b + 4*b*n))*Hypergeometric2F1[1, -1/2*1/n, 1 -
 1/(2*n), -((b*x^n)/a)])/(4*a^3*b*n^2*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{Ab-aB}{2abn\sqrt{x}\left(a+bx^n\right)^2}-\frac{(aB-A(4bn+b))\int \frac{1}{x^{3/2}(bx^n+a)^2}dx}{4abn}$$

$$\Big\downarrow \text{888}$$

$$\frac{(aB-A(4bn+b))\,\text{Hypergeometric2F1}\left(2,-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{2a^3bn\sqrt{x}}+\frac{Ab-aB}{2abn\sqrt{x}\left(a+bx^n\right)^2}$$

input
```
Int[(A + B*x^n)/(x^(3/2)*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*Sqrt[x]*(a + b*x^n)^2) + ((a*B - A*(b + 4*b*n))*Hyper
geometric2F1[2, -1/2*1/n, 1 - 1/(2*n), -((b*x^n)/a)])/(2*a^3*b*n*Sqrt[x])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{3}{2}}\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^3*x^2*x^(3*n) + 3*a*b^2*x^2*x^(2*n) + 3*a^
2*b*x^2*x^n + a^3*x^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**(3/2)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
6*((4*n^2 + n)*A*a*b - B*a^2*n)*integrate(1/(((24*n^2 + 10*n + 1)*b^5*x^(4
*n) + 4*(24*n^2 + 10*n + 1)*a*b^4*x^(3*n) + 6*(24*n^2 + 10*n + 1)*a^2*b^3*
x^(2*n) + 4*(24*n^2 + 10*n + 1)*a^3*b^2*x^n + (24*n^2 + 10*n + 1)*a^4*b)*x
^(3/2)), x) - 2*(B*b*(6*n + 1)*x*x^n + (A*b*(4*n + 1) + 6*B*a*n)*x)/(((24*
n^2 + 10*n + 1)*b^4*x^(3*n) + 3*(24*n^2 + 10*n + 1)*a*b^3*x^(2*n) + 3*(24*
n^2 + 10*n + 1)*a^2*b^2*x^n + (24*n^2 + 10*n + 1)*a^3*b)*x^(3/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx=\int \frac{A+B\,x^n}{x^{3/2}\,{\left(a+b\,x^n\right)}^3}\,dx$$

input
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^(3/2)*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{3/2}\left(a+bx^n\right)^3}\,dx=\int \frac{1}{x^{2n+\frac{1}{2}}b^2x+2x^{n+\frac{1}{2}}abx+\sqrt{x}\,a^2x}dx$$

input
```
int((A+B*x^n)/x^(3/2)/(a+b*x^n)^3,x)
```

output
```
int(1/(x**((4*n + 1)/2)*b**2*x + 2*x**((2*n + 1)/2)*a*b*x + sqrt(x)*a**2*x
),x)
```

3.328 $\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx = \frac{Ab-aB}{2abnx^{3/2}(a+bx^n)^2} + \frac{(3aB-Ab(3+4n))\,\text{Hypergeometric2F1}\left(2,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{6a^3bnx^{3/2}}$$

output
```
1/2*(A*b-B*a)/a/b/n/x^(3/2)/(a+b*x^n)^2+1/6*(3*B*a-A*b*(3+4*n))*hypergeom(
[2, -3/2/n],[1-3/2/n],-b*x^n/a)/a^3/b/n/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx = \frac{\frac{6a^2(Ab-aB)n}{(a+bx^n)^2} + \frac{3a(-3aB+Ab(3+4n))}{a+bx^n} + (3+2n)(3aB-Ab(3+4n))\,\text{Hypergeometric2F}}{12a^3bn^2x^{3/2}}$$

input
```
Integrate[(A + B*x^n)/(x^(5/2)*(a + b*x^n)^3),x]
```

output
```
((6*a^2*(A*b - a*B)*n)/(a + b*x^n)^2 + (3*a*(-3*a*B + A*b*(3 + 4*n)))/(a +
 b*x^n) + (3 + 2*n)*(3*a*B - A*b*(3 + 4*n))*Hypergeometric2F1[1, -3/(2*n),
 1 - 3/(2*n), -((b*x^n)/a)])/(12*a^3*b*n^2*x^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^3}dx$$

$$\Big\downarrow \text{ 957}$$

$$\frac{Ab-aB}{2abnx^{3/2}\left(a+bx^n\right)^2}-\frac{(3aB-Ab(4n+3))\int \frac{1}{x^{5/2}\left(bx^n+a\right)^2}dx}{4abn}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(3aB-Ab(4n+3))\,\text{Hypergeometric2F1}\left(2,-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{6a^3bnx^{3/2}}+\frac{Ab-aB}{2abnx^{3/2}\left(a+bx^n\right)^2}$$

input
```
Int[(A + B*x^n)/(x^(5/2)*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*x^(3/2)*(a + b*x^n)^2) + ((3*a*B - A*b*(3 + 4*n))*Hyp
ergeometric2F1[2, -3/(2*n), 1 - 3/(2*n), -((b*x^n)/a)])/(6*a^3*b*n*x^(3/2)
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{5}{2}}\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^{n}+A}{\left(bx^{n}+a\right)^{3}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^3*x^3*x^(3*n) + 3*a*b^2*x^3*x^(2*n) + 3*a^
2*b*x^3*x^n + a^3*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((A+B*x**n)/x**(5/2)/(a+b*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^3x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
2*((4*n^2 + 3*n)*A*a*b - 3*B*a^2*n)*integrate(1/(((8*n^2 + 10*n + 3)*b^5*x
^(4*n) + 4*(8*n^2 + 10*n + 3)*a*b^4*x^(3*n) + 6*(8*n^2 + 10*n + 3)*a^2*b^3
*x^(2*n) + 4*(8*n^2 + 10*n + 3)*a^3*b^2*x^n + (8*n^2 + 10*n + 3)*a^4*b)*x^
(5/2)), x) - 2/3*(3*B*b*(2*n + 1)*x*x^n + (A*b*(4*n + 3) + 6*B*a*n)*x)/(((
8*n^2 + 10*n + 3)*b^4*x^(3*n) + 3*(8*n^2 + 10*n + 3)*a*b^3*x^(2*n) + 3*(8*
n^2 + 10*n + 3)*a^2*b^2*x^n + (8*n^2 + 10*n + 3)*a^3*b)*x^(5/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{5/2}(a+bx^n)^3}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^3x^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^3}\,dx=\int \frac{A+B\,x^n}{x^{5/2}\,\left(a+b\,x^n\right)^3}\,dx$$

input
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^(5/2)*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{5/2}\left(a+bx^n\right)^3}\,dx=\int \frac{1}{x^{2n+\frac{1}{2}}b^2x^2+2x^{n+\frac{1}{2}}ab\,x^2+\sqrt{x}\,a^2x^2}dx$$

input
```
int((A+B*x^n)/x^(5/2)/(a+b*x^n)^3,x)
```

output
```
int(1/(x**((4*n + 1)/2)*b**2*x**2 + 2*x**((2*n + 1)/2)*a*b*x**2 + sqrt(x)*
a**2*x**2),x)
```

3.329 $\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^3} \, dx = \frac{Ab-aB}{2abnx^{5/2}(a+bx^n)^2} + \frac{(5aB-Ab(5+4n)) \text{Hypergeometric2F1}\left(2,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{10a^3bnx^{5/2}}$$

output
```
1/2*(A*b-B*a)/a/b/n/x^(5/2)/(a+b*x^n)^2+1/10*(5*B*a-A*b*(5+4*n))*hypergeom
([2, -5/2/n],[1-5/2/n],-b*x^n/a)/a^3/b/n/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^3} \, dx = \frac{\frac{10a^2(Ab-aB)n}{(a+bx^n)^2} + \frac{5a(-5aB+Ab(5+4n))}{a+bx^n} + (5+2n)(5aB-Ab(5+4n)) \text{Hypergeometric2F}}{20a^3bn^2x^{5/2}}$$

input
```
Integrate[(A + B*x^n)/(x^(7/2)*(a + b*x^n)^3),x]
```

output
```
((10*a^2*(A*b - a*B)*n)/(a + b*x^n)^2 + (5*a*(-5*a*B + A*b*(5 + 4*n)))/(a
+ b*x^n) + (5 + 2*n)*(5*a*B - A*b*(5 + 4*n))*Hypergeometric2F1[1, -5/(2*n)
, 1 - 5/(2*n), -((b*x^n)/a)])/(20*a^3*b*n^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^{7/2}(a+bx^n)^3}dx$$
$$\Big\downarrow \text{957}$$
$$\frac{Ab-aB}{2abnx^{5/2}(a+bx^n)^2}-\frac{(5aB-Ab(4n+5))\int \frac{1}{x^{7/2}(bx^n+a)^2}dx}{4abn}$$
$$\Big\downarrow \text{888}$$
$$\frac{(5aB-Ab(4n+5))\,\text{Hypergeometric2F1}\left(2,-\frac{5}{2n},1-\frac{5}{2n},-\frac{bx^n}{a}\right)}{10a^3bnx^{5/2}}+\frac{Ab-aB}{2abnx^{5/2}(a+bx^n)^2}$$

input
```
Int[(A + B*x^n)/(x^(7/2)*(a + b*x^n)^3),x]
```

output
```
(A*b - a*B)/(2*a*b*n*x^(5/2)*(a + b*x^n)^2) + ((5*a*B - A*b*(5 + 4*n))*Hyp
ergeometric2F1[2, -5/(2*n), 1 - 5/(2*n), -((b*x^n)/a)])/(10*a^3*b*n*x^(5/2
))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{\frac{7}{2}}\left(a+b\,x^{n}\right)^{3}}d x$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x)
```

output
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*sqrt(x)/(b^3*x^4*x^(3*n) + 3*a*b^2*x^4*x^(2*n) + 3*a^
2*b*x^4*x^n + a^3*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*x**n)/x**(7/2)/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
6*((4*n^2 + 5*n)*A*a*b - 5*B*a^2*n)*integrate(1/((((24*n^2 + 50*n + 25)*b^5
*x^(4*n) + 4*(24*n^2 + 50*n + 25)*a*b^4*x^(3*n) + 6*(24*n^2 + 50*n + 25)*a
^2*b^3*x^(2*n) + 4*(24*n^2 + 50*n + 25)*a^3*b^2*x^n + (24*n^2 + 50*n + 25)
*a^4*b)*x^(7/2)), x) - 2*(B*b*(6*n + 5)*x*x^n + (A*b*(4*n + 5) + 6*B*a*n)*
x)/((((24*n^2 + 50*n + 25)*b^4*x^(3*n) + 3*(24*n^2 + 50*n + 25)*a*b^3*x^(2*
n) + 3*(24*n^2 + 50*n + 25)*a^2*b^2*x^n + (24*n^2 + 50*n + 25)*a^3*b)*x^(7
/2))
```

Giac [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^3x^{\frac{7}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^3*x^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\int \frac{A+B\,x^n}{x^{7/2}\,\left(a+b\,x^n\right)^3}\,dx$$

input
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)^3),x)
```

output
```
int((A + B*x^n)/(x^(7/2)*(a + b*x^n)^3), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^{7/2}\left(a+bx^n\right)^3}\,dx=\int \frac{1}{x^{2n+\frac{1}{2}}b^2x^3+2x^{n+\frac{1}{2}}ab\,x^3+\sqrt{x}\,a^2x^3}dx$$

input
```
int((A+B*x^n)/x^(7/2)/(a+b*x^n)^3,x)
```

output
```
int(1/(x**((4*n + 1)/2)*b**2*x**3 + 2*x**((2*n + 1)/2)*a*b*x**3 + sqrt(x)*
a**2*x**3),x)
```

3.330 $\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{2Bx^3(a+bx^n)^{3/2}}{3b(2+n)} + \frac{\left(A-\frac{2aB}{b(2+n)}\right)x^3\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2/3*B*x^3*(a+b*x^n)^(3/2)/b/(2+n)+1/3*(A-2*a*B/b/(2+n))*x^3*(a+b*x^n)^(1/2
)*hypergeom([-1/2, 3/n],[(3+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{x^3\sqrt{a+bx^n}\left(A(3+n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3+n}{n},2\right.\right.}{3(3+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x^2*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(x^3*Sqrt[a + b*x^n]*(A*(3 + n)*Hypergeometric2F1[-1/2, 3/n, (3 + n)/n, -(
(b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[-1/2, (3 + n)/n, 2 + 3/n, -((b*x^n
)/a)]))/(3*(3 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{b(n+2)}\right)\int x^2\sqrt{bx^n+a}dx+\frac{2Bx^3(a+bx^n)^{3/2}}{3b(n+2)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{b(n+2)}\right)\int x^2\sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{3/2}}{3b(n+2)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3\sqrt{a+bx^n}\left(A-\frac{2aB}{b(n+2)}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{3/2}}{3b(n+2)}$$

input
```
Int[x^2*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*x^3*(a + b*x^n)^(3/2))/(3*b*(2 + n)) + ((A - (2*a*B)/(b*(2 + n)))*x^3
*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3
*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^2\sqrt{a+b\,x^n}\,(A+B\,x^n)\,dx$$

input
```
int(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{Aa^{\frac{3}{n}}a^{\frac{1}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(3/n)*a**(1/2 - 3/n)*x**3*gamma(3/n)*hyper((-1/2, 3/n), (1 + 3/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + B*a**(-1/2 - 3/n)*a**(1 + 3/
n)*x**(n + 3)*gamma(1 + 3/n)*hyper((-1/2, 1 + 3/n), (2 + 3/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx=\int x^2\,(A+B\,x^n)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+bx^n}(A+Bx^n)\,dx$$
$$=\frac{2x^n\sqrt{x^nb+a}\,bn\,x^3+12x^n\sqrt{x^nb+a}\,b\,x^3+8\sqrt{x^nb+a}\,an\,x^3+12\sqrt{x^nb+a}\,a\,x^3+3\left(\int\frac{\sqrt{x^nb+a}}{x^nb\,n^2+8x^nbn+12x^nb}\right.}{3}$$

input
```
int(x^2*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n*x**3 + 12*x**n*sqrt(x**n*b + a)*b*x**3 + 8*sq
rt(x**n*b + a)*a*n*x**3 + 12*sqrt(x**n*b + a)*a*x**3 + 3*int((sqrt(x**n*b
+ a)*x**2)/(x**n*b*n**2 + 8*x**n*b*n + 12*x**n*b + a*n**2 + 8*a*n + 12*a),
x)*a**2*n**4 + 24*int((sqrt(x**n*b + a)*x**2)/(x**n*b*n**2 + 8*x**n*b*n +
12*x**n*b + a*n**2 + 8*a*n + 12*a),x)*a**2*n**3 + 36*int((sqrt(x**n*b + a)
*x**2)/(x**n*b*n**2 + 8*x**n*b*n + 12*x**n*b + a*n**2 + 8*a*n + 12*a),x)*a
**2*n**2)/(3*(n**2 + 8*n + 12))
```

3.331 $\int x\sqrt{a+bx^n}(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{2Bx^2(a+bx^n)^{3/2}}{b(4+3n)} + \frac{\left(A-\frac{4aB}{4b+3bn}\right)x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x^2*(a+b*x^n)^(3/2)/b/(4+3*n)+1/2*(A-4*a*B/(3*b*n+4*b))*x^2*(a+b*x^n)^
(1/2)*hypergeom([-1/2, 2/n],[(2+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{x^2\sqrt{a+bx^n}\left(A(2+n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2+n}{n},2\right.\right.}{2(2+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(x^2*Sqrt[a + b*x^n]*(A*(2 + n)*Hypergeometric2F1[-1/2, 2/n, (2 + n)/n, -(
(b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[-1/2, (2 + n)/n, 2*(1 + n^(-1)), -
((b*x^n)/a)]))/(2*(2 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{3bn+4b}\right)\int x\sqrt{bx^n+a}dx+\frac{2Bx^2(a+bx^n)^{3/2}}{b(3n+4)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{4aB}{3bn+4b}\right)\int x\sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{3/2}}{b(3n+4)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt{a+bx^n}\left(A-\frac{4aB}{3bn+4b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{3/2}}{b(3n+4)}$$

input
```
Int[x*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*x^2*(a + b*x^n)^(3/2))/(b*(4 + 3*n)) + ((A - (4*a*B)/(4*b + 3*b*n))*x
^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 2/n, (2 + n)/n, -((b*x^n)/a)])/
(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})\,dx$$

input
```
int(x*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int(x*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{Aa^{\frac{2}{n}}a^{\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(2/n)*a**(1/2 - 2/n)*x**2*gamma(2/n)*hyper((-1/2, 2/n), (1 + 2/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + B*a**(-1/2 - 2/n)*a**(1 + 2/
n)*x**(n + 2)*gamma(1 + 2/n)*hyper((-1/2, 1 + 2/n), (2 + 2/n,), b*x**n*exp
_polar(I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx=\int x\,(A+B\,x^n)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int(x*(A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int(x*(A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^n}(A+Bx^n)\,dx$$
$$=\frac{2x^n\sqrt{x^nb+a}\,bn\,x^2+8x^n\sqrt{x^nb+a}\,b\,x^2+8\sqrt{x^nb+a}\,an\,x^2+8\sqrt{x^nb+a}\,a\,x^2+9\left(\int\frac{\sqrt{x^nb+a}}{3x^nb\,n^2+16x^nbn+16x^nb+}\right.}{}$$

input
```
int(x*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n*x**2 + 8*x**n*sqrt(x**n*b + a)*b*x**2 + 8*sqr
t(x**n*b + a)*a*n*x**2 + 8*sqrt(x**n*b + a)*a*x**2 + 9*int((sqrt(x**n*b +
a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n + 16*x**n*b + 3*a*n**2 + 16*a*n + 16*a)
,x)*a**2*n**4 + 48*int((sqrt(x**n*b + a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n +
 16*x**n*b + 3*a*n**2 + 16*a*n + 16*a),x)*a**2*n**3 + 48*int((sqrt(x**n*b
+ a)*x)/(3*x**n*b*n**2 + 16*x**n*b*n + 16*x**n*b + 3*a*n**2 + 16*a*n + 16*
a),x)*a**2*n**2)/(3*n**2 + 16*n + 16)
```

3.332 $\int \sqrt{a+bx^n}(A+Bx^n)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \sqrt{a+bx^n}(A+Bx^n)\, dx = \frac{2Bx(a+bx^n)^{3/2}}{b(2+3n)} + \frac{\left(A-\frac{2aB}{2b+3bn}\right)x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x*(a+b*x^n)^(3/2)/b/(2+3*n)+(A-2*a*B/(3*b*n+2*b))*x*(a+b*x^n)^(1/2)*hy
pergeom([-1/2, 1/n],[1+1/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \sqrt{a+bx^n}(A+Bx^n)\, dx = \frac{x\sqrt{a+bx^n}\left(2B(a+bx^n)\sqrt{1+\frac{bx^n}{a}}+(-2aB+Ab(2+3n))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(2+3n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(x*Sqrt[a + b*x^n]*(2*B*(a + b*x^n)*Sqrt[1 + (b*x^n)/a] + (-2*a*B + A*b*(2
 + 3*n))*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(b*(2
 + 3*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(A-\frac{2aB}{3bn+2b}\right)\int \sqrt{bx^n+a}dx+\frac{2Bx(a+bx^n)^{3/2}}{b(3n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{3bn+2b}\right)\int \sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{3/2}}{b(3n+2)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt{a+bx^n}\left(A-\frac{2aB}{3bn+2b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{3/2}}{b(3n+2)}$$

input
```
Int[Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*x*(a + b*x^n)^(3/2))/(b*(2 + 3*n)) + ((A - (2*a*B)/(2*b + 3*b*n))*x*S
qrt[a + b*x^n]*Hypergeometric2F1[-1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/
Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{a+b\,x^{n}}\,(A+B\,x^{n})\,dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{Aa^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(1/n)*a**(1/2 - 1/n)*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(-1/2 - 1/n)*a**(1 + 1/n)*
x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^n}(A+Bx^n)\,dx=\int (A+B\,x^n)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int((A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int((A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+bx^n}(A+Bx^n)\ dx = \frac{2x^n\sqrt{x^nb+a}\,bnx+4x^n\sqrt{x^nb+a}\,bx+8\sqrt{x^nb+a}\,anx+4\sqrt{x^nb+a}\,ax+9\left(\int \frac{\sqrt{x^nb+a}}{3x^nb\,n^2+8x^nbn+4x^nb+3a\,n^2+8a}\right.}{3n^2}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n*x + 4*x**n*sqrt(x**n*b + a)*b*x + 8*sqrt(x**n
*b + a)*a*n*x + 4*sqrt(x**n*b + a)*a*x + 9*int(sqrt(x**n*b + a)/(3*x**n*b*
n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**4 + 24*i
nt(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 3*a*n**2 + 8*
a*n + 4*a),x)*a**2*n**3 + 12*int(sqrt(x**n*b + a)/(3*x**n*b*n**2 + 8*x**n*
b*n + 4*x**n*b + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**2)/(3*n**2 + 8*n + 4)
```

3.333 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x} dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x} dx = \frac{2A\sqrt{a+bx^n}}{n} + \frac{2B(a+bx^n)^{3/2}}{3bn} - \frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*A*(a+b*x^n)^(1/2)/n+2/3*B*(a+b*x^n)^(3/2)/b/n-2*a^(1/2)*A*arctanh((a+b*x
^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x} dx = \frac{2\sqrt{a+bx^n}(3Ab+B(a+bx^n)) - 6\sqrt{a}Ab\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3bn}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/x,x]
```

output
```
(2*Sqrt[a + b*x^n]*(3*A*b + B*(a + b*x^n)) - 6*Sqrt[a]*A*b*ArcTanh[Sqrt[a
+ b*x^n]/Sqrt[a]])/(3*b*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x} \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int x^{-n}\sqrt{bx^n+a}(Bx^n+A)\,dx^n}{n}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{A\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2B(a+bx^n)^{3/2}}{3b}}{n}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{A\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2B(a+bx^n)^{3/2}}{3b}}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{A\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2B(a+bx^n)^{3/2}}{3b}}{n}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{A\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2B(a+bx^n)^{3/2}}{3b}}{n}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/x,x]
```

output
```
((2*B*(a + b*x^n)^(3/2))/(3*b) + A*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x^n]/Sqrt[a]]))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{2B(a+b\,x^n)^{\frac{3}{2}}}{3}+2Ab\sqrt{a+b\,x^n}-2A\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{bn}$	55
default	$\frac{\frac{2B(a+b\,x^n)^{\frac{3}{2}}}{3}+2Ab\sqrt{a+b\,x^n}-2A\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{bn}$	55
risch	$\frac{2\left(Bb\,\mathrm{e}^{n\ln(x)}+3Ab+Ba\right)\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{3bn}-\frac{2A\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)}{n}$	62

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(1/3*B*(a+b*x^n)^(3/2)+A*b*(a+b*x^n)^(1/2)-A*a^(1/2)*b*arctanh((a+b*
x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = \left[\frac{3\,A\sqrt{a}b\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(Bbx^n+Ba+3\,Ab)\sqrt{bx^n+a}}{3\,bn}, \frac{2\left(3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right)+(\ldots\right.}{3\,bn\ldots}\right.$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*A*sqrt(a)*b*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2
*(B*b*x^n + B*a + 3*A*b)*sqrt(b*x^n + a))/(b*n), 2/3*(3*A*sqrt(-a)*b*arcta
n(sqrt(-a)/sqrt(b*x^n + a)) + (B*b*x^n + B*a + 3*A*b)*sqrt(b*x^n + a))/(b*
n)]
```

Sympy [A] (verification not implemented)

Time = 13.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2Aa\,\mathrm{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2A\sqrt{a+bx^n} + \frac{2B(a+bx^n)^{\frac{3}{2}}}{3b} & \text{for } b \neq 0 \\ A\sqrt{a}\log\left(B\sqrt{a}x^n\right) + B\sqrt{a}x^n & \text{otherwise} \end{cases}}{n} & \text{for } n \neq 0 \\ \left(A\sqrt{a+b} + B\sqrt{a+b}\right)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/x,x)
```

output
```
Piecewise((Piecewise((2*A*a*atan(sqrt(a + b*x**n)/sqrt(-a))/sqrt(-a) + 2*A
*sqrt(a + b*x**n) + 2*B*(a + b*x**n)**(3/2)/(3*b), Ne(b, 0)), (A*sqrt(a)*l
og(B*sqrt(a)*x**n) + B*sqrt(a)*x**n, True))/n, Ne(n, 0)), ((A*sqrt(a + b)
+ B*sqrt(a + b))*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = A\left(\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\sqrt{bx^n+a}}{n}\right) + \frac{2\,(bx^n+a)^{\frac{3}{2}}B}{3\,bn}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x,x, algorithm="maxima")
```

output
```
A*(sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n
+ 2*sqrt(b*x^n + a)/n) + 2/3*(b*x^n + a)^(3/2)*B/(b*n)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{x}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x}\,dx = \frac{2x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,a+3\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)a^2n}{3n}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b + a)*a + 3*int(sqrt(x**n*b + a)
/(x**n*b*x + a*x),x)*a**2*n)/(3*n)
```

3.334 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = -\frac{2B(a+bx^n)^{3/2}}{b(2-3n)x} - \frac{\left(A-\frac{2aB}{2b-3bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(3/2)/b/(2-3*n)/x-(A-2*a*B/(-3*b*n+2*b))*(a+b*x^n)^(1/2)*hy
pergeom([-1/2, -1/n],[-(1-n)/n],-b*x^n/a)/x/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \frac{\sqrt{a+bx^n}\left((A-An)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{-1+n}{n},\right.\right.}{(-1+n)x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/x^2,x]
```

output
```
(Sqrt[a + b*x^n]*((A - A*n)*Hypergeometric2F1[-1/2, -n^(-1), (-1 + n)/n, -
((b*x^n)/a)] + B*x^n*Hypergeometric2F1[-1/2, (-1 + n)/n, 2 - n^(-1), -((b*
x^n)/a)]))/((-1 + n)*x*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{2b-3bn}\right)\int \frac{\sqrt{bx^n+a}}{x^2}dx-\frac{2B(a+bx^n)^{3/2}}{b(2-3n)x}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-3bn}\right)\int \frac{\sqrt{\frac{bx^n}{a}+1}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{b(2-3n)x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-3bn}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{b(2-3n)x}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/x^2,x]
```

output
```
(-2*B*(a + b*x^n)^(3/2))/(b*(2 - 3*n)*x) - ((A - (2*a*B)/(2*b - 3*b*n))*Sq
rt[a + b*x^n]*Hypergeometric2F1[-1/2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)]
)/(x*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{x^{2}}dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \frac{Aa^{-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{n} \\ 1-\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)} + \frac{Ba^{-\frac{1}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}x^{n-1}\Gamma\left(1-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, 1-\frac{1}{n} \\ 2-\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/x**2,x)
```

output
```
A*a**(1/2 + 1/n)*gamma(-1/n)*hyper((-1/2, -1/n), (1 - 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + B*a**(-1/2 + 1/n)*a**(1 - 1/n
)*x**(n - 1)*gamma(1 - 1/n)*hyper((-1/2, 1 - 1/n), (2 - 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 - 1/n))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{x^2}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^2,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^2}\,dx$$
$$= \frac{2x^n\sqrt{x^nb+a}\,bn - 4x^n\sqrt{x^nb+a}\,b + 8\sqrt{x^nb+a}\,an - 4\sqrt{x^nb+a}\,a + 9\left(\int \frac{\sqrt{x^nb+a}}{3x^nbn^2x^2-8x^nbnx^2+4x^nbx^2+3an^2x^2\cdots}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^2,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n - 4*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b +
 a)*a*n - 4*sqrt(x**n*b + a)*a + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**
2 - 8*x**n*b*n*x**2 + 4*x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**
2),x)*a**2*n**4*x - 24*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**2 - 8*x**n*b
*n*x**2 + 4*x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**2),x)*a**2*n
**3*x + 12*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**2 - 8*x**n*b*n*x**2 + 4*
x**n*b*x**2 + 3*a*n**2*x**2 - 8*a*n*x**2 + 4*a*x**2),x)*a**2*n**2*x)/(x*(3
*n**2 - 8*n + 4))
```

3.335 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3} \, dx = -\frac{2B(a+bx^n)^{3/2}}{b(4-3n)x^2} - \frac{\left(A-\frac{4aB}{4b-3bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(3/2)/b/(4-3*n)/x^2-1/2*(A-4*a*B/(-3*b*n+4*b))*(a+b*x^n)^(1
/2)*hypergeom([-1/2, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3} \, dx = \frac{\sqrt{a+bx^n}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{-2}{}\right.\right.}{2(-2+n)x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/x^3,x]
```

output
```
(Sqrt[a + b*x^n]*(-(A*(-2 + n)*Hypergeometric2F1[-1/2, -2/n, (-2 + n)/n, -
((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[-1/2, (-2 + n)/n, 2 - 2/n, -((b*
x^n)/a)]))/(2*(-2 + n)*x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}dx$$

$$\Big\downarrow \text{ 959}$$

$$\left(A-\frac{4aB}{4b-3bn}\right)\int \frac{\sqrt{bx^n+a}}{x^3}dx-\frac{2B(a+bx^n)^{3/2}}{b(4-3n)x^2}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-3bn}\right)\int \frac{\sqrt{\frac{bx^n}{a}+1}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{b(4-3n)x^2}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-3bn}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{b(4-3n)x^2}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/x^3,x]
```

output
```
(-2*B*(a + b*x^n)^(3/2))/(b*(4 - 3*n)*x^2) - ((A - (4*a*B)/(4*b - 3*b*n))*
Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -2/n, -((2 - n)/n), -((b*x^n)/a)])
/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{x^{3}}dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx = \frac{Aa^{-\frac{2}{n}}a^{\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{Ba^{-\frac{1}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/x**3,x)
```

output
```
A*a**(1/2 + 2/n)*gamma(-2/n)*hyper((-1/2, -2/n), (1 - 2/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + B*a**(-1/2 + 2/n)*a**(1 -
2/n)*x**(n - 2)*gamma(1 - 2/n)*hyper((-1/2, 1 - 2/n), (2 - 2/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(2 - 2/n))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{x^3}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^3,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^3}\,dx$$
$$= \frac{2x^n\sqrt{x^nb+a}\,bn-8x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-8\sqrt{x^nb+a}\,a+9\left(\int \frac{\sqrt{x^nb+a}}{3x^nbn^2x^3-16x^nbnx^3+16x^nbx^3+3an^2x^3}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^3,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n - 8*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b +
 a)*a*n - 8*sqrt(x**n*b + a)*a + 9*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**
3 - 16*x**n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a
*x**3),x)*a**2*n**4*x**2 - 48*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**3 - 1
6*x**n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a*x**3
),x)*a**2*n**3*x**2 + 48*int(sqrt(x**n*b + a)/(3*x**n*b*n**2*x**3 - 16*x**
n*b*n*x**3 + 16*x**n*b*x**3 + 3*a*n**2*x**3 - 16*a*n*x**3 + 16*a*x**3),x)*
a**2*n**2*x**2)/(x**2*(3*n**2 - 16*n + 16))
```

3.336 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4} dx = -\frac{2B(a+bx^n)^{3/2}}{3b(2-n)x^3} - \frac{\left(A-\frac{2aB}{b(2-n)}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2/3*B*(a+b*x^n)^(3/2)/b/(2-n)/x^3-1/3*(A-2*a*B/b/(2-n))*(a+b*x^n)^(1/2)*h
ypergeom([-1/2, -3/n],[-(3-n)/n],-b*x^n/a)/x^3/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4} dx = \frac{\sqrt{a+bx^n}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{-3}{\ldots}\right.\right.}{3(-3+n)x^3\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/x^4,x]
```

output
```
(Sqrt[a + b*x^n]*(-(A*(-3 + n)*Hypergeometric2F1[-1/2, -3/n, (-3 + n)/n, -
((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[-1/2, (-3 + n)/n, 2 - 3/n, -((b*
x^n)/a)]))/(3*(-3 + n)*x^3*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{b(2-n)}\right)\int \frac{\sqrt{bx^n+a}}{x^4}dx-\frac{2B(a+bx^n)^{3/2}}{3b(2-n)x^3}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{b(2-n)}\right)\int \frac{\sqrt{\frac{bx^n}{a}+1}}{x^4}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{3b(2-n)x^3}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{a+bx^n}\left(A-\frac{2aB}{b(2-n)}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{3b(2-n)x^3}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/x^4,x]
```

output
```
(-2*B*(a + b*x^n)^(3/2))/(3*b*(2 - n)*x^3) - ((A - (2*a*B)/(b*(2 - n)))*Sq
rt[a + b*x^n]*Hypergeometric2F1[-1/2, -3/n, -((3 - n)/n), -((b*x^n)/a)])/(
3*x^3*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{x^{4}}dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx = \frac{Aa^{-\frac{3}{n}}a^{\frac{1}{2}+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{n}\\1-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)} + \frac{Ba^{-\frac{1}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}x^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/x**4,x)
```

output
```
A*a**(1/2 + 3/n)*gamma(-3/n)*hyper((-1/2, -3/n), (1 - 3/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + B*a**(-1/2 + 3/n)*a**(1 -
3/n)*x**(n - 3)*gamma(1 - 3/n)*hyper((-1/2, 1 - 3/n), (2 - 3/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(2 - 3/n))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{x^4}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^4,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{x^4}\,dx$$
$$= \frac{2x^n\sqrt{x^nb+a}\,bn-12x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-12\sqrt{x^nb+a}\,a+3\left(\int \frac{\sqrt{x^nb+a}}{x^nb\,n^2x^4-8x^nbn\,x^4+12x^nb\,x^4+a\,n^2x}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/x^4,x)
```

output
```
(2*x**n*sqrt(x**n*b + a)*b*n - 12*x**n*sqrt(x**n*b + a)*b + 8*sqrt(x**n*b
+ a)*a*n - 12*sqrt(x**n*b + a)*a + 3*int(sqrt(x**n*b + a)/(x**n*b*n**2*x**
4 - 8*x**n*b*n*x**4 + 12*x**n*b*x**4 + a*n**2*x**4 - 8*a*n*x**4 + 12*a*x**
4),x)*a**2*n**4*x**3 - 24*int(sqrt(x**n*b + a)/(x**n*b*n**2*x**4 - 8*x**n*
b*n*x**4 + 12*x**n*b*x**4 + a*n**2*x**4 - 8*a*n*x**4 + 12*a*x**4),x)*a**2*
n**3*x**3 + 36*int(sqrt(x**n*b + a)/(x**n*b*n**2*x**4 - 8*x**n*b*n*x**4 +
12*x**n*b*x**4 + a*n**2*x**4 - 8*a*n*x**4 + 12*a*x**4),x)*a**2*n**2*x**3)/
(3*x**3*(n**2 - 8*n + 12))
```

3.337 $\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{2Bx^3(a+bx^n)^{5/2}}{b(6+5n)}+\frac{a\left(A-\frac{6aB}{6b+5bn}\right)x^3\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x^3*(a+b*x^n)^(5/2)/b/(6+5*n)+1/3*a*(A-6*a*B/(5*b*n+6*b))*x^3*(a+b*x^n
)^(1/2)*hypergeom([-3/2, 3/n],[(3+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{ax^3\sqrt{a+bx^n}\left(A(3+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\right.}{3(3+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x^2*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(a*x^3*Sqrt[a + b*x^n]*(A*(3 + n)*Hypergeometric2F1[-3/2, 3/n, (3 + n)/n,
-((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[-3/2, (3 + n)/n, 2 + 3/n, -((b*x
^n)/a)]))/(3*(3 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{5bn+6b}\right)\int x^2(bx^n+a)^{3/2}dx+\frac{2Bx^3(a+bx^n)^{5/2}}{b(5n+6)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{6aB}{5bn+6b}\right)\int x^2\left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{5/2}}{b(5n+6)}$$

$$\Big\downarrow \text{888}$$

$$\frac{ax^3\sqrt{a+bx^n}\left(A-\frac{6aB}{5bn+6b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{5/2}}{b(5n+6)}$$

input
```
Int[x^2*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*x^3*(a + b*x^n)^(5/2))/(b*(6 + 5*n)) + (a*(A - (6*a*B)/(6*b + 5*b*n))
*x^3*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 3/n, (3 + n)/n, -((b*x^n)/a)]
)/(3*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^2(a+b\,x^n)^{\frac{3}{2}}\,(A+B\,x^n)\,dx$$

input
```
int(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+bx^n)^{3/2}\,(A+Bx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.51 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{Aaa^{\frac{3}{n}}a^{\frac{1}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{Aa^{-\frac{1}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}bx^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{Baa^{-\frac{1}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{3}{n}}a^{2+\frac{3}{n}}bx^{2n+3}\Gamma\left(2+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{3}{n}\\3+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
A*a*a**(3/n)*a**(1/2 - 3/n)*x**3*gamma(3/n)*hyper((-1/2, 3/n), (1 + 3/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + A*a**(-1/2 - 3/n)*a**(1 +
3/n)*b*x**(n + 3)*gamma(1 + 3/n)*hyper((-1/2, 1 + 3/n), (2 + 3/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(2 + 3/n)) + B*a*a**(-1/2 - 3/n)*a**(1 + 3/n)*
x**(n + 3)*gamma(1 + 3/n)*hyper((-1/2, 1 + 3/n), (2 + 3/n,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 + 3/n)) + B*a**(-3/2 - 3/n)*a**(2 + 3/n)*b*x**(2*n
 + 3)*gamma(2 + 3/n)*hyper((-1/2, 2 + 3/n), (3 + 3/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(3 + 3/n))
```

Maxima [F]

$$\int x^2(a+bx^n)^{3/2}\,(A+Bx^n)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^n)^{3/2}\,(A+Bx^n)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^n)^{3/2}\,(A+Bx^n)\,dx=\int x^2\,(A+B\,x^n)\,\left(a+b\,x^n\right)^{3/2}\,dx$$

input
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2x^3+48x^{2n}\sqrt{x^nb+a}\,b^2n\,x^3+72x^{2n}\sqrt{x^nb+a}\,b^2x^3+22x^n\sqrt{x^nb+a}\,ab\,n^2x}{}$$

input
```
int(x^2*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2*x**3 + 48*x**(2*n)*sqrt(x**n*b + a)
*b**2*n*x**3 + 72*x**(2*n)*sqrt(x**n*b + a)*b**2*x**3 + 22*x**n*sqrt(x**n*
b + a)*a*b*n**2*x**3 + 156*x**n*sqrt(x**n*b + a)*a*b*n*x**3 + 144*x**n*sqr
t(x**n*b + a)*a*b*x**3 + 46*sqrt(x**n*b + a)*a**2*n**2*x**3 + 108*sqrt(x**
n*b + a)*a**2*n*x**3 + 72*sqrt(x**n*b + a)*a**2*x**3 + 75*int((sqrt(x**n*b
 + a)*x**2)/(5*x**n*b*n**3 + 46*x**n*b*n**2 + 108*x**n*b*n + 72*x**n*b + 5
*a*n**3 + 46*a*n**2 + 108*a*n + 72*a),x)*a**3*n**6 + 690*int((sqrt(x**n*b
+ a)*x**2)/(5*x**n*b*n**3 + 46*x**n*b*n**2 + 108*x**n*b*n + 72*x**n*b + 5*
a*n**3 + 46*a*n**2 + 108*a*n + 72*a),x)*a**3*n**5 + 1620*int((sqrt(x**n*b
+ a)*x**2)/(5*x**n*b*n**3 + 46*x**n*b*n**2 + 108*x**n*b*n + 72*x**n*b + 5*
a*n**3 + 46*a*n**2 + 108*a*n + 72*a),x)*a**3*n**4 + 1080*int((sqrt(x**n*b
+ a)*x**2)/(5*x**n*b*n**3 + 46*x**n*b*n**2 + 108*x**n*b*n + 72*x**n*b + 5*
a*n**3 + 46*a*n**2 + 108*a*n + 72*a),x)*a**3*n**3)/(3*(5*n**3 + 46*n**2 +
108*n + 72))
```

3.338 $\int x(a+bx^n)^{3/2}(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 102

$$\int x(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{2Bx^2(a+bx^n)^{5/2}}{b(4+5n)} + \frac{a\left(A-\frac{4aB}{4b+5bn}\right)x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x^2*(a+b*x^n)^(5/2)/b/(4+5*n)+1/2*a*(A-4*a*B/(5*b*n+4*b))*x^2*(a+b*x^n
)^(1/2)*hypergeom([-3/2, 2/n],[(2+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int x(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{ax^2\sqrt{a+bx^n}\left(A(2+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\right.}{2(2+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(a*x^2*Sqrt[a + b*x^n]*(A*(2 + n)*Hypergeometric2F1[-3/2, 2/n, (2 + n)/n,
-((b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[-3/2, (2 + n)/n, 2*(1 + n^(-1)),
 -((b*x^n)/a)]))/(2*(2 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^{3/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{5bn+4b}\right)\int x(bx^n+a)^{3/2}dx+\frac{2Bx^2(a+bx^n)^{5/2}}{b(5n+4)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{4aB}{5bn+4b}\right)\int x\left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{5/2}}{b(5n+4)}$$

$$\Big\downarrow \text{888}$$

$$\frac{ax^2\sqrt{a+bx^n}\left(A-\frac{4aB}{5bn+4b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{5/2}}{b(5n+4)}$$

input
```
Int[x*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*x^2*(a + b*x^n)^(5/2))/(b*(4 + 5*n)) + (a*(A - (4*a*B)/(4*b + 5*b*n))
*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 2/n, (2 + n)/n, -((b*x^n)/a)]
)/(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x(a+b\,x^{n})^{\frac{3}{2}}\left(A+B\,x^{n}\right)dx$$

input
```
int(x*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int(x*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.45

$$\int x(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\frac{Aaa^{\frac{2}{n}}a^{\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}+\frac{Aa^{-\frac{1}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}bx^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}+\frac{Baa^{-\frac{1}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}+\frac{Ba^{-\frac{3}{2}-\frac{2}{n}}a^{2+\frac{2}{n}}bx^{2n+2}\Gamma\left(2+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{2}{n}\\3+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
A*a*a**(2/n)*a**(1/2 - 2/n)*x**2*gamma(2/n)*hyper((-1/2, 2/n), (1 + 2/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + A*a**(-1/2 - 2/n)*a**(1 +
2/n)*b*x**(n + 2)*gamma(1 + 2/n)*hyper((-1/2, 1 + 2/n), (2 + 2/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(2 + 2/n)) + B*a*a**(-1/2 - 2/n)*a**(1 + 2/n)*
x**(n + 2)*gamma(1 + 2/n)*hyper((-1/2, 1 + 2/n), (2 + 2/n,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 + 2/n)) + B*a**(-3/2 - 2/n)*a**(2 + 2/n)*b*x**(2*n
 + 2)*gamma(2 + 2/n)*hyper((-1/2, 2 + 2/n), (3 + 2/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(3 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\int x\left(A+B\,x^n\right)\,\left(a+b\,x^n\right)^{3/2}\,dx$$

input
```
int(x*(A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int(x*(A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int x(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2x^2+32x^{2n}\sqrt{x^nb+a}\,b^2n\,x^2+32x^{2n}\sqrt{x^nb+a}\,b^2x^2+22x^n\sqrt{x^nb+a}\,ab\,n^2x}{}$$

input
```
int(x*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2*x**2 + 32*x**(2*n)*sqrt(x**n*b + a)
*b**2*n*x**2 + 32*x**(2*n)*sqrt(x**n*b + a)*b**2*x**2 + 22*x**n*sqrt(x**n*
b + a)*a*b*n**2*x**2 + 104*x**n*sqrt(x**n*b + a)*a*b*n*x**2 + 64*x**n*sqrt
(x**n*b + a)*a*b*x**2 + 46*sqrt(x**n*b + a)*a**2*n**2*x**2 + 72*sqrt(x**n*
b + a)*a**2*n*x**2 + 32*sqrt(x**n*b + a)*a**2*x**2 + 225*int((sqrt(x**n*b
+ a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a
*n**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**6 + 1380*int((sqrt(x**n*b +
 a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*
n**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**5 + 2160*int((sqrt(x**n*b +
a)*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*n
**3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**4 + 960*int((sqrt(x**n*b + a)
*x)/(15*x**n*b*n**3 + 92*x**n*b*n**2 + 144*x**n*b*n + 64*x**n*b + 15*a*n**
3 + 92*a*n**2 + 144*a*n + 64*a),x)*a**3*n**3)/(15*n**3 + 92*n**2 + 144*n +
 64)
```

3.339 $\int (a+bx^n)^{3/2} (A+Bx^n)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{2Bx(a+bx^n)^{5/2}}{b(2+5n)}+\frac{a\left(A-\frac{2aB}{2b+5bn}\right)x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x*(a+b*x^n)^(5/2)/b/(2+5*n)+a*(A-2*a*B/(5*b*n+2*b))*x*(a+b*x^n)^(1/2)*
hypergeom([-3/2, 1/n],[1+1/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{x\sqrt{a+bx^n}\left(2B(a+bx^n)^2+\frac{a(-2aB+Ab(2+5n))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}\right)}{b(2+5n)}$$

input
```
Integrate[(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(x*Sqrt[a + b*x^n]*(2*B*(a + b*x^n)^2 + (a*(-2*a*B + A*b*(2 + 5*n))*Hyperg
eometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/Sqrt[1 + (b*x^n)/a]))
/(b*(2 + 5*n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(A-\frac{2aB}{5bn+2b}\right)\int (bx^n+a)^{3/2}dx+\frac{2Bx(a+bx^n)^{5/2}}{b(5n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{2aB}{5bn+2b}\right)\int \left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{5/2}}{b(5n+2)}$$

$$\Big\downarrow \text{778}$$

$$\frac{ax\sqrt{a+bx^n}\left(A-\frac{2aB}{5bn+2b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{5/2}}{b(5n+2)}$$

input
```
Int[(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*x*(a + b*x^n)^(5/2))/(b*(2 + 5*n)) + (a*(A - (2*a*B)/(2*b + 5*b*n))*x
*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)]
)/Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{3}{2}} (A+B\,x^{n})\,dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx^n)^{3/2} (A+Bx^n)\ dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.73

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{Aaa^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Aa^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}bx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{Baa^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{1}{n}}a^{2+\frac{1}{n}}bx^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
A*a*a**(1/n)*a**(1/2 - 1/n)*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,), b*
x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + A*a**(-1/2 - 1/n)*a**(1 + 1/n
)*b*x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + B*a*a**(-1/2 - 1/n)*a**(1 + 1/n)*x**
(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(2 + 1/n)) + B*a**(-3/2 - 1/n)*a**(2 + 1/n)*b*x**(2*n +
1)*gamma(2 + 1/n)*hyper((-1/2, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*pi
)/a)/(n*gamma(3 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (A+B\,x^n)\,(a+b\,x^n)^{3/2}\,dx$$

input
```
int((A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int((A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int (a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2x+16x^{2n}\sqrt{x^nb+a}\,b^2nx+8x^{2n}\sqrt{x^nb+a}\,b^2x+22x^n\sqrt{x^nb+a}\,ab\,n^2x+52}{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2*x + 16*x**(2*n)*sqrt(x**n*b + a)*b*
*2*n*x + 8*x**(2*n)*sqrt(x**n*b + a)*b**2*x + 22*x**n*sqrt(x**n*b + a)*a*b
*n**2*x + 52*x**n*sqrt(x**n*b + a)*a*b*n*x + 16*x**n*sqrt(x**n*b + a)*a*b*
x + 46*sqrt(x**n*b + a)*a**2*n**2*x + 36*sqrt(x**n*b + a)*a**2*n*x + 8*sqr
t(x**n*b + a)*a**2*x + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*
b*n**2 + 36*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)
*a**3*n**6 + 690*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 3
6*x**n*b*n + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**5
 + 540*int(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n
 + 8*x**n*b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**4 + 120*int
(sqrt(x**n*b + a)/(15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*
b + 15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*n**3)/(15*n**3 + 46*n**2
 + 36*n + 8)
```

3.340 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x} \, dx = \frac{2aA\sqrt{a+bx^n}}{n} + \frac{2A(a+bx^n)^{3/2}}{3n} + \frac{2B(a+bx^n)^{5/2}}{5bn} - \frac{2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a*A*(a+b*x^n)^(1/2)/n+2/3*A*(a+b*x^n)^(3/2)/n+2/5*B*(a+b*x^n)^(5/2)/b/n-
2*a^(3/2)*A*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x} \, dx = \frac{2\left(15aAb\sqrt{a+bx^n} + 5Ab(a+bx^n)^{3/2} + 3B(a+bx^n)^{5/2} - 15a^{3/2}Abarctan\right.}{15bn}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/x,x]
```

output
```
(2*(15*a*A*b*Sqrt[a + b*x^n] + 5*A*b*(a + b*x^n)^(3/2) + 3*B*(a + b*x^n)^(
5/2) - 15*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]]))/(15*b*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {948, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{-n}(bx^n+a)^{3/2}(Bx^n+A)\,dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{A\int x^{-n}(bx^n+a)^{3/2}dx^n+\frac{2B(a+bx^n)^{5/2}}{5b}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{A\left(a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2B(a+bx^n)^{5/2}}{5b}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{A\left(a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2B(a+bx^n)^{5/2}}{5b}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{A\left(a\left(\frac{2a\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2B(a+bx^n)^{5/2}}{5b}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{A\left(a\left(2\sqrt{a+bx^n}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2B(a+bx^n)^{5/2}}{5b}}{n}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/x,x]
```

output
```
((2*B*(a + b*x^n)^(5/2))/(5*b) + A*((2*(a + b*x^n)^(3/2))/3 + a*(2*Sqrt[a
+ b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{2B(a+b\,x^{n})^{\frac{5}{2}}}{5}+\frac{2Ab(a+b\,x^{n})^{\frac{3}{2}}}{3}+2Aab\sqrt{a+b\,x^{n}}-2A\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{bn}$
default	$\frac{\frac{2B(a+b\,x^{n})^{\frac{5}{2}}}{5}+\frac{2Ab(a+b\,x^{n})^{\frac{3}{2}}}{3}+2Aab\sqrt{a+b\,x^{n}}-2A\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{bn}$
risch	$\frac{2(3b^{2}B\,{\mathrm e}^{2n\ln(x)}+5A\,b^{2}{\mathrm e}^{n\ln(x)}+6Bab\,{\mathrm e}^{n\ln(x)}+20abA+3a^{2}B)\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{15bn}-\frac{2a^{\frac{3}{2}}A\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln(x)}}}{\sqrt{a}}\right)}{n}$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(1/5*B*(a+b*x^n)^(5/2)+1/3*A*b*(a+b*x^n)^(3/2)+A*a*b*(a+b*x^n)^(1/2)
-A*a^(3/2)*b*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\,(3\,Bb^2x^{2\,n}+3\,Ba^2+20\,Aab+(6\,B}{15\,bn}\right.$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*A*a^(3/2)*b*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) +
 2*(3*B*b^2*x^(2*n) + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^n)*sqrt(b
*x^n + a))/(b*n), 2/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^n + a))
 + (3*B*b^2*x^(2*n) + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^n)*sqrt(b
*x^n + a))/(b*n)]
```

Sympy [A] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa\sqrt{a+bx^n} + \frac{2A(a+bx^n)^{\frac{3}{2}}}{3} + \frac{2B(a+bx^n)^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ Aa^{\frac{3}{2}}\log\left(Ba^{\frac{3}{2}}x^n\right) + Ba^{\frac{3}{2}}x^n & \text{otherwise} \end{cases}}{n} \\ \left(Aa\sqrt{a+b} + Ab\sqrt{a+b} + Ba\sqrt{a+b} + Bb\sqrt{a+b}\right)\log(x) \end{cases}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/x,x)
```

output
```
Piecewise((Piecewise((2*A*a**2*atan(sqrt(a + b*x**n)/sqrt(-a))/sqrt(-a) +
2*A*a*sqrt(a + b*x**n) + 2*A*(a + b*x**n)**(3/2)/3 + 2*B*(a + b*x**n)**(5/
2)/(5*b), Ne(b, 0)), (A*a**(3/2)*log(B*a**(3/2)*x**n) + B*a**(3/2)*x**n, T
rue))/n, Ne(n, 0)), ((A*a*sqrt(a + b) + A*b*sqrt(a + b) + B*a*sqrt(a + b)
+ B*b*sqrt(a + b))*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \frac{1}{3}\left(\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left((bx^n+a)^{\frac{3}{2}} + 3\sqrt{bx^n+a}a\right)}{n}\right)A + \frac{2\,(bx^n+a)^{\frac{5}{2}}B}{5\,bn}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x,x, algorithm="maxima")
```

output
```
1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a))
)/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)*A + 2/5*(b*x^n + a)^(
5/2)*B/(b*n)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{x}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x}\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2+22x^n\sqrt{x^nb+a}\,ab+46\sqrt{x^nb+a}\,a^2+15\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)}{15n}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x,x)
```

output

```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b + 46*sqrt
(x**n*b + a)*a**2 + 15*int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a**3*n)/(1
5*n)
```

3.341 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = -\frac{2B(a+bx^n)^{5/2}}{b(2-5n)x} - \frac{a\left(A-\frac{2aB}{2b-5bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(5/2)/b/(2-5*n)/x-a*(A-2*a*B/(-5*b*n+2*b))*(a+b*x^n)^(1/2)*
hypergeom([-3/2, -1/n],[-(1-n)/n],-b*x^n/a)/x/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \frac{a\sqrt{a+bx^n}\left((A-An)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{H}\right.}{(-1+n)x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/x^2,x]
```

output
```
(a*Sqrt[a + b*x^n]*((A - A*n)*Hypergeometric2F1[-3/2, -n^(-1), (-1 + n)/n,
 -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[-3/2, (-1 + n)/n, 2 - n^(-1), -((
b*x^n)/a)]))/((-1 + n)*x*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{2b-5bn}\right)\int \frac{(bx^n+a)^{3/2}}{x^2}dx-\frac{2B(a+bx^n)^{5/2}}{b(2-5n)x}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-5bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(2-5n)x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-5bn}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(2-5n)x}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/x^2,x]
```

output
```
(-2*B*(a + b*x^n)^(5/2))/(b*(2 - 5*n)*x) - (a*(A - (2*a*B)/(2*b - 5*b*n))*
Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -n^(-1), -((1 - n)/n), -((b*x^n)/a
)])/(x*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{3}{2}}\,(A+B\,x^{n})}{x^{2}}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.39

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \frac{Aaa^{-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)} + \frac{Aa^{-\frac{1}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}bx^{n-1}\Gamma\left(1-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)} + \frac{Baa^{-\frac{1}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}x^{n-1}\Gamma\left(1-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{1}{n}}a^{2-\frac{1}{n}}bx^{2n-1}\Gamma\left(2-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{1}{n}\\3-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/x**2,x)
```

output
```
A*a*a**(1/2 + 1/n)*gamma(-1/n)*hyper((-1/2, -1/n), (1 - 1/n,), b*x**n*exp_
polar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + A*a**(-1/2 + 1/n)*a**(1 - 1
/n)*b*x**(n - 1)*gamma(1 - 1/n)*hyper((-1/2, 1 - 1/n), (2 - 1/n,), b*x**n*
exp_polar(I*pi)/a)/(n*gamma(2 - 1/n)) + B*a*a**(-1/2 + 1/n)*a**(1 - 1/n)*x
**(n - 1)*gamma(1 - 1/n)*hyper((-1/2, 1 - 1/n), (2 - 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 - 1/n)) + B*a**(-3/2 + 1/n)*a**(2 - 1/n)*b*x**(2*n
- 1)*gamma(2 - 1/n)*hyper((-1/2, 2 - 1/n), (3 - 1/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(3 - 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{x^2}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^2,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^2}\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2-16x^{2n}\sqrt{x^nb+a}\,b^2n+8x^{2n}\sqrt{x^nb+a}\,b^2+22x^n\sqrt{x^nb}}{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^2,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 16*x**(2*n)*sqrt(x**n*b + a)*b**2
*n + 8*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b*n**2
- 52*x**n*sqrt(x**n*b + a)*a*b*n + 16*x**n*sqrt(x**n*b + a)*a*b + 46*sqrt(
x**n*b + a)*a**2*n**2 - 36*sqrt(x**n*b + a)*a**2*n + 8*sqrt(x**n*b + a)*a*
*2 + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 - 46*x**n*b*n**2*x**2 +
 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 - 46*a*n**2*x**2 + 36*a
*n*x**2 - 8*a*x**2),x)*a**3*n**6*x - 690*int(sqrt(x**n*b + a)/(15*x**n*b*n
**3*x**2 - 46*x**n*b*n**2*x**2 + 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n
**3*x**2 - 46*a*n**2*x**2 + 36*a*n*x**2 - 8*a*x**2),x)*a**3*n**5*x + 540*i
nt(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 - 46*x**n*b*n**2*x**2 + 36*x**n*b
*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 - 46*a*n**2*x**2 + 36*a*n*x**2 -
8*a*x**2),x)*a**3*n**4*x - 120*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**2 -
 46*x**n*b*n**2*x**2 + 36*x**n*b*n*x**2 - 8*x**n*b*x**2 + 15*a*n**3*x**2 -
 46*a*n**2*x**2 + 36*a*n*x**2 - 8*a*x**2),x)*a**3*n**3*x)/(x*(15*n**3 - 46
*n**2 + 36*n - 8))
```

3.342 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = -\frac{2B(a+bx^n)^{5/2}}{b(4-5n)x^2} - \frac{a\left(A-\frac{4aB}{4b-5bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(5/2)/b/(4-5*n)/x^2-1/2*a*(A-4*a*B/(-5*b*n+4*b))*(a+b*x^n)^
(1/2)*hypergeom([-3/2, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \frac{a\sqrt{a+bx^n}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2B\right.}{2(-2+n)x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/x^3,x]
```

output
```
(a*Sqrt[a + b*x^n]*(-(A*(-2 + n)*Hypergeometric2F1[-3/2, -2/n, (-2 + n)/n,
 -((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[-3/2, (-2 + n)/n, 2 - 2/n, -((
b*x^n)/a)]))/(2*(-2 + n)*x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{4b-5bn}\right)\int \frac{(bx^n+a)^{3/2}}{x^3}dx-\frac{2B(a+bx^n)^{5/2}}{b(4-5n)x^2}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-5bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(4-5n)x^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-5bn}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(4-5n)x^2}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/x^3,x]
```

output
```
(-2*B*(a + b*x^n)^(5/2))/(b*(4 - 5*n)*x^2) - (a*(A - (4*a*B)/(4*b - 5*b*n)
)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -2/n, -((2 - n)/n), -((b*x^n)/a)
])/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{3}{2}}\,(A+B\,x^n)}{x^3}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \frac{Aaa^{-\frac{2}{n}}a^{\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{Aa^{-\frac{1}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}bx^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)} + \frac{Baa^{-\frac{1}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{2}{n}}a^{2-\frac{2}{n}}bx^{2n-2}\Gamma\left(2-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},2-\frac{2}{n}\\3-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/x**3,x)
```

output
```
A*a*a**(1/2 + 2/n)*gamma(-2/n)*hyper((-1/2, -2/n), (1 - 2/n,), b*x**n*exp_
polar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + A*a**(-1/2 + 2/n)*a**(1
- 2/n)*b*x**(n - 2)*gamma(1 - 2/n)*hyper((-1/2, 1 - 2/n), (2 - 2/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(2 - 2/n)) + B*a*a**(-1/2 + 2/n)*a**(1 - 2/n
)*x**(n - 2)*gamma(1 - 2/n)*hyper((-1/2, 1 - 2/n), (2 - 2/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 - 2/n)) + B*a**(-3/2 + 2/n)*a**(2 - 2/n)*b*x**(2
*n - 2)*gamma(2 - 2/n)*hyper((-1/2, 2 - 2/n), (3 - 2/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(3 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{x^3}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^3,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^3}\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2 - 32x^{2n}\sqrt{x^nb+a}\,b^2n + 32x^{2n}\sqrt{x^nb+a}\,b^2 + 22x^n\sqrt{x^n}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^3,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 32*x**(2*n)*sqrt(x**n*b + a)*b**2
*n + 32*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b*n**2
 - 104*x**n*sqrt(x**n*b + a)*a*b*n + 64*x**n*sqrt(x**n*b + a)*a*b + 46*sqr
t(x**n*b + a)*a**2*n**2 - 72*sqrt(x**n*b + a)*a**2*n + 32*sqrt(x**n*b + a)
*a**2 + 225*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**
3 + 144*x**n*b*n*x**3 - 64*x**n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 +
 144*a*n*x**3 - 64*a*x**3),x)*a**3*n**6*x**2 - 1380*int(sqrt(x**n*b + a)/(
15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**3 + 144*x**n*b*n*x**3 - 64*x**n*b*
x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 + 144*a*n*x**3 - 64*a*x**3),x)*a**3
*n**5*x**2 + 2160*int(sqrt(x**n*b + a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n*
*2*x**3 + 144*x**n*b*n*x**3 - 64*x**n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*
x**3 + 144*a*n*x**3 - 64*a*x**3),x)*a**3*n**4*x**2 - 960*int(sqrt(x**n*b +
 a)/(15*x**n*b*n**3*x**3 - 92*x**n*b*n**2*x**3 + 144*x**n*b*n*x**3 - 64*x*
*n*b*x**3 + 15*a*n**3*x**3 - 92*a*n**2*x**3 + 144*a*n*x**3 - 64*a*x**3),x)
*a**3*n**3*x**2)/(x**2*(15*n**3 - 92*n**2 + 144*n - 64))
```

3.343 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4} dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4} dx = -\frac{2B(a+bx^n)^{5/2}}{b(6-5n)x^3} - \frac{a\left(A-\frac{6aB}{6b-5bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(5/2)/b/(6-5*n)/x^3-1/3*a*(A-6*a*B/(-5*b*n+6*b))*(a+b*x^n)^
(1/2)*hypergeom([-3/2, -3/n],[-(3-n)/n],-b*x^n/a)/x^3/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4} dx = \frac{a\sqrt{a+bx^n}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3B\right.}{3(-3+n)x^3\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/x^4,x]
```

output
```
(a*Sqrt[a + b*x^n]*(-(A*(-3 + n)*Hypergeometric2F1[-3/2, -3/n, (-3 + n)/n,
 -((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[-3/2, (-3 + n)/n, 2 - 3/n, -((
b*x^n)/a)]))/(3*(-3 + n)*x^3*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{6b-5bn}\right)\int \frac{(bx^n+a)^{3/2}}{x^4}dx-\frac{2B(a+bx^n)^{5/2}}{b(6-5n)x^3}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{6aB}{6b-5bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{x^4}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(6-5n)x^3}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a\sqrt{a+bx^n}\left(A-\frac{6aB}{6b-5bn}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{b(6-5n)x^3}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/x^4,x]
```

output
```
(-2*B*(a + b*x^n)^(5/2))/(b*(6 - 5*n)*x^3) - (a*(A - (6*a*B)/(6*b - 5*b*n)
)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -3/n, -((3 - n)/n), -((b*x^n)/a)
])/(3*x^3*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{3}{2}}\,(A+B\,x^n)}{x^4}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \frac{Aaa^{-\frac{3}{n}}a^{\frac{1}{2}+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{n}\\1-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)} + \frac{Aa^{-\frac{1}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}bx^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)} + \frac{Baa^{-\frac{1}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}x^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{3}{n}}a^{2-\frac{3}{n}}bx^{2n-3}\Gamma\left(2-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{3}{n}\\3-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{3}{n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/x**4,x)
```

output
```
A*a*a**(1/2 + 3/n)*gamma(-3/n)*hyper((-1/2, -3/n), (1 - 3/n,), b*x**n*exp_
polar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + A*a**(-1/2 + 3/n)*a**(1
- 3/n)*b*x**(n - 3)*gamma(1 - 3/n)*hyper((-1/2, 1 - 3/n), (2 - 3/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(2 - 3/n)) + B*a*a**(-1/2 + 3/n)*a**(1 - 3/n
)*x**(n - 3)*gamma(1 - 3/n)*hyper((-1/2, 1 - 3/n), (2 - 3/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 - 3/n)) + B*a**(-3/2 + 3/n)*a**(2 - 3/n)*b*x**(2
*n - 3)*gamma(2 - 3/n)*hyper((-1/2, 2 - 3/n), (3 - 3/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(3 - 3/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^4,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{x^4}\,dx = \frac{6x^{2n}\sqrt{x^nb+a}\,b^2n^2 - 48x^{2n}\sqrt{x^nb+a}\,b^2n + 72x^{2n}\sqrt{x^nb+a}\,b^2 + 22x^n\sqrt{x^n}}{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/x^4,x)
```

output
```
(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 48*x**(2*n)*sqrt(x**n*b + a)*b**2
*n + 72*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a*b*n**2
 - 156*x**n*sqrt(x**n*b + a)*a*b*n + 144*x**n*sqrt(x**n*b + a)*a*b + 46*sq
rt(x**n*b + a)*a**2*n**2 - 108*sqrt(x**n*b + a)*a**2*n + 72*sqrt(x**n*b +
a)*a**2 + 75*int(sqrt(x**n*b + a)/(5*x**n*b*n**3*x**4 - 46*x**n*b*n**2*x**
4 + 108*x**n*b*n*x**4 - 72*x**n*b*x**4 + 5*a*n**3*x**4 - 46*a*n**2*x**4 +
108*a*n*x**4 - 72*a*x**4),x)*a**3*n**6*x**3 - 690*int(sqrt(x**n*b + a)/(5*
x**n*b*n**3*x**4 - 46*x**n*b*n**2*x**4 + 108*x**n*b*n*x**4 - 72*x**n*b*x**
4 + 5*a*n**3*x**4 - 46*a*n**2*x**4 + 108*a*n*x**4 - 72*a*x**4),x)*a**3*n**
5*x**3 + 1620*int(sqrt(x**n*b + a)/(5*x**n*b*n**3*x**4 - 46*x**n*b*n**2*x*
*4 + 108*x**n*b*n*x**4 - 72*x**n*b*x**4 + 5*a*n**3*x**4 - 46*a*n**2*x**4 +
 108*a*n*x**4 - 72*a*x**4),x)*a**3*n**4*x**3 - 1080*int(sqrt(x**n*b + a)/(
5*x**n*b*n**3*x**4 - 46*x**n*b*n**2*x**4 + 108*x**n*b*n*x**4 - 72*x**n*b*x
**4 + 5*a*n**3*x**4 - 46*a*n**2*x**4 + 108*a*n*x**4 - 72*a*x**4),x)*a**3*n
**3*x**3)/(3*x**3*(5*n**3 - 46*n**2 + 108*n - 72))
```

3.344 $\int x^2(a+bx^n)^{5/2}\,(A+Bx^n)\;dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int x^2(a+bx^n)^{5/2}\,(A+Bx^n)\;dx=\frac{2Bx^3(a+bx^n)^{7/2}}{b(6+7n)}+\frac{a^2\left(A-\frac{6aB}{6b+7bn}\right)x^3\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x^3*(a+b*x^n)^(7/2)/b/(6+7*n)+1/3*a^2*(A-6*a*B/(7*b*n+6*b))*x^3*(a+b*x
^n)^(1/2)*hypergeom([-5/2, 3/n],[(3+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^n)^{5/2}\,(A+Bx^n)\;dx=\frac{a^2x^3\sqrt{a+bx^n}\left(A(3+n)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\right.}{3(3+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x^2*(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(a^2*x^3*Sqrt[a + b*x^n]*(A*(3 + n)*Hypergeometric2F1[-5/2, 3/n, (3 + n)/n
, -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[-5/2, (3 + n)/n, 2 + 3/n, -((b
*x^n)/a)]))/(3*(3 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)^{5/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{7bn+6b}\right)\int x^2(bx^n+a)^{5/2}\,dx+\frac{2Bx^3(a+bx^n)^{7/2}}{b(7n+6)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{6aB}{7bn+6b}\right)\int x^2\left(\frac{bx^n}{a}+1\right)^{5/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{7/2}}{b(7n+6)}$$

$$\Big\downarrow \text{888}$$

$$\frac{a^2x^3\sqrt{a+bx^n}\left(A-\frac{6aB}{7bn+6b}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^3(a+bx^n)^{7/2}}{b(7n+6)}$$

input
```
Int[x^2*(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(2*B*x^3*(a + b*x^n)^(7/2))/(b*(6 + 7*n)) + (a^2*(A - (6*a*B)/(6*b + 7*b*n
))*x^3*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 3/n, (3 + n)/n, -((b*x^n)/a
)])/(3*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^2(a+b\,x^n)^{\frac{5}{2}}\,(A+B\,x^n)\,dx$$

input
```
int(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output
```
int(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+bx^n)^{5/2}\,(A+Bx^n)\;dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.91 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.81

$$\int x^2(a+bx^n)^{5/2}(A+Bx^n)\,dx = \frac{Aa^2a^{\frac{3}{n}}a^{\frac{1}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{2Aaa^{-\frac{1}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}bx^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{Aa^{-\frac{3}{2}-\frac{3}{n}}a^{2+\frac{3}{n}}b^2x^{2n+3}\Gamma\left(2+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{3}{n}\\3+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{3}{n}\right)} + \frac{Ba^2a^{-\frac{1}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)} + \frac{2Baa^{-\frac{3}{2}-\frac{3}{n}}a^{2+\frac{3}{n}}bx^{2n+3}\Gamma\left(2+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{3}{n}\\3+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{3}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{3}{n}}a^{3+\frac{3}{n}}b^2x^{3n+3}\Gamma\left(3+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3+\frac{3}{n}\\4+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(a+b*x**n)**(5/2)*(A+B*x**n),x)
```

output
```
A*a**2*a**(3/n)*a**(1/2 - 3/n)*x**3*gamma(3/n)*hyper((-1/2, 3/n), (1 + 3/n
,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + 2*A*a*a**(-1/2 - 3/n)*a
**(1 + 3/n)*b*x**(n + 3)*gamma(1 + 3/n)*hyper((-1/2, 1 + 3/n), (2 + 3/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 3/n)) + A*a**(-3/2 - 3/n)*a**(2 +
3/n)*b**2*x**(2*n + 3)*gamma(2 + 3/n)*hyper((-1/2, 2 + 3/n), (3 + 3/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 3/n)) + B*a**2*a**(-1/2 - 3/n)*a**(1
 + 3/n)*x**(n + 3)*gamma(1 + 3/n)*hyper((-1/2, 1 + 3/n), (2 + 3/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(2 + 3/n)) + 2*B*a*a**(-3/2 - 3/n)*a**(2 + 3/
n)*b*x**(2*n + 3)*gamma(2 + 3/n)*hyper((-1/2, 2 + 3/n), (3 + 3/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(3 + 3/n)) + B*a**(-5/2 - 3/n)*a**(3 + 3/n)*b*
*2*x**(3*n + 3)*gamma(3 + 3/n)*hyper((-1/2, 3 + 3/n), (4 + 3/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(4 + 3/n))
```

Maxima [F]

$$\int x^2(a+bx^n)^{5/2}\left(A+Bx^n\right)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^n)^{5/2}\left(A+Bx^n\right)\,dx=\int (Bx^n+A)(bx^n+a)^{\frac{5}{2}}x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^n)^{5/2}\left(A+Bx^n\right)\,dx=\int x^2\,\left(A+B\,x^n\right)\,{\left(a+b\,x^n\right)}^{5/2}\,dx$$

input
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(5/2),x)
```

output
```
int(x^2*(A + B*x^n)*(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int x^2(a+bx^n)^{5/2}\left(A+Bx^n\right)\,dx=\frac{30x^{3n}\sqrt{x^nb+a}\,b^3n^3x^3+276x^{3n}\sqrt{x^nb+a}\,b^3n^2x^3+648x^{3n}\sqrt{x^nb+a}\,b^3n\,x^3+432x^{3n}\sqrt{x^nb+}}{}$$

input
```
int(x^2*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output

```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3*x**3 + 276*x**(3*n)*sqrt(x**n*b +
a)*b**3*n**2*x**3 + 648*x**(3*n)*sqrt(x**n*b + a)*b**3*n*x**3 + 432*x**(3*
n)*sqrt(x**n*b + a)*b**3*x**3 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3*
x**3 + 1164*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**2*x**3 + 2448*x**(2*n)*sqr
t(x**n*b + a)*a*b**2*n*x**3 + 1296*x**(2*n)*sqrt(x**n*b + a)*a*b**2*x**3 +
 244*x**n*sqrt(x**n*b + a)*a**2*b*n**3*x**3 + 1920*x**n*sqrt(x**n*b + a)*a
**2*b*n**2*x**3 + 2952*x**n*sqrt(x**n*b + a)*a**2*b*n*x**3 + 1296*x**n*sqr
t(x**n*b + a)*a**2*b*x**3 + 352*sqrt(x**n*b + a)*a**3*n**3*x**3 + 1032*sqr
t(x**n*b + a)*a**3*n**2*x**3 + 1152*sqrt(x**n*b + a)*a**3*n*x**3 + 432*sqr
t(x**n*b + a)*a**3*x**3 + 3675*int((sqrt(x**n*b + a)*x**2)/(35*x**n*b*n**4
 + 352*x**n*b*n**3 + 1032*x**n*b*n**2 + 1152*x**n*b*n + 432*x**n*b + 35*a*
n**4 + 352*a*n**3 + 1032*a*n**2 + 1152*a*n + 432*a),x)*a**4*n**8 + 36960*i
nt((sqrt(x**n*b + a)*x**2)/(35*x**n*b*n**4 + 352*x**n*b*n**3 + 1032*x**n*b
*n**2 + 1152*x**n*b*n + 432*x**n*b + 35*a*n**4 + 352*a*n**3 + 1032*a*n**2
+ 1152*a*n + 432*a),x)*a**4*n**7 + 108360*int((sqrt(x**n*b + a)*x**2)/(35*
x**n*b*n**4 + 352*x**n*b*n**3 + 1032*x**n*b*n**2 + 1152*x**n*b*n + 432*x**
n*b + 35*a*n**4 + 352*a*n**3 + 1032*a*n**2 + 1152*a*n + 432*a),x)*a**4*n**
6 + 120960*int((sqrt(x**n*b + a)*x**2)/(35*x**n*b*n**4 + 352*x**n*b*n**3 +
 1032*x**n*b*n**2 + 1152*x**n*b*n + 432*x**n*b + 35*a*n**4 + 352*a*n**3 +
1032*a*n**2 + 1152*a*n + 432*a),x)*a**4*n**5 + 45360*int((sqrt(x**n*b +...
```

3.345 $\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx=\frac{2Bx^2(a+bx^n)^{7/2}}{b(4+7n)}+\frac{a^2\left(A-\frac{4aB}{4b+7bn}\right)x^2\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x^2*(a+b*x^n)^(7/2)/b/(4+7*n)+1/2*a^2*(A-4*a*B/(7*b*n+4*b))*x^2*(a+b*x
^n)^(1/2)*hypergeom([-5/2, 2/n],[(2+n)/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx=\frac{a^2x^2\sqrt{a+bx^n}\left(A(2+n)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\right.}{2(2+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[x*(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(a^2*x^2*Sqrt[a + b*x^n]*(A*(2 + n)*Hypergeometric2F1[-5/2, 2/n, (2 + n)/n
, -((b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[-5/2, (2 + n)/n, 2*(1 + n^(-1)
), -((b*x^n)/a)]))/(2*(2 + n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{7bn+4b}\right)\int x(bx^n+a)^{5/2}dx+\frac{2Bx^2(a+bx^n)^{7/2}}{b(7n+4)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{4aB}{7bn+4b}\right)\int x\left(\frac{bx^n}{a}+1\right)^{5/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{7/2}}{b(7n+4)}$$

$$\Big\downarrow \text{888}$$

$$\frac{a^2x^2\sqrt{a+bx^n}\left(A-\frac{4aB}{7bn+4b}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx^2(a+bx^n)^{7/2}}{b(7n+4)}$$

input
```
Int[x*(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(2*B*x^2*(a + b*x^n)^(7/2))/(b*(4 + 7*n)) + (a^2*(A - (4*a*B)/(4*b + 7*b*n
))*x^2*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, 2/n, (2 + n)/n, -((b*x^n)/a
)])/(2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x(a+b\,x^{n})^{\frac{5}{2}}\left(A+B\,x^{n}\right)dx$$

input
```
int(x*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output
```
int(x*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+bx^n)^{5/2}\left(A+Bx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.03 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.81

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx = \frac{Aa^2a^{\frac{2}{n}}a^{\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{2Aaa^{-\frac{1}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}bx^{n+2}\Gamma\left(1+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{Aa^{-\frac{3}{2}-\frac{2}{n}}a^{2+\frac{2}{n}}b^2x^{2n+2}\Gamma\left(2+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{2}{n}\\3+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{2}{n}\right)} + \frac{Ba^2a^{-\frac{1}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{2Baa^{-\frac{3}{2}-\frac{2}{n}}a^{2+\frac{2}{n}}bx^{2n+2}\Gamma\left(2+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{2}{n}\\3+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{2}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{2}{n}}a^{3+\frac{2}{n}}b^2x^{3n+2}\Gamma\left(3+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3+\frac{2}{n}\\4+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**(5/2)*(A+B*x**n),x)
```

output
```
A*a**2*a**(2/n)*a**(1/2 - 2/n)*x**2*gamma(2/n)*hyper((-1/2, 2/n), (1 + 2/n
,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + 2*A*a*a**(-1/2 - 2/n)*a
**(1 + 2/n)*b*x**(n + 2)*gamma(1 + 2/n)*hyper((-1/2, 1 + 2/n), (2 + 2/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 2/n)) + A*a**(-3/2 - 2/n)*a**(2 +
2/n)*b**2*x**(2*n + 2)*gamma(2 + 2/n)*hyper((-1/2, 2 + 2/n), (3 + 2/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 2/n)) + B*a**2*a**(-1/2 - 2/n)*a**(1
 + 2/n)*x**(n + 2)*gamma(1 + 2/n)*hyper((-1/2, 1 + 2/n), (2 + 2/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(2 + 2/n)) + 2*B*a*a**(-3/2 - 2/n)*a**(2 + 2/
n)*b*x**(2*n + 2)*gamma(2 + 2/n)*hyper((-1/2, 2 + 2/n), (3 + 2/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(3 + 2/n)) + B*a**(-5/2 - 2/n)*a**(3 + 2/n)*b*
*2*x**(3*n + 2)*gamma(3 + 2/n)*hyper((-1/2, 3 + 2/n), (4 + 2/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(4 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)*x, x)
```

Giac [F]

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{5}{2}}x\,dx$$

input
```
integrate(x*(a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx=\int x\,(A+B\,x^n)\,(a+b\,x^n)^{5/2}\,dx$$

input
```
int(x*(A + B*x^n)*(a + b*x^n)^(5/2),x)
```

output
```
int(x*(A + B*x^n)*(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int x(a+bx^n)^{5/2}(A+Bx^n)\,dx=\frac{30x^{3n}\sqrt{x^nb+a}\,b^3n^3x^2+184x^{3n}\sqrt{x^nb+a}\,b^3n^2x^2+288x^{3n}\sqrt{x^nb+a}\,b^3n\,x^2+128x^{3n}\sqrt{x^nb+}}{}$$

input
```
int(x*(a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output

```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3*x**2 + 184*x**(3*n)*sqrt(x**n*b +
a)*b**3*n**2*x**2 + 288*x**(3*n)*sqrt(x**n*b + a)*b**3*n*x**2 + 128*x**(3*
n)*sqrt(x**n*b + a)*b**3*x**2 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3*
x**2 + 776*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**2*x**2 + 1088*x**(2*n)*sqrt
(x**n*b + a)*a*b**2*n*x**2 + 384*x**(2*n)*sqrt(x**n*b + a)*a*b**2*x**2 + 2
44*x**n*sqrt(x**n*b + a)*a**2*b*n**3*x**2 + 1280*x**n*sqrt(x**n*b + a)*a**
2*b*n**2*x**2 + 1312*x**n*sqrt(x**n*b + a)*a**2*b*n*x**2 + 384*x**n*sqrt(x
**n*b + a)*a**2*b*x**2 + 352*sqrt(x**n*b + a)*a**3*n**3*x**2 + 688*sqrt(x*
*n*b + a)*a**3*n**2*x**2 + 512*sqrt(x**n*b + a)*a**3*n*x**2 + 128*sqrt(x**
n*b + a)*a**3*x**2 + 11025*int((sqrt(x**n*b + a)*x)/(105*x**n*b*n**4 + 704
*x**n*b*n**3 + 1376*x**n*b*n**2 + 1024*x**n*b*n + 256*x**n*b + 105*a*n**4
+ 704*a*n**3 + 1376*a*n**2 + 1024*a*n + 256*a),x)*a**4*n**8 + 73920*int((s
qrt(x**n*b + a)*x)/(105*x**n*b*n**4 + 704*x**n*b*n**3 + 1376*x**n*b*n**2 +
 1024*x**n*b*n + 256*x**n*b + 105*a*n**4 + 704*a*n**3 + 1376*a*n**2 + 1024
*a*n + 256*a),x)*a**4*n**7 + 144480*int((sqrt(x**n*b + a)*x)/(105*x**n*b*n
**4 + 704*x**n*b*n**3 + 1376*x**n*b*n**2 + 1024*x**n*b*n + 256*x**n*b + 10
5*a*n**4 + 704*a*n**3 + 1376*a*n**2 + 1024*a*n + 256*a),x)*a**4*n**6 + 107
520*int((sqrt(x**n*b + a)*x)/(105*x**n*b*n**4 + 704*x**n*b*n**3 + 1376*x**
n*b*n**2 + 1024*x**n*b*n + 256*x**n*b + 105*a*n**4 + 704*a*n**3 + 1376*a*n
**2 + 1024*a*n + 256*a),x)*a**4*n**5 + 26880*int((sqrt(x**n*b + a)*x)/(...
```

3.346 $\int (a+bx^n)^{5/2} (A+Bx^n) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int (a+bx^n)^{5/2} (A+Bx^n) \, dx = \frac{2Bx(a+bx^n)^{7/2}}{b(2+7n)} + \frac{a^2\left(A-\frac{2aB}{2b+7bn}\right)x\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*x*(a+b*x^n)^(7/2)/b/(2+7*n)+a^2*(A-2*a*B/(7*b*n+2*b))*x*(a+b*x^n)^(1/2
)*hypergeom([-5/2, 1/n],[1+1/n],-b*x^n/a)/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int (a+bx^n)^{5/2} (A+Bx^n) \, dx = \frac{x\sqrt{a+bx^n}\left(B(a+bx^n)^3 - \frac{a^2\left(aB-\frac{1}{2}Ab(2+7n)\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{1+\frac{bx^n}{a}}}\right)}{b+\frac{7bn}{2}}$$

input
```
Integrate[(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(x*Sqrt[a + b*x^n]*(B*(a + b*x^n)^3 - (a^2*(a*B - (A*b*(2 + 7*n))/2)*Hyper
geometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/Sqrt[1 + (b*x^n)/a])
)/(b + (7*b*n)/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^{5/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(A-\frac{2aB}{7bn+2b}\right)\int (bx^n+a)^{5/2}dx+\frac{2Bx(a+bx^n)^{7/2}}{b(7n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{2aB}{7bn+2b}\right)\int \left(\frac{bx^n}{a}+1\right)^{5/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{7/2}}{b(7n+2)}$$

$$\Big\downarrow \text{778}$$

$$\frac{a^2x\sqrt{a+bx^n}\left(A-\frac{2aB}{7bn+2b}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2Bx(a+bx^n)^{7/2}}{b(7n+2)}$$

input
```
Int[(a + b*x^n)^(5/2)*(A + B*x^n),x]
```

output
```
(2*B*x*(a + b*x^n)^(7/2))/(b*(2 + 7*n)) + (a^2*(A - (2*a*B)/(2*b + 7*b*n))
*x*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, n^(-1), 1 + n^(-1), -((b*x^n)/a
)])/Sqrt[1 + (b*x^n)/a]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{\frac{5}{2}} (A+B\,x^{n})\,dx$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output
```
int((a+b*x^n)^(5/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+bx^n)^{5/2} (A+Bx^n)\ dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.24

$$\int (a+bx^n)^{5/2}(A+Bx^n)\,dx = \frac{Aa^2a^{\frac{1}{n}}a^{\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2Aaa^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}bx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{Aa^{-\frac{3}{2}-\frac{1}{n}}a^{2+\frac{1}{n}}b^2x^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{Ba^2a^{-\frac{1}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{2Baa^{-\frac{3}{2}-\frac{1}{n}}a^{2+\frac{1}{n}}bx^{2n+1}\Gamma\left(2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{1}{n}\\3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{1}{n}}a^{3+\frac{1}{n}}b^2x^{3n+1}\Gamma\left(3+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3+\frac{1}{n}\\4+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)*(A+B*x**n),x)
```

output
```
A*a**2*a**(1/n)*a**(1/2 - 1/n)*x*gamma(1/n)*hyper((-1/2, 1/n), (1 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 2*A*a*a**(-1/2 - 1/n)*a**(
1 + 1/n)*b*x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*
x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + A*a**(-3/2 - 1/n)*a**(2 + 1/n
)*b**2*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-1/2, 2 + 1/n), (3 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + B*a**2*a**(-1/2 - 1/n)*a**(1 +
1/n)*x**(n + 1)*gamma(1 + 1/n)*hyper((-1/2, 1 + 1/n), (2 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 2*B*a*a**(-3/2 - 1/n)*a**(2 + 1/n)*
b*x**(2*n + 1)*gamma(2 + 1/n)*hyper((-1/2, 2 + 1/n), (3 + 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + B*a**(-5/2 - 1/n)*a**(3 + 1/n)*b**2*
x**(3*n + 1)*gamma(3 + 1/n)*hyper((-1/2, 3 + 1/n), (4 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(4 + 1/n))
```

Maxima [F]

$$\int (a + bx^n)^{5/2} (A + Bx^n) \, dx = \int (Bx^n + A)(bx^n + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int (a + bx^n)^{5/2} (A + Bx^n) \, dx = \int (Bx^n + A)(bx^n + a)^{\frac{5}{2}} \, dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^{5/2}(A+Bx^n)\,dx=\int (A+B\,x^n)\,(a+b\,x^n)^{5/2}\,dx$$

input
```
int((A + B*x^n)*(a + b*x^n)^(5/2),x)
```

output
```
int((A + B*x^n)*(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int (a+bx^n)^{5/2}(A+Bx^n)\,dx=\frac{30x^{3n}\sqrt{x^nb+a}\,b^3n^3x+92x^{3n}\sqrt{x^nb+a}\,b^3n^2x+72x^{3n}\sqrt{x^nb+a}\,b^3nx+16x^{3n}\sqrt{x^nb+a}\,b^3x+}{}$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n),x)
```

output

```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3*x + 92*x**(3*n)*sqrt(x**n*b + a)*b
**3*n**2*x + 72*x**(3*n)*sqrt(x**n*b + a)*b**3*n*x + 16*x**(3*n)*sqrt(x**n
*b + a)*b**3*x + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3*x + 388*x**(2*n
)*sqrt(x**n*b + a)*a*b**2*n**2*x + 272*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n*
x + 48*x**(2*n)*sqrt(x**n*b + a)*a*b**2*x + 244*x**n*sqrt(x**n*b + a)*a**2
*b*n**3*x + 640*x**n*sqrt(x**n*b + a)*a**2*b*n**2*x + 328*x**n*sqrt(x**n*b
 + a)*a**2*b*n*x + 48*x**n*sqrt(x**n*b + a)*a**2*b*x + 352*sqrt(x**n*b + a
)*a**3*n**3*x + 344*sqrt(x**n*b + a)*a**3*n**2*x + 128*sqrt(x**n*b + a)*a*
*3*n*x + 16*sqrt(x**n*b + a)*a**3*x + 11025*int(sqrt(x**n*b + a)/(105*x**n
*b*n**4 + 352*x**n*b*n**3 + 344*x**n*b*n**2 + 128*x**n*b*n + 16*x**n*b + 1
05*a*n**4 + 352*a*n**3 + 344*a*n**2 + 128*a*n + 16*a),x)*a**4*n**8 + 36960
*int(sqrt(x**n*b + a)/(105*x**n*b*n**4 + 352*x**n*b*n**3 + 344*x**n*b*n**2
 + 128*x**n*b*n + 16*x**n*b + 105*a*n**4 + 352*a*n**3 + 344*a*n**2 + 128*a
*n + 16*a),x)*a**4*n**7 + 36120*int(sqrt(x**n*b + a)/(105*x**n*b*n**4 + 35
2*x**n*b*n**3 + 344*x**n*b*n**2 + 128*x**n*b*n + 16*x**n*b + 105*a*n**4 +
352*a*n**3 + 344*a*n**2 + 128*a*n + 16*a),x)*a**4*n**6 + 13440*int(sqrt(x*
*n*b + a)/(105*x**n*b*n**4 + 352*x**n*b*n**3 + 344*x**n*b*n**2 + 128*x**n*
b*n + 16*x**n*b + 105*a*n**4 + 352*a*n**3 + 344*a*n**2 + 128*a*n + 16*a),x
)*a**4*n**5 + 1680*int(sqrt(x**n*b + a)/(105*x**n*b*n**4 + 352*x**n*b*n**3
 + 344*x**n*b*n**2 + 128*x**n*b*n + 16*x**n*b + 105*a*n**4 + 352*a*n**3...
```

3.347 $\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \frac{2a^2A\sqrt{a+bx^n}}{n}+\frac{2aA(a+bx^n)^{3/2}}{3n}+\frac{2A(a+bx^n)^{5/2}}{5n}+\frac{2B(a+bx^n)^{7/2}}{7bn}-\frac{2a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{n}$$

output
```
2*a^2*A*(a+b*x^n)^(1/2)/n+2/3*a*A*(a+b*x^n)^(3/2)/n+2/5*A*(a+b*x^n)^(5/2)/
n+2/7*B*(a+b*x^n)^(7/2)/b/n-2*a^(5/2)*A*arctanh((a+b*x^n)^(1/2)/a^(1/2))/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \frac{2\left(105a^2Ab\sqrt{a+bx^n}+35aAb(a+bx^n)^{3/2}+21Ab(a+bx^n)^{5/2}+15B(a+\right.}{105bn}$$

input
```
Integrate[((a + b*x^n)^(5/2)*(A + B*x^n))/x,x]
```

output
```
(2*(105*a^2*A*b*Sqrt[a + b*x^n] + 35*a*A*b*(a + b*x^n)^(3/2) + 21*A*b*(a +
 b*x^n)^(5/2) + 15*B*(a + b*x^n)^(7/2) - 105*a^(5/2)*A*b*ArcTanh[Sqrt[a +
b*x^n]/Sqrt[a]]))/(105*b*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {948, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{-n}(bx^n+a)^{5/2}(Bx^n+A)dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{A\int x^{-n}(bx^n+a)^{5/2}dx^n+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{A\left(a\int x^{-n}(bx^n+a)^{3/2}dx^n+\frac{2}{5}(a+bx^n)^{5/2}\right)+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{A\left(a\left(a\int x^{-n}\sqrt{bx^n+a}dx^n+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}\right)+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{A\left(a\left(a\left(a\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}\right)+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{A\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+2\sqrt{a+bx^n}\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}\right)+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{A\left(a\left(a\left(2\sqrt{a+bx^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^n)^{3/2}\right)+\frac{2}{5}(a+bx^n)^{5/2}\right)+\frac{2B(a+bx^n)^{7/2}}{7b}}{n}$$

input
```
Int[((a + b*x^n)^(5/2)*(A + B*x^n))/x,x]
```

output
```
((2*B*(a + b*x^n)^(7/2))/(7*b) + A*((2*(a + b*x^n)^(5/2))/5 + a*((2*(a + b
*x^n)^(3/2))/3 + a*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/
Sqrt[a]]))))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{2B(a+b\,x^n)^{\frac{7}{2}}}{7}+\frac{2Ab(a+b\,x^n)^{\frac{5}{2}}}{5}+\frac{2Aab(a+b\,x^n)^{\frac{3}{2}}}{3}+2A\,a^2b\sqrt{a+b\,x^n}-2A\,a^{\frac{5}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{bn}$
default	$\frac{\frac{2B(a+b\,x^n)^{\frac{7}{2}}}{7}+\frac{2Ab(a+b\,x^n)^{\frac{5}{2}}}{5}+\frac{2Aab(a+b\,x^n)^{\frac{3}{2}}}{3}+2A\,a^2b\sqrt{a+b\,x^n}-2A\,a^{\frac{5}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{bn}$
risch	$\frac{2\left(15B\,b^3\mathrm{e}^{3n\ln(x)}+21A\,b^3\mathrm{e}^{2n\ln(x)}+45Ba\,b^2\mathrm{e}^{2n\ln(x)}+77Aa\,b^2\mathrm{e}^{n\ln(x)}+45B\,a^2b\,\mathrm{e}^{n\ln(x)}+161A\,a^2b+15B\,a^3\right)\sqrt{a+b\,\mathrm{e}^{n}}}{105bn}$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(1/7*B*(a+b*x^n)^(7/2)+1/5*A*b*(a+b*x^n)^(5/2)+1/3*A*a*b*(a+b*x^n)^(
3/2)+A*a^2*b*(a+b*x^n)^(1/2)-A*a^(5/2)*b*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\left(15\,Bb^3x^{3\,n}+15\,Ba^3+161\,Aa^2b\right.}{105\,bn}\right.$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x,x, algorithm="fricas")
```

output
```
[1/105*(105*A*a^(5/2)*b*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)
 + 2*(15*B*b^3*x^(3*n) + 15*B*a^3 + 161*A*a^2*b + 3*(15*B*a*b^2 + 7*A*b^3)
*x^(2*n) + (45*B*a^2*b + 77*A*a*b^2)*x^n)*sqrt(b*x^n + a))/(b*n), 2/105*(1
05*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^n + a)) + (15*B*b^3*x^(3*n) +
 15*B*a^3 + 161*A*a^2*b + 3*(15*B*a*b^2 + 7*A*b^3)*x^(2*n) + (45*B*a^2*b +
 77*A*a*b^2)*x^n)*sqrt(b*x^n + a))/(b*n)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(99) = 198.

Time = 28.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2Aa^3\operatorname{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa^2\sqrt{a+bx^n} + \frac{2Aa(a+bx^n)^{\frac{3}{2}}}{3} + \frac{2A(a+bx^n)^{\frac{5}{2}}}{5} + \frac{2B(a+}{7b} \\ Aa^{\frac{5}{2}}\log\left(Ba^{\frac{5}{2}}x^n\right) + Ba^{\frac{5}{2}}x^n \end{cases}}{n} \\ \left(Aa^2\sqrt{a+b} + 2Aab\sqrt{a+b} + Ab^2\sqrt{a+b} + Ba^2\sqrt{a+b} + 2Bab\sqrt{a+b}\right. \end{cases}$$

input
```
integrate((a+b*x**n)**(5/2)*(A+B*x**n)/x,x)
```

output
```
Piecewise((Piecewise((2*A*a**3*atan(sqrt(a + b*x**n)/sqrt(-a))/sqrt(-a) +
2*A*a**2*sqrt(a + b*x**n) + 2*A*a*(a + b*x**n)**(3/2)/3 + 2*A*(a + b*x**n)
**(5/2)/5 + 2*B*(a + b*x**n)**(7/2)/(7*b), Ne(b, 0)), (A*a**(5/2)*log(B*a*
*(5/2)*x***n) + B*a**(5/2)*x***n, True))/n, Ne(n, 0)), ((A*a**2*sqrt(a + b)
+ 2*A*a*b*sqrt(a + b) + A*b**2*sqrt(a + b) + B*a**2*sqrt(a + b) + 2*B*a*b*
sqrt(a + b) + B*b**2*sqrt(a + b))*log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \frac{2\,(bx^n+a)^{\frac{7}{2}}B}{7\,bn} + \frac{1}{15}\left(\frac{15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n} + \frac{2\left(3\,(bx^n+a)^{\frac{5}{2}}+5\,(bx^n+a)^{\frac{3}{2}}a+15\,\sqrt{bx^n+a}a^2\right)}{n}\right)A$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x,x, algorithm="maxima")
```

output
```
2/7*(b*x^n + a)^(7/2)*B/(b*n) + 1/15*(15*a^(5/2)*log((sqrt(b*x^n + a) - sq
rt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2*(3*(b*x^n + a)^(5/2) + 5*(b*x^n
+ a)^(3/2)*a + 15*sqrt(b*x^n + a)*a^2)/n)*A
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{5/2}}{x}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x}\,dx = \frac{30x^{3n}\sqrt{x^n b+a}\,b^3+132x^{2n}\sqrt{x^n b+a}\,a\,b^2+244x^n\sqrt{x^n b+a}\,a^2b+352\sqrt{x^n}}{105n}$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x,x)
```

output
```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2
+ 244*x**n*sqrt(x**n*b + a)*a**2*b + 352*sqrt(x**n*b + a)*a**3 + 105*int(s
qrt(x**n*b + a)/(x**n*b*x + a*x),x)*a**4*n)/(105*n)
```

3.348 $\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = -\frac{2B(a+bx^n)^{7/2}}{b(2-7n)x} - \frac{a^2\left(A-\frac{2aB}{2b-7bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(7/2)/b/(2-7*n)/x-a^2*(A-2*a*B/(-7*b*n+2*b))*(a+b*x^n)^(1/2
)*hypergeom([-5/2, -1/n],[-(1-n)/n],-b*x^n/a)/x/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \frac{a^2\sqrt{a+bx^n}\left((A-An)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\right.}{(-1+n)x\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(5/2)*(A + B*x^n))/x^2,x]
```

output
```
(a^2*Sqrt[a + b*x^n]*((A - A*n)*Hypergeometric2F1[-5/2, -n^(-1), (-1 + n)/
n, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[-5/2, (-1 + n)/n, 2 - n^(-1), -
((b*x^n)/a)]))/((-1 + n)*x*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{2b-7bn}\right)\int \frac{(bx^n+a)^{5/2}}{x^2}dx-\frac{2B(a+bx^n)^{7/2}}{b(2-7n)x}$$

$$\Big\downarrow \text{889}$$

$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-7bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{5/2}}{x^2}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(2-7n)x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a^2\sqrt{a+bx^n}\left(A-\frac{2aB}{2b-7bn}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(2-7n)x}$$

input
```
Int[((a + b*x^n)^(5/2)*(A + B*x^n))/x^2,x]
```

output
```
(-2*B*(a + b*x^n)^(7/2))/(b*(2 - 7*n)*x) - (a^2*(A - (2*a*B)/(2*b - 7*b*n)
)*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -n^(-1), -((1 - n)/n), -((b*x^n)
/a)])/(x*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{5}{2}}\,(A+B\,x^{n})}{x^{2}}dx$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x)
```

output
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.70

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \frac{Aa^2a^{-\frac{1}{n}}a^{\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$
$$+\frac{2Aaa^{-\frac{1}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}bx^{n-1}\Gamma\left(1-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)}$$
$$+\frac{Aa^{-\frac{3}{2}+\frac{1}{n}}a^{2-\frac{1}{n}}b^2x^{2n-1}\Gamma\left(2-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{1}{n}\\3-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{1}{n}\right)}$$
$$+\frac{Ba^2a^{-\frac{1}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}x^{n-1}\Gamma\left(1-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)}$$
$$+\frac{2Baa^{-\frac{3}{2}+\frac{1}{n}}a^{2-\frac{1}{n}}bx^{2n-1}\Gamma\left(2-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{1}{n}\\3-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{1}{n}\right)}$$
$$+\frac{Ba^{-\frac{5}{2}+\frac{1}{n}}a^{3-\frac{1}{n}}b^2x^{3n-1}\Gamma\left(3-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3-\frac{1}{n}\\4-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)*(A+B*x**n)/x**2,x)
```

output
```
A*a**2*a**(1/2 + 1/n)*gamma(-1/n)*hyper((-1/2, -1/n), (1 - 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + 2*A*a*a**(-1/2 + 1/n)*a*
*(1 - 1/n)*b*x**(n - 1)*gamma(1 - 1/n)*hyper((-1/2, 1 - 1/n), (2 - 1/n,),
b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - 1/n)) + A*a**(-3/2 + 1/n)*a**(2 - 1
/n)*b**2*x**(2*n - 1)*gamma(2 - 1/n)*hyper((-1/2, 2 - 1/n), (3 - 1/n,), b*
x**n*exp_polar(I*pi)/a)/(n*gamma(3 - 1/n)) + B*a**2*a**(-1/2 + 1/n)*a**(1
- 1/n)*x**(n - 1)*gamma(1 - 1/n)*hyper((-1/2, 1 - 1/n), (2 - 1/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(2 - 1/n)) + 2*B*a*a**(-3/2 + 1/n)*a**(2 - 1/n
)*b*x**(2*n - 1)*gamma(2 - 1/n)*hyper((-1/2, 2 - 1/n), (3 - 1/n,), b*x**n*
exp_polar(I*pi)/a)/(n*gamma(3 - 1/n)) + B*a**(-5/2 + 1/n)*a**(3 - 1/n)*b**
2*x**(3*n - 1)*gamma(3 - 1/n)*hyper((-1/2, 3 - 1/n), (4 - 1/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(4 - 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{5/2}}{x^2}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^2,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^2,x)
```

output

```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3 - 92*x**(3*n)*sqrt(x**n*b + a)*b**
3*n**2 + 72*x**(3*n)*sqrt(x**n*b + a)*b**3*n - 16*x**(3*n)*sqrt(x**n*b + a
)*b**3 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3 - 388*x**(2*n)*sqrt(x**
n*b + a)*a*b**2*n**2 + 272*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n - 48*x**(2*n
)*sqrt(x**n*b + a)*a*b**2 + 244*x**n*sqrt(x**n*b + a)*a**2*b*n**3 - 640*x*
*n*sqrt(x**n*b + a)*a**2*b*n**2 + 328*x**n*sqrt(x**n*b + a)*a**2*b*n - 48*
x**n*sqrt(x**n*b + a)*a**2*b + 352*sqrt(x**n*b + a)*a**3*n**3 - 344*sqrt(x
**n*b + a)*a**3*n**2 + 128*sqrt(x**n*b + a)*a**3*n - 16*sqrt(x**n*b + a)*a
**3 + 11025*int(sqrt(x**n*b + a)/(105*x**n*b*n**4*x**2 - 352*x**n*b*n**3*x
**2 + 344*x**n*b*n**2*x**2 - 128*x**n*b*n*x**2 + 16*x**n*b*x**2 + 105*a*n*
*4*x**2 - 352*a*n**3*x**2 + 344*a*n**2*x**2 - 128*a*n*x**2 + 16*a*x**2),x)
*a**4*n**8*x - 36960*int(sqrt(x**n*b + a)/(105*x**n*b*n**4*x**2 - 352*x**n
*b*n**3*x**2 + 344*x**n*b*n**2*x**2 - 128*x**n*b*n*x**2 + 16*x**n*b*x**2 +
 105*a*n**4*x**2 - 352*a*n**3*x**2 + 344*a*n**2*x**2 - 128*a*n*x**2 + 16*a
*x**2),x)*a**4*n**7*x + 36120*int(sqrt(x**n*b + a)/(105*x**n*b*n**4*x**2 -
 352*x**n*b*n**3*x**2 + 344*x**n*b*n**2*x**2 - 128*x**n*b*n*x**2 + 16*x**n
*b*x**2 + 105*a*n**4*x**2 - 352*a*n**3*x**2 + 344*a*n**2*x**2 - 128*a*n*x*
*2 + 16*a*x**2),x)*a**4*n**6*x - 13440*int(sqrt(x**n*b + a)/(105*x**n*b*n*
*4*x**2 - 352*x**n*b*n**3*x**2 + 344*x**n*b*n**2*x**2 - 128*x**n*b*n*x**2
+ 16*x**n*b*x**2 + 105*a*n**4*x**2 - 352*a*n**3*x**2 + 344*a*n**2*x**2 ...
```

3.349 $\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3} dx = -\frac{2B(a+bx^n)^{7/2}}{b(4-7n)x^2} - \frac{a^2\left(A-\frac{4aB}{4b-7bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(7/2)/b/(4-7*n)/x^2-1/2*a^2*(A-4*a*B/(-7*b*n+4*b))*(a+b*x^n
)^(1/2)*hypergeom([-5/2, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3} dx = \frac{a^2\sqrt{a+bx^n}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2\right.}{2(-2+n)x^2\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(5/2)*(A + B*x^n))/x^3,x]
```

output
```
(a^2*Sqrt[a + b*x^n]*(-(A*(-2 + n)*Hypergeometric2F1[-5/2, -2/n, (-2 + n)/
n, -((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[-5/2, (-2 + n)/n, 2 - 2/n, -
((b*x^n)/a)]))/(2*(-2 + n)*x^2*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{4b-7bn}\right)\int \frac{(bx^n+a)^{5/2}}{x^3}dx-\frac{2B(a+bx^n)^{7/2}}{b(4-7n)x^2}$$

$$\Big\downarrow \text{889}$$

$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-7bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{5/2}}{x^3}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(4-7n)x^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a^2\sqrt{a+bx^n}\left(A-\frac{4aB}{4b-7bn}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(4-7n)x^2}$$

input
```
Int[((a + b*x^n)^(5/2)*(A + B*x^n))/x^3,x]
```

output
```
(-2*B*(a + b*x^n)^(7/2))/(b*(4 - 7*n)*x^2) - (a^2*(A - (4*a*B)/(4*b - 7*b*
n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -2/n, -((2 - n)/n), -((b*x^n)/
a)])/(2*x^2*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{5}{2}}\,(A+B\,x^{n})}{x^{3}}dx$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x)
```

output
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.58 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \frac{Aa^2a^{-\frac{2}{n}}a^{\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{2Aaa^{-\frac{1}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}bx^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)} + \frac{Aa^{-\frac{3}{2}+\frac{2}{n}}a^{2-\frac{2}{n}}b^2x^{2n-2}\Gamma\left(2-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{2}{n}\\3-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{2}{n}\right)} + \frac{Ba^2a^{-\frac{1}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)} + \frac{2Baa^{-\frac{3}{2}+\frac{2}{n}}a^{2-\frac{2}{n}}bx^{2n-2}\Gamma\left(2-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{2}{n}\\3-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{2}{n}\right)} + \frac{Ba^{-\frac{5}{2}+\frac{2}{n}}a^{3-\frac{2}{n}}b^2x^{3n-2}\Gamma\left(3-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3-\frac{2}{n}\\4-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)*(A+B*x**n)/x**3,x)
```

output
```
A*a**2*a**(1/2 + 2/n)*gamma(-2/n)*hyper((-1/2, -2/n), (1 - 2/n,), b*x**n*e
xp_polar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + 2*A*a*a**(-1/2 + 2/n)
*a**(1 - 2/n)*b*x**(n - 2)*gamma(1 - 2/n)*hyper((-1/2, 1 - 2/n), (2 - 2/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - 2/n)) + A*a**(-3/2 + 2/n)*a**(2
- 2/n)*b**2*x**(2*n - 2)*gamma(2 - 2/n)*hyper((-1/2, 2 - 2/n), (3 - 2/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 - 2/n)) + B*a**2*a**(-1/2 + 2/n)*a**
(1 - 2/n)*x**(n - 2)*gamma(1 - 2/n)*hyper((-1/2, 1 - 2/n), (2 - 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(2 - 2/n)) + 2*B*a*a**(-3/2 + 2/n)*a**(2 -
2/n)*b*x**(2*n - 2)*gamma(2 - 2/n)*hyper((-1/2, 2 - 2/n), (3 - 2/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(3 - 2/n)) + B*a**(-5/2 + 2/n)*a**(3 - 2/n)*
b**2*x**(3*n - 2)*gamma(3 - 2/n)*hyper((-1/2, 3 - 2/n), (4 - 2/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(4 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{5/2}}{x^3}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^3,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^3,x)
```

output

```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3 - 184*x**(3*n)*sqrt(x**n*b + a)*b*
*3*n**2 + 288*x**(3*n)*sqrt(x**n*b + a)*b**3*n - 128*x**(3*n)*sqrt(x**n*b
+ a)*b**3 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3 - 776*x**(2*n)*sqrt(
x**n*b + a)*a*b**2*n**2 + 1088*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n - 384*x*
*(2*n)*sqrt(x**n*b + a)*a*b**2 + 244*x**n*sqrt(x**n*b + a)*a**2*b*n**3 - 1
280*x**n*sqrt(x**n*b + a)*a**2*b*n**2 + 1312*x**n*sqrt(x**n*b + a)*a**2*b*
n - 384*x**n*sqrt(x**n*b + a)*a**2*b + 352*sqrt(x**n*b + a)*a**3*n**3 - 68
8*sqrt(x**n*b + a)*a**3*n**2 + 512*sqrt(x**n*b + a)*a**3*n - 128*sqrt(x**n
*b + a)*a**3 + 11025*int(sqrt(x**n*b + a)/(105*x**n*b*n**4*x**3 - 704*x**n
*b*n**3*x**3 + 1376*x**n*b*n**2*x**3 - 1024*x**n*b*n*x**3 + 256*x**n*b*x**
3 + 105*a*n**4*x**3 - 704*a*n**3*x**3 + 1376*a*n**2*x**3 - 1024*a*n*x**3 +
 256*a*x**3),x)*a**4*n**8*x**2 - 73920*int(sqrt(x**n*b + a)/(105*x**n*b*n*
*4*x**3 - 704*x**n*b*n**3*x**3 + 1376*x**n*b*n**2*x**3 - 1024*x**n*b*n*x**
3 + 256*x**n*b*x**3 + 105*a*n**4*x**3 - 704*a*n**3*x**3 + 1376*a*n**2*x**3
 - 1024*a*n*x**3 + 256*a*x**3),x)*a**4*n**7*x**2 + 144480*int(sqrt(x**n*b
+ a)/(105*x**n*b*n**4*x**3 - 704*x**n*b*n**3*x**3 + 1376*x**n*b*n**2*x**3
- 1024*x**n*b*n*x**3 + 256*x**n*b*x**3 + 105*a*n**4*x**3 - 704*a*n**3*x**3
 + 1376*a*n**2*x**3 - 1024*a*n*x**3 + 256*a*x**3),x)*a**4*n**6*x**2 - 1075
20*int(sqrt(x**n*b + a)/(105*x**n*b*n**4*x**3 - 704*x**n*b*n**3*x**3 + 137
6*x**n*b*n**2*x**3 - 1024*x**n*b*n*x**3 + 256*x**n*b*x**3 + 105*a*n**4*...
```

3.350 $\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = -\frac{2B(a+bx^n)^{7/2}}{b(6-7n)x^3} - \frac{a^2\left(A-\frac{6aB}{6b-7bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(7/2)/b/(6-7*n)/x^3-1/3*a^2*(A-6*a*B/(-7*b*n+6*b))*(a+b*x^n
)^(1/2)*hypergeom([-5/2, -3/n],[-(3-n)/n],-b*x^n/a)/x^3/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = \frac{a^2\sqrt{a+bx^n}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3\right.}{3(-3+n)x^3\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[((a + b*x^n)^(5/2)*(A + B*x^n))/x^4,x]
```

output
```
(a^2*Sqrt[a + b*x^n]*(-(A*(-3 + n)*Hypergeometric2F1[-5/2, -3/n, (-3 + n)/
n, -((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[-5/2, (-3 + n)/n, 2 - 3/n, -
((b*x^n)/a)]))/(3*(-3 + n)*x^3*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}dx$$
$$\Big\downarrow \text{959}$$
$$\left(A-\frac{6aB}{6b-7bn}\right)\int \frac{(bx^n+a)^{5/2}}{x^4}dx-\frac{2B(a+bx^n)^{7/2}}{b(6-7n)x^3}$$
$$\Big\downarrow \text{889}$$
$$\frac{a^2\sqrt{a+bx^n}\left(A-\frac{6aB}{6b-7bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{5/2}}{x^4}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(6-7n)x^3}$$
$$\Big\downarrow \text{888}$$
$$-\frac{a^2\sqrt{a+bx^n}\left(A-\frac{6aB}{6b-7bn}\right)\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{7/2}}{b(6-7n)x^3}$$

input
```
Int[((a + b*x^n)^(5/2)*(A + B*x^n))/x^4,x]
```

output
```
(-2*B*(a + b*x^n)^(7/2))/(b*(6 - 7*n)*x^3) - (a^2*(A - (6*a*B)/(6*b - 7*b*
n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-5/2, -3/n, -((3 - n)/n), -((b*x^n)/
a)])/(3*x^3*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{5}{2}}\,(A+B\,x^n)}{x^4}dx$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x)
```

output
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.82 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.65

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = \frac{Aa^2a^{-\frac{3}{n}}a^{\frac{1}{2}+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{n}\\1-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)}$$
$$+\frac{2Aaa^{-\frac{1}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}bx^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}$$
$$+\frac{Aa^{-\frac{3}{2}+\frac{3}{n}}a^{2-\frac{3}{n}}b^2x^{2n-3}\Gamma\left(2-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{3}{n}\\3-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{3}{n}\right)}$$
$$+\frac{Ba^2a^{-\frac{1}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}x^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}$$
$$+\frac{2Baa^{-\frac{3}{2}+\frac{3}{n}}a^{2-\frac{3}{n}}bx^{2n-3}\Gamma\left(2-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{3}{n}\\3-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-\frac{3}{n}\right)}$$
$$+\frac{Ba^{-\frac{5}{2}+\frac{3}{n}}a^{3-\frac{3}{n}}b^2x^{3n-3}\Gamma\left(3-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},3-\frac{3}{n}\\4-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(4-\frac{3}{n}\right)}$$

input
```
integrate((a+b*x**n)**(5/2)*(A+B*x**n)/x**4,x)
```

output
```
A*a**2*a**(1/2 + 3/n)*gamma(-3/n)*hyper((-1/2, -3/n), (1 - 3/n,), b*x**n*e
xp_polar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + 2*A*a*a**(-1/2 + 3/n)
*a**(1 - 3/n)*b*x**(n - 3)*gamma(1 - 3/n)*hyper((-1/2, 1 - 3/n), (2 - 3/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - 3/n)) + A*a**(-3/2 + 3/n)*a**(2
- 3/n)*b**2*x**(2*n - 3)*gamma(2 - 3/n)*hyper((-1/2, 2 - 3/n), (3 - 3/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 - 3/n)) + B*a**2*a**(-1/2 + 3/n)*a**
(1 - 3/n)*x**(n - 3)*gamma(1 - 3/n)*hyper((-1/2, 1 - 3/n), (2 - 3/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(2 - 3/n)) + 2*B*a*a**(-3/2 + 3/n)*a**(2 -
3/n)*b*x**(2*n - 3)*gamma(2 - 3/n)*hyper((-1/2, 2 - 3/n), (3 - 3/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(3 - 3/n)) + B*a**(-5/2 + 3/n)*a**(3 - 3/n)*
b**2*x**(3*n - 3)*gamma(3 - 3/n)*hyper((-1/2, 3 - 3/n), (4 - 3/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(4 - 3/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(5/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx=\int \frac{(A+B\,x^n)\,(a+b\,x^n)^{5/2}}{x^4}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^4,x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(5/2))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{5/2}(A+Bx^n)}{x^4}\,dx=\text{Too large to display}$$

input
```
int((a+b*x^n)^(5/2)*(A+B*x^n)/x^4,x)
```

output
```
(30*x**(3*n)*sqrt(x**n*b + a)*b**3*n**3 - 276*x**(3*n)*sqrt(x**n*b + a)*b*
*3*n**2 + 648*x**(3*n)*sqrt(x**n*b + a)*b**3*n - 432*x**(3*n)*sqrt(x**n*b
+ a)*b**3 + 132*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n**3 - 1164*x**(2*n)*sqrt
(x**n*b + a)*a*b**2*n**2 + 2448*x**(2*n)*sqrt(x**n*b + a)*a*b**2*n - 1296*
x**(2*n)*sqrt(x**n*b + a)*a*b**2 + 244*x**n*sqrt(x**n*b + a)*a**2*b*n**3 -
 1920*x**n*sqrt(x**n*b + a)*a**2*b*n**2 + 2952*x**n*sqrt(x**n*b + a)*a**2*
b*n - 1296*x**n*sqrt(x**n*b + a)*a**2*b + 352*sqrt(x**n*b + a)*a**3*n**3 -
 1032*sqrt(x**n*b + a)*a**3*n**2 + 1152*sqrt(x**n*b + a)*a**3*n - 432*sqrt
(x**n*b + a)*a**3 + 3675*int(sqrt(x**n*b + a)/(35*x**n*b*n**4*x**4 - 352*x
**n*b*n**3*x**4 + 1032*x**n*b*n**2*x**4 - 1152*x**n*b*n*x**4 + 432*x**n*b*
x**4 + 35*a*n**4*x**4 - 352*a*n**3*x**4 + 1032*a*n**2*x**4 - 1152*a*n*x**4
 + 432*a*x**4),x)*a**4*n**8*x**3 - 36960*int(sqrt(x**n*b + a)/(35*x**n*b*n
**4*x**4 - 352*x**n*b*n**3*x**4 + 1032*x**n*b*n**2*x**4 - 1152*x**n*b*n*x*
*4 + 432*x**n*b*x**4 + 35*a*n**4*x**4 - 352*a*n**3*x**4 + 1032*a*n**2*x**4
 - 1152*a*n*x**4 + 432*a*x**4),x)*a**4*n**7*x**3 + 108360*int(sqrt(x**n*b
+ a)/(35*x**n*b*n**4*x**4 - 352*x**n*b*n**3*x**4 + 1032*x**n*b*n**2*x**4 -
 1152*x**n*b*n*x**4 + 432*x**n*b*x**4 + 35*a*n**4*x**4 - 352*a*n**3*x**4 +
 1032*a*n**2*x**4 - 1152*a*n*x**4 + 432*a*x**4),x)*a**4*n**6*x**3 - 120960
*int(sqrt(x**n*b + a)/(35*x**n*b*n**4*x**4 - 352*x**n*b*n**3*x**4 + 1032*x
**n*b*n**2*x**4 - 1152*x**n*b*n*x**4 + 432*x**n*b*x**4 + 35*a*n**4*x**4...
```

3.351 $\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{2Bx^3\sqrt{a+bx^n}}{b(6+n)} + \frac{\left(A-\frac{6aB}{b(6+n)}\right)x^3\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3\sqrt{a+bx^n}}$$

output
```
2*B*x^3*(a+b*x^n)^(1/2)/b/(6+n)+1/3*(A-6*a*B/b/(6+n))*x^3*(1+b*x^n/a)^(1/2
)*hypergeom([1/2, 3/n],[(3+n)/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{x^3\sqrt{1+\frac{bx^n}{a}}\left(A(3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{n},2+\frac{3}{n}\right.\right.}{3(3+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x^2*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(x^3*Sqrt[1 + (b*x^n)/a]*(A*(3 + n)*Hypergeometric2F1[1/2, 3/n, (3 + n)/n,
 -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[1/2, (3 + n)/n, 2 + 3/n, -((b*x
^n)/a)]))/(3*(3 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{b(n+6)}\right)\int \frac{x^2}{\sqrt{bx^n+a}}dx+\frac{2Bx^3\sqrt{a+bx^n}}{b(n+6)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(n+6)}\right)\int \frac{x^2}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2Bx^3\sqrt{a+bx^n}}{b(n+6)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(n+6)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3\sqrt{a+bx^n}}+\frac{2Bx^3\sqrt{a+bx^n}}{b(n+6)}$$

input
```
Int[(x^2*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*B*x^3*Sqrt[a + b*x^n])/(b*(6 + n)) + ((A - (6*a*B)/(b*(6 + n)))*x^3*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*
Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(A + B\,x^n)}{\sqrt{a + b\,x^n}}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(A + Bx^n)}{\sqrt{a + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{3}{n}}a^{-\frac{1}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(3/n)*a**(-1/2 - 3/n)*x**3*gamma(3/n)*hyper((1/2, 3/n), (1 + 3/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + B*a**(-3/2 - 3/n)*a**(1 + 3/
n)*x**(n + 3)*gamma(1 + 3/n)*hyper((1/2, 1 + 3/n), (2 + 3/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{(Bx^n+A)x^2}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x^2/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{(Bx^n+A)x^2}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{x^2\,(A+B\,x^n)}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(1/2),x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{x^nb+a}\,x^3+\left(\int \frac{\sqrt{x^nb+a}\,x^2}{x^nbn+6x^nb+an+6a}dx\right)a\,n^2+6\left(\int \frac{\sqrt{x^nb+a}\,x^2}{x^nbn+6x^nb+an+6a}dx\right)an}{n+6}$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*x**3 + int((sqrt(x**n*b + a)*x**2)/(x**n*b*n + 6*x**n*
b + a*n + 6*a),x)*a*n**2 + 6*int((sqrt(x**n*b + a)*x**2)/(x**n*b*n + 6*x**
n*b + a*n + 6*a),x)*a*n)/(n + 6)
```

3.352 $\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{2Bx^2\sqrt{a+bx^n}}{b(4+n)} + \frac{\left(A-\frac{4aB}{b(4+n)}\right)x^2\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

output
```
2*B*x^2*(a+b*x^n)^(1/2)/b/(4+n)+1/2*(A-4*a*B/b/(4+n))*x^2*(1+b*x^n/a)^(1/2
)*hypergeom([1/2, 2/n],[(2+n)/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{x^2\sqrt{1+\frac{bx^n}{a}}\left(A(2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{n},2(1+\right.\right.}{2(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*(A*(2 + n)*Hypergeometric2F1[1/2, 2/n, (2 + n)/n,
 -((b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[1/2, (2 + n)/n, 2*(1 + n^(-1)),
 -((b*x^n)/a)]))/(2*(2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{b(n+4)}\right)\int \frac{x}{\sqrt{bx^n+a}}dx+\frac{2Bx^2\sqrt{a+bx^n}}{b(n+4)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(n+4)}\right)\int \frac{x}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2Bx^2\sqrt{a+bx^n}}{b(n+4)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(n+4)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}+\frac{2Bx^2\sqrt{a+bx^n}}{b(n+4)}$$

input
```
Int[(x*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*B*x^2*Sqrt[a + b*x^n])/(b*(4 + n)) + ((A - (4*a*B)/(b*(4 + n)))*x^2*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*
Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(A+B\,x^{n})}{\sqrt{a+b\,x^{n}}}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{2}{n}}a^{-\frac{1}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(2/n)*a**(-1/2 - 2/n)*x**2*gamma(2/n)*hyper((1/2, 2/n), (1 + 2/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + B*a**(-3/2 - 2/n)*a**(1 + 2/
n)*x**(n + 2)*gamma(1 + 2/n)*hyper((1/2, 1 + 2/n), (2 + 2/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{(Bx^n+A)x}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{(Bx^n+A)x}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{x\,(A+B\,x^n)}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n)^(1/2),x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}\,x^2+\left(\int \frac{\sqrt{x^nb+a}\,x}{x^nbn+4x^nb+an+4a}dx\right)a\,n^2+4\left(\int \frac{\sqrt{x^nb+a}\,x}{x^nbn+4x^nb+an+4a}dx\right)an}{n+4}$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*x**2 + int((sqrt(x**n*b + a)*x)/(x**n*b*n + 4*x**n*b +
 a*n + 4*a),x)*a*n**2 + 4*int((sqrt(x**n*b + a)*x)/(x**n*b*n + 4*x**n*b +
a*n + 4*a),x)*a*n)/(n + 4)
```

3.353 $\int \frac{A+Bx^n}{\sqrt{a+bx^n}} dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}} dx = \frac{2Bx\sqrt{a+bx^n}}{b(2+n)} + \frac{\left(A-\frac{2aB}{b(2+n)}\right)x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

output
```
2*B*x*(a+b*x^n)^(1/2)/b/(2+n)+(A-2*a*B/b/(2+n))*x*(1+b*x^n/a)^(1/2)*hyperg
eom([1/2, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}} dx = \frac{x\left(2B(a+bx^n)+(-2aB+Ab(2+n))\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)\right)}{b(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/Sqrt[a + b*x^n],x]
```

output
```
(x*(2*B*(a + b*x^n) + (-2*a*B + A*b*(2 + n))*Sqrt[1 + (b*x^n)/a]*Hypergeom
etric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)]))/(b*(2 + n)*Sqrt[a + b*x^
n])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{913}$$

$$\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{\sqrt{bx^n+a}}dx+\frac{2Bx\sqrt{a+bx^n}}{b(n+2)}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2Bx\sqrt{a+bx^n}}{b(n+2)}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}+\frac{2Bx\sqrt{a+bx^n}}{b(n+2)}$$

input
```
Int[(A + B*x^n)/Sqrt[a + b*x^n],x]
```

output
```
(2*B*x*Sqrt[a + b*x^n])/(b*(2 + n)) + ((A - (2*a*B)/(b*(2 + n)))*x*Sqrt[1
+ (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/Sqr
t[a + b*x^n]
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{\sqrt{a + b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^n}{\sqrt{a + bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{1}{n}}a^{-\frac{1}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(1/n)*a**(-1/2 - 1/n)*x*gamma(1/n)*hyper((1/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(-3/2 - 1/n)*a**(1 + 1/n)*
x**(n + 1)*gamma(1 + 1/n)*hyper((1/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}\,dx=\int \frac{Bx^n+A}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}\,dx=\int \frac{A+B\,x^n}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n)^(1/2),x)
```

output
```
int((A + B*x^n)/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}\,x+\left(\int \frac{\sqrt{x^nb+a}}{x^nbn+2x^nb+an+2a}dx\right)a\,n^2+2\left(\int \frac{\sqrt{x^nb+a}}{x^nbn+2x^nb+an+2a}dx\right)an}{n+2}$$

input
```
int((A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a)*x + int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n +
2*a),x)*a*n**2 + 2*int(sqrt(x**n*b + a)/(x**n*b*n + 2*x**n*b + a*n + 2*a),
x)*a*n)/(n + 2)
```

3.354 $\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\frac{2B\sqrt{a+bx^n}}{bn}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
2*B*(a+b*x^n)^(1/2)/b/n-2*A*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\frac{2B\sqrt{a+bx^n}}{bn}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[(A + B*x^n)/(x*Sqrt[a + b*x^n]),x]
```

output
```
(2*B*Sqrt[a + b*x^n])/(b*n) - (2*A*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt
[a]*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{\sqrt{bx^n+a}}dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{A\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n+\frac{2B\sqrt{a+bx^n}}{b}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2A\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{b}+\frac{2B\sqrt{a+bx^n}}{b}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2B\sqrt{a+bx^n}}{b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}}}{n}$$

input
```
Int[(A + B*x^n)/(x*Sqrt[a + b*x^n]),x]
```

output
```
((2*B*Sqrt[a + b*x^n])/b - (2*A*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/Sqrt[a])
/n
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2B\sqrt{a+b\,x^{n}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}}}{bn}$	42
default	$\frac{2B\sqrt{a+b\,x^{n}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{\sqrt{a}}}{bn}$	42
risch	$\frac{2B\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{bn}-\frac{2A\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln\left(x\right)}}}{\sqrt{a}}\right)}{n\sqrt{a}}$	47

input
```
int((A+B*x^n)/x/(a+b*x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(B*(a+b*x^n)^(1/2)-A*b/a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.18

$$\int \frac{A + Bx^n}{x\sqrt{a + bx^n}}\,dx = \left[\frac{A\sqrt{a}b\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right) + 2\sqrt{bx^n + a}Ba}{abn}, \frac{2\left(A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^n+a}}\right) + \sqrt{bx^n + a}Ba\right)}{abn}\right]$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
[(A*sqrt(a)*b*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(
b*x^n + a)*B*a)/(a*b*n), 2*(A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^n + a))
+ sqrt(b*x^n + a)*B*a)/(a*b*n)]
```

Sympy [A] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

$$\int \frac{A + Bx^n}{x\sqrt{a + bx^n}}\,dx = \begin{cases} \frac{\begin{cases} \frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2B\sqrt{a+bx^n}}{b} & \text{for}\ b \neq 0 \\ \frac{A\log(Bx^n)+Bx^n}{\sqrt{a}} & \text{otherwise} \end{cases}}{n} & \text{for}\ n \neq 0 \\ -\frac{(-A-B)\log(x)}{\sqrt{a+b}} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n)**(1/2),x)
```

output
```
Piecewise((Piecewise((2*A*atan(sqrt(a + b*x**n)/sqrt(-a))/sqrt(-a) + 2*B*s
qrt(a + b*x**n)/b, Ne(b, 0)), ((A*log(B*x**n) + B*x**n)/sqrt(a), True))/n,
 Ne(n, 0)), (-(-A - B)*log(x)/sqrt(a + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\frac{A\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}n}+\frac{2\sqrt{bx^n+a}B}{bn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
A*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*n)
 + 2*sqrt(b*x^n + a)*B/(b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\int \frac{Bx^n+A}{\sqrt{bx^n+a}x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\int \frac{A+B\,x^n}{x\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/(x*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x\sqrt{a+bx^n}}\,dx=\frac{2\sqrt{x^nb+a}+\left(\int \frac{\sqrt{x^nb+a}}{x^nbx+ax}dx\right)an}{n}$$

input
```
int((A+B*x^n)/x/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)*a*n)/n
```

3.355 $\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 102

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx=-\frac{2B\sqrt{a+bx^n}}{b(2-n)x}-\frac{\left(A-\frac{2aB}{b(2-n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

output
```
-2*B*(a+b*x^n)^(1/2)/b/(2-n)/x-(A-2*a*B/b/(2-n))*(1+b*x^n/a)^(1/2)*hyperge
om([1/2, -1/n],[-(1-n)/n],-b*x^n/a)/x/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx$$

$$=\frac{\sqrt{1+\frac{bx^n}{a}}\left((A-An)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-1+n}{n},2-\right.\right.}{(-1+n)x\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^2*Sqrt[a + b*x^n]),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*((A - A*n)*Hypergeometric2F1[1/2, -n^(-1), (-1 + n)/n
, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[1/2, (-1 + n)/n, 2 - n^(-1), -((
b*x^n)/a)]))/((-1 + n)*x*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{b(2-n)}\right)\int \frac{1}{x^2\sqrt{bx^n+a}}dx-\frac{2B\sqrt{a+bx^n}}{b(2-n)x}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(2-n)}\right)\int \frac{1}{x^2\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{b(2-n)x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(2-n)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{b(2-n)x}$$

input
```
Int[(A + B*x^n)/(x^2*Sqrt[a + b*x^n]),x]
```

output
```
(-2*B*Sqrt[a + b*x^n])/(b*(2 - n)*x) - ((A - (2*a*B)/(b*(2 - n)))*Sqrt[1 +
 (b*x^n)/a]*Hypergeometric2F1[1/2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
x*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2}\sqrt{a+b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx = \frac{Aa^{-\frac{1}{n}}a^{-\frac{1}{2}+\frac{1}{n}}\Gamma\left(-\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{1}{n}\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{1}{n}}a^{1-\frac{1}{n}}x^{n-1}\Gamma\left(1-\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(-1/2 + 1/n)*gamma(-1/n)*hyper((1/2, -1/n), (1 - 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + B*a**(-3/2 + 1/n)*a**(1 - 1/n
)*x**(n - 1)*gamma(1 - 1/n)*hyper((1/2, 1 - 1/n), (2 - 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(2 - 1/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx=\int \frac{Bx^n+A}{\sqrt{bx^n+a}x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx=\int \frac{A+B\,x^n}{x^2\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^2\sqrt{a+bx^n}}\,dx$$
$$=\frac{2\sqrt{x^nb+a}+\left(\int\frac{\sqrt{x^nb+a}}{x^nbn\,x^2-2x^nb\,x^2+an\,x^2-2a\,x^2}dx\right)a\,n^2x-2\left(\int\frac{\sqrt{x^nb+a}}{x^nbn\,x^2-2x^nb\,x^2+an\,x^2-2a\,x^2}dx\right)anx}{x\,(n-2)}$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*n*x**2 - 2*x**n*b*x**2
+ a*n*x**2 - 2*a*x**2),x)*a*n**2*x - 2*int(sqrt(x**n*b + a)/(x**n*b*n*x**2
 - 2*x**n*b*x**2 + a*n*x**2 - 2*a*x**2),x)*a*n*x)/(x*(n - 2))
```

3.356 $\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx=-\frac{2B\sqrt{a+bx^n}}{b(4-n)x^2}-\frac{\left(A-\frac{4aB}{b(4-n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

output
```
-2*B*(a+b*x^n)^(1/2)/b/(4-n)/x^2-1/2*(A-4*a*B/b/(4-n))*(1+b*x^n/a)^(1/2)*h
ypergeom([1/2, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx$$
$$=\frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-2+n}{n},\right.\right.}{2(-2+n)x^2\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^3*Sqrt[a + b*x^n]),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-2 + n)*Hypergeometric2F1[1/2, -2/n, (-2 + n)/n
, -((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[1/2, (-2 + n)/n, 2 - 2/n, -((
b*x^n)/a)]))/(2*(-2 + n)*x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{b(4-n)}\right)\int \frac{1}{x^3\sqrt{bx^n+a}}dx-\frac{2B\sqrt{a+bx^n}}{b(4-n)x^2}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(4-n)}\right)\int \frac{1}{x^3\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{b(4-n)x^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(4-n)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{b(4-n)x^2}$$

input
```
Int[(A + B*x^n)/(x^3*Sqrt[a + b*x^n]),x]
```

output
```
(-2*B*Sqrt[a + b*x^n])/(b*(4 - n)*x^2) - ((A - (4*a*B)/(b*(4 - n)))*Sqrt[1
 + (b*x^n)/a]*Hypergeometric2F1[1/2, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2
*x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{x^{3}\sqrt{a + b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^{n}}{x^{3}\sqrt{a + bx^{n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx = \frac{Aa^{-\frac{2}{n}}a^{-\frac{1}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(-1/2 + 2/n)*gamma(-2/n)*hyper((1/2, -2/n), (1 - 2/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + B*a**(-3/2 + 2/n)*a**(1 -
2/n)*x**(n - 2)*gamma(1 - 2/n)*hyper((1/2, 1 - 2/n), (2 - 2/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 - 2/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}\,x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx = \int \frac{A+B\,x^n}{x^3\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^3\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}+\left(\int \frac{\sqrt{x^nb+a}}{x^nbn\,x^3-4x^nb\,x^3+an\,x^3-4a\,x^3}dx\right)a\,n^2x^2-4\left(\int \frac{\sqrt{x^nb+a}}{x^nbn\,x^3-4x^nb\,x^3+an\,x^3-4a\,x^3}dx\right)an\,x^2}{x^2\,(n-4)}$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*n*x**3 - 4*x**n*b*x**3
+ a*n*x**3 - 4*a*x**3),x)*a*n**2*x**2 - 4*int(sqrt(x**n*b + a)/(x**n*b*n*x
**3 - 4*x**n*b*x**3 + a*n*x**3 - 4*a*x**3),x)*a*n*x**2)/(x**2*(n - 4))
```

3.357 $\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = -\frac{2B\sqrt{a+bx^n}}{b(6-n)x^3} - \frac{\left(A-\frac{6aB}{b(6-n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{a+bx^n}}$$

output
```
-2*B*(a+b*x^n)^(1/2)/b/(6-n)/x^3-1/3*(A-6*a*B/b/(6-n))*(1+b*x^n/a)^(1/2)*h
ypergeom([1/2, -3/n],[-(3-n)/n],-b*x^n/a)/x^3/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{-3+n}{n},\right.\right.}{3(-3+n)x^3\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^4*Sqrt[a + b*x^n]),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-3 + n)*Hypergeometric2F1[1/2, -3/n, (-3 + n)/n
, -((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[1/2, (-3 + n)/n, 2 - 3/n, -((
b*x^n)/a)]))/(3*(-3 + n)*x^3*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + Bx^n}{x^4\sqrt{a + bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A - \frac{6aB}{b(6-n)}\right)\int \frac{1}{x^4\sqrt{bx^n + a}}dx - \frac{2B\sqrt{a + bx^n}}{b(6-n)x^3}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a} + 1}\left(A - \frac{6aB}{b(6-n)}\right)\int \frac{1}{x^4\sqrt{\frac{bx^n}{a} + 1}}dx}{\sqrt{a + bx^n}} - \frac{2B\sqrt{a + bx^n}}{b(6-n)x^3}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a} + 1}\left(A - \frac{6aB}{b(6-n)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{3}{n}, -\frac{3-n}{n}, -\frac{bx^n}{a}\right)}{3x^3\sqrt{a + bx^n}} - \frac{2B\sqrt{a + bx^n}}{b(6-n)x^3}$$

input
```
Int[(A + B*x^n)/(x^4*Sqrt[a + b*x^n]),x]
```

output
```
(-2*B*Sqrt[a + b*x^n])/(b*(6 - n)*x^3) - ((A - (6*a*B)/(b*(6 - n)))*Sqrt[1
 + (b*x^n)/a]*Hypergeometric2F1[1/2, -3/n, -((3 - n)/n), -((b*x^n)/a)])/(3
*x^3*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{4}\sqrt{a+b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \frac{Aa^{-\frac{3}{n}}a^{-\frac{1}{2}+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{3}{n}\\1-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}x^{n-3}\Gamma\left(1-\frac{3}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(-1/2 + 3/n)*gamma(-3/n)*hyper((1/2, -3/n), (1 - 3/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + B*a**(-3/2 + 3/n)*a**(1 -
3/n)*x**(n - 3)*gamma(1 - 3/n)*hyper((1/2, 1 - 3/n), (2 - 3/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 - 3/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \int \frac{A+B\,x^n}{x^4\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^4\sqrt{a+bx^n}}\,dx = \frac{2\sqrt{x^nb+a}+\left(\int \frac{\sqrt{x^nb+a}}{x^nbn\,x^4-6x^nb\,x^4+an\,x^4-6a\,x^4}dx\right)a\,n^2x^3-6\left(\int \frac{\sqrt{x^nb+a}}{x^nbn\,x^4-6x^nb\,x^4+an\,x^4-6a\,x^4}dx\right)an\,x^3}{x^3\,(n-6)}$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*b + a) + int(sqrt(x**n*b + a)/(x**n*b*n*x**4 - 6*x**n*b*x**4
+ a*n*x**4 - 6*a*x**4),x)*a*n**2*x**3 - 6*int(sqrt(x**n*b + a)/(x**n*b*n*x
**4 - 6*x**n*b*x**4 + a*n*x**4 - 6*a*x**4),x)*a*n*x**3)/(x**3*(n - 6))
```

3.358 $\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}} \, dx = \frac{2Bx^3}{b(6-n)\sqrt{a+bx^n}} + \frac{\left(\frac{A}{a}-\frac{6B}{b(6-n)}\right)x^3\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3\sqrt{a+bx^n}}$$

output
```
2*B*x^3/b/(6-n)/(a+b*x^n)^(1/2)+1/3*(A/a-6*B/b/(6-n))*x^3*(1+b*x^n/a)^(1/2
)*hypergeom([3/2, 3/n],[(3+n)/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}} \, dx = \frac{x^3\sqrt{1+\frac{bx^n}{a}}\left(A(3+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometr}\right.}{3a(3+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x^2*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(x^3*Sqrt[1 + (b*x^n)/a]*(A*(3 + n)*Hypergeometric2F1[3/2, 3/n, (3 + n)/n,
 -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[3/2, (3 + n)/n, 2 + 3/n, -((b*x
^n)/a)]))/(3*a*(3 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{b(6-n)}\right)\int \frac{x^2}{(bx^n+a)^{3/2}}dx+\frac{2Bx^3}{b(6-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(6-n)}\right)\int \frac{x^2}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}+\frac{2Bx^3}{b(6-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(6-n)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a\sqrt{a+bx^n}}+\frac{2Bx^3}{b(6-n)\sqrt{a+bx^n}}$$

input
```
Int[(x^2*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*B*x^3)/(b*(6 - n)*Sqrt[a + b*x^n]) + ((A - (6*a*B)/(b*(6 - n)))*x^3*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*
a*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(A+B\,x^n)}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.95 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{3}{n}}a^{-\frac{3}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(3/n)*a**(-3/2 - 3/n)*x**3*gamma(3/n)*hyper((3/2, 3/n), (1 + 3/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + B*a**(-5/2 - 3/n)*a**(1 + 3/
n)*x**(n + 3)*gamma(1 + 3/n)*hyper((3/2, 1 + 3/n), (2 + 3/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)x^2}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{x^2\,(A+B\,x^n)}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(3/2),x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{x^n b+a}\,x^2}{x^n b+a}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((sqrt(x**n*b + a)*x**2)/(x**n*b + a),x)
```

3.359 $\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2Bx^2}{b(4-n)\sqrt{a+bx^n}} + \frac{\left(\frac{A}{a}-\frac{4B}{b(4-n)}\right)x^2\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2\sqrt{a+bx^n}}$$

output
```
2*B*x^2/b/(4-n)/(a+b*x^n)^(1/2)+1/2*(A/a-4*B/b/(4-n))*x^2*(1+b*x^n/a)^(1/2
)*hypergeom([3/2, 2/n],[(2+n)/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{bx^n}{a}}\left(A(2+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometri}\right.}{2a(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*(A*(2 + n)*Hypergeometric2F1[3/2, 2/n, (2 + n)/n,
 -((b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[3/2, (2 + n)/n, 2*(1 + n^(-1)),
 -((b*x^n)/a)]))/(2*a*(2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{b(4-n)}\right)\int \frac{x}{(bx^n+a)^{3/2}}dx+\frac{2Bx^2}{b(4-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(4-n)}\right)\int \frac{x}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}+\frac{2Bx^2}{b(4-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(4-n)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a\sqrt{a+bx^n}}+\frac{2Bx^2}{b(4-n)\sqrt{a+bx^n}}$$

input
```
Int[(x*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*B*x^2)/(b*(4 - n)*Sqrt[a + b*x^n]) + ((A - (4*a*B)/(b*(4 - n)))*x^2*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*
a*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(A+B\,x^{n})}{(a+b\,x^{n})^{\frac{3}{2}}}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{2}{n}}a^{-\frac{3}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(2/n)*a**(-3/2 - 2/n)*x**2*gamma(2/n)*hyper((3/2, 2/n), (1 + 2/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + B*a**(-5/2 - 2/n)*a**(1 + 2/
n)*x**(n + 2)*gamma(1 + 2/n)*hyper((3/2, 1 + 2/n), (2 + 2/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{x\,(A+B\,x^n)}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n)^(3/2),x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{x^n b+a}\,x}{x^n b+a}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((sqrt(x**n*b + a)*x)/(x**n*b + a),x)
```

3.360 $\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx = \frac{2Bx}{b(2-n)\sqrt{a+bx^n}} + \frac{\left(\frac{A}{a}-\frac{2B}{b(2-n)}\right)x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{\sqrt{a+bx^n}}$$

output
```
2*B*x/b/(2-n)/(a+b*x^n)^(1/2)+(A/a-2*B/b/(2-n))*x*(1+b*x^n/a)^(1/2)*hyperg
eom([3/2, 1/n],[1+1/n],-b*x^n/a)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx = \frac{-2aBx+(2aB+Ab(-2+n))x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{ab(-2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(a + b*x^n)^(3/2),x]
```

output
```
(-2*a*B*x + (2*a*B + A*b*(-2 + n))*x*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1
[3/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*b*(-2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \; 910$$

$$\frac{(2aB-Ab(2-n))\int \frac{1}{\sqrt{bx^n+a}}dx}{abn}+\frac{2x(Ab-aB)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \; 779$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-n))\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}}dx}{abn\sqrt{a+bx^n}}+\frac{2x(Ab-aB)}{abn\sqrt{a+bx^n}}$$

$$\Big\downarrow \; 778$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{abn\sqrt{a+bx^n}}+\frac{2x(Ab-aB)}{abn\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/(a + b*x^n)^(3/2),x]
```

output
```
(2*(A*b - a*B)*x)/(a*b*n*Sqrt[a + b*x^n]) + ((2*a*B - A*b*(2 - n))*x*Sqrt[
1 + (b*x^n)/a]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(
a*b*n*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{\left(a + b\,x^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^n}{(a + bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{1}{n}}a^{-\frac{3}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(1/n)*a**(-3/2 - 1/n)*x*gamma(1/n)*hyper((3/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(-5/2 - 1/n)*a**(1 + 1/n)*
x**(n + 1)*gamma(1 + 1/n)*hyper((3/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx=\int \frac{A+B\,x^n}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n)^(3/2),x)
```

output
```
int((A + B*x^n)/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{3/2}}\,dx=\int \frac{\sqrt{x^nb+a}}{x^nb+a}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b + a),x)
```

3.361 $\int \frac{A+Bx^n}{x(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx=\frac{2(Ab-aB)}{abn\sqrt{a+bx^n}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
2*(A*b-B*a)/a/b/n/(a+b*x^n)^(1/2)-2*A*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(
3/2)/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx=-\frac{2(-Ab+aB)}{abn\sqrt{a+bx^n}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[(A + B*x^n)/(x*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*(-(A*b) + a*B))/(a*b*n*Sqrt[a + b*x^n]) - (2*A*ArcTanh[Sqrt[a + b*x^n]
/Sqrt[a]])/(a^(3/2)*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {948, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 948$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{(bx^n+a)^{3/2}}dx^n}{n}$$

$$\Big\downarrow \ 87$$

$$\frac{\frac{A\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{a}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2A\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{ab}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{2(Ab-aB)}{ab\sqrt{a+bx^n}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}}{n}$$

input
```
Int[(A + B*x^n)/(x*(a + b*x^n)^(3/2)),x]
```

output
```
((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^n]) - (2*A*ArcTanh[Sqrt[a + b*x^n]/Sqrt
[a]])/a^(3/2))/n
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2(-Ab+Ba)}{a\sqrt{a+b\,x^{n}}}}{bn}$	53
default	$\frac{-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2(-Ab+Ba)}{a\sqrt{a+b\,x^{n}}}}{bn}$	53

input
```
int((A+B*x^n)/x/(a+b*x^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(-A*b/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))-(-A*b+B*a)/a/(a+b*x^n
)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.98

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx = \left[\frac{\left(A\sqrt{a}b^2x^n+Aa^{\frac{3}{2}}b\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)-2\left(Ba^2-Aab\right)\sqrt{bx^n+a}}{a^2b^2nx^n+a^3bn}, \frac{2\left(\left(A\sqrt{\right.\right.}{}\right.$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
[((A*sqrt(a)*b^2*x^n + A*a^(3/2)*b)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a)
 + 2*a)/x^n) - 2*(B*a^2 - A*a*b)*sqrt(b*x^n + a))/(a^2*b^2*n*x^n + a^3*b*n
), 2*((A*sqrt(-a)*b^2*x^n + A*sqrt(-a)*a*b)*arctan(sqrt(-a)/sqrt(b*x^n + a
)) - (B*a^2 - A*a*b)*sqrt(b*x^n + a))/(a^2*b^2*n*x^n + a^3*b*n)]
```

Sympy [A] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}-\frac{-Ab+Ba}{an\sqrt{a+bx^n}}\right)}{b} & \text{for}\ b\neq 0\\ \frac{A\log\left(Bx^n\right)+Bx^n}{a^{\frac{3}{2}}n} & \text{otherwise}\end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n)**(3/2),x)
```

output
```
Piecewise((2*(A*b*atan(sqrt(a + b*x**n)/sqrt(-a))/(a*n*sqrt(-a)) - (-A*b +
 B*a)/(a*n*sqrt(a + b*x**n)))/b, Ne(b, 0)), ((A*log(B*x**n) + B*x**n)/(a**
(3/2)*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx = A\left(\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n}+\frac{2}{\sqrt{bx^n+a}an}\right)-\frac{2\,B}{\sqrt{bx^n+a}bn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
A*(log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n
) + 2/(sqrt(b*x^n + a)*a*n)) - 2*B/(sqrt(b*x^n + a)*b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{A+B\,x^n}{x\,\left(a+b\,x^n\right)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/(x*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{\sqrt{x^n b+a}}{x^n b x+a x}dx$$

input
```
int((A+B*x^n)/x/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x + a*x),x)
```

3.362 $\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx = -\frac{2B}{b(2+n)x\sqrt{a+bx^n}} - \frac{\left(\frac{A}{a}-\frac{2B}{b(2+n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x\sqrt{a+bx^n}}$$

output
```
-2*B/b/(2+n)/x/(a+b*x^n)^(1/2)-(A/a-2*B/b/(2+n))*(1+b*x^n/a)^(1/2)*hyperge
om([3/2, -1/n],[-(1-n)/n],-b*x^n/a)/x/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx^n}{a}}\left((A-An)\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeome}\right.}{a(-1+n)x\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^2*(a + b*x^n)^(3/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*((A - A*n)*Hypergeometric2F1[3/2, -n^(-1), (-1 + n)/n
, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[3/2, (-1 + n)/n, 2 - n^(-1), -((
b*x^n)/a)]))/(a*(-1 + n)*x*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{x^2 (bx^n+a)^{3/2}}dx-\frac{2B}{b(n+2)x\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{x^2\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}-\frac{2B}{b(n+2)x\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax\sqrt{a+bx^n}}-\frac{2B}{b(n+2)x\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/(x^2*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*B)/(b*(2 + n)*x*Sqrt[a + b*x^n]) - ((A - (2*a*B)/(b*(2 + n)))*Sqrt[1 +
 (b*x^n)/a]*Hypergeometric2F1[3/2, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2} \left(a+b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^{3/2}} \, dx = \frac{A a^{-\frac{1}{n}} a^{-\frac{3}{2}+\frac{1}{n}} \Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1 - \frac{1}{n}\right)} + \frac{B a^{-\frac{5}{2}+\frac{1}{n}} a^{1-\frac{1}{n}} x^{n-1} \Gamma\left(1 - \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, 1 - \frac{1}{n} \\ 2 - \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2 - \frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(-3/2 + 1/n)*gamma(-1/n)*hyper((3/2, -1/n), (1 - 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + B*a**(-5/2 + 1/n)*a**(1 - 1/n
)*x**(n - 1)*gamma(1 - 1/n)*hyper((3/2, 1 - 1/n), (2 - 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(2 - 1/n))
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^{3/2}} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx = \int \frac{A+B\,x^n}{x^2\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{x^n b+a}}{x^n b\,x^2+a\,x^2}dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x**2 + a*x**2),x)
```

3.363 $\int \frac{A+Bx^n}{x^3(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{3/2}}\,dx = -\frac{2B}{b(4+n)x^2\sqrt{a+bx^n}} - \frac{\left(\frac{A}{a}-\frac{4B}{b(4+n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2\sqrt{a+bx^n}}$$

output
```
-2*B/b/(4+n)/x^2/(a+b*x^n)^(1/2)-1/2*(A/a-4*B/b/(4+n))*(1+b*x^n/a)^(1/2)*h
ypergeom([3/2, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hyperge}\right.}{2a(-2+n)x^2\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^3*(a + b*x^n)^(3/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-2 + n)*Hypergeometric2F1[3/2, -2/n, (-2 + n)/n
, -((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[3/2, (-2 + n)/n, 2 - 2/n, -((
b*x^n)/a)]))/(2*a*(-2 + n)*x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3 (a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{b(n+4)}\right) \int \frac{1}{x^3 (bx^n+a)^{3/2}}dx-\frac{2B}{b(n+4)x^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(n+4)}\right)\int \frac{1}{x^3\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}-\frac{2B}{b(n+4)x^2\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{b(n+4)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a+bx^n}}-\frac{2B}{b(n+4)x^2\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/(x^3*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*B)/(b*(4 + n)*x^2*Sqrt[a + b*x^n]) - ((A - (4*a*B)/(b*(4 + n)))*Sqrt[1
 + (b*x^n)/a]*Hypergeometric2F1[3/2, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2
*a*x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{3} \left(a+b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^3 \left(a+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{3/2}}\,dx=\frac{Aa^{-\frac{2}{n}}a^{-\frac{3}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}+\frac{Ba^{-\frac{5}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(-3/2 + 2/n)*gamma(-2/n)*hyper((3/2, -2/n), (1 - 2/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + B*a**(-5/2 + 2/n)*a**(1 -
2/n)*x**(n - 2)*gamma(1 - 2/n)*hyper((3/2, 1 - 2/n), (2 - 2/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 - 2/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{3/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{3/2}}\,dx=\int \frac{A+B\,x^n}{x^3\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{3/2}}\,dx=\int \frac{\sqrt{x^n b+a}}{x^n b\,x^3+a\,x^3}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x**3 + a*x**3),x)
```

3.364 $\int \frac{A+Bx^n}{x^4(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{3/2}}\,dx = -\frac{2B}{b(6+n)x^3\sqrt{a+bx^n}} - \frac{\left(\frac{A}{a}-\frac{6B}{b(6+n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3x^3\sqrt{a+bx^n}}$$

output
```
-2*B/b/(6+n)/x^3/(a+b*x^n)^(1/2)-1/3*(A/a-6*B/b/(6+n))*(1+b*x^n/a)^(1/2)*h
ypergeom([3/2, -3/n],[-(3-n)/n],-b*x^n/a)/x^3/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hyperge}\right.}{3a(-3+n)x^3\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^4*(a + b*x^n)^(3/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-3 + n)*Hypergeometric2F1[3/2, -3/n, (-3 + n)/n
, -((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[3/2, (-3 + n)/n, 2 - 3/n, -((
b*x^n)/a)]))/(3*a*(-3 + n)*x^3*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{6aB}{b(n+6)}\right)\int \frac{1}{x^4\left(bx^n+a\right)^{3/2}}dx-\frac{2B}{b(n+6)x^3\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(n+6)}\right)\int \frac{1}{x^4\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}-\frac{2B}{b(n+6)x^3\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{6aB}{b(n+6)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3ax^3\sqrt{a+bx^n}}-\frac{2B}{b(n+6)x^3\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/(x^4*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*B)/(b*(6 + n)*x^3*Sqrt[a + b*x^n]) - ((A - (6*a*B)/(b*(6 + n)))*Sqrt[1
 + (b*x^n)/a]*Hypergeometric2F1[3/2, -3/n, -((3 - n)/n), -((b*x^n)/a)])/(3
*a*x^3*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{4} \left(a+b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{3/2}}\,dx = \frac{Aa^{-\frac{3}{n}}a^{-\frac{3}{2}+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},-\frac{3}{n}\\1-\frac{3}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)} + \frac{Ba^{-\frac{5}{2}+\frac{3}{n}}a^{1-\frac{3}{n}}x^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(-3/2 + 3/n)*gamma(-3/n)*hyper((3/2, -3/n), (1 - 3/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + B*a**(-5/2 + 3/n)*a**(1 -
3/n)*x**(n - 3)*gamma(1 - 3/n)*hyper((3/2, 1 - 3/n), (2 - 3/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 - 3/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{3/2}}\,dx = \int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{A + Bx^n}{x^4 (a + bx^n)^{3/2}} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^n}{x^4 (a + bx^n)^{3/2}} \, dx = \int \frac{A + B\,x^n}{x^4 \, (a + b\,x^n)^{3/2}} \, dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{x^4 (a + bx^n)^{3/2}} \, dx = \int \frac{\sqrt{x^n b + a}}{x^n b\, x^4 + a\, x^4} dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(3/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**n*b*x**4 + a*x**4),x)
```

3.365 $\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2Bx^3}{3b(2-n)(a+bx^n)^{3/2}} - \frac{(2aB-Ab(2-n))x^3\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2b(2-n)\sqrt{a+bx^n}}$$

output
```
2/3*B*x^3/b/(2-n)/(a+b*x^n)^(3/2)-1/3*(2*B*a-A*b*(2-n))*x^3*(1+b*x^n/a)^(1
/2)*hypergeom([5/2, 3/n],[(3+n)/n],-b*x^n/a)/a^2/b/(2-n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{x^3\sqrt{1+\frac{bx^n}{a}}\left(A(3+n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometr}\right.}{3a^2(3+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x^2*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(x^3*Sqrt[1 + (b*x^n)/a]*(A*(3 + n)*Hypergeometric2F1[5/2, 3/n, (3 + n)/n,
 -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[5/2, (3 + n)/n, 2 + 3/n, -((b*x
^n)/a)]))/(3*a^2*(3 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2Bx^3}{3b(2-n)(a+bx^n)^{3/2}}-\frac{(2aB-Ab(2-n))\int \frac{x^2}{(bx^n+a)^{5/2}}dx}{b(2-n)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2Bx^3}{3b(2-n)(a+bx^n)^{3/2}}-\frac{\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-n))\int \frac{x^2}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2b(2-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2Bx^3}{3b(2-n)(a+bx^n)^{3/2}}-\frac{x^3\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-n))\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a^2b(2-n)\sqrt{a+bx^n}}$$

input
```
Int[(x^2*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*B*x^3)/(3*b*(2 - n)*(a + b*x^n)^(3/2)) - ((2*a*B - A*b*(2 - n))*x^3*Sqr
t[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*
a^2*b*(2 - n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x^2(A + B\,x^n)}{(a + b\,x^n)^{\frac{5}{2}}} dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{3}{n}}a^{-\frac{5}{2}-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{3}{n}\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)} + \frac{Ba^{-\frac{7}{2}-\frac{3}{n}}a^{1+\frac{3}{n}}x^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(3/n)*a**(-5/2 - 3/n)*x**3*gamma(3/n)*hyper((5/2, 3/n), (1 + 3/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + B*a**(-7/2 - 3/n)*a**(1 + 3/
n)*x**(n + 3)*gamma(1 + 3/n)*hyper((5/2, 1 + 3/n), (2 + 3/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int \frac{x^2(A + Bx^n)}{(a + bx^n)^{5/2}} \, dx = \int \frac{(Bx^n + A)x^2}{(bx^n + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2(A + Bx^n)}{(a + bx^n)^{5/2}} \, dx = \int \frac{(Bx^n + A)x^2}{(bx^n + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x^2/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A + Bx^n)}{(a + bx^n)^{5/2}} \, dx = \int \frac{x^2 \, (A + B \, x^n)}{(a + b \, x^n)^{5/2}} \, dx$$

input
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(5/2),x)
```

output
```
int((x^2*(A + B*x^n))/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^n b+a}\,x^2}{x^{2n}b^2+2x^n ab+a^2}dx$$

input
```
int(x^2*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((sqrt(x**n*b + a)*x**2)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.366 $\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}} \, dx = \frac{2Bx^2}{b(4-3n)(a+bx^n)^{3/2}} + \frac{\left(A-\frac{4aB}{4b-3bn}\right)x^2\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}$$

output
```
2*B*x^2/b/(4-3*n)/(a+b*x^n)^(3/2)+1/2*(A-4*a*B/(-3*b*n+4*b))*x^2*(1+b*x^n/
a)^(1/2)*hypergeom([5/2, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}} \, dx = \frac{x^2\sqrt{1+\frac{bx^n}{a}}\left(A(2+n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hypergeometri}\right.}{2a^2(2+n)\sqrt{a+bx^n}}$$

input
```
Integrate[(x*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^n)/a]*(A*(2 + n)*Hypergeometric2F1[5/2, 2/n, (2 + n)/n,
 -((b*x^n)/a)] + 2*B*x^n*Hypergeometric2F1[5/2, (2 + n)/n, 2*(1 + n^(-1)),
 -((b*x^n)/a)]))/(2*a^2*(2 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{4b-3bn}\right)\int \frac{x}{(bx^n+a)^{5/2}}dx+\frac{2Bx^2}{b(4-3n)\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{4b-3bn}\right)\int \frac{x}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}+\frac{2Bx^2}{b(4-3n)\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{4b-3bn}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a^2\sqrt{a+bx^n}}+\frac{2Bx^2}{b(4-3n)\,(a+bx^n)^{3/2}}$$

input
```
Int[(x*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*B*x^2)/(b*(4 - 3*n)*(a + b*x^n)^(3/2)) + ((A - (4*a*B)/(4*b - 3*b*n))*x
^2*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 2/n, (2 + n)/n, -((b*x^n)/a)
])/(2*a^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{x(A + B\,x^n)}{(a + b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int(x*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.50 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{2}{n}}a^{-\frac{5}{2}-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{2}{n}\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{Ba^{-\frac{7}{2}-\frac{2}{n}}a^{1+\frac{2}{n}}x^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(2/n)*a**(-5/2 - 2/n)*x**2*gamma(2/n)*hyper((5/2, 2/n), (1 + 2/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + B*a**(-7/2 - 2/n)*a**(1 + 2/
n)*x**(n + 2)*gamma(1 + 2/n)*hyper((5/2, 1 + 2/n), (2 + 2/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)x}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*x/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{x\,(A+B\,x^n)}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((x*(A + B*x^n))/(a + b*x^n)^(5/2),x)
```

output
```
int((x*(A + B*x^n))/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{x(A + Bx^n)}{(a + bx^n)^{5/2}} \, dx = \int \frac{\sqrt{x^n b + a}\, x}{x^{2n} b^2 + 2x^n ab + a^2} dx$$

input
```
int(x*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((sqrt(x**n*b + a)*x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.367 $\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx = \frac{2Bx}{b(2-3n)\,(a+bx^n)^{3/2}} + \frac{\left(A-\frac{2aB}{2b-3bn}\right)x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2\sqrt{a+bx^n}}$$

output
```
2*B*x/b/(2-3*n)/(a+b*x^n)^(3/2)+(A-2*a*B/(-3*b*n+2*b))*x*(1+b*x^n/a)^(1/2)
*hypergeom([5/2, 1/n],[1+1/n],-b*x^n/a)/a^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx = \frac{x\left(B-\frac{(2aB+Ab(-2+3n))(a+bx^n)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{2a^2}\right)}{\left(b-\frac{3bn}{2}\right)(a+bx^n)^{3/2}}$$

input
```
Integrate[(A + B*x^n)/(a + b*x^n)^(5/2),x]
```

output
```
(x*(B - ((2*a*B + A*b*(-2 + 3*n))*(a + b*x^n)*Sqrt[1 + (b*x^n)/a]*Hypergeo
metric2F1[5/2, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(2*a^2)))/((b - (3*b*n)/
2)*(a + b*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {910, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}dx$$

$$\Big\downarrow \text{910}$$

$$\frac{(2aB-Ab(2-3n))\int \frac{1}{(bx^n+a)^{3/2}}dx}{3abn}+\frac{2x(Ab-aB)}{3abn\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-3n))\int \frac{1}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{3a^2bn\sqrt{a+bx^n}}+\frac{2x(Ab-aB)}{3abn\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt{\frac{bx^n}{a}+1}(2aB-Ab(2-3n))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{3a^2bn\sqrt{a+bx^n}}+\frac{2x(Ab-aB)}{3abn\,(a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/(a + b*x^n)^(5/2),x]
```

output
```
(2*(A*b - a*B)*x)/(3*a*b*n*(a + b*x^n)^(3/2)) + ((2*a*B - A*b*(2 - 3*n))*x
*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, n^(-1), 1 + n^(-1), -((b*x^n)/
a)])/(3*a^2*b*n*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 910
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Simp[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1), x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/
n + p, 0])
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{\left(a + b\,x^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^n}{(a + bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.59 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{1}{n}}a^{-\frac{5}{2}-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{1}{n}\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{7}{2}-\frac{1}{n}}a^{1+\frac{1}{n}}x^{n+1}\Gamma\left(1+\frac{1}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(1/n)*a**(-5/2 - 1/n)*x*gamma(1/n)*hyper((5/2, 1/n), (1 + 1/n,), b*x*
*n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + B*a**(-7/2 - 1/n)*a**(1 + 1/n)*
x**(n + 1)*gamma(1 + 1/n)*hyper((5/2, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx=\int \frac{A+B\,x^n}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/(a + b*x^n)^(5/2),x)
```

output
```
int((A + B*x^n)/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(a+bx^n)^{5/2}}\,dx=\int \frac{\sqrt{x^nb+a}}{x^{2n}b^2+2x^nab+a^2}dx$$

input
```
int((A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.368 $\int \frac{A+Bx^n}{x(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}\,dx=\frac{2(Ab-aB)}{3abn\left(a+bx^n\right)^{3/2}}+\frac{2A}{a^2n\sqrt{a+bx^n}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
2/3*(A*b-B*a)/a/b/n/(a+b*x^n)^(3/2)+2*A/a^2/n/(a+b*x^n)^(1/2)-2*A*arctanh(
(a+b*x^n)^(1/2)/a^(1/2))/a^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}\,dx=-\frac{2(-aAb+a^2B-3Ab(a+bx^n))}{3a^2bn\left(a+bx^n\right)^{3/2}}-\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

input
```
Integrate[(A + B*x^n)/(x*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*(-(a*A*b) + a^2*B - 3*A*b*(a + b*x^n)))/(3*a^2*b*n*(a + b*x^n)^(3/2))
- (2*A*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(Bx^n+A)}{(bx^n+a)^{5/2}}dx^n}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{A\int \frac{x^{-n}}{(bx^n+a)^{3/2}}dx^n}{a}+\frac{2(Ab-aB)}{3ab(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{A\left(\frac{\int \frac{x^{-n}}{\sqrt{bx^n+a}}dx^n}{a}+\frac{2}{a\sqrt{a+bx^n}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{A\left(\frac{2\int \frac{1}{\frac{x^{2n}}{b}-\frac{a}{b}}d\sqrt{bx^n+a}}{ab}+\frac{2}{a\sqrt{a+bx^n}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{A\left(\frac{2}{a\sqrt{a+bx^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(Ab-aB)}{3ab(a+bx^n)^{3/2}}}{n}$$

input
```
Int[(A + B*x^n)/(x*(a + b*x^n)^(5/2)),x]
```

output
```
((2*(A*b - a*B))/(3*a*b*(a + b*x^n)^(3/2)) + (A*(2/(a*Sqrt[a + b*x^n]) - (
2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/a^(3/2)))/a)/n
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{2(-Ab+Ba)}{3a(a+b\,x^n)^{\frac{3}{2}}}+\frac{2Ab}{a^2\sqrt{a+b\,x^n}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{bn}$	68
default	$\frac{-\frac{2(-Ab+Ba)}{3a(a+b\,x^n)^{\frac{3}{2}}}+\frac{2Ab}{a^2\sqrt{a+b\,x^n}}-\frac{2Ab\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,x^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{bn}$	68

input
```
int((A+B*x^n)/x/(a+b*x^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/n/b*(-1/3*(-A*b+B*a)/a/(a+b*x^n)^(3/2)+A*b/a^2/(a+b*x^n)^(1/2)-A*b/a^(5/
2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.27

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}\,dx = \left[\frac{3\left(A\sqrt{a}b^3x^{2n}+2\,Aa^{\frac{3}{2}}b^2x^n+Aa^{\frac{5}{2}}b\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2\,a}{x^n}\right)+2\left(3\,Aab^2x^n-Ba\right.}{3\left(a^3b^3nx^{2n}+2\,a^4b^2nx^n+a^5bn\right)}\right.$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(A*sqrt(a)*b^3*x^(2*n) + 2*A*a^(3/2)*b^2*x^n + A*a^(5/2)*b)*log((b
*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(3*A*a*b^2*x^n - B*a^3 +
4*A*a^2*b)*sqrt(b*x^n + a))/(a^3*b^3*n*x^(2*n) + 2*a^4*b^2*n*x^n + a^5*b*n
), 2/3*(3*(A*sqrt(-a)*b^3*x^(2*n) + 2*A*sqrt(-a)*a*b^2*x^n + A*sqrt(-a)*a^
2*b)*arctan(sqrt(-a)/sqrt(b*x^n + a)) + (3*A*a*b^2*x^n - B*a^3 + 4*A*a^2*b
)*sqrt(b*x^n + a))/(a^3*b^3*n*x^(2*n) + 2*a^4*b^2*n*x^n + a^5*b*n)]
```

Sympy [A] (verification not implemented)

Time = 29.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab}{a^2n\sqrt{a+bx^n}}+\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^n}}{\sqrt{-a}}\right)}{a^2n\sqrt{-a}}-\frac{-Ab+Ba}{3an(a+bx^n)^{\frac{3}{2}}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^n\right)+Bx^n}{a^{\frac{5}{2}}n} & \text{otherwise} \end{cases}$$

input
```
integrate((A+B*x**n)/x/(a+b*x**n)**(5/2),x)
```

output
```
Piecewise((2*(A*b/(a**2*n*sqrt(a + b*x**n)) + A*b*atan(sqrt(a + b*x**n)/sq
rt(-a))/(a**2*n*sqrt(-a)) - (-A*b + B*a)/(3*a*n*(a + b*x**n)**(3/2)))/b, N
e(b, 0)), ((A*log(B*x**n) + B*x**n)/(a**(5/2)*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^n}{x\left(a+bx^n\right)^{5/2}}\,dx = \frac{1}{3}A\left(\frac{3\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}n}+\frac{2\left(3\,bx^n+4\,a\right)}{\left(bx^n+a\right)^{\frac{3}{2}}a^2n}\right)-\frac{2\,B}{3\left(bx^n+a\right)^{\frac{3}{2}}bn}$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
1/3*A*(3*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(
5/2)*n) + 2*(3*b*x^n + 4*a)/((b*x^n + a)^(3/2)*a^2*n)) - 2/3*B/((b*x^n + a
)^(3/2)*b*n)
```

Giac [F]

$$\int \frac{A+Bx^n}{x(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((A+B*x^n)/x/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x(a+bx^n)^{5/2}}\,dx = \int \frac{A+B\,x^n}{x\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/(x*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/(x*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^nb+a}}{x^{2n}b^2x+2x^nabx+a^2x}dx$$

input
```
int((A+B*x^n)/x/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x + 2*x**n*a*b*x + a**2*x),x)
```

3.369 $\int \frac{A+Bx^n}{x^2(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^{5/2}}\,dx=-\frac{2B}{b(2+3n)x\left(a+bx^n\right)^{3/2}}-\frac{\left(A-\frac{2aB}{2b+3bn}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2x\sqrt{a+bx^n}}$$

output
```
-2*B/b/(2+3*n)/x/(a+b*x^n)^(3/2)-(A-2*a*B/(3*b*n+2*b))*(1+b*x^n/a)^(1/2)*h
ypergeom([5/2, -1/n],[-(1-n)/n],-b*x^n/a)/a^2/x/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^n}{x^2\left(a+bx^n\right)^{5/2}}\,dx=\frac{\sqrt{1+\frac{bx^n}{a}}\left((A-An)\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeome}\right.}{a^2(-1+n)x\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^2*(a + b*x^n)^(5/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*((A - A*n)*Hypergeometric2F1[5/2, -n^(-1), (-1 + n)/n
, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[5/2, (-1 + n)/n, 2 - n^(-1), -((
b*x^n)/a)]))/(a^2*(-1 + n)*x*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^{5/2}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{3bn+2b}\right)\int \frac{1}{x^2 (bx^n+a)^{5/2}}dx-\frac{2B}{b(3n+2)x (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{3bn+2b}\right)\int \frac{1}{x^2\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}-\frac{2B}{b(3n+2)x (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{3bn+2b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2x\sqrt{a+bx^n}}-\frac{2B}{b(3n+2)x (a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/(x^2*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*B)/(b*(2 + 3*n)*x*(a + b*x^n)^(3/2)) - ((A - (2*a*B)/(2*b + 3*b*n))*Sq
rt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -n^(-1), -((1 - n)/n), -((b*x^n)/
a)])/(a^2*x*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{2} \left(a+b\,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^2 (a+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 47.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^{5/2}} \, dx = \frac{A a^{-\frac{1}{n}} a^{-\frac{5}{2}+\frac{1}{n}} \Gamma\left(-\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{5}{2}, -\frac{1}{n} \\ 1 - \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{nx\Gamma\left(1 - \frac{1}{n}\right)} + \frac{B a^{-\frac{7}{2}+\frac{1}{n}} a^{1-\frac{1}{n}} x^{n-1} \Gamma\left(1 - \frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{5}{2}, 1 - \frac{1}{n} \\ 2 - \frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2 - \frac{1}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**2/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(-5/2 + 1/n)*gamma(-1/n)*hyper((5/2, -1/n), (1 - 1/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(1/n)*n*x*gamma(1 - 1/n)) + B*a**(-7/2 + 1/n)*a**(1 - 1/n
)*x**(n - 1)*gamma(1 - 1/n)*hyper((5/2, 1 - 1/n), (2 - 1/n,), b*x**n*exp_p
olar(I*pi)/a)/(n*gamma(2 - 1/n))
```

Maxima [F]

$$\int \frac{A + Bx^n}{x^2 (a + bx^n)^{5/2}} \, dx = \int \frac{Bx^n + A}{(bx^n + a)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{5/2}}\,dx = \int \frac{A+B\,x^n}{x^2\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/(x^2*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^2(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{x^nb+a}}{x^{2n}b^2x^2+2x^nab\,x^2+a^2x^2}\,dx$$

input
```
int((A+B*x^n)/x^2/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x**2 + 2*x**n*a*b*x**2 + a**2*x**2),x)
```

3.370 $\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{5/2}}\,dx = -\frac{2B}{b(4+3n)x^2\left(a+bx^n\right)^{3/2}} - \frac{\left(A-\frac{4aB}{4b+3bn}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}$$

output
```
-2*B/b/(4+3*n)/x^2/(a+b*x^n)^(3/2)-1/2*(A-4*a*B/(3*b*n+4*b))*(1+b*x^n/a)^(
1/2)*hypergeom([5/2, -2/n],[-(2-n)/n],-b*x^n/a)/a^2/x^2/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^n}{x^3\left(a+bx^n\right)^{5/2}}\,dx = \frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-2+n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2Bx^n\,\text{Hyperge}\right.}{2a^2(-2+n)x^2\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^3*(a + b*x^n)^(5/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-2 + n)*Hypergeometric2F1[5/2, -2/n, (-2 + n)/n
, -((b*x^n)/a)]) + 2*B*x^n*Hypergeometric2F1[5/2, (-2 + n)/n, 2 - 2/n, -((
b*x^n)/a)]))/(2*a^2*(-2 + n)*x^2*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^3 (a+bx^n)^{5/2}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{4aB}{3bn+4b}\right)\int \frac{1}{x^3 (bx^n+a)^{5/2}}dx-\frac{2B}{b(3n+4)x^2 (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{3bn+4b}\right)\int \frac{1}{x^3\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}-\frac{2B}{b(3n+4)x^2 (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{4aB}{3bn+4b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2x^2\sqrt{a+bx^n}}-\frac{2B}{b(3n+4)x^2 (a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/(x^3*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*B)/(b*(4 + 3*n)*x^2*(a + b*x^n)^(3/2)) - ((A - (4*a*B)/(4*b + 3*b*n))*
Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -2/n, -((2 - n)/n), -((b*x^n)/a
)])/(2*a^2*x^2*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{x^{3} \left(a + b\,x^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^n}{x^3 \left(a + bx^n\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 112.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx = \frac{Aa^{-\frac{2}{n}}a^{-\frac{5}{2}+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},-\frac{2}{n}\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{Ba^{-\frac{7}{2}+\frac{2}{n}}a^{1-\frac{2}{n}}x^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)}$$

input
```
integrate((A+B*x**n)/x**3/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(-5/2 + 2/n)*gamma(-2/n)*hyper((5/2, -2/n), (1 - 2/n,), b*x**n*exp_po
lar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + B*a**(-7/2 + 2/n)*a**(1 -
2/n)*x**(n - 2)*gamma(1 - 2/n)*hyper((5/2, 1 - 2/n), (2 - 2/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 - 2/n))
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx=\int \frac{A+B\,x^n}{x^3\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/(x^3*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^3(a+bx^n)^{5/2}}\,dx=\int \frac{\sqrt{x^nb+a}}{x^{2n}b^2x^3+2x^nab\,x^3+a^2x^3}dx$$

input
```
int((A+B*x^n)/x^3/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x**3 + 2*x**n*a*b*x**3 + a**2*x**3),x)
```

3.371 $\int \frac{A+Bx^n}{x^4(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{5/2}}\,dx=-\frac{2B}{3b(2+n)x^3\left(a+bx^n\right)^{3/2}}+\frac{(2aB-Ab(2+n))\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3a^2b(2+n)x^3\sqrt{a+bx^n}}$$

output
```
-2/3*B/b/(2+n)/x^3/(a+b*x^n)^(3/2)+1/3*(2*B*a-A*b*(2+n))*(1+b*x^n/a)^(1/2)
*hypergeom([5/2, -3/n],[-(3-n)/n],-b*x^n/a)/a^2/b/(2+n)/x^3/(a+b*x^n)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{5/2}}\,dx=\frac{\sqrt{1+\frac{bx^n}{a}}\left(-A(-3+n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{n},\frac{-3+n}{n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hyperge}\right.}{3a^2(-3+n)x^3\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(x^4*(a + b*x^n)^(5/2)),x]
```

output
```
(Sqrt[1 + (b*x^n)/a]*(-(A*(-3 + n)*Hypergeometric2F1[5/2, -3/n, (-3 + n)/n
, -((b*x^n)/a)]) + 3*B*x^n*Hypergeometric2F1[5/2, (-3 + n)/n, 2 - 3/n, -((
b*x^n)/a)]))/(3*a^2*(-3 + n)*x^3*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^{5/2}} \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{x^4 (bx^n+a)^{5/2}}dx-\frac{2B}{3b(n+2)x^3 (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\int \frac{1}{x^4\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}-\frac{2B}{3b(n+2)x^3 (a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB}{b(n+2)}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{n},-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{3a^2x^3\sqrt{a+bx^n}}-\frac{2B}{3b(n+2)x^3 (a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/(x^4*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*B)/(3*b*(2 + n)*x^3*(a + b*x^n)^(3/2)) - ((A - (2*a*B)/(b*(2 + n)))*Sq
rt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -3/n, -((3 - n)/n), -((b*x^n)/a)]
)/(3*a^2*x^3*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{x^{4} \left(a+b\,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{x^4 (a+bx^n)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4(a+bx^n)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*x**n)/x**4/(a+b*x**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{x^4(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{x^4(a+bx^n)^{5/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{A+B\,x^n}{x^4\,\left(a+b\,x^n\right)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/(x^4*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{x^4\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{\sqrt{x^n b+a}}{x^{2n}b^2x^4+2x^n ab\,x^4+a^2x^4}dx$$

input
```
int((A+B*x^n)/x^4/(a+b*x^n)^(5/2),x)
```

output
```
int(sqrt(x**n*b + a)/(x**(2*n)*b**2*x**4 + 2*x**n*a*b*x**4 + a**2*x**4),x)
```

3.372 $\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx=\frac{2B(ex)^{5/2}(a+bx^n)^{3/2}}{be(5+3n)}+\frac{2\left(A-\frac{5aB}{5b+3bn}\right)(ex)^{5/2}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*(e*x)^(5/2)*(a+b*x^n)^(3/2)/b/e/(5+3*n)+2/5*(A-5*a*B/(3*b*n+5*b))*(e*x
)^(5/2)*(a+b*x^n)^(1/2)*hypergeom([-1/2, 5/2/n],[1+5/2/n],-b*x^n/a)/e/(1+b
*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx=\frac{2x(ex)^{3/2}\sqrt{a+bx^n}\left(A(5+2n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)+5Bx^n\,\text{Hypergeo}\right.}{5(5+2n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[a + b*x^n]*(A*(5 + 2*n)*Hypergeometric2F1[-1/2, 5/(2
*n), 1 + 5/(2*n), -((b*x^n)/a)] + 5*B*x^n*Hypergeometric2F1[-1/2, (5/2 + n
)/n, 2 + 5/(2*n), -((b*x^n)/a)]))/(5*(5 + 2*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{5aB}{3bn+5b}\right)\int (ex)^{3/2}\sqrt{bx^n+a}dx+\frac{2B(ex)^{5/2}\left(a+bx^n\right)^{3/2}}{be(3n+5)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{5aB}{3bn+5b}\right)\int (ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{5/2}\left(a+bx^n\right)^{3/2}}{be(3n+5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{5/2}\sqrt{a+bx^n}\left(A-\frac{5aB}{3bn+5b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{5/2}\left(a+bx^n\right)^{3/2}}{be(3n+5)}$$

input
```
Int[(e*x)^(3/2)*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*(e*x)^(5/2)*(a + b*x^n)^(3/2))/(b*e*(5 + 3*n)) + (2*(A - (5*a*B)/(5*b
 + 3*b*n))*(e*x)^(5/2)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 5/(2*n), 1
+ 5/(2*n), -((b*x^n)/a)])/(5*e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^{\frac{3}{2}} \sqrt{a+b\,x^{n}}\,(A+B\,x^{n})\,dx$$

input
```
int((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\ dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\ dx = \frac{Aa^{\frac{5}{2n}}a^{\frac{1}{2}-\frac{5}{2n}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{2n}\\1+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{5}{2n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{5}{2n}}a^{1+\frac{5}{2n}}e^{\frac{3}{2}}x^{n+\frac{5}{2}}\Gamma\left(1+\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{5}{2n}\\2+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{5}{2n}\right)}$$

input
```
integrate((e*x)**(3/2)*(a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(5/(2*n))*a**(1/2 - 5/(2*n))*e**(3/2)*x**(5/2)*gamma(5/(2*n))*hyper((
-1/2, 5/(2*n)), (1 + 5/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 5/(
2*n))) + B*a**(-1/2 - 5/(2*n))*a**(1 + 5/(2*n))*e**(3/2)*x**(n + 5/2)*gamm
a(1 + 5/(2*n))*hyper((-1/2, 1 + 5/(2*n)), (2 + 5/(2*n),), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(2 + 5/(2*n)))
```

Maxima [F]

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (e\,x)^{3/2}\,(A+B\,x^n)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int((e*x)^(3/2)*(A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int((e*x)^(3/2)*(A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int (ex)^{3/2}\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{\sqrt{e}\,e\Big(2x^{n+\frac{1}{2}}\sqrt{x^n b+a}\,bn\,x^2+10x^{n+\frac{1}{2}}\sqrt{x^n b+a}\,b\,x^2+8\sqrt{x}\,\sqrt{x^n b+a}\,an\,x^2+10\sqrt{x}\,\sqrt{x^n b+a}}{}$$

input

```
int((e*x)^(3/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output

```
(sqrt(e)*e*(2*x**((2*n + 1)/2)*sqrt(x**n*b + a)*b*n*x**2 + 10*x**((2*n + 1
)/2)*sqrt(x**n*b + a)*b*x**2 + 8*sqrt(x)*sqrt(x**n*b + a)*a*n*x**2 + 10*sq
rt(x)*sqrt(x**n*b + a)*a*x**2 + 9*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n
*b*n**2 + 20*x**n*b*n + 25*x**n*b + 3*a*n**2 + 20*a*n + 25*a),x)*a**2*n**4
 + 60*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n*b*n**2 + 20*x**n*b*n + 25*x
**n*b + 3*a*n**2 + 20*a*n + 25*a),x)*a**2*n**3 + 75*int((sqrt(x)*sqrt(x**n
*b + a)*x)/(3*x**n*b*n**2 + 20*x**n*b*n + 25*x**n*b + 3*a*n**2 + 20*a*n +
25*a),x)*a**2*n**2))/(3*n**2 + 20*n + 25)
```

3.373 $\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{2B(ex)^{3/2}\left(a+bx^n\right)^{3/2}}{3be(1+n)} + \frac{2\left(A-\frac{aB}{b+bn}\right)(ex)^{3/2}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2/3*B*(e*x)^(3/2)*(a+b*x^n)^(3/2)/b/e/(1+n)+2/3*(A-a*B/(b*n+b))*(e*x)^(3/2
)*(a+b*x^n)^(1/2)*hypergeom([-1/2, 3/2/n],[1+3/2/n],-b*x^n/a)/e/(1+b*x^n/a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{2x\sqrt{ex}\sqrt{a+bx^n}\left(A(3+2n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(\right.\right.}{3(3+2n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[e*x]*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[a + b*x^n]*(A*(3 + 2*n)*Hypergeometric2F1[-1/2, 3/(2*n
), 1 + 3/(2*n), -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[-1/2, (3/2 + n)/
n, 2 + 3/(2*n), -((b*x^n)/a)]))/(3*(3 + 2*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{bn+b}\right)\int \sqrt{ex}\sqrt{bx^n+a}dx+\frac{2B(ex)^{3/2}\left(a+bx^n\right)^{3/2}}{3be(n+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{aB}{bn+b}\right)\int \sqrt{ex}\sqrt{\frac{bx^n}{a}+1}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{3/2}\left(a+bx^n\right)^{3/2}}{3be(n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{3/2}\sqrt{a+bx^n}\left(A-\frac{aB}{bn+b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{3/2}\left(a+bx^n\right)^{3/2}}{3be(n+1)}$$

input
```
Int[Sqrt[e*x]*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*(e*x)^(3/2)*(a + b*x^n)^(3/2))/(3*b*e*(1 + n)) + (2*(A - (a*B)/(b + b
*n))*(e*x)^(3/2)*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 3/(2*n), 1 + 3/(2
*n), -((b*x^n)/a)])/(3*e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{ex}\,\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})\,dx$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.62 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.29

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{Aa^{\frac{3}{2n}}a^{\frac{1}{2}-\frac{3}{2n}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{3}{2n} \\ 1+\frac{3}{2n} \end{matrix}\middle| \frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},1+\frac{3}{2n} \\ 2+\frac{3}{2n} \end{matrix}\middle| \frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(3/(2*n))*a**(1/2 - 3/(2*n))*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper((-
1/2, 3/(2*n)), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/(2
*n))) + B*a**(-1/2 - 3/(2*n))*a**(1 + 3/(2*n))*sqrt(e)*x**(n + 3/2)*gamma(
1 + 3/(2*n))*hyper((-1/2, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \int (Bx^n+A)\sqrt{bx^n+a}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*sqrt(e*x), x)
```

Giac [F]

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \int (Bx^n+A)\sqrt{bx^n+a}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx = \int \sqrt{e\,x}\,\left(A+B\,x^n\right)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int((e*x)^(1/2)*(A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int((e*x)^(1/2)*(A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{ex}\sqrt{a+bx^n}(A+Bx^n)\,dx$$
$$=\frac{\sqrt{e}\left(2x^{n+\frac{1}{2}}\sqrt{x^nb+a}\,bnx+6x^{n+\frac{1}{2}}\sqrt{x^nb+a}\,bx+8\sqrt{x}\,\sqrt{x^nb+a}\,anx+6\sqrt{x}\,\sqrt{x^nb+a}\,ax+3\left(\int \frac{}{x^nbn^2+4}\right.\right.}{}$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
(sqrt(e)*(2*x**((2*n + 1)/2)*sqrt(x**n*b + a)*b*n*x + 6*x**((2*n + 1)/2)*s
qrt(x**n*b + a)*b*x + 8*sqrt(x)*sqrt(x**n*b + a)*a*n*x + 6*sqrt(x)*sqrt(x*
*n*b + a)*a*x + 3*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*n**2 + 4*x**n*b*n
 + 3*x**n*b + a*n**2 + 4*a*n + 3*a),x)*a**2*n**4 + 12*int((sqrt(x)*sqrt(x*
*n*b + a))/(x**n*b*n**2 + 4*x**n*b*n + 3*x**n*b + a*n**2 + 4*a*n + 3*a),x)
*a**2*n**3 + 9*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*n**2 + 4*x**n*b*n +
3*x**n*b + a*n**2 + 4*a*n + 3*a),x)*a**2*n**2))/(3*(n**2 + 4*n + 3))
```

3.374 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{2B\sqrt{ex}(a+bx^n)^{3/2}}{be(1+3n)} + \frac{2\left(A-\frac{aB}{b+3bn}\right)\sqrt{ex}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*(e*x)^(1/2)*(a+b*x^n)^(3/2)/b/e/(1+3*n)+2*(A-a*B/(3*b*n+b))*(e*x)^(1/2
)*(a+b*x^n)^(1/2)*hypergeom([-1/2, 1/2/n],[1+1/2/n],-b*x^n/a)/e/(1+b*x^n/a
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{2x\sqrt{a+bx^n}\Big((A+2An)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\Big(-\frac{1}{2}}{(1+2n)\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/Sqrt[e*x],x]
```

output
```
(2*x*Sqrt[a + b*x^n]*((A + 2*A*n)*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[-1/2, (1/2 + n)/n, (4 + n
^(-1))/2, -((b*x^n)/a)]))/((1 + 2*n)*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{3bn+b}\right)\int \frac{\sqrt{bx^n+a}}{\sqrt{ex}}dx+\frac{2B\sqrt{ex}(a+bx^n)^{3/2}}{be(3n+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{aB}{3bn+b}\right)\int \frac{\sqrt{\frac{bx^n}{a}+1}}{\sqrt{ex}}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B\sqrt{ex}(a+bx^n)^{3/2}}{be(3n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}\sqrt{a+bx^n}\left(A-\frac{aB}{3bn+b}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B\sqrt{ex}(a+bx^n)^{3/2}}{be(3n+1)}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/Sqrt[e*x],x]
```

output
```
(2*B*Sqrt[e*x]*(a + b*x^n)^(3/2))/(b*e*(1 + 3*n)) + (2*(A - (a*B)/(b + 3*b
*n))*Sqrt[e*x]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^(-1
))/2, -((b*x^n)/a)])/(e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{\sqrt{ex}}dx$$

input

```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

output

```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{1}{2n}}a^{\frac{1}{2}-\frac{1}{2n}}\sqrt{x}\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2n}\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{Ba^{-\frac{1}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}x^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/(e*x)**(1/2),x)
```

output
```
A*a**(1/(2*n))*a**(1/2 - 1/(2*n))*sqrt(x)*gamma(1/(2*n))*hyper((-1/2, 1/(2
*n)), (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(2*
n))) + B*a**(-1/2 - 1/(2*n))*a**(1 + 1/(2*n))*x**(n + 1/2)*gamma(1 + 1/(2*
n))*hyper((-1/2, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(
sqrt(e)*n*gamma(2 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{\sqrt{e}\left(2x^{n+\frac{1}{2}}\sqrt{x^nb+a}\,bn+2x^{n+\frac{1}{2}}\sqrt{x^nb+a}\,b+8\sqrt{x}\,\sqrt{x^nb+a}\,an+2\sqrt{x}\,\sqrt{x^nb+a}\,a+9\left(\int \frac{}{3x^nbn^2x+4x^nbn}\right.\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*(2*x**((2*n + 1)/2)*sqrt(x**n*b + a)*b*n + 2*x**((2*n + 1)/2)*sqr
t(x**n*b + a)*b + 8*sqrt(x)*sqrt(x**n*b + a)*a*n + 2*sqrt(x)*sqrt(x**n*b +
 a)*a + 9*int((sqrt(x)*sqrt(x**n*b + a))/(3*x**n*b*n**2*x + 4*x**n*b*n*x +
 x**n*b*x + 3*a*n**2*x + 4*a*n*x + a*x),x)*a**2*n**4 + 12*int((sqrt(x)*sqr
t(x**n*b + a))/(3*x**n*b*n**2*x + 4*x**n*b*n*x + x**n*b*x + 3*a*n**2*x + 4
*a*n*x + a*x),x)*a**2*n**3 + 3*int((sqrt(x)*sqrt(x**n*b + a))/(3*x**n*b*n*
*2*x + 4*x**n*b*n*x + x**n*b*x + 3*a*n**2*x + 4*a*n*x + a*x),x)*a**2*n**2)
)/(e*(3*n**2 + 4*n + 1))
```

3.375 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = -\frac{2B(a+bx^n)^{3/2}}{be(1-3n)\sqrt{ex}} - \frac{2\left(A-\frac{aB}{b-3bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(3/2)/b/e/(1-3*n)/(e*x)^(1/2)-2*(A-a*B/(-3*b*n+b))*(a+b*x^n
)^(1/2)*hypergeom([-1/2, -1/2/n],[1-1/2/n],-b*x^n/a)/e/(e*x)^(1/2)/(1+b*x^
n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^n}\left(Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1-\frac{1}{2n},2-\frac{1}{2n},-\frac{bx^n}{a}\right)+A(1-2\right.}{(-1+2n)(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/(e*x)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^n]*(B*x^n*Hypergeometric2F1[-1/2, 1 - 1/(2*n), 2 - 1/(2*
n), -((b*x^n)/a)] + A*(1 - 2*n)*Hypergeometric2F1[-1/2, -1/2*1/n, 1 - 1/(2
*n), -((b*x^n)/a)]))/((-1 + 2*n)*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{b-3bn}\right)\int \frac{\sqrt{bx^n+a}}{(ex)^{3/2}}dx-\frac{2B(a+bx^n)^{3/2}}{be(1-3n)\sqrt{ex}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^n}\left(A-\frac{aB}{b-3bn}\right)\int \frac{\sqrt{\frac{bx^n}{a}+1}}{(ex)^{3/2}}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{be(1-3n)\sqrt{ex}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{a+bx^n}\left(A-\frac{aB}{b-3bn}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{be(1-3n)\sqrt{ex}}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/(e*x)^(3/2),x]
```

output
```
(-2*B*(a + b*x^n)^(3/2))/(b*e*(1 - 3*n)*Sqrt[e*x]) - (2*(A - (a*B)/(b - 3*
b*n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -1/2*1/n, 1 - 1/(2*n), -((b*
x^n)/a)])/(e*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx=\frac{Aa^{-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}\Gamma\left(-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{2n}\\1-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\sqrt{x}\Gamma\left(1-\frac{1}{2n}\right)}+\frac{Ba^{-\frac{1}{2}+\frac{1}{2n}}a^{1-\frac{1}{2n}}x^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{2n}\\2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/(e*x)**(3/2),x)
```

output
```
A*a**(1/2 + 1/(2*n))*gamma(-1/(2*n))*hyper((-1/2, -1/(2*n)), (1 - 1/(2*n),
), b*x**n*exp_polar(I*pi)/a)/(a**(1/(2*n))*e**(3/2)*n*sqrt(x)*gamma(1 - 1/
(2*n))) + B*a**(-1/2 + 1/(2*n))*a**(1 - 1/(2*n))*x**(n - 1/2)*gamma(1 - 1/
(2*n))*hyper((-1/2, 1 - 1/(2*n)), (2 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a
)/(e**(3/2)*n*gamma(2 - 1/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(2x^n\sqrt{x^nb+a}\,bn-2x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-2\sqrt{x^nb+a}\,a+9\sqrt{}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(2*x**n*sqrt(x**n*b + a)*b*n - 2*x**n*sqrt(x**n*b + a)*b + 8*sqrt
(x**n*b + a)*a*n - 2*sqrt(x**n*b + a)*a + 9*sqrt(x)*int((sqrt(x)*sqrt(x**n
*b + a))/(3*x**n*b*n**2*x**2 - 4*x**n*b*n*x**2 + x**n*b*x**2 + 3*a*n**2*x*
*2 - 4*a*n*x**2 + a*x**2),x)*a**2*n**4 - 12*sqrt(x)*int((sqrt(x)*sqrt(x**n
*b + a))/(3*x**n*b*n**2*x**2 - 4*x**n*b*n*x**2 + x**n*b*x**2 + 3*a*n**2*x*
*2 - 4*a*n*x**2 + a*x**2),x)*a**2*n**3 + 3*sqrt(x)*int((sqrt(x)*sqrt(x**n*
b + a))/(3*x**n*b*n**2*x**2 - 4*x**n*b*n*x**2 + x**n*b*x**2 + 3*a*n**2*x**
2 - 4*a*n*x**2 + a*x**2),x)*a**2*n**2))/(sqrt(x)*e**2*(3*n**2 - 4*n + 1))
```

3.376 $\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 128

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = -\frac{2B(a+bx^n)^{3/2}}{3be(1-n)(ex)^{3/2}} + \frac{2(aB-Ab(1-n))\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3be(1-n)(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2/3*B*(a+b*x^n)^(3/2)/b/e/(1-n)/(e*x)^(3/2)+2/3*(B*a-A*b*(1-n))*(a+b*x^n)
^(1/2)*hypergeom([-1/2, -3/2/n],[1-3/2/n],-b*x^n/a)/b/e/(1-n)/(e*x)^(3/2)/
(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \frac{2x\sqrt{a+bx^n}\left(3Bx^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1-\frac{3}{2n},2-\frac{3}{2n},-\frac{bx^n}{a}\right)+A(3-\right.}{3(-3+2n)(ex)^{5/2}\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^n]*(A + B*x^n))/(e*x)^(5/2),x]
```

output
```
(2*x*Sqrt[a + b*x^n]*(3*B*x^n*Hypergeometric2F1[-1/2, 1 - 3/(2*n), 2 - 3/(
2*n), -((b*x^n)/a)] + A*(3 - 2*n)*Hypergeometric2F1[-1/2, -3/(2*n), 1 - 3/
(2*n), -((b*x^n)/a)]))/(3*(-3 + 2*n)*(e*x)^(5/2)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 959}$$

$$-\frac{(aB-Ab(1-n))\int \frac{\sqrt{bx^n+a}}{(ex)^{5/2}}dx}{b(1-n)}-\frac{2B(a+bx^n)^{3/2}}{3be(1-n)(ex)^{3/2}}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{\sqrt{a+bx^n}(aB-Ab(1-n))\int \frac{\sqrt{\frac{bx^n}{a}+1}}{(ex)^{5/2}}dx}{b(1-n)\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{3be(1-n)(ex)^{3/2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2\sqrt{a+bx^n}(aB-Ab(1-n))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3be(1-n)(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{3/2}}{3be(1-n)(ex)^{3/2}}$$

input
```
Int[(Sqrt[a + b*x^n]*(A + B*x^n))/(e*x)^(5/2),x]
```

output
```
(-2*B*(a + b*x^n)^(3/2))/(3*b*e*(1 - n)*(e*x)^(3/2)) + (2*(a*B - A*b*(1 -
n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, -3/(2*n), 1 - 3/(2*n), -((b*x^
n)/a)])/(3*b*e*(1 - n)*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{a+b\,x^{n}}\,(A+B\,x^{n})}{(ex)^{\frac{5}{2}}}dx$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

output
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \frac{Aa^{-\frac{3}{2n}}a^{\frac{1}{2}+\frac{3}{2n}}\Gamma\left(-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{2n}\\1-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}nx^{\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right)} + \frac{Ba^{-\frac{1}{2}+\frac{3}{2n}}a^{1-\frac{3}{2n}}x^{n-\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{2n}\\2-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}n\Gamma\left(2-\frac{3}{2n}\right)}$$

input
```
integrate((a+b*x**n)**(1/2)*(A+B*x**n)/(e*x)**(5/2),x)
```

output
```
A*a**(1/2 + 3/(2*n))*gamma(-3/(2*n))*hyper((-1/2, -3/(2*n)), (1 - 3/(2*n),
), b*x**n*exp_polar(I*pi)/a)/(a**(3/(2*n))*e**(5/2)*n*x**(3/2)*gamma(1 - 3
/(2*n))) + B*a**(-1/2 + 3/(2*n))*a**(1 - 3/(2*n))*x**(n - 3/2)*gamma(1 - 3
/(2*n))*hyper((-1/2, 1 - 3/(2*n)), (2 - 3/(2*n),), b*x**n*exp_polar(I*pi)/
a)/(e**(5/2)*n*gamma(2 - 3/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/(e*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{bx^n+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)/(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \int \frac{(A+B\,x^n)\,\sqrt{a+b\,x^n}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(1/2))/(e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^n}(A+Bx^n)}{(ex)^{5/2}}\,dx = \frac{\sqrt{e}\left(2x^n\sqrt{x^nb+a}\,bn-6x^n\sqrt{x^nb+a}\,b+8\sqrt{x^nb+a}\,an-6\sqrt{x^nb+a}\,a+3\sqrt{}\right.}{}$$

input
```
int((a+b*x^n)^(1/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*(2*x**n*sqrt(x**n*b + a)*b*n - 6*x**n*sqrt(x**n*b + a)*b + 8*sqrt
(x**n*b + a)*a*n - 6*sqrt(x**n*b + a)*a + 3*sqrt(x)*int((sqrt(x)*sqrt(x**n
*b + a))/(x**n*b*n**2*x**3 - 4*x**n*b*n*x**3 + 3*x**n*b*x**3 + a*n**2*x**3
 - 4*a*n*x**3 + 3*a*x**3),x)*a**2*n**4*x - 12*sqrt(x)*int((sqrt(x)*sqrt(x*
*n*b + a))/(x**n*b*n**2*x**3 - 4*x**n*b*n*x**3 + 3*x**n*b*x**3 + a*n**2*x*
*3 - 4*a*n*x**3 + 3*a*x**3),x)*a**2*n**3*x + 9*sqrt(x)*int((sqrt(x)*sqrt(x
**n*b + a))/(x**n*b*n**2*x**3 - 4*x**n*b*n*x**3 + 3*x**n*b*x**3 + a*n**2*x
**3 - 4*a*n*x**3 + 3*a*x**3),x)*a**2*n**2*x))/(3*sqrt(x)*e**3*x*(n**2 - 4*
n + 3))
```

3.377 $\int (ex)^{3/2} (a+bx^n)^{3/2} (A+Bx^n) \, dx$

Optimal result

Integrand size = 26, antiderivative size = 123

$$\int (ex)^{3/2}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{2B(ex)^{5/2}(a+bx^n)^{5/2}}{5be(1+n)} - \frac{2a(aB-Ab(1+n))(ex)^{5/2}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5be(1+n)\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2/5*B*(e*x)^(5/2)*(a+b*x^n)^(5/2)/b/e/(1+n)-2/5*a*(B*a-A*b*(1+n))*(e*x)^(5
/2)*(a+b*x^n)^(1/2)*hypergeom([-3/2, 5/2/n],[1+5/2/n],-b*x^n/a)/b/e/(1+n)/
(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int (ex)^{3/2}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \frac{2ax(ex)^{3/2}\sqrt{a+bx^n}\left(A(5+2n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)+5B\right.}{5(5+2n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*a*x*(e*x)^(3/2)*Sqrt[a + b*x^n]*(A*(5 + 2*n)*Hypergeometric2F1[-3/2, 5/
(2*n), 1 + 5/(2*n), -((b*x^n)/a)] + 5*B*x^n*Hypergeometric2F1[-3/2, (5/2 +
 n)/n, 2 + 5/(2*n), -((b*x^n)/a)]))/(5*(5 + 2*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} (a+bx^n)^{3/2} (A+Bx^n) \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{bn+b}\right) \int (ex)^{3/2} (bx^n+a)^{3/2}dx+\frac{2B(ex)^{5/2} (a+bx^n)^{5/2}}{5be(n+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{aB}{bn+b}\right)\int (ex)^{3/2} \left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{5/2} (a+bx^n)^{5/2}}{5be(n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2a(ex)^{5/2}\sqrt{a+bx^n}\left(A-\frac{aB}{bn+b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{5/2} (a+bx^n)^{5/2}}{5be(n+1)}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*(e*x)^(5/2)*(a + b*x^n)^(5/2))/(5*b*e*(1 + n)) + (2*a*(A - (a*B)/(b +
 b*n))*(e*x)^(5/2)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 5/(2*n), 1 + 5/
(2*n), -((b*x^n)/a)])/(5*e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^{\frac{3}{2}} (a + b\,x^{n})^{\frac{3}{2}} (A + B\,x^{n})\,dx$$

input
```
int((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^{3/2}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{3/2}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{3/2}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*(e*x)^(3/2), x)
```

Giac [F]

$$\int (ex)^{3/2} (a+bx^n)^{3/2} (A+Bx^n) \, dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} (a+bx^n)^{3/2} (A+Bx^n) \, dx = \int (e\,x)^{3/2}\,\left(A+B\,x^n\right)\,\left(a+b\,x^n\right)^{3/2} \, dx$$

input
```
int((e*x)^(3/2)*(A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int((e*x)^(3/2)*(A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int (ex)^{3/2} (a+bx^n)^{3/2} (A+Bx^n) \, dx = \frac{\sqrt{e}\,e\Big(6x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2n^2x^2+40x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2n\,x^2+50x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2x}{}$$

input
```
int((e*x)^(3/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
(sqrt(e)*e*(6*x**((4*n + 1)/2)*sqrt(x**n*b + a)*b**2*n**2*x**2 + 40*x**((4
*n + 1)/2)*sqrt(x**n*b + a)*b**2*n*x**2 + 50*x**((4*n + 1)/2)*sqrt(x**n*b
+ a)*b**2*x**2 + 22*x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b*n**2*x**2 + 130*
x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b*n*x**2 + 100*x**((2*n + 1)/2)*sqrt(x
**n*b + a)*a*b*x**2 + 46*sqrt(x)*sqrt(x**n*b + a)*a**2*n**2*x**2 + 90*sqrt
(x)*sqrt(x**n*b + a)*a**2*n*x**2 + 50*sqrt(x)*sqrt(x**n*b + a)*a**2*x**2 +
 45*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n*b*n**3 + 23*x**n*b*n**2 + 45*
x**n*b*n + 25*x**n*b + 3*a*n**3 + 23*a*n**2 + 45*a*n + 25*a),x)*a**3*n**6
+ 345*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n*b*n**3 + 23*x**n*b*n**2 + 4
5*x**n*b*n + 25*x**n*b + 3*a*n**3 + 23*a*n**2 + 45*a*n + 25*a),x)*a**3*n**
5 + 675*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n*b*n**3 + 23*x**n*b*n**2 +
 45*x**n*b*n + 25*x**n*b + 3*a*n**3 + 23*a*n**2 + 45*a*n + 25*a),x)*a**3*n
**4 + 375*int((sqrt(x)*sqrt(x**n*b + a)*x)/(3*x**n*b*n**3 + 23*x**n*b*n**2
 + 45*x**n*b*n + 25*x**n*b + 3*a*n**3 + 23*a*n**2 + 45*a*n + 25*a),x)*a**3
*n**3))/(5*(3*n**3 + 23*n**2 + 45*n + 25))
```

3.378 $\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{2B(ex)^{3/2}(a+bx^n)^{5/2}}{be(3+5n)}+\frac{2a\left(A-\frac{3aB}{3b+5bn}\right)(ex)^{3/2}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*(e*x)^(3/2)*(a+b*x^n)^(5/2)/b/e/(3+5*n)+2/3*a*(A-3*a*B/(5*b*n+3*b))*(e
*x)^(3/2)*(a+b*x^n)^(1/2)*hypergeom([-3/2, 3/2/n],[1+3/2/n],-b*x^n/a)/e/(1
+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx=\frac{2ax\sqrt{ex}\sqrt{a+bx^n}\left(A(3+2n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeom}\right.}{3(3+2n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*a*x*Sqrt[e*x]*Sqrt[a + b*x^n]*(A*(3 + 2*n)*Hypergeometric2F1[-3/2, 3/(2
*n), 1 + 3/(2*n), -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[-3/2, (3/2 + n
)/n, 2 + 3/(2*n), -((b*x^n)/a)]))/(3*(3 + 2*n)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{3aB}{5bn+3b}\right)\int \sqrt{ex}(bx^n+a)^{3/2}dx+\frac{2B(ex)^{3/2}(a+bx^n)^{5/2}}{be(5n+3)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{3aB}{5bn+3b}\right)\int \sqrt{ex}\left(\frac{bx^n}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{3/2}(a+bx^n)^{5/2}}{be(5n+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2a(ex)^{3/2}\sqrt{a+bx^n}\left(A-\frac{3aB}{5bn+3b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B(ex)^{3/2}(a+bx^n)^{5/2}}{be(5n+3)}$$

input
```
Int[Sqrt[e*x]*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*(e*x)^(3/2)*(a + b*x^n)^(5/2))/(b*e*(3 + 5*n)) + (2*a*(A - (3*a*B)/(3
*b + 5*b*n))*(e*x)^(3/2)*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 3/(2*n),
1 + 3/(2*n), -((b*x^n)/a)])/(3*e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{ex}\,(a+b\,x^{n})^{\frac{3}{2}}\,(A+B\,x^{n})\,dx$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ex}(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.00 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.65

$$\int \sqrt{ex}(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx = \frac{Aaa^{\frac{3}{2n}}a^{\frac{1}{2}-\frac{3}{2n}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{2n}\\1+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Aa^{-\frac{1}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}b\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)} + \frac{Baa^{-\frac{1}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{3}{2n}}a^{2+\frac{3}{2n}}b\sqrt{e}x^{2n+\frac{3}{2}}\Gamma\left(2+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{3}{2n}\\3+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
A*a*a**(3/(2*n))*a**(1/2 - 3/(2*n))*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper(
(-1/2, 3/(2*n)), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/
(2*n))) + A*a**(-1/2 - 3/(2*n))*a**(1 + 3/(2*n))*b*sqrt(e)*x**(n + 3/2)*ga
mma(1 + 3/(2*n))*hyper((-1/2, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 3/(2*n))) + B*a*a**(-1/2 - 3/(2*n))*a**(1 + 3/(2*
n))*sqrt(e)*x**(n + 3/2)*gamma(1 + 3/(2*n))*hyper((-1/2, 1 + 3/(2*n)), (2
+ 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 3/(2*n))) + B*a**(-3/2
 - 3/(2*n))*a**(2 + 3/(2*n))*b*sqrt(e)*x**(2*n + 3/2)*gamma(2 + 3/(2*n))*h
yper((-1/2, 2 + 3/(2*n)), (3 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gam
ma(3 + 3/(2*n)))
```

Maxima [F]

$$\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*sqrt(e*x), x)
```

Giac [F]

$$\int \sqrt{ex}(a+bx^n)^{3/2}(A+Bx^n)\,dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\int \sqrt{e\,x}\,\left(A+B\,x^n\right)\,\left(a+b\,x^n\right)^{3/2}\,dx$$

input
```
int((e*x)^(1/2)*(A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int((e*x)^(1/2)*(A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{ex}(a+bx^n)^{3/2}\left(A+Bx^n\right)\,dx=\frac{\sqrt{e}\,\left(6x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2n^2x+24x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2nx+18x^{2n+\frac{1}{2}}\sqrt{x^n b+a}\,b^2x+22x^{n+\frac{1}{2}}\sqrt{x^n\cdots}\right.}{}$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
(sqrt(e)*(6*x**((4*n + 1)/2)*sqrt(x**n*b + a)*b**2*n**2*x + 24*x**((4*n +
1)/2)*sqrt(x**n*b + a)*b**2*n*x + 18*x**((4*n + 1)/2)*sqrt(x**n*b + a)*b**
2*x + 22*x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b*n**2*x + 78*x**((2*n + 1)/2
)*sqrt(x**n*b + a)*a*b*n*x + 36*x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b*x +
46*sqrt(x)*sqrt(x**n*b + a)*a**2*n**2*x + 54*sqrt(x)*sqrt(x**n*b + a)*a**2
*n*x + 18*sqrt(x)*sqrt(x**n*b + a)*a**2*x + 75*int((sqrt(x)*sqrt(x**n*b +
a))/(5*x**n*b*n**3 + 23*x**n*b*n**2 + 27*x**n*b*n + 9*x**n*b + 5*a*n**3 +
23*a*n**2 + 27*a*n + 9*a),x)*a**3*n**6 + 345*int((sqrt(x)*sqrt(x**n*b + a)
)/(5*x**n*b*n**3 + 23*x**n*b*n**2 + 27*x**n*b*n + 9*x**n*b + 5*a*n**3 + 23
*a*n**2 + 27*a*n + 9*a),x)*a**3*n**5 + 405*int((sqrt(x)*sqrt(x**n*b + a))/
(5*x**n*b*n**3 + 23*x**n*b*n**2 + 27*x**n*b*n + 9*x**n*b + 5*a*n**3 + 23*a
*n**2 + 27*a*n + 9*a),x)*a**3*n**4 + 135*int((sqrt(x)*sqrt(x**n*b + a))/(5
*x**n*b*n**3 + 23*x**n*b*n**2 + 27*x**n*b*n + 9*x**n*b + 5*a*n**3 + 23*a*n
**2 + 27*a*n + 9*a),x)*a**3*n**3))/(3*(5*n**3 + 23*n**2 + 27*n + 9))
```

3.379 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{2B\sqrt{ex}(a+bx^n)^{5/2}}{be(1+5n)} + \frac{2a\left(A-\frac{aB}{b+5bn}\right)\sqrt{ex}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*(e*x)^(1/2)*(a+b*x^n)^(5/2)/b/e/(1+5*n)+2*a*(A-a*B/(5*b*n+b))*(e*x)^(1
/2)*(a+b*x^n)^(1/2)*hypergeom([-3/2, 1/2/n],[1+1/2/n],-b*x^n/a)/e/(1+b*x^n
/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{2ax\sqrt{a+bx^n}\left((A+2An)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)\right.}{(1+2n)\sqrt{ex}\sqrt{1+}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/Sqrt[e*x],x]
```

output
```
(2*a*x*Sqrt[a + b*x^n]*((A + 2*A*n)*Hypergeometric2F1[-3/2, 1/(2*n), (2 +
n^(-1))/2, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[-3/2, (1/2 + n)/n, (4 +
 n^(-1))/2, -((b*x^n)/a)]))/((1 + 2*n)*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{5bn+b}\right)\int \frac{(bx^n+a)^{3/2}}{\sqrt{ex}}dx+\frac{2B\sqrt{ex}(a+bx^n)^{5/2}}{be(5n+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{aB}{5bn+b}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{\sqrt{ex}}dx}{\sqrt{\frac{bx^n}{a}+1}}+\frac{2B\sqrt{ex}(a+bx^n)^{5/2}}{be(5n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2a\sqrt{ex}\sqrt{a+bx^n}\left(A-\frac{aB}{5bn+b}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{\frac{bx^n}{a}+1}}+\frac{2B\sqrt{ex}(a+bx^n)^{5/2}}{be(5n+1)}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/Sqrt[e*x],x]
```

output
```
(2*B*Sqrt[e*x]*(a + b*x^n)^(5/2))/(b*e*(1 + 5*n)) + (2*a*(A - (a*B)/(b + 5
*b*n))*Sqrt[e*x]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2 + n^(
-1))/2, -((b*x^n)/a)])/(e*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^{\frac{3}{2}}\,(A+B\,x^n)}{\sqrt{ex}}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.92 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{Aaa^{\frac{1}{2n}}a^{\frac{1}{2}-\frac{1}{2n}}\sqrt{x}\,\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{2n}\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{Aa^{-\frac{1}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}bx^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)} + \frac{Baa^{-\frac{1}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}x^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{1}{2n}}a^{2+\frac{1}{2n}}bx^{2n+\frac{1}{2}}\Gamma\left(2+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2+\frac{1}{2n}\\3+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(3+\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/(e*x)**(1/2),x)
```

output
```
A*a*a**(1/(2*n))*a**(1/2 - 1/(2*n))*sqrt(x)*gamma(1/(2*n))*hyper((-1/2, 1/
(2*n)), (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(
2*n))) + A*a**(-1/2 - 1/(2*n))*a**(1 + 1/(2*n))*b*x**(n + 1/2)*gamma(1 + 1
/(2*n))*hyper((-1/2, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/
a)/(sqrt(e)*n*gamma(2 + 1/(2*n))) + B*a*a**(-1/2 - 1/(2*n))*a**(1 + 1/(2*n
))*x**(n + 1/2)*gamma(1 + 1/(2*n))*hyper((-1/2, 1 + 1/(2*n)), (2 + 1/(2*n)
,), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(2 + 1/(2*n))) + B*a**(-3/2
- 1/(2*n))*a**(2 + 1/(2*n))*b*x**(2*n + 1/2)*gamma(2 + 1/(2*n))*hyper((-1/
2, 2 + 1/(2*n)), (3 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamm
a(3 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/sqrt(e*x), x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/sqrt(e*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{\sqrt{ex}}\,dx = \frac{\sqrt{e}\left(6x^{2n+\frac{1}{2}}\sqrt{x^nb+a}\,b^2n^2+8x^{2n+\frac{1}{2}}\sqrt{x^nb+a}\,b^2n+2x^{2n+\frac{1}{2}}\sqrt{x^nb+a}\,b^2+\right.}{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*(6*x**((4*n + 1)/2)*sqrt(x**n*b + a)*b**2*n**2 + 8*x**((4*n + 1)/
2)*sqrt(x**n*b + a)*b**2*n + 2*x**((4*n + 1)/2)*sqrt(x**n*b + a)*b**2 + 22
*x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b*n**2 + 26*x**((2*n + 1)/2)*sqrt(x**
n*b + a)*a*b*n + 4*x**((2*n + 1)/2)*sqrt(x**n*b + a)*a*b + 46*sqrt(x)*sqrt
(x**n*b + a)*a**2*n**2 + 18*sqrt(x)*sqrt(x**n*b + a)*a**2*n + 2*sqrt(x)*sq
rt(x**n*b + a)*a**2 + 225*int((sqrt(x)*sqrt(x**n*b + a))/(15*x**n*b*n**3*x
 + 23*x**n*b*n**2*x + 9*x**n*b*n*x + x**n*b*x + 15*a*n**3*x + 23*a*n**2*x
+ 9*a*n*x + a*x),x)*a**3*n**6 + 345*int((sqrt(x)*sqrt(x**n*b + a))/(15*x**
n*b*n**3*x + 23*x**n*b*n**2*x + 9*x**n*b*n*x + x**n*b*x + 15*a*n**3*x + 23
*a*n**2*x + 9*a*n*x + a*x),x)*a**3*n**5 + 135*int((sqrt(x)*sqrt(x**n*b + a
))/(15*x**n*b*n**3*x + 23*x**n*b*n**2*x + 9*x**n*b*n*x + x**n*b*x + 15*a*n
**3*x + 23*a*n**2*x + 9*a*n*x + a*x),x)*a**3*n**4 + 15*int((sqrt(x)*sqrt(x
**n*b + a))/(15*x**n*b*n**3*x + 23*x**n*b*n**2*x + 9*x**n*b*n*x + x**n*b*x
 + 15*a*n**3*x + 23*a*n**2*x + 9*a*n*x + a*x),x)*a**3*n**3))/(e*(15*n**3 +
 23*n**2 + 9*n + 1))
```

3.380 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}} dx = -\frac{2B(a+bx^n)^{5/2}}{be(1-5n)\sqrt{ex}} - \frac{2a\left(A-\frac{aB}{b-5bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(5/2)/b/e/(1-5*n)/(e*x)^(1/2)-2*a*(A-a*B/(-5*b*n+b))*(a+b*x
^n)^(1/2)*hypergeom([-3/2, -1/2/n],[1-1/2/n],-b*x^n/a)/e/(e*x)^(1/2)/(1+b*
x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}} dx = \frac{2ax\sqrt{a+bx^n}\left(Bx^n\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1-\frac{1}{2n},2-\frac{1}{2n},-\frac{bx^n}{a}\right)+A(1\right.}{(-1+2n)(ex)^{3/2}\sqrt{1+}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/(e*x)^(3/2),x]
```

output
```
(2*a*x*Sqrt[a + b*x^n]*(B*x^n*Hypergeometric2F1[-3/2, 1 - 1/(2*n), 2 - 1/(
2*n), -((b*x^n)/a)] + A*(1 - 2*n)*Hypergeometric2F1[-3/2, -1/2*1/n, 1 - 1/
(2*n), -((b*x^n)/a)]))/((-1 + 2*n)*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{b-5bn}\right)\int \frac{(bx^n+a)^{3/2}}{(ex)^{3/2}}dx-\frac{2B(a+bx^n)^{5/2}}{be(1-5n)\sqrt{ex}}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{aB}{b-5bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{(ex)^{3/2}}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{be(1-5n)\sqrt{ex}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2a\sqrt{a+bx^n}\left(A-\frac{aB}{b-5bn}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{be(1-5n)\sqrt{ex}}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/(e*x)^(3/2),x]
```

output
```
(-2*B*(a + b*x^n)^(5/2))/(b*e*(1 - 5*n)*Sqrt[e*x]) - (2*a*(A - (a*B)/(b -
5*b*n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -1/2*1/n, 1 - 1/(2*n), -((
b*x^n)/a)])/(e*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{3}{2}}\,(A+B\,x^{n})}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.64 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.72

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \frac{Aaa^{-\frac{1}{2n}}a^{\frac{1}{2}+\frac{1}{2n}}\Gamma\left(-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{2n}\\1-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\sqrt{x}\Gamma\left(1-\frac{1}{2n}\right)} + \frac{Aa^{-\frac{1}{2}+\frac{1}{2n}}a^{1-\frac{1}{2n}}bx^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{2n}\\2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)} + \frac{Baa^{-\frac{1}{2}+\frac{1}{2n}}a^{1-\frac{1}{2n}}x^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{1}{2n}\\2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{1}{2n}}a^{2-\frac{1}{2n}}bx^{2n-\frac{1}{2}}\Gamma\left(2-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{1}{2n}\\3-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(3-\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/(e*x)**(3/2),x)
```

output
```
A*a*a**(1/2 + 1/(2*n))*gamma(-1/(2*n))*hyper((-1/2, -1/(2*n)), (1 - 1/(2*n
),), b*x**n*exp_polar(I*pi)/a)/(a**(1/(2*n))*e**(3/2)*n*sqrt(x)*gamma(1 -
1/(2*n))) + A*a**(-1/2 + 1/(2*n))*a**(1 - 1/(2*n))*b*x**(n - 1/2)*gamma(1
- 1/(2*n))*hyper((-1/2, 1 - 1/(2*n)), (2 - 1/(2*n),), b*x**n*exp_polar(I*p
i)/a)/(e**(3/2)*n*gamma(2 - 1/(2*n))) + B*a*a**(-1/2 + 1/(2*n))*a**(1 - 1/
(2*n))*x**(n - 1/2)*gamma(1 - 1/(2*n))*hyper((-1/2, 1 - 1/(2*n)), (2 - 1/(
2*n),), b*x**n*exp_polar(I*pi)/a)/(e**(3/2)*n*gamma(2 - 1/(2*n))) + B*a**(
-3/2 + 1/(2*n))*a**(2 - 1/(2*n))*b*x**(2*n - 1/2)*gamma(2 - 1/(2*n))*hyper
((-1/2, 2 - 1/(2*n)), (3 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(e**(3/2)*
n*gamma(3 - 1/(2*n)))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/(e*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/(e*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\,\Big(6x^{2n}\sqrt{x^nb+a}\,b^2n^2-8x^{2n}\sqrt{x^nb+a}\,b^2n+2x^{2n}\sqrt{x^nb+a}\,b^2+22x^n\sqrt{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 8*x**(2*n)*sqrt(x**n*b +
 a)*b**2*n + 2*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)*a
*b*n**2 - 26*x**n*sqrt(x**n*b + a)*a*b*n + 4*x**n*sqrt(x**n*b + a)*a*b + 4
6*sqrt(x**n*b + a)*a**2*n**2 - 18*sqrt(x**n*b + a)*a**2*n + 2*sqrt(x**n*b
+ a)*a**2 + 225*sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(15*x**n*b*n**3*x**
2 - 23*x**n*b*n**2*x**2 + 9*x**n*b*n*x**2 - x**n*b*x**2 + 15*a*n**3*x**2 -
 23*a*n**2*x**2 + 9*a*n*x**2 - a*x**2),x)*a**3*n**6 - 345*sqrt(x)*int((sqr
t(x)*sqrt(x**n*b + a))/(15*x**n*b*n**3*x**2 - 23*x**n*b*n**2*x**2 + 9*x**n
*b*n*x**2 - x**n*b*x**2 + 15*a*n**3*x**2 - 23*a*n**2*x**2 + 9*a*n*x**2 - a
*x**2),x)*a**3*n**5 + 135*sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(15*x**n*
b*n**3*x**2 - 23*x**n*b*n**2*x**2 + 9*x**n*b*n*x**2 - x**n*b*x**2 + 15*a*n
**3*x**2 - 23*a*n**2*x**2 + 9*a*n*x**2 - a*x**2),x)*a**3*n**4 - 15*sqrt(x)
*int((sqrt(x)*sqrt(x**n*b + a))/(15*x**n*b*n**3*x**2 - 23*x**n*b*n**2*x**2
 + 9*x**n*b*n*x**2 - x**n*b*x**2 + 15*a*n**3*x**2 - 23*a*n**2*x**2 + 9*a*n
*x**2 - a*x**2),x)*a**3*n**3))/(sqrt(x)*e**2*(15*n**3 - 23*n**2 + 9*n - 1)
)
```

3.381 $\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx = -\frac{2B(a+bx^n)^{5/2}}{be(3-5n)(ex)^{3/2}} - \frac{2a\left(A-\frac{3aB}{3b-5bn}\right)\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}}$$

output
```
-2*B*(a+b*x^n)^(5/2)/b/e/(3-5*n)/(e*x)^(3/2)-2/3*a*(A-3*a*B/(-5*b*n+3*b))*
(a+b*x^n)^(1/2)*hypergeom([-3/2, -3/2/n],[1-3/2/n],-b*x^n/a)/e/(e*x)^(3/2)
/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx = \frac{2ax\sqrt{a+bx^n}\left(3Bx^n\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1-\frac{3}{2n},2-\frac{3}{2n},-\frac{bx^n}{a}\right)+A(\right.}{3(-3+2n)(ex)^{5/2}\sqrt{1+}}$$

input
```
Integrate[((a + b*x^n)^(3/2)*(A + B*x^n))/(e*x)^(5/2),x]
```

output
```
(2*a*x*Sqrt[a + b*x^n]*(3*B*x^n*Hypergeometric2F1[-3/2, 1 - 3/(2*n), 2 - 3
/(2*n), -((b*x^n)/a)] + A*(3 - 2*n)*Hypergeometric2F1[-3/2, -3/(2*n), 1 -
3/(2*n), -((b*x^n)/a)]))/(3*(-3 + 2*n)*(e*x)^(5/2)*Sqrt[1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{3aB}{3b-5bn}\right)\int \frac{(bx^n+a)^{3/2}}{(ex)^{5/2}}dx-\frac{2B(a+bx^n)^{5/2}}{be(3-5n)(ex)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt{a+bx^n}\left(A-\frac{3aB}{3b-5bn}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^{3/2}}{(ex)^{5/2}}dx}{\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{be(3-5n)(ex)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2a\sqrt{a+bx^n}\left(A-\frac{3aB}{3b-5bn}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}}-\frac{2B(a+bx^n)^{5/2}}{be(3-5n)(ex)^{3/2}}$$

input
```
Int[((a + b*x^n)^(3/2)*(A + B*x^n))/(e*x)^(5/2),x]
```

output
```
(-2*B*(a + b*x^n)^(5/2))/(b*e*(3 - 5*n)*(e*x)^(3/2)) - (2*a*(A - (3*a*B)/(
3*b - 5*b*n))*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, -3/(2*n), 1 - 3/(2*n
), -((b*x^n)/a)])/(3*e*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{\frac{3}{2}}\,(A+B\,x^{n})}{(ex)^{\frac{5}{2}}}dx$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

output
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.23 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.63

$$\begin{aligned}\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx &= \frac{Aaa^{-\frac{3}{2n}}a^{\frac{1}{2}+\frac{3}{2n}}\Gamma\left(-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{2n}\\1-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}nx^{\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right)}\\
&+\frac{Aa^{-\frac{1}{2}+\frac{3}{2n}}a^{1-\frac{3}{2n}}bx^{n-\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{2n}\\2-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}n\Gamma\left(2-\frac{3}{2n}\right)}\\
&+\frac{Baa^{-\frac{1}{2}+\frac{3}{2n}}a^{1-\frac{3}{2n}}x^{n-\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},1-\frac{3}{2n}\\2-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}n\Gamma\left(2-\frac{3}{2n}\right)}\\
&+\frac{Ba^{-\frac{3}{2}+\frac{3}{2n}}a^{2-\frac{3}{2n}}bx^{2n-\frac{3}{2}}\Gamma\left(2-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},2-\frac{3}{2n}\\3-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}n\Gamma\left(3-\frac{3}{2n}\right)}\end{aligned}$$

input
```
integrate((a+b*x**n)**(3/2)*(A+B*x**n)/(e*x)**(5/2),x)
```

output
```
A*a*a**(1/2 + 3/(2*n))*gamma(-3/(2*n))*hyper((-1/2, -3/(2*n)), (1 - 3/(2*n
),), b*x**n*exp_polar(I*pi)/a)/(a**(3/(2*n))*e**(5/2)*n*x**(3/2)*gamma(1 -
 3/(2*n))) + A*a**(-1/2 + 3/(2*n))*a**(1 - 3/(2*n))*b*x**(n - 3/2)*gamma(1
 - 3/(2*n))*hyper((-1/2, 1 - 3/(2*n)), (2 - 3/(2*n),), b*x**n*exp_polar(I*
pi)/a)/(e**(5/2)*n*gamma(2 - 3/(2*n))) + B*a*a**(-1/2 + 3/(2*n))*a**(1 - 3
/(2*n))*x**(n - 3/2)*gamma(1 - 3/(2*n))*hyper((-1/2, 1 - 3/(2*n)), (2 - 3/
(2*n),), b*x**n*exp_polar(I*pi)/a)/(e**(5/2)*n*gamma(2 - 3/(2*n))) + B*a**
(-3/2 + 3/(2*n))*a**(2 - 3/(2*n))*b*x**(2*n - 3/2)*gamma(2 - 3/(2*n))*hype
r((-1/2, 2 - 3/(2*n)), (3 - 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(e**(5/2)
*n*gamma(3 - 3/(2*n)))
```

Maxima [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/(e*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)/(e*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx=\int \frac{(A+B\,x^n)\,(a+b\,x^n)^{3/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^n)*(a + b*x^n)^(3/2))/(e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^{3/2}(A+Bx^n)}{(ex)^{5/2}}\,dx=\frac{\sqrt{e}\,\left(6x^{2n}\sqrt{x^nb+a}\,b^2n^2-24x^{2n}\sqrt{x^nb+a}\,b^2n+18x^{2n}\sqrt{x^nb+a}\,b^2+22x^{n}\right.}{}$$

input
```
int((a+b*x^n)^(3/2)*(A+B*x^n)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*(6*x**(2*n)*sqrt(x**n*b + a)*b**2*n**2 - 24*x**(2*n)*sqrt(x**n*b
+ a)*b**2*n + 18*x**(2*n)*sqrt(x**n*b + a)*b**2 + 22*x**n*sqrt(x**n*b + a)
*a*b*n**2 - 78*x**n*sqrt(x**n*b + a)*a*b*n + 36*x**n*sqrt(x**n*b + a)*a*b
+ 46*sqrt(x**n*b + a)*a**2*n**2 - 54*sqrt(x**n*b + a)*a**2*n + 18*sqrt(x**
n*b + a)*a**2 + 75*sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(5*x**n*b*n**3*x
**3 - 23*x**n*b*n**2*x**3 + 27*x**n*b*n*x**3 - 9*x**n*b*x**3 + 5*a*n**3*x*
*3 - 23*a*n**2*x**3 + 27*a*n*x**3 - 9*a*x**3),x)*a**3*n**6*x - 345*sqrt(x)
*int((sqrt(x)*sqrt(x**n*b + a))/(5*x**n*b*n**3*x**3 - 23*x**n*b*n**2*x**3
+ 27*x**n*b*n*x**3 - 9*x**n*b*x**3 + 5*a*n**3*x**3 - 23*a*n**2*x**3 + 27*a
*n*x**3 - 9*a*x**3),x)*a**3*n**5*x + 405*sqrt(x)*int((sqrt(x)*sqrt(x**n*b
+ a))/(5*x**n*b*n**3*x**3 - 23*x**n*b*n**2*x**3 + 27*x**n*b*n*x**3 - 9*x**
n*b*x**3 + 5*a*n**3*x**3 - 23*a*n**2*x**3 + 27*a*n*x**3 - 9*a*x**3),x)*a**
3*n**4*x - 135*sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(5*x**n*b*n**3*x**3
- 23*x**n*b*n**2*x**3 + 27*x**n*b*n*x**3 - 9*x**n*b*x**3 + 5*a*n**3*x**3 -
 23*a*n**2*x**3 + 27*a*n*x**3 - 9*a*x**3),x)*a**3*n**3*x))/(3*sqrt(x)*e**3
*x*(5*n**3 - 23*n**2 + 27*n - 9))
```

3.382 $\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{(ex)^{3/2}\left(A+Bx^n\right)}{\sqrt{a+bx^n}}\,dx=\frac{2B(ex)^{5/2}\sqrt{a+bx^n}}{be(5+n)}+\frac{2\left(A-\frac{5aB}{b(5+n)}\right)(ex)^{5/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(5/2)*(a+b*x^n)^(1/2)/b/e/(5+n)+2/5*(A-5*a*B/b/(5+n))*(e*x)^(5/2
)*(1+b*x^n/a)^(1/2)*hypergeom([1/2, 5/2/n],[1+5/2/n],-b*x^n/a)/e/(a+b*x^n)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{(ex)^{3/2}\left(A+Bx^n\right)}{\sqrt{a+bx^n}}\,dx=\frac{2x(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\left(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)+5B\right.}{5(5+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*(A*(5 + 2*n)*Hypergeometric2F1[1/2, 5
/(2*n), 1 + 5/(2*n), -((b*x^n)/a)] + 5*B*x^n*Hypergeometric2F1[1/2, (5/2 +
 n)/n, 2 + 5/(2*n), -((b*x^n)/a)]))/(5*(5 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$$

$$\Big\downarrow \text{ 959}$$

$$\left(A-\frac{5aB}{b(n+5)}\right)\int \frac{(ex)^{3/2}}{\sqrt{bx^n+a}}dx+\frac{2B(ex)^{5/2}\sqrt{a+bx^n}}{be(n+5)}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{b(n+5)}\right)\int \frac{(ex)^{3/2}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}\sqrt{a+bx^n}}{be(n+5)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2(ex)^{5/2}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{b(n+5)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}\sqrt{a+bx^n}}{be(n+5)}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*B*(e*x)^(5/2)*Sqrt[a + b*x^n])/(b*e*(5 + n)) + (2*(A - (5*a*B)/(b*(5 +
n)))*(e*x)^(5/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 5/(2*n), 1 + 5
/(2*n), -((b*x^n)/a)])/(5*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (A+B\,x^n)}{\sqrt{a+b\,x^n}}dx$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{5}{2n}}a^{-\frac{1}{2}-\frac{5}{2n}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{2n}\\1+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{5}{2n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{5}{2n}}a^{1+\frac{5}{2n}}e^{\frac{3}{2}}x^{n+\frac{5}{2}}\Gamma\left(1+\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{5}{2n}\\2+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{5}{2n}\right)}$$

input
```
integrate((e*x)**(3/2)*(A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(5/(2*n))*a**(-1/2 - 5/(2*n))*e**(3/2)*x**(5/2)*gamma(5/(2*n))*hyper(
(1/2, 5/(2*n)), (1 + 5/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 5/(
2*n))) + B*a**(-3/2 - 5/(2*n))*a**(1 + 5/(2*n))*e**(3/2)*x**(n + 5/2)*gamm
a(1 + 5/(2*n))*hyper((1/2, 1 + 5/(2*n)), (2 + 5/(2*n),), b*x**n*exp_polar(
I*pi)/a)/(n*gamma(2 + 5/(2*n)))
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{(e\,x)^{3/2}\,(A+B\,x^n)}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(1/2),x)
```

output
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2} (A + Bx^n)}{\sqrt{a + bx^n}} \, dx = \frac{\sqrt{e}\, e \left(2\sqrt{x}\, \sqrt{x^n b + a}\, x^2 + \left(\int \frac{\sqrt{x}\, \sqrt{x^n b + a}\, x}{x^n b n + 5x^n b + a n + 5a} dx\right) a\, n^2 + 5\left(\int \frac{\sqrt{x}\, \sqrt{x^n b + a}\, x}{x^n b n + 5x^n b + a n + 5a} dx\right.\right.}{n + 5}$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(sqrt(e)*e*(2*sqrt(x)*sqrt(x**n*b + a)*x**2 + int((sqrt(x)*sqrt(x**n*b + a
)*x)/(x**n*b*n + 5*x**n*b + a*n + 5*a),x)*a*n**2 + 5*int((sqrt(x)*sqrt(x**
n*b + a)*x)/(x**n*b*n + 5*x**n*b + a*n + 5*a),x)*a*n))/(n + 5)
```

3.383 $\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{2B(ex)^{3/2}\sqrt{a+bx^n}}{be(3+n)} + \frac{2\left(A-\frac{3aB}{b(3+n)}\right)(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(3/2)*(a+b*x^n)^(1/2)/b/e/(3+n)+2/3*(A-3*a*B/b/(3+n))*(e*x)^(3/2
)*(1+b*x^n/a)^(1/2)*hypergeom([1/2, 3/2/n],[1+3/2/n],-b*x^n/a)/e/(a+b*x^n)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$$
$$= \frac{2x\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\left(A(3+2n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{3(3+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, 3/(
2*n), 1 + 3/(2*n), -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[1/2, (3/2 + n
)/n, 2 + 3/(2*n), -((b*x^n)/a)]))/(3*(3 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$$
$$\Big\downarrow \text{959}$$
$$\left(A-\frac{3aB}{b(n+3)}\right)\int \frac{\sqrt{ex}}{\sqrt{bx^n+a}}dx+\frac{2B(ex)^{3/2}\sqrt{a+bx^n}}{be(n+3)}$$
$$\Big\downarrow \text{889}$$
$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(n+3)}\right)\int \frac{\sqrt{ex}}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2B(ex)^{3/2}\sqrt{a+bx^n}}{be(n+3)}$$
$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(n+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{a+bx^n}}+\frac{2B(ex)^{3/2}\sqrt{a+bx^n}}{be(n+3)}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*B*(e*x)^(3/2)*Sqrt[a + b*x^n])/(b*e*(3 + n)) + (2*(A - (3*a*B)/(b*(3 +
n)))*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 3/(2*n), 1 + 3
/(2*n), -((b*x^n)/a)])/(3*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{ex}\,(A+B\,x^n)}{\sqrt{a+b\,x^n}}dx$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{3}{2n}}a^{-\frac{1}{2}-\frac{3}{2n}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{2n}\\1+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(3/(2*n))*a**(-1/2 - 3/(2*n))*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper((
1/2, 3/(2*n)), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/(2
*n))) + B*a**(-3/2 - 3/(2*n))*a**(1 + 3/(2*n))*sqrt(e)*x**(n + 3/2)*gamma(
1 + 3/(2*n))*hyper((1/2, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{\sqrt{bx^n+a}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx=\int \frac{\sqrt{e\,x}\,\left(A+B\,x^n\right)}{\sqrt{a+b\,x^n}}\,dx$$

input
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(1/2),x)
```

output
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{\sqrt{e}\left(2\sqrt{x}\,\sqrt{x^n b+a}\,x+\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bn+3x^n b+an+3a}dx\right)a\,n^2+3\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bn+3x^n b+an+3a}dx\right)an\right)}{n+3}$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(x)*sqrt(x**n*b + a)*x + int((sqrt(x)*sqrt(x**n*b + a))/(x
**n*b*n + 3*x**n*b + a*n + 3*a),x)*a*n**2 + 3*int((sqrt(x)*sqrt(x**n*b + a
))/(x**n*b*n + 3*x**n*b + a*n + 3*a),x)*a*n))/(n + 3)
```

3.384 $\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \frac{2B\sqrt{ex}\sqrt{a+bx^n}}{be(1+n)} + \frac{2\left(A-\frac{aB}{b+bn}\right)\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(1/2)*(a+b*x^n)^(1/2)/b/e/(1+n)+2*(A-a*B/(b*n+b))*(e*x)^(1/2)*(1
+b*x^n/a)^(1/2)*hypergeom([1/2, 1/2/n],[1+1/2/n],-b*x^n/a)/e/(a+b*x^n)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left((A+2An)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)+Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\ldots\right.\right.}{(1+2n)\sqrt{ex}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[e*x]*Sqrt[a + b*x^n]),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*((A + 2*A*n)*Hypergeometric2F1[1/2, 1/(2*n), (2 +
 n^(-1))/2, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[1/2, (1/2 + n)/n, (4 +
 n^(-1))/2, -((b*x^n)/a)]))/((1 + 2*n)*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{\sqrt{ex}\sqrt{bx^n+a}}dx+\frac{2B\sqrt{ex}\sqrt{a+bx^n}}{be(n+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}\sqrt{a+bx^n}}{be(n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}\sqrt{a+bx^n}}{be(n+1)}$$

input
```
Int[(A + B*x^n)/(Sqrt[e*x]*Sqrt[a + b*x^n]),x]
```

output
```
(2*B*Sqrt[e*x]*Sqrt[a + b*x^n])/(b*e*(1 + n)) + (2*(A - (a*B)/(b + b*n))*S
qrt[e*x]*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, 1/(2*n), (2 + n^(-1))/
2, -((b*x^n)/a)])/(e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{\sqrt{ex}\,\sqrt{a + b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \frac{Aa^{\frac{1}{2n}}a^{-\frac{1}{2}-\frac{1}{2n}}\sqrt{x}\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{2n}\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{Ba^{-\frac{3}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}x^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(1/2)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(1/(2*n))*a**(-1/2 - 1/(2*n))*sqrt(x)*gamma(1/(2*n))*hyper((1/2, 1/(2
*n)), (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(2*
n))) + B*a**(-3/2 - 1/(2*n))*a**(1 + 1/(2*n))*x**(n + 1/2)*gamma(1 + 1/(2*
n))*hyper((1/2, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(s
qrt(e)*n*gamma(2 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{ex}\sqrt{a+bx^n}}\,dx = \int \frac{A+B\,x^n}{\sqrt{e\,x}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx^n}{\sqrt{ex}\sqrt{a + bx^n}}\,dx$$
$$= \frac{\sqrt{e}\left(2\sqrt{x}\,\sqrt{x^n b + a} + \left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bnx+x^n bx+anx+ax}dx\right) a\,n^2 + \left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bnx+x^n bx+anx+ax}dx\right) an\right)}{e\,(n+1)}$$

input

```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(1/2),x)
```

output

```
(sqrt(e)*(2*sqrt(x)*sqrt(x**n*b + a) + int((sqrt(x)*sqrt(x**n*b + a))/(x**
n*b*n*x + x**n*b*x + a*n*x + a*x),x)*a*n**2 + int((sqrt(x)*sqrt(x**n*b + a
))/(x**n*b*n*x + x**n*b*x + a*n*x + a*x),x)*a*n))/(e*(n + 1))
```

3.385 $\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx = -\frac{2B\sqrt{a+bx^n}}{be(1-n)\sqrt{ex}} - \frac{2\left(A-\frac{aB}{b(1-n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{a+bx^n}}$$

output
```
-2*B*(a+b*x^n)^(1/2)/b/e/(1-n)/(e*x)^(1/2)-2*(A-a*B/b/(1-n))*(1+b*x^n/a)^(
1/2)*hypergeom([1/2, -1/2/n],[1-1/2/n],-b*x^n/a)/e/(e*x)^(1/2)/(a+b*x^n)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left(Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{1}{2n},2-\frac{1}{2n},-\frac{bx^n}{a}\right)+A(1-2n)\,\text{Hy}\right.}{(-1+2n)(ex)^{3/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(3/2)*Sqrt[a + b*x^n]),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(B*x^n*Hypergeometric2F1[1/2, 1 - 1/(2*n), 2 - 1/
(2*n), -((b*x^n)/a)] + A*(1 - 2*n)*Hypergeometric2F1[1/2, -1/2*1/n, 1 - 1/
(2*n), -((b*x^n)/a)]))/((-1 + 2*n)*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{b-bn}\right)\int \frac{1}{(ex)^{3/2}\sqrt{bx^n+a}}dx-\frac{2B\sqrt{a+bx^n}}{be(1-n)\sqrt{ex}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-bn}\right)\int \frac{1}{(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{be(1-n)\sqrt{ex}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-bn}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{be(1-n)\sqrt{ex}}$$

input
```
Int[(A + B*x^n)/((e*x)^(3/2)*Sqrt[a + b*x^n]),x]
```

output
```
(-2*B*Sqrt[a + b*x^n])/(b*e*(1 - n)*Sqrt[e*x]) - (2*(A - (a*B)/(b - b*n))*
Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -1/2*1/n, 1 - 1/(2*n), -((b*x^n
)/a)])/(e*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A + B\,x^{n}}{\left(e x\right)^{\frac{3}{2}} \sqrt{a + b\,x^{n}}}dx$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A + Bx^{n}}{(ex)^{3/2}\sqrt{a + bx^{n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.90 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx = \frac{Aa^{-\frac{1}{2n}}a^{-\frac{1}{2}+\frac{1}{2n}}\Gamma\left(-\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{1}{2n}\\1-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\sqrt{x}\Gamma\left(1-\frac{1}{2n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{1}{2n}}a^{1-\frac{1}{2n}}x^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},1-\frac{1}{2n}\\2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(3/2)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(-1/2 + 1/(2*n))*gamma(-1/(2*n))*hyper((1/2, -1/(2*n)), (1 - 1/(2*n),
), b*x**n*exp_polar(I*pi)/a)/(a**(1/(2*n))*e**(3/2)*n*sqrt(x)*gamma(1 - 1/
(2*n))) + B*a**(-3/2 + 1/(2*n))*a**(1 - 1/(2*n))*x**(n - 1/2)*gamma(1 - 1/
(2*n))*hyper((1/2, 1 - 1/(2*n)), (2 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a)
/(e**(3/2)*n*gamma(2 - 1/(2*n)))
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx=\int \frac{Bx^n+A}{\sqrt{bx^n+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx=\int \frac{A+B\,x^n}{{(e\,x)}^{3/2}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}\sqrt{a+bx^n}}\,dx=\frac{\sqrt{e}\,\left(2\sqrt{x^nb+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{x^nb+a}}{x^nbn\,x^2-x^nb\,x^2+an\,x^2-a\,x^2}dx\right)a\,n^2-\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{x^nb-}}{x^nbn\,x^2-x^nb\,x^2+}\right.\right.}{\sqrt{x}\,e^2\,(n-1)}$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(x**n*b + a) + sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(x**
n*b*n*x**2 - x**n*b*x**2 + a*n*x**2 - a*x**2),x)*a*n**2 - sqrt(x)*int((sqr
t(x)*sqrt(x**n*b + a))/(x**n*b*n*x**2 - x**n*b*x**2 + a*n*x**2 - a*x**2),x
)*a*n))/(sqrt(x)*e**2*(n - 1))
```

3.386 $\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx = -\frac{2B\sqrt{a+bx^n}}{be(3-n)(ex)^{3/2}} - \frac{2\left(A-\frac{3aB}{b(3-n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}\sqrt{a+bx^n}}$$

output
```
-2*B*(a+b*x^n)^(1/2)/b/e/(3-n)/(e*x)^(3/2)-2/3*(A-3*a*B/b/(3-n))*(1+b*x^n/
a)^(1/2)*hypergeom([1/2, -3/2/n],[1-3/2/n],-b*x^n/a)/e/(e*x)^(3/2)/(a+b*x^
n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left(3Bx^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{3}{2n},2-\frac{3}{2n},-\frac{bx^n}{a}\right)+A(3-2n)\,\text{H}\right.}{3(-3+2n)(ex)^{5/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(5/2)*Sqrt[a + b*x^n]),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(3*B*x^n*Hypergeometric2F1[1/2, 1 - 3/(2*n), 2 -
3/(2*n), -((b*x^n)/a)] + A*(3 - 2*n)*Hypergeometric2F1[1/2, -3/(2*n), 1 -
3/(2*n), -((b*x^n)/a)]))/(3*(-3 + 2*n)*(e*x)^(5/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{3aB}{b(3-n)}\right)\int \frac{1}{(ex)^{5/2}\sqrt{bx^n+a}}dx-\frac{2B\sqrt{a+bx^n}}{be(3-n)(ex)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(3-n)}\right)\int \frac{1}{(ex)^{5/2}\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{be(3-n)(ex)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(3-n)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}\sqrt{a+bx^n}}-\frac{2B\sqrt{a+bx^n}}{be(3-n)(ex)^{3/2}}$$

input
```
Int[(A + B*x^n)/((e*x)^(5/2)*Sqrt[a + b*x^n]),x]
```

output
```
(-2*B*Sqrt[a + b*x^n])/(b*e*(3 - n)*(e*x)^(3/2)) - (2*(A - (3*a*B)/(b*(3 -
 n)))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1/2, -3/(2*n), 1 - 3/(2*n), -(
(b*x^n)/a)])/(3*e*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(e x \right)^{\frac{5}{2}} \sqrt{a +b\,x^{n}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx = \frac{Aa^{-\frac{3}{2n}}a^{-\frac{1}{2}+\frac{3}{2n}}\Gamma\left(-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},-\frac{3}{2n}\\1-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}nx^{\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right)} + \frac{Ba^{-\frac{3}{2}+\frac{3}{2n}}a^{1-\frac{3}{2n}}x^{n-\frac{3}{2}}\Gamma\left(1-\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},1-\frac{3}{2n}\\2-\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{5}{2}}n\Gamma\left(2-\frac{3}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(5/2)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(-1/2 + 3/(2*n))*gamma(-3/(2*n))*hyper((1/2, -3/(2*n)), (1 - 3/(2*n),
), b*x**n*exp_polar(I*pi)/a)/(a**(3/(2*n))*e**(5/2)*n*x**(3/2)*gamma(1 - 3
/(2*n))) + B*a**(-3/2 + 3/(2*n))*a**(1 - 3/(2*n))*x**(n - 3/2)*gamma(1 - 3
/(2*n))*hyper((1/2, 1 - 3/(2*n)), (2 - 3/(2*n),), b*x**n*exp_polar(I*pi)/a
)/(e**(5/2)*n*gamma(2 - 3/(2*n)))
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx = \int \frac{Bx^n+A}{\sqrt{bx^n+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*(e*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx=\int \frac{Bx^n+A}{\sqrt{bx^n+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/(sqrt(b*x^n + a)*(e*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx=\int \frac{A+B\,x^n}{{(e\,x)}^{5/2}\,\sqrt{a+b\,x^n}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(1/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\sqrt{a+bx^n}}\,dx=\frac{\sqrt{e}\left(2\sqrt{x^n b+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bn\,x^3-3x^n b\,x^3+an\,x^3-3a\,x^3}dx\right)a\,n^2x-3\sqrt{x}\left(\int \frac{\sqrt{x}\cdots}{x^n bn\,x^3-3x^n\cdots}\cdots\right.\right.}{\sqrt{x}\,e^3x\,(n-3)}$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(1/2),x)
```

output
```
(sqrt(e)*(2*sqrt(x**n*b + a) + sqrt(x)*int((sqrt(x)*sqrt(x**n*b + a))/(x**
n*b*n*x**3 - 3*x**n*b*x**3 + a*n*x**3 - 3*a*x**3),x)*a*n**2*x - 3*sqrt(x)*
int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*n*x**3 - 3*x**n*b*x**3 + a*n*x**3 -
 3*a*x**3),x)*a*n*x))/(sqrt(x)*e**3*x*(n - 3))
```

3.387 $\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2B(ex)^{5/2}}{be(5-n)\sqrt{a+bx^n}} + \frac{2\left(\frac{A}{a}-\frac{5B}{b(5-n)}\right)(ex)^{5/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(5/2)/b/e/(5-n)/(a+b*x^n)^(1/2)+2/5*(A/a-5*B/b/(5-n))*(e*x)^(5/2
)*(1+b*x^n/a)^(1/2)*hypergeom([3/2, 5/2/n],[1+5/2/n],-b*x^n/a)/e/(a+b*x^n)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2x(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\left(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)+5B\right.}{5a(5+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*(A*(5 + 2*n)*Hypergeometric2F1[3/2, 5
/(2*n), 1 + 5/(2*n), -((b*x^n)/a)] + 5*B*x^n*Hypergeometric2F1[3/2, (5/2 +
 n)/n, 2 + 5/(2*n), -((b*x^n)/a)]))/(5*a*(5 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{5aB}{b(5-n)}\right)\int \frac{(ex)^{3/2}}{(bx^n+a)^{3/2}}dx+\frac{2B(ex)^{5/2}}{be(5-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{b(5-n)}\right)\int \frac{(ex)^{3/2}}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}}{be(5-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{5/2}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{b(5-n)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5ae\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}}{be(5-n)\sqrt{a+bx^n}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*B*(e*x)^(5/2))/(b*e*(5 - n)*Sqrt[a + b*x^n]) + (2*(A - (5*a*B)/(b*(5 -
n)))*(e*x)^(5/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 5/(2*n), 1 + 5
/(2*n), -((b*x^n)/a)])/(5*a*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (A + B\,x^{n})}{(a + b\,x^{n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.21

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{5}{2n}}a^{-\frac{3}{2}-\frac{5}{2n}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{2n}\\1+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{5}{2n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{5}{2n}}a^{1+\frac{5}{2n}}e^{\frac{3}{2}}x^{n+\frac{5}{2}}\Gamma\left(1+\frac{5}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{5}{2n}\\2+\frac{5}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{5}{2n}\right)}$$

input
```
integrate((e*x)**(3/2)*(A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(5/(2*n))*a**(-3/2 - 5/(2*n))*e**(3/2)*x**(5/2)*gamma(5/(2*n))*hyper(
(3/2, 5/(2*n)), (1 + 5/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 5/(
2*n))) + B*a**(-5/2 - 5/(2*n))*a**(1 + 5/(2*n))*e**(3/2)*x**(n + 5/2)*gamm
a(1 + 5/(2*n))*hyper((3/2, 1 + 5/(2*n)), (2 + 5/(2*n),), b*x**n*exp_polar(
I*pi)/a)/(n*gamma(2 + 5/(2*n)))
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(e\,x)^{3/2}\,(A+B\,x^n)}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(3/2),x)
```

output
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2} (A + Bx^n)}{(a + bx^n)^{3/2}} \, dx = \sqrt{e} \left(\int \frac{\sqrt{x}\, \sqrt{x^n b + a}\, x}{x^n b + a} dx \right) e$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a)*x)/(x**n*b + a),x)*e
```

3.388 $\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2B(ex)^{3/2}}{be(3-n)\sqrt{a+bx^n}} + \frac{2\left(\frac{A}{a}-\frac{3B}{b(3-n)}\right)(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(3/2)/b/e/(3-n)/(a+b*x^n)^(1/2)+2/3*(A/a-3*B/b/(3-n))*(e*x)^(3/2
)*(1+b*x^n/a)^(1/2)*hypergeom([3/2, 3/2/n],[1+3/2/n],-b*x^n/a)/e/(a+b*x^n)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2x\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\left(A(3+2n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hy}\right.}{3a(3+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a]*(A*(3 + 2*n)*Hypergeometric2F1[3/2, 3/(
2*n), 1 + 3/(2*n), -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[3/2, (3/2 + n
)/n, 2 + 3/(2*n), -((b*x^n)/a)]))/(3*a*(3 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{3aB}{b(3-n)}\right)\int \frac{\sqrt{ex}}{(bx^n+a)^{3/2}}dx+\frac{2B(ex)^{3/2}}{be(3-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(3-n)}\right)\int \frac{\sqrt{ex}}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}+\frac{2B(ex)^{3/2}}{be(3-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(3-n)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3ae\sqrt{a+bx^n}}+\frac{2B(ex)^{3/2}}{be(3-n)\sqrt{a+bx^n}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*B*(e*x)^(3/2))/(b*e*(3 - n)*Sqrt[a + b*x^n]) + (2*(A - (3*a*B)/(b*(3 -
n)))*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 3/(2*n), 1 + 3
/(2*n), -((b*x^n)/a)])/(3*a*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{ex}\,(A+B\,x^n)}{(a+b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{3}{2n}}a^{-\frac{3}{2}-\frac{3}{2n}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{3}{2n}\\1+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(3/(2*n))*a**(-3/2 - 3/(2*n))*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper((
3/2, 3/(2*n)), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/(2
*n))) + B*a**(-5/2 - 3/(2*n))*a**(1 + 3/(2*n))*sqrt(e)*x**(n + 3/2)*gamma(
1 + 3/(2*n))*hyper((3/2, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \int \frac{\sqrt{e\,x}\,(A+B\,x^n)}{(a+b\,x^n)^{3/2}}\,dx$$

input
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(3/2),x)
```

output
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{x^nb+a}}{x^nb+a}dx\right)$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b + a),x)
```

3.389 $\int \frac{A+Bx^n}{\sqrt{ex}(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx=\frac{2B\sqrt{ex}}{be(1-n)\sqrt{a+bx^n}}+\frac{2\left(\frac{A}{a}-\frac{B}{b(1-n)}\right)\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{e\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(1/2)/b/e/(1-n)/(a+b*x^n)^(1/2)+2*(A/a-B/b/(1-n))*(e*x)^(1/2)*(1
+b*x^n/a)^(1/2)*hypergeom([3/2, 1/2/n],[1+1/2/n],-b*x^n/a)/e/(a+b*x^n)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx=\frac{2x\sqrt{1+\frac{bx^n}{a}}\left((A+2An)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)+Bx^n\,\text{Hy}\right.}{(a+2an)\sqrt{ex}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[e*x]*(a + b*x^n)^(3/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*((A + 2*A*n)*Hypergeometric2F1[3/2, 1/(2*n), (2 +
 n^(-1))/2, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[3/2, (1/2 + n)/n, (4 +
 n^(-1))/2, -((b*x^n)/a)]))/((a + 2*a*n)*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{b-bn}\right)\int \frac{1}{\sqrt{ex}\,(bx^n+a)^{3/2}}dx+\frac{2B\sqrt{ex}}{be(1-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-bn}\right)\int \frac{1}{\sqrt{ex}\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}}{be(1-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-bn}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{ae\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}}{be(1-n)\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/(Sqrt[e*x]*(a + b*x^n)^(3/2)),x]
```

output
```
(2*B*Sqrt[e*x])/(b*e*(1 - n)*Sqrt[a + b*x^n]) + (2*(A - (a*B)/(b - b*n))*S
qrt[e*x]*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, 1/(2*n), (2 + n^(-1))/
2, -((b*x^n)/a)])/(a*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\sqrt{e x}\,\left(a+b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.83 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{1}{2n}}a^{-\frac{3}{2}-\frac{1}{2n}}\sqrt{x}\,\Gamma\left(\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{1}{2n}\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}\,n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{Ba^{-\frac{5}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}x^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}\,n\Gamma\left(2+\frac{1}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(1/2)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(1/(2*n))*a**(-3/2 - 1/(2*n))*sqrt(x)*gamma(1/(2*n))*hyper((3/2, 1/(2
*n)), (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(2*
n))) + B*a**(-5/2 - 1/(2*n))*a**(1 + 1/(2*n))*x**(n + 1/2)*gamma(1 + 1/(2*
n))*hyper((3/2, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(s
qrt(e)*n*gamma(2 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx=\int \frac{A+B\,x^n}{\sqrt{e\,x}\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{3/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n bx+ax}dx\right)}{e}$$

input
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*x + a*x),x))/e
```

3.390 $\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx=-\frac{2B}{be(1+n)\sqrt{ex}\sqrt{a+bx^n}}-\frac{2\left(\frac{A}{a}-\frac{B}{b+bn}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}\sqrt{a+bx^n}}$$

output
```
-2*B/b/e/(1+n)/(e*x)^(1/2)/(a+b*x^n)^(1/2)-2*(A/a-B/(b*n+b))*(1+b*x^n/a)^(
1/2)*hypergeom([3/2, -1/2/n],[1-1/2/n],-b*x^n/a)/e/(e*x)^(1/2)/(a+b*x^n)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx=\frac{2x\sqrt{1+\frac{bx^n}{a}}\left(Bx^n\,\text{Hypergeometric2F1}\left(\frac{3}{2},1-\frac{1}{2n},2-\frac{1}{2n},-\frac{bx^n}{a}\right)+A(1-2n)\right.}{a(-1+2n)(ex)^{3/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(3/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(B*x^n*Hypergeometric2F1[3/2, 1 - 1/(2*n), 2 - 1/
(2*n), -((b*x^n)/a)] + A*(1 - 2*n)*Hypergeometric2F1[3/2, -1/2*1/n, 1 - 1/
(2*n), -((b*x^n)/a)]))/(a*(-1 + 2*n)*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{(ex)^{3/2}(bx^n+a)^{3/2}}dx-\frac{2B}{be(n+1)\sqrt{ex}\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{(ex)^{3/2}\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}-\frac{2B}{be(n+1)\sqrt{ex}\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{ae\sqrt{ex}\sqrt{a+bx^n}}-\frac{2B}{be(n+1)\sqrt{ex}\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*B)/(b*e*(1 + n)*Sqrt[e*x]*Sqrt[a + b*x^n]) - (2*(A - (a*B)/(b + b*n))*
Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -1/2*1/n, 1 - 1/(2*n), -((b*x^n
)/a)])/(a*e*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(e x\right)^{\frac{3}{2}} \left(a +b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{(ex)^{3/2}\,(a+bx^n)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx = \frac{Aa^{-\frac{1}{2n}}a^{-\frac{3}{2}+\frac{1}{2n}}\Gamma\left(-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},-\frac{1}{2n}\\1-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\sqrt{x}\Gamma\left(1-\frac{1}{2n}\right)} + \frac{Ba^{-\frac{5}{2}+\frac{1}{2n}}a^{1-\frac{1}{2n}}x^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},1-\frac{1}{2n}\\2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(3/2)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(-3/2 + 1/(2*n))*gamma(-1/(2*n))*hyper((3/2, -1/(2*n)), (1 - 1/(2*n),
), b*x**n*exp_polar(I*pi)/a)/(a**(1/(2*n))*e**(3/2)*n*sqrt(x)*gamma(1 - 1/
(2*n))) + B*a**(-5/2 + 1/(2*n))*a**(1 - 1/(2*n))*x**(n - 1/2)*gamma(1 - 1/
(2*n))*hyper((3/2, 1 - 1/(2*n)), (2 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a)
/(e**(3/2)*n*gamma(2 - 1/(2*n)))
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx = \int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx=\int \frac{A+B\,x^n}{(e\,x)^{3/2}\,(a+b\,x^n)^{3/2}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{3/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^n b\,x^2+a\,x^2}dx\right)}{e^2}$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*x**2 + a*x**2),x))/e**2
```

3.391 $\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{3/2}}\,dx = -\frac{2B}{be(3+n)(ex)^{3/2}\sqrt{a+bx^n}} - \frac{2\left(\frac{A}{a}-\frac{3B}{b(3+n)}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}\sqrt{a+bx^n}}$$

output
```
-2*B/b/e/(3+n)/(e*x)^(3/2)/(a+b*x^n)^(1/2)-2/3*(A/a-3*B/b/(3+n))*(1+b*x^n/
a)^(1/2)*hypergeom([3/2, -3/2/n],[1-3/2/n],-b*x^n/a)/e/(e*x)^(3/2)/(a+b*x^
n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{3/2}}\,dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left(3Bx^n\,\text{Hypergeometric2F1}\left(\frac{3}{2},1-\frac{3}{2n},2-\frac{3}{2n},-\frac{bx^n}{a}\right)+A(3-2n\right.}{3a(-3+2n)(ex)^{5/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(3/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(3*B*x^n*Hypergeometric2F1[3/2, 1 - 3/(2*n), 2 -
3/(2*n), -((b*x^n)/a)] + A*(3 - 2*n)*Hypergeometric2F1[3/2, -3/(2*n), 1 -
3/(2*n), -((b*x^n)/a)]))/(3*a*(-3 + 2*n)*(e*x)^(5/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{3aB}{b(n+3)}\right)\int \frac{1}{(ex)^{5/2}(bx^n+a)^{3/2}}dx-\frac{2B}{be(n+3)(ex)^{3/2}\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(n+3)}\right)\int \frac{1}{(ex)^{5/2}\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^n}}-\frac{2B}{be(n+3)(ex)^{3/2}\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{3aB}{b(n+3)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3ae(ex)^{3/2}\sqrt{a+bx^n}}-\frac{2B}{be(n+3)(ex)^{3/2}\sqrt{a+bx^n}}$$

input
```
Int[(A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(3/2)),x]
```

output
```
(-2*B)/(b*e*(3 + n)*(e*x)^(3/2)*Sqrt[a + b*x^n]) - (2*(A - (3*a*B)/(b*(3 +
 n)))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[3/2, -3/(2*n), 1 - 3/(2*n), -(
(b*x^n)/a)])/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(e x \right)^{\frac{5}{2}} \left(a +b\,x^{n}\right)^{\frac{3}{2}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*x**n)/(e*x)**(5/2)/(a+b*x**n)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*(e*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(3/2)*(e*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{A+B\,x^n}{{\left(e\,x\right)}^{5/2}\,{\left(a+b\,x^n\right)}^{3/2}} \,d x$$

input
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(3/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{3/2}}\,dx=\frac{\sqrt{e}\,\left(\int \frac{\sqrt{x}\,\sqrt{x^{n} b+a}}{x^{n} b\,x^{3}+a\,x^{3}}d x \right)}{e^{3}}$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(3/2),x)
```

output
```
(sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a))/(x**n*b*x**3 + a*x**3),x))/e**3
```

3.392 $\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2B(ex)^{5/2}}{be(5-3n)(a+bx^n)^{3/2}} - \frac{2(5aB-Ab(5-3n))(ex)^{5/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2be(5-3n)\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(5/2)/b/e/(5-3*n)/(a+b*x^n)^(3/2)-2/5*(5*B*a-A*b*(5-3*n))*(e*x)^
(5/2)*(1+b*x^n/a)^(1/2)*hypergeom([5/2, 5/2/n],[1+5/2/n],-b*x^n/a)/a^2/b/e
/(5-3*n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2x(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\left(A(5+2n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)+5B\right.}{5a^2(5+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*(A*(5 + 2*n)*Hypergeometric2F1[5/2, 5
/(2*n), 1 + 5/(2*n), -((b*x^n)/a)] + 5*B*x^n*Hypergeometric2F1[5/2, (5/2 +
 n)/n, 2 + 5/(2*n), -((b*x^n)/a)]))/(5*a^2*(5 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{5aB}{5b-3bn}\right)\int \frac{(ex)^{3/2}}{(bx^n+a)^{5/2}}dx+\frac{2B(ex)^{5/2}}{be(5-3n)(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{5b-3bn}\right)\int \frac{(ex)^{3/2}}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}}{be(5-3n)(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{5/2}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{5aB}{5b-3bn}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{5}{2n},1+\frac{5}{2n},-\frac{bx^n}{a}\right)}{5a^2e\sqrt{a+bx^n}}+\frac{2B(ex)^{5/2}}{be(5-3n)(a+bx^n)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*B*(e*x)^(5/2))/(b*e*(5 - 3*n)*(a + b*x^n)^(3/2)) + (2*(A - (5*a*B)/(5*b
 - 3*b*n))*(e*x)^(5/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 5/(2*n),
 1 + 5/(2*n), -((b*x^n)/a)])/(5*a^2*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (A + B\,x^n)}{(a + b\,x^n)^{\frac{5}{2}}} dx$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx=\int \frac{(Bx^n+A)(ex)^{\frac{3}{2}}}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^(3/2)/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx=\int \frac{(e\,x)^{3/2}\,(A+B\,x^n)}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(5/2),x)
```

output
```
int(((e*x)^(3/2)*(A + B*x^n))/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx=\sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{x^nb+a}\,x}{x^{2n}b^2+2x^nab+a^2}dx\right)e$$

input
```
int((e*x)^(3/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a)*x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**
2),x)*e
```

3.393 $\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2B(ex)^{3/2}}{3be(1-n)\,(a+bx^n)^{3/2}} - \frac{2(aB-Ab(1-n))(ex)^{3/2}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2be(1-n)\sqrt{a+bx^n}}$$

output
```
2/3*B*(e*x)^(3/2)/b/e/(1-n)/(a+b*x^n)^(3/2)-2/3*(B*a-A*b*(1-n))*(e*x)^(3/2
)*(1+b*x^n/a)^(1/2)*hypergeom([5/2, 3/2/n],[1+3/2/n],-b*x^n/a)/a^2/b/e/(1-
n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2x\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\left(A(3+2n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)+3Bx^n\,\text{Hy}\right.}{3a^2(3+2n)\sqrt{a+bx^n}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a]*(A*(3 + 2*n)*Hypergeometric2F1[5/2, 3/(
2*n), 1 + 3/(2*n), -((b*x^n)/a)] + 3*B*x^n*Hypergeometric2F1[5/2, (3/2 + n
)/n, 2 + 3/(2*n), -((b*x^n)/a)]))/(3*a^2*(3 + 2*n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B(ex)^{3/2}}{3be(1-n)(a+bx^n)^{3/2}}-\frac{(aB-Ab(1-n))\int \frac{\sqrt{ex}}{(bx^n+a)^{5/2}}dx}{b(1-n)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B(ex)^{3/2}}{3be(1-n)(a+bx^n)^{3/2}}-\frac{\sqrt{\frac{bx^n}{a}+1}(aB-Ab(1-n))\int \frac{\sqrt{ex}}{\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2b(1-n)\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B(ex)^{3/2}}{3be(1-n)(a+bx^n)^{3/2}}-\frac{2(ex)^{3/2}\sqrt{\frac{bx^n}{a}+1}(aB-Ab(1-n))\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{3}{2n},1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2be(1-n)\sqrt{a+bx^n}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*B*(e*x)^(3/2))/(3*b*e*(1 - n)*(a + b*x^n)^(3/2)) - (2*(a*B - A*b*(1 - n
))*(e*x)^(3/2)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 3/(2*n), 1 + 3/(
2*n), -((b*x^n)/a)])/(3*a^2*b*e*(1 - n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{\sqrt{ex}\,(A+B\,x^n)}{(a+b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 48.80 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{3}{2n}}a^{-\frac{5}{2}-\frac{3}{2n}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{3}{2n}\\1+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)} + \frac{Ba^{-\frac{7}{2}-\frac{3}{2n}}a^{1+\frac{3}{2n}}\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(3/(2*n))*a**(-5/2 - 3/(2*n))*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper((
5/2, 3/(2*n)), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/(2
*n))) + B*a**(-7/2 - 3/(2*n))*a**(1 + 3/(2*n))*sqrt(e)*x**(n + 3/2)*gamma(
1 + 3/(2*n))*hyper((5/2, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)\sqrt{ex}}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(e*x)/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{\sqrt{e\,x}\,(A+B\,x^n)}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(5/2),x)
```

output
```
int(((e*x)^(1/2)*(A + B*x^n))/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \sqrt{e}\left(\int \frac{\sqrt{x}\,\sqrt{x^n b+a}}{x^{2n}b^2+2x^n ab+a^2}dx\right)$$

input
```
int((e*x)^(1/2)*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
sqrt(e)*int((sqrt(x)*sqrt(x**n*b + a))/(x**(2*n)*b**2 + 2*x**n*a*b + a**2)
,x)
```

3.394 $\int \frac{A+Bx^n}{\sqrt{ex}(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\frac{2B\sqrt{ex}}{be(1-3n)\,(a+bx^n)^{3/2}}-\frac{2(aB-Ab(1-3n))\sqrt{ex}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^2be(1-3n)\sqrt{a+bx^n}}$$

output
```
2*B*(e*x)^(1/2)/b/e/(1-3*n)/(a+b*x^n)^(3/2)-2*(B*a-A*b*(1-3*n))*(e*x)^(1/2
)*(1+b*x^n/a)^(1/2)*hypergeom([5/2, 1/2/n],[1+1/2/n],-b*x^n/a)/a^2/b/e/(1-
3*n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\frac{2x\sqrt{1+\frac{bx^n}{a}}\left((A+2An)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)+Bx^n\,\text{Hy}\right.}{a^2(1+2n)\sqrt{ex}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/(Sqrt[e*x]*(a + b*x^n)^(5/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*((A + 2*A*n)*Hypergeometric2F1[5/2, 1/(2*n), (2 +
 n^(-1))/2, -((b*x^n)/a)] + B*x^n*Hypergeometric2F1[5/2, (1/2 + n)/n, (4 +
 n^(-1))/2, -((b*x^n)/a)]))/(a^2*(1 + 2*n)*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{b-3bn}\right)\int \frac{1}{\sqrt{ex}\,(bx^n+a)^{5/2}}dx+\frac{2B\sqrt{ex}}{be(1-3n)\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-3bn}\right)\int \frac{1}{\sqrt{ex}\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}}{be(1-3n)\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2\sqrt{ex}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{b-3bn}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{a^2e\sqrt{a+bx^n}}+\frac{2B\sqrt{ex}}{be(1-3n)\,(a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/(Sqrt[e*x]*(a + b*x^n)^(5/2)),x]
```

output
```
(2*B*Sqrt[e*x])/(b*e*(1 - 3*n)*(a + b*x^n)^(3/2)) + (2*(A - (a*B)/(b - 3*b
*n))*Sqrt[e*x]*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, 1/(2*n), (2 + n^
(-1))/2, -((b*x^n)/a)])/(a^2*e*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\sqrt{e x}\,\left(a+b\,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 133.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.17

$$\int \frac{A + Bx^n}{\sqrt{ex}\,(a + bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{1}{2n}}a^{-\frac{5}{2}-\frac{1}{2n}}\sqrt{x}\,\Gamma\left(\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{1}{2n}\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{Ba^{-\frac{7}{2}-\frac{1}{2n}}a^{1+\frac{1}{2n}}x^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{2},1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)}$$

input
```
integrate((A+B*x**n)/(e*x)**(1/2)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(1/(2*n))*a**(-5/2 - 1/(2*n))*sqrt(x)*gamma(1/(2*n))*hyper((5/2, 1/(2
*n)), (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(2*
n))) + B*a**(-7/2 - 1/(2*n))*a**(1 + 1/(2*n))*x**(n + 1/2)*gamma(1 + 1/(2*
n))*hyper((5/2, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(s
qrt(e)*n*gamma(2 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{A + Bx^n}{\sqrt{ex}\,(a + bx^n)^{5/2}}\,dx = \int \frac{Bx^n + A}{(bx^n + a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\int \frac{Bx^n+A}{(bx^n+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\int \frac{A+B\,x^n}{\sqrt{e\,x}\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(1/2)*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{\sqrt{ex}\,(a+bx^n)^{5/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x^n b+a}}{x^{2n+\frac{1}{2}}b^2+2x^{n+\frac{1}{2}}ab+\sqrt{x}\,a^2}dx\right)}{e}$$

input
```
int((A+B*x^n)/(e*x)^(1/2)/(a+b*x^n)^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(x**n*b + a)/(x**((4*n + 1)/2)*b**2 + 2*x**((2*n + 1)/2)*
a*b + sqrt(x)*a**2),x))/e
```

3.395 $\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{5/2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}} \, dx = -\frac{2B}{be(1+3n)\sqrt{ex}\left(a+bx^n\right)^{3/2}} - \frac{2\left(A-\frac{aB}{b+3bn}\right)\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{a^2e\sqrt{ex}\sqrt{a+bx^n}}$$

output
```
-2*B/b/e/(1+3*n)/(e*x)^(1/2)/(a+b*x^n)^(3/2)-2*(A-a*B/(3*b*n+b))*(1+b*x^n/
a)^(1/2)*hypergeom([5/2, -1/2/n],[1-1/2/n],-b*x^n/a)/a^2/e/(e*x)^(1/2)/(a+
b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}} \, dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left(Bx^n\,\text{Hypergeometric2F1}\left(\frac{5}{2},1-\frac{1}{2n},2-\frac{1}{2n},-\frac{bx^n}{a}\right)+A(1-2n)\right.}{a^2(-1+2n)(ex)^{3/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(5/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(B*x^n*Hypergeometric2F1[5/2, 1 - 1/(2*n), 2 - 1/
(2*n), -((b*x^n)/a)] + A*(1 - 2*n)*Hypergeometric2F1[5/2, -1/2*1/n, 1 - 1/
(2*n), -((b*x^n)/a)]))/(a^2*(-1 + 2*n)*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{3bn+b}\right)\int \frac{1}{(ex)^{3/2}(bx^n+a)^{5/2}}dx-\frac{2B}{be(3n+1)\sqrt{ex}\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{3bn+b}\right)\int \frac{1}{(ex)^{3/2}\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}-\frac{2B}{be(3n+1)\sqrt{ex}\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{3bn+b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{1}{2n},1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{a^2e\sqrt{ex}\sqrt{a+bx^n}}-\frac{2B}{be(3n+1)\sqrt{ex}\,(a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*B)/(b*e*(1 + 3*n)*Sqrt[e*x]*(a + b*x^n)^(3/2)) - (2*(A - (a*B)/(b + 3*
b*n))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -1/2*1/n, 1 - 1/(2*n), -(
(b*x^n)/a)])/(a^2*e*Sqrt[e*x]*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{(ex)^{\frac{3}{2}}\left(a+b\,x^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*x**n)/(e*x)**(3/2)/(a+b*x**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{5/2}}\,dx=\int \frac{A+B\,x^n}{(e\,x)^{3/2}\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(3/2)*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{3/2}(a+bx^n)^{5/2}}\,dx=\frac{\sqrt{e}\left(\int \frac{\sqrt{x^n b+a}}{x^{2n+\frac{1}{2}}b^2x+2x^{n+\frac{1}{2}}abx+\sqrt{x}\,a^2x}dx\right)}{e^2}$$

input
```
int((A+B*x^n)/(e*x)^(3/2)/(a+b*x^n)^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(x**n*b + a)/(x**((4*n + 1)/2)*b**2*x + 2*x**((2*n + 1)/2
)*a*b*x + sqrt(x)*a**2*x),x))/e**2
```

3.396 $\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx = -\frac{2B}{3be(1+n)(ex)^{3/2}(a+bx^n)^{3/2}} + \frac{2(aB-Ab(1+n))\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2be(1+n)(ex)^{3/2}\sqrt{a+bx^n}}$$

output
```
-2/3*B/b/e/(1+n)/(e*x)^(3/2)/(a+b*x^n)^(3/2)+2/3*(B*a-A*b*(1+n))*(1+b*x^n/
a)^(1/2)*hypergeom([5/2, -3/2/n],[1-3/2/n],-b*x^n/a)/a^2/b/e/(1+n)/(e*x)^(
3/2)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx = \frac{2x\sqrt{1+\frac{bx^n}{a}}\left(3Bx^n\,\text{Hypergeometric2F1}\left(\frac{5}{2},1-\frac{3}{2n},2-\frac{3}{2n},-\frac{bx^n}{a}\right)+A(3-2n)\right.}{3a^2(-3+2n)(ex)^{5/2}\sqrt{a+bx^n}}$$

input
```
Integrate[(A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(5/2)),x]
```

output
```
(2*x*Sqrt[1 + (b*x^n)/a]*(3*B*x^n*Hypergeometric2F1[5/2, 1 - 3/(2*n), 2 -
3/(2*n), -((b*x^n)/a)] + A*(3 - 2*n)*Hypergeometric2F1[5/2, -3/(2*n), 1 -
3/(2*n), -((b*x^n)/a)]))/(3*a^2*(-3 + 2*n)*(e*x)^(5/2)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{(ex)^{5/2}(bx^n+a)^{5/2}}dx-\frac{2B}{3be(n+1)(ex)^{3/2}(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\int \frac{1}{(ex)^{5/2}\left(\frac{bx^n}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^n}}-\frac{2B}{3be(n+1)(ex)^{3/2}(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{aB}{bn+b}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2n},1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3a^2e(ex)^{3/2}\sqrt{a+bx^n}}-\frac{2B}{3be(n+1)(ex)^{3/2}(a+bx^n)^{3/2}}$$

input
```
Int[(A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(5/2)),x]
```

output
```
(-2*B)/(3*b*e*(1 + n)*(e*x)^(3/2)*(a + b*x^n)^(3/2)) - (2*(A - (a*B)/(b +
b*n))*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[5/2, -3/(2*n), 1 - 3/(2*n), -(
(b*x^n)/a)])/(3*a^2*e*(e*x)^(3/2)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{A+B\,x^{n}}{\left(e x\right)^{\frac{5}{2}} \left(a +b\,x^{n}\right)^{\frac{5}{2}}}d x$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x)
```

output
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((A+B*x**n)/(e*x)**(5/2)/(a+b*x**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*(e*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}\left(a+bx^n\right)^{5/2}}\,dx=\int \frac{Bx^n+A}{\left(bx^n+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)/((b*x^n + a)^(5/2)*(e*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx = \int \frac{A+B\,x^n}{(e\,x)^{5/2}\,(a+b\,x^n)^{5/2}}\,dx$$

input
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(5/2)),x)
```

output
```
int((A + B*x^n)/((e*x)^(5/2)*(a + b*x^n)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx^n}{(ex)^{5/2}(a+bx^n)^{5/2}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{x^n b+a}}{x^{2n+\frac{1}{2}}b^2x^2+2x^{n+\frac{1}{2}}ab\,x^2+\sqrt{x}\,a^2x^2}dx\right)}{e^3}$$

input
```
int((A+B*x^n)/(e*x)^(5/2)/(a+b*x^n)^(5/2),x)
```

output
```
(sqrt(e)*int(sqrt(x**n*b + a)/(x**((4*n + 1)/2)*b**2*x**2 + 2*x**((2*n + 1
)/2)*a*b*x**2 + sqrt(x)*a**2*x**2),x))/e**3
```

3.397 $\int (ex)^m (a+bx^n)^2 (a(1+m)+b(1+m+3n)x^n)\,d$

Optimal result

Integrand size = 32, antiderivative size = 20

$$\int (ex)^m (a+bx^n)^2 (a(1+m)+b(1+m+3n)x^n)\,dx=\frac{(ex)^{1+m}(a+bx^n)^3}{e}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^3/e
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (ex)^m (a+bx^n)^2 (a(1+m)+b(1+m+3n)x^n)\,dx=x(ex)^m(a+bx^n)^3$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^2*(a*(1 + m) + b*(1 + m + 3*n)*x^n),x]
```

output
```
x*(e*x)^m*(a + b*x^n)^3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(20) = 40.

Time = 0.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^2 (a(m+1)+b(m+3n+1)x^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3(m+1)(ex)^m+3a^2b(m+n+1)x^n(ex)^m+3ab^2(m+2n+1)x^{2n}(ex)^m+b^3(m+3n+1)x^{3n}(ex)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(ex)^{m+1}}{e}+3a^2bx^{n+1}(ex)^m+3ab^2x^{2n+1}(ex)^m+b^3x^{3n+1}(ex)^m$$

input
```
Int[(e*x)^m*(a + b*x^n)^2*(a*(1 + m) + b*(1 + m + 3*n)*x^n),x]
```

output
```
3*a^2*b*x^(1 + n)*(e*x)^m + 3*a*b^2*x^(1 + 2*n)*(e*x)^m + b^3*x^(1 + 3*n)*
(e*x)^m + (a^3*(e*x)^(1 + m))/e
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.73 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.95

method	result	size
parallelrisch	$x\,x^{3n}(ex)^{m}\,b^{3}+3x\,x^{2n}(ex)^{m}\,a\,b^{2}+3x\,x^{n}(ex)^{m}\,a^{2}b+x(ex)^{m}\,a^{3}$	59
risch	$x(b^{3}x^{3n}+3a\,b^{2}x^{2n}+3b\,a^{2}x^{n}+a^{3})\,e^{m}x^{m}\mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\,\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}$	82
orering	Expression too large to display	1654

input
```
int((e*x)^m*(a+b*x^n)^2*(a*(1+m)+b*(1+m+3*n)*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(x^n)^3*(e*x)^m*b^3+3*x*(x^n)^2*(e*x)^m*a*b^2+3*x*x^n*(e*x)^m*a^2*b+x*(e
*x)^m*a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(20) = 40$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int (ex)^{m}\,(a+bx^{n})^{2}\,(a(1+m)+b(1+m+3n)x^{n})\;dx$$
$$= b^{3}xx^{3\,n}e^{(m\log(e)+m\log(x))}+3\,ab^{2}xx^{2\,n}e^{(m\log(e)+m\log(x))} + 3\,a^{2}bxx^{n}e^{(m\log(e)+m\log(x))}+a^{3}xe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(a*(1+m)+b*(1+m+3*n)*x^n),x, algorithm="fric
as")
```

output
```
b^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*a*b^2*x*x^(2*n)*e^(m*log(e) + m*
log(x)) + 3*a^2*b*x*x^n*e^(m*log(e) + m*log(x)) + a^3*x*e^(m*log(e) + m*lo
g(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(15) = 30$.

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.00

$$\int (ex)^m (a + bx^n)^2 (a(1 + m) + b(1 + m + 3n)x^n) \, dx$$
$$= a^3x(ex)^m + 3a^2bxx^n(ex)^m + 3ab^2xx^{2n}(ex)^m + b^3xx^{3n}(ex)^m$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(a*(1+m)+b*(1+m+3*n)*x**n),x)
```

output
```
a**3*x*(e*x)**m + 3*a**2*b*x*x**n*(e*x)**m + 3*a*b**2*x*x**(2*n)*(e*x)**m
+ b**3*x*x**(3*n)*(e*x)**m
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 293, normalized size of antiderivative = 14.65

$$\int (ex)^m (a + bx^n)^2 (a(1 + m) + b(1 + m + 3n)x^n) \, dx$$
$$= \frac{b^3e^m mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,b^3e^m nxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,ab^2e^m mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{6\,ab^2e^m nxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2be^m mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{3\,a^2be^m nxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{b^3e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,ab^2e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,a^2be^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}\,a^3m}{e(m+1)} + \frac{(ex)^{m+1}\,a^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(a*(1+m)+b*(1+m+3*n)*x^n),x, algorithm="maxi
ma")
```

output
```
b^3*e^m*m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*b^3*e^m*n*x*e^(m*l
og(x) + 3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*e^m*m*x*e^(m*log(x) + 2*n*log(
x))/(m + 2*n + 1) + 6*a*b^2*e^m*n*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1
) + 3*a^2*b*e^m*m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 3*a^2*b*e^m*n*x*
e^(m*log(x) + n*log(x))/(m + n + 1) + b^3*e^m*x*e^(m*log(x) + 3*n*log(x))/
(m + 3*n + 1) + 3*a*b^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*
a^2*b*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*a^3*m/(e*(
m + 1)) + (e*x)^(m + 1)*a^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(20) = 40$.

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 9.25

$$\int (ex)^m (a + bx^n)^2 (a(1 + m) + b(1 + m + 3n)x^n) \, dx = b^3 x x^{3n} e^{(m \log(e) + m \log(x))} + 3 ab^2 x x^{2n} e^{(m \log(e) + m \log(x))} + b^3 x x^{2n} e^{(m \log(e) + m \log(x))} + 3 a^2 b x x^n e^{(m \log(e) + m \log(x))} + 3 ab^2 x x^n e^{(m \log(e) + m \log(x))} + b^3 x x^n e^{(m \log(e) + m \log(x))} + a^3 x e^{(m \log(e) + m \log(x))} + 3 a^2 b x e^{(m \log(e) + m \log(x))} + 3 ab^2 x e^{(m \log(e) + m \log(x))} + b^3 x e^{(m \log(e) + m \log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(a*(1+m)+b*(1+m+3*n)*x^n),x, algorithm="giac
")
```

output
```
b^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*a*b^2*x*x^(2*n)*e^(m*log(e) + m*
log(x)) + b^3*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*a^2*b*x*x^n*e^(m*log(e
) + m*log(x)) + 3*a*b^2*x*x^n*e^(m*log(e) + m*log(x)) + b^3*x*x^n*e^(m*log
(e) + m*log(x)) + a^3*x*e^(m*log(e) + m*log(x)) + 3*a^2*b*x*e^(m*log(e) +
m*log(x)) + 3*a*b^2*x*e^(m*log(e) + m*log(x)) + b^3*x*e^(m*log(e) + m*log(
x))
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int (ex)^m (a+bx^n)^2 (a(1+m)+b(1+m+3n)x^n) \, dx = (e\,x)^m \left(a^3\,x+b^3\,x\,x^{3\,n}+3\,a^2\,b\,x\,x^n+3\,a\,b^2\,x\,x^{2\,n}\right)$$

input
```
int((e*x)^m*(a*(m + 1) + b*x^n*(m + 3*n + 1))*(a + b*x^n)^2,x)
```

output
```
(e*x)^m*(a^3*x + b^3*x*x^(3*n) + 3*a^2*b*x*x^n + 3*a*b^2*x*x^(2*n))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int (ex)^m (a+bx^n)^2 (a(1+m)+b(1+m+3n)x^n) \, dx = x^m e^m x\left(x^{3n}b^3+3x^{2n}a\,b^2+3x^n a^2 b+a^3\right)$$

input
```
int((e*x)^m*(a+b*x^n)^2*(a*(1+m)+b*(1+m+3*n)*x^n),x)
```

output
```
x**m*e**m*x*(x**(3*n)*b**3 + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b + a**3)
```

3.398 $\int (ex)^m (a+bx^n) (a(1+m)+b(1+m+2n)x^n) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 20

$$\int (ex)^m (a+bx^n) (a(1+m)+b(1+m+2n)x^n) \, dx = \frac{(ex)^{1+m} (a+bx^n)^2}{e}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^2/e
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int (ex)^m (a+bx^n) (a(1+m)+b(1+m+2n)x^n) \, dx = x(ex)^m (a+bx^n)^2$$

input
```
Integrate[(e*x)^m*(a + b*x^n)*(a*(1 + m) + b*(1 + m + 2*n)*x^n),x]
```

output
```
x*(e*x)^m*(a + b*x^n)^2
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a + bx^n) (a(m+1) + b(m+2n+1)x^n) \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(a^2(m+1)(ex)^m + 2ab(m+n+1)x^n(ex)^m + b^2(m+2n+1)x^{2n}(ex)^m\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2(ex)^{m+1}}{e} + 2abx^{n+1}(ex)^m + b^2x^{2n+1}(ex)^m$$

input
```
Int[(e*x)^m*(a + b*x^n)*(a*(1 + m) + b*(1 + m + 2*n)*x^n),x]
```

output
```
2*a*b*x^(1 + n)*(e*x)^m + b^2*x^(1 + 2*n)*(e*x)^m + (a^2*(e*x)^(1 + m))/e
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

method	result
parallelrisch	$x\,x^{2n}(ex)^m\,b^2+2x\,x^n(ex)^m\,ab+x(ex)^m\,a^2$
risch	$(b^2x^{2n}+2ab\,x^n+a^2)\,x\,e^m x^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}$
orering	$\frac{x(3m^2+6mn+2n^2+3m+3n+1)(ex)^m(a+b\,x^n)(a(1+m)+b(1+m+2n)x^n)}{(m^2+2mn+2m+2n+1)(1+m+n)}-\frac{3x^2(m+n)\left(\frac{(ex)^m m(a+b\,x^n)(a(1+m)+b(1+m}{x}\right.}{}$

input
```
int((e*x)^m*(a+b*x^n)*(a*(1+m)+b*(1+m+2*n)*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(x^n)^2*(e*x)^m*b^2+2*x*x^n*(e*x)^m*a*b+x*(e*x)^m*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int (ex)^m\,(a+bx^n)\,(a(1+m)+b(1+m+2n)x^n)\;dx$$
$$=b^2xx^{2\,n}e^{(m\log(e)+m\log(x))}+2\,abxx^ne^{(m\log(e)+m\log(x))}+a^2xe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(a*(1+m)+b*(1+m+2*n)*x^n),x, algorithm="fricas
")
```

output
```
b^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(e) + m*log(x)
) + a^2*x*e^(m*log(e) + m*log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(15) = 30.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int (ex)^m (a+bx^n) (a(1+m)+b(1+m+2n)x^n) \, dx = a^2x(ex)^m + 2abxx^n(ex)^m + b^2xx^{2n}(ex)^m$$

input
```
integrate((e*x)**m*(a+b*x**n)*(a*(1+m)+b*(1+m+2*n)*x**n),x)
```

output
```
a**2*x*(e*x)**m + 2*a*b*x*x**n*(e*x)**m + b**2*x*x**(2*n)*(e*x)**m
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(20) = 40.

Time = 0.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 9.90

$$\begin{aligned}\int (ex)^m (a+bx^n) (a(1+m)+b(1+m+2n)x^n) \, dx &= \frac{b^2e^m m x e^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{2b^2e^m n x e^{(m\log(x)+2n\log(x))}}{m+2n+1} \\ &\quad + \frac{2abe^m m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{2abe^m n x e^{(m\log(x)+n\log(x))}}{m+n+1} \\ &\quad + \frac{b^2e^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{2abe^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}a^2m}{e(m+1)} + \frac{(ex)^{m+1}a^2}{e(m+1)}\end{aligned}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(a*(1+m)+b*(1+m+2*n)*x^n),x, algorithm="maxima
")
```

output
```
b^2*e^m*m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*b^2*e^m*n*x*e^(m*l
og(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*e^m*m*x*e^(m*log(x) + n*log(x))/
(m + n + 1) + 2*a*b*e^m*n*x*e^(m*log(x) + n*log(x))/(m + n + 1) + b^2*e^m*
x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*e^m*x*e^(m*log(x) + n*lo
g(x))/(m + n + 1) + (e*x)^(m + 1)*a^2*m/(e*(m + 1)) + (e*x)^(m + 1)*a^2/(e
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 5.10

$$\int (ex)^m (a+bx^n)(a(1+m)+b(1+m+2n)x^n)\,dx$$
$$= b^2xx^{2n}e^{(m\log(e)+m\log(x))} + 2\,abxx^ne^{(m\log(e)+m\log(x))} + b^2xx^ne^{(m\log(e)+m\log(x))} + a^2xe^{(m\log(e)+m\log(x))} + 2\,abxe^{(m\log(e)+m\log(x))} + b^2xe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(a*(1+m)+b*(1+m+2*n)*x^n),x, algorithm="giac")
```

output
```
b^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 2*a*b*x*x^n*e^(m*log(e) + m*log(x)
) + b^2*x*x^n*e^(m*log(e) + m*log(x)) + a^2*x*e^(m*log(e) + m*log(x)) + 2*
a*b*x*e^(m*log(e) + m*log(x)) + b^2*x*e^(m*log(e) + m*log(x))
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int (ex)^m (a+bx^n)(a(1+m)+b(1+m+2n)x^n)\,dx = (e\,x)^m\,\left(a^2\,x+b^2\,x\,x^{2\,n}+2\,a\,b\,x\,x^n\right)$$

input
```
int((e*x)^m*(a*(m + 1) + b*x^n*(m + 2*n + 1))*(a + b*x^n),x)
```

output
```
(e*x)^m*(a^2*x + b^2*x*x^(2*n) + 2*a*b*x*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int (ex)^m (a+bx^n)(a(1+m)+b(1+m+2n)x^n)\,dx = x^me^mx\left(x^{2n}b^2+2x^nab+a^2\right)$$

input
```
int((e*x)^m*(a+b*x^n)*(a*(1+m)+b*(1+m+2*n)*x^n),x)
```

output
```
x**m*e**m*x*(x**(2*n)*b**2 + 2*x**n*a*b + a**2)
```

3.399 $\int (ex)^m (a(1+m)+b(1+m+n)x^n) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int (ex)^m (a(1+m)+b(1+m+n)x^n) \, dx = \frac{a(ex)^{1+m}}{e} + \frac{bx^n(ex)^{1+m}}{e}$$

output
```
a*(e*x)^(1+m)/e+b*x^n*(e*x)^(1+m)/e
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int (ex)^m (a(1+m)+b(1+m+n)x^n) \, dx = x(ex)^m (a+bx^n)$$

input
```
Integrate[(e*x)^m*(a*(1 + m) + b*(1 + m + n)*x^n),x]
```

output
```
x*(e*x)^m*(a + b*x^n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a(m+1)+b(m+n+1)x^n) \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int (a(m+1)(ex)^m+b(m+n+1)x^n(ex)^m) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(ex)^{m+1}}{e}+bx^{n+1}(ex)^m$$

input
```
Int[(e*x)^m*(a*(1 + m) + b*(1 + m + n)*x^n),x]
```

output
```
b*x^(1 + n)*(e*x)^m + (a*(e*x)^(1 + m))/e
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
parallelrisch	$x\,x^{n}(ex)^{m} b+x(ex)^{m} a$	21
norman	$ax\,\mathrm{e}^{m\ln(ex)}+bx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{m\ln(ex)}$	27
risch	$(a+b\,x^{n})\,x\,e^{m}x^{m}\mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\,\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}$	56
orering	$\frac{x(2m+n+1)(ex)^{m}(a(1+m)+b(1+m+n)x^{n})}{m^{2}+mn+2m+n+1}-\frac{x^{2}\left(\frac{(ex)^{m}m(a(1+m)+b(1+m+n)x^{n})}{x}+\frac{(ex)^{m}b(1+m+n)x^{n}n}{x}\right)}{m^{2}+mn+2m+n+1}$	107

input
```
int((e*x)^m*(a*(1+m)+b*(1+m+n)*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*x^n*(e*x)^m*b+x*(e*x)^m*a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (ex)^m\,(a(1+m)+b(1+m+n)x^n)\;dx=bxx^ne^{(m\log(e)+m\log(x))}+axe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m+n)*x^n),x, algorithm="fricas")
```

output
```
b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (ex)^m\,(a(1+m)+b(1+m+n)x^n)\;dx=ax(ex)^m+bxx^n(ex)^m$$

input
```
integrate((e*x)**m*(a*(1+m)+b*(1+m+n)*x**n),x)
```

output
```
a*x*(e*x)**m + b*x*x**n*(e*x)**m
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

$$\int (ex)^m (a(1+m)+b(1+m+n)x^n) \, dx = \frac{be^m mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{be^m nxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{be^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}am}{e(m+1)} + \frac{(ex)^{m+1}a}{e(m+1)}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m+n)*x^n),x, algorithm="maxima")
```

output
```
b*e^m*m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + b*e^m*n*x*e^(m*log(x) + n*
log(x))/(m + n + 1) + b*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^
(m + 1)*a*m/(e*(m + 1)) + (e*x)^(m + 1)*a/(e*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int (ex)^m (a(1+m)+b(1+m+n)x^n) \, dx = bxx^n e^{(m\log(e)+m\log(x))} + axe^{(m\log(e)+m\log(x))} + bxe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m+n)*x^n),x, algorithm="giac")
```

output
```
b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)) + b*x*e^(m*l
og(e) + m*log(x))
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int (ex)^m \left(a(1+m)+b(1+m+n)x^n\right)\,dx=(e\,x)^m\,\left(a\,x+b\,x^{n+1}\right)$$

input
```
int((a*(m + 1) + b*x^n*(m + n + 1))*(e*x)^m,x)
```

output
```
(e*x)^m*(a*x + b*x^(n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.54

$$\int (ex)^m \left(a(1+m)+b(1+m+n)x^n\right)\,dx=x^m e^m x(x^n b+a)$$

input
```
int((e*x)^m*(a*(1+m)+b*(1+m+n)*x^n),x)
```

output
```
x**m*e**m*x*(x**n*b + a)
```

3.400 $\int \frac{(ex)^m(a(1+m)+b(1+m)x^n)}{a+bx^n} dx$

Optimal result

Integrand size = 29, antiderivative size = 11

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n} \, dx = \frac{(ex)^{1+m}}{e}$$

output
```
(e*x)^(1+m)/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n} \, dx = x(ex)^m$$

input
```
Integrate[((e*x)^m*(a*(1 + m) + b*(1 + m)*x^n))/(a + b*x^n),x]
```

output
```
x*(e*x)^m
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {281, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a(m+1)+b(m+1)x^n)}{a+bx^n} \, dx$$

$$\Big\downarrow \text{281}$$

$$(m+1)\int (ex)^m dx$$

$$\Big\downarrow \text{17}$$

$$\frac{(ex)^{m+1}}{e}$$

input
```
Int[((e*x)^m*(a*(1 + m) + b*(1 + m)*x^n))/(a + b*x^n),x]
```

output
```
(e*x)^(1 + m)/e
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$(ex)^{m} x$	8
parallelrisch	$(ex)^{m} x$	8
norman	$x\,{\mathrm e}^{m \ln \left(e x \right)}$	10
orering	$\frac{x (e x)^{m} \left(a (1+m)+b (1+m) x^{n}\right)}{(1+m) \left(a +b \,x^{n}\right)}$	36
risch	$x\,e^{m} x^{m} {\mathrm e}^{\frac{i \,\mathrm{csgn}\left(i e x \right) \pi m \left(\mathrm{csgn}\left(i e x \right)-\mathrm{csgn}\left(i x \right)\right) \left(-\mathrm{csgn}\left(i e x \right)+\mathrm{csgn}\left(i e \right)\right)}{2}}$	49

input
```
int((e*x)^m*(a*(1+m)+b*(1+m)*x^n)/(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
(e*x)^m*x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n}\,dx = (ex)^m x$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m)*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
(e*x)^m*x
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n}\,dx = x(ex)^m$$

input
```
integrate((e*x)**m*(a*(1+m)+b*(1+m)*x**n)/(a+b*x**n),x)
```

output
```
x*(e*x)**m
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n}\,dx = e^m x x^m$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m)*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
e^m*x*x^m
```

Giac [F]

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n}\,dx = \int \frac{(b(m+1)x^n+a(m+1))(ex)^m}{bx^n+a}\,dx$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m)*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((b*(m + 1)*x^n + a*(m + 1))*(e*x)^m/(b*x^n + a), x)
```

Mupad [B] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n}\,dx = x\,(e\,x)^m$$

input
```
int(((e*x)^m*(a*(m + 1) + b*x^n*(m + 1)))/(a + b*x^n),x)
```

output
```
x*(e*x)^m
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m (a(1+m)+b(1+m)x^n)}{a+bx^n} \, dx = x^m e^m x$$

input
```
int((e*x)^m*(a*(1+m)+b*(1+m)*x^n)/(a+b*x^n),x)
```

output
```
x**m*e**m*x
```

3.401 $\int \frac{(ex)^m(a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 20

$$\int \frac{(ex)^m (a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2} dx = \frac{(ex)^{1+m}}{e(a+bx^n)}$$

output
```
(e*x)^(1+m)/e/(a+b*x^n)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.85

$$\int \frac{(ex)^m (a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2} dx$$
$$= \frac{x(ex)^m \left((1+m-n)\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right) + n\,\text{Hypergeometric2F1}\left(2, \frac{1+m}{n}, \frac{1+m+}{n}\right.\right.}{a(1+m)}$$

input
```
Integrate[((e*x)^m*(a*(1 + m) + b*(1 + m - n)*x^n))/(a + b*x^n)^2,x]
```

output
```
(x*(e*x)^m*((1 + m - n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((
b*x^n)/a)] + n*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
]))/(a*(1 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m\left(a(m+1)+b(m-n+1)x^n\right)}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{951}$$

$$\frac{(ex)^{m+1}}{e\left(a+bx^n\right)}$$

input
```
Int[((e*x)^m*(a*(1 + m) + b*(1 + m - n)*x^n))/(a + b*x^n)^2,x]
```

output
```
(e*x)^(1 + m)/(e*(a + b*x^n))
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{x(ex)^m}{a+b\,x^n}$	17
risch	$\frac{x\,e^m x^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}}{a+b\,x^n}$	58

input
```
int((e*x)^m*(a*(1+m)+b*(1+m-n)*x^n)/(a+b*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(e*x)^m/(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx=\frac{xe^{(m\log(e)+m\log(x))}}{bx^n+a}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-n)*x^n)/(a+b*x^n)^2,x, algorithm="fricas
")
```

output
```
x*e^(m*log(e) + m*log(x))/(b*x^n + a)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx=\frac{x(ex)^m}{a+bx^n}$$

input
```
integrate((e*x)**m*(a*(1+m)+b*(1+m-n)*x**n)/(a+b*x**n)**2,x)
```

output
```
x*(e*x)**m/(a + b*x**n)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m (a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx = \frac{e^m x x^m}{bx^n+a}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-n)*x^n)/(a+b*x^n)^2,x, algorithm="maxima
")
```

output
```
e^m*x*x^m/(b*x^n + a)
```

Giac [F]

$$\int \frac{(ex)^m (a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx = \int \frac{(b(m-n+1)x^n+a(m+1))(ex)^m}{(bx^n+a)^2}\,dx$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-n)*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*(m - n + 1)*x^n + a*(m + 1))*(e*x)^m/(b*x^n + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx = \frac{x\,(e\,x)^m}{a+b\,x^n}$$

input
```
int(((e*x)^m*(a*(m + 1) + b*x^n*(m - n + 1)))/(a + b*x^n)^2,x)
```

output
```
(x*(e*x)^m)/(a + b*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-n)x^n)}{(a+bx^n)^2}\,dx = \frac{x^m e^m x}{x^n b + a}$$

input
```
int((e*x)^m*(a*(1+m)+b*(1+m-n)*x^n)/(a+b*x^n)^2,x)
```

output
```
(x**m*e**m*x)/(x**n*b + a)
```

3.402 $\int \frac{(ex)^m(a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 20

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx=\frac{(ex)^{1+m}}{e\,(a+bx^n)^2}$$

output
```
(e*x)^(1+m)/e/(a+b*x^n)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx$$
$$=\frac{x(ex)^m\left((1+m-2n)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+2n\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},1+\right.\right.}{a^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a*(1 + m) + b*(1 + m - 2*n)*x^n))/(a + b*x^n)^3,x]
```

output
```
(x*(e*x)^m*((1 + m - 2*n)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)] + 2*n*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n
)/a)]))/(a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a(m+1)+b(m-2n+1)x^n)}{(a+bx^n)^3} dx$$

$$\Big\downarrow \text{951}$$

$$\frac{(ex)^{m+1}}{e(a+bx^n)^2}$$

input
```
Int[((e*x)^m*(a*(1 + m) + b*(1 + m - 2*n)*x^n))/(a + b*x^n)^3,x]
```

output
```
(e*x)^(1 + m)/(e*(a + b*x^n)^2)
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{x(ex)^m}{(a+b\,x^n)^2}$	17
risch	$\frac{x\,e^m x^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(iex)\pi m(\mathrm{csgn}(iex)-\mathrm{csgn}(ix))(-\,\mathrm{csgn}(iex)+\mathrm{csgn}(ie))}{2}}}{(a+b\,x^n)^2}$	58

input
```
int((e*x)^m*(a*(1+m)+b*(1+m-2*n)*x^n)/(a+b*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(e*x)^m/(a+b*x^n)^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx = \frac{xe^{(m\log(e)+m\log(x))}}{b^2x^{2\,n}+2\,abx^n+a^2}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-2*n)*x^n)/(a+b*x^n)^3,x, algorithm="fric
as")
```

output
```
x*e^(m*log(e) + m*log(x))/(b^2*x^(2*n) + 2*a*b*x^n + a^2)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{(ex)^m\,(a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx = \frac{x(ex)^m}{a^2+2abx^n+b^2x^{2n}}$$

input
```
integrate((e*x)**m*(a*(1+m)+b*(1+m-2*n)*x**n)/(a+b*x**n)**3,x)
```

output
```
x*(e*x)**m/(a**2 + 2*a*b*x**n + b**2*x**(2*n))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{(ex)^m (a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3} dx = \frac{e^m x x^m}{b^2 x^{2n} + 2 abx^n + a^2}$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-2*n)*x^n)/(a+b*x^n)^3,x, algorithm="maxi
ma")
```

output
```
e^m*x*x^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)
```

Giac [F]

$$\int \frac{(ex)^m (a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3} dx = \int \frac{(b(m-2n+1)x^n + a(m+1))(ex)^m}{(bx^n+a)^3} dx$$

input
```
integrate((e*x)^m*(a*(1+m)+b*(1+m-2*n)*x^n)/(a+b*x^n)^3,x, algorithm="giac
")
```

output
```
integrate((b*(m - 2*n + 1)*x^n + a*(m + 1))*(e*x)^m/(b*x^n + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{(ex)^m (a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx = \frac{x\,(e\,x)^m}{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}$$

input
```
int(((e*x)^m*(a*(m + 1) + b*x^n*(m - 2*n + 1)))/(a + b*x^n)^3,x)
```

output
```
(x*(e*x)^m)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{(ex)^m (a(1+m)+b(1+m-2n)x^n)}{(a+bx^n)^3}\,dx = \frac{x^m e^m x}{x^{2n}b^2+2x^n ab+a^2}$$

input
```
int((e*x)^m*(a*(1+m)+b*(1+m-2*n)*x^n)/(a+b*x^n)^3,x)
```

output
```
(x**m*e**m*x)/(x**(2*n)*b**2 + 2*x**n*a*b + a**2)
```

3.403 $\int (cx)^m (a+bx^n)^3 (A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\,dx = \frac{a^3A(cx)^{1+m}}{c(1+m)} + \frac{a^2(3Ab+aB)x^n(cx)^{1+m}}{c(1+m+n)} + \frac{3ab(Ab+aB)x^{2n}(cx)^{1+m}}{c(1+m+2n)} + \frac{b^2(Ab+3aB)x^{3n}(cx)^{1+m}}{c(1+m+3n)} + \frac{b^3Bx^{4n}(cx)^{1+m}}{c(1+m+4n)}$$

output
```
a^3*A*(c*x)^(1+m)/c/(1+m)+a^2*(3*A*b+B*a)*x^n*(c*x)^(1+m)/c/(1+m+n)+3*a*b*
(A*b+B*a)*x^(2*n)*(c*x)^(1+m)/c/(1+m+2*n)+b^2*(A*b+3*B*a)*x^(3*n)*(c*x)^(1
+m)/c/(1+m+3*n)+b^3*B*x^(4*n)*(c*x)^(1+m)/c/(1+m+4*n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\,dx = x(cx)^m \left(\frac{a^3A}{1+m} + \frac{a^2(3Ab+aB)x^n}{1+m+n} + \frac{3ab(Ab+aB)x^{2n}}{1+m+2n} + \frac{b^2(Ab+3aB)x^{3n}}{1+m+3n} + \frac{b^3Bx^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
x*(c*x)^m*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^n)/(1 + m + n) + (3*a*b*
(A*b + a*B)*x^(2*n))/(1 + m + 2*n) + (b^2*(A*b + 3*a*B)*x^(3*n))/(1 + m +
3*n) + (b^3*B*x^(4*n))/(1 + m + 4*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^3A(cx)^m+a^2x^n(cx)^m(aB+3Ab)+b^2x^{3n}(cx)^m(3aB+Ab)+3abx^{2n}(cx)^m(aB+Ab)+b^3Bx^{4n}(cx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3A(cx)^{m+1}}{c(m+1)}+\frac{a^2x^{n+1}(cx)^m(aB+3Ab)}{m+n+1}+\frac{b^2x^{3n+1}(cx)^m(3aB+Ab)}{m+3n+1}+\frac{3abx^{2n+1}(cx)^m(aB+Ab)}{m+2n+1}+\frac{b^3Bx^{4n+1}(cx)^m}{m+4n+1}$$

input
```
Int[(c*x)^m*(a + b*x^n)^3*(A + B*x^n),x]
```

output
```
(a^2*(3*A*b + a*B)*x^(1 + n)*(c*x)^m)/(1 + m + n) + (3*a*b*(A*b + a*B)*x^(
1 + 2*n)*(c*x)^m)/(1 + m + 2*n) + (b^2*(A*b + 3*a*B)*x^(1 + 3*n)*(c*x)^m)/
(1 + m + 3*n) + (b^3*B*x^(1 + 4*n)*(c*x)^m)/(1 + m + 4*n) + (a^3*A*(c*x)^(
1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.13 (sec) , antiderivative size = 1576, normalized size of antiderivative = 10.58

method	result	size
risch	Expression too large to display	1576
parallelrisch	Expression too large to display	2207
orering	Expression too large to display	2976

input
```
int((c*x)^m*(a+b*x^n)^3*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(6*B*b^3*m^3*n*(x^n)^4+11*B*b^3*m^2*n^2*(x^n)^4+6*B*b^3*m*n^3*(x^n)^4+27
*A*a^2*b*x^n*n+(x^n)^4*b^3*B+(x^n)^3*b^3*A+18*B*b^3*m^2*n*(x^n)^4+22*B*b^3
*m*n^2*(x^n)^4+3*A*a*b^2*m^4*(x^n)^2+21*A*b^3*m^2*n*(x^n)^3+28*A*b^3*m*n^2
*(x^n)^3+72*B*a^2*b*m^2*n*(x^n)^2+114*B*a^2*b*m*n^2*(x^n)^2+63*B*a*b^2*m*n
*(x^n)^3+81*A*a^2*b*m^2*n*x^n+156*A*a^2*b*m*n^2*x^n+72*A*a*b^2*m*n*(x^n)^2
+72*B*a^2*b*m*n*(x^n)^2+81*A*a^2*b*m*n*x^n+A*a^3*m^4+7*A*b^3*m^3*n*(x^n)^3
+14*A*b^3*m^2*n^2*(x^n)^3+8*A*b^3*m*n^3*(x^n)^3+3*B*a*b^2*m^4*(x^n)^3+36*B
*a^2*b*m*n^3*(x^n)^2+63*B*a*b^2*m^2*n*(x^n)^3+84*B*a*b^2*m*n^2*(x^n)^3+27*
A*a^2*b*m^3*n*x^n+78*A*a^2*b*m^2*n^2*x^n+72*A*a^2*b*m*n^3*x^n+72*A*a*b^2*m
^2*n*(x^n)^2+114*A*a*b^2*m*n^2*(x^n)^2+21*B*a*b^2*m^3*n*(x^n)^3+42*B*a*b^2
*m^2*n^2*(x^n)^3+24*B*a*b^2*m*n^3*(x^n)^3+24*A*a*b^2*m^3*n*(x^n)^2+57*A*a*
b^2*m^2*n^2*(x^n)^2+24*A*a^3*n^4+4*A*a^3*m^3+50*A*a^3*n^3+6*A*a^3*m^2+4*a^
3*A*m+10*a^3*A*n+10*A*a^3*m^3*n+35*A*a^3*m^2*n^2+50*A*a^3*m*n^3+30*A*a^3*m
^2*n+70*A*a^3*m*n^2+30*A*a^3*m*n+35*A*a^3*n^2+a^3*A+x^n*B*a^3+36*A*a*b^2*m
*n^3*(x^n)^2+24*B*a^2*b*m^3*n*(x^n)^2+57*B*a^2*b*m^2*n^2*(x^n)^2+21*B*a*b^
2*(x^n)^3*n+18*A*a^2*b*m^2*x^n+78*A*a^2*b*n^2*x^n+12*A*a*b^2*(x^n)^2*m+24*
A*a*b^2*(x^n)^2*n+27*B*a^3*m*n*x^n+12*B*a^2*b*(x^n)^2*m+24*B*a^2*b*(x^n)^2
*n+12*A*a^2*b*x^n*m+21*A*b^3*m*n*(x^n)^3+9*B*a^3*m^3*n*x^n+26*B*a^3*m^2*n^
2*x^n+24*B*a^3*m*n^3*x^n+12*B*a^2*b*m^3*(x^n)^2+36*B*a^2*b*n^3*(x^n)^2+3*B
*a^2*b*m^4*(x^n)^2+12*B*a*b^2*m^3*(x^n)^3+24*B*a*b^2*n^3*(x^n)^3+18*B*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(149) = 298$.

Time = 0.14 (sec) , antiderivative size = 1104, normalized size of antiderivative = 7.41

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a+b*x^n)^3*(A+B*x^n),x, algorithm="fricas")
```

output
```
((B*b^3*m^4 + 4*B*b^3*m^3 + 6*B*b^3*m^2 + 4*B*b^3*m + B*b^3 + 6*(B*b^3*m +
 B*b^3)*n^3 + 11*(B*b^3*m^2 + 2*B*b^3*m + B*b^3)*n^2 + 6*(B*b^3*m^3 + 3*B*
b^3*m^2 + 3*B*b^3*m + B*b^3)*n)*x*x^(4*n)*e^(m*log(c) + m*log(x)) + ((3*B*
a*b^2 + A*b^3)*m^4 + 3*B*a*b^2 + A*b^3 + 4*(3*B*a*b^2 + A*b^3)*m^3 + 8*(3*
B*a*b^2 + A*b^3 + (3*B*a*b^2 + A*b^3)*m)*n^3 + 6*(3*B*a*b^2 + A*b^3)*m^2 +
 14*(3*B*a*b^2 + A*b^3 + (3*B*a*b^2 + A*b^3)*m^2 + 2*(3*B*a*b^2 + A*b^3)*m
)*n^2 + 4*(3*B*a*b^2 + A*b^3)*m + 7*(3*B*a*b^2 + A*b^3 + (3*B*a*b^2 + A*b^
3)*m^3 + 3*(3*B*a*b^2 + A*b^3)*m^2 + 3*(3*B*a*b^2 + A*b^3)*m)*n)*x*x^(3*n)
*e^(m*log(c) + m*log(x)) + 3*((B*a^2*b + A*a*b^2)*m^4 + B*a^2*b + A*a*b^2
+ 4*(B*a^2*b + A*a*b^2)*m^3 + 12*(B*a^2*b + A*a*b^2 + (B*a^2*b + A*a*b^2)*
m)*n^3 + 6*(B*a^2*b + A*a*b^2)*m^2 + 19*(B*a^2*b + A*a*b^2 + (B*a^2*b + A*
a*b^2)*m^2 + 2*(B*a^2*b + A*a*b^2)*m)*n^2 + 4*(B*a^2*b + A*a*b^2)*m + 8*(B
*a^2*b + A*a*b^2 + (B*a^2*b + A*a*b^2)*m^3 + 3*(B*a^2*b + A*a*b^2)*m^2 + 3
*(B*a^2*b + A*a*b^2)*m)*n)*x*x^(2*n)*e^(m*log(c) + m*log(x)) + ((B*a^3 + 3
*A*a^2*b)*m^4 + B*a^3 + 3*A*a^2*b + 4*(B*a^3 + 3*A*a^2*b)*m^3 + 24*(B*a^3
+ 3*A*a^2*b + (B*a^3 + 3*A*a^2*b)*m)*n^3 + 6*(B*a^3 + 3*A*a^2*b)*m^2 + 26*
(B*a^3 + 3*A*a^2*b + (B*a^3 + 3*A*a^2*b)*m^2 + 2*(B*a^3 + 3*A*a^2*b)*m)*n^
2 + 4*(B*a^3 + 3*A*a^2*b)*m + 9*(B*a^3 + 3*A*a^2*b + (B*a^3 + 3*A*a^2*b)*m
^3 + 3*(B*a^3 + 3*A*a^2*b)*m^2 + 3*(B*a^3 + 3*A*a^2*b)*m)*n)*x*x^n*e^(m*lo
g(c) + m*log(x)) + (A*a^3*m^4 + 24*A*a^3*n^4 + 4*A*a^3*m^3 + 6*A*a^3*m^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16781 vs. 2(134) = 268.

Time = 6.76 (sec) , antiderivative size = 16781, normalized size of antiderivative = 112.62

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(a+b*x**n)**3*(A+B*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)**3*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((A*a**3*lo
g(x) + 3*A*a**2*b*x**n/n + 3*A*a*b**2*x**(2*n)/(2*n) + A*b**3*x**(3*n)/(3*
n) + B*a**3*x**n/n + 3*B*a**2*b*x**(2*n)/(2*n) + B*a*b**2*x**(3*n)/n + B*b
**3*x**(4*n)/(4*n))/c, Eq(m, -1)), (A*a**3*Piecewise((0**(-4*n - 1)*x, Eq(
c, 0)), (Piecewise((-1/(4*n*(c*x)**(4*n)), Ne(n, 0)), (log(c*x), True))/c,
 True)) + 3*A*a**2*b*Piecewise((-x*x**n*(c*x)**(-4*n - 1)/(3*n), Ne(n, 0))
, (x*x**n*(c*x)**(-4*n - 1)*log(x), True)) + 3*A*a*b**2*Piecewise((-x*x**(
2*n)*(c*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(c*x)**(-4*n - 1)*log
(x), True)) + A*b**3*Piecewise((-x*x**(3*n)*(c*x)**(-4*n - 1)/n, Ne(n, 0))
, (x*x**(3*n)*(c*x)**(-4*n - 1)*log(x), True)) + B*a**3*Piecewise((-x*x**n
*(c*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(c*x)**(-4*n - 1)*log(x), Tru
e)) + 3*B*a**2*b*Piecewise((-x*x**(2*n)*(c*x)**(-4*n - 1)/(2*n), Ne(n, 0))
, (x*x**(2*n)*(c*x)**(-4*n - 1)*log(x), True)) + 3*B*a*b**2*Piecewise((-x*
x**(3*n)*(c*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(c*x)**(-4*n - 1)*log
(x), True)) + B*b**3*x*x**(4*n)*(c*x)**(-4*n - 1)*log(x), Eq(m, -4*n - 1))
, (A*a**3*Piecewise((0**(-3*n - 1)*x, Eq(c, 0)), (Piecewise((-1/(3*n*(c*x)
**(3*n)), Ne(n, 0)), (log(c*x), True))/c, True)) + 3*A*a**2*b*Piecewise((-
x*x**n*(c*x)**(-3*n - 1)/(2*n), Ne(n, 0)), (x*x**n*(c*x)**(-3*n - 1)*log(x
), True)) + 3*A*a*b**2*Piecewise((-x*x**(2*n)*(c*x)**(-3*n - 1)/n, Ne(n, 0
)), (x*x**(2*n)*(c*x)**(-3*n - 1)*log(x), True)) + A*b**3*x*x**(3*n)*(c...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.47

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\, dx = \frac{Bb^3c^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,Bab^2c^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Ab^3c^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Ba^2bc^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Aab^2c^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ba^3c^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{3\,Aa^2bc^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(cx)^{m+1}Aa^3}{c(m+1)}$$

input
```
integrate((c*x)^m*(a+b*x^n)^3*(A+B*x^n),x, algorithm="maxima")
```

output
```
B*b^3*c^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 3*B*a*b^2*c^m*x*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 1) + A*b^3*c^m*x*e^(m*log(x) + 3*n*log(x)
)/(m + 3*n + 1) + 3*B*a^2*b*c^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1)
+ 3*A*a*b^2*c^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*a^3*c^m*x*e^
(m*log(x) + n*log(x))/(m + n + 1) + 3*A*a^2*b*c^m*x*e^(m*log(x) + n*log(x)
)/(m + n + 1) + (c*x)^(m + 1)*A*a^3/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7893 vs. $2(149) = 298$.

Time = 0.18 (sec) , antiderivative size = 7893, normalized size of antiderivative = 52.97

$$\int (cx)^m (a + bx^n)^3 (A + Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a+b*x^n)^3*(A+B*x^n),x, algorithm="giac")
```

output
```
(B*b^3*m^4*x*x^(4*n)*e^(m*log(c) + m*log(x)) + 6*B*b^3*m^3*n*x*x^(4*n)*e^(
m*log(c) + m*log(x)) + 11*B*b^3*m^2*n^2*x*x^(4*n)*e^(m*log(c) + m*log(x))
+ 6*B*b^3*m*n^3*x*x^(4*n)*e^(m*log(c) + m*log(x)) + 3*B*a*b^2*m^4*x*x^(3*n
)*e^(m*log(c) + m*log(x)) + A*b^3*m^4*x*x^(3*n)*e^(m*log(c) + m*log(x)) +
B*b^3*m^4*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 21*B*a*b^2*m^3*n*x*x^(3*n)*e
^(m*log(c) + m*log(x)) + 7*A*b^3*m^3*n*x*x^(3*n)*e^(m*log(c) + m*log(x)) +
 6*B*b^3*m^3*n*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 42*B*a*b^2*m^2*n^2*x*x^
(3*n)*e^(m*log(c) + m*log(x)) + 14*A*b^3*m^2*n^2*x*x^(3*n)*e^(m*log(c) + m
*log(x)) + 11*B*b^3*m^2*n^2*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 24*B*a*b^2
*m*n^3*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 8*A*b^3*m*n^3*x*x^(3*n)*e^(m*lo
g(c) + m*log(x)) + 6*B*b^3*m*n^3*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 3*B*a
^2*b*m^4*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 3*A*a*b^2*m^4*x*x^(2*n)*e^(m*
log(c) + m*log(x)) + 3*B*a*b^2*m^4*x*x^(2*n)*e^(m*log(c) + m*log(x)) + A*b
^3*m^4*x*x^(2*n)*e^(m*log(c) + m*log(x)) + B*b^3*m^4*x*x^(2*n)*e^(m*log(c)
 + m*log(x)) + 24*B*a^2*b*m^3*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 24*A*a
*b^2*m^3*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 21*B*a*b^2*m^3*n*x*x^(2*n)*
e^(m*log(c) + m*log(x)) + 7*A*b^3*m^3*n*x*x^(2*n)*e^(m*log(c) + m*log(x))
+ 6*B*b^3*m^3*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 57*B*a^2*b*m^2*n^2*x*x
^(2*n)*e^(m*log(c) + m*log(x)) + 57*A*a*b^2*m^2*n^2*x*x^(2*n)*e^(m*log(c)
+ m*log(x)) + 42*B*a*b^2*m^2*n^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 14...
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.78

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\,dx = \frac{A\,a^3\,x\,(c\,x)^m}{m+1} + \frac{b^2\,x\,x^{3\,n}\,(c\,x)^m\,(A\,b+3\,B\,a)\,(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{a^2\,x\,x^n\,(c\,x)^m\,(3\,A\,b+B\,a)\,(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{B\,b^3\,x\,x^{4\,n}\,(c\,x)^m\,(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{3\,a\,b\,x\,x^{2\,n}\,(c\,x)^m\,(A\,b+B\,a)\,(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$

input
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^3,x)
```

output
```
(A*a^3*x*(c*x)^m)/(m + 1) + (b^2*x*x^(3*n)*(c*x)^m*(A*b + 3*B*a)*(3*m + 7*
n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14*n^2 + 8*n^3 + 1))/(4*m
+ 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^
3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (a^2*x*x^n*(c*x)^m*
(3*A*b + B*a)*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*
n^2 + 24*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 +
1) + (B*b^3*x*x^(4*n)*(c*x)^m*(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3
*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2
*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4
+ 35*m^2*n^2 + 1) + (3*a*b*x*x^(2*n)*(c*x)^m*(A*b + B*a)*(3*m + 8*n + 16*m
*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.68

$$\int (cx)^m (a+bx^n)^3 (A+Bx^n)\, dx$$

$$= \frac{x^m c^m x(144x^{2n}a^2b^2m^2n + 72x^{2n}a^2b^2m\,n^3 + 228x^{2n}a^2b^2m\,n^2 + 144x^{2n}a^2b^2mn + 36x^n a^3b\,m^3n + 104x^n a^3}{}$$

input
```
int((c*x)^m*(a+b*x^n)^3*(A+B*x^n),x)
```

output
```
(x**m*c**m*x*(x**(4*n)*b**4*m**4 + 6*x**(4*n)*b**4*m**3*n + 4*x**(4*n)*b**
4*m**3 + 11*x**(4*n)*b**4*m**2*n**2 + 18*x**(4*n)*b**4*m**2*n + 6*x**(4*n)
*b**4*m**2 + 6*x**(4*n)*b**4*m*n**3 + 22*x**(4*n)*b**4*m*n**2 + 18*x**(4*n
)*b**4*m*n + 4*x**(4*n)*b**4*m + 6*x**(4*n)*b**4*n**3 + 11*x**(4*n)*b**4*n
**2 + 6*x**(4*n)*b**4*n + x**(4*n)*b**4 + 4*x**(3*n)*a*b**3*m**4 + 28*x**(
3*n)*a*b**3*m**3*n + 16*x**(3*n)*a*b**3*m**3 + 56*x**(3*n)*a*b**3*m**2*n**
2 + 84*x**(3*n)*a*b**3*m**2*n + 24*x**(3*n)*a*b**3*m**2 + 32*x**(3*n)*a*b*
*3*m*n**3 + 112*x**(3*n)*a*b**3*m*n**2 + 84*x**(3*n)*a*b**3*m*n + 16*x**(3
*n)*a*b**3*m + 32*x**(3*n)*a*b**3*n**3 + 56*x**(3*n)*a*b**3*n**2 + 28*x**(
3*n)*a*b**3*n + 4*x**(3*n)*a*b**3 + 6*x**(2*n)*a**2*b**2*m**4 + 48*x**(2*n
)*a**2*b**2*m**3*n + 24*x**(2*n)*a**2*b**2*m**3 + 114*x**(2*n)*a**2*b**2*m
**2*n**2 + 144*x**(2*n)*a**2*b**2*m**2*n + 36*x**(2*n)*a**2*b**2*m**2 + 72
*x**(2*n)*a**2*b**2*m*n**3 + 228*x**(2*n)*a**2*b**2*m*n**2 + 144*x**(2*n)*
a**2*b**2*m*n + 24*x**(2*n)*a**2*b**2*m + 72*x**(2*n)*a**2*b**2*n**3 + 114
*x**(2*n)*a**2*b**2*n**2 + 48*x**(2*n)*a**2*b**2*n + 6*x**(2*n)*a**2*b**2
+ 4*x**n*a**3*b*m**4 + 36*x**n*a**3*b*m**3*n + 16*x**n*a**3*b*m**3 + 104*x
**n*a**3*b*m**2*n**2 + 108*x**n*a**3*b*m**2*n + 24*x**n*a**3*b*m**2 + 96*x
**n*a**3*b*m*n**3 + 208*x**n*a**3*b*m*n**2 + 108*x**n*a**3*b*m*n + 16*x**n
*a**3*b*m + 96*x**n*a**3*b*n**3 + 104*x**n*a**3*b*n**2 + 36*x**n*a**3*b*n
+ 4*x**n*a**3*b + a**4*m**4 + 10*a**4*m**3*n + 4*a**4*m**3 + 35*a**4*m*...
```

3.404 $\int (cx)^m (a+bx^n)^2 (A+Bx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n)\,dx = \frac{a^2A(cx)^{1+m}}{c(1+m)} + \frac{a(2Ab+aB)x^n(cx)^{1+m}}{c(1+m+n)} + \frac{b(Ab+2aB)x^{2n}(cx)^{1+m}}{c(1+m+2n)} + \frac{b^2Bx^{3n}(cx)^{1+m}}{c(1+m+3n)}$$

output
```
a^2*A*(c*x)^(1+m)/c/(1+m)+a*(2*A*b+B*a)*x^n*(c*x)^(1+m)/c/(1+m+n)+b*(A*b+2
*B*a)*x^(2*n)*(c*x)^(1+m)/c/(1+m+2*n)+b^2*B*x^(3*n)*(c*x)^(1+m)/c/(1+m+3*n
)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n)\,dx = x(cx)^m \left(\frac{a^2A}{1+m} + \frac{a(2Ab+aB)x^n}{1+m+n} + \frac{b(Ab+2aB)x^{2n}}{1+m+2n} + \frac{b^2Bx^{3n}}{1+m+3n}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
x*(c*x)^m*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^n)/(1 + m + n) + (b*(A*b +
 2*a*B)*x^(2*n))/(1 + m + 2*n) + (b^2*B*x^(3*n))/(1 + m + 3*n))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(a^2A(cx)^m+bx^{2n}(cx)^m(2aB+Ab)+ax^n(cx)^m(aB+2Ab)+b^2Bx^{3n}(cx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2A(cx)^{m+1}}{c(m+1)}+\frac{ax^{n+1}(cx)^m(aB+2Ab)}{m+n+1}+\frac{bx^{2n+1}(cx)^m(2aB+Ab)}{m+2n+1}+\frac{b^2Bx^{3n+1}(cx)^m}{m+3n+1}$$

input
```
Int[(c*x)^m*(a + b*x^n)^2*(A + B*x^n),x]
```

output
```
(a*(2*A*b + a*B)*x^(1 + n)*(c*x)^m)/(1 + m + n) + (b*(A*b + 2*a*B)*x^(1 +
2*n)*(c*x)^m)/(1 + m + 2*n) + (b^2*B*x^(1 + 3*n)*(c*x)^m)/(1 + m + 3*n) +
(a^2*A*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.53 (sec) , antiderivative size = 699, normalized size of antiderivative = 6.30

method	result
risch	$\frac{x(3B\,b^2m^2x^{3n}+3A\,b^2m^2x^{2n}+3m\,b^2B\,x^{3n}+3A\,b^2x^{2n}m+B\,b^2m^3x^{3n}+A\,b^2m^3x^{2n}+a^2A+6Aab\,m^2x^n+10B\,a^2mn\,x^n+6Aa}{}$
parallelrisch	$\frac{8Bx\,x^{2n}(cx)^m abn+3Bx\,x^{3n}(cx)^m b^2m^2n+2Bx\,x^{3n}(cx)^m b^2m\,n^2+4Ax\,x^{2n}(cx)^m b^2m^2n+3Ax\,x^{2n}(cx)^m b^2m\,n^2+6Bx\,x^{3n}(}{}$
orering	Expression too large to display

input
```
int((c*x)^m*(a+b*x^n)^2*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(3*B*b^2*m^2*n*(x^n)^3+2*B*b^2*m*n^2*(x^n)^3+a^2*A+6*A*a*b*m^2*x^n+10*B*
a^2*m*n*x^n+6*B*a*b*(x^n)^2*m+6*A*a*b*x^n*m+16*B*a*b*m*n*(x^n)^2+20*A*a*b*
m*n*x^n+8*B*a*b*m^2*n*(x^n)^2+6*B*a*b*m*n^2*(x^n)^2+10*A*a*b*m^2*n*x^n+2*B
*a*b*(x^n)^2+2*a*b*A*x^n+2*B*(x^n)^3*b^2*n^2+3*A*(x^n)^2*b^2*n^2+3*B*(x^n)
^3*b^2*n+4*A*(x^n)^2*b^2*n+6*B*x^n*a^2*n^2+5*B*x^n*a^2*n+6*A*a^2*m^2*n+11*
A*a^2*m*n^2+12*A*a^2*m*n+a^2*B*x^n+6*A*a^2*n^3+11*A*a^2*n^2+6*A*a^2*n+b^2*
B*(x^n)^3+A*a^2*m^3+3*A*a^2*m^2+3*B*b^2*m^2*(x^n)^3+3*A*b^2*m^2*(x^n)^2+B*
a^2*m^3*x^n+3*m*b^2*B*(x^n)^3+3*A*b^2*(x^n)^2*m+3*B*a^2*x^n*m+B*b^2*m^3*(x
^n)^3+A*b^2*m^3*(x^n)^2+3*B*a^2*m^2*x^n+A*b^2*(x^n)^2+3*a^2*A*m+2*A*a*b*m^
3*x^n+8*A*b^2*m*n*(x^n)^2+5*B*a^2*m^2*n*x^n+6*B*a^2*m*n^2*x^n+6*B*a*b*m^2*
(x^n)^2+6*B*(x^n)^2*a*b*n^2+12*A*x^n*a*b*n^2+12*A*a*b*m*n^2*x^n+8*B*(x^n)^
2*a*b*n+10*A*x^n*a*b*n+4*A*b^2*m^2*n*(x^n)^2+3*A*b^2*m*n^2*(x^n)^2+2*B*a*b
*m^3*(x^n)^2+6*B*b^2*m*n*(x^n)^3)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*c^m*x^
m*exp(1/2*I*Pi*csgn(I*c*x)*m*(csgn(I*c*x)-csgn(I*x))*(-csgn(I*c*x)+csgn(I*
c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.75

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n)\,dx = \frac{(Bb^2m^3+3\,Bb^2m^2+3\,Bb^2m+Bb^2+2\,(Bb^2m+Bb^2)n^2+3\,(Bb^2m^2+2\,Bb^2m+Bb^2)n)xx^{3\,n}e^{(m\log(c)}}{}$$

input
```
integrate((c*x)^m*(a+b*x^n)^2*(A+B*x^n),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 3*B*b^2*m^2 + 3*B*b^2*m + B*b^2 + 2*(B*b^2*m + B*b^2)*n^2 +
3*(B*b^2*m^2 + 2*B*b^2*m + B*b^2)*n)*x*x^(3*n)*e^(m*log(c) + m*log(x)) + (
(2*B*a*b + A*b^2)*m^3 + 2*B*a*b + A*b^2 + 3*(2*B*a*b + A*b^2)*m^2 + 3*(2*B
*a*b + A*b^2 + (2*B*a*b + A*b^2)*m)*n^2 + 3*(2*B*a*b + A*b^2)*m + 4*(2*B*a
*b + A*b^2 + (2*B*a*b + A*b^2)*m^2 + 2*(2*B*a*b + A*b^2)*m)*n)*x*x^(2*n)*e
^(m*log(c) + m*log(x)) + ((B*a^2 + 2*A*a*b)*m^3 + B*a^2 + 2*A*a*b + 3*(B*a
^2 + 2*A*a*b)*m^2 + 6*(B*a^2 + 2*A*a*b + (B*a^2 + 2*A*a*b)*m)*n^2 + 3*(B*a
^2 + 2*A*a*b)*m + 5*(B*a^2 + 2*A*a*b + (B*a^2 + 2*A*a*b)*m^2 + 2*(B*a^2 +
2*A*a*b)*m)*n)*x*x^n*e^(m*log(c) + m*log(x)) + (A*a^2*m^3 + 6*A*a^2*n^3 +
3*A*a^2*m^2 + 3*A*a^2*m + A*a^2 + 11*(A*a^2*m + A*a^2)*n^2 + 6*(A*a^2*m^2
+ 2*A*a^2*m + A*a^2)*n)*x*e^(m*log(c) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 +
4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m
 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5882 vs. $2(99) = 198$.

Time = 3.83 (sec) , antiderivative size = 5882, normalized size of antiderivative = 52.99

$$\int (cx)^m (a + bx^n)^2 (A + Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(a+b*x**n)**2*(A+B*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)**2*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((A*a**2*lo
g(x) + 2*A*a*b*x**n/n + A*b**2*x**(2*n)/(2*n) + B*a**2*x**n/n + B*a*b*x**(
2*n)/n + B*b**2*x**(3*n)/(3*n))/c, Eq(m, -1)), (A*a**2*Piecewise((0**(-3*n
 - 1)*x, Eq(c, 0)), (Piecewise((-1/(3*n*(c*x)**(3*n)), Ne(n, 0)), (log(c*x
), True))/c, True)) + 2*A*a*b*Piecewise((-x*x**n*(c*x)**(-3*n - 1)/(2*n),
Ne(n, 0)), (x*x**n*(c*x)**(-3*n - 1)*log(x), True)) + A*b**2*Piecewise((-x
*x**(2*n)*(c*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(c*x)**(-3*n - 1)*lo
g(x), True)) + B*a**2*Piecewise((-x*x**n*(c*x)**(-3*n - 1)/(2*n), Ne(n, 0)
), (x*x**n*(c*x)**(-3*n - 1)*log(x), True)) + 2*B*a*b*Piecewise((-x*x**(2*
n)*(c*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(c*x)**(-3*n - 1)*log(x), T
rue)) + B*b**2*x*x**(3*n)*(c*x)**(-3*n - 1)*log(x), Eq(m, -3*n - 1)), (A*a
**2*Piecewise((0**(-2*n - 1)*x, Eq(c, 0)), (Piecewise((-1/(2*n*(c*x)**(2*n
)), Ne(n, 0)), (log(c*x), True))/c, True)) + 2*A*a*b*Piecewise((-x*x**n*(c
*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(c*x)**(-2*n - 1)*log(x), True)) + A
*b**2*x*x**(2*n)*(c*x)**(-2*n - 1)*log(x) + B*a**2*Piecewise((-x*x**n*(c*x
)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(c*x)**(-2*n - 1)*log(x), True)) + 2*B
*a*b*x*x**(2*n)*(c*x)**(-2*n - 1)*log(x) + B*b**2*Piecewise((x*x**(3*n)*(c
*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(c*x)**(-2*n - 1)*log(x), True))
, Eq(m, -2*n - 1)), (A*a**2*Piecewise((0**(-n - 1)*x, Eq(c, 0)), (Piecewis
e((-1/(n*(c*x)**n), Ne(n, 0)), (log(c*x), True))/c, True)) + 2*A*a*b*x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.40

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n)\, dx = \frac{Bb^2c^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{2\,Babc^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ab^2c^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ba^2c^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{2\,Aabc^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(cx)^{m+1}Aa^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(a+b*x^n)^2*(A+B*x^n),x, algorithm="maxima")
```

output
```
B*b^2*c^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 2*B*a*b*c^m*x*e^(m*l
og(x) + 2*n*log(x))/(m + 2*n + 1) + A*b^2*c^m*x*e^(m*log(x) + 2*n*log(x))/
(m + 2*n + 1) + B*a^2*c^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a*b*
c^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (c*x)^(m + 1)*A*a^2/(c*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(111) = 222$.

Time = 0.15 (sec) , antiderivative size = 2951, normalized size of antiderivative = 26.59

$$\int (cx)^m (a + bx^n)^2 (A + Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a+b*x^n)^2*(A+B*x^n),x, algorithm="giac")
```

output
```
(B*b^2*m^3*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 3*B*b^2*m^2*n*x*x^(3*n)*e^(
m*log(c) + m*log(x)) + 2*B*b^2*m*n^2*x*x^(3*n)*e^(m*log(c) + m*log(x)) + 2
*B*a*b*m^3*x*x^(2*n)*e^(m*log(c) + m*log(x)) + A*b^2*m^3*x*x^(2*n)*e^(m*lo
g(c) + m*log(x)) + B*b^2*m^3*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 8*B*a*b*m
^2*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 4*A*b^2*m^2*n*x*x^(2*n)*e^(m*log(
c) + m*log(x)) + 3*B*b^2*m^2*n*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 6*B*a*b
*m*n^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + 3*A*b^2*m*n^2*x*x^(2*n)*e^(m*lo
g(c) + m*log(x)) + 2*B*b^2*m*n^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + B*a^2
*m^3*x*x^n*e^(m*log(c) + m*log(x)) + 2*A*a*b*m^3*x*x^n*e^(m*log(c) + m*log
(x)) + 2*B*a*b*m^3*x*x^n*e^(m*log(c) + m*log(x)) + A*b^2*m^3*x*x^n*e^(m*lo
g(c) + m*log(x)) + B*b^2*m^3*x*x^n*e^(m*log(c) + m*log(x)) + 5*B*a^2*m^2*n
*x*x^n*e^(m*log(c) + m*log(x)) + 10*A*a*b*m^2*n*x*x^n*e^(m*log(c) + m*log(
x)) + 8*B*a*b*m^2*n*x*x^n*e^(m*log(c) + m*log(x)) + 4*A*b^2*m^2*n*x*x^n*e^
(m*log(c) + m*log(x)) + 3*B*b^2*m^2*n*x*x^n*e^(m*log(c) + m*log(x)) + 6*B*
a^2*m*n^2*x*x^n*e^(m*log(c) + m*log(x)) + 12*A*a*b*m*n^2*x*x^n*e^(m*log(c)
 + m*log(x)) + 6*B*a*b*m*n^2*x*x^n*e^(m*log(c) + m*log(x)) + 3*A*b^2*m*n^2
*x*x^n*e^(m*log(c) + m*log(x)) + 2*B*b^2*m*n^2*x*x^n*e^(m*log(c) + m*log(x
)) + A*a^2*m^3*x*e^(m*log(c) + m*log(x)) + B*a^2*m^3*x*e^(m*log(c) + m*log
(x)) + 2*A*a*b*m^3*x*e^(m*log(c) + m*log(x)) + 2*B*a*b*m^3*x*e^(m*log(c) +
 m*log(x)) + A*b^2*m^3*x*e^(m*log(c) + m*log(x)) + B*b^2*m^3*x*e^(m*log...
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.39

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n) \, dx$$
$$= \frac{A a^2 x (c x)^m}{m+1} + \frac{a x x^n (c x)^m (2 A b + B a) (m^2 + 5 m n + 2 m + 6 n^2 + 5 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1} + \frac{b x x^{2 n} (c x)^m (A b + 2 B a) (m^2 + 4 m n + 2 m + 3 n^2 + 4 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1} + \frac{B b^2 x x^{3 n} (c x)^m (m^2 + 3 m n + 2 m + 2 n^2 + 3 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1}$$

input
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^2,x)
```

output
```
(A*a^2*x*(c*x)^m)/(m + 1) + (a*x*x^n*(c*x)^m*(2*A*b + B*a)*(2*m + 5*n + 5*
m*n + m^2 + 6*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 +
 m^3 + 11*n^2 + 6*n^3 + 1) + (b*x*x^(2*n)*(c*x)^m*(A*b + 2*B*a)*(2*m + 4*n
 + 4*m*n + m^2 + 3*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*
m^2 + m^3 + 11*n^2 + 6*n^3 + 1) + (B*b^2*x*x^(3*n)*(c*x)^m*(2*m + 3*n + 3*
m*n + m^2 + 2*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 +
 m^3 + 11*n^2 + 6*n^3 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 480, normalized size of antiderivative = 4.32

$$\int (cx)^m (a+bx^n)^2 (A+Bx^n) \, dx$$
$$= \frac{x^m c^m x (2x^{3n} b^3 n^2 + 3x^{3n} b^3 n + a^3 + 3x^{2n} a b^2 + 3x^n a^2 b + x^{3n} b^3 + 6a^3 n^3 + 11a^3 n^2 + 6a^3 n + 9x^{2n} a b^2 n^2 +}{}$$

input
```
int((c*x)^m*(a+b*x^n)^2*(A+B*x^n),x)
```

output
```
(x**m*c**m*x*(x**(3*n)*b**3*m**3 + 3*x**(3*n)*b**3*m**2*n + 3*x**(3*n)*b**
3*m**2 + 2*x**(3*n)*b**3*m*n**2 + 6*x**(3*n)*b**3*m*n + 3*x**(3*n)*b**3*m
+ 2*x**(3*n)*b**3*n**2 + 3*x**(3*n)*b**3*n + x**(3*n)*b**3 + 3*x**(2*n)*a*
b**2*m**3 + 12*x**(2*n)*a*b**2*m**2*n + 9*x**(2*n)*a*b**2*m**2 + 9*x**(2*n
)*a*b**2*m*n**2 + 24*x**(2*n)*a*b**2*m*n + 9*x**(2*n)*a*b**2*m + 9*x**(2*n
)*a*b**2*n**2 + 12*x**(2*n)*a*b**2*n + 3*x**(2*n)*a*b**2 + 3*x**n*a**2*b*m
**3 + 15*x**n*a**2*b*m**2*n + 9*x**n*a**2*b*m**2 + 18*x**n*a**2*b*m*n**2 +
 30*x**n*a**2*b*m*n + 9*x**n*a**2*b*m + 18*x**n*a**2*b*n**2 + 15*x**n*a**2
*b*n + 3*x**n*a**2*b + a**3*m**3 + 6*a**3*m**2*n + 3*a**3*m**2 + 11*a**3*m
*n**2 + 12*a**3*m*n + 3*a**3*m + 6*a**3*n**3 + 11*a**3*n**2 + 6*a**3*n + a
**3))/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*
n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n + 1)
```

3.405 $\int (cx)^m (a+bx^n) (A+Bx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (cx)^m (a+bx^n) (A+Bx^n) \, dx = \frac{aA(cx)^{1+m}}{c(1+m)} + \frac{(Ab+aB)x^n(cx)^{1+m}}{c(1+m+n)} + \frac{bBx^{2n}(cx)^{1+m}}{c(1+m+2n)}$$

output
```
a*A*(c*x)^(1+m)/c/(1+m)+(A*b+B*a)*x^n*(c*x)^(1+m)/c/(1+m+n)+b*B*x^(2*n)*(c
*x)^(1+m)/c/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int (cx)^m (a+bx^n) (A+Bx^n) \, dx = x(cx)^m \left(\frac{aA}{1+m} + \frac{(Ab+aB)x^n}{1+m+n} + \frac{bBx^{2n}}{1+m+2n}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^n)*(A + B*x^n),x]
```

output
```
x*(c*x)^m*((a*A)/(1 + m) + ((A*b + a*B)*x^n)/(1 + m + n) + (b*B*x^(2*n))/(
1 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (a+bx^n) (A+Bx^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^n(cx)^m(aB+Ab)+aA(cx)^m+bBx^{2n}(cx)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{n+1}(cx)^m(aB+Ab)}{m+n+1}+\frac{aA(cx)^{m+1}}{c(m+1)}+\frac{bBx^{2n+1}(cx)^m}{m+2n+1}$$

input
```
Int[(c*x)^m*(a + b*x^n)*(A + B*x^n),x]
```

output
```
((A*b + a*B)*x^(1 + n)*(c*x)^m)/(1 + m + n) + (b*B*x^(1 + 2*n)*(c*x)^m)/(1
 + m + 2*n) + (a*A*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.18

method	result
risch	$\frac{x\left(Bb\,m^2x^{2n}+Bbmn\,x^{2n}+Ab\,m^2x^n+2Abmn\,x^n+Ba\,m^2x^n+2Bamn\,x^n+2B\,x^{2n}bm+bnB\,x^{2n}+Aa\,m^2+3Aamn+2Aa\,n^2+\right.}{(1}$
parallelrisch	$\frac{Ax(cx)^m a\,m^2+2Ax(cx)^m a\,n^2+Bx\,x^{2n}(cx)^m b+Ax\,x^n(cx)^m b+2Ax(cx)^m am+3Ax(cx)^m an+Bx\,x^n(cx)^m a+Bx\,x^{2n}(cx)^m}{}$
orering	$\frac{x(3m^2+6mn+2n^2+3m+3n+1)(cx)^m(a+b\,x^n)(A+B\,x^n)}{(m^2+2mn+2m+2n+1)(1+m+n)}-\frac{3x^2(m+n)\left(\frac{(cx)^m m(a+b\,x^n)(A+B\,x^n)}{x}+\frac{(cx)^m b\,x^n n(A+B\,x^n)}{x}\right.}{(m^2+2mn+2m+2n+1)(1+m+n)}$

input
```
int((c*x)^m*(a+b*x^n)*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b*m^2*(x^n)^2+B*b*m*n*(x^n)^2+A*b*m^2*x^n+2*A*b*m*n*x^n+B*a*m^2*x^n+2
*B*a*m*n*x^n+2*B*(x^n)^2*b*m+B*(x^n)^2*b*n+A*a*m^2+3*A*a*m*n+2*A*a*n^2+2*A
*x^n*b*m+2*A*x^n*b*n+2*B*x^n*a*m+2*B*x^n*a*n+B*(x^n)^2*b+2*A*a*m+3*A*a*n+A
*x^n*b+B*x^n*a+A*a)/(1+m)/(1+m+n)/(1+m+2*n)*c^m*x^m*exp(1/2*I*Pi*csgn(I*c*
x)*m*(csgn(I*c*x)-csgn(I*x))*(-csgn(I*c*x)+csgn(I*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.57

$$\int (cx)^m\,(a+bx^n)\,(A+Bx^n)\,dx$$
$$=\frac{(Bbm^2+2\,Bbm+Bb+(Bbm+Bb)n)xx^{2\,n}e^{(m\log(c)+m\log(x))}+((Ba+Ab)m^2+Ba+Ab+2\,(Ba+A}{m^3+2\,(m+1)}$$

input
```
integrate((c*x)^m*(a+b*x^n)*(A+B*x^n),x, algorithm="fricas")
```

output
```
((B*b*m^2 + 2*B*b*m + B*b + (B*b*m + B*b)*n)*x*x^(2*n)*e^(m*log(c) + m*log
(x)) + ((B*a + A*b)*m^2 + B*a + A*b + 2*(B*a + A*b)*m + 2*(B*a + A*b + (B*
a + A*b)*m)*n)*x*x^n*e^(m*log(c) + m*log(x)) + (A*a*m^2 + 2*A*a*n^2 + 2*A*
a*m + A*a + 3*(A*a*m + A*a)*n)*x*e^(m*log(c) + m*log(x)))/(m^3 + 2*(m + 1)
*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(61) = 122$.

Time = 1.93 (sec) , antiderivative size = 1498, normalized size of antiderivative = 20.81

$$\int (cx)^m (a + bx^n) (A + Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(a+b*x**n)*(A+B*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((A*a*log(x) +
 A*b*x**n/n + B*a*x**n/n + B*b*x**(2*n)/(2*n))/c, Eq(m, -1)), (A*a*Piecewi
se((0**(-2*n - 1)*x, Eq(c, 0)), (Piecewise((-1/(2*n*(c*x)**(2*n)), Ne(n, 0
)), (log(c*x), True))/c, True)) + A*b*Piecewise((-x*x**n*(c*x)**(-2*n - 1)
/n, Ne(n, 0)), (x*x**n*(c*x)**(-2*n - 1)*log(x), True)) + B*a*Piecewise((-
x*x**n*(c*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(c*x)**(-2*n - 1)*log(x), T
rue)) + B*b*x*x**(2*n)*(c*x)**(-2*n - 1)*log(x), Eq(m, -2*n - 1)), (A*a*Pi
ecewise((0**(-n - 1)*x, Eq(c, 0)), (Piecewise((-1/(n*(c*x)**n), Ne(n, 0)),
 (log(c*x), True))/c, True)) + A*b*x*x**n*(c*x)**(-n - 1)*log(x) + B*a*x*x
**n*(c*x)**(-n - 1)*log(x) + B*b*Piecewise((x*x**(2*n)*(c*x)**(-n - 1)/n,
Ne(n, 0)), (x*x**(2*n)*(c*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (A*
a*m**2*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1) + 3*A*a*m*n*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2
 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*a*m*x*(c*x)**m/(m**3 + 3*m**2*n +
 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*a*n**2*x*(c*x)*
*m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
+ 3*A*a*n*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m +
2*n**2 + 3*n + 1) + A*a*x*(c*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 +
6*m*n + 3*m + 2*n**2 + 3*n + 1) + A*b*m**2*x*x**n*(c*x)**m/(m**3 + 3*m**2*
n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*b*m*n*x*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int (cx)^m (a+bx^n) (A+Bx^n) \, dx = \frac{Bbc^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Bac^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Abc^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(cx)^{m+1}Aa}{c(m+1)}$$

input
```
integrate((c*x)^m*(a+b*x^n)*(A+B*x^n),x, algorithm="maxima")
```

output
```
B*b*c^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*a*c^m*x*e^(m*log(x)
+ n*log(x))/(m + n + 1) + A*b*c^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) +
(c*x)^(m + 1)*A*a/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(72) = 144.

Time = 0.13 (sec) , antiderivative size = 763, normalized size of antiderivative = 10.60

$$\int (cx)^m (a+bx^n) (A+Bx^n) \, dx = \text{Too large to display}$$

input
```
integrate((c*x)^m*(a+b*x^n)*(A+B*x^n),x, algorithm="giac")
```

output
```
(B*b*m^2*x*x^(2*n)*e^(m*log(c) + m*log(x)) + B*b*m*n*x*x^(2*n)*e^(m*log(c)
 + m*log(x)) + B*a*m^2*x*x^n*e^(m*log(c) + m*log(x)) + A*b*m^2*x*x^n*e^(m*
log(c) + m*log(x)) + B*b*m^2*x*x^n*e^(m*log(c) + m*log(x)) + 2*B*a*m*n*x*x
^n*e^(m*log(c) + m*log(x)) + 2*A*b*m*n*x*x^n*e^(m*log(c) + m*log(x)) + B*b
*m*n*x*x^n*e^(m*log(c) + m*log(x)) + A*a*m^2*x*e^(m*log(c) + m*log(x)) + B
*a*m^2*x*e^(m*log(c) + m*log(x)) + A*b*m^2*x*e^(m*log(c) + m*log(x)) + B*b
*m^2*x*e^(m*log(c) + m*log(x)) + 3*A*a*m*n*x*e^(m*log(c) + m*log(x)) + 2*B
*a*m*n*x*e^(m*log(c) + m*log(x)) + 2*A*b*m*n*x*e^(m*log(c) + m*log(x)) + B
*b*m*n*x*e^(m*log(c) + m*log(x)) + 2*A*a*n^2*x*e^(m*log(c) + m*log(x)) + 2
*B*b*m*x*x^(2*n)*e^(m*log(c) + m*log(x)) + B*b*n*x*x^(2*n)*e^(m*log(c) + m
*log(x)) + 2*B*a*m*x*x^n*e^(m*log(c) + m*log(x)) + 2*A*b*m*x*x^n*e^(m*log(
c) + m*log(x)) + 2*B*b*m*x*x^n*e^(m*log(c) + m*log(x)) + 2*B*a*n*x*x^n*e^(
m*log(c) + m*log(x)) + 2*A*b*n*x*x^n*e^(m*log(c) + m*log(x)) + B*b*n*x*x^n
*e^(m*log(c) + m*log(x)) + 2*A*a*m*x*e^(m*log(c) + m*log(x)) + 2*B*a*m*x*e
^(m*log(c) + m*log(x)) + 2*A*b*m*x*e^(m*log(c) + m*log(x)) + 2*B*b*m*x*e^(
m*log(c) + m*log(x)) + 3*A*a*n*x*e^(m*log(c) + m*log(x)) + 2*B*a*n*x*e^(m*
log(c) + m*log(x)) + 2*A*b*n*x*e^(m*log(c) + m*log(x)) + B*b*n*x*e^(m*log(
c) + m*log(x)) + B*b*x*x^(2*n)*e^(m*log(c) + m*log(x)) + B*a*x*x^n*e^(m*lo
g(c) + m*log(x)) + A*b*x*x^n*e^(m*log(c) + m*log(x)) + B*b*x*x^n*e^(m*log(
c) + m*log(x)) + A*a*x*e^(m*log(c) + m*log(x)) + B*a*x*e^(m*log(c) + m*...
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int (cx)^m (a+bx^n)(A+Bx^n)\,dx = (c\,x)^m\left(\frac{A\,a\,x}{m+1}+\frac{x\,x^n\,(A\,b+B\,a)\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{B\,b\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

input
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n),x)
```

output
```
(c*x)^m*((A*a*x)/(m + 1) + (x*x^n*(A*b + B*a)*(m + 2*n + 1))/(2*m + 3*n +
3*m*n + m^2 + 2*n^2 + 1) + (B*b*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n
+ m^2 + 2*n^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.50

$$\int (cx)^m (a+bx^n) (A+Bx^n)\, dx$$
$$= \frac{x^m c^m x(x^{2n}b^2m^2 + x^{2n}b^2mn + 2x^{2n}b^2m + x^{2n}b^2n + x^{2n}b^2 + 2x^n ab\, m^2 + 4x^n abmn + 4x^n abm + 4x^n abn}{m^3 + 3m^2n + 2m\, n^2 + 3m^2 + 6mn + 2n^2 + 3m + 3}$$

input
```
int((c*x)^m*(a+b*x^n)*(A+B*x^n),x)
```

output
```
(x**m*c**m*x*(x**(2*n)*b**2*m**2 + x**(2*n)*b**2*m*n + 2*x**(2*n)*b**2*m +
 x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m**2 + 4*x**n*a*b*m*n + 4*x*
*n*a*b*m + 4*x**n*a*b*n + 2*x**n*a*b + a**2*m**2 + 3*a**2*m*n + 2*a**2*m +
 2*a**2*n**2 + 3*a**2*n + a**2))/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1)
```

3.406 $\int (cx)^m (A + Bx^n) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int (cx)^m (A + Bx^n) \, dx = \frac{A(cx)^{1+m}}{c(1+m)} + \frac{Bx^n(cx)^{1+m}}{c(1+m+n)}$$

output
```
A*(c*x)^(1+m)/c/(1+m)+B*x^n*(c*x)^(1+m)/c/(1+m+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

$$\int (cx)^m (A + Bx^n) \, dx = x(cx)^m \left(\frac{A}{1+m} + \frac{Bx^n}{1+m+n}\right)$$

input
```
Integrate[(c*x)^m*(A + B*x^n),x]
```

output
```
x*(c*x)^m*(A/(1 + m) + (B*x^n)/(1 + m + n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m (A+Bx^n)\, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int (A(cx)^m+Bx^n(cx)^m)\, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{A(cx)^{m+1}}{c(m+1)}+\frac{Bx^{n+1}(cx)^m}{m+n+1}$$

input
```
Int[(c*x)^m*(A + B*x^n),x]
```

output
```
(B*x^(1 + n)*(c*x)^m)/(1 + m + n) + (A*(c*x)^(1 + m))/(c*(1 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{Ax\,\mathrm{e}^{m\ln(cx)}}{1+m}+\frac{Bx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{m\ln(cx)}}{1+m+n}$	38
parallelrisch	$\frac{Bx\,x^{n}(cx)^{m}m+Ax(cx)^{m}m+Ax(cx)^{m}n+Bx\,x^{n}(cx)^{m}+Ax(cx)^{m}}{(1+m)(1+m+n)}$	63
risch	$\frac{x(B\,x^{n}m+Am+An+B\,x^{n}+A)c^{m}x^{m}\mathrm{e}^{\frac{i\pi\ \mathrm{csgn}(icx)m(\mathrm{csgn}(icx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(icx)+\mathrm{csgn}(ic))}{2}}}{(1+m)(1+m+n)}$	79
orering	$\frac{x(2m+n+1)(cx)^{m}(A+B\,x^{n})}{m^{2}+mn+2m+n+1}-\frac{x^{2}\left(\frac{(cx)^{m}m(A+B\,x^{n})}{x}+\frac{(cx)^{m}B\,x^{n}n}{x}\right)}{m^{2}+mn+2m+n+1}$	87

input
```
int((c*x)^m*(A+B*x^n),x,method=_RETURNVERBOSE)
```

output
```
A/(1+m)*x*exp(m*ln(c*x))+B/(1+m+n)*x*exp(n*ln(x))*exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int (cx)^m\left(A+Bx^n\right)\,dx = \frac{(Bm+B)xx^ne^{(m\log(c)+m\log(x))}+(Am+An+A)xe^{(m\log(c)+m\log(x))}}{m^2+(m+1)n+2\,m+1}$$

input
```
integrate((c*x)^m*(A+B*x^n),x, algorithm="fricas")
```

output
```
((B*m + B)*x*x^n*e^(m*log(c) + m*log(x)) + (A*m + A*n + A)*x*e^(m*log(c) +
 m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(31) = 62$.

Time = 0.82 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.97

$$\int (cx)^m\left(A+Bx^n\right)\,dx$$
$$= \begin{cases} \frac{(A+B)\log(x)}{c} & \text{for } m=-1\ / \\ \frac{A\log(x)+\frac{Bx^n}{n}}{c} & \text{for } m=-1 \\ A\left(\begin{cases} 0^{-n-1}x & \text{for } c=0 \\ \frac{\begin{cases} -\frac{(cx)^{-n}}{n} & \text{for } n\neq 0 \\ \log(cx) & \text{otherwise} \end{cases}}{c} & \text{otherwise} \end{cases}\right) + Bxx^n(cx)^{-n-1}\log(x) & \text{for } m=-n\ - \\ \frac{Amx(cx)^m}{m^2+mn+2m+n+1} + \frac{Anx(cx)^m}{m^2+mn+2m+n+1} + \frac{Ax(cx)^m}{m^2+mn+2m+n+1} + \frac{Bmxx^n(cx)^m}{m^2+mn+2m+n+1} + \frac{Bxx^n(cx)^m}{m^2+mn+2m+n+1} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(A+B*x**n),x)
```

output
```
Piecewise(((A + B)*log(x)/c, Eq(m, -1) & Eq(n, 0)), ((A*log(x) + B*x**n/n)
/c, Eq(m, -1)), (A*Piecewise((0**(-n - 1)*x, Eq(c, 0)), (Piecewise((-1/(n*
(c*x)**n), Ne(n, 0)), (log(c*x), True))/c, True)) + B*x*x**n*(c*x)**(-n -
1)*log(x), Eq(m, -n - 1)), (A*m*x*(c*x)**m/(m**2 + m*n + 2*m + n + 1) + A*
n*x*(c*x)**m/(m**2 + m*n + 2*m + n + 1) + A*x*(c*x)**m/(m**2 + m*n + 2*m +
 n + 1) + B*m*x*x**n*(c*x)**m/(m**2 + m*n + 2*m + n + 1) + B*x*x**n*(c*x)*
*m/(m**2 + m*n + 2*m + n + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int (cx)^m\left(A+Bx^n\right)\,dx = \frac{Bc^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(cx)^{m+1}A}{c(m+1)}$$

input
```
integrate((c*x)^m*(A+B*x^n),x, algorithm="maxima")
```

output
```
B*c^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (c*x)^(m + 1)*A/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(39) = 78$.

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.00

$$\int (cx)^m (A + Bx^n)\, dx$$
$$= \frac{Bmxx^n e^{(m\log(c)+m\log(x))} + Amxe^{(m\log(c)+m\log(x))} + Bmxe^{(m\log(c)+m\log(x))} + Anxe^{(m\log(c)+m\log(x))} + Bxx}{m^2 + mn + 2\,m + n + 1}$$

input
```
integrate((c*x)^m*(A+B*x^n),x, algorithm="giac")
```

output
```
(B*m*x*x^n*e^(m*log(c) + m*log(x)) + A*m*x*e^(m*log(c) + m*log(x)) + B*m*x
*e^(m*log(c) + m*log(x)) + A*n*x*e^(m*log(c) + m*log(x)) + B*x*x^n*e^(m*lo
g(c) + m*log(x)) + A*x*e^(m*log(c) + m*log(x)) + B*x*e^(m*log(c) + m*log(x
)))/(m^2 + m*n + 2*m + n + 1)
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int (cx)^m (A + Bx^n)\, dx = (c\,x)^m \left(\frac{A\,x}{m+1} + \frac{B\,x^{n+1}}{m+n+1}\right)$$

input
```
int((c*x)^m*(A + B*x^n),x)
```

output
```
(c*x)^m*((A*x)/(m + 1) + (B*x^(n + 1))/(m + n + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int (cx)^m (A+Bx^n)\,dx=\frac{x^m c^m x(x^n bm+x^n b+am+an+a)}{m^2+mn+2m+n+1}$$

input
```
int((c*x)^m*(A+B*x^n),x)
```

output
```
(x**m*c**m*x*(x**n*b*m + x**n*b + a*m + a*n + a))/(m**2 + m*n + 2*m + n +
1)
```

3.407 $\int \frac{(cx)^m(A+Bx^n)}{a+bx^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(cx)^m(A+Bx^n)}{a+bx^n}\,dx = \frac{B(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aB)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{abc(1+m)}$$

output
```
B*(c*x)^(1+m)/b/c/(1+m)+(A*b-B*a)*(c*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m
+n)/n],-b*x^n/a)/a/b/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^m(A+Bx^n)}{a+bx^n}\,dx = \frac{x(cx)^m\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(x*(c*x)^m*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, -((b*x^n)/a)]))/(a*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{a+bx^n} \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(Ab-aB)\int \frac{(cx)^m}{bx^n+a}dx}{b}+\frac{B(cx)^{m+1}}{bc(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+1}}{bc(m+1)}$$

input
```
Int[((c*x)^m*(A + B*x^n))/(a + b*x^n),x]
```

output
```
(B*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*B)*(c*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b*c*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(cx)^m (A + B\,x^n)}{a + b\,x^n}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n),x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(cx)^m (A + Bx^n)}{a + bx^n}\,dx = \int \frac{(Bx^n + A)(cx)^m}{bx^n + a}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(c*x)^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.90

$$\int \frac{(cx)^m (A + Bx^n)}{a + bx^n} dx$$
$$= \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-1-\frac{1}{n}}c^m m x^{m+1}\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-1-\frac{1}{n}}c^m x^{m+1}\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{Ba^{-\frac{m}{n}-2-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m m x^{m+n+1}\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$
$$+ \frac{Ba^{-\frac{m}{n}-2-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$
$$+ \frac{Ba^{-\frac{m}{n}-2-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*m*x**(m + 1)*lerchphi(b*x**n*exp
_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n))
 + A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**m*x**(m + 1)*lerchphi(b*x**n*ex
p_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)
) + B*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*m*x**(m + n + 1)*lerchph
i(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*g
amma(m/n + 2 + 1/n)) + B*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m
 + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n +
 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + B*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1
/n)*c**m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/
n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (A+Bx^n)}{a+bx^n}\,dx = \int \frac{(Bx^n+A)(cx)^m}{bx^n+a}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
B*c^m*x*x^m/(b*(m + 1)) - (B*a*c^m - A*b*c^m)*integrate(x^m/(b^2*x^n + a*b
), x)
```

Giac [F]

$$\int \frac{(cx)^m (A+Bx^n)}{a+bx^n}\,dx = \int \frac{(Bx^n+A)(cx)^m}{bx^n+a}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^n)}{a+bx^n}\,dx = \int \frac{(c\,x)^m\,(A+B\,x^n)}{a+b\,x^n}\,dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n),x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.17

$$\int \frac{(cx)^m (A + Bx^n)}{a + bx^n} \, dx = \frac{x^m c^m x}{m + 1}$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n),x)
```

output
```
(x**m*c**m*x)/(m + 1)
```

3.408 $\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{(Ab-aB)(cx)^{1+m}}{abcn(a+bx^n)} + \frac{(aB(1+m)-Ab(1+m-n))(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2bc(1+m)n}$$

output
```
(A*b-B*a)*(c*x)^(1+m)/a/b/c/n/(a+b*x^n)+(a*B*(1+m)-A*b*(1+m-n))*(c*x)^(1+m
)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/b/c/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2}\,dx = \frac{x(cx)^m\left(aB\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+}{n}\right.\right.}{a^2b(1+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
(x*(c*x)^m*(a*B*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)] + (A*b - a*B)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]))/(a^2*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)-Ab(m-n+1))\int \frac{(cx)^m}{bx^n+a}dx}{abn}+\frac{(cx)^{m+1}(Ab-aB)}{abcn\,(a+bx^n)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}(aB(m+1)-Ab(m-n+1))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{a^2bc(m+1)n}+\frac{(cx)^{m+1}(Ab-aB)}{abcn\,(a+bx^n)}$$

input
```
Int[((c*x)^m*(A + B*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*(c*x)^(1 + m))/(a*b*c*n*(a + b*x^n)) + ((a*B*(1 + m) - A*b*(1
 + m - n))*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
(b*x^n)/a)])/(a^2*b*c*(1 + m)*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(cx)^m (A+B x^n)}{(a+b x^n)^2}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2} dx = \int \frac{(Bx^n+A)(cx)^m}{(bx^n+a)^2} dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(c*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 2382, normalized size of antiderivative = 22.47

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n)**2,x)
```

output
```
A*(-a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*c**m*m**2*x**(m + 1)*lerchphi(b*x
**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n +
1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n -
 2 - 1/n)*c**m*m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n
 + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*c**m*m*n*x**(m + 1)*ga
mma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 +
1/n)) - 2*a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*c**m*m*x**(m + 1)*lerchphi(
b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n
 + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/
n - 2 - 1/n)*c**m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/
n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*c**m*n*x**(m + 1)*gam
ma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1
/n)) - a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*c**m*x**(m + 1)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1
+ 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) - a**(m/n + 1/n)*a**(-m/n - 2 -
 1/n)*b*c**m*m**2*x**n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/
n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma
(m/n + 1 + 1/n)) + a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*b*c**m*m*n*x**n*x...
```

Maxima [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2} dx = \int \frac{(Bx^n+A)(cx)^m}{(bx^n+a)^2} dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-(B*a*c^m - A*b*c^m)*x*x^m/(a*b^2*n*x^n + a^2*b*n) - (A*b*c^m*(m - n + 1)
- B*a*c^m*(m + 1))*integrate(x^m/(a*b^2*n*x^n + a^2*b*n), x)
```

Giac [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2} dx = \int \frac{(Bx^n+A)(cx)^m}{(bx^n+a)^2} dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^2} dx = \int \frac{(c\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^2} dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^2,x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^2} \, dx = c^m \left(\int \frac{x^m}{x^n b + a} dx \right)$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^2,x)
```

output
```
c**m*int(x**m/(x**n*b + a),x)
```

3.409 $\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{(Ab-aB)(cx)^{1+m}}{2abcn\,(a+bx^n)^2} + \frac{(aB(1+m)-Ab(1+m-2n))(cx)^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{2a^3bc(1+m)n}$$

output
```
1/2*(A*b-B*a)*(c*x)^(1+m)/a/b/c/n/(a+b*x^n)^2+1/2*(a*B*(1+m)-A*b*(1+m-2*n)
)*(c*x)^(1+m)*hypergeom([2, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/b/c/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^3}\,dx = \frac{x(cx)^m\left(aB\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+m+}{n}\right.\right.}{a^3b(1+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
(x*(c*x)^m*(a*B*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)] + (A*b - a*B)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]))/(a^3*b*(1 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^3}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(aB(m+1)-Ab(m-2n+1))\int \frac{(cx)^m}{(bx^n+a)^2}dx}{2abn}+\frac{(cx)^{m+1}(Ab-aB)}{2abcn(a+bx^n)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}(aB(m+1)-Ab(m-2n+1))\text{Hypergeometric2F1}\left(2,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{2a^3bc(m+1)n}+\frac{(cx)^{m+1}(Ab-aB)}{2abcn(a+bx^n)^2}$$

input
```
Int[((c*x)^m*(A + B*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*(c*x)^(1 + m))/(2*a*b*c*n*(a + b*x^n)^2) + ((a*B*(1 + m) - A*
b*(1 + m - 2*n))*(c*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(2*a^3*b*c*(1 + m)*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(cx)^m (A + B\,x^n)}{(a + b\,x^n)^3}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3}\,dx = \int \frac{(Bx^n + A)(cx)^m}{(bx^n + a)^3}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(c*x)^m/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
+ a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 112.94 (sec) , antiderivative size = 8303, normalized size of antiderivative = 74.13

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3} \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n)**3,x)
```

output
```
A*(a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*c**m*m**3*x**(m + 1)*lerchphi(b
*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**2*n**4*gamma
(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n)) - 3*a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*c**m
*m**2*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(
m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n +
 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - a**2*a**(m/n + 1/
n)*a**(-m/n - 3 - 1/n)*c**m*m**2*n*x**(m + 1)*gamma(m/n + 1/n)/(2*a**2*n**
4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*n**
4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 3*a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/
n)*c**m*m**2*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*g
amma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x**n*gamma(
m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 2*a**2*a**(m
/n + 1/n)*a**(-m/n - 3 - 1/n)*c**m*m*n**2*x**(m + 1)*lerchphi(b*x**n*exp_p
olar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 +
1/n) + 4*a*b*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m
/n + 1 + 1/n)) + 3*a**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*c**m*m*n**2*x**
(m + 1)*gamma(m/n + 1/n)/(2*a**2*n**4*gamma(m/n + 1 + 1/n) + 4*a*b*n**4*x*
*n*gamma(m/n + 1 + 1/n) + 2*b**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - 6*a
**2*a**(m/n + 1/n)*a**(-m/n - 3 - 1/n)*c**m*m*n*x**(m + 1)*lerchphi(b*x...
```

Maxima [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3} \, dx = \int \frac{(Bx^n + A)(cx)^m}{(bx^n + a)^3} \, dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
-((m^2 - m*(n - 2) - n + 1)*B*a*c^m - (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1
)*A*b*c^m)*integrate(1/2*x^m/(a^2*b^2*n^2*x^n + a^3*b*n^2), x) + 1/2*((B*a
^2*c^m*(m - n + 1) - A*a*b*c^m*(m - 3*n + 1))*x*x^m - (A*b^2*c^m*(m - 2*n
+ 1) - B*a*b*c^m*(m + 1))*x*e^(m*log(x) + n*log(x)))/(a^2*b^3*n^2*x^(2*n)
+ 2*a^3*b^2*n^2*x^n + a^4*b*n^2)
```

Giac [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3} \, dx = \int \frac{(Bx^n + A)(cx)^m}{(bx^n + a)^3} \, dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3} \, dx = \int \frac{(c\,x)^m \,(A + B\,x^n)}{(a + b\,x^n)^3} \, dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^3,x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^3} dx = c^m \left(\int \frac{x^m}{x^{2n}b^2 + 2x^n ab + a^2} dx \right)$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^3,x)
```

output
```
c**m*int(x**m/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.410 $\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \frac{2B(cx)^{1+m}(a+bx^n)^{5/2}}{bc(2+2m+5n)} + \frac{a\left(\frac{A}{1+m}-\frac{2aB}{b(2+2m+5n)}\right)(cx)^{1+m}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c\sqrt{1+\frac{bx^n}{a}}}$$

output
```
2*B*(c*x)^(1+m)*(a+b*x^n)^(5/2)/b/c/(2+2*m+5*n)+a*(A/(1+m)-2*a*B/b/(2+2*m+
5*n))*(c*x)^(1+m)*(a+b*x^n)^(1/2)*hypergeom([-3/2, (1+m)/n],[(1+m+n)/n],-b
*x^n/a)/c/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \frac{ax(cx)^m\sqrt{a+bx^n}\left(A(1+m+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+B(1+m)x^{\cdots}\right.}{(1+m)(1+m+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(a*x*(c*x)^m*Sqrt[a + b*x^n]*(A*(1 + m + n)*Hypergeometric2F1[-3/2, (1 + m
)/n, (1 + m + n)/n, -((b*x^n)/a)] + B*(1 + m)*x^n*Hypergeometric2F1[-3/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*Sqrt[
1 + (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^n\right)^{3/2} \left(A+Bx^n\right) \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B(cx)^{m+1} \left(a+bx^n\right)^{5/2}}{bc(2m+5n+2)}-\frac{(2aB(m+1)-Ab(2m+5n+2)) \int (cx)^m \left(bx^n+a\right)^{3/2}dx}{b(2m+5n+2)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B(cx)^{m+1} \left(a+bx^n\right)^{5/2}}{bc(2m+5n+2)}-\frac{a\sqrt{a+bx^n}(2aB(m+1)-Ab(2m+5n+2)) \int (cx)^m \left(\frac{bx^n}{a}+1\right)^{3/2}dx}{b(2m+5n+2)\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B(cx)^{m+1} \left(a+bx^n\right)^{5/2}}{bc(2m+5n+2)}-\frac{a(cx)^{m+1}\sqrt{a+bx^n}(2aB(m+1)-Ab(2m+5n+2)) \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{bc(m+1)(2m+5n+2)\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(c*x)^m*(a + b*x^n)^(3/2)*(A + B*x^n),x]
```

output
```
(2*B*(c*x)^(1 + m)*(a + b*x^n)^(5/2))/(b*c*(2 + 2*m + 5*n)) - (a*(2*a*B*(1
 + m) - A*b*(2 + 2*m + 5*n))*(c*x)^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2
F1[-3/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(b*c*(1 + m)*(2 + 2*m +
5*n)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (cx)^m \left(a+b\,x^n\right)^{\frac{3}{2}} \left(A+B\,x^n\right) dx$$

input
```
int((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output
```
int((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.22 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.75

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \frac{Aa a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+\frac{1}{2}-\frac{1}{n}} c^m x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{Aa^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}} a^{\frac{m}{n}+1+\frac{1}{n}} b c^m x^{m+n+1} \Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{n}+1+\frac{1}{n} \\ \frac{m}{n}+2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)} + \frac{Baa^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}} a^{\frac{m}{n}+1+\frac{1}{n}} c^m x^{m+n+1} \Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{n}+1+\frac{1}{n} \\ \frac{m}{n}+2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)} + \frac{Ba^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}} a^{\frac{m}{n}+2+\frac{1}{n}} b c^m x^{m+2n+1} \Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{n}+2+\frac{1}{n} \\ \frac{m}{n}+3+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(a+b*x**n)**(3/2)*(A+B*x**n),x)
```

output
```
A*a*a**(m/n + 1/n)*a**(-m/n + 1/2 - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*
hyper((-1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*ga
mma(m/n + 1 + 1/n)) + A*a**(-m/n - 1/2 - 1/n)*a**(m/n + 1 + 1/n)*b*c**m*x*
*(m + n + 1)*gamma(m/n + 1 + 1/n)*hyper((-1/2, m/n + 1 + 1/n), (m/n + 2 +
1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n)) + B*a*a**(-m/n -
 1/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m + n + 1)*gamma(m/n + 1 + 1/n)*hy
per((-1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*
gamma(m/n + 2 + 1/n)) + B*a**(-m/n - 3/2 - 1/n)*a**(m/n + 2 + 1/n)*b*c**m*
x**(m + 2*n + 1)*gamma(m/n + 2 + 1/n)*hyper((-1/2, m/n + 2 + 1/n), (m/n +
3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 3 + 1/n))
```

Maxima [F]

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}(cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m (a+bx^n)^{3/2} (A+Bx^n) \, dx = \int (Bx^n+A)(bx^n+a)^{\frac{3}{2}}(cx)^m \, dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m (a + bx^n)^{3/2} (A + Bx^n) \, dx = \int (c\,x)^m \left(A + B\,x^n\right) \left(a + b\,x^n\right)^{3/2} dx$$

input
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int (cx)^m (a + bx^n)^{3/2} (A + Bx^n) \, dx = \text{too large to display}$$

input
```
int((c*x)^m*(a+b*x^n)^(3/2)*(A+B*x^n),x)
```

output

```
(c**m*(8*x**(m + 2*n)*sqrt(x**n*b + a)*b**2*m**2*x + 16*x**(m + 2*n)*sqrt(
x**n*b + a)*b**2*m*n*x + 16*x**(m + 2*n)*sqrt(x**n*b + a)*b**2*m*x + 6*x**
(m + 2*n)*sqrt(x**n*b + a)*b**2*n**2*x + 16*x**(m + 2*n)*sqrt(x**n*b + a)*
b**2*n*x + 8*x**(m + 2*n)*sqrt(x**n*b + a)*b**2*x + 16*x**(m + n)*sqrt(x**
n*b + a)*a*b*m**2*x + 52*x**(m + n)*sqrt(x**n*b + a)*a*b*m*n*x + 32*x**(m
+ n)*sqrt(x**n*b + a)*a*b*m*x + 22*x**(m + n)*sqrt(x**n*b + a)*a*b*n**2*x
+ 52*x**(m + n)*sqrt(x**n*b + a)*a*b*n*x + 16*x**(m + n)*sqrt(x**n*b + a)*
a*b*x + 8*x**m*sqrt(x**n*b + a)*a**2*m**2*x + 36*x**m*sqrt(x**n*b + a)*a**
2*m*n*x + 16*x**m*sqrt(x**n*b + a)*a**2*m*x + 46*x**m*sqrt(x**n*b + a)*a**
2*n**2*x + 36*x**m*sqrt(x**n*b + a)*a**2*n*x + 8*x**m*sqrt(x**n*b + a)*a**
2*x + 120*int((x**m*sqrt(x**n*b + a))/(8*x**n*b*m**3 + 36*x**n*b*m**2*n +
24*x**n*b*m**2 + 46*x**n*b*m*n**2 + 72*x**n*b*m*n + 24*x**n*b*m + 15*x**n*
b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b + 8*a*m**3 + 36*a*m**2*n
+ 24*a*m**2 + 46*a*m*n**2 + 72*a*m*n + 24*a*m + 15*a*n**3 + 46*a*n**2 + 36
*a*n + 8*a),x)*a**3*m**3*n**3 + 540*int((x**m*sqrt(x**n*b + a))/(8*x**n*b*
m**3 + 36*x**n*b*m**2*n + 24*x**n*b*m**2 + 46*x**n*b*m*n**2 + 72*x**n*b*m*
n + 24*x**n*b*m + 15*x**n*b*n**3 + 46*x**n*b*n**2 + 36*x**n*b*n + 8*x**n*b
 + 8*a*m**3 + 36*a*m**2*n + 24*a*m**2 + 46*a*m*n**2 + 72*a*m*n + 24*a*m +
15*a*n**3 + 46*a*n**2 + 36*a*n + 8*a),x)*a**3*m**2*n**4 + 360*int((x**m*sq
rt(x**n*b + a))/(8*x**n*b*m**3 + 36*x**n*b*m**2*n + 24*x**n*b*m**2 + 46...
```

3.411 $\int (cx)^m \sqrt{a+bx^n}(A+Bx^n)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int (cx)^m \sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{2B(cx)^{1+m}(a+bx^n)^{3/2}}{bc(2+2m+3n)} + \frac{\left(\frac{A}{1+m}-\frac{2aB}{b(2+2m+3n)}\right)(cx)^{1+m}\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c\sqrt{1+\frac{bx^n}{a}}}$$

output

```
2*B*(c*x)^(1+m)*(a+b*x^n)^(3/2)/b/c/(2+2*m+3*n)+(A/(1+m)-2*a*B/b/(2+2*m+3*
n))*(c*x)^(1+m)*(a+b*x^n)^(1/2)*hypergeom([-1/2, (1+m)/n],[(1+m+n)/n],-b*x
^n/a)/c/(1+b*x^n/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int (cx)^m \sqrt{a+bx^n}(A+Bx^n)\,dx = \frac{x(cx)^m\sqrt{a+bx^n}\left(A(1+m+n)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+B(1+m)x^n\,\text{Hypergeor}\right.}{(1+m)(1+m+n)\sqrt{1+\frac{bx^n}{a}}}$$

input
```
Integrate[(c*x)^m*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(x*(c*x)^m*Sqrt[a + b*x^n]*(A*(1 + m + n)*Hypergeometric2F1[-1/2, (1 + m)/
n, (1 + m + n)/n, -((b*x^n)/a)] + B*(1 + m)*x^n*Hypergeometric2F1[-1/2, (1
 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*Sqrt[1
+ (b*x^n)/a])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m\sqrt{a+bx^n}(A+Bx^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2B(cx)^{m+1}(a+bx^n)^{3/2}}{bc(2m+3n+2)}-\frac{(2aB(m+1)-Ab(2m+3n+2))\int (cx)^m\sqrt{bx^n+a}dx}{b(2m+3n+2)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2B(cx)^{m+1}(a+bx^n)^{3/2}}{bc(2m+3n+2)}-\frac{\sqrt{a+bx^n}(2aB(m+1)-Ab(2m+3n+2))\int (cx)^m\sqrt{\frac{bx^n}{a}+1}dx}{b(2m+3n+2)\sqrt{\frac{bx^n}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2B(cx)^{m+1}(a+bx^n)^{3/2}}{bc(2m+3n+2)}-\frac{(cx)^{m+1}\sqrt{a+bx^n}(2aB(m+1)-Ab(2m+3n+2))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{bc(m+1)(2m+3n+2)\sqrt{\frac{bx^n}{a}+1}}$$

input
```
Int[(c*x)^m*Sqrt[a + b*x^n]*(A + B*x^n),x]
```

output
```
(2*B*(c*x)^(1 + m)*(a + b*x^n)^(3/2))/(b*c*(2 + 2*m + 3*n)) - ((2*a*B*(1 +
 m) - A*b*(2 + 2*m + 3*n))*(c*x)^(1 + m)*Sqrt[a + b*x^n]*Hypergeometric2F1
[-1/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(b*c*(1 + m)*(2 + 2*m + 3*
n)*Sqrt[1 + (b*x^n)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (cx)^m \sqrt{a+b\,x^n}\,(A+B\,x^n)\,dx$$

input
```
int((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
int((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (cx)^m \sqrt{a+bx^n}(A+Bx^n)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.31

$$\int (cx)^m \sqrt{a+bx^n}(A+Bx^n)\, dx = \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+\frac{1}{2}-\frac{1}{n}}c^m x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(a+b*x**n)**(1/2)*(A+B*x**n),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n + 1/2 - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*hy
per((-1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamm
a(m/n + 1 + 1/n)) + B*a**(-m/n - 1/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m
+ n + 1)*gamma(m/n + 1 + 1/n)*hyper((-1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,
), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int (cx)^m\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (Bx^n+A)\sqrt{bx^n+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*sqrt(b*x^n + a)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{a+bx^n}(A+Bx^n)\,dx=\int (c\,x)^m\,\left(A+B\,x^n\right)\,\sqrt{a+b\,x^n}\,dx$$

input
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(A + B*x^n)*(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int (cx)^m\sqrt{a+bx^n}(A+Bx^n)\ dx$$
$$= \frac{c^m\left(4x^{m+n}\sqrt{x^nb+a}\,bmx+2x^{m+n}\sqrt{x^nb+a}\,bnx+4x^{m+n}\sqrt{x^nb+a}\,bx+4x^m\sqrt{x^nb+a}\,amx+8x^m\sqrt{x^n}\right.}{}$$

input
```
int((c*x)^m*(a+b*x^n)^(1/2)*(A+B*x^n),x)
```

output
```
(c**m*(4*x**(m + n)*sqrt(x**n*b + a)*b*m*x + 2*x**(m + n)*sqrt(x**n*b + a)
*b*n*x + 4*x**(m + n)*sqrt(x**n*b + a)*b*x + 4*x**m*sqrt(x**n*b + a)*a*m*x
 + 8*x**m*sqrt(x**n*b + a)*a*n*x + 4*x**m*sqrt(x**n*b + a)*a*x + 12*int((x
**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*
b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8
*a*n + 4*a),x)*a**2*m**2*n**2 + 24*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m
**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b +
4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*m*n**3 + 24*i
nt((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*
x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**
2 + 8*a*n + 4*a),x)*a**2*m*n**2 + 9*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*
m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b +
 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**4 + 24*in
t((x**m*sqrt(x**n*b + a))/(4*x**n*b*m**2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x
**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2
 + 8*a*n + 4*a),x)*a**2*n**3 + 12*int((x**m*sqrt(x**n*b + a))/(4*x**n*b*m*
*2 + 8*x**n*b*m*n + 8*x**n*b*m + 3*x**n*b*n**2 + 8*x**n*b*n + 4*x**n*b + 4
*a*m**2 + 8*a*m*n + 8*a*m + 3*a*n**2 + 8*a*n + 4*a),x)*a**2*n**2))/(4*m**2
 + 8*m*n + 8*m + 3*n**2 + 8*n + 4)
```

3.412 $\int \frac{(cx)^m(A+Bx^n)}{\sqrt{a+bx^n}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{2B(cx)^{1+m}\sqrt{a+bx^n}}{bc(2+2m+n)} + \frac{\left(\frac{A}{1+m}-\frac{2aB}{b(2+2m+n)}\right)(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{c\sqrt{a+bx^n}}$$

output
```
2*B*(c*x)^(1+m)*(a+b*x^n)^(1/2)/b/c/(2+2*m+n)+(A/(1+m)-2*a*B/b/(2+2*m+n))*
(c*x)^(1+m)*(1+b*x^n/a)^(1/2)*hypergeom([1/2, (1+m)/n],[(1+m+n)/n],-b*x^n/
a)/c/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^n}{a}}\left(A(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+B(1+m)x^n\,\text{Hypergeome}\right.}{(1+m)(1+m+n)\sqrt{a+bx^n}}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*(A*(1 + m + n)*Hypergeometric2F1[1/2, (1 +
m)/n, (1 + m + n)/n, -((b*x^n)/a)] + B*(1 + m)*x^n*Hypergeometric2F1[1/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*Sqrt[
a + b*x^n])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(A-\frac{2aB(m+1)}{b(2m+n+2)}\right)\int \frac{(cx)^m}{\sqrt{bx^n+a}}dx+\frac{2B(cx)^{m+1}\sqrt{a+bx^n}}{bc(2m+n+2)}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB(m+1)}{b(2m+n+2)}\right)\int \frac{(cx)^m}{\sqrt{\frac{bx^n}{a}+1}}dx}{\sqrt{a+bx^n}}+\frac{2B(cx)^{m+1}\sqrt{a+bx^n}}{bc(2m+n+2)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}\left(A-\frac{2aB(m+1)}{b(2m+n+2)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{c(m+1)\sqrt{a+bx^n}}+\frac{2B(cx)^{m+1}\sqrt{a+bx^n}}{bc(2m+n+2)}$$

input
```
Int[((c*x)^m*(A + B*x^n))/Sqrt[a + b*x^n],x]
```

output
```
(2*B*(c*x)^(1 + m)*Sqrt[a + b*x^n])/(b*c*(2 + 2*m + n)) + ((A - (2*a*B*(1
+ m))/(b*(2 + 2*m + n)))*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2
F1[1/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(c*(1 + m)*Sqrt[a + b*x^n
])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(cx)^{m} (A + B\,x^{n})}{\sqrt{a + b\,x^{n}}}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.34

$$\begin{aligned}\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}\,dx &= \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{1}{2}-\frac{1}{n}}c^m x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} \\ &+ \frac{Ba^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n)**(1/2),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 1/2 - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*hy
per((1/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(m/n + 1 + 1/n)) + B*a**(-m/n - 3/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m +
 n + 1)*gamma(m/n + 1 + 1/n)*hyper((1/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (A + Bx^n)}{\sqrt{a + bx^n}} \, dx = \int \frac{(Bx^n + A)(cx)^m}{\sqrt{bx^n + a}} \, dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(c*x)^m/sqrt(b*x^n + a), x)
```

Giac [F]

$$\int \frac{(cx)^m (A + Bx^n)}{\sqrt{a + bx^n}} \, dx = \int \frac{(Bx^n + A)(cx)^m}{\sqrt{bx^n + a}} \, dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/sqrt(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^n)}{\sqrt{a + bx^n}} \, dx = \int \frac{(c\,x)^m \,(A + B\,x^n)}{\sqrt{a + b\,x^n}} \, dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(1/2),x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m (A+Bx^n)}{\sqrt{a+bx^n}}\,dx = \frac{c^m\left(2x^m\sqrt{x^n b+a}\,x+2\left(\int \frac{x^m\sqrt{x^n b+a}}{2x^n bm+x^n bn+2x^n b+2am+an+2a}dx\right)amn+\left(\int \frac{x^m\sqrt{x^n b+a}}{2x^n bm+x^n bn+2x^n b+2am+an+2a}dx\right)a\,n^2 \cdots}{2m+n+2}$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(1/2),x)
```

output
```
(c**m*(2*x**m*sqrt(x**n*b + a)*x + 2*int((x**m*sqrt(x**n*b + a))/(2*x**n*b
*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*m*n + int((x**m*sqrt(x*
*n*b + a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*m + a*n + 2*a),x)*a*n**
2 + 2*int((x**m*sqrt(x**n*b + a))/(2*x**n*b*m + x**n*b*n + 2*x**n*b + 2*a*
m + a*n + 2*a),x)*a*n))/(2*m + n + 2)
```

3.413 $\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{2B(cx)^{1+m}}{bc(2+2m-n)\sqrt{a+bx^n}} - \frac{(2aB(1+m)-Ab(2+2m-n))(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{abc(1+m)(2+2m-n)\sqrt{a+bx^n}}$$

output
```
2*B*(c*x)^(1+m)/b/c/(2+2*m-n)/(a+b*x^n)^(1/2)-(2*a*B*(1+m)-A*b*(2+2*m-n))*
(c*x)^(1+m)*(1+b*x^n/a)^(1/2)*hypergeom([3/2, (1+m)/n],[(1+m+n)/n],-b*x^n/
a)/a/b/c/(1+m)/(2+2*m-n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^{3/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^n}{a}}\left(A(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+B\right.}{a(1+m)(1+m+n)\sqrt{a+}}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*(A*(1 + m + n)*Hypergeometric2F1[3/2, (1 +
m)/n, (1 + m + n)/n, -((b*x^n)/a)] + B*(1 + m)*x^n*Hypergeometric2F1[3/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)]))/(a*(1 + m)*(1 + m + n)*Sqr
t[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{3/2}} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB(m+1)-Ab(2m-n+2))\int \frac{(cx)^m}{\sqrt{bx^n+a}}dx}{abn}+\frac{2(cx)^{m+1}(Ab-aB)}{abcn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}(2aB(m+1)-Ab(2m-n+2))\int \frac{(cx)^m}{\sqrt{\frac{bx^n}{a}+1}}dx}{abn\sqrt{a+bx^n}}+\frac{2(cx)^{m+1}(Ab-aB)}{abcn\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}(2aB(m+1)-Ab(2m-n+2))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{abc(m+1)n\sqrt{a+bx^n}}+\frac{2(cx)^{m+1}(Ab-aB)}{abcn\sqrt{a+bx^n}}$$

input
```
Int[((c*x)^m*(A + B*x^n))/(a + b*x^n)^(3/2),x]
```

output
```
(2*(A*b - a*B)*(c*x)^(1 + m))/(a*b*c*n*Sqrt[a + b*x^n]) + ((2*a*B*(1 + m)
- A*b*(2 + 2*m - n))*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometric2F1[1
/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b*c*(1 + m)*n*Sqrt[a + b*x
^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(cx)^m (A + B\,x^n)}{(a + b\,x^n)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.15

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{3/2}}\,dx = \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{3}{2}-\frac{1}{n}}c^m x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 3/2 - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*hy
per((3/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(m/n + 1 + 1/n)) + B*a**(-m/n - 5/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m +
 n + 1)*gamma(m/n + 1 + 1/n)*hyper((3/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{3/2}}\,dx = \int \frac{(Bx^n + A)(cx)^m}{(bx^n + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{3/2}}\,dx = \int \frac{(Bx^n + A)(cx)^m}{(bx^n + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{3/2}}\,dx = \int \frac{(c\,x)^m\,(A + B\,x^n)}{(a + b\,x^n)^{3/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(3/2),x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{3/2}} \, dx = c^m \left(\int \frac{x^m \sqrt{x^n b+a}}{x^n b+a} dx \right)$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
c**m*int((x**m*sqrt(x**n*b + a))/(x**n*b + a),x)
```

3.414 $\int \frac{(cx)^m(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{(cx)^m\,(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{2B(cx)^{1+m}}{bc(2+2m-3n)\,(a+bx^n)^{3/2}} - \frac{(2aB(1+m)-Ab(2+2m-3n))(cx)^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2bc(1+m)(2+2m-3n)\sqrt{a+bx^n}}$$

output
```
2*B*(c*x)^(1+m)/b/c/(2+2*m-3*n)/(a+b*x^n)^(3/2)-(2*a*B*(1+m)-A*b*(2+2*m-3*
n))*(c*x)^(1+m)*(1+b*x^n/a)^(1/2)*hypergeom([5/2, (1+m)/n],[(1+m+n)/n],-b*
x^n/a)/a^2/b/c/(1+m)/(2+2*m-3*n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^m\,(A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^n}{a}}\left(A(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+B\right.}{a^2(1+m)(1+m+n)\sqrt{a}}$$

input
```
Integrate[((c*x)^m*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^n)/a]*(A*(1 + m + n)*Hypergeometric2F1[5/2, (1 +
m)/n, (1 + m + n)/n, -((b*x^n)/a)] + B*(1 + m)*x^n*Hypergeometric2F1[5/2,
(1 + m + n)/n, (1 + m + 2*n)/n, -((b*x^n)/a)]))/(a^2*(1 + m)*(1 + m + n)*S
qrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {957, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(2aB(m+1)-Ab(2m-3n+2))\int \frac{(cx)^m}{(bx^n+a)^{3/2}}dx}{3abn}+\frac{2(cx)^{m+1}(Ab-aB)}{3abcn\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}(2aB(m+1)-Ab(2m-3n+2))\int \frac{(cx)^m}{\left(\frac{bx^n}{a}+1\right)^{3/2}}dx}{3a^2bn\sqrt{a+bx^n}}+\frac{2(cx)^{m+1}(Ab-aB)}{3abcn\,(a+bx^n)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}\sqrt{\frac{bx^n}{a}+1}(2aB(m+1)-Ab(2m-3n+2))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{3a^2bc(m+1)n\sqrt{a+bx^n}}+\frac{2(cx)^{m+1}(Ab-aB)}{3abcn\,(a+bx^n)^{3/2}}$$

input
```
Int[((c*x)^m*(A + B*x^n))/(a + b*x^n)^(5/2),x]
```

output
```
(2*(A*b - a*B)*(c*x)^(1 + m))/(3*a*b*c*n*(a + b*x^n)^(3/2)) + ((2*a*B*(1 +
 m) - A*b*(2 + 2*m - 3*n))*(c*x)^(1 + m)*Sqrt[1 + (b*x^n)/a]*Hypergeometri
c2F1[3/2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(3*a^2*b*c*(1 + m)*n*Sq
rt[a + b*x^n])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(cx)^m (A + B\,x^n)}{(a + b\,x^n)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.15

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \frac{Aa^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}-\frac{5}{2}-\frac{1}{n}}c^m x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{Ba^{-\frac{m}{n}-\frac{7}{2}-\frac{1}{n}}a^{\frac{m}{n}+1+\frac{1}{n}}c^m x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{m}{n}+1+\frac{1}{n}\\ \frac{m}{n}+2+\frac{1}{n}\end{matrix}\middle|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((c*x)**m*(A+B*x**n)/(a+b*x**n)**(5/2),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 5/2 - 1/n)*c**m*x**(m + 1)*gamma(m/n + 1/n)*hy
per((5/2, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma
(m/n + 1 + 1/n)) + B*a**(-m/n - 7/2 - 1/n)*a**(m/n + 1 + 1/n)*c**m*x**(m +
 n + 1)*gamma(m/n + 1 + 1/n)*hyper((5/2, m/n + 1 + 1/n), (m/n + 2 + 1/n,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)(cx)^m}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(Bx^n+A)(cx)^m}{(bx^n+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(c*x)^m/(b*x^n + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx^n)}{(a+bx^n)^{5/2}}\,dx = \int \frac{(c\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^{5/2}}\,dx$$

input
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(5/2),x)
```

output
```
int(((c*x)^m*(A + B*x^n))/(a + b*x^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m (A + Bx^n)}{(a + bx^n)^{5/2}} \, dx = c^m \left(\int \frac{x^m \sqrt{x^n b + a}}{x^{2n} b^2 + 2x^n ab + a^2} dx \right)$$

input
```
int((c*x)^m*(A+B*x^n)/(a+b*x^n)^(5/2),x)
```

output
```
c**m*int((x**m*sqrt(x**n*b + a))/(x**(2*n)*b**2 + 2*x**n*a*b + a**2),x)
```

3.415 $\int x^2(a+bx^n)^p\left(c+dx^n\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int x^2(a+bx^n)^p\left(c+dx^n\right)\,dx=\frac{dx^3(a+bx^n)^{1+p}}{b(3+n+np)}+\frac{1}{3}\left(c-\frac{3ad}{b(3+n+np)}\right)x^3(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x^3*(a+b*x^n)^(p+1)/b/(n*p+n+3)+1/3*(c-3*a*d/b/(n*p+n+3))*x^3*(a+b*x^n)^
p*hypergeom([-p, 3/n],[(3+n)/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^n)^p\left(c+dx^n\right)\,dx$$
$$=\frac{x^3(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(3+n)\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{bx^n}{a}\right)+3dx^n\,\text{Hypergeometric2F1}\right.}{3(3+n)}$$

input
```
Integrate[x^2*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^3*(a + b*x^n)^p*(c*(3 + n)*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*x
^n)/a)] + 3*d*x^n*Hypergeometric2F1[(3 + n)/n, -p, 2 + 3/n, -((b*x^n)/a)])
)/(3*(3 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{3ad}{b(np+n+3)}\right)\int x^2(bx^n+a)^p\,dx+\frac{dx^3(a+bx^n)^{p+1}}{b(np+n+3)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{3ad}{b(np+n+3)}\right)\int x^2\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^3(a+bx^n)^{p+1}}{b(np+n+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{3ad}{b(np+n+3)}\right)\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{bx^n}{a}\right)+\frac{dx^3(a+bx^n)^{p+1}}{b(np+n+3)}$$

input
```
Int[x^2*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^3*(a + b*x^n)^(1 + p))/(b*(3 + n + n*p)) + ((c - (3*a*d)/(b*(3 + n +
n*p)))*x^3*(a + b*x^n)^p*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*x^n)/a
)])/(3*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{2}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^2*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^2*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{2}(a+bx^{n})^{p}\left(c+dx^{n}\right)\,dx=\int(dx^{n}+c)(bx^{n}+a)^{p}x^{2}\,dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^2*x^n + c*x^2)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.73 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int x^2(a+bx^n)^p\left(c+dx^n\right)\,dx=\frac{a^{\frac{3}{n}}a^{p-\frac{3}{n}}cx^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{n},-p\\1+\frac{3}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)}+\frac{a^{1+\frac{3}{n}}a^{p-1-\frac{3}{n}}dx^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(3/n)*a**(p - 3/n)*c*x**3*gamma(3/n)*hyper((3/n, -p), (1 + 3/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + a**(1 + 3/n)*a**(p - 1 - 3/n)*d*
x**(n + 3)*gamma(1 + 3/n)*hyper((-p, 1 + 3/n), (2 + 3/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int x^2(a+bx^n)^p\left(c+dx^n\right)\,dx=\int(dx^n+c)(bx^n+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^n)^p\,(c+dx^n)\;dx=\int x^2\,(a+b\,x^n)^p\,(c+d\,x^n)\;dx$$

input
```
int(x^2*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^2*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^2(a+bx^n)^p\,(c+dx^n)\;dx=\text{Too large to display}$$

input
```
int(x^2*(a+b*x^n)^p*(c+d*x^n),x)
```

output

```
(x**n*(x**n*b + a)**p*b*d*n*p*x**3 + 3*x**n*(x**n*b + a)**p*b*d*x**3 + (x*
*n*b + a)**p*a*d*n*p*x**3 + (x**n*b + a)**p*b*c*n*p*x**3 + (x**n*b + a)**p
*b*c*n*x**3 + 3*(x**n*b + a)**p*b*c*x**3 - 3*int(((x**n*b + a)**p*x**2)/(x
**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a
*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**2*d*n**3*p**3 - 3*int
(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p +
 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x
)*a**2*d*n**3*p**2 - 18*int(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**
n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p
 + 6*a*n*p + 3*a*n + 9*a),x)*a**2*d*n**2*p**2 - 9*int(((x**n*b + a)**p*x**
2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*
b + a*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**2*d*n**2*p - 27*
int(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*
p + 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a
),x)*a**2*d*n*p + int(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n*
*2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a
*n*p + 3*a*n + 9*a),x)*a*b*c*n**4*p**4 + 2*int(((x**n*b + a)**p*x**2)/(x**
n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n
**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a*b*c*n**4*p**3 + int(((x*
*n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3...
```

3.416 $\int x(a+bx^n)^p\left(c+dx^n\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\frac{dx^2(a+bx^n)^{1+p}}{b(2+n+np)}+\frac{1}{2}\left(c-\frac{2ad}{b(2+n+np)}\right)x^2(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x^2*(a+b*x^n)^(p+1)/b/(n*p+n+2)+1/2*(c-2*a*d/b/(n*p+n+2))*x^2*(a+b*x^n)^
p*hypergeom([-p, 2/n],[(2+n)/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx$$
$$=\frac{x^2(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(2+n)\,\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{bx^n}{a}\right)+2dx^n\,\text{Hypergeometric2F1}\right.}{2(2+n)}$$

input
```
Integrate[x*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^2*(a + b*x^n)^p*(c*(2 + n)*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*x
^n)/a)] + 2*d*x^n*Hypergeometric2F1[(2 + n)/n, -p, 2*(1 + n^(-1)), -((b*x^
n)/a)]))/(2*(2 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c+dx^n)\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{2ad}{b(np+n+2)}\right)\int x(bx^n+a)^p\,dx+\frac{dx^2(a+bx^n)^{p+1}}{b(np+n+2)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{2ad}{b(np+n+2)}\right)\int x\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^2(a+bx^n)^{p+1}}{b(np+n+2)}$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{2ad}{b(np+n+2)}\right)\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{bx^n}{a}\right)+\frac{dx^2(a+bx^n)^{p+1}}{b(np+n+2)}$$

input
```
Int[x*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^2*(a + b*x^n)^(1 + p))/(b*(2 + n + n*p)) + ((c - (2*a*d)/(b*(2 + n +
n*p)))*x^2*(a + b*x^n)^p*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*x^n)/a
)])/(2*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int(x*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x(a+bx^n)^p\,(c+dx^n)\;dx=\int(dx^n+c)(bx^n+a)^px\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x*x^n + c*x)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\frac{a^{\frac{2}{n}}a^{p-\frac{2}{n}}cx^2\Gamma\left(\frac{2}{n}\right){}_2F_1\left(\begin{matrix}\frac{2}{n},-p\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)}+\frac{a^{1+\frac{2}{n}}a^{p-1-\frac{2}{n}}dx^{n+2}\Gamma\left(1+\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(2/n)*a**(p - 2/n)*c*x**2*gamma(2/n)*hyper((2/n, -p), (1 + 2/n,), b*x**
n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + a**(1 + 2/n)*a**(p - 1 - 2/n)*d*
x**(n + 2)*gamma(1 + 2/n)*hyper((-p, 1 + 2/n), (2 + 2/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\int(dx^n+c)(bx^n+a)^px\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x, x)
```

Giac [F]

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\int (dx^n+c)(bx^n+a)^p x\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\int x\,\left(a+b\,x^n\right)^p\,\left(c+d\,x^n\right)\,dx$$

input
```
int(x*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**n*(x**n*b + a)**p*b*d*n*p*x**2 + 2*x**n*(x**n*b + a)**p*b*d*x**2 + (x*
*n*b + a)**p*a*d*n*p*x**2 + (x**n*b + a)**p*b*c*n*p*x**2 + (x**n*b + a)**p
*b*c*n*x**2 + 2*(x**n*b + a)**p*b*c*x**2 - 2*int(((x**n*b + a)**p*x)/(x**n
*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n*
*2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*d*n**3*p**3 - 2*int(((
x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**
n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2
*d*n**3*p**2 - 8*int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p
 + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p
 + 2*a*n + 4*a),x)*a**2*d*n**2*p**2 - 4*int(((x**n*b + a)**p*x)/(x**n*b*n*
*2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p*
*2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*d*n**2*p - 8*int(((x**n*b +
 a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n +
4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*d*n*p +
 int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p
+ 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),
x)*a*b*c*n**4*p**4 + 2*int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*
n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4
*a*n*p + 2*a*n + 4*a),x)*a*b*c*n**4*p**3 + int(((x**n*b + a)**p*x)/(x**n*b
*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n...
```

3.417 $\int (a+bx^n)^p (c+dx^n)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c-\frac{ad}{b+bn+bnp}\right) x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x*(a+b*x^n)^(p+1)/b/(n*p+n+1)+(c-a*d/(b*n*p+b*n+b))*x*(a+b*x^n)^p*hyperg
eom([-p, 1/n],[1+1/n],-b*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06

$$\int (a+bx^n)^p (c+dx^n)\, dx = \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(d(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p + (-ad+bc(1+n+np))\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1\right.\right.}{b(1+n+np)}$$

input
```
Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
 n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n
 + n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{913}$$

$$\left(c-\frac{ad}{bnp+bn+b}\right)\int (bx^n+a)^p\,dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{779}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\int \left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

$$\Big\downarrow \text{778}$$

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right)\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)+\frac{dx(a+bx^n)^{p+1}}{b(np+n+1)}$$

input
```
Int[(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p
))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]
)/(1 + (b*x^n)/a)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int (a+bx^n)^p (c+dx^n)\,dx = \frac{a^{\frac{1}{n}} a^{p-\frac{1}{n}} c x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{1+\frac{1}{n}} a^{p-1-\frac{1}{n}} d x^{n+1} \Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, 1+\frac{1}{n} \\ 2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(1/n)*a**(p - 1/n)*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(1 + 1/n)*a**(p - 1 - 1/n)*d*x**
(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I
*pi)/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (c+dx^n)\,dx = \int (dx^n+c)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (c+dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (c+dx^n) \, dx = \int (a+b\,x^n)^p\,(c+d\,x^n) \, dx$$

input
```
int((a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (a+bx^n)^p (c+dx^n) \, dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n),x)
```

output

```
(x**n*(x**n*b + a)**p*b*d*n*p*x + x**n*(x**n*b + a)**p*b*d*x + (x**n*b + a
)**p*a*d*n*p*x + (x**n*b + a)**p*b*c*n*p*x + (x**n*b + a)**p*b*c*n*x + (x*
*n*b + a)**p*b*c*x - int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p
 + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a
*n + a),x)*a**2*d*n**3*p**3 - int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n
*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*
a*n*p + a*n + a),x)*a**2*d*n**3*p**2 - 2*int((x**n*b + a)**p/(x**n*b*n**2*
p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*
n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n**2*p**2 - int((x**n*b + a)**p/(x**
n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*
p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n**2*p - int((x**n*b + a)**
p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a
*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*n*p + int((x**n*b + a
)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b
+ a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n**4*p**4 + 2*int((
x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n
 + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n**4*p**3
 + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p +
x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a*b*c*n
**4*p**2 + 3*int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*...
```

3.418 $\int \frac{(a+bx^n)^p(c+dx^n)}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x}\,dx = \frac{d(a+bx^n)^{1+p}}{bn(1+p)} - \frac{c(a+bx^n)^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^n}{a}\right)}{an(1+p)}$$

output
```
d*(a+b*x^n)^(p+1)/b/n/(p+1)-c*(a+b*x^n)^(p+1)*hypergeom([1, p+1],[2+p],1+b
*x^n/a)/a/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x}\,dx = \frac{(a+bx^n)^{1+p}\left(ad-bc\,\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^n}{a}\right)\right)}{abn(1+p)}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/x,x]
```

output
```
((a + b*x^n)^(1 + p)*(a*d - b*c*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*
x^n)/a]))/(a*b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {948, 90, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{x}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int x^{-n}(bx^n+a)^p(dx^n+c)\,dx^n}{n}$$
$$\Big\downarrow \text{90}$$
$$\frac{c\int x^{-n}(bx^n+a)^p\,dx^n+\frac{d(a+bx^n)^{p+1}}{b(p+1)}}{n}$$
$$\Big\downarrow \text{75}$$
$$\frac{\frac{d(a+bx^n)^{p+1}}{b(p+1)}-\frac{c(a+bx^n)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^n}{a}+1\right)}{a(p+1)}}{n}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/x,x]
```

output
```
((d*(a + b*x^n)^(1 + p))/(b*(1 + p)) - (c*(a + b*x^n)^(1 + p)*Hypergeometr
ic2F1[1, 1 + p, 2 + p, 1 + (b*x^n)/a])/(a*(1 + p)))/n
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(c+d\,x^{n})}{x}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/x,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x}\,dx=\int \frac{(dx^n+c)(bx^n+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p/x, x)
```

Sympy [A] (verification not implemented)

Time = 12.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x}\,dx = -\frac{b^p c x^{np}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ax^{-n}e^{i\pi}}{b}\right)}{n\Gamma(1-p)} - d\left(\begin{cases}-(a+b)^p\log(x) & \text{for } n=0\\ -\frac{\begin{cases}a^p x^n & \text{for } b=0\\ \frac{\begin{cases}\frac{(a+bx^n)^{p+1}}{p+1} & \text{for } p\neq -1\\ \log(a+bx^n) & \text{otherwise}\end{cases}}{b} & \text{otherwise}\end{cases}}{n} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/x,x)
```

output
```
-b**p*c*x**(n*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*
x**n))/(n*gamma(1 - p)) - d*Piecewise((-(a + b)**p*log(x), Eq(n, 0)), (-Pi
ecewise((a**p*x**n, Eq(b, 0)), (Piecewise(((a + b*x**n)**(p + 1)/(p + 1),
Ne(p, -1)), (log(a + b*x**n), True))/b, True))/n, True))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x} \, dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x} \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x,x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x} \, dx$$
$$= \frac{x^n(x^n b+a)^p\,bdp + (x^n b+a)^p\,adp + (x^n b+a)^p\,bcp + (x^n b+a)^p\,bc + \left(\int \frac{(x^n b+a)^p}{x^n bx+ax} dx\right) abcn\,p^2 + \left(\int \frac{(x^n b+}{x^n bx+}\right.}{bnp\,(p+1)}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/x,x)
```

output
```
(x**n*(x**n*b + a)**p*b*d*p + (x**n*b + a)**p*a*d*p + (x**n*b + a)**p*b*c*
p + (x**n*b + a)**p*b*c + int((x**n*b + a)**p/(x**n*b*x + a*x),x)*a*b*c*n*
p**2 + int((x**n*b + a)**p/(x**n*b*x + a*x),x)*a*b*c*n*p)/(b*n*p*(p + 1))
```

3.419 $\int \frac{(a+bx^n)^p(c+dx^n)}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^2}\,dx = -\frac{d(a+bx^n)^{1+p}}{b(1-n-np)x} - \frac{\left(c-\frac{ad}{b(1-n-np)}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x}$$

output
```
-d*(a+b*x^n)^(p+1)/b/(-n*p-n+1)/x-(c-a*d/b/(-n*p-n+1))*(a+b*x^n)^p*hyperge
om([-p, -1/n],[-(1-n)/n],-b*x^n/a)/x/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^2}\,dx = \frac{(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left((c-cn)\,\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,\frac{-1+n}{n},-\frac{bx^n}{a}\right)+dx^n\,\text{Hypergeometric2F1}\left(\right.\right.}{(-1+n)x}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/x^2,x]
```

output
```
((a + b*x^n)^p*((c - c*n)*Hypergeometric2F1[-n^(-1), -p, (-1 + n)/n, -((b*
x^n)/a)] + d*x^n*Hypergeometric2F1[(-1 + n)/n, -p, 2 - n^(-1), -((b*x^n)/a
)]))/((-1 + n)*x*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{x^2}dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad}{b(n(-p)-n+1)}\right)\int \frac{(bx^n+a)^p}{x^2}dx-\frac{d(a+bx^n)^{p+1}}{bx(n(-p)-n+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{b(n(-p)-n+1)}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{x^2}dx-\frac{d(a+bx^n)^{p+1}}{bx(n(-p)-n+1)}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{b(n(-p)-n+1)}\right)\text{Hypergeometric2F1}\left(-\frac{1}{n},-p,-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{x}-\frac{d(a+bx^n)^{p+1}}{bx(n(-p)-n+1)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/x^2,x]
```

output
```
-((d*(a + b*x^n)^(1 + p))/(b*(1 - n - n*p)*x)) - ((c - (a*d)/(b*(1 - n - n
*p)))*(a + b*x^n)^p*Hypergeometric2F1[-n^(-1), -p, -((1 - n)/n), -((b*x^n)
/a)])/(x*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(c+d\,x^{n})}{x^{2}}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/x^2,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^2}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2}\,dx = \frac{a^{1-\frac{1}{n}}a^{p-1+\frac{1}{n}}dx^{n-1}\Gamma\left(1-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1-\frac{1}{n}\\2-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{1}{n}\right)} + \frac{a^{-\frac{1}{n}}a^{p+\frac{1}{n}}c\Gamma\left(-\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-\frac{1}{n},-p\\1-\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx\Gamma\left(1-\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/x**2,x)
```

output
```
a**(1 - 1/n)*a**(p - 1 + 1/n)*d*x**(n - 1)*gamma(1 - 1/n)*hyper((-p, 1 - 1
/n), (2 - 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - 1/n)) + a**(p + 1/
n)*c*gamma(-1/n)*hyper((-1/n, -p), (1 - 1/n,), b*x**n*exp_polar(I*pi)/a)/(
a**(1/n)*n*x*gamma(1 - 1/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2} \, dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2} \, dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^2} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2} \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^2} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^2,x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^2} \, dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/x^2,x)
```

output
```
(x**n*(x**n*b + a)**p*b*d*n*p - x**n*(x**n*b + a)**p*b*d + (x**n*b + a)**p
*a*d*n*p + (x**n*b + a)**p*b*c*n*p + (x**n*b + a)**p*b*c*n - (x**n*b + a)*
*p*b*c + int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**2 + x**n*b*n**2*p*x**2 -
 2*x**n*b*n*p*x**2 - x**n*b*n*x**2 + x**n*b*x**2 + a*n**2*p**2*x**2 + a*n*
*2*p*x**2 - 2*a*n*p*x**2 - a*n*x**2 + a*x**2),x)*a**2*d*n**3*p**3*x + int(
(x**n*b + a)**p/(x**n*b*n**2*p**2*x**2 + x**n*b*n**2*p*x**2 - 2*x**n*b*n*p
*x**2 - x**n*b*n*x**2 + x**n*b*x**2 + a*n**2*p**2*x**2 + a*n**2*p*x**2 - 2
*a*n*p*x**2 - a*n*x**2 + a*x**2),x)*a**2*d*n**3*p**2*x - 2*int((x**n*b + a
)**p/(x**n*b*n**2*p**2*x**2 + x**n*b*n**2*p*x**2 - 2*x**n*b*n*p*x**2 - x**
n*b*n*x**2 + x**n*b*x**2 + a*n**2*p**2*x**2 + a*n**2*p*x**2 - 2*a*n*p*x**2
 - a*n*x**2 + a*x**2),x)*a**2*d*n**2*p**2*x - int((x**n*b + a)**p/(x**n*b*
n**2*p**2*x**2 + x**n*b*n**2*p*x**2 - 2*x**n*b*n*p*x**2 - x**n*b*n*x**2 +
x**n*b*x**2 + a*n**2*p**2*x**2 + a*n**2*p*x**2 - 2*a*n*p*x**2 - a*n*x**2 +
 a*x**2),x)*a**2*d*n**2*p*x + int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**2 +
 x**n*b*n**2*p*x**2 - 2*x**n*b*n*p*x**2 - x**n*b*n*x**2 + x**n*b*x**2 + a*
n**2*p**2*x**2 + a*n**2*p*x**2 - 2*a*n*p*x**2 - a*n*x**2 + a*x**2),x)*a**2
*d*n*p*x + int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**2 + x**n*b*n**2*p*x**2
 - 2*x**n*b*n*p*x**2 - x**n*b*n*x**2 + x**n*b*x**2 + a*n**2*p**2*x**2 + a*
n**2*p*x**2 - 2*a*n*p*x**2 - a*n*x**2 + a*x**2),x)*a*b*c*n**4*p**4*x + 2*i
nt((x**n*b + a)**p/(x**n*b*n**2*p**2*x**2 + x**n*b*n**2*p*x**2 - 2*x**n...
```

3.420 $\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx = -\frac{d(a+bx^n)^{1+p}}{b(2-n-np)x^2} - \frac{\left(c-\frac{2ad}{b(2-n-np)}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2}$$

output
```
-d*(a+b*x^n)^(p+1)/b/(-n*p-n+2)/x^2-1/2*(c-2*a*d/b/(-n*p-n+2))*(a+b*x^n)^p
*hypergeom([-p, -2/n],[-(2-n)/n],-b*x^n/a)/x^2/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx = \frac{(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(-c(-2+n)\,\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2dx^n\,\text{Hypergeometric}\right.}{2(-2+n)x^2}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/x^3,x]
```

output
```
((a + b*x^n)^p*(-(c*(-2 + n)*Hypergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*
x^n)/a)]) + 2*d*x^n*Hypergeometric2F1[(-2 + n)/n, -p, 2 - 2/n, -((b*x^n)/a
)]))/(2*(-2 + n)*x^2*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{x^3}dx$$
$$\Big\downarrow \text{959}$$
$$\left(c-\frac{2ad}{b(n(-p)-n+2)}\right)\int \frac{(bx^n+a)^p}{x^3}dx-\frac{d(a+bx^n)^{p+1}}{bx^2(n(-p)-n+2)}$$
$$\Big\downarrow \text{889}$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{2ad}{b(n(-p)-n+2)}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{x^3}dx-\frac{d(a+bx^n)^{p+1}}{bx^2(n(-p)-n+2)}$$
$$\Big\downarrow \text{888}$$
$$-\frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{2ad}{b(n(-p)-n+2)}\right)\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2x^2}-\frac{d(a+bx^n)^{p+1}}{bx^2(n(-p)-n+2)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/x^3,x]
```

output
```
-((d*(a + b*x^n)^(1 + p))/(b*(2 - n - n*p)*x^2)) - ((c - (2*a*d)/(b*(2 - n
 - n*p)))*(a + b*x^n)^p*Hypergeometric2F1[-2/n, -p, -((2 - n)/n), -((b*x^n
)/a)])/(2*x^2*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^3}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/x^3,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x^3}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^3,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{x^3}\,dx = \frac{a^{1-\frac{2}{n}}a^{p-1+\frac{2}{n}}dx^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-p,1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)} + \frac{a^{-\frac{2}{n}}a^{p+\frac{2}{n}}c\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{2}{n},-p\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/x**3,x)
```

output
```
a**(1 - 2/n)*a**(p - 1 + 2/n)*d*x**(n - 2)*gamma(1 - 2/n)*hyper((-p, 1 - 2
/n), (2 - 2/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - 2/n)) + a**(p + 2/
n)*c*gamma(-2/n)*hyper((-2/n, -p), (1 - 2/n,), b*x**n*exp_polar(I*pi)/a)/(
a**(2/n)*n*x**2*gamma(1 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p(c+dx^n)}{x^3}\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^3}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^3,x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{x^3} \, dx = \text{too large to display}$$

input

```
int((a+b*x^n)^p*(c+d*x^n)/x^3,x)
```

output

```
(x**n*(x**n*b + a)**p*b*d*n*p - 2*x**n*(x**n*b + a)**p*b*d + (x**n*b + a)*
*p*a*d*n*p + (x**n*b + a)**p*b*c*n*p + (x**n*b + a)**p*b*c*n - 2*(x**n*b +
 a)**p*b*c + 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*
x**3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x
**3 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*d*n**3
*p**3*x**2 + 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*
x**3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x
**3 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*d*n**3
*p**2*x**2 - 8*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*
x**3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x
**3 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*d*n**2
*p**2*x**2 - 4*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*
x**3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x
**3 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*d*n**2
*p*x**2 + 8*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**
3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3
 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*d*n*p*x**
2 + int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3 - 4*x*
*n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 + a*n**
2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a*b*c*n**4*p**4*x**...
```

3.421 $\int (ex)^{3/2} (a + bx^n)^p (c + dx^n) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int (ex)^{3/2} (a + bx^n)^p (c + dx^n) \, dx = \frac{2d(ex)^{5/2} (a + bx^n)^{1+p}}{be(5 + 2n + 2np)} + \frac{2\left(c - \frac{5ad}{b(5+2n(1+p))}\right) (ex)^{5/2} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2n}, -p, 1 + \frac{5}{2n}, -\frac{bx^n}{a}\right)}{5e}$$

output
```
2*d*(e*x)^(5/2)*(a+b*x^n)^(p+1)/b/e/(2*n*p+2*n+5)+2/5*(c-5*a*d/b/(5+2*n*(p
+1)))*(e*x)^(5/2)*(a+b*x^n)^p*hypergeom([-p, 5/2/n],[1+5/2/n],-b*x^n/a)/e/
((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int (ex)^{3/2} (a + bx^n)^p (c + dx^n) \, dx = \frac{2x(ex)^{3/2} (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \left(5dx^n \text{Hypergeometric2F1}\left(1 + \frac{5}{2n}, -p, 2 + \frac{5}{2n}, -\frac{bx^n}{a}\right) + c(5 -\right.}{5(5 + 2n)}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(2*x*(e*x)^(3/2)*(a + b*x^n)^p*(5*d*x^n*Hypergeometric2F1[1 + 5/(2*n), -p,
 2 + 5/(2*n), -((b*x^n)/a)] + c*(5 + 2*n)*Hypergeometric2F1[5/(2*n), -p, 1
 + 5/(2*n), -((b*x^n)/a)]))/(5*(5 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} (c + dx^n) (a + bx^n)^p \, dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2d(ex)^{5/2} (a + bx^n)^{p+1}}{be(2np + 2n + 5)} - \frac{(5ad - bc(2n(p+1) + 5)) \int (ex)^{3/2} (bx^n + a)^p dx}{b(2np + 2n + 5)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2d(ex)^{5/2} (a + bx^n)^{p+1}}{be(2np + 2n + 5)} - \frac{(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (5ad - bc(2n(p+1) + 5)) \int (ex)^{3/2} \left(\frac{bx^n}{a} + 1\right)^p dx}{b(2np + 2n + 5)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2d(ex)^{5/2} (a + bx^n)^{p+1}}{be(2np + 2n + 5)} - \frac{2(ex)^{5/2} (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (5ad - bc(2n(p+1) + 5)) \text{Hypergeometric2F1}\left(\frac{5}{2n}, -p, 1 + \frac{5}{2n}, -\frac{bx^n}{a}\right)}{5be(2np + 2n + 5)}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(2*d*(e*x)^(5/2)*(a + b*x^n)^(1 + p))/(b*e*(5 + 2*n + 2*n*p)) - (2*(5*a*d
- b*c*(5 + 2*n*(1 + p)))*(e*x)^(5/2)*(a + b*x^n)^p*Hypergeometric2F1[5/(2*
n), -p, 1 + 5/(2*n), -((b*x^n)/a)])/(5*b*e*(5 + 2*n + 2*n*p)*(1 + (b*x^n)/
a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^{\frac{3}{2}} (a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(ex)^{\frac{3}{2}} (bx^n+a)^p \, dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*e*x*x^n + c*e*x)*sqrt(e*x)*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(ex)^{\frac{3}{2}} (bx^n+a)^p \, dx$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(e*x)^(3/2)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,0,7,2,1,2,2]%%%}+%%%{-4,[0,0,0,7,2,1,1,2]%%%}+%%%{
-2,[0,0,0
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \int (e\,x)^{3/2} \, (a+b\,x^n)^p \, (c+d\,x^n) \, dx$$

input
```
int((e*x)^(3/2)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((e*x)^(3/2)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (ex)^{3/2} (a+bx^n)^p (c+dx^n) \, dx = \text{too large to display}$$

input
```
int((e*x)^(3/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(2*sqrt(e)*e*(2*x**((2*n + 1)/2)*(x**n*b + a)**p*b*d*n*p*x**2 + 5*x**((2*n
 + 1)/2)*(x**n*b + a)**p*b*d*x**2 + 2*sqrt(x)*(x**n*b + a)**p*a*d*n*p*x**2
 + 2*sqrt(x)*(x**n*b + a)**p*b*c*n*p*x**2 + 2*sqrt(x)*(x**n*b + a)**p*b*c*
n*x**2 + 5*sqrt(x)*(x**n*b + a)**p*b*c*x**2 - 20*int((sqrt(x)*(x**n*b + a)
**p*x)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 20*x**n*b*n*p + 10*x**n*b*n
 + 25*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 20*a*n*p + 10*a*n + 25*a),x)*a
**2*d*n**3*p**3 - 20*int((sqrt(x)*(x**n*b + a)**p*x)/(4*x**n*b*n**2*p**2 +
 4*x**n*b*n**2*p + 20*x**n*b*n*p + 10*x**n*b*n + 25*x**n*b + 4*a*n**2*p**2
 + 4*a*n**2*p + 20*a*n*p + 10*a*n + 25*a),x)*a**2*d*n**3*p**2 - 100*int((s
qrt(x)*(x**n*b + a)**p*x)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 20*x**n*
b*n*p + 10*x**n*b*n + 25*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 20*a*n*p +
10*a*n + 25*a),x)*a**2*d*n**2*p**2 - 50*int((sqrt(x)*(x**n*b + a)**p*x)/(4
*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 20*x**n*b*n*p + 10*x**n*b*n + 25*x**
n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 20*a*n*p + 10*a*n + 25*a),x)*a**2*d*n**
2*p - 125*int((sqrt(x)*(x**n*b + a)**p*x)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n
**2*p + 20*x**n*b*n*p + 10*x**n*b*n + 25*x**n*b + 4*a*n**2*p**2 + 4*a*n**2
*p + 20*a*n*p + 10*a*n + 25*a),x)*a**2*d*n*p + 8*int((sqrt(x)*(x**n*b + a)
**p*x)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 20*x**n*b*n*p + 10*x**n*b*n
 + 25*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 20*a*n*p + 10*a*n + 25*a),x)*a
*b*c*n**4*p**4 + 16*int((sqrt(x)*(x**n*b + a)**p*x)/(4*x**n*b*n**2*p**2...
```

3.422 $\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{2d(ex)^{3/2}\,(a+bx^n)^{1+p}}{be(3+2n+2np)}+\frac{2\left(c-\frac{3ad}{b(3+2n(1+p))}\right)(ex)^{3/2}\,(a+bx^n)^p\,\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e}$$

output
```
2*d*(e*x)^(3/2)*(a+b*x^n)^(p+1)/b/e/(2*n*p+2*n+3)+2/3*(c-3*a*d/b/(3+2*n*(p
+1)))*(e*x)^(3/2)*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],-b*x^n/a)/e/
((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{2x\sqrt{ex}(a+bx^n)^p\,\left(1+\frac{bx^n}{a}\right)^{-p}\left(3dx^n\,\text{Hypergeometric2F1}\left(1+\frac{3}{2n},-p,2+\frac{3}{2n},-\frac{bx^n}{a}\right)+c(3+2n)\,\text{Hyperg}\right.}{3(3+2n)}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(2*x*Sqrt[e*x]*(a + b*x^n)^p*(3*d*x^n*Hypergeometric2F1[1 + 3/(2*n), -p, 2
 + 3/(2*n), -((b*x^n)/a)] + c*(3 + 2*n)*Hypergeometric2F1[3/(2*n), -p, 1 +
 3/(2*n), -((b*x^n)/a)]))/(3*(3 + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}(c+dx^n)\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2d(ex)^{3/2}\,(a+bx^n)^{p+1}}{be(2np+2n+3)}-\frac{(3ad-bc(2n(p+1)+3))\int \sqrt{ex}(bx^n+a)^p dx}{b(2np+2n+3)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2d(ex)^{3/2}\,(a+bx^n)^{p+1}}{be(2np+2n+3)}-\frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(3ad-bc(2n(p+1)+3))\int \sqrt{ex}\left(\frac{bx^n}{a}+1\right)^p dx}{b(2np+2n+3)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2d(ex)^{3/2}\,(a+bx^n)^{p+1}}{be(2np+2n+3)}-\frac{2(ex)^{3/2}\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(3ad-bc(2n(p+1)+3))\,\text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},-\frac{bx^n}{a}\right)}{3be(2np+2n+3)}$$

input
```
Int[Sqrt[e*x]*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(2*d*(e*x)^(3/2)*(a + b*x^n)^(1 + p))/(b*e*(3 + 2*n + 2*n*p)) - (2*(3*a*d
- b*c*(3 + 2*n*(1 + p)))*(e*x)^(3/2)*(a + b*x^n)^p*Hypergeometric2F1[3/(2*
n), -p, 1 + 3/(2*n), -((b*x^n)/a)])/(3*b*e*(3 + 2*n + 2*n*p)*(1 + (b*x^n)/
a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \sqrt{ex}\,(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)\sqrt{ex}(bx^n+a)^p\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*sqrt(e*x)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{a^{\frac{3}{2n}}a^{p-\frac{3}{2n}}c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{2n}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2n},-p\\1+\frac{3}{2n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{2n}\right)}+\frac{a^{1+\frac{3}{2n}}a^{p-1-\frac{3}{2n}}d\sqrt{e}x^{n+\frac{3}{2}}\Gamma\left(1+\frac{3}{2n}\right){}_2F_1\left(\left.\begin{matrix}-p,1+\frac{3}{2n}\\2+\frac{3}{2n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{2n}\right)}$$

input
```
integrate((e*x)**(1/2)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(3/(2*n))*a**(p - 3/(2*n))*c*sqrt(e)*x**(3/2)*gamma(3/(2*n))*hyper((3/(
2*n), -p), (1 + 3/(2*n),), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/(2*n))
) + a**(1 + 3/(2*n))*a**(p - 1 - 3/(2*n))*d*sqrt(e)*x**(n + 3/2)*gamma(1 +
 3/(2*n))*hyper((-p, 1 + 3/(2*n)), (2 + 3/(2*n),), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(2 + 3/(2*n)))
```

Maxima [F]

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)\sqrt{ex}(bx^n+a)^p\,dx$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*sqrt(e*x)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,0,5,2,1,2,2]%%%}+%%%{-4,[0,0,0,5,2,1,1,2]%%%}+%%%{
-2,[0,0,0
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx=\int \sqrt{e\,x}\,{\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right)\;dx$$

input
```
int((e*x)^(1/2)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((e*x)^(1/2)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int \sqrt{ex}(a+bx^n)^p\,(c+dx^n)\;dx = \text{Too large to display}$$

input
```
int((e*x)^(1/2)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(2*sqrt(e)*(2*x**((2*n + 1)/2)*(x**n*b + a)**p*b*d*n*p*x + 3*x**((2*n + 1)
/2)*(x**n*b + a)**p*b*d*x + 2*sqrt(x)*(x**n*b + a)**p*a*d*n*p*x + 2*sqrt(x
)*(x**n*b + a)**p*b*c*n*p*x + 2*sqrt(x)*(x**n*b + a)**p*b*c*n*x + 3*sqrt(x
)*(x**n*b + a)**p*b*c*x - 12*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*
p**2 + 4*x**n*b*n**2*p + 12*x**n*b*n*p + 6*x**n*b*n + 9*x**n*b + 4*a*n**2*
p**2 + 4*a*n**2*p + 12*a*n*p + 6*a*n + 9*a),x)*a**2*d*n**3*p**3 - 12*int((
sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 12*x**n*b
*n*p + 6*x**n*b*n + 9*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 12*a*n*p + 6*a
*n + 9*a),x)*a**2*d*n**3*p**2 - 36*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b
*n**2*p**2 + 4*x**n*b*n**2*p + 12*x**n*b*n*p + 6*x**n*b*n + 9*x**n*b + 4*a
*n**2*p**2 + 4*a*n**2*p + 12*a*n*p + 6*a*n + 9*a),x)*a**2*d*n**2*p**2 - 18
*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 12*
x**n*b*n*p + 6*x**n*b*n + 9*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 12*a*n*p
 + 6*a*n + 9*a),x)*a**2*d*n**2*p - 27*int((sqrt(x)*(x**n*b + a)**p)/(4*x**
n*b*n**2*p**2 + 4*x**n*b*n**2*p + 12*x**n*b*n*p + 6*x**n*b*n + 9*x**n*b +
4*a*n**2*p**2 + 4*a*n**2*p + 12*a*n*p + 6*a*n + 9*a),x)*a**2*d*n*p + 8*int
((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2 + 4*x**n*b*n**2*p + 12*x**n
*b*n*p + 6*x**n*b*n + 9*x**n*b + 4*a*n**2*p**2 + 4*a*n**2*p + 12*a*n*p + 6
*a*n + 9*a),x)*a*b*c*n**4*p**4 + 16*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*
b*n**2*p**2 + 4*x**n*b*n**2*p + 12*x**n*b*n*p + 6*x**n*b*n + 9*x**n*b +...
```

3.423 $\int \frac{(a+bx^n)^p(c+dx^n)}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \frac{2d\sqrt{ex}(a+bx^n)^{1+p}}{be(1+2n+2np)} - \frac{2(ad-b(c+2cn(1+p)))\sqrt{ex}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{be(1+2n(1+p))}$$

output
```
2*d*(e*x)^(1/2)*(a+b*x^n)^(p+1)/b/e/(2*n*p+2*n+1)-2*(a*d-b*(c+2*c*n*(p+1))
)*(e*x)^(1/2)*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],-b*x^n/a)/b/e/(1
+2*n*(p+1))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \frac{2x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(dx^n\,\text{Hypergeometric2F1}\left(1+\frac{1}{2n},-p,2+\frac{1}{2n},-\frac{bx^n}{a}\right)+c(1+2n)\,\text{Hypergeome}\right.}{(1+2n)\sqrt{ex}}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/Sqrt[e*x],x]
```

output
```
(2*x*(a + b*x^n)^p*(d*x^n*Hypergeometric2F1[1 + 1/(2*n), -p, 2 + 1/(2*n),
-((b*x^n)/a)] + c*(1 + 2*n)*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n), -(
(b*x^n)/a)]))/((1 + 2*n)*Sqrt[e*x]*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2d\sqrt{ex}(a+bx^n)^{p+1}}{be(2np+2n+1)} - \frac{(ad-b(2cn(p+1)+c))\int \frac{(bx^n+a)^p}{\sqrt{ex}}dx}{b(2n(p+1)+1)}$$

$$\Big\downarrow \text{889}$$

$$\frac{2d\sqrt{ex}(a+bx^n)^{p+1}}{be(2np+2n+1)} - \frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(ad-b(2cn(p+1)+c))\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{\sqrt{ex}}dx}{b(2n(p+1)+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2d\sqrt{ex}(a+bx^n)^{p+1}}{be(2np+2n+1)} - \frac{2\sqrt{ex}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(ad-b(2cn(p+1)+c))\text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),-\frac{bx^n}{a}\right)}{be(2n(p+1)+1)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/Sqrt[e*x],x]
```

output
```
(2*d*Sqrt[e*x]*(a + b*x^n)^(1 + p))/(b*e*(1 + 2*n + 2*n*p)) - (2*(a*d - b*
(c + 2*c*n*(1 + p)))*Sqrt[e*x]*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p
, (2 + n^(-1))/2, -((b*x^n)/a)])/(b*e*(1 + 2*n*(1 + p))*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(c+d\,x^{n})}{\sqrt{e x}}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx=\int \frac{(dx^n+c)(bx^n+a)^p}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*sqrt(e*x)*(b*x^n + a)^p/(e*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \frac{a^{\frac{1}{2n}}a^{p-\frac{1}{2n}}c\sqrt{x}\Gamma\left(\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2n},-p\\1+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(1+\frac{1}{2n}\right)} + \frac{a^{1+\frac{1}{2n}}a^{p-1-\frac{1}{2n}}dx^{n+\frac{1}{2}}\Gamma\left(1+\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}-p,1+\frac{1}{2n}\\2+\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{\sqrt{e}n\Gamma\left(2+\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/(e*x)**(1/2),x)
```

output
```
a**(1/(2*n))*a**(p - 1/(2*n))*c*sqrt(x)*gamma(1/(2*n))*hyper((1/(2*n), -p)
, (1 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(e)*n*gamma(1 + 1/(2*n)))
 + a**(1 + 1/(2*n))*a**(p - 1 - 1/(2*n))*d*x**(n + 1/2)*gamma(1 + 1/(2*n))
*hyper((-p, 1 + 1/(2*n)), (2 + 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(sqrt(
e)*n*gamma(2 + 1/(2*n)))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{\sqrt{ex}}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/sqrt(e*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,0,2,1,0,1,2]%%%}+%%%{-2,[0,0,0,2,1,0,0,2]%%%} / %%
%{4,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(1/2),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{\sqrt{ex}}\,dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(1/2),x)
```

output

```
(2*sqrt(e)*(2*x**((2*n + 1)/2)*(x**n*b + a)**p*b*d*n*p + x**((2*n + 1)/2)*
(x**n*b + a)**p*b*d + 2*sqrt(x)*(x**n*b + a)**p*a*d*n*p + 2*sqrt(x)*(x**n*
b + a)**p*b*c*n*p + 2*sqrt(x)*(x**n*b + a)**p*b*c*n + sqrt(x)*(x**n*b + a)
**p*b*c - 4*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x + 4*x**n*b
*n**2*p*x + 4*x**n*b*n*p*x + 2*x**n*b*n*x + x**n*b*x + 4*a*n**2*p**2*x + 4
*a*n**2*p*x + 4*a*n*p*x + 2*a*n*x + a*x),x)*a**2*d*n**3*p**3 - 4*int((sqrt
(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x + 4*x**n*b*n**2*p*x + 4*x**n*b*
n*p*x + 2*x**n*b*n*x + x**n*b*x + 4*a*n**2*p**2*x + 4*a*n**2*p*x + 4*a*n*p
*x + 2*a*n*x + a*x),x)*a**2*d*n**3*p**2 - 4*int((sqrt(x)*(x**n*b + a)**p)/
(4*x**n*b*n**2*p**2*x + 4*x**n*b*n**2*p*x + 4*x**n*b*n*p*x + 2*x**n*b*n*x
+ x**n*b*x + 4*a*n**2*p**2*x + 4*a*n**2*p*x + 4*a*n*p*x + 2*a*n*x + a*x),x
)*a**2*d*n**2*p**2 - 2*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x
 + 4*x**n*b*n**2*p*x + 4*x**n*b*n*p*x + 2*x**n*b*n*x + x**n*b*x + 4*a*n**2
*p**2*x + 4*a*n**2*p*x + 4*a*n*p*x + 2*a*n*x + a*x),x)*a**2*d*n**2*p - int
((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x + 4*x**n*b*n**2*p*x + 4*x
**n*b*n*p*x + 2*x**n*b*n*x + x**n*b*x + 4*a*n**2*p**2*x + 4*a*n**2*p*x + 4
*a*n*p*x + 2*a*n*x + a*x),x)*a**2*d*n*p + 8*int((sqrt(x)*(x**n*b + a)**p)/
(4*x**n*b*n**2*p**2*x + 4*x**n*b*n**2*p*x + 4*x**n*b*n*p*x + 2*x**n*b*n*x
+ x**n*b*x + 4*a*n**2*p**2*x + 4*a*n**2*p*x + 4*a*n*p*x + 2*a*n*x + a*x),x
)*a*b*c*n**4*p**4 + 16*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**...
```

3.424 $\int \frac{(a+bx^n)^p(c+dx^n)}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(a+bx^n)^p(c+dx^n)}{(ex)^{3/2}}\,dx = -\frac{2d(a+bx^n)^{1+p}}{be(1-2n-2np)\sqrt{ex}} - \frac{2\left(c-\frac{ad}{b-2bn(1+p)}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}}$$

output
```
-2*d*(a+b*x^n)^(p+1)/b/e/(-2*n*p-2*n+1)/(e*x)^(1/2)-2*(c-a*d/(b-2*b*n*(p+1
)))*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],-b*x^n/a)/e/(e*x)^(1/2)/(
(1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^p(c+dx^n)}{(ex)^{3/2}}\,dx = \frac{2x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(dx^n\,\text{Hypergeometric2F1}\left(1-\frac{1}{2n},-p,2-\frac{1}{2n},-\frac{bx^n}{a}\right.\right.}{(-1+2n)(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/(e*x)^(3/2),x]
```

output
```
(2*x*(a + b*x^n)^p*(d*x^n*Hypergeometric2F1[1 - 1/(2*n), -p, 2 - 1/(2*n),
-((b*x^n)/a)] + c*(1 - 2*n)*Hypergeometric2F1[-1/2*1/n, -p, 1 - 1/(2*n), -
((b*x^n)/a)]))/((-1 + 2*n)*(e*x)^(3/2)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad}{b-2bn(p+1)}\right)\int \frac{(bx^n+a)^p}{(ex)^{3/2}}dx-\frac{2d(a+bx^n)^{p+1}}{be\sqrt{ex}(-2np-2n+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{b-2bn(p+1)}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(ex)^{3/2}}dx-\frac{2d(a+bx^n)^{p+1}}{be\sqrt{ex}(-2np-2n+1)}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{b-2bn(p+1)}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},-\frac{bx^n}{a}\right)}{e\sqrt{ex}}-\frac{2d(a+bx^n)^{p+1}}{be\sqrt{ex}(-2np-2n+1)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/(e*x)^(3/2),x]
```

output
```
(-2*d*(a + b*x^n)^(1 + p))/(b*e*(1 - 2*n - 2*n*p)*Sqrt[e*x]) - (2*(c - (a*
d)/(b - 2*b*n*(1 + p)))*(a + b*x^n)^p*Hypergeometric2F1[-1/2*1/n, -p, 1 -
1/(2*n), -((b*x^n)/a)])/(e*Sqrt[e*x]*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(c+d\,x^{n})}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{3/2}}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*sqrt(e*x)*(b*x^n + a)^p/(e^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{3/2}}\,dx = \frac{a^{1-\frac{1}{2n}}a^{p-1+\frac{1}{2n}}dx^{n-\frac{1}{2}}\Gamma\left(1-\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}-p,1-\frac{1}{2n}\\ 2-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\Gamma\left(2-\frac{1}{2n}\right)} + \frac{a^{-\frac{1}{2n}}a^{p+\frac{1}{2n}}c\Gamma\left(-\frac{1}{2n}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2n},-p\\ 1-\frac{1}{2n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{e^{\frac{3}{2}}n\sqrt{x}\Gamma\left(1-\frac{1}{2n}\right)}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/(e*x)**(3/2),x)
```

output
```
a**(1 - 1/(2*n))*a**(p - 1 + 1/(2*n))*d*x**(n - 1/2)*gamma(1 - 1/(2*n))*hy
per((-p, 1 - 1/(2*n)), (2 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(e**(3/2)
*n*gamma(2 - 1/(2*n))) + a**(p + 1/(2*n))*c*gamma(-1/(2*n))*hyper((-1/(2*n
), -p), (1 - 1/(2*n),), b*x**n*exp_polar(I*pi)/a)/(a**(1/(2*n))*e**(3/2)*n
*sqrt(x)*gamma(1 - 1/(2*n)))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{3/2}}\, dx = \int \frac{(dx^n+c)(bx^n+a)^p}{(ex)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/(e*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,0,2,1,0,1,2]%%%}+%%%{-2,[0,0,0,2,1,0,0,2]%%%} / %%
%{4,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{3/2}}\, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{(e\,x)^{3/2}}\, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(3/2),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a + bx^n)^p (c + dx^n)}{(ex)^{3/2}} \, dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(3/2),x)
```

output
```
(2*sqrt(e)*(2*x***n*(x***n*b + a)**p*b*d*n*p - x***n*(x***n*b + a)**p*b*d + 2*
(x**n*b + a)**p*a*d*n*p + 2*(x**n*b + a)**p*b*c*n*p + 2*(x**n*b + a)**p*b*
c*n - (x**n*b + a)**p*b*c + 4*sqrt(x)*int((sqrt(x)*(x**n*b + a)**p)/(4*x**
n*b*n**2*p**2*x**2 + 4*x**n*b*n**2*p*x**2 - 4*x**n*b*n*p*x**2 - 2*x**n*b*n
*x**2 + x**n*b*x**2 + 4*a*n**2*p**2*x**2 + 4*a*n**2*p*x**2 - 4*a*n*p*x**2
- 2*a*n*x**2 + a*x**2),x)*a**2*d*n**3*p**3 + 4*sqrt(x)*int((sqrt(x)*(x**n*
b + a)**p)/(4*x**n*b*n**2*p**2*x**2 + 4*x**n*b*n**2*p*x**2 - 4*x**n*b*n*p*
x**2 - 2*x**n*b*n*x**2 + x**n*b*x**2 + 4*a*n**2*p**2*x**2 + 4*a*n**2*p*x**
2 - 4*a*n*p*x**2 - 2*a*n*x**2 + a*x**2),x)*a**2*d*n**3*p**2 - 4*sqrt(x)*in
t((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x**2 + 4*x**n*b*n**2*p*x**
2 - 4*x**n*b*n*p*x**2 - 2*x**n*b*n*x**2 + x**n*b*x**2 + 4*a*n**2*p**2*x**2
 + 4*a*n**2*p*x**2 - 4*a*n*p*x**2 - 2*a*n*x**2 + a*x**2),x)*a**2*d*n**2*p*
*2 - 2*sqrt(x)*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p**2*x**2 + 4*
x**n*b*n**2*p*x**2 - 4*x**n*b*n*p*x**2 - 2*x**n*b*n*x**2 + x**n*b*x**2 + 4
*a*n**2*p**2*x**2 + 4*a*n**2*p*x**2 - 4*a*n*p*x**2 - 2*a*n*x**2 + a*x**2),
x)*a**2*d*n**2*p + sqrt(x)*int((sqrt(x)*(x**n*b + a)**p)/(4*x**n*b*n**2*p*
*2*x**2 + 4*x**n*b*n**2*p*x**2 - 4*x**n*b*n*p*x**2 - 2*x**n*b*n*x**2 + x**
n*b*x**2 + 4*a*n**2*p**2*x**2 + 4*a*n**2*p*x**2 - 4*a*n*p*x**2 - 2*a*n*x**
2 + a*x**2),x)*a**2*d*n*p + 8*sqrt(x)*int((sqrt(x)*(x**n*b + a)**p)/(4*x**
n*b*n**2*p**2*x**2 + 4*x**n*b*n**2*p*x**2 - 4*x**n*b*n*p*x**2 - 2*x**n*...
```

3.425 $\int \frac{(a+bx^n)^p(c+dx^n)}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{(ex)^{5/2}}\,dx = -\frac{2d(a+bx^n)^{1+p}}{be(3-2n(1+p))(ex)^{3/2}} - \frac{2\left(c-\frac{3ad}{b(3-2n(1+p))}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2n},-p,1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}}$$

output
```
-2*d*(a+b*x^n)^(p+1)/b/e/(3-2*n*(p+1))/(e*x)^(3/2)-2/3*(c-3*a*d/b/(3-2*n*(
p+1)))*(a+b*x^n)^p*hypergeom([-p, -3/2/n],[1-3/2/n],-b*x^n/a)/e/(e*x)^(3/2
)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{(ex)^{5/2}}\,dx = \frac{2x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(3dx^n\,\text{Hypergeometric2F1}\left(1-\frac{3}{2n},-p,2-\frac{3}{2n},-\frac{bx}{a}\right.\right.}{3(-3+2n)(ex)^{5}}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n))/(e*x)^(5/2),x]
```

output
```
(2*x*(a + b*x^n)^p*(3*d*x^n*Hypergeometric2F1[1 - 3/(2*n), -p, 2 - 3/(2*n)
, -((b*x^n)/a)] + c*(3 - 2*n)*Hypergeometric2F1[-3/(2*n), -p, 1 - 3/(2*n),
 -((b*x^n)/a)]))/(3*(-3 + 2*n)*(e*x)^(5/2)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^n)(a+bx^n)^p}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{3ad}{b(3-2n(p+1))}\right)\int \frac{(bx^n+a)^p}{(ex)^{5/2}}dx-\frac{2d(a+bx^n)^{p+1}}{be(ex)^{3/2}(-2np-2n+3)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{3ad}{b(3-2n(p+1))}\right)\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{(ex)^{5/2}}dx-\frac{2d(a+bx^n)^{p+1}}{be(ex)^{3/2}(-2np-2n+3)}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{3ad}{b(3-2n(p+1))}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2n},-p,1-\frac{3}{2n},-\frac{bx^n}{a}\right)}{3e(ex)^{3/2}}-\frac{2d(a+bx^n)^{p+1}}{be(ex)^{3/2}(-2np-2n+3)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n))/(e*x)^(5/2),x]
```

output
```
(-2*d*(a + b*x^n)^(1 + p))/(b*e*(3 - 2*n - 2*n*p)*(e*x)^(3/2)) - (2*(c - (
3*a*d)/(b*(3 - 2*n*(1 + p))))*(a + b*x^n)^p*Hypergeometric2F1[-3/(2*n), -p
, 1 - 3/(2*n), -((b*x^n)/a)])/(3*e*(e*x)^(3/2)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(c+d\,x^{n})}{(ex)^{\frac{5}{2}}}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{(ex)^{5/2}}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*sqrt(e*x)*(b*x^n + a)^p/(e^3*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{(ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)/(e*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^n)^p\,(c+dx^n)}{(ex)^{5/2}}\,dx = \int \frac{(dx^n+c)(bx^n+a)^p}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p/(e*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,0,2,1,0,1,2]%%%}+%%%{-2,[0,0,0,2,1,0,0,2]%%%} / %%
%{4,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{5/2}} \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{(e\,x)^{5/2}} \,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(5/2),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/(e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)}{(ex)^{5/2}} \, dx = \text{too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)/(e*x)^(5/2),x)
```

output
```
(2*sqrt(e)*(2*x***n*(x***n*b + a)**p*b*d*n*p - 3*x**n*(x***n*b + a)**p*b*d +
2*(x***n*b + a)**p*a*d*n*p + 2*(x***n*b + a)**p*b*c*n*p + 2*(x***n*b + a)**p*
b*c*n - 3*(x***n*b + a)**p*b*c + 12*sqrt(x)*int((sqrt(x)*(x***n*b + a)**p)/(
4*x***n*b*n**2*p**2*x**3 + 4*x***n*b*n**2*p*x**3 - 12*x***n*b*n*p*x**3 - 6*x*
*n*b*n*x**3 + 9*x***n*b*x**3 + 4*a*n**2*p**2*x**3 + 4*a*n**2*p*x**3 - 12*a*
n*p*x**3 - 6*a*n*x**3 + 9*a*x**3),x)*a**2*d*n**3*p**3*x + 12*sqrt(x)*int((
sqrt(x)*(x***n*b + a)**p)/(4*x***n*b*n**2*p**2*x**3 + 4*x***n*b*n**2*p*x**3 -
 12*x***n*b*n*p*x**3 - 6*x***n*b*n*x**3 + 9*x***n*b*x**3 + 4*a*n**2*p**2*x**3
 + 4*a*n**2*p*x**3 - 12*a*n*p*x**3 - 6*a*n*x**3 + 9*a*x**3),x)*a**2*d*n**3
*p**2*x - 36*sqrt(x)*int((sqrt(x)*(x***n*b + a)**p)/(4*x***n*b*n**2*p**2*x**
3 + 4*x***n*b*n**2*p*x**3 - 12*x***n*b*n*p*x**3 - 6*x***n*b*n*x**3 + 9*x***n*b
*x**3 + 4*a*n**2*p**2*x**3 + 4*a*n**2*p*x**3 - 12*a*n*p*x**3 - 6*a*n*x**3
+ 9*a*x**3),x)*a**2*d*n**2*p**2*x - 18*sqrt(x)*int((sqrt(x)*(x***n*b + a)**
p)/(4*x***n*b*n**2*p**2*x**3 + 4*x***n*b*n**2*p*x**3 - 12*x***n*b*n*p*x**3 -
6*x***n*b*n*x**3 + 9*x***n*b*x**3 + 4*a*n**2*p**2*x**3 + 4*a*n**2*p*x**3 - 1
2*a*n*p*x**3 - 6*a*n*x**3 + 9*a*x**3),x)*a**2*d*n**2*p*x + 27*sqrt(x)*int(
(sqrt(x)*(x***n*b + a)**p)/(4*x***n*b*n**2*p**2*x**3 + 4*x***n*b*n**2*p*x**3
- 12*x***n*b*n*p*x**3 - 6*x***n*b*n*x**3 + 9*x***n*b*x**3 + 4*a*n**2*p**2*x**
3 + 4*a*n**2*p*x**3 - 12*a*n*p*x**3 - 6*a*n*x**3 + 9*a*x**3),x)*a**2*d*n*p
*x + 8*sqrt(x)*int((sqrt(x)*(x***n*b + a)**p)/(4*x***n*b*n**2*p**2*x**3 +...
```

3.426 $\int x^2(a+bx^n)^p\left(3a+b(5+2p)x^n\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 99

$$\int x^2(a+bx^n)^p\left(3a+b(5+2p)x^n\right)\,dx = \frac{(5+2p)x^3(a+bx^n)^{1+p}}{3+n+np} - \frac{a(2-n)(1+p)x^3(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3+n+np}$$

output
```
(5+2*p)*x^3*(a+b*x^n)^(p+1)/(n*p+n+3)-a*(2-n)*(p+1)*x^3*(a+b*x^n)^p*hyperg
eom([-p, 3/n],[(3+n)/n],-b*x^n/a)/(n*p+n+3)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int x^2(a+bx^n)^p\left(3a+b(5+2p)x^n\right)\,dx = \frac{x^3(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(3+n)\,\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{3+n}{n},-\frac{bx^n}{a}\right)+b(5+2p)x^n\,\text{Hypergeomet}\right.}{3+n}$$

input
```
Integrate[x^2*(a + b*x^n)^p*(3*a + b*(5 + 2*p)*x^n),x]
```

output
```
(x^3*(a + b*x^n)^p*(a*(3 + n)*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*x
^n)/a)] + b*(5 + 2*p)*x^n*Hypergeometric2F1[(3 + n)/n, -p, 2 + 3/n, -((b*x
^n)/a)]))/((3 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^n)^p(3a+b(2p+5)x^n)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2p+5)x^3(a+bx^n)^{p+1}}{np+n+3}-\frac{3a(2-n)(p+1)\int x^2(bx^n+a)^p dx}{np+n+3}$$

$$\Big\downarrow \text{889}$$

$$\frac{(2p+5)x^3(a+bx^n)^{p+1}}{np+n+3}-\frac{3a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x^2\left(\frac{bx^n}{a}+1\right)^p dx}{np+n+3}$$

$$\Big\downarrow \text{888}$$

$$\frac{(2p+5)x^3(a+bx^n)^{p+1}}{np+n+3}-\frac{a(2-n)(p+1)x^3(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{n},-p,\frac{n+3}{n},-\frac{bx^n}{a}\right)}{np+n+3}$$

input
```
Int[x^2*(a + b*x^n)^p*(3*a + b*(5 + 2*p)*x^n),x]
```

output
```
((5 + 2*p)*x^3*(a + b*x^n)^(1 + p))/(3 + n + n*p) - (a*(2 - n)*(1 + p)*x^3
*(a + b*x^n)^p*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*x^n)/a)])/((3 +
n + n*p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^2(a+b\,x^n)^p\,(3a+b(5+2p)\,x^n)\,dx$$

input
```
int(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x)
```

output
```
int(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x)
```

Fricas [F]

$$\int x^2(a+bx^n)^p\,(3a+b(5+2p)x^n)\;dx=\int(b(2\,p+5)x^n+3\,a)(bx^n+a)^px^2\,dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x, algorithm="fricas")
```

output
```
integral(((2*b*p + 5*b)*x^2*x^n + 3*a*x^2)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.87 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.77

$$\int x^2(a+bx^n)^p\left(3a+b(5+2p)x^n\right)\,dx$$
$$=\frac{3aa^{\frac{3}{n}}a^{p-\frac{3}{n}}x^3\Gamma\left(\frac{3}{n}\right){}_2F_1\left(\begin{matrix}\frac{3}{n},-p\\1+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{3}{n}\right)}+\frac{2a^{1+\frac{3}{n}}a^{p-1-\frac{3}{n}}bpx^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}+\frac{5a^{1+\frac{3}{n}}a^{p-1-\frac{3}{n}}bx^{n+3}\Gamma\left(1+\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{3}{n}\\2+\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{3}{n}\right)}$$

input
```
integrate(x**2*(a+b*x**n)**p*(3*a+b*(5+2*p)*x**n),x)
```

output
```
3*a*a**(3/n)*a**(p - 3/n)*x**3*gamma(3/n)*hyper((3/n, -p), (1 + 3/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 3/n)) + 2*a**(1 + 3/n)*a**(p - 1 - 3/n
)*b*p*x**(n + 3)*gamma(1 + 3/n)*hyper((-p, 1 + 3/n), (2 + 3/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 + 3/n)) + 5*a**(1 + 3/n)*a**(p - 1 - 3/n)*b*x*
*(n + 3)*gamma(1 + 3/n)*hyper((-p, 1 + 3/n), (2 + 3/n,), b*x**n*exp_polar(
I*pi)/a)/(n*gamma(2 + 3/n))
```

Maxima [F]

$$\int x^2(a+bx^n)^p \left(3a+b(5+2p)x^n\right) \, dx = \int (b(2\,p+5)x^n+3\,a)(bx^n+a)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x, algorithm="maxima")
```

output
```
integrate((b*(2*p + 5)*x^n + 3*a)*(b*x^n + a)^p*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^n)^p \left(3a+b(5+2p)x^n\right) \, dx = \int (b(2\,p+5)x^n+3\,a)(bx^n+a)^p x^2 \, dx$$

input
```
integrate(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x, algorithm="giac")
```

output
```
integrate((b*(2*p + 5)*x^n + 3*a)*(b*x^n + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^n)^p \left(3a+b(5+2p)x^n\right) \, dx = \int x^2 \left(3\,a+b\,x^n\,(2\,p+5)\right) \left(a+b\,x^n\right)^p \, dx$$

input
```
int(x^2*(3*a + b*x^n*(2*p + 5))*(a + b*x^n)^p,x)
```

output
```
int(x^2*(3*a + b*x^n*(2*p + 5))*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int x^2(a+bx^n)^p\left(3a+b(5+2p)x^n\right)dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*x^n)^p*(3*a+b*(5+2*p)*x^n),x)
```

output
```
(2*x**n*(x**n*b + a)**p*b*n*p**2*x**3 + 5*x**n*(x**n*b + a)**p*b*n*p*x**3
+ 6*x**n*(x**n*b + a)**p*b*p*x**3 + 15*x**n*(x**n*b + a)**p*b*x**3 + 2*(x*
*n*b + a)**p*a*n*p**2*x**3 + 8*(x**n*b + a)**p*a*n*p*x**3 + 3*(x**n*b + a)
**p*a*n*x**3 + 9*(x**n*b + a)**p*a*x**3 + 3*int(((x**n*b + a)**p*x**2)/(x*
*n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*
n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**2*n**4*p**4 + 6*int(((
x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*
x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a
**2*n**4*p**3 + 3*int(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n*
*2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a
*n*p + 3*a*n + 9*a),x)*a**2*n**4*p**2 - 6*int(((x**n*b + a)**p*x**2)/(x**n
*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n*
*2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**2*n**3*p**4 + 6*int(((x*
*n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x*
*n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**
2*n**3*p**3 + 21*int(((x**n*b + a)**p*x**2)/(x**n*b*n**2*p**2 + x**n*b*n**
2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n**2*p**2 + a*n**2*p + 6*a*
n*p + 3*a*n + 9*a),x)*a**2*n**3*p**2 + 9*int(((x**n*b + a)**p*x**2)/(x**n*
b*n**2*p**2 + x**n*b*n**2*p + 6*x**n*b*n*p + 3*x**n*b*n + 9*x**n*b + a*n**
2*p**2 + a*n**2*p + 6*a*n*p + 3*a*n + 9*a),x)*a**2*n**3*p - 36*int(((x*...
```

3.427 $\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 98

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx = \frac{2(2+p)x^2(a+bx^n)^{1+p}}{2+n+np} - \frac{a(2-n)(1+p)x^2(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2+n+np}$$

output
```
2*(2+p)*x^2*(a+b*x^n)^(p+1)/(n*p+n+2)-a*(2-n)*(p+1)*x^2*(a+b*x^n)^p*hyperg
eom([-p, 2/n],[(2+n)/n],-b*x^n/a)/(n*p+n+2)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx = \frac{x^2(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(2+n)\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{2+n}{n},-\frac{bx^n}{a}\right)+2b(2+p)x^n\text{Hypergeomet}\right.}{2+n}$$

input
```
Integrate[x*(a + b*x^n)^p*(2*a + b*(4 + 2*p)*x^n),x]
```

output
```
(x^2*(a + b*x^n)^p*(a*(2 + n)*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*x
^n)/a)] + 2*b*(2 + p)*x^n*Hypergeometric2F1[(2 + n)/n, -p, 2*(1 + n^(-1)),
 -((b*x^n)/a)]))/((2 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^n)^p\left(2a+b(2p+4)x^n\right)\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{2(p+2)x^2(a+bx^n)^{p+1}}{np+n+2}-\frac{2a(2-n)(p+1)\int x(bx^n+a)^p\,dx}{np+n+2}$$

$$\Big\downarrow \text{889}$$

$$\frac{2(p+2)x^2(a+bx^n)^{p+1}}{np+n+2}-\frac{2a(2-n)(p+1)\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x\left(\frac{bx^n}{a}+1\right)^p dx}{np+n+2}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(p+2)x^2(a+bx^n)^{p+1}}{np+n+2}-\frac{a(2-n)(p+1)x^2(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{2}{n},-p,\frac{n+2}{n},-\frac{bx^n}{a}\right)}{np+n+2}$$

input
```
Int[x*(a + b*x^n)^p*(2*a + b*(4 + 2*p)*x^n),x]
```

output
```
(2*(2 + p)*x^2*(a + b*x^n)^(1 + p))/(2 + n + n*p) - (a*(2 - n)*(1 + p)*x^2
*(a + b*x^n)^p*Hypergeometric2F1[2/n, -p, (2 + n)/n, -((b*x^n)/a)])/((2 +
n + n*p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x(a+b\,x^n)^p\,(2a+b(4+2p)\,x^n)\,dx$$

input
```
int(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x)
```

output
```
int(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x)
```

Fricas [F]

$$\int x(a+bx^n)^p\,(2a+b(4+2p)x^n)\;dx=\int 2\,(b(p+2)x^n+a)(bx^n+a)^p x\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x, algorithm="fricas")
```

output
```
integral(2*((b*p + 2*b)*x*x^n + a*x)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.79

$$\int x(a+bx^n)^p\,(2a+b(4+2p)x^n)\;dx$$
$$= \frac{2aa^{\frac{2}{n}}a^{p-\frac{2}{n}}x^2\Gamma\left(\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{n},-p\\1+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{2}{n}\right)} + \frac{2a^{1+\frac{2}{n}}a^{p-1-\frac{2}{n}}bpx^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-p,1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)} + \frac{4a^{1+\frac{2}{n}}a^{p-1-\frac{2}{n}}bx^{n+2}\Gamma\left(1+\frac{2}{n}\right)\,{}_2F_1\left(\begin{matrix}-p,1+\frac{2}{n}\\2+\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{2}{n}\right)}$$

input
```
integrate(x*(a+b*x**n)**p*(2*a+b*(4+2*p)*x**n),x)
```

output
```
2*a*a**(2/n)*a**(p - 2/n)*x**2*gamma(2/n)*hyper((2/n, -p), (1 + 2/n,), b*x
**n*exp_polar(I*pi)/a)/(n*gamma(1 + 2/n)) + 2*a**(1 + 2/n)*a**(p - 1 - 2/n
)*b*p*x**(n + 2)*gamma(1 + 2/n)*hyper((-p, 1 + 2/n), (2 + 2/n,), b*x**n*ex
p_polar(I*pi)/a)/(n*gamma(2 + 2/n)) + 4*a**(1 + 2/n)*a**(p - 1 - 2/n)*b*x*
*(n + 2)*gamma(1 + 2/n)*hyper((-p, 1 + 2/n), (2 + 2/n,), b*x**n*exp_polar(
I*pi)/a)/(n*gamma(2 + 2/n))
```

Maxima [F]

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx=\int 2\,(b(p+2)x^n+a)(bx^n+a)^px\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x, algorithm="maxima")
```

output
```
2*integrate((b*(p + 2)*x^n + a)*(b*x^n + a)^p*x, x)
```

Giac [F]

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx=\int 2\,(b(p+2)x^n+a)(bx^n+a)^px\,dx$$

input
```
integrate(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x, algorithm="giac")
```

output
```
integrate(2*(b*(p + 2)*x^n + a)*(b*x^n + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx=\int x\,\left(2\,a+b\,x^n\,\left(2\,p+4\right)\right)\,\left(a+b\,x^n\right)^p\,dx$$

input
```
int(x*(2*a + b*x^n*(2*p + 4))*(a + b*x^n)^p,x)
```

output
```
int(x*(2*a + b*x^n*(2*p + 4))*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int x(a+bx^n)^p\left(2a+b(4+2p)x^n\right)\,dx=\text{Too large to display}$$

input
```
int(x*(a+b*x^n)^p*(2*a+b*(4+2*p)*x^n),x)
```

output
```
(2*(x**n*(x**n*b + a)**p*b*n*p**2*x**2 + 2*x**n*(x**n*b + a)**p*b*n*p*x**2
 + 2*x**n*(x**n*b + a)**p*b*p*x**2 + 4*x**n*(x**n*b + a)**p*b*x**2 + (x**n
*b + a)**p*a*n*p**2*x**2 + 3*(x**n*b + a)**p*a*n*p*x**2 + (x**n*b + a)**p*
a*n*x**2 + 2*(x**n*b + a)**p*a*x**2 + int(((x**n*b + a)**p*x)/(x**n*b*n**2
*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2
 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*n**4*p**4 + 2*int(((x**n*b +
a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4
*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*n**4*p**
3 + int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n
*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*
a),x)*a**2*n**4*p**2 - 2*int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*
b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p +
 4*a*n*p + 2*a*n + 4*a),x)*a**2*n**3*p**4 + 4*int(((x**n*b + a)**p*x)/(x**
n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n
**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*n**3*p**2 + 2*int(((x
**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x**n*b*n*p + 2*x**n
*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a*n + 4*a),x)*a**2*
n**3*p - 8*int(((x**n*b + a)**p*x)/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 4*x
**n*b*n*p + 2*x**n*b*n + 4*x**n*b + a*n**2*p**2 + a*n**2*p + 4*a*n*p + 2*a
*n + 4*a),x)*a**2*n**2*p**3 - 8*int(((x**n*b + a)**p*x)/(x**n*b*n**2*p*...
```

3.428 $\int (a+bx^n)^p (a+b(3+2p)x^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int (a+bx^n)^p (a+b(3+2p)x^n)\,dx = \frac{(3+2p)x(a+bx^n)^{1+p}}{1+n+np} - \frac{a(2-n)(1+p)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)}{1+n+np}$$

output
```
(3+2*p)*x*(a+b*x^n)^(p+1)/(n*p+n+1)-a*(2-n)*(p+1)*x*(a+b*x^n)^p*hypergeom(
[-p, 1/n],[1+1/n],-b*x^n/a)/(n*p+n+1)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int (a+bx^n)^p (a+b(3+2p)x^n)\,dx = \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left((3+2p)(a+bx^n)\left(1+\frac{bx^n}{a}\right)^p+a(-2+n)(1+p)\,\text{Hypergeometric2F1}\left(\frac{1}{n},-p,\right.\right.}{1+n+np}$$

input
```
Integrate[(a + b*x^n)^p*(a + b*(3 + 2*p)*x^n),x]
```

output
```
(x*(a + b*x^n)^p*((3 + 2*p)*(a + b*x^n)*(1 + (b*x^n)/a)^p + a*(-2 + n)*(1
+ p)*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/((1 + n + n
*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {913, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^n)^p\left(a+b(2p+3)x^n\right)dx$$

$$\Big\downarrow \text{913}$$

$$\frac{(2p+3)x(a+bx^n)^{p+1}}{np+n+1}-\frac{a(2-n)(p+1)\int (bx^n+a)^p dx}{np+n+1}$$

$$\Big\downarrow \text{779}$$

$$\frac{(2p+3)x(a+bx^n)^{p+1}}{np+n+1}-\frac{a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \left(\frac{bx^n}{a}+1\right)^p dx}{np+n+1}$$

$$\Big\downarrow \text{778}$$

$$\frac{(2p+3)x(a+bx^n)^{p+1}}{np+n+1}-\frac{a(2-n)(p+1)x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)}{np+n+1}$$

input
```
Int[(a + b*x^n)^p*(a + b*(3 + 2*p)*x^n),x]
```

output
```
((3 + 2*p)*x*(a + b*x^n)^(1 + p))/(1 + n + n*p) - (a*(2 - n)*(1 + p)*x*(a
+ b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/((1 +
n + n*p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{p}\,(a+b(3+2p)\,x^{n})\,dx$$

input
```
int((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x)
```

output
```
int((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x)
```

Fricas [F]

$$\int (a+bx^n)^p\,(a+b(3+2p)x^n)\;dx=\int (b(2\,p+3)x^n+a)(bx^n+a)^p\,dx$$

input
```
integrate((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x, algorithm="fricas")
```

output
```
integral(((2*b*p + 3*b)*x^n + a)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.70 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

$$\int (a+bx^n)^p\,(a+b(3+2p)x^n)\,dx = \frac{aa^{\frac{1}{n}}a^{p-\frac{1}{n}}x\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}bpx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{3a^{1+\frac{1}{n}}a^{p-1-\frac{1}{n}}bx^{n+1}\Gamma\left(1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(a+b*(3+2*p)*x**n),x)
```

output
```
a*a**(1/n)*a**(p - 1/n)*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*e
xp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 2*a**(1 + 1/n)*a**(p - 1 - 1/n)*b*p
*x**(n + 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_pol
ar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**(1 + 1/n)*a**(p - 1 - 1/n)*b*x**(n +
 1)*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(2 + 1/n))
```

Maxima [F]

$$\int (a+bx^n)^p (a+b(3+2p)x^n) \, dx = \int (b(2p+3)x^n+a)(bx^n+a)^p \, dx$$

input
```
integrate((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x, algorithm="maxima")
```

output
```
integrate((b*(2*p + 3)*x^n + a)*(b*x^n + a)^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+bx^n)^p (a+b(3+2p)x^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,2]%%%}+%%%{2,[0,0,2,2,1,1,2]%%%}+%%%{1,[0,0
,2,2,1,0,
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^n)^p (a+b(3+2p)x^n) \, dx = \int (a+b\,x^n)^p\,(a+b\,x^n\,(2\,p+3)) \, dx$$

input
```
int((a + b*x^n)^p*(a + b*x^n*(2*p + 3)),x)
```

output
```
int((a + b*x^n)^p*(a + b*x^n*(2*p + 3)), x)
```

Reduce [F]

$$\int (a+bx^n)^p (a+b(3+2p)x^n)\ dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(a+b*(3+2*p)*x^n),x)
```

output
```
(2*x**n*(x**n*b + a)**p*b*n*p**2*x + 3*x**n*(x**n*b + a)**p*b*n*p*x + 2*x*
*n*(x**n*b + a)**p*b*p*x + 3*x**n*(x**n*b + a)**p*b*x + 2*(x**n*b + a)**p*
a*n*p**2*x + 4*(x**n*b + a)**p*a*n*p*x + (x**n*b + a)**p*a*n*x + (x**n*b +
 a)**p*a*x + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**
n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),
x)*a**2*n**4*p**4 + 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*
p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p +
a*n + a),x)*a**2*n**4*p**3 + int((x**n*b + a)**p/(x**n*b*n**2*p**2 + x**n*
b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**2*p + 2*a
*n*p + a*n + a),x)*a**2*n**4*p**2 - 2*int((x**n*b + a)**p/(x**n*b*n**2*p**
2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**2 + a*n**
2*p + 2*a*n*p + a*n + a),x)*a**2*n**3*p**4 - 2*int((x**n*b + a)**p/(x**n*b
*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n**2*p**
2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**3*p**3 + int((x**n*b + a)**p/
(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*n
**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**3*p**2 + int((x**n*b +
 a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*
b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**3*p - 4*int((x*
*n*b + a)**p/(x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n +
 x**n*b + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*n**2*p**3...
```

3.429 $\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx=\frac{2(a+bx^n)^{1+p}}{n}$$

output
```
2*(a+b*x^n)^(p+1)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx=\frac{2(a+bx^n)^{1+p}}{n}$$

input
```
Integrate[b*(2 + 2*p)*x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(2*(a + b*x^n)^(1 + p))/n
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {27, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int b(2p+2)x^{n-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{27}$$

$$2b(p+1)\int x^{n-1}(bx^n+a)^p\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{2(a+bx^n)^{p+1}}{n}$$

input
```
Int[b*(2 + 2*p)*x^(-1 + n)*(a + b*x^n)^p,x]
```

output
```
(2*(a + b*x^n)^(1 + p))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

method	result	size
risch	$\frac{(2+2p)(a+b\,x^{n})(a+b\,x^{n})^{p}}{n(p+1)}$	31

input
```
int(b*(2+2*p)*x^(-1+n)*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(2+2*p)*(a+b*x^n)/n/(p+1)*(a+b*x^n)^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx=\frac{2\,(bx^n+a)(bx^n+a)^p}{n}$$

input
```
integrate(b*(2+2*p)*x^(-1+n)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
2*(b*x^n + a)*(b*x^n + a)^p/n
```

Sympy [A] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 5.31

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx$$
$$=b(2p+2)\left(\begin{cases}\frac{\log(x)}{a} & \text{for } b=0\wedge n=0\wedge p=-1\\ \frac{a^pxx^{n-1}}{n} & \text{for } b=0\\ (a+b)^p\log(x) & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+x^n\right)}{bn} & \text{for } p=-1\\ \frac{a(a+bx^n)^p}{bnp+bn}+\frac{bx^n(a+bx^n)^p}{bnp+bn} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(b*(2+2*p)*x**(-1+n)*(a+b*x**n)**p,x)
```

output
```
b*(2*p + 2)*Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*x
*x**(n - 1)/n, Eq(b, 0)), ((a + b)**p*log(x), Eq(n, 0)), (log(a/b + x**n)/
(b*n), Eq(p, -1)), (a*(a + b*x**n)**p/(b*n*p + b*n) + b*x**n*(a + b*x**n)*
*p/(b*n*p + b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx = \frac{2\,(bx^n+a)^{p+1}}{n}$$

input
```
integrate(b*(2+2*p)*x^(-1+n)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
2*(b*x^n + a)^(p + 1)/n
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx = \frac{2\,(bx^n+a)^{p+1}}{n}$$

input
```
integrate(b*(2+2*p)*x^(-1+n)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
2*(b*x^n + a)^(p + 1)/n
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx=\frac{2\,(a+b\,x^n)^{p+1}}{n}$$

input
```
int(b*x^(n - 1)*(2*p + 2)*(a + b*x^n)^p,x)
```

output
```
(2*(a + b*x^n)^(p + 1))/n
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int b(2+2p)x^{-1+n}(a+bx^n)^p\,dx=\frac{2(x^nb+a)^p\,(x^nb+a)}{n}$$

input
```
int(b*(2+2*p)*x^(-1+n)*(a+b*x^n)^p,x)
```

output
```
(2*(x**n*b + a)**p*(x**n*b + a))/n
```

3.430 $\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 90

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx = -\frac{x^{1-n}(a+bx^n)^{1+p}}{1-n} + \frac{b(2-n)(1+p)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},-\frac{bx^n}{a}\right)}{1-n}$$

output
```
-x^(1-n)*(a+b*x^n)^(p+1)/(1-n)+b*(2-n)*(p+1)*x*(a+b*x^n)^p*hypergeom([-p,
1/n],[1+1/n],-b*x^n/a)/(1-n)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx = \frac{x^{1-n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a\,\text{Hypergeometric2F1}\left(-1+\frac{1}{n},-p,\frac{1}{n},-\frac{bx^n}{a}\right)+b(-1+n)(1+2p)x^n\,\text{Hyper}\right.}{-1+n}$$

input
```
Integrate[((a + b*x^n)^p*(-a + b*(1 + 2*p)*x^n))/x^n,x]
```

output
```
(x^(1 - n)*(a + b*x^n)^p*(a*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -((
b*x^n)/a)] + b*(-1 + n)*(1 + 2*p)*x^n*Hypergeometric2F1[n^(-1), -p, 1 + n^
(-1), -((b*x^n)/a)]))/((-1 + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n}(a+bx^n)^p\left(b(2p+1)x^n-a\right)dx$$

$$\Big\downarrow \; 959$$

$$\frac{(2p+1)x^{1-n}(a+bx^n)^{p+1}}{np+1}-\frac{a(2-n)(p+1)\int x^{-n}(bx^n+a)^p dx}{np+1}$$

$$\Big\downarrow \; 889$$

$$\frac{(2p+1)x^{1-n}(a+bx^n)^{p+1}}{np+1}-\frac{a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x^{-n}\left(\frac{bx^n}{a}+1\right)^p dx}{np+1}$$

$$\Big\downarrow \; 888$$

$$\frac{(2p+1)x^{1-n}(a+bx^n)^{p+1}}{np+1}-\frac{a(2-n)(p+1)x^{1-n}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n}-1,-p,\frac{1}{n},-\frac{bx^n}{a}\right)}{(1-n)(np+1)}$$

input
```
Int[((a + b*x^n)^p*(-a + b*(1 + 2*p)*x^n))/x^n,x]
```

output
```
((1 + 2*p)*x^(1 - n)*(a + b*x^n)^(1 + p))/(1 + n*p) - (a*(2 - n)*(1 + p)*x
^(1 - n)*(a + b*x^n)^p*Hypergeometric2F1[-1 + n^(-1), -p, n^(-1), -((b*x^n
)/a)])/((1 - n)*(1 + n*p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (a+b\,x^{n})^{p} \left(-a+b(2p+1)\,x^{n}\right) x^{-n}dx$$

input
```
int((a+b*x^n)^p*(-a+b*(2*p+1)*x^n)/(x^n),x)
```

output
```
int((a+b*x^n)^p*(-a+b*(2*p+1)*x^n)/(x^n),x)
```

Fricas [F]

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx=\int\frac{(b(2\,p+1)x^n-a)(bx^n+a)^p}{x^n}\,dx$$

input
```
integrate((a+b*x^n)^p*(-a+b*(1+2*p)*x^n)/(x^n),x, algorithm="fricas")
```

output
```
integral(((2*b*p + b)*x^n - a)*(b*x^n + a)^p/x^n, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.89

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx = -\frac{aa^{-1+\frac{1}{n}}a^{p+1-\frac{1}{n}}b^{-1+\frac{1}{n}}b^{1-\frac{1}{n}}x^{1-n}\Gamma\left(-1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,-1+\frac{1}{n}\\ \frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{1}{n}\right)} + \frac{2a^{\frac{1}{n}}a^{p-\frac{1}{n}}bpx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\ 1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{a^{\frac{1}{n}}a^{p-\frac{1}{n}}bx\Gamma\left(\frac{1}{n}\right){}_2F_1\left(\begin{matrix}\frac{1}{n},-p\\ 1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(-a+b*(1+2*p)*x**n)/(x**n),x)
```

output
```
-a*a**(-1 + 1/n)*a**(p + 1 - 1/n)*b**(-1 + 1/n)*b**(1 - 1/n)*x**(1 - n)*ga
mma(-1 + 1/n)*hyper((-p, -1 + 1/n), (1/n,), b*x**n*exp_polar(I*pi)/a)/(n*g
amma(1/n)) + 2*a**(1/n)*a**(p - 1/n)*b*p*x*gamma(1/n)*hyper((1/n, -p), (1
+ 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**(1/n)*a**(p - 1
/n)*b*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(n*gamma(1 + 1/n))
```

Maxima [F]

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx = \int \frac{(b(2\,p+1)x^n-a)(bx^n+a)^p}{x^n}\,dx$$

input
```
integrate((a+b*x^n)^p*(-a+b*(1+2*p)*x^n)/(x^n),x, algorithm="maxima")
```

output
```
integrate((b*(2*p + 1)*x^n - a)*(b*x^n + a)^p/x^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*x^n)^p*(-a+b*(1+2*p)*x^n)/(x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,2]%%%}+%%%{2,[0,0,2,2,1,1,2]%%%}+%%%{1,[0,0
,2,2,1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx=-\int\frac{(a+b\,x^n)^p\,\left(a-b\,x^n\,(2\,p+1)\right)}{x^n}\,dx$$

input
```
int(-((a + b*x^n)^p*(a - b*x^n*(2*p + 1)))/x^n,x)
```

output
```
-int(((a + b*x^n)^p*(a - b*x^n*(2*p + 1)))/x^n, x)
```

Reduce [F]

$$\int x^{-n}(a+bx^n)^p\left(-a+b(1+2p)x^n\right)\,dx=\text{Too large to display}$$

input
```
int((a+b*x^n)^p*(-a+b*(1+2*p)*x^n)/(x^n),x)
```

output
```
(2*x**n*(x**n*b + a)**p*b*n*p**2*x - x**n*(x**n*b + a)**p*b*n*p*x - x**n*(
x**n*b + a)**p*b*n*x + 2*x**n*(x**n*b + a)**p*b*p*x + x**n*(x**n*b + a)**p
*b*x + 2*(x**n*b + a)**p*a*n*p**2*x - (x**n*b + a)**p*a*x + x**n*int((x**n
*b + a)**p/(x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p -
x**(2*n)*b*n + x**(2*n)*b + x**n*a*n**2*p**2 - x**n*a*n**2*p + 2*x**n*a*n*
p - x**n*a*n + x**n*a),x)*a**2*n**4*p**4 - x**n*int((x**n*b + a)**p/(x**(2
*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p - x**(2*n)*b*n + x*
*(2*n)*b + x**n*a*n**2*p**2 - x**n*a*n**2*p + 2*x**n*a*n*p - x**n*a*n + x*
*n*a),x)*a**2*n**4*p**2 - 2*x**n*int((x**n*b + a)**p/(x**(2*n)*b*n**2*p**2
 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b + x**n
*a*n**2*p**2 - x**n*a*n**2*p + 2*x**n*a*n*p - x**n*a*n + x**n*a),x)*a**2*n
**3*p**4 + 2*x**n*int((x**n*b + a)**p/(x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n
**2*p + 2*x**(2*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b + x**n*a*n**2*p**2 -
x**n*a*n**2*p + 2*x**n*a*n*p - x**n*a*n + x**n*a),x)*a**2*n**3*p**3 + 3*x*
*n*int((x**n*b + a)**p/(x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2
*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b + x**n*a*n**2*p**2 - x**n*a*n**2*p +
 2*x**n*a*n*p - x**n*a*n + x**n*a),x)*a**2*n**3*p**2 - x**n*int((x**n*b +
a)**p/(x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p - x**(2
*n)*b*n + x**(2*n)*b + x**n*a*n**2*p**2 - x**n*a*n**2*p + 2*x**n*a*n*p - x
**n*a*n + x**n*a),x)*a**2*n**3*p - 4*x**n*int((x**n*b + a)**p/(x**(2*n)...
```

3.431 $\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = -\frac{2p(a+bx^n)^{1+p}}{(2-n-np)x^2} + \frac{a(2-n)(1+p)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{(2-n-np)x^2}$$

output
```
-2*p*(a+b*x^n)^(p+1)/(-n*p-n+2)/x^2+a*(2-n)*(p+1)*(a+b*x^n)^p*hypergeom([-
p, -2/n],[-(2-n)/n],-b*x^n/a)/(-n*p-n+2)/x^2/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = \frac{(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(-2+n)\,\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,\frac{-2+n}{n},-\frac{bx^n}{a}\right)+2bpx^n\,\text{Hypergeometric2}\right.}{(-2+n)x^2}$$

input
```
Integrate[((a + b*x^n)^p*(-2*a + 2*b*p*x^n))/x^3,x]
```

output
```
((a + b*x^n)^p*(a*(-2 + n)*Hypergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*x^
n)/a)] + 2*b*p*x^n*Hypergeometric2F1[(-2 + n)/n, -p, 2 - 2/n, -((b*x^n)/a)
]))/((-2 + n)*x^2*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p (2bpx^n-2a)}{x^3} dx$$

$$\Big\downarrow \text{959}$$

$$-\frac{2a(2-n)(p+1)\int \frac{(bx^n+a)^p}{x^3}dx}{n(-p)-n+2}-\frac{2p(a+bx^n)^{p+1}}{x^2(n(-p)-n+2)}$$

$$\Big\downarrow \text{889}$$

$$-\frac{2a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{x^3}dx}{n(-p)-n+2}-\frac{2p(a+bx^n)^{p+1}}{x^2(n(-p)-n+2)}$$

$$\Big\downarrow \text{888}$$

$$\frac{a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{2}{n},-p,-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{x^2(n(-p)-n+2)}-\frac{2p(a+bx^n)^{p+1}}{x^2(n(-p)-n+2)}$$

input
```
Int[((a + b*x^n)^p*(-2*a + 2*b*p*x^n))/x^3,x]
```

output
```
(-2*p*(a + b*x^n)^(1 + p))/((2 - n - n*p)*x^2) + (a*(2 - n)*(1 + p)*(a + b
*x^n)^p*Hypergeometric2F1[-2/n, -p, -((2 - n)/n), -((b*x^n)/a)])/((2 - n -
 n*p)*x^2*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^{n})^{p}\,(-2a+2bp\,x^{n})}{x^{3}}dx$$

input
```
int((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x)
```

output
```
int((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = \int \frac{2\,(bpx^n-a)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x, algorithm="fricas")
```

output
```
integral(2*(b*p*x^n - a)*(b*x^n + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = -\frac{2aa^{-\frac{2}{n}}a^{p+\frac{2}{n}}\Gamma\left(-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-\frac{2}{n},-p\\1-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^2\Gamma\left(1-\frac{2}{n}\right)} + \frac{2a^{1-\frac{2}{n}}a^{p-1+\frac{2}{n}}bpx^{n-2}\Gamma\left(1-\frac{2}{n}\right){}_2F_1\left(\begin{matrix}-p,1-\frac{2}{n}\\2-\frac{2}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{2}{n}\right)}$$

input
```
integrate((a+b*x**n)**p*(-2*a+2*b*p*x**n)/x**3,x)
```

output
```
-2*a*a**(p + 2/n)*gamma(-2/n)*hyper((-2/n, -p), (1 - 2/n,), b*x**n*exp_pol
ar(I*pi)/a)/(a**(2/n)*n*x**2*gamma(1 - 2/n)) + 2*a**(1 - 2/n)*a**(p - 1 +
2/n)*b*p*x**(n - 2)*gamma(1 - 2/n)*hyper((-p, 1 - 2/n), (2 - 2/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(2 - 2/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = \int \frac{2\,(bpx^n-a)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x, algorithm="maxima")
```

output
```
2*integrate((b*p*x^n - a)*(b*x^n + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = \int \frac{2\,(bpx^n-a)(bx^n+a)^p}{x^3}\,dx$$

input
```
integrate((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate(2*(b*p*x^n - a)*(b*x^n + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p(-2a+2bpx^n)}{x^3}\,dx = \int -\frac{(2\,a-2\,b\,p\,x^n)\,(a+b\,x^n)^p}{x^3}\,dx$$

input
```
int(-((2*a - 2*b*p*x^n)*(a + b*x^n)^p)/x^3,x)
```

output
```
int(-((2*a - 2*b*p*x^n)*(a + b*x^n)^p)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p\,(-2a+2bpx^n)}{x^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(-2*a+2*b*p*x^n)/x^3,x)
```

output
```
(2*(x**n*(x**n*b + a)**p*b*n*p**2 - 2*x**n*(x**n*b + a)**p*b*p + (x**n*b +
 a)**p*a*n*p**2 - (x**n*b + a)**p*a*n*p - (x**n*b + a)**p*a*n + 2*(x**n*b
+ a)**p*a - int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**
3 - 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3
 + a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**4*p**4
*x**2 - 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3
- 4*x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 +
 a*n**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**4*p**3*x
**2 - int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3 - 4*
x**n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 + a*n
**2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**4*p**2*x**2
+ 2*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3 - 4*x*
*n*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 + a*n**
2*p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**3*p**4*x**2 +
8*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3 - 4*x**n
*b*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 + a*n**2*
p*x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**3*p**3*x**2 + 8*
int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**3 + x**n*b*n**2*p*x**3 - 4*x**n*b
*n*p*x**3 - 2*x**n*b*n*x**3 + 4*x**n*b*x**3 + a*n**2*p**2*x**3 + a*n**2*p*
x**3 - 4*a*n*p*x**3 - 2*a*n*x**3 + 4*a*x**3),x)*a**2*n**3*p**2*x**2 + 2...
```

3.432 $\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 107

$$\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx = \frac{(1-2p)(a+bx^n)^{1+p}}{(3-n-np)x^3} + \frac{a(2-n)(1+p)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{n},-p,-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{(3-n-np)x^3}$$

output
```
(1-2*p)*(a+b*x^n)^(p+1)/(-n*p-n+3)/x^3+a*(2-n)*(p+1)*(a+b*x^n)^p*hypergeom
([-p, -3/n],[-(3-n)/n],-b*x^n/a)/(-n*p-n+3)/x^3/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx = \frac{(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(-3+n)\,\text{Hypergeometric2F1}\left(-\frac{3}{n},-p,\frac{-3+n}{n},-\frac{bx^n}{a}\right)+b(-1+2p)x^n\,\text{Hyperge}\right.}{(-3+n)x^3}$$

input
```
Integrate[((a + b*x^n)^p*(-3*a + b*(-1 + 2*p)*x^n))/x^4,x]
```

output
```
((a + b*x^n)^p*(a*(-3 + n)*Hypergeometric2F1[-3/n, -p, (-3 + n)/n, -((b*x^
n)/a)] + b*(-1 + 2*p)*x^n*Hypergeometric2F1[(-3 + n)/n, -p, 2 - 3/n, -((b*
x^n)/a)]))/((-3 + n)*x^3*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p (b(2p-1)x^n-3a)}{x^4}dx$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(1-2p)(a+bx^n)^{p+1}}{x^3(n(-p)-n+3)}-\frac{3a(2-n)(p+1)\int \frac{(bx^n+a)^p}{x^4}dx}{n(-p)-n+3}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(1-2p)(a+bx^n)^{p+1}}{x^3(n(-p)-n+3)}-\frac{3a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^n}{a}+1\right)^p}{x^4}dx}{n(-p)-n+3}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{a(2-n)(p+1)(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{n},-p,-\frac{3-n}{n},-\frac{bx^n}{a}\right)}{x^3(n(-p)-n+3)}+\frac{(1-2p)(a+bx^n)^{p+1}}{x^3(n(-p)-n+3)}$$

input
```
Int[((a + b*x^n)^p*(-3*a + b*(-1 + 2*p)*x^n))/x^4,x]
```

output
```
((1 - 2*p)*(a + b*x^n)^(1 + p))/((3 - n - n*p)*x^3) + (a*(2 - n)*(1 + p)*(
a + b*x^n)^p*Hypergeometric2F1[-3/n, -p, -((3 - n)/n), -((b*x^n)/a)])/((3
- n - n*p)*x^3*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p\,(-3a+b(-1+2p)\,x^n)}{x^4}dx$$

input
```
int((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x)
```

output
```
int((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx = \int \frac{(b(2p-1)x^n-3a)(bx^n+a)^p}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x, algorithm="fricas")
```

output
```
integral(((2*b*p - b)*x^n - 3*a)*(b*x^n + a)^p/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.02 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx &= -\frac{3aa^{-\frac{3}{n}}a^{p+\frac{3}{n}}\Gamma\left(-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-\frac{3}{n},-p\\1-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{nx^3\Gamma\left(1-\frac{3}{n}\right)} \\ &\quad+\frac{2a^{1-\frac{3}{n}}a^{p-1+\frac{3}{n}}bpx^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)} \\ &\quad-\frac{a^{1-\frac{3}{n}}a^{p-1+\frac{3}{n}}bx^{n-3}\Gamma\left(1-\frac{3}{n}\right){}_2F_1\left(\begin{matrix}-p,1-\frac{3}{n}\\2-\frac{3}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-\frac{3}{n}\right)}\end{aligned}$$

input
```
integrate((a+b*x**n)**p*(-3*a+b*(-1+2*p)*x**n)/x**4,x)
```

output
```
-3*a*a**(p + 3/n)*gamma(-3/n)*hyper((-3/n, -p), (1 - 3/n,), b*x**n*exp_pol
ar(I*pi)/a)/(a**(3/n)*n*x**3*gamma(1 - 3/n)) + 2*a**(1 - 3/n)*a**(p - 1 +
3/n)*b*p*x**(n - 3)*gamma(1 - 3/n)*hyper((-p, 1 - 3/n), (2 - 3/n,), b*x**n
*exp_polar(I*pi)/a)/(n*gamma(2 - 3/n)) - a**(1 - 3/n)*a**(p - 1 + 3/n)*b*x
**(n - 3)*gamma(1 - 3/n)*hyper((-p, 1 - 3/n), (2 - 3/n,), b*x**n*exp_polar
(I*pi)/a)/(n*gamma(2 - 3/n))
```

Maxima [F]

$$\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx=\int \frac{(b(2p-1)x^n-3a)(bx^n+a)^p}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*(2*p - 1)*x^n - 3*a)*(b*x^n + a)^p/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p(-3a+b(-1+2p)x^n)}{x^4}\,dx=\int \frac{(b(2p-1)x^n-3a)(bx^n+a)^p}{x^4}\,dx$$

input
```
integrate((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate((b*(2*p - 1)*x^n - 3*a)*(b*x^n + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p\,(-3a+b(-1+2p)x^n)}{x^4}\,dx = \int -\frac{(3\,a-b\,x^n\,(2\,p-1))\,\,(a+b\,x^n)^p}{x^4}\,dx$$

input
```
int(-((3*a - b*x^n*(2*p - 1))*(a + b*x^n)^p)/x^4,x)
```

output
```
int(-((3*a - b*x^n*(2*p - 1))*(a + b*x^n)^p)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p\,(-3a+b(-1+2p)x^n)}{x^4}\,dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(-3*a+b*(-1+2*p)*x^n)/x^4,x)
```

output

```
(2*x**n*(x**n*b + a)**p*b*n*p**2 - x**n*(x**n*b + a)**p*b*n*p - 6*x**n*(x*
*n*b + a)**p*b*p + 3*x**n*(x**n*b + a)**p*b + 2*(x**n*b + a)**p*a*n*p**2 -
 4*(x**n*b + a)**p*a*n*p - 3*(x**n*b + a)**p*a*n + 9*(x**n*b + a)**p*a - 3
*int((x**n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*
b*n*p*x**4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p
*x**4 - 6*a*n*p*x**4 - 3*a*n*x**4 + 9*a*x**4),x)*a**2*n**4*p**4*x**3 - 6*i
nt((x**n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*b*
n*p*x**4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p*x
**4 - 6*a*n*p*x**4 - 3*a*n*x**4 + 9*a*x**4),x)*a**2*n**4*p**3*x**3 - 3*int
((x**n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*b*n*
p*x**4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p*x**
4 - 6*a*n*p*x**4 - 3*a*n*x**4 + 9*a*x**4),x)*a**2*n**4*p**2*x**3 + 6*int((
x**n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*b*n*p*
x**4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p*x**4
- 6*a*n*p*x**4 - 3*a*n*x**4 + 9*a*x**4),x)*a**2*n**3*p**4*x**3 + 30*int((x
**n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*b*n*p*x
**4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p*x**4 -
 6*a*n*p*x**4 - 3*a*n*x**4 + 9*a*x**4),x)*a**2*n**3*p**3*x**3 + 33*int((x*
*n*b + a)**p/(x**n*b*n**2*p**2*x**4 + x**n*b*n**2*p*x**4 - 6*x**n*b*n*p*x*
*4 - 3*x**n*b*n*x**4 + 9*x**n*b*x**4 + a*n**2*p**2*x**4 + a*n**2*p*x**4...
```

3.433 $\int (ex)^m (a+bx^n)^p (c+dx^n)\, dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int (ex)^m (a+bx^n)^p (c+dx^n)\, dx = \frac{d(ex)^{1+m}(a+bx^n)^{1+p}}{be(1+m+n+np)} + \frac{\left(\frac{c}{1+m}-\frac{ad}{b(1+m+n+np)}\right)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{e}$$

output
```
d*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b/e/(n*p+m+n+1)+(c/(1+m)-a*d/b/(n*p+m+n+1))*
(e*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/e/((
1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int (ex)^m (a+bx^n)^p (c+dx^n)\, dx = \frac{x(ex)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m+n)\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+d(1+m)x\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(c*(1 + m + n)*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)] + d*(1 + m)*x^n*Hypergeometric2F1[(1 + m + n)/
n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*(1 + (b*x^n)/
a)^p)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (c+dx^n) (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int (ex)^m (bx^n+a)^p dx+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int (ex)^m \left(\frac{bx^n}{a}+1\right)^p dx+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right) \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{e(m+1)}+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^n)^(1 + p))/(b*e*(1 + m + n + n*p)) + ((c - (a*d
*(1 + m))/(b*(1 + m + n + n*p)))*(e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometri
c2F1[(1 + m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/(e*(1 + m)*(1 + (b*x^n)/
a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (c+d\,x^n)\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(bx^n+a)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 34.22 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.31

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+p-\frac{1}{n}}ce^m x^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{a^{\frac{m}{n}+1+\frac{1}{n}}a^{-\frac{m}{n}+p-1-\frac{1}{n}}de^m x^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+1+\frac{1}{n} \\ \frac{m}{n}+2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c*e**m*x**(m + 1)*gamma(m/n + 1/n)*hype
r((-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/
n + 1 + 1/n)) + a**(m/n + 1 + 1/n)*a**(-m/n + p - 1 - 1/n)*d*e**m*x**(m +
n + 1)*gamma(m/n + 1 + 1/n)*hyper((-p, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int (ex)^m (a + bx^n)^p (c + dx^n) \, dx = \int (dx^n + c)(bx^n + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*(e*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^m (a + bx^n)^p (c + dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,0,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,0,1,1,1,0,1]%%%
}+%%%{1,[
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a + bx^n)^p (c + dx^n) \, dx = \int (e\,x)^m \left(a + b\,x^n\right)^p \left(c + d\,x^n\right) \,dx$$

input
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(e**m*(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d
*n*p*x + x**(m + n)*(x**n*b + a)**p*b*d*x + x**m*(x**n*b + a)**p*a*d*n*p*x
 + x**m*(x**n*b + a)**p*b*c*m*x + x**m*(x**n*b + a)**p*b*c*n*p*x + x**m*(x
**n*b + a)**p*b*c*n*x + x**m*(x**n*b + a)**p*b*c*x - int((x**m*(x**n*b + a
)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**
2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m
*n*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2
*d*m**3*n*p - 2*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p +
 x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p
 + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 +
a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n**2*p**2 - int((x**m*(x**n*b
 + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b
*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2
*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*
a**2*d*m**2*n**2*p - 3*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*
m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**
n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*
p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n*p - int((x**m*(x**n*
b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*
b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2...
```

3.434 $\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 27, antiderivative size = 93

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx = -\frac{cx^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)} - \frac{dx^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

output
```
-c*(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))-d*(a+b*x^n)^p*hypergeom([-p, -p
],[1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx = \frac{(a+bx^n)^p\left(-\frac{cx^{-n(1+p)}(a+bx^n)}{a(1+p)}-\frac{dx^{-np}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{p}\right)}{n}$$

input
```
Integrate[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(-(((c*(a + b*x^n))/(a*(1 + p)*x^(n*(1 + p)))) - (d*Hypergeo
metric2F1[-p, -p, 1 - p, -((b*x^n)/a)])/(p*x^(n*p)*(1 + (b*x^n)/a)^p)))/n
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int x^{-n(p+1)-1}(bx^n+a)^{p+1}dx}{b}-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}$$

$$\Big\downarrow \text{882}$$

$$\frac{dx^{-n(p+1)}\left(\frac{x^n}{a+bx^n}\right)^{p+1}(a+bx^n)^{p+1}\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-2}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{bn}-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}-\frac{dx^{-n(p+1)}(a+bx^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^n}{bx^n+a}\right)}{bn(p+1)}$$

input
```
Int[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((b*c - a*d)*(a + b*x^n)^(1 + p))/(a*b*n*(1 + p)*x^(n*(1 + p)))) - (d*(a
 + b*x^n)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*x^n)/(a + b*x^n)])/(
b*n*(1 + p)*x^(n*(1 + p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-1-n(p+1)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p - n - 1)*x^n + c*x^(-n*p - n - 1))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 64.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\frac{a^pa^{-p-1}b^{p+1}c\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-1)}{n\Gamma\left(-p\right)}+\frac{a^pdx^{-np}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1-p\right)}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**p*a**(-p - 1)*b**(p + 1)*c*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 1)/(n*g
amma(-p)) + a**p*d*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_polar(I*
pi)/a)/(n*x**(n*p)*gamma(1 - p))
```

Maxima [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int (dx^n+c)(bx^n+a)^px^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,0,2,0,2,2,1,0]%%%}+%%%{2,[0,0,0,2,0,2,1,1,0]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p+1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{-x^n(x^nb+a)^p\,bc-(x^nb+a)^p\,ac+x^{np+n}\left(\int\frac{(x^nb+a)^p}{x^{np}x}dx\right)adnp+x^{np+n}\left(\int\frac{(x^nb+a)^p}{x^{np}x}dx\right)adn}{x^{np+n}an\,(p+1)}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
( - x**n*(x**n*b + a)**p*b*c - (x**n*b + a)**p*a*c + x**(n*p + n)*int((x**
n*b + a)**p/(x**(n*p)*x),x)*a*d*n*p + x**(n*p + n)*int((x**n*b + a)**p/(x*
*(n*p)*x),x)*a*d*n)/(x**(n*p + n)*a*n*(p + 1))
```

3.435 $\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))$

Optimal result

Integrand size = 34, antiderivative size = 22

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p)))x^n)\,dx=\frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^(p+1)/e
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.00

$$\begin{aligned}&\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p)))x^n)\,dx\\&=x(ex)^m(a+bx^n)^p\Bigg(1\\&\quad+\frac{bx^n}{a}\Bigg)^{-p}\left(a\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)\right.\\&\quad\left.+\frac{b(1+m+n+np)x^n\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n}\right)\end{aligned}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n*(1 + p))*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n
, -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n
, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^p (a(m+1)+bx^n(m+n(p+1)+1)) \, dx$$

$$\Big\downarrow \text{951}$$

$$\frac{(ex)^{m+1} (a+bx^n)^{p+1}}{e}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n*(1 + p))*x^n),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(22) = 44$.

Time = 8.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

method	result	size
parallelrisch	$\frac{x\,x^{n}(ex)^{m}(a+b\,x^{n})^{p}b^{2}+x(ex)^{m}(a+b\,x^{n})^{p}ab}{b}$	46

input
```
int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(1+m+n*(p+1))*x^n),x,method=_RETURNVERB
OSE)
```

output
```
(x*x^n*(e*x)^m*(a+b*x^n)^p*b^2+x*(e*x)^m*(a+b*x^n)^p*a*b)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n)\,dx = \left(bxx^n e^{(m\log(e)+m\log(x))}+axe^{(m\log(e)+m\log(x))}\right)(bx^n+a)^p$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(1+m+n*(p+1))*x^n),x, algorithm="
fricas")
```

output
```
(b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a
)^p
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 2.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n)\,dx = ax(ex)^m (a+bx^n)^p + bxx^n(ex)^m (a+bx^n)^p$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(1+m+n*(p+1))*x**n),x)
```

output
```
a*x*(e*x)**m*(a + b*x**n)**p + b*x*x**n*(e*x)**m*(a + b*x**n)**p
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n)\,dx = \left(ae^m xx^m + be^m xe^{(m\log(x)+n\log(x))}\right)(bx^n+a)^p$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(1+m+n*(p+1))*x^n),x, algorithm="
maxima")
```

output
```
(a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n)\,dx = (bx^n+a)^p bxx^n e^{(m\log(e)+m\log(x))} + (bx^n+a)^p axe^{(m\log(e)+m\log(x))}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(1+m+n*(p+1))*x^n),x, algorithm="
giac")
```

output
```
(b*x^n + a)^p*b*x*x^n*e^(m*log(e) + m*log(x)) + (b*x^n + a)^p*a*x*e^(m*log
(e) + m*log(x))
```

Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n) \, dx = \left(a\,x\,(e\,x)^m+b\,x^{n+1}\,(e\,x)^m\right)\,(a+b\,x^n)^p$$

input
```
int((e*x)^m*(a*(m + 1) + b*x^n*(m + n*(p + 1) + 1))*(a + b*x^n)^p,x)
```

output
```
(a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^m (a+bx^n)^p (a(1+m)+b(1+m+n(1+p))x^n) \, dx = x^m e^m (x^n b+a)^p x (x^n b+a)$$

input
```
int((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(1+m+n*(p+1))*x^n),x)
```

output
```
x**m*e**m*(x**n*b + a)**p*x*(x**n*b + a)
```

3.436 $\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx=\frac{bx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

output
```
1/3*b*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/3*d*x^3*hy
pergeom([1, 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx=\frac{bcx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)-adx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

input
```
Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hype
rgeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}dx$$
$$\Big\downarrow \text{ 1010}$$
$$\frac{b\int \frac{x^2}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{x^2}{dx^n+c}dx}{bc-ad}$$
$$\Big\downarrow \text{ 888}$$
$$\frac{bx^3 \,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3 \,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

input
```
Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d
)) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
- a*d))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx$$

input
```
integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x^2/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^2/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^2}{x^{2n}bd+x^n ad+x^n bc+ac}dx$$

input
```
int(x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x**2/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.437 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)} - \frac{dx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

output
```
1/2*b*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/2*d*x^2*hy
pergeom([1, 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \frac{bcx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right) - adx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

input
```
Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hype
rgeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{1010, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b\int \frac{x}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{x}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{bx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

input
```
Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d
)) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
- a*d))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{x}{(a+b\,x^{n})\,(c+d\,x^{n})}dx$$

input
```
int(x/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$$

input
```
integrate(x/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x}{x^{2n}bd+x^n ad+x^n bc+ac}dx$$

input
```
int(x/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(x/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.438 $\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

output
```
b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/
n],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\frac{x\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)\right)}{ac(-bc+ad)}$$

input
```
Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d)
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {917, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}dx$$
$$\Big\downarrow \text{ 917}$$
$$\frac{b\int \frac{1}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^n+c}dx}{bc-ad}$$
$$\Big\downarrow \text{ 778}$$
$$\frac{bx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

input
```
Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d
)) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c
- a*d))
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})\,(c+d\,x^{n})}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

input
```
integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx=\int \frac{1}{x^{2n}bd+x^n ad+x^n bc+ac}dx$$

input
```
int(1/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.439 $\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx=\frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}$$

output
```
ln(x)/a/c-b*ln(a+b*x^n)/a/(-a*d+b*c)/n+d*ln(c+d*x^n)/c/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}\,dx=\frac{bc\log(x^n)-ad\log(x^n)-bc\log(a+bx^n)+ad\log(c+dx^n)}{abc^2n-a^2cdn}$$

input
```
Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Log[x^n] - a*d*Log[x^n] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a
*b*c^2*n - a^2*c*d*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^{-n}}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{ 93}$$

$$\frac{\int \left(\frac{x^{-n}}{ac}+\frac{b^2}{a(ad-bc)(bx^n+a)}+\frac{d^2}{c(bc-ad)(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b\log(a+bx^n)}{a(bc-ad)}+\frac{d\log(c+dx^n)}{c(bc-ad)}+\frac{\log(x^n)}{ac}}{n}$$

input
```
Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(Log[x^n]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)) + (d*Log[c + d*x^n])/
(c*(b*c - a*d)))/n
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{\ln(x) a d n-\ln(x) b c n+b\ln(a+b\,x^{n}) c-d\ln(c+d\,x^{n}) a}{(a d-c b) a c n}$	58
derivativedivides	$\frac{\frac{\ln(x^{n})}{a c}-\frac{d\ln(c+d\,x^{n})}{c(a d-c b)}+\frac{b\ln(a+b\,x^{n})}{a(a d-c b)}}{n}$	64
default	$\frac{\frac{\ln(x^{n})}{a c}-\frac{d\ln(c+d\,x^{n})}{c(a d-c b)}+\frac{b\ln(a+b\,x^{n})}{a(a d-c b)}}{n}$	64
norman	$\frac{\ln(x)}{a c}+\frac{b\ln(a+b\,{\mathrm e}^{n\ln(x)})}{(a d-c b) a n}-\frac{d\ln(c+d\,{\mathrm e}^{n\ln(x)})}{c n(a d-c b)}$	68
risch	$-\frac{\ln(x) b}{(a d-c b) a}+\frac{\ln(x) d}{c(a d-c b)}+\frac{b\ln(x^{n}+\frac{a}{b})}{(a d-c b) a n}-\frac{d\ln(x^{n}+\frac{c}{d})}{c n(a d-c b)}$	94

input
```
int(1/x/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*a*d*n-ln(x)*b*c*n+b*ln(a+b*x^n)*c-d*ln(c+d*x^n)*a)/(a*d-b*c)/a/c/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx=-\frac{bc\log\left(bx^n+a\right)-ad\log\left(dx^n+c\right)-(bc-ad)n\log\left(x\right)}{(abc^2-a^2cd)n}$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^
2 - a^2*c*d)*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(46) = 92$.

Time = 1.78 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.52

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx$$
$$= \begin{cases} \frac{\frac{\log(x)}{c} - \frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for}\ b=0 \\ \frac{\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for}\ d=0 \\ \frac{-\frac{x^{-n}}{cn} + \frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2n}}{b} & \text{for}\ a=0 \\ \frac{cdn\log(x)}{bc^3n+bc^2dnx^n} - \frac{cd\log\left(\frac{c}{d}+x^n\right)}{bc^3n+bc^2dnx^n} + \frac{cd}{bc^3n+bc^2dnx^n} + \frac{d^2nx^n\log(x)}{bc^3n+bc^2dnx^n} - \frac{d^2x^n\log\left(\frac{c}{d}+x^n\right)}{bc^3n+bc^2dnx^n} & \text{for}\ a=\frac{bc}{d} \\ \frac{-\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2n}}{d} & \text{for}\ c=0 \\ \frac{\log(x)}{(a+b)(c+d)} & \text{for}\ n=0 \\ \frac{adn\log(x)}{a^2cdn-abc^2n} - \frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n} - \frac{bcn\log(x)}{a^2cdn-abc^2n} + \frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), ((log(x)/a - l
og(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-1/(c*n*x**n) + d*log(x**(-n) + d/c)
/(c**2*n))/b, Eq(a, 0)), (c*d*n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - c*d*
log(c/d + x**n)/(b*c**3*n + b*c**2*d*n*x**n) + c*d/(b*c**3*n + b*c**2*d*n*
x**n) + d**2*n*x**n*log(x)/(b*c**3*n + b*c**2*d*n*x**n) - d**2*x**n*log(c/
d + x**n)/(b*c**3*n + b*c**2*d*n*x**n), Eq(a, b*c/d)), ((-1/(a*n*x**n) + b
*log(x**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(
n, 0)), (a*d*n*log(x)/(a**2*c*d*n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**
2*c*d*n - a*b*c**2*n) - b*c*n*log(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a
/b + x**n)/(a**2*c*d*n - a*b*c**2*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = -\frac{b\log\left(\frac{bx^n+a}{b}\right)}{abcn-a^2dn}+\frac{d\log\left(\frac{dx^n+c}{d}\right)}{bc^2n-acdn}+\frac{\log\left(x\right)}{ac}$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n
- a*c*d*n) + log(x)/(a*c)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{d\,x\,\left(a^2\,d\,n-a\,b\,c\,n\right)}\right)}{a^2\,d\,n-a\,b\,c\,n} + \frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^n+b\,c\,n\,x^n}{b\,x\,\left(b\,c^2\,n-a\,c\,d\,n\right)}\right)}{b\,c^2\,n-a\,c\,d\,n}+\frac{\ln\left(x\right)\,\left(n-1\right)}{a\,c\,n}$$

input
```
int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
(b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b
*c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n +
 b*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n -
 1))/(a*c*n)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \frac{\log(x^n b+a)\,bc-\log(x^n d+c)\,ad+\log(x)\,adn-\log(x)\,bcn}{acn\,(ad-bc)}$$

input
```
int(1/x/(a+b*x^n)/(c+d*x^n),x)
```

output
```
(log(x**n*b + a)*b*c - log(x**n*d + c)*a*d + log(x)*a*d*n - log(x)*b*c*n)/
(a*c*n*(a*d - b*c))
```

3.440 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)x} + \frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)x}$$

output
```
-b*hypergeom([1, -1/n],[-(1-n)/n],-b*x^n/a)/a/(-a*d+b*c)/x+d*hypergeom([1,
 -1/n],[-(1-n)/n],-d*x^n/c)/c/(-a*d+b*c)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx = \frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)-ad\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{dx^n}{c}\right)}{ac(-bc+ad)x}$$

input
```
Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hyperge
ometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1010, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} \, dx$$

$$\Big\downarrow \text{ 1010}$$

$$\frac{b \int \frac{1}{x^2 (bx^n+a)}dx}{bc-ad} - \frac{d \int \frac{1}{x^2 (dx^n+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{d \,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)} - \frac{b \,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
-((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c -
a*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(
c*(b*c - a*d)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{1}{x^2 (a+b x^n) (c+d x^n)}dx$$

input
```
int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^n) (c+dx^n)} dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^2} dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx$$

input
```
integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{(bx^n + a)(dx^n + c)x^2} dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{(bx^n + a)(dx^n + c)x^2} dx$$

input
```
integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^2 \left(a + b x^n\right) \left(c + d x^n\right)} dx$$

input
```
int(1/(x^2*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/(x^2*(a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^{2n} bd\, x^2 + x^n ad\, x^2 + x^n bc\, x^2 + ac\, x^2} dx$$

input
```
int(1/x^2/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(x**(2*n)*b*d*x**2 + x**n*a*d*x**2 + x**n*b*c*x**2 + a*c*x**2),x)
```

3.441 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2} + \frac{d\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}$$

output
```
-1/2*b*hypergeom([1, -2/n],[-(2-n)/n],-b*x^n/a)/a/(-a*d+b*c)/x^2+1/2*d*hyp
ergeom([1, -2/n],[-(2-n)/n],-d*x^n/c)/c/(-a*d+b*c)/x^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx = \frac{bc\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{bx^n}{a}\right)-ad\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},\frac{-2+n}{n},-\frac{dx^n}{c}\right)}{2ac(-bc+ad)x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeome
tric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1010, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^n) (c+dx^n)} \, dx$$

$$\Big\downarrow \text{1010}$$

$$\frac{b \int \frac{1}{x^3 (bx^n+a)}dx}{bc-ad} - \frac{d \int \frac{1}{x^3 (dx^n+c)}dx}{bc-ad}$$

$$\Big\downarrow \text{888}$$

$$\frac{d \,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)} - \frac{b \,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

output
```
-1/2*(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c -
a*d)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*
c*(b*c - a*d)*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{1}{x^3 (a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n)(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\left(a+bx^n\right)\left(c+dx^n\right)}\,dx = \int \frac{1}{(bx^n+a)(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^3 (a + b x^n) (c + d x^n)} dx$$

input
```
int(1/(x^3*(a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(1/(x^3*(a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a + bx^n) (c + dx^n)} dx = \int \frac{1}{x^{2n} bd\, x^3 + x^n ad\, x^3 + x^n bc\, x^3 + ac\, x^3} dx$$

input
```
int(1/x^3/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int(1/(x**(2*n)*b*d*x**3 + x**n*a*d*x**3 + x**n*b*c*x**3 + a*c*x**3),x)
```

3.442 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(3-2n)-bc(3-n))x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} + \frac{d^2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}$$

output
```
b*x^3/a/(-a*d+b*c)/n/(a+b*x^n)+1/3*b*(a*d*(3-2*n)-b*c*(3-n))*x^3*hypergeom
([1, 3/n],[(3+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/3*d^2*x^3*hypergeom([1,
 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x^3\left(bc(ad(3-2n)+bc(-3+n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{3+n}{n},-\frac{bx^n}{a}\right)+a\left(3bc(bc-ad)+ad\right.\right.}{3a^2c(bc-ad)^2n(a+bx^n)}$$

input

```
Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

output

```
(x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1,
3/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)
*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d
)^2*n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{bx^3}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{x^2(bd(3-n)x^n+bc(3-n)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{bx^3}{an(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{b(bc(3-n)-ad(3-2n))x^2}{(bc-ad)(bx^n+a)}+\frac{ad^2nx^2}{(ad-bc)(dx^n+c)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^3}{an(bc-ad)\,(a+bx^n)}-\frac{-\frac{ad^2nx^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{dx^n}{c}\right)}{3c(bc-ad)}-\frac{bx^3(ad(3-2n)-bc(3-n))\,\text{Hypergeometric2F1}\left(1,\frac{3}{n},\frac{n+3}{n},-\frac{bx^n}{a}\right)}{3a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-1/3*(b*(a*d*(3 - 2*n) - b*c*(3 -
 n))*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d
)) - (a*d^2*n*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c
*(b*c - a*d)))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\,x^n\right)^2 \left(c+d\,x^n\right)}dx$$

input
```
int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c
 + a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^n)^2\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)} dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(
x^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^
2*n)*x^n), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{x^2}{(bx^n+a)^2(dx^n+c)} dx$$

input
```
integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{x^2}{(a+b\,x^n)^2\,(c+d\,x^n)} dx$$

input
```
int(x^2/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x^2/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x^2}{x^{3n}b^2d + 2x^{2n}abd + x^{2n}b^2c + x^n a^2 d + 2x^n abc + a^2 c} dx$$

input
```
int(x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x**2/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n*a**2
*d + 2*x**n*a*b*c + a**2*c),x)
```

3.443 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 143

$$\begin{aligned}\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{bx^2}{a(bc-ad)n(a+bx^n)} \\ &+ \frac{b(2ad(1-n)-bc(2-n))x^2 \operatorname{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} \\ &+ \frac{d^2x^2 \operatorname{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}\end{aligned}$$

output
```
b*x^2/a/(-a*d+b*c)/n/(a+b*x^n)+1/2*b*(2*a*d*(1-n)-b*c*(2-n))*x^2*hypergeom
([1, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/2*d^2*x^2*hypergeom([1,
 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x^2\left(bc(bc(-2+n)-2ad(-1+n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{2+n}{n},-\frac{bx^n}{a}\right)+a\big(2bc(bc-ad)+a\right.}{2a^2c(bc-ad)^2n(a+bx^n)}$$

input
```
Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
 2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*
d)^2*n*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$$

$$\Big\downarrow \ 1006$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)}-\frac{\int \frac{x(bd(2-n)x^n+bc(2-n)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \ 1067$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{anxd^2}{(ad-bc)(dx^n+c)}+\frac{b(bc(2-n)-2ad(1-n))x}{(bc-ad)(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$\frac{bx^2}{an(bc-ad)(a+bx^n)} - \frac{-\frac{ad^2nx^2 \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)} - \frac{bx^2(2ad(1-n)-bc(2-n)) \text{Hypergeometric2F1}\left(1,\frac{2}{n},\frac{n+2}{n},-\frac{bx^n}{a}\right)}{2a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-1/2*(b*(2*a*d*(1 - n) - b*c*(2 -
 n))*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d
)) - (a*d^2*n*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c
*(b*c - a*d)))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{x}{\left(a+b\,x^{n}\right)^{2}\left(c+d\,x^{n}\right)}dx$$

input
```
int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^n)^2\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)} \, dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*
d*n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n -
 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(bx^n+a)^2(dx^n+c)} \, dx$$

input
```
integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{x}{(a+b\,x^n)^2\,(c+d\,x^n)} \, dx$$

input
```
int(x/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x}{x^{3n}b^2d+2x^{2n}abd+x^{2n}b^2c+x^na^2d+2x^nabc+a^2c}dx$$

input
```
int(x/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(x/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n*a**2*d
+ 2*x**n*a*b*c + a**2*c),x)
```

3.444 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2}$$

output
```
b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n
],[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d
*x^n/c)/c/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} \, dx = \frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(1-2n)+bc(-1+n)) \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a^2n} + \frac{d^2 \text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n)
)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {931, 1020, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} \, dx$$

$$\Big\downarrow \text{931}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)} - \frac{\int \frac{bd(1-n)x^n+adn+b(c-cn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{bx}{an(bc-ad)(a+bx^n)} - \frac{-\frac{ad^2n\int \frac{1}{dx^n+c}dx}{bc-ad} - \frac{b(ad(1-2n)-bc(1-n))\int \frac{1}{bx^n+a}dx}{bc-ad}}{an(bc-ad)}$$

$$\Big\downarrow \text{778}$$

$$\frac{bx}{an(bc-ad)\left(a+bx^n\right)}-\frac{-\frac{ad^2nx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)}-\frac{bx(ad(1-2n)-bc(1-n))\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) - (-((b*(a*d*(1 - 2*n) - b*c*(1 - n))*
x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)))
 - (a*d^2*n*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(
b*c - a*d)))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

Maple [F]

$$\int \frac{1}{(a+b\,x^{n})^{2}\,(c+d\,x^{n})}dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
 a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a
^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n
+ a^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n)
```

Giac [F]

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{1}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{(a+bx^n)^2 (c+dx^n)} dx = \int \frac{1}{x^{3n}b^2d + 2x^{2n}abd + x^{2n}b^2c + x^n a^2 d + 2x^n abc + a^2c} dx$$

input
```
int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n*a**2*d
+ 2*x**n*a*b*c + a**2*c),x)
```

3.445 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{b}{a(bc-ad)n\left(a+bx^n\right)}+\frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log\left(a+bx^n\right)}{a^2(bc-ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

output
```
b/a/(-a*d+b*c)/n/(a+b*x^n)+ln(x)/a^2/c-b*(-2*a*d+b*c)*ln(a+b*x^n)/a^2/(-a*
d+b*c)^2/n-d^2*ln(c+d*x^n)/c/(-a*d+b*c)^2/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = -\frac{b}{a(-bc+ad)n\left(a+bx^n\right)}+\frac{\log\left(x^n\right)}{a^2cn} + \frac{b(-bc+2ad)\log\left(a+bx^n\right)}{a^2(-bc+ad)^2n}-\frac{d^2\log\left(c+dx^n\right)}{c(bc-ad)^2n}$$

input
```
Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
-(b/(a*(-(b*c) + a*d)*n*(a + b*x^n))) + Log[x^n]/(a^2*c*n) + (b*(-(b*c) +
2*a*d)*Log[a + b*x^n])/(a^2*(-(b*c) + a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*
(b*c - a*d)^2*n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}}{\left(bx^n+a\right)^2\left(dx^n+c\right)}dx^n}{n}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{x^{-n}}{a^2c}+\frac{b^2(2ad-bc)}{a^2(ad-bc)^2(bx^n+a)}-\frac{d^3}{c(bc-ad)^2(dx^n+c)}+\frac{b^2}{a(ad-bc)(bx^n+a)^2}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2}+\frac{\log(x^n)}{a^2c}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2}+\frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

input
```
Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^n)) + Log[x^n]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a
 + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\ln(x^n)}{a^2 c}-\frac{d^2 \ln(c+d x^n)}{(ad-cb)^2 c}-\frac{b}{a(ad-cb)(a+b x^n)}+\frac{b(2ad-cb)\ln(a+b x^n)}{a^2(ad-cb)^2}}{n}$
default	$\frac{\frac{\ln(x^n)}{a^2 c}-\frac{d^2 \ln(c+d x^n)}{(ad-cb)^2 c}-\frac{b}{a(ad-cb)(a+b x^n)}+\frac{b(2ad-cb)\ln(a+b x^n)}{a^2(ad-cb)^2}}{n}$
norman	$\frac{\frac{b^2 e^{n\ln(x)}}{n a^2(ad-cb)}+\frac{\ln(x)}{ac}+\frac{b\ln(x)e^{n\ln(x)}}{a^2 c}}{a+b e^{n\ln(x)}}+\frac{b(2ad-cb)\ln\left(a+b e^{n\ln(x)}\right)}{(a^2d^2-2abcd+b^2c^2)a^2 n}-\frac{d^2\ln\left(c+d e^{n\ln(x)}\right)}{cn(a^2d^2-2abcd+b^2c^2)}$
parallelrisch	$\frac{x^n \ln(x)a^2 b d^2 n-2x^n \ln(x)a b^2 cdn+x^n \ln(x)b^3c^2n+\ln(x)a^3d^2n-2\ln(x)a^2bcdn+\ln(x)a b^2c^2n+2\ln(a+b x^n)x^n a b^2 cd}{(a^2d^2-2abcd+b^2}$
risch	$\frac{\ln(x)d^2}{c(a^2d^2-2abcd+b^2c^2)}-\frac{2\ln(x)bd}{(a^2d^2-2abcd+b^2c^2)a}+\frac{\ln(x)b^2c}{(a^2d^2-2abcd+b^2c^2)a^2}-\frac{b}{(ad-cb)an(a+b x^n)}-\frac{d^2\ln(x^n}{cn(a^2d^2-2a}$

input
```
int(1/x/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/a^2/c*ln(x^n)-d^2/(a*d-b*c)^2/c*ln(c+d*x^n)-b/a/(a*d-b*c)/(a+b*x^n)
+b*(2*a*d-b*c)/a^2/(a*d-b*c)^2*ln(a+b*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(101) = 202$.

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.22

$$\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx = \frac{ab^2c^2 - a^2bcd + (b^3c^2 - 2\,ab^2cd + a^2bd^2)nx^n\log(x) + (ab^2c^2 - 2\,a^2bcd + a^3d^2)n\log(x) - (ab^2c^2 - 2\,a^2b}{(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2)nx^n + (a^3b^2c^3 - 2\,a}$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x)
+ (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d
+ (b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*
log(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^
3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = -\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n-2\,abc^2dn+a^2cd^2n} - \frac{\left(b^2c-2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n} + \frac{b}{a^2bcn-a^3dn+(ab^2cn-a^2bdn)x^n} + \frac{\log\left(x\right)}{a^2c}$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
-d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
 - 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)
 + b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x}\,dx$$

input
```
integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx=\int \frac{1}{x\,{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)} \,d x$$

input
```
int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.67

$$\int \frac{1}{x\left(a+bx^n\right)^2\left(c+dx^n\right)}\,dx = \frac{2x^n\log(x^n b+a)\,a\,b^2cd-x^n\log(x^n b+a)\,b^3c^2-x^n\log(x^n d+c)\,a^2b\,d^2+x^n\log(x)\,a^2b\,d^2n-2x^n\log(x)\,a\,b}{a^2cn}$$

input
```
int(1/x/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(2*x**n*log(x**n*b + a)*a*b**2*c*d - x**n*log(x**n*b + a)*b**3*c**2 - x**n
*log(x**n*d + c)*a**2*b*d**2 + x**n*log(x)*a**2*b*d**2*n - 2*x**n*log(x)*a
*b**2*c*d*n + x**n*log(x)*b**3*c**2*n + x**n*a*b**2*c*d - x**n*b**3*c**2 +
 2*log(x**n*b + a)*a**2*b*c*d - log(x**n*b + a)*a*b**2*c**2 - log(x**n*d +
 c)*a**3*d**2 + log(x)*a**3*d**2*n - 2*log(x)*a**2*b*c*d*n + log(x)*a*b**2
*c**2*n)/(a**2*c*n*(x**n*a**2*b*d**2 - 2*x**n*a*b**2*c*d + x**n*b**3*c**2
+ a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2))
```

3.446 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{b(bc(1+n)-ad(1+2n))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2x}$$

output
```
b/a/(-a*d+b*c)/n/x/(a+b*x^n)-b*(b*c*(1+n)-a*d*(1+2*n))*hypergeom([1, -1/n]
,[-(1-n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x-d^2*hypergeom([1, -1/n],[-(1-n)
/n],-d*x^n/c)/c/(-a*d+b*c)^2/x
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{bc(-bc(1+n)+ad(1+2n))(a+bx^n)\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},\frac{-1+n}{n},-\frac{bx^n}{a}\right)-a\big(bc(-bc+ad)+ad}{a^2c(bc-ad)^2nx(a+bx^n)}$$

input
```
Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^
(-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x
^n)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c
- a*d)^2*n*x*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)}\,dx$$

$$\Big\downarrow \ 1006$$

$$\frac{b}{anx(bc-ad)(a+bx^n)}-\frac{\int \frac{-bd(n+1)x^n+adn-bc(n+1)}{x^2(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \ 1067$$

$$\frac{b}{anx(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{and^2}{(ad-bc)x^2(dx^n+c)}+\frac{b(ad(2n+1)-bc(n+1))}{(bc-ad)x^2(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$\frac{b}{anx(bc-ad)\,(a+bx^n)}-\frac{\frac{ad^2n\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{cx(bc-ad)}+\frac{b(bc(n+1)-ad(2n+1))\,\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{bx^n}{a}\right)}{ax(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - ((b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hype
rgeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a*d)*x) +
 (a*d^2*n*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b
*c - a*d)*x))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^2 (a + b\,x^n)^2 (c + d\,x^n)} dx$$

input
```
int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n)
+ (2*a*b*c + a^2*d)*x^2*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*inte
grate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*
c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x
*x^n + (a^2*b*c*n - a^3*d*n)*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)x^2} \, dx$$

input
```
integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{1}{x^2 \, (a + b\, x^n)^2 \, (c + d\, x^n)} \, dx$$

input
```
int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^n)^2 (c + dx^n)} \, dx$$
$$= \int \frac{1}{x^{3n} b^2 d\, x^2 + 2x^{2n} abd\, x^2 + x^{2n} b^2 c\, x^2 + x^n a^2 d\, x^2 + 2x^n abc\, x^2 + a^2 c\, x^2} dx$$

input
```
int(1/x^2/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(x**(3*n)*b**2*d*x**2 + 2*x**(2*n)*a*b*d*x**2 + x**(2*n)*b**2*c*x**2
 + x**n*a**2*d*x**2 + 2*x**n*a*b*c*x**2 + a**2*c*x**2),x)
```

3.447 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \frac{b}{a(bc-ad)nx^2 (a+bx^n)} + \frac{b(2ad(1+n)-bc(2+n))\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2} - \frac{d^2\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2c(bc-ad)^2x^2}$$

output
```
b/a/(-a*d+b*c)/n/x^2/(a+b*x^n)+1/2*b*(2*a*d*(1+n)-b*c*(2+n))*hypergeom([1,
 -2/n],[-(2-n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x^2-1/2*d^2*hypergeom([1, -
2/n],[-(2-n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/x^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} dx$$
$$= \frac{bc(2ad(1+n) - bc(2+n))(a + bx^n) \text{Hypergeometric2F1}\left(1, -\frac{2}{n}, \frac{-2+n}{n}, -\frac{bx^n}{a}\right) - a\left(2bc(-bc+ad) + ad^2\right.}{2a^2c(bc-ad)^2nx^2(a+bx^n)}$$

input
```
Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n,
(-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d
)^2*n*x^2*(a + b*x^n))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} dx$$

$$\Big\downarrow \text{1006}$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)} - \frac{\int \frac{-bd(n+2)x^n + adn - bc(n+2)}{x^3(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)} - \frac{\int \left(\frac{and^2}{(ad-bc)x^3(dx^n+c)} + \frac{b(2ad(n+1)-bc(n+2))}{(bc-ad)x^3(bx^n+a)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b}{anx^2(bc-ad)(a+bx^n)} - \frac{\frac{ad^2n\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)} - \frac{b(2ad(n+1)-bc(n+2))\,\text{Hypergeometric2F1}\left(1,-\frac{2}{n},-\frac{2-n}{n},-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}}{an(bc-ad)}$$

input
```
Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) - (-1/2*(b*(2*a*d*(1 + n) - b*c*(2 + n
))*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(a*(b*c - a*d)*
x^2) + (a*d^2*n*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2
*c*(b*c - a*d)*x^2))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{1}{x^3 \left(a+b\,x^n\right)^2 \left(c+d\,x^n\right)}dx$$

input
```
int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{1}{(bx^n+a)^2(dx^n+c)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n)
+ (2*a*b*c + a^2*d)*x^3*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)x^3} dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*inte
grate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*
c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x
^2*x^n + (a^2*b*c*n - a^3*d*n)*x^2)
```

Giac [F]

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} dx = \int \frac{1}{(bx^n + a)^2 (dx^n + c)x^3} dx$$

input
```
integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^n)^2 (c + dx^n)} dx = \int \frac{1}{x^3 (a + b x^n)^2 (c + d x^n)} dx$$

input
```
int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)}\,dx$$
$$= \int \frac{1}{x^{3n}b^2d\,x^3+2x^{2n}abd\,x^3+x^{2n}b^2c\,x^3+x^na^2d\,x^3+2x^nabc\,x^3+a^2c\,x^3}dx$$

input
```
int(1/x^3/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int(1/(x**(3*n)*b**2*d*x**3 + 2*x**(2*n)*a*b*d*x**3 + x**(2*n)*b**2*c*x**3
 + x**n*a**2*d*x**3 + 2*x**n*a*b*c*x**3 + a**2*c*x**3),x)
```

3.448 $\int \frac{x}{(1-x)(1+x)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{1}{2(1+x)} + \frac{\operatorname{arctanh}(x)}{2}$$

output
```
1/(2+2*x)+1/2*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{1}{4}\left(\frac{2}{1+x} - \log(1-x) + \log(1+x)\right)$$

input
```
Integrate[x/((1 - x)*(1 + x)^2),x]
```

output
```
(2/(1 + x) - Log[1 - x] + Log[1 + x])/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x)(x+1)^2} \, dx$$
$$\Big\downarrow \; 86$$
$$\int \left(-\frac{1}{2 (x^2-1)}-\frac{1}{2(x+1)^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\text{arctanh}(x)}{2}+\frac{1}{2(x+1)}$$

input
```
Int[x/((1 - x)*(1 + x)^2),x]
```

output
```
1/(2*(1 + x)) + ArcTanh[x]/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

method	result	size
default	$\frac{1}{2+2x}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
norman	$\frac{1}{2+2x}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
risch	$\frac{1}{2+2x}+\frac{\ln(x+1)}{4}-\frac{\ln(x-1)}{4}$	21
parallelrisch	$-\frac{\ln(x-1)x-\ln(x+1)x-2+\ln(x-1)-\ln(x+1)}{4(x+1)}$	33

input
```
int(x/(1-x)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x+1)+1/4*ln(x+1)-1/4*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{(x+1)\log{(x+1)}-(x+1)\log{(x-1)}+2}{4\,(x+1)}$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="fricas")
```

output
```
1/4*((x + 1)*log(x + 1) - (x + 1)*log(x - 1) + 2)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = -\frac{\log(x-1)}{4} + \frac{\log(x+1)}{4} + \frac{1}{2x+2}$$

input
```
integrate(x/(1-x)/(1+x)**2,x)
```

output
```
-log(x - 1)/4 + log(x + 1)/4 + 1/(2*x + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{1}{2(x+1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="maxima")
```

output
```
1/2/(x + 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{(1-x)(1+x)^2}\,dx = \frac{1}{2(x+1)} - \frac{1}{4}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

input
```
integrate(x/(1-x)/(1+x)^2,x, algorithm="giac")
```

output
```
1/2/(x + 1) - 1/4*log(abs(-2/(x + 1) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{\operatorname{atanh}(x)}{2} + \frac{1}{2\,(x+1)}$$

input
```
int(-x/((x - 1)*(x + 1)^2),x)
```

output
```
atanh(x)/2 + 1/(2*(x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{x}{(1-x)(1+x)^2} \, dx = \frac{-\log(x-1)\,x - \log(x-1) + \log(x+1)\,x + \log(x+1) - 2x}{4x+4}$$

input
```
int(x/(1-x)/(1+x)^2,x)
```

output
```
( - log(x - 1)*x - log(x - 1) + log(x + 1)*x + log(x + 1) - 2*x)/(4*(x + 1
))
```

3.449 $\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4(1+x^2)}+\frac{\operatorname{arctanh}(x)}{4}$$

output
```
-1/4*x/(x^2+1)+1/4*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{1}{8}\left(-\frac{2x}{1+x^2}-\log(1-x)+\log(1+x)\right)$$

input
```
Integrate[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

output
```
((-2*x)/(1 + x^2) - Log[1 - x] + Log[1 + x])/8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {373, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^2)(x^2+1)^2}dx$$

$$\Big\downarrow \text{ 373}$$

$$\frac{1}{4}\int \frac{1}{1-x^2}dx-\frac{x}{4(x^2+1)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(x)}{4}-\frac{x}{4(x^2+1)}$$

input
```
Int[x^2/((1 - x^2)*(1 + x^2)^2),x]
```

output
```
-1/4*x/(1 + x^2) + ArcTanh[x]/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{x}{4(x^2+1)}+\frac{\ln(x+1)}{8}-\frac{\ln(x-1)}{8}$	24
norman	$-\frac{x}{4(x^2+1)}+\frac{\ln(x+1)}{8}-\frac{\ln(x-1)}{8}$	24
risch	$-\frac{x}{4(x^2+1)}+\frac{\ln(x+1)}{8}-\frac{\ln(x-1)}{8}$	24
parallelrisch	$-\frac{\ln(x-1)x^2-\ln(x+1)x^2+\ln(x-1)-\ln(x+1)+2x}{8(x^2+1)}$	41

input
```
int(x^2/(-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/(x^2+1)+1/8*ln(x+1)-1/8*ln(x-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{(x^2+1)\log(x+1)-(x^2+1)\log(x-1)-2x}{8(x^2+1)}$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/8*((x^2 + 1)*log(x + 1) - (x^2 + 1)*log(x - 1) - 2*x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4x^2+4} - \frac{\log(x-1)}{8} + \frac{\log(x+1)}{8}$$

input
```
integrate(x**2/(-x**2+1)/(x**2+1)**2,x)
```

output
```
-x/(4*x**2 + 4) - log(x - 1)/8 + log(x + 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{x}{4(x^2+1)} + \frac{1}{8}\log(x+1) - \frac{1}{8}\log(x-1)$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/4*x/(x^2 + 1) + 1/8*log(x + 1) - 1/8*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = -\frac{1}{4\left(x+\frac{1}{x}\right)} + \frac{1}{16}\log\left(\left|x+\frac{1}{x}+2\right|\right) - \frac{1}{16}\log\left(\left|x+\frac{1}{x}-2\right|\right)$$

input
```
integrate(x^2/(-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/4/(x + 1/x) + 1/16*log(abs(x + 1/x + 2)) - 1/16*log(abs(x + 1/x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{\operatorname{atanh}(x)}{4} - \frac{x}{4(x^2+1)}$$

input
```
int(-x^2/((x^2 - 1)*(x^2 + 1)^2),x)
```

output
```
atanh(x)/4 - x/(4*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{x^2}{(1-x^2)(1+x^2)^2}\,dx = \frac{-\log(x-1)\,x^2 - \log(x-1) + \log(x+1)\,x^2 + \log(x+1) - 2x}{8x^2+8}$$

input
```
int(x^2/(-x^2+1)/(x^2+1)^2,x)
```

output
```
( - log(x - 1)*x**2 - log(x - 1) + log(x + 1)*x**2 + log(x + 1) - 2*x)/(8*
(x**2 + 1))
```

3.450 $\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx = -\frac{x}{6(1+x^3)} + \frac{\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}} + \frac{\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log(1-x) - \frac{1}{36}\log(1+x) + \frac{1}{72}\log\left(1-x+x^2\right) + \frac{1}{24}\log\left(1+x+x^2\right)$$

output
```
-1/6*x/(x^3+1)+1/36*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1
+2*x)*3^(1/2))*3^(1/2)-1/12*ln(1-x)-1/36*ln(1+x)+1/72*ln(x^2-x+1)+1/24*ln(
x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2} dx = \frac{1}{72}\left(-\frac{12x}{1+x^3} - 2\sqrt{3}\arctan\left(\frac{-1+2x}{\sqrt{3}}\right) + 6\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) - 6\log(1-x) - 2\log(1+x) + \log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

input
```
Integrate[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

output
```
((-12*x)/(1 + x^3) - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 6*Sqrt[3]*ArcT
an[(1 + 2*x)/Sqrt[3]] - 6*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] + 3
*Log[1 + x + x^2])/72
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {971, 1020, 750, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^3)(x^3+1)^2}dx$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{6}\int \frac{2x^3+1}{(1-x^3)(x^3+1)}dx-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{1-x^3}dx-\frac{1}{2}\int \frac{1}{x^3+1}dx\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\int \frac{1}{x+1}dx\right)+\frac{3}{2}\left(\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx+\frac{1}{3}\int \frac{1}{1-x}dx\right)\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(-\frac{1}{3}\int \frac{2-x}{x^2-x+1}dx-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\int \frac{x+2}{x^2+x+1}dx-\frac{1}{3}\log(1-x)\right)\right)-\frac{x}{6(x^3+1)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int -\frac{1-2x}{x^2-x+1}dx-\frac{3}{2}\int \frac{1}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x}{x^2+\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$\big\downarrow$ 25

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^2-x+1}dx-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^2+x+1}dx+\frac{1}{2}\int \frac{2x}{x^2+\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$\big\downarrow$ 1083

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(3\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx-3\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$\big\downarrow$ 217

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{2}\int \frac{1-2x}{x^2-x+1}dx-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{2x+1}{x^2+x+1}dx+\sqrt{3}\,\text{arct}\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

$\big\downarrow$ 1103

$$\frac{1}{6}\left(\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{2}\log\left(x^2-x+1\right)-\sqrt{3}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)\right)-\frac{1}{3}\log(x+1)\right)+\frac{3}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{2}\log\left(x\right.\right.\right.\right.$$
$$\frac{x}{6\left(x^3+1\right)}$$

input
```
Int[x^3/((1 - x^3)*(1 + x^3)^2),x]
```

output
```
-1/6*x/(1 + x^3) + ((-1/3*Log[1 + x] + (-(Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3
]]) + Log[1 - x + x^2]/2)/3)/2 + (3*(-1/3*Log[1 - x] + (Sqrt[3]*ArcTan[(1
+ 2*x)/Sqrt[3]] + Log[1 + x + x^2]/2)/3))/2)/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{x}{6(x^3+1)}+\frac{\ln(4x^2-4x+4)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\ln(4x^2+4x+4)}{24}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x-1)}{12}-\frac{\ln(x+1}{36}$
default	$\frac{-2x-2}{36x^2-36x+36}+\frac{\ln(x^2-x+1)}{72}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{36}+\frac{\ln(x^2+x+1)}{24}+\frac{\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{1}{18x+18}-\frac{\ln(x+1)}{36}$

input
```
int(x^3/(-x^3+1)/(x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*x/(x^3+1)+1/72*ln(4*x^2-4*x+4)-1/36*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))+1/24*ln(4*x^2+4*x+4)+1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/12*ln(x
-1)-1/36*ln(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx$$
$$= \frac{6\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3(x^3+1)\log(x^2+x+1}{72(x^3+1)}$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="fricas")
```

output
```
1/72*(6*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*(x^3 +
 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*(x^3 + 1)*log(x^2 + x + 1) + (x^3 +
1)*log(x^2 - x + 1) - 2*(x^3 + 1)*log(x + 1) - 6*(x^3 + 1)*log(x - 1) - 12
*x)/(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{x}{6x^3+6} - \frac{\log(x-1)}{12} - \frac{\log(x+1)}{36} + \frac{\log(x^2-x+1)}{72} + \frac{\log(x^2+x+1)}{24} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{36} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

input
```
integrate(x**3/(-x**3+1)/(x**3+1)**2,x)
```

output
```
-x/(6*x**3 + 6) - log(x - 1)/12 - log(x + 1)/36 + log(x**2 - x + 1)/72 + l
og(x**2 + x + 1)/24 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/36 + sqrt(3)
*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log(x+1) - \frac{1}{12}\log(x-1)$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x
 + 1) - 1/36*log(x + 1) - 1/12*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{36}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x}{6(x^3+1)} + \frac{1}{24}\log\left(x^2+x+1\right) + \frac{1}{72}\log\left(x^2-x+1\right) - \frac{1}{36}\log\left(|x+1|\right) - \frac{1}{12}\log\left(|x-1|\right)$$

input
```
integrate(x^3/(-x^3+1)/(x^3+1)^2,x, algorithm="giac")
```

output
```
1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/36*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/6*x/(x^3 + 1) + 1/24*log(x^2 + x + 1) + 1/72*log(x^2 - x
 + 1) - 1/36*log(abs(x + 1)) - 1/12*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = -\frac{\ln(x-1)}{12} - \frac{\ln(x+1)}{36} - \frac{x}{6\,(x^3+1)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{24}+\frac{\sqrt{3}\,1\mathrm{i}}{24}\right) + \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{72}+\frac{\sqrt{3}\,1\mathrm{i}}{72}\right)$$

input
```
int(-x^3/((x^3 - 1)*(x^3 + 1)^2),x)
```

output
```
log(x + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 + 1/24) - log(x + 1)/36 - x
/(6*(x^3 + 1)) - log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/24 - 1/24) -
log(x - 1)/12 + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 + 1/72) - l
og(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/72 - 1/72)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{(1-x^3)(1+x^3)^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^3 - 2\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right) + 6\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x^3 + 6\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right) + \log(x^2-x+1)\,x^3}{}$$

input
```
int(x^3/(-x^3+1)/(x^3+1)^2,x)
```

output
```
( - 2*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**3 - 2*sqrt(3)*atan((2*x - 1)/sqrt
(3)) + 6*sqrt(3)*atan((2*x + 1)/sqrt(3))*x**3 + 6*sqrt(3)*atan((2*x + 1)/s
qrt(3)) + log(x**2 - x + 1)*x**3 + log(x**2 - x + 1) + 3*log(x**2 + x + 1)
*x**3 + 3*log(x**2 + x + 1) - 6*log(x - 1)*x**3 - 6*log(x - 1) - 2*log(x +
 1)*x**3 - 2*log(x + 1) - 12*x)/(72*(x**3 + 1))
```

3.451 $\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = -\frac{x}{8(1+x^4)}+\frac{\arctan(x)}{8}+\frac{\arctan\left(1-\sqrt{2}x\right)}{16\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}x\right)}{16\sqrt{2}}+\frac{\operatorname{arctanh}(x)}{8}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}x}{1+x^2}\right)}{16\sqrt{2}}$$

output
```
-1/8*x/(x^4+1)+1/8*arctan(x)-1/32*arctan(-1+x*2^(1/2))*2^(1/2)-1/32*arctan
(1+x*2^(1/2))*2^(1/2)+1/8*arctanh(x)-1/32*arctanh(2^(1/2)*x/(x^2+1))*2^(1/
2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = \frac{1}{64}\left(-\frac{8x}{1+x^4}+8\arctan(x)+2\sqrt{2}\arctan\left(1-\sqrt{2}x\right) - 2\sqrt{2}\arctan\left(1+\sqrt{2}x\right)-4\log(1-x)+4\log(1+x) + \sqrt{2}\log\left(1-\sqrt{2}x+x^2\right)-\sqrt{2}\log\left(1+\sqrt{2}x+x^2\right)\right)$$

input
```
Integrate[x^4/((1 - x^4)*(1 + x^4)^2),x]
```

output
```
((-8*x)/(1 + x^4) + 8*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt
[2]*ArcTan[1 + Sqrt[2]*x] - 4*Log[1 - x] + 4*Log[1 + x] + Sqrt[2]*Log[1 -
Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/64
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {971, 1020, 755, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^4)(x^4+1)^2}\,dx$$

$$\Big\downarrow \text{971}$$

$$\frac{1}{8}\int \frac{3x^4+1}{(1-x^4)(x^4+1)}dx-\frac{x}{8(x^4+1)}$$

$$\Big\downarrow \text{1020}$$

$$\frac{1}{8}\left(2\int \frac{1}{1-x^4}dx-\int \frac{1}{x^4+1}dx\right)-\frac{x}{8(x^4+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{8}\left(2\int \frac{1}{1-x^4}dx-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{x}{8(x^4+1)}$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{8}\left(2\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{1}{2}\int \frac{1}{x^2+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{x}{8(x^4+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{8}\left(2\left(\frac{1}{2}\int \frac{1}{1-x^2}dx+\frac{\arctan(x)}{2}\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx\right)-\frac{x}{8(x^4+1)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx-\frac{1}{2}\int \frac{x^2+1}{x^4+1}dx+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)\right)-\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 1476

$$\frac{1}{8}\left(\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}x+1}dx-\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}x+1}dx\right)-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)\right)-\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 1082

$$\frac{1}{8}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+\frac{1}{2}\left(\frac{\int \frac{1}{-\left(\sqrt{2}x+1\right)^2-1}d\left(\sqrt{2}x+1\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\left(1-\sqrt{2}x\right)^2-1}d\left(1-\sqrt{2}x\right)}{\sqrt{2}}\right)+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctan}}{2}\right.\right.$$
$$\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 217

$$\frac{1}{8}\left(-\frac{1}{2}\int \frac{1-x^2}{x^4+1}dx+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)\right)-\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 1479

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\text{arcta}}{}\right.\right.$$
$$\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 25

$$\frac{1}{8}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+1\right)}{x^2+\sqrt{2}x+1}dx}{2\sqrt{2}}\right)+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan}{}\right.\right.$$
$$\frac{x}{8(x^4+1)}$$

$\Big\downarrow$ 27

$$\frac{1}{8}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2x}{x^2-\sqrt{2}x+1}dx}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}x+1}{x^2+\sqrt{2}x+1}dx\right)+2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\text{ar}}{}\right.\right.$$
$$\frac{x}{8\left(x^4+1\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{8}\left(2\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}\right)+\frac{1}{2}\left(\frac{\arctan\left(1-\sqrt{2}x\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}x+1\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-\frac{\text{log}}{}\right.\right.$$
$$\frac{x}{8\left(x^4+1\right)}$$

input
```
Int[x^4/((1 - x^4)*(1 + x^4)^2),x]
```

output
```
-1/8*x/(1 + x^4) + ((ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*x]
/Sqrt[2])/2 + 2*(ArcTan[x]/2 + ArcTanh[x]/2) + (Log[1 - Sqrt[2]*x + x^2]/(
2*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]))/2)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
risch	$-\frac{x}{8(x^4+1)}+\frac{\ln(x+1)}{16}+\frac{\arctan(x)}{8}-\frac{\ln(x-1)}{16}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(x-\_R\right)\right)}{32}$	47
default	$-\frac{x}{8(x^4+1)}-\frac{\sqrt{2}\left(\ln\left(\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}\right)+2\arctan\left(x\sqrt{2}+1\right)+2\arctan\left(x\sqrt{2}-1\right)\right)}{64}+\frac{\arctan(x)}{8}+\frac{\ln(x+1)}{16}-\frac{\ln(x-1)}{16}$	79

input
```
int(x^4/(-x^4+1)/(x^4+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x/(x^4+1)+1/16*ln(x+1)+1/8*arctan(x)-1/16*ln(x-1)+1/32*sum(_R*ln(x-_R
),_R=RootOf(_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = -\frac{2\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x+1\right)+2\sqrt{2}(x^4+1)\arctan\left(\sqrt{2}x-1\right)+\sqrt{2}(x^4+1)\log\left(x^2+\sqrt{2}x+1\right)-}{6}$$

input

```
integrate(x^4/(-x^4+1)/(x^4+1)^2,x, algorithm="fricas")
```

output

```
-1/64*(2*sqrt(2)*(x^4 + 1)*arctan(sqrt(2)*x + 1) + 2*sqrt(2)*(x^4 + 1)*arc
tan(sqrt(2)*x - 1) + sqrt(2)*(x^4 + 1)*log(x^2 + sqrt(2)*x + 1) - sqrt(2)*
(x^4 + 1)*log(x^2 - sqrt(2)*x + 1) - 8*(x^4 + 1)*arctan(x) - 4*(x^4 + 1)*l
og(x + 1) + 4*(x^4 + 1)*log(x - 1) + 8*x)/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 145.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = -\frac{x}{8x^4+8}-\frac{\log(x-1)}{16}+\frac{\log(x+1)}{16}-\frac{i\log(x-i)}{16}+\frac{i\log(x+i)}{16} - \operatorname{RootSum}\left(1048576t^4+1,\left(t\mapsto t\log\left(-\frac{50331648t^5}{17}+\frac{496t}{17}+x\right)\right)\right)$$

input

```
integrate(x**4/(-x**4+1)/(x**4+1)**2,x)
```

output

```
-x/(8*x**4 + 8) - log(x - 1)/16 + log(x + 1)/16 - I*log(x - I)/16 + I*log(
x + I)/16 - RootSum(1048576*_t**4 + 1, Lambda(_t, _t*log(-50331648*_t**5/1
7 + 496*_t/17 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = -\frac{1}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{64}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{64}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{8(x^4+1)} + \frac{1}{8}\arctan(x) + \frac{1}{16}\log(x+1) - \frac{1}{16}\log(x-1)$$

input
```
integrate(x^4/(-x^4+1)/(x^4+1)^2,x, algorithm="maxima")
```

output
```
-1/32*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/32*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 1/64*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/64*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/8*x/(x^4 + 1) + 1/8*arctan(x) + 1/16*
log(x + 1) - 1/16*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.14

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = -\frac{1}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{1}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{64}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{64}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right) - \frac{x}{8(x^4+1)} + \frac{1}{8}\arctan(x) + \frac{1}{16}\log(|x+1|) - \frac{1}{16}\log(|x-1|)$$

input
```
integrate(x^4/(-x^4+1)/(x^4+1)^2,x, algorithm="giac")
```

output
```
-1/32*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/32*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) - 1/64*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/64*
sqrt(2)*log(x^2 - sqrt(2)*x + 1) - 1/8*x/(x^4 + 1) + 1/8*arctan(x) + 1/16*
log(abs(x + 1)) - 1/16*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = \frac{\mathrm{atan}(x)}{8} - \frac{x}{8\,(x^4+1)} - \frac{\mathrm{atan}(x\,1\mathrm{i})\;1\mathrm{i}}{8} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,x\right)\;1\mathrm{i}}{16} - \frac{(-1)^{3/4}\,\mathrm{atan}\left((-1)^{3/4}\,x\right)\;1\mathrm{i}}{16}$$

input
```
int(-x^4/((x^4 - 1)*(x^4 + 1)^2),x)
```

output
```
atan(x)/8 - (atan(x*1i)*1i)/8 + ((-1)^(1/4)*atan((-1)^(1/4)*x)*1i)/16 - ((
-1)^(3/4)*atan((-1)^(3/4)*x)*1i)/16 - x/(8*(x^4 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.12

$$\int \frac{x^4}{(1-x^4)(1+x^4)^2}\,dx = \frac{2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right)x^4 + 2\sqrt{2}\,atan\left(\frac{\sqrt{2}-2x}{\sqrt{2}}\right) - 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right)x^4 - 2\sqrt{2}\,atan\left(\frac{\sqrt{2}+2x}{\sqrt{2}}\right) + 8atan(x)\,x^4 +}{}$$

input
```
int(x^4/(-x^4+1)/(x^4+1)^2,x)
```

output
```
(2*sqrt(2)*atan((sqrt(2) - 2*x)/sqrt(2))*x**4 + 2*sqrt(2)*atan((sqrt(2) -
2*x)/sqrt(2)) - 2*sqrt(2)*atan((sqrt(2) + 2*x)/sqrt(2))*x**4 - 2*sqrt(2)*a
tan((sqrt(2) + 2*x)/sqrt(2)) + 8*atan(x)*x**4 + 8*atan(x) + sqrt(2)*log( -
 sqrt(2)*x + x**2 + 1)*x**4 + sqrt(2)*log( - sqrt(2)*x + x**2 + 1) - sqrt(
2)*log(sqrt(2)*x + x**2 + 1)*x**4 - sqrt(2)*log(sqrt(2)*x + x**2 + 1) - 4*
log(x - 1)*x**4 - 4*log(x - 1) + 4*log(x + 1)*x**4 + 4*log(x + 1) - 8*x)/(
64*(x**4 + 1))
```

3.452 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n} dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n} dx = -\frac{(bc-ad)^3x^n}{d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{2n}}{2d^3n} - \frac{b^2(bc-3ad)x^{3n}}{3d^2n} + \frac{b^3x^{4n}}{4dn} + \frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}$$

output
```
-(-a*d+b*c)^3*x^n/d^4/n+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x^(2*n)/d^3/n-
1/3*b^2*(-3*a*d+b*c)*x^(3*n)/d^2/n+1/4*b^3*x^(4*n)/d/n+c*(-a*d+b*c)^3*ln(c
+d*x^n)/d^5/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n} dx = \frac{dx^n(12a^3d^3+18a^2bd^2(-2c+dx^n)+6ab^2d(6c^2-3cdx^n+2d^2x^{2n})+b^3(-12c^3+6c^2dx^n-4cd^2x^{2n}+3c}{12d^5n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(d*x^n*(12*a^3*d^3 + 18*a^2*b*d^2*(-2*c + d*x^n) + 6*a*b^2*d*(6*c^2 - 3*c*
d*x^n + 2*d^2*x^(2*n)) + b^3*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*
d^3*x^(3*n))) + 12*c*(b*c - a*d)^3*Log[c + d*x^n])/(12*d^5*n)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^3}{c+dx^n}\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int \frac{x^n(bx^n+a)^3}{dx^n+c}dx^n}{n}$$
$$\Big\downarrow \text{86}$$
$$\frac{\int \left(\frac{b(b^2c^2-3abdc+3a^2d^2)x^n}{d^3}-\frac{b^2(bc-3ad)x^{2n}}{d^2}+\frac{b^3x^{3n}}{d}+\frac{(ad-bc)^3}{d^4}+\frac{c(bc-ad)^3}{d^4(dx^n+c)}\right)dx^n}{n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{bx^{2n}(3a^2d^2-3abcd+b^2c^2)}{2d^3}-\frac{b^2x^{3n}(bc-3ad)}{3d^2}+\frac{c(bc-ad)^3\log(c+dx^n)}{d^5}-\frac{x^n(bc-ad)^3}{d^4}+\frac{b^3x^{4n}}{4d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(-(((b*c - a*d)^3*x^n)/d^4) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*n)
)/(2*d^3) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2) + (b^3*x^(4*n))/(4*d) + (c
*(b*c - a*d)^3*Log[c + d*x^n])/d^5)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.45

method	result
norman	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\mathrm{e}^{n\ln(x)}}{d^4n}+\frac{b^3\mathrm{e}^{4n\ln(x)}}{4dn}+\frac{b(3a^2d^2-3abcd+b^2c^2)\mathrm{e}^{2n\ln(x)}}{2d^3n}+\frac{b^2(3ad-cb)\mathrm{e}^{3n\ln(x)}}{3d^2n}-\frac{c(a^3d^3-}{}$
risch	$\frac{b^3x^{4n}}{4dn}+\frac{b^2x^{3n}a}{dn}-\frac{b^3x^{3n}c}{3d^2n}+\frac{3b\,x^{2n}a^2}{2dn}-\frac{3b^2x^{2n}ac}{2d^2n}+\frac{b^3x^{2n}c^2}{2d^3n}+\frac{x^na^3}{dn}-\frac{3x^na^2bc}{d^2n}+\frac{3x^na\,b^2c^2}{d^3n}-\frac{x^nb^3c^3}{d^4n}-\frac{c\ln(x}{a}$

input
```
int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/n*exp(n*ln(x))+1/4*b^3
/d/n*exp(n*ln(x))^4+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3/n*exp(n*ln(x))
^2+1/3*b^2*(3*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/d^5/n*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.36

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{3\,b^3d^4x^{4\,n} - 4\,(b^3cd^3 - 3\,ab^2d^4)x^{3\,n} + 6\,(b^3c^2d^2 - 3\,ab^2cd^3 + 3\,a^2bd^4)x^{2\,n} - 12\,(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bc}{12\,d^5n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2
*d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*
d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b
*c^2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(114) = 228.

Time = 5.13 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.67

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{2n-1}}{2n}+\frac{a^2bxx^nx^{2n-1}}{n}+\frac{3ab^2xx^{2n}x^{2n-1}}{4n}+\frac{b^3xx^{3n}x^{2n-1}}{5n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ -\frac{a^3c\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{a^3x^n}{dn}+\frac{3a^2bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{3a^2bcx^n}{d^2n}+\frac{3a^2bx^{2n}}{2dn}-\frac{3ab^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{3ab^2c^2x^n}{d^3n}-\frac{3ab^2cx^{2n}}{2d^2n}+\frac{ab^2x^{3}}{dn} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(2*n - 1
)/(2*n) + a**2*b*x*x**n*x**(2*n - 1)/n + 3*a*b**2*x*x**(2*n)*x**(2*n - 1)/
(4*n) + b**3*x*x**(3*n)*x**(2*n - 1)/(5*n))/c, Eq(d, 0)), ((a + b)**3*log(
x)/(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d**2*n) + a**3*x**n/(d*n)
 + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*b*c*x**n/(d**2*n) + 3*a
**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**n)/(d**4*n) + 3*a*b**2
*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n) + a*b**2*x**(3*n)/(d*
n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x**n/(d**4*n) + b**3*c
**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) + b**3*x**(4*n)/(4*d*
n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.78

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= a^3\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) + \frac{1}{12}b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right) - \frac{1}{2}ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{3}{2}a^2b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x
^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) -
12*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^
2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*
x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{(bx^n+a)^3 x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.67

$$\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{3x^{4n}b^3d^4+12x^{3n}a\,b^2d^4-4x^{3n}b^3c\,d^3+18x^{2n}a^2b\,d^4-18x^{2n}a\,b^2c\,d^3+6x^{2n}b^3c^2d^2+12x^na^3d^4-36x^na^2bc}{}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
(3*x**(4*n)*b**3*d**4 + 12*x**(3*n)*a*b**2*d**4 - 4*x**(3*n)*b**3*c*d**3 +
 18*x**(2*n)*a**2*b*d**4 - 18*x**(2*n)*a*b**2*c*d**3 + 6*x**(2*n)*b**3*c**
2*d**2 + 12*x**n*a**3*d**4 - 36*x**n*a**2*b*c*d**3 + 36*x**n*a*b**2*c**2*d
**2 - 12*x**n*b**3*c**3*d - 12*log(x**n*d + c)*a**3*c*d**3 + 36*log(x**n*d
 + c)*a**2*b*c**2*d**2 - 36*log(x**n*d + c)*a*b**2*c**3*d + 12*log(x**n*d
+ c)*b**3*c**4)/(12*d**5*n)
```

3.453 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{(bc-ad)^2x^n}{d^3n} - \frac{b(bc-2ad)x^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} - \frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}$$

output
```
(-a*d+b*c)^2*x^n/d^3/n-1/2*b*(-2*a*d+b*c)*x^(2*n)/d^2/n+1/3*b^2*x^(3*n)/d/
n-c*(-a*d+b*c)^2*ln(c+d*x^n)/d^4/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{dx^n(6a^2d^2+6abd(-2c+dx^n)+b^2(6c^2-3cdx^n+2d^2x^{2n}))-6c(bc-ad)^2\log(c+dx^n)}{6d^4n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(d*x^n*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^n) + b^2*(6*c^2 - 3*c*d*x^n + 2*d^
2*x^(2*n))) - 6*c*(b*c - a*d)^2*Log[c + d*x^n])/(6*d^4*n)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^2}{c+dx^n}\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n(bx^n+a)^2}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{b(bc-2ad)x^n}{d^2}+\frac{b^2x^{2n}}{d}+\frac{(ad-bc)^2}{d^3}-\frac{c(bc-ad)^2}{d^3(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4}+\frac{x^n(bc-ad)^2}{d^3}-\frac{bx^{2n}(bc-2ad)}{2d^2}+\frac{b^2x^{3n}}{3d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n
))/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

method	result	size
norman	$\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) {\mathrm e}^{n \ln \left(x \right)}}{d^{3} n}+\frac{b^{2} {\mathrm e}^{3 n \ln \left(x \right)}}{3 d n}+\frac{b \left(2 a d -c b \right) {\mathrm e}^{2 n \ln \left(x \right)}}{2 d^{2} n}-\frac{c \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln \left(c +d \,{\mathrm e}^{n \ln \left(x \right)}\right)}{d^{4} n}$	118
risch	$\frac{b^{2} x^{3 n}}{3 d n}+\frac{b \,x^{2 n} a}{d n}-\frac{b^{2} x^{2 n} c}{2 d^{2} n}+\frac{x^{n} a^{2}}{d n}-\frac{2 x^{n} a b c}{d^{2} n}+\frac{x^{n} b^{2} c^{2}}{d^{3} n}-\frac{c \ln \left(x^{n}+\frac{c}{d}\right) a^{2}}{d^{2} n}+\frac{2 c^{2} \ln \left(x^{n}+\frac{c}{d}\right) a b}{d^{3} n}-\frac{c^{3} \ln \left(x^{n}+\frac{c}{d}\right) b^{2}}{d^{4} n}$	161

input
```
int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/3*b^2/d/n*exp(n*ln(x))^
3+1/2*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^2-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4
/n*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{2\,b^2d^3x^{3\,n} - 3\,(b^2cd^2 - 2\,abd^3)x^{2\,n} + 6\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^n - 6\,(b^2c^3 - 2\,abc^2d + a^2cd^2)\log\left(dx^n + \right.}{6\,d^4n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d
*x^n + c))/(d^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(75) = 150.

Time = 3.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)^2\log(x)}{c} & \text{for } d \\ \frac{\frac{a^2xx^{2n-1}}{2n} + \frac{2abxx^nx^{2n-1}}{3n} + \frac{b^2xx^{2n}x^{2n-1}}{4n}}{c} & \text{for } d \\ \frac{(a+b)^2\log(x)}{c+d} & \text{for } n \\ -\frac{a^2c\log\left(\frac{c}{d}+x^n\right)}{d^2n} + \frac{a^2x^n}{dn} + \frac{2abc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{2abcx^n}{d^2n} + \frac{abx^{2n}}{dn} - \frac{b^2c^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{b^2c^2x^n}{d^3n} - \frac{b^2cx^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} & \text{other} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(2*n - 1
)/(2*n) + 2*a*b*x*x**n*x**(2*n - 1)/(3*n) + b**2*x*x**(2*n)*x**(2*n - 1)/(
4*n))/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (-a**2*c*log(c/
d + x**n)/(d**2*n) + a**2*x**n/(d*n) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n)
 - 2*a*b*c*x**n/(d**2*n) + a*b*x**(2*n)/(d*n) - b**2*c**3*log(c/d + x**n)/
(d**4*n) + b**2*c**2*x**n/(d**3*n) - b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**
(3*n)/(3*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = a^2\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) - \frac{1}{6}b^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + ab\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
 + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) +
a*b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx = \int \frac{(bx^n+a)^2x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{x^{2\,n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$=\frac{2x^{3n}b^2d^3+6x^{2n}ab\,d^3-3x^{2n}b^2c\,d^2+6x^na^2d^3-12x^nabc\,d^2+6x^nb^2c^2d-6\log(x^nd+c)\,a^2c\,d^2+12\log(}{6d^4n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(2*x**(3*n)*b**2*d**3 + 6*x**(2*n)*a*b*d**3 - 3*x**(2*n)*b**2*c*d**2 + 6*x
**n*a**2*d**3 - 12*x**n*a*b*c*d**2 + 6*x**n*b**2*c**2*d - 6*log(x**n*d + c
)*a**2*c*d**2 + 12*log(x**n*d + c)*a*b*c**2*d - 6*log(x**n*d + c)*b**2*c**
3)/(6*d**4*n)
```

3.454 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx = -\frac{(bc-ad)x^n}{d^2n} + \frac{bx^{2n}}{2dn} + \frac{c(bc-ad)\log(c+dx^n)}{d^3n}$$

output
```
-(-a*d+b*c)*x^n/d^2/n+1/2*b*x^(2*n)/d/n+c*(-a*d+b*c)*ln(c+d*x^n)/d^3/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n} dx = \frac{dx^n(-2bc+2ad+bdx^n)+2c(bc-ad)\log(c+dx^n)}{2d^3n}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3
*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)}{c+dx^n} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n(bx^n+a)}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{bx^n}{d}+\frac{ad-bc}{d^2}+\frac{c(bc-ad)}{d^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c(bc-ad)\log(c+dx^n)}{d^3}-\frac{x^n(bc-ad)}{d^2}+\frac{bx^{2n}}{2d}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(-(((b*c - a*d)*x^n)/d^2) + (b*x^(2*n))/(2*d) + (c*(b*c - a*d)*Log[c + d*x
^n])/d^3)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
norman	$\frac{(ad-cb)\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{b\,\mathrm{e}^{2n\ln(x)}}{2dn}-\frac{c(ad-cb)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^3n}$	65
risch	$\frac{b\,x^{2n}}{2dn}+\frac{x^n a}{dn}-\frac{x^n cb}{d^2n}-\frac{c\ln\left(x^n+\frac{c}{d}\right)a}{d^2n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)b}{d^3n}$	81

input
```
int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(a*d-b*c)/n*exp(n*ln(x))+1/2*b/d/n*exp(n*ln(x))^2-c*(a*d-b*c)/d^3/n*
ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\frac{bd^2x^{2\,n}-2\,(bcd-ad^2)x^n+2\,(bc^2-acd)\log\left(dx^n+c\right)}{2\,d^3n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n +
 c))/(d^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(48) = 96$.

Time = 1.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0 \wedge n=0 \\ \frac{\frac{axx^{2n-1}}{2n}+\frac{bxx^nx^{2n-1}}{3n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ -\frac{ac\log\left(\frac{c}{d}+x^n\right)}{d^2n}+\frac{ax^n}{dn}+\frac{bc^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{bcx^n}{d^2n}+\frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(2*n - 1)/(2*n
) + b*x*x**n*x**(2*n - 1)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq
(n, 0)), (-a*c*log(c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d +
x**n)/(d**3*n) - b*c*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx = a\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)+\frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c
)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{(bx^n+a)x^{2\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx=\int \frac{x^{2\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx =\frac{x^{2n}b\,d^2+2x^na\,d^2-2x^nbcd-2\log(x^nd+c)\,acd+2\log(x^nd+c)\,b\,c^2}{2d^3n}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x)
```

output
```
(x**(2*n)*b*d**2 + 2*x**n*a*d**2 - 2*x**n*b*c*d - 2*log(x**n*d + c)*a*c*d
+ 2*log(x**n*d + c)*b*c**2)/(2*d**3*n)
```

3.455 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx$

Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx = -\frac{a\log(a+bx^n)}{b(bc-ad)n} + \frac{c\log(c+dx^n)}{d(bc-ad)n}$$

output
```
-a*ln(a+b*x^n)/b/(-a*d+b*c)/n+c*ln(c+d*x^n)/d/(-a*d+b*c)/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx = -\frac{ad\log(a+bx^n) - bc\log(c+dx^n)}{b^2cdn - abd^2n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
-((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{c}{(bc-ad)(dx^n+c)}-\frac{a}{(bc-ad)(bx^n+a)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{c\log(c+dx^n)}{d(bc-ad)}-\frac{a\log(a+bx^n)}{b(bc-ad)}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(-((a*Log[a + b*x^n])/(b*(b*c - a*d))) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
))/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

method	result	size
norman	$\frac{a\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{\left(ad-cb\right)bn}-\frac{c\ln\left(c+d\,{\mathrm e}^{n\ln\left(x\right)}\right)}{dn\left(ad-cb\right)}$	59
risch	$\frac{\ln\left(x\right)}{db}+\frac{\ln\left(x\right)c}{d\left(ad-cb\right)}-\frac{\ln\left(x\right)a}{\left(ad-cb\right)b}-\frac{c\ln\left(x^{n}+\frac{c}{d}\right)}{dn\left(ad-cb\right)}+\frac{a\ln\left(x^{n}+\frac{a}{b}\right)}{\left(ad-cb\right)bn}$	103

input
```
int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a/(a*d-b*c)/b/n*ln(a+b*exp(n*ln(x)))-c/d/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx=-\frac{ad\log\left(bx^n+a\right)-bc\log\left(dx^n+c\right)}{(b^2cd-abd^2)n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = -\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn-abdn}+\frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn-ad^2n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n
- a*d^2*n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{\log(x^n b+a)\,ad - \log(x^n d+c)\,bc}{bdn\,(ad-bc)}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x)
```

output
```
(log(x**n*b + a)*a*d - log(x**n*d + c)*b*c)/(b*d*n*(a*d - b*c))
```

3.456 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx=\frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

output
```
a/b/(-a*d+b*c)/n/(a+b*x^n)+c*ln(a+b*x^n)/(-a*d+b*c)^2/n-c*ln(c+d*x^n)/(-a*
d+b*c)^2/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx=\frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (
c*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^2 (c+dx^n)} dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^2(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{a}{(bc-ad)(bx^n+a)^2}+\frac{bc}{(bc-ad)^2(bx^n+a)}-\frac{cd}{(bc-ad)^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a}{b(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2}-\frac{c\log(c+dx^n)}{(bc-ad)^2}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^n)) + (c*Log[a + b*x^n])/(b*c - a*d)^2 - (c*Log
[c + d*x^n])/(b*c - a*d)^2)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.43

method	result	size
risch	$-\frac{a}{(ad-cb)bn(a+b\,x^{n})}-\frac{c\ln\left(x^{n}+\frac{c}{d}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{c\ln\left(x^{n}+\frac{a}{b}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	107
norman	$\frac{\mathrm{e}^{n\ln(x)}}{(ad-cb)n\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}+\frac{c\ln\left(a+b\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}-\frac{c\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{n(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	109

input
```
int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*d-b*c)/b/n/(a+b*x^n)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+c/d)+c/n
/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(x^n+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{abc-a^2d+(b^2cx^n+abc)\log(bx^n+a)-(b^2cx^n+abc)\log(dx^n+c)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^n+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*
log(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2
- 2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{c\log\left(\frac{bx^n+a}{b}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2n-2\,abcdn+a^2d^2n} + \frac{a}{ab^2cn-a^2bdn+(b^3cn-ab^2dn)x^n}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n
+ c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n +
 (b^3*c*n - a*b^2*d*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x^n\log(x^nb+a)\,bc - x^n\log(x^nd+c)\,bc + x^nad - x^nbc + \log(x^nb+a)\,ac - \log(x^nd+c)\,ac}{n\,(x^na^2b\,d^2 - 2x^na\,b^2cd + x^nb^3c^2 + a^3d^2 - 2a^2bcd + a\,b^2c^2)}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*b*c - x**n*log(x**n*d + c)*b*c + x**n*a*d - x**n*b*c
 + log(x**n*b + a)*a*c - log(x**n*d + c)*a*c)/(n*(x**n*a**2*b*d**2 - 2*x**
n*a*b**2*c*d + x**n*b**3*c**2 + a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2))
```

3.457 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \frac{a}{2b(bc-ad)n\,(a+bx^n)^2} - \frac{c}{(bc-ad)^2n\,(a+bx^n)} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

output
```
1/2*a/b/(-a*d+b*c)/n/(a+b*x^n)^2-c/(-a*d+b*c)^2/n/(a+b*x^n)-c*d*ln(a+b*x^n
)/(-a*d+b*c)^3/n+c*d*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx = \frac{-abc-a^2d-2b^2cx^n}{2b(bc-ad)^2n\,(a+bx^n)^2} - \frac{cd\log(a+bx^n)}{(bc-ad)^3n} + \frac{cd\log(c+dx^n)}{(bc-ad)^3n}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(-(a*b*c) - a^2*d - 2*b^2*c*x^n)/(2*b*(b*c - a*d)^2*n*(a + b*x^n)^2) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*
n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^3(c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^3(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{cd^2}{(bc-ad)^3(dx^n+c)}-\frac{bcd}{(bc-ad)^3(bx^n+a)}+\frac{bc}{(bc-ad)^2(bx^n+a)^2}-\frac{a}{(bc-ad)(bx^n+a)^3}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2}-\frac{c}{(bc-ad)^2(a+bx^n)}-\frac{cd\log(a+bx^n)}{(bc-ad)^3}+\frac{cd\log(c+dx^n)}{(bc-ad)^3}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*
Log[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{2 b^{2} c \,x^{n}+a^{2} d +a b c}{2 \left(a d -c b \right)^{2} b n \left(a +b \,x^{n}\right)^{2}}-\frac{c d \ln \left(x^{n}+\frac{c}{d}\right)}{n \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right)}+\frac{c d \ln \left(x^{n}+\frac{a}{b}\right)}{n \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right)}$
norman	$\frac{-\frac{b c \,{\mathrm e}^{n \ln \left(x \right)}}{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) n}+\frac{a \left(-a b d -b^{2} c \right)}{2 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) b^{2} n}}{\left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)^{2}}+\frac{c d \ln \left(a +b \,{\mathrm e}^{n \ln \left(x \right)}\right)}{n \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right)}-\frac{c d \ln \left(c +d \,{\mathrm e}^{n \ln \left(x \right)}\right)}{n \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right)}$

input
```
int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b^2*c*x^n+a^2*d+a*b*c)/(a*d-b*c)^2/b/n/(a+b*x^n)^2-c*d/n/(a^3*d^3-
3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)+c*d/n/(a^3*d^3-3*a^2*b*c*
d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(103) = 206.

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.54

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = -\frac{ab^2c^2 - a^3d^2 + 2\,(b^3c^2 - ab^2cd)x^n + 2\,(b^3cdx^{2\,n} + 2\,ab^2cdx^n + a^2bcd)\log\left(bx^n + a\right) - 2\,(b^3cdx^{2\,n} +}{2\,((b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b^3d^3)nx^{2\,n} + 2\,(ab^5c^3 - 3\,a^2b^4c^2d + 3\,a^3b^3cd^2 - a^4b^2d^3)nx^n + (a^2b^4}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*
n) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*
a*b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a
^2*b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3
*a^3*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a
^4*b^2*c*d^2 - a^5*b*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.31

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = -\frac{cd\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n}+\frac{cd\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} - \frac{2\,b^2cx^n+abc+a^2d}{2\,(a^2b^3c^2n-2\,a^3b^2cdn+a^4bd^2n+(b^5c^2n-2\,ab^4cdn+a^2b^3d^2n)x^{2\,n}+2\,(ab^4c^2n-2\,a^2b^3cdn+a^3b^2d^2}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
-c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a
^3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*
c*d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n -
2*a^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n
)*x^(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{2n-1}}{(a+b\,x^n)^3\,(c+d\,x^n)}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.10

$$\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{2x^{2n}\log(x^nb+a)\,ab^3cd-2x^{2n}\log(x^nd+c)\,ab^3cd+x^{2n}ab^3cd-x^{2n}b^4c^2+4x^n\log(x^nb+a)\,a^2b^2cd-4x^n}{2abn\,(x^{2n}a^3b^2d^3-3x^{2n}a^2b^3cd^2+3x^{2n}ab^4c^2d-x^{2n}b^5c^3+2x^na^4bd^3-6x^na^3b^2cd^2+}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
(2*x**(2*n)*log(x**n*b + a)*a*b**3*c*d - 2*x**(2*n)*log(x**n*d + c)*a*b**3
*c*d + x**(2*n)*a*b**3*c*d - x**(2*n)*b**4*c**2 + 4*x**n*log(x**n*b + a)*a
**2*b**2*c*d - 4*x**n*log(x**n*d + c)*a**2*b**2*c*d + 2*log(x**n*b + a)*a*
*3*b*c*d - 2*log(x**n*d + c)*a**3*b*c*d - a**4*d**2 + a**3*b*c*d)/(2*a*b*n
*(x**(2*n)*a**3*b**2*d**3 - 3*x**(2*n)*a**2*b**3*c*d**2 + 3*x**(2*n)*a*b**
4*c**2*d - x**(2*n)*b**5*c**3 + 2*x**n*a**4*b*d**3 - 6*x**n*a**3*b**2*c*d*
*2 + 6*x**n*a**2*b**3*c**2*d - 2*x**n*a*b**4*c**3 + a**5*d**3 - 3*a**4*b*c
*d**2 + 3*a**3*b**2*c**2*d - a**2*b**3*c**3))
```

3.458 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{c(bc-ad)^3x^n}{d^5n} - \frac{(bc-ad)^3x^{2n}}{2d^4n} + \frac{b(b^2c^2-3abcd+3a^2d^2)\,x^{3n}}{3d^3n} - \frac{b^2(bc-3ad)x^{4n}}{4d^2n} + \frac{b^3x^{5n}}{5dn} - \frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6n}$$

output
```
c*(-a*d+b*c)^3*x^n/d^5/n-1/2*(-a*d+b*c)^3*x^(2*n)/d^4/n+1/3*b*(3*a^2*d^2-3
*a*b*c*d+b^2*c^2)*x^(3*n)/d^3/n-1/4*b^2*(-3*a*d+b*c)*x^(4*n)/d^2/n+1/5*b^3
*x^(5*n)/d/n-c^2*(-a*d+b*c)^3*ln(c+d*x^n)/d^6/n
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \frac{dx^n(30a^3d^3(-2c+dx^n)+30a^2bd^2(6c^2-3cdx^n+2d^2x^{2n})+15ab^2d(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}}{60d^6n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
(d*x^n*(30*a^3*d^3*(-2*c + d*x^n) + 30*a^2*b*d^2*(6*c^2 - 3*c*d*x^n + 2*d^
2*x^(2*n)) + 15*a*b^2*d*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x
^(3*n)) + b^3*(60*c^4 - 30*c^3*d*x^n + 20*c^2*d^2*x^(2*n) - 15*c*d^3*x^(3*
n) + 12*d^4*x^(4*n))) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^3}{c+dx^n}\,dx$$
$$\Big\downarrow \; 948$$
$$\frac{\int \frac{x^{2n}(bx^n+a)^3}{dx^n+c}dx^n}{n}$$
$$\Big\downarrow \; 99$$
$$\frac{\int \left(\frac{(ad-bc)^3x^n}{d^4}+\frac{b(b^2c^2-3abdc+3a^2d^2)x^{2n}}{d^3}-\frac{b^2(bc-3ad)x^{3n}}{d^2}+\frac{b^3x^{4n}}{d}+\frac{c(bc-ad)^3}{d^5}-\frac{c^2(bc-ad)^3}{d^5(dx^n+c)}\right)dx^n}{n}$$
$$\Big\downarrow \; 2009$$
$$\frac{\frac{bx^{3n}(3a^2d^2-3abcd+b^2c^2)}{3d^3}-\frac{b^2x^{4n}(bc-3ad)}{4d^2}-\frac{c^2(bc-ad)^3\log(c+dx^n)}{d^6}+\frac{cx^n(bc-ad)^3}{d^5}-\frac{x^{2n}(bc-ad)^3}{2d^4}+\frac{b^3x^{5n}}{5d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

output
```
((c*(b*c - a*d)^3*x^n)/d^5 - ((b*c - a*d)^3*x^(2*n))/(2*d^4) + (b*(b^2*c^2
 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3) - (b^2*(b*c - 3*a*d)*x^(4*n))/(
4*d^2) + (b^3*x^(5*n))/(5*d) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/d^6)/n
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(150) = 300$.

Time = 0.97 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.16

method	result
risch	$\frac{b^{3} x^{5 n}}{5 d n}+\frac{3 b^{2} x^{4 n} a}{4 d n}-\frac{b^{3} x^{4 n} c}{4 d^{2} n}+\frac{b\, x^{3 n} a^{2}}{d n}-\frac{b^{2} x^{3 n} a c}{d^{2} n}+\frac{b^{3} x^{3 n} c^{2}}{3 d^{3} n}+\frac{x^{2 n} a^{3}}{2 d n}-\frac{3 x^{2 n} a^{2} b c}{2 d^{2} n}+\frac{3 x^{2 n} a\, b^{2} c^{2}}{2 d^{3} n}-\frac{x^{2 n} b^{3} c^{3}}{2 d^{4} n}-$

input
```
int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x
^n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*
a^3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^
2*b^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/
d^5/n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^
4/d^5/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= \frac{12\,b^3d^5x^{5\,n} - 15\,(b^3cd^4 - 3\,ab^2d^5)x^{4\,n} + 20\,(b^3c^2d^3 - 3\,ab^2cd^4 + 3\,a^2bd^5)x^{3\,n} - 30\,(b^3c^3d^2 - 3\,ab^2c^2d^3 + 3}{}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*
c^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2
*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*
d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a
^2*b*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. 2(138) = 276.

Time = 7.75 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.71

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= \begin{cases} \frac{(a+b)^3\log(x)}{c} \\ \frac{\frac{a^3xx^{3n-1}}{3n}+\frac{3a^2bxx^nx^{3n-1}}{4n}+\frac{3ab^2xx^{2n}x^{3n-1}}{5n}+\frac{b^3xx^{3n}x^{3n-1}}{6n}}{c} \\ \frac{(a+b)^3\log(x)}{c+d} \\ \frac{a^3c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{a^3cx^n}{d^2n} + \frac{a^3x^{2n}}{2dn} - \frac{3a^2bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{3a^2bc^2x^n}{d^3n} - \frac{3a^2bcx^{2n}}{2d^2n} + \frac{a^2bx^{3n}}{dn} + \frac{3ab^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n} - \frac{3ab^2c^3x^n}{d^4n} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x*x**(3*n - 1
)/(3*n) + 3*a**2*b*x*x**n*x**(3*n - 1)/(4*n) + 3*a*b**2*x*x**(2*n)*x**(3*n
 - 1)/(5*n) + b**3*x*x**(3*n)*x**(3*n - 1)/(6*n))/c, Eq(d, 0)), ((a + b)**
3*log(x)/(c + d), Eq(n, 0)), (a**3*c**2*log(c/d + x**n)/(d**3*n) - a**3*c*
x**n/(d**2*n) + a**3*x**(2*n)/(2*d*n) - 3*a**2*b*c**3*log(c/d + x**n)/(d**
4*n) + 3*a**2*b*c**2*x**n/(d**3*n) - 3*a**2*b*c*x**(2*n)/(2*d**2*n) + a**2
*b*x**(3*n)/(d*n) + 3*a*b**2*c**4*log(c/d + x**n)/(d**5*n) - 3*a*b**2*c**3
*x**n/(d**4*n) + 3*a*b**2*c**2*x**(2*n)/(2*d**3*n) - a*b**2*c*x**(3*n)/(d*
*2*n) + 3*a*b**2*x**(4*n)/(4*d*n) - b**3*c**5*log(c/d + x**n)/(d**6*n) + b
**3*c**4*x**n/(d**5*n) - b**3*c**3*x**(2*n)/(2*d**4*n) + b**3*c**2*x**(3*n
)/(3*d**3*n) - b**3*c*x**(4*n)/(4*d**2*n) + b**3*x**(5*n)/(5*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = -\frac{1}{60}\,b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n}-\frac{12\,d^4x^{5\,n}-15\,cd^3x^{4\,n}+20\,c^2d^2x^{3\,n}-30\,c^3dx^{2\,n}+60\,c^4x^n}{d^5n}\right) + \frac{1}{4}\,ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right) - \frac{1}{2}\,a^2b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a^3\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
-1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*
x^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1
/4*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(
3*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^
n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) +
 1/2*a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n)
)
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{(bx^n+a)^3x^{3\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.79

$$\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$$
$$= \frac{12x^{5n}b^3d^5 + 45x^{4n}a\,b^2d^5 - 15x^{4n}b^3c\,d^4 + 60x^{3n}a^2b\,d^5 - 60x^{3n}a\,b^2c\,d^4 + 20x^{3n}b^3c^2d^3 + 30x^{2n}a^3d^5 - 90x^2}{}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
(12*x**(5*n)*b**3*d**5 + 45*x**(4*n)*a*b**2*d**5 - 15*x**(4*n)*b**3*c*d**4
 + 60*x**(3*n)*a**2*b*d**5 - 60*x**(3*n)*a*b**2*c*d**4 + 20*x**(3*n)*b**3*
c**2*d**3 + 30*x**(2*n)*a**3*d**5 - 90*x**(2*n)*a**2*b*c*d**4 + 90*x**(2*n
)*a*b**2*c**2*d**3 - 30*x**(2*n)*b**3*c**3*d**2 - 60*x**n*a**3*c*d**4 + 18
0*x**n*a**2*b*c**2*d**3 - 180*x**n*a*b**2*c**3*d**2 + 60*x**n*b**3*c**4*d
+ 60*log(x**n*d + c)*a**3*c**2*d**3 - 180*log(x**n*d + c)*a**2*b*c**3*d**2
 + 180*log(x**n*d + c)*a*b**2*c**4*d - 60*log(x**n*d + c)*b**3*c**5)/(60*d
**6*n)
```

3.459 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}$$

output
```
-c*(-a*d+b*c)^2*x^n/d^4/n+1/2*(-a*d+b*c)^2*x^(2*n)/d^3/n-1/3*b*(-2*a*d+b*c
)*x^(3*n)/d^2/n+1/4*b^2*x^(4*n)/d/n+c^2*(-a*d+b*c)^2*ln(c+d*x^n)/d^5/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.06

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{dx^n(6a^2d^2(-2c+dx^n)+4abd(6c^2-3cdx^n+2d^2x^{2n})+b^2(-12c^3+6c^2dx^n-4cd^2x^{2n}+3d^3x^{3n}))+12c}{12d^5n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(d*x^n*(6*a^2*d^2*(-2*c + d*x^n) + 4*a*b*d*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2
*n)) + b^2*(-12*c^3 + 6*c^2*d*x^n - 4*c*d^2*x^(2*n) + 3*d^3*x^(3*n))) + 12
*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(12*d^5*n)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^2}{c+dx^n}\,dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^2}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{(ad-bc)^2x^n}{d^3}-\frac{b(bc-2ad)x^{2n}}{d^2}+\frac{b^2x^{3n}}{d}-\frac{c(bc-ad)^2}{d^4}+\frac{c^2(bc-ad)^2}{d^4(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5}-\frac{cx^n(bc-ad)^2}{d^4}+\frac{x^{2n}(bc-ad)^2}{2d^3}-\frac{bx^{3n}(bc-2ad)}{3d^2}+\frac{b^2x^{4n}}{4d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

output
```
(-((c*(b*c - a*d)^2*x^n)/d^4) + ((b*c - a*d)^2*x^(2*n))/(2*d^3) - (b*(b*c
- 2*a*d)*x^(3*n))/(3*d^2) + (b^2*x^(4*n))/(4*d) + (c^2*(b*c - a*d)^2*Log[c
 + d*x^n])/d^5)/n
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.33

method	result
norman	$\frac{b^{2} \mathrm{e}^{4 n \ln \left(x \right)}}{4 d n}+\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \mathrm{e}^{2 n \ln \left(x \right)}}{2 d^{3} n}+\frac{b \left(2 a d -c b \right) \mathrm{e}^{3 n \ln \left(x \right)}}{3 d^{2} n}-\frac{c \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \mathrm{e}^{n \ln \left(x \right)}}{d^{4} n}+\frac{c^{2} \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln}{d^{5} n}$
risch	$\frac{b^{2} x^{4 n}}{4 d n}+\frac{2 b \,x^{3 n} a}{3 d n}-\frac{b^{2} x^{3 n} c}{3 d^{2} n}+\frac{x^{2 n} a^{2}}{2 d n}-\frac{x^{2 n} a b c}{d^{2} n}+\frac{x^{2 n} b^{2} c^{2}}{2 d^{3} n}-\frac{c \,x^{n} a^{2}}{d^{2} n}+\frac{2 c^{2} x^{n} a b}{d^{3} n}-\frac{c^{3} x^{n} b^{2}}{d^{4} n}+\frac{c^{2} \ln \left(x^{n}+\frac{c}{d}\right) a^{2}}{d^{3} n}-\frac{2 c^{3}}{}$

input
```
int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(
x))^2+1/3*b*(2*a*d-b*c)/d^2/n*exp(n*ln(x))^3-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/d^4/n*exp(n*ln(x))+c^2/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*ln(c+d*exp(n*ln(
x)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{3\,b^2d^4x^{4\,n}-4\,(b^2cd^3-2\,abd^4)x^{3\,n}+6\,(b^2c^2d^2-2\,abcd^3+a^2d^4)x^{2\,n}-12\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^n+}{12\,d^5n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*
c*d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5
*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(99) = 198.

Time = 5.12 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.35

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)^2\log(x)}{c} \\ \frac{\frac{a^2xx^{3n-1}}{3n}+\frac{abxx^nx^{3n-1}}{2n}+\frac{b^2xx^{2n}x^{3n-1}}{5n}}{c} \\ \frac{(a+b)^2\log(x)}{c+d} \\ \frac{a^2c^2\log\left(\frac{c}{d}+x^n\right)}{d^3n}-\frac{a^2cx^n}{d^2n}+\frac{a^2x^{2n}}{2dn}-\frac{2abc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n}+\frac{2abc^2x^n}{d^3n}-\frac{abcx^{2n}}{d^2n}+\frac{2abx^{3n}}{3dn}+\frac{b^2c^4\log\left(\frac{c}{d}+x^n\right)}{d^5n}-\frac{b^2c^3x^n}{d^4n}+\frac{b^2c^2}{2d} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x*x**(3*n - 1
)/(3*n) + a*b*x*x**n*x**(3*n - 1)/(2*n) + b**2*x*x**(2*n)*x**(3*n - 1)/(5*
n))/c, Eq(d, 0)), ((a + b)**2*log(x)/(c + d), Eq(n, 0)), (a**2*c**2*log(c/
d + x**n)/(d**3*n) - a**2*c*x**n/(d**2*n) + a**2*x**(2*n)/(2*d*n) - 2*a*b*
c**3*log(c/d + x**n)/(d**4*n) + 2*a*b*c**2*x**n/(d**3*n) - a*b*c*x**(2*n)/
(d**2*n) + 2*a*b*x**(3*n)/(3*d*n) + b**2*c**4*log(c/d + x**n)/(d**5*n) - b
**2*c**3*x**n/(d**4*n) + b**2*c**2*x**(2*n)/(2*d**3*n) - b**2*c*x**(3*n)/(
3*d**2*n) + b**2*x**(4*n)/(4*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.63

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx = \frac{1}{12}\,b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right) - \frac{1}{3}\,ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a^2\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(
3*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1
/2*a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{(bx^n+a)^2x^{3\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx=\int \frac{x^{3\,n-1}\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.51

$$\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$$
$$=\frac{3x^{4n}b^2d^4+8x^{3n}ab\,d^4-4x^{3n}b^2c\,d^3+6x^{2n}a^2d^4-12x^{2n}abc\,d^3+6x^{2n}b^2c^2d^2-12x^na^2c\,d^3+24x^nab\,c^2d^2-}{12d^5n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(3*x**(4*n)*b**2*d**4 + 8*x**(3*n)*a*b*d**4 - 4*x**(3*n)*b**2*c*d**3 + 6*x
**(2*n)*a**2*d**4 - 12*x**(2*n)*a*b*c*d**3 + 6*x**(2*n)*b**2*c**2*d**2 - 1
2*x**n*a**2*c*d**3 + 24*x**n*a*b*c**2*d**2 - 12*x**n*b**2*c**3*d + 12*log(
x**n*d + c)*a**2*c**2*d**2 - 24*log(x**n*d + c)*a*b*c**3*d + 12*log(x**n*d
 + c)*b**2*c**4)/(12*d**5*n)
```

3.460 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx=\frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}$$

output
```
c*(-a*d+b*c)*x^n/d^3/n-1/2*(-a*d+b*c)*x^(2*n)/d^2/n+1/3*b*x^(3*n)/d/n-c^2*
(-a*d+b*c)*ln(c+d*x^n)/d^4/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx =\frac{dx^n(3ad(-2c+dx^n)+b(6c^2-3cdx^n+2d^2x^{2n}))+6c^2(-bc+ad)\log(c+dx^n)}{6d^4n}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
(d*x^n*(3*a*d*(-2*c + d*x^n) + b*(6*c^2 - 3*c*d*x^n + 2*d^2*x^(2*n))) + 6*
c^2*(-(b*c) + a*d)*Log[c + d*x^n])/(6*d^4*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)}{c+dx^n} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)}{dx^n+c}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{(ad-bc)x^n}{d^2}+\frac{bx^{2n}}{d}+\frac{c(bc-ad)}{d^3}-\frac{c^2(bc-ad)}{d^3(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4}+\frac{cx^n(bc-ad)}{d^3}-\frac{x^{2n}(bc-ad)}{2d^2}+\frac{bx^{3n}}{3d}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

output
```
((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*
d) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{b\,\mathrm{e}^{3n\ln(x)}}{3dn}+\frac{(ad-cb)\mathrm{e}^{2n\ln(x)}}{2d^2n}-\frac{c(ad-cb)\mathrm{e}^{n\ln(x)}}{d^3n}+\frac{c^2(ad-cb)\ln\left(c+d\,\mathrm{e}^{n\ln(x)}\right)}{d^4n}$	91
risch	$\frac{b\,x^{3n}}{3dn}+\frac{x^{2n}a}{2dn}-\frac{x^{2n}cb}{2d^2n}-\frac{c\,x^n a}{d^2n}+\frac{c^2x^n b}{d^3n}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)a}{d^3n}-\frac{c^3\ln\left(x^n+\frac{c}{d}\right)b}{d^4n}$	115

input
```
int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/d/n*exp(n*ln(x))^3+1/2/d^2*(a*d-b*c)/n*exp(n*ln(x))^2-c*(a*d-b*c)/d^
3/n*exp(n*ln(x))+c^2/d^4*(a*d-b*c)/n*ln(c+d*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{2\,bd^3x^{3\,n}-3\,(bcd^2-ad^3)x^{2\,n}+6\,(bc^2d-acd^2)x^n-6\,(bc^3-ac^2d)\log\left(dx^n+c\right)}{6\,d^4n}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)
*x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(71) = 142$.

Time = 3.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0 \wedge n=0 \\ \frac{\frac{axx^{3n-1}}{3n}+\frac{bxx^nx^{3n-1}}{4n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{acx^n}{d^2n} + \frac{ax^{2n}}{2dn} - \frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{bc^2x^n}{d^3n} - \frac{bcx^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

output
```
Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x*x**(3*n - 1)/(3*n
) + b*x*x**n*x**(3*n - 1)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq
(n, 0)), (a*c**2*log(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)
/(2*d*n) - b*c**3*log(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x*
*(2*n)/(2*d**2*n) + b*x**(3*n)/(3*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = -\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n} - \frac{2\,d^2x^{3\,n} - 3\,cdx^{2\,n} + 6\,c^2x^n}{d^3n}\right) + \frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2\,n} - 2\,cx^n}{d^2n}\right)$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n)
+ 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*
n) - 2*c*x^n)/(d^2*n))
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)}{c+d\,x^n}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx = \frac{2x^{3n}b\,d^3+3x^{2n}a\,d^3-3x^{2n}bc\,d^2-6x^nac\,d^2+6x^nb\,c^2d+6\log(x^nd+c)\,a\,c^2d-6\log(x^nd+c)\,b\,c^3}{6d^4n}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x)
```

output
```
(2*x**(3*n)*b*d**3 + 3*x**(2*n)*a*d**3 - 3*x**(2*n)*b*c*d**2 - 6*x**n*a*c*
d**2 + 6*x**n*b*c**2*d + 6*log(x**n*d + c)*a*c**2*d - 6*log(x**n*d + c)*b*
c**3)/(6*d**4*n)
```

3.461 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{x^n}{bdn} + \frac{a^2\log(a+bx^n)}{b^2(bc-ad)n} - \frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}$$

output
```
x^n/b/d/n+a^2*ln(a+b*x^n)/b^2/(-a*d+b*c)/n-c^2*ln(c+d*x^n)/d^2/(-a*d+b*c)/
n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2d^2\log(a+bx^n)+b(d(bc-ad)x^n-bc^2\log(c+dx^n))}{b^2d^2(bc-ad)n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(a^2*d^2*Log[a + b*x^n] + b*(d*(b*c - a*d)*x^n - b*c^2*Log[c + d*x^n]))/(b
^2*d^2*(b*c - a*d)*n)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\log(a+bx^n)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)}+\frac{x^n}{bd}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])
/(d^2*(b*c - a*d)))/n
```

Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{e^{n\ln(x)}}{bdn}+\frac{c^2\ln\left(c+d\,e^{n\ln(x)}\right)}{d^2n(ad-cb)}-\frac{a^2\ln\left(a+b\,e^{n\ln(x)}\right)}{(ad-cb)b^2n}$	78
risch	$-\frac{\ln(x)a}{b^2d}-\frac{\ln(x)c}{b\,d^2}+\frac{x^n}{bdn}-\frac{\ln(x)c^2}{d^2(ad-cb)}+\frac{\ln(x)a^2}{(ad-cb)b^2}+\frac{c^2\ln\left(x^n+\frac{c}{d}\right)}{d^2n(ad-cb)}-\frac{a^2\ln\left(x^n+\frac{a}{b}\right)}{(ad-cb)b^2n}$	137

input
```
int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(c+d*exp(n*ln(x)))-a^2/(a*d-b*c
)/b^2/n*ln(a+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx=\frac{a^2d^2\log(bx^n+a)-b^2c^2\log(dx^n+c)+(b^2cd-abd^2)x^n}{(b^3cd^2-ab^2d^3)n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
(a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n
)/((b^3*c*d^2 - a*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn-ab^2dn} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n-ad^3n} + \frac{x^n}{bdn}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c
*d^2*n - a*d^3*n) + x^n/(b*d*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{x^{3\,n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx = \frac{x^n ab\,d^2 - x^n b^2cd - \log(x^n b+a)\,a^2d^2 + \log(x^n d+c)\,b^2c^2}{b^2d^2n\,(ad-bc)}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x)
```

output
```
(x**n*a*b*d**2 - x**n*b**2*c*d - log(x**n*b + a)*a**2*d**2 + log(x**n*d +
c)*b**2*c**2)/(b**2*d**2*n*(a*d - b*c))
```

3.462 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 95

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} - \frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}$$

output
```
-a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)-a*(-a*d+2*b*c)*ln(a+b*x^n)/b^2/(-a*d+b*c)^
2/n+c^2*ln(c+d*x^n)/d/(-a*d+b*c)^2/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2}{b^2(bc-ad)n(a+bx^n)} + \frac{a(-2bc+ad)\log(a+bx^n)}{b^2(bc-ad)^2n} + \frac{c^2\log(c+dx^n)}{d(-bc+ad)^2n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
-(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) + (a*(-2*b*c + a*d)*Log[a + b*x^n])
/(b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(-(b*c) + a*d)^2*n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^2(c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^2(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)^2}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^n+a)}+\frac{c^2}{(bc-ad)^2(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2))/n
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

method	result
norman	$\frac{a^2}{(ad-cb)b^2n\left(a+b\,e^{n\ln(x)}\right)}+\frac{c^2\ln\left(c+d\,e^{n\ln(x)}\right)}{dn(a^2d^2-2abcd+b^2c^2)}+\frac{a(ad-2cb)\ln\left(a+b\,e^{n\ln(x)}\right)}{(a^2d^2-2abcd+b^2c^2)b^2n}$
risch	$\frac{\ln(x)}{b^2d}-\frac{\ln(x)c^2}{d(a^2d^2-2abcd+b^2c^2)}-\frac{\ln(x)a^2d}{(a^2d^2-2abcd+b^2c^2)b^2}+\frac{2\ln(x)ac}{(a^2d^2-2abcd+b^2c^2)b}+\frac{a^2}{(ad-cb)b^2n(a+b\,x^n)}+\frac{c^2\ln(x^n+}{dn(a^2d^2-2abca}$

input
```
int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
a^2/(a*d-b*c)/b^2/n/(a+b*exp(n*ln(x)))+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)
*ln(c+d*exp(n*ln(x)))+a*(a*d-2*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(a
+b*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.75

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = -\frac{a^2bcd - a^3d^2 + (2\,a^2bcd - a^3d^2 + (2\,ab^2cd - a^2bd^2)x^n)\log\left(bx^n + a\right) - (b^3c^2x^n + ab^2c^2)\log\left(dx^n + c\right)}{(b^5c^2d - 2\,ab^4cd^2 + a^2b^3d^3)nx^n + (ab^4c^2d - 2\,a^2b^3cd^2 + a^3b^2d^3)n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
-(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)
*x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2
*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 +
 a^3*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^2dn - 2\,abcd^2n + a^2d^3n} - \frac{a^2}{ab^3cn - a^2b^2dn + (b^4cn - ab^3dn)x^n} - \frac{(2\,abc - a^2d)\log\left(\frac{bx^n+a}{b}\right)}{b^4c^2n - 2\,ab^3cdn + a^2b^2d^2n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*
b^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log
((b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.04

$$\int \frac{x^{-1+3n}}{(a+bx^n)^2 (c+dx^n)}\,dx$$
$$= \frac{x^n \log(x^n b+a)\, a^2 b\, d^2 - 2x^n \log(x^n b+a)\, a\, b^2 cd + x^n \log(x^n d+c)\, b^3 c^2 - x^n a^2 b\, d^2 + x^n a\, b^2 cd + \log(x^n b +}{b^2 dn\,(x^n a^2 b\, d^2 - 2x^n a\, b^2 cd + x^n b^3 c^2 + a^3 d^2 - 2a^2 bcd + a}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
(x**n*log(x**n*b + a)*a**2*b*d**2 - 2*x**n*log(x**n*b + a)*a*b**2*c*d + x*
*n*log(x**n*d + c)*b**3*c**2 - x**n*a**2*b*d**2 + x**n*a*b**2*c*d + log(x*
*n*b + a)*a**3*d**2 - 2*log(x**n*b + a)*a**2*b*c*d + log(x**n*d + c)*a*b**
2*c**2)/(b**2*d*n*(x**n*a**2*b*d**2 - 2*x**n*a*b**2*c*d + x**n*b**3*c**2 +
 a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2))
```

3.463 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = -\frac{a^2}{2b^2(bc-ad)n(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2n(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3n}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3n}$$

output
```
-1/2*a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)^2+a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/n/(a
+b*x^n)+c^2*ln(a+b*x^n)/(-a*d+b*c)^3/n-c^2*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{\frac{a(-bc+ad)(-3abc+a^2d-4b^2cx^n+2abdx^n)}{b^2(a+bx^n)^2}+2c^2\log(a+bx^n)-2c^2\log(c+dx^n)}{2(bc-ad)^3n}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
((a*(-(b*c) + a*d)*(-3*a*b*c + a^2*d - 4*b^2*c*x^n + 2*a*b*d*x^n))/(b^2*(a
 + b*x^n)^2) + 2*c^2*Log[a + b*x^n] - 2*c^2*Log[c + d*x^n])/(2*(b*c - a*d)
^3*n)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {948, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^3(c+dx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^3(dx^n+c)}dx^n}{n}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{a^2}{b(bc-ad)(bx^n+a)^3}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^n+a)^2}+\frac{bc^2}{(bc-ad)^3(bx^n+a)}-\frac{c^2d}{(bc-ad)^3(dx^n+c)}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c -
a*d)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*
x^n])/(b*c - a*d)^3)/n
```

Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{a\left(2abd\,x^{n}-4b^{2}c\,x^{n}+a^{2}d-3abc\right)}{2\left(ad-cb\right)^{2}b^{2}n\left(a+b\,x^{n}\right)^{2}}+\frac{c^{2}\ln\left(x^{n}+\frac{c}{d}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{c^{2}\ln\left(x^{n}+\frac{a}{b}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$
norman	$\frac{\frac{\left(-ad+2cb\right)a\,{\mathrm e}^{n\ln\left(x\right)}}{nb\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{a^{2}\left(-ad+3cb\right)}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)b^{2}n}}{\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)^{2}}+\frac{c^{2}\ln\left(c+d\,{\mathrm e}^{n\ln\left(x\right)}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{c^{2}\ln\left(a+b\,{\mathrm e}^{n\ln\left(x\right)}\right)}{n\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$

input
```
int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(2*a*b*d*x^n-4*b^2*c*x^n+a^2*d-3*a*b*c)/(a*d-b*c)^2/b^2/n/(a+b*x^n)
^2+c^2/n/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+c/d)-c^2/n/(
a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(x^n+a/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(118) = 236.

Time = 0.11 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.51

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{3a^2b^2c^2 - 4a^3bcd + a^4d^2 + 2(2ab^3c^2 - 3a^2b^2cd + a^3bd^2)x^n + 2(b^4c^2x^{2n} + 2ab^3c^2x^n + a^2b^2c^2)\log(bx^n}{2((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)nx^{2n} + 2(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)nx^n +}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*
d + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*l
og(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*
x^n + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(
2*n) + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x
^n + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(118) = 236$.

Time = 0.04 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \frac{c^2\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n-3\,ab^2c^2dn+3\,a^2bcd^2n-a^3d^3n} + \frac{3\,a^2bc-a^3d+2\,(2\,ab^2c-a^2bd)x^n}{2\,(a^2b^4c^2n-2\,a^3b^3cdn+a^4b^2d^2n+(b^6c^2n-2\,ab^5cdn+a^2b^4d^2n)x^{2\,n}+2\,(ab^5c^2n-2\,a^2b^4cdn+a^3b^3d}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c
*d^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n
)/(a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*
c*d*n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^
3*d^2*n)*x^n)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^3(dx^n+c)}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^3\left(c+dx^n\right)}\,dx=\int \frac{x^{3\,n-1}}{\left(a+b\,x^n\right)^3\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.80

$$\int \frac{x^{-1+3n}}{\left(a+bx^n\right)^3\left(c+dx^n\right)}\,dx = \frac{-2x^{2n}\log(x^nb+a)\,b^3c^2+2x^{2n}\log(x^nd+c)\,b^3c^2+x^{2n}a^2b\,d^2-3x^{2n}a\,b^2cd+2x^{2n}b^3c^2-4x^n\log(x^nb+a)}{2bn\left(x^{2n}a^3b^2d^3-3x^{2n}a^2b^3c\,d^2+3x^{2n}a\,b^4c^2d-x^{2n}b^5c^3+2x^na^4b\,d^3-6x^na^3b^2c\right.}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
( - 2*x**(2*n)*log(x**n*b + a)*b**3*c**2 + 2*x**(2*n)*log(x**n*d + c)*b**3
*c**2 + x**(2*n)*a**2*b*d**2 - 3*x**(2*n)*a*b**2*c*d + 2*x**(2*n)*b**3*c**
2 - 4*x**n*log(x**n*b + a)*a*b**2*c**2 + 4*x**n*log(x**n*d + c)*a*b**2*c**
2 - 2*log(x**n*b + a)*a**2*b*c**2 + 2*log(x**n*d + c)*a**2*b*c**2 + a**3*c
*d - a**2*b*c**2)/(2*b*n*(x**(2*n)*a**3*b**2*d**3 - 3*x**(2*n)*a**2*b**3*c
*d**2 + 3*x**(2*n)*a*b**4*c**2*d - x**(2*n)*b**5*c**3 + 2*x**n*a**4*b*d**3
 - 6*x**n*a**3*b**2*c*d**2 + 6*x**n*a**2*b**3*c**2*d - 2*x**n*a*b**4*c**3
+ a**5*d**3 - 3*a**4*b*c*d**2 + 3*a**3*b**2*c**2*d - a**2*b**3*c**3))
```

3.464 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

output
```
1/14*x^14*(c*x+b)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(14) = 28.

Time = 0.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 12.29

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

input
```
Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

output
```
(b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (
143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7
*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^2
4)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)
/14
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx$$
$$\Big\downarrow \; 83$$
$$\frac{1}{14}x^{14}(b+cx)^{14}$$

input
```
Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

output
```
(x^14*(b + c*x)^14)/14
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
orering	$\frac{x^{14}(cx+b)^{14}}{14}$
gosper	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c$
default	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c$
norman	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c$
risch	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c$
parallelrisch	$143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}b^{14}x^{14}+b^{13}c$

input
```
int(x^13*(c*x+b)^13*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/14*x^14*(c*x+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\begin{aligned}\int x^{13}(b+cx)^{13}(b+2cx)\,dx &= \frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}\\&\quad+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}\\&\quad+\frac{429}{2}\,b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}\,b^{10}c^4x^{18}\\&\quad+26\,b^{11}c^3x^{17}+\frac{13}{2}\,b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}\end{aligned}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 12.50

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} + 143b^5c^9x^{23} + \frac{143b^4c^{10}x^{24}}{2} + 26b^3c^{11}x^{25} + \frac{13b^2c^{12}x^{26}}{2} + bc^{13}x^{27} + \frac{c^{14}x^{28}}{14}$$

input
```
integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

output
```
b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**
17 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2
+ 1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 1
43*b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c*
*13*x**27 + c**14*x**28/14
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}b^{10}c^4x^{18} + 26\,b^{11}c^3x^{17}+\frac{13}{2}b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}b^{14}x^{14}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

output
```
1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143
/2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*
x^21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^
11*c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{1}{14}\left(cx^2+bx\right)^{14}$$

input
```
integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

output
```
1/14*(c*x^2 + b*x)^14
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 154, normalized size of antiderivative = 11.00

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

input
```
int(x^13*(b + c*x)^13*(b + 2*c*x),x)
```

output
```
(b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2
*x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (
429*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b
^5*c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26
)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 10.93

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx = \frac{x^{14}(c^{14}x^{14}+14b\,c^{13}x^{13}+91b^2c^{12}x^{12}+364b^3c^{11}x^{11}+1001b^4c^{10}x^{10}+2002b^5c^9x^9+3003b^6c^8x^8+3432b^7c}{14}$$

input
```
int(x^13*(c*x+b)^13*(2*c*x+b),x)
```

output
```
(x**14*(b**14 + 14*b**13*c*x + 91*b**12*c**2*x**2 + 364*b**11*c**3*x**3 +
1001*b**10*c**4*x**4 + 2002*b**9*c**5*x**5 + 3003*b**8*c**6*x**6 + 3432*b*
*7*c**7*x**7 + 3003*b**6*c**8*x**8 + 2002*b**5*c**9*x**9 + 1001*b**4*c**10
*x**10 + 364*b**3*c**11*x**11 + 91*b**2*c**12*x**12 + 14*b*c**13*x**13 + c
**14*x**14))/14
```

3.465 $\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx=\frac{1}{28}x^{28}(b+cx^2)^{14}$$

output
```
1/28*x^28*(c*x^2+b)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{27}(b+cx^2)^{13}(b+2cx^2)\,dx=\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

input
```
Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

output
```
(b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (
858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b
^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/
2 + (c^14*x^56)/28
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{354, 83\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{27} \left(b+cx^2\right)^{13} \left(b+2cx^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^{26} \left(cx^2+b\right)^{13} \left(2cx^2+b\right) dx^2$$

$$\Big\downarrow \text{83}$$

$$\frac{1}{28} x^{28} \left(b+cx^2\right)^{14}$$

input
```
Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]
```

output
```
(x^28*(b + c*x^2)^14)/28
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
orering	$\frac{x^{28}\left(c\,x^{2}+b\right)^{14}}{28}$
gosper	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{13}{4}b^{2}c^{12}x^{52}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{143}{4}b^{4}c^{10}x^{48}+13b^{3}c^{11}x^{50}+$
default	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{13}{4}b^{2}c^{12}x^{52}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{143}{4}b^{4}c^{10}x^{48}+13b^{3}c^{11}x^{50}+$
risch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{13}{4}b^{2}c^{12}x^{52}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{143}{4}b^{4}c^{10}x^{48}+13b^{3}c^{11}x^{50}+$
parallelrisch	$\frac{1}{2}b\,c^{13}x^{54}+\frac{1}{28}c^{14}x^{56}+\frac{13}{4}b^{2}c^{12}x^{52}+\frac{429}{4}b^{6}c^{8}x^{44}+\frac{143}{2}b^{5}c^{9}x^{46}+\frac{143}{4}b^{4}c^{10}x^{48}+13b^{3}c^{11}x^{50}+$

input
```
int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
1/28*x^28*(c*x^2+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6x^{40} + \frac{143}{2}\,b^9c^5x^{38} + \frac{143}{4}\,b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}\,b^{12}c^2x^{32} + \frac{1}{2}\,b^{13}cx^{30} + \frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 11.38

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + \frac{143b^5c^9x^{46}}{2} + \frac{143b^4c^{10}x^{48}}{4} + 13b^3c^{11}x^{50} + \frac{13b^2c^{12}x^{52}}{4} + \frac{bc^{13}x^{54}}{2} + \frac{c^{14}x^{56}}{28}$$

input
```
integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

output
```
b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x
**34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**4
0/4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/
2 + 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 +
 b*c**13*x**54/2 + c**14*x**56/28
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8c^6x^{40}+\frac{143}{2}\,b^9c^5x^{38}+\frac{143}{4}\,b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}\,b^{12}c^2x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8c^6x^{40} + \frac{143}{2}b^9c^5x^{38} + \frac{143}{4}b^{10}c^4x^{36} + 13\,b^{11}c^3x^{34} + \frac{13}{4}b^{12}c^2x^{32} + \frac{1}{2}b^{13}cx^{30} + \frac{1}{28}b^{14}x^{28}$$

input
```
integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

output
```
1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
 143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7
*c^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36
+ 13*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx = \frac{b^{14}\,x^{28}}{28} + \frac{b^{13}\,c\,x^{30}}{2} + \frac{13\,b^{12}\,c^2\,x^{32}}{4} + 13\,b^{11}\,c^3\,x^{34} + \frac{143\,b^{10}\,c^4\,x^{36}}{4} + \frac{143\,b^9\,c^5\,x^{38}}{2} + \frac{429\,b^8\,c^6\,x^{40}}{4} + \frac{858\,b^7\,c^7\,x^{42}}{7} + \frac{429\,b^6\,c^8\,x^{44}}{4} + \frac{143\,b^5\,c^9\,x^{46}}{2} + \frac{143\,b^4\,c^{10}\,x^{48}}{4} + 13\,b^3\,c^{11}\,x^{50} + \frac{13\,b^2\,c^{12}\,x^{52}}{4} + \frac{b\,c^{13}\,x^{54}}{2} + \frac{c^{14}\,x^{56}}{28}$$

input
```
int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)
```

output
```
(b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*
b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5
*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44
)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (1
3*b^2*c^12*x^52)/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 9.69

$$\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$
$$= \frac{x^{28}(c^{14}x^{28} + 14b\,c^{13}x^{26} + 91b^2c^{12}x^{24} + 364b^3c^{11}x^{22} + 1001b^4c^{10}x^{20} + 2002b^5c^9x^{18} + 3003b^6c^8x^{16} + 3432b}{28}$$

input
```
int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x)
```

output
```
(x**28*(b**14 + 14*b**13*c*x**2 + 91*b**12*c**2*x**4 + 364*b**11*c**3*x**6
 + 1001*b**10*c**4*x**8 + 2002*b**9*c**5*x**10 + 3003*b**8*c**6*x**12 + 34
32*b**7*c**7*x**14 + 3003*b**6*c**8*x**16 + 2002*b**5*c**9*x**18 + 1001*b*
*4*c**10*x**20 + 364*b**3*c**11*x**22 + 91*b**2*c**12*x**24 + 14*b*c**13*x
**26 + c**14*x**28))/28
```

3.466 $\int x^{41}(b+cx^3)^{13}(b+2cx^3)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

output
```
1/42*x^42*(c*x^3+b)^14
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx=\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

input
```
Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

output
```
(b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^5
1)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2
 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (1
43*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^1
3*x^81)/3 + (c^14*x^84)/42
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{41} \left(b+cx^3\right)^{13} \left(b+2cx^3\right) \, dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3} \int x^{39} \left(cx^3+b\right)^{13} \left(2cx^3+b\right)dx^3$$

$$\Big\downarrow \text{ 83}$$

$$\frac{1}{42} x^{42} \left(b+cx^3\right)^{14}$$

input
```
Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

output
```
(x^42*(b + c*x^3)^14)/42
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
orering	$\frac{x^{42} \left(c \,x^{3}+b \right)^{14}}{42}$
gosper	$\frac{1}{42} c^{14} x^{84}+\frac{143}{2} b^{6} c^{8} x^{66}+\frac{143}{3} b^{5} c^{9} x^{69}+\frac{143}{6} b^{4} c^{10} x^{72}+\frac{26}{3} b^{3} c^{11} x^{75}+\frac{13}{6} b^{2} c^{12} x^{78}+\frac{1}{3} b \,c^{13} x^{81}+$
default	$\frac{1}{42} c^{14} x^{84}+\frac{143}{2} b^{6} c^{8} x^{66}+\frac{143}{3} b^{5} c^{9} x^{69}+\frac{143}{6} b^{4} c^{10} x^{72}+\frac{26}{3} b^{3} c^{11} x^{75}+\frac{13}{6} b^{2} c^{12} x^{78}+\frac{1}{3} b \,c^{13} x^{81}+$
risch	$\frac{1}{42} c^{14} x^{84}+\frac{143}{2} b^{6} c^{8} x^{66}+\frac{143}{3} b^{5} c^{9} x^{69}+\frac{143}{6} b^{4} c^{10} x^{72}+\frac{26}{3} b^{3} c^{11} x^{75}+\frac{13}{6} b^{2} c^{12} x^{78}+\frac{1}{3} b \,c^{13} x^{81}+$
parallelrisch	$\frac{1}{42} c^{14} x^{84}+\frac{143}{2} b^{6} c^{8} x^{66}+\frac{143}{3} b^{5} c^{9} x^{69}+\frac{143}{6} b^{4} c^{10} x^{72}+\frac{26}{3} b^{3} c^{11} x^{75}+\frac{13}{6} b^{2} c^{12} x^{78}+\frac{1}{3} b \,c^{13} x^{81}+$

input
```
int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
1/42*x^42*(c*x^3+b)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 185, normalized size of antiderivative = 11.56

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+\frac{143b^5c^9x^{69}}{3}+\frac{143b^4c^{10}x^{72}}{6}+\frac{26b^3c^{11}x^{75}}{3}+\frac{13b^2c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

input
```
integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

output
```
b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x
**51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x*
*60/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**6
9/3 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78
/6 + b*c**13*x**81/3 + c**14*x**84/42
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\begin{aligned}\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx &= \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}\\&+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}\\&+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}\\&+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}\\&+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}\end{aligned}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6x^{60}+\frac{143}{3}\,b^9c^5x^{57}+\frac{143}{6}\,b^{10}c^4x^{54}+\frac{26}{3}\,b^{11}c^3x^{51}+\frac{13}{6}\,b^{12}c^2x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

input
```
integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

output
```
1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
 + 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b
^7*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^5
4 + 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*
x^42
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx = \frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

input
```
int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

output
```
(b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*
b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9
*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*
x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75
)/3 + (13*b^2*c^12*x^78)/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 9.69

$$\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$
$$=\frac{x^{42}(c^{14}x^{42}+14b\,c^{13}x^{39}+91b^2c^{12}x^{36}+364b^3c^{11}x^{33}+1001b^4c^{10}x^{30}+2002b^5c^9x^{27}+3003b^6c^8x^{24}+3432b}{42}$$

input
```
int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x)
```

output
```
(x**42*(b**14 + 14*b**13*c*x**3 + 91*b**12*c**2*x**6 + 364*b**11*c**3*x**9
 + 1001*b**10*c**4*x**12 + 2002*b**9*c**5*x**15 + 3003*b**8*c**6*x**18 + 3
432*b**7*c**7*x**21 + 3003*b**6*c**8*x**24 + 2002*b**5*c**9*x**27 + 1001*b
**4*c**10*x**30 + 364*b**3*c**11*x**33 + 91*b**2*c**12*x**36 + 14*b*c**13*
x**39 + c**14*x**42))/42
```

3.467 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 21

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

output
```
1/14*x^(14*n)*(b+c*x^n)^14/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14n-1}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int x^{13n}(cx^n+b)^{13}(2cx^n+b)\,dx^n}{n}$$
$$\Big\downarrow \text{83}$$
$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

input
```
Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]
```

output
```
(x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 214.13 (sec) , antiderivative size = 230, normalized size of antiderivative = 10.95

method	result
risch	$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}+\frac{26b^3c^{11}x^{25n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{143b^5c^9x^{23n}}{n}+\frac{429b^6c^8x^{22n}}{2n}+\frac{1716b^7c^7x}{7n}$
parallelrisch	$\frac{c^{14}x^{-1+14n}x^{14n}x+14b\,c^{13}x^{-1+14n}x^{13n}x+91b^2c^{12}x^{-1+14n}x^{12n}x+364b^3c^{11}x^{-1+14n}x^{11n}x+1001b^4c^{10}x^{-1+14n}x^{10n}x+20}{}$
orering	Expression too large to display

input
```
int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^1
1/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^
8/n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^
5/n*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^
2/n*(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$=\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^2c^{12}x^{26\,n}+364\,b^3c^{11}x^{25\,n}+1001\,b^4c^{10}x^{24\,n}+2002\,b^5c^9x^{23\,n}+3003\,b^6c^8x^{22\,n}}{}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.

Time = 13.79 (sec) , antiderivative size = 360, normalized size of antiderivative = 17.14

$$\int x^{-1+14n}(b+cx^n)^{13}\,(b+2cx^n)\;dx$$
$$=\begin{cases} \frac{b^{14}xx^{14n-1}}{14n}+\frac{b^{13}cxx^nx^{14n-1}}{n}+\frac{13b^{12}c^2xx^{2n}x^{14n-1}}{2n}+\frac{26b^{11}c^3xx^{3n}x^{14n-1}}{n}+\frac{143b^{10}c^4xx^{4n}x^{14n-1}}{2n}+\frac{143b^9c^5xx^{5n}x^{14n-1}}{n}+42 \\ (b+c)^{13}\,(b+2c)\log{(x)} \end{cases}$$

input
```
integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
```

output
```
Piecewise((b**14*x*x**(14*n - 1)/(14*n) + b**13*c*x*x**n*x**(14*n - 1)/n +
 13*b**12*c**2*x*x**(2*n)*x**(14*n - 1)/(2*n) + 26*b**11*c**3*x*x**(3*n)*x
**(14*n - 1)/n + 143*b**10*c**4*x*x**(4*n)*x**(14*n - 1)/(2*n) + 143*b**9*
c**5*x*x**(5*n)*x**(14*n - 1)/n + 429*b**8*c**6*x*x**(6*n)*x**(14*n - 1)/(
2*n) + 1716*b**7*c**7*x*x**(7*n)*x**(14*n - 1)/(7*n) + 429*b**6*c**8*x*x**
(8*n)*x**(14*n - 1)/(2*n) + 143*b**5*c**9*x*x**(9*n)*x**(14*n - 1)/n + 143
*b**4*c**10*x*x**(10*n)*x**(14*n - 1)/(2*n) + 26*b**3*c**11*x*x**(11*n)*x*
*(14*n - 1)/n + 13*b**2*c**12*x*x**(12*n)*x**(14*n - 1)/(2*n) + b*c**13*x*
x**(13*n)*x**(14*n - 1)/n + c**14*x*x**(14*n)*x**(14*n - 1)/(14*n), Ne(n,
0)), ((b + c)**13*(b + 2*c)*log(x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{-1+14n}(b+cx^n)^{13}\left(b+2cx^n\right)\,dx = \frac{c^{14}x^{28\,n}}{14\,n} + \frac{bc^{13}x^{27\,n}}{n} + \frac{13\,b^2c^{12}x^{26\,n}}{2\,n} + \frac{26\,b^3c^{11}x^{25\,n}}{n} + \frac{143\,b^4c^{10}x^{24\,n}}{2\,n} + \frac{143\,b^5c^9x^{23\,n}}{n} + \frac{429\,b^6c^8x^{22\,n}}{2\,n} + \frac{1716\,b^7c^7x^{21\,n}}{7\,n} + \frac{429\,b^8c^6x^{20\,n}}{2\,n} + \frac{143\,b^9c^5x^{19\,n}}{n} + \frac{143\,b^{10}c^4x^{18\,n}}{2\,n} + \frac{26\,b^{11}c^3x^{17\,n}}{n} + \frac{13\,b^{12}c^2x^{16\,n}}{2\,n} + \frac{b^{13}cx^{15\,n}}{n} + \frac{b^{14}x^{14\,n}}{14\,n}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

output
```
1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b
^3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n +
429/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20
*n)/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x
^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14
*n)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 9.00

$$\int x^{-1+14n}(b+cx^n)^{13}\left(b+2cx^n\right)\,dx$$
$$= \frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n}}{}$$

input
```
integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

output
```
1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*
c^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*
c^8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^
5*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*
x^(16*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 229, normalized size of antiderivative = 10.90

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx = \frac{b^{14}x^{14n}}{14n} + \frac{c^{14}x^{28n}}{14n} + \frac{13b^{12}c^2x^{16n}}{2n} + \frac{26b^{11}c^3x^{17n}}{n} + \frac{143b^{10}c^4x^{18n}}{2n} + \frac{143b^9c^5x^{19n}}{n} + \frac{429b^8c^6x^{20n}}{2n} + \frac{1716b^7c^7x^{21n}}{7n} + \frac{429b^6c^8x^{22n}}{2n} + \frac{143b^5c^9x^{23n}}{n} + \frac{143b^4c^{10}x^{24n}}{2n} + \frac{26b^3c^{11}x^{25n}}{n} + \frac{13b^2c^{12}x^{26n}}{2n} + \frac{b^{13}c\,x^{15n}}{n} + \frac{b\,c^{13}x^{27n}}{n}$$

input
```
int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

output
```
(b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(
2*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9
*c^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/
(7*n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4
*c^10*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/
(2*n) + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 8.86

$$\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx = \frac{x^{14n}(x^{14n}c^{14}+14x^{13n}b\,c^{13}+91x^{12n}b^2c^{12}+364x^{11n}b^3c^{11}+1001x^{10n}b^4c^{10}+2002x^{9n}b^5c^9+3003x^{8n}b^6c^8+}{1}$$

input
```
int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x)
```

output
```
(x**(14*n)*(x**(14*n)*c**14 + 14*x**(13*n)*b*c**13 + 91*x**(12*n)*b**2*c**
12 + 364*x**(11*n)*b**3*c**11 + 1001*x**(10*n)*b**4*c**10 + 2002*x**(9*n)*
b**5*c**9 + 3003*x**(8*n)*b**6*c**8 + 3432*x**(7*n)*b**7*c**7 + 3003*x**(6
*n)*b**8*c**6 + 2002*x**(5*n)*b**9*c**5 + 1001*x**(4*n)*b**10*c**4 + 364*x
**(3*n)*b**11*c**3 + 91*x**(2*n)*b**12*c**2 + 14*x**n*b**13*c + b**14))/(1
4*n)
```

3.468 $\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 13

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=x^m(a+bx^n)^p$$

output
```
x^m*(a+b*x^n)^p
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 8.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$=\frac{x^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(a(m+n)\,\text{Hypergeometric2F1}\left(\frac{m}{n},1-p,\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(m+np)x^n\,\text{Hyperg}\right.}{a(m+n)}$$

input
```
Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

output
```
(x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((
b*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n,
-((b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}(a+bx^n)^{p-1}\left(am+bx^n(m+np)\right)\,dx$$

$$\Big\downarrow \text{951}$$

$$x^m(a+bx^n)^p$$

input
```
Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

output
```
x^m*(a + b*x^n)^p
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(13) = 26$.

Time = 8.84 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.85

method	result	size
parallelrisch	$\frac{x\,x^{n} x^{m-1} (a+b\,x^{n})^{p-1} b^{2}+x\,x^{m-1} (a+b\,x^{n})^{p-1} a b}{b}$	50

input
```
int(x^(m-1)*(a+b*x^n)^(p-1)*(a*m+b*(n*p+m)*x^n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*x^(m-1)*(a+b*x^n)^(p-1)*b^2+x*x^(m-1)*(a+b*x^n)^(p-1)*a*b)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(13) = 26$.

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx = \left(bxx^{m-1}x^n + axx^{m-1}\right)(bx^n+a)^{p-1}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="fric
as")
```

output
```
(b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(10) = 20$.

Time = 2.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.00

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$= axx^{m-1}(a+bx^n)^{p-1} + bxx^nx^{m-1}(a+bx^n)^{p-1}$$

input
```
integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

output
```
a*x*x**(m - 1)*(a + b*x**n)**(p - 1) + b*x*x**n*x**(m - 1)*(a + b*x**n)**(
p - 1)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=e^{(p\log(bx^n+a)+m\log(x))}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxi
ma")
```

output
```
e^(p*log(b*x^n + a) + m*log(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(13) = 26.

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 5.38

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx$$
$$=bxx^ne^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}+axe^{(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x))}$$

input
```
integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac
")
```

output
```
b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^
(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=\left(a\,x^m+b\,x^{m+n}\right)\,\left(a+b\,x^n\right)^{p-1}$$

input
```
int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
```

output
```
(a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)\,dx=x^m(x^nb+a)^p$$

input
```
int(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x)
```

output
```
x**m*(x**n*b + a)**p
```

3.469 $\int \frac{b+2cx}{x(b+cx)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{b+2cx}{x(b+cx)} \, dx = \log(x(b+cx))$$

output
```
ln(x*(c*x+b))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)} \, dx = \log(x) + \log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
```

output
```
Log[x] + Log[b + c*x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{x(b+cx)} \, dx$$
$$\Big\downarrow \; 86$$
$$\int \left(\frac{c}{b+cx}+\frac{1}{x}\right) dx$$
$$\Big\downarrow \; 2009$$
$$\log(b+cx)+\log(x)$$

input
```
Int[(b + 2*c*x)/(x*(b + c*x)),x]
```

output
```
Log[x] + Log[b + c*x]
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
risch	$\ln\left(c\,x^{2}+bx\right)$	11

input
```
int((2*c*x+b)/x/(c*x+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x)
```

output
```
log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(cx+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
```

output
```
log(c*x + b) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log\left(|cx+b|\right)+\log\left(|x|\right)$$

input
```
integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
```

output
```
log(abs(c*x + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \ln\left(x\left(b+c\,x\right)\right)$$

input
```
int((b + 2*c*x)/(x*(b + c*x)),x)
```

output
```
log(x*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{b+2cx}{x(b+cx)}\,dx = \log(cx+b)+\log(x)$$

input
```
int((2*c*x+b)/x/(c*x+b),x)
```

output
```
log(b + c*x) + log(x)
```

3.470 $\int \frac{b+2cx^2}{x(b+cx^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^2}{x(b+cx^2)} dx = \log(x) + \frac{1}{2}\log\left(b+cx^2\right)$$

output
```
ln(x)+1/2*ln(c*x^2+b)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x(b+cx^2)} dx = \log(x) + \frac{1}{2}\log\left(b+cx^2\right)$$

input
```
Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

output
```
Log[x] + Log[b + c*x^2]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^2\left(cx^2+b\right)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{cx^2+b}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\log\left(b+cx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]
```

output
```
(Log[x^2] + Log[b + c*x^2])/2
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^{2}+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(c\,x^{2}+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(c\,x^{2}+b)}{2}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^{2}+b)}{2}$	14

input
```
int((2*c*x^2+b)/x/(c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/2*ln(c*x^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx=\frac{1}{2}\,\log\left(cx^2+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
```

output
```
1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^2}{x\,(b+cx^2)}\,dx = \log\left(x\right) + \frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

input
```
integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

output
```
log(x) + log(b/c + x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^2}{x\,(b+cx^2)}\,dx = \frac{1}{2}\log\left(cx^2+b\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
```

output
```
1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{b+2cx^2}{x\,(b+cx^2)}\,dx = \frac{1}{2}\log\left(x^2\right) + \frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

input
```
integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
```

output
```
1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \frac{\ln(c\,x^2+b)}{2}+\ln(x)$$

input
```
int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
```

output
```
log(b + c*x^2)/2 + log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^2}{x(b+cx^2)}\,dx = \frac{\log(c\,x^2+b)}{2}+\log(x)$$

input
```
int((2*c*x^2+b)/x/(c*x^2+b),x)
```

output
```
(log(b + c*x**2) + 2*log(x))/2
```

3.471 $\int \frac{b+2cx^3}{x(b+cx^3)} dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^3}{x(b+cx^3)} dx = \log(x) + \frac{1}{3}\log\left(b+cx^3\right)$$

output
```
ln(x)+1/3*ln(c*x^3+b)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x(b+cx^3)} dx = \log(x) + \frac{1}{3}\log\left(b+cx^3\right)$$

input
```
Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

output
```
Log[x] + Log[b + c*x^3]/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x(b+cx^3)}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^3(cx^3+b)}dx^3$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{3}\int \left(\frac{c}{cx^3+b}+\frac{1}{x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\log\left(b+cx^3\right)+\log\left(x^3\right)\right)$$

input
```
Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

output
```
(Log[x^3] + Log[b + c*x^3])/3
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
parallelrisch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14

input
```
int((2*c*x^3+b)/x/(c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
ln(x)+1/3*ln(c*x^3+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\,\log\left(cx^3+b\right)+\log\left(x\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

output
```
1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

input
```
integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

output
```
log(x) + log(b/c + x**3)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

output
```
1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx=\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

input
```
integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

output
```
1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{\ln(c\,x^3+b)}{3} + \ln(x)$$

input
```
int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
```

output
```
log(b + c*x^3)/3 + log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.53

$$\int \frac{b+2cx^3}{x(b+cx^3)}\,dx = \frac{\log\left(b^{\frac{2}{3}} - c^{\frac{1}{3}}b^{\frac{1}{3}}x + c^{\frac{2}{3}}x^2\right)}{3} + \frac{\log\left(b^{\frac{1}{3}} + c^{\frac{1}{3}}x\right)}{3} + \log(x)$$

input
```
int((2*c*x^3+b)/x/(c*x^3+b),x)
```

output
```
(log(b**(2/3) - c**(1/3)*b**(1/3)*x + c**(2/3)*x**2) + log(b**(1/3) + c**(
1/3)*x) + 3*log(x))/3
```

3.472 $\int \frac{b+2cx^n}{x(b+cx^n)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{b+2cx^n}{x(b+cx^n)} \, dx = \log(x) + \frac{\log(b+cx^n)}{n}$$

output
```
ln(x)+ln(b+c*x^n)/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{b+2cx^n}{x(b+cx^n)} \, dx = \frac{\log(x^n) + \log(n(b+cx^n))}{n}$$

input
```
Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

output
```
(Log[x^n] + Log[n*(b + c*x^n)])/n
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^n}{x(b+cx^n)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{-n}(2cx^n+b)}{cx^n+b}dx^n}{n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(x^{-n}+\frac{c}{cx^n+b}\right)dx^n}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(b+cx^n)+\log(x^n)}{n}$$

input
```
Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

output
```
(Log[x^n] + Log[b + c*x^n])/n
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{\ln(x^n(b+cx^n))}{n}$	17
default	$\frac{\ln(x^n(b+cx^n))}{n}$	17
norman	$\ln(x)+\frac{\ln\left(b+c\,e^{n\ln(x)}\right)}{n}$	18
risch	$\ln(x)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18
parallelrisch	$\frac{n\ln(x)+\ln(b+cx^n)}{n}$	18

input
```
int((b+2*c*x^n)/x/(b+c*x^n),x,method=_RETURNVERBOSE)
```

output
```
1/n*ln(x^n*(b+c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx=\frac{n\log(x)+\log(cx^n+b)}{n}$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
```

output
```
(n*log(x) + log(c*x^n + b))/n
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx = \begin{cases} \log(x) & \text{for } c=0 \wedge (c=0 \vee n=0) \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

input
```
integrate((b+2*c*x**n)/x/(b+c*x**n),x)
```

output
```
Piecewise((log(x), Eq(c, 0) & (Eq(c, 0) | Eq(n, 0))), ((b + 2*c)*log(x)/(b
 + c), Eq(n, 0)), (log(x) + log(b/c + x**n)/n, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{b+2cx^n}{x(b+cx^n)}\,dx = b\left(\frac{\log(x)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
```

output
```
b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Giac [F]

$$\int \frac{b + 2cx^n}{x(b + cx^n)} \, dx = \int \frac{2\,cx^n + b}{(cx^n + b)x} \, dx$$

input
```
integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{b + 2cx^n}{x(b + cx^n)} \, dx = \ln(x) + \frac{\ln(b + c\,x^n)}{n}$$

input
```
int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
```

output
```
log(x) + log(b + c*x^n)/n
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{b + 2cx^n}{x(b + cx^n)} \, dx = \frac{\log(x^n c + b) + \log(x)\,n}{n}$$

input
```
int((b+2*c*x^n)/x/(b+c*x^n),x)
```

output
```
(log(x**n*c + b) + log(x)*n)/n
```

3.473 $\int \frac{b+2cx}{x^8(b+cx)^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx=-\frac{1}{7x^7(b+cx)^7}$$

output
```
-1/7/x^7/(c*x+b)^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx=-\frac{1}{7x^7(b+cx)^7}$$

input
```
Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{x^8(b+cx)^8} \, dx$$
$$\Big\downarrow \, 83$$
$$-\frac{1}{7x^7(b+cx)^7}$$

input
```
Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

output
```
-1/7*1/(x^7*(b + c*x)^7)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
parallelrisch	$-\frac{1}{7x^7(cx+b)^7}$
orering	$-\frac{1}{7x^7(cx+b)^7}$
default	$\frac{132c^7}{b^{13}(cx+b)}+\frac{66c^7}{b^{12}(cx+b)^2}+\frac{30c^7}{b^{11}(cx+b)^3}+\frac{12c^7}{b^{10}(cx+b)^4}+\frac{4c^7}{b^9(cx+b)^5}+\frac{c^7}{b^8(cx+b)^6}+\frac{c^7}{7b^7(cx+b)^7}-\frac{1}{7b^7x^7}-\frac{132}{b^{13}}$

input
```
int((2*c*x+b)/x^8/(c*x+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7/(c*x+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.

Time = 0.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.21

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

input
```
integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

output
```
-1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**1
0 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x
**14)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.79

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7)}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

output
```
-1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4
*c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,(cx^2+bx)^7}$$

input
```
integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

output
```
-1/7/(c*x^2 + b*x)^7
```

Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7\,x^7\,(b+c\,x)^7}$$

input
```
int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
```

output
```
-1/(7*x^7*(b + c*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 5.57

$$\int \frac{b+2cx}{x^8(b+cx)^8}\,dx = -\frac{1}{7x^7\,(c^7x^7+7b\,c^6x^6+21b^2c^5x^5+35b^3c^4x^4+35b^4c^3x^3+21b^5c^2x^2+7b^6cx+b^7)}$$

input
```
int((2*c*x+b)/x^8/(c*x+b)^8,x)
```

output
```
( - 1)/(7*x**7*(b**7 + 7*b**6*c*x + 21*b**5*c**2*x**2 + 35*b**4*c**3*x**3
+ 35*b**3*c**4*x**4 + 21*b**2*c**5*x**5 + 7*b*c**6*x**6 + c**7*x**7))
```

3.474 $\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

output
```
-1/14/x^14/(c*x^2+b)^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8} dx = -\frac{1}{14x^{14}(b+cx^2)^7}$$

input
```
Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{2cx^2+b}{x^{16}\left(cx^2+b\right)^8}dx^2$$

$$\Big\downarrow \; 83$$

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

input
```
Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

output
```
-1/14*1/(x^14*(b + c*x^2)^7)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
norman	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
risch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
parallelrisch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
orering	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
default	$-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}-\frac{15c^4}{b^{11}x^6}+\frac{6c^3}{b^{10}x^8}-\frac{2c^2}{b^9x^{10}}+\frac{c}{2b^8x^{12}}-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^2+b\right)^5}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\right.}{}$

input
```
int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/14/x^14/(c*x^2+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx =$$
$$-\frac{1}{14\left(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14}\right)}$$

input
```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx = -\frac{1}{14b^7x^{14}+98b^6cx^{16}+294b^5c^2x^{18}+490b^4c^3x^{20}+490b^3c^4x^{22}+294b^2c^5x^{24}+98bc^6x^{26}+14c^7x^{28}}$$

input
```
integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

output
```
-1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*
x**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c
**7*x**28)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^2}{x^{15}(b+cx^2)^8}\,dx = -\frac{1}{14\,(c^7x^{28}+7\,bc^6x^{26}+21\,b^2c^5x^{24}+35\,b^3c^4x^{22}+35\,b^4c^3x^{20}+21\,b^5c^2x^{18}+7\,b^6cx^{16}+b^7x^{14})}$$

input
```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

output
```
-1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^
4*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\left(cx^4+bx^2\right)^7}$$

input
```
integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

output
```
-1/14/(c*x^4 + b*x^2)^7
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=-\frac{1}{14\,x^{14}\left(c\,x^2+b\right)^7}$$

input
```
int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
```

output
```
-1/(14*x^14*(b + c*x^2)^7)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.00

$$\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx=$$
$$-\frac{1}{14x^{14}\left(c^7x^{14}+7b\,c^6x^{12}+21b^2c^5x^{10}+35b^3c^4x^8+35b^4c^3x^6+21b^5c^2x^4+7b^6c\,x^2+b^7\right)}$$

input
```
int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x)
```

output
```
( - 1)/(14*x**14*(b**7 + 7*b**6*c*x**2 + 21*b**5*c**2*x**4 + 35*b**4*c**3*
x**6 + 35*b**3*c**4*x**8 + 21*b**2*c**5*x**10 + 7*b*c**6*x**12 + c**7*x**1
4))
```

3.475 $\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} \, dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

output
```
-1/21/x^21/(c*x^3+b)^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8} \, dx = -\frac{1}{21x^{21}(b+cx^3)^7}$$

input
```
Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{3}\int \frac{2cx^3+b}{x^{24}(cx^3+b)^8}dx^3$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{1}{21x^{21}(b+cx^3)^7}$$

input
```
Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

output
```
-1/21*1/(x^21*(b + c*x^3)^7)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
gosper	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
risch	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
parallelrisch	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
orering	$-\frac{1}{21x^{21}(c\,x^3+b)^7}$
default	$-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}-\frac{10c^4}{b^{11}x^9}+\frac{4c^3}{b^{10}x^{12}}-\frac{4c^2}{3b^9x^{15}}+\frac{c}{3b^8x^{18}}-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^3+b\right)^5}-\frac{66b}{c\left(c\,x^3+b\right)^2}-\frac{12b^3}{c\left(c\,x^3+b\right)^4}\right.}{}$

input
```
int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21/x^21/(c*x^3+b)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx = -\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 1.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx = -\frac{1}{21b^7x^{21}+147b^6cx^{24}+441b^5c^2x^{27}+735b^4c^3x^{30}+735b^3c^4x^{33}+441b^2c^5x^{36}+147bc^6x^{39}+21c^7x^{42}}$$

input
```
integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

output
```
-1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3
*x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21
*c**7*x**42)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.06

$$\int \frac{b+2cx^3}{x^{22}(b+cx^3)^8}\,dx = -\frac{1}{21\,(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21})}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

output
```
-1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^
4*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21\left(cx^6+bx^3\right)^7}$$

input
```
integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

output
```
-1/21/(c*x^6 + b*x^3)^7
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21\,x^{21}\left(c\,x^3+b\right)^7}$$

input
```
int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
```

output
```
-1/(21*x^21*(b + c*x^3)^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 5.00

$$\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx=-\frac{1}{21x^{21}\left(c^7x^{21}+7b\,c^6x^{18}+21b^2c^5x^{15}+35b^3c^4x^{12}+35b^4c^3x^9+21b^5c^2x^6+7b^6c\,x^3+b^7\right)}$$

input
```
int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x)
```

output
```
( - 1)/(21*x**21*(b**7 + 7*b**6*c*x**3 + 21*b**5*c**2*x**6 + 35*b**4*c**3*
x**9 + 35*b**3*c**4*x**12 + 21*b**2*c**5*x**15 + 7*b*c**6*x**18 + c**7*x**
21))
```

3.476 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx$

Optimal result

Integrand size = 25, antiderivative size = 21

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

output
```
-1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx = -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

input
```
Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {948, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-7n-1}(b+2cx^n)}{(b+cx^n)^8} dx$$

$$\Big\downarrow \ 948$$

$$\frac{\int \frac{x^{-8n}(2cx^n+b)}{(cx^n+b)^8}dx^n}{n}$$

$$\Big\downarrow \ 83$$

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

input
```
Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]
```

output
```
-1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 11.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
parallelrisch	$-\frac{x^{-1-7n} x}{7n(b+c x^{n})^{7}}$
risch	$-\frac{132 c^{6} x^{-n}}{b^{13} n}+\frac{66 c^{5} x^{-2n}}{b^{12} n}-\frac{30 c^{4} x^{-3n}}{b^{11} n}+\frac{12 c^{3} x^{-4n}}{b^{10} n}-\frac{4 c^{2} x^{-5n}}{b^{9} n}+\frac{c x^{-6n}}{b^{8} n}-\frac{x^{-7n}}{7 b^{7} n}+\frac{c^{7}(924 x^{6n} c^{6}+6006 x^{5n} b c^{5}+$

input
```
int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*x^(-1-7*n)*x/n/(b+c*x^n)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx =$$

$$-\frac{1}{7\left(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+21\,b^2c^5nx^{12\,n}+35\,b^3c^4nx^{11\,n}+35\,b^4c^3nx^{10\,n}+21\,b^5c^2nx^{9\,n}+7\,b^6cnx^{8\,n}+b\right.}$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

output
```
-1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3
*c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n
*x^(8*n) + b^7*n*x^(7*n))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(19) = 38$.

Time = 50.85 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \begin{cases} -\frac{xx^{-7n-1}}{7b^7n+49b^6cnx^n+147b^5c^2nx^{2n}+245b^4c^3nx^{3n}+245b^3c^4nx^{4n}+147b^2c^5nx^{5n}+49bc^6nx^{6n}+7c^7nx^{7n}} & \text{for } n\neq 0 \\ \frac{(b+2c)\log(x)}{(b+c)^8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

output
```
Piecewise(((-x*x**(-7*n - 1)/(7*b**7*n + 49*b**6*c*n*x**n + 147*b**5*c**2*n
*x**(2*n) + 245*b**4*c**3*n*x**(3*n) + 245*b**3*c**4*n*x**(4*n) + 147*b**2
*c**5*n*x**(5*n) + 49*b*c**6*n*x**(6*n) + 7*c**7*n*x**(7*n)), Ne(n, 0)), (
(b + 2*c)*log(x)/(b + c)**8, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 612, normalized size of antiderivative = 29.14

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx =$$
$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^9}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5}\right.$$
$$+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12\,n}+2342340\,bc^{11}x^{11\,n}+6426420\,b^2c^{10}x^{10\,n}+9579570\,b^3c^9x^{9\,n}+8270262\,b^4c^8x^{8\,n}}{b^{13}c^7nx^{13\,n}+7\,b^{14}c^6nx^{12\,n}+21\,b^{15}c^5nx^{11\,n}}\right.$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

output
```
-1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^
11*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 401801
4*b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*
b^8*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^
2*x^(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x
^(13*n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*
x^(10*n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) +
360360*c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c
*((360360*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10
*n) + 9579570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*
x^(7*n) + 934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^
(4*n) + 1001*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b
^12)/(b^13*c^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n)
 + 35*b^16*c^4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n)
+ 7*b^19*c*n*x^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c
^6*log((c*x^n + b)/c)/(b^14*n))
```

Giac [F]

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx=\int \frac{(2\,cx^n+b)x^{-7\,n-1}}{(cx^n+b)^8}\,dx$$

input
```
integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

output
```
integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx=$$

$$-\frac{1}{7\,x^{7\,n}\,\left(b^7\,n+c^7\,n\,x^{7\,n}+7\,b^6\,c\,n\,x^n+7\,b\,c^6\,n\,x^{6\,n}+21\,b^5\,c^2\,n\,x^{2\,n}+35\,b^4\,c^3\,n\,x^{3\,n}+35\,b^3\,c^4\,n\,x^{4\,n}+21\,b\right.}$$

input
```
int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)
```

output
```
-1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +
 21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b
^2*c^5*n*x^(5*n)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

$$\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx = -\frac{1}{7x^{7n}n\left(x^{7n}c^7+7x^{6n}b\,c^6+21x^{5n}b^2c^5+35x^{4n}b^3c^4+35x^{3n}b^4c^3+21x^{2n}b^5c^2+7x^nb^6c+b^7\right)}$$

input
```
int(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x)
```

output
```
( - 1)/(7*x**(7*n)*n*(x**(7*n)*c**7 + 7*x**(6*n)*b*c**6 + 21*x**(5*n)*b**2
*c**5 + 35*x**(4*n)*b**3*c**4 + 35*x**(3*n)*b**4*c**3 + 21*x**(2*n)*b**5*c
**2 + 7*x**n*b**6*c + b**7))
```

3.477 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=-\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
-1/8*(x^16+1)^(1/2)-1/24*(x^16+1)^(3/2)+1/8*arctanh(1/2*(x^16+1)^(1/2)*2^(
1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=\frac{1}{24}\left(-4-x^{16}\right)\sqrt{1+x^{16}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

output
```
((-4 - x^16)*Sqrt[1 + x^16])/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(4*Sqrt[
2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {948, 90, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{31}\sqrt{x^{16}+1}}{1-x^{16}} \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{1}{16}\int \frac{x^{16}\sqrt{x^{16}+1}}{1-x^{16}}dx^{16}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{16}\left(\int \frac{\sqrt{x^{16}+1}}{1-x^{16}}dx^{16}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{16}\left(2\int \frac{1}{\left(1-x^{16}\right)\sqrt{x^{16}+1}}dx^{16}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{16}\left(4\int \frac{1}{2-x^{32}}d\sqrt{x^{16}+1}-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{16}\left(2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)-\frac{2}{3}\left(x^{16}+1\right)^{3/2}-2\sqrt{x^{16}+1}\right)$$

input
```
Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

output
```
(-2*Sqrt[1 + x^16] - (2*(1 + x^16)^(3/2))/3 + 2*Sqrt[2]*ArcTanh[Sqrt[1 + x
^16]/Sqrt[2]])/16
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

method	result	siz
pseudoelliptic	$-\frac{x^{16}\sqrt{x^{16}+1}}{24}-\frac{\sqrt{x^{16}+1}}{6}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}-1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(x^{8}+1\right)\sqrt{2}}{2\sqrt{x^{16}+1}}\right)}{16}$	69
trager	$\left(-\frac{x^{16}}{24}-\frac{1}{6}\right)\sqrt{x^{16}+1}+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+4\sqrt{x^{16}+1}+3\operatorname{RootOf}\left(\_Z^{2}-2\right)}{(x-1)(x+1)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	87
risch	$-\frac{\left(x^{16}+4\right)\sqrt{x^{16}+1}}{24}-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^{2}-2\right)x^{16}+4\sqrt{x^{16}+1}-3\operatorname{RootOf}\left(\_Z^{2}-2\right)}{(x-1)(x+1)\left(x^{2}+1\right)\left(x^{4}+1\right)\left(x^{8}+1\right)}\right)}{16}$	87

input
```
int(x^31*(x^16+1)^(1/2)/(-x^16+1),x,method=_RETURNVERBOSE)
```

output
```
-1/24*x^16*(x^16+1)^(1/2)-1/6*(x^16+1)^(1/2)-1/16*2^(1/2)*arctanh(1/2*(x^8
-1)*2^(1/2)/(x^16+1)^(1/2))+1/16*2^(1/2)*arctanh(1/2*(x^8+1)*2^(1/2)/(x^16
+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx=-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
```

output
```
-1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(
x^16 + 1) + 3)/(x^16 - 1))
```

Sympy [A] (verification not implemented)

Time = 34.77 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{(x^{16}+1)^{\frac{3}{2}}}{24} - \frac{\sqrt{x^{16}+1}}{8} - \frac{\sqrt{2}\left(\log\left(\sqrt{x^{16}+1}-\sqrt{2}\right)-\log\left(\sqrt{x^{16}+1}+\sqrt{2}\right)\right)}{16}$$

input
```
integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

output
```
-(x**16 + 1)**(3/2)/24 - sqrt(x**16 + 1)/8 - sqrt(2)*(log(sqrt(x**16 + 1)
- sqrt(2)) - log(sqrt(x**16 + 1) + sqrt(2)))/16
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

output
```
-1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqr
t(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}} - \frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right) - \frac{1}{8}\sqrt{x^{16}+1}$$

input
```
integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

output
```
-1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
 + 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^{16}+1}}{2}\right)}{8} - \frac{\sqrt{x^{16}+1}}{8} - \frac{\left(x^{16}+1\right)^{3/2}}{24}$$

input
```
int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^
16 + 1)^(3/2)/24
```

Reduce [F]

$$\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx = -\frac{\sqrt{x^{16}+1}\,x^{16}}{24} + \frac{\sqrt{x^{16}+1}}{12} - 2\left(\int \frac{\sqrt{x^{16}+1}\,x^{31}}{x^{32}-1}dx\right)$$

input
```
int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)
```

output
```
( - sqrt(x**16 + 1)*x***16 + 2*sqrt(x**16 + 1) - 48*int((sqrt(x**16 + 1)*x*
*31)/(x**32 - 1),x))/24
```

3.478 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

output
```
2*c^(1/2)*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)-2*d
^(1/2)*arctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 10.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx=-\frac{2\sqrt{d}\sqrt{bc-ad}\sqrt{c+\frac{d}{x}}\,x\sqrt{\frac{b(d+cx)}{(bc-ad)x}}\,\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bd+bcx}+\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}$$

input
```
Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

output
```
(-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d
)*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2
*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {948, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\sqrt{c+\frac{d}{x}}x}{\sqrt{a+\frac{b}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{140}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-\int \frac{cx}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{27}$$

$$-d\int \frac{1}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}-c\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{66}$$

$$-2d\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-c\int \frac{x}{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \text{104}$$

$$-2c\int \frac{1}{\frac{c}{x^2}-a}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}-2d\int \frac{1}{b-\frac{d}{x^2}}d\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

input
```
Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

output
```
(2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[
a] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/
Sqrt[b]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(69) = 138$.

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{\sqrt{\frac{ax+b}{x}}\, x \sqrt{\frac{cx+d}{x}}\, \left(\ln\left(\frac{adx+cbx+2\sqrt{bd}\,\sqrt{(cx+d)(ax+b)}+2bd}{x}\right)\sqrt{ac}\, d-\ln\left(\frac{2acx+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}+ad+cb}{2\sqrt{ac}}\right)\sqrt{bd}\, c\right)}{\sqrt{bd}\,\sqrt{ac}\,\sqrt{(cx+d)(ax+b)}}$	143

input
```
int((c+d/x)^(1/2)/(a+b/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+c*b*x+2*(b*d)^(1/2)*((c*
x+d)*(a*x+b))^(1/2)+2*b*d)/x)*(a*c)^(1/2)*d-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*
x+b))^(1/2)*(a*c)^(1/2)+a*d+c*b)/(a*c)^(1/2))*(b*d)^(1/2)*c)/(b*d)^(1/2)/(
a*c)^(1/2)/((c*x+d)*(a*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 705, normalized size of antiderivative = 7.58

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \left[\frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 - 4\left(2\,a^2cx^2 + \left(abc + a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2cd\right)x\right) + \frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2 + (b^2c^2 + 6\,abcd + a^2d^2)x^2 - 4\left(2\,b^2dx + (b^2c + abd)x^2\right)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 8\,(b^2c}{x^2}\right. - \sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx + bc + ad}\right) + \frac{1}{2}\sqrt{\frac{d}{b}}\log\left(-\frac{8\,b^2d^2 + (b^2c^2 + 6\,abcd + a^2d^2)x^2 - 4\left(2\,b^2dx + (b^2c + abd)x^2\right)\sqrt{\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 8\,(b^2c}{x^2}\right. + \frac{1}{2}\sqrt{\frac{c}{a}}\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 - 4\left(2\,a^2cx^2 + \left(abc + a^2d\right)x\right)\sqrt{\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2cd\right)x\right), -\sqrt{-\frac{c}{a}}\arctan\left(\frac{2\,ax\sqrt{-\frac{c}{a}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx + bc + ad}\right) + \sqrt{-\frac{d}{b}}\arctan\left(\frac{2\,bx\sqrt{-\frac{d}{b}}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,bd + (bc + ad)x}\right)\right]$$

input
```
integrate((c+d/x)^(1/2)/(a+b/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a
^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x
) - 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 +
6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(
-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x
 + b*c + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/
x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(2*
b*x*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*b*d + (b*c + a*d)*x)
) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(
2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d
)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*arctan(2*a*x*sqrt(-c/a)*sqrt(
(a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) + sqrt(-d/b)*arctan(
2*b*x*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*b*d + (b*c + a*d)*
x))]
```

Sympy [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
integrate((c+d/x)**(1/2)/(a+b/x)**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx=\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$$

input
```
integrate((c+d/x)^(1/2)/(a+b/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

input
```
integrate((c+d/x)^(1/2)/(a+b/x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx = \int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

input
```
int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
```

output
```
int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}x}\,dx$$

$$= \frac{2\sqrt{c}\sqrt{a}\log\left(\frac{\sqrt{c}\sqrt{ax+b}+\sqrt{a}\sqrt{cx+d}}{\sqrt{ad-bc}}\right)b+\sqrt{d}\sqrt{b}\log\left(\sqrt{c}\sqrt{ax+b}+\sqrt{a}\sqrt{cx+d}-\sqrt{2\sqrt{d}\sqrt{c}\sqrt{b}\sqrt{a}+ad+}\right.}{}$$

input
```
int((c+d/x)^(1/2)/(a+b/x)^(1/2)/x,x)
```

output
```
(2*sqrt(c)*sqrt(a)*log((sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d))/sqr
t(a*d - b*c))*b + sqrt(d)*sqrt(b)*log(sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt
(c*x + d) - sqrt(2*sqrt(d)*sqrt(c)*sqrt(b)*sqrt(a) + a*d + b*c))*a + sqrt(
d)*sqrt(b)*log(sqrt(c)*sqrt(a*x + b) + sqrt(a)*sqrt(c*x + d) + sqrt(2*sqrt
(d)*sqrt(c)*sqrt(b)*sqrt(a) + a*d + b*c))*a - sqrt(d)*sqrt(b)*log(2*sqrt(c
)*sqrt(a)*sqrt(c*x + d)*sqrt(a*x + b) + 2*sqrt(d)*sqrt(c)*sqrt(b)*sqrt(a)
+ 2*a*c*x)*a)/(a*b)
```

3.479 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 252

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx &= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} \\ &+ \frac{5(bc-ad)(7bc+ad)\,(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)\,(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} \\ &+ \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{5(bc-ad)^3(7bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}\end{aligned}$$

output
```
-5/64*(-a*d+b*c)^2*(a*d+7*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^4/n+5/9
6*(-a*d+b*c)*(a*d+7*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d^3/n-1/24*(a*d
+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x
^n)^(1/2)/b/d/n+5/64*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1
/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(9/2)/n
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(15a^3d^3+a^2bd^2(-191c+118dx^n)+ab^2d(265c^2-172\right.}{}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 1
18*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*
c^3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^
(7/2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqr
t[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {948, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\int \frac{(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{8bd}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\int\frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6d}\right)}{8bd}}{n}$$

$\big\downarrow$ 60

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6d}\right)}{8bd}}{n}$$

$\big\downarrow$ 60

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4d}\right)}{6d}\right)}{8bd}}{n}$$

$\big\downarrow$ 66

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4d}\right)}{6d}\right)}{8bd}}{n}$$

$\big\downarrow$ 221

$$\frac{\frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+}}{\sqrt{b}\sqrt{c+}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6d}\right)}{8bd}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
(((a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d) - ((7*b*c + a*d)*(((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*d) - (5*(b*c - a*d)*(((a + b*x^n)^(3/2)*Sqrt[c
 + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/d - (
(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/
(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^{n})^{\frac{5}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.41

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \left[-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}\right.}{}\right.$$

$$-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right.}{}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt
(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4
*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c
*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2
*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7
*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*
sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt
(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191
*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2
*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*s
qrt(d*x^n + c))/(b^2*d^5*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}\,(a+b\,x^n)^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Too large to display}$$

input

```
int(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output

```
(48*x**(3*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b**2*d**3 + 48*x**(3*n)*s
qrt(x**n*d + c)*sqrt(x**n*b + a)*b**3*c*d**2 + 136*x**(2*n)*sqrt(x**n*d +
c)*sqrt(x**n*b + a)*a**2*b*d**3 + 80*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b
 + a)*a*b**2*c*d**2 - 56*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**3*c
**2*d + 118*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*d**3 - 54*x**n*sqr
t(x**n*d + c)*sqrt(x**n*b + a)*a**2*b*c*d**2 - 102*x**n*sqrt(x**n*d + c)*s
qrt(x**n*b + a)*a*b**2*c**2*d + 70*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*
b**3*c**3 - 236*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*c*d**2 + 344*sqrt(x
**n*d + c)*sqrt(x**n*b + a)*a**2*b*c**2*d - 140*sqrt(x**n*d + c)*sqrt(x**n
*b + a)*a*b**2*c**3 + 15*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/
(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b
*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**5*d**5*n + 75*i
nt((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**
(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x
+ a**2*c*d*x + a*b*c**2*x),x)*a**4*b*c*d**4*n - 210*int((x**(2*n)*sqrt(x**
n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x*
*n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c*
*2*x),x)*a**3*b**2*c**2*d**3*n + 30*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x*
*n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x +
 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2...
```

3.480 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 199

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\,\sqrt{c+dx^n}}{3bdn} - \frac{(bc-ad)^2(5bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}$$

output
```
1/8*(-a*d+b*c)*(a*d+5*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^3/n-1/12*(a
*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d^2/n+1/3*(a+b*x^n)^(5/2)*(c+d
*x^n)^(1/2)/b/d/n-1/8*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(
1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(7/2)/n
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(3a^2d^2+2abd(-11c+7dx^n)+b^2(15c^2-10cdx^n+8d\right.}{24b^2d^{7/2}n\sqrt{c+dx}}$$

input
```
Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x
^n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*
b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x
^n])/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\int \frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6bd}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6bd}}{n}$$

$\Big\downarrow$ 60

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4d}\right)}{6bd}}{n}$$

$\Big\downarrow$ 66

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4d}\right)}{6bd}}{n}$$

$\Big\downarrow$ 221

$$\frac{\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6bd}}{n}$$

input
```
Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d) - ((5*b*c + a*d)*(((a + b*x^n
)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[c +
 d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}(a+b\,x^{n})^{\frac{3}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.36

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \left[\frac{3\left(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3\right)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n} + b^2c^2 + 6\,abcd + c\right.}{}\right.$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*
x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d
+ a*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3
*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n
 + c))/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a
^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 +
3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^
n + c))/(b^2*d^4*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{(bx^n+a)^{\frac{3}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{(bx^n+a)^{\frac{3}{2}}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}\,{(a+b\,x^n)}^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{8x^{2n}\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,ab\,d^2+8x^{2n}\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,b^2cd+14x^n\sqrt{x^nd+c}}{}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
(8*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*d**2 + 8*x**(2*n)*sqrt(x
**n*d + c)*sqrt(x**n*b + a)*b**2*c*d + 14*x**n*sqrt(x**n*d + c)*sqrt(x**n*
b + a)*a**2*d**2 + 4*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*c*d - 10*x
**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**2*c**2 - 28*sqrt(x**n*d + c)*sqrt
(x**n*b + a)*a**2*c*d + 20*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*c**2 + 3*
int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x*
*(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x
 + a**2*c*d*x + a*b*c**2*x),x)*a**4*d**4*n + 12*int((x**(2*n)*sqrt(x**n*d
+ c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a
**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x
),x)*a**3*b*c*d**3*n - 18*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))
/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*
b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2*b**2*c**2*d*
*2*n - 12*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d
**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b
**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a*b**3*c**3*d*n + 15*int((x**(2*n
)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*
c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*
x + a*b*c**2*x),x)*b**4*c**4*n)/(24*d**2*n*(a*d + b*c))
```

3.481 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 146

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}$$

output
```
-1/4*(a*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^2/n+1/2*(a+b*x^n)^(3/
2)*(c+d*x^n)^(1/2)/b/d/n+1/4*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*x
^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)(-3bc+ad+2bdx^n)+(bc-ad)^{3/2}(3bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
 a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {948, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4bd}}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4bd}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4bd}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4bd}}{n}$$

input
```
Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
(((a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d) - ((3*b*c + a*d)*((Sqrt[a + b
*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/
(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/n
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}\sqrt{a+b\,x^{n}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.46

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$= \left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\right.}{16\,b^2d^3n}\right.$$
$$-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)-2\,(2\,b^2d^2x^n-3\,b^2cd+}{8\,b^2d^3n}$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqr
t(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(
2*b^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2
*d^3*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x
^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{2\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$= \frac{2x^n\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,ad+2x^n\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,bc-4\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,ac+\left(\int \frac{}{x^{2n}ab\,d^2x+x^{2n}b^2cdx}\right.}{}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
(2*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*d + 2*x**n*sqrt(x**n*d + c)*sq
rt(x**n*b + a)*b*c - 4*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*c + int((x**(2*
n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2
*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d
*x + a*b*c**2*x),x)*a**3*d**3*n + 3*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x*
*n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x +
 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2*b*
c*d**2*n - int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*
d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*
b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a*b**2*c**2*d*n - 3*int((x**(2*n
)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*
c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*
x + a*b*c**2*x),x)*b**3*c**3*n)/(4*d*n*(a*d + b*c))
```

3.482 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 89

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

output
```
(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n-(a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^
(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n) - \sqrt{bc-ad}(bc+ad)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[
(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {948, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{\int \frac{x^n}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}}{n}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{bd}}{n}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}}}{n}$$

input
```
Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)))/n
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.16

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$
$$= \left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}bd+(bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}bdx^n+(bc+\right.\right.}{4\,b^2d^2n}\right.$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n +
 (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b
*d^2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c
 + a*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x^n)))/(b^2*d^2*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n}\sqrt{x^nd+c}\,\sqrt{x^nb+a}}{x^{2n}bdx+x^nadx+x^nbcx+acx}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*b*d*x + x**n*a*
d*x + x**n*b*c*x + a*c*x),x)
```

3.483 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}n}$$

output
```
2*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+2*arctanh(d^(1/2)*(a+b*
x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/d^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\text{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqr
t[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c
- a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {948, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{2\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{b}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^n}}{b(bc-ad)\sqrt{a+bx^n}}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
((2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]))/n
```

Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{\left(a+b\,x^{n}\right)^{\frac{3}{2}} \sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(75) = 150$.

Time = 0.19 (sec) , antiderivative size = 408, normalized size of antiderivative = 4.48

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}abd + \left((b^2c-abd)\sqrt{bd}x^n + (abc-a^2d)\sqrt{bd}\right)\log\left(8\right.}{2\left((b\right.}\right.$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)
*x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*
c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n
 + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)
*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*
a*b*d - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arct
an(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqr
t(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4
*c*d - a*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

Sympy [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(2*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{(a+b\,x^n)^{3/2}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2n}\sqrt{x^n d+c}\,\sqrt{x^n b+a}}{x^{3n}b^2dx+2x^{2n}abdx+x^{2n}b^2cx+x^n a^2dx+2x^n abcx+a^2cx}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(3*n)*b**2*d*x + 2*x*
*(2*n)*a*b*d*x + x**(2*n)*b**2*c*x + x**n*a**2*d*x + 2*x**n*a*b*c*x + a**2
*c*x),x)
```

3.484 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}} - \frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}$$

output
```
2/3*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(3/2)-2/3*(-a*d+3*b*c)*(c+d
*x^n)^(1/2)/b/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2\sqrt{c+dx^n}(-2ac-3bcx^n+adx^n)}{3(bc-ad)^2n\,(a+bx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a +
 b*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {948, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n-1}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^n}{(bx^n+a)^{5/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{(3bc-ad)\int \frac{1}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b(bc-ad)}+\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{48}$$

$$\frac{\frac{2a\sqrt{c+dx^n}}{3b(bc-ad)(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2\sqrt{a+bx^n}}}{n}$$

input
```
Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
((2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^n]))/n
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+2n}}{(a+b\,x^n)^{\frac{5}{2}}\,\sqrt{c+d\,x^n}}dx$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.42

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c}}{3\,((b^4c^2-2\,ab^3cd+a^2b^2d^2)nx^{2\,n}+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)nx^n+(a^2b^2c^2-2\,a^3bcd+a^4d^2)n)}$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
-2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\int \frac{x^{2\,n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.52

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx=\frac{2\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,(x^nad-3x^nbc-2ac)}{3n\,(x^{2n}a^2b^2d^2-2x^{2n}a\,b^3cd+x^{2n}b^4c^2+2x^na^3b\,d^2-4x^na^2b^2cd+2x^na\,b^3c^2}$$

input
```
int(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
(2*sqrt(x**n*d + c)*sqrt(x**n*b + a)*(x**n*a*d - 3*x**n*b*c - 2*a*c))/(3*n
*(x**(2*n)*a**2*b**2*d**2 - 2*x**(2*n)*a*b**3*c*d + x**(2*n)*b**4*c**2 + 2
*x**n*a**3*b*d**2 - 4*x**n*a**2*b**2*c*d + 2*x**n*a*b**3*c**2 + a**4*d**2
- 2*a**3*b*c*d + a**2*b**2*c**2))
```

3.485 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 356

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{(9bc+11ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{(a+bx^n)^{9/2}\sqrt{c+dx^n}}{5b^2dn} - \frac{(bc-ad)^3\left(63b^2c^2+14abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{128b^{5/2}d^{11/2}n}$$

output
```
1/128*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*
x^n)^(1/2)/b^2/d^5/n-1/192*(-a*d+b*c)*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a
+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^4/n+1/240*(3*a^2*d^2+14*a*b*c*d+63*b^2
*c^2)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d^3/n-1/40*(11*a*d+9*b*c)*(a+b*x
^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/5*(a+b*x^n)^(9/2)*(c+d*x^n)^(1/2)/b^
2/d/n-1/128*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)
*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(11/2)/n
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\sqrt{c+dx^n}\left(-\frac{24(3bc+ad)(a+bx^n)^4}{bd}+64x^n(a+bx^n)^4+\frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\left(\right.}{}\right.}{320bdn\sqrt{a+bx}}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
(Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*
x^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a
+ b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d
^3*(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c
 + d*x^n))/(b*c - a*d)])))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {948, 101, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{(bx^n+a)^{5/2}(3(3bc+ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{5bd}+\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\int \frac{(bx^n+a)^{5/2}(3(3bc+ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{10bd}}{n}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\int \frac{(bx^n+a)^{5/2}}{\sqrt{dx^n+c}}dx^n}{8bd}}{10bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\int \frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6d}\right)}{8bd}}{10bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-a\cdots}{\cdots}\right.}{6d}\right.}{8bd}}{10bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-a\cdots}{\cdots}\right.}{\cdots}\right.}{8bd}}{10bd}}{n}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad\cdots}{\cdots}\right)}{\cdots}\right)}{8bd}}{10bd}}{n}$$

$\Big\downarrow$ 221

$$\frac{\frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bd}-\frac{\frac{3(ad+3bc)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad\cdots}{\cdots}\right)}{\cdots}\right)}{8bd}}{10bd}}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

output
```
((x^n*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d) - ((3*(3*b*c + a*d)*(a +
b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d) - ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*d) - (5*(b*c - a*d)*(((a + b*
x^n)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^n]*Sqrt[
c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sq
rt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/(10*b*d))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{5}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.17

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*
b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(3
84*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2
*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x
^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2
*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^
5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c
*d^4 - 3*a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d
)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310
*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 -
144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*
d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 4
81*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^6*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{5}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \text{Too large to display}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
(384*x**(4*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b**3*d**4 + 384*x**(4*n)
*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**4*c*d**3 + 1008*x**(3*n)*sqrt(x**n*d
 + c)*sqrt(x**n*b + a)*a**2*b**2*d**4 + 576*x**(3*n)*sqrt(x**n*d + c)*sqrt
(x**n*b + a)*a*b**3*c*d**3 - 432*x**(3*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a
)*b**4*c**2*d**2 + 744*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*b*d
**4 - 440*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**2*b**2*c*d**3 - 68
0*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b**3*c**2*d**2 + 504*x**(2*
n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**4*c**3*d + 30*x**n*sqrt(x**n*d + c
)*sqrt(x**n*b + a)*a**4*d**4 - 932*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*
a**3*b*c*d**3 + 536*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**2*b**2*c**2*
d**2 + 868*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b**3*c**3*d - 630*x**n
*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**4*c**4 - 60*sqrt(x**n*d + c)*sqrt(x*
*n*b + a)*a**4*c*d**3 + 1924*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*b*c**2
*d**2 - 2996*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**2*b**2*c**3*d + 1260*sqr
t(x**n*d + c)*sqrt(x**n*b + a)*a*b**3*c**4 - 45*int((x**(2*n)*sqrt(x**n*d
+ c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a
**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x
),x)*a**6*d**6*n - 120*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x
**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c
*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**5*b*c*d**5*n -...
```

3.486 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+9ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4b^2dn} + \frac{(bc-ad)^2\left(35b^2c^2+10abcd+3a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{5/2}d^{9/2}n}$$

output

```
-1/64*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^
n)^(1/2)/b^2/d^4/n+1/96*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*x^n)^(3/2)*
(c+d*x^n)^(1/2)/b^2/d^3/n-1/24*(9*a*d+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/
2)/b^2/d^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d/n+1/64*(-a*d+b*c)^2
*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)
/(c+d*x^n)^(1/2))/b^(5/2)/d^(9/2)/n
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.84

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(9a^3d^3+3a^2bd^2(5c-2dx^n)-ab^2d(145c^2-92cdx^n\right.}{}$$

input
```
Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
(-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2
*d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 -
 70*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)
*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*A
rcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt
[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {948, 101, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{(bx^n+a)^{3/2}((7bc+3ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{4bd}+\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\int \frac{(bx^n+a)^{3/2}((7bc+3ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{8bd}}{n}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\int \frac{(bx^n+a)^{3/2}}{\sqrt{dx^n+c}}dx^n}{6bd}}{8bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4d}\right)}{6bd}}{8bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \cdots}{\cdots}\right.}{4d}\cdots\right.}{6bd}\cdots}{8bd}\cdots}{n}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{\cdots}{b\cdots}}{\cdots}\right.}{4d}\cdots\right.}{6bd}\cdots}{8bd}\cdots}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bd}-\frac{\frac{(3ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\mathrm{arc}\cdots}{\cdots}\right.}{4d}\cdots\right.}{6bd}\cdots}{8bd}\cdots}{n}$$

input
```
Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

output
```
((x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(4*b*d) - (((7*b*c + 3*a*d)*(a +
b*x^n)^(5/2)*Sqrt[c + d*x^n])/(3*b*d) - ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*d) - (3*(b*c - a*d)*((Sqrt[a
+ b*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n
])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/(8*b*d
))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}(a+b\,x^{n})^{\frac{3}{2}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.11

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \left[\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+\right.}{}\right.$$

$$-\frac{3\,(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd})\sqrt{bx^n+a}\sqrt{dx^n}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right.}{}$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2
*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqr
t(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^
4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*
d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^
2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^
4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*s
qrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(
b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2
)*x^n)) - 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a
^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35
*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d
*x^n + c))/(b^3*d^5*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,(a+b\,x^n)^{3/2}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \text{Too large to display}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
(48*x**(3*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b**2*d**3 + 48*x**(3*n)*s
qrt(x**n*d + c)*sqrt(x**n*b + a)*b**3*c*d**2 + 72*x**(2*n)*sqrt(x**n*d + c
)*sqrt(x**n*b + a)*a**2*b*d**3 + 16*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b
+ a)*a*b**2*c*d**2 - 56*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**3*c*
*2*d + 6*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*d**3 - 86*x**n*sqrt(x
**n*d + c)*sqrt(x**n*b + a)*a**2*b*c*d**2 - 22*x**n*sqrt(x**n*d + c)*sqrt(
x**n*b + a)*a*b**2*c**2*d + 70*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**3
*c**3 - 12*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a**3*c*d**2 + 184*sqrt(x**n*d
 + c)*sqrt(x**n*b + a)*a**2*b*c**2*d - 140*sqrt(x**n*d + c)*sqrt(x**n*b +
a)*a*b**2*c**3 - 9*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2
*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x
 + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**5*d**5*n - 21*int((x*
*(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*
b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2
*c*d*x + a*b*c**2*x),x)*a**4*b*c*d**4*n - 66*int((x**(2*n)*sqrt(x**n*d + c
)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2
*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x
)*a**3*b**2*c**2*d**3*n + 126*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b +
 a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**
n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2*b**3*...
```

3.487 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 218

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+7ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12b^2d^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3b^2dn} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}$$

output

```
1/8*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^3/
n-1/12*(7*a*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/3*(a+b*x^
n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d/n-1/8*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c
^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(7/
2)/n
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.88

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$= \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3a^2d^2+2abd(-2c+dx^n)+b^2(15c^2-10cdx^n+8d^2x^{2n})\right)-3(bc-ad)^{3/2}\,(5b^2}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n
) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^
2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sq
rt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n]
)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {948, 101, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$\Big\downarrow \ 948$$
$$\frac{\int \frac{x^{2n}\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{n}$$
$$\Big\downarrow \ 101$$
$$\frac{\frac{\int -\frac{\sqrt{bx^n+a}((5bc+3ad)x^n+2ac)}{2\sqrt{dx^n+c}}dx^n}{3bd}+\frac{x^n(a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^n (a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\int \frac{\sqrt{bx^n+a}((5bc+3ad)x^n+2ac)}{\sqrt{dx^n+c}}dx^n}{6bd}}{n}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{x^n (a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\int \frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}dx^n}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{x^n (a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}\right)}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{x^n (a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}\right)}{4bd}}{6bd}}{n}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{x^n (a+bx^n)^{3/2}\sqrt{c+dx^n}}{3bd}-\frac{\frac{(3ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}\right)}{4bd}}{6bd}}{n}$$

input
```
Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

output
```
((x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d) - (((5*b*c + 3*a*d)*(a +
b*x^n)^(3/2)*Sqrt[c + d*x^n])/(2*b*d) - (3*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^
2)*((Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sq
rt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/(6
*b*d))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}\sqrt{a+b\,x^{n}}}{\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.16

$$\int \frac{x^{-1+3n}\sqrt{a+bx^{n}}}{\sqrt{c+dx^{n}}}\,dx = \left[-\frac{3\,(5\,b^{3}c^{3}-3\,ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3})\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{2\,n}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}+4\left(2\,\sqrt{bd}bdx^{n}+(bc\right.\right.}{}\right.$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x
^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d +
 a*b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a
^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sq
rt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^4*n)]
```

Sympy [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

Maxima [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx=\int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{\sqrt{bx^n+a}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

input
```
int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

output
```
int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$
$$= \frac{8x^{2n}\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,ab\,d^2+8x^{2n}\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,b^2cd+2x^n\sqrt{x^nd+c}\,\sqrt{x^nb+a}\,a^2d^2-8x^n\sqrt{x^n}}{}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output

```
(8*x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*d**2 + 8*x**(2*n)*sqrt(x
**n*d + c)*sqrt(x**n*b + a)*b**2*c*d + 2*x**n*sqrt(x**n*d + c)*sqrt(x**n*b
 + a)*a**2*d**2 - 8*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*c*d - 10*x*
*n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*b**2*c**2 - 4*sqrt(x**n*d + c)*sqrt(x
**n*b + a)*a**2*c*d + 20*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*b*c**2 - 3*in
t((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(
2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x +
 a**2*c*d*x + a*b*c**2*x),x)*a**4*d**4*n - 6*int((x**(2*n)*sqrt(x**n*d + c
)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2
*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x
)*a**3*b*c*d**3*n - 12*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x
**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c
*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2*b**2*c**2*d**2*
n + 6*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*
x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*
c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a*b**3*c**3*d*n + 15*int((x**(2*n)*sq
rt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*
x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x +
a*b*c**2*x),x)*b**4*c**4*n)/(24*b*d**2*n*(a*d + b*c))
```

3.488 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 154

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = -\frac{(3bc+5ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2b^2dn} + \frac{(3b^2c^2+2abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}$$

output
```
-1/4*(5*a*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/2*(a+b*x^n)
^(3/2)*(c+d*x^n)^(1/2)/b^2/d/n+1/4*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctanh
(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(5/2)/n
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.02

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^n}(c+dx^n)\left(-3bc-3ad+2bdx^n\right)+\sqrt{bc-ad}(3b^2c^2+2abcd+3a^2d^2)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\text{arcsinh}\left(\cdots\right.}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

input
```
Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt
[b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c
- a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)
*n*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {948, 101, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{101}$$

$$\frac{\frac{\int -\frac{3(bc+ad)x^n+2ac}{2\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\int \frac{3(bc+ad)x^n+2ac}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{4bd}}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2bd}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{bd}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bd}-\frac{\frac{\left(4abcd-3(ad+bc)^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}}+\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{bd}}{4bd}}{n}$$

input
```
Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

output
```
((x^n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d) - ((3*(b*c + a*d)*Sqrt[a +
b*x^n]*Sqrt[c + d*x^n])/(b*d) + ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)))/(4*b
*d))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.34

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \left[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}bdx^n+(bc+ad)\sqrt{bd}\right)\right.}{16\,b^3d^3n}\right.$$
$$-\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{-bd}\arctan\left(\frac{\left(2\,\sqrt{-bd}bdx^n+(bc+ad)\sqrt{-bd}\right)\sqrt{bx^n+a}\sqrt{dx^n+c}}{2\,(b^2d^2x^{2\,n}+abcd+(b^2cd+abd^2)x^n)}\right)-2\,(2\,b^2d^2x^n-3\,b^2cd}{8\,b^3d^3n}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sq
rt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*
(2*b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(
b^3*d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/
2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x
^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*
d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3
*n)]
```

Sympy [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{\sqrt{bx^n+a}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^n}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$$
$$= \frac{2x^n\sqrt{x^n d+c}\,\sqrt{x^n b+a}\,ad+2x^n\sqrt{x^n d+c}\,\sqrt{x^n b+a}\,bc-4\sqrt{x^n d+c}\,\sqrt{x^n b+a}\,ac-3\left(\int \frac{}{x^{2n}ab\,d^2x+x^{2n}b^2cd}\right.}{}$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x)
```

output

```
(2*x**n*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*d + 2*x**n*sqrt(x**n*d + c)*sq
rt(x**n*b + a)*b*c - 4*sqrt(x**n*d + c)*sqrt(x**n*b + a)*a*c - 3*int((x**(
2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b*
*2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c
*d*x + a*b*c**2*x),x)*a**3*d**3*n - 5*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(
x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x
 + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a**2*
b*c*d**2*n - 5*int((x**(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*
a*b*d**2*x + x**(2*n)*b**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x
**n*b**2*c**2*x + a**2*c*d*x + a*b*c**2*x),x)*a*b**2*c**2*d*n - 3*int((x**
(2*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(2*n)*a*b*d**2*x + x**(2*n)*b
**2*c*d*x + x**n*a**2*d**2*x + 2*x**n*a*b*c*d*x + x**n*b**2*c**2*x + a**2*
c*d*x + a*b*c**2*x),x)*b**3*c**3*n)/(4*b*d*n*(a*d + b*c))
```

3.489 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 133

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

output
```
-2*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+(a+b*x^n)^(1/2)*(c
+d*x^n)^(1/2)/b^2/d/n-(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/
(c+d*x^n)^(1/2))/b^(5/2)/d^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.39

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{-b\sqrt{d}(c+dx^n)\left(-3a^2d+b^2cx^n+ab(c-dx^n)\right)+\sqrt{bc-ad}(b^2c^2+2abcd-}{b^3d^{3/2}(-bc+ad)n\sqrt{a+bx^n}\sqrt{c+}}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
(-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[
b*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c +
d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(
b^3*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {948, 100, 27, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{2\int -\frac{a(bc-ad)-b(bc-ad)x^n}{2\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{(bc-ad)(a-bx^n)}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{a-bx^n}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n}{2d}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{66}$$

$$\frac{-\frac{\frac{(3ad+bc)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}}{d}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)\sqrt{a+bx^n}}-\frac{\frac{(3ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}d^{3/2}}-\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{d}}{b^2}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*Sqrt[a + b*x^n]) - (-((Sqrt[a +
 b*x^n]*Sqrt[c + d*x^n])/d) + ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b]*d^(3/2)))/b^2)/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}}{(a+b\,x^{n})^{\frac{3}{2}}\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(113) = 226$.

Time = 0.24 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.06

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \left[\frac{4\,(ab^2cd - 3\,a^2bd^2 + (b^3cd - ab^2d^2)x^n)\sqrt{bx^n+a}\sqrt{dx^n+c} + \left((b^3c^2 + 2\,ab\right.}{}\right.$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
 a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n
 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqr
t(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^
5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2
*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n
 + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2
 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n +
 (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n)
+ a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a
*b^4*c*d^2 - a^2*b^3*d^3)*n)]
```

Sympy [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(a+bx^n)^{\frac{3}{2}}\sqrt{c+dx^n}}\,dx$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

output
```
Integral(x**(3*n - 1)/((a + b*x**n)**(3/2)*sqrt(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3\,n-1}}{(a+b\,x^n)^{3/2}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n}\sqrt{x^n d+c}\,\sqrt{x^n b+a}}{x^{3n}b^2dx+2x^{2n}abdx+x^{2n}b^2cx+x^n a^2dx+2x^n abcx+a^2cx}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x)
```

output
```
int((x**(3*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(3*n)*b**2*d*x + 2*x*
*(2*n)*a*b*d*x + x**(2*n)*b**2*c*x + x**n*a**2*d*x + 2*x**n*a*b*c*x + a**2
*c*x),x)
```

3.490 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 147

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}n}$$

output
```
-2/3*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(3/2)+4/3*a*(-2*a*d+3*
b*c)*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)+2*arctanh(d^(1/2)*
(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(1/2)/n
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.48

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{2\sqrt{c+dx^n}\left(\frac{a^2}{-bc+ad} + \frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2} - \frac{3(a+bx^n)\left(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}} - \sqrt{d}\sqrt{a+bx^n}\right.}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right.}{3b^2n\,(a+bx^n)^{3/2}}$$

input
```
Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
(2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n
))/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n)
)/(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])
/Sqrt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]))
)/(3*b^2*n*(a + b*x^n)^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 100, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n-1}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}dx$$

$$\Big\downarrow \; 948$$

$$\frac{\int \frac{x^{2n}}{(bx^n+a)^{5/2}\sqrt{dx^n+c}}dx^n}{n}$$

$$\Big\downarrow \; 100$$

$$\frac{\frac{2\int -\frac{a(3bc-ad)-3b(bc-ad)x^n}{2(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{a(3bc-ad)-3b(bc-ad)x^n}{(bx^n+a)^{3/2}\sqrt{dx^n+c}}dx^n}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \; 87$$

$$\frac{-\frac{-3(bc-ad)\int \frac{1}{\sqrt{bx^n+a}\sqrt{dx^n+c}}dx^n-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \; 66$$

$$-\frac{\frac{-6(bc-ad)\int \frac{1}{b-dx^{2n}}d\frac{\sqrt{bx^n+a}}{\sqrt{dx^n+c}}-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)(a+bx^n)^{3/2}}-\frac{-\frac{6(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{\sqrt{b}\sqrt{d}}-\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{(bc-ad)\sqrt{a+bx^n}}}{3b^2(bc-ad)}}{n}$$

input
```
Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*(a + b*x^n)^(3/2)) - ((-4*a*(
3*b*c - 2*a*d)*Sqrt[c + d*x^n])/((b*c - a*d)*Sqrt[a + b*x^n]) - (6*(b*c -
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(Sqrt[b
]*Sqrt[d]))/(3*b^2*(b*c - a*d)))/n
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{x^{-1+3n}}{(a+b\,x^{n})^{\frac{5}{2}}\,\sqrt{c+d\,x^{n}}}dx$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(123) = 246.

Time = 0.29 (sec) , antiderivative size = 769, normalized size of antiderivative = 5.23

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Too large to display}$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas"
)
```

output
```
[1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n
)*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)
*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2
*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)
*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/
((b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*
a^2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^
4*b^3*d^3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^
2*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b
*d^2)*sqrt(-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*a
rctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*
sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((
b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^
2*b^5*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*
b^3*d^3)*n)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

input
```
integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

input
```
int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

output
```
int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \int \frac{x^{3n}\sqrt{x^n d+c}\,\sqrt{x^n b+a}}{x^{4n}b^3dx+3x^{3n}a\,b^2dx+x^{3n}b^3cx+3x^{2n}a^2bdx+3x^{2n}a\,b^2cx+x^n a^3dx+3$$

input
```
int(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x)
```

output
```
int((x**(3*n)*sqrt(x**n*d + c)*sqrt(x**n*b + a))/(x**(4*n)*b**3*d*x + 3*x*
*(3*n)*a*b**2*d*x + x**(3*n)*b**3*c*x + 3*x**(2*n)*a**2*b*d*x + 3*x**(2*n)
*a*b**2*c*x + x**n*a**3*d*x + 3*x**n*a**2*b*c*x + a**3*c*x),x)
```

3.491 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \frac{b(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)} - \frac{d(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
b*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)/e/
(1+m)-d*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b
*c)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \frac{x(ex)^m\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)\right.}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)
/a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))
/(a*c*(-(b*c) + a*d)*(1 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{1010, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}dx$$

$$\Big\downarrow \ 1010$$

$$\frac{b\int \frac{(ex)^m}{bx^n+a}dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{dx^n+c}dx}{bc-ad}$$

$$\Big\downarrow \ 888$$

$$\frac{b(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1
+ m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1010
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n,
m}, x] && NeQ[b*c - a*d, 0]
```

Maple [F]

$$\int \frac{(ex)^m}{(a+b\,x^n)\,(c+d\,x^n)}dx$$

input
```
int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^n)\,(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$$

input
```
integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = \int \frac{(e\,x)^m}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

input
```
int((e*x)^m/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int((e*x)^m/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx = e^m\left(\int \frac{x^m}{x^{2n}bd+x^n ad+x^n bc+ac}dx\right)$$

input
```
int((e*x)^m/(a+b*x^n)/(c+d*x^n),x)
```

output
```
e**m*int(x**m/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.492 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)} dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)} dx = \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2e(1+m)n} + \frac{d^2(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2e(1+m)}$$

output
```
b*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+b*(a*d*(1+m-2*n)-b*c*(1+m-n))*(e*
x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(
1+m)/n+d^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*
d+b*c)^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1+m-2n)-bc(1+m-n))\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a^2(1+m)n}+\frac{d^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\right.}{c+cm}\right.}{(bc-ad)^2}$$

input
```
Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) -
b*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1006, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx$$

$$\Big\downarrow \; 1006$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}-\frac{\int \frac{(ex)^m(bd(m-n+1)x^n+bc(m-n+1)+adn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \; 1067$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)(a+bx^n)}-\frac{\int \left(\frac{b(bc(m-n+1)-ad(m-2n+1))(ex)^m}{(bc-ad)(bx^n+a)}+\frac{ad^2n(ex)^m}{(ad-bc)(dx^n+c)}\right)dx}{an(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(ex)^{m+1}}{aen(bc-ad)\,(a+bx^n)} - \frac{-\frac{ad^2n(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)} - \frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},\right.}{ae(m+1)(bc-ad)}}{an(bc-ad)}$$

input
```
Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
(b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) - (-((b*(a*d*(1 + m - 2*
n) - b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m))) - (a*d^2*n*(e*x)^(1 + m
)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c -
a*d)*e*(1 + m)))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 1006
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

input
```
int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a
*b*c + a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{(a+bx^n)^2\,(c+dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
d^2*e^m*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x^n), x) + b*e^m*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*
c*n - a^2*b*d*n)*x^n) - (b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*
integrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n -
2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \int \frac{(e\,x)^m}{(a+b\,x^n)^2\,(c+d\,x^n)}\,dx$$

input
```
int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{(a+bx^n)^2 (c+dx^n)} \, dx = e^m \left(\int \frac{x^m}{x^{3n}b^2d + 2x^{2n}abd + x^{2n}b^2c + x^n a^2 d + 2x^n abc + a^2 c} dx \right)$$

input
```
int((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
e**m*int(x**m/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n
*a**2*d + 2*x**n*a*b*c + a**2*c),x)
```

3.493 $\int \frac{x^m}{\sqrt{a+bx^n}(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(1+m)\sqrt{a+bx^n}}$$

output
```
x^(1+m)*(1+b*x^n/a)^(1/2)*AppellF1((1+m)/n,1/2,1,(1+m+n)/n,-b*x^n/a,-d*x^n
/c)/c/(1+m)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+cm)\sqrt{a+bx^n}}$$

input
```
Integrate[x^m/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/n,
-((b*x^n)/a), -((d*x^n)/c)])/((c + c*m)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^n}{a}+1}(dx^n+c)}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(m+1)\sqrt{a+bx^n}}$$

input
```
Int[x^m/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/n,
-((b*x^n)/a), -((d*x^n)/c)])/(c*(1 + m)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{a+b\,x^n}\,(c+d\,x^n)}dx$$

input
```
int(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)*x^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx$$

input
```
integrate(x**m/(a+b*x**n)**(1/2)/(c+d*x**n),x)
```

output
```
Integral(x**m/(sqrt(a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m}{\sqrt{a+b\,x^n}\,(c+d\,x^n)}\,dx$$

input
```
int(x^m/((a + b*x^n)^(1/2)*(c + d*x^n)),x)
```

output
```
int(x^m/((a + b*x^n)^(1/2)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^m\sqrt{x^n b+a}}{x^{2n}bd+x^n ad+x^n bc+ac}dx$$

input
```
int(x^m/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int((x**m*sqrt(x**n*b + a))/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.494 $\int \frac{x^n}{\sqrt{a+bx^n}(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \frac{x^{1+n}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(1+n)\sqrt{a+bx^n}}$$

output
```
x^(1+n)*(1+b*x^n/a)^(1/2)*AppellF1(1+1/n,1/2,1,2+1/n,-b*x^n/a,-d*x^n/c)/c/
(1+n)/(a+b*x^n)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \frac{x^{1+n}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(1+n)\sqrt{a+bx^n}}$$

input
```
Integrate[x^n/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(x^(1 + n)*Sqrt[1 + (b*x^n)/a]*AppellF1[1 + n^(-1), 1/2, 1, 2 + n^(-1), -(
(b*x^n)/a), -((d*x^n)/c)])/(c*(1 + n)*Sqrt[a + b*x^n])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{\sqrt{\frac{bx^n}{a}+1}\int \frac{x^n}{\sqrt{\frac{bx^n}{a}+1}(dx^n+c)}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n+1}\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},1,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c(n+1)\sqrt{a+bx^n}}$$

input
```
Int[x^n/(Sqrt[a + b*x^n]*(c + d*x^n)),x]
```

output
```
(x^(1 + n)*Sqrt[1 + (b*x^n)/a]*AppellF1[1 + n^(-1), 1/2, 1, 2 + n^(-1), -(
(b*x^n)/a), -((d*x^n)/c)])/(c*(1 + n)*Sqrt[a + b*x^n])
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^n}{\sqrt{a+b\,x^n}\,(c+d\,x^n)}dx$$

input
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)*x^n/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx$$

input
```
integrate(x**n/(a+b*x**n)**(1/2)/(c+d*x**n),x)
```

output
```
Integral(x**n/(sqrt(a + b*x**n)*(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate(x^n/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(x^n/(sqrt(b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)}\,dx$$

input
```
int(x^n/((a + b*x^n)^(1/2)*(c + d*x^n)),x)
```

output
```
int(x^n/((a + b*x^n)^(1/2)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)}\,dx = \int \frac{x^n\sqrt{x^n b+a}}{x^{2n}bd+x^n ad+x^n bc+ac}dx$$

input
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n),x)
```

output
```
int((x**n*sqrt(x**n*b + a))/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.495 $\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 121

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n} \, dx = \frac{x^{-np}(a+bx^n)^p \operatorname{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{dnp} - \frac{x^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \operatorname{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{dnp}$$

output
```
(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d/n/p/(x^(
n*p))-(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/d/n/p/(x^(n*p))/((1+b
*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = -\frac{x^{-n(-1+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(-1+p)}$$

input
```
Integrate[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c*n*(-1 + p)*x^(n*(-1 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-1)-1}(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-pn+n-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n-np}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(1-p)}$$

input
```
Int[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
(x^(n - n*p)*(a + b*x^n)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^n)/a), -((
d*x^n)/c)])/(c*n*(1 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p-1)}(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{x^n(x^n b+a)^p}{x^{np+n}dx+x^{np}cx}dx$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*d*x + x**(n*p)*c*x),x)
```

3.496 $\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 357

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \frac{d\left(3b^2c^2+\frac{ad(1+m+n)(ad(1+m+2n)-3bc(1+m+n(3+p)))}{(1+m+n(2+p))(1+m+n(3+p))}\right)(ex)^{1+m}(a+bx^n)^{1+p}}{b^3e(1+m+n+np)} - \frac{d^2(ad(1+m+2n)-3bc(1+m+n(3+p)))x^n(ex)^{1+m}(a+bx^n)^{1+p}}{b^2e(1+m+n(2+p))(1+m+n(3+p))} + \frac{d^3x^{2n}(ex)^{1+m}(a+bx^n)^{1+p}}{be(1+m+n(3+p))} + \frac{\left(\frac{c^3}{1+m}-\frac{ad\left(3b^2c^2+\frac{ad(1+m+n)(ad(1+m+2n)-3bc(1+m+n(3+p)))}{(1+m+n(2+p))(1+m+n(3+p))}\right)}{b^3(1+m+n+np)}\right)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2}}{e}$$

output
```
d*(3*b^2*c^2+a*d*(1+m+n)*(a*d*(1+m+2*n)-3*b*c*(1+m+n*(3+p)))/(1+m+n*(2+p))
/(1+m+n*(3+p)))*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b^3/e/(n*p+m+n+1)-d^2*(a*d*(1+
m+2*n)-3*b*c*(1+m+n*(3+p)))*x^n*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b^2/e/(1+m+n*(
2+p))/(1+m+n*(3+p))+d^3*x^(2*n)*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b/e/(1+m+n*(3+
p))+(c^3/(1+m)-a*d*(3*b^2*c^2+a*d*(1+m+n)*(a*d*(1+m+2*n)-3*b*c*(1+m+n*(3+p
)))/(1+m+n*(2+p))/(1+m+n*(3+p)))/b^3/(n*p+m+n+1))*(e*x)^(1+m)*(a+b*x^n)^p*
hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/e/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 6.69 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.59

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{c^3 \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + dx^n \left(\frac{3c^2 \text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + dx^n \left(\frac{3c \text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n} + \frac{dx^n \text{Hypergeometric2F1}\left(\frac{1+m+3n}{n},-p,\frac{1+}{}\right.}{1+m+3n}\right.\right.\right.$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*((c^3*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n
)/n, -((b*x^n)/a)])/(1 + m) + d*x^n*((3*c^2*Hypergeometric2F1[(1 + m + n)/
n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + d*x^n*((3*c*Hypergeom
etric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n
) + (d*x^n*Hypergeometric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^
n)/a)])/(1 + m + 3*n)))))/(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1008, 25, 1066, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (c+dx^n)^3 (a+bx^n)^p \, dx$$
$$\Big\downarrow \text{ 1008}$$

$$\frac{\int -(ex)^m (bx^n+a)^p (dx^n+c) (d(ad(m+2n+1)-bc(m+n(p+5)+1))x^n+c(ad(m+1)-bc(m+n(p+3)}{b(m+n(p+3)+1)}$$
$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}$$

$\downarrow$ 25

$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}-$$
$$\frac{\int (ex)^m (bx^n+a)^p (dx^n+c) (d(ad(m+2n+1)-bc(m+n(p+5)+1))x^n+c(ad(m+1)-bc(m+n(p+3)+}{b(m+n(p+3)+1)}$$

$\downarrow$ 1066

$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}-$$
$$\frac{\int (ex)^m(bx^n+a)^p(d(bcn(p+2)(ad(m+1)-bc(m+n(p+3)+1))+(bc-ad)(m+1)(ad(m+2n+1)-bc(m+n(p+3)+1))+(bc-ad)n(ad(m+2n+1)-bc}{b(m+n(p+2)+1)}$$

$\downarrow$ 959

$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}-$$
$$\left(c(bcn(p+2)(ad(m+1)-bc(m+n(p+3)+1))+(m+1)(bc-ad)(ad(m+2n+1)-bc(m+n(p+3)+1)))-\frac{ad(m+1)(bcn(p+2)(ad(m+1)-bc(m+n(p+3)+1)}{}\right.$$

$\downarrow$ 889

$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}-$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(bcn(p+2)(ad(m+1)-bc(m+n(p+3)+1))+(m+1)(bc-ad)(ad(m+2n+1)-bc(m+n(p+3)+1)))-\frac{ad(m+1)(bcn(p+2)(ad(}{}\right.$$

$\downarrow$ 888

$$\frac{d(ex)^{m+1}(c+dx^n)^2(a+bx^n)^{p+1}}{be(m+n(p+3)+1)}-$$
$$(ex)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(bcn(p+2)(ad(m+1)-bc(m+n(p+3)+1))+(m+1)(bc-ad)(ad(m+2n+1)-bc(m+n(p+3)+1)))-\frac{ad(m+1)(bcn(p+2)(ad(m+1)-}{}\right.$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*e*(1 + m + n*(3 + p
))) - ((d*(a*d*(1 + m + 2*n) - b*c*(1 + m + n*(5 + p)))*(e*x)^(1 + m)*(a +
 b*x^n)^(1 + p)*(c + d*x^n))/(b*e*(1 + m + n*(2 + p))) + ((d*(b*c*n*(2 + p
)*(a*d*(1 + m) - b*c*(1 + m + n*(3 + p))) + (b*c - a*d)*(1 + m)*(a*d*(1 +
m + 2*n) - b*c*(1 + m + n*(3 + p))) + (b*c - a*d)*n*(a*d*(1 + m + 2*n) - b
*c*(1 + m + n*(5 + p))))*(e*x)^(1 + m)*(a + b*x^n)^(1 + p))/(b*e*(1 + m +
n + n*p)) + ((c*(b*c*n*(2 + p)*(a*d*(1 + m) - b*c*(1 + m + n*(3 + p))) + (
b*c - a*d)*(1 + m)*(a*d*(1 + m + 2*n) - b*c*(1 + m + n*(3 + p)))) - (a*d*(
1 + m)*(b*c*n*(2 + p)*(a*d*(1 + m) - b*c*(1 + m + n*(3 + p))) + (b*c - a*d
)*(1 + m)*(a*d*(1 + m + 2*n) - b*c*(1 + m + n*(3 + p))) + (b*c - a*d)*n*(a
*d*(1 + m + 2*n) - b*c*(1 + m + n*(5 + p)))))/(b*(1 + m + n + n*p)))*(e*x)
^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -((
b*x^n)/a)])/(e*(1 + m)*(1 + (b*x^n)/a)^p))/(b*(1 + m + n*(2 + p))))/(b*(1
+ m + n*(3 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (c+d\,x^n)^3\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3\,dx = \int (dx^n+c)^3 (bx^n+a)^p (ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p
*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \int (dx^n+c)^3 (bx^n+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p*(e*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,0,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,0,2,3,4,3,0]%%%
}+%%%{6,[
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \int (e\,x)^m\,(a+b\,x^n)^p\,(c+d\,x^n)^3 \,dx$$

input
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^3,x)
```

output
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^3, x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^3 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output

```
(e**m*(x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*m**3*x + 3*x**(m + 3*n)*(x**
n*b + a)**p*b**3*d**3*m**2*n*p*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**
3*m**2*n*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*m**2*x + 3*x**(m + 3
*n)*(x**n*b + a)**p*b**3*d**3*m*n**2*p**2*x + 6*x**(m + 3*n)*(x**n*b + a)*
*p*b**3*d**3*m*n**2*p*x + 2*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*m*n**2*
x + 6*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*m*n*p*x + 6*x**(m + 3*n)*(x**
n*b + a)**p*b**3*d**3*m*n*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*m*x
 + x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*n**3*p**3*x + 3*x**(m + 3*n)*(x*
*n*b + a)**p*b**3*d**3*n**3*p**2*x + 2*x**(m + 3*n)*(x**n*b + a)**p*b**3*d
**3*n**3*p*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*n**2*p**2*x + 6*x*
*(m + 3*n)*(x**n*b + a)**p*b**3*d**3*n**2*p*x + 2*x**(m + 3*n)*(x**n*b + a
)**p*b**3*d**3*n**2*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*n*p*x + 3
*x**(m + 3*n)*(x**n*b + a)**p*b**3*d**3*n*x + x**(m + 3*n)*(x**n*b + a)**p
*b**3*d**3*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d**3*m**2*n*p*x + 2*x**
(m + 2*n)*(x**n*b + a)**p*a*b**2*d**3*m*n**2*p**2*x + x**(m + 2*n)*(x**n*b
 + a)**p*a*b**2*d**3*m*n**2*p*x + 2*x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d*
*3*m*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d**3*n**3*p**3*x + x**(m
+ 2*n)*(x**n*b + a)**p*a*b**2*d**3*n**3*p**2*x + 2*x**(m + 2*n)*(x**n*b +
a)**p*a*b**2*d**3*n**2*p**2*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d**3*n
**2*p*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d**3*n*p*x + 3*x**(m + 2*...
```

3.497 $\int (ex)^m (a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 225

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{d(ad(1+m+n)-2bc(1+m+n(2+p)))(ex)^{1+m}(a+bx^n)^{1+p}}{b^2e(1+m+n+np)(1+m+n(2+p))} + \frac{d^2x^n(ex)^{1+m}(a+bx^n)^{1+p}}{be(1+m+n(2+p))} + \frac{\left(\frac{c^2}{1+m}+\frac{ad(ad(1+m+n)-2bc(1+m+n(2+p)))}{b^2(1+m+n+np)(1+m+n(2+p))}\right)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\right.}{e}$$

output
```
-d*(a*d*(1+m+n)-2*b*c*(1+m+n*(2+p)))*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b^2/e/(n*
p+m+n+1)/(1+m+n*(2+p))+d^2*x^n*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b/e/(1+m+n*(2+p
))+(c^2/(1+m)+a*d*(a*d*(1+m+n)-2*b*c*(1+m+n*(2+p)))/b^2/(n*p+m+n+1)/(1+m+n
*(2+p)))*(e*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^
n/a)/e/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.71

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^2 \, dx$$
$$= x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{c^2 \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + dx^n \left(\frac{2c \text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + \frac{dx^n \text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n}\right)\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*((c^2*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n
)/n, -((b*x^n)/a)])/(1 + m) + d*x^n*((2*c*Hypergeometric2F1[(1 + m + n)/n,
 -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + (d*x^n*Hypergeometric2F
1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*n))))/(1
 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1008, 25, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (c+dx^n)^2 (a+bx^n)^p \, dx$$
$$\Big\downarrow \; 1008$$

$$\frac{\int -(ex)^m (bx^n+a)^p (d(ad(m+n+1)-bc(m+n(p+3)+1))x^n+c(ad(m+1)-bc(m+n(p+2)+1)))\,dx}{b(m+n(p+2)+1)} - \frac{d(ex)^{m+1}(c+dx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)}$$

$$\Big\downarrow \; 25$$

$$\frac{d(ex)^{m+1}(c+dx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\int (ex)^m (bx^n+a)^p (d(ad(m+n+1)-bc(m+n(p+3)+1))x^n+c(ad(m+1)-bc(m+n(p+2)+1)))\,dx}{b(m+n(p+2)+1)}$$

$$\Big\downarrow \; 959$$

$$\frac{d(ex)^{m+1}(c+dx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\left(c(ad(m+1)-bc(m+n(p+2)+1))-\frac{ad(m+1)(ad(m+n+1)-bc(m+n(p+3)+1))}{b(m+np+n+1)}\right)\int (ex)^m (bx^n+a)^p\,dx + \frac{d(ex)^{m+1}(a}{}}{b(m+n(p+2)+1)}$$

$$\Big\downarrow \; 889$$

$$\frac{d(ex)^{m+1}(c+dx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(ad(m+1)-bc(m+n(p+2)+1))-\frac{ad(m+1)(ad(m+n+1)-bc(m+n(p+3)+1))}{b(m+np+n+1)}\right)\int (ex)^m \Big(\frac{b}{}}{b(m+n(p+2)+1)}$$

$$\Big\downarrow \; 888$$

$$\frac{d(ex)^{m+1}(c+dx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\frac{(ex)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(ad(m+1)-bc(m+n(p+2)+1))-\frac{ad(m+1)(ad(m+n+1)-bc(m+n(p+3)+1))}{b(m+np+n+1)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n}{n}\right.}{e(m+1)}}{b(m+n(p+2)+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*e*(1 + m + n*(2 + p))
) - ((d*(a*d*(1 + m + n) - b*c*(1 + m + n*(3 + p)))*(e*x)^(1 + m)*(a + b*x
^n)^(1 + p))/(b*e*(1 + m + n + n*p)) + ((c*(a*d*(1 + m) - b*c*(1 + m + n*(
2 + p))) - (a*d*(1 + m)*(a*d*(1 + m + n) - b*c*(1 + m + n*(3 + p))))/(b*(1
 + m + n + n*p)))*(e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n,
 -p, (1 + m + n)/n, -((b*x^n)/a)])/(e*(1 + m)*(1 + (b*x^n)/a)^p))/(b*(1 +
m + n*(2 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (c+d\,x^n)^2\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^2\,dx = \int (dx^n+c)^2 (bx^n+a)^p (ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^2 \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^2 \, dx = \int (dx^n + c)^2 (bx^n + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*(e*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,0,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,0,1,2,3,2,0]%
%%}+%%%{-
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^2 \, dx = \int (e\,x)^m \, (a+b\,x^n)^p \, (c+d\,x^n)^2 \, dx$$

input
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^2,x)
```

output
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^2, x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^2 \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(e**m*(x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*m**2*x + 2*x**(m + 2*n)*(x**
n*b + a)**p*b**2*d**2*m*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*m*n
*x + 2*x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*m*x + x**(m + 2*n)*(x**n*b +
 a)**p*b**2*d**2*n**2*p**2*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*n**2
*p*x + 2*x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*n*p*x + x**(m + 2*n)*(x**n
*b + a)**p*b**2*d**2*n*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d**2*x + x**(
m + n)*(x**n*b + a)**p*a*b*d**2*m*n*p*x + x**(m + n)*(x**n*b + a)**p*a*b*d
**2*n**2*p**2*x + x**(m + n)*(x**n*b + a)**p*a*b*d**2*n*p*x + 2*x**(m + n)
*(x**n*b + a)**p*b**2*c*d*m**2*x + 4*x**(m + n)*(x**n*b + a)**p*b**2*c*d*m
*n*p*x + 4*x**(m + n)*(x**n*b + a)**p*b**2*c*d*m*n*x + 4*x**(m + n)*(x**n*
b + a)**p*b**2*c*d*m*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*d*n**2*p**2*x
 + 4*x**(m + n)*(x**n*b + a)**p*b**2*c*d*n**2*p*x + 4*x**(m + n)*(x**n*b +
 a)**p*b**2*c*d*n*p*x + 4*x**(m + n)*(x**n*b + a)**p*b**2*c*d*n*x + 2*x**(
m + n)*(x**n*b + a)**p*b**2*c*d*x - x**m*(x**n*b + a)**p*a**2*d**2*m*n*p*x
 - x**m*(x**n*b + a)**p*a**2*d**2*n**2*p*x - x**m*(x**n*b + a)**p*a**2*d**
2*n*p*x + 2*x**m*(x**n*b + a)**p*a*b*c*d*m*n*p*x + 2*x**m*(x**n*b + a)**p*
a*b*c*d*n**2*p**2*x + 4*x**m*(x**n*b + a)**p*a*b*c*d*n**2*p*x + 2*x**m*(x*
*n*b + a)**p*a*b*c*d*n*p*x + x**m*(x**n*b + a)**p*b**2*c**2*m**2*x + 2*x**
m*(x**n*b + a)**p*b**2*c**2*m*n*p*x + 3*x**m*(x**n*b + a)**p*b**2*c**2*m*n
*x + 2*x**m*(x**n*b + a)**p*b**2*c**2*m*x + x**m*(x**n*b + a)**p*b**2*c...
```

3.498 $\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \frac{d(ex)^{1+m}(a+bx^n)^{1+p}}{be(1+m+n+np)} + \frac{\left(\frac{c}{1+m}-\frac{ad}{b(1+m+n+np)}\right)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{e}$$

output
```
d*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b/e/(n*p+m+n+1)+(c/(1+m)-a*d/b/(n*p+m+n+1))*
(e*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/e/((
1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \frac{x(ex)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+d(1+m)x\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(c*(1 + m + n)*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)] + d*(1 + m)*x^n*Hypergeometric2F1[(1 + m + n)/
n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*(1 + (b*x^n)/
a)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (c+dx^n) (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int (ex)^m (bx^n+a)^p dx+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int (ex)^m \left(\frac{bx^n}{a}+1\right)^p dx+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right) \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{e(m+1)}+\frac{d(ex)^{m+1} (a+bx^n)^{p+1}}{be(m+np+n+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^n)^(1 + p))/(b*e*(1 + m + n + n*p)) + ((c - (a*d
*(1 + m))/(b*(1 + m + n + n*p)))*(e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometri
c2F1[(1 + m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/(e*(1 + m)*(1 + (b*x^n)/
a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int (ex)^m (a + b x^n)^p (c + d x^n) \, dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(bx^n+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.31

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+p-\frac{1}{n}} c e^m x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)} + \frac{a^{\frac{m}{n}+1+\frac{1}{n}} a^{-\frac{m}{n}+p-1-\frac{1}{n}} d e^m x^{m+n+1} \Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+1+\frac{1}{n} \\ \frac{m}{n}+2+\frac{1}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c*e**m*x**(m + 1)*gamma(m/n + 1/n)*hype
r((-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/
n + 1 + 1/n)) + a**(m/n + 1 + 1/n)*a**(-m/n + p - 1 - 1/n)*d*e**m*x**(m +
n + 1)*gamma(m/n + 1 + 1/n)*hyper((-p, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b
*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(bx^n+a)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*(e*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,0,1,2,1,0,1]%%%}+%%%{2,[0,0,2,2,0,1,1,1,0,1]%%%
}+%%%{1,[
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (c+dx^n) \, dx = \int (e\,x)^m\,(a+b\,x^n)^p\,(c+d\,x^n) \, dx$$

input
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (ex)^m (a + bx^n)^p (c + dx^n) \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(e**m*(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d
*n*p*x + x**(m + n)*(x**n*b + a)**p*b*d*x + x**m*(x**n*b + a)**p*a*d*n*p*x
 + x**m*(x**n*b + a)**p*b*c*m*x + x**m*(x**n*b + a)**p*b*c*n*p*x + x**m*(x
**n*b + a)**p*b*c*n*x + x**m*(x**n*b + a)**p*b*c*x - int((x**m*(x**n*b + a
)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**
2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m
*n*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2
*d*m**3*n*p - 2*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p +
 x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p
 + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 +
a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n**2*p**2 - int((x**m*(x**n*b
 + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b
*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2
*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*
a**2*d*m**2*n**2*p - 3*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*
m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**
n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*
p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n*p - int((x**m*(x**n*
b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*
b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2...
```

3.499 $\int (ex)^m (a+bx^n)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int (ex)^m (a+bx^n)^p \, dx = \frac{(ex)^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/e/(1
+m)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int (ex)^m (a+bx^n)^p \, dx = \frac{x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,1+\frac{1+m}{n},-\frac{bx^n}{a}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, 1 + (1 + m)/n, -
((b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/
n, -((b*x^n)/a)])/(e*(1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m (a + b\,x^n)^p\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (ex)^m (a + bx^n)^p \, dx = \int (bx^n + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(e*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int (ex)^m (a+bx^n)^p \, dx = \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+p-\frac{1}{n}} e^m x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p,x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*e**m*x**(m + 1)*gamma(m/n + 1/n)*hyper(
(-p, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n
+ 1 + 1/n))
```

Maxima [F]

$$\int (ex)^m (a+bx^n)^p \, dx = \int (bx^n+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (a+bx^n)^p \, dx = \int (bx^n+a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p \, dx = \int (e\,x)^m\,(a+b\,x^n)^p\,dx$$

input
```
int((e*x)^m*(a + b*x^n)^p,x)
```

output
```
int((e*x)^m*(a + b*x^n)^p, x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p \, dx$$
$$= \frac{e^m \left(x^m(x^n b+a)^p x+\left(\int \frac{x^m(x^n b+a)^p}{x^n bm+x^n bnp+x^n b+am+anp+a}dx\right) amnp+\left(\int \frac{x^m(x^n b+a)^p}{x^n bm+x^n bnp+x^n b+am+anp+a}dx\right) a\,n^2p^2\right.}{np+m+1}$$

input
```
int((e*x)^m*(a+b*x^n)^p,x)
```

output
```
(e**m*(x**m*(x**n*b + a)**p*x + int((x**m*(x**n*b + a)**p)/(x**n*b*m + x**
n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*m*n*p + int((x**m*(x**n*b + a)**p
)/(x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),x)*a*n**2*p**2 + int(
(x**m*(x**n*b + a)**p)/(x**n*b*m + x**n*b*n*p + x**n*b + a*m + a*n*p + a),
x)*a*n*p))/(m + n*p + 1)
```

3.500 $\int \frac{(ex)^m(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \frac{(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ce(1+m)}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^p*AppellF1((1+m)/n,-p,1,(1+m+n)/n,-b*x^n/a,-d*x^n/c)
/c/e/(1+m)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \frac{x(ex)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c+cm}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 1, (1 + m + n)/n, -((b*x^
n)/a), -((d*x^n)/c)])/((c + c*m)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n} \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \frac{(ex)^m \left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{n},-p,1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{ce(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 1, (1 + m + n)/n, -((
b*x^n)/a), -((d*x^n)/c)])/(c*e*(1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^p(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p (ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p (ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(e\,x)^m\,(a+b\,x^n)^p}{c+d\,x^n}\,dx$$

input
```
int(((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{c+dx^n}\,dx = e^m \left(\int \frac{x^m (x^n b+a)^p}{x^n d+c}dx\right)$$

input
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
e**m*int((x**m*(x**n*b + a)**p)/(x**n*d + c),x)
```

3.501 $\int \frac{(ex)^m(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \frac{(ex)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m}{n},-p,2,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2e(1+m)}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^p*AppellF1((1+m)/n,-p,2,(1+m+n)/n,-b*x^n/a,-d*x^n/c)
/c^2/e/(1+m)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \frac{x(ex)^m (a+bx^n)^p \left(\frac{a+bx^n}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1+m}{n},-p,2,1+\frac{1+m}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 2, 1 + (1 + m)/n, -((b*x^
n)/a), -((d*x^n)/c)])/(c^2*(1 + m)*((a + b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2} \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int \frac{(ex)^m \left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{n},-p,2,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2 e(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 2, (1 + m + n)/n, -((
b*x^n)/a), -((d*x^n)/c)])/(c^2*e*(1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p (ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(e*x)^m/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p (ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p (ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(e\,x)^m\,(a+b\,x^n)^p}{(c+d\,x^n)^2}\,dx$$

input
```
int(((e*x)^m*(a + b*x^n)^p)/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(a + b*x^n)^p)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^p}{(c+dx^n)^2}\,dx = e^m \left(\int \frac{x^m (x^n b+a)^p}{x^{2n} d^2 + 2x^n cd + c^2}dx\right)$$

input
```
int((e*x)^m*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
e**m*int((x**m*(x**n*b + a)**p)/(x**(2*n)*d**2 + 2*x**n*c*d + c**2),x)
```

3.502 $\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\,dx=\frac{dx^{1+m+2n}(a+bx^n)^{1+p}}{b(1+m+n(3+p))}+\left(\frac{c}{1+m+2n}-\frac{ad}{b(1+m+n(3+p))}\right)x^{1+m+2n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x^(1+m+2*n)*(a+b*x^n)^(p+1)/b/(1+m+n*(3+p))+(c/(1+m+2*n)-a*d/b/(1+m+n*(3
+p)))*x^(1+m+2*n)*(a+b*x^n)^p*hypergeom([-p, (1+m+2*n)/n],[(1+m+3*n)/n],-b
*x^n/a)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\,dx$$
$$=\frac{x^{1+m+2n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m+3n)\,\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)+d(1+\right.}{(1+m+2n)(1+m+3n)}$$

input
```
Integrate[x^(m + 2*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m + 2*n)*(a + b*x^n)^p*(c*(1 + m + 3*n)*Hypergeometric2F1[(1 + m +
 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)] + d*(1 + m + 2*n)*x^n*Hypergeo
metric2F1[(1 + m + 3*n)/n, -p, (1 + m + 4*n)/n, -((b*x^n)/a)]))/((1 + m +
2*n)*(1 + m + 3*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2n}(c+dx^n)\,(a+bx^n)^p\,dx$$
$$\Big\downarrow \; 959$$
$$\left(c-\frac{ad(m+2n+1)}{b(m+n(p+3)+1)}\right)\int x^{m+2n}(bx^n+a)^p\,dx+\frac{dx^{m+2n+1}(a+bx^n)^{p+1}}{b(m+n(p+3)+1)}$$
$$\Big\downarrow \; 889$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+2n+1)}{b(m+n(p+3)+1)}\right)\int x^{m+2n}\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^{m+2n+1}(a+bx^n)^{p+1}}{b(m+n(p+3)+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+2n+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+2n+1)}{b(m+n(p+3)+1)}\right)\text{Hypergeometric2F1}\left(\frac{m+2n+1}{n},-p,\frac{m+3n+1}{n},-\frac{bx^n}{a}\right)}{m+2n+1}+\frac{dx^{m+2n+1}(a+bx^n)^{p+1}}{b(m+n(p+3)+1)}$$

input
```
Int[x^(m + 2*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m + 2*n)*(a + b*x^n)^(1 + p))/(b*(1 + m + n*(3 + p))) + ((c - (a
*d*(1 + m + 2*n))/(b*(1 + m + n*(3 + p))))*x^(1 + m + 2*n)*(a + b*x^n)^p*H
ypergeometric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/((1
+ m + 2*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{m+2n}(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{m+2\,n}\,dx$$

input
```
integrate(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p*x^(m + 2*n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{a^{\frac{m}{n}+2+\frac{1}{n}}a^{-\frac{m}{n}+p-2-\frac{1}{n}}cx^{m+2n+1}\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+2+\frac{1}{n}\\ \frac{m}{n}+3+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}$$
$$+\frac{a^{\frac{m}{n}+3+\frac{1}{n}}a^{-\frac{m}{n}+p-3-\frac{1}{n}}dx^{m+3n+1}\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+3+\frac{1}{n}\\ \frac{m}{n}+4+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+4+\frac{1}{n}\right)}$$

input
```
integrate(x**(m+2*n)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 2 + 1/n)*a**(-m/n + p - 2 - 1/n)*c*x**(m + 2*n + 1)*gamma(m/n +
2 + 1/n)*hyper((-p, m/n + 2 + 1/n), (m/n + 3 + 1/n,), b*x**n*exp_polar(I*p
i)/a)/(n*gamma(m/n + 3 + 1/n)) + a**(m/n + 3 + 1/n)*a**(-m/n + p - 3 - 1/n
)*d*x**(m + 3*n + 1)*gamma(m/n + 3 + 1/n)*hyper((-p, m/n + 3 + 1/n), (m/n
+ 4 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 4 + 1/n))
```

Maxima [F]

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^px^{m+2\,n}\,dx$$

input
```
integrate(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(m + 2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,0,1,0,1]%%%}+%%%{4,[0,0,2,2,1,1,0,1,0,1]%%%
}+%%%{3,[
```

Mupad [F(-1)]

Timed out.

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx=\int x^{m+2\,n}\,(a+b\,x^n)^p\,(c+d\,x^n)\;dx$$

input
```
int(x^(m + 2*n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^(m + 2*n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^{m+2n}(a+bx^n)^p\,(c+dx^n)\;dx=\text{too large to display}$$

input
```
int(x^(m+2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output

```
(x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m**3*x + 3*x**(m + 3*n)*(x**n*b + a)*
*p*b**3*d*m**2*n*p*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m**2*n*x + 3*
x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m**2*x + 3*x**(m + 3*n)*(x**n*b + a)**
p*b**3*d*m*n**2*p**2*x + 6*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m*n**2*p*x
+ 2*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m*n**2*x + 6*x**(m + 3*n)*(x**n*b
+ a)**p*b**3*d*m*n*p*x + 6*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*m*n*x + 3*x
**(m + 3*n)*(x**n*b + a)**p*b**3*d*m*x + x**(m + 3*n)*(x**n*b + a)**p*b**3
*d*n**3*p**3*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*n**3*p**2*x + 2*x**
(m + 3*n)*(x**n*b + a)**p*b**3*d*n**3*p*x + 3*x**(m + 3*n)*(x**n*b + a)**p
*b**3*d*n**2*p**2*x + 6*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*n**2*p*x + 2*x
**(m + 3*n)*(x**n*b + a)**p*b**3*d*n**2*x + 3*x**(m + 3*n)*(x**n*b + a)**p
*b**3*d*n*p*x + 3*x**(m + 3*n)*(x**n*b + a)**p*b**3*d*n*x + x**(m + 3*n)*(
x**n*b + a)**p*b**3*d*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d*m**2*n*p*x
 + 2*x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d*m*n**2*p**2*x + x**(m + 2*n)*(x
**n*b + a)**p*a*b**2*d*m*n**2*p*x + 2*x**(m + 2*n)*(x**n*b + a)**p*a*b**2*
d*m*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d*n**3*p**3*x + x**(m + 2*
n)*(x**n*b + a)**p*a*b**2*d*n**3*p**2*x + 2*x**(m + 2*n)*(x**n*b + a)**p*a
*b**2*d*n**2*p**2*x + x**(m + 2*n)*(x**n*b + a)**p*a*b**2*d*n**2*p*x + x**
(m + 2*n)*(x**n*b + a)**p*a*b**2*d*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b*
*3*c*m**3*x + 3*x**(m + 2*n)*(x**n*b + a)**p*b**3*c*m**2*n*p*x + 4*x**(...
```

3.503 $\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\,dx=\frac{dx^{1+m+n}(a+bx^n)^{1+p}}{b(1+m+n(2+p))}+\left(\frac{c}{1+m+n}-\frac{ad}{b(1+m+n(2+p))}\right)x^{1+m+n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x^(1+m+n)*(a+b*x^n)^(p+1)/b/(1+m+n*(2+p))+(c/(1+m+n)-a*d/b/(1+m+n*(2+p))
)*x^(1+m+n)*(a+b*x^n)^p*hypergeom([-p, (1+m+n)/n],[(1+m+2*n)/n],-b*x^n/a)/
((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int x^{m+n}(a+bx^n)^p(c+dx^n)\,dx = \frac{x^{1+m+n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m+2n)\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)+d(1+\right.}{(1+m+n)(1+m+2n)}$$

input
```
Integrate[x^(m + n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m + n)*(a + b*x^n)^p*(c*(1 + m + 2*n)*Hypergeometric2F1[(1 + m + n
)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)] + d*(1 + m + n)*x^n*Hypergeometric
2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)]))/((1 + m + n)*(1
+ m + 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+n}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m+n+1)}{b(m+n(p+2)+1)}\right)\int x^{m+n}(bx^n+a)^p\,dx+\frac{dx^{m+n+1}(a+bx^n)^{p+1}}{b(m+n(p+2)+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+n+1)}{b(m+n(p+2)+1)}\right)\int x^{m+n}\left(\frac{bx^n}{a}+1\right)^p\,dx+\frac{dx^{m+n+1}(a+bx^n)^{p+1}}{b(m+n(p+2)+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+n+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+n+1)}{b(m+n(p+2)+1)}\right)\text{Hypergeometric2F1}\left(\frac{m+n+1}{n},-p,\frac{m+2n+1}{n},-\frac{bx^n}{a}\right)}{m+n+1}+\frac{dx^{m+n+1}(a+bx^n)^{p+1}}{b(m+n(p+2)+1)}$$

input
```
Int[x^(m + n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m + n)*(a + b*x^n)^(1 + p))/(b*(1 + m + n*(2 + p))) + ((c - (a*d
*(1 + m + n))/(b*(1 + m + n*(2 + p))))*x^(1 + m + n)*(a + b*x^n)^p*Hyperge
ometric2F1[(1 + m + n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/((1 + m + n)
*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{m+n}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{m+n}(a+bx^{n})^{p}\,(c+dx^{n})\,\,dx=\int (dx^{n}+c)(bx^{n}+a)^{p}x^{m+n}\,dx$$

input
```
integrate(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^n + c)*(b*x^n + a)^p*x^(m + n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.36

$$\begin{aligned}&\int x^{m+n}(a+bx^{n})^{p}\,(c+dx^{n})\,\,dx\\&=\frac{a^{\frac{m}{n}+1+\frac{1}{n}}a^{-\frac{m}{n}+p-1-\frac{1}{n}}cx^{m+n+1}\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}-p,\frac{m}{n}+1+\frac{1}{n}\\\frac{m}{n}+2+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}\\&+\frac{a^{\frac{m}{n}+2+\frac{1}{n}}a^{-\frac{m}{n}+p-2-\frac{1}{n}}dx^{m+2n+1}\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}-p,\frac{m}{n}+2+\frac{1}{n}\\\frac{m}{n}+3+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+3+\frac{1}{n}\right)}\end{aligned}$$

input
```
integrate(x**(m+n)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 1 + 1/n)*a**(-m/n + p - 1 - 1/n)*c*x**(m + n + 1)*gamma(m/n + 1
+ 1/n)*hyper((-p, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**n*exp_polar(I*pi)
/a)/(n*gamma(m/n + 2 + 1/n)) + a**(m/n + 2 + 1/n)*a**(-m/n + p - 2 - 1/n)*
d*x**(m + 2*n + 1)*gamma(m/n + 2 + 1/n)*hyper((-p, m/n + 2 + 1/n), (m/n +
3 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 3 + 1/n))
```

Maxima [F]

$$\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^px^{m+n}\,dx$$

input
```
integrate(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(m + n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,0,1,2,1,0,1]%%%}+%%%{3,[0,0,2,2,0,1,1,1,0,1]%%%
}+%%%{2,[
```

Mupad [F(-1)]

Timed out.

$$\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\;dx=\int x^{m+n}\,(a+b\,x^n)^p\,(c+d\,x^n)\;dx$$

input
```
int(x^(m + n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^(m + n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^{m+n}(a+bx^n)^p\,(c+dx^n)\;dx=\text{too large to display}$$

input
```
int(x^(m+n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output

```
(x**(m + 2*n)*(x**n*b + a)**p*b**2*d*m**2*x + 2*x**(m + 2*n)*(x**n*b + a)*
*p*b**2*d*m*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*m*n*x + 2*x**(m +
2*n)*(x**n*b + a)**p*b**2*d*m*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*n**2
*p**2*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*n**2*p*x + 2*x**(m + 2*n)*(x
**n*b + a)**p*b**2*d*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*n*x + x**
(m + 2*n)*(x**n*b + a)**p*b**2*d*x + x**(m + n)*(x**n*b + a)**p*a*b*d*m*n*
p*x + x**(m + n)*(x**n*b + a)**p*a*b*d*n**2*p**2*x + x**(m + n)*(x**n*b +
a)**p*a*b*d*n*p*x + x**(m + n)*(x**n*b + a)**p*b**2*c*m**2*x + 2*x**(m + n
)*(x**n*b + a)**p*b**2*c*m*n*p*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*m*n
*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*m*x + x**(m + n)*(x**n*b + a)**p*
b**2*c*n**2*p**2*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*n**2*p*x + 2*x**(
m + n)*(x**n*b + a)**p*b**2*c*n*p*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*
n*x + x**(m + n)*(x**n*b + a)**p*b**2*c*x - x**m*(x**n*b + a)**p*a**2*d*m*
n*p*x - x**m*(x**n*b + a)**p*a**2*d*n**2*p*x - x**m*(x**n*b + a)**p*a**2*d
*n*p*x + x**m*(x**n*b + a)**p*a*b*c*m*n*p*x + x**m*(x**n*b + a)**p*a*b*c*n
**2*p**2*x + 2*x**m*(x**n*b + a)**p*a*b*c*n**2*p*x + x**m*(x**n*b + a)**p*
a*b*c*n*p*x + int((x**m*(x**n*b + a)**p)/(x**n*b*m**3 + 3*x**n*b*m**2*n*p
+ 3*x**n*b*m**2*n + 3*x**n*b*m**2 + 3*x**n*b*m*n**2*p**2 + 6*x**n*b*m*n**2
*p + 2*x**n*b*m*n**2 + 6*x**n*b*m*n*p + 6*x**n*b*m*n + 3*x**n*b*m + x**n*b
*n**3*p**3 + 3*x**n*b*n**3*p**2 + 2*x**n*b*n**3*p + 3*x**n*b*n**2*p**2 ...
```

3.504 $\int x^m(a+bx^n)^p(c+dx^n)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int x^m(a+bx^n)^p(c+dx^n)\,dx = \frac{dx^{1+m}(a+bx^n)^{1+p}}{b(1+m+n+np)} + \left(\frac{c}{1+m}-\frac{ad}{b(1+m+n+np)}\right)x^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)$$

output
```
d*x^(1+m)*(a+b*x^n)^(p+1)/b/(n*p+m+n+1)+(c/(1+m)-a*d/b/(n*p+m+n+1))*x^(1+m
)*(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/((1+b*x^n/a)^p
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int x^m (a+bx^n)^p (c+dx^n)\, dx$$
$$= \frac{x^{1+m}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(c(1+m+n)\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+d(1+m)x^n \text{ H}\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[x^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m)*(a + b*x^n)^p*(c*(1 + m + n)*Hypergeometric2F1[(1 + m)/n, -p, (
1 + m + n)/n, -((b*x^n)/a)] + d*(1 + m)*x^n*Hypergeometric2F1[(1 + m + n)/
n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m + n)*(1 + (b*x^n)/
a)^p)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (c+dx^n) (a+bx^n)^p \, dx$$
$$\Big\downarrow \text{959}$$
$$\left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int x^m (bx^n+a)^p dx+\frac{dx^{m+1}(a+bx^n)^{p+1}}{b(m+np+n+1)}$$
$$\Big\downarrow \text{889}$$
$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\int x^m \left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^{m+1}(a+bx^n)^{p+1}}{b(m+np+n+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m+1)}{b(m+np+n+1)}\right)\text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{m+1}+\frac{dx^{m+1}(a+bx^n)^{p+1}}{b(m+np+n+1)}$$

input
```
Int[x^m*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m)*(a + b*x^n)^(1 + p))/(b*(1 + m + n + n*p)) + ((c - (a*d*(1 +
m))/(b*(1 + m + n + n*p)))*x^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 +
m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/((1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^m (a + b\,x^n)^p\,(c + d\,x^n)\,dx$$

input
```
int(x^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^m*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^m (a + bx^n)^p\,(c + dx^n)\;dx = \int (dx^n + c)(bx^n + a)^p x^m\,dx$$

input
```
integrate(x^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^m*x^n + c*x^m)*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.37

$$\int x^m (a + bx^n)^p\,(c + dx^n)\;dx$$
$$= \frac{a^{\frac{m}{n}+\frac{1}{n}} a^{-\frac{m}{n}+p-\frac{1}{n}} c x^{m+1} \Gamma\left(\frac{m}{n}+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+\frac{1}{n} \\ \frac{m}{n}+1+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{a^{\frac{m}{n}+1+\frac{1}{n}} a^{-\frac{m}{n}+p-1-\frac{1}{n}} d x^{m+n+1} \Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{n}+1+\frac{1}{n} \\ \frac{m}{n}+2+\frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate(x**m*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*c*x**(m + 1)*gamma(m/n + 1/n)*hyper((-p
, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1
 + 1/n)) + a**(m/n + 1 + 1/n)*a**(-m/n + p - 1 - 1/n)*d*x**(m + n + 1)*gam
ma(m/n + 1 + 1/n)*hyper((-p, m/n + 1 + 1/n), (m/n + 2 + 1/n,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int x^m (a + b x^n)^p (c + d x^n) \, dx = \int (dx^n + c)(bx^n + a)^p x^m \, dx$$

input
```
integrate(x^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^m, x)
```

Giac [F]

$$\int x^m (a + b x^n)^p (c + d x^n) \, dx = \int (dx^n + c)(bx^n + a)^p x^m \, dx$$

input
```
integrate(x^m*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(a+bx^n)^p\left(c+dx^n\right)\,dx=\int x^m\,{\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right)\,dx$$

input
```
int(x^m*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^m*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^m(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{too large to display}$$

input
```
int(x^m*(a+b*x^n)^p*(c+d*x^n),x)
```

output

```
(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d*n*p*x
 + x**(m + n)*(x**n*b + a)**p*b*d*x + x**m*(x**n*b + a)**p*a*d*n*p*x + x**
m*(x**n*b + a)**p*b*c*m*x + x**m*(x**n*b + a)**p*b*c*n*p*x + x**m*(x**n*b
+ a)**p*b*c*n*x + x**m*(x**n*b + a)**p*b*c*x - int((x**m*(x**n*b + a)**p)/
(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2
 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p +
 a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**
3*n*p - 2*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*
b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**
n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2
*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n**2*p**2 - int((x**m*(x**n*b + a)*
*p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*
p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n
*p + a*m*n + 2*a*m + a*n**2*p**2 + a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d
*m**2*n**2*p - 3*int((x**m*(x**n*b + a)**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p
+ x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2*p**2 + x**n*b*n**2*p + 2*x**n*b*n*
p + x**n*b*n + x**n*b + a*m**2 + 2*a*m*n*p + a*m*n + 2*a*m + a*n**2*p**2 +
 a*n**2*p + 2*a*n*p + a*n + a),x)*a**2*d*m**2*n*p - int((x**m*(x**n*b + a)
**p)/(x**n*b*m**2 + 2*x**n*b*m*n*p + x**n*b*m*n + 2*x**n*b*m + x**n*b*n**2
*p**2 + x**n*b*n**2*p + 2*x**n*b*n*p + x**n*b*n + x**n*b + a*m**2 + 2*a...
```

3.505 $\int x^{m-n}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int x^{m-n}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{cx^{1+m-n}(a+bx^n)^{1+p}}{a(1+m-n)}+\frac{(ad(1+m-n)-bc(1+m+np))x^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},\right.}{a(1+m)(1+m-n)}$$

output
```
c*x^(1+m-n)*(a+b*x^n)^(p+1)/a/(1+m-n)+(a*d*(1+m-n)-b*c*(n*p+m+1))*x^(1+m)*
(a+b*x^n)^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(1+m)/(1+m-n)/
((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int x^{m-n}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{x^{1+m-n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(d(1+m-n)x^n\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+c(1+m\right.}{(1+m)(1+m-n)}$$

input
```
Integrate[x^(m - n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m - n)*(a + b*x^n)^p*(d*(1 + m - n)*x^n*Hypergeometric2F1[(1 + m)/
n, -p, (1 + m + n)/n, -((b*x^n)/a)] + c*(1 + m)*Hypergeometric2F1[(1 + m -
 n)/n, -p, (1 + m)/n, -((b*x^n)/a)]))/((1 + m)*(1 + m - n)*(1 + (b*x^n)/a)
^p)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-n}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m-n+1)}{b(m+np+1)}\right)\int x^{m-n}(bx^n+a)^p dx+\frac{dx^{m-n+1}(a+bx^n)^{p+1}}{b(m+np+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-n+1)}{b(m+np+1)}\right)\int x^{m-n}\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^{m-n+1}(a+bx^n)^{p+1}}{b(m+np+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m-n+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-n+1)}{b(m+np+1)}\right)\text{Hypergeometric2F1}\left(\frac{m-n+1}{n},-p,\frac{m+1}{n},-\frac{bx^n}{a}\right)}{m-n+1}+\frac{dx^{m-n+1}(a+bx^n)^{p+1}}{b(m+np+1)}$$

input
```
Int[x^(m - n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m - n)*(a + b*x^n)^(1 + p))/(b*(1 + m + n*p)) + ((c - (a*d*(1 +
m - n))/(b*(1 + m + n*p)))*x^(1 + m - n)*(a + b*x^n)^p*Hypergeometric2F1[(
1 + m - n)/n, -p, (1 + m)/n, -((b*x^n)/a)])/((1 + m - n)*(1 + (b*x^n)/a)^p
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{m-n}(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{m-n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int(dx^n+c)(bx^n+a)^px^{m-n}\,dx$$

input
```
integrate(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(m - n)*x^n + c*x^(m - n))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.85 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19

$$\int x^{m-n}(a+bx^n)^p\left(c+dx^n\right)\,dx$$
$$=\frac{a^{\frac{m}{n}+\frac{1}{n}}a^{-\frac{m}{n}+p-\frac{1}{n}}dx^{m+1}\Gamma\left(\frac{m}{n}+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}+\frac{1}{n}\\ \frac{m}{n}+1+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+\frac{a^{\frac{m}{n}-1+\frac{1}{n}}a^{-\frac{m}{n}+p+1-\frac{1}{n}}cx^{m-n+1}\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right){}_2F_1\left(\begin{matrix}-p,\frac{m}{n}-1+\frac{1}{n}\\ \frac{m}{n}+\frac{1}{n}\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}$$

input
```
integrate(x**(m-n)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n + 1/n)*a**(-m/n + p - 1/n)*d*x**(m + 1)*gamma(m/n + 1/n)*hyper((-p
, m/n + 1/n), (m/n + 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1
 + 1/n)) + a**(m/n - 1 + 1/n)*a**(-m/n + p + 1 - 1/n)*c*x**(m - n + 1)*gam
ma(m/n - 1 + 1/n)*hyper((-p, m/n - 1 + 1/n), (m/n + 1/n,), b*x**n*exp_pola
r(I*pi)/a)/(n*gamma(m/n + 1/n))
```

Maxima [F]

$$\int x^{m-n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int (dx^n+c)(bx^n+a)^px^{m-n}\,dx$$

input
```
integrate(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(m - n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{m-n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,0,1,0,1]%%%}+%%%{1,[0,0,2,2,1,1,0,1,0,1]%%%
}+%%%{1,[
```

Mupad [F(-1)]

Timed out.

$$\int x^{m-n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int x^{m-n}\,{\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right)\,dx$$

input
```
int(x^(m - n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^(m - n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^{m-n}(a+bx^n)^p\,(c+dx^n)\;dx = \text{too large to display}$$

input
```
int(x^(m-n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d*n*p*x
 - x**(m + n)*(x**n*b + a)**p*b*d*n*x + x**(m + n)*(x**n*b + a)**p*b*d*x +
 x**m*(x**n*b + a)**p*a*d*n*p*x + x**m*(x**n*b + a)**p*b*c*m*x + x**m*(x**
n*b + a)**p*b*c*n*p*x + x**m*(x**n*b + a)**p*b*c*x - x**n*int((x**m*(x**n*
b + a)**p)/(x**(2*n)*b*m**2 + 2*x**(2*n)*b*m*n*p - x**(2*n)*b*m*n + 2*x**(
2*n)*b*m + x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p - x
**(2*n)*b*n + x**(2*n)*b + x**n*a*m**2 + 2*x**n*a*m*n*p - x**n*a*m*n + 2*x
**n*a*m + x**n*a*n**2*p**2 - x**n*a*n**2*p + 2*x**n*a*n*p - x**n*a*n + x**
n*a),x)*a**2*d*m**3*n*p - 2*x**n*int((x**m*(x**n*b + a)**p)/(x**(2*n)*b*m*
*2 + 2*x**(2*n)*b*m*n*p - x**(2*n)*b*m*n + 2*x**(2*n)*b*m + x**(2*n)*b*n**
2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b
+ x**n*a*m**2 + 2*x**n*a*m*n*p - x**n*a*m*n + 2*x**n*a*m + x**n*a*n**2*p**
2 - x**n*a*n**2*p + 2*x**n*a*n*p - x**n*a*n + x**n*a),x)*a**2*d*m**2*n**2*
p**2 + 2*x**n*int((x**m*(x**n*b + a)**p)/(x**(2*n)*b*m**2 + 2*x**(2*n)*b*m
*n*p - x**(2*n)*b*m*n + 2*x**(2*n)*b*m + x**(2*n)*b*n**2*p**2 - x**(2*n)*b
*n**2*p + 2*x**(2*n)*b*n*p - x**(2*n)*b*n + x**(2*n)*b + x**n*a*m**2 + 2*x
**n*a*m*n*p - x**n*a*m*n + 2*x**n*a*m + x**n*a*n**2*p**2 - x**n*a*n**2*p +
 2*x**n*a*n*p - x**n*a*n + x**n*a),x)*a**2*d*m**2*n**2*p - 3*x**n*int((x**
m*(x**n*b + a)**p)/(x**(2*n)*b*m**2 + 2*x**(2*n)*b*m*n*p - x**(2*n)*b*m*n
+ 2*x**(2*n)*b*m + x**(2*n)*b*n**2*p**2 - x**(2*n)*b*n**2*p + 2*x**(2*n...
```

3.506 $\int x^{m-2n}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int x^{m-2n}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{cx^{1+m-2n}(a+bx^n)^{1+p}}{a(1+m-2n)}+\frac{(ad(1+m-2n)-bc(1+m-n(1-p)))x^{1+m-n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m-n}{n}\right.}{a(1+m-2n)(1+m-n)}$$

output
```
c*x^(1+m-2*n)*(a+b*x^n)^(p+1)/a/(1+m-2*n)+(a*d*(1+m-2*n)-b*c*(1+m-n*(1-p))
)*x^(1+m-n)*(a+b*x^n)^p*hypergeom([-p, (1+m-n)/n],[(1+m)/n],-b*x^n/a)/a/(1
+m-2*n)/(1+m-n)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int x^{m-2n}(a+bx^n)^p\,(c+dx^n)\;dx$$

$$=\frac{x^{1+m-2n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m-n)\,\text{Hypergeometric2F1}\left(\frac{1+m-2n}{n},-p,\frac{1+m-n}{n},-\frac{bx^n}{a}\right)+d(1+\right.}{(1+m-2n)(1+m-n)}$$

input
```
Integrate[x^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m - 2*n)*(a + b*x^n)^p*(c*(1 + m - n)*Hypergeometric2F1[(1 + m - 2
*n)/n, -p, (1 + m - n)/n, -((b*x^n)/a)] + d*(1 + m - 2*n)*x^n*Hypergeometr
ic2F1[(1 + m - n)/n, -p, (1 + m)/n, -((b*x^n)/a)]))/((1 + m - 2*n)*(1 + m
- n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2n}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m-2n+1)}{b(m-n(1-p)+1)}\right)\int x^{m-2n}(bx^n+a)^p dx+\frac{dx^{m-2n+1}(a+bx^n)^{p+1}}{b(m-n(1-p)+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-2n+1)}{b(m-n(1-p)+1)}\right)\int x^{m-2n}\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^{m-2n+1}(a+bx^n)^{p+1}}{b(m-n(1-p)+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m-2n+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-2n+1)}{b(m-n(1-p)+1)}\right)\text{Hypergeometric2F1}\left(\frac{m-2n+1}{n},-p,\frac{m-n+1}{n},-\frac{bx^n}{a}\right)}{m-2n+1}+\frac{dx^{m-2n+1}(a+bx^n)^{p+1}}{b(m-n(1-p)+1)}$$

input
```
Int[x^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m - 2*n)*(a + b*x^n)^(1 + p))/(b*(1 + m - n*(1 - p))) + ((c - (a
*d*(1 + m - 2*n))/(b*(1 + m - n*(1 - p))))*x^(1 + m - 2*n)*(a + b*x^n)^p*H
ypergeometric2F1[(1 + m - 2*n)/n, -p, (1 + m - n)/n, -((b*x^n)/a)])/((1 +
m - 2*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{m-2n}(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{m-2n}(a+bx^n)^p\,(c+dx^n)\,\,dx=\int (dx^n+c)(bx^n+a)^p x^{m-2\,n}\,dx$$

input
```
integrate(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(m - 2*n)*x^n + c*x^(m - 2*n))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.15

$$\int x^{m-2n}(a+bx^n)^p\,(c+dx^n)\,\,dx = \frac{a^{\frac{m}{n}-2+\frac{1}{n}}a^{-\frac{m}{n}+p+2-\frac{1}{n}}cx^{m-2n+1}\Gamma\left(\frac{m}{n}-2+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{m}{n}-2+\frac{1}{n}\\ \frac{m}{n}-1+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right)} + \frac{a^{\frac{m}{n}-1+\frac{1}{n}}a^{-\frac{m}{n}+p+1-\frac{1}{n}}dx^{m-n+1}\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{m}{n}-1+\frac{1}{n}\\ \frac{m}{n}+\frac{1}{n}\end{matrix}\right|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}$$

input
```
integrate(x**(m-2*n)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n - 2 + 1/n)*a**(-m/n + p + 2 - 1/n)*c*x**(m - 2*n + 1)*gamma(m/n -
2 + 1/n)*hyper((-p, m/n - 2 + 1/n), (m/n - 1 + 1/n,), b*x**n*exp_polar(I*p
i)/a)/(n*gamma(m/n - 1 + 1/n)) + a**(m/n - 1 + 1/n)*a**(-m/n + p + 1 - 1/n
)*d*x**(m - n + 1)*gamma(m/n - 1 + 1/n)*hyper((-p, m/n - 1 + 1/n), (m/n +
1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1/n))
```

Maxima [F]

$$\int x^{m-2n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int(dx^n+c)(bx^n+a)^p x^{m-2\,n}\,dx$$

input
```
integrate(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(m - 2*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{m-2n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,0,1,0,1]%%%}+%%%{-1,[0,0,2,2,1,0,0,1,0,1]%%
%}+%%%{2,
```

Mupad [F(-1)]

Timed out.

$$\int x^{m-2n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int x^{m-2\,n}\,{\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right)\,dx$$

input
```
int(x^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^{m-2n}(a+bx^n)^p(c+dx^n)\,dx = \text{too large to display}$$

input
```
int(x^(m-2*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d*n*p*x
 - 2*x**(m + n)*(x**n*b + a)**p*b*d*n*x + x**(m + n)*(x**n*b + a)**p*b*d*x
 + x**m*(x**n*b + a)**p*a*d*n*p*x + x**m*(x**n*b + a)**p*b*c*m*x + x**m*(x
**n*b + a)**p*b*c*n*p*x - x**m*(x**n*b + a)**p*b*c*n*x + x**m*(x**n*b + a)
**p*b*c*x - x**(2*n)*int((x**m*(x**n*b + a)**p)/(x**(3*n)*b*m**2 + 2*x**(3
*n)*b*m*n*p - 3*x**(3*n)*b*m*n + 2*x**(3*n)*b*m + x**(3*n)*b*n**2*p**2 - 3
*x**(3*n)*b*n**2*p + 2*x**(3*n)*b*n**2 + 2*x**(3*n)*b*n*p - 3*x**(3*n)*b*n
 + x**(3*n)*b + x**(2*n)*a*m**2 + 2*x**(2*n)*a*m*n*p - 3*x**(2*n)*a*m*n +
2*x**(2*n)*a*m + x**(2*n)*a*n**2*p**2 - 3*x**(2*n)*a*n**2*p + 2*x**(2*n)*a
*n**2 + 2*x**(2*n)*a*n*p - 3*x**(2*n)*a*n + x**(2*n)*a),x)*a**2*d*m**3*n*p
 - 2*x**(2*n)*int((x**m*(x**n*b + a)**p)/(x**(3*n)*b*m**2 + 2*x**(3*n)*b*m
*n*p - 3*x**(3*n)*b*m*n + 2*x**(3*n)*b*m + x**(3*n)*b*n**2*p**2 - 3*x**(3*
n)*b*n**2*p + 2*x**(3*n)*b*n**2 + 2*x**(3*n)*b*n*p - 3*x**(3*n)*b*n + x**(
3*n)*b + x**(2*n)*a*m**2 + 2*x**(2*n)*a*m*n*p - 3*x**(2*n)*a*m*n + 2*x**(2
*n)*a*m + x**(2*n)*a*n**2*p**2 - 3*x**(2*n)*a*n**2*p + 2*x**(2*n)*a*n**2 +
 2*x**(2*n)*a*n*p - 3*x**(2*n)*a*n + x**(2*n)*a),x)*a**2*d*m**2*n**2*p**2
+ 5*x**(2*n)*int((x**m*(x**n*b + a)**p)/(x**(3*n)*b*m**2 + 2*x**(3*n)*b*m*
n*p - 3*x**(3*n)*b*m*n + 2*x**(3*n)*b*m + x**(3*n)*b*n**2*p**2 - 3*x**(3*n
)*b*n**2*p + 2*x**(3*n)*b*n**2 + 2*x**(3*n)*b*n*p - 3*x**(3*n)*b*n + x**(3
*n)*b + x**(2*n)*a*m**2 + 2*x**(2*n)*a*m*n*p - 3*x**(2*n)*a*m*n + 2*x**...
```

3.507 $\int x^{m-3n}(a+bx^n)^p\left(c+dx^n\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int x^{m-3n}(a+bx^n)^p\left(c+dx^n\right)\,dx=\frac{cx^{1+m-3n}(a+bx^n)^{1+p}}{a(1+m-3n)}+\frac{(ad(1+m-3n)-bc(1+m-n(2-p)))x^{1+m-2n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m-2}{n}\right.}{a(1+m-3n)(1+m-2n)}$$

output
```
c*x^(1+m-3*n)*(a+b*x^n)^(p+1)/a/(1+m-3*n)+(a*d*(1+m-3*n)-b*c*(1+m-n*(-p+2)
))*x^(1+m-2*n)*(a+b*x^n)^p*hypergeom([-p, (1+m-2*n)/n],[(1+m-n)/n],-b*x^n/
a)/a/(1+m-3*n)/(1+m-2*n)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^{m-3n}(a+bx^n)^p\left(c+dx^n\right)\,dx$$
$$=\frac{x^{1+m-3n}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(1+m-2n)\,\text{Hypergeometric2F1}\left(\frac{1+m-3n}{n},-p,\frac{1+m-2n}{n},-\frac{bx^n}{a}\right)+d(1\right.}{(1+m-3n)(1+m-2n)}$$

input
```
Integrate[x^(m - 3*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(x^(1 + m - 3*n)*(a + b*x^n)^p*(c*(1 + m - 2*n)*Hypergeometric2F1[(1 + m -
 3*n)/n, -p, (1 + m - 2*n)/n, -((b*x^n)/a)] + d*(1 + m - 3*n)*x^n*Hypergeo
metric2F1[(1 + m - 2*n)/n, -p, (1 + m - n)/n, -((b*x^n)/a)]))/((1 + m - 3*
n)*(1 + m - 2*n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3n}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\left(c-\frac{ad(m-3n+1)}{b(m-n(2-p)+1)}\right)\int x^{m-3n}(bx^n+a)^p dx+\frac{dx^{m-3n+1}(a+bx^n)^{p+1}}{b(m-n(2-p)+1)}$$

$$\Big\downarrow \text{889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-3n+1)}{b(m-n(2-p)+1)}\right)\int x^{m-3n}\left(\frac{bx^n}{a}+1\right)^p dx+\frac{dx^{m-3n+1}(a+bx^n)^{p+1}}{b(m-n(2-p)+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m-3n+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad(m-3n+1)}{b(m-n(2-p)+1)}\right)\text{Hypergeometric2F1}\left(\frac{m-3n+1}{n},-p,\frac{m-2n+1}{n},-\frac{bx^n}{a}\right)}{m-3n+1}+\frac{dx^{m-3n+1}(a+bx^n)^{p+1}}{b(m-n(2-p)+1)}$$

input
```
Int[x^(m - 3*n)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(1 + m - 3*n)*(a + b*x^n)^(1 + p))/(b*(1 + m - n*(2 - p))) + ((c - (a
*d*(1 + m - 3*n))/(b*(1 + m - n*(2 - p))))*x^(1 + m - 3*n)*(a + b*x^n)^p*H
ypergeometric2F1[(1 + m - 3*n)/n, -p, (1 + m - 2*n)/n, -((b*x^n)/a)])/((1
+ m - 3*n)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{m-3n}(a+b\,x^n)^p\,(c+d\,x^n)\,dx$$

input
```
int(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{m-3n}(a+bx^n)^p(c+dx^n)\,dx=\int (dx^n+c)(bx^n+a)^p x^{m-3\,n}\,dx$$

input
```
integrate(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(m - 3*n)*x^n + c*x^(m - 3*n))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.73 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.16

$$\int x^{m-3n}(a+bx^n)^p(c+dx^n)\,dx$$
$$=\frac{a^{\frac{m}{n}-3+\frac{1}{n}}a^{-\frac{m}{n}+p+3-\frac{1}{n}}cx^{m-3n+1}\Gamma\left(\frac{m}{n}-3+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{m}{n}-3+\frac{1}{n}\\ \frac{m}{n}-2+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}-2+\frac{1}{n}\right)}+\frac{a^{\frac{m}{n}-2+\frac{1}{n}}a^{-\frac{m}{n}+p+2-\frac{1}{n}}dx^{m-2n+1}\Gamma\left(\frac{m}{n}-2+\frac{1}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,\frac{m}{n}-2+\frac{1}{n}\\ \frac{m}{n}-1+\frac{1}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}-1+\frac{1}{n}\right)}$$

input
```
integrate(x**(m-3*n)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(m/n - 3 + 1/n)*a**(-m/n + p + 3 - 1/n)*c*x**(m - 3*n + 1)*gamma(m/n -
3 + 1/n)*hyper((-p, m/n - 3 + 1/n), (m/n - 2 + 1/n,), b*x**n*exp_polar(I*p
i)/a)/(n*gamma(m/n - 2 + 1/n)) + a**(m/n - 2 + 1/n)*a**(-m/n + p + 2 - 1/n
)*d*x**(m - 2*n + 1)*gamma(m/n - 2 + 1/n)*hyper((-p, m/n - 2 + 1/n), (m/n
- 1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n - 1 + 1/n))
```

Maxima [F]

$$\int x^{m-3n}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{m-3\,n}\,dx$$

input
```
integrate(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(m - 3*n), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{m-3n}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,2,0,1,0,1]%%%}+%%%{-1,[0,0,2,2,1,1,0,1,0,1]%%
%}+%%%{-2
```

Mupad [F(-1)]

Timed out.

$$\int x^{m-3n}(a+bx^n)^p\,(c+dx^n)\;dx=\int x^{m-3\,n}\,{\left(a+b\,x^n\right)}^p\,\left(c+d\,x^n\right)\,dx$$

input
```
int(x^(m - 3*n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int(x^(m - 3*n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int x^{m-3n}(a+bx^n)^p\,(c+dx^n)\;dx = \text{too large to display}$$

input
```
int(x^(m-3*n)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**(m + n)*(x**n*b + a)**p*b*d*m*x + x**(m + n)*(x**n*b + a)**p*b*d*n*p*x
 - 3*x**(m + n)*(x**n*b + a)**p*b*d*n*x + x**(m + n)*(x**n*b + a)**p*b*d*x
 + x**m*(x**n*b + a)**p*a*d*n*p*x + x**m*(x**n*b + a)**p*b*c*m*x + x**m*(x
**n*b + a)**p*b*c*n*p*x - 2*x**m*(x**n*b + a)**p*b*c*n*x + x**m*(x**n*b +
a)**p*b*c*x - x**(3*n)*int((x**m*(x**n*b + a)**p)/(x**(4*n)*b*m**2 + 2*x**
(4*n)*b*m*n*p - 5*x**(4*n)*b*m*n + 2*x**(4*n)*b*m + x**(4*n)*b*n**2*p**2 -
 5*x**(4*n)*b*n**2*p + 6*x**(4*n)*b*n**2 + 2*x**(4*n)*b*n*p - 5*x**(4*n)*b
*n + x**(4*n)*b + x**(3*n)*a*m**2 + 2*x**(3*n)*a*m*n*p - 5*x**(3*n)*a*m*n
+ 2*x**(3*n)*a*m + x**(3*n)*a*n**2*p**2 - 5*x**(3*n)*a*n**2*p + 6*x**(3*n)
*a*n**2 + 2*x**(3*n)*a*n*p - 5*x**(3*n)*a*n + x**(3*n)*a),x)*a**2*d*m**3*n
*p - 2*x**(3*n)*int((x**m*(x**n*b + a)**p)/(x**(4*n)*b*m**2 + 2*x**(4*n)*b
*m*n*p - 5*x**(4*n)*b*m*n + 2*x**(4*n)*b*m + x**(4*n)*b*n**2*p**2 - 5*x**(
4*n)*b*n**2*p + 6*x**(4*n)*b*n**2 + 2*x**(4*n)*b*n*p - 5*x**(4*n)*b*n + x*
*(4*n)*b + x**(3*n)*a*m**2 + 2*x**(3*n)*a*m*n*p - 5*x**(3*n)*a*m*n + 2*x**
(3*n)*a*m + x**(3*n)*a*n**2*p**2 - 5*x**(3*n)*a*n**2*p + 6*x**(3*n)*a*n**2
 + 2*x**(3*n)*a*n*p - 5*x**(3*n)*a*n + x**(3*n)*a),x)*a**2*d*m**2*n**2*p**
2 + 8*x**(3*n)*int((x**m*(x**n*b + a)**p)/(x**(4*n)*b*m**2 + 2*x**(4*n)*b*
m*n*p - 5*x**(4*n)*b*m*n + 2*x**(4*n)*b*m + x**(4*n)*b*n**2*p**2 - 5*x**(4
*n)*b*n**2*p + 6*x**(4*n)*b*n**2 + 2*x**(4*n)*b*n*p - 5*x**(4*n)*b*n + x**
(4*n)*b + x**(3*n)*a*m**2 + 2*x**(3*n)*a*m*n*p - 5*x**(3*n)*a*m*n + 2*x...
```

3.508 $\int x^p(b+cx)^p(b+2cx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

output
```
x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

input
```
Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

output
```
(x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^p(b+2cx)(b+cx)^p\,dx$$

$$\Big\downarrow \, 83$$

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

input
```
Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
```

output
```
(x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{x^{p+1}(cx+b)^{p+1}}{p+1}$	21
risch	$\frac{x(cx+b)x^{p}(cx+b)^{p}}{p+1}$	23
orering	$\frac{x(cx+b)x^{p}(cx+b)^{p}}{p+1}$	23
parallelrisch	$\frac{x^{2}x^{p}(cx+b)^{p}bc+x\,x^{p}(cx+b)^{p}b^{2}}{b(p+1)}$	42

input
```
int(x^p*(c*x+b)^p*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{(cx^2+bx)(cx+b)^px^p}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 1.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int x^p(b+cx)^p(b+2cx)\,dx=\begin{cases}\frac{bxx^p(b+cx)^p}{p+1}+\frac{cx^2x^p(b+cx)^p}{p+1} & \text{for } p\neq -1\\ \log(x)+\log\left(\frac{b}{c}+x\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
```

output
```
Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1
), Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{(cx^2+bx)e^{(p\log(cx+b)+p\log(x))}}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
```

output
```
(c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{(cx+b)^p cx^2x^p + (cx+b)^p bxx^p}{p+1}$$

input
```
integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
```

output
```
((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{x\,x^p\,{(b+c\,x)}^p\,(b+c\,x)}{p+1}$$

input
```
int(x^p*(b + c*x)^p*(b + 2*c*x),x)
```

output
```
(x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^p(b+cx)^p(b+2cx)\,dx = \frac{x^p(cx+b)^p\,x(cx+b)}{p+1}$$

input
```
int(x^p*(c*x+b)^p*(2*c*x+b),x)
```

output
```
(x**p*(b + c*x)**p*x*(b + c*x))/(p + 1)
```

3.509 $\int x^{-1+2(1+p)}(b+cx^2)^p(b+2cx^2)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+2(1+p)}(b+cx^2)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}(b+cx^2)^{1+p}}{2(1+p)}$$

output
```
x^(2+2*p)*(c*x^2+b)^(p+1)/(2+2*p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+2(1+p)}(b+cx^2)^p\left(b+2cx^2\right)\,dx$$
$$=\frac{x^{2+2p}(b+cx^2)^p\left(1+\frac{cx^2}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^2}{b}\right)+2c(1+p)x^2\,\text{Hype}\right.}{2(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

output
```
(x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
-((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^2)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2(p+1)-1}\left(b+2cx^2\right)\left(b+cx^2\right)^p\,dx$$

$$\Big\downarrow \; 356$$

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

input
```
Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

output
```
(x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{2+2p}\left(c\,x^{2}+b\right)^{p+1}}{2+2p}$	26
risch	$\frac{x\left(c\,x^{2}+b\right)x^{2p+1}\left(c\,x^{2}+b\right)^{p}}{2+2p}$	32
orering	$\frac{x\left(c\,x^{2}+b\right)x^{2p+1}\left(c\,x^{2}+b\right)^{p}}{2+2p}$	32
parallelrisch	$\frac{x^{3}x^{2p+1}\left(c\,x^{2}+b\right)^{p}bc+x\,x^{2p+1}\left(c\,x^{2}+b\right)^{p}b^{2}}{2b(p+1)}$	55

input
```
int(x^(2*p+1)*(c*x^2+b)^p*(2*c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(2+2*p)/(p+1)*(c*x^2+b)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{(cx^3+bx)(cx^2+b)^px^{2\,p+1}}{2\,(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

output
```
1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(20) = 40$.

Time = 43.67 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int x^{-1+2(1+p)}(b+cx^2)^p(b+2cx^2)\,dx = \begin{cases} \frac{bxx^{2p+1}(b+cx^2)^p}{2p+2}+\frac{cx^3x^{2p+1}(b+cx^2)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x)+\frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2}+\frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
```

output
```
Piecewise((b*x*x**(2*p + 1)*(b + c*x**2)**p/(2*p + 2) + c*x**3*x**(2*p + 1
)*(b + c*x**2)**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 +
 log(x + sqrt(-b/c))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+2(1+p)}(b+cx^2)^p(b+2cx^2)\,dx = \frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2p\log(x))}}{2(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
```

output
```
1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx = \frac{(cx^2+b)^p cx^3 e^{(2\,p\log(x)+\log(x))} + (cx^2+b)^p bxe^{(2\,p\log(x)+\log(x))}}{2\,(p+1)}$$

input
```
integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
```

output
```
1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*
p*log(x) + log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx = \left(c\,x^2+b\right)^p\left(\frac{c\,x^{2\,p+1}\,x^3}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

input
```
int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
```

output
```
(b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2)
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2p}(c\,x^2+b)^p\,x^2(c\,x^2+b)}{2p+2}$$

input
```
int(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x)
```

output
```
(x**(2*p)*(b + c*x**2)**p*x**2*(b + c*x**2))/(2*(p + 1))
```

3.510 $\int x^{-1+3(1+p)}(b+cx^3)^p\,(b+2cx^3)\;dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

output
```
x^(3*p+3)*(c*x^3+b)^(p+1)/(3*p+3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.59

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx$$
$$=\frac{x^{3+3p}\left(b+cx^3\right)^p\left(1+\frac{cx^3}{b}\right)^{-p}\left(b(2+p)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^3}{b}\right)+2c(1+p)x^3\,\text{Hype}\right.}{3(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

output
```
(x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p,
-((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x
^3)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3(p+1)-1}\left(b+2cx^3\right)\left(b+cx^3\right)^p\,dx$$

$$\Big\downarrow \text{951}$$

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

input
```
Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

output
```
(x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x^{3p+3}\left(c\,x^{3}+b\right)^{p+1}}{3p+3}$	26
risch	$\frac{x\left(c\,x^{3}+b\right)x^{2+3p}\left(c\,x^{3}+b\right)^{p}}{3p+3}$	32
orering	$\frac{x\left(c\,x^{3}+b\right)x^{2+3p}\left(c\,x^{3}+b\right)^{p}}{3p+3}$	32
parallelrisch	$\frac{x^{4}x^{2+3p}\left(c\,x^{3}+b\right)^{p}c^{2}+x\,x^{2+3p}\left(c\,x^{3}+b\right)^{p}bc}{3c(p+1)}$	55

input
```
int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^(3*p+3)/(p+1)*(c*x^3+b)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^4+bx)(cx^3+b)^p x^{3\,p+2}}{3\,(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
```

output
```
1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3\,p\log(x))}}{3\,(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
```

output
```
1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{(cx^3+b)^p cx^4e^{(3\,p\log(x)+2\,\log(x))}+(cx^3+b)^p bxe^{(3\,p\log(x)+2\,\log(x))}}{3\,(p+1)}$$

input
```
integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
```

output
```
1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(
3*p*log(x) + 2*log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\left(c\,x^3+b\right)^p\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

input
```
int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
```

output
```
(b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)\,dx=\frac{x^{3p}(c\,x^3+b)^p\,x^3(c\,x^3+b)}{3p+3}$$

input
```
int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x)
```

output
```
(x**(3*p)*(b + c*x**3)**p*x**3*(b + c*x**3))/(3*(p + 1))
```

3.511 $\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 27

$$\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n)\,dx = \frac{x^{n(1+p)}(b+cx^n)^{1+p}}{n(1+p)}$$

output
```
x^(n*(p+1))*(b+c*x^n)^(p+1)/n/(p+1)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int x^{-1+n(1+p)}(b+cx^n)^p (b+2cx^n)\,dx$$
$$= \frac{(b+cx^n)^p \left(1+\frac{cx^n}{b}\right)^{-p} \left(b(2+p)x^{n(1+p)}\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx^n}{b}\right)+2c(1+p)x^{n(2+p)}\right.}{n(1+p)(2+p)}$$

input
```
Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

output
```
((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p
, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3
 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{951\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n(p+1)-1}(b+2cx^n)(b+cx^n)^p\,dx$$

$$\Big\downarrow \text{951}$$

$$\frac{x^{n(p+1)}(b+cx^n)^{p+1}}{n(p+1)}$$

input
```
Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
```

output
```
(x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Defintions of rubi rules used

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(27) = 54$.

Time = 2.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

method	result	size
parallelrisch	$\frac{x\,x^{n} x^{n p+n-1} \left(b+c\,x^{n}\right)^{p} c^{2}+x\,x^{n p+n-1} \left(b+c\,x^{n}\right)^{p} b c}{c n \left(p+1\right)}$	60

input
```
int(x^(-1+n*(p+1))*(b+c*x^n)^p*(b+2*c*x^n),x,method=_RETURNVERBOSE)
```

output
```
(x*x^n*x^(n*p+n-1)*(b+c*x^n)^p*c^2+x*x^(n*p+n-1)*(b+c*x^n)^p*b*c)/c/n/(p+1
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx=\frac{(cxx^n+bx)(cx^n+b)^p x^{np+n-1}}{np+n}$$

input
```
integrate(x^(-1+n*(p+1))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
```

output
```
(c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx = \frac{b^{p+1}c^{-p-1}c^{p+1}x^{np+n}\Gamma(p+1)\,{}_2F_1\left(\left.\begin{matrix}-p,p+1\\p+2\end{matrix}\right|\frac{cx^ne^{i\pi}}{b}\right)}{n\Gamma\,(p+2)} + \frac{2b^{p+2}cc^{-p-2}c^{p+2}x^{np+2n}\Gamma(p+2)\,{}_2F_1\left(\left.\begin{matrix}-p,p+2\\p+3\end{matrix}\right|\frac{cx^ne^{i\pi}}{b}\right)}{b^2n\Gamma\,(p+3)}$$

input
```
integrate(x**(-1+n*(p+1))*(b+c*x**n)**p*(b+2*c*x**n),x)
```

output
```
b**(p + 1)*c**(-p - 1)*c**(p + 1)*x**(n*p + n)*gamma(p + 1)*hyper((-p, p +
 1), (p + 2,), c*x**n*exp_polar(I*pi)/b)/(n*gamma(p + 2)) + 2*b**(p + 2)*c
*c**(-p - 2)*c**(p + 2)*x**(n*p + 2*n)*gamma(p + 2)*hyper((-p, p + 2), (p
+ 3,), c*x**n*exp_polar(I*pi)/b)/(b**2*n*gamma(p + 3))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx = \frac{(cx^{2\,n}+bx^n)e^{(np\log(x)+p\log(cx^n+b))}}{n(p+1)}$$

input
```
integrate(x^(-1+n*(p+1))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
```

output
```
(c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx = \frac{(cx^n+b)^p cxx^n e^{(np\log(x)+n\log(x)-\log(x))} + (cx^n+b)^p bxe^{(np\log(x)+n\log(x)-\log(x))}}{np+n}$$

input
```
integrate(x^(-1+n*(p+1))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
```

output
```
((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*
b*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx = \left(\frac{b\,x\,x^{n\,(p+1)-1}}{n\,(p+1)} + \frac{c\,x\,x^n\,x^{n\,(p+1)-1}}{n\,(p+1)}\right)\,(b+c\,x^n)^p$$

input
```
int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
```

output
```
((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
 1)))*(b + c*x^n)^p
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^{-1+n(1+p)}(b+cx^n)^p\,(b+2cx^n)\;dx=\frac{x^{np+n}(x^nc+b)^p\,(x^nc+b)}{n\,(p+1)}$$

input
```
int(x^(-1+n*(p+1))*(b+c*x^n)^p*(b+2*c*x^n),x)
```

output
```
(x**(n*p + n)*(x**n*c + b)**p*(x**n*c + b))/(n*(p + 1))
```

3.512 $\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^4 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 313

$$
\begin{aligned}
\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^4 \, dx &= -\frac{6c^2d^2x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)} - \frac{2bc^3(bc-2ad(3+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3n(1+p)(2+p)(3+p)} \\
&+ \frac{2c^3(bc-2ad(3+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{a^2n(2+p)(3+p)} - \frac{c^4x^{-n(3+p)}(a+bx^n)^{1+p}}{an(3+p)} \\
&+ \frac{d^4x^{n(1-p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{n(1-p)} \\
&- \frac{4cd^3x^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}
\end{aligned}
$$

output
```
-6*c^2*d^2*(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))-2*b*c^3*(b*c-2*a*d*(3+p
))*(a+b*x^n)^(p+1)/a^3/n/(p+1)/(2+p)/(3+p)/(x^(n*(p+1)))+2*c^3*(b*c-2*a*d*
(3+p))*(a+b*x^n)^(p+1)/a^2/n/(2+p)/(3+p)/(x^(n*(2+p)))-c^4*(a+b*x^n)^(p+1)
/a/n/(3+p)/(x^(n*(3+p)))+d^4*x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p, 1-p],[
-p+2],-b*x^n/a)/n/(1-p)/((1+b*x^n/a)^p)-4*c*d^3*(a+b*x^n)^p*hypergeom([-p,
 -p],[1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.78

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx$$
$$= \frac{x^{-n(3+p)}(a+bx^n)^p\left(-\frac{c^4\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^n}{a}\right)}{3+p}+dx^n\left(-\frac{4c^3\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}}{2+p}\right.\right.}{}$$

input
```
Integrate[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^4,x]
```

output
```
((a + b*x^n)^p*(-((c^4*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^n)/a)]
)/((3 + p)*(1 + (b*x^n)/a)^p)) + d*x^n*((-4*c^3*Hypergeometric2F1[-2 - p,
-p, -1 - p, -((b*x^n)/a)])/((2 + p)*(1 + (b*x^n)/a)^p) + d*x^n*((-6*c^2*(a
 + b*x^n))/(a*(1 + p)) - (d^2*x^(2*n)*Hypergeometric2F1[1 - p, -p, 2 - p,
-((b*x^n)/a)])/((-1 + p)*(1 + (b*x^n)/a)^p) - (4*c*d*x^n*Hypergeometric2F1
[-p, -p, 1 - p, -((b*x^n)/a)])/(p*(1 + (b*x^n)/a)^p)))))/(n*x^(n*(3 + p)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1008, 1066}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+3)-1}(c+dx^n)^4\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-n(p+3)-1}(bx^n+a)^p\,(dx^n+c)^2\,(dn(4bc+adp)x^n+cn(bc+ad(p+3)))\,dx}{bn}+\frac{dx^{-n(p+3)}(c+dx^n)^3\,(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{1066}$$

Indeterminate

input
```
Int[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^4,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int x^{-1-n(3+p)} (a+b\,x^n)^p\,(c+d\,x^n)^4\,dx$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x)
```

output
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x)
```

Fricas [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx=\int (dx^n+c)^4(bx^n+a)^px^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((d^4*x^(-n*p - 3*n - 1)*x^(4*n) + 4*c*d^3*x^(-n*p - 3*n - 1)*x^(3
*n) + 6*c^2*d^2*x^(-n*p - 3*n - 1)*x^(2*n) + 4*c^3*d*x^(-n*p - 3*n - 1)*x^
n + c^4*x^(-n*p - 3*n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p*(c+d*x**n)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx=\int (dx^n+c)^4(bx^n+a)^px^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^4*(b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[3,0,8,5,3,5,5,4,0]%%%}+%%%{-5,[3,0,8,5,3,5,4,4,0]%%%}+
%%%{-10,[
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^4}{x^{n\,(p+3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^4)/x^(n*(p + 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^4)/x^(n*(p + 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^4\,dx = \text{Too large to display}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^4,x)
```

output

```
(x**(4*n)*(x**n*b + a)**p*a**3*d**4*p**4 + 6*x**(4*n)*(x**n*b + a)**p*a**3
*d**4*p**3 + 11*x**(4*n)*(x**n*b + a)**p*a**3*d**4*p**2 + 6*x**(4*n)*(x**n
*b + a)**p*a**3*d**4*p - 4*x**(3*n)*(x**n*b + a)**p*a**3*c*d**3*p**3 - 24*
x**(3*n)*(x**n*b + a)**p*a**3*c*d**3*p**2 - 44*x**(3*n)*(x**n*b + a)**p*a*
*3*c*d**3*p - 24*x**(3*n)*(x**n*b + a)**p*a**3*c*d**3 - 6*x**(3*n)*(x**n*b
 + a)**p*a**2*b*c**2*d**2*p**3 - 30*x**(3*n)*(x**n*b + a)**p*a**2*b*c**2*d
**2*p**2 - 36*x**(3*n)*(x**n*b + a)**p*a**2*b*c**2*d**2*p + 4*x**(3*n)*(x*
*n*b + a)**p*a*b**2*c**3*d*p**2 + 12*x**(3*n)*(x**n*b + a)**p*a*b**2*c**3*
d*p - 2*x**(3*n)*(x**n*b + a)**p*b**3*c**4*p - 6*x**(2*n)*(x**n*b + a)**p*
a**3*c**2*d**2*p**3 - 30*x**(2*n)*(x**n*b + a)**p*a**3*c**2*d**2*p**2 - 36
*x**(2*n)*(x**n*b + a)**p*a**3*c**2*d**2*p - 4*x**(2*n)*(x**n*b + a)**p*a*
*2*b*c**3*d*p**3 - 12*x**(2*n)*(x**n*b + a)**p*a**2*b*c**3*d*p**2 + 2*x**(
2*n)*(x**n*b + a)**p*a*b**2*c**4*p**2 - 4*x**n*(x**n*b + a)**p*a**3*c**3*d
*p**3 - 16*x**n*(x**n*b + a)**p*a**3*c**3*d*p**2 - 12*x**n*(x**n*b + a)**p
*a**3*c**3*d*p - x**n*(x**n*b + a)**p*a**2*b*c**4*p**3 - x**n*(x**n*b + a)
**p*a**2*b*c**4*p**2 - (x**n*b + a)**p*a**3*c**4*p**3 - 3*(x**n*b + a)**p*
a**3*c**4*p**2 - 2*(x**n*b + a)**p*a**3*c**4*p + x**(n*p + 3*n)*int((x**n*
(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**4*d**4*n*p**5 + 6
*x**(n*p + 3*n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*
x),x)*a**4*d**4*n*p**4 + 11*x**(n*p + 3*n)*int((x**n*(x**n*b + a)**p)/(...
```

3.513 $\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 225

$$\begin{aligned}\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^3 \, dx &= -\frac{c(2b^2c^2-3abcd(3+p)+3a^2d^2(6+5p+p^2))\, x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3n(1+p)(2+p)(3+p)} \\ &\quad + \frac{c^2(2bc-3ad(3+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{a^2n(2+p)(3+p)} - \frac{c^3x^{-n(3+p)}(a+bx^n)^{1+p}}{an(3+p)} \\ &\quad - \frac{d^3x^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}\end{aligned}$$

output
```
-c*(2*b^2*c^2-3*a*b*c*d*(3+p)+3*a^2*d^2*(p^2+5*p+6))*(a+b*x^n)^(p+1)/a^3/n
/(p+1)/(2+p)/(3+p)/(x^(n*(p+1)))+c^2*(2*b*c-3*a*d*(3+p))*(a+b*x^n)^(p+1)/a
^2/n/(2+p)/(3+p)/(x^(n*(2+p)))-c^3*(a+b*x^n)^(p+1)/a/n/(3+p)/(x^(n*(3+p)))
-d^3*(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^
n/a)^p)
```

Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.84

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx$$

$$= \frac{x^{-n(3+p)}(a+bx^n)^p\left(-\frac{c^3\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^n}{a}\right)}{3+p}+dx^n\left(-\frac{3c^2\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F}}{2+p}\right.\right.}{n}$$

input
```
Integrate[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^p*(-((c^3*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^n)/a)]
)/((3 + p)*(1 + (b*x^n)/a)^p)) + d*x^n*((-3*c^2*Hypergeometric2F1[-2 - p,
-p, -1 - p, -((b*x^n)/a)])/((2 + p)*(1 + (b*x^n)/a)^p) + d*x^n*((-3*c*(a +
 b*x^n))/(a*(1 + p)) - (d*x^n*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a
)])/(p*(1 + (b*x^n)/a)^p)))))/(n*x^(n*(3 + p)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1008}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+3)-1}(c+dx^n)^3\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\text{Indeterminate}$$

input
```
Int[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(3+p)}(a+b\,x^n)^p\,(c+d\,x^n)^3\,dx$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\int (dx^n+c)^3(bx^n+a)^p x^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(-n*p - 3*n - 1)*x^(3*n) + 3*c*d^2*x^(-n*p - 3*n - 1)*x^(2
*n) + 3*c^2*d*x^(-n*p - 3*n - 1)*x^n + c^3*x^(-n*p - 3*n - 1))*(b*x^n + a)
^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\int (dx^n+c)^3(bx^n+a)^p x^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,4,3,3,0]%%%}+%%
%{6,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\int\frac{(a+b\,x^n)^p\,(c+d\,x^n)^3}{x^{n\,(p+3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx$$
$$=\frac{-3x^{3n}(x^nb+a)^p\,a^2bc\,d^2p^2-15x^{3n}(x^nb+a)^p\,a^2bc\,d^2p-18x^{3n}(x^nb+a)^p\,a^2bc\,d^2+3x^{3n}(x^nb+a)^p\,a\,b^2c^2}{}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
( - 3*x**(3*n)*(x**n*b + a)**p*a**2*b*c*d**2*p**2 - 15*x**(3*n)*(x**n*b +
a)**p*a**2*b*c*d**2*p - 18*x**(3*n)*(x**n*b + a)**p*a**2*b*c*d**2 + 3*x**(
3*n)*(x**n*b + a)**p*a*b**2*c**2*d*p + 9*x**(3*n)*(x**n*b + a)**p*a*b**2*c
**2*d - 2*x**(3*n)*(x**n*b + a)**p*b**3*c**3 - 3*x**(2*n)*(x**n*b + a)**p*
a**3*c*d**2*p**2 - 15*x**(2*n)*(x**n*b + a)**p*a**3*c*d**2*p - 18*x**(2*n)
*(x**n*b + a)**p*a**3*c*d**2 - 3*x**(2*n)*(x**n*b + a)**p*a**2*b*c**2*d*p*
*2 - 9*x**(2*n)*(x**n*b + a)**p*a**2*b*c**2*d*p + 2*x**(2*n)*(x**n*b + a)*
*p*a*b**2*c**3*p - 3*x**n*(x**n*b + a)**p*a**3*c**2*d*p**2 - 12*x**n*(x**n
*b + a)**p*a**3*c**2*d*p - 9*x**n*(x**n*b + a)**p*a**3*c**2*d - x**n*(x**n
*b + a)**p*a**2*b*c**3*p**2 - x**n*(x**n*b + a)**p*a**2*b*c**3*p - (x**n*b
 + a)**p*a**3*c**3*p**2 - 3*(x**n*b + a)**p*a**3*c**3*p - 2*(x**n*b + a)**
p*a**3*c**3 + x**(n*p + 3*n)*int((x**n*b + a)**p/(x**(n*p)*x),x)*a**3*d**3
*n*p**3 + 6*x**(n*p + 3*n)*int((x**n*b + a)**p/(x**(n*p)*x),x)*a**3*d**3*n
*p**2 + 11*x**(n*p + 3*n)*int((x**n*b + a)**p/(x**(n*p)*x),x)*a**3*d**3*n*
p + 6*x**(n*p + 3*n)*int((x**n*b + a)**p/(x**(n*p)*x),x)*a**3*d**3*n)/(x**
(n*p + 3*n)*a**3*n*(p**3 + 6*p**2 + 11*p + 6))
```

3.514 $\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 160

$$\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{(2b^2c^2-2abcd(3+p)+a^2d^2(6+5p+p^2))\, x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3n(1+p)(2+p)(3+p)} + \frac{2c(bc-ad(3+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{a^2n(2+p)(3+p)} - \frac{c^2x^{-n(3+p)}(a+bx^n)^{1+p}}{an(3+p)}$$

output
```
-(2*b^2*c^2-2*a*b*c*d*(3+p)+a^2*d^2*(p^2+5*p+6))*(a+b*x^n)^(p+1)/a^3/n/(p+
1)/(2+p)/(3+p)/(x^(n*(p+1)))+2*c*(b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a^2/n/(2+
p)/(3+p)/(x^(n*(2+p)))-c^2*(a+b*x^n)^(p+1)/a/n/(3+p)/(x^(n*(3+p)))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.64

$$\int x^{-1-n(3+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{c^2x^{-n(3+p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(1+\frac{dx^n}{c}\right)^{3+p} \text{Hypergeometric2F1}\left(-3-p,-p,-2-p,\frac{-\frac{bx^n}{a}+\frac{dx^n}{c}}{1+\frac{dx^n}{c}}\right)}{n(3+p)}$$

input
```
Integrate[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-((c^2*(a + b*x^n)^p*(1 + (d*x^n)/c)^(3 + p)*Hypergeometric2F1[-3 - p, -p,
 -2 - p, (-((b*x^n)/a) + (d*x^n)/c)/(1 + (d*x^n)/c)])/(n*(3 + p)*x^(n*(3 +
 p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1008, 25, 959, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+3)-1}(c+dx^n)^2 (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{1008}$$

$$-\frac{\int -x^{-n(p+3)-1}(bx^n+a)^p \left(cn(bc-ad(p+3))-ad^2n(p+2)x^n\right)dx}{bn}-\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int x^{-n(p+3)-1}(bx^n+a)^p \left(cn(bc-ad(p+3))-ad^2n(p+2)x^n\right)dx}{bn}-\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{n\left(a^2d^2(p^2+5p+6)-2abcd(p+3)+2b^2c^2\right)\int x^{-n(p+3)-1}(bx^n+a)^p dx}{2b}+\frac{ad^2(p+2)x^{-n(p+3)}(a+bx^n)^{p+1}}{2b}}{bn}-\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{803}$$

$$\frac{\frac{n(a^2d^2(p^2+5p+6)-2abcd(p+3)+2b^2c^2)\left(-\frac{2b\int x^{-n(p+2)-1}(bx^n+a)^pdx}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}\right)}{2b}+\frac{ad^2(p+2)x^{-n(p+3)}(a+bx^n)^{p+1}}{2b}}{bn}-\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{803}$$

$$\frac{\frac{n(a^2d^2(p^2+5p+6)-2abcd(p+3)+2b^2c^2)\left(-\frac{2b\left(-\frac{b\int x^{-n(p+1)-1}(bx^n+a)^pdx}{a(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}\right)}{2b}+\frac{ad^2(p+2)x}{}}{bn}$$

$$\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{796}$$

$$\frac{\frac{n(a^2d^2(p^2+5p+6)-2abcd(p+3)+2b^2c^2)\left(-\frac{2b\left(\frac{bx^{-n(p+1)}(a+bx^n)^{p+1}}{a^2n(p+1)(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}\right)}{2b}+\frac{ad^2(p+2)x^{-n(p+}}{2b}}{bn}$$

$$\frac{dx^{-n(p+3)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

input
```
Int[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-((d*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*n*x^(n*(3 + p)))) + ((a*d^2*(2 +
p)*(a + b*x^n)^(1 + p))/(2*b*x^(n*(3 + p))) + (n*(2*b^2*c^2 - 2*a*b*c*d*(3
 + p) + a^2*d^2*(6 + 5*p + p^2))*(-((a + b*x^n)^(1 + p)/(a*n*(3 + p)*x^(n*
(3 + p)))) - (2*b*((b*(a + b*x^n)^(1 + p))/(a^2*n*(1 + p)*(2 + p)*x^(n*(1
+ p))) - (a + b*x^n)^(1 + p)/(a*n*(2 + p)*x^(n*(2 + p)))))/(a*(3 + p))))/(
2*b))/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(163) = 326$.

Time = 8.20 (sec) , antiderivative size = 730, normalized size of antiderivative = 4.56

method	result
parallelrisch	$-\frac{x\,x^{3n} x^{-np-3n-1} (a+b\,x^{n})^{p} a^{2} b^{2} d^{2} p^{2}+5x\,x^{3n} x^{-np-3n-1} (a+b\,x^{n})^{p} a^{2} b^{2} d^{2} p-2x\,x^{3n} x^{-np-3n-1} (a+b\,x^{n})^{p} a\,b^{3} c d p+x\,x^{2n}}{}$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-(x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^2*b^2*d^2*p^2+5*x*(x^n)^3*x^(-n*p
-3*n-1)*(a+b*x^n)^p*a^2*b^2*d^2*p-2*x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*a
*b^3*c*d*p+x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*d^2*p^2+2*x*(x^n)^2*
x^(-n*p-3*n-1)*(a+b*x^n)^p*a^2*b^2*c*d*p^2+6*x*(x^n)^3*x^(-n*p-3*n-1)*(a+b
*x^n)^p*a^2*b^2*d^2-6*x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*a*b^3*c*d+2*x*(
x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*b^4*c^2+5*x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*
x^n)^p*a^3*b*d^2*p+6*x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^2*b^2*c*d*p-2*
x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*x^n)^p*a*b^3*c^2*p+2*x*x^n*x^(-n*p-3*n-1)*(a
+b*x^n)^p*a^3*b*c*d*p^2+x*x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^2*b^2*c^2*p^2+6
*x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*d^2+8*x*x^n*x^(-n*p-3*n-1)*(a+
b*x^n)^p*a^3*b*c*d*p+x*x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^2*b^2*c^2*p+x*x^(-
n*p-3*n-1)*(a+b*x^n)^p*a^3*b*c^2*p^2+6*x*x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^
3*b*c*d+3*x*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*c^2*p+2*x*x^(-n*p-3*n-1)*(a+b
*x^n)^p*a^3*b*c^2)/(p^2+5*p+6)/n/(p+1)/a^3/b
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.88

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^2\;dx =$$

$$-\frac{((a^2bd^2p^2+2\,b^3c^2-6\,ab^2cd+6\,a^2bd^2-(2\,ab^2cd-5\,a^2bd^2)p)xx^{-np-3\,n-1}x^{3\,n}+(6\,a^3d^2+(2\,a^2bcd+a$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
-((a^2*b*d^2*p^2 + 2*b^3*c^2 - 6*a*b^2*c*d + 6*a^2*b*d^2 - (2*a*b^2*c*d -
5*a^2*b*d^2)*p)*x*x^(-n*p - 3*n - 1)*x^(3*n) + (6*a^3*d^2 + (2*a^2*b*c*d +
 a^3*d^2)*p^2 - (2*a*b^2*c^2 - 6*a^2*b*c*d - 5*a^3*d^2)*p)*x*x^(-n*p - 3*n
 - 1)*x^(2*n) + (6*a^3*c*d + (a^2*b*c^2 + 2*a^3*c*d)*p^2 + (a^2*b*c^2 + 8*
a^3*c*d)*p)*x*x^(-n*p - 3*n - 1)*x^n + (a^3*c^2*p^2 + 3*a^3*c^2*p + 2*a^3*
c^2)*x*x^(-n*p - 3*n - 1))*(b*x^n + a)^p/(a^3*n*p^3 + 6*a^3*n*p^2 + 11*a^3
*n*p + 6*a^3*n)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\left(c+dx^n\right)^2\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\left(c+dx^n\right)^2\,dx=\int (dx^n+c)^2(bx^n+a)^px^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\left(c+dx^n\right)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p+3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 3) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.92

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{(x^nb+a)^p\,(-x^{3n}a^2b\,d^2p^2-5x^{3n}a^2b\,d^2p-6x^{3n}a^2b\,d^2+2x^{3n}a\,b^2cdp+6x^{3n}a\,b^2cd-2x^{3n}b^3c^2-x^{2n}a^3d^2p}{}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
((x**n*b + a)**p*( - x**(3*n)*a**2*b*d**2*p**2 - 5*x**(3*n)*a**2*b*d**2*p
- 6*x**(3*n)*a**2*b*d**2 + 2*x**(3*n)*a*b**2*c*d*p + 6*x**(3*n)*a*b**2*c*d
 - 2*x**(3*n)*b**3*c**2 - x**(2*n)*a**3*d**2*p**2 - 5*x**(2*n)*a**3*d**2*p
 - 6*x**(2*n)*a**3*d**2 - 2*x**(2*n)*a**2*b*c*d*p**2 - 6*x**(2*n)*a**2*b*c
*d*p + 2*x**(2*n)*a*b**2*c**2*p - 2*x**n*a**3*c*d*p**2 - 8*x**n*a**3*c*d*p
 - 6*x**n*a**3*c*d - x**n*a**2*b*c**2*p**2 - x**n*a**2*b*c**2*p - a**3*c**
2*p**2 - 3*a**3*c**2*p - 2*a**3*c**2))/(x**(n*p + 3*n)*a**3*n*(p**3 + 6*p*
*2 + 11*p + 6))
```

3.515 $\int x^{-1-n(3+p)}(a+bx^n)^p(c+dx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 137

$$\int x^{-1-n(3+p)}(a+bx^n)^p(c+dx^n)\,dx = -\frac{b(2bc-ad(3+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3n(1+p)(2+p)(3+p)} + \frac{(2bc-ad(3+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{a^2n(2+p)(3+p)} - \frac{cx^{-n(3+p)}(a+bx^n)^{1+p}}{an(3+p)}$$

output
```
-b*(2*b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a^3/n/(p+1)/(2+p)/(3+p)/(x^(n*(p+1)))
+(2*b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a^2/n/(2+p)/(3+p)/(x^(n*(2+p)))-c*(a+b*
x^n)^(p+1)/a/n/(3+p)/(x^(n*(3+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

$$\int x^{-1-n(3+p)}(a+bx^n)^p(c+dx^n)\,dx =$$

$$-\frac{x^{-n(3+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(2+p)\,\text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^n}{a}\right)+d(3+p)x\right.}{n(2+p)(3+p)}$$

input
```
Integrate[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(c*(2 + p)*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^n
)/a)] + d*(3 + p)*x^n*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^n)/a)])
)/(n*(2 + p)*(3 + p)*x^(n*(3 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {959, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+3)-1}\left(c+dx^n\right)\left(a+bx^n\right)^p\,dx$$

$$\Big\downarrow \; 959$$

$$\frac{(2bc-ad(p+3))\int x^{-n(p+3)-1}\left(bx^n+a\right)^pdx}{2b}-\frac{dx^{-n(p+3)}\left(a+bx^n\right)^{p+1}}{2bn}$$

$$\Big\downarrow \; 803$$

$$\frac{(2bc-ad(p+3))\left(-\frac{2b\int x^{-n(p+2)-1}\left(bx^n+a\right)^pdx}{a(p+3)}-\frac{x^{-n(p+3)}\left(a+bx^n\right)^{p+1}}{an(p+3)}\right)}{2b}-\frac{dx^{-n(p+3)}\left(a+bx^n\right)^{p+1}}{2bn}$$

$$\Big\downarrow \; 803$$

$$\frac{(2bc-ad(p+3))\left(-\frac{2b\left(-\frac{b\int x^{-n(p+1)-1}\left(bx^n+a\right)^pdx}{a(p+2)}-\frac{x^{-n(p+2)}\left(a+bx^n\right)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}\left(a+bx^n\right)^{p+1}}{an(p+3)}\right)}{2b}-\frac{dx^{-n(p+3)}\left(a+bx^n\right)^{p+1}}{2bn}$$

$$\Big\downarrow \; 796$$

$$\frac{(2bc-ad(p+3))\left(-\frac{2b\left(\frac{bx^{-n(p+1)}(a+bx^n)^{p+1}}{a^2n(p+1)(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}\right)}{2b}-\frac{dx^{-n(p+3)}(a+bx^n)^{p+1}}{2bn}$$

input
```
Int[x^(-1 - n*(3 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-1/2*(d*(a + b*x^n)^(1 + p))/(b*n*x^(n*(3 + p))) + ((2*b*c - a*d*(3 + p))*
(-((a + b*x^n)^(1 + p)/(a*n*(3 + p)*x^(n*(3 + p)))) - (2*b*((b*(a + b*x^n)
^(1 + p))/(a^2*n*(1 + p)*(2 + p)*x^(n*(1 + p))) - (a + b*x^n)^(1 + p)/(a*n
*(2 + p)*x^(n*(2 + p)))))/(a*(3 + p))))/(2*b)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(140) = 280.

Time = 2.20 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.56

method	result
parallelrisch	$\frac{x\,x^{3n}x^{-np-3n-1}(a+b\,x^{n})^{p}a\,b^{3}dp-x\,x^{2n}x^{-np-3n-1}(a+b\,x^{n})^{p}a^{2}b^{2}d\,p^{2}+3x\,x^{3n}x^{-np-3n-1}(a+b\,x^{n})^{p}a\,b^{3}d-2x\,x^{3n}x^{-np-3}}{}$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
(x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*a*b^3*d*p-x*(x^n)^2*x^(-n*p-3*n-1)*(
a+b*x^n)^p*a^2*b^2*d*p^2+3*x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*a*b^3*d-2*
x*(x^n)^3*x^(-n*p-3*n-1)*(a+b*x^n)^p*b^4*c-3*x*(x^n)^2*x^(-n*p-3*n-1)*(a+b
*x^n)^p*a^2*b^2*d*p+2*x*(x^n)^2*x^(-n*p-3*n-1)*(a+b*x^n)^p*a*b^3*c*p-x*x^n
*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*d*p^2-x*x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a
^2*b^2*c*p^2-4*x*x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*d*p-x*x^n*x^(-n*p-3*
n-1)*(a+b*x^n)^p*a^2*b^2*c*p-x*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*c*p^2-3*x*
x^n*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*d-3*x*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*
b*c*p-2*x*x^(-n*p-3*n-1)*(a+b*x^n)^p*a^3*b*c)/b/(3+p)/(p^2+3*p+2)/a^3/n
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.61

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)\,dx$$
$$=\frac{((ab^2dp-2\,b^3c+3\,ab^2d)xx^{-np-3\,n-1}x^{3\,n}-(a^2bdp^2-(2\,ab^2c-3\,a^2bd)p)xx^{-np-3\,n-1}x^{2\,n}-(3\,a^3d+(a^2b}{a^3np^3+6\,a^3np^2+11\,a}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
((a*b^2*d*p - 2*b^3*c + 3*a*b^2*d)*x*x^(-n*p - 3*n - 1)*x^(3*n) - (a^2*b*d
*p^2 - (2*a*b^2*c - 3*a^2*b*d)*p)*x*x^(-n*p - 3*n - 1)*x^(2*n) - (3*a^3*d
+ (a^2*b*c + a^3*d)*p^2 + (a^2*b*c + 4*a^3*d)*p)*x*x^(-n*p - 3*n - 1)*x^n
- (a^3*c*p^2 + 3*a^3*c*p + 2*a^3*c)*x*x^(-n*p - 3*n - 1))*(b*x^n + a)^p/(a
^3*n*p^3 + 6*a^3*n*p^2 + 11*a^3*n*p + 6*a^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(110) = 220$.

Time = 95.26 (sec) , antiderivative size = 648, normalized size of antiderivative = 4.73

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)\;dx = \frac{a^2a^pa^{-p-3}b^{p+3}cp^2\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} + \frac{3a^2a^pa^{-p-3}b^{p+3}cp\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} + \frac{2a^2a^pa^{-p-3}b^{p+3}c\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} - \frac{2aa^pa^{-p-3}bb^{p+3}cpx^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} - \frac{2aa^pa^{-p-3}bb^{p+3}cx^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} - \frac{aa^pa^{-p-2}b^{p+2}dpx^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)} - \frac{aa^pa^{-p-2}b^{p+2}dx^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)} + \frac{2a^pa^{-p-3}b^2b^{p+3}cx^{2n}\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^2n\Gamma\left(-p\right)+2abnx^n\Gamma\left(-p\right)+b^2nx^{2n}\Gamma\left(-p\right)} + \frac{a^pa^{-p-2}b^{p+2}d\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{n\Gamma\left(-p\right)}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**2*a**p*a**(-p - 3)*b**(p + 3)*c*p**2*(a/(b*x**n) + 1)**(p + 3)*gamma(-p
 - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)*gamma(-
p)) + 3*a**2*a**p*a**(-p - 3)*b**(p + 3)*c*p*(a/(b*x**n) + 1)**(p + 3)*gam
ma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)*ga
mma(-p)) + 2*a**2*a**p*a**(-p - 3)*b**(p + 3)*c*(a/(b*x**n) + 1)**(p + 3)*
gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)
*gamma(-p)) - 2*a*a**p*a**(-p - 3)*b*b**(p + 3)*c*p*x**n*(a/(b*x**n) + 1)*
*(p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n
*x**(2*n)*gamma(-p)) - 2*a*a**p*a**(-p - 3)*b*b**(p + 3)*c*x**n*(a/(b*x**n
) + 1)**(p + 3)*gamma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) +
 b**2*n*x**(2*n)*gamma(-p)) - a*a**p*a**(-p - 2)*b**(p + 2)*d*p*(a/(b*x**n
) + 1)**(p + 1)*gamma(-p - 2)/(b*n*x**n*gamma(-p)) - a*a**p*a**(-p - 2)*b*
*(p + 2)*d*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(b*n*x**n*gamma(-p)) +
2*a**p*a**(-p - 3)*b**2*b**(p + 3)*c*x**(2*n)*(a/(b*x**n) + 1)**(p + 3)*ga
mma(-p - 3)/(a**2*n*gamma(-p) + 2*a*b*n*x**n*gamma(-p) + b**2*n*x**(2*n)*g
amma(-p)) + a**p*a**(-p - 2)*b**(p + 2)*d*(a/(b*x**n) + 1)**(p + 1)*gamma(
-p - 2)/(n*gamma(-p))
```

Maxima [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int(dx^n+c)(bx^n+a)^p x^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\left(c+dx^n\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-2,[0,0,2,2,1,1,1,0,1]%%%}+%%%{-2,[0,0,2,2,1,1,0,0,1]%%%}+
%%%{1,[0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p+3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 3) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{(x^nb+a)^p\,(x^{3n}a\,b^2dp+3x^{3n}a\,b^2d-2x^{3n}b^3c-x^{2n}a^2bd\,p^2-3x^{2n}a^2bdp+2x^{2n}a\,b^2cp-x^na^3d\,p^2-4x^na^3}{x^{np+3n}a^3n\,(p^3+6p^2+11p+6)}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
((x**n*b + a)**p*(x**(3*n)*a*b**2*d*p + 3*x**(3*n)*a*b**2*d - 2*x**(3*n)*b
**3*c - x**(2*n)*a**2*b*d*p**2 - 3*x**(2*n)*a**2*b*d*p + 2*x**(2*n)*a*b**2
*c*p - x**n*a**3*d*p**2 - 4*x**n*a**3*d*p - 3*x**n*a**3*d - x**n*a**2*b*c*
p**2 - x**n*a**2*b*c*p - a**3*c*p**2 - 3*a**3*c*p - 2*a**3*c))/(x**(n*p +
3*n)*a**3*n*(p**3 + 6*p**2 + 11*p + 6))
```

3.516 $\int x^{-1-n(3+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx=-\frac{2b^2x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3n(1+p)(2+p)(3+p)}+\frac{2bx^{-n(2+p)}(a+bx^n)^{1+p}}{a^2n(2+p)(3+p)}-\frac{x^{-n(3+p)}(a+bx^n)^{1+p}}{an(3+p)}$$

output
```
-2*b^2*(a+b*x^n)^(p+1)/a^3/n/(p+1)/(2+p)/(3+p)/(x^(n*(p+1)))+2*b*(a+b*x^n)
^(p+1)/a^2/n/(2+p)/(3+p)/(x^(n*(2+p)))-(a+b*x^n)^(p+1)/a/n/(3+p)/(x^(n*(3+
p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx=-\frac{x^{-n(3+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^n}{a}\right)}{n(3+p)}$$

input
```
Integrate[x^(-1 - n*(3 + p))*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^n)/a)])/(n*(
3 + p)*x^(n*(3 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+3)-1}(a+bx^n)^p \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{2b\int x^{-n(p+2)-1}(bx^n+a)^p dx}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}$$

$$\Big\downarrow \text{803}$$

$$-\frac{2b\left(-\frac{b\int x^{-n(p+1)-1}(bx^n+a)^p dx}{a(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}$$

$$\Big\downarrow \text{796}$$

$$-\frac{2b\left(\frac{bx^{-n(p+1)}(a+bx^n)^{p+1}}{a^2n(p+1)(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{a(p+3)}-\frac{x^{-n(p+3)}(a+bx^n)^{p+1}}{an(p+3)}$$

input
```
Int[x^(-1 - n*(3 + p))*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*n*(3 + p)*x^(n*(3 + p)))) - (2*b*((b*(a + b*x^n)^
(1 + p))/(a^2*n*(1 + p)*(2 + p)*x^(n*(1 + p))) - (a + b*x^n)^(1 + p)/(a*n*
(2 + p)*x^(n*(2 + p)))))/(a*(3 + p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-1-n(3+p)}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx = \frac{(2\,ab^2pxx^{-np-3\,n-1}x^{2\,n}-2\,b^3xx^{-np-3\,n-1}x^{3\,n}-(a^2bp^2+a^2bp)xx^{-np-3\,n-1}x^n-(a^3p^2+3\,a^3p+2\,a^3)xx^{-n}}{a^3np^3+6\,a^3np^2+11\,a^3np+6\,a^3n}$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
(2*a*b^2*p*x*x^(-n*p - 3*n - 1)*x^(2*n) - 2*b^3*x*x^(-n*p - 3*n - 1)*x^(3*
n) - (a^2*b*p^2 + a^2*b*p)*x*x^(-n*p - 3*n - 1)*x^n - (a^3*p^2 + 3*a^3*p +
 2*a^3)*x*x^(-n*p - 3*n - 1))*(b*x^n + a)^p/(a^3*n*p^3 + 6*a^3*n*p^2 + 11*
a^3*n*p + 6*a^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(92) = 184$.

Time = 42.67 (sec) , antiderivative size = 464, normalized size of antiderivative = 4.00

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx = \frac{a^2b^{p+3}p^2\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)} + \frac{3a^2b^{p+3}p\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)} + \frac{2a^2b^{p+3}\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)} - \frac{2abb^{p+3}px^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)} - \frac{2abb^{p+3}x^n\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)} + \frac{2b^2b^{p+3}x^{2n}\left(\frac{ax^{-n}}{b}+1\right)^{p+3}\Gamma(-p-3)}{a^5n\Gamma(-p)+2a^4bnx^n\Gamma(-p)+a^3b^2nx^{2n}\Gamma(-p)}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p,x)
```

output
```
a**2*b**(p + 3)*p**2*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma
(-p) + 2*a**4*b*n*x**n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p)) + 3*a**
2*b**(p + 3)*p*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma(-p) +
 2*a**4*b*n*x**n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p)) + 2*a**2*b**(
p + 3)*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma(-p) + 2*a**4*
b*n*x**n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p)) - 2*a*b*b**(p + 3)*p*
x**n*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma(-p) + 2*a**4*b*
n*x**n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p)) - 2*a*b*b**(p + 3)*x**n
*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma(-p) + 2*a**4*b*n*x*
*n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p)) + 2*b**2*b**(p + 3)*x**(2*n
)*(a/(b*x**n) + 1)**(p + 3)*gamma(-p - 3)/(a**5*n*gamma(-p) + 2*a**4*b*n*x
**n*gamma(-p) + a**3*b**2*n*x**(2*n)*gamma(-p))
```

Maxima [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Giac [F]

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+3)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p + 3) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p + 3) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^{-1-n(3+p)}(a+bx^n)^p\,dx$$
$$=\frac{(x^nb+a)^p\,(-2x^{3n}b^3+2x^{2n}a\,b^2p-x^na^2b\,p^2-x^na^2bp-a^3p^2-3a^3p-2a^3)}{x^{np+3n}a^3n\,(p^3+6p^2+11p+6)}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*( - 2*x**(3*n)*b**3 + 2*x**(2*n)*a*b**2*p - x**n*a**2*b*p
**2 - x**n*a**2*b*p - a**3*p**2 - 3*a**3*p - 2*a**3))/(x**(n*p + 3*n)*a**3
*n*(p**3 + 6*p**2 + 11*p + 6))
```

3.517 $\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 302

$$\begin{aligned}\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx &= -\frac{d^2x^{-n(1+p)}(a+bx^n)^{1+p}}{ac^3n(1+p)}-\frac{bdx^{-n(1+p)}(a+bx^n)^{1+p}}{a^2c^2n(1+p)(2+p)}-\frac{2b^2x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3cn(1+p)(2+p)(3+p)}\\ &\quad+\frac{dx^{-n(2+p)}(a+bx^n)^{1+p}}{ac^2n(2+p)}+\frac{2bx^{-n(2+p)}(a+bx^n)^{1+p}}{a^2cn(2+p)(3+p)}-\frac{x^{-n(3+p)}(a+bx^n)^{1+p}}{acn(3+p)}\\ &\quad+\frac{d^3x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^4np}\end{aligned}$$

output
```
-d^2*(a+b*x^n)^(p+1)/a/c^3/n/(p+1)/(x^(n*(p+1)))-b*d*(a+b*x^n)^(p+1)/a^2/c
^2/n/(p+1)/(2+p)/(x^(n*(p+1)))-2*b^2*(a+b*x^n)^(p+1)/a^3/c/n/(p+1)/(2+p)/(
3+p)/(x^(n*(p+1)))+d*(a+b*x^n)^(p+1)/a/c^2/n/(2+p)/(x^(n*(2+p)))+2*b*(a+b*
x^n)^(p+1)/a^2/c/n/(2+p)/(3+p)/(x^(n*(2+p)))-(a+b*x^n)^(p+1)/a/c/n/(3+p)/(
x^(n*(3+p)))+d^3*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b
*x^n))/c^4/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 4.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.14

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= \frac{x^{-n(3+p)}(a+bx^n)^{-1+p}\left(c(-1+p)(a+bx^n)\left(c^3p(2+3p+p^2)-3c^2dp(1+p)x^n+6cd^2px^{2n}-6d^3x^{3n}\right)\Phi\right.}{}$$

input
```
Integrate[(x^(-1 - n*(3 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
((a + b*x^n)^(-1 + p)*(c*(-1 + p)*(a + b*x^n)*(c^3*p*(2 + 3*p + p^2) - 3*c
^2*d*p*(1 + p)*x^n + 6*c*d^2*p*x^(2*n) - 6*d^3*x^(3*n))*HurwitzLerchPhi[((
b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + (b*c - a*d)*x^n*(c + d*x^n)*((c^
2*(2 + 6*p + 3*p^2) - c*d*(5 + 9*p)*x^n + 11*d^2*x^(2*n))*Hypergeometric2F
1[2, 1 - p, 2 - p, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 3*(c + c*p - 2*d*x
^n)*(c + d*x^n)*HypergeometricPFQ[{2, 2, 1 - p}, {1, 2 - p}, ((b*c - a*d)*
x^n)/(c*(a + b*x^n))] + (c + d*x^n)^2*HypergeometricPFQ[{2, 2, 2, 1 - p},
{1, 1, 2 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))/(c^5*n*(-1 + p)*(1 +
p)*(2 + p)*(3 + p)*x^(n*(3 + p)))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 7.06 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+3)-1}(a+bx^n)^p}{c+dx^n}\,dx$$
$$\Big\downarrow \; 1013$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int\frac{x^{-n(p+3)-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{-n(p+3)}(a+bx^n)^{p-1}\Big(c(1-p)\left(a+bx^n\right)\left(c^3p(p+1)(p+2)-3c^2dp(p+1)x^n+6cd^2px^{2n}-6d^3x^{3n}\right)\Phi\Big(\frac{(bc-ad)x}{c(bx^n+a}}{}$$

input
```
Int[(x^(-1 - n*(3 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
((a + b*x^n)^(-1 + p)*(c*(1 - p)*(a + b*x^n)*(c^3*p*(1 + p)*(2 + p) - 3*c^
2*d*p*(1 + p)*x^n + 6*c*d^2*p*x^(2*n) - 6*d^3*x^(3*n))*HurwitzLerchPhi[((b
*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - (b*c - a*d)*x^n*(c + d*x^n)*((c^2
*(2 + 6*p + 3*p^2) - c*d*(5 + 9*p)*x^n + 11*d^2*x^(2*n))*Hypergeometric2F1
[2, 1 - p, 2 - p, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 3*(c*(1 + p) - 2*d*
x^n)*(c + d*x^n)*HypergeometricPFQ[{2, 2, 1 - p}, {1, 2 - p}, ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + (c + d*x^n)^2*HypergeometricPFQ[{2, 2, 2, 1 - p},
 {1, 1, 2 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))/(c^5*n*(1 - p)*(1 +
p)*(2 + p)*(3 + p)*x^(n*(3 + p)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(3+p)}(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 3*n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+3)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 3) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 3) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Too large to display}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*a**2*b*d**2*p**2 + 5*x**(3*n)*(x**n*b + a)**p*a*
*2*b*d**2*p + 6*x**(3*n)*(x**n*b + a)**p*a**2*b*d**2 - x**(3*n)*(x**n*b +
a)**p*a*b**2*c*d*p**2 - 3*x**(3*n)*(x**n*b + a)**p*a*b**2*c*d*p - 2*x**(3*
n)*(x**n*b + a)**p*b**3*c**2*p - x**(2*n)*(x**n*b + a)**p*a**3*d**2*p**3 -
 5*x**(2*n)*(x**n*b + a)**p*a**3*d**2*p**2 - 6*x**(2*n)*(x**n*b + a)**p*a*
*3*d**2*p + x**(2*n)*(x**n*b + a)**p*a**2*b*c*d*p**3 + 3*x**(2*n)*(x**n*b
+ a)**p*a**2*b*c*d*p**2 + 2*x**(2*n)*(x**n*b + a)**p*a*b**2*c**2*p**2 + x*
*n*(x**n*b + a)**p*a**3*c*d*p**3 + 4*x**n*(x**n*b + a)**p*a**3*c*d*p**2 +
3*x**n*(x**n*b + a)**p*a**3*c*d*p - x**n*(x**n*b + a)**p*a**2*b*c**2*p**3
- x**n*(x**n*b + a)**p*a**2*b*c**2*p**2 - (x**n*b + a)**p*a**3*c**2*p**3 -
 3*(x**n*b + a)**p*a**3*c**2*p**2 - 2*(x**n*b + a)**p*a**3*c**2*p - x**(n*
p + 3*n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x +
x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**4*d**3*n*p**4 - 6*x**(n*p + 3*n
)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p
 + n)*b*c*x + x**(n*p)*a*c*x),x)*a**4*d**3*n*p**3 - 11*x**(n*p + 3*n)*int(
(x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*
b*c*x + x**(n*p)*a*c*x),x)*a**4*d**3*n*p**2 - 6*x**(n*p + 3*n)*int((x**n*b
 + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x +
 x**(n*p)*a*c*x),x)*a**4*d**3*n*p + x**(n*p + 3*n)*int((x**n*b + a)**p/(x*
*(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)...
```

3.518 $\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 507

$$\begin{aligned}\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx &= -\frac{2b^2(bc-ad(4+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^3c^2(bc-ad)n(1+p)(2+p)(3+p)} \\ &\quad -\frac{d^2(4bc-ad(4+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{ac^4(bc-ad)n(1+p)} \\ &\quad -\frac{bd(4bc-ad(4+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^2c^3(bc-ad)n(1+p)(2+p)} \\ &\quad +\frac{2b(bc-ad(4+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{a^2c^2(bc-ad)n(2+p)(3+p)}+\frac{d(4bc-ad(4+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{ac^3(bc-ad)n(2+p)} \\ &\quad -\frac{d(bc-ad(4+p))x^{-n(2+p)}(a+bx^n)^{1+p}}{ac^2(bc-ad)n(3+p)\,(c+dx^n)}-\frac{x^{-n(3+p)}(a+bx^n)^{1+p}}{acn(3+p)\,(c+dx^n)} \\ &\quad +\frac{d^3(4bc-ad(4+p))x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^5(bc-ad)np}\end{aligned}$$

output
```
-2*b^2*(b*c-a*d*(4+p))*(a+b*x^n)^(p+1)/a^3/c^2/(-a*d+b*c)/n/(p+1)/(2+p)/(3
+p)/(x^(n*(p+1)))-d^2*(4*b*c-a*d*(4+p))*(a+b*x^n)^(p+1)/a/c^4/(-a*d+b*c)/n
/(p+1)/(x^(n*(p+1)))-b*d*(4*b*c-a*d*(4+p))*(a+b*x^n)^(p+1)/a^2/c^3/(-a*d+b
*c)/n/(p+1)/(2+p)/(x^(n*(p+1)))+2*b*(b*c-a*d*(4+p))*(a+b*x^n)^(p+1)/a^2/c^
2/(-a*d+b*c)/n/(2+p)/(3+p)/(x^(n*(2+p)))+d*(4*b*c-a*d*(4+p))*(a+b*x^n)^(p+
1)/a/c^3/(-a*d+b*c)/n/(2+p)/(x^(n*(2+p)))-d*(b*c-a*d*(4+p))*(a+b*x^n)^(p+1
)/a/c^2/(-a*d+b*c)/n/(3+p)/(x^(n*(2+p)))/(c+d*x^n)-(a+b*x^n)^(p+1)/a/c/n/(
3+p)/(x^(n*(3+p)))/(c+d*x^n)+d^3*(4*b*c-a*d*(4+p))*(a+b*x^n)^p*hypergeom([
1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^5/(-a*d+b*c)/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 5.79 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx =$$

$$-\frac{x^{-n(3+p)}(a+bx^n)^p\left(c(c^4(2+2p-7p^2+3p^4)+4c^3d(-1+p)^2\left(-1-p+p^2\right)x^n-6c^2d^2(-1+p)^2(-\right.}{}$$

input
```
Integrate[(x^(-1 - n*(3 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(c*(c^4*(2 + 2*p - 7*p^2 + 3*p^4) + 4*c^3*d*(-1 + p)^2*(-
1 - p + p^2)*x^n - 6*c^2*d^2*(-1 + p)^2*(-2 + 3*p)*x^(2*n) + 44*c*d^3*(-1
+ p)^2*x^(3*n) - 2*d^4*(-13 + 25*p)*x^(4*n))*HurwitzLerchPhi[((b*c - a*d)*
x^n)/(c*(a + b*x^n)), 1, 1 - p] - c*(c^4*p^2*(-1 + 4*p + 3*p^2) + 4*c^3*d*
p^2*(-1 - 2*p + p^2)*x^n - 6*c^2*d^2*p^2*(-5 + 3*p)*x^(2*n) + 4*c*d^3*(-6
- 11*p + 11*p^2)*x^(3*n) - 2*d^4*(12 + 25*p)*x^(4*n))*HurwitzLerchPhi[((b*
c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + ((b*c - a*d)*x^n*(c + d*x^n)^2*((c
^2*(-1 + 6*p + 6*p^2) - 2*c*d*(1 + 12*p)*x^n + 35*d^2*x^(2*n))*Hypergeomet
ricPFQ[{2, 2, 1 - p}, {1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - (c
+ d*x^n)*(2*(c + 2*c*p - 5*d*x^n)*HypergeometricPFQ[{2, 2, 2, 1 - p}, {1,
1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - (c + d*x^n)*Hypergeometric
PFQ[{2, 2, 2, 2, 1 - p}, {1, 1, 1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n
))])))/((2 - 3*p + p^2)*(a + b*x^n))))/(c^6*n*(1 + p)*(2 + p)*(3 + p)*x^(n
*(3 + p))*(c + d*x^n)))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 9.07 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+3)-1}(a+bx^n)^p}{(c+dx^n)^2}dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-n(p+3)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-n(p+3)}(a+bx^n)^p\left(-\frac{x^n(bc-ad)(c+dx^n)^2\left(\left(c^2\left(-6p^2-6p+1\right)+2cd(12p+1)x^n-35d^2x^{2n}\right)\,{}_3F_2\left(2,2,1-p;1,3-p;\frac{(bc-ad)x^n}{c(bx^n+a)}\right)+(c+dx^n)(\cdots}{(p^2-3p+2)\cdots}\right.}{\cdots}$$

input
```
Int[(x^(-1 - n*(3 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(c*(c^4*(2 + 2*p - 7*p^2 + 3*p^4) - 4*c^3*d*(1 - p)^2*(1
+ p - p^2)*x^n + 6*c^2*d^2*(2 - 3*p)*(1 - p)^2*x^(2*n) + 44*c*d^3*(1 - p)^
2*x^(3*n) + 2*d^4*(13 - 25*p)*x^(4*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(
c*(a + b*x^n)), 1, 1 - p] + c*(c^4*p^2*(1 - 4*p - 3*p^2) + 4*c^3*d*p^2*(1
+ 2*p - p^2)*x^n - 6*c^2*d^2*(5 - 3*p)*p^2*x^(2*n) + 4*c*d^3*(6 + 11*p - 1
1*p^2)*x^(3*n) + 2*d^4*(12 + 25*p)*x^(4*n))*HurwitzLerchPhi[((b*c - a*d)*x
^n)/(c*(a + b*x^n)), 1, -p] - ((b*c - a*d)*x^n*(c + d*x^n)^2*((c^2*(1 - 6*
p - 6*p^2) + 2*c*d*(1 + 12*p)*x^n - 35*d^2*x^(2*n))*HypergeometricPFQ[{2,
2, 1 - p}, {1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (c + d*x^n)*(2
*(c*(1 + 2*p) - 5*d*x^n)*HypergeometricPFQ[{2, 2, 2, 1 - p}, {1, 1, 3 - p}
, ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - (c + d*x^n)*HypergeometricPFQ[{2, 2
, 2, 2, 1 - p}, {1, 1, 1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))/((
2 - 3*p + p^2)*(a + b*x^n))))/(c^7*n*(1 + p)*(2 + p)*(3 + p)*x^(n*(3 + p))
*(1 + (d*x^n)/c)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(3+p)} (a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 3*n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(3+p))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 3) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+3)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 3) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 3) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
( - 6*x**(4*n)*(x**n*b + a)**p*a*b**3*c*d**2*p**2 - 30*x**(4*n)*(x**n*b +
a)**p*a*b**3*c*d**2*p - 24*x**(4*n)*(x**n*b + a)**p*a*b**3*c*d**2 + 6*x**(
4*n)*(x**n*b + a)**p*b**4*c**2*d*p**2 + 6*x**(4*n)*(x**n*b + a)**p*b**4*c*
*2*d*p + 2*x**(3*n)*(x**n*b + a)**p*a**3*b*d**3*p**4 + 18*x**(3*n)*(x**n*b
 + a)**p*a**3*b*d**3*p**3 + 52*x**(3*n)*(x**n*b + a)**p*a**3*b*d**3*p**2 +
 48*x**(3*n)*(x**n*b + a)**p*a**3*b*d**3*p - 2*x**(3*n)*(x**n*b + a)**p*a*
*2*b**2*c*d**2*p**4 - 20*x**(3*n)*(x**n*b + a)**p*a**2*b**2*c*d**2*p**3 -
62*x**(3*n)*(x**n*b + a)**p*a**2*b**2*c*d**2*p**2 - 72*x**(3*n)*(x**n*b +
a)**p*a**2*b**2*c*d**2*p + 2*x**(3*n)*(x**n*b + a)**p*a*b**3*c**2*d*p**3 +
 28*x**(3*n)*(x**n*b + a)**p*a*b**3*c**2*d*p**2 + 18*x**(3*n)*(x**n*b + a)
**p*a*b**3*c**2*d*p - 24*x**(3*n)*(x**n*b + a)**p*a*b**3*c**2*d + 6*x**(3*
n)*(x**n*b + a)**p*b**4*c**3*p**2 + 6*x**(3*n)*(x**n*b + a)**p*b**4*c**3*p
 - x**(2*n)*(x**n*b + a)**p*a**4*d**3*p**5 - 10*x**(2*n)*(x**n*b + a)**p*a
**4*d**3*p**4 - 35*x**(2*n)*(x**n*b + a)**p*a**4*d**3*p**3 - 50*x**(2*n)*(
x**n*b + a)**p*a**4*d**3*p**2 - 24*x**(2*n)*(x**n*b + a)**p*a**4*d**3*p +
x**(2*n)*(x**n*b + a)**p*a**3*b*c*d**2*p**5 + 11*x**(2*n)*(x**n*b + a)**p*
a**3*b*c*d**2*p**4 + 41*x**(2*n)*(x**n*b + a)**p*a**3*b*c*d**2*p**3 + 67*x
**(2*n)*(x**n*b + a)**p*a**3*b*c*d**2*p**2 + 36*x**(2*n)*(x**n*b + a)**p*a
**3*b*c*d**2*p - x**(2*n)*(x**n*b + a)**p*a**2*b**2*c**2*d*p**4 - 12*x**(2
*n)*(x**n*b + a)**p*a**2*b**2*c**2*d*p**3 - 11*x**(2*n)*(x**n*b + a)**p...
```

3.519 $\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^3\,dx$

Optimal result

Integrand size = 29, antiderivative size = 186

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^3\,dx = \frac{d^3x^{-np}(a+bx^n)^{1+p}}{bn} + \frac{c^2(bc-3ad(2+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^2n(1+p)(2+p)} - \frac{c^3x^{-n(2+p)}(a+bx^n)^{1+p}}{an(2+p)} - \frac{d^2(3bc+adp)x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{bnp}$$

output

```
d^3*(a+b*x^n)^(p+1)/b/n/(x^(n*p))+c^2*(b*c-3*a*d*(2+p))*(a+b*x^n)^(p+1)/a^
2/n/(p+1)/(2+p)/(x^(n*(p+1)))-c^3*(a+b*x^n)^(p+1)/a/n/(2+p)/(x^(n*(2+p)))-
d^2*(a*d*p+3*b*c)*(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/b/n/p/(x^
(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^3\;dx$$
$$=\frac{x^{-n(2+p)}(a+bx^n)^p\left(-\frac{3c^2dx^n(a+bx^n)}{a(1+p)}-\frac{c^3\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^n}{a}\right)}{2+p}-\frac{d^3x^{3n}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hype}}{}\right.}{n}$$

input
```
Integrate[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^p*((-3*c^2*d*x^n*(a + b*x^n))/(a*(1 + p)) - (c^3*Hypergeometr
ic2F1[-2 - p, -p, -1 - p, -((b*x^n)/a)])/((2 + p)*(1 + (b*x^n)/a)^p) - (d^
3*x^(3*n)*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^n)/a)])/((-1 + p)*(1
+ (b*x^n)/a)^p) - (3*c*d^2*x^(2*n)*Hypergeometric2F1[-p, -p, 1 - p, -((b*x
^n)/a)])/(p*(1 + (b*x^n)/a)^p)))/(n*x^(n*(2 + p)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {1008, 1066}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+2)-1}(c+dx^n)^3\,(a+bx^n)^p\;dx$$
$$\Big\downarrow \; 1008$$
$$\frac{\int x^{-n(p+2)-1}(bx^n+a)^p\,(dx^n+c)\,(dn(3bc+adp)x^n+cn(bc+ad(p+2)))\,dx}{bn}+\frac{dx^{-n(p+2)}(c+dx^n)^2\,(a+bx^n)^{p+1}}{bn}$$
$$\Big\downarrow \; 1066$$

Indeterminate

input
```
Int[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int x^{-1-n(2+p)} (a+b\,x^n)^p\,(c+d\,x^n)^3\,dx$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\int (dx^n+c)^3(bx^n+a)^px^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(-n*p - 2*n - 1)*x^(3*n) + 3*c*d^2*x^(-n*p - 2*n - 1)*x^(2
*n) + 3*c^2*d*x^(-n*p - 2*n - 1)*x^n + c^3*x^(-n*p - 2*n - 1))*(b*x^n + a)
^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx=\int (dx^n+c)^3(bx^n+a)^px^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p*x^(-n*(p + 2) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,4,3,3,0]%%%}+%%
%{6,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^3 \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^3}{x^{n\,(p+2)+1}} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^3 \, dx$$
$$= \frac{x^{3n}(x^nb+a)^p\,a^2d^3p^3 + 3x^{3n}(x^nb+a)^p\,a^2d^3p^2 + 2x^{3n}(x^nb+a)^p\,a^2d^3p - 3x^{2n}(x^nb+a)^p\,a^2c\,d^2p^2 - 9x^{2n}(}{}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*a**2*d**3*p**3 + 3*x**(3*n)*(x**n*b + a)**p*a**2
*d**3*p**2 + 2*x**(3*n)*(x**n*b + a)**p*a**2*d**3*p - 3*x**(2*n)*(x**n*b +
 a)**p*a**2*c*d**2*p**2 - 9*x**(2*n)*(x**n*b + a)**p*a**2*c*d**2*p - 6*x**
(2*n)*(x**n*b + a)**p*a**2*c*d**2 - 3*x**(2*n)*(x**n*b + a)**p*a*b*c**2*d*
p**2 - 6*x**(2*n)*(x**n*b + a)**p*a*b*c**2*d*p + x**(2*n)*(x**n*b + a)**p*
b**2*c**3*p - 3*x**n*(x**n*b + a)**p*a**2*c**2*d*p**2 - 6*x**n*(x**n*b + a
)**p*a**2*c**2*d*p - x**n*(x**n*b + a)**p*a*b*c**3*p**2 - (x**n*b + a)**p*
a**2*c**3*p**2 - (x**n*b + a)**p*a**2*c**3*p + x**(n*p + 2*n)*int((x**n*(x
**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**4 + 3*x
**(n*p + 2*n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x)
,x)*a**3*d**3*n*p**3 + 2*x**(n*p + 2*n)*int((x**n*(x**n*b + a)**p)/(x**(n*
p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**2 + 3*x**(n*p + 2*n)*int((x**
n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*d**2*n*p*
*3 + 9*x**(n*p + 2*n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n
*p)*a*x),x)*a**2*b*c*d**2*n*p**2 + 6*x**(n*p + 2*n)*int((x**n*(x**n*b + a)
**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*d**2*n*p)/(x**(n*p + 2*
n)*a**2*n*p*(p**2 + 3*p + 2))
```

3.520 $\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 145

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \frac{c(bc-2ad(2+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^2n(1+p)(2+p)} - \frac{c^2x^{-n(2+p)}(a+bx^n)^{1+p}}{an(2+p)} - \frac{d^2x^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

output
```
c*(b*c-2*a*d*(2+p))*(a+b*x^n)^(p+1)/a^2/n/(p+1)/(2+p)/(x^(n*(p+1)))-c^2*(a
+b*x^n)^(p+1)/a/n/(2+p)/(x^(n*(2+p)))-d^2*(a+b*x^n)^p*hypergeom([-p, -p],[
1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.01

$$\int x^{-1-n(2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{x^{-n(2+p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(ac^2p(1+p) \text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^n}{a}\right) + d(2+\right.}{anp(1+p)(2-}$$

input
```
Integrate[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(a*c^2*p*(1 + p)*Hypergeometric2F1[-2 - p, -p, -1 - p, -(
(b*x^n)/a)] + d*(2 + p)*x^n*(2*c*p*(a + b*x^n)*(1 + (b*x^n)/a)^p + a*d*(1
+ p)*x^n*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a)])))/(a*n*p*(1 + p)*
(2 + p)*x^(n*(2 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {1008}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+2)-1}(c+dx^n)^2(a+bx^n)^p\,dx$$

$$\Big\downarrow \; 1008$$

$$\text{Indeterminate}$$

input
```
Int[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
Indeterminate
```

Defintions of rubi rules used

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(2+p)} (a+b\,x^n)^p\,(c+d\,x^n)^2\,dx$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-n(2+p)} (a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2 (bx^n+a)^p x^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p - 2*n - 1)*x^(2*n) + 2*c*d*x^(-n*p - 2*n - 1)*x^n +
c^2*x^(-n*p - 2*n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)} (a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int (dx^n+c)^2(bx^n+a)^p x^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p + 2) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p+2)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{-2x^{2n}(x^nb+a)^p\,abcdp-4x^{2n}(x^nb+a)^p\,abcd+x^{2n}(x^nb+a)^p\,b^2c^2-2x^n(x^nb+a)^p\,a^2cdp-4x^n(x^nb+}{}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
( - 2*x**(2*n)*(x**n*b + a)**p*a*b*c*d*p - 4*x**(2*n)*(x**n*b + a)**p*a*b*
c*d + x**(2*n)*(x**n*b + a)**p*b**2*c**2 - 2*x**n*(x**n*b + a)**p*a**2*c*d
*p - 4*x**n*(x**n*b + a)**p*a**2*c*d - x**n*(x**n*b + a)**p*a*b*c**2*p - (
x**n*b + a)**p*a**2*c**2*p - (x**n*b + a)**p*a**2*c**2 + x**(n*p + 2*n)*in
t((x**n*b + a)**p/(x**(n*p)*x),x)*a**2*d**2*n*p**2 + 3*x**(n*p + 2*n)*int(
(x**n*b + a)**p/(x**(n*p)*x),x)*a**2*d**2*n*p + 2*x**(n*p + 2*n)*int((x**n
*b + a)**p/(x**(n*p)*x),x)*a**2*d**2*n)/(x**(n*p + 2*n)*a**2*n*(p**2 + 3*p
 + 2))
```

3.521 $\int x^{-1-n(2+p)}(a+bx^n)^p(c+dx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 81

$$\int x^{-1-n(2+p)}(a+bx^n)^p(c+dx^n)\,dx=\frac{(bc-ad(2+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^2n(1+p)(2+p)}-\frac{cx^{-n(2+p)}(a+bx^n)^{1+p}}{an(2+p)}$$

output
```
(b*c-a*d*(2+p))*(a+b*x^n)^(p+1)/a^2/n/(p+1)/(2+p)/(x^(n*(p+1)))-c*(a+b*x^n
)^(p+1)/a/n/(2+p)/(x^(n*(2+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int x^{-1-n(2+p)}(a+bx^n)^p(c+dx^n)\,dx=\frac{(a+bx^n)^p\left(-\frac{dx^{-n(1+p)}(a+bx^n)}{a(1+p)}-\frac{cx^{-n(2+p)}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^n}{a}\right)}{2+p}\right)}{n}$$

input
```
Integrate[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(-((d*(a + b*x^n))/(a*(1 + p)*x^(n*(1 + p)))) - (c*Hypergeo
metric2F1[-2 - p, -p, -1 - p, -((b*x^n)/a)])/((2 + p)*x^(n*(2 + p))*(1 + (
b*x^n)/a)^p)))/n
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {959, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+2)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(bc-ad(p+2))\int x^{-n(p+2)-1}(bx^n+a)^p\,dx}{b}-\frac{dx^{-n(p+2)}(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{803}$$

$$\frac{(bc-ad(p+2))\left(-\frac{b\int x^{-n(p+1)-1}(bx^n+a)^p dx}{a(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{b}-\frac{dx^{-n(p+2)}(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{796}$$

$$\frac{(bc-ad(p+2))\left(\frac{bx^{-n(p+1)}(a+bx^n)^{p+1}}{a^2n(p+1)(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}\right)}{b}-\frac{dx^{-n(p+2)}(a+bx^n)^{p+1}}{bn}$$

input
```
Int[x^(-1 - n*(2 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-((d*(a + b*x^n)^(1 + p))/(b*n*x^(n*(2 + p)))) + ((b*c - a*d*(2 + p))*((b*
(a + b*x^n)^(1 + p))/(a^2*n*(1 + p)*(2 + p)*x^(n*(1 + p))) - (a + b*x^n)^(
1 + p)/(a*n*(2 + p)*x^(n*(2 + p)))))/b
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(83) = 166.

Time = 2.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.32

method	result
parallelrisch	$-\frac{x\,x^{2n} x^{-np-2n-1} (a+b\,x^{n})^{p} a\,b^{2} d p+2 x\,x^{2n} x^{-np-2n-1} (a+b\,x^{n})^{p} a\,b^{2} d-x\,x^{2n} x^{-np-2n-1} (a+b\,x^{n})^{p} b^{3} c+x\,x^{n} x^{-np-2n-1} (}{}$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
-(x*(x^n)^2*x^(-n*p-2*n-1)*(a+b*x^n)^p*a*b^2*d*p+2*x*(x^n)^2*x^(-n*p-2*n-1
)*(a+b*x^n)^p*a*b^2*d-x*(x^n)^2*x^(-n*p-2*n-1)*(a+b*x^n)^p*b^3*c+x*x^n*x^(
-n*p-2*n-1)*(a+b*x^n)^p*a^2*b*d*p+x*x^n*x^(-n*p-2*n-1)*(a+b*x^n)^p*a*b^2*c
*p+2*x*x^n*x^(-n*p-2*n-1)*(a+b*x^n)^p*a^2*b*d+x*x^(-n*p-2*n-1)*(a+b*x^n)^p
*a^2*b*c*p+x*x^(-n*p-2*n-1)*(a+b*x^n)^p*a^2*b*c)/b/(2+p)/a^2/n/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.62

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\;dx = -\frac{((abdp-b^2c+2\,abd)xx^{-np-2\,n-1}x^{2\,n}+(2\,a^2d+(abc+a^2d)p)xx^{-np-2\,n-1}x^n+(a^2cp+a^2c)xx^{-np-2\,n-}}{a^2np^2+3\,a^2np+2\,a^2n}$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
-((a*b*d*p - b^2*c + 2*a*b*d)*x*x^(-n*p - 2*n - 1)*x^(2*n) + (2*a^2*d + (a
*b*c + a^2*d)*p)*x*x^(-n*p - 2*n - 1)*x^n + (a^2*c*p + a^2*c)*x*x^(-n*p -
2*n - 1))*(b*x^n + a)^p/(a^2*n*p^2 + 3*a^2*n*p + 2*a^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(63) = 126.

Time = 91.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.27

$$\begin{aligned}\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\;dx = &-\frac{aa^pa^{-p-2}b^{p+2}cpx^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)}\\ &-\frac{aa^pa^{-p-2}b^{p+2}cx^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{bn\Gamma\left(-p\right)}\\ &+\frac{a^pa^{-p-2}b^{p+2}c\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{n\Gamma\left(-p\right)}\\ &+\frac{a^pa^{-p-1}b^{p+1}d\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-1)}{n\Gamma\left(-p\right)}\end{aligned}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
-a*a**p*a**(-p - 2)*b**(p + 2)*c*p*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)
/(b*n*x**n*gamma(-p)) - a*a**p*a**(-p - 2)*b**(p + 2)*c*(a/(b*x**n) + 1)**
(p + 1)*gamma(-p - 2)/(b*n*x**n*gamma(-p)) + a**p*a**(-p - 2)*b**(p + 2)*c
*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(n*gamma(-p)) + a**p*a**(-p - 1)*
b**(p + 1)*d*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 1)/(n*gamma(-p))
```

Maxima [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int(dx^n+c)(bx^n+a)^p x^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p + 2) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,0,2,2,1,1,1,0,1]%%%}+%%%{-1,[0,0,2,2,1,1,0,0,1]%%%}+
%%%{1,[0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\,\,dx=\int\frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p+2)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,(c+dx^n)\,\,dx$$
$$=\frac{(x^nb+a)^p\,(-x^{2n}abdp-2x^{2n}abd+x^{2n}b^2c-x^na^2dp-2x^na^2d-x^nabcp-a^2cp-a^2c)}{x^{np+2n}a^2n\,(p^2+3p+2)}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
((x**n*b + a)**p*( - x**(2*n)*a*b*d*p - 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c
 - x**n*a**2*d*p - 2*x**n*a**2*d - x**n*a*b*c*p - a**2*c*p - a**2*c))/(x**
(n*p + 2*n)*a**2*n*(p**2 + 3*p + 2))
```

3.522 $\int x^{-1-n(2+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 70

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx=\frac{bx^{-n(1+p)}(a+bx^n)^{1+p}}{a^2n(1+p)(2+p)}-\frac{x^{-n(2+p)}(a+bx^n)^{1+p}}{an(2+p)}$$

output
```
b*(a+b*x^n)^(p+1)/a^2/n/(p+1)/(2+p)/(x^(n*(p+1)))-(a+b*x^n)^(p+1)/a/n/(2+p
)/(x^(n*(2+p)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx = -\frac{x^{-n(2+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^n}{a}\right)}{n(2+p)}$$

input
```
Integrate[x^(-1 - n*(2 + p))*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^n)/a)])/(n*(
2 + p)*x^(n*(2 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+2)-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{b\int x^{-n(p+1)-1}(bx^n+a)^p\,dx}{a(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}$$

$$\Big\downarrow \text{796}$$

$$\frac{bx^{-n(p+1)}(a+bx^n)^{p+1}}{a^2n(p+1)(p+2)}-\frac{x^{-n(p+2)}(a+bx^n)^{p+1}}{an(p+2)}$$

input
```
Int[x^(-1 - n*(2 + p))*(a + b*x^n)^p,x]
```

output
```
(b*(a + b*x^n)^(1 + p))/(a^2*n*(1 + p)*(2 + p)*x^(n*(1 + p))) - (a + b*x^n
)^(1 + p)/(a*n*(2 + p)*x^(n*(2 + p)))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-1-n(2+p)}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx = -\frac{(abpxx^{-np-2n-1}x^{n}-b^{2}xx^{-np-2n-1}x^{2n}+(a^{2}p+a^{2})xx^{-np-2n-1})(bx^{n}+a)^{p}}{a^{2}np^{2}+3\,a^{2}np+2\,a^{2}n}$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(a*b*p*x*x^(-n*p - 2*n - 1)*x^n - b^2*x*x^(-n*p - 2*n - 1)*x^(2*n) + (a^2
*p + a^2)*x*x^(-n*p - 2*n - 1))*(b*x^n + a)^p/(a^2*n*p^2 + 3*a^2*n*p + 2*a
^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(53) = 106$.

Time = 42.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx = -\frac{b^{p+2}px^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{abn\Gamma(-p)} - \frac{b^{p+2}x^{-n}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{abn\Gamma(-p)} + \frac{b^{p+2}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-2)}{a^2n\Gamma(-p)}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p,x)
```

output
```
-b**(p + 2)*p*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(a*b*n*x**n*gamma(-p
)) - b**(p + 2)*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(a*b*n*x**n*gamma(
-p)) + b**(p + 2)*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 2)/(a**2*n*gamma(-p
))
```

Maxima [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1), x)
```

Giac [F]

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx=\int (bx^n+a)^p x^{-n(p+2)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p+2)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p + 2) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p + 2) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^{-1-n(2+p)}(a+bx^n)^p\,dx=\frac{(x^nb+a)^p\,(x^{2n}b^2-x^nabp-a^2p-a^2)}{x^{np+2n}a^2n\,(p^2+3p+2)}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**(2*n)*b**2 - x**n*a*b*p - a**2*p - a**2))/(x**(n*p +
2*n)*a**2*n*(p**2 + 3*p + 2))
```

3.523 $\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 174

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \frac{dx^{-n(1+p)}(a+bx^n)^{1+p}}{ac^2n(1+p)}+\frac{bx^{-n(1+p)}(a+bx^n)^{1+p}}{a^2cn(1+p)(2+p)}-\frac{x^{-n(2+p)}(a+bx^n)^{1+p}}{acn(2+p)} - \frac{d^2x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^3np}$$

output
```
d*(a+b*x^n)^(p+1)/a/c^2/n/(p+1)/(x^(n*(p+1)))+b*(a+b*x^n)^(p+1)/a^2/c/n/(p
+1)/(2+p)/(x^(n*(p+1)))-(a+b*x^n)^(p+1)/a/c/n/(2+p)/(x^(n*(2+p)))-d^2*(a+b
*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^3/n/p/(x^(n*
p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.99 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = -\frac{x^{-n(2+p)}(a+bx^n)^p\left(-c(c^2p(1+p)-2cdpx^n+2d^2x^{2n})\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,-p\right)+\frac{(bc-ad)x^n(c+dx^n)\left(-\left((c+2cp\right.\right.}{}\right.}{c^4n(1+p)(2+p)}$$

input
```
Integrate[(x^(-1 - n*(2 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(-(c*(c^2*p*(1 + p) - 2*c*d*p*x^n + 2*d^2*x^(2*n))*Hurwit
zLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]) + ((b*c - a*d)*x^n*(c
 + d*x^n)*(-((c + 2*c*p - 3*d*x^n)*Hypergeometric2F1[2, 1 - p, 2 - p, ((b*
c - a*d)*x^n)/(c*(a + b*x^n))]) + (c + d*x^n)*HypergeometricPFQ[{2, 2, 1 -
 p}, {1, 2 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))/((-1 + p)*(a + b*x^n
))))/(c^4*n*(1 + p)*(2 + p)*x^(n*(2 + p))))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.92 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+2)-1}(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-n(p+2)-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{-n(p+2)}(a+bx^n)^p\left(c\left(c^2p(p+1)-2cdpx^n+2d^2x^{2n}\right)\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)-\frac{x^n(bc-ad)(c+dx^n)\left((c(2p+1)-3dx^n)\,\text{Hype}\right.}{}\right.}{c^4n(p+1)(p+2)}$$

input
```
Int[(x^(-1 - n*(2 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(c*(c^2*p*(1 + p) - 2*c*d*p*x^n + 2*d^2*x^(2*n))*HurwitzLer
chPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - ((b*c - a*d)*x^n*(c + d*
x^n)*((c*(1 + 2*p) - 3*d*x^n)*Hypergeometric2F1[2, 1 - p, 2 - p, ((b*c - a
*d)*x^n)/(c*(a + b*x^n))] - (c + d*x^n)*HypergeometricPFQ[{2, 2, 1 - p}, {
1, 2 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))/((1 - p)*(a + b*x^n))))/(c
^4*n*(1 + p)*(2 + p)*x^(n*(2 + p)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(2+p)}(a+b\,x^{n})^{p}}{c+d\,x^{n}}dx$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 2*n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+2)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 2) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 2) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= \frac{-x^{2n}(x^nb+a)^p\,abdp-2x^{2n}(x^nb+a)^p\,abd+x^{2n}(x^nb+a)^p\,b^2cp+x^n(x^nb+a)^p\,a^2d\,p^2+2x^n(x^nb+a)^p\,a}{}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
( - x**(2*n)*(x**n*b + a)**p*a*b*d*p - 2*x**(2*n)*(x**n*b + a)**p*a*b*d +
x**(2*n)*(x**n*b + a)**p*b**2*c*p + x**n*(x**n*b + a)**p*a**2*d*p**2 + 2*x
**n*(x**n*b + a)**p*a**2*d*p - x**n*(x**n*b + a)**p*a*b*c*p**2 - (x**n*b +
 a)**p*a**2*c*p**2 - (x**n*b + a)**p*a**2*c*p + x**(n*p + 2*n)*int((x**n*b
 + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x +
 x**(n*p)*a*c*x),x)*a**3*d**2*n*p**3 + 3*x**(n*p + 2*n)*int((x**n*b + a)**
p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*
p)*a*c*x),x)*a**3*d**2*n*p**2 + 2*x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(
n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*
x),x)*a**3*d**2*n*p - x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b
*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*b
*c*d*n*p**3 - 3*x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x +
 x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*b*c*d*n
*p**2 - 2*x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n
*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*b*c*d*n*p)/(x
**(n*p + 2*n)*a**2*c**2*n*p*(p**2 + 3*p + 2))
```

3.524 $\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 314

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2} dx = \frac{b(bc-ad(3+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{a^2c^2(bc-ad)n(1+p)(2+p)} + \frac{d(3bc-ad(3+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{ac^3(bc-ad)n(1+p)} - \frac{d(bc-ad(3+p))x^{-n(1+p)}(a+bx^n)^{1+p}}{ac^2(bc-ad)n(2+p)(c+dx^n)} - \frac{x^{-n(2+p)}(a+bx^n)^{1+p}}{acn(2+p)(c+dx^n)} - \frac{d^2(3bc-ad(3+p))x^{-np}(a+bx^n)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^4(bc-ad)np}$$

output
```
b*(b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a^2/c^2/(-a*d+b*c)/n/(p+1)/(2+p)/(x^(n*(
p+1)))+d*(3*b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a/c^3/(-a*d+b*c)/n/(p+1)/(x^(n*
(p+1)))-d*(b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(2+p)/(x^(n*(
p+1)))/(c+d*x^n)-(a+b*x^n)^(p+1)/a/c/n/(2+p)/(x^(n*(2+p)))/(c+d*x^n)-d^2*(
3*b*c-a*d*(3+p))*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b
*x^n))/c^4/(-a*d+b*c)/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.08 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.26

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2} dx =$$
$$-\frac{x^{-n(2+p)}(a+bx^n)^{-1+p}\left(c(2-3p+p^2)(a+bx^n)\left(c^3(-1+p)^2(1+2p)+3c^2d(-1+p)^3x^n-9cd^2(-1\right.\right.}{}$$

input
```
Integrate[(x^(-1 - n*(2 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^(-1 + p)*(c*(2 - 3*p + p^2)*(a + b*x^n)*(c^3*(-1 + p)^2*(1
+ 2*p) + 3*c^2*d*(-1 + p)^3*x^n - 9*c*d^2*(-1 + p)^2*x^(2*n) + d^3*(-5 + 1
1*p)*x^(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p]
 - c*(2 - 3*p + p^2)*(a + b*x^n)*(2*c^3*p^3 + 3*c^2*d*(-2 + p)*p^2*x^n - 3
*c*d^2*(-2 - 3*p + 3*p^2)*x^(2*n) + d^3*(6 + 11*p)*x^(3*n))*HurwitzLerchPh
i[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - (b*c - a*d)*x^n*(c + d*x^n)^
2*((-3*c*p + 6*d*x^n)*HypergeometricPFQ[{2, 2, 1 - p}, {1, 3 - p}, ((b*c -
 a*d)*x^n)/(c*(a + b*x^n))] + (c + d*x^n)*HypergeometricPFQ[{2, 2, 2, 1 -
p}, {1, 1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))/(c^5*n*(-2 + p)*(
-1 + p)*(1 + p)*(2 + p)*x^(n*(2 + p))*(c + d*x^n)))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.75 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+2)-1}(a+bx^n)^p}{(c+dx^n)^2} dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int\frac{x^{-n(p+2)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-n(p+2)}(a+bx^n)^{p-1}\left(x^n(bc-ad)\left(c+dx^n\right)^2\left(3(cp-2dx^n)\ {}_3F_2\left(2,2,1-p;1,3-p;\frac{(bc-ad)x^n}{c(bx^n+a)}\right)-(c+dx^n)\ {}_4\right.\right.}{}$$

input
```
Int[(x^(-1 - n*(2 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^(-1 + p)*(c*(1 - p)*(2 - p)*(a + b*x^n)*(c^3*(1 - p)^2*(1 +
 2*p) - 3*c^2*d*(1 - p)^3*x^n - 9*c*d^2*(1 - p)^2*x^(2*n) - d^3*(5 - 11*p)
*x^(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - c
*(1 - p)*(2 - p)*(a + b*x^n)*(2*c^3*p^3 - 3*c^2*d*(2 - p)*p^2*x^n + 3*c*d^
2*(2 + 3*p - 3*p^2)*x^(2*n) + d^3*(6 + 11*p)*x^(3*n))*HurwitzLerchPhi[((b*
c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + (b*c - a*d)*x^n*(c + d*x^n)^2*(3*(
c*p - 2*d*x^n)*HypergeometricPFQ[{2, 2, 1 - p}, {1, 3 - p}, ((b*c - a*d)*x
^n)/(c*(a + b*x^n))] - (c + d*x^n)*HypergeometricPFQ[{2, 2, 2, 1 - p}, {1,
 1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))/(c^6*n*(4 - 5*p^2 + p^4)
*x^(n*(2 + p))*(1 + (d*x^n)/c)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
 + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(2+p)}(a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 2*n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(2+p))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+2)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 2) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+2)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 2) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 2) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*a**2*b*d**3*p**2 + 5*x**(3*n)*(x**n*b + a)**p*a*
*2*b*d**3*p + 6*x**(3*n)*(x**n*b + a)**p*a**2*b*d**3 - x**(3*n)*(x**n*b +
a)**p*a*b**2*c*d**2*p**2 - 4*x**(3*n)*(x**n*b + a)**p*a*b**2*c*d**2*p - x*
*(3*n)*(x**n*b + a)**p*b**3*c**2*d*p - x**(2*n)*(x**n*b + a)**p*a**3*d**3*
p**3 - 5*x**(2*n)*(x**n*b + a)**p*a**3*d**3*p**2 - 6*x**(2*n)*(x**n*b + a)
**p*a**3*d**3*p + x**(2*n)*(x**n*b + a)**p*a**2*b*c*d**2*p**3 + 5*x**(2*n)
*(x**n*b + a)**p*a**2*b*c*d**2*p**2 + 5*x**(2*n)*(x**n*b + a)**p*a**2*b*c*
d**2*p + 6*x**(2*n)*(x**n*b + a)**p*a**2*b*c*d**2 - 4*x**(2*n)*(x**n*b + a
)**p*a*b**2*c**2*d*p - x**(2*n)*(x**n*b + a)**p*b**3*c**3*p + x**n*(x**n*b
 + a)**p*a**3*c*d**2*p**3 + 3*x**n*(x**n*b + a)**p*a**3*c*d**2*p**2 - x**n
*(x**n*b + a)**p*a**2*b*c**2*d*p**3 - x**n*(x**n*b + a)**p*a**2*b*c**2*d*p
**2 - 3*x**n*(x**n*b + a)**p*a**2*b*c**2*d*p + x**n*(x**n*b + a)**p*a*b**2
*c**3*p**2 - (x**n*b + a)**p*a**3*c**2*d*p**3 - (x**n*b + a)**p*a**3*c**2*
d*p**2 + (x**n*b + a)**p*a**2*b*c**3*p**2 + (x**n*b + a)**p*a**2*b*c**3*p
- x**(n*p + 3*n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p*x -
 x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x**(n*p +
 2*n)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a*b*c*d**2*x - 2*x**(n*p + 2*n)*b**2
*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + x**(n*p + n)*a*b*c**2*d*p*x -
 2*x**(n*p + n)*a*b*c**2*d*x - x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c*
*2*d*p*x - x**(n*p)*a*b*c**3*x),x)*a**5*d**6*n*p**5 - 6*x**(n*p + 3*n)*...
```

3.525 $\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 210

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^3 \, dx = \frac{d^3x^{n(1-p)}(a+bx^n)^{1+p}}{2bn} - \frac{3c^2dx^{-np}(a+bx^n)^{1+p}}{anp} - \frac{c^3x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)} + \frac{d(6b^2c^2+6abcdp-a^2d^2(1-p)p)\,x^{n(1-p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p\right.}{2abn(1-p)p}$$

output
```
1/2*d^3*x^(n*(1-p))*(a+b*x^n)^(p+1)/b/n-3*c^2*d*(a+b*x^n)^(p+1)/a/n/p/(x^(
n*p))-c^3*(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))+1/2*d*(6*b^2*c^2+6*a*b*c
*d*p-a^2*d^2*(1-p)*p)*x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p, 1-p],[-p+2],-
b*x^n/a)/a/b/n/(1-p)/p/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.93

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx$$
$$=\frac{x^{-n(1+p)}(a+bx^n)^p\left(-\frac{3cd^2x^{2n}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{-1+p}-\frac{d^3x^{3n}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2\right.}{-2+p}\right.}{n}$$

input
```
Integrate[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^p*((-3*c*d^2*x^(2*n)*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^n)/a)])/((-1 + p)*(1 + (b*x^n)/a)^p) - (d^3*x^(3*n)*Hypergeometric2F1[2
 - p, -p, 3 - p, -((b*x^n)/a)])/((-2 + p)*(1 + (b*x^n)/a)^p) + c^2*(-((a*c
 + b*c*x^n)/(a + a*p)) - (3*d*x^n*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^
n)/a)])/(p*(1 + (b*x^n)/a)^p))))/(n*x^(n*(1 + p)))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1008, 1066, 954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(c+dx^n)^3\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-n(p+1)-1}(bx^n+a)^p\,(dx^n+c)\,(dn(4bc-ad(1-p))x^n+cn(2bc+ad(p+1)))\,dx}{2bn}+\frac{dx^{-n(p+1)}(c+dx^n)^2\,(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{1066}$$

$$\frac{\frac{\int x^{-n(p+1)-1}(bx^n+a)^p\left(dn^2\left(6b^2c^2+6abdpc-a^2d^2(1-p)p\right)x^n+cn^2\left(2b^2c^2+5abd(p+1)c-a^2d^2\left(1-p^2\right)\right)\right)dx}{bn}+\frac{dx^{-n(p+1)}(c+dx^n)(4bc-ad(1-p)}{b}}{2bn}$$

$$\frac{dx^{-n(p+1)}(c+dx^n)^2(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{954}$$

$$\frac{\frac{\frac{dn^2\left(-a^2d^2(1-p)p+6abcdp+6b^2c^2\right)\int x^{-n(p+1)-1}(bx^n+a)^{p+1}dx}{b}-\frac{nx^{-n(p+1)}(bc-ad)\left(-a^2d^2(1-p)p+abcd(5p+1)+2b^2c^2\right)(a+bx^n)^{p+1}}{ab(p+1)}}{bn}+\frac{dx^{-n(p+1)}(c}{}}{2bn}$$

$$\frac{dx^{-n(p+1)}(c+dx^n)^2(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{\frac{dnx^{-n(p+1)}\left(-a^2d^2(1-p)p+6abcdp+6b^2c^2\right)\left(\frac{x^n}{a+bx^n}\right)^{p+1}(a+bx^n)^{p+1}\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-2}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{b}-\frac{nx^{-n(p+1)}(bc-ad)\left(-a^2d^2(1-p)p+abcd(5p+1)+2b^2c^2\right.}{ab(p+1)}}{bn}}{2bn}$$

$$\frac{dx^{-n(p+1)}(c+dx^n)^2(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{-\frac{dnx^{-n(p+1)}\left(-a^2d^2(1-p)p+6abcdp+6b^2c^2\right)(a+bx^n)^{p+1}\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^n}{bx^n+a}\right)}{b(p+1)}-\frac{nx^{-n(p+1)}(bc-ad)\left(-a^2d^2(1-p)p+abcd(5p+1)+2b^2c\right.}{ab(p+1)}}{bn}}{2bn}$$

$$\frac{dx^{-n(p+1)}(c+dx^n)^2(a+bx^n)^{p+1}}{2bn}$$

input
```
Int[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(d*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(2*b*n*x^(n*(1 + p))) + ((d*(4*b*c -
 a*d*(1 - p))*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*x^(n*(1 + p))) + (-(((b*
c - a*d)*n*(2*b^2*c^2 - a^2*d^2*(1 - p)*p + a*b*c*d*(1 + 5*p))*(a + b*x^n)
^(1 + p))/(a*b*(1 + p)*x^(n*(1 + p)))) - (d*n*(6*b^2*c^2 + 6*a*b*c*d*p - a
^2*d^2*(1 - p)*p)*(a + b*x^n)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*
x^n)/(a + b*x^n)])/(b*(1 + p)*x^(n*(1 + p))))/(b*n))/(2*b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int x^{-1-n(p+1)} (a+b\,x^{n})^{p} \left(c+d\,x^{n}\right)^{3} dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\left(c+dx^n\right)^3\,dx=\int (dx^n+c)^3(bx^n+a)^px^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(-n*p - n - 1)*x^(3*n) + 3*c*d^2*x^(-n*p - n - 1)*x^(2*n)
+ 3*c^2*d*x^(-n*p - n - 1)*x^n + c^3*x^(-n*p - n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\left(c+dx^n\right)^3\,dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^3\,dx = \int (dx^n+c)^3(bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,2,4,2,4,4,3,0]%%%}+%%%{4,[2,0,2,4,2,4,3,3,0]%%%}+%%
%{6,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^3\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^3}{x^{n\,(p+1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*(p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^3\,dx$$
$$=\frac{x^{3n}(x^nb+a)^p\,ab\,d^3p^2+x^{3n}(x^nb+a)^p\,ab\,d^3p+x^{2n}(x^nb+a)^p\,a^2d^3p^3+x^{2n}(x^nb+a)^p\,a^2d^3p^2+6x^{2n}(x^nb}{}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*a*b*d**3*p**2 + x**(3*n)*(x**n*b + a)**p*a*b*d**
3*p + x**(2*n)*(x**n*b + a)**p*a**2*d**3*p**3 + x**(2*n)*(x**n*b + a)**p*a
**2*d**3*p**2 + 6*x**(2*n)*(x**n*b + a)**p*a*b*c*d**2*p**2 + 6*x**(2*n)*(x
**n*b + a)**p*a*b*c*d**2*p - 6*x**n*(x**n*b + a)**p*a*b*c**2*d*p - 6*x**n*
(x**n*b + a)**p*a*b*c**2*d - 2*x**n*(x**n*b + a)**p*b**2*c**3*p - 2*(x**n*
b + a)**p*a*b*c**3*p + x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p +
n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**4 - x**(n*p + n)*int((x**n*(x**n*
b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**2 + 6*x**(n
*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*
*2*b*c*d**2*n*p**3 + 6*x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p +
n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*d**2*n*p**2 + 6*x**(n*p + n)*int((x**n*
(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*b**2*c**2*d*n*p**2
 + 6*x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*
a*x),x)*a*b**2*c**2*d*n*p)/(2*x**(n*p + n)*a*b*n*p*(p + 1))
```

3.526 $\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 134

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \frac{d^2x^{-np}(a+bx^n)^{1+p}}{bn} - \frac{c^2x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)} - \frac{d(2bc+adp)x^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{bnp}$$

output
```
d^2*(a+b*x^n)^(p+1)/b/n/(x^(n*p))-c^2*(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1
)))-d*(a*d*p+2*b*c)*(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/b/n/p/(
x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.11

$$\int x^{-1-n(1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{x^{-n(1+p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(ad^2p(1+p)x^{2n} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right) + c(-1\right.}{an(-1+p)p(1}$$

input
```
Integrate[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(a*d^2*p*(1 + p)*x^(2*n)*Hypergeometric2F1[1 - p, -p, 2 -
 p, -((b*x^n)/a)] + c*(-1 + p)*(c*p*(a + b*x^n)*(1 + (b*x^n)/a)^p + 2*a*d*
(1 + p)*x^n*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a)])))/(a*n*(-1 + p
)*p*(1 + p)*x^(n*(1 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1008, 954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(c+dx^n)^2(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-n(p+1)-1}(bx^n+a)^p\left(dn(2bc+adp)x^n+cn(bc+ad(p+1))\right)dx}{bn}+\frac{dx^{-n(p+1)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{954}$$

$$\frac{\frac{dn(adp+2bc)\int x^{-n(p+1)-1}(bx^n+a)^{p+1}dx}{b}-\frac{x^{-n(p+1)}(bc-ad)(adp+bc)(a+bx^n)^{p+1}}{ab(p+1)}}{bn}+\frac{dx^{-n(p+1)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{dx^{-n(p+1)}(adp+2bc)\left(\frac{x^n}{a+bx^n}\right)^{p+1}(a+bx^n)^{p+1}\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-2}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{b}-\frac{x^{-n(p+1)}(bc-ad)(adp+bc)(a+bx^n)^{p+1}}{ab(p+1)}}{bn}+\frac{dx^{-n(p+1)}(c+dx^n)(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{-\frac{dx^{-n(p+1)}(adp+2bc)(a+bx^n)^{p+1}\,\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^n}{bx^n+a}\right)}{b(p+1)}-\frac{x^{-n(p+1)}(bc-ad)(adp+bc)(a+bx^n)^{p+1}}{ab(p+1)}}{bn}+\frac{dx^{-n(p+1)}\left(c+dx^n\right)\left(a+bx^n\right)^{p+1}}{bn}$$

input
```
Int[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*n*x^(n*(1 + p))) + (-(((b*c - a*d)*
(b*c + a*d*p)*(a + b*x^n)^(1 + p))/(a*b*(1 + p)*x^(n*(1 + p)))) - (d*(2*b*
c + a*d*p)*(a + b*x^n)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*x^n)/(a
 + b*x^n)])/(b*(1 + p)*x^(n*(1 + p))))/(b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(p+1)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{2}\,dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-n(1+p)}(a+bx^{n})^{p}\,(c+dx^{n})^{2}\,dx=\int (dx^{n}+c)^{2}(bx^{n}+a)^{p}x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p - n - 1)*x^(2*n) + 2*c*d*x^(-n*p - n - 1)*x^n + c^2*
x^(-n*p - n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2(bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,1,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,1,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p+1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{x^{2n}(x^nb+a)^p\,a\,d^2p^2+x^{2n}(x^nb+a)^p\,a\,d^2p-2x^n(x^nb+a)^p\,acdp-2x^n(x^nb+a)^p\,acd-x^n(x^nb+a)^p\,b\,c}{}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*a*d**2*p**2 + x**(2*n)*(x**n*b + a)**p*a*d**2*p
- 2*x**n*(x**n*b + a)**p*a*c*d*p - 2*x**n*(x**n*b + a)**p*a*c*d - x**n*(x*
*n*b + a)**p*b*c**2*p - (x**n*b + a)**p*a*c**2*p + x**(n*p + n)*int((x**n*
(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*d**2*n*p**3 + x
**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x
)*a**2*d**2*n*p**2 + 2*x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p +
n)*b*x + x**(n*p)*a*x),x)*a*b*c*d*n*p**2 + 2*x**(n*p + n)*int((x**n*(x**n*
b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*b*c*d*n*p)/(x**(n*p + n)
*a*n*p*(p + 1))
```

3.527 $\int x^{-1-n(1+p)}(a+bx^n)^p(c+dx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 93

$$\int x^{-1-n(1+p)}(a+bx^n)^p(c+dx^n)\,dx = -\frac{cx^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)} - \frac{dx^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

output
```
-c*(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))-d*(a+b*x^n)^p*hypergeom([-p, -p
],[1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int x^{-1-n(1+p)}(a+bx^n)^p(c+dx^n)\,dx = \frac{(a+bx^n)^p\left(-\frac{cx^{-n(1+p)}(a+bx^n)}{a(1+p)}-\frac{dx^{-np}\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{p}\right)}{n}$$

input
```
Integrate[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(-((c*(a + b*x^n))/(a*(1 + p)*x^(n*(1 + p)))) - (d*Hypergeo
metric2F1[-p, -p, 1 - p, -((b*x^n)/a)])/(p*x^(n*p)*(1 + (b*x^n)/a)^p)))/n
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{954}$$

$$\frac{d\int x^{-n(p+1)-1}(bx^n+a)^{p+1}dx}{b}-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}$$

$$\Big\downarrow \text{882}$$

$$\frac{dx^{-n(p+1)}\left(\frac{x^n}{a+bx^n}\right)^{p+1}(a+bx^n)^{p+1}\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-2}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{bn}-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-n(p+1)}(bc-ad)(a+bx^n)^{p+1}}{abn(p+1)}-\frac{dx^{-n(p+1)}(a+bx^n)^{p+1}\text{Hypergeometric2F1}\left(1,-p-1,-p,\frac{bx^n}{bx^n+a}\right)}{bn(p+1)}$$

input
```
Int[x^(-1 - n*(1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((b*c - a*d)*(a + b*x^n)^(1 + p))/(a*b*n*(1 + p)*x^(n*(1 + p)))) - (d*(a
 + b*x^n)^(1 + p)*Hypergeometric2F1[1, -1 - p, -p, (b*x^n)/(a + b*x^n)])/(
b*n*(1 + p)*x^(n*(1 + p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

Maple [F]

$$\int x^{-1-n(p+1)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p - n - 1)*x^n + c*x^(-n*p - n - 1))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 84.95 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{a^p a^{-p-1}b^{p+1}c\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-1)}{n\Gamma\left(-p\right)}+\frac{a^p dx^{-np}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1-p\right)}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**p*a**(-p - 1)*b**(p + 1)*c*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 1)/(n*g
amma(-p)) + a**p*d*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_polar(I*
pi)/a)/(n*x**(n*p)*gamma(1 - p))
```

Maxima [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,0,2,0,2,2,1,0]%%%}+%%%{2,[0,0,0,2,0,2,1,1,0]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p+1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,(c+dx^n)\,\,dx$$
$$=\frac{-x^n(x^nb+a)^p\,bc-(x^nb+a)^p\,ac+x^{np+n}\left(\int\frac{(x^nb+a)^p}{x^{np}x}dx\right)adnp+x^{np+n}\left(\int\frac{(x^nb+a)^p}{x^{np}x}dx\right)adn}{x^{np+n}an\,(p+1)}$$

input

```
int(x^(-1-n*(p+1))*(a+b*x^n)^p*(c+d*x^n),x)
```

output

```
( - x**n*(x**n*b + a)**p*b*c - (x**n*b + a)**p*a*c + x**(n*p + n)*int((x**
n*b + a)**p/(x**(n*p)*x),x)*a*d*n*p + x**(n*p + n)*int((x**n*b + a)**p/(x*
*(n*p)*x),x)*a*d*n)/(x**(n*p + n)*a*n*(p + 1))
```

3.528 $\int x^{-1-n(1+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx=-\frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)}$$

output
```
-(a+b*x^n)^(p+1)/a/n/(p+1)/(x^(n*(p+1)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx=-\frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{an(1+p)}$$

input
```
Integrate[x^(-1 - n*(1 + p))*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p+1)-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{x^{-n(p+1)}(a+bx^n)^{p+1}}{an(p+1)}$$

input
```
Int[x^(-1 - n*(1 + p))*(a + b*x^n)^p,x]
```

output
```
-((a + b*x^n)^(1 + p)/(a*n*(1 + p)*x^(n*(1 + p))))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(33) = 66.

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

method	result	size
parallelrisch	$-\frac{x\,x^{n}x^{-np-n-1}(a+b\,x^{n})^{p}b^{2}+x\,x^{-np-n-1}(a+b\,x^{n})^{p}ab}{n(p+1)ab}$	70

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-(x*x^n*x^(-n*p-n-1)*(a+b*x^n)^p*b^2+x*x^(-n*p-n-1)*(a+b*x^n)^p*a*b)/n/(p+
1)/a/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx = -\frac{(bxx^{-np-n-1}x^n + axx^{-np-n-1})(bx^n+a)^p}{anp+an}$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
-(b*x*x^(-n*p - n - 1)*x^n + a*x*x^(-n*p - n - 1))*(b*x^n + a)^p/(a*n*p +
a*n)
```

Sympy [A] (verification not implemented)

Time = 41.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx = \frac{b^{p+1}\left(\frac{ax^{-n}}{b}+1\right)^{p+1}\Gamma(-p-1)}{an\Gamma\left(-p\right)}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p,x)
```

output
```
b**(p + 1)*(a/(b*x**n) + 1)**(p + 1)*gamma(-p - 1)/(a*n*gamma(-p))
```

Maxima [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx=\int (bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx=\int (bx^n+a)^p x^{-n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+p)}(a+bx^n)^p\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p+1)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p + 1) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p + 1) + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int x^{-1-n(1+p)}(a+bx^n)^p \, dx = -\frac{(x^n b+a)^p (x^n b+a)}{x^{np+n} a n (p+1)}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p,x)
```

output
```
( - (x**n*b + a)**p*(x**n*b + a))/(x**(n*p + n)*a*n*(p + 1))
```

3.529 $\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 96

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n} \, dx = -\frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{acn(1+p)} + \frac{dx^{-np}(a+bx^n)^p \operatorname{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^2np}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/(p+1)/(x^(n*(p+1)))+d*(a+b*x^n)^p*hypergeom([1, -p]
,[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^2/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.66 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.46

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n} \, dx = \frac{x^{-n(1+p)}(a+bx^n)^p \left(c(cp-dx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,-p\right) + \frac{(bc-ad)x^n(c+dx^n) \operatorname{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{(-1+p)(a+bx^n)}\right)}{c^3n(1+p)}$$

input
```
Integrate[(x^(-1 - n*(1 + p))*(a + b*x^n)^p/(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(c*(c*p - d*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
b*x^n)), 1, -p] + ((b*c - a*d)*x^n*(c + d*x^n)*Hypergeometric2F1[2, 1 - p,
 2 - p, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((-1 + p)*(a + b*x^n))))/(c^3*
n*(1 + p)*x^(n*(1 + p)))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+1)-1}(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-n(p+1)-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{-n(p+1)}(a+bx^n)^p\left(c(cp-dx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)-\frac{x^n(bc-ad)(c+dx^n)\text{ Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(bx^n+a)}\right)}{(1-p)(a+bx^n)}\right)}{c^3n(p+1)}$$

input
```
Int[(x^(-1 - n*(1 + p)))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
((a + b*x^n)^p*(c*(c*p - d*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
b*x^n)), 1, -p] - ((b*c - a*d)*x^n*(c + d*x^n)*Hypergeometric2F1[2, 1 - p,
 2 - p, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 - p)*(a + b*x^n))))/(c^3*n
*(1 + p)*x^(n*(1 + p)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p+1)}(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+1)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= \frac{x^n(x^nb+a)^p\,b-(x^nb+a)^p\,ap-x^{np+n}\left(\int \frac{(x^nb+a)^p}{x^{np+2n}bdx+x^{np+n}adx+x^{np+n}bcx+x^{np}acx}dx\right)a^2dn\,p^2-x^{np+n}\left(\int \frac{}{x^{np+}}\right.}{}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
(x**n*(x**n*b + a)**p*b - (x**n*b + a)**p*a*p - x**(n*p + n)*int((x**n*b +
 a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x
**(n*p)*a*c*x),x)*a**2*d*n*p**2 - x**(n*p + n)*int((x**n*b + a)**p/(x**(n*
p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x)
,x)*a**2*d*n*p + x**(n*p + n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x +
x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*b*c*n*p**2
+ x**(n*p + n)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*
d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*b*c*n*p)/(x**(n*p + n)*a*c
*n*p*(p + 1))
```

3.530 $\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 191

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = -\frac{d(bc-ad(2+p))x^{-np}(a+bx^n)^{1+p}}{ac^2(bc-ad)n(1+p)\,(c+dx^n)} - \frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{acn(1+p)\,(c+dx^n)} + \frac{d(2bc-ad(2+p))x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^3(bc-ad)np}$$

output

```
-d*(b*c-a*d*(2+p))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(p+1)/(x^(n*p))/(c+d
*x^n)-(a+b*x^n)^(p+1)/a/c/n/(p+1)/(x^(n*(p+1)))/(c+d*x^n)+d*(2*b*c-a*d*(2+
p))*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^3/(-
a*d+b*c)/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.78 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.48

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = -\frac{x^{-n(1+p)}(a+bx^n)^{-1+p}\left(c(2-3p+p^2)(a+bx^n)\left(c^2(-1+p)^2+2cd(-1+p)^2x^n-d^2(-1+3p)x^{2n}\right)\Phi\right.}{}$$

input
```
Integrate[(x^(-1 - n*(1 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^(-1 + p)*(c*(2 - 3*p + p^2)*(a + b*x^n)*(c^2*(-1 + p)^2 + 2
*c*d*(-1 + p)^2*x^n - d^2*(-1 + 3*p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)
*x^n)/(c*(a + b*x^n)), 1, 1 - p] - c*(2 - 3*p + p^2)*(a + b*x^n)*(c^2*p^2
+ 2*c*d*(-1 - p + p^2)*x^n - d^2*(2 + 3*p)*x^(2*n))*HurwitzLerchPhi[((b*c
- a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + (b*c - a*d)*x^n*(c + d*x^n)^2*Hyperg
eometricPFQ[{2, 2, 1 - p}, {1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))])
)/(c^4*n*(-2 + p)*(-1 + p)*(1 + p)*x^(n*(1 + p))*(c + d*x^n)))
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.60 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+1)-1}(a+bx^n)^p}{(c+dx^n)^2}dx$$

$$\Big\downarrow \ 1013$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-n(p+1)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \textbf{1012}$$

$$-\frac{x^{-n(p+1)}(a+bx^n)^{p-1}\left(x^n(bc-ad)\left(c+dx^n\right)^2 {}_3F_2\left(2,2,1-p;1,3-p;\frac{(bc-ad)x^n}{c(bx^n+a)}\right)-c(1-p)(2-p)\left(a+bx^n\right)\right.}{}$$

input
```
Int[(x^(-1 - n*(1 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^(-1 + p)*(c*(1 - p)*(2 - p)*(a + b*x^n)*(c^2*(1 - p)^2 + 2*
c*d*(1 - p)^2*x^n + d^2*(1 - 3*p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^
n)/(c*(a + b*x^n)), 1, 1 - p] - c*(1 - p)*(2 - p)*(a + b*x^n)*(c^2*p^2 - 2
*c*d*(1 + p - p^2)*x^n - d^2*(2 + 3*p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*
d)*x^n)/(c*(a + b*x^n)), 1, -p] + (b*c - a*d)*x^n*(c + d*x^n)^2*Hypergeome
tricPFQ[{2, 2, 1 - p}, {1, 3 - p}, ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))/(c
^5*n*(1 - p)*(2 - p)*(1 + p)*x^(n*(1 + p))*(1 + (d*x^n)/c)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p+1)}(a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+1)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
( - 2*x**(2*n)*(x**n*b + a)**p*b**2*d + 2*x**n*(x**n*b + a)**p*a*b*d*p - 2
*x**n*(x**n*b + a)**p*b**2*c - (x**n*b + a)**p*a**2*d*p**2 - (x**n*b + a)*
*p*a**2*d*p + 2*(x**n*b + a)**p*a*b*c*p - x**(n*p + 2*n)*int((x**n*b + a)*
*p/(x**(n*p + 3*n)*a*b*d**3*p*x + x**(n*p + 3*n)*a*b*d**3*x - 2*x**(n*p +
3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + x**(n*p + 2*n)*a**2*d*
*3*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - 4*x**(n*p + 2*n)*b**2*c**2*d*x +
2*x**(n*p + n)*a**2*c*d**2*p*x + 2*x**(n*p + n)*a**2*c*d**2*x + x**(n*p +
n)*a*b*c**2*d*p*x - 3*x**(n*p + n)*a*b*c**2*d*x - 2*x**(n*p + n)*b**2*c**3
*x + x**(n*p)*a**2*c**2*d*p*x + x**(n*p)*a**2*c**2*d*x - 2*x**(n*p)*a*b*c*
*3*x),x)*a**4*d**4*n*p**4 - 4*x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(n*p
+ 3*n)*a*b*d**3*p*x + x**(n*p + 3*n)*a*b*d**3*x - 2*x**(n*p + 3*n)*b**2*c*
d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + x**(n*p + 2*n)*a**2*d**3*x + 2*x**
(n*p + 2*n)*a*b*c*d**2*p*x - 4*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p +
n)*a**2*c*d**2*p*x + 2*x**(n*p + n)*a**2*c*d**2*x + x**(n*p + n)*a*b*c**2*
d*p*x - 3*x**(n*p + n)*a*b*c**2*d*x - 2*x**(n*p + n)*b**2*c**3*x + x**(n*p
)*a**2*c**2*d*p*x + x**(n*p)*a**2*c**2*d*x - 2*x**(n*p)*a*b*c**3*x),x)*a**
4*d**4*n*p**3 - 5*x**(n*p + 2*n)*int((x**n*b + a)**p/(x**(n*p + 3*n)*a*b*d
**3*p*x + x**(n*p + 3*n)*a*b*d**3*x - 2*x**(n*p + 3*n)*b**2*c*d**2*x + x**
(n*p + 2*n)*a**2*d**3*p*x + x**(n*p + 2*n)*a**2*d**3*x + 2*x**(n*p + 2*n)*
a*b*c*d**2*p*x - 4*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a**2*c...
```

3.531 $\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3} dx$

Optimal result

Integrand size = 29, antiderivative size = 308

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3} dx = -\frac{d(2bc-ad(3+p))x^{-np}(a+bx^n)^{1+p}}{2ac^2(bc-ad)n(1+p)(c+dx^n)^2} - \frac{x^{-n(1+p)}(a+bx^n)^{1+p}}{acn(1+p)(c+dx^n)^2} - \frac{d(2b^2c^2-abcd(9+5p)+a^2d^2(6+5p+p^2))x^{-np}(a+bx^n)^{1+p}}{2ac^3(bc-ad)^2n(1+p)(c+dx^n)} + \frac{d(6b^2c^2-6abcd(2+p)+a^2d^2(6+5p+p^2))x^{-np}(a+bx^n)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)}{c(a+bx}\right.}{2c^4(bc-ad)^2np}$$

output
```
-1/2*d*(2*b*c-a*d*(3+p))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(p+1)/(x^(n*p)
)/(c+d*x^n)^2-(a+b*x^n)^(p+1)/a/c/n/(p+1)/(x^(n*(p+1)))/(c+d*x^n)^2-1/2*d*
(2*b^2*c^2-a*b*c*d*(9+5*p)+a^2*d^2*(p^2+5*p+6))*(a+b*x^n)^(p+1)/a/c^3/(-a*
d+b*c)^2/n/(p+1)/(x^(n*p))/(c+d*x^n)+1/2*d*(6*b^2*c^2-6*a*b*c*d*(2+p)+a^2*
d^2*(p^2+5*p+6))*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b
*x^n))/c^4/(-a*d+b*c)^2/n/p/(x^(n*p))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.84 (sec) , antiderivative size = 2698, normalized size of antiderivative = 8.76

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^(-1 - n*(1 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^(-1 + p)*(-2*c*(-6 + 11*p - 6*p^2 + p^3)*(a + b*x^n)*(c^3*(-1
 + p)^3 + 3*c^2*d*(-1 + p)^3*x^n + 3*c*d^2*(-1 + 2*p - 4*p^2 + p^3)*x^(2*n
) - d^3*(1 - p + 6*p^2)*x^(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
 b*x^n)), 1, 1 - p] + c*(-6 + 11*p - 6*p^2 + p^3)*(a + b*x^n)*(c^3*(-2 + p
)^3 + 3*c^2*d*(-2 + p)^3*x^n + 3*c*d^2*(-2 + p)^3*x^(2*n) - d^3*(8 - 13*p
+ 6*p^2)*x^(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2
- p] - 6*a*c^4*p^3*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -
p] + 11*a*c^4*p^4*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p
] - 6*a*c^4*p^5*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]
+ a*c^4*p^6*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + 36
*a*c^3*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 3
0*a*c^3*d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]
- 30*a*c^3*d*p^2*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1,
 -p] - 6*b*c^4*p^3*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)),
1, -p] + 12*a*c^3*d*p^3*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, -p] + 11*b*c^4*p^4*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b
*x^n)), 1, -p] + 27*a*c^3*d*p^4*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(
a + b*x^n)), 1, -p] - 6*b*c^4*p^5*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c
*(a + b*x^n)), 1, -p] - 18*a*c^3*d*p^5*x^n*HurwitzLerchPhi[((b*c - a*d)*x^
n)/(c*(a + b*x^n)), 1, -p] + b*c^4*p^6*x^n*HurwitzLerchPhi[((b*c - a*d)...
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 5.65 (sec) , antiderivative size = 2724, normalized size of antiderivative = 8.84, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p+1)-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-n(p+1)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \text{1012}$$

$$x^{-n(p+1)}(bx^n+a)^{p-1}\left(-bc^4p^6\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)x^n-3ac^3dp^6\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)x^n+6bc^4p^5\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,\right.\right.$$

input
```
Int[(x^(-1 - n*(1 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^(-1 + p)*(2*c*(1 - p)*(2 - p)*(3 - p)*(a + b*x^n)*(c^3*(1 - p
)^3 + 3*c^2*d*(1 - p)^3*x^n + 3*c*d^2*(1 - 2*p + 4*p^2 - p^3)*x^(2*n) + d^
3*(1 - p + 6*p^2)*x^(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n
)), 1, 1 - p] - c*(1 - p)*(2 - p)*(3 - p)*(a + b*x^n)*(c^3*(2 - p)^3 + 3*c
^2*d*(2 - p)^3*x^n + 3*c*d^2*(2 - p)^3*x^(2*n) + d^3*(8 - 13*p + 6*p^2)*x^
(3*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + 6*a*
c^4*p^3*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 11*a*c
^4*p^4*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + 6*a*c^4
*p^5*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - a*c^4*p^6
*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 36*a*c^3*d*x^
n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + 30*a*c^3*d*p
*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + 30*a*c^3*
d*p^2*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] + 6*b*
c^4*p^3*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 12
*a*c^3*d*p^3*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]
 - 11*b*c^4*p^4*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1,
-p] - 27*a*c^3*d*p^4*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n))
, 1, -p] + 6*b*c^4*p^5*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n
)), 1, -p] + 18*a*c^3*d*p^5*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
b*x^n)), 1, -p] - b*c^4*p^6*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a...
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p+1)}(a+b\,x^n)^p}{(c+d\,x^n)^3}dx$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - n - 1)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(p+1))*(a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p+1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p + 1) - 1)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p+1)+1}\,(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)^3),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p + 1) + 1)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(p+1))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
(6*x**(3*n)*(x**n*b + a)**p*a**4*b**3*d**6*p**4 + 48*x**(3*n)*(x**n*b + a)
**p*a**4*b**3*d**6*p**3 + 138*x**(3*n)*(x**n*b + a)**p*a**4*b**3*d**6*p**2
 + 168*x**(3*n)*(x**n*b + a)**p*a**4*b**3*d**6*p + 72*x**(3*n)*(x**n*b + a
)**p*a**4*b**3*d**6 - 54*x**(3*n)*(x**n*b + a)**p*a**3*b**4*c*d**5*p**3 -
282*x**(3*n)*(x**n*b + a)**p*a**3*b**4*c*d**5*p**2 - 480*x**(3*n)*(x**n*b
+ a)**p*a**3*b**4*c*d**5*p - 264*x**(3*n)*(x**n*b + a)**p*a**3*b**4*c*d**5
 + 120*x**(3*n)*(x**n*b + a)**p*a**2*b**5*c**2*d**4*p**2 + 408*x**(3*n)*(x
**n*b + a)**p*a**2*b**5*c**2*d**4*p + 336*x**(3*n)*(x**n*b + a)**p*a**2*b*
*5*c**2*d**4 - 72*x**(3*n)*(x**n*b + a)**p*a*b**6*c**3*d**3*p - 144*x**(3*
n)*(x**n*b + a)**p*a*b**6*c**3*d**3 - 6*x**(2*n)*(x**n*b + a)**p*a**5*b**2
*d**6*p**5 - 48*x**(2*n)*(x**n*b + a)**p*a**5*b**2*d**6*p**4 - 138*x**(2*n
)*(x**n*b + a)**p*a**5*b**2*d**6*p**3 - 168*x**(2*n)*(x**n*b + a)**p*a**5*
b**2*d**6*p**2 - 72*x**(2*n)*(x**n*b + a)**p*a**5*b**2*d**6*p + 78*x**(2*n
)*(x**n*b + a)**p*a**4*b**3*c*d**5*p**4 + 426*x**(2*n)*(x**n*b + a)**p*a**
4*b**3*c*d**5*p**3 + 768*x**(2*n)*(x**n*b + a)**p*a**4*b**3*c*d**5*p**2 +
528*x**(2*n)*(x**n*b + a)**p*a**4*b**3*c*d**5*p + 144*x**(2*n)*(x**n*b + a
)**p*a**4*b**3*c*d**5 - 12*x**(2*n)*(x**n*b + a)**p*a**3*b**4*c**2*d**4*p*
*4 - 372*x**(2*n)*(x**n*b + a)**p*a**3*b**4*c**2*d**4*p**3 - 1152*x**(2*n)
*(x**n*b + a)**p*a**3*b**4*c**2*d**4*p**2 - 1104*x**(2*n)*(x**n*b + a)**p*
a**3*b**4*c**2*d**4*p - 384*x**(2*n)*(x**n*b + a)**p*a**3*b**4*c**2*d**...
```

3.532 $\int x^{-1-np}(a+bx^n)^p\left(c+dx^n\right)^3\,dx$

Optimal result

Integrand size = 27, antiderivative size = 250

$$\int x^{-1-np}(a+bx^n)^p\left(c+dx^n\right)^3\,dx$$
$$=\frac{d^3x^{n(2-p)}(a+bx^n)^{1+p}}{3bn}-\frac{c^3x^{-np}(a+bx^n)^{1+p}}{anp}+\frac{c^2(bc+3adp)x^{n-np}(a+bx^n)^{1+p}}{a^2n(1-p)p}$$
$$-\frac{\left(6b^3c^3+18ab^2c^2dp-9a^2bcd^2(1-p)p+a^3d^3p(2-3p+p^2)\right)x^{n(2-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeor}}{3a^2bn(1-p)(2-p)p}$$

output
```
1/3*d^3*x^(n*(-p+2))*(a+b*x^n)^(p+1)/b/n-c^3*(a+b*x^n)^(p+1)/a/n/p/(x^(n*p
))+c^2*(3*a*d*p+b*c)*x^(-n*p+n)*(a+b*x^n)^(p+1)/a^2/n/(1-p)/p-1/3*(6*b^3*c
^3+18*a*b^2*c^2*d*p-9*a^2*b*c*d^2*(1-p)*p+a^3*d^3*p*(p^2-3*p+2))*x^(n*(-p+
2))*(a+b*x^n)^p*hypergeom([-p, -p+2],[3-p],-b*x^n/a)/a^2/b/n/(1-p)/(-p+2)/
p/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.81

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\,dx =$$
$$-\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(3c^2dp(6-5p+p^2)\,x^n\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)+(-\right.}{}$$

input
```
Integrate[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
-(((a + b*x^n)^p*(3*c^2*d*p*(6 - 5*p + p^2)*x^n*Hypergeometric2F1[1 - p, -
p, 2 - p, -((b*x^n)/a)] + (-1 + p)*(3*c*d^2*(-3 + p)*p*x^(2*n)*Hypergeomet
ric2F1[2 - p, -p, 3 - p, -((b*x^n)/a)] + (-2 + p)*(d^3*p*x^(3*n)*Hypergeom
etric2F1[3 - p, -p, 4 - p, -((b*x^n)/a)] + c^3*(-3 + p)*Hypergeometric2F1[
-p, -p, 1 - p, -((b*x^n)/a)]))))/(n*(-3 + p)*(-2 + p)*(-1 + p)*p*x^(n*p)*(
1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1008, 1066, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(c+dx^n)^3\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-np-1}(bx^n+a)^p\,(dx^n+c)\,(dn(5bc-ad(2-p))x^n+cn(3bc+adp))\,dx}{3bn}+\frac{dx^{-np}(c+dx^n)^2\,(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{1066}$$

$$\frac{\frac{\int x^{-np-1}(bx^n+a)^p\left(dn^2\left(11b^2c^2-abd(7-8p)c+a^2d^2\left(p^2-3p+2\right)\right)x^n+cn^2\left(6b^2c^2+7abdpc-a^2d^2(2-p)p\right)\right)dx}{2bn}+\frac{dx^{-np}(c+dx^n)(5bc-ad(2-p))(}{2b}}{3bn}$$

$$\frac{dx^{-np}(c+dx^n)^2(a+bx^n)^{p+1}}{3bn}$$

$\Big\downarrow$ 959

$$\frac{\frac{\frac{n^2\left(a^3d^3p\left(p^2-3p+2\right)-9a^2bcd^2(1-p)p+18ab^2c^2dp+6b^3c^3\right)\int x^{-np-1}(bx^n+a)^p dx}{b}+\frac{dnx^{-np}\left(a^2d^2\left(p^2-3p+2\right)-abcd(7-8p)+11b^2c^2\right)(a+bx^n)^{p+1}}{b}}{2bn}+\frac{dx^{-}}{}}{3bn}$$

$$\frac{dx^{-np}(c+dx^n)^2(a+bx^n)^{p+1}}{3bn}$$

$\Big\downarrow$ 882

$$\frac{\frac{\frac{nx^{-np}\left(a^3d^3p\left(p^2-3p+2\right)-9a^2bcd^2(1-p)p+18ab^2c^2dp+6b^3c^3\right)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-1}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{b}+\frac{dnx^{-np}\left(a^2d^2\left(p^2-3p+2\right)-abcd(7-8p)+1\right.}{b}}{2bn}}{3bn}$$

$$\frac{dx^{-np}(c+dx^n)^2(a+bx^n)^{p+1}}{3bn}$$

$\Big\downarrow$ 74

$$\frac{\frac{\frac{dnx^{-np}\left(a^2d^2\left(p^2-3p+2\right)-abcd(7-8p)+11b^2c^2\right)(a+bx^n)^{p+1}}{b}-\frac{nx^{-np}\left(a^3d^3p\left(p^2-3p+2\right)-9a^2bcd^2(1-p)p+18ab^2c^2dp+6b^3c^3\right)(a+bx^n)^p\,\text{Hypergeometric2F1}}{bp}}{2bn}}{3bn}$$

$$\frac{dx^{-np}(c+dx^n)^2(a+bx^n)^{p+1}}{3bn}$$

input
```
Int[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n)^3,x]
```

output
```
(d*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(3*b*n*x^(n*p)) + ((d*(5*b*c - a*d*(
2 - p))*(a + b*x^n)^(1 + p)*(c + d*x^n))/(2*b*x^(n*p)) + ((d*n*(11*b^2*c^2
 - a*b*c*d*(7 - 8*p) + a^2*d^2*(2 - 3*p + p^2))*(a + b*x^n)^(1 + p))/(b*x^
(n*p)) - (n*(6*b^3*c^3 + 18*a*b^2*c^2*d*p - 9*a^2*b*c*d^2*(1 - p)*p + a^3*
d^3*p*(2 - 3*p + p^2))*(a + b*x^n)^p*Hypergeometric2F1[1, -p, 1 - p, (b*x^
n)/(a + b*x^n)])/(b*p*x^(n*p)))/(2*b*n))/(3*b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int x^{-np-1}(a+b\,x^{n})^{p}\left(c+d\,x^{n}\right)^{3}dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

Fricas [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\;dx=\int\,(dx^n+c)^3(bx^n+a)^px^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^(-n*p - 1)*x^(3*n) + 3*c*d^2*x^(-n*p - 1)*x^(2*n) + 3*c^2*
d*x^(-n*p - 1)*x^n + c^3*x^(-n*p - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\;dx=\text{Timed out}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p*(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\,dx = \int (dx^n+c)^3(bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^3*(b*x^n + a)^p*x^(-n*p - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2,0,6,4,2,4,4,3,0]%%%}+%%%{4,[2,0,6,4,2,4,3,3,0]%%%}+%%
%{6,[2,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^3}{x^{n\,p+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*p + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^3)/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^3\,dx$$
$$=\frac{2x^{3n}(x^nb+a)^p\,b^2d^3p+x^{2n}(x^nb+a)^p\,ab\,d^3p^2+9x^{2n}(x^nb+a)^p\,b^2c\,d^2p+x^n(x^nb+a)^p\,a^2d^3p^3-2x^n(x^nb}{}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^3,x)
```

output
```
(2*x**(3*n)*(x**n*b + a)**p*b**2*d**3*p + x**(2*n)*(x**n*b + a)**p*a*b*d**
3*p**2 + 9*x**(2*n)*(x**n*b + a)**p*b**2*c*d**2*p + x**n*(x**n*b + a)**p*a
**2*d**3*p**3 - 2*x**n*(x**n*b + a)**p*a**2*d**3*p**2 + 9*x**n*(x**n*b + a
)**p*a*b*c*d**2*p**2 + 18*x**n*(x**n*b + a)**p*b**2*c**2*d*p - 6*(x**n*b +
 a)**p*b**2*c**3 + x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x +
 x**(n*p)*a*x),x)*a**3*d**3*n*p**4 - 3*x**(n*p)*int((x**n*(x**n*b + a)**p)
/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**3 + 2*x**(n*p)*int((x
**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**3*n*p**2
 + 9*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x)
,x)*a**2*b*c*d**2*n*p**3 - 9*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p
+ n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*d**2*n*p**2 + 18*x**(n*p)*int((x**n*(
x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*b**2*c**2*d*n*p**2
+ 6*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),
x)*b**3*c**3*n*p)/(6*x**(n*p)*b**2*n*p)
```

3.533 $\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx$

Optimal result

Integrand size = 27, antiderivative size = 171

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx=-\frac{c^2x^{-np}(a+bx^n)^{1+p}}{anp}+\frac{d^2x^{n-np}(a+bx^n)^{1+p}}{2bn}+\frac{(2b^2c^2+4abcdp-a^2d^2(1-p)p)\,x^{n-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\right.}{2abn(1-p)p}$$

output
```
-c^2*(a+b*x^n)^(p+1)/a/n/p/(x^(n*p))+1/2*d^2*x^(-n*p+n)*(a+b*x^n)^(p+1)/b/
n+1/2*(2*b^2*c^2+4*a*b*c*d*p-a^2*d^2*(1-p)*p)*x^(-n*p+n)*(a+b*x^n)^p*hyper
geom([-p, 1-p],[-p+2],-b*x^n/a)/a/b/n/(1-p)/p/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx=$$

$$-\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(2cd(-2+p)px^n\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)+(-1+p\right.}{n(-}$$

input
```
Integrate[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(2*c*d*(-2 + p)*p*x^n*Hypergeometric2F1[1 - p, -p, 2 - p,
 -((b*x^n)/a)] + (-1 + p)*(d^2*p*x^(2*n)*Hypergeometric2F1[2 - p, -p, 3 -
p, -((b*x^n)/a)] + c^2*(-2 + p)*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)
/a)])))/(n*(-2 + p)*(-1 + p)*p*x^(n*p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1008, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(c+dx^n)^2(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-np-1}(bx^n+a)^p(dn(3bc-ad(1-p))x^n+cn(2bc+adp))\,dx}{2bn}+\frac{dx^{-np}(c+dx^n)(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{n(-a^2d^2(1-p)p+4abcdp+2b^2c^2)\int x^{-np-1}(bx^n+a)^p dx}{b}+\frac{dx^{-np}(3bc-ad(1-p))(a+bx^n)^{p+1}}{b}}{2bn}+\frac{dx^{-np}(c+dx^n)(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{x^{-np}(-a^2d^2(1-p)p+4abcdp+2b^2c^2)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-1}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{b}+\frac{dx^{-np}(3bc-ad(1-p))(a+bx^n)^{p+1}}{b}}{2bn}+\frac{dx^{-np}(c+dx^n)(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{dx^{-np}(3bc-ad(1-p))(a+bx^n)^{p+1}}{b}-\frac{x^{-np}(-a^2d^2(1-p)p+4abcdp+2b^2c^2)(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^n}{bx^n+a}\right)}{bp}}{2bn}+\frac{dx^{-np}(c+dx^n)\,(a+bx^n)^{p+1}}{2bn}$$

input

```
Int[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output

```
(d*(a + b*x^n)^(1 + p)*(c + d*x^n))/(2*b*n*x^(n*p)) + ((d*(3*b*c - a*d*(1
- p))*(a + b*x^n)^(1 + p))/(b*x^(n*p)) - ((2*b^2*c^2 + 4*a*b*c*d*p - a^2*d
^2*(1 - p)*p)*(a + b*x^n)^p*Hypergeometric2F1[1, -p, 1 - p, (b*x^n)/(a + b
*x^n)])/(b*p*x^(n*p)))/(2*b*n)
```

Defintions of rubi rules used

rule 74

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-np-1}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{2}\,dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int (dx^n+c)^2(bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p - 1)*x^(2*n) + 2*c*d*x^(-n*p - 1)*x^n + c^2*x^(-n*p
- 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2(bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*p - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,p+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*p + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{x^{2n}(x^nb+a)^p\,b\,d^2p+x^n(x^nb+a)^p\,a\,d^2p^2+4x^n(x^nb+a)^p\,bcdp-2(x^nb+a)^p\,b\,c^2+x^{np}\Big(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}}{}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b*d**2*p + x**n*(x**n*b + a)**p*a*d**2*p**2 + 4*
x**n*(x**n*b + a)**p*b*c*d*p - 2*(x**n*b + a)**p*b*c**2 + x**(n*p)*int((x*
*n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*d**2*n*p**3
- x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)
*a**2*d**2*n*p**2 + 4*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*
x + x**(n*p)*a*x),x)*a*b*c*d*n*p**2 + 2*x**(n*p)*int((x**n*(x**n*b + a)**p
)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*b**2*c**2*n*p)/(2*x**(n*p)*b*n*p)
```

3.534 $\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx = \frac{dx^{-np}(a+bx^n)^{1+p}}{bn} - \frac{(bc+adp)x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{bnp}$$

output
```
d*(a+b*x^n)^(p+1)/b/n/(x^(n*p))-(a*d*p+b*c)*(a+b*x^n)^p*hypergeom([-p, -p]
,[1-p],-b*x^n/a)/b/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx = -\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(dpx^n\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)+c(-1+p)\,\text{Hyperge}\right.}{n(-1+p)p}$$

input
```
Integrate[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(d*p*x^n*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^n)/a)
] + c*(-1 + p)*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a)]))/(n*(-1 + p
)*p*x^(n*p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(c+dx^n)\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(adp+bc)\int x^{-np-1}(bx^n+a)^p\,dx}{b}+\frac{dx^{-np}(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{x^{-np}(adp+bc)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-1}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{bn}+\frac{dx^{-np}(a+bx^n)^{p+1}}{bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{dx^{-np}(a+bx^n)^{p+1}}{bn}-\frac{x^{-np}(adp+bc)\,(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^n}{bx^n+a}\right)}{bnp}$$

input
```
Int[x^(-1 - n*p)*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*(a + b*x^n)^(1 + p))/(b*n*x^(n*p)) - ((b*c + a*d*p)*(a + b*x^n)^p*Hyper
geometric2F1[1, -p, 1 - p, (b*x^n)/(a + b*x^n)])/(b*n*p*x^(n*p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{-np-1}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-np}(a+bx^n)^p\left(c+dx^n\right)\,dx=\int (dx^n+c)(bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p - 1)*x^n + c*x^(-n*p - 1))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int x^{-1-np}(a+bx^n)^p\left(c+dx^n\right)\,dx = \frac{a^p c x^{-np}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(1-p\right)} + \frac{a^{1-p}a^{2p-1}b^{1-p}b^{p-1}dx^{-np+n}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-p\right)}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**p*c*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_polar(I*pi)/a)/(n*x*
*(n*p)*gamma(1 - p)) + a**(1 - p)*a**(2*p - 1)*b**(1 - p)*b**(p - 1)*d*x**
(-n*p + n)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**n*exp_polar(I*pi
)/a)/(n*gamma(2 - p))
```

Maxima [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*p - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,0,2,2,1,1,1,0,1]%%%}+%%%{1,[0,0,2,2,1,1,0,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\;dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,p+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*p + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(a+bx^n)^p\,(c+dx^n)\,dx$$
$$=\frac{x^n(x^nb+a)^p\,dp-(x^nb+a)^p\,c+x^{np}\left(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)adn\,p^2+x^{np}\left(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)bcnp}{x^{np}np}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**n*(x**n*b + a)**p*d*p - (x**n*b + a)**p*c + x**(n*p)*int((x**n*(x**n*b
 + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*d*n*p**2 + x**(n*p)*int((
x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*b*c*n*p)/(x**(n
*p)*n*p)
```

3.535 $\int x^{-1-np}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int x^{-1-np}(a+bx^n)^p\,dx = -\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

output
```
-(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/n/p/(x^(n*p))/((1+b*x^n/a)
^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int x^{-1-np}(a+bx^n)^p\,dx = -\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{np}$$

input
```
Integrate[x^(-1 - n*p)*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^n)/a)])/(n*p*x^(n
*p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-np-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{882}$$

$$\frac{x^{-np}\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p-1}}{1-\frac{bx^n}{bx^n+a}}d\frac{x^n}{bx^n+a}}{n}$$

$$\Big\downarrow \text{74}$$

$$-\frac{x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{bx^n}{bx^n+a}\right)}{np}$$

input
```
Int[x^(-1 - n*p)*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[1, -p, 1 - p, (b*x^n)/(a + b*x^n)])/(n*
p*x^(n*p)))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

Maple [F]

$$\int x^{-np-1}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p,x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-np}(a+bx^n)^p\;dx=\int (bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int x^{-1-np}(a+bx^n)^p\,dx = \frac{a^p x^{-np}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma(1-p)}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p,x)
```

output
```
a**p*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**n*exp_polar(I*pi)/a)/(n*x**(
n*p)*gamma(1 - p))
```

Maxima [F]

$$\int x^{-1-np}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1), x)
```

Giac [F]

$$\int x^{-1-np}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-np-1}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-np}(a+bx^n)^p \, dx = \int \frac{(a+b\,x^n)^p}{x^{n\,p+1}} \, dx$$

input
```
int((a + b*x^n)^p/x^(n*p + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-np}(a+bx^n)^p \, dx = \int \frac{(x^n b+a)^p}{x^{np}x} dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p,x)
```

output
```
int((x**n*b + a)**p/(x**(n*p)*x),x)
```

3.536 $\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx=-\frac{x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{cnp}$$

output
```
-(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c/n/p/(x^
(n*p))
```

Mathematica [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.47

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = -\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(1+\frac{dx^n}{c}\right)^p\text{Hypergeometric2F1}\left(-p,-p,1-p,\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{cnp}$$

input
```
Integrate[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, ((-(b*
c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*n*p*x^(n*p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-np-1}(a+bx^n)^p}{c+dx^n}dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-np-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-np}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{dx^n}{c}+1\right)^p \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{\frac{bx^n}{a}-\frac{dx^n}{c}}{\frac{dx^n}{c}+1}\right)}{cnp}$$

input
```
Int[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[-p, -p, 1 - p, -(((b*
x^n)/a - (d*x^n)/c)/(1 + (d*x^n)/c))])/(c*n*p*x^(n*p)*(1 + (b*x^n)/a)^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-np-1}(a+b\,x^{n})^{p}}{c+d\,x^{n}}dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx=\int \frac{(bx^n+a)^px^{-np-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,p+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{c+dx^n}\,dx = \frac{-(x^n b+a)^p\,b + x^{np}\left(\int \frac{(x^n b+a)^p}{x^{np+2n}bdx+x^{np+n}adx+x^{np+n}bcx+x^{np}acx}dx\right)a^2dnp - x^{np}\left(\int \frac{(x^n b+a)^p}{x^{np+2n}bdx+x^{np+n}adx+x^{np+n}bcx+}\right.}{x^{np}adnp}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
( - (x**n*b + a)**p*b + x**(n*p)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x
 + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*d*n*p
 - x**(n*p)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x
 + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*b*c*n*p)/(x**(n*p)*a*d*n*p)
```

3.537 $\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 124

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2} \, dx = -\frac{x^{-np}(a+bx^n)^{1+p}}{acnp\,(c+dx^n)} + \frac{(bc-ad(1+p))x^{n-np}(a+bx^n)^{-1+p}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^3n(1-p)p}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/p/(x^(n*p))/(c+d*x^n)+(b*c-a*d*(p+1))*x^(-n*p+n)*(a
+b*x^n)^(-1+p)*hypergeom([2, 1-p],[-p+2],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^3/n
/(1-p)/p
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2} \, dx = \frac{x^{-np}(a+bx^n)^p\left(-dpx^n\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)+(c+d(1+p)x^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,-p\right)\right)}{c^2n\,(c+dx^n)}$$

input
```
Integrate[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
((a + b*x^n)^p*(-(d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)
), 1, 1 - p]) + (c + d*(1 + p)*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(
a + b*x^n)), 1, -p]))/(c^2*n*x^(n*p)*(c + d*x^n))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-np-1}(a+bx^n)^p}{(c+dx^n)^2}dx$$

$$\Big\downarrow \text{ 1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-np-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{ 1012}$$

$$-\frac{x^{-np}(a+bx^n)^p\left(dpx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,1-p\right)-(c+d(p+1)x^n)\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)\right)}{c^3n\left(\frac{dx^n}{c}+1\right)}$$

input
```
Int[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)
), 1, 1 - p] - (c + d*(1 + p)*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
 + b*x^n)), 1, -p]))/(c^3*n*x^(n*p)*(1 + (d*x^n)/c)))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-np-1}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{2}}dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx=\int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,p+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output

```
(x**n*(x**n*b + a)**p*b*d - (x**n*b + a)**p*a*d*p + (x**n*b + a)**p*b*c -
x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p*x - x**
(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x**(n*p + 2*n
)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a*b*c*d**2*x - 2*x**(n*p + 2*n)*b**2*c**
2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + x**(n*p + n)*a*b*c**2*d*p*x - 2*x
**(n*p + n)*a*b*c**2*d*x - x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c**2*d
*p*x - x**(n*p)*a*b*c**3*x),x)*a**3*d**4*n*p**3 - x**(n*p + n)*int((x**n*(
x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p*x - x**(n*p + 3*n)*b**2*c*d**2*
x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - x**(n
*p + 2*n)*a*b*c*d**2*x - 2*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a
**2*c*d**2*p*x + x**(n*p + n)*a*b*c**2*d*p*x - 2*x**(n*p + n)*a*b*c**2*d*x
 - x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c**2*d*p*x - x**(n*p)*a*b*c**3
*x),x)*a**3*d**4*n*p**2 + x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p
 + 3*n)*a*b*d**3*p*x - x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*
d**3*p*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a*b*c*d**2*x -
 2*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + x**(n*p
 + n)*a*b*c**2*d*p*x - 2*x**(n*p + n)*a*b*c**2*d*x - x**(n*p + n)*b**2*c**
3*x + x**(n*p)*a**2*c**2*d*p*x - x**(n*p)*a*b*c**3*x),x)*a**2*b*c*d**3*n*p
**3 + 3*x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p
*x - x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x*...
```

3.538 $\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 226

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = -\frac{x^{-np}(a+bx^n)^{1+p}}{acnp\,(c+dx^n)^2} - \frac{d(2bc-ad(2+p))x^{n-np}(a+bx^n)^{1+p}}{2ac^2(bc-ad)np\,(c+dx^n)^2} + \frac{(2b^2c^2-4abcd(1+p)+a^2d^2(2+3p+p^2))\,x^{n-np}(a+bx^n)^{-1+p}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\right.}{2c^4(bc-ad)n(1-p)p}$$

output
```
-(a+b*x^n)^(p+1)/a/c/n/p/(x^(n*p))/(c+d*x^n)^2-1/2*d*(2*b*c-a*d*(2+p))*x^(
-n*p+n)*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/p/(c+d*x^n)^2+1/2*(2*b^2*c^2-4*
a*b*c*d*(p+1)+a^2*d^2*(p^2+3*p+2))*x^(-n*p+n)*(a+b*x^n)^(-1+p)*hypergeom([
2, 1-p],[-p+2],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^4/(-a*d+b*c)/n/(1-p)/p
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$
$$= \frac{x^{-np}(a+bx^n)^p\left(-2dpx^n(2c+d(1+p)x^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)+d^2(-1+p)px^{2n}\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,2-p\right.\right.}{2c^3n\,(c+dx^n)^2}$$

input
```
Integrate[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
((a + b*x^n)^p*(-2*d*p*x^n*(2*c + d*(1 + p)*x^n)*HurwitzLerchPhi[((b*c - a
*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] + d^2*(-1 + p)*p*x^(2*n)*HurwitzLerchP
hi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + (2*c^2 + 4*c*d*(1 + p)*x
^n + d^2*(2 + 3*p + p^2)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
+ b*x^n)), 1, -p]))/(2*c^3*n*x^(n*p)*(c + d*x^n)^2)
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.86 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-np-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-np-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-np}(a+bx^n)^p\left(-\left(2c^2+4cd(p+1)x^n+d^2(p+1)(p+2)x^{2n}\right)\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)+d^2(1-p)px^{2n}\Phi\left(\frac{(bc-ad)}{c(bx^n+}\right.}{2c^5n\left(\frac{dx^n}{c}+1\right)^2}$$

input
```
Int[(x^(-1 - n*p)*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
-1/2*((a + b*x^n)^p*(2*d*p*x^n*(2*c + d*(1 + p)*x^n)*HurwitzLerchPhi[((b*c
 - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] + d^2*(1 - p)*p*x^(2*n)*HurwitzLer
chPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - (2*c^2 + 4*c*d*(1 + p
)*x^n + d^2*(1 + p)*(2 + p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*
(a + b*x^n)), 1, -p]))/(c^5*n*x^(n*p)*(1 + (d*x^n)/c)^2)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-np-1}(a+b\,x^n)^p}{(c+d\,x^n)^3}dx$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p - 1)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2
*d*x^n + c^3), x)
```

Sympy [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{x^{-np-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

input
```
integrate(x**(-n*p-1)*(a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Integral(x**(-n*p - 1)*(a + b*x**n)**p/(c + d*x**n)**3, x)
```

Maxima [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-np-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*p - 1)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,p+1}\,(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)^3),x)
```

output
```
int((a + b*x^n)^p/(x^(n*p + 1)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{x^{-1-np}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{too large to display}$$

input
```
int(x^(-n*p-1)*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output

```
( - 2*x**(2*n)*(x**n*b + a)**p*b**2*d**2 + 2*x**n*(x**n*b + a)**p*a*b*d**2
*p - 4*x**n*(x**n*b + a)**p*b**2*c*d - (x**n*b + a)**p*a**2*d**2*p**2 - (x
**n*b + a)**p*a**2*d**2*p + 4*(x**n*b + a)**p*a*b*c*d*p - 2*(x**n*b + a)**
p*b**2*c**2 + 2*x**(n*p + 2*n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + 4*n)*
a**5*b*d**8*p**5*x + 5*x**(n*p + 4*n)*a**5*b*d**8*p**4*x + 9*x**(n*p + 4*n
)*a**5*b*d**8*p**3*x + 7*x**(n*p + 4*n)*a**5*b*d**8*p**2*x + 2*x**(n*p + 4
*n)*a**5*b*d**8*p*x - 11*x**(n*p + 4*n)*a**4*b**2*c*d**7*p**4*x - 38*x**(n
*p + 4*n)*a**4*b**2*c*d**7*p**3*x - 43*x**(n*p + 4*n)*a**4*b**2*c*d**7*p**
2*x - 16*x**(n*p + 4*n)*a**4*b**2*c*d**7*p*x + 42*x**(n*p + 4*n)*a**3*b**3
*c**2*d**6*p**3*x + 92*x**(n*p + 4*n)*a**3*b**3*c**2*d**6*p**2*x + 54*x**(
n*p + 4*n)*a**3*b**3*c**2*d**6*p*x + 4*x**(n*p + 4*n)*a**3*b**3*c**2*d**6*
x - 68*x**(n*p + 4*n)*a**2*b**4*c**3*d**5*p**2*x - 84*x**(n*p + 4*n)*a**2*
b**4*c**3*d**5*p*x - 16*x**(n*p + 4*n)*a**2*b**4*c**3*d**5*x + 48*x**(n*p
+ 4*n)*a*b**5*c**4*d**4*p*x + 24*x**(n*p + 4*n)*a*b**5*c**4*d**4*x - 12*x*
*(n*p + 4*n)*b**6*c**5*d**3*x + x**(n*p + 3*n)*a**6*d**8*p**5*x + 5*x**(n*
p + 3*n)*a**6*d**8*p**4*x + 9*x**(n*p + 3*n)*a**6*d**8*p**3*x + 7*x**(n*p
+ 3*n)*a**6*d**8*p**2*x + 2*x**(n*p + 3*n)*a**6*d**8*p*x + 3*x**(n*p + 3*n
)*a**5*b*c*d**7*p**5*x + 4*x**(n*p + 3*n)*a**5*b*c*d**7*p**4*x - 11*x**(n*
p + 3*n)*a**5*b*c*d**7*p**3*x - 22*x**(n*p + 3*n)*a**5*b*c*d**7*p**2*x - 1
0*x**(n*p + 3*n)*a**5*b*c*d**7*p*x - 33*x**(n*p + 3*n)*a**4*b**2*c**2*d...
```

3.539 $\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \frac{c^2x^{n(1-p)}(a+bx^n)^{1+p}}{an(1-p)} + \frac{d^2x^{n(2-p)}(a+bx^n)^{1+p}}{3bn} - \frac{(6b^2c^2-6abcd(1-p)+a^2d^2(2-3p+p^2))\,x^{n(2-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,\right.}{3abn(1-p)(2-p)}$$

output
```
c^2*x^(n*(1-p))*(a+b*x^n)^(p+1)/a/n/(1-p)+1/3*d^2*x^(n*(-p+2))*(a+b*x^n)^(
p+1)/b/n-1/3*(6*b^2*c^2-6*a*b*c*d*(1-p)+a^2*d^2*(p^2-3*p+2))*x^(n*(-p+2))*
(a+b*x^n)^p*hypergeom([-p, -p+2],[3-p],-b*x^n/a)/a/b/n/(1-p)/(-p+2)/((1+b*
x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = -\frac{x^{n-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c^2(6-5p+p^2)\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)+d(-1+\right.}{n(}$$

input
```
Integrate[x^(-1 - n*(-1 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-((x^(n - n*p)*(a + b*x^n)^p*(c^2*(6 - 5*p + p^2)*Hypergeometric2F1[1 - p,
 -p, 2 - p, -((b*x^n)/a)] + d*(-1 + p)*x^n*(2*c*(-3 + p)*Hypergeometric2F1
[2 - p, -p, 3 - p, -((b*x^n)/a)] + d*(-2 + p)*x^n*Hypergeometric2F1[3 - p,
 -p, 4 - p, -((b*x^n)/a)])))/(n*(-3 + p)*(-2 + p)*(-1 + p)*(1 + (b*x^n)/a)
^p))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1008, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-1}(c+dx^n)^2(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{-pn+n-1}(bx^n+a)^p\left(dn(4bc-ad(2-p))x^n+cn(3bc-ad(1-p))\right)dx}{3bn}+\frac{dx^{n(1-p)}(c+dx^n)(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{n\left(a^2d^2\left(p^2-3p+2\right)-6abcd(1-p)+6b^2c^2\right)\int x^{-pn+n-1}(bx^n+a)^pdx}{2b}+\frac{dx^{n-np}(4bc-ad(2-p))(a+bx^n)^{p+1}}{2b}}{3bn}+\frac{dx^{n(1-p)}(c+dx^n)(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{ax^{-np}\left(a^2d^2\left(p^2-3p+2\right)-6abcd(1-p)+6b^2c^2\right)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^n}{bx^n+a}}{2b}+\frac{dx^{n-np}(4bc-ad(2-p))(a+bx^n)^{p+1}}{2b}}{3bn}+\frac{dx^{n(1-p)}(c+dx^n)(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{ax^{n-np}\left(a^2d^2\left(p^2-3p+2\right)-6abcd(1-p)+6b^2c^2\right)(a+bx^n)^{p-1}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^n}{bx^n+a}\right)}{2b(1-p)}+\frac{dx^{n-np}(4bc-ad(2-p))(a+bx^n)^{p+1}}{2b}}{3bn}+$$

$$\frac{dx^{n(1-p)}(c+dx^n)\,(a+bx^n)^{p+1}}{3bn}$$

input
```
Int[x^(-1 - n*(-1 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*x^(n*(1 - p))*(a + b*x^n)^(1 + p)*(c + d*x^n))/(3*b*n) + ((d*(4*b*c - a
*d*(2 - p))*x^(n - n*p)*(a + b*x^n)^(1 + p))/(2*b) + (a*(6*b^2*c^2 - 6*a*b
*c*d*(1 - p) + a^2*d^2*(2 - 3*p + p^2))*x^(n - n*p)*(a + b*x^n)^(-1 + p)*H
ypergeometric2F1[2, 1 - p, 2 - p, (b*x^n)/(a + b*x^n)])/(2*b*(1 - p)))/(3*
b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(p-1)}(a+b\,x^{n})^{p}\left(c+d\,x^{n}\right)^{2}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-n(-1+p)}(a+bx^{n})^{p}\left(c+dx^{n}\right)^{2}\,dx=\int (dx^{n}+c)^{2}(bx^{n}+a)^{p}x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p + n - 1)*x^(2*n) + 2*c*d*x^(-n*p + n - 1)*x^n + c^2*
x^(-n*p + n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2(bx^n+a)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p - 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p-1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{2x^{3n}(x^nb+a)^p\,b^2d^2+x^{2n}(x^nb+a)^p\,ab\,d^2p+6x^{2n}(x^nb+a)^p\,b^2cd+x^n(x^nb+a)^p\,a^2d^2p^2-2x^n(x^nb+a)}{}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(2*x**(3*n)*(x**n*b + a)**p*b**2*d**2 + x**(2*n)*(x**n*b + a)**p*a*b*d**2*
p + 6*x**(2*n)*(x**n*b + a)**p*b**2*c*d + x**n*(x**n*b + a)**p*a**2*d**2*p
**2 - 2*x**n*(x**n*b + a)**p*a**2*d**2*p + 6*x**n*(x**n*b + a)**p*a*b*c*d*
p + 6*x**n*(x**n*b + a)**p*b**2*c**2 + x**(n*p)*int((x**n*(x**n*b + a)**p)
/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**2*n*p**3 - 3*x**(n*p)*int((x
**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d**2*n*p**2
 + 2*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x)
,x)*a**3*d**2*n*p + 6*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*
x + x**(n*p)*a*x),x)*a**2*b*c*d*n*p**2 - 6*x**(n*p)*int((x**n*(x**n*b + a)
**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*d*n*p + 6*x**(n*p)*int(
(x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*b**2*c**2*n*
p)/(6*x**(n*p)*b**2*n)
```

3.540 $\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\frac{dx^{n(1-p)}(a+bx^n)^{1+p}}{2bn}+\frac{(2bc-ad(1-p))x^{n(1-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{2bn(1-p)}$$

output
```
1/2*d*x^(n*(1-p))*(a+b*x^n)^(p+1)/b/n+1/2*(2*b*c-a*d*(1-p))*x^(n*(1-p))*(a
+b*x^n)^p*hypergeom([-p, 1-p],[-p+2],-b*x^n/a)/b/n/(1-p)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx=-\frac{x^{n-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(-2+p)\,\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)+d(-1+p)x^n\right.}{n(-2+p)(-1+p)}$$

input
```
Integrate[x^(-1 - n*(-1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-((x^(n - n*p)*(a + b*x^n)^p*(c*(-2 + p)*Hypergeometric2F1[1 - p, -p, 2 -
p, -((b*x^n)/a)] + d*(-1 + p)*x^n*Hypergeometric2F1[2 - p, -p, 3 - p, -((b
*x^n)/a)]))/(n*(-2 + p)*(-1 + p)*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(2bc-ad(1-p))\int x^{-pn+n-1}(bx^n+a)^p\,dx}{2b}+\frac{dx^{n(1-p)}(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{ax^{-np}(2bc-ad(1-p))\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^n}{bx^n+a}}{2bn}+\frac{dx^{n(1-p)}(a+bx^n)^{p+1}}{2bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{ax^{n-np}(2bc-ad(1-p))(a+bx^n)^{p-1}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^n}{bx^n+a}\right)}{2bn(1-p)}+\frac{dx^{n(1-p)}(a+bx^n)^{p+1}}{2bn}$$

input
```
Int[x^(-1 - n*(-1 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(n*(1 - p))*(a + b*x^n)^(1 + p))/(2*b*n) + (a*(2*b*c - a*d*(1 - p))*x
^(n - n*p)*(a + b*x^n)^(-1 + p)*Hypergeometric2F1[2, 1 - p, 2 - p, (b*x^n)
/(a + b*x^n)])/(2*b*n*(1 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{-1-n(p-1)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p + n - 1)*x^n + c*x^(-n*p + n - 1))*(b*x^n + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{a^{1-p}a^{2p-1}b^{1-p}b^{p-1}cx^{-np+n}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(2-p\right)}+\frac{a^{2-p}a^{2p-2}b^{2-p}b^{p-2}dx^{-np+2n}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma\left(3-p\right)}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(1 - p)*a**(2*p - 1)*b**(1 - p)*b**(p - 1)*c*x**(-n*p + n)*gamma(1 - p)
*hyper((-p, 1 - p), (2 - p,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 - p)) +
 a**(2 - p)*a**(2*p - 2)*b**(2 - p)*b**(p - 2)*d*x**(-n*p + 2*n)*gamma(2 -
 p)*hyper((-p, 2 - p), (3 - p,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 - p)
)
```

Maxima [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n) \, dx = \int (dx^n+c)(bx^n+a)^p x^{-n(p-1)-1} \, dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p - 1) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{2,[0,0,2,2,1,1,1,0,1]%%%}+%%%{2,[0,0,2,2,1,1,0,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p (c+dx^n) \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p-1)+1}} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{x^{2n}(x^nb+a)^p\,bd+x^n(x^nb+a)^p\,adp+2x^n(x^nb+a)^p\,bc+x^{np}\left(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)a^2dn\,p^2-x^{np}\left(\int\frac{x^n}{x^{np+}}\right.}{2x^{np}bn}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b*d + x**n*(x**n*b + a)**p*a*d*p + 2*x**n*(x**n*
b + a)**p*b*c + x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x*
*(n*p)*a*x),x)*a**2*d*n*p**2 - x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*
p + n)*b*x + x**(n*p)*a*x),x)*a**2*d*n*p + 2*x**(n*p)*int((x**n*(x**n*b +
a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a*b*c*n*p)/(2*x**(n*p)*b*n)
```

3.541 $\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx = \frac{x^{n(1-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{n(1-p)}$$

output
```
x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p, 1-p],[-p+2],-b*x^n/a)/n/(1-p)/((1+b
*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx = \frac{x^{n-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{n-np}$$

input
```
Integrate[x^(-1 - n*(-1 + p))*(a + b*x^n)^p,x]
```

output
```
(x^(n - n*p)*(a + b*x^n)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^n)/a
)])/((n - n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-1)-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{882}$$

$$\frac{ax^{-np}\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^2}d\frac{x^n}{bx^n+a}}{n}$$

$$\Big\downarrow \text{74}$$

$$\frac{ax^{n-np}(a+bx^n)^{p-1}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{bx^n}{bx^n+a}\right)}{n(1-p)}$$

input
```
Int[x^(-1 - n*(-1 + p))*(a + b*x^n)^p,x]
```

output
```
(a*x^(n - n*p)*(a + b*x^n)^(-1 + p)*Hypergeometric2F1[2, 1 - p, 2 - p, (b*
x^n)/(a + b*x^n)])/(n*(1 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

Maple [F]

$$\int x^{-1-n(p-1)}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-1+p)}(a+bx^{n})^{p}\;dx=\int(bx^{n}+a)^{p}x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx = \frac{a^p x^{-np+n}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix} -p, 1-p \\ 2-p \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma(2-p)}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p,x)
```

output
```
a**p*x**(-n*p + n)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**n*exp_po
lar(I*pi)/a)/(n*gamma(2 - p))
```

Maxima [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx=\int\frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p - 1) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-1+p)}(a+bx^n)^p\,dx=\frac{x^n(x^nb+a)^p+x^{np}\left(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)anp}{x^{np}n}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p,x)
```

output
```
(x**n*(x**n*b + a)**p + x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*
b*x + x**(n*p)*a*x),x)*a*n*p)/(x**(n*p)*n)
```

3.542 $\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 121

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \frac{x^{-np}(a+bx^n)^p \,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{dnp} - \frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{dnp}$$

output

```
(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d/n/p/(x^(
n*p))-(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/d/n/p/(x^(n*p))/((1+b
*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = -\frac{x^{-n(-1+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(-1+p)}$$

input
```
Integrate[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c*n*(-1 + p)*x^(n*(-1 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-1)-1}(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \ 1013$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-pn+n-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \ 1012$$

$$\frac{x^{n-np}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(1-p,-p,1,2-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(1-p)}$$

input
```
Int[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
(x^(n - n*p)*(a + b*x^n)^p*AppellF1[1 - p, -p, 1, 2 - p, -((b*x^n)/a), -((
d*x^n)/c)])/(c*n*(1 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p-1)} (a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{x^n(x^nb+a)^p}{x^{np+n}dx+x^{np}cx}dx$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*d*x + x**(n*p)*c*x),x)
```

3.543 $\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 71

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \frac{ax^{n(1-p)}(a+bx^n)^{-1+p}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^2n(1-p)}$$

output
```
a*x^(n*(1-p))*(a+b*x^n)^(-1+p)*hypergeom([2, 1-p],[-p+2],(-a*d+b*c)*x^n/c/
(a+b*x^n))/c^2/n/(1-p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.45

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = -\frac{x^{n-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(1+\frac{dx^n}{c}\right)^p\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{(-bc+ad)x^n}{a(c+dx^n)}\right)}{cn(-1+p)(c+dx^n)}$$

input
```
Integrate[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-((x^(n - n*p)*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[1 - p, -p
, 2 - p, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*n*(-1 + p)*(1 + (b*x^n)
/a)^p*(c + d*x^n)))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-1)-1}(a+bx^n)^p}{(c+dx^n)^2}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-pn+n-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n-np}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{dx^n}{c}+1\right)^{p-1}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{\frac{bx^n}{a}-\frac{dx^n}{c}}{\frac{dx^n}{c}+1}\right)}{c^2n(1-p)}$$

input
```
Int[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
(x^(n - n*p)*(a + b*x^n)^p*(1 + (d*x^n)/c)^(-1 + p)*Hypergeometric2F1[1 -
p, -p, 2 - p, -(((b*x^n)/a - (d*x^n)/c)/(1 + (d*x^n)/c))])/(c^2*n*(1 - p)*
(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p-1)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{2}}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^{n})^{p}}{(c+dx^{n})^{2}}\,dx=\int \frac{(bx^{n}+a)^{p}x^{-n(p-1)-1}}{(dx^{n}+c)^{2}}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x
)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx=\text{too large to display}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output

```
( - x**n*(x**n*b + a)**p*b + x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(
n*p + 3*n)*a*b*d**3*p*x - x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a*
*2*d**3*p*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a*b*c*d**2*
x - 2*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + x**(
n*p + n)*a*b*c**2*d*p*x - 2*x**(n*p + n)*a*b*c**2*d*x - x**(n*p + n)*b**2*
c**3*x + x**(n*p)*a**2*c**2*d*p*x - x**(n*p)*a*b*c**3*x),x)*a**3*d**3*n*p*
*2 - x**(n*p + n)*int((x**n*(x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p*x
- x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x**(n*p
+ 2*n)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a*b*c*d**2*x - 2*x**(n*p + 2*n)*b**
2*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + x**(n*p + n)*a*b*c**2*d*p*x
- 2*x**(n*p + n)*a*b*c**2*d*x - x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c
**2*d*p*x - x**(n*p)*a*b*c**3*x),x)*a**2*b*c*d**2*n*p**2 - x**(n*p + n)*in
t((x**n*(x**n*b + a)**p)/(x**(n*p + 3*n)*a*b*d**3*p*x - x**(n*p + 3*n)*b**
2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*
x - x**(n*p + 2*n)*a*b*c*d**2*x - 2*x**(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n
*p + n)*a**2*c*d**2*p*x + x**(n*p + n)*a*b*c**2*d*p*x - 2*x**(n*p + n)*a*b
*c**2*d*x - x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c**2*d*p*x - x**(n*p)
*a*b*c**3*x),x)*a**2*b*c*d**2*n*p + x**(n*p + n)*int((x**n*(x**n*b + a)**p
)/(x**(n*p + 3*n)*a*b*d**3*p*x - x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p +
2*n)*a**2*d**3*p*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - x**(n*p + 2*n)*a...
```

3.544 $\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx=\frac{x^{n(1-p)}(a+bx^n)^{1+p}}{acn(1-p)\,(c+dx^n)^2}-\frac{a(2bc-ad(1+p))x^{n(2-p)}(a+bx^n)^{-2+p}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^4n(1-p)(2-p)}$$

output
```
x^(n*(1-p))*(a+b*x^n)^(p+1)/a/c/n/(1-p)/(c+d*x^n)^2-a*(2*b*c-a*d*(p+1))*x^
(n*(-p+2))*(a+b*x^n)^(-2+p)*hypergeom([3, -p+2],[3-p],(-a*d+b*c)*x^n/c/(a+
b*x^n))/c^4/n/(1-p)/(-p+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.17

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$
$$=\frac{px^{n-np}(a+bx^n)^p\left(2(c+dpx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)-d(-1+p)x^n\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,2-p\right)-(2c+d(1-\right.}{2c^2n\,(c+dx^n)^2}$$

input
```
Integrate[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
(p*x^(n - n*p)*(a + b*x^n)^p*(2*(c + d*p*x^n)*HurwitzLerchPhi[((b*c - a*d)
*x^n)/(c*(a + b*x^n)), 1, 1 - p] - d*(-1 + p)*x^n*HurwitzLerchPhi[((b*c -
a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - (2*c + d*(1 + p)*x^n)*HurwitzLerchP
hi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]))/(2*c^2*n*(c + d*x^n)^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.56 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-1)-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \ 1013$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-pn+n-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \ 1012$$

$$\frac{px^{n-np}(a+bx^n)^p\left(d(1-p)x^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,2-p\right)+2(c+dpx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,1-p\right)-(2c+d(p+1)x^n)\,\Phi}{2c^4n\left(\frac{dx^n}{c}+1\right)^2}$$

input
```
Int[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
(p*x^(n - n*p)*(a + b*x^n)^p*(2*(c + d*p*x^n)*HurwitzLerchPhi[((b*c - a*d)
*x^n)/(c*(a + b*x^n)), 1, 1 - p] + d*(1 - p)*x^n*HurwitzLerchPhi[((b*c - a
*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - (2*c + d*(1 + p)*x^n)*HurwitzLerchPh
i[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]))/(2*c^4*n*(1 + (d*x^n)/c)^2)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p-1)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{3}}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}\,(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^3),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx=\text{too large to display}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b**2*d - x**n*(x**n*b + a)**p*a*b*d*p + 2*x**n*(
x**n*b + a)**p*b**2*c - 2*x**(n*p + 2*n)*int((x**n*(x**n*b + a)**p)/(x**(n
*p + 4*n)*a**5*b*d**8*p**5*x + 5*x**(n*p + 4*n)*a**5*b*d**8*p**4*x + 9*x**
(n*p + 4*n)*a**5*b*d**8*p**3*x + 7*x**(n*p + 4*n)*a**5*b*d**8*p**2*x + 2*x
**(n*p + 4*n)*a**5*b*d**8*p*x - 11*x**(n*p + 4*n)*a**4*b**2*c*d**7*p**4*x
- 38*x**(n*p + 4*n)*a**4*b**2*c*d**7*p**3*x - 43*x**(n*p + 4*n)*a**4*b**2*
c*d**7*p**2*x - 16*x**(n*p + 4*n)*a**4*b**2*c*d**7*p*x + 42*x**(n*p + 4*n)
*a**3*b**3*c**2*d**6*p**3*x + 92*x**(n*p + 4*n)*a**3*b**3*c**2*d**6*p**2*x
 + 54*x**(n*p + 4*n)*a**3*b**3*c**2*d**6*p*x + 4*x**(n*p + 4*n)*a**3*b**3*
c**2*d**6*x - 68*x**(n*p + 4*n)*a**2*b**4*c**3*d**5*p**2*x - 84*x**(n*p +
4*n)*a**2*b**4*c**3*d**5*p*x - 16*x**(n*p + 4*n)*a**2*b**4*c**3*d**5*x + 4
8*x**(n*p + 4*n)*a*b**5*c**4*d**4*p*x + 24*x**(n*p + 4*n)*a*b**5*c**4*d**4
*x - 12*x**(n*p + 4*n)*b**6*c**5*d**3*x + x**(n*p + 3*n)*a**6*d**8*p**5*x
+ 5*x**(n*p + 3*n)*a**6*d**8*p**4*x + 9*x**(n*p + 3*n)*a**6*d**8*p**3*x +
7*x**(n*p + 3*n)*a**6*d**8*p**2*x + 2*x**(n*p + 3*n)*a**6*d**8*p*x + 3*x**
(n*p + 3*n)*a**5*b*c*d**7*p**5*x + 4*x**(n*p + 3*n)*a**5*b*c*d**7*p**4*x -
 11*x**(n*p + 3*n)*a**5*b*c*d**7*p**3*x - 22*x**(n*p + 3*n)*a**5*b*c*d**7*
p**2*x - 10*x**(n*p + 3*n)*a**5*b*c*d**7*p*x - 33*x**(n*p + 3*n)*a**4*b**2
*c**2*d**6*p**4*x - 72*x**(n*p + 3*n)*a**4*b**2*c**2*d**6*p**3*x - 37*x**(
n*p + 3*n)*a**4*b**2*c**2*d**6*p**2*x + 6*x**(n*p + 3*n)*a**4*b**2*c**2...
```

3.545 $\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 243

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \frac{x^{n(1-p)}(a+bx^n)^{1+p}}{acn(1-p)\,(c+dx^n)^3} + \frac{d(3bc-ad(2+p))x^{n(2-p)}(a+bx^n)^{1+p}}{3ac^2(bc-ad)n(1-p)\,(c+dx^n)^3} - \frac{a(6b^2c^2-6abcd(1+p)+a^2d^2(2+3p+p^2))\,x^{n(2-p)}(a+bx^n)^{-2+p}\,\text{Hypergeometric2F1}\left(3,2-p,3-p\right.}{3c^5(bc-ad)n(1-p)(2-p)}$$

output
```
x^(n*(1-p))*(a+b*x^n)^(p+1)/a/c/n/(1-p)/(c+d*x^n)^3+1/3*d*(3*b*c-a*d*(2+p)
)*x^(n*(-p+2))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(1-p)/(c+d*x^n)^3-1/3*a*
(6*b^2*c^2-6*a*b*c*d*(p+1)+a^2*d^2*(p^2+3*p+2))*x^(n*(-p+2))*(a+b*x^n)^(-2
+p)*hypergeom([3, -p+2],[3-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^5/(-a*d+b*c)/n
/(1-p)/(-p+2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.24 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.07

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx$$
$$= \frac{px^{n-np}(a+bx^n)^p\left(3(2c^2+4cdpx^n+d^2p(1+p)x^{2n})\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)-3d(-1+p)x^n(2c+dpx^n)\,\Phi\right.}{}$$

input
```
Integrate[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p/(c + d*x^n)^4,x]
```

output
```
(p*x^(n - n*p)*(a + b*x^n)^p*(3*(2*c^2 + 4*c*d*p*x^n + d^2*p*(1 + p)*x^(2*
n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 3*d*(-1
 + p)*x^n*(2*c + d*p*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)
), 1, 2 - p] + 2*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x
^n)), 1, 3 - p] - 3*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
+ b*x^n)), 1, 3 - p] + d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(
c*(a + b*x^n)), 1, 3 - p] - 6*c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
+ b*x^n)), 1, -p] - 6*c*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*
x^n)), 1, -p] - 6*c*d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, -p] - 2*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, -p] - 3*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*
x^n)), 1, -p] - d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
b*x^n)), 1, -p]))/(6*c^3*n*(c + d*x^n)^3)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.08 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-1)-1}(a+bx^n)^p}{(c+dx^n)^4}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{-pn+n-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^4}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{px^{n-np}(a+bx^n)^p\left(3\left(2c^2+4cdpx^n+d^2p(p+1)x^{2n}\right)\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,1-p\right)-6c^2\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)+d^2p^2x^{2n}}{}$$

input
```
Int[(x^(-1 - n*(-1 + p))*(a + b*x^n)^p)/(c + d*x^n)^4,x]
```

output
```
(p*x^(n - n*p)*(a + b*x^n)^p*(3*(2*c^2 + 4*c*d*p*x^n + d^2*p*(1 + p)*x^(2*
n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] + 3*d*(1
- p)*x^n*(2*c + d*p*x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n))
, 1, 2 - p] + 2*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, 3 - p] - 3*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
 b*x^n)), 1, 3 - p] + d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c
*(a + b*x^n)), 1, 3 - p] - 6*c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
 b*x^n)), 1, -p] - 6*c*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x
^n)), 1, -p] - 6*c*d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n
)), 1, -p] - 2*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n
)), 1, -p] - 3*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x
^n)), 1, -p] - d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b
*x^n)), 1, -p]))/(6*c^6*n*(1 + (d*x^n)/c)^3)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(p-1)} (a+b\,x^{n})^{p}}{(c+d\,x^{n})^{4}}dx$$

input
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
int(x^(-1-n*(p-1))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + n - 1)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6
*c^2*d^2*x^(2*n) + 4*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-1+p))*(a+b*x**n)**p/(c+d*x**n)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^4, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-1)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 1) - 1)/(d*x^n + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-1)+1}\,(c+d\,x^n)^4}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^4),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 1) + 1)*(c + d*x^n)^4), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-1+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-1+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
( - 2*x**(3*n)*(x**n*b + a)**p*a*b**3*d**3*p - 4*x**(3*n)*(x**n*b + a)**p*
a*b**3*d**3 + 2*x**(3*n)*(x**n*b + a)**p*b**4*c*d**2 + 2*x**(2*n)*(x**n*b
+ a)**p*a**2*b**2*d**3*p**2 + 4*x**(2*n)*(x**n*b + a)**p*a**2*b**2*d**3*p
- 8*x**(2*n)*(x**n*b + a)**p*a*b**3*c*d**2*p - 12*x**(2*n)*(x**n*b + a)**p
*a*b**3*c*d**2 + 6*x**(2*n)*(x**n*b + a)**p*b**4*c**2*d - x**n*(x**n*b + a
)**p*a**3*b*d**3*p**3 - 6*x**n*(x**n*b + a)**p*a**3*b*d**3*p**2 - 11*x**n*
(x**n*b + a)**p*a**3*b*d**3*p - 6*x**n*(x**n*b + a)**p*a**3*b*d**3 + 10*x*
*n*(x**n*b + a)**p*a**2*b**2*c*d**2*p**2 + 40*x**n*(x**n*b + a)**p*a**2*b*
*2*c*d**2*p + 24*x**n*(x**n*b + a)**p*a**2*b**2*c*d**2 - 30*x**n*(x**n*b +
 a)**p*a*b**3*c**2*d*p - 48*x**n*(x**n*b + a)**p*a*b**3*c**2*d + 24*x**n*(
x**n*b + a)**p*b**4*c**3 - 6*x**(n*p + 3*n)*int((x**(3*n)*(x**n*b + a)**p)
/(x**(n*p + 5*n)*a**8*b*d**12*p**8*x + 16*x**(n*p + 5*n)*a**8*b*d**12*p**7
*x + 110*x**(n*p + 5*n)*a**8*b*d**12*p**6*x + 424*x**(n*p + 5*n)*a**8*b*d*
*12*p**5*x + 1001*x**(n*p + 5*n)*a**8*b*d**12*p**4*x + 1480*x**(n*p + 5*n)
*a**8*b*d**12*p**3*x + 1336*x**(n*p + 5*n)*a**8*b*d**12*p**2*x + 672*x**(n
*p + 5*n)*a**8*b*d**12*p*x + 144*x**(n*p + 5*n)*a**8*b*d**12*x - 26*x**(n*
p + 5*n)*a**7*b**2*c*d**11*p**7*x - 340*x**(n*p + 5*n)*a**7*b**2*c*d**11*p
**6*x - 1868*x**(n*p + 5*n)*a**7*b**2*c*d**11*p**5*x - 5584*x**(n*p + 5*n)
*a**7*b**2*c*d**11*p**4*x - 9794*x**(n*p + 5*n)*a**7*b**2*c*d**11*p**3*x -
 10060*x**(n*p + 5*n)*a**7*b**2*c*d**11*p**2*x - 5592*x**(n*p + 5*n)*a*...
```

3.546 $\int x^{-1-n(-2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int x^{-1-n(-2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \frac{c^2x^{n(2-p)}(a+bx^n)^{1+p}}{an(2-p)} + \frac{d^2x^{n(3-p)}(a+bx^n)^{1+p}}{4bn} - \frac{(12b^2c^2-8abcd(2-p)+a^2d^2(6-5p+p^2))\, x^{n(3-p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(3-p\right.}{4abn(2-p)(3-p)}$$

output
```
c^2*x^(n*(-p+2))*(a+b*x^n)^(p+1)/a/n/(-p+2)+1/4*d^2*x^(n*(3-p))*(a+b*x^n)^
(p+1)/b/n-1/4*(12*b^2*c^2-8*a*b*c*d*(-p+2)+a^2*d^2*(p^2-5*p+6))*x^(n*(3-p)
)*(a+b*x^n)^p*hypergeom([-p, 3-p],[4-p],-b*x^n/a)/a/b/n/(-p+2)/(3-p)/((1+b
*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int x^{-1-n(-2+p)}(a+bx^n)^p (c+dx^n)^2 \, dx =$$
$$-\frac{x^{-n(-2+p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(c^2(12-7p+p^2)\, \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)+d(\right.}{}$$

input
```
Integrate[x^(-1 - n*(-2 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(c^2*(12 - 7*p + p^2)*Hypergeometric2F1[2 - p, -p, 3 - p,
 -((b*x^n)/a)] + d*(-2 + p)*x^n*(2*c*(-4 + p)*Hypergeometric2F1[3 - p, -p,
 4 - p, -((b*x^n)/a)] + d*(-3 + p)*x^n*Hypergeometric2F1[4 - p, -p, 5 - p,
 -((b*x^n)/a)])))/(n*(-4 + p)*(-3 + p)*(-2 + p)*x^(n*(-2 + p))*(1 + (b*x^n
)/a)^p))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1008, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-2)-1}(c+dx^n)^2\,(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{n(2-p)-1}(bx^n+a)^p\,(dn(5bc-ad(3-p))x^n+cn(4bc-ad(2-p)))\,dx}{4bn}+\frac{dx^{n(2-p)}(c+dx^n)\,(a+bx^n)^{p+1}}{4bn}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{n\left(a^2d^2\left(p^2-5p+6\right)-8abcd(2-p)+12b^2c^2\right)\int x^{n(2-p)-1}(bx^n+a)^p dx}{3b}+\frac{dx^{n(2-p)}(5bc-ad(3-p))(a+bx^n)^{p+1}}{3b}}{4bn}+\frac{dx^{n(2-p)}(c+dx^n)\,(a+bx^n)^{p+1}}{4bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{a^2x^{-np}\left(a^2d^2\left(p^2-5p+6\right)-8abcd(2-p)+12b^2c^2\right)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{1-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^n}{bx^n+a}}{3b}+\frac{dx^{n(2-p)}(5bc-ad(3-p))(a+bx^n)^{p+1}}{3b}}{4bn}+\frac{dx^{n(2-p)}(c+dx^n)\,(a+bx^n)^{p+1}}{4bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{a^2x^{2n-np}\left(a^2d^2(p^2-5p+6)-8abcd(2-p)+12b^2c^2\right)(a+bx^n)^{p-2}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{bx^n}{bx^n+a}\right)}{3b(2-p)}+\frac{dx^{n(2-p)}(5bc-ad(3-p))(a+bx^n)^p}{3b}}{4bn}$$
$$\frac{dx^{n(2-p)}\left(c+dx^n\right)\left(a+bx^n\right)^{p+1}}{4bn}$$

input
```
Int[x^(-1 - n*(-2 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*x^(n*(2 - p))*(a + b*x^n)^(1 + p)*(c + d*x^n))/(4*b*n) + ((d*(5*b*c - a
*d*(3 - p))*x^(n*(2 - p))*(a + b*x^n)^(1 + p))/(3*b) + (a^2*(12*b^2*c^2 -
8*a*b*c*d*(2 - p) + a^2*d^2*(6 - 5*p + p^2))*x^(2*n - n*p)*(a + b*x^n)^(-2
 + p)*Hypergeometric2F1[3, 2 - p, 3 - p, (b*x^n)/(a + b*x^n)])/(3*b*(2 - p
)))/(4*b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(-2+p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{2}\,dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-n(-2+p)}(a+bx^{n})^{p}\left(c+dx^{n}\right)^{2}\,dx=\int (dx^{n}+c)^{2}(bx^{n}+a)^{p}x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p + 2*n - 1)*x^(2*n) + 2*c*d*x^(-n*p + 2*n - 1)*x^n +
c^2*x^(-n*p + 2*n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2(bx^n+a)^p x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p - 2) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p-2)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$$
$$=\frac{6x^{4n}(x^nb+a)^p\,b^3d^2+2x^{3n}(x^nb+a)^p\,a\,b^2d^2p+16x^{3n}(x^nb+a)^p\,b^3cd+x^{2n}(x^nb+a)^p\,a^2b\,d^2p^2-3x^{2n}(x^n}{}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output

```
(6*x**(4*n)*(x**n*b + a)**p*b**3*d**2 + 2*x**(3*n)*(x**n*b + a)**p*a*b**2*
d**2*p + 16*x**(3*n)*(x**n*b + a)**p*b**3*c*d + x**(2*n)*(x**n*b + a)**p*a
**2*b*d**2*p**2 - 3*x**(2*n)*(x**n*b + a)**p*a**2*b*d**2*p + 8*x**(2*n)*(x
**n*b + a)**p*a*b**2*c*d*p + 12*x**(2*n)*(x**n*b + a)**p*b**3*c**2 + x**n*
(x**n*b + a)**p*a**3*d**2*p**3 - 5*x**n*(x**n*b + a)**p*a**3*d**2*p**2 + 6
*x**n*(x**n*b + a)**p*a**3*d**2*p + 8*x**n*(x**n*b + a)**p*a**2*b*c*d*p**2
 - 16*x**n*(x**n*b + a)**p*a**2*b*c*d*p + 12*x**n*(x**n*b + a)**p*a*b**2*c
**2*p + x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a
*x),x)*a**4*d**2*n*p**4 - 6*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p +
 n)*b*x + x**(n*p)*a*x),x)*a**4*d**2*n*p**3 + 11*x**(n*p)*int((x**n*(x**n*
b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**4*d**2*n*p**2 - 6*x**(n
*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**4*d
**2*n*p + 8*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*
p)*a*x),x)*a**3*b*c*d*n*p**3 - 24*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**
(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*b*c*d*n*p**2 + 16*x**(n*p)*int((x**n
*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*b*c*d*n*p + 12
*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*
a**2*b**2*c**2*n*p**2 - 12*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p +
n)*b*x + x**(n*p)*a*x),x)*a**2*b**2*c**2*n*p)/(24*x**(n*p)*b**3*n)
```

3.547 $\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\,dx=\frac{dx^{n(2-p)}(a+bx^n)^{1+p}}{3bn}+\frac{(3bc-ad(2-p))x^{n(2-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)}{3bn(2-p)}$$

output
```
1/3*d*x^(n*(-p+2))*(a+b*x^n)^(p+1)/b/n+1/3*(3*b*c-a*d*(-p+2))*x^(n*(-p+2))
*(a+b*x^n)^p*hypergeom([-p, -p+2],[3-p],-b*x^n/a)/b/n/(-p+2)/((1+b*x^n/a)^
p)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\,dx=-\frac{x^{-n(-2+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(c(-3+p)\,\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)+d(-2+p\right.}{n(-3+p)(-2+p)}$$

input
```
Integrate[x^(-1 - n*(-2 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(c*(-3 + p)*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^n)
/a)] + d*(-2 + p)*x^n*Hypergeometric2F1[3 - p, -p, 4 - p, -((b*x^n)/a)]))/
(n*(-3 + p)*(-2 + p)*x^(n*(-2 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-2)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(3bc-ad(2-p))\int x^{n(2-p)-1}(bx^n+a)^p\,dx}{3b}+\frac{dx^{n(2-p)}(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{a^2x^{-np}(3bc-ad(2-p))\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{1-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^n}{bx^n+a}}{3bn}+\frac{dx^{n(2-p)}(a+bx^n)^{p+1}}{3bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{a^2x^{2n-np}(3bc-ad(2-p))(a+bx^n)^{p-2}\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{bx^n}{bx^n+a}\right)}{3bn(2-p)}+\frac{dx^{n(2-p)}(a+bx^n)^{p+1}}{3bn}$$

input
```
Int[x^(-1 - n*(-2 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(n*(2 - p))*(a + b*x^n)^(1 + p))/(3*b*n) + (a^2*(3*b*c - a*d*(2 - p))
*x^(2*n - n*p)*(a + b*x^n)^(-2 + p)*Hypergeometric2F1[3, 2 - p, 3 - p, (b*
x^n)/(a + b*x^n)])/(3*b*n*(2 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{-1-n(-2+p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int(dx^n+c)(bx^n+a)^px^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p + 2*n - 1)*x^n + c*x^(-n*p + 2*n - 1))*(b*x^n + a)^p,
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 86.90 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\;dx$$
$$=\frac{a^{2-p}a^{2p-2}b^{2-p}b^{p-2}cx^{-np+2n}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma(3-p)}+\frac{a^{3-p}a^{2p-3}b^{3-p}b^{p-3}dx^{-np+3n}\Gamma(3-p)\,{}_2F_1\left(\begin{matrix}-p,3-p\\4-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma(4-p)}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
a**(2 - p)*a**(2*p - 2)*b**(2 - p)*b**(p - 2)*c*x**(-n*p + 2*n)*gamma(2 -
p)*hyper((-p, 2 - p), (3 - p,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 - p))
 + a**(3 - p)*a**(2*p - 3)*b**(3 - p)*b**(p - 3)*d*x**(-n*p + 3*n)*gamma(3
 - p)*hyper((-p, 3 - p), (4 - p,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(4 -
p))
```

Maxima [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\;dx = \int (dx^n+c)(bx^n+a)^p x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p - 2) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\;dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{3,[0,0,2,2,1,1,1,0,1]%%%}+%%%{3,[0,0,2,2,1,1,0,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\;dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p-2)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,(c+dx^n)\ dx$$
$$=\frac{2x^{3n}(x^nb+a)^p\,b^2d+x^{2n}(x^nb+a)^p\,abdp+3x^{2n}(x^nb+a)^p\,b^2c+x^n(x^nb+a)^p\,a^2d\,p^2-2x^n(x^nb+a)^p\,a^2d}{}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(2*x**(3*n)*(x**n*b + a)**p*b**2*d + x**(2*n)*(x**n*b + a)**p*a*b*d*p + 3*
x**(2*n)*(x**n*b + a)**p*b**2*c + x**n*(x**n*b + a)**p*a**2*d*p**2 - 2*x**
n*(x**n*b + a)**p*a**2*d*p + 3*x**n*(x**n*b + a)**p*a*b*c*p + x**(n*p)*int
((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*d*n*p**3
 - 3*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x)
,x)*a**3*d*n*p**2 + 2*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*
x + x**(n*p)*a*x),x)*a**3*d*n*p + 3*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x
**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*n*p**2 - 3*x**(n*p)*int((x**n*
(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*b*c*n*p)/(6*x**
(n*p)*b**2*n)
```

3.548 $\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx = \frac{x^{n(2-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)}{n(2-p)}$$

output
```
x^(n*(-p+2))*(a+b*x^n)^p*hypergeom([-p, -p+2],[3-p],-b*x^n/a)/n/(-p+2)/((1
+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx = -\frac{x^{-n(-2+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)}{n(-2+p)}$$

input
```
Integrate[x^(-1 - n*(-2 + p))*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^n)/a)])/(n*(-2
 + p)*x^(n*(-2 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-2)-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{882}$$

$$\frac{a^2x^{-np}\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{1-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^3}d\frac{x^n}{bx^n+a}}{n}$$

$$\Big\downarrow \text{74}$$

$$\frac{a^2x^{2n-np}(a+bx^n)^{p-2}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{bx^n}{bx^n+a}\right)}{n(2-p)}$$

input
```
Int[x^(-1 - n*(-2 + p))*(a + b*x^n)^p,x]
```

output
```
(a^2*x^(2*n - n*p)*(a + b*x^n)^(-2 + p)*Hypergeometric2F1[3, 2 - p, 3 - p,
 (b*x^n)/(a + b*x^n)])/(n*(2 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

Maple [F]

$$\int x^{-1-n(-2+p)}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-2+p)}(a+bx^{n})^{p}\,dx=\int(bx^{n}+a)^{p}x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx = \frac{a^p x^{-np+2n}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma(3-p)}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p,x)
```

output
```
a**p*x**(-n*p + 2*n)*gamma(2 - p)*hyper((-p, 2 - p), (3 - p,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(3 - p))
```

Maxima [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1), x)
```

Giac [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-2)-1}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p - 2) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p - 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-2+p)}(a+bx^n)^p\,dx$$
$$=\frac{x^{2n}(x^nb+a)^p\,b+x^n(x^nb+a)^p\,ap+x^{np}\left(\int \frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)a^2n\,p^2-x^{np}\left(\int \frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx\right)a^2np}{2x^{np}bn}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p,x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b + x**n*(x**n*b + a)**p*a*p + x**(n*p)*int((x**
n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*n*p**2 - x**(
n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**2*
n*p)/(2*x**(n*p)*b*n)
```

3.549 $\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n} dx$

Optimal result

Integrand size = 29, antiderivative size = 192

$$\begin{aligned}\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n} dx &= -\frac{cx^{-np}(a+bx^n)^p \,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^2np} \\ &\quad + \frac{x^{n(1-p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{dn(1-p)} \\ &\quad + \frac{cx^{-np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{d^2np}\end{aligned}$$

output
```
-c*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d^2/n/p
/(x^(n*p))+x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p, 1-p],[-p+2],-b*x^n/a)/d/
n/(1-p)/((1+b*x^n/a)^p)+c*(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n/a)/d
^2/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= -\frac{x^{-n(-2+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(2-p,-p,1,3-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(-2+p)}$$

input
```
Integrate[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*AppellF1[2 - p, -p, 1, 3 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c*n*(-2 + p)*x^(n*(-2 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-2)-1}(a+bx^n)^p}{c+dx^n}\,dx$$
$$\Big\downarrow \text{1013}$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(2-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^{n(2-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(2-p,-p,1,3-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(2-p)}$$

input
```
Int[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
(x^(n*(2 - p))*(a + b*x^n)^p*AppellF1[2 - p, -p, 1, 3 - p, -((b*x^n)/a), -
((d*x^n)/c)])/(c*n*(2 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-2+p)}(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= \frac{x^n(x^nb+a)^p\,dp-(x^nb+a)^p\,cp+(x^nb+a)^p\,c-x^{np}\left(\int \frac{(x^nb+a)^p}{x^{np+2n}bdx+x^{np+n}adx+x^{np+n}bcx+x^{np}acx}dx\right)a\,c^2n\,p^2+}{}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output

```
(x**n*(x**n*b + a)**p*d*p - (x**n*b + a)**p*c*p + (x**n*b + a)**p*c - x**(
n*p)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(
n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*c**2*n*p**2 + x**(n*p)*int((x**n*b +
 a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x
**(n*p)*a*c*x),x)*a*c**2*n*p + x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x*
*(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*
c*x),x)*a*d**2*n*p**2 - x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p +
 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)
*b*c*d*n*p)/(x**(n*p)*d**2*n*p)
```

3.550 $\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 196

$$\begin{aligned}\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx &= \frac{x^{-np}(a+bx^n)^p\,\text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^2np}\\ &\quad-\frac{ax^{n(1-p)}(a+bx^n)^{-1+p}\,\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{cdn(1-p)}\\ &\quad-\frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{d^2np}\end{aligned}$$

output
```
(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d^2/n/p/(x
^(n*p))-a*x^(n*(1-p))*(a+b*x^n)^(-1+p)*hypergeom([2, 1-p],[-p+2],(-a*d+b*c
)*x^n/c/(a+b*x^n))/c/d/n/(1-p)-(a+b*x^n)^p*hypergeom([-p, -p],[1-p],-b*x^n
/a)/d^2/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.40

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$$
$$= -\frac{x^{-n(-2+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(2-p,-p,2,3-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2n(-2+p)}$$

input
```
Integrate[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*AppellF1[2 - p, -p, 2, 3 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c^2*n*(-2 + p)*x^(n*(-2 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.40, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-2)-1}(a+bx^n)^p}{(c+dx^n)^2}\,dx$$
$$\Big\downarrow \text{1013}$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(2-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^{n(2-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(2-p,-p,2,3-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2n(2-p)}$$

input
```
Int[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
(x^(n*(2 - p))*(a + b*x^n)^p*AppellF1[2 - p, -p, 2, 3 - p, -((b*x^n)/a), -
((d*x^n)/c)])/(c^2*n*(2 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-2+p)}(a+b\,x^n)^p}{(c+d\,x^n)^2}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2} \, dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^2} \, dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2} \, dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^2} \, dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{x^{2n}(x^nb+a)^p}{x^{np+2n}d^2x+2x^{np+n}cdx+x^{np}c^2x}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p + 2*n)*d**2*x + 2*x**(n*p + n)*c*d
*x + x**(n*p)*c**2*x),x)
```

3.551 $\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \frac{a^2x^{n(2-p)}(a+bx^n)^{-2+p}\,\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^3n(2-p)}$$

output
```
a^2*x^(n*(-p+2))*(a+b*x^n)^(-2+p)*hypergeom([3, -p+2],[3-p],(-a*d+b*c)*x^n
/c/(a+b*x^n))/c^3/n/(-p+2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = -\frac{x^{-n(-2+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(1+\frac{dx^n}{c}\right)^{-2+p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,\frac{-\frac{bx^n}{a}+\frac{dx^n}{c}}{1+\frac{dx^n}{c}}\right)}{c^3n(-2+p)}$$

input
```
Integrate[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
-(((a + b*x^n)^p*(1 + (d*x^n)/c)^(-2 + p)*Hypergeometric2F1[2 - p, -p, 3 -
 p, (-((b*x^n)/a) + (d*x^n)/c)/(1 + (d*x^n)/c)])/(c^3*n*(-2 + p)*x^(n*(-2
+ p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-2)-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$

$$\Big\downarrow \text{ 1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(2-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{x^{n(2-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{dx^n}{c}+1\right)^{p-2}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{\frac{bx^n}{a}-\frac{dx^n}{c}}{\frac{dx^n}{c}+1}\right)}{c^3n(2-p)}$$

input
```
Int[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
(x^(n*(2 - p))*(a + b*x^n)^p*(1 + (d*x^n)/c)^(-2 + p)*Hypergeometric2F1[2
- p, -p, 3 - p, -(((b*x^n)/a - (d*x^n)/c)/(1 + (d*x^n)/c))])/(c^3*n*(2 - p
)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-2+p)} (a+b\,x^{n})^{p}}{(c+d\,x^{n})^{3}}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) +
 3*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}\,(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^3),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
( - x**(2*n)*(x**n*b + a)**p*a**2*b*d**2*p**2 - 3*x**(2*n)*(x**n*b + a)**p
*a**2*b*d**2*p - 2*x**(2*n)*(x**n*b + a)**p*a**2*b*d**2 + 4*x**(2*n)*(x**n
*b + a)**p*a*b**2*c*d*p + 5*x**(2*n)*(x**n*b + a)**p*a*b**2*c*d - 3*x**(2*
n)*(x**n*b + a)**p*b**3*c**2 - x**n*(x**n*b + a)**p*a**3*d**2*p**2 - 3*x**
n*(x**n*b + a)**p*a**3*d**2*p - 2*x**n*(x**n*b + a)**p*a**3*d**2 + x**n*(x
**n*b + a)**p*a**2*b*c*d*p**2 + 6*x**n*(x**n*b + a)**p*a**2*b*c*d*p + 2*x*
*n*(x**n*b + a)**p*a**2*b*c*d - 3*x**n*(x**n*b + a)**p*a*b**2*c**2*p + 2*x
**(n*p + 2*n)*int((x**(3*n)*(x**n*b + a)**p)/(x**(n*p + 4*n)*a**3*b*d**6*p
**3*x + 4*x**(n*p + 4*n)*a**3*b*d**6*p**2*x + 5*x**(n*p + 4*n)*a**3*b*d**6
*p*x + 2*x**(n*p + 4*n)*a**3*b*d**6*x - 7*x**(n*p + 4*n)*a**2*b**2*c*d**5*
p**2*x - 15*x**(n*p + 4*n)*a**2*b**2*c*d**5*p*x - 8*x**(n*p + 4*n)*a**2*b*
*2*c*d**5*x + 12*x**(n*p + 4*n)*a*b**3*c**2*d**4*p*x + 12*x**(n*p + 4*n)*a
*b**3*c**2*d**4*x - 6*x**(n*p + 4*n)*b**4*c**3*d**3*x + x**(n*p + 3*n)*a**
4*d**6*p**3*x + 4*x**(n*p + 3*n)*a**4*d**6*p**2*x + 5*x**(n*p + 3*n)*a**4*
d**6*p*x + 2*x**(n*p + 3*n)*a**4*d**6*x + 3*x**(n*p + 3*n)*a**3*b*c*d**5*p
**3*x + 5*x**(n*p + 3*n)*a**3*b*c*d**5*p**2*x - 2*x**(n*p + 3*n)*a**3*b*c*
d**5*x - 21*x**(n*p + 3*n)*a**2*b**2*c**2*d**4*p**2*x - 33*x**(n*p + 3*n)*
a**2*b**2*c**2*d**4*p*x - 12*x**(n*p + 3*n)*a**2*b**2*c**2*d**4*x + 36*x**
(n*p + 3*n)*a*b**3*c**3*d**3*p*x + 30*x**(n*p + 3*n)*a*b**3*c**3*d**3*x -
18*x**(n*p + 3*n)*b**4*c**4*d**2*x + 3*x**(n*p + 2*n)*a**4*c*d**5*p**3*...
```

3.552 $\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \frac{x^{n(2-p)}(a+bx^n)^{1+p}}{acn(2-p)\,(c+dx^n)^3} - \frac{a^2(3bc-ad(1+p))x^{n(3-p)}(a+bx^n)^{-3+p}\,\text{Hypergeometric2F1}\left(4,3-p,4-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^5n(2-p)(3-p)}$$

output
```
x^(n*(-p+2))*(a+b*x^n)^(p+1)/a/c/n/(-p+2)/(c+d*x^n)^3-a^2*(3*b*c-a*d*(p+1)
)*x^(n*(3-p))*(a+b*x^n)^(-3+p)*hypergeom([4, 3-p],[4-p],(-a*d+b*c)*x^n/c/(
a+b*x^n))/c^5/n/(-p+2)/(3-p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.42 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.17

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = -\frac{(-1+p)px^{-n(-2+p)}(a+bx^n)^p\left(3(2c+dpx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)-3(c+d(-1+p)x^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)}\right.\right.}{}$$

input
```
Integrate[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^4,x]
```

output
```
-1/6*((-1 + p)*p*(a + b*x^n)^p*(3*(2*c + d*p*x^n)*HurwitzLerchPhi[((b*c -
a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 3*(c + d*(-1 + p)*x^n)*HurwitzLerch
Phi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - 2*d*x^n*HurwitzLerchPhi
[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + d*p*x^n*HurwitzLerchPhi[((
b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 3*c*HurwitzLerchPhi[((b*c - a
*d)*x^n)/(c*(a + b*x^n)), 1, -p] - d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)
/(c*(a + b*x^n)), 1, -p] - d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
 + b*x^n)), 1, -p]))/(c^2*n*x^(n*(-2 + p))*(c + d*x^n)^3)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.99 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-2)-1}(a+bx^n)^p}{(c+dx^n)^4}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(2-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^4}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(1-p)px^{n(2-p)}(a+bx^n)^p\left(-2dx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)+dpx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)-dx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,-p\right)\right.}{}$$

input
```
Int[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^4,x]
```

output
```
((1 - p)*p*x^(n*(2 - p))*(a + b*x^n)^p*(3*(2*c + d*p*x^n)*HurwitzLerchPhi[
((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 3*(c - d*(1 - p)*x^n)*Hurwi
tzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - 2*d*x^n*HurwitzL
erchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + d*p*x^n*HurwitzLerc
hPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 3*c*HurwitzLerchPhi[((
b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - d*x^n*HurwitzLerchPhi[((b*c - a*
d)*x^n)/(c*(a + b*x^n)), 1, -p] - d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n
)/(c*(a + b*x^n)), 1, -p]))/(6*c^5*n*(1 + (d*x^n)/c)^3)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-2+p)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{4}}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) +
 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p/(c+d*x**n)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^4, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}\,(c+d\,x^n)^4}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^4),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^4), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
(x**(3*n)*(x**n*b + a)**p*a**2*b**2*d**3*p**2 + 5*x**(3*n)*(x**n*b + a)**p
*a**2*b**2*d**3*p + 6*x**(3*n)*(x**n*b + a)**p*a**2*b**2*d**3 - 5*x**(3*n)
*(x**n*b + a)**p*a*b**3*c*d**2*p - 11*x**(3*n)*(x**n*b + a)**p*a*b**3*c*d*
*2 + 4*x**(3*n)*(x**n*b + a)**p*b**4*c**2*d - x**(2*n)*(x**n*b + a)**p*a**
3*b*d**3*p**3 - 5*x**(2*n)*(x**n*b + a)**p*a**3*b*d**3*p**2 - 6*x**(2*n)*(
x**n*b + a)**p*a**3*b*d**3*p + 8*x**(2*n)*(x**n*b + a)**p*a**2*b**2*c*d**2
*p**2 + 26*x**(2*n)*(x**n*b + a)**p*a**2*b**2*c*d**2*p + 18*x**(2*n)*(x**n
*b + a)**p*a**2*b**2*c*d**2 - 19*x**(2*n)*(x**n*b + a)**p*a*b**3*c**2*d*p
- 33*x**(2*n)*(x**n*b + a)**p*a*b**3*c**2*d + 12*x**(2*n)*(x**n*b + a)**p*
b**4*c**3 - x**n*(x**n*b + a)**p*a**4*d**3*p**3 - 6*x**n*(x**n*b + a)**p*a
**4*d**3*p**2 - 11*x**n*(x**n*b + a)**p*a**4*d**3*p - 6*x**n*(x**n*b + a)*
*p*a**4*d**3 + x**n*(x**n*b + a)**p*a**3*b*c*d**2*p**3 + 13*x**n*(x**n*b +
 a)**p*a**3*b*c*d**2*p**2 + 30*x**n*(x**n*b + a)**p*a**3*b*c*d**2*p + 24*x
**n*(x**n*b + a)**p*a**3*b*c*d**2 - 7*x**n*(x**n*b + a)**p*a**2*b**2*c**2*
d*p**2 - 31*x**n*(x**n*b + a)**p*a**2*b**2*c**2*d*p - 18*x**n*(x**n*b + a)
**p*a**2*b**2*c**2*d + 12*x**n*(x**n*b + a)**p*a*b**3*c**3*p + 6*x**(n*p +
 3*n)*int((x**(3*n)*(x**n*b + a)**p)/(x**(n*p + 5*n)*a**8*b*d**12*p**8*x +
 16*x**(n*p + 5*n)*a**8*b*d**12*p**7*x + 110*x**(n*p + 5*n)*a**8*b*d**12*p
**6*x + 424*x**(n*p + 5*n)*a**8*b*d**12*p**5*x + 1001*x**(n*p + 5*n)*a**8*
b*d**12*p**4*x + 1480*x**(n*p + 5*n)*a**8*b*d**12*p**3*x + 1336*x**(n*p...
```

3.553 $\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 245

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \frac{x^{n(2-p)}(a+bx^n)^{1+p}}{acn(2-p)(c+dx^n)^4}+\frac{d(4bc-ad(2+p))x^{n(3-p)}(a+bx^n)^{1+p}}{4ac^2(bc-ad)n(2-p)(c+dx^n)^4} - \frac{a^2(12b^2c^2-8abcd(1+p)+a^2d^2(2+3p+p^2))\,x^{n(3-p)}(a+bx^n)^{-3+p}\,\text{Hypergeometric2F1}\left(4,3-p,4-\ldots\right.}{4c^6(bc-ad)n(2-p)(3-p)}$$

output
```
x^(n*(-p+2))*(a+b*x^n)^(p+1)/a/c/n/(-p+2)/(c+d*x^n)^4+1/4*d*(4*b*c-a*d*(2+
p))*x^(n*(3-p))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(-p+2)/(c+d*x^n)^4-1/4*
a^2*(12*b^2*c^2-8*a*b*c*d*(p+1)+a^2*d^2*(p^2+3*p+2))*x^(n*(3-p))*(a+b*x^n)
^(-3+p)*hypergeom([4, 3-p],[4-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^6/(-a*d+b*c
)/n/(-p+2)/(3-p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.98 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.95

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx =$$

$$-\frac{(-1+p)px^{-n(-2+p)}(a+bx^n)^p\left(4(6c^2+6cdpx^n+d^2p(1+p)x^{2n})\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)-6(2c^2+4cd\right.}{}$$

input
```
Integrate[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^5,x]
```

output
```
-1/24*((-1 + p)*p*(a + b*x^n)^p*(4*(6*c^2 + 6*c*d*p*x^n + d^2*p*(1 + p)*x^
(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 6*(2
*c^2 + 4*c*d*(-1 + p)*x^n + d^2*(-1 + p)*p*x^(2*n))*HurwitzLerchPhi[((b*c
- a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - 16*c*d*x^n*HurwitzLerchPhi[((b*c
- a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 8*c*d*p*x^n*HurwitzLerchPhi[((b*c
 - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 8*d^2*x^(2*n)*HurwitzLerchPhi[((
b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 12*d^2*p*x^(2*n)*HurwitzLerch
Phi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 4*d^2*p^2*x^(2*n)*Hurwi
tzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 6*d^2*x^(2*n)*Hu
rwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] + 5*d^2*p*x^(2*
n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - d^2*p^2*
x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 12*
c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 8*c*d*x^n*
HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 8*c*d*p*x^n*Hu
rwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 2*d^2*x^(2*n)*Hu
rwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 3*d^2*p*x^(2*n)*
HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - d^2*p^2*x^(2*n
)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]))/(c^3*n*x^(n*
(-2 + p))*(c + d*x^n)^4)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 2.21 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-2)-1}(a+bx^n)^p}{(c+dx^n)^5}dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(2-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^5}dx$$

$$\Big\downarrow \text{1012}$$

$$\underline{(1-p)px^{n(2-p)}(a+bx^n)^p\left(4\left(6c^2+6cdpx^n+d^2p(p+1)x^{2n}\right)\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,1-p\right)-6\left(2c^2-4cd(1-p)x^n-d\right.}$$

input
```
Int[(x^(-1 - n*(-2 + p))*(a + b*x^n)^p)/(c + d*x^n)^5,x]
```

output
```
((1 - p)*p*x^(n*(2 - p))*(a + b*x^n)^p*(4*(6*c^2 + 6*c*d*p*x^n + d^2*p*(1
+ p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p]
 - 6*(2*c^2 - 4*c*d*(1 - p)*x^n - d^2*(1 - p)*p*x^(2*n))*HurwitzLerchPhi[(
(b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] - 16*c*d*x^n*HurwitzLerchPhi[(
(b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 8*c*d*p*x^n*HurwitzLerchPhi[
((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 8*d^2*x^(2*n)*HurwitzLerchP
hi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 12*d^2*p*x^(2*n)*Hurwitz
LerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 4*d^2*p^2*x^(2*n)*
HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 6*d^2*x^(2*
n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] + 5*d^2*p*
x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - d^2
*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p]
- 12*c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 8*c*d
*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 8*c*d*p*x
^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 2*d^2*x^(2*
n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - 3*d^2*p*x^(
2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - d^2*p^2*x
^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p]))/(24*c^7
*n*(1 + (d*x^n)/c)^4)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-2+p)}(a+b\,x^n)^p}{(c+d\,x^n)^5}dx$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

output
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 2*n - 1)/(d^5*x^(5*n) + 5*c*d^4*x^(4*n) +
 10*c^2*d^3*x^(3*n) + 10*c^3*d^2*x^(2*n) + 5*c^4*d*x^n + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-2+p))*(a+b*x**n)**p/(c+d*x**n)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^5, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-2)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 2) - 1)/(d*x^n + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-2)+1}\,(c+d\,x^n)^5}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^5),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 2) + 1)*(c + d*x^n)^5), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-2+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-2+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

output
```
too large to display
```

3.554 $\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 188

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\frac{c^2x^{n(3-p)}(a+bx^n)^{1+p}}{an(3-p)}+\frac{d^2x^{n(4-p)}(a+bx^n)^{1+p}}{5bn}-\frac{(20b^2c^2-10abcd(3-p)+a^2d^2(12-7p+p^2))\,x^{n(4-p)}(a+bx^n)^p\,\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\,(4-$$
$$5abn(3-p)(4-p)$$

output
```
c^2*x^(n*(3-p))*(a+b*x^n)^(p+1)/a/n/(3-p)+1/5*d^2*x^(n*(4-p))*(a+b*x^n)^(p
+1)/b/n-1/5*(20*b^2*c^2-10*a*b*c*d*(3-p)+a^2*d^2*(p^2-7*p+12))*x^(n*(4-p))
*(a+b*x^n)^p*hypergeom([-p, 4-p],[5-p],-b*x^n/a)/a/b/n/(3-p)/(4-p)/((1+b*x
^n/a)^p)
```

Mathematica [A] (verified)

Time = 5.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=$$
$$-\frac{x^{-n(-3+p)}(a+bx^n)^p\,\left(1+\frac{bx^n}{a}\right)^{-p}\left(c^2(20-9p+p^2)\,\text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{bx^n}{a}\right)+d(-$$

input
```
Integrate[x^(-1 - n*(-3 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*(c^2*(20 - 9*p + p^2)*Hypergeometric2F1[3 - p, -p, 4 - p,
 -((b*x^n)/a)] + d*(-3 + p)*x^n*(2*c*(-5 + p)*Hypergeometric2F1[4 - p, -p,
 5 - p, -((b*x^n)/a)] + d*(-4 + p)*x^n*Hypergeometric2F1[5 - p, -p, 6 - p,
 -((b*x^n)/a)])))/(n*(-5 + p)*(-4 + p)*(-3 + p)*x^(n*(-3 + p))*(1 + (b*x^n
)/a)^p))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1008, 959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-3)-1}(c+dx^n)^2(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{1008}$$

$$\frac{\int x^{n(3-p)-1}(bx^n+a)^p(dn(6bc-ad(4-p))x^n+cn(5bc-ad(3-p)))\,dx}{5bn}+\frac{dx^{n(3-p)}(c+dx^n)(a+bx^n)^{p+1}}{5bn}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{n(a^2d^2(p^2-7p+12)-10abcd(3-p)+20b^2c^2)\int x^{n(3-p)-1}(bx^n+a)^p dx}{4b}+\frac{dx^{n(3-p)}(6bc-ad(4-p))(a+bx^n)^{p+1}}{4b}}{5bn}+\frac{dx^{n(3-p)}(c+dx^n)(a+bx^n)^{p+1}}{5bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{\frac{a^3x^{-np}(a^2d^2(p^2-7p+12)-10abcd(3-p)+20b^2c^2)\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{2-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^4}d\frac{x^n}{bx^n+a}}{4b}+\frac{dx^{n(3-p)}(6bc-ad(4-p))(a+bx^n)^{p+1}}{4b}}{5bn}+\frac{dx^{n(3-p)}(c+dx^n)(a+bx^n)^{p+1}}{5bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{\frac{a^3x^{3n-np}(a^2d^2(p^2-7p+12)-10abcd(3-p)+20b^2c^2)(a+bx^n)^{p-3}\,\text{Hypergeometric2F1}\left(4,3-p,4-p,\frac{bx^n}{bx^n+a}\right)}{4b(3-p)}+\frac{dx^{n(3-p)}(6bc-ad(4-p))(a+bx^n}{4b}}{5bn}$$

$$\frac{dx^{n(3-p)}(c+dx^n)\,(a+bx^n)^{p+1}}{5bn}$$

input
```
Int[x^(-1 - n*(-3 + p))*(a + b*x^n)^p*(c + d*x^n)^2,x]
```

output
```
(d*x^(n*(3 - p))*(a + b*x^n)^(1 + p)*(c + d*x^n))/(5*b*n) + ((d*(6*b*c - a
*d*(4 - p))*x^(n*(3 - p))*(a + b*x^n)^(1 + p))/(4*b) + (a^3*(20*b^2*c^2 -
10*a*b*c*d*(3 - p) + a^2*d^2*(12 - 7*p + p^2))*x^(3*n - n*p)*(a + b*x^n)^(
-3 + p)*Hypergeometric2F1[4, 3 - p, 4 - p, (b*x^n)/(a + b*x^n)])/(4*b*(3 -
 p)))/(5*b*n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1008
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c
 - a*d, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Maple [F]

$$\int x^{-1-n(-3+p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{2}\,dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

Fricas [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx=\int (dx^n+c)^2(bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^(-n*p + 3*n - 1)*x^(2*n) + 2*c*d*x^(-n*p + 3*n - 1)*x^n +
c^2*x^(-n*p + 3*n - 1))*(b*x^n + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p*(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \int (dx^n+c)^2(bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)^2*(b*x^n + a)^p*x^(-n*(p - 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,1,3,3,2,0]%%%}+%%%{-3,[1,0,4,3,1,3,2,2,0]%%%}+
%%%{-3,[1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^2}{x^{n\,(p-3)+1}} \,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^2)/x^(n*(p - 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n)^2,x)
```

output
```
(24*x**(5*n)*(x**n*b + a)**p*b**4*d**2 + 6*x**(4*n)*(x**n*b + a)**p*a*b**3
*d**2*p + 60*x**(4*n)*(x**n*b + a)**p*b**4*c*d + 2*x**(3*n)*(x**n*b + a)**
p*a**2*b**2*d**2*p**2 - 8*x**(3*n)*(x**n*b + a)**p*a**2*b**2*d**2*p + 20*x
**(3*n)*(x**n*b + a)**p*a*b**3*c*d*p + 40*x**(3*n)*(x**n*b + a)**p*b**4*c*
*2 + x**(2*n)*(x**n*b + a)**p*a**3*b*d**2*p**3 - 7*x**(2*n)*(x**n*b + a)**
p*a**3*b*d**2*p**2 + 12*x**(2*n)*(x**n*b + a)**p*a**3*b*d**2*p + 10*x**(2*
n)*(x**n*b + a)**p*a**2*b**2*c*d*p**2 - 30*x**(2*n)*(x**n*b + a)**p*a**2*b
**2*c*d*p + 20*x**(2*n)*(x**n*b + a)**p*a*b**3*c**2*p + x**n*(x**n*b + a)*
*p*a**4*d**2*p**4 - 9*x**n*(x**n*b + a)**p*a**4*d**2*p**3 + 26*x**n*(x**n*
b + a)**p*a**4*d**2*p**2 - 24*x**n*(x**n*b + a)**p*a**4*d**2*p + 10*x**n*(
x**n*b + a)**p*a**3*b*c*d*p**3 - 50*x**n*(x**n*b + a)**p*a**3*b*c*d*p**2 +
 60*x**n*(x**n*b + a)**p*a**3*b*c*d*p + 20*x**n*(x**n*b + a)**p*a**2*b**2*
c**2*p**2 - 40*x**n*(x**n*b + a)**p*a**2*b**2*c**2*p + x**(n*p)*int((x**n*
(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**5*d**2*n*p**5 - 1
0*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)
*a**5*d**2*n*p**4 + 35*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b
*x + x**(n*p)*a*x),x)*a**5*d**2*n*p**3 - 50*x**(n*p)*int((x**n*(x**n*b + a
)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**5*d**2*n*p**2 + 24*x**(n*p)*
int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**5*d**2*
n*p + 10*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*...
```

3.555 $\int x^{-1-n(-3+p)}(a+bx^n)^p (c+dx^n)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 118

$$\int x^{-1-n(-3+p)}(a+bx^n)^p (c+dx^n)\,dx = \frac{dx^{n(3-p)}(a+bx^n)^{1+p}}{4bn} + \frac{(4bc-ad(3-p))x^{n(3-p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{bx^n}{a}\right)}{4bn(3-p)}$$

output
```
1/4*d*x^(n*(3-p))*(a+b*x^n)^(p+1)/b/n+1/4*(4*b*c-a*d*(3-p))*x^(n*(3-p))*(a
+b*x^n)^p*hypergeom([-p, 3-p],[4-p],-b*x^n/a)/b/n/(3-p)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.90

$$\int x^{-1-n(-3+p)}(a+bx^n)^p (c+dx^n)\,dx = -\frac{x^{-n(-3+p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(c(-4+p)\,\text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{bx^n}{a}\right)+d(-3+p\right.}{n(-4+p)(-3+p)}$$

input
```
Integrate[x^(-1 - n*(-3 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*(c*(-4 + p)*Hypergeometric2F1[3 - p, -p, 4 - p, -((b*x^n)
/a)] + d*(-3 + p)*x^n*Hypergeometric2F1[4 - p, -p, 5 - p, -((b*x^n)/a)]))/
(n*(-4 + p)*(-3 + p)*x^(n*(-3 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {959, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-3)-1}(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{959}$$

$$\frac{(4bc-ad(3-p))\int x^{n(3-p)-1}(bx^n+a)^p\,dx}{4b}+\frac{dx^{n(3-p)}(a+bx^n)^{p+1}}{4bn}$$

$$\Big\downarrow \text{882}$$

$$\frac{a^3x^{-np}(4bc-ad(3-p))\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int\frac{\left(\frac{x^n}{bx^n+a}\right)^{2-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^4}d\frac{x^n}{bx^n+a}}{4bn}+\frac{dx^{n(3-p)}(a+bx^n)^{p+1}}{4bn}$$

$$\Big\downarrow \text{74}$$

$$\frac{a^3x^{3n-np}(4bc-ad(3-p))(a+bx^n)^{p-3}\,\text{Hypergeometric2F1}\left(4,3-p,4-p,\frac{bx^n}{bx^n+a}\right)}{4bn(3-p)}+\frac{dx^{n(3-p)}(a+bx^n)^{p+1}}{4bn}$$

input
```
Int[x^(-1 - n*(-3 + p))*(a + b*x^n)^p*(c + d*x^n),x]
```

output
```
(d*x^(n*(3 - p))*(a + b*x^n)^(1 + p))/(4*b*n) + (a^3*(4*b*c - a*d*(3 - p))
*x^(3*n - n*p)*(a + b*x^n)^(-3 + p)*Hypergeometric2F1[4, 3 - p, 4 - p, (b*
x^n)/(a + b*x^n)])/(4*b*n*(3 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int x^{-1-n(-3+p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})\,dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

Fricas [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((d*x^(-n*p + 3*n - 1)*x^n + c*x^(-n*p + 3*n - 1))*(b*x^n + a)^p,
x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p*(c+d*x**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,(c+dx^n)\;dx=\int (dx^n+c)(bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^n + a)^p*x^(-n*(p - 3) - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p(c+dx^n)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{4,[0,0,2,2,1,1,1,0,1]%%%}+%%%{4,[0,0,2,2,1,1,0,0,1]%%%}+%%
%{1,[0,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p(c+dx^n)\,dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)}{x^{n\,(p-3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n))/x^(n*(p - 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p(c+dx^n)\,dx$$
$$= \frac{6x^{4n}(x^nb+a)^p\,b^3d+2x^{3n}(x^nb+a)^p\,a\,b^2dp+8x^{3n}(x^nb+a)^p\,b^3c+x^{2n}(x^nb+a)^p\,a^2bd\,p^2-3x^{2n}(x^nb+a}{}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p*(c+d*x^n),x)
```

output
```
(6*x**(4*n)*(x**n*b + a)**p*b**3*d + 2*x**(3*n)*(x**n*b + a)**p*a*b**2*d*p
 + 8*x**(3*n)*(x**n*b + a)**p*b**3*c + x**(2*n)*(x**n*b + a)**p*a**2*b*d*p
**2 - 3*x**(2*n)*(x**n*b + a)**p*a**2*b*d*p + 4*x**(2*n)*(x**n*b + a)**p*a
*b**2*c*p + x**n*(x**n*b + a)**p*a**3*d*p**3 - 5*x**n*(x**n*b + a)**p*a**3
*d*p**2 + 6*x**n*(x**n*b + a)**p*a**3*d*p + 4*x**n*(x**n*b + a)**p*a**2*b*
c*p**2 - 8*x**n*(x**n*b + a)**p*a**2*b*c*p + x**(n*p)*int((x**n*(x**n*b +
a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**4*d*n*p**4 - 6*x**(n*p)*int
((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**4*d*n*p**3
 + 11*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x
),x)*a**4*d*n*p**2 - 6*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b
*x + x**(n*p)*a*x),x)*a**4*d*n*p + 4*x**(n*p)*int((x**n*(x**n*b + a)**p)/(
x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*b*c*n*p**3 - 12*x**(n*p)*int((x**
n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*b*c*n*p**2 +
8*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)
*a**3*b*c*n*p)/(24*x**(n*p)*b**3*n)
```

3.556 $\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = \frac{x^{n(3-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{bx^n}{a}\right)}{n(3-p)}$$

output
```
x^(n*(3-p))*(a+b*x^n)^p*hypergeom([-p, 3-p],[4-p],-b*x^n/a)/n/(3-p)/((1+b*
x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = -\frac{x^{-n(-3+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{bx^n}{a}\right)}{n(-3+p)}$$

input
```
Integrate[x^(-1 - n*(-3 + p))*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[3 - p, -p, 4 - p, -((b*x^n)/a)])/(n*(-3
 + p)*x^(n*(-3 + p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(p-3)-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{882}$$

$$\frac{a^3x^{-np}\left(\frac{x^n}{a+bx^n}\right)^p(a+bx^n)^p\int \frac{\left(\frac{x^n}{bx^n+a}\right)^{2-p}}{\left(1-\frac{bx^n}{bx^n+a}\right)^4}d\frac{x^n}{bx^n+a}}{n}$$

$$\Big\downarrow \text{74}$$

$$\frac{a^3x^{3n-np}(a+bx^n)^{p-3}\,\text{Hypergeometric2F1}\left(4,3-p,4-p,\frac{bx^n}{bx^n+a}\right)}{n(3-p)}$$

input
```
Int[x^(-1 - n*(-3 + p))*(a + b*x^n)^p,x]
```

output
```
(a^3*x^(3*n - n*p)*(a + b*x^n)^(-3 + p)*Hypergeometric2F1[4, 3 - p, 4 - p,
 (b*x^n)/(a + b*x^n)])/(n*(3 - p))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

Maple [F]

$$\int x^{-1-n(-3+p)} (a + b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = \frac{a^p x^{-np+3n}\Gamma(3-p)\,{}_2F_1\left(\begin{matrix}-p,3-p\\4-p\end{matrix}\middle|\frac{bx^ne^{i\pi}}{a}\right)}{n\Gamma(4-p)}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p,x)
```

output
```
a**p*x**(-n*p + 3*n)*gamma(3 - p)*hyper((-p, 3 - p), (4 - p,), b*x**n*exp_
polar(I*pi)/a)/(n*gamma(4 - p))
```

Maxima [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1), x)
```

Giac [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx = \int (bx^n+a)^p x^{-n(p-3)-1}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx=\int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(n*(p - 3) + 1),x)
```

output
```
int((a + b*x^n)^p/x^(n*(p - 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-3+p)}(a+bx^n)^p\,dx$$
$$=\frac{2x^{3n}(x^nb+a)^p\,b^2+x^{2n}(x^nb+a)^p\,abp+x^n(x^nb+a)^p\,a^2p^2-2x^n(x^nb+a)^p\,a^2p+x^{np}\Big(\int\frac{x^n(x^nb+a)^p}{x^{np+n}bx+x^{np}ax}dx}{6x^{np}b^2n}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p,x)
```

output
```
(2*x**(3*n)*(x**n*b + a)**p*b**2 + x**(2*n)*(x**n*b + a)**p*a*b*p + x**n*(
x**n*b + a)**p*a**2*p**2 - 2*x**n*(x**n*b + a)**p*a**2*p + x**(n*p)*int((x
**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a**3*n*p**3 - 3*
x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**(n*p)*a*x),x)*a
**3*n*p**2 + 2*x**(n*p)*int((x**n*(x**n*b + a)**p)/(x**(n*p + n)*b*x + x**
(n*p)*a*x),x)*a**3*n*p)/(6*x**(n*p)*b**2*n)
```

3.557 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 267

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \frac{c^2x^{-np}(a+bx^n)^p \operatorname{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^3np} - \frac{cx^{n(1-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{d^2n(1-p)} + \frac{x^{n(2-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^n}{a}\right)}{dn(2-p)} - \frac{c^2x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{d^3np}$$

output

```
c^2*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d^3/n/
p/(x^(n*p))-c*x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p, 1-p],[-p+2],-b*x^n/a)
/d^2/n/(1-p)/((1+b*x^n/a)^p)+x^(n*(-p+2))*(a+b*x^n)^p*hypergeom([-p, -p+2]
,[3-p],-b*x^n/a)/d/n/(-p+2)/((1+b*x^n/a)^p)-c^2*(a+b*x^n)^p*hypergeom([-p,
 -p],[1-p],-b*x^n/a)/d^3/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = -\frac{x^{-n(-3+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(3-p,-p,1,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(-3+p)}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
-(((a + b*x^n)^p*AppellF1[3 - p, -p, 1, 4 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c*n*(-3 + p)*x^(n*(-3 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{c+dx^n}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{dx^n+c}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n(3-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(3-p,-p,1,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cn(3-p)}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n),x]
```

output
```
(x^(n*(3 - p))*(a + b*x^n)^p*AppellF1[3 - p, -p, 1, 4 - p, -((b*x^n)/a), -
((d*x^n)/c)])/(c*n*(3 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^n)^p}{c+d\,x^n}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{dx^n+c}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{c+dx^n}\,dx$$
$$= \frac{x^{2n}(x^nb+a)^p\,b\,d^2p + x^n(x^nb+a)^p\,a\,d^2p^2 - 2x^n(x^nb+a)^p\,bcdp - (x^nb+a)^p\,acd\,p^2 + (x^nb+a)^p\,acdp +}{}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n),x)
```

output
```
(x**(2*n)*(x**n*b + a)**p*b*d**2*p + x**n*(x**n*b + a)**p*a*d**2*p**2 - 2*
x**n*(x**n*b + a)**p*b*c*d*p - (x**n*b + a)**p*a*c*d*p**2 + (x**n*b + a)**
p*a*c*d*p + 2*(x**n*b + a)**p*b*c**2*p - 2*(x**n*b + a)**p*b*c**2 - x**(n*
p)*int((x**n*b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*
p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*c**2*d*n*p**3 + x**(n*p)*int((x**n*
b + a)**p/(x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x
+ x**(n*p)*a*c*x),x)*a**2*c**2*d*n*p**2 + 2*x**(n*p)*int((x**n*b + a)**p/(
x**(n*p + 2*n)*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*
a*c*x),x)*a*b*c**3*n*p**2 - 2*x**(n*p)*int((x**n*b + a)**p/(x**(n*p + 2*n)
*b*d*x + x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*b*
c**3*n*p + x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p + 2*n)*b*d*x +
 x**(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*d**3*n*
p**3 - x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p + 2*n)*b*d*x + x**
(n*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a**2*d**3*n*p**2
 - 2*x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p + 2*n)*b*d*x + x**(n
*p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*a*b*c*d**2*n*p**2
+ 2*x**(n*p)*int((x**(2*n)*(x**n*b + a)**p)/(x**(n*p + 2*n)*b*d*x + x**(n*
p + n)*a*d*x + x**(n*p + n)*b*c*x + x**(n*p)*a*c*x),x)*b**2*c**2*d*n*p)/(2
*x**(n*p)*b*d**3*n*p)
```

3.558 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 264

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = -\frac{2cx^{-np}(a+bx^n)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^3np} + \frac{ax^{n(1-p)}(a+bx^n)^{-1+p}\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^2n(1-p)} + \frac{x^{n(1-p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^n}{a}\right)}{d^2n(1-p)} + \frac{2cx^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{d^3np}$$

output
```
-2*c*(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d^3/n
/p/(x^(n*p))+a*x^(n*(1-p))*(a+b*x^n)^(-1+p)*hypergeom([2, 1-p],[-p+2],(-a*
d+b*c)*x^n/c/(a+b*x^n))/d^2/n/(1-p)+x^(n*(1-p))*(a+b*x^n)^p*hypergeom([-p,
 1-p],[-p+2],-b*x^n/a)/d^2/n/(1-p)/((1+b*x^n/a)^p)+2*c*(a+b*x^n)^p*hyperge
om([-p, -p],[1-p],-b*x^n/a)/d^3/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx$$
$$= -\frac{x^{-n(-3+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(3-p,-p,2,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2n(-3+p)}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
-(((a + b*x^n)^p*AppellF1[3 - p, -p, 2, 4 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c^2*n*(-3 + p)*x^(n*(-3 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{(c+dx^n)^2}\,dx$$
$$\Big\downarrow \text{1013}$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^2}dx$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^{n(3-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(3-p,-p,2,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2n(3-p)}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^2,x]
```

output
```
(x^(n*(3 - p))*(a + b*x^n)^p*AppellF1[3 - p, -p, 2, 4 - p, -((b*x^n)/a), -
((d*x^n)/c)])/(c^2*n*(3 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{2}}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d^2*x^(2*n) + 2*c*d*x^n + c^2),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^2}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)^2}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^2),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^2,x)
```

output

```
(x**(2*n)*(x**n*b + a)**p*a*d**2*p**2 + x**(2*n)*(x**n*b + a)**p*a*d**2*p
- 2*x**(2*n)*(x**n*b + a)**p*b*c*d*p - x**n*(x**n*b + a)**p*a*c*d*p**2 + x
**n*(x**n*b + a)**p*a*c*d*p + 2*x**n*(x**n*b + a)**p*a*c*d + (x**n*b + a)*
*p*a*c**2*p**2 - 3*(x**n*b + a)**p*a*c**2*p + 2*(x**n*b + a)**p*a*c**2 + x
**(n*p + n)*int((x**n*b + a)**p/(x**(n*p + 3*n)*a*b*d**3*p*x + x**(n*p + 3
*n)*a*b*d**3*x - 2*x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3
*p*x + x**(n*p + 2*n)*a**2*d**3*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - 4*x*
*(n*p + 2*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + 2*x**(n*p +
n)*a**2*c*d**2*x + x**(n*p + n)*a*b*c**2*d*p*x - 3*x**(n*p + n)*a*b*c**2*d
*x - 2*x**(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c**2*d*p*x + x**(n*p)*a**2
*c**2*d*x - 2*x**(n*p)*a*b*c**3*x),x)*a**3*c**3*d**2*n*p**4 - 2*x**(n*p +
n)*int((x**n*b + a)**p/(x**(n*p + 3*n)*a*b*d**3*p*x + x**(n*p + 3*n)*a*b*d
**3*x - 2*x**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + x*
*(n*p + 2*n)*a**2*d**3*x + 2*x**(n*p + 2*n)*a*b*c*d**2*p*x - 4*x**(n*p + 2
*n)*b**2*c**2*d*x + 2*x**(n*p + n)*a**2*c*d**2*p*x + 2*x**(n*p + n)*a**2*c
*d**2*x + x**(n*p + n)*a*b*c**2*d*p*x - 3*x**(n*p + n)*a*b*c**2*d*x - 2*x*
*(n*p + n)*b**2*c**3*x + x**(n*p)*a**2*c**2*d*p*x + x**(n*p)*a**2*c**2*d*x
 - 2*x**(n*p)*a*b*c**3*x),x)*a**3*c**3*d**2*n*p**3 - x**(n*p + n)*int((x**
n*b + a)**p/(x**(n*p + 3*n)*a*b*d**3*p*x + x**(n*p + 3*n)*a*b*d**3*x - 2*x
**(n*p + 3*n)*b**2*c*d**2*x + x**(n*p + 2*n)*a**2*d**3*p*x + x**(n*p + ...
```

3.559 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 273

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \frac{x^{-np}(a+bx^n)^p \text{Hypergeometric2F1}\left(1,-p,1-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{d^3np} - \frac{ax^{n(1-p)}(a+bx^n)^{-1+p}\text{Hypergeometric2F1}\left(2,1-p,2-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{cd^2n(1-p)} - \frac{a^2x^{n(2-p)}(a+bx^n)^{-2+p}\text{Hypergeometric2F1}\left(3,2-p,3-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^2dn(2-p)} - \frac{x^{-np}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^n}{a}\right)}{d^3np}$$

output
```
(a+b*x^n)^p*hypergeom([1, -p],[1-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/d^3/n/p/(x
^(n*p))-a*x^(n*(1-p))*(a+b*x^n)^(-1+p)*hypergeom([2, 1-p],[-p+2],(-a*d+b*c
)*x^n/c/(a+b*x^n))/c/d^2/n/(1-p)-a^2*x^(n*(-p+2))*(a+b*x^n)^(-2+p)*hyperge
om([3, -p+2],[3-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^2/d/n/(-p+2)-(a+b*x^n)^p*
hypergeom([-p, -p],[1-p],-b*x^n/a)/d^3/n/p/(x^(n*p))/((1+b*x^n/a)^p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.29

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$
$$= -\frac{x^{-n(-3+p)}(a+bx^n)^p\left(\frac{a+bx^n}{a}\right)^{-p}\text{AppellF1}\left(3-p,-p,3,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3n(-3+p)}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
-(((a + b*x^n)^p*AppellF1[3 - p, -p, 3, 4 - p, -((b*x^n)/a), -((d*x^n)/c)]
)/(c^3*n*(-3 + p)*x^(n*(-3 + p))*((a + b*x^n)/a)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{(c+dx^n)^3}\,dx$$
$$\Big\downarrow \text{1013}$$
$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^3}dx$$
$$\Big\downarrow \text{1012}$$
$$\frac{x^{n(3-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{AppellF1}\left(3-p,-p,3,4-p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3n(3-p)}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^3,x]
```

output
```
(x^(n*(3 - p))*(a + b*x^n)^p*AppellF1[3 - p, -p, 3, 4 - p, -((b*x^n)/a), -
((d*x^n)/c)])/(c^3*n*(3 - p)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{3}}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) +
 3*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^3, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^3}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)^3}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^3),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^3}\,dx = \int \frac{x^{3n}(x^n b+a)^p}{x^{np+3n}d^3x+3x^{np+2n}c\,d^2x+3x^{np+n}c^2dx+x^{np}c^3x}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^3,x)
```

output
```
int((x**(3*n)*(x**n*b + a)**p)/(x**(n*p + 3*n)*d**3*x + 3*x**(n*p + 2*n)*c
*d**2*x + 3*x**(n*p + n)*c**2*d*x + x**(n*p)*c**3*x),x)
```

3.560 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \frac{a^3x^{n(3-p)}(a+bx^n)^{-3+p}\,\text{Hypergeometric2F1}\left(4,3-p,4-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^4n(3-p)}$$

output
```
a^3*x^(n*(3-p))*(a+b*x^n)^(-3+p)*hypergeom([4, 3-p],[4-p],(-a*d+b*c)*x^n/c
/(a+b*x^n))/c^4/n/(3-p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.41

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = -\frac{x^{-n(-3+p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(1+\frac{dx^n}{c}\right)^{-3+p}\text{Hypergeometric2F1}\left(3-p,-p,4-p,\frac{-\frac{bx^n}{a}+\frac{dx^n}{c}}{1+\frac{dx^n}{c}}\right)}{c^4n(-3+p)}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^4,x]
```

output
```
-(((a + b*x^n)^p*(1 + (d*x^n)/c)^(-3 + p)*Hypergeometric2F1[3 - p, -p, 4 -
 p, (-((b*x^n)/a) + (d*x^n)/c)/(1 + (d*x^n)/c)])/(c^4*n*(-3 + p)*x^(n*(-3
+ p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{(c+dx^n)^4}dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^4}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n(3-p)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{dx^n}{c}+1\right)^{p-3}\text{Hypergeometric2F1}\left(3-p,-p,4-p,-\frac{\frac{bx^n}{a}-\frac{dx^n}{c}}{\frac{dx^n}{c}+1}\right)}{c^4n(3-p)}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^4,x]
```

output
```
(x^(n*(3 - p))*(a + b*x^n)^p*(1 + (d*x^n)/c)^(-3 + p)*Hypergeometric2F1[3
- p, -p, 4 - p, -(((b*x^n)/a - (d*x^n)/c)/(1 + (d*x^n)/c))])/(c^4*n*(3 - p
)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{4}}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^{n})^{p}}{(c+dx^{n})^{4}}\,dx = \int \frac{(bx^{n}+a)^{p}x^{-n(p-3)-1}}{(dx^{n}+c)^{4}}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) +
 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^4, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^4}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)^4}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^4),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^4), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^4}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^4,x)
```

output
```
( - x**(3*n)*(x***n*b + a)**p*a**3*b*d**3*p**3 - 7*x**(3*n)*(x***n*b + a)**p
*a**3*b*d**3*p**2 - 16*x**(3*n)*(x***n*b + a)**p*a**3*b*d**3*p - 12*x**(3*n
)*(x***n*b + a)**p*a**3*b*d**3 + 9*x**(3*n)*(x***n*b + a)**p*a**2*b**2*c*d**
2*p**2 + 33*x**(3*n)*(x***n*b + a)**p*a**2*b**2*c*d**2*p + 30*x**(3*n)*(x**
n*b + a)**p*a**2*b**2*c*d**2 - 16*x**(3*n)*(x***n*b + a)**p*a*b**3*c**2*d*p
 - 28*x**(3*n)*(x***n*b + a)**p*a*b**3*c**2*d + 8*x**(3*n)*(x***n*b + a)**p*
b**4*c**3 - x**(2*n)*(x***n*b + a)**p*a**4*d**3*p**3 - 7*x**(2*n)*(x***n*b +
 a)**p*a**4*d**3*p**2 - 16*x**(2*n)*(x***n*b + a)**p*a**4*d**3*p - 12*x**(2
*n)*(x***n*b + a)**p*a**4*d**3 + x**(2*n)*(x***n*b + a)**p*a**3*b*c*d**2*p**
3 + 12*x**(2*n)*(x***n*b + a)**p*a**3*b*c*d**2*p**2 + 29*x**(2*n)*(x***n*b +
 a)**p*a**3*b*c*d**2*p + 18*x**(2*n)*(x***n*b + a)**p*a**3*b*c*d**2 - 5*x**
(2*n)*(x***n*b + a)**p*a**2*b**2*c**2*d*p**2 - 17*x**(2*n)*(x***n*b + a)**p*
a**2*b**2*c**2*d*p - 6*x**(2*n)*(x***n*b + a)**p*a**2*b**2*c**2*d + 4*x**(2
*n)*(x***n*b + a)**p*a*b**3*c**3*p + x***n*(x***n*b + a)**p*a**4*c*d**2*p**3
+ 3*x***n*(x***n*b + a)**p*a**4*c*d**2*p**2 - 4*x***n*(x***n*b + a)**p*a**4*c*
d**2*p - 12*x***n*(x***n*b + a)**p*a**4*c*d**2 - x***n*(x***n*b + a)**p*a**3*b
*c**2*d*p**3 - 7*x***n*(x***n*b + a)**p*a**3*b*c**2*d*p**2 + 12*x***n*(x***n*b
 + a)**p*a**3*b*c**2*d*p + 12*x***n*(x***n*b + a)**p*a**3*b*c**2*d + 4*x***n*
(x***n*b + a)**p*a**2*b**2*c**3*p**2 - 8*x***n*(x***n*b + a)**p*a**2*b**2*c**
3*p - 6*x**(n*p + 3*n)*int((x**(3*n)*(x***n*b + a)**p)/(x**(n*p + 5*n)*a...
```

3.561 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5} dx$

Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5} dx = \frac{x^{n(3-p)}(a+bx^n)^{1+p}}{acn(3-p)(c+dx^n)^4} - \frac{a^3(4bc-ad(1+p))x^{n(4-p)}(a+bx^n)^{-4+p}\,\text{Hypergeometric2F1}\left(5,4-p,5-p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{c^6n(3-p)(4-p)}$$

output
```
x^(n*(3-p))*(a+b*x^n)^(p+1)/a/c/n/(3-p)/(c+d*x^n)^4-a^3*(4*b*c-a*d*(p+1))*
x^(n*(4-p))*(a+b*x^n)^(-4+p)*hypergeom([5, 4-p],[5-p],(-a*d+b*c)*x^n/c/(a+
b*x^n))/c^6/n/(3-p)/(4-p)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.58 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.01

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5} dx$$

$$= \frac{p(2-3p+p^2)\,x^{-n(-3+p)}(a+bx^n)^p\left(4(3c+dpx^n)\,\Phi\left(\frac{(bc-ad)x^n}{c(a+bx^n)},1,1-p\right)-6(2c+d(-1+p)x^n)\,\Phi\left(\frac{(bc-a\cdots}{c(a+\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^5,x]
```

output
```
(p*(2 - 3*p + p^2)*(a + b*x^n)^p*(4*(3*c + d*p*x^n)*HurwitzLerchPhi[((b*c
- a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 6*(2*c + d*(-1 + p)*x^n)*HurwitzL
erchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + 4*c*HurwitzLerchPhi
[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 8*d*x^n*HurwitzLerchPhi[((
b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 4*d*p*x^n*HurwitzLerchPhi[((b
*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 3*d*x^n*HurwitzLerchPhi[((b*c
- a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - d*p*x^n*HurwitzLerchPhi[((b*c - a
*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 4*c*HurwitzLerchPhi[((b*c - a*d)*x^n
)/(c*(a + b*x^n)), 1, -p] - d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a
+ b*x^n)), 1, -p] - d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, -p]))/(24*c^2*n*x^(n*(-3 + p))*(c + d*x^n)^4)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.38 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{(c+dx^n)^5}\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^5}dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(1-p)(2-p)px^{n(3-p)}(a+bx^n)^p\left(-8dx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)+4dpx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)+3dx^n\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+}\right.\right.}{}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^5,x]
```

output
```
((1 - p)*(2 - p)*p*x^(n*(3 - p))*(a + b*x^n)^p*(4*(3*c + d*p*x^n)*HurwitzL
erchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 6*(2*c - d*(1 - p)*
x^n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + 4*c*Hu
rwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 8*d*x^n*Hurwi
tzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 4*d*p*x^n*Hurwit
zLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 3*d*x^n*HurwitzLe
rchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - d*p*x^n*HurwitzLerch
Phi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 4*c*HurwitzLerchPhi[((b
*c - a*d)*x^n)/(c*(a + b*x^n)), 1, -p] - d*x^n*HurwitzLerchPhi[((b*c - a*d
)*x^n)/(c*(a + b*x^n)), 1, -p] - d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)
/(c*(a + b*x^n)), 1, -p]))/(24*c^6*n*(1 + (d*x^n)/c)^4)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^{n})^{p}}{(c+d\,x^{n})^{5}}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d^5*x^(5*n) + 5*c*d^4*x^(4*n) +
 10*c^2*d^3*x^(3*n) + 10*c^3*d^2*x^(2*n) + 5*c^4*d*x^n + c^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^5, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^5}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)^5}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^5),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^5), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^5}\,dx = \text{too large to display}$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^5,x)
```

output
```
too large to display
```

3.562 $\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6} dx$

Optimal result

Integrand size = 29, antiderivative size = 245

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6} dx = \frac{x^{n(3-p)}(a+bx^n)^{1+p}}{acn(3-p)(c+dx^n)^5} + \frac{d(5bc-ad(2+p))x^{n(4-p)}(a+bx^n)^{1+p}}{5ac^2(bc-ad)n(3-p)(c+dx^n)^5} - \frac{a^3(20b^2c^2-10abcd(1+p)+a^2d^2(2+3p+p^2))x^{n(4-p)}(a+bx^n)^{-4+p}\text{Hypergeometric2F1}\left(5,4-p,5\right.}{5c^7(bc-ad)n(3-p)(4-p)}$$

output
```
x^(n*(3-p))*(a+b*x^n)^(p+1)/a/c/n/(3-p)/(c+d*x^n)^5+1/5*d*(5*b*c-a*d*(2+p)
)*x^(n*(4-p))*(a+b*x^n)^(p+1)/a/c^2/(-a*d+b*c)/n/(3-p)/(c+d*x^n)^5-1/5*a^3
*(20*b^2*c^2-10*a*b*c*d*(p+1)+a^2*d^2*(p^2+3*p+2))*x^(n*(4-p))*(a+b*x^n)^(
-4+p)*hypergeom([5, 4-p],[5-p],(-a*d+b*c)*x^n/c/(a+b*x^n))/c^7/(-a*d+b*c)/
n/(3-p)/(4-p)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.41 (sec) , antiderivative size = 966, normalized size of antiderivative = 3.94

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^6,x]
```

output
```
(p*(2 - 3*p + p^2)*(a + b*x^n)^p*(5*(12*c^2 + 8*c*d*p*x^n + d^2*p*(1 + p)*
x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 1 - p] - 10
*(6*c^2 + 6*c*d*(-1 + p)*x^n + d^2*(-1 + p)*p*x^(2*n))*HurwitzLerchPhi[((b
*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + 20*c^2*HurwitzLerchPhi[((b*c -
 a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 80*c*d*x^n*HurwitzLerchPhi[((b*c -
 a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 40*c*d*p*x^n*HurwitzLerchPhi[((b*c
 - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 20*d^2*x^(2*n)*HurwitzLerchPhi[(
(b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 30*d^2*p*x^(2*n)*HurwitzLerc
hPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 10*d^2*p^2*x^(2*n)*Hur
witzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 30*c*d*x^n*Hur
witzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 10*c*d*p*x^n*H
urwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 30*d^2*x^(2*
n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] + 25*d^2*p
*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 5*
d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 -
p] + 12*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1,
5 - p] - 7*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n))
, 1, 5 - p] + d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*
x^n)), 1, 5 - p] - 20*c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)
), 1, -p] - 10*c*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)...
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.18 (sec) , antiderivative size = 977, normalized size of antiderivative = 3.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n(p-3)-1}(a+bx^n)^p}{(c+dx^n)^6}\,dx$$

$$\Big\downarrow \ 1013$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int \frac{x^{n(3-p)-1}\left(\frac{bx^n}{a}+1\right)^p}{(dx^n+c)^6}dx$$

$$\Big\downarrow \ 1012$$

$$\frac{(1-p)(2-p)px^{n(3-p)}(bx^n+a)^p\left(-80cd\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)x^n+40cdp\Phi\left(\frac{(bc-ad)x^n}{c(bx^n+a)},1,3-p\right)x^n+30cd\Phi\left(\frac{(b}{c}\right.\right.}{}$$

input
```
Int[(x^(-1 - n*(-3 + p))*(a + b*x^n)^p)/(c + d*x^n)^6,x]
```

output
```
((1 - p)*(2 - p)*p*x^(n*(3 - p))*(a + b*x^n)^p*(5*(12*c^2 + 8*c*d*p*x^n +
d^2*p*(1 + p)*x^(2*n))*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)),
1, 1 - p] - 10*(6*c^2 - 6*c*d*(1 - p)*x^n - d^2*(1 - p)*p*x^(2*n))*Hurwitz
LerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 2 - p] + 20*c^2*HurwitzLerc
hPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 80*c*d*x^n*HurwitzLerc
hPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 40*c*d*p*x^n*HurwitzLe
rchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 20*d^2*x^(2*n)*Hurwi
tzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] - 30*d^2*p*x^(2*n)
*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 10*d^2*p^2
*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 3 - p] + 30
*c*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] - 10
*c*d*p*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p] -
30*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1, 4 - p
] + 25*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^n)), 1,
 4 - p] - 5*d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a + b*x^
n)), 1, 4 - p] + 12*d^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*(a +
b*x^n)), 1, 5 - p] - 7*d^2*p*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c*
(a + b*x^n)), 1, 5 - p] + d^2*p^2*x^(2*n)*HurwitzLerchPhi[((b*c - a*d)*x^n
)/(c*(a + b*x^n)), 1, 5 - p] - 20*c^2*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c
*(a + b*x^n)), 1, -p] - 10*c*d*x^n*HurwitzLerchPhi[((b*c - a*d)*x^n)/(c...
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-1-n(-3+p)}(a+b\,x^n)^p}{(c+d\,x^n)^6}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x)
```

Fricas [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^6}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-n*p + 3*n - 1)/(d^6*x^(6*n) + 6*c*d^5*x^(5*n) +
 15*c^2*d^4*x^(4*n) + 20*c^3*d^3*x^(3*n) + 15*c^4*d^2*x^(2*n) + 6*c^5*d*x^
n + c^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \text{Timed out}$$

input
```
integrate(x**(-1-n*(-3+p))*(a+b*x**n)**p/(c+d*x**n)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^6}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^6, x)
```

Giac [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \int \frac{(bx^n+a)^p x^{-n(p-3)-1}}{(dx^n+c)^6}\,dx$$

input
```
integrate(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-n*(p - 3) - 1)/(d*x^n + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx = \int \frac{(a+b\,x^n)^p}{x^{n\,(p-3)+1}\,(c+d\,x^n)^6}\,dx$$

input
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^6),x)
```

output
```
int((a + b*x^n)^p/(x^(n*(p - 3) + 1)*(c + d*x^n)^6), x)
```

Reduce [F]

$$\int \frac{x^{-1-n(-3+p)}(a+bx^n)^p}{(c+dx^n)^6}\,dx=\int \frac{x^{-1-n(-3+p)}(x^nb+a)^p}{(x^nd+c)^6}dx$$

input
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x)
```

output
```
int(x^(-1-n*(-3+p))*(a+b*x^n)^p/(c+d*x^n)^6,x)
```

3.563 $\int x^m(2+bx^n)^p (3+dx^n)^q \, dx$

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int x^m(2+bx^n)^p (3+dx^n)^q \, dx = \frac{2^p 3^q x^{1+m} \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{2},-\frac{dx^n}{3}\right)}{1+m}$$

output
```
2^p*3^q*x^(1+m)*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-1/2*b*x^n,-1/3*d*x^n)/(1
+m)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^m(2+bx^n)^p (3+dx^n)^q \, dx = \frac{2^p 3^q x^{1+m} \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-q,1+\frac{1+m}{n},-\frac{bx^n}{2},-\frac{dx^n}{3}\right)}{1+m}$$

input
```
Integrate[x^m*(2 + b*x^n)^p*(3 + d*x^n)^q,x]
```

output
```
(2^p*3^q*x^(1 + m)*AppellF1[(1 + m)/n, -p, -q, 1 + (1 + m)/n, -1/2*(b*x^n)
, -1/3*(d*x^n)])/(1 + m)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (bx^n + 2)^p (dx^n + 3)^q \, dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{2^p 3^q x^{m+1} \text{AppellF1}\left(\frac{m+1}{n}, -p, -q, \frac{m+n+1}{n}, -\frac{bx^n}{2}, -\frac{dx^n}{3}\right)}{m+1}$$

input
```
Int[x^m*(2 + b*x^n)^p*(3 + d*x^n)^q,x]
```

output
```
(2^p*3^q*x^(1 + m)*AppellF1[(1 + m)/n, -p, -q, (1 + m + n)/n, -1/2*(b*x^n)
, -1/3*(d*x^n)])/(1 + m)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Maple [F]

$$\int x^m (2 + b\,x^n)^p\,(3 + d\,x^n)^q\,dx$$

input
```
int(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x)
```

output
```
int(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x)
```

Fricas [F]

$$\int x^m (2 + bx^n)^p\,(3 + dx^n)^q\;dx = \int (bx^n + 2)^p (dx^n + 3)^q x^m\,dx$$

input
```
integrate(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + 2)^p*(d*x^n + 3)^q*x^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m (2 + bx^n)^p\,(3 + dx^n)^q\;dx = \text{Timed out}$$

input
```
integrate(x**m*(2+b*x**n)**p*(3+d*x**n)**q,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m(2+bx^n)^p(3+dx^n)^q\,dx = \int (bx^n+2)^p(dx^n+3)^q x^m\,dx$$

input
```
integrate(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + 2)^p*(d*x^n + 3)^q*x^m, x)
```

Giac [F]

$$\int x^m(2+bx^n)^p(3+dx^n)^q\,dx = \int (bx^n+2)^p(dx^n+3)^q x^m\,dx$$

input
```
integrate(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + 2)^p*(d*x^n + 3)^q*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m(2+bx^n)^p(3+dx^n)^q\,dx = \int x^m\,(b\,x^n+2)^p\,(d\,x^n+3)^q\,dx$$

input
```
int(x^m*(b*x^n + 2)^p*(d*x^n + 3)^q,x)
```

output
```
int(x^m*(b*x^n + 2)^p*(d*x^n + 3)^q, x)
```

Reduce [F]

$$\int x^m (2 + bx^n)^p (3 + dx^n)^q \, dx = \text{too large to display}$$

input
```
int(x^m*(2+b*x^n)^p*(3+d*x^n)^q,x)
```

output
```
(3*x**m*(x**n*d + 3)**q*(x**n*b + 2)**p*b*x + 2*x**m*(x**n*d + 3)**q*(x**n
*b + 2)**p*d*x - 9*int((x**(m + 2*n)*(x**n*d + 3)**q*(x**n*b + 2)**p)/(3*x
**(2*n)*b**2*d*m + 3*x**(2*n)*b**2*d*n*p + 3*x**(2*n)*b**2*d + 2*x**(2*n)*
b*d**2*m + 2*x**(2*n)*b*d**2*n*q + 2*x**(2*n)*b*d**2 + 9*x**n*b**2*m + 9*x
**n*b**2*n*p + 9*x**n*b**2 + 12*x**n*b*d*m + 6*x**n*b*d*n*p + 6*x**n*b*d*n
*q + 12*x**n*b*d + 4*x**n*d**2*m + 4*x**n*d**2*n*q + 4*x**n*d**2 + 18*b*m
+ 18*b*n*p + 18*b + 12*d*m + 12*d*n*q + 12*d),x)*b**3*d*m*n*q - 9*int((x**
(m + 2*n)*(x**n*d + 3)**q*(x**n*b + 2)**p)/(3*x**(2*n)*b**2*d*m + 3*x**(2*
n)*b**2*d*n*p + 3*x**(2*n)*b**2*d + 2*x**(2*n)*b*d**2*m + 2*x**(2*n)*b*d**
2*n*q + 2*x**(2*n)*b*d**2 + 9*x**n*b**2*m + 9*x**n*b**2*n*p + 9*x**n*b**2
+ 12*x**n*b*d*m + 6*x**n*b*d*n*p + 6*x**n*b*d*n*q + 12*x**n*b*d + 4*x**n*d
**2*m + 4*x**n*d**2*n*q + 4*x**n*d**2 + 18*b*m + 18*b*n*p + 18*b + 12*d*m
+ 12*d*n*q + 12*d),x)*b**3*d*n**2*p*q - 9*int((x**(m + 2*n)*(x**n*d + 3)**
q*(x**n*b + 2)**p)/(3*x**(2*n)*b**2*d*m + 3*x**(2*n)*b**2*d*n*p + 3*x**(2*
n)*b**2*d + 2*x**(2*n)*b*d**2*m + 2*x**(2*n)*b*d**2*n*q + 2*x**(2*n)*b*d**
2 + 9*x**n*b**2*m + 9*x**n*b**2*n*p + 9*x**n*b**2 + 12*x**n*b*d*m + 6*x**n
*b*d*n*p + 6*x**n*b*d*n*q + 12*x**n*b*d + 4*x**n*d**2*m + 4*x**n*d**2*n*q
+ 4*x**n*d**2 + 18*b*m + 18*b*n*p + 18*b + 12*d*m + 12*d*n*q + 12*d),x)*b*
*3*d*n*q - 6*int((x**(m + 2*n)*(x**n*d + 3)**q*(x**n*b + 2)**p)/(3*x**(2*n
)*b**2*d*m + 3*x**(2*n)*b**2*d*n*p + 3*x**(2*n)*b**2*d + 2*x**(2*n)*b*d...
```

3.564 $\int (ex)^m (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^q \, dx = \frac{(ex)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-b*x^
n/a,-d*x^n/c)/e/(1+m)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^q \, dx = \frac{x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (1 + m
+ n)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m)*(1 + (b*x^n)/a)^p*(1 + (d*x^
n)/c)^q)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^n\right)^p \left(c+dx^n\right)^q \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \int (ex)^m \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{n},-p,-q,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (1
+ m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(1 + m)*(1 + (b*x^n)/a)^p*(1 +
 (d*x^n)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (c+d\,x^n)^q\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^q\,dx = \int (bx^n+a)^p (dx^n+c)^q (ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^q \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^q \, dx = \int (bx^n + a)^p (dx^n + c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (a + bx^n)^p (c + dx^n)^q \, dx = \int (bx^n + a)^p (dx^n + c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^q \, dx = \int (e\,x)^m \, (a+b\,x^n)^p \, (c+d\,x^n)^q \, dx$$

input
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int((e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (c+dx^n)^q \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
(e**m*(x**m*(x**n*d + c)**q*(x**n*b + a)**p*a*d*x + x**m*(x**n*d + c)**q*(
x**n*b + a)**p*b*c*x - int((x**(m + 2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/
(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n
)*b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*
m + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*
n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*m + x**n*b**2*c**
2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d*n*q + a**2*c*d + a*b*c**2*m
 + a*b*c**2*n*p + a*b*c**2),x)*a**2*b*d**3*m*n*p - int((x**(m + 2*n)*(x**n
*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2*n*q +
 x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n
)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + 2*x*
*n*a*b*c*d*m + x**n*a*b*c*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n
*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d
*n*q + a**2*c*d + a*b*c**2*m + a*b*c**2*n*p + a*b*c**2),x)*a**2*b*d**3*n**
2*p*q - int((x**(m + 2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d
**2*m + x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m +
x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*d
**2*n*q + x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*n*p + x**n*a*b*
c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b*
*2*c**2 + a**2*c*d*m + a**2*c*d*n*q + a**2*c*d + a*b*c**2*m + a*b*c**2*...
```

3.565 $\int x^{-1-n(3+2p)}(a+bx^n)^p (c+dx^n)^p \, dx$

Optimal result

Integrand size = 31, antiderivative size = 264

$$\int x^{-1-n(3+2p)}(a+bx^n)^p (c+dx^n)^p \, dx = \frac{(bc+ad)(2+p)x^{-2n(1+p)}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{2a^2c^2n(1+p)(3+2p)} - \frac{x^{-n(3+2p)}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{acn(3+2p)} - \frac{(2abcd(1+p)+b^2c^2(2+p)+a^2d^2(2+p))\, x^{-n(1+2p)}(a+bx^n)^{1+p}(c+dx^n)^p \left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{-p} \text{Hypergeom}}{2a^3c^2n(1+2p)(3+2p)}$$

output

```
1/2*(a*d+b*c)*(2+p)*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a^2/c^2/n/(p+1)/(3+2*p
)/(x^(2*n*(p+1)))-(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c/n/(3+2*p)/(x^(n*(3+2
*p)))-1/2*(2*a*b*c*d*(p+1)+b^2*c^2*(2+p)+a^2*d^2*(2+p))*(a+b*x^n)^(p+1)*(c
+d*x^n)^p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*x^n/c/(a+b*x^n))/a^3/c^
2/n/(1+2*p)/(3+2*p)/(x^(n*(1+2*p)))/((a*(c+d*x^n)/c/(a+b*x^n))^p)
```

Mathematica [F]

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx$$

input
```
Integrate[x^(-1 - n*(3 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
Integrate[x^(-1 - n*(3 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p+3)-1}(a+bx^n)^p\,(c+dx^n)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x^{-2pn-3n-1}\left(\frac{bx^n}{a}+1\right)^p(dx^n+c)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^p\left(\frac{dx^n}{c}+1\right)^{-p}\int x^{-2pn-3n-1}\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^p dx$$

$$\Big\downarrow \text{7299}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^p\left(\frac{dx^n}{c}+1\right)^{-p}\int x^{-2pn-3n-1}\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^p dx$$

input
```
Int[x^(-1 - n*(3 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{-1-n(3+2p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\;dx=\int (bx^n+a)^p(dx^n+c)^px^{-n(2\,p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - 3*n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\ dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(3+2*p))*(a+b*x**n)**p*(c+d*x**n)**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\ dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p + 3) - 1), x)
```

Giac [F]

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\ dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2p+3)-1}\,dx$$

input
```
integrate(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p + 3) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^p}{x^{n\,(2\,p+3)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 3) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 3) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(3+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\text{too large to display}$$

input
```
int(x^(-1-n*(3+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output

```
( - x**(3*n)*(x**n*d + c)**p*(x**n*b + a)**p*a**2*b*d**3*p - 2*x**(3*n)*(x
**n*d + c)**p*(x**n*b + a)**p*a**2*b*d**3 + 2*x**(3*n)*(x**n*d + c)**p*(x*
*n*b + a)**p*a*b**2*c*d**2*p - x**(3*n)*(x**n*d + c)**p*(x**n*b + a)**p*b*
*3*c**2*d*p - 2*x**(3*n)*(x**n*d + c)**p*(x**n*b + a)**p*b**3*c**2*d + x**
(2*n)*(x**n*d + c)**p*(x**n*b + a)**p*a**3*d**3*p**2 + 2*x**(2*n)*(x**n*d
+ c)**p*(x**n*b + a)**p*a**3*d**3*p - x**(2*n)*(x**n*d + c)**p*(x**n*b + a
)**p*a**2*b*c*d**2*p**2 + 2*x**(2*n)*(x**n*d + c)**p*(x**n*b + a)**p*a**2*
b*c*d**2*p - x**(2*n)*(x**n*d + c)**p*(x**n*b + a)**p*a*b**2*c**2*d*p**2 +
 2*x**(2*n)*(x**n*d + c)**p*(x**n*b + a)**p*a*b**2*c**2*d*p + x**(2*n)*(x*
*n*d + c)**p*(x**n*b + a)**p*b**3*c**3*p**2 + 2*x**(2*n)*(x**n*d + c)**p*(
x**n*b + a)**p*b**3*c**3*p - 2*x**n*(x**n*d + c)**p*(x**n*b + a)**p*a**3*c
*d**2*p**2 - x**n*(x**n*d + c)**p*(x**n*b + a)**p*a**3*c*d**2*p - 4*x**n*(
x**n*d + c)**p*(x**n*b + a)**p*a**2*b*c**2*d*p**2 - 2*x**n*(x**n*d + c)**p
*(x**n*b + a)**p*a**2*b*c**2*d*p - 2*x**n*(x**n*d + c)**p*(x**n*b + a)**p*
a*b**2*c**3*p**2 - x**n*(x**n*d + c)**p*(x**n*b + a)**p*a*b**2*c**3*p - 4*
(x**n*d + c)**p*(x**n*b + a)**p*a**3*c**2*d*p**2 - 6*(x**n*d + c)**p*(x**n
*b + a)**p*a**3*c**2*d*p - 2*(x**n*d + c)**p*(x**n*b + a)**p*a**3*c**2*d -
 4*(x**n*d + c)**p*(x**n*b + a)**p*a**2*b*c**3*p**2 - 6*(x**n*d + c)**p*(x
**n*b + a)**p*a**2*b*c**3*p - 2*(x**n*d + c)**p*(x**n*b + a)**p*a**2*b*c**
3 + 4*x**(2*n*p + 3*n)*int(((x**n*d + c)**p*(x**n*b + a)**p)/(4*x**(2*n...
```

3.566 $\int x^{-1-n(2+2p)}(a+bx^n)^p (c+dx^n)^p \, dx$

Optimal result

Integrand size = 31, antiderivative size = 167

$$\int x^{-1-n(2+2p)}(a+bx^n)^p (c+dx^n)^p \, dx = -\frac{x^{-2n(1+p)}(a+bx^n)^{1+p}(c+dx^n)^{1+p}}{2acn(1+p)} + \frac{(bc+ad)x^{-n(1+2p)}(a+bx^n)^{1+p}(c+dx^n)^p \left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{-p} \text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(bc-ad)}{c(a+b}\right.}{2a^2cn(1+2p)}$$

output
```
-1/2*(a+b*x^n)^(p+1)*(c+d*x^n)^(p+1)/a/c/n/(p+1)/(x^(2*n*(p+1)))+1/2*(a*d+
b*c)*(a+b*x^n)^(p+1)*(c+d*x^n)^p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*
x^n/c/(a+b*x^n))/a^2/c/n/(1+2*p)/(x^(n*(1+2*p)))/((a*(c+d*x^n)/c/(a+b*x^n)
)^p)
```

Mathematica [F]

$$\int x^{-1-n(2+2p)}(a+bx^n)^p (c+dx^n)^p \, dx = \int x^{-1-n(2+2p)}(a+bx^n)^p (c+dx^n)^p \, dx$$

input
```
Integrate[x^(-1 - n*(2 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
Integrate[x^(-1 - n*(2 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p+2)-1}(a+bx^n)^p (c+dx^n)^p \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^{-2n(p+1)-1} \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^p \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \int x^{-2n(p+1)-1} \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^p dx$$

$$\Big\downarrow \text{7299}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \int x^{-2n(p+1)-1} \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^p dx$$

input
```
Int[x^(-1 - n*(2 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{-1-n(2+2p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(2+2p)}(a+bx^{n})^{p}\,(c+dx^{n})^{p}\;dx=\int (bx^{n}+a)^{p}(dx^{n}+c)^{p}x^{-2\,n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - 2*n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-n(2+2p)}(a+bx^{n})^{p}\,(c+dx^{n})^{p}\;dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(2+2*p))*(a+b*x**n)**p*(c+d*x**n)**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-n(2+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int (bx^n+a)^p(dx^n+c)^p x^{-2\,n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*(p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(2+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int (bx^n+a)^p(dx^n+c)^p x^{-2\,n(p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*(p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(2+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^p}{x^{n\,(2\,p+2)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 2) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 2) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(2+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\text{too large to display}$$

input
```
int(x^(-1-n*(2+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
(x**(2*n)*(x**n*d + c)**p*(x**n*b + a)**p*b*d - x**n*(x**n*d + c)**p*(x**n
*b + a)**p*a*d*p - x**n*(x**n*d + c)**p*(x**n*b + a)**p*b*c*p - 2*(x**n*d
+ c)**p*(x**n*b + a)**p*a*c*p - (x**n*d + c)**p*(x**n*b + a)**p*a*c - 2*x*
*(2*n*p + 2*n)*int(((x**n*d + c)**p*(x**n*b + a)**p)/(2*x**(2*n*p + 2*n)*a
*b*d**2*p*x + x**(2*n*p + 2*n)*a*b*d**2*x + 2*x**(2*n*p + 2*n)*b**2*c*d*p*
x + x**(2*n*p + 2*n)*b**2*c*d*x + 2*x**(2*n*p + n)*a**2*d**2*p*x + x**(2*n
*p + n)*a**2*d**2*x + 4*x**(2*n*p + n)*a*b*c*d*p*x + 2*x**(2*n*p + n)*a*b*
c*d*x + 2*x**(2*n*p + n)*b**2*c**2*p*x + x**(2*n*p + n)*b**2*c**2*x + 2*x*
*(2*n*p)*a**2*c*d*p*x + x**(2*n*p)*a**2*c*d*x + 2*x**(2*n*p)*a*b*c**2*p*x
+ x**(2*n*p)*a*b*c**2*x),x)*a**3*d**3*n*p**3 - 3*x**(2*n*p + 2*n)*int(((x*
*n*d + c)**p*(x**n*b + a)**p)/(2*x**(2*n*p + 2*n)*a*b*d**2*p*x + x**(2*n*p
 + 2*n)*a*b*d**2*x + 2*x**(2*n*p + 2*n)*b**2*c*d*p*x + x**(2*n*p + 2*n)*b*
*2*c*d*x + 2*x**(2*n*p + n)*a**2*d**2*p*x + x**(2*n*p + n)*a**2*d**2*x + 4
*x**(2*n*p + n)*a*b*c*d*p*x + 2*x**(2*n*p + n)*a*b*c*d*x + 2*x**(2*n*p + n
)*b**2*c**2*p*x + x**(2*n*p + n)*b**2*c**2*x + 2*x**(2*n*p)*a**2*c*d*p*x +
 x**(2*n*p)*a**2*c*d*x + 2*x**(2*n*p)*a*b*c**2*p*x + x**(2*n*p)*a*b*c**2*x
),x)*a**3*d**3*n*p**2 - x**(2*n*p + 2*n)*int(((x**n*d + c)**p*(x**n*b + a)
**p)/(2*x**(2*n*p + 2*n)*a*b*d**2*p*x + x**(2*n*p + 2*n)*a*b*d**2*x + 2*x*
*(2*n*p + 2*n)*b**2*c*d*p*x + x**(2*n*p + 2*n)*b**2*c*d*x + 2*x**(2*n*p +
n)*a**2*d**2*p*x + x**(2*n*p + n)*a**2*d**2*x + 4*x**(2*n*p + n)*a*b*c*...
```

3.567 $\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx$

Optimal result

Integrand size = 31, antiderivative size = 106

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx = -\frac{x^{-n(1+2p)}(a+bx^n)^{1+p}\,(c+dx^n)^p\left(\frac{a(c+dx^n)}{c(a+bx^n)}\right)^{-p}\text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(bc-ad)x^n}{c(a+bx^n)}\right)}{an(1+2p)}$$

output
```
-(a+b*x^n)^(p+1)*(c+d*x^n)^p*hypergeom([-p, -1-2*p],[-2*p],(-a*d+b*c)*x^n/
c/(a+b*x^n))/a/n/(1+2*p)/(x^(n*(1+2*p)))/((a*(c+d*x^n)/c/(a+b*x^n))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx = -\frac{x^{-n(1+2p)}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}(c+dx^n)^{1+p}\left(1+\frac{dx^n}{c}\right)^p\text{Hypergeometric2F1}\left(-1-2p,-p,-2p,\frac{(-bc\cdots}{a(c\cdots}\right.}{cn(1+2p)}$$

input
```
Integrate[x^(-1 - n*(1 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*(c + d*x^n)^(1 + p)*(1 + (d*x^n)/c)^p*Hypergeometric2F1[-
1 - 2*p, -p, -2*p, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*n*(1 + 2*p)*x
^(n*(1 + 2*p))*(1 + (b*x^n)/a)^p))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p+1)-1}(a+bx^n)^p(c+dx^n)^p\,dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x^{-2pn-n-1}\left(\frac{bx^n}{a}+1\right)^p(dx^n+c)^p dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^p\left(\frac{dx^n}{c}+1\right)^{-p}\int x^{-2pn-n-1}\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^p dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-n(2p+1)}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^p\left(\frac{dx^n}{c}+1\right)^{p+1}\text{Hypergeometric2F1}\left(-2p-1,-p,-2p,-\frac{\frac{bx^n}{a}-\frac{dx^n}{c}}{\frac{dx^n}{c}+1}\right)}{n(2p+1)}$$

input
```
Int[x^(-1 - n*(1 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*(c + d*x^n)^p*(1 + (d*x^n)/c)^(1 + p)*Hypergeometric2F1[-
1 - 2*p, -p, -2*p, -(((b*x^n)/a - (d*x^n)/c)/(1 + (d*x^n)/c))])/(n*(1 + 2*
p)*x^(n*(1 + 2*p))*(1 + (b*x^n)/a)^p))
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-n(2p+1)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(2*p+1))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(x^(-1-n*(2*p+1))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(1+2p)}(a+bx^{n})^{p}\,(c+dx^{n})^{p}\;dx=\int(bx^{n}+a)^{p}(dx^{n}+c)^{p}x^{-n(2\,p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(1+2*p))*(a+b*x**n)**p*(c+d*x**n)**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p + 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2p+1)-1}\,dx$$

input
```
integrate(x^(-1-n*(1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p + 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^p}{x^{n\,(2\,p+1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p + 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\text{too large to display}$$

input
```
int(x^(-1-n*(1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output

```
( - 2*x**n*(x**n*d + c)**p*(x**n*b + a)**p*b*d - (x**n*d + c)**p*(x**n*b +
 a)**p*a*d - (x**n*d + c)**p*(x**n*b + a)**p*b*c + 2*x**(2*n*p + n)*int(((
x**n*d + c)**p*(x**n*b + a)**p)/(2*x**(2*n*p + 2*n)*a*b*d**2*p*x + x**(2*n
*p + 2*n)*a*b*d**2*x + 2*x**(2*n*p + 2*n)*b**2*c*d*p*x + x**(2*n*p + 2*n)*
b**2*c*d*x + 2*x**(2*n*p + n)*a**2*d**2*p*x + x**(2*n*p + n)*a**2*d**2*x +
 4*x**(2*n*p + n)*a*b*c*d*p*x + 2*x**(2*n*p + n)*a*b*c*d*x + 2*x**(2*n*p +
 n)*b**2*c**2*p*x + x**(2*n*p + n)*b**2*c**2*x + 2*x**(2*n*p)*a**2*c*d*p*x
 + x**(2*n*p)*a**2*c*d*x + 2*x**(2*n*p)*a*b*c**2*p*x + x**(2*n*p)*a*b*c**2
*x),x)*a**3*d**3*n*p**2 + x**(2*n*p + n)*int(((x**n*d + c)**p*(x**n*b + a)
**p)/(2*x**(2*n*p + 2*n)*a*b*d**2*p*x + x**(2*n*p + 2*n)*a*b*d**2*x + 2*x*
*(2*n*p + 2*n)*b**2*c*d*p*x + x**(2*n*p + 2*n)*b**2*c*d*x + 2*x**(2*n*p +
n)*a**2*d**2*p*x + x**(2*n*p + n)*a**2*d**2*x + 4*x**(2*n*p + n)*a*b*c*d*p
*x + 2*x**(2*n*p + n)*a*b*c*d*x + 2*x**(2*n*p + n)*b**2*c**2*p*x + x**(2*n
*p + n)*b**2*c**2*x + 2*x**(2*n*p)*a**2*c*d*p*x + x**(2*n*p)*a**2*c*d*x +
2*x**(2*n*p)*a*b*c**2*p*x + x**(2*n*p)*a*b*c**2*x),x)*a**3*d**3*n*p - 2*x*
*(2*n*p + n)*int(((x**n*d + c)**p*(x**n*b + a)**p)/(2*x**(2*n*p + 2*n)*a*b
*d**2*p*x + x**(2*n*p + 2*n)*a*b*d**2*x + 2*x**(2*n*p + 2*n)*b**2*c*d*p*x
+ x**(2*n*p + 2*n)*b**2*c*d*x + 2*x**(2*n*p + n)*a**2*d**2*p*x + x**(2*n*p
 + n)*a**2*d**2*x + 4*x**(2*n*p + n)*a*b*c*d*p*x + 2*x**(2*n*p + n)*a*b*c*
d*x + 2*x**(2*n*p + n)*b**2*c**2*p*x + x**(2*n*p + n)*b**2*c**2*x + 2*x...
```

3.568 $\int x^{-1-2np}(a+bx^n)^p (c+dx^n)^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int x^{-1-2np}(a+bx^n)^p (c+dx^n)^p \, dx = -\frac{x^{-2np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^p \left(1+\frac{dx^n}{c}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{2np}$$

output
```
-1/2*(a+b*x^n)^p*(c+d*x^n)^p*AppellF1(-2*p,-p,-p,1-2*p,-b*x^n/a,-d*x^n/c)/
n/p/(x^(2*n*p))/((1+b*x^n/a)^p)/((1+d*x^n/c)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int x^{-1-2np}(a+bx^n)^p (c+dx^n)^p \, dx = -\frac{x^{-2np}(a+bx^n)^p \left(\frac{a+bx^n}{a}\right)^{-p} (c+dx^n)^p \left(\frac{c+dx^n}{c}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{2np}$$

input
```
Integrate[x^(-1 - 2*n*p)*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
-1/2*((a + b*x^n)^p*(c + d*x^n)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -((b*x^n
)/a), -((d*x^n)/c)])/(n*p*x^(2*n*p)*((a + b*x^n)/a)^p*((c + d*x^n)/c)^p)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2np-1}(a+bx^n)^p (c+dx^n)^p \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^{-2np-1}\left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^p dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \int x^{-2np-1}\left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^p dx$$

$$\Big\downarrow \text{1012}$$

$$-\frac{x^{-2np}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{2np}$$

input
```
Int[x^(-1 - 2*n*p)*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
-1/2*((a + b*x^n)^p*(c + d*x^n)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -((b*x^n
)/a), -((d*x^n)/c)])/(n*p*x^(2*n*p)*(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-2np-1}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{p}\,dx$$

input
```
int(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-2np}(a+bx^n)^p\,(c+dx^n)^p\,\,dx=\int(bx^n+a)^p(dx^n+c)^px^{-2\,np-1}\,dx$$

input
```
integrate(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - 1), x)
```

Sympy [F]

$$\int x^{-1-2np}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int x^{-2np-1}(a+bx^n)^p\,(c+dx^n)^p\,dx$$

input
```
integrate(x**(-2*n*p-1)*(a+b*x**n)**p*(c+d*x**n)**p,x)
```

output
```
Integral(x**(-2*n*p - 1)*(a + b*x**n)**p*(c + d*x**n)**p, x)
```

Maxima [F]

$$\int x^{-1-2np}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int (bx^n+a)^p(dx^n+c)^p x^{-2\,np-1}\,dx$$

input
```
integrate(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - 1), x)
```

Giac [F]

$$\int x^{-1-2np}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int (bx^n+a)^p(dx^n+c)^p x^{-2\,np-1}\,dx$$

input
```
integrate(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2np}(a+bx^n)^p(c+dx^n)^p\,dx=\int\frac{(a+b\,x^n)^p\,(c+d\,x^n)^p}{x^{2\,n\,p+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(2*n*p + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(2*n*p + 1), x)
```

Reduce [F]

$$\int x^{-1-2np}(a+bx^n)^p(c+dx^n)^p\,dx=\int\frac{(x^nd+c)^p\,(x^nb+a)^p}{x^{2np}x}dx$$

input
```
int(x^(-2*n*p-1)*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(((x**n*d + c)**p*(x**n*b + a)**p)/(x**(2*n*p)*x),x)
```

3.569 $\int x^{-1-n(-1+2p)}(a+bx^n)^p (c+dx^n)^p \, dx$

Optimal result

Integrand size = 31, antiderivative size = 103

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p (c+dx^n)^p \, dx = \frac{x^{n(1-2p)}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^p \left(1+\frac{dx^n}{c}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{n(1-2p)}$$

output
```
x^(n*(1-2*p))*(a+b*x^n)^p*(c+d*x^n)^p*AppellF1(1-2*p,-p,-p,2-2*p,-b*x^n/a,
-d*x^n/c)/n/(1-2*p)/((1+b*x^n/a)^p)/((1+d*x^n/c)^p)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p (c+dx^n)^p \, dx = \frac{x^{n-2np}(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^p \left(1+\frac{dx^n}{c}\right)^{-p} \text{AppellF1}\left(1-2p,-p,-p,2-2p,-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{n-2np}$$

input
```
Integrate[x^(-1 - n*(-1 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
(x^(n - 2*n*p)*(a + b*x^n)^p*(c + d*x^n)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2
*p, -((b*x^n)/a), -((d*x^n)/c)])/((n - 2*n*p)*(1 + (b*x^n)/a)^p*(1 + (d*x^
n)/c)^p)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n(2p-1)-1}(a+bx^n)^p (c+dx^n)^p \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^{-2pn+n-1}\left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^p dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \int x^{-2pn+n-1}\left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^p dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{x^{n-2np}(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^p \left(\frac{dx^n}{c}+1\right)^{-p} \operatorname{AppellF1}\left(1-2p,-p,-p,2(1-p),-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{n(1-2p)}$$

input
```
Int[x^(-1 - n*(-1 + 2*p))*(a + b*x^n)^p*(c + d*x^n)^p,x]
```

output
```
(x^(n - 2*n*p)*(a + b*x^n)^p*(c + d*x^n)^p*AppellF1[1 - 2*p, -p, -p, 2*(1
- p), -((b*x^n)/a), -((d*x^n)/c)])/(n*(1 - 2*p)*(1 + (b*x^n)/a)^p*(1 + (d*
x^n)/c)^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-n(-1+2p)}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
int(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-n(-1+2p)}(a+bx^{n})^{p}\,(c+dx^{n})^{p}\,dx=\int(bx^{n}+a)^{p}(dx^{n}+c)^{p}x^{-n(2\,p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^p*x^(-2*n*p + n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p\,(c+dx^n)^p\;dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n*(-1+2*p))*(a+b*x**n)**p*(c+d*x**n)**p,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p\,(c+dx^n)^p\;dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2\,p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p - 1) - 1), x)
```

Giac [F]

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p\,(c+dx^n)^p\;dx = \int (bx^n+a)^p(dx^n+c)^p x^{-n(2\,p-1)-1}\,dx$$

input
```
integrate(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^p*x^(-n*(2*p - 1) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^p}{x^{n\,(2\,p-1)+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p - 1) + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^p)/x^(n*(2*p - 1) + 1), x)
```

Reduce [F]

$$\int x^{-1-n(-1+2p)}(a+bx^n)^p\,(c+dx^n)^p\,dx=\text{Too large to display}$$

input
```
int(x^(-1-n*(-1+2*p))*(a+b*x^n)^p*(c+d*x^n)^p,x)
```

output
```
(x**n*(x**n*d + c)**p*(x**n*b + a)**p*a*d + x**n*(x**n*d + c)**p*(x**n*b +
 a)**p*b*c - 2*(x**n*d + c)**p*(x**n*b + a)**p*a*c - 4*x**(2*n*p)*int(((x*
*n*d + c)**p*(x**n*b + a)**p)/(x**(2*n*p + 2*n)*a*b*d**2*x + x**(2*n*p + 2
*n)*b**2*c*d*x + x**(2*n*p + n)*a**2*d**2*x + 2*x**(2*n*p + n)*a*b*c*d*x +
 x**(2*n*p + n)*b**2*c**2*x + x**(2*n*p)*a**2*c*d*x + x**(2*n*p)*a*b*c**2*
x),x)*a**3*c**2*d*n*p - 4*x**(2*n*p)*int(((x**n*d + c)**p*(x**n*b + a)**p)
/(x**(2*n*p + 2*n)*a*b*d**2*x + x**(2*n*p + 2*n)*b**2*c*d*x + x**(2*n*p +
n)*a**2*d**2*x + 2*x**(2*n*p + n)*a*b*c*d*x + x**(2*n*p + n)*b**2*c**2*x +
 x**(2*n*p)*a**2*c*d*x + x**(2*n*p)*a*b*c**2*x),x)*a**2*b*c**3*n*p + x**(2
*n*p)*int((x**(2*n)*(x**n*d + c)**p*(x**n*b + a)**p)/(x**(2*n*p + 2*n)*a*b
*d**2*x + x**(2*n*p + 2*n)*b**2*c*d*x + x**(2*n*p + n)*a**2*d**2*x + 2*x**
(2*n*p + n)*a*b*c*d*x + x**(2*n*p + n)*b**2*c**2*x + x**(2*n*p)*a**2*c*d*x
 + x**(2*n*p)*a*b*c**2*x),x)*a**3*d**3*n*p + 3*x**(2*n*p)*int((x**(2*n)*(x
**n*d + c)**p*(x**n*b + a)**p)/(x**(2*n*p + 2*n)*a*b*d**2*x + x**(2*n*p +
2*n)*b**2*c*d*x + x**(2*n*p + n)*a**2*d**2*x + 2*x**(2*n*p + n)*a*b*c*d*x
+ x**(2*n*p + n)*b**2*c**2*x + x**(2*n*p)*a**2*c*d*x + x**(2*n*p)*a*b*c**2
*x),x)*a**2*b*c*d**2*n*p + 3*x**(2*n*p)*int((x**(2*n)*(x**n*d + c)**p*(x**
n*b + a)**p)/(x**(2*n*p + 2*n)*a*b*d**2*x + x**(2*n*p + 2*n)*b**2*c*d*x +
x**(2*n*p + n)*a**2*d**2*x + 2*x**(2*n*p + n)*a*b*c*d*x + x**(2*n*p + n)*b
**2*c**2*x + x**(2*n*p)*a**2*c*d*x + x**(2*n*p)*a*b*c**2*x),x)*a*b**2*c...
```

3.570 $\int x^{-1+3n}(a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\int x^{-1+3n}(a+bx^n)^p (c+dx^n)^q \, dx = -\frac{(bc(2+p)+ad(4+p+2q))(a+bx^n)^{1+p}(c+dx^n)^{1+q}}{b^2d^2n(2+p+q)(3+p+q)} + \frac{(a+bx^n)^{2+p}(c+dx^n)^{1+q}}{b^2dn(3+p+q)} + \frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2))(a+bx^n)^{1+p}(c+dx^n)^{1+q}\,\text{Hypergeome}}{b^2d^2(bc-ad)n(1+p)(2+p+q)(3+p+q)}$$

output
```
-(b*c*(2+p)+a*d*(4+p+2*q))*(a+b*x^n)^(p+1)*(c+d*x^n)^(1+q)/b^2/d^2/n/(2+p+
q)/(3+p+q)+(a+b*x^n)^(2+p)*(c+d*x^n)^(1+q)/b^2/d/n/(3+p+q)+(b^2*c^2*(p^2+3
*p+2)+2*a*b*c*d*(p+1)*(1+q)+a^2*d^2*(q^2+3*q+2))*(a+b*x^n)^(p+1)*(c+d*x^n)
^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b*x^n)/(-a*d+b*c))/b^2/d^2/(-a*d+b
*c)/n/(p+1)/(2+p+q)/(3+p+q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

$$\int x^{-1+3n}(a+bx^n)^p(c+dx^n)^q\,dx$$
$$=\frac{(a+bx^n)^{1+p}(c+dx^n)^q\left(-\frac{(bc(2+p)+ad(2+q))(c+dx^n)}{bd(2+p+q)}+x^n(c+dx^n)+\frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q\cdots}{b\cdots}\right.}{bdn(3+p+q)}$$

input
```
Integrate[x^(-1 + 3*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^(1 + p)*(c + d*x^n)^q*(-(((b*c*(2 + p) + a*d*(2 + q))*(c + d*
x^n))/(b*d*(2 + p + q))) + x^n*(c + d*x^n) + ((b^2*c^2*(2 + 3*p + p^2) + 2
*a*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*Hypergeometric2F1[1 +
p, -q, 2 + p, (d*(a + b*x^n))/(-(b*c) + a*d)])/(b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^n))/(b*c - a*d))^q)))/(b*d*n*(3 + p + q))
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {948, 101, 25, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1}(a+bx^n)^p(c+dx^n)^q\,dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int x^{2n}(bx^n+a)^p(dx^n+c)^q\,dx^n}{n}$$
$$\Big\downarrow \text{101}$$
$$\frac{\frac{\int -(bx^n+a)^p(dx^n+c)^q((bc(p+2)+ad(q+2))x^n+ac)dx^n}{bd(p+q+3)}+\frac{x^n(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+3)}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{x^n(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+3)}-\frac{\int(bx^n+a)^p(dx^n+c)^q((bc(p+2)+ad(q+2))x^n+ac)dx^n}{bd(p+q+3)}}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{x^n(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+3)}-\frac{\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\int(bx^n+a)^p(dx^n+c)^qdx^n+\frac{(a+bx^n)^{p+1}(c+dx^n)^{q+1}(ad(q+2)+bc(p+2))}{bd(p+q+2)}}{bd(p+q+3)}}{n}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{x^n(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+3)}-\frac{(c+dx^n)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\int(bx^n+a)^p\left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^qdx^n+\frac{(a+bx^n)}{}}{bd(p+q+3)}}{n}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{x^n(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+3)}-\frac{\frac{(a+bx^n)^{p+1}(c+dx^n)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{}{}\right.}{b(p+1)}}{bd(p+q+3)}}{n}$$

input
```
Int[x^(-1 + 3*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((x^n*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + q))/(b*d*(3 + p + q)) - (((b*c*
(2 + p) + a*d*(2 + q))*(a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + q))/(b*d*(2 +
p + q)) + ((a*c - ((b*c*(1 + p) + a*d*(1 + q))*(b*c*(2 + p) + a*d*(2 + q))
)/(b*d*(2 + p + q)))*(a + b*x^n)^(1 + p)*(c + d*x^n)^q*Hypergeometric2F1[1
 + p, -q, 2 + p, -((d*(a + b*x^n))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d*x^
n))/(b*c - a*d))^q))/(b*d*(3 + p + q)))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1+3n}(a+b\,x^{n})^{p}\left(c+d\,x^{n}\right)^{q}dx$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1+3n}(a+bx^n)^p\left(c+dx^n\right)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{3\,n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*x^(3*n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+3n}(a+bx^n)^p\left(c+dx^n\right)^q\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+3*n)*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1+3n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{3n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(3*n - 1), x)
```

Giac [F]

$$\int x^{-1+3n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{3n-1}\,dx$$

input
```
integrate(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+3n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int x^{3n-1}\,(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(3*n - 1)*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int(x^(3*n - 1)*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int x^{-1+3n}(a+bx^n)^p\,(c+dx^n)^q\,dx = \text{too large to display}$$

input
```
int(x^(-1+3*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
(x**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*d**3*p**2*q + 2*x**(3*n)*
(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*d**3*p*q**2 + 3*x**(3*n)*(x**n*d +
c)**q*(x**n*b + a)**p*a*b**2*d**3*p*q + x**(3*n)*(x**n*d + c)**q*(x**n*b +
 a)**p*a*b**2*d**3*q**3 + 3*x**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**
2*d**3*q**2 + 2*x**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*d**3*q + x
**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**3*c*d**2*p**3 + 2*x**(3*n)*(x**
n*d + c)**q*(x**n*b + a)**p*b**3*c*d**2*p**2*q + 3*x**(3*n)*(x**n*d + c)**
q*(x**n*b + a)**p*b**3*c*d**2*p**2 + x**(3*n)*(x**n*d + c)**q*(x**n*b + a)
**p*b**3*c*d**2*p*q**2 + 3*x**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**3*c
*d**2*p*q + 2*x**(3*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**3*c*d**2*p + x**
(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a**2*b*d**3*p**2*q + x**(2*n)*(x**n*
d + c)**q*(x**n*b + a)**p*a**2*b*d**3*p*q**2 + x**(2*n)*(x**n*d + c)**q*(x
**n*b + a)**p*a**2*b*d**3*p*q + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a
*b**2*c*d**2*p**3 + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*c*d**2
*p**2*q + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*c*d**2*p**2 + x*
*(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b**2*c*d**2*p*q**2 + x**(2*n)*(x*
*n*d + c)**q*(x**n*b + a)**p*a*b**2*c*d**2*q**3 + x**(2*n)*(x**n*d + c)**q
*(x**n*b + a)**p*a*b**2*c*d**2*q**2 + x**(2*n)*(x**n*d + c)**q*(x**n*b + a
)**p*b**3*c**2*d*p**2*q + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**3*c*
*2*d*p*q**2 + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**3*c**2*d*p*q ...
```

3.571 $\int x^{-1+2n}(a+bx^n)^p(c+dx^n)^q\,dx$

Optimal result

Integrand size = 26, antiderivative size = 135

$$\int x^{-1+2n}(a+bx^n)^p(c+dx^n)^q\,dx=\frac{(a+bx^n)^{1+p}(c+dx^n)^{1+q}}{bdn(2+p+q)}-\frac{(bc(1+p)+ad(1+q))(a+bx^n)^{1+p}(c+dx^n)^{1+q}\,\text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^n)}{bc-ad}\right)}{bd(bc-ad)n(1+p)(2+p+q)}$$

output
```
(a+b*x^n)^(p+1)*(c+d*x^n)^(1+q)/b/d/n/(2+p+q)-(b*c*(p+1)+a*d*(1+q))*(a+b*x
^n)^(p+1)*(c+d*x^n)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b*x^n)/(-a*d+b*
c))/b/d/(-a*d+b*c)/n/(p+1)/(2+p+q)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int x^{-1+2n}(a+bx^n)^p(c+dx^n)^q\,dx=\frac{(a+bx^n)^{1+p}(c+dx^n)^q\left(b(c+dx^n)-\frac{(bc(1+p)+ad(1+q))\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,\frac{d(a+bx^n)}{-bc+ad}\right)}{1+p}\right)}{b^2dn(2+p+q)}$$

input
```
Integrate[x^(-1 + 2*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^(1 + p)*(c + d*x^n)^q*(b*(c + d*x^n) - ((b*c*(1 + p) + a*d*(1
 + q))*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(a + b*x^n))/(-(b*c) + a*d)]
)/((1 + p)*((b*(c + d*x^n))/(b*c - a*d))^q)))/(b^2*d*n*(2 + p + q))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {948, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1}(a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^n(bx^n+a)^p (dx^n+c)^q dx^n}{n}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+2)}-\frac{(ad(q+1)+bc(p+1))\int (bx^n+a)^p(dx^n+c)^qdx^n}{bd(p+q+2)}}{n}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+2)}-\frac{(c+dx^n)^q(ad(q+1)+bc(p+1))\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\int (bx^n+a)^p\left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^qdx^n}{bd(p+q+2)}}{n}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{(a+bx^n)^{p+1}(c+dx^n)^{q+1}}{bd(p+q+2)}-\frac{(a+bx^n)^{p+1}(c+dx^n)^q(ad(q+1)+bc(p+1))\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^n+a)}{bc-ad}\right)}{b^2d(p+1)(p+q+2)}}{n}$$

input
```
Int[x^(-1 + 2*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(((a + b*x^n)^(1 + p)*(c + d*x^n)^(1 + q))/(b*d*(2 + p + q)) - ((b*c*(1 +
p) + a*d*(1 + q))*(a + b*x^n)^(1 + p)*(c + d*x^n)^q*Hypergeometric2F1[1 +
p, -q, 2 + p, -((d*(a + b*x^n))/(b*c - a*d))])/(b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^n))/(b*c - a*d))^q))/n
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1+2n}(a+b\,x^{n})^{p}\,(c+d\,x^{n})^{q}\,dx$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1+2n}(a+bx^{n})^{p}\,(c+dx^{n})^{q}\;dx=\int (bx^{n}+a)^{p}(dx^{n}+c)^{q}x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*x^(2*n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+2n}(a+bx^{n})^{p}\,(c+dx^{n})^{q}\;dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+2*n)*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1+2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(2*n - 1), x)
```

Giac [F]

$$\int x^{-1+2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{2\,n-1}\,dx$$

input
```
integrate(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int x^{2\,n-1}\,(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(2*n - 1)*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int(x^(2*n - 1)*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int x^{-1+2n}(a+bx^n)^p\,(c+dx^n)^q\,dx = \text{too large to display}$$

input
```
int(x^(-1+2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
(x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*a*b*d**2*p*q + x**(2*n)*(x**n*d
+ c)**q*(x**n*b + a)**p*a*b*d**2*q**2 + x**(2*n)*(x**n*d + c)**q*(x**n*b +
 a)**p*a*b*d**2*q + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**2*c*d*p**2
 + x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p*b**2*c*d*p*q + x**(2*n)*(x**n*
d + c)**q*(x**n*b + a)**p*b**2*c*d*p + x**n*(x**n*d + c)**q*(x**n*b + a)**
p*a**2*d**2*p*q + x**n*(x**n*d + c)**q*(x**n*b + a)**p*a*b*c*d*p**2 + x**n
*(x**n*d + c)**q*(x**n*b + a)**p*a*b*c*d*q**2 + x**n*(x**n*d + c)**q*(x**n
*b + a)**p*b**2*c**2*p*q - (x**n*d + c)**q*(x**n*b + a)**p*a**2*c*d*p - (x
**n*d + c)**q*(x**n*b + a)**p*a*b*c**2*q - int((x**(2*n)*(x**n*d + c)**q*(
x**n*b + a)**p)/(x**(2*n)*a*b*d**2*p**2*q*x + 2*x**(2*n)*a*b*d**2*p*q**2*x
 + 3*x**(2*n)*a*b*d**2*p*q*x + x**(2*n)*a*b*d**2*q**3*x + 3*x**(2*n)*a*b*d
**2*q**2*x + 2*x**(2*n)*a*b*d**2*q*x + x**(2*n)*b**2*c*d*p**3*x + 2*x**(2*
n)*b**2*c*d*p**2*q*x + 3*x**(2*n)*b**2*c*d*p**2*x + x**(2*n)*b**2*c*d*p*q*
*2*x + 3*x**(2*n)*b**2*c*d*p*q*x + 2*x**(2*n)*b**2*c*d*p*x + x**n*a**2*d**
2*p**2*q*x + 2*x**n*a**2*d**2*p*q**2*x + 3*x**n*a**2*d**2*p*q*x + x**n*a**
2*d**2*q**3*x + 3*x**n*a**2*d**2*q**2*x + 2*x**n*a**2*d**2*q*x + x**n*a*b*
c*d*p**3*x + 3*x**n*a*b*c*d*p**2*q*x + 3*x**n*a*b*c*d*p**2*x + 3*x**n*a*b*
c*d*p*q**2*x + 6*x**n*a*b*c*d*p*q*x + 2*x**n*a*b*c*d*p*x + x**n*a*b*c*d*q*
*3*x + 3*x**n*a*b*c*d*q**2*x + 2*x**n*a*b*c*d*q*x + x**n*b**2*c**2*p**3*x
+ 2*x**n*b**2*c**2*p**2*q*x + 3*x**n*b**2*c**2*p**2*x + x**n*b**2*c**2*...
```

3.572 $\int x^{-1+n}(a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 24, antiderivative size = 70

$$\int x^{-1+n}(a+bx^n)^p (c+dx^n)^q \, dx = \frac{(a+bx^n)^{1+p}(c+dx^n)^{1+q}\text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^n)}{bc-ad}\right)}{(bc-ad)n(1+p)}$$

output
```
(a+b*x^n)^(p+1)*(c+d*x^n)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(a+b*x^n)/(-
a*d+b*c))/(-a*d+b*c)/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

$$\int x^{-1+n}(a+bx^n)^p (c+dx^n)^q \, dx = \frac{(a+bx^n)^{1+p}(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,\frac{d(a+bx^n)}{-bc+ad}\right)}{bn(1+p)}$$

input
```
Integrate[x^(-1 + n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^(1 + p)*(c + d*x^n)^q*Hypergeometric2F1[1 + p, -q, 2 + p, (d*
(a + b*x^n))/(-(b*c) + a*d)])/(b*n*(1 + p)*((b*(c + d*x^n))/(b*c - a*d))^q
)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {946, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1}(a+bx^n)^p\,(c+dx^n)^q\;dx$$

$$\Big\downarrow \; 946$$

$$\frac{\int (bx^n+a)^p\,(dx^n+c)^q\,dx^n}{n}$$

$$\Big\downarrow \; 80$$

$$\frac{(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\int (bx^n+a)^p\left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^n}{n}$$

$$\Big\downarrow \; 79$$

$$\frac{(a+bx^n)^{p+1}\,(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^n+a)}{bc-ad}\right)}{bn(p+1)}$$

input
```
Int[x^(-1 + n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^(1 + p)*(c + d*x^n)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((
d*(a + b*x^n))/(b*c - a*d))])/(b*n*(1 + p)*((b*(c + d*x^n))/(b*c - a*d))^q
)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [F]

$$\int x^{-1+n}(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\int (bx^n+a)^p(dx^n+c)^q x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*x^(n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1+n)*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\int (bx^n+a)^p(dx^n+c)^q x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(n - 1), x)
```

Giac [F]

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^q x^{n-1}\,dx$$

input
```
integrate(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int x^{n-1}\,(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(n - 1)*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int(x^(n - 1)*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int x^{-1+n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\text{too large to display}$$

input
```
int(x^(-1+n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
(x**n*(x**n*d + c)**q*(x**n*b + a)**p*a*d*q + x**n*(x**n*d + c)**q*(x**n*b
 + a)**p*b*c*p + (x**n*d + c)**q*(x**n*b + a)**p*a*c*p + (x**n*d + c)**q*(
x**n*b + a)**p*a*c*q + int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**
(2*n)*a*b*d**2*p*q*x + x**(2*n)*a*b*d**2*q**2*x + x**(2*n)*a*b*d**2*q*x +
x**(2*n)*b**2*c*d*p**2*x + x**(2*n)*b**2*c*d*p*q*x + x**(2*n)*b**2*c*d*p*x
 + x**n*a**2*d**2*p*q*x + x**n*a**2*d**2*q**2*x + x**n*a**2*d**2*q*x + x**
n*a*b*c*d*p**2*x + 2*x**n*a*b*c*d*p*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d*
q**2*x + x**n*a*b*c*d*q*x + x**n*b**2*c**2*p**2*x + x**n*b**2*c**2*p*q*x +
 x**n*b**2*c**2*p*x + a**2*c*d*p*q*x + a**2*c*d*q**2*x + a**2*c*d*q*x + a*
b*c**2*p**2*x + a*b*c**2*p*q*x + a*b*c**2*p*x),x)*a**3*d**3*n*p**2*q**2 +
int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*p*q*x +
x**(2*n)*a*b*d**2*q**2*x + x**(2*n)*a*b*d**2*q*x + x**(2*n)*b**2*c*d*p**2*
x + x**(2*n)*b**2*c*d*p*q*x + x**(2*n)*b**2*c*d*p*x + x**n*a**2*d**2*p*q*x
 + x**n*a**2*d**2*q**2*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p**2*x + 2*x*
*n*a*b*c*d*p*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d*q**2*x + x**n*a*b*c*d*q
*x + x**n*b**2*c**2*p**2*x + x**n*b**2*c**2*p*q*x + x**n*b**2*c**2*p*x + a
**2*c*d*p*q*x + a**2*c*d*q**2*x + a**2*c*d*q*x + a*b*c**2*p**2*x + a*b*c**
2*p*q*x + a*b*c**2*p*x),x)*a**3*d**3*n*p*q**3 + int((x**(2*n)*(x**n*d + c)
**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*p*q*x + x**(2*n)*a*b*d**2*q**2*x +
 x**(2*n)*a*b*d**2*q*x + x**(2*n)*b**2*c*d*p**2*x + x**(2*n)*b**2*c*d*p...
```

3.573 $\int \frac{(a+bx^n)^p(c+dx^n)^q}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{(a+bx^n)^p(c+dx^n)^q}{x}\,dx = -\frac{(a+bx^n)^{1+p}(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,1,-q,2+p,1+\frac{bx^n}{a},-\frac{d(a+bx^n)}{bc-ad}\right)}{an(1+p)}$$

output
```
-(a+b*x^n)^(p+1)*(c+d*x^n)^q*AppellF1(p+1,-q,1,2+p,-d*(a+b*x^n)/(-a*d+b*c)
,1+b*x^n/a)/a/n/(p+1)/((b*(c+d*x^n)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^n)^p(c+dx^n)^q}{x}\,dx = \frac{\left(1+\frac{ax^{-n}}{b}\right)^{-p}\left(1+\frac{cx^{-n}}{d}\right)^{-q}(a+bx^n)^p(c+dx^n)^q\,\text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{ax^{-n}}{b},-\frac{cx^{-n}}{d}\right)}{n(p+q)}$$

input
```
Integrate[((a + b*x^n)^p*(c + d*x^n)^q)/x,x]
```

output
```
((a + b*x^n)^p*(c + d*x^n)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^
n)), -(c/(d*x^n))]]/(n*(p + q)*(1 + a/(b*x^n))^p*(1 + c/(d*x^n))^q)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} dx$$

$$\Big\downarrow \ 948$$

$$\frac{\int x^{-n}(bx^n+a)^p (dx^n+c)^q dx^n}{n}$$

$$\Big\downarrow \ 154$$

$$\frac{(c+dx^n)^q \left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q} \int x^{-n}(bx^n+a)^p \left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^n}{n}$$

$$\Big\downarrow \ 153$$

$$-\frac{(a+bx^n)^{p+1} (c+dx^n)^q \left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d(bx^n+a)}{bc-ad},\frac{bx^n+a}{a}\right)}{an(p+1)}$$

input
```
Int[((a + b*x^n)^p*(c + d*x^n)^q)/x,x]
```

output
```
-(((a + b*x^n)^(1 + p)*(c + d*x^n)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a
 + b*x^n))/(b*c - a*d)), (a + b*x^n)/a])/(a*n*(1 + p)*((b*(c + d*x^n))/(b*
c - a*d))^q))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^q}{x}dx$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^q/x,x)
```

output
```
int((a+b*x^n)^p*(c+d*x^n)^q/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \int \frac{(bx^n+a)^p (dx^n+c)^q}{x} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q/x,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q/x, x)
```

Sympy [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx$$

input
```
integrate((a+b*x**n)**p*(c+d*x**n)**q/x,x)
```

output
```
Integral((a + b*x**n)**p*(c + d*x**n)**q/x, x)
```

Maxima [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \int \frac{(bx^n+a)^p (dx^n+c)^q}{x} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q/x,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q/x, x)
```

Giac [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \int \frac{(bx^n+a)^p (dx^n+c)^q}{x} \, dx$$

input
```
integrate((a+b*x^n)^p*(c+d*x^n)^q/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^q}{x} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x,x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^n)^p (c+dx^n)^q}{x} \, dx = \text{Too large to display}$$

input
```
int((a+b*x^n)^p*(c+d*x^n)^q/x,x)
```

output

```
((x**n*d + c)**q*(x**n*b + a)**p*a*d + (x**n*d + c)**q*(x**n*b + a)**p*b*c
 + int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*q*x + x**(2*n)
*b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d*q*x +
 x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)*a**3*c*d**2*n*q**2 +
 2*int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*q*x + x**(2*n)
*b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d*q*x +
 x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)*a**2*b*c**2*d*n*p*q
+ int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*q*x + x**(2*n)*
b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d*q*x +
x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)*a*b**2*c**3*n*p**2 -
int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*q*x + x*
*(2*n)*b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x + x**n*a*b*c*d
*q*x + x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)*a**2*b*d**3*n*
p*q - int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*q*
x + x**(2*n)*b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x + x**n*a
*b*c*d*q*x + x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)*a*b**2*c
*d**2*n*p**2 - int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*
b*d**2*q*x + x**(2*n)*b**2*c*d*p*x + x**n*a**2*d**2*q*x + x**n*a*b*c*d*p*x
 + x**n*a*b*c*d*q*x + x**n*b**2*c**2*p*x + a**2*c*d*q*x + a*b*c**2*p*x),x)
*a*b**2*c*d**2*n*q**2 - int((x**(2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/...
```

3.574 $\int x^{-1-n}(a+bx^n)^p (c+dx^n)^q \,dx$

Optimal result

Integrand size = 26, antiderivative size = 97

$$\int x^{-1-n}(a+bx^n)^p (c+dx^n)^q \,dx = \frac{b(a+bx^n)^{1+p}(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,2,-q,2+p,1+\frac{bx^n}{a},-\frac{d(a+bx^n)}{bc-ad}\right)}{a^2n(1+p)}$$

output
```
b*(a+b*x^n)^(p+1)*(c+d*x^n)^q*AppellF1(p+1,-q,2,2+p,-d*(a+b*x^n)/(-a*d+b*c
),1+b*x^n/a)/a^2/n/(p+1)/((b*(c+d*x^n)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int x^{-1-n}(a+bx^n)^p (c+dx^n)^q \,dx = \frac{x^{-n}\left(1+\frac{ax^{-n}}{b}\right)^{-p}\left(1+\frac{cx^{-n}}{d}\right)^{-q}(a+bx^n)^p(c+dx^n)^q\operatorname{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{ax^{-n}}{b},-\right.}{n(-1+p+q)}$$

input
```
Integrate[x^(-1 - n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^p*(c + d*x^n)^q*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/(b
*x^n)), -(c/(d*x^n))])/(n*(-1 + p + q)*x^n*(1 + a/(b*x^n))^p*(1 + c/(d*x^n
))^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1}(a+bx^n)^p\left(c+dx^n\right)^q\,dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{-2n}(bx^n+a)^p\left(dx^n+c\right)^q\,dx^n}{n}$$

$$\Big\downarrow \text{154}$$

$$\frac{(c+dx^n)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\int x^{-2n}(bx^n+a)^p\left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^n}{n}$$

$$\Big\downarrow \text{153}$$

$$\frac{b(a+bx^n)^{p+1}\left(c+dx^n\right)^q\left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d(bx^n+a)}{bc-ad},\frac{bx^n+a}{a}\right)}{a^2n(p+1)}$$

input
```
Int[x^(-1 - n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(b*(a + b*x^n)^(1 + p)*(c + d*x^n)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
 + b*x^n))/(b*c - a*d)), (a + b*x^n)/a])/(a^2*n*(1 + p)*((b*(c + d*x^n))/(
b*c - a*d))^q)
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] && !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !G
tQ[Simplify[b/(b*c - a*d)], 0] && !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1-n}(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\int (bx^n+a)^p(dx^n+c)^qx^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*x^(-n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-n)*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\int (bx^n+a)^p(dx^n+c)^qx^{-n-1}\,dx$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(-n - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,0,4,3,8,2,3,3,9,3]%%%}+%%%{6,[0,0,4,3,7,3,3,3,8,4]%%
%}+%%%{-1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\int\frac{(a+b\,x^n)^p\,(c+d\,x^n)^q}{x^{n+1}}\,dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x^(n + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x^(n + 1), x)
```

Reduce [F]

$$\int x^{-1-n}(a+bx^n)^p\,(c+dx^n)^q\;dx=\text{too large to display}$$

input
```
int(x^(-1-n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output

```
( - x**n*(x**n*d + c)**q*(x**n*b + a)**p*a*b*d**2*p - x**n*(x**n*d + c)**q
*(x**n*b + a)**p*b**2*c*d*q - (x**n*d + c)**q*(x**n*b + a)**p*a**2*d**2*q*
*2 + (x**n*d + c)**q*(x**n*b + a)**p*a**2*d**2*q - 2*(x**n*d + c)**q*(x**n
*b + a)**p*a*b*c*d*p*q + (x**n*d + c)**q*(x**n*b + a)**p*a*b*c*d*p + (x**n
*d + c)**q*(x**n*b + a)**p*a*b*c*d*q - (x**n*d + c)**q*(x**n*b + a)**p*b**
2*c**2*p**2 + (x**n*d + c)**q*(x**n*b + a)**p*b**2*c**2*p + x**n*int(((x**
n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a**2*b*d**3*q**2*x - x**(2*n)*a**2*
b*d**3*q*x + 2*x**(2*n)*a*b**2*c*d**2*p*q*x - x**(2*n)*a*b**2*c*d**2*p*x -
 x**(2*n)*a*b**2*c*d**2*q*x + x**(2*n)*b**3*c**2*d*p**2*x - x**(2*n)*b**3*
c**2*d*p*x + x**n*a**3*d**3*q**2*x - x**n*a**3*d**3*q*x + 2*x**n*a**2*b*c*
d**2*p*q*x - x**n*a**2*b*c*d**2*p*x + x**n*a**2*b*c*d**2*q**2*x - 2*x**n*a
**2*b*c*d**2*q*x + x**n*a*b**2*c**2*d*p**2*x + 2*x**n*a*b**2*c**2*d*p*q*x
- 2*x**n*a*b**2*c**2*d*p*x - x**n*a*b**2*c**2*d*q*x + x**n*b**3*c**3*p**2*
x - x**n*b**3*c**3*p*x + a**3*c*d**2*q**2*x - a**3*c*d**2*q*x + 2*a**2*b*c
**2*d*p*q*x - a**2*b*c**2*d*p*x - a**2*b*c**2*d*q*x + a*b**2*c**3*p**2*x -
 a*b**2*c**3*p*x),x)*a**5*d**5*n*q**5 - 2*x**n*int(((x**n*d + c)**q*(x**n*
b + a)**p)/(x**(2*n)*a**2*b*d**3*q**2*x - x**(2*n)*a**2*b*d**3*q*x + 2*x**
(2*n)*a*b**2*c*d**2*p*q*x - x**(2*n)*a*b**2*c*d**2*p*x - x**(2*n)*a*b**2*c
*d**2*q*x + x**(2*n)*b**3*c**2*d*p**2*x - x**(2*n)*b**3*c**2*d*p*x + x**n*
a**3*d**3*q**2*x - x**n*a**3*d**3*q*x + 2*x**n*a**2*b*c*d**2*p*q*x - x*...
```

3.575 $\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 26, antiderivative size = 100

$$\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx = -\frac{b^2(a+bx^n)^{1+p}(c+dx^n)^q \left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,3,-q,2+p,1+\frac{bx^n}{a},-\frac{d(a+bx^n)}{bc-ad}\right)}{a^3 n(1+p)}$$

output
```
-b^2*(a+b*x^n)^(p+1)*(c+d*x^n)^q*AppellF1(p+1,-q,3,2+p,-d*(a+b*x^n)/(-a*d+
b*c),1+b*x^n/a)/a^3/n/(p+1)/((b*(c+d*x^n)/(-a*d+b*c))^q)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.10

$$\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx = \frac{x^{-2n}\left(1+\frac{ax^{-n}}{b}\right)^{-p}\left(1+\frac{cx^{-n}}{d}\right)^{-q}(a+bx^n)^p(c+dx^n)^q \text{AppellF1}\left(2-p-q,-p,-q,3-p-q,-\frac{ax^{-n}}{b},-\right.}{n(-2+p+q)}$$

input
```
Integrate[x^(-1 - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((a + b*x^n)^p*(c + d*x^n)^q*AppellF1[2 - p - q, -p, -q, 3 - p - q, -(a/(b
*x^n)), -(c/(d*x^n))])/(n*(-2 + p + q)*x^(2*n)*(1 + a/(b*x^n))^p*(1 + c/(d
*x^n))^q)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {948, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1}(a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int x^{-3n}(bx^n+a)^p (dx^n+c)^q \, dx^n}{n}$$

$$\Big\downarrow \text{154}$$

$$\frac{(c+dx^n)^q \left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q} \int x^{-3n}(bx^n+a)^p \left(\frac{bdx^n}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^n}{n}$$

$$\Big\downarrow \text{153}$$

$$-\frac{b^2(a+bx^n)^{p+1} (c+dx^n)^q \left(\frac{b(c+dx^n)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,3,p+2,-\frac{d(bx^n+a)}{bc-ad},\frac{bx^n+a}{a}\right)}{a^3n(p+1)}$$

input
```
Int[x^(-1 - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
-((b^2*(a + b*x^n)^(1 + p)*(c + d*x^n)^q*AppellF1[1 + p, -q, 3, 2 + p, -((
d*(a + b*x^n))/(b*c - a*d)), (a + b*x^n)/a])/(a^3*n*(1 + p)*((b*(c + d*x^n
))/(b*c - a*d))^q))
```

Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int x^{-1-2n}(a+b\,x^n)^p\,(c+d\,x^n)^q\,dx$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int x^{-1-2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^qx^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*x^(-2*n - 1), x)
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1-2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(-1-2*n)*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int x^{-1-2n}(a+bx^n)^p\,(c+dx^n)^q\,dx=\int (bx^n+a)^p(dx^n+c)^qx^{-2\,n-1}\,dx$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*x^(-2*n - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[0,0,4,3,8,2,3,3,9,3]%%%}+%%%{6,[0,0,4,3,7,3,3,3,8,4]%%
%}+%%%{-1
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx = \int \frac{(a+b\,x^n)^p\,(c+d\,x^n)^q}{x^{2\,n+1}} \, dx$$

input
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x^(2*n + 1),x)
```

output
```
int(((a + b*x^n)^p*(c + d*x^n)^q)/x^(2*n + 1), x)
```

Reduce [F]

$$\int x^{-1-2n}(a+bx^n)^p (c+dx^n)^q \, dx = \text{too large to display}$$

input
```
int(x^(-1-2*n)*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output

```
( - (x**n*d + c)**q*(x**n*b + a)**p + x**(2*n)*int(((x**n*d + c)**q*(x**n*
b + a)**p)/(x**(3*n)*a*b*d**2*q*x - 2*x**(3*n)*a*b*d**2*x + x**(3*n)*b**2*
c*d*p*x - 2*x**(3*n)*b**2*c*d*x + x**(2*n)*a**2*d**2*q*x - 2*x**(2*n)*a**2
*d**2*x + x**(2*n)*a*b*c*d*p*x + x**(2*n)*a*b*c*d*q*x - 4*x**(2*n)*a*b*c*d
*x + x**(2*n)*b**2*c**2*p*x - 2*x**(2*n)*b**2*c**2*x + x**n*a**2*c*d*q*x -
 2*x**n*a**2*c*d*x + x**n*a*b*c**2*p*x - 2*x**n*a*b*c**2*x),x)*a**2*d**2*n
*q**2 - 2*x**(2*n)*int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(3*n)*a*b*d**
2*q*x - 2*x**(3*n)*a*b*d**2*x + x**(3*n)*b**2*c*d*p*x - 2*x**(3*n)*b**2*c*
d*x + x**(2*n)*a**2*d**2*q*x - 2*x**(2*n)*a**2*d**2*x + x**(2*n)*a*b*c*d*p
*x + x**(2*n)*a*b*c*d*q*x - 4*x**(2*n)*a*b*c*d*x + x**(2*n)*b**2*c**2*p*x
- 2*x**(2*n)*b**2*c**2*x + x**n*a**2*c*d*q*x - 2*x**n*a**2*c*d*x + x**n*a*
b*c**2*p*x - 2*x**n*a*b*c**2*x),x)*a**2*d**2*n*q + 2*x**(2*n)*int(((x**n*d
 + c)**q*(x**n*b + a)**p)/(x**(3*n)*a*b*d**2*q*x - 2*x**(3*n)*a*b*d**2*x +
 x**(3*n)*b**2*c*d*p*x - 2*x**(3*n)*b**2*c*d*x + x**(2*n)*a**2*d**2*q*x -
2*x**(2*n)*a**2*d**2*x + x**(2*n)*a*b*c*d*p*x + x**(2*n)*a*b*c*d*q*x - 4*x
**(2*n)*a*b*c*d*x + x**(2*n)*b**2*c**2*p*x - 2*x**(2*n)*b**2*c**2*x + x**n
*a**2*c*d*q*x - 2*x**n*a**2*c*d*x + x**n*a*b*c**2*p*x - 2*x**n*a*b*c**2*x)
,x)*a*b*c*d*n*p*q - 2*x**(2*n)*int(((x**n*d + c)**q*(x**n*b + a)**p)/(x**(
3*n)*a*b*d**2*q*x - 2*x**(3*n)*a*b*d**2*x + x**(3*n)*b**2*c*d*p*x - 2*x**(
3*n)*b**2*c*d*x + x**(2*n)*a**2*d**2*q*x - 2*x**(2*n)*a**2*d**2*x + x**...
```

3.576 $\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx=\frac{1}{3}x^3(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

output
```
1/3*x^3*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],b^2*x^(2*
n)/a^2)/((1-b^2*x^(2*n)/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\,dx=\frac{1}{3}x^3(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/
(2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(bx^n-a)^p(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p(a+bx^n)^p\left(b^2x^{2n}-a^2\right)^{-p}\int x^2\left(b^2x^{2n}-a^2\right)^p dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\int x^2\left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2n},-p,1+\frac{3}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*
n), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int x^2(-a+b\,x^n)^p\,(a+b\,x^n)^p\,dx$$

input
```
int(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
int(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx$$

input
```
integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Giac [F]

$$\int x^2(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px^2\,dx$$

input
```
integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx=\int x^2\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int(x^2*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int(x^2*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

Reduce [F]

$$\int x^2(-a+bx^n)^p(a+bx^n)^p\,dx$$
$$=\frac{(x^nb+a)^p\,(x^nb-a)^p\,x^3-4\left(\int\frac{(x^nb+a)^p(x^nb-a)^px^2}{2x^{2n}b^2np+3x^{2n}b^2-2a^2np-3a^2}dx\right)a^2n^2p^2-6\left(\int\frac{(x^nb+a)^p(x^nb-a)^px^2}{2x^{2n}b^2np+3x^{2n}b^2-2a^2np-3a^2}dx\right)a}{2np+3}$$

input
```
int(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b - a)**p*x**3 - 4*int(((x**n*b + a)**p*(x**n*b - a
)**p*x**2)/(2*x**(2*n)*b**2*n*p + 3*x**(2*n)*b**2 - 2*a**2*n*p - 3*a**2),x
)*a**2*n**2*p**2 - 6*int(((x**n*b + a)**p*(x**n*b - a)**p*x**2)/(2*x**(2*n
)*b**2*n*p + 3*x**(2*n)*b**2 - 2*a**2*n*p - 3*a**2),x)*a**2*n*p)/(2*n*p +
3)
```

3.577 $\int x(-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{b^2x^{2n}}{a^2}\right)$$

output
```
1/2*x^2*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],b^2*x^(2*n)/a
^2)/((1-b^2*x^(2*n)/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int x(-a+bx^n)^p (a+bx^n)^p \, dx = \frac{1}{2}x^2(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(
-1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(bx^n-a)^p(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p(a+bx^n)^p\left(b^2x^{2n}-a^2\right)^{-p}\int x\left(b^2x^{2n}-a^2\right)^p dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{n},-p,1+\frac{1}{n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int x(-a+b\,x^{n})^{p}\,(a+b\,x^{n})^{p}\,dx$$

input
```
int(x*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
int(x*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x(-a+bx^{n})^{p}\,(a+bx^{n})^{p}\,dx=\int(bx^{n}+a)^{p}(bx^{n}-a)^{p}x\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Sympy [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx$$

input
```
integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Giac [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\;dx=\int (bx^n+a)^p(bx^n-a)^px\,dx$$

input
```
integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx=\int x\,(a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int(x*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int(x*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

Reduce [F]

$$\int x(-a+bx^n)^p\,(a+bx^n)^p\,dx$$
$$=\frac{(x^nb+a)^p\,(x^nb-a)^p\,x^2-2\left(\int\frac{(x^nb+a)^p(x^nb-a)^px}{x^{2n}b^2np+x^{2n}b^2-a^2np-a^2}dx\right)a^2n^2p^2-2\left(\int\frac{(x^nb+a)^p(x^nb-a)^px}{x^{2n}b^2np+x^{2n}b^2-a^2np-a^2}dx\right)a^2np}{2np+2}$$

input
```
int(x*(-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b - a)**p*x**2 - 2*int(((x**n*b + a)**p*(x**n*b - a
)**p*x)/(x**(2*n)*b**2*n*p + x**(2*n)*b**2 - a**2*n*p - a**2),x)*a**2*n**2
*p**2 - 2*int(((x**n*b + a)**p*(x**n*b - a)**p*x)/(x**(2*n)*b**2*n*p + x**
(2*n)*b**2 - a**2*n*p - a**2),x)*a**2*n*p)/(2*(n*p + 1))
```

3.578 $\int (-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

output
```
x*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2
)/((1-b^2*x^(2*n)/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = x(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,1+\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n)
, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {785, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx^n-a)^p (a+bx^n)^p \, dx$$

$$\Big\downarrow \text{785}$$

$$(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int \left(b^2x^{2n}-a^2\right)^p dx$$

$$\Big\downarrow \text{779}$$

$$(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p dx$$

$$\Big\downarrow \text{778}$$

$$x(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2n},-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2x^{2n}}{a^2}\right)$$

input
```
Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

output
```
(x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1)
)/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 785
```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p])   Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Fre
eQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Maple [F]

$$\int (-a+b\,x^{n})^{p} (a+b\,x^{n})^{p}\,dx$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
int((-a+b*x^n)^p*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p (bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Sympy [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (-a+bx^n)^p (a+bx^n)^p \, dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

Maxima [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p (bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Giac [F]

$$\int (-a+bx^n)^p (a+bx^n)^p \, dx = \int (bx^n+a)^p (bx^n-a)^p \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-a+bx^n)^p\,(a+bx^n)^p\ dx=\int (a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

input
```
int((a + b*x^n)^p*(b*x^n - a)^p,x)
```

output
```
int((a + b*x^n)^p*(b*x^n - a)^p, x)
```

Reduce [F]

$$\int (-a+bx^n)^p\,(a+bx^n)^p\ dx = \frac{(x^nb+a)^p\,(x^nb-a)^p\,x-4\left(\int \frac{(x^nb+a)^p(x^nb-a)^p}{2x^{2n}b^2np+x^{2n}b^2-2a^2np-a^2}dx\right)a^2n^2p^2-2\left(\int \frac{(x^nb+a)^p(x^nb-a)^p}{2x^{2n}b^2np+x^{2n}b^2-2a^2np-a^2}dx\right)a^2np}{2np+1}$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p*(x**n*b - a)**p*x - 4*int(((x**n*b + a)**p*(x**n*b - a)**
p)/(2*x**(2*n)*b**2*n*p + x**(2*n)*b**2 - 2*a**2*n*p - a**2),x)*a**2*n**2*
p**2 - 2*int(((x**n*b + a)**p*(x**n*b - a)**p)/(2*x**(2*n)*b**2*n*p + x**(
2*n)*b**2 - 2*a**2*n*p - a**2),x)*a**2*n*p)/(2*n*p + 1)
```

3.579 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x}\,dx = -\frac{(-a+bx^n)^p (a+bx^n)^p (a^2-b^2x^{2n}) \text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

output
```
-1/2*(-a+b*x^n)^p*(a+b*x^n)^p*(a^2-b^2*x^(2*n))*hypergeom([1, p+1],[2+p],1
-b^2*x^(2*n)/a^2)/a^2/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x}\,dx = \frac{(-a+bx^n)^p (a+bx^n)^p (-a^2+b^2x^{2n}) \text{Hypergeometric2F1}\left(1,1+p,2+p,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

input
```
Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

output
```
((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {799, 136, 243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n-a)^p (a+bx^n)^p}{x} \, dx$$

$$\Big\downarrow \text{799}$$

$$\frac{\int x^{-n}(bx^n-a)^p (bx^n+a)^p dx^n}{n}$$

$$\Big\downarrow \text{136}$$

$$\frac{(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int x^{-n}\left(b^2x^{2n}-a^2\right)^p dx^n}{n}$$

$$\Big\downarrow \text{243}$$

$$\frac{(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int x^{-n}\left(b^2x^{2n}-a^2\right)^p dx^{2n}}{2n}$$

$$\Big\downarrow \text{75}$$

$$\frac{\left(b^2x^{2n}-a^2\right)(bx^n-a)^p (a+bx^n)^p \text{Hypergeometric2F1}\left(1,p+1,p+2,1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

input
```
Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

output
```
((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1
+ p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 136
```
Int[((f_.)*(x_))^(p_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_),
x_] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[m]/(a*c + b*d*x^2)^Fr
acPart[m])   Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b, c, d, f,
 m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 799
```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1
*x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
 && EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Maple [F]

$$\int \frac{(-a+b\,x^{n})^{p}\,(a+b\,x^{n})^{p}}{x}dx$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x,x)
```

output
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x,x)
```

Fricas [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx=\int \frac{(bx^n+a)^p(bx^n-a)^p}{x}\,dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Sympy [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx=\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

Maxima [F]

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x}\,dx=\int \frac{(bx^n+a)^p(bx^n-a)^p}{x}\,dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Giac [F]

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x} \, dx = \int \frac{(bx^n + a)^p (bx^n - a)^p}{x} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x} \, dx = \int \frac{(a + b\,x^n)^p \,(b\,x^n - a)^p}{x} \, dx$$

input
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x,x)
```

output
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x, x)
```

Reduce [F]

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x} \, dx = \frac{(x^n b + a)^p \,(x^n b - a)^p - 2\left(\int \frac{(x^n b + a)^p (x^n b - a)^p}{x^{2n} b^2 x - a^2 x} dx\right) a^2 np}{2np}$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x,x)
```

output
```
((x**n*b + a)**p*(x**n*b - a)**p - 2*int(((x**n*b + a)**p*(x**n*b - a)**p)
/(x**(2*n)*b**2*x - a**2*x),x)*a**2*n*p)/(2*n*p)
```

3.580 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 76

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

output
```
-(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],b^2*x^(2*n)/a^2
)/x/((1-b^2*x^(2*n)/a^2)^p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{(-a+bx^n)^p\,(a+bx^n)^p}{x^2}\,dx = -\frac{(-a+bx^n)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

input
```
Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

output
```
-(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/2*1/n, -p}, {1 - 1/(
2*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {890, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^n-a)^p (a+bx^n)^p}{x^2} dx$$

$$\Big\downarrow \text{890}$$

$$(bx^n-a)^p (a+bx^n)^p \left(b^2x^{2n}-a^2\right)^{-p} \int \frac{\left(b^2x^{2n}-a^2\right)^p}{x^2}dx$$

$$\Big\downarrow \text{889}$$

$$(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \int \frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2}dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2n},-p,1-\frac{1}{2n},\frac{b^2x^{2n}}{a^2}\right)}{x}$$

input
```
Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

output
```
-(((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/2*1/n, -p, 1 - 1/(2*n
), (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 890
```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^Fra
cPart[p]/(a1*a2 + b1*b2*x^(2*n))^FracPart[p])   Int[(c*x)^m*(a1*a2 + b1*b2*
x^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
 a1*b2, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{(-a + b\,x^n)^p\,(a + b\,x^n)^p}{x^2}dx$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x)
```

output
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(-a + bx^n)^p\,(a + bx^n)^p}{x^2}\,dx = \int \frac{(bx^n + a)^p(bx^n - a)^p}{x^2}\,dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
```

output
```
Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

Maxima [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} \, dx = \int \frac{(bx^n+a)^p (bx^n-a)^p}{x^2} \, dx$$

input
```
integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} dx = \int \frac{(a+b\,x^n)^p (b\,x^n-a)^p}{x^2} dx$$

input
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2,x)
```

output
```
int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2, x)
```

Reduce [F]

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} dx = \frac{(x^n b+a)^p (x^n b-a)^p - 4\left(\int \frac{(x^n b+a)^p (x^n b-a)^p}{2x^{2n}b^2np\,x^2-x^{2n}b^2x^2-2a^2np\,x^2+a^2x^2}dx\right) a^2n^2p^2x + 2\left(\int \frac{(x^n b+a)^p (x^n b-a)^p}{2x^{2n}b^2np\,x^2-x^{2n}b^2x^2-2a^2np\,x^2\cdots}\right.}{x\,(2np-1)}$$

input
```
int((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x)
```

output
```
((x**n*b + a)**p*(x**n*b - a)**p - 4*int(((x**n*b + a)**p*(x**n*b - a)**p)
/(2*x**(2*n)*b**2*n*p*x**2 - x**(2*n)*b**2*x**2 - 2*a**2*n*p*x**2 + a**2*x
**2),x)*a**2*n**2*p**2*x + 2*int(((x**n*b + a)**p*(x**n*b - a)**p)/(2*x**(
2*n)*b**2*n*p*x**2 - x**(2*n)*b**2*x**2 - 2*a**2*n*p*x**2 + a**2*x**2),x)*
a**2*n*p*x)/(x*(2*n*p - 1))
```

3.581 $\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}(c+d(e+fx)^2)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 255

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx = \frac{2b\sqrt{eg+fgx}}{dfg} + \frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}$$

output
```
2*b*(f*g*x+e*g)^(1/2)/d/f/g+1/2*(-a*d+b*c)*arctan(1-2^(1/2)*d^(1/4)*(f*g*x
+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b
*c)*arctan(1+2^(1/2)*d^(1/4)*(f*g*x+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^
(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*(f*
g*x+e*g)^(1/2)/(c^(1/2)*g^(1/2)+d^(1/2)*g^(1/2)*(f*x+e)))*2^(1/2)/c^(3/4)/
d^(5/4)/f/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{4bc^{3/4}\sqrt[4]{d}(e+fx)+\sqrt{2}(bc-ad)\sqrt{e+fx}\arctan\left(\frac{\sqrt{c}-\sqrt{d}(e+fx)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{e+fx}}\right)-\sqrt{2}(bc-ad)\sqrt{e+fx}\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{2c^{3/4}d^{5/4}f\sqrt{g(e+fx)}}$$

input
```
Integrate[(a + b*(e + f*x)^2)/(Sqrt[e*g + f*g*x]*(c + d*(e + f*x)^2)),x]
```

output
```
(4*b*c^(3/4)*d^(1/4)*(e + f*x) + Sqrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTan[
(Sqrt[c] - Sqrt[d]*(e + f*x))/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e + f*x])] - S
qrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e +
 f*x])/(Sqrt[c] + Sqrt[d]*(e + f*x))])/(2*c^(3/4)*d^(5/4)*f*Sqrt[g*(e + f*
x)])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {1014, 363, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$\Big\downarrow \text{1014}$$
$$\frac{\sqrt{e+fx}\int \frac{b(e+fx)^2+a}{\sqrt{e+fx}(d(e+fx)^2+c)}d(e+fx)}{f\sqrt{eg+fgx}}$$
$$\Big\downarrow \text{363}$$

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{e+fx}\left(d(e+fx)^2+c\right)}d(e+fx)}{d}\right)}{f\sqrt{eg+fgx}}$$

$\big\downarrow$ 266

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\int \frac{1}{d(e+fx)^2+c}d\sqrt{e+fx}}{d}\right)}{f\sqrt{eg+fgx}}$$

$\big\downarrow$ 755

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}(e+fx)}{d(e+fx)^2+c}d\sqrt{e+fx}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}(e+fx)+\sqrt{c}}{d(e+fx)^2+c}d\sqrt{e+fx}}{2\sqrt{c}}\right)}{d}\right)}{f\sqrt{eg+fgx}}$$

$\big\downarrow$ 1476

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\frac{\int \frac{1}{e+fx-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{e+fx}}{2\sqrt{d}}+\frac{\int \frac{1}{e+fx+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{e+fx}}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}(e+fx)}{d(e+fx)^2+c}d\sqrt{e+fx}}{2\sqrt{c}}\right)}{d}\right)}{f\sqrt{eg+fgx}}$$

$\big\downarrow$ 1082

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\frac{\int\frac{1}{-e-fx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-e-fx-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{c}-\sqrt{d}(e+fx)}{d(e+fx)^2+c}d\sqrt{e+fx}}{2\sqrt{c}}\right)}{d}\right)}{f\sqrt{eg+fgx}}$$

$\Big\downarrow$ 217

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}(e+fx)}{d(e+fx)^2+c}d\sqrt{e+fx}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{f\sqrt{eg+fgx}}$$

$\Big\downarrow$ 1479

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{d}\left(e+fx-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{e+fx}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(e+fx+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{e+fx}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\arctan\left(\cdots\right.}{\cdots}\cdots\right)}{d}\cdots\right)}{f\sqrt{eg+fgx}\cdots}$$

$\downarrow$ 25

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{d}\left(e+fx-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{e+fx}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(e+fx+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{e+fx}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{2}\cdots}\cdots\right)}{d}\cdots\right)}{f\sqrt{eg+fgx}\cdots}$$

$\downarrow$ 27

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{e+fx}}{e+fx-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{e+fx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}+\sqrt[4]{c}}{e+fx+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{e+fx}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{e+fx}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{\cdots}}\cdots\right)}{d}\cdots\right)}{f\sqrt{eg+fgx}\cdots}$$

$\Big\downarrow$ 1103

$$\frac{\sqrt{e+fx}\left(\frac{2b\sqrt{e+fx}}{d}-\frac{2(bc-ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{e+fx}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{e+fx}+\sqrt{c}+\sqrt{d}(e+fx)\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\cdots\right)}{\cdots}}{2\sqrt{c}}\cdots\right)}{d}\cdots\right)}{f\sqrt{eg+fgx}\cdots}$$

input
```
Int[(a + b*(e + f*x)^2)/(Sqrt[e*g + f*g*x]*(c + d*(e + f*x)^2)),x]
```

output
```
(Sqrt[e + f*x]*((2*b*Sqrt[e + f*x])/d - (2*(b*c - a*d)*((-(ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[e + f*x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[e + f*x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*
Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e + f*x] + Sqr
t[d]*(e + f*x)]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[e + f*x] + Sqrt[d]*(e + f*x)]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2
*Sqrt[c])))/d))/(f*Sqrt[e*g + f*g*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1014
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q
_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a +
b*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && LinearPairQ[u, v, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{4dc}}{fg}$
default	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{4dc}}{fg}$
risch	$\frac{2b(fx+e)}{df\sqrt{g(fx+e)}}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\right.}{4dfgc}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{-\sqrt{\frac{c\,g^2}{d}}-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}+(-fx-e)g}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}-\sqrt{\frac{c\,g^2}{d}}+(-fx-e)g}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}-1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}\right)\right.}{4dfgc}$

input
```
int((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x,method=_RETURNVERB
OSE)
```

output
```
2/g/f*(b/d*(f*g*x+e*g)^(1/2)+1/8*(a*d-b*c)/d*(c*g^2/d)^(1/4)/c*2^(1/2)*(ln
((f*g*x+e*g+(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2))/(f*
g*x+e*g-(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2)))+2*arct
an(2^(1/2)/(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)+1)-2*arctan(-2^(1/2)/(c*g^2/d
)^(1/4)*(f*g*x+e*g)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.65

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{dfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}\log\left(cdfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}-\sqrt{fgx+eg}\right.}{}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x, algorithm="
fricas")
```

output
```
1/2*(d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) -
 sqrt(f*g*x + e*g)*(b*c - a*d)) + I*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(I*c
*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) - I*d*f*g
*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
/(c^3*d^5*f^4*g^2))^(1/4)*log(-I*c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f
*g*x + e*g)*(b*c - a*d)) - d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(-c*d*f*g*(-(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^
3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) + 4*sqrt(f*g*x + e*
g)*b)/(d*f*g)
```

Sympy [F]

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx=\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{g\,(e+fx)}\,(c+de^2+2defx+df^2x^2)}\,dx$$

input
```
integrate((a+b*(f*x+e)**2)/(f*g*x+e*g)**(1/2)/(c+d*(f*x+e)**2),x)
```

output
```
Integral((a + b*e**2 + 2*b*e*f*x + b*f**2*x**2)/(sqrt(g*(e + f*x))*(c + d*
e**2 + 2*d*e*f*x + d*f**2*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx=$$

$$-\frac{\frac{\left(\frac{\sqrt{2}g^2\log\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}g^2\log\left(-\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}+\frac{2\sqrt{2}g\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}d^{\ldots}\right.}{2\sqrt{\ldots}}\right.}{\sqrt{\sqrt{c}\sqrt{d}g\sqrt{\ldots}}}\ldots\right.}{d}\ldots}{4\,fg}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x, algorithm="
maxima")
```

output
```
-1/4*((sqrt(2)*g^2*log(sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (
f*g*x + e*g)*sqrt(d) + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) - sqrt(2)*g^2*lo
g(-sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (f*g*x + e*g)*sqrt(d)
 + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) + 2*sqrt(2)*g*arctan(1/2*sqrt(2)*(sq
rt(2)*(c*g^2)^(1/4)*d^(1/4) + 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqrt(c)*sq
rt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)) + 2*sqrt(2)*g*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c*g^2)^(1/4)*d^(1/4) - 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqr
t(c)*sqrt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)))*(b*c - a*d)/d - 8*sqrt
(f*g*x + e*g)*b/d)/(f*g)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.47

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{2\sqrt{fgx+eg}b}{dfg} - \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}bc-(cd^3g^2)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}bc-(cd^3g^2)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}-2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}bc-(cd^3g^2)^{\frac{1}{4}}ad\right)\log\left(fgx+eg+\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$
$$+ \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}bc-(cd^3g^2)^{\frac{1}{4}}ad\right)\log\left(fgx+eg-\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x, algorithm="
giac")
```

output
```
2*sqrt(f*g*x + e*g)*b/(d*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b*c - (c*d^
3*g^2)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) + 2*sqrt(f*g
*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b
*c - (c*d^3*g^2)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) -
 2*sqrt(f*g*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/4*sqrt(2)*((c*d^3*g
^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g + sqrt(2)*sqrt(f*g*
x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g) + 1/4*sqrt(2)*((c*d^
3*g^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g - sqrt(2)*sqrt(f
*g*x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g)
```

Mupad [B] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 1044, normalized size of antiderivative = 4.09

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*(e + f*x)^2)/((e*g + f*g*x)^(1/2)*(c + d*(e + f*x)^2)),x)
```

output
```
(2*b*(e*g + f*g*x)^(1/2))/(d*f*g) - (atan((((a*d - b*c)*((16*(e*g + f*g*x)
^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(
2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a
^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 -
a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(2*(-c)^(3
/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^
2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d^2*g^3 - a*c*d^3*g
^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*
f*g^(1/2)) - ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*
d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b
*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2))))
*(a*d - b*c)*1i)/((-c)^(3/4)*d^(5/4)*f*g^(1/2)) - (atan((((a*d - b*c)*((16
*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2
- ((b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*
g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g
*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + ((b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/
(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(
a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - ((b*c^2*d^2*g^3 -...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{\sqrt{g}\left(2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a-2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b+2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(2\sqrt{c}\right.\right.}{}$$

input
```
int((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x)
```

output
```
(sqrt(g)*(2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(
1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d - 2*d**(3/4)*c**(1
/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(1/4)*c**(1/4)*sqrt(2))/(d*
*(1/4)*c**(1/4)*sqrt(2)))*b*c + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d
)*sqrt(e + f*x) + d**(1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*
a*d - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) + d**(1/4)
*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c + 8*sqrt(e + f*x)*b*c*
d - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(
2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d + d**(3/4)*c**(1/4)*sqrt(2)*lo
g( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(
d)*f*x)*b*c + d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x
)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d - d**(3/4)*c**(1/4)*sqr
t(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + s
qrt(d)*f*x)*b*c))/(4*c*d**2*f*g)
```

3.582 $\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}(c+d(e+fx)^2)}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 255

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx = \frac{2b\sqrt{eg+fgx}}{dfg} + \frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}$$

output
```
2*b*(f*g*x+e*g)^(1/2)/d/f/g+1/2*(-a*d+b*c)*arctan(1-2^(1/2)*d^(1/4)*(f*g*x
+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b
*c)*arctan(1+2^(1/2)*d^(1/4)*(f*g*x+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^
(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*(f*
g*x+e*g)^(1/2)/(c^(1/2)*g^(1/2)+d^(1/2)*g^(1/2)*(f*x+e)))*2^(1/2)/c^(3/4)/
d^(5/4)/f/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx = \frac{4bc^{3/4}\sqrt[4]{d}(e+fx)+\sqrt{2}(bc-ad)\sqrt{e+fx}\arctan\left(\frac{\sqrt{c}-\sqrt{d}(e+fx)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{e+fx}}\right)-\sqrt{2}(bc-ad)\sqrt{e+fx}\,\text{arctanh}\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{2c^{3/4}d^{5/4}f\sqrt{g(e+fx)}}$$

input
```
Integrate[(a + b*e^2 + 2*b*e*f*x + b*f^2*x^2)/(Sqrt[e*g + f*g*x]*(c + d*(e
 + f*x)^2)),x]
```

output
```
(4*b*c^(3/4)*d^(1/4)*(e + f*x) + Sqrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTan[
(Sqrt[c] - Sqrt[d]*(e + f*x))/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e + f*x])] - S
qrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e +
 f*x])/(Sqrt[c] + Sqrt[d]*(e + f*x))])/(2*c^(3/4)*d^(5/4)*f*Sqrt[g*(e + f*
x)])
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$, Rules used = {2084, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$

$$\Big\downarrow \text{ 2084}$$

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}dx$$

$$\Big\downarrow \text{ 2159}$$

$$\int \left(\frac{a-\frac{bc}{d}}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}+\frac{b}{d\sqrt{eg+fgx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}+1\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{2b\sqrt{eg+fgx}}{dfg}$$

input
```
Int[(a + b*e^2 + 2*b*e*f*x + b*f^2*x^2)/(Sqrt[e*g + f*g*x]*(c + d*(e + f*x
)^2)),x]
```

output
```
(2*b*Sqrt[e*g + f*g*x])/(d*f*g) + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[g*(e + f*x)])/(c^(1/4)*Sqrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]
) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[g*(e + f*x)])/(c^(1/4)*S
qrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) + ((b*c - a*d)*Log[Sqrt[c]*S
qrt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f
*x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) - ((b*c - a*d)*Log[Sqrt[c]*Sq
rt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f*
x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2084
```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] &&  !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x]) &&  !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*
x)^2*((a_.) + (b_.)*x + (c_.)*x^2)^(t_.) /; FreeQ[{a, b, c, d, e, f, g, t},
 x]] &&  !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*x)^2*((a_.
) + (c_.)*x^2)^(t_.) /; FreeQ[{a, c, d, e, f, g, t}, x]]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\ldots\right.\right.}{4dc}}{fg}$
default	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\ldots\right.\right.}{4dc}}{fg}$
risch	$\frac{2b(fx+e)}{df\sqrt{g(fx+e)}}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\ldots\right.}{4dfgc}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{-\sqrt{\frac{c g^2}{d}}-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}+(-fx-e)g}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}-\sqrt{\frac{c g^2}{d}}+(-fx-e)g}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}-1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}\ldots\right.\right.}{4dfgc}$

input
```
int((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x,meth
od=_RETURNVERBOSE)
```

output
```
2/g/f*(b/d*(f*g*x+e*g)^(1/2)+1/8*(a*d-b*c)/d*(c*g^2/d)^(1/4)/c*2^(1/2)*(ln
((f*g*x+e*g+(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2))/(f*
g*x+e*g-(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2)))+2*arct
an(2^(1/2)/(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)+1)-2*arctan(-2^(1/2)/(c*g^2/d
)^(1/4)*(f*g*x+e*g)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.65

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{dfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}\log\left(cdfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}-\sqrt{fgx+eg}\right.}{}$$

input

```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),
x, algorithm="fricas")
```

output

```
1/2*(d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) -
 sqrt(f*g*x + e*g)*(b*c - a*d)) + I*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(I*c
*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) - I*d*f*g
*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
/(c^3*d^5*f^4*g^2))^(1/4)*log(-I*c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f
*g*x + e*g)*(b*c - a*d)) - d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(-c*d*f*g*(-(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^
3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) + 4*sqrt(f*g*x + e*
g)*b)/(d*f*g)
```

Sympy [F]

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx=\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{g\,(e+fx)}\,(c+de^2+2defx+df^2x^2)}\,dx$$

input
```
integrate((b*f**2*x**2+2*b*e*f*x+b*e**2+a)/(f*g*x+e*g)**(1/2)/(c+d*(f*x+e)
**2),x)
```

output
```
Integral((a + b*e**2 + 2*b*e*f*x + b*f**2*x**2)/(sqrt(g*(e + f*x))*(c + d*
e**2 + 2*d*e*f*x + d*f**2*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx=$$
$$-\frac{\frac{\left(\frac{\sqrt{2}g^2\log\left(\sqrt{2}\left(cg^2\right)^{\frac14}\sqrt{fgx+eg}\,d^{\frac14}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac34}d^{\frac14}}-\frac{\sqrt{2}g^2\log\left(-\sqrt{2}\left(cg^2\right)^{\frac14}\sqrt{fgx+eg}\,d^{\frac14}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac34}d^{\frac14}}+\frac{2\sqrt{2}g\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(cg^2\right)^{\frac14}d^{\cdots}\cdots\right.}{2\sqrt{\cdots}}\right.}{\sqrt{\sqrt{c}\sqrt{d}g}\sqrt{\cdots}}\cdots}{d}\cdots}{4\,fg}$$

input
```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),
x, algorithm="maxima")
```

output
```
-1/4*((sqrt(2)*g^2*log(sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (
f*g*x + e*g)*sqrt(d) + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) - sqrt(2)*g^2*lo
g(-sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (f*g*x + e*g)*sqrt(d)
 + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) + 2*sqrt(2)*g*arctan(1/2*sqrt(2)*(sq
rt(2)*(c*g^2)^(1/4)*d^(1/4) + 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqrt(c)*sq
rt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)) + 2*sqrt(2)*g*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c*g^2)^(1/4)*d^(1/4) - 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqr
t(c)*sqrt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)))*(b*c - a*d)/d - 8*sqrt
(f*g*x + e*g)*b/d)/(f*g)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.47

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$

$$= \frac{2\sqrt{fgx+eg}b}{dfg} - \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}\,bc-(cd^3g^2)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}\,bc-(cd^3g^2)^{\frac{1}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}-2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}\,bc-(cd^3g^2)^{\frac{1}{4}}\,ad\right)\log\left(fgx+eg+\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$
$$+ \frac{\sqrt{2}\left((cd^3g^2)^{\frac{1}{4}}\,bc-(cd^3g^2)^{\frac{1}{4}}\,ad\right)\log\left(fgx+eg-\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$

input
```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),
x, algorithm="giac")
```

output
```
2*sqrt(f*g*x + e*g)*b/(d*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b*c - (c*d^
3*g^2)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) + 2*sqrt(f*g
*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b
*c - (c*d^3*g^2)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) -
 2*sqrt(f*g*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/4*sqrt(2)*((c*d^3*g
^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g + sqrt(2)*sqrt(f*g*
x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g) + 1/4*sqrt(2)*((c*d^
3*g^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g - sqrt(2)*sqrt(f
*g*x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1044, normalized size of antiderivative = 4.09

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*e^2 + b*f^2*x^2 + 2*b*e*f*x)/((e*g + f*g*x)^(1/2)*(c + d*(e + f
*x)^2)),x)
```

output
```
(2*b*(e*g + f*g*x)^(1/2))/(d*f*g) - (atan((((a*d - b*c)*((16*(e*g + f*g*x)
^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(
2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a
^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 -
a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(2*(-c)^(3
/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^
2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d^2*g^3 - a*c*d^3*g
^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*
f*g^(1/2)) - ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*
d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b
*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))
*(a*d - b*c)*1i)/((-c)^(3/4)*d^(5/4)*f*g^(1/2)) - (atan((((a*d - b*c)*((16
*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2
- ((b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*
g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g
*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + ((b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/
(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(
a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - ((b*c^2*d^2*g^3 -...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+d(e+fx)^2)}\,dx$$
$$= \frac{\sqrt{g}\left(2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a-2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b+2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{c}}{}\right.}{}$$

input
```
int((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(c+d*(f*x+e)^2),x)
```

output
```
(sqrt(g)*(2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(
1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d - 2*d**(3/4)*c**(1
/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(1/4)*c**(1/4)*sqrt(2))/(d*
*(1/4)*c**(1/4)*sqrt(2)))*b*c + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d
)*sqrt(e + f*x) + d**(1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*
a*d - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) + d**(1/4)
*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c + 8*sqrt(e + f*x)*b*c*
d - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(
2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d + d**(3/4)*c**(1/4)*sqrt(2)*lo
g( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(
d)*f*x)*b*c + d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x
)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d - d**(3/4)*c**(1/4)*sqr
t(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + s
qrt(d)*f*x)*b*c))/(4*c*d**2*f*g)
```

3.583 $\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}(c+de^2+2defx+df^2x^2)}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 255

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \frac{2b\sqrt{eg+fgx}}{dfg}+\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}$$

output
```
2*b*(f*g*x+e*g)^(1/2)/d/f/g+1/2*(-a*d+b*c)*arctan(1-2^(1/2)*d^(1/4)*(f*g*x
+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b
*c)*arctan(1+2^(1/2)*d^(1/4)*(f*g*x+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^
(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*(f*
g*x+e*g)^(1/2)/(c^(1/2)*g^(1/2)+d^(1/2)*g^(1/2)*(f*x+e)))*2^(1/2)/c^(3/4)/
d^(5/4)/f/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$= \frac{4bc^{3/4}\sqrt[4]{d}(e+fx)+\sqrt{2}(bc-ad)\sqrt{e+fx}\arctan\left(\frac{\sqrt{c}-\sqrt{d}(e+fx)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{e+fx}}\right)-\sqrt{2}(bc-ad)\sqrt{e+fx}\,\mathrm{arctanh}\left(\frac{\sqrt{}}{}\right.}{2c^{3/4}d^{5/4}f\sqrt{g(e+fx)}}$$

input
```
Integrate[(a + b*(e + f*x)^2)/(Sqrt[e*g + f*g*x]*(c + d*e^2 + 2*d*e*f*x +
d*f^2*x^2)),x]
```

output
```
(4*b*c^(3/4)*d^(1/4)*(e + f*x) + Sqrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTan[
(Sqrt[c] - Sqrt[d]*(e + f*x))/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e + f*x])] - S
qrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e +
 f*x])/(Sqrt[c] + Sqrt[d]*(e + f*x))])/(2*c^(3/4)*d^(5/4)*f*Sqrt[g*(e + f*
x)])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.064$, Rules used = {2084, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$\Big\downarrow \; 2084$$
$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}dx$$
$$\Big\downarrow \; 2159$$

$$\int \left(\frac{a-\frac{bc}{d}}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}+\frac{b}{d\sqrt{eg+fgx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}+1\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{2b\sqrt{eg+fgx}}{dfg}$$

input
```
Int[(a + b*(e + f*x)^2)/(Sqrt[e*g + f*g*x]*(c + d*e^2 + 2*d*e*f*x + d*f^2*
x^2)),x]
```

output
```
(2*b*Sqrt[e*g + f*g*x])/(d*f*g) + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[g*(e + f*x)])/(c^(1/4)*Sqrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]
) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[g*(e + f*x)])/(c^(1/4)*S
qrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) + ((b*c - a*d)*Log[Sqrt[c]*S
qrt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f
*x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) - ((b*c - a*d)*Log[Sqrt[c]*Sq
rt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f*
x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2084
```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] && !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x]) && !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*
x)^2*((a_.) + (b_.)*x + (c_.)*x^2)^(t_.) /; FreeQ[{a, b, c, d, e, f, g, t},
 x]] && !MatchQ[z^m*u^p*v^q, ((d_.) + (e_.)*x)^m*((f_.) + (g_.)*x)^2*((a_.
) + (c_.)*x^2)^(t_.) /; FreeQ[{a, c, d, e, f, g, t}, x]]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{4dc}}{fg}$
default	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\right.\right.}{4dc}}{fg}$
risch	$\frac{2b(fx+e)}{df\sqrt{g(fx+e)}}+\frac{(ad-cb)\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}{fgx+eg-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c\,g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\right.}{4dfgc}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{-\sqrt{\frac{c\,g^2}{d}}-\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}+(-fx-e)g}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}-\sqrt{\frac{c\,g^2}{d}}+(-fx-e)g}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}-1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}+\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}{\left(\frac{c\,g^2}{d}\right)^{\frac{1}{4}}}\right.\right.}{4dfgc}$

input
```
int((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d*e^2+c),x,meth
od=_RETURNVERBOSE)
```

output
```
2/g/f*(b/d*(f*g*x+e*g)^(1/2)+1/8*(a*d-b*c)/d*(c*g^2/d)^(1/4)/c*2^(1/2)*(ln
((f*g*x+e*g+(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2))/(f*
g*x+e*g-(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2)))+2*arct
an(2^(1/2)/(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)+1)-2*arctan(-2^(1/2)/(c*g^2/d
)^(1/4)*(f*g*x+e*g)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.65

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$= \frac{dfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}\log\left(cdfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}-\sqrt{fgx+eg}\right.}{}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d*e^2+c),
x, algorithm="fricas")
```

output
```
1/2*(d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) -
 sqrt(f*g*x + e*g)*(b*c - a*d)) + I*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(I*c
*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) - I*d*f*g
*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
/(c^3*d^5*f^4*g^2))^(1/4)*log(-I*c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f
*g*x + e*g)*(b*c - a*d)) - d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(-c*d*f*g*(-(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^
3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) + 4*sqrt(f*g*x + e*
g)*b)/(d*f*g)
```

Sympy [F]

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{g\,(e+fx)}\,(c+de^2+2defx+df^2x^2)}\,dx$$

input
```
integrate((a+b*(f*x+e)**2)/(f*g*x+e*g)**(1/2)/(d*f**2*x**2+2*d*e*f*x+d*e**
2+c),x)
```

output
```
Integral((a + b*e**2 + 2*b*e*f*x + b*f**2*x**2)/(sqrt(g*(e + f*x))*(c + d*
e**2 + 2*d*e*f*x + d*f**2*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = -\frac{\frac{\left(\frac{\sqrt{2}g^2\log\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}g^2\log\left(-\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}+\frac{2\sqrt{2}g\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}d^{\cdots}\right.}{2\sqrt{\cdots}}\right.}{\sqrt{\sqrt{c}\sqrt{d}g}\sqrt{\cdots}}\right.}{d}}{4\,fg}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d*e^2+c),
x, algorithm="maxima")
```

output
```
-1/4*((sqrt(2)*g^2*log(sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (
f*g*x + e*g)*sqrt(d) + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) - sqrt(2)*g^2*lo
g(-sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (f*g*x + e*g)*sqrt(d)
 + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) + 2*sqrt(2)*g*arctan(1/2*sqrt(2)*(sq
rt(2)*(c*g^2)^(1/4)*d^(1/4) + 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqrt(c)*sq
rt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)) + 2*sqrt(2)*g*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c*g^2)^(1/4)*d^(1/4) - 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqr
t(c)*sqrt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)))*(b*c - a*d)/d - 8*sqrt
(f*g*x + e*g)*b/d)/(f*g)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.47

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$

$$= \frac{2\sqrt{fgx+eg}b}{dfg} - \frac{\sqrt{2}\left(\left(cd^3g^2\right)^{\frac{1}{4}}bc - \left(cd^3g^2\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left(\left(cd^3g^2\right)^{\frac{1}{4}}bc - \left(cd^3g^2\right)^{\frac{1}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}-2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left(\left(cd^3g^2\right)^{\frac{1}{4}}bc - \left(cd^3g^2\right)^{\frac{1}{4}}ad\right)\log\left(fgx+eg+\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$
$$+ \frac{\sqrt{2}\left(\left(cd^3g^2\right)^{\frac{1}{4}}bc - \left(cd^3g^2\right)^{\frac{1}{4}}ad\right)\log\left(fgx+eg-\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$

input
```
integrate((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d*e^2+c),
x, algorithm="giac")
```

output
```
2*sqrt(f*g*x + e*g)*b/(d*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b*c - (c*d^
3*g^2)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) + 2*sqrt(f*g
*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b
*c - (c*d^3*g^2)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) -
 2*sqrt(f*g*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/4*sqrt(2)*((c*d^3*g
^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g + sqrt(2)*sqrt(f*g*
x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g) + 1/4*sqrt(2)*((c*d^
3*g^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g - sqrt(2)*sqrt(f
*g*x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 1044, normalized size of antiderivative = 4.09

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*(e + f*x)^2)/((e*g + f*g*x)^(1/2)*(c + d*e^2 + d*f^2*x^2 + 2*d*
e*f*x)),x)
```

output
```
(2*b*(e*g + f*g*x)^(1/2))/(d*f*g) - (atan((((a*d - b*c)*((16*(e*g + f*g*x)
^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(
2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a
^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 -
a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(2*(-c)^(3
/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^
2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d^2*g^3 - a*c*d^3*g
^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*
f*g^(1/2)) - ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*
d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b
*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))
*(a*d - b*c)*1i)/((-c)^(3/4)*d^(5/4)*f*g^(1/2)) - (atan((((a*d - b*c)*((16
*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2
- ((b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*
g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g
*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + ((b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/
(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(
a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - ((b*c^2*d^2*g^3 -...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{a+b(e+fx)^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$= \frac{\sqrt{g}\left(2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a-2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b+2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{c}}{}\right.\right.}{}$$

input
```
int((a+b*(f*x+e)^2)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d*e^2+c),x)
```

output
```
(sqrt(g)*(2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(
1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d - 2*d**(3/4)*c**(1
/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(1/4)*c**(1/4)*sqrt(2))/(d*
*(1/4)*c**(1/4)*sqrt(2)))*b*c + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d
)*sqrt(e + f*x) + d**(1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*
a*d - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) + d**(1/4)
*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c + 8*sqrt(e + f*x)*b*c*
d - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(
2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d + d**(3/4)*c**(1/4)*sqrt(2)*lo
g( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(
d)*f*x)*b*c + d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x
)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d - d**(3/4)*c**(1/4)*sqr
t(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + s
qrt(d)*f*x)*b*c))/(4*c*d**2*f*g)
```

3.584 $\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}(c+de^2+2defx+df^2x^2)}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 255

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \frac{2b\sqrt{eg+fgx}}{dfg}+\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}} - \frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{eg+fgx}}{\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}$$

output
```
2*b*(f*g*x+e*g)^(1/2)/d/f/g+1/2*(-a*d+b*c)*arctan(1-2^(1/2)*d^(1/4)*(f*g*x
+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b
*c)*arctan(1+2^(1/2)*d^(1/4)*(f*g*x+e*g)^(1/2)/c^(1/4)/g^(1/2))*2^(1/2)/c^
(3/4)/d^(5/4)/f/g^(1/2)-1/2*(-a*d+b*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*(f*
g*x+e*g)^(1/2)/(c^(1/2)*g^(1/2)+d^(1/2)*g^(1/2)*(f*x+e)))*2^(1/2)/c^(3/4)/
d^(5/4)/f/g^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$=\frac{4bc^{3/4}\sqrt[4]{d}(e+fx)+\sqrt{2}(bc-ad)\sqrt{e+fx}\arctan\left(\frac{\sqrt{c}-\sqrt{d}(e+fx)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{e+fx}}\right)-\sqrt{2}(bc-ad)\sqrt{e+fx}\,\text{arctanh}\Bigg(}{2c^{3/4}d^{5/4}f\sqrt{g(e+fx)}}$$

input
```
Integrate[(a + b*e^2 + 2*b*e*f*x + b*f^2*x^2)/(Sqrt[e*g + f*g*x]*(c + d*e^
2 + 2*d*e*f*x + d*f^2*x^2)),x]
```

output
```
(4*b*c^(3/4)*d^(1/4)*(e + f*x) + Sqrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTan[
(Sqrt[c] - Sqrt[d]*(e + f*x))/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e + f*x])] - S
qrt[2]*(b*c - a*d)*Sqrt[e + f*x]*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[e +
 f*x])/(Sqrt[c] + Sqrt[d]*(e + f*x))])/(2*c^(3/4)*d^(5/4)*f*Sqrt[g*(e + f*
x)])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.035, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$\Big\downarrow \ 2159$$
$$\int \left(\frac{a-\frac{bc}{d}}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}+\frac{b}{d\sqrt{eg+fgx}}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{g(e+fx)}}{\sqrt[4]{c}\sqrt{g}}+1\right)}{\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{g(e+fx)}+\sqrt{c}\sqrt{g}+\sqrt{d}\sqrt{g}(e+fx)\right)}{2\sqrt{2}c^{3/4}d^{5/4}f\sqrt{g}}+\frac{2b\sqrt{eg+fgx}}{dfg}$$

input
```
Int[(a + b*e^2 + 2*b*e*f*x + b*f^2*x^2)/(Sqrt[e*g + f*g*x]*(c + d*e^2 + 2*
d*e*f*x + d*f^2*x^2)),x]
```

output
```
(2*b*Sqrt[e*g + f*g*x])/(d*f*g) + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)
*Sqrt[g*(e + f*x)])/(c^(1/4)*Sqrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]
) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[g*(e + f*x)])/(c^(1/4)*S
qrt[g])])/(Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) + ((b*c - a*d)*Log[Sqrt[c]*S
qrt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f
*x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g]) - ((b*c - a*d)*Log[Sqrt[c]*Sq
rt[g] + Sqrt[d]*Sqrt[g]*(e + f*x) + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[g*(e + f*
x)]])/(2*Sqrt[2]*c^(3/4)*d^(5/4)*f*Sqrt[g])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\sqrt{\cdots}\right)\right)}{4dc}}{fg}$
default	$\frac{\frac{2b\sqrt{fgx+eg}}{d}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\arctan\left(-\sqrt{\cdots}\right)\right)}{4dc}}{fg}$
risch	$\frac{2b(fx+e)}{df\sqrt{g(fx+e)}}+\frac{(ad-cb)\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fgx+eg+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}{fgx+eg-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{fgx+eg}\,\sqrt{2}+\sqrt{\frac{c g^2}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{fgx+eg}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}+1\right)-2\cdots\right)}{4dfgc}$
pseudoelliptic	$\frac{\left(\ln\left(\frac{-\sqrt{\frac{c g^2}{d}}-\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}+(-fx-e)g}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}\sqrt{g(fx+e)}\,\sqrt{2}-\sqrt{\frac{c g^2}{d}}+(-fx-e)g}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}-1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{g(fx+e)}+\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}{\left(\frac{c g^2}{d}\right)^{\frac{1}{4}}}\cdots\right)\right)\cdots}{4dfgc}$

input
```
int((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d
*e^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/g/f*(b/d*(f*g*x+e*g)^(1/2)+1/8*(a*d-b*c)/d*(c*g^2/d)^(1/4)/c*2^(1/2)*(ln
((f*g*x+e*g+(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2))/(f*
g*x+e*g-(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)*2^(1/2)+(c*g^2/d)^(1/2)))+2*arct
an(2^(1/2)/(c*g^2/d)^(1/4)*(f*g*x+e*g)^(1/2)+1)-2*arctan(-2^(1/2)/(c*g^2/d
)^(1/4)*(f*g*x+e*g)^(1/2)+1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.65

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$
$$= \frac{dfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}\log\left(cdfg\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5f^4g^2}\right)^{\frac{1}{4}}-\sqrt{fgx+eg}\right.}{}$$

input
```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e
*f*x+d*e^2+c),x, algorithm="fricas")
```

output
```
1/2*(d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) -
 sqrt(f*g*x + e*g)*(b*c - a*d)) + I*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(I*c
*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) - I*d*f*g
*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
/(c^3*d^5*f^4*g^2))^(1/4)*log(-I*c*d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4) - sqrt(f
*g*x + e*g)*(b*c - a*d)) - d*f*g*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^
2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5*f^4*g^2))^(1/4)*log(-c*d*f*g*(-(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^
3*d^5*f^4*g^2))^(1/4) - sqrt(f*g*x + e*g)*(b*c - a*d)) + 4*sqrt(f*g*x + e*
g)*b)/(d*f*g)
```

Sympy [F]

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{g\,(e+fx)}\,(c+de^2+2defx+df^2x^2)}\,dx$$

input
```
integrate((b*f**2*x**2+2*b*e*f*x+b*e**2+a)/(f*g*x+e*g)**(1/2)/(d*f**2*x**2
+2*d*e*f*x+d*e**2+c),x)
```

output
```
Integral((a + b*e**2 + 2*b*e*f*x + b*f**2*x**2)/(sqrt(g*(e + f*x))*(c + d*
e**2 + 2*d*e*f*x + d*f**2*x**2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.14

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = -\frac{\frac{\left(\frac{\sqrt{2}g^2\log\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}g^2\log\left(-\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}\sqrt{fgx+eg}d^{\frac{1}{4}}+(fgx+eg)\sqrt{d}+\sqrt{c}g\right)}{\left(cg^2\right)^{\frac{3}{4}}d^{\frac{1}{4}}}+\frac{2\sqrt{2}g\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(cg^2\right)^{\frac{1}{4}}d^{\cdots}\right.}{2\sqrt{\cdots}}\right.}{\sqrt{\sqrt{c}\sqrt{d}g}\sqrt{\cdots}}\cdots}{d}\cdots}{4\,fg}$$

input
```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e
*f*x+d*e^2+c),x, algorithm="maxima")
```

output
```
-1/4*((sqrt(2)*g^2*log(sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (
f*g*x + e*g)*sqrt(d) + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) - sqrt(2)*g^2*lo
g(-sqrt(2)*(c*g^2)^(1/4)*sqrt(f*g*x + e*g)*d^(1/4) + (f*g*x + e*g)*sqrt(d)
 + sqrt(c)*g)/((c*g^2)^(3/4)*d^(1/4)) + 2*sqrt(2)*g*arctan(1/2*sqrt(2)*(sq
rt(2)*(c*g^2)^(1/4)*d^(1/4) + 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqrt(c)*sq
rt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)) + 2*sqrt(2)*g*arctan(-1/2*sqrt
(2)*(sqrt(2)*(c*g^2)^(1/4)*d^(1/4) - 2*sqrt(f*g*x + e*g)*sqrt(d))/sqrt(sqr
t(c)*sqrt(d)*g))/(sqrt(sqrt(c)*sqrt(d)*g)*sqrt(c)))*(b*c - a*d)/d - 8*sqrt
(f*g*x + e*g)*b/d)/(f*g)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.47

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx$$

$$= \frac{2\sqrt{fgx+eg}b}{dfg} - \frac{\sqrt{2}\left((cd^3g^2)^{\frac14}bc-(cd^3g^2)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac14}+2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac14}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac14}bc-(cd^3g^2)^{\frac14}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{cg^2}{d}\right)^{\frac14}-2\sqrt{fgx+eg}\right)}{2\left(\frac{cg^2}{d}\right)^{\frac14}}\right)}{2\,cd^2fg}$$
$$- \frac{\sqrt{2}\left((cd^3g^2)^{\frac14}bc-(cd^3g^2)^{\frac14}ad\right)\log\left(fgx+eg+\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac14}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$
$$+ \frac{\sqrt{2}\left((cd^3g^2)^{\frac14}bc-(cd^3g^2)^{\frac14}ad\right)\log\left(fgx+eg-\sqrt{2}\sqrt{fgx+eg}\left(\frac{cg^2}{d}\right)^{\frac14}+\sqrt{\frac{cg^2}{d}}\right)}{4\,cd^2fg}$$

input
```
integrate((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e
*f*x+d*e^2+c),x, algorithm="giac")
```

output
```
2*sqrt(f*g*x + e*g)*b/(d*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b*c - (c*d^
3*g^2)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) + 2*sqrt(f*g
*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/2*sqrt(2)*((c*d^3*g^2)^(1/4)*b
*c - (c*d^3*g^2)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c*g^2/d)^(1/4) -
 2*sqrt(f*g*x + e*g))/(c*g^2/d)^(1/4))/(c*d^2*f*g) - 1/4*sqrt(2)*((c*d^3*g
^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g + sqrt(2)*sqrt(f*g*
x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g) + 1/4*sqrt(2)*((c*d^
3*g^2)^(1/4)*b*c - (c*d^3*g^2)^(1/4)*a*d)*log(f*g*x + e*g - sqrt(2)*sqrt(f
*g*x + e*g)*(c*g^2/d)^(1/4) + sqrt(c*g^2/d))/(c*d^2*f*g)
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 1044, normalized size of antiderivative = 4.09

$$\int \frac{a+be^2+2befx+bf^2x^2}{\sqrt{eg+fgx}\,(c+de^2+2defx+df^2x^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*e^2 + b*f^2*x^2 + 2*b*e*f*x)/((e*g + f*g*x)^(1/2)*(c + d*e^2 +
d*f^2*x^2 + 2*d*e*f*x)),x)
```

output
```
(2*b*(e*g + f*g*x)^(1/2))/(d*f*g) - (atan((((a*d - b*c)*((16*(e*g + f*g*x)
^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(
2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a
^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 -
a*c*d^3*g^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2)))*1i)/(2*(-c)^(3
/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^
2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - (16*(b*c^2*d^2*g^3 - a*c*d^3*g
^3)*(a*d - b*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*
f*g^(1/2)) - ((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*
d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + (16*(b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b
*c))/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2))))
*(a*d - b*c)*1i)/((-c)^(3/4)*d^(5/4)*f*g^(1/2)) - (atan((((a*d - b*c)*((16
*(e*g + f*g*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2
- ((b*c^2*d^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*
g^(1/2))))/(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)) + ((a*d - b*c)*((16*(e*g + f*g
*x)^(1/2)*(a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 + ((b*c^2*d
^2*g^3 - a*c*d^3*g^3)*(a*d - b*c)*16i)/((-c)^(3/4)*d^(5/4)*f^2*g^(1/2))))/
(2*(-c)^(3/4)*d^(5/4)*f*g^(1/2)))/(((a*d - b*c)*((16*(e*g + f*g*x)^(1/2)*(
a^2*d^3*g^2 + b^2*c^2*d*g^2 - 2*a*b*c*d^2*g^2))/f^2 - ((b*c^2*d^2*g^3 -...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{a + be^2 + 2befx + bf^2x^2}{\sqrt{eg + fgx}\,(c + de^2 + 2defx + df^2x^2)}\,dx$$
$$= \frac{\sqrt{g}\left(2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)a - 2d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{d}\,\sqrt{fx+e}-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)b + 2d^{\frac{7}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{2\sqrt{c}\ldots}{}\right.\right.}{}$$

input
```
int((b*f^2*x^2+2*b*e*f*x+b*e^2+a)/(f*g*x+e*g)^(1/2)/(d*f^2*x^2+2*d*e*f*x+d
*e^2+c),x)
```

output

```
(sqrt(g)*(2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(
1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*a*d - 2*d**(3/4)*c**(1
/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) - d**(1/4)*c**(1/4)*sqrt(2))/(d*
*(1/4)*c**(1/4)*sqrt(2)))*b*c + 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d
)*sqrt(e + f*x) + d**(1/4)*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*
a*d - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((2*sqrt(d)*sqrt(e + f*x) + d**(1/4)
*c**(1/4)*sqrt(2))/(d**(1/4)*c**(1/4)*sqrt(2)))*b*c + 8*sqrt(e + f*x)*b*c*
d - d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(
2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d + d**(3/4)*c**(1/4)*sqrt(2)*lo
g( - d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(
d)*f*x)*b*c + d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x
)*sqrt(2) + sqrt(c) + sqrt(d)*e + sqrt(d)*f*x)*a*d - d**(3/4)*c**(1/4)*sqr
t(2)*log(d**(1/4)*c**(1/4)*sqrt(e + f*x)*sqrt(2) + sqrt(c) + sqrt(d)*e + s
qrt(d)*f*x)*b*c))/(4*c*d**2*f*g)
```

3.585 $\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 246

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = -\frac{e\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}} + \frac{e\arctan\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}} - \frac{e\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}bc^{5/6}d^{2/3}} + \frac{e\text{arctanh}\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}bc^{5/6}d^{2/3}}$$

output
```
-1/18*e*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x
+a)^3)^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)+1/18*e*arctan(1/3*(c+d*(b*
x+a)^3)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)-1/6*e*arc
tanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x+a)^3)^(1/2))*2^(1
/3)/b/c^(5/6)/d^(2/3)+1/18*e*arctanh((c+d*(b*x+a)^3)^(1/2)/c^(1/2))*2^(1/3
)/b/c^(5/6)/d^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 15.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.36

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \frac{e(a+bx)^2\left(\frac{c+d(a+bx)^3}{c}\right)^{3/2}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d(a+bx)^3}{c},-\frac{d(a+bx)^3}{4c}\right)}{8b\,(c+d(a+bx)^3)^{3/2}}$$

input
```
Integrate[(a*e + b*e*x)/(Sqrt[c + d*(a + b*x)^3]*(4*c + d*(a + b*x)^3)),x]
```

output
```
(e*(a + b*x)^2*((c + d*(a + b*x)^3)/c)^(3/2)*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*(a + b*x)^3)/c), -1/4*(d*(a + b*x)^3)/c])/(8*b*(c + d*(a + b*x)^3)^(3/2
))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1014, 986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ae+bex}{\sqrt{d(a+bx)^3+c}\,(d(a+bx)^3+4c)}\,dx$$

$$\Big\downarrow \text{1014}$$

$$\frac{e\int \frac{a+bx}{\sqrt{d(a+bx)^3+c}(d(a+bx)^3+4c)}d(a+bx)}{b}$$

$$\Big\downarrow \text{986}$$

$$\frac{e\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{d}(a+bx)+\sqrt[3]{c}\right)}{\sqrt{d(a+bx)^3+c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\arctan\left(\frac{\sqrt{d(a+bx)^3+c}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{d(a+bx)^3+c}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\text{arctanh}\left(\sqrt{\right.}{9\ 2^{2/3}c^{5}}\right.}{b}$$

input
```
Int[(a*e + b*e*x)/(Sqrt[c + d*(a + b*x)^3]*(4*c + d*(a + b*x)^3)),x]
```

output
```
(e*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*(a + b*x)))/Sq
rt[c + d*(a + b*x)^3]]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c +
 d*(a + b*x)^3]/(Sqrt[3]*Sqrt[c])]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - A
rcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*(a + b*x)))/Sqrt[c + d*(a + b*x
)^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c + d*(a + b*x)^3]/Sqrt[c
]]/(9*2^(2/3)*c^(5/6)*d^(2/3))))/b
```

Defintions of rubi rules used

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 1014
```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q
_.), x_Symbol] :> Simp[u^m/(Coefficient[v, x, 1]*v^m)   Subst[Int[x^m*(a +
b*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x]
 && LinearPairQ[u, v, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.65 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.23

method	result
default	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^3 d\_Z^3+3da\,b^2\_Z^2+3a^2bd\_Z+a^3d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}-\left(-c\,d^2\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(\cdots\right.}{i\sqrt{3}\left(\cdots\right.}}}{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^3 d\_Z^3+3da\,b^2\_Z^2+3a^2bd\_Z+a^3d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}-\left(-c\,d^2\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(\cdots\right.}{i\sqrt{3}\left(\cdots\right.}}}{\cdots}\right.\right.}{\cdots}$

input
```
int((b*e*x+a*e)/(c+d*(b*x+a)^3)^(1/2)/(4*c+d*(b*x+a)^3),x,method=_RETURNVE
RBOSE)
```

output
```
1/18*e/c/d^2/b*2^(1/2)*sum(-2*I/(_alpha*b+a)/d*3^(1/2)*(-c*d^2)^(1/3)*(1/6
*I*b*(2*a/b+2*x-1/d/b*(I*3^(1/2)*(-c*d^2)^(1/3)-(-c*d^2)^(1/3)))*d*3^(1/2)
/(-c*d^2)^(1/3))^(1/2)*(b*(x-1/d/b*(-c*d^2)^(1/3)+a/b)*d/(I*3^(1/2)*(-c*d^
2)^(1/3)-3*(-c*d^2)^(1/3)))^(1/2)*(-1/6*I*b*(2*a/b+2*x-1/d/b*(-(-c*d^2)^(1
/3)-I*3^(1/2)*(-c*d^2)^(1/3)))*d*3^(1/2)/(-c*d^2)^(1/3))^(1/2)/(b^3*d*x^3+
3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)*(2*d^2*(_alpha^2*b^2+2*_alpha*a*b
+a^2)+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*b*d+I*(-c*d^2)^(1/3)*3^(1/2)*a*d-I*(
-c*d^2)^(2/3)*3^(1/2)-(-c*d^2)^(1/3)*_alpha*b*d-(-c*d^2)^(1/3)*a*d-(-c*d^2
)^(2/3))*EllipticPi(((x-(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)-a)/b)/((-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(-1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b))^(1/2),1/6/d*(2*I*
3^(1/2)*(-c*d^2)^(1/3)*_alpha^2*b^2*d+4*I*3^(1/2)*(-c*d^2)^(1/3)*_alpha*a*
b*d-I*3^(1/2)*(-c*d^2)^(2/3)*_alpha*b+2*I*3^(1/2)*(-c*d^2)^(1/3)*a^2*d-I*3
^(1/2)*(-c*d^2)^(2/3)*a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha*b-3*(-c*d^2)
^(2/3)*a-3*c*d)/c,(((-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-
a)/b-(-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b)/((-1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(1/d*(-c*d^2)^(1/3)-a)/
b))^(1/2)),_alpha=RootOf(_Z^3*b^3*d+3*_Z^2*a*b^2*d+3*_Z*a^2*b*d+a^3*d+4*c)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(c+d*(b*x+a)^3)^(1/2)/(4*c+d*(b*x+a)^3),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(c+d*(b*x+a)**3)**(1/2)/(4*c+d*(b*x+a)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \int \frac{bex+ae}{\left((bx+a)^3d+4\,c\right)\sqrt{(bx+a)^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(c+d*(b*x+a)^3)^(1/2)/(4*c+d*(b*x+a)^3),x, algorithm
="maxima")
```

output
```
integrate((b*e*x + a*e)/(((b*x + a)^3*d + 4*c)*sqrt((b*x + a)^3*d + c)), x
)
```

Giac [F]

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \int \frac{bex+ae}{\left((bx+a)^3d+4\,c\right)\sqrt{(bx+a)^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(c+d*(b*x+a)^3)^(1/2)/(4*c+d*(b*x+a)^3),x, algorithm
="giac")
```

output
```
integrate((b*e*x + a*e)/(((b*x + a)^3*d + 4*c)*sqrt((b*x + a)^3*d + c)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx = \text{Hanged}$$

input
```
int((a*e + b*e*x)/((4*c + d*(a + b*x)^3)*(c + d*(a + b*x)^3)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{ae+bex}{\sqrt{c+d(a+bx)^3}\,(4c+d(a+bx)^3)}\,dx$$
$$= e\Bigg(\Bigg(\Bigg(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a$$
$$+\Bigg(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}\,x}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a$$

input
```
int((b*e*x+a*e)/(c+d*(b*x+a)^3)^(1/2)/(4*c+d*(b*x+a)^3),x)
```

output
```
e*(int(sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)/(a*
*6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**2*x**
3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*d**2*
x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2),x)*
a + int((sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)*x
)/(a**6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**
2*x**3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*
d**2*x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2
),x)*b)
```

3.586 $\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\left(4c+d(a+bx)^3\right)}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 246

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\left(4c+d(a+bx)^3\right)}\,dx = -\frac{e\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}} + \frac{e\arctan\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}} - \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}bc^{5/6}d^{2/3}} + \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}bc^{5/6}d^{2/3}}$$

output
```
-1/18*e*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x
+a)^3)^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)+1/18*e*arctan(1/3*(c+d*(b*
x+a)^3)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)-1/6*e*arc
tanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x+a)^3)^(1/2))*2^(1
/3)/b/c^(5/6)/d^(2/3)+1/18*e*arctanh((c+d*(b*x+a)^3)^(1/2)/c^(1/2))*2^(1/3
)/b/c^(5/6)/d^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.36

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx = \frac{e(a+bx)^2\left(\frac{c+d(a+bx)^3}{c}\right)^{3/2}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d(a+bx)^3}{c},-\frac{d(a+bx)^3}{4c}\right)}{8b\,(c+d(a+bx)^3)^{3/2}}$$

input

```
Integrate[(a*e + b*e*x)/(Sqrt[c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^
3*d*x^3]*(4*c + d*(a + b*x)^3)),x]
```

output

```
(e*(a + b*x)^2*((c + d*(a + b*x)^3)/c)^(3/2)*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*(a + b*x)^3)/c), -1/4*(d*(a + b*x)^3)/c])/(8*b*(c + d*(a + b*x)^3)^(3/2
))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {2511, 27, 986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ae+bex}{\sqrt{a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3+c}\,(d(a+bx)^3+4c)}\,dx$$

$$\Big\downarrow \; 2511$$

$$\frac{\int \frac{e^3(ae+bxe)}{(4ce^3+d(ae+bxe)^3)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{be}$$

$$\Big\downarrow \; 27$$

$$\frac{e^2 \int \frac{ae+bxe}{\left(4ce^3+d(ae+bxe)^3\right)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{b}$$

$$\Big\downarrow \text{ 986}$$

$$\frac{e^2 \left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{d}(ae+bex)+\sqrt[3]{c}e\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}+\frac{\arctan\left(\frac{\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}e-\sqrt[3]{2}\sqrt[3]{d}(ae+bex)\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}e}+\frac{\text{arc}}{}\right.}{b}$$

input
```
Int[(a*e + b*e*x)/(Sqrt[c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*d*x^
3]*(4*c + d*(a + b*x)^3)),x]
```

output
```
(e^2*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)*e + 2^(1/3)*d^(1/3)*(a*e + b*e
*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2
/3)*e) + ArcTan[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/(Sqrt[3]*Sqrt[c])]/(3*2^
(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)*e) - ArcTanh[(c^(1/6)*(c^(1/3)*e - 2^(1/3)*d
^(1/3)*(a*e + b*e*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(3*2^(2/3)*c
^(5/6)*d^(2/3)*e) + ArcTanh[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/Sqrt[c]]/(9*
2^(2/3)*c^(5/6)*d^(2/3)*e)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2511
```
Int[(Pn_)^(p_.)*(Qn_)^(q_.)*((g_) + (h_.)*(x_))^(m_.), x_Symbol] :> With[{P
x = Pn /. x -> (x - g)/h, Qx = Qn /. x -> (x - g)/h}, Simp[1/h   Subst[Int[
x^m*ExpandToSum[Px, x]^p*ExpandToSum[Qx, x]^q, x], x, g + h*x], x] /; Binom
ialQ[Px, x] && BinomialQ[Qx, x]] /; FreeQ[{g, h, m, p, q}, x] && PolyQ[Pn,
x] && PolyQ[Qn, x] && EqQ[Expon[Pn, x], Expon[Qn, x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.23

method	result
default	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b^{3}d\_Z^{3}+3da\,b^{2}\_Z^{2}+3a^{2}bd\_Z+a^{3}d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}-\left(-c\,d^{2}\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b(\cdots}{i\sqrt{3}(\cdots}}}{\cdots}$
elliptic	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\operatorname{RootOf}\left(b^{3}d\_Z^{3}+3da\,b^{2}\_Z^{2}+3a^{2}bd\_Z+a^{3}d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}-\left(-c\,d^{2}\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b(\cdots}{i\sqrt{3}(\cdots}}}{\cdots}$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/(4*c+d
*(b*x+a)^3),x,method=_RETURNVERBOSE)
```

output
```
1/18*e/c/d^2/b*2^(1/2)*sum(-2*I/(_alpha*b+a)/d*3^(1/2)*(-c*d^2)^(1/3)*(1/6
*I*b*(2*a/b+2*x-1/d/b*(I*3^(1/2)*(-c*d^2)^(1/3)-(-c*d^2)^(1/3)))*d*3^(1/2)
/(-c*d^2)^(1/3))^(1/2)*(b*(x-1/d/b*(-c*d^2)^(1/3)+a/b)*d/(I*3^(1/2)*(-c*d^
2)^(1/3)-3*(-c*d^2)^(1/3)))^(1/2)*(-1/6*I*b*(2*a/b+2*x-1/d/b*(-(-c*d^2)^(1
/3)-I*3^(1/2)*(-c*d^2)^(1/3)))*d*3^(1/2)/(-c*d^2)^(1/3))^(1/2)/(b^3*d*x^3+
3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)*(2*d^2*(_alpha^2*b^2+2*_alpha*a*b
+a^2)+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*b*d+I*(-c*d^2)^(1/3)*3^(1/2)*a*d-I*(
-c*d^2)^(2/3)*3^(1/2)-(-c*d^2)^(1/3)*_alpha*b*d-(-c*d^2)^(1/3)*a*d-(-c*d^2
)^(2/3))*EllipticPi(((x-(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)-a)/b)/((-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(-1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b))^(1/2),1/6/d*(2*I*
3^(1/2)*(-c*d^2)^(1/3)*_alpha^2*b^2*d+4*I*3^(1/2)*(-c*d^2)^(1/3)*_alpha*a*
b*d-I*3^(1/2)*(-c*d^2)^(2/3)*_alpha*b+2*I*3^(1/2)*(-c*d^2)^(1/3)*a^2*d-I*3
^(1/2)*(-c*d^2)^(2/3)*a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha*b-3*(-c*d^2)
^(2/3)*a-3*c*d)/c,(((-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-
a)/b-(-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b)/((-1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(1/d*(-c*d^2)^(1/3)-a)/
b))^(1/2)),_alpha=RootOf(_Z^3*b^3*d+3*_Z^2*a*b^2*d+3*_Z*a^2*b*d+a^3*d+4*c)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx=\text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(4*c+d*(b*x+a)^3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b**3*d*x**3+3*a*b**2*d*x**2+3*a**2*b*d*x+a**3*d+c)*
*(1/2)/(4*c+d*(b*x+a)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx = \int \frac{bex+ae}{\sqrt{b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+c}\left((bx+a)^3d+4\,c\right)}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(4*c+d*(b*x+a)^3),x, algorithm="maxima")
```

output
```
integrate((b*e*x + a*e)/(sqrt(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^
3*d + c)*((b*x + a)^3*d + 4*c)), x)
```

Giac [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx = \int \frac{bex+ae}{\sqrt{b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+c}\left((bx+a)^3d+4\,c\right)}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(4*c+d*(b*x+a)^3),x, algorithm="giac")
```

output
```
integrate((b*e*x + a*e)/(sqrt(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^
3*d + c)*((b*x + a)^3*d + 4*c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx = \text{Hanged}$$

input
```
int((a*e + b*e*x)/((4*c + d*(a + b*x)^3)*(c + a^3*d + b^3*d*x^3 + 3*a^2*b*
d*x + 3*a*b^2*d*x^2)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+d(a+bx)^3)}\,dx$$
$$= e\left(\left(\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.\right.\right.$$
$$+\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}\,x}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.$$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/(4*c+d
*(b*x+a)^3),x)
```

output
```
e*(int(sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)/(a*
*6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**2*x**
3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*d**2*
x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2),x)*
a + int((sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)*x
)/(a**6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**
2*x**3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*
d**2*x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2
),x)*b)
```

3.587 $\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx$

Optimal result

Integrand size = 61, antiderivative size = 246

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = -\frac{e\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}}+\frac{e\arctan\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}} - \frac{e\,\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}bc^{5/6}d^{2/3}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}bc^{5/6}d^{2/3}}$$

output
```
-1/18*e*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x
+a)^3)^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)+1/18*e*arctan(1/3*(c+d*(b*
x+a)^3)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)-1/6*e*arc
tanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x+a)^3)^(1/2))*2^(1
/3)/b/c^(5/6)/d^(2/3)+1/18*e*arctanh((c+d*(b*x+a)^3)^(1/2)/c^(1/2))*2^(1/3
)/b/c^(5/6)/d^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.36

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = \frac{e(a+bx)^2\left(\frac{c+d(a+bx)^3}{c}\right)^{3/2}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d(a+bx)^3}{c},-\frac{d(a+bx)^3}{4c}\right)}{8b\left(c+d(a+bx)^3\right)^{3/2}}$$

input
```
Integrate[(a*e + b*e*x)/((4*c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*
d*x^3)*Sqrt[c + d*(a + b*x)^3]),x]
```

output
```
(e*(a + b*x)^2*((c + d*(a + b*x)^3)/c)^(3/2)*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*(a + b*x)^3)/c), -1/4*(d*(a + b*x)^3)/c])/(8*b*(c + d*(a + b*x)^3)^(3/2
))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = {2511, 27, 986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ae+bex}{(a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3+4c)\sqrt{d(a+bx)^3+c}}\,dx$$

$$\Big\downarrow \text{ 2511}$$

$$\frac{\int \frac{e^3(ae+bxe)}{(4ce^3+d(ae+bxe)^3)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{be}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^2 \int \frac{ae+bxe}{(4ce^3+d(ae+bxe)^3)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{b}$$

$\big\downarrow$ 986

$$\frac{e^2\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{d}(ae+bex)+\sqrt[3]{c}e\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}+\frac{\arctan\left(\frac{\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}e-\sqrt[3]{2}\sqrt[3]{d}(ae+bex)\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}e}+ \text{arc}\ldots\right.}{b}$$

input
```
Int[(a*e + b*e*x)/((4*c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*d*x^3)
*Sqrt[c + d*(a + b*x)^3]),x]
```

output
```
(e^2*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)*e + 2^(1/3)*d^(1/3)*(a*e + b*e
*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2
/3)*e) + ArcTan[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/(Sqrt[3]*Sqrt[c])]/(3*2^
(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)*e) - ArcTanh[(c^(1/6)*(c^(1/3)*e - 2^(1/3)*d
^(1/3)*(a*e + b*e*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(3*2^(2/3)*c
^(5/6)*d^(2/3)*e) + ArcTanh[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/Sqrt[c]]/(9*
2^(2/3)*c^(5/6)*d^(2/3)*e)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2511
```
Int[(Pn_)^(p_.)*(Qn_)^(q_.)*((g_) + (h_.)*(x_))^(m_.), x_Symbol] :> With[{P
x = Pn /. x -> (x - g)/h, Qx = Qn /. x -> (x - g)/h}, Simp[1/h   Subst[Int[
x^m*ExpandToSum[Px, x]^p*ExpandToSum[Qx, x]^q, x], x, g + h*x], x] /; Binom
ialQ[Px, x] && BinomialQ[Qx, x]] /; FreeQ[{g, h, m, p, q}, x] && PolyQ[Pn,
x] && PolyQ[Qn, x] && EqQ[Expon[Pn, x], Expon[Qn, x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.23

method	result
default	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^3 d\_Z^3+3dab^2\_Z^2+3a^2bd\_Z+a^3d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}-\left(-c\,d^2\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(\cdots\right.}{i\sqrt{3}\left(\cdots\right.}}\cdots}{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^3 d\_Z^3+3dab^2\_Z^2+3a^2bd\_Z+a^3d+4c\right)}\left(-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}-\left(-c\,d^2\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{b\left(\cdots\right.}{i\sqrt{3}\left(\cdots\right.}}\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c)/(c+d*(b*x+
a)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*e/c/d^2/b*2^(1/2)*sum(-2*I/(_alpha*b+a)/d*3^(1/2)*(-c*d^2)^(1/3)*(1/6
*I*b*(2*a/b+2*x-1/d/b*(I*3^(1/2)*(-c*d^2)^(1/3)-(-c*d^2)^(1/3)))*d*3^(1/2)
/(-c*d^2)^(1/3))^(1/2)*(b*(x-1/d/b*(-c*d^2)^(1/3)+a/b)*d/(I*3^(1/2)*(-c*d^
2)^(1/3)-3*(-c*d^2)^(1/3)))^(1/2)*(-1/6*I*b*(2*a/b+2*x-1/d/b*(-(-c*d^2)^(1
/3)-I*3^(1/2)*(-c*d^2)^(1/3)))*d*3^(1/2)/(-c*d^2)^(1/3))^(1/2)/(b^3*d*x^3+
3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)*(2*d^2*(_alpha^2*b^2+2*_alpha*a*b
+a^2)+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*b*d+I*(-c*d^2)^(1/3)*3^(1/2)*a*d-I*(
-c*d^2)^(2/3)*3^(1/2)-(-c*d^2)^(1/3)*_alpha*b*d-(-c*d^2)^(1/3)*a*d-(-c*d^2
)^(2/3))*EllipticPi(((x-(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)-a)/b)/((-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(-1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b))^(1/2),1/6/d*(2*I*
3^(1/2)*(-c*d^2)^(1/3)*_alpha^2*b^2*d+4*I*3^(1/2)*(-c*d^2)^(1/3)*_alpha*a*
b*d-I*3^(1/2)*(-c*d^2)^(2/3)*_alpha*b+2*I*3^(1/2)*(-c*d^2)^(1/3)*a^2*d-I*3
^(1/2)*(-c*d^2)^(2/3)*a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha*b-3*(-c*d^2)
^(2/3)*a-3*c*d)/c,(((-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-
a)/b-(-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b)/((-1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(1/d*(-c*d^2)^(1/3)-a)/
b))^(1/2)),_alpha=RootOf(_Z^3*b^3*d+3*_Z^2*a*b^2*d+3*_Z*a^2*b*d+a^3*d+4*c)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c)/(c+d
*(b*x+a)^3)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b**3*d*x**3+3*a*b**2*d*x**2+3*a**2*b*d*x+a**3*d+4*c
)/(c+d*(b*x+a)**3)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = \int \frac{bex+ae}{(b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+4\,c)\sqrt{(bx+a)^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c)/(c+d
*(b*x+a)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*e*x + a*e)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d
+ 4*c)*sqrt((b*x + a)^3*d + c)), x)
```

Giac [F]

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\sqrt{c+d(a+bx)^3}}\,dx = \int \frac{bex+ae}{(b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+4\,c)\sqrt{(bx+a)^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c)/(c+d
*(b*x+a)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*e*x + a*e)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d
+ 4*c)*sqrt((b*x + a)^3*d + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\,\sqrt{c+d(a+bx)^3}}\,dx=\text{Hanged}$$

input
```
int((a*e + b*e*x)/((c + d*(a + b*x)^3)^(1/2)*(4*c + a^3*d + b^3*d*x^3 + 3*
a^2*b*d*x + 3*a*b^2*d*x^2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{ae+bex}{(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)\,\sqrt{c+d(a+bx)^3}}\,dx$$
$$=e\left(\left(\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.\right.\right.$$
$$+\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}\,x}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.$$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c)/(c+d*(b*x+
a)^3)^(1/2),x)
```

output

```
e*(int(sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)/(a*
*6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**2*x**
3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*d**2*
x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2),x)*
a + int((sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)*x
)/(a**6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**
2*x**3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*
d**2*x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2
),x)*b)
```

3.588 $\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\left(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx\right)}$

Optimal result

Integrand size = 83, antiderivative size = 246

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx$$
$$= -\frac{e\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}}+\frac{e\arctan\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}bc^{5/6}d^{2/3}}$$
$$-\frac{e\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}(a+bx)\right)}{\sqrt{c+d(a+bx)^3}}\right)}{3\ 2^{2/3}bc^{5/6}d^{2/3}}+\frac{e\text{arctanh}\left(\frac{\sqrt{c+d(a+bx)^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}bc^{5/6}d^{2/3}}$$

output

```
-1/18*e*arctan(3^(1/2)*c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x
+a)^3)^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)+1/18*e*arctan(1/3*(c+d*(b*
x+a)^3)^(1/2)*3^(1/2)/c^(1/2))*2^(1/3)*3^(1/2)/b/c^(5/6)/d^(2/3)-1/6*e*arc
tanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*(b*x+a))/(c+d*(b*x+a)^3)^(1/2))*2^(1
/3)/b/c^(5/6)/d^(2/3)+1/18*e*arctanh((c+d*(b*x+a)^3)^(1/2)/c^(1/2))*2^(1/3
)/b/c^(5/6)/d^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.36

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \frac{e(a+bx)^2\left(\frac{c+d(a+bx)^3}{c}\right)^{3/2}\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{d(a+bx)^3}{c},-\frac{d(a+bx)^3}{4c}\right)}{8b\,(c+d(a+bx)^3)^{3/2}}$$

input
```
Integrate[(a*e + b*e*x)/(Sqrt[c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^
3*d*x^3]*(4*c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*d*x^3)),x]
```

output
```
(e*(a + b*x)^2*((c + d*(a + b*x)^3)/c)^(3/2)*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*(a + b*x)^3)/c), -1/4*(d*(a + b*x)^3/c])/(8*b*(c + d*(a + b*x)^3)^(3/2
))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2511, 27, 986}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ae+bex}{\sqrt{a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3+c}\,(a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3+4c)}\,dx$$

$$\Big\downarrow \text{2511}$$

$$\frac{\int \frac{e^3(ae+bxe)}{(4ce^3+d(ae+bxe)^3)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{be}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{ae+bxe}{(4ce^3+d(ae+bxe)^3)\sqrt{\frac{d(ae+bxe)^3}{e^3}+c}}d(ae+bxe)}{b}$$

$$\Big\downarrow \text{986}$$

$$\frac{e^2\left(-\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{d}(ae+bex)+\sqrt[3]{c}e\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}+\frac{\arctan\left(\frac{\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}e}-\frac{\text{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}e-\sqrt[3]{2}\sqrt[3]{d}(ae+bex)\right)}{e\sqrt{\frac{d(ae+bex)^3}{e^3}+c}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}e}+\text{arc}\right.}{b}$$

input
```
Int[(a*e + b*e*x)/(Sqrt[c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*d*x^
3]*(4*c + a^3*d + 3*a^2*b*d*x + 3*a*b^2*d*x^2 + b^3*d*x^3)),x]
```

output
```
(e^2*(-1/3*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)*e + 2^(1/3)*d^(1/3)*(a*e + b*e
*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(2^(2/3)*Sqrt[3]*c^(5/6)*d^(2
/3)*e) + ArcTan[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/(Sqrt[3]*Sqrt[c])]/(3*2^
(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)*e) - ArcTanh[(c^(1/6)*(c^(1/3)*e - 2^(1/3)*d
^(1/3)*(a*e + b*e*x)))/(e*Sqrt[c + (d*(a*e + b*e*x)^3)/e^3])]/(3*2^(2/3)*c
^(5/6)*d^(2/3)*e) + ArcTanh[Sqrt[c + (d*(a*e + b*e*x)^3)/e^3]/Sqrt[c]]/(9*
2^(2/3)*c^(5/6)*d^(2/3)*e)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 986
```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[q*(ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]]/(9*2^(2/3)*b
*Rt[c, 2])), x] + (-Simp[q*(ArcTanh[Rt[c, 2]*((1 - 2^(1/3)*q*x)/Sqrt[c + d*
x^3])]/(3*2^(2/3)*b*Rt[c, 2])), x] + Simp[q*(ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])), x] - Simp[q*(ArcTan[Sqrt[3]*R
t[c, 2]*((1 + 2^(1/3)*q*x)/Sqrt[c + d*x^3])]/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2])
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

rule 2511
```
Int[(Pn_)^(p_.)*(Qn_)^(q_.)*((g_) + (h_.)*(x_))^(m_.), x_Symbol] :> With[{P
x = Pn /. x -> (x - g)/h, Qx = Qn /. x -> (x - g)/h}, Simp[1/h   Subst[Int[
x^m*ExpandToSum[Px, x]^p*ExpandToSum[Qx, x]^q, x], x, g + h*x], x] /; Binom
ialQ[Px, x] && BinomialQ[Qx, x]] /; FreeQ[{g, h, m, p, q}, x] && PolyQ[Pn,
x] && PolyQ[Qn, x] && EqQ[Expon[Pn, x], Expon[Qn, x]]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 795, normalized size of antiderivative = 3.23

method	result
default	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^{3}d\_Z^{3}+3dab^{2}\_Z^{2}+3a^{2}bd\_Z+a^{3}d+4c\right)}\left(-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}-\left(-cd^{2}\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b(\cdots}{i\sqrt{3}(\cdots}}\cdots}{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{e\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b^{3}d\_Z^{3}+3dab^{2}\_Z^{2}+3a^{2}bd\_Z+a^{3}d+4c\right)}\left(-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{6}\sqrt{\frac{ib\left(\frac{2a}{b}+2x-\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}-\left(-cd^{2}\right)^{\frac{1}{3}}}{db}\right)d\sqrt{3}}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{b(\cdots}{i\sqrt{3}(\cdots}}\cdots}{\cdots}\right.\right.}{\cdots}$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/(b^3*d
*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c),x,method=_RETURNVERBOSE)
```

output
```
1/18*e/c/d^2/b*2^(1/2)*sum(-2*I/(_alpha*b+a)/d*3^(1/2)*(-c*d^2)^(1/3)*(1/6
*I*b*(2*a/b+2*x-1/d/b*(I*3^(1/2)*(-c*d^2)^(1/3)-(-c*d^2)^(1/3)))*d*3^(1/2)
/(-c*d^2)^(1/3))^(1/2)*(b*(x-1/d/b*(-c*d^2)^(1/3)+a/b)*d/(I*3^(1/2)*(-c*d^
2)^(1/3)-3*(-c*d^2)^(1/3)))^(1/2)*(-1/6*I*b*(2*a/b+2*x-1/d/b*(-(-c*d^2)^(1
/3)-I*3^(1/2)*(-c*d^2)^(1/3)))*d*3^(1/2)/(-c*d^2)^(1/3))^(1/2)/(b^3*d*x^3+
3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)*(2*d^2*(_alpha^2*b^2+2*_alpha*a*b
+a^2)+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*b*d+I*(-c*d^2)^(1/3)*3^(1/2)*a*d-I*(
-c*d^2)^(2/3)*3^(1/2)-(-c*d^2)^(1/3)*_alpha*b*d-(-c*d^2)^(1/3)*a*d-(-c*d^2
)^(2/3))*EllipticPi(((x-(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1
/3)-a)/b)/((-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(-1/
2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b))^(1/2),1/6/d*(2*I*
3^(1/2)*(-c*d^2)^(1/3)*_alpha^2*b^2*d+4*I*3^(1/2)*(-c*d^2)^(1/3)*_alpha*a*
b*d-I*3^(1/2)*(-c*d^2)^(2/3)*_alpha*b+2*I*3^(1/2)*(-c*d^2)^(1/3)*a^2*d-I*3
^(1/2)*(-c*d^2)^(2/3)*a+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha*b-3*(-c*d^2)
^(2/3)*a-3*c*d)/c,(((-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-
a)/b-(-1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b)/((-1/2/d*
(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)-a)/b-(1/d*(-c*d^2)^(1/3)-a)/
b))^(1/2)),_alpha=RootOf(_Z^3*b^3*d+3*_Z^2*a*b^2*d+3*_Z*a^2*b*d+a^3*d+4*c)
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \text{Timed out}$$

input
```
integrate((b*e*x+a*e)/(b**3*d*x**3+3*a*b**2*d*x**2+3*a**2*b*d*x+a**3*d+c)*
*(1/2)/(b**3*d*x**3+3*a*b**2*d*x**2+3*a**2*b*d*x+a**3*d+4*c),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \int \frac{bex+ae}{(b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+4\,c)\sqrt{b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c),x, algorithm="maxima")
```

output
```
integrate((b*e*x + a*e)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d
+ 4*c)*sqrt(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d + c)), x)
```

Giac [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \int \frac{bex+ae}{(b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+4\,c)\sqrt{b^3dx^3+3\,ab^2dx^2+3\,a^2bdx+a^3d+c}}\,dx$$

input
```
integrate((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/
(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c),x, algorithm="giac")
```

output
```
integrate((b*e*x + a*e)/((b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d
+ 4*c)*sqrt(b^3*d*x^3 + 3*a*b^2*d*x^2 + 3*a^2*b*d*x + a^3*d + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx = \text{Hanged}$$

input
```
int((a*e + b*e*x)/((4*c + a^3*d + b^3*d*x^3 + 3*a^2*b*d*x + 3*a*b^2*d*x^2)
*(c + a^3*d + b^3*d*x^3 + 3*a^2*b*d*x + 3*a*b^2*d*x^2)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{ae+bex}{\sqrt{c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3}\,(4c+a^3d+3a^2bdx+3ab^2dx^2+b^3dx^3)}\,dx$$
$$= e\left(\left(\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.\right.\right.$$
$$+\left(\int \frac{\sqrt{b^3d\,x^3+3a\,b^2d\,x^2+3a^2bdx+a^3d+c}\,x}{b^6d^2x^6+6a\,b^5d^2x^5+15a^2b^4d^2x^4+20a^3b^3d^2x^3+15a^4b^2d^2x^2+6a^5b\,d^2x+a^6d^2+5b^3cd\,x^3+15a\right.$$

input
```
int((b*e*x+a*e)/(b^3*d*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+c)^(1/2)/(b^3*d
*x^3+3*a*b^2*d*x^2+3*a^2*b*d*x+a^3*d+4*c),x)
```

output
```
e*(int(sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)/(a*
*6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**2*x**
3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*d**2*
x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2),x)*
a + int((sqrt(a**3*d + 3*a**2*b*d*x + 3*a*b**2*d*x**2 + b**3*d*x**3 + c)*x
)/(a**6*d**2 + 6*a**5*b*d**2*x + 15*a**4*b**2*d**2*x**2 + 20*a**3*b**3*d**
2*x**3 + 5*a**3*c*d + 15*a**2*b**4*d**2*x**4 + 15*a**2*b*c*d*x + 6*a*b**5*
d**2*x**5 + 15*a*b**2*c*d*x**2 + b**6*d**2*x**6 + 5*b**3*c*d*x**3 + 4*c**2
),x)*b)
```

3.589 $\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 36

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{b-\frac{a}{x}}x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

output
```
2*(b-a/x)^(1/2)*x^(1+m)/(1+2*m)/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{\sqrt{b-\frac{a}{x}}x^{1+m}}{\left(\frac{1}{2}+m\right)\sqrt{a-bx}}$$

input
```
Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

output
```
(Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} \, dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}} \int \frac{x^{m-\frac{1}{2}}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}} \int x^{m-\frac{1}{2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
orering	$\frac{2x\sqrt{b-\frac{a}{x}}\,x^{m}}{(1+2m)\sqrt{-bx+a}}$	32
gosper	$\frac{2x^{1+m}\sqrt{-\frac{-bx+a}{x}}}{(1+2m)\sqrt{-bx+a}}$	36

input
```
int((b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x/(1+2*m)*(b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\,xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

input
```
integrate((b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)*x**m/(-b*x+a)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{x}x^m}{2i\,m+i}$$

input
```
integrate((b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*x^m/(2*I*m + I)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\,x^{m+1}\,\sqrt{b-\frac{a}{x}}}{(2\,m+1)\,\sqrt{a-b\,x}}$$

input
```
int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{x}\,a^m\left(\sqrt{x}\,\sqrt{b}\,\sqrt{bx-a}+bx\right)^{2m}i}{b^m\left(\sqrt{a}\,\sqrt{bx-a}+\sqrt{x}\,\sqrt{b}\,\sqrt{a}\right)^{2m}(2m+1)}$$

input
```
int((b-a/x)^(1/2)*x^m/(-b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(x)*a**m*(sqrt(x)*sqrt(b)*sqrt( - a + b*x) + b*x)**(2*m)*i)/(b**
m*(sqrt(a)*sqrt( - a + b*x) + sqrt(x)*sqrt(b)*sqrt(a))**(2*m)*(2*m + 1))
```

3.590 $\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{b-\frac{a}{x}}x^3}{5\sqrt{a-bx}}$$

output
```
2/5*(b-a/x)^(1/2)*x^3/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 9.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2x^2\sqrt{a-bx}}{5\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

output
```
(-2*x^2*Sqrt[a - b*x])/(5*Sqrt[b - a/x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{x^{3/2}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int x^{3/2}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
orering	$\frac{2\sqrt{b-\frac{a}{x}}\, x^{3}}{5\sqrt{-bx+a}}$	24
gosper	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x^{3}}{5\sqrt{-bx+a}}$	27
default	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x^{3}}{5\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\, \sqrt{-\left(-bx+a\right)x}\, \sqrt{\frac{x\left(-bx+a\right)}{bx-a}}\, \left(bx-a\right)x^{3}}{5\left(-bx+a\right)^{\frac{3}{2}} \sqrt{\left(bx-a\right)x}\, \sqrt{-x}}$	80

input
```
int((b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b-a/x)^(1/2)*x^3/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx=-\frac{2\sqrt{-bx+a}x^3\sqrt{\frac{bx-a}{x}}}{5\,(bx-a)}$$

input
```
integrate((b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \int \frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)*x**2/(-b*x+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2}{5}i\,x^{\frac{5}{2}}$$

input
```
integrate((b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*I*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2}{5}\sqrt{-x}x^2\mathrm{sgn}(x)$$

input
```
integrate((b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/5*sqrt(-x)*x^2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

input
```
int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{x}\,i\,x^2}{5}$$

input
```
int((b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(x)*i*x**2)/5
```

3.591 $\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{b-\frac{a}{x}}x^2}{3\sqrt{a-bx}}$$

output
```
2/3*(b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 5.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=-\frac{2x\sqrt{a-bx}}{3\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

output
```
(-2*x*Sqrt[a - b*x])/(3*Sqrt[b - a/x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{x}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \sqrt{x}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
orering	$\frac{2\sqrt{b-\frac{a}{x}}\, x^{2}}{3\sqrt{-bx+a}}$	24
gosper	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x^{2}}{3\sqrt{-bx+a}}$	27
default	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x^{2}}{3\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x\left(-bx+a\right)}{bx-a}}\,\left(bx-a\right)x^{2}}{3\left(-bx+a\right)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}}$	80

input
```
int((b-a/x)^(1/2)*x/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b-a/x)^(1/2)*x^2/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx=-\frac{2\,\sqrt{-bx+a}x^2\sqrt{\frac{bx-a}{x}}}{3\,(bx-a)}$$

input
```
integrate((b-a/x)^(1/2)*x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b - \frac{a}{x}}x}{\sqrt{a - bx}}\,dx = \int \frac{x\sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)*x/(-b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b - \frac{a}{x}}x}{\sqrt{a - bx}}\,dx = -\frac{2}{3}i\,x^{\frac{3}{2}}$$

input
```
integrate((b-a/x)^(1/2)*x/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*I*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{b - \frac{a}{x}}x}{\sqrt{a - bx}}\,dx = \frac{2}{3}\sqrt{-x}x\mathrm{sgn}(x)$$

input
```
integrate((b-a/x)^(1/2)*x/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(-x)*x*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

input
```
int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{x}\,ix}{3}$$

input
```
int((b-a/x)^(1/2)*x/(-b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(x)*i*x)/3
```

3.592 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} \, dx = \frac{2\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}$$

output
```
2*(b-a/x)^(1/2)*x/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} \, dx = -\frac{2\sqrt{a-bx}}{\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

output
```
(-2*Sqrt[a - b*x])/Sqrt[b - a/x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {942, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{942}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{\sqrt{x}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{\sqrt{x}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
orering	$\frac{2\sqrt{b-\frac{a}{x}}\, x}{\sqrt{-bx+a}}$	22
gosper	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x}{\sqrt{-bx+a}}$	25
default	$\frac{2\sqrt{-\frac{-bx+a}{x}}\, x}{\sqrt{-bx+a}}$	25
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	78

input
```
int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b-a/x)^(1/2)*x/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\, dx = -\frac{2\sqrt{-bx+a}x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -2i\sqrt{x}$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*I*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = 2\sqrt{-x}\mathrm{sgn}(x)$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(-x)*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
```

output
```
(2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=-2\sqrt{x}\,i$$

input
```
int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)
```

output
```
- 2*sqrt(x)*i
```

3.593 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

output
```
-2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{x^{3/2}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{x^{3/2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

method	result	size
orering	$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{-bx+a}}$	21
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	77

input
```
int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=\frac{2\,\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=\int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=\frac{2i}{\sqrt{x}}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*I/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx=\frac{2\,\mathrm{sgn}(x)}{\sqrt{-x}}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*sgn(x)/sqrt(-x)
```

Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = -\frac{2\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
```

output
```
-(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2\sqrt{x}\,i}{x}$$

input
```
int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)
```

output
```
(2*sqrt(x)*i)/x
```

3.594 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

output
```
-2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 3.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\left(b-\frac{a}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

input
```
Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

output
```
(2*(b - a/x)^(3/2))/(3*(a - b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{x^{5/2}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{x^{5/2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
orering	$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{-bx+a}}$	24
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{3(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}\,x}$	80

input
```
int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\left(bx^2-ax\right)}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2i}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*I/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\,\mathrm{sgn}(x)}{3\,\sqrt{-x}x}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sgn(x)/(sqrt(-x)*x)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = -\frac{2\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
```

output
```
-(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\sqrt{x}\,i}{3x^2}$$

input
```
int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)
```

output
```
(2*sqrt(x)*i)/(3*x**2)
```

3.595 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\operatorname{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

output
```
(a+b/x)^m*x*(d*x+c)^n*AppellF1(1-m,-m,-n,2-m,-a*x/b,-d*x/c)/(1-m)/((1+a*x/
b)^m)/((1+d*x/c)^n)
```

Mathematica [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

input
```
Integrate[(a + b/x)^m*(c + d*x)^n,x]
```

output
```
Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {942, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n \, dx$$

$$\Big\downarrow \text{942}$$

$$x^m \left(a+\frac{b}{x}\right)^m (ax+b)^{-m} \int x^{-m}(b+ax)^m(c+dx)^n dx$$

$$\Big\downarrow \text{152}$$

$$x^m \left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} \int x^{-m}\left(\frac{ax}{b}+1\right)^m (c+dx)^n dx$$

$$\Big\downarrow \text{152}$$

$$x^m \left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \int x^{-m}\left(\frac{ax}{b}+1\right)^m \left(\frac{dx}{c}+1\right)^n dx$$

$$\Big\downarrow \text{150}$$

$$\frac{x\left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \text{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

input
```
Int[(a + b/x)^m*(c + d*x)^n,x]
```

output
```
((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d
*x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{m} \left(dx+c\right)^{n}dx$$

input
```
int((a+b/x)^m*(d*x+c)^n,x)
```

output
```
int((a+b/x)^m*(d*x+c)^n,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

input
```
integrate((a+b/x)**m*(d*x+c)**n,x)
```

output
```
Integral((a + b/x)**m*(c + d*x)**n, x)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^n*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^n\,dx$$

input
```
int((a + b/x)^m*(c + d*x)^n,x)
```

output
```
int((a + b/x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\text{too large to display}$$

input
```
int((a+b/x)^m*(d*x+c)^n,x)
```

output

```
( - (c + d*x)**n*(a*x + b)**m*c*m - (c + d*x)**n*(a*x + b)**m*c*n + (c + d
*x)**n*(a*x + b)**m*d*m*x - (c + d*x)**n*(a*x + b)**m*d*n*x + 2*x**m*int((
(c + d*x)**n*(a*x + b)**m*x)/(x**m*a*c*m*n*x + x**m*a*c*m*x - x**m*a*c*n**
2*x - x**m*a*c*n*x + x**m*a*d*m*n*x**2 + x**m*a*d*m*x**2 - x**m*a*d*n**2*x
**2 - x**m*a*d*n*x**2 + x**m*b*c*m*n + x**m*b*c*m - x**m*b*c*n**2 - x**m*b
*c*n + x**m*b*d*m*n*x + x**m*b*d*m*x - x**m*b*d*n**2*x - x**m*b*d*n*x),x)*
a*c*d*m**2*n**2 + 2*x**m*int(((c + d*x)**n*(a*x + b)**m*x)/(x**m*a*c*m*n*x
 + x**m*a*c*m*x - x**m*a*c*n**2*x - x**m*a*c*n*x + x**m*a*d*m*n*x**2 + x**
m*a*d*m*x**2 - x**m*a*d*n**2*x**2 - x**m*a*d*n*x**2 + x**m*b*c*m*n + x**m*
b*c*m - x**m*b*c*n**2 - x**m*b*c*n + x**m*b*d*m*n*x + x**m*b*d*m*x - x**m*
b*d*n**2*x - x**m*b*d*n*x),x)*a*c*d*m**2*n - 2*x**m*int(((c + d*x)**n*(a*x
 + b)**m*x)/(x**m*a*c*m*n*x + x**m*a*c*m*x - x**m*a*c*n**2*x - x**m*a*c*n*
x + x**m*a*d*m*n*x**2 + x**m*a*d*m*x**2 - x**m*a*d*n**2*x**2 - x**m*a*d*n*
x**2 + x**m*b*c*m*n + x**m*b*c*m - x**m*b*c*n**2 - x**m*b*c*n + x**m*b*d*m
*n*x + x**m*b*d*m*x - x**m*b*d*n**2*x - x**m*b*d*n*x),x)*a*c*d*m*n**3 - 2*
x**m*int(((c + d*x)**n*(a*x + b)**m*x)/(x**m*a*c*m*n*x + x**m*a*c*m*x - x*
*m*a*c*n**2*x - x**m*a*c*n*x + x**m*a*d*m*n*x**2 + x**m*a*d*m*x**2 - x**m*
a*d*n**2*x**2 - x**m*a*d*n*x**2 + x**m*b*c*m*n + x**m*b*c*m - x**m*b*c*n**
2 - x**m*b*c*n + x**m*b*d*m*n*x + x**m*b*d*m*x - x**m*b*d*n**2*x - x**m*b*
d*n*x),x)*a*c*d*m*n**2 + x**m*int(((c + d*x)**n*(a*x + b)**m*x)/(x**m*a...
```

3.596 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 187

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx = \frac{c^3\left(a+\frac{b}{x}\right)^{1+m}x^2}{b(1-m)}+\frac{d^2(12ac-bd(3-m))\left(a+\frac{b}{x}\right)^{1+m}x^3}{12a^2}+\frac{d^3\left(a+\frac{b}{x}\right)^{1+m}x^4}{4a} - \frac{b(24a^3c^3-bd(36a^2c^2-bd(12ac-bd(3-m))(2-m))(1-m))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(3,1\right.}{12a^5(1-m^2)}$$

output
```
c^3*(a+b/x)^(1+m)*x^2/b/(1-m)+1/12*d^2*(12*a*c-b*d*(3-m))*(a+b/x)^(1+m)*x^
3/a^2+1/4*d^3*(a+b/x)^(1+m)*x^4/a-1/12*b*(24*a^3*c^3-b*d*(36*a^2*c^2-b*d*(
12*a*c-b*d*(3-m))*(2-m))*(1-m))*(a+b/x)^(1+m)*hypergeom([3, 1+m],[2+m],1+b
/a/x)/a^5/(-m^2+1)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3 \, dx$$
$$= \frac{\left(a+\frac{b}{x}\right)^m (b+ax) \left(-a^3(1+m)x^2(12a^2c^3-b^2d^3(3-4m+m^2)\,x-3abd^2(-1+m)x(4c+dx))+b^2(24\right.}{}$$

input
```
Integrate[(a + b/x)^m*(c + d*x)^3,x]
```

output
```
((a + b/x)^m*(b + a*x)*(-(a^3*(1 + m)*x^2*(12*a^2*c^3 - b^2*d^3*(3 - 4*m +
 m^2)*x - 3*a*b*d^2*(-1 + m)*x*(4*c + d*x))) + b^2*(24*a^3*c^3 + 36*a^2*b*
c^2*d*(-1 + m) + 12*a*b^2*c*d^2*(2 - 3*m + m^2) + b^3*d^3*(-6 + 11*m - 6*m
^2 + m^3))*Hypergeometric2F1[3, 1 + m, 2 + m, 1 + b/(a*x)]))/(12*a^5*b*(-1
 + m)*(1 + m)*x)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {941, 948, 111, 162, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{941}$$

$$\int x^3 \left(\frac{c}{x}+d\right)^3 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{948}$$

$$-\int \left(a+\frac{b}{x}\right)^m \left(\frac{c}{x}+d\right)^3 x^5 d\frac{1}{x}$$

$$\Big\downarrow \text{111}$$

$$\frac{\int \left(a+\frac{b}{x}\right)^m \left(\frac{c}{x}+d\right) \left(d(4ac-bd(1-m))+\frac{c(2ac+bd(m+1))}{x}\right) x^5 d\frac{1}{x}}{b(1-m)}+\frac{cx^4\left(\frac{c}{x}+d\right)^2\left(a+\frac{b}{x}\right)^{m+1}}{b(1-m)}$$

$$\Big\downarrow \text{162}$$

$$\frac{\frac{\left(24a^3c^3-36a^2bc^2d(1-m)+12ab^2cd^2\left(m^2-3m+2\right)-b^3d^3\left(-m^3+6m^2-11m+6\right)\right)\int \left(a+\frac{b}{x}\right)^m x^3 d\frac{1}{x}}{12a^2}-\frac{dx^4\left(a+\frac{b}{x}\right)^{m+1}\left(\frac{24a^2c^2-12abcd(1-m)+b^2}{x}\right.}{12a^2}}{b(1-m)}$$

$$\frac{cx^4\left(\frac{c}{x}+d\right)^2\left(a+\frac{b}{x}\right)^{m+1}}{b(1-m)}$$

$$\Big\downarrow \text{75}$$

$$\frac{-\frac{dx^4\left(a+\frac{b}{x}\right)^{m+1}\left(\frac{24a^2c^2-12abcd(1-m)+b^2d^2\left(m^2-4m+3\right)}{x}+3ad(4ac-bd(1-m))\right)}{12a^2}-\frac{b^2\left(a+\frac{b}{x}\right)^{m+1}\left(24a^3c^3-36a^2bc^2d(1-m)+12ab^2cd^2\left(m^2-\right.\right.}{}}{b(1-m)}$$

$$\frac{cx^4\left(\frac{c}{x}+d\right)^2\left(a+\frac{b}{x}\right)^{m+1}}{b(1-m)}$$

input
```
Int[(a + b/x)^m*(c + d*x)^3,x]
```

output
```
(c*(a + b/x)^(1 + m)*(d + c/x)^2*x^4)/(b*(1 - m)) + (-1/12*(d*(a + b/x)^(1
 + m)*(3*a*d*(4*a*c - b*d*(1 - m)) + (24*a^2*c^2 - 12*a*b*c*d*(1 - m) + b^
2*d^2*(3 - 4*m + m^2))/x)*x^4)/a^2 - (b^2*(24*a^3*c^3 - 36*a^2*b*c^2*d*(1
- m) + 12*a*b^2*c*d^2*(2 - 3*m + m^2) - b^3*d^3*(6 - 11*m + 6*m^2 - m^3))*
(a + b/x)^(1 + m)*Hypergeometric2F1[3, 1 + m, 2 + m, 1 + b/(a*x)])/(12*a^5
*(1 + m)))/(b*(1 - m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 111

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 941

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^3\,dx$$

input
```
int((a+b/x)^m*(d*x+c)^3,x)
```

output
```
int((a+b/x)^m*(d*x+c)^3,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx=\int (dx+c)^3\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 155.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx = \frac{b^m c^3 x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{3b^m c^2 d x^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)} + \frac{3b^m c d^2 x^{3-m}\Gamma(3-m)\,{}_2F_1\left(\begin{matrix}-m,3-m\\4-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(4-m)} + \frac{b^m d^3 x^{4-m}\Gamma(4-m)\,{}_2F_1\left(\begin{matrix}-m,4-m\\5-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(5-m)}$$

input
```
integrate((a+b/x)**m*(d*x+c)**3,x)
```

output
```
b**m*c**3*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_pol
ar(I*pi)/b)/gamma(2 - m) + 3*b**m*c**2*d*x**(2 - m)*gamma(2 - m)*hyper((-m
, 2 - m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + 3*b**m*c*d**2*x*
*(3 - m)*gamma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/
gamma(4 - m) + b**m*d**3*x**(4 - m)*gamma(4 - m)*hyper((-m, 4 - m), (5 - m
,), a*x*exp_polar(I*pi)/b)/gamma(5 - m)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx = \int (dx+c)^3\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx=\int (dx+c)^3\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^3\,dx$$

input
```
int((a + b/x)^m*(c + d*x)^3,x)
```

output
```
int((a + b/x)^m*(c + d*x)^3, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^3\,dx$$
$$=\frac{24(ax+b)^m a^3c^3x+36(ax+b)^m a^3c^2d\,x^2+24(ax+b)^m a^3c\,d^2x^3+6(ax+b)^m a^3d^3x^4+36(ax+b)^m a}{}$$

input
```
int((a+b/x)^m*(d*x+c)^3,x)
```

output

```
(24*(a*x + b)**m*a**3*c**3*x + 36*(a*x + b)**m*a**3*c**2*d*x**2 + 24*(a*x
+ b)**m*a**3*c*d**2*x**3 + 6*(a*x + b)**m*a**3*d**3*x**4 + 36*(a*x + b)**m
*a**2*b*c**2*d*m*x + 12*(a*x + b)**m*a**2*b*c*d**2*m*x**2 + 2*(a*x + b)**m
*a**2*b*d**3*m*x**3 + 12*(a*x + b)**m*a*b**2*c*d**2*m**2*x - 24*(a*x + b)*
*m*a*b**2*c*d**2*m*x + (a*x + b)**m*a*b**2*d**3*m**2*x**2 - 3*(a*x + b)**m
*a*b**2*d**3*m*x**2 + (a*x + b)**m*b**3*d**3*m**3*x - 5*(a*x + b)**m*b**3*
d**3*m**2*x + 6*(a*x + b)**m*b**3*d**3*m*x + 24*x**m*int((a*x + b)**m/(x**
m*a*x + x**m*b),x)*a**3*b*c**3*m + 36*x**m*int((a*x + b)**m/(x**m*a*x + x*
*m*b),x)*a**2*b**2*c**2*d*m**2 - 36*x**m*int((a*x + b)**m/(x**m*a*x + x**m
*b),x)*a**2*b**2*c**2*d*m + 12*x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x
)*a*b**3*c*d**2*m**3 - 36*x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*a*b
**3*c*d**2*m**2 + 24*x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*a*b**3*c
*d**2*m + x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*b**4*d**3*m**4 - 6*
x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*b**4*d**3*m**3 + 11*x**m*int(
(a*x + b)**m/(x**m*a*x + x**m*b),x)*b**4*d**3*m**2 - 6*x**m*int((a*x + b)*
*m/(x**m*a*x + x**m*b),x)*b**4*d**3*m)/(24*x**m*a**3)
```

3.597 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx=\frac{c^2\left(a+\frac{b}{x}\right)^{1+m}x^2}{b(1-m)}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a}$$
$$-\frac{b(6a^2c^2-6abcd(1-m)+b^2d^2(2-3m+m^2))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(3,1+m,2+m,1+\right.}{3a^4\left(1-m^2\right)}$$

output
```
c^2*(a+b/x)^(1+m)*x^2/b/(1-m)+1/3*d^2*(a+b/x)^(1+m)*x^3/a-1/3*b*(6*a^2*c^2
-6*a*b*c*d*(1-m)+b^2*d^2*(m^2-3*m+2))*(a+b/x)^(1+m)*hypergeom([3, 1+m],[2+
m],1+b/a/x)/a^4/(-m^2+1)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$$
$$=\frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(a^2d(1+m)x^2(bd(-2+m)+2a(3c+dx))-b(6a^2c^2+6abcd(-1+m)+b^2d^2(2-3\right.}{6a^4(1+m)x}$$

input
```
Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

output
```
((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
 - b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeome
tric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {941, 948, 100, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{941}$$

$$\int x^2 \left(\frac{c}{x}+d\right)^2 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{948}$$

$$-\int \left(a+\frac{b}{x}\right)^m \left(\frac{c}{x}+d\right)^2 x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{100}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\int \left(a+\frac{b}{x}\right)^m \left(\frac{3ac^2}{x}+d(6ac-bd(2-m))\right)x^3d\frac{1}{x}}{3a}$$

$$\Big\downarrow \text{87}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\frac{(6a^2c^2-bd(1-m)(6ac-bd(2-m)))\int \left(a+\frac{b}{x}\right)^m x^2d\frac{1}{x}}{2a}-\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{2a}}{3a}$$

$$\Big\downarrow \text{75}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-bd(1-m)(6ac-bd(2-m)))\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{2a^3(m+1)}-\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{2a}}{3a}$$

input
```
Int[(a + b/x)^m*(c + d*x)^2,x]
```

output
```
(d^2*(a + b/x)^(1 + m)*x^3)/(3*a) - (-1/2*(d*(6*a*c - b*d*(2 - m))*(a + b/
x)^(1 + m)*x^2)/a + (b*(6*a^2*c^2 - b*d*(6*a*c - b*d*(2 - m))*(1 - m))*(a
+ b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1
+ m)))/(3*a)
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{m} (dx+c)^{2} \, dx$$

input
```
int((a+b/x)^m*(d*x+c)^2,x)
```

output
```
int((a+b/x)^m*(d*x+c)^2,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^{m} (c+dx)^{2} \, dx = \int (dx+c)^{2} \left(a+\frac{b}{x}\right)^{m} \, dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.93 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx = \frac{b^m c^2 x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{2b^m c d x^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)} + \frac{b^m d^2 x^{3-m}\Gamma(3-m)\,{}_2F_1\left(\begin{matrix}-m,3-m\\4-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(4-m)}$$

input
```
integrate((a+b/x)**m*(d*x+c)**2,x)
```

output
```
b**m*c**2*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_pol
ar(I*pi)/b)/gamma(2 - m) + 2*b**m*c*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2
 - m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + b**m*d**2*x**(3 - m
)*gamma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/gamma(4
 - m)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx = \int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \int (dx+c)^2 \left(a+\frac{b}{x}\right)^m \, dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx = \int \left(a+\frac{b}{x}\right)^m (c+d\,x)^2 \, dx$$

input
```
int((a + b/x)^m*(c + d*x)^2,x)
```

output
```
int((a + b/x)^m*(c + d*x)^2, x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx$$
$$= \frac{6(ax+b)^m a^2c^2x + 6(ax+b)^m a^2cd\,x^2 + 2(ax+b)^m a^2d^2x^3 + 6(ax+b)^m abcdmx + (ax+b)^m ab\,d^2m\,x}{}$$

input
```
int((a+b/x)^m*(d*x+c)^2,x)
```

output
```
(6*(a*x + b)**m*a**2*c**2*x + 6*(a*x + b)**m*a**2*c*d*x**2 + 2*(a*x + b)**
m*a**2*d**2*x**3 + 6*(a*x + b)**m*a*b*c*d*m*x + (a*x + b)**m*a*b*d**2*m*x*
*2 + (a*x + b)**m*b**2*d**2*m**2*x - 2*(a*x + b)**m*b**2*d**2*m*x + 6*x**m
*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*a**2*b*c**2*m + 6*x**m*int((a*x +
 b)**m/(x**m*a*x + x**m*b),x)*a*b**2*c*d*m**2 - 6*x**m*int((a*x + b)**m/(x
**m*a*x + x**m*b),x)*a*b**2*c*d*m + x**m*int((a*x + b)**m/(x**m*a*x + x**m
*b),x)*b**3*d**2*m**3 - 3*x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*b**
3*d**2*m**2 + 2*x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*b**3*d**2*m)/
(6*x**m*a**2)
```

3.598 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

output
```
1/2*d*(a+b/x)^(1+m)*x^2/a-1/2*b*(2*a*c-b*d*(1-m))*(a+b/x)^(1+m)*hypergeom(
[2, 1+m],[2+m],1+b/a/x)/a^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{\left(a+\frac{b}{x}\right)^m (b+ax)\left(a^2d(1+m)x^2+b(-2ac-bd(-1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{a\ldots}\right.\right.}{2a^3(1+m)x}$$

input
```
Integrate[(a + b/x)^m*(c + d*x),x]
```

output
```
((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {941, 948, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{941}$$

$$\int x\left(\frac{c}{x}+d\right)\left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{948}$$

$$-\int \left(a+\frac{b}{x}\right)^m\left(\frac{c}{x}+d\right)x^3d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{(2ac-bd(1-m))\int \left(a+\frac{b}{x}\right)^m x^2d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{75}$$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

input
```
Int[(a + b/x)^m*(c + d*x),x]
```

output
```
(d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 +
m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)\,dx$$

input
```
int((a+b/x)^m*(d*x+c),x)
```

output
```
int((a+b/x)^m*(d*x+c),x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

output
```
integral((d*x + c)*((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\frac{b^m c x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{b^m d x^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

input
```
integrate((a+b/x)**m*(d*x+c),x)
```

output
```
b**m*c*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(
I*pi)/b)/gamma(2 - m) + b**m*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2 - m),
(3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(a + b/x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)\,dx$$

input
```
int((a + b/x)^m*(c + d*x),x)
```

output
```
int((a + b/x)^m*(c + d*x), x)
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$$
$$=\frac{2(ax+b)^m acx+(ax+b)^m ad\,x^2+(ax+b)^m bdmx+2x^m\left(\int \frac{(ax+b)^m}{x^m ax+x^m b}dx\right)abcm+x^m\left(\int \frac{(ax+b)^m}{x^m ax+x^m b}dx\right)}{2x^m a}$$

input
```
int((a+b/x)^m*(d*x+c),x)
```

output
```
(2*(a*x + b)**m*a*c*x + (a*x + b)**m*a*d*x**2 + (a*x + b)**m*b*d*m*x + 2*x
**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*a*b*c*m + x**m*int((a*x + b)**
m/(x**m*a*x + x**m*b),x)*b**2*d*m**2 - x**m*int((a*x + b)**m/(x**m*a*x + x
**m*b),x)*b**2*d*m)/(2*x**m*a)
```

3.599 $\int \left(a+\frac{b}{x}\right)^m dx$

Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \left(a+\frac{b}{x}\right)^m dx = -\frac{b\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{a^2(1+m)}$$

output
```
-b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],1+b/a/x)/a^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x}\right)^m dx = -\frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} \text{Hypergeometric2F1}\left(1-m,-m,2-m,-\frac{ax}{b}\right)}{-1+m}$$

input
```
Integrate[(a + b/x)^m,x]
```

output
```
-(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m
)*(1 + (a*x)/b)^m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {773, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \; 773$$

$$-\int \left(a+\frac{b}{x}\right)^m x^2d\frac{1}{x}$$

$$\Big\downarrow \; 75$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{a^2(m+1)}$$

input
```
Int[(a + b/x)^m,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a
^2*(1 + m)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m dx$$

input
```
int((a+b/x)^m,x)
```

output
```
int((a+b/x)^m,x)
```

Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^m dx = \frac{b^m x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

input
```
integrate((a+b/x)**m,x)
```

output
```
b**m*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*
pi)/b)/gamma(2 - m)
```

Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m, x)
```

Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m, x)
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(a+\frac{b}{x}\right)^m dx = -\frac{x\left(a+\frac{b}{x}\right)^m {}_2F_1\left(1-m,-m;\ 2-m;\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^m (m-1)}$$

input
```
int((a + b/x)^m,x)
```

output
```
-(x*(a + b/x)^m*hypergeom([1 - m, -m], 2 - m, -(a*x)/b))/(((a*x)/b + 1)^m*
(m - 1))
```

Reduce [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \frac{(ax+b)^m x + x^m \left(\int \frac{(ax+b)^m}{x^m a x + x^m b} dx\right) b m}{x^m}$$

input
```
int((a+b/x)^m,x)
```

output
```
((a*x + b)**m*x + x**m*int((a*x + b)**m/(x**m*a*x + x**m*b),x)*b*m)/x**m
```

3.600 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=-\frac{c\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)}+\frac{\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{b}{ax}\right)}{ad(1+m)}$$

output
```
-c*(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)
/(1+m)+(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],1+b/a/x)/a/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(ac\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,\text{Hypergeometric2F}\right.}{ad(-ac+bd)(1+m)x}$$

input
```
Integrate[(a + b/x)^m/(c + d*x),x]
```

output
```
((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x
))/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/
(a*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {941, 948, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m x}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{97}$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^m}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{\int \left(a+\frac{b}{x}\right)^m xd\frac{1}{x}}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^m}{\frac{c}{x}+d}d\frac{1}{x}}{d}+\frac{\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{b}{ax}+1\right)}{ad(m+1)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x),x]
```

output
```
-((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a
*c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] && !IntegerQ[p]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{dx+c}dx$$

input
```
int((a+b/x)^m/(d*x+c),x)
```

output
```
int((a+b/x)^m/(d*x+c),x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d*x + c), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{c+dx}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c),x)
```

output
```
Integral((a + b/x)**m/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c), x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{c+d\,x}\,dx$$

input
```
int((a + b/x)^m/(c + d*x),x)
```

output
```
int((a + b/x)^m/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{(ax+b)^m}{x^mc+x^mdx}dx$$

input
```
int((a+b/x)^m/(d*x+c),x)
```

output
```
int((a*x + b)**m/(x**m*c + x**m*d*x),x)
```

3.601 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

output
```
-b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2
/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+m)}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/
(-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {941, 946, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^2\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{78}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

input
```
Int[(a + b/x)^m/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a
*c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}dx$$

input
```
int((a+b/x)^m/(d*x+c)^2,x)
```

output
```
int((a+b/x)^m/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^2,x)
```

output
```
int((a + b/x)^m/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((a+b/x)^m/(d*x+c)^2,x)
```

output

```
((a*x + b)**m*a*x + x**m*int((a*x + b)**m/(x**m*a**2*c**3*x + 2*x**m*a**2*
c**2*d*x**2 + x**m*a**2*c*d**2*x**3 + x**m*a*b*c**3 - x**m*a*b*c**2*d*m*x
+ 2*x**m*a*b*c**2*d*x - 2*x**m*a*b*c*d**2*m*x**2 + x**m*a*b*c*d**2*x**2 -
x**m*a*b*d**3*m*x**3 - x**m*b**2*c**2*d*m - 2*x**m*b**2*c*d**2*m*x - x**m*
b**2*d**3*m*x**2),x)*a**2*b*c**3*m + x**m*int((a*x + b)**m/(x**m*a**2*c**3
*x + 2*x**m*a**2*c**2*d*x**2 + x**m*a**2*c*d**2*x**3 + x**m*a*b*c**3 - x**
m*a*b*c**2*d*m*x + 2*x**m*a*b*c**2*d*x - 2*x**m*a*b*c*d**2*m*x**2 + x**m*a
*b*c*d**2*x**2 - x**m*a*b*d**3*m*x**3 - x**m*b**2*c**2*d*m - 2*x**m*b**2*c
*d**2*m*x - x**m*b**2*d**3*m*x**2),x)*a**2*b*c**2*d*m*x - x**m*int((a*x +
b)**m/(x**m*a**2*c**3*x + 2*x**m*a**2*c**2*d*x**2 + x**m*a**2*c*d**2*x**3
+ x**m*a*b*c**3 - x**m*a*b*c**2*d*m*x + 2*x**m*a*b*c**2*d*x - 2*x**m*a*b*c
*d**2*m*x**2 + x**m*a*b*c*d**2*x**2 - x**m*a*b*d**3*m*x**3 - x**m*b**2*c**
2*d*m - 2*x**m*b**2*c*d**2*m*x - x**m*b**2*d**3*m*x**2),x)*a*b**2*c**2*d*m
**2 - x**m*int((a*x + b)**m/(x**m*a**2*c**3*x + 2*x**m*a**2*c**2*d*x**2 +
x**m*a**2*c*d**2*x**3 + x**m*a*b*c**3 - x**m*a*b*c**2*d*m*x + 2*x**m*a*b*c
**2*d*x - 2*x**m*a*b*c*d**2*m*x**2 + x**m*a*b*c*d**2*x**2 - x**m*a*b*d**3*
m*x**3 - x**m*b**2*c**2*d*m - 2*x**m*b**2*c*d**2*m*x - x**m*b**2*d**3*m*x*
*2),x)*a*b**2*c**2*d*m - x**m*int((a*x + b)**m/(x**m*a**2*c**3*x + 2*x**m*
a**2*c**2*d*x**2 + x**m*a**2*c*d**2*x**3 + x**m*a*b*c**3 - x**m*a*b*c**2*d
*m*x + 2*x**m*a*b*c**2*d*x - 2*x**m*a*b*c*d**2*m*x**2 + x**m*a*b*c*d**2...
```

3.602 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}}{bc(1-m)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(3,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{c(ac-bd)^3(1-m)(1+m)}$$

output
```
(a+b/x)^(1+m)/b/c/(1-m)/(d+c/x)^2-b*(2*a*c-b*d*(1+m))*(a+b/x)^(1+m)*hyperg
eom([3, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^3/(1-m)/(1+m)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(-\frac{dx^2}{(c+dx)^2}+\frac{b(-2ac+bd(1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{bc+acx}{acx-bdx}\right)}{(ac-bd)^2(1+m)}\right)}{2c(ac-bd)}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

output
```
((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hyp
ergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*
d)^2*(1 + m))))/(2*c*(a*c - b*d))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {941, 948, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^3\left(\frac{c}{x}+d\right)^3}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^3 x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(2ac-bd(m+1))\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}}{2c(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x)^3,x]
```

output
```
-1/2*(d*(a + b/x)^(1 + m))/(c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(
1 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))
/(a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{3}}dx$$

input
```
int((a+b/x)^m/(d*x+c)^3,x)
```

output
```
int((a+b/x)^m/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{3}}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(c+dx\right)^{3}}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**3,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^3}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^3,x)
```

output
```
int((a + b/x)^m/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int((a+b/x)^m/(d*x+c)^3,x)
```

output
```
(2*(a*x + b)**m*a**2*c*x + (a*x + b)**m*a**2*d*x**2 - (a*x + b)**m*a*b*d*m
*x + 8*x**m*int((a*x + b)**m/(12*x**m*a**6*c**8*x + 36*x**m*a**6*c**7*d*x*
*2 + 36*x**m*a**6*c**6*d**2*x**3 + 12*x**m*a**6*c**5*d**3*x**4 + 12*x**m*a
**5*b*c**8 - 48*x**m*a**5*b*c**7*d*m*x + 12*x**m*a**5*b*c**7*d*x - 144*x**
m*a**5*b*c**6*d**2*m*x**2 - 36*x**m*a**5*b*c**6*d**2*x**2 - 144*x**m*a**5*
b*c**5*d**3*m*x**3 - 60*x**m*a**5*b*c**5*d**3*x**3 - 48*x**m*a**5*b*c**4*d
**4*m*x**4 - 24*x**m*a**5*b*c**4*d**4*x**4 - 48*x**m*a**4*b**2*c**7*d*m -
24*x**m*a**4*b**2*c**7*d + 68*x**m*a**4*b**2*c**6*d**2*m**2*x - 60*x**m*a*
*4*b**2*c**6*d**2*m*x - 56*x**m*a**4*b**2*c**6*d**2*x + 204*x**m*a**4*b**2
*c**5*d**3*m**2*x**2 + 108*x**m*a**4*b**2*c**5*d**3*m*x**2 - 24*x**m*a**4*
b**2*c**5*d**3*x**2 + 204*x**m*a**4*b**2*c**4*d**4*m**2*x**3 + 204*x**m*a*
*4*b**2*c**4*d**4*m*x**3 + 24*x**m*a**4*b**2*c**4*d**4*x**3 + 68*x**m*a**4
*b**2*c**3*d**5*m**2*x**4 + 84*x**m*a**4*b**2*c**3*d**5*m*x**4 + 16*x**m*a
**4*b**2*c**3*d**5*x**4 + 68*x**m*a**3*b**3*c**6*d**2*m**2 + 84*x**m*a**3*
b**3*c**6*d**2*m + 16*x**m*a**3*b**3*c**6*d**2 - 42*x**m*a**3*b**3*c**5*d*
*3*m**3*x + 112*x**m*a**3*b**3*c**5*d**3*m**2*x + 198*x**m*a**3*b**3*c**5*
d**3*m*x + 44*x**m*a**3*b**3*c**5*d**3*x - 126*x**m*a**3*b**3*c**4*d**4*m*
*3*x**2 - 72*x**m*a**3*b**3*c**4*d**4*m**2*x**2 + 90*x**m*a**3*b**3*c**4*d
**4*m*x**2 + 36*x**m*a**3*b**3*c**4*d**4*x**2 - 126*x**m*a**3*b**3*c**3*d*
*5*m**3*x**3 - 208*x**m*a**3*b**3*c**3*d**5*m**2*x**3 - 78*x**m*a**3*b*...
```

3.603 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 173

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} + \frac{\left(a+\frac{b}{x}\right)^{1+m}}{bc^2(1-m)\left(d+\frac{c}{x}\right)^2} - \frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(3,1+m,2+m,\frac{c(a}{ac}\right.}{3c^2(ac-bd)^4(1-m^2)}$$

output
```
1/3*d^2*(a+b/x)^(1+m)/c^2/(a*c-b*d)/(d+c/x)^3+(a+b/x)^(1+m)/b/c^2/(1-m)/(d
+c/x)^2-1/3*b*(6*a^2*c^2-6*a*b*c*d*(1+m)+b^2*d^2*(m^2+3*m+2))*(a+b/x)^(1+m
)*hypergeom([3, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c^2/(a*c-b*d)^4/(-m^2+1)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(\frac{2d^2(ac-bd)x^3}{(c+dx)^3} + \frac{d(-6ac+bd(4+m))x^2}{(c+dx)^2} - \frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\text{Hypergeometric2F1}\left(2,1+m,2+m\right.}{(ac-bd)^2(1+m)}\right.}{6c^2(ac-bd)^2}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

output
```
((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d
*(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(
2 + 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x -
b*d*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {941, 948, 100, 25, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} \, dx$$

$$\Big\downarrow \text{ 941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^4\left(\frac{c}{x}+d\right)^4}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^4x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-\frac{\int -\frac{\left(a+\frac{b}{x}\right)^m\left(d(3ac-bd(m+1))-\frac{3c(ac-bd)}{x}\right)}{\left(\frac{c}{x}+d\right)^3}d\frac{1}{x}}{3c^2(ac-bd)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^m\left(d(3ac-bd(m+1))-\frac{3c(ac-bd)}{x}\right)}{\left(\frac{c}{x}+d\right)^3}d\frac{1}{x}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

$$\Big\downarrow \; 87$$

$$\frac{-\frac{\left(6a^2c^2-6abcd(m+1)+b^2d^2(m^2+3m+2)\right)\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}}{2(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{2\left(\frac{c}{x}+d\right)^2(ac-bd)}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

$$\Big\downarrow \; 78$$

$$\frac{-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2(m^2+3m+2)\right)\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{2\left(\frac{c}{x}+d\right)^2(ac-bd)}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x)^4,x]
```

output
```
(d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) + (-1/2*(d*(6*a*c
- b*d*(4 + m))*(a + b/x)^(1 + m))/((a*c - b*d)*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hyperge
ometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(2*(a*c - b*d)^3*(
1 + m)))/(3*c^2*(a*c - b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{4}}dx$$

input
```
int((a+b/x)^m/(d*x+c)^4,x)
```

output
```
int((a+b/x)^m/(d*x+c)^4,x)
```

Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*
x + c^4), x)
```

Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**4,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**4, x)
```

Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^4}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^4,x)
```

output
```
int((a + b/x)^m/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\text{too large to display}$$

input
```
int((a+b/x)^m/(d*x+c)^4,x)
```

output

```
(24*(a*x + b)**m*a**4*c**3*x + 6*(a*x + b)**m*a**4*c**2*d*x**2 + 2*(a*x +
b)**m*a**4*c*d**2*x**3 - 30*(a*x + b)**m*a**3*b*c**2*d*m*x - 48*(a*x + b)*
*m*a**3*b*c**2*d*x - 8*(a*x + b)**m*a**3*b*c*d**2*m*x**2 - 12*(a*x + b)**m
*a**3*b*c*d**2*x**2 - 2*(a*x + b)**m*a**3*b*d**3*m*x**3 - 4*(a*x + b)**m*a
**3*b*d**3*x**3 + 10*(a*x + b)**m*a**2*b**2*c*d**2*m**2*x + 40*(a*x + b)**
m*a**2*b**2*c*d**2*m*x + 24*(a*x + b)**m*a**2*b**2*c*d**2*x + 2*(a*x + b)*
*m*a**2*b**2*d**3*m**2*x**2 + 4*(a*x + b)**m*a**2*b**2*d**3*m*x**2 - (a*x
+ b)**m*a*b**3*d**3*m**3*x - 6*(a*x + b)**m*a*b**3*d**3*m**2*x - 11*(a*x +
 b)**m*a*b**3*d**3*m*x - 6*(a*x + b)**m*a*b**3*d**3*x + 13824*x**m*int((a*
x + b)**m/(576*x**m*a**9*c**12*x + 2304*x**m*a**9*c**11*d*x**2 + 3456*x**m
*a**9*c**10*d**2*x**3 + 2304*x**m*a**9*c**9*d**3*x**4 + 576*x**m*a**9*c**8
*d**4*x**5 + 576*x**m*a**8*b*c**12 - 2880*x**m*a**8*b*c**11*d*m*x - 1728*x
**m*a**8*b*c**11*d*x - 11520*x**m*a**8*b*c**10*d**2*m*x**2 - 12672*x**m*a*
*8*b*c**10*d**2*x**2 - 17280*x**m*a**8*b*c**9*d**3*m*x**3 - 21888*x**m*a**
8*b*c**9*d**3*x**3 - 11520*x**m*a**8*b*c**8*d**4*m*x**4 - 15552*x**m*a**8*
b*c**8*d**4*x**4 - 2880*x**m*a**8*b*c**7*d**5*m*x**5 - 4032*x**m*a**8*b*c*
*7*d**5*x**5 - 2880*x**m*a**7*b**2*c**11*d*m - 4032*x**m*a**7*b**2*c**11*d
 + 5904*x**m*a**7*b**2*c**10*d**2*m**2*x + 5472*x**m*a**7*b**2*c**10*d**2*
m*x - 4464*x**m*a**7*b**2*c**10*d**2*x + 23616*x**m*a**7*b**2*c**9*d**3*m*
*2*x**2 + 50688*x**m*a**7*b**2*c**9*d**3*m*x**2 + 22464*x**m*a**7*b**2*...
```

3.604 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}} dx = \frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

output
```
(b-a/x^2)^(1/2)*x^(1+m)/m/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}} dx = \frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}} \int \frac{x^{m-1}\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}} \int x^{m-1}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
orering	$\frac{x\sqrt{b-\frac{a}{x^{2}}}\, x^{m}}{m\sqrt{-b\,x^{2}+a}}$	29
gosper	$\frac{x^{1+m}\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{m\sqrt{-b\,x^{2}+a}}$	35
risch	$\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\, x^{m}}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}} m}$	67

input
```
int((b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/m*(b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

input
```
integrate((b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)*x**m/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{i\,x^m}{m}$$

input
```
integrate((b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*x^m/m
```

Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-b\,x^2}}$$

input
```
int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
(x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{a^{\frac{m}{2}}\left(\sqrt{b}\,\sqrt{b\,x^2-a}\,x+b\,x^2\right)^m i}{b^{\frac{m}{2}}\left(\sqrt{a}\,\sqrt{b\,x^2-a}+\sqrt{b}\,\sqrt{a}\,x\right)^m m}$$

input
```
int((b-a/x^2)^(1/2)*x^m/(-b*x^2+a)^(1/2),x)
```

output
```
( - a**(m/2)*(sqrt(b)*sqrt( - a + b*x**2)*x + b*x**2)**m*i)/(b**(m/2)*(sqr
t(a)*sqrt( - a + b*x**2) + sqrt(b)*sqrt(a)*x)**m*m)
```

3.605 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^3}{2\sqrt{a-bx^2}}$$

output
```
1/2*(b-a/x^2)^(1/2)*x^3/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}x\sqrt{a-bx^2}}{2b}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

output
```
-1/2*(Sqrt[b - a/x^2]*x*Sqrt[a - b*x^2])/b
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$
$$\Big\downarrow \; 1017$$
$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{x\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$
$$\Big\downarrow \; 283$$
$$\frac{x\sqrt{b-\frac{a}{x^2}}\int xdx}{\sqrt{a-bx^2}}$$
$$\Big\downarrow \; 15$$
$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
orering	$\frac{\sqrt{b-\frac{a}{x^{2}}}\, x^{3}}{2 \sqrt{-b \,x^{2}+a}}$	26
gosper	$\frac{\sqrt{-\frac{-b \,x^{2}+a}{x^{2}}}\, x^{3}}{2 \sqrt{-b \,x^{2}+a}}$	31
default	$\frac{\sqrt{-\frac{-b \,x^{2}+a}{x^{2}}}\, x^{3}}{2 \sqrt{-b \,x^{2}+a}}$	31
risch	$\frac{i \sqrt{-\frac{-b \,x^{2}+a}{x^{2}}}\, x^{3} \left(b \,x^{2}-a \right) \sqrt{\frac{-b \,x^{2}+a}{b \,x^{2}-a}}}{2 \left(-b \,x^{2}+a \right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b-a/x^2)^(1/2)*x^3/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}\,x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^2-a)}$$

input
```
integrate((b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx = \int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)*x**2/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx = -\frac{1}{2}i\,x^2$$

input
```
integrate((b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{bx^2-a}\sqrt{-bx^2+a}\mathrm{sgn}(x)}{2\,b} - \frac{\sqrt{-a}\sqrt{a}\mathrm{sgn}(x)}{2\,b}$$

input
```
integrate((b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 - a)*sqrt(-b*x^2 + a)*sgn(x)/b - 1/2*sqrt(-a)*sqrt(a)*sgn(x
)/b
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = \frac{x^3\,\sqrt{b-\frac{a}{x^2}}}{2\,\sqrt{a-b\,x^2}}$$

input
```
int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
(x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{i\,x^2}{2}$$

input
```
int((b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2),x)
```

output
```
( - i*x**2)/2
```

3.606 $\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

output
```
(b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 283, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{ 283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int 1dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
orering	$\frac{\sqrt{b-\frac{a}{x^{2}}}\, x^{2}}{\sqrt{-b\,x^{2}+a}}$	25
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\, x^{2}}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\, x^{2} \left(b\,x^{2}-a\right) \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)*x/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b-a/x^2)^(1/2)*x^2/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{-bx^2+a}x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

input
```
integrate((b-a/x^2)^(1/2)*x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx=\int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)*x/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx=-i\sqrt{x^2}$$

input
```
integrate((b-a/x^2)^(1/2)*x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*sqrt(x^2)
```

Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate((b-a/x^2)^(1/2)*x/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

input
```
int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=-ix$$

input
```
int((b-a/x^2)^(1/2)*x/(-b*x^2+a)^(1/2),x)
```

output
```
 - i*x
```

3.607 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx = \frac{\sqrt{b-\frac{a}{x^2}}x\log(x)}{\sqrt{a-bx^2}}$$

output
```
(b-a/x^2)^(1/2)*x*ln(x)/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} \, dx = \frac{\sqrt{b-\frac{a}{x^2}}x\log(x)}{\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {942, 283, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{942}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\ln\left(x\right)}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\,\ln\left(x\right)}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*x^2+a)/x^2)^(1/2)*x/(-b*x^2+a)^(1/2)*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = -\arctan\left(\frac{\sqrt{-bx^2+a}(x^3-x)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a-b)x^2-a}\right)$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-b*x^2 + a)*(x^3 - x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a - b)*
x^2 - a))
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = -i\log(x)$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*log(x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \frac{1}{2}i\left(\log\left(x^2|b|\right)-\log\left(|a|\right)\right)\text{sgn}(x)$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*I*(log(x^2*abs(b)) - log(abs(a)))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=-\log\left(\frac{\sqrt{b}\,\sqrt{b\,x^2-a}\,x+b\,x^2}{\sqrt{a}\,\sqrt{b\,x^2-a}+\sqrt{b}\,\sqrt{a}\,x}\right)i$$

input
```
int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
- log((sqrt(b)*sqrt( - a + b*x**2)*x + b*x**2)/(sqrt(a)*sqrt( - a + b*x**
2) + sqrt(b)*sqrt(a)*x))*i
```

3.608 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 26

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

output
```
-(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

output
```
-(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x^2\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x^2}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

output
```
-(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
orering	$-\frac{\sqrt{b-\frac{a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	23
gosper	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	28
default	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	28
risch	$-\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	60

input
```
int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{i}{\sqrt{x^2}}$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
I/sqrt(x^2)
```

Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{-bx^2+ax}}\,dx$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}$$

input
```
int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
```

output
```
-(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{i}{x}$$

input
```
int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)
```

output
```
i/x
```

3.609 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

output
```
-1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

output
```
-1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x^3\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x^3}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

output
```
-1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
orering	$-\frac{\sqrt{b-\frac{a}{x^{2}}}}{2x\sqrt{-b\,x^{2}+a}}$	26
gosper	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2x\sqrt{-b\,x^{2}+a}}$	31
default	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2x\sqrt{-b\,x^{2}+a}}$	31
risch	$-\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{2x\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x^2-1)\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^3-ax)}$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{i}{2\,x^2}$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*I/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{1}{2}\,b\left(-\frac{i}{a}+\frac{i}{bx^2}\right)\mathrm{sgn}(x)$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*(-I/a + I/(b*x^2))*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

input
```
int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
```

output
```
-(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{i(-2b\,x^2+a)}{2a\,x^2}$$

input
```
int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x)
```

output
```
(i*(a - 2*b*x**2))/(2*a*x**2)
```

3.610 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 581

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x} + \frac{2(c+dx)^{3/2}\,(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x}$$
$$+ \frac{2(ac^2-3bd^2)\sqrt{c+dx}(b+ax^2)}{5a^{3/2}\sqrt{a+\frac{b}{x^2}}x\left(\sqrt{ac^2+bd^2}+\sqrt{a}(c+dx)\right)}$$
$$- \frac{2(ac^2-3bd^2)\,(ac^2+bd^2)^{3/4}\sqrt{\frac{d^2(b+ax^2)}{(ac^2+bd^2)\left(1+\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}\right)^2}}\left(1+\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}\right)E\left(2\arctan\left(\frac{\sqrt[4]{a}\sqrt{c+dx}}{\sqrt[4]{ac^2+bd^2}}\right)|\frac{1}{2}\left(1+\right.\right.}{5a^{7/4}d^2\sqrt{a+\frac{b}{x^2}}x}$$
$$+ \frac{(ac^2+bd^2)^{3/4}\left(ac^2-3bd^2-\sqrt{a}c\sqrt{ac^2+bd^2}\right)\sqrt{\frac{d^2(b+ax^2)}{(ac^2+bd^2)\left(1+\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}\right)^2}}\left(1+\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}\right)\text{EllipticF}\left(2\arctan\right.}{5a^{7/4}d^2\sqrt{a+\frac{b}{x^2}}x}$$

output
```
2/5*c*(d*x+c)^(1/2)*(a*x^2+b)/a/(a+b/x^2)^(1/2)/x+2/5*(d*x+c)^(3/2)*(a*x^2
+b)/a/(a+b/x^2)^(1/2)/x+2/5*(a*c^2-3*b*d^2)*(d*x+c)^(1/2)*(a*x^2+b)/a^(3/2
)/(a+b/x^2)^(1/2)/x/((a*c^2+b*d^2)^(1/2)+a^(1/2)*(d*x+c))-2/5*(a*c^2-3*b*d
^2)*(a*c^2+b*d^2)^(3/4)*(d^2*(a*x^2+b)/(a*c^2+b*d^2)/(1+a^(1/2)*(d*x+c)/(a
*c^2+b*d^2)^(1/2))^2)^(1/2)*(1+a^(1/2)*(d*x+c)/(a*c^2+b*d^2)^(1/2))*Ellipt
icE(sin(2*arctan(a^(1/4)*(d*x+c)^(1/2)/(a*c^2+b*d^2)^(1/4))),1/2*(2+2*a^(1
/2)*c/(a*c^2+b*d^2)^(1/2))^(1/2))/a^(7/4)/d^2/(a+b/x^2)^(1/2)/x+1/5*(a*c^2
+b*d^2)^(3/4)*(a*c^2-3*b*d^2-a^(1/2)*c*(a*c^2+b*d^2)^(1/2))*(d^2*(a*x^2+b)
/(a*c^2+b*d^2)/(1+a^(1/2)*(d*x+c)/(a*c^2+b*d^2)^(1/2))^2)^(1/2)*(1+a^(1/2)
*(d*x+c)/(a*c^2+b*d^2)^(1/2))*InverseJacobiAM(2*arctan(a^(1/4)*(d*x+c)^(1/
2)/(a*c^2+b*d^2)^(1/4)),1/2*(2+2*a^(1/2)*c/(a*c^2+b*d^2)^(1/2))^(1/2))/a^(
7/4)/d^2/(a+b/x^2)^(1/2)/x
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.08 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{c+dx}\left(\frac{2(2c+dx)(b+ax^2)}{a}+\frac{2\left(d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}\,(ac^2-3bd^2)(b+ax^2)+\sqrt{a}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/}\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[
b]*d)/Sqrt[a]]*(a*c^2 - 3*b*d^2)*(b + a*x^2) + Sqrt[a]*((-I)*a^(3/2)*c^3 +
 a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3)*Sqrt[(d*((I*Sqrt
[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c + d*
x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]/
Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)] - Sqr
t[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)*Sqrt[(d*((I*S
qrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a
]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)]))/
(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a + b/x^2]*
x)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1780, 596, 687, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}dx$$

$$\Big\downarrow \text{ 1780}$$

$$\frac{\sqrt{ax^2+b}\int \frac{x(c+dx)^{3/2}}{\sqrt{ax^2+b}}dx}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{ 596}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\int \frac{(bd-acx)\sqrt{c+dx}}{\sqrt{ax^2+b}}dx}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{2\int\frac{a\left(4bcd-\left(ac^2-3bd^2\right)x\right)}{2\sqrt{c+dx}\sqrt{ax^2+b}}dx}{3a}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{1}{3}\int\frac{4bcd-\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\sqrt{ax^2+b}}dx-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{599}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\int-\frac{c\left(ac^2+bd^2\right)-\left(ac^2-3bd^2\right)(c+dx)}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{3d^2}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{2\int\frac{c\left(ac^2+bd^2\right)-\left(ac^2-3bd^2\right)(c+dx)}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{3d^2}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\sqrt{ac^2+bd^2}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\int \frac{1}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{\sqrt{a}}-\frac{\left(ac^2-3bd^2\right)\sqrt{ac^2 \cdots}}{\cdots}\right)}{3d^2}\cdots\right)}{5a}\cdots\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$\downarrow$ 1416

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\left(ac^2+bd^2\right)^{3/4}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)\sqrt{\frac{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}{\left(\frac{ac^2}{d^2}+b\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)^2}}\ \text{EllipticF}\cdots}{2a^{3/4}\sqrt{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{x\sqrt{a+\cdots}}$$

$\downarrow$ 1509

$$\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\left(ac^2+bd^2\right)^{3/4}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)\sqrt{\frac{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}{\left(\frac{ac^2}{d^2}+b\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)^2}}\text{EllipticF}}{2a^{3/4}\sqrt{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}\right.}{}}{}\right.}{}$$

input
```
Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[b + a*x^2]*((2*(c + d*x)^(3/2)*Sqrt[b + a*x^2])/(5*a) - (3*((-2*c*Sq
rt[c + d*x]*Sqrt[b + a*x^2])/3 - (2*(-(((a*c^2 - 3*b*d^2)*Sqrt[a*c^2 + b*d
^2]*(-((Sqrt[c + d*x]*Sqrt[b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c
 + d*x)^2)/d^2])/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 +
b*d^2]))) + ((a*c^2 + b*d^2)^(1/4)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b
*d^2])*Sqrt[(b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c + d*x)^2)/d^2
)/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b*d^2])^2)]*Ell
ipticE[2*ArcTan[(a^(1/4)*Sqrt[c + d*x])/(a*c^2 + b*d^2)^(1/4)], (1 + (Sqrt
[a]*c)/Sqrt[a*c^2 + b*d^2])/2])/(a^(1/4)*Sqrt[b + (a*c^2)/d^2 - (2*a*c*(c
+ d*x))/d^2 + (a*(c + d*x)^2)/d^2])))/Sqrt[a]) + ((a*c^2 + b*d^2)^(3/4)*(a
*c^2 - 3*b*d^2 - Sqrt[a]*c*Sqrt[a*c^2 + b*d^2])*(1 + (Sqrt[a]*(c + d*x))/S
qrt[a*c^2 + b*d^2])*Sqrt[(b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c
+ d*x)^2)/d^2)/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b*
d^2])^2)]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[c + d*x])/(a*c^2 + b*d^2)^(1/4)
], (1 + (Sqrt[a]*c)/Sqrt[a*c^2 + b*d^2])/2])/(2*a^(3/4)*Sqrt[b + (a*c^2)/d
^2 - (2*a*c*(c + d*x))/d^2 + (a*(c + d*x)^2)/d^2])))/(3*d^2)))/(5*a)))/(Sq
rt[a + b/x^2]*x)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 596
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] - Simp[n/(b*(n
 + 2*p + 2))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p*(a*d - b*c*x), x], x] /;
 FreeQ[{a, b, c, d, p}, x] && GtQ[n, 0] && NeQ[n + 2*p + 2, 0]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1780
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*
n))^FracPart[p])   Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x]
/; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] &&  !IntegerQ[p] &&  !
IntegerQ[q] && PosQ[n]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.07

method	result
risch	$\frac{2(dx+2c)(a\,x^2+b)\sqrt{dx+c}}{5a\sqrt{\frac{a\,x^2+b}{x^2}}\,x}+\frac{\left(\frac{2\left(a\,c^2-3b\,d^2\right)\left(\frac{c}{d}-\frac{\sqrt{-ab}}{a}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}}\left(\left(-\frac{c}{d}-\frac{\sqrt{-ab}}{a}\right)\mathrm{EllipticE}\left(\sqrt{\frac{x\cdots}{\frac{c}{d}-\cdots}}\cdots\right.\right.}{\sqrt{a\,d\,x^3+a\,c\,x^2+b\,d\,x+c\,b}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(d*x+2*c)*(a*x^2+b)*(d*x+c)^(1/2)/a/((a*x^2+b)/x^2)^(1/2)/x+1/5/a*(2*(
a*c^2-3*b*d^2)*(c/d-1/a*(-a*b)^(1/2))*((x+c/d)/(c/d-1/a*(-a*b)^(1/2)))^(1/
2)*((x-1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x+1/a*(-a*b)^(1/
2))/(-c/d+1/a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b*d*x+b*c)^(1/2)*((-c/
d-1/a*(-a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/
d+1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2))+1/a*(-a*b)^(1/2)*Ellip
ticF(((x+c/d)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/a*(-a*b)^(1/2))/(-c/d
-1/a*(-a*b)^(1/2)))^(1/2)))-8*d*b*c*(c/d-1/a*(-a*b)^(1/2))*((x+c/d)/(c/d-1
/a*(-a*b)^(1/2)))^(1/2)*((x-1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/
2)*((x+1/a*(-a*b)^(1/2))/(-c/d+1/a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b
*d*x+b*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/
a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2)))/((a*x^2+b)/x^2)^(1/2)/x*(
(a*x^2+b)*(d*x+c))^(1/2)/(d*x+c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.40

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx =$$
$$-\frac{2\left(\left(ac^3+9\,bcd^2\right)\sqrt{ad}\text{weierstrassPInverse}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\frac{3\,dx+c}{3\,d}\right)+3\left(ac^2d-3\,bd^3\right)\sqrt{ad}\text{weier}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*((a*c^3 + 9*b*c*d^2)*sqrt(a*d)*weierstrassPInverse(4/3*(a*c^2 - 3*b*
d^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d) + 3*(a
*c^2*d - 3*b*d^3)*sqrt(a*d)*weierstrassZeta(4/3*(a*c^2 - 3*b*d^2)/(a*d^2),
 -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), weierstrassPInverse(4/3*(a*c^2 - 3*b*d
^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d)) - 3*(a
*d^3*x^2 + 2*a*c*d^2*x)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2))/(a^2*d^2)
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)/sqrt(a + b/x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Giac [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(c+d\,x)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
int((c + d*x)^(3/2)/(a + b/x^2)^(1/2),x)
```

output
```
int((c + d*x)^(3/2)/(a + b/x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{10\sqrt{dx+c}\,\sqrt{a\,x^2+b}\,a\,c^2+4\sqrt{dx+c}\,\sqrt{a\,x^2+b}\,acdx-6\sqrt{dx+c}\,\sqrt{a\,x^2+b}\,b\,d^2-3\Big(}{}$$

input
```
int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x)
```

output
```
(10*sqrt(c + d*x)*sqrt(a*x**2 + b)*a*c**2 + 4*sqrt(c + d*x)*sqrt(a*x**2 +
b)*a*c*d*x - 6*sqrt(c + d*x)*sqrt(a*x**2 + b)*b*d**2 - 3*int((sqrt(c + d*x
)*sqrt(a*x**2 + b)*x**2)/(a*c*x**2 + a*d*x**3 + b*c + b*d*x),x)*a**2*c**2*
d + 9*int((sqrt(c + d*x)*sqrt(a*x**2 + b)*x**2)/(a*c*x**2 + a*d*x**3 + b*c
 + b*d*x),x)*a*b*d**3 - 9*int((sqrt(c + d*x)*sqrt(a*x**2 + b))/(a*c*x**2 +
 a*d*x**3 + b*c + b*d*x),x)*a*b*c**2*d + 3*int((sqrt(c + d*x)*sqrt(a*x**2
+ b))/(a*c*x**2 + a*d*x**3 + b*c + b*d*x),x)*b**2*d**3)/(10*a**2*c)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file