Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.6/59-1.1.3.6-a

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May 18, 2024 Compiled on May 18, 2024 at 11:56pm

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**91**]. This is test number [59].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	95.60 (87)	4.40 (4)
Rubi	94.51 (86)	5.49 (5)
Reduce	38.46 (35)	61.54 (56)
Maple	35.16 (32)	64.84 (59)
Mupad	35.16 (32)	64.84 (59)
Giac	35.16 (32)	64.84 (59)
Maxima	35.16 (32)	64.84 (59)
Fricas	30.77 (28)	69.23 (63)
Sympy	26.37 (24)	73.63 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

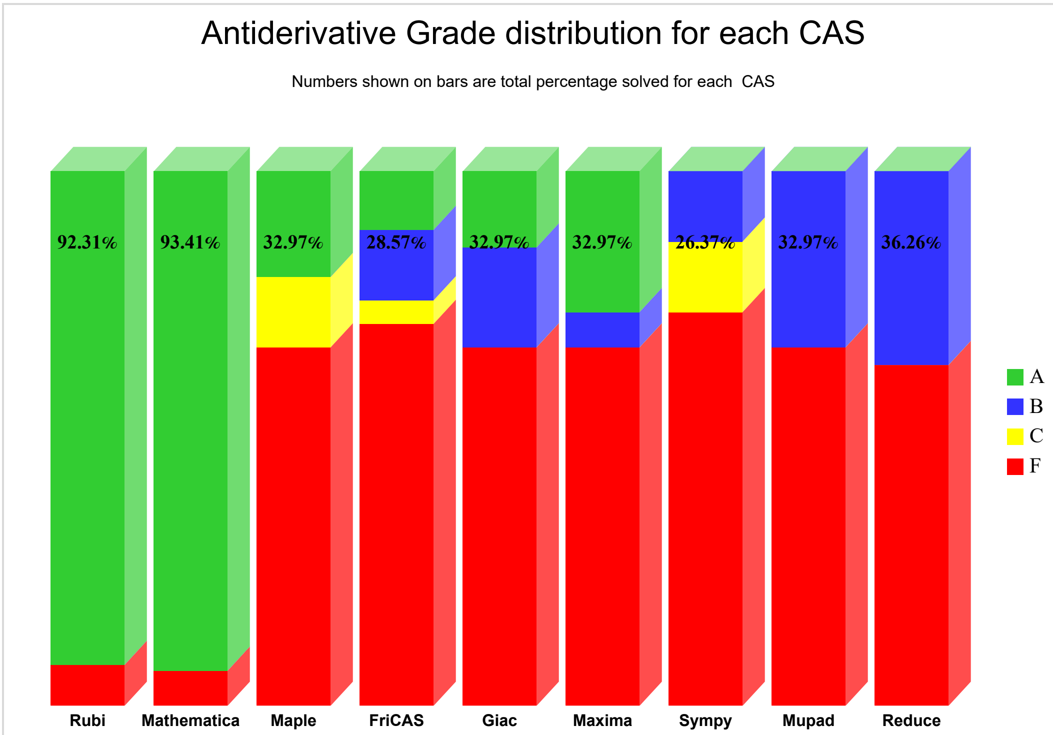
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

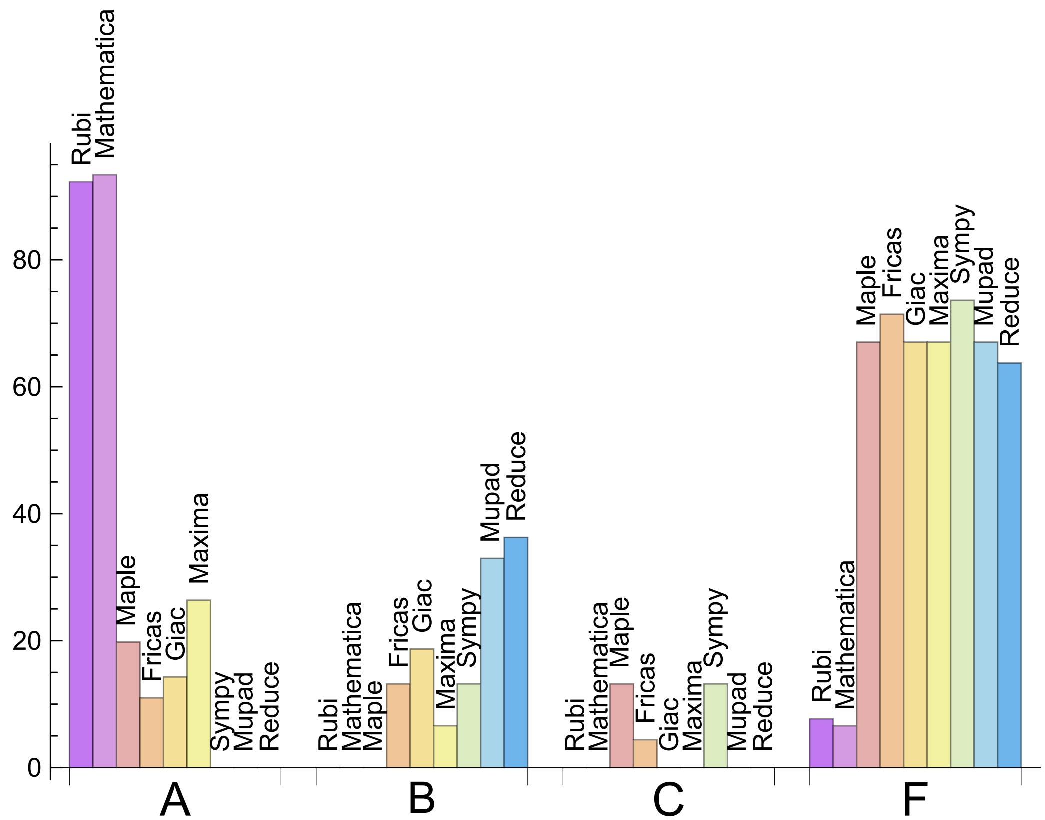
System	% A grade	% B grade	% C grade	% F grade
Mathematica	93.407	0.000	0.000	6.593
Rubi	92.308	0.000	0.000	7.692
Maxima	26.374	6.593	0.000	67.033
Maple	19.780	0.000	13.187	67.033
Giac	14.286	18.681	0.000	67.033
Fricas	10.989	13.187	4.396	71.429
Mupad	0.000	32.967	0.000	67.033
Reduce	0.000	36.264	0.000	63.736
Sympy	0.000	13.187	13.187	73.626

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	4	100.00	0.00	0.00
Rubi	5	100.00	0.00	0.00
Reduce	56	100.00	0.00	0.00
Maple	59	100.00	0.00	0.00
Mupad	59	0.00	100.00	0.00
Giac	59	94.92	1.69	3.39
Maxima	59	100.00	0.00	0.00
Fricas	63	90.48	6.35	3.17
Sympy	67	2.99	68.66	28.36

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.09
Giac	0.24
Mathematica	0.47
Maple	0.69
Rubi	0.97
Mupad	8.20
Reduce	11.67
Fricas	19.63
Sympy	23.29

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	195.71	0.82	162.00	0.78
Rubi	274.41	1.04	238.00	1.00
Maxima	306.09	1.35	275.50	1.10
Reduce	1365.00	5.20	150.00	0.71
Fricas	1861.29	7.13	728.00	5.34
Maple	2203.94	7.98	250.50	0.95
Mupad	9172.00	33.47	1160.00	5.22
Giac	13049.38	44.35	591.00	1.55
Sympy	42369.04	144.73	5529.00	41.28

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

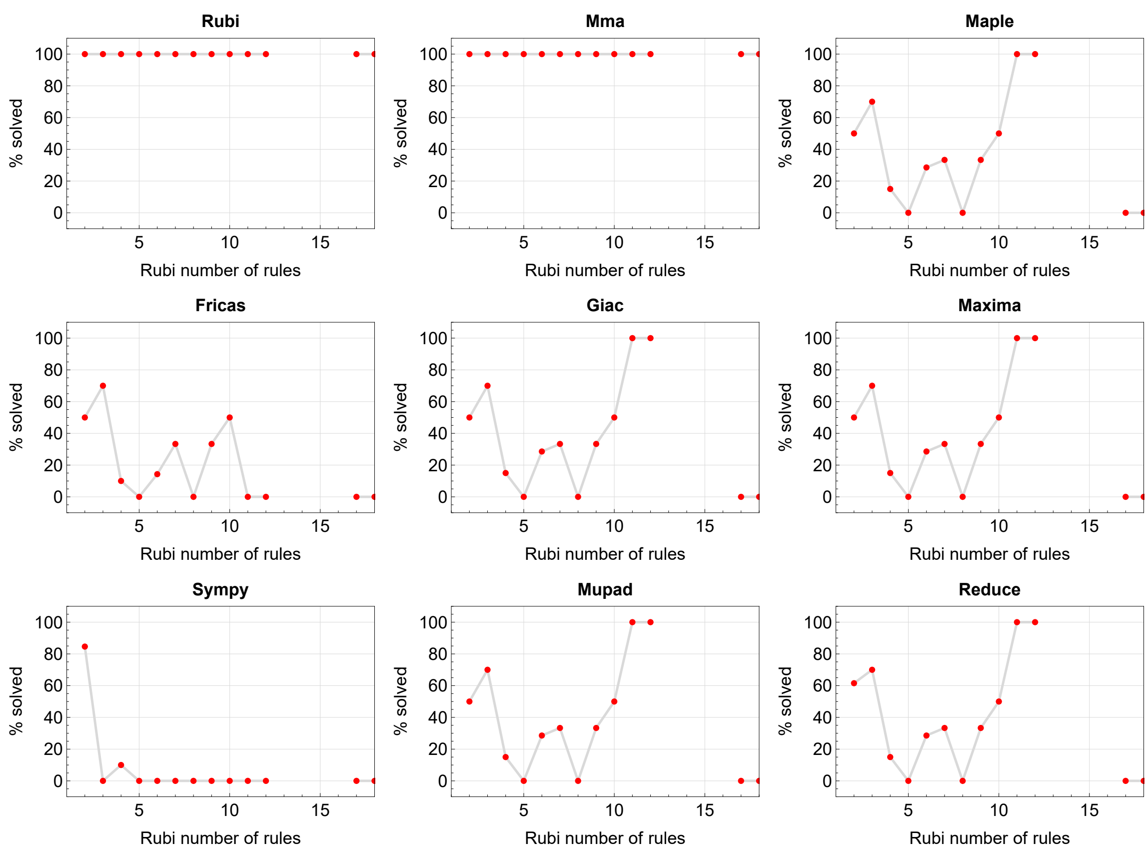


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

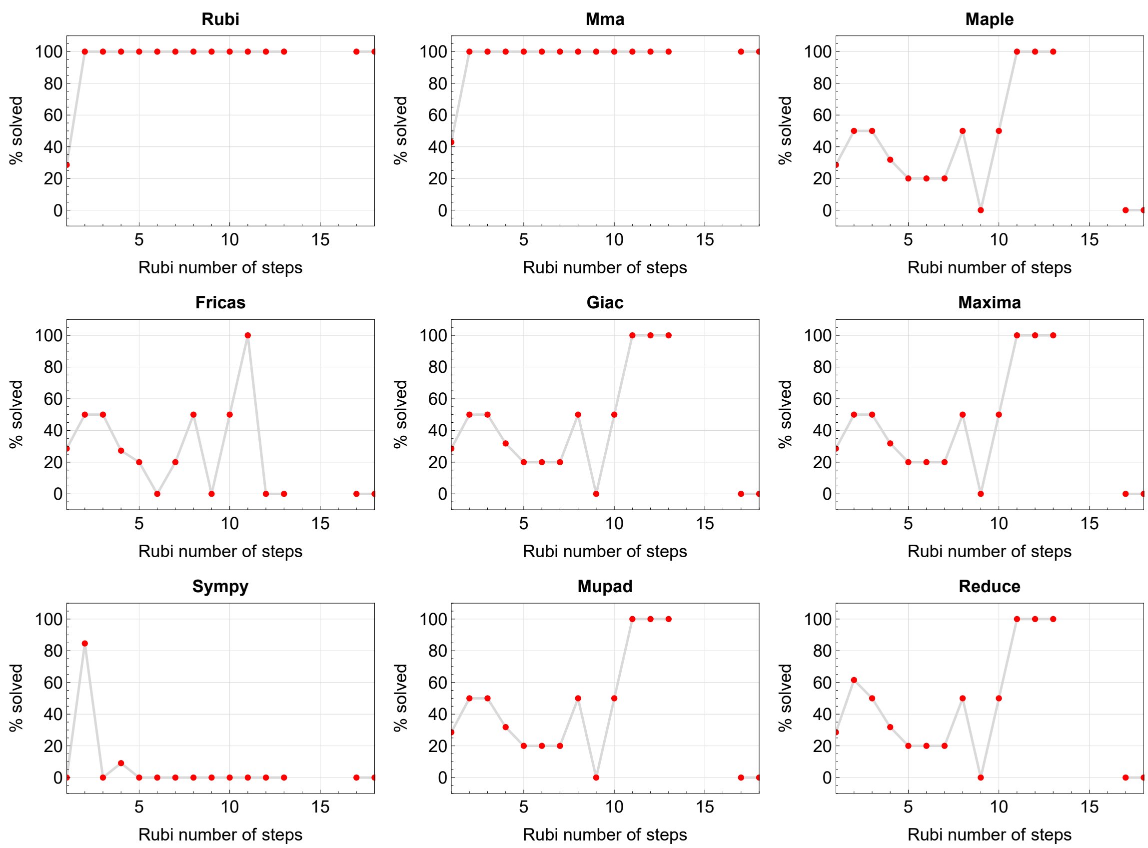


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

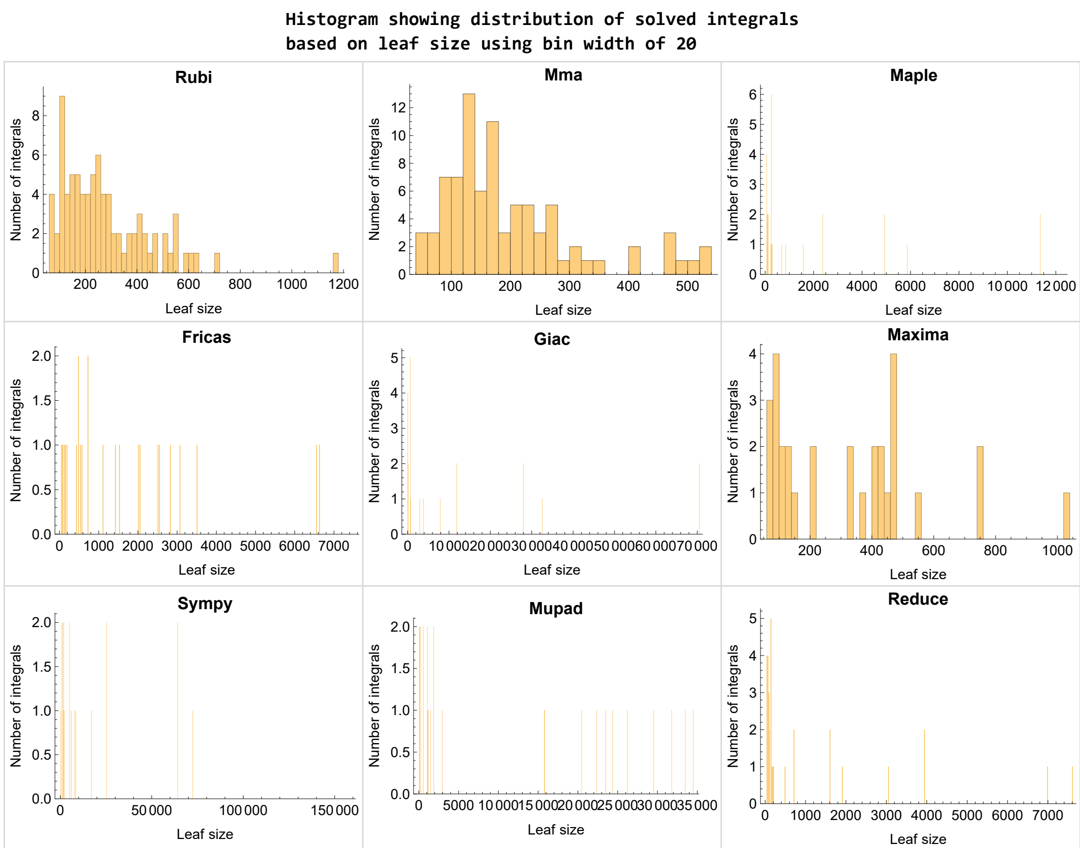


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

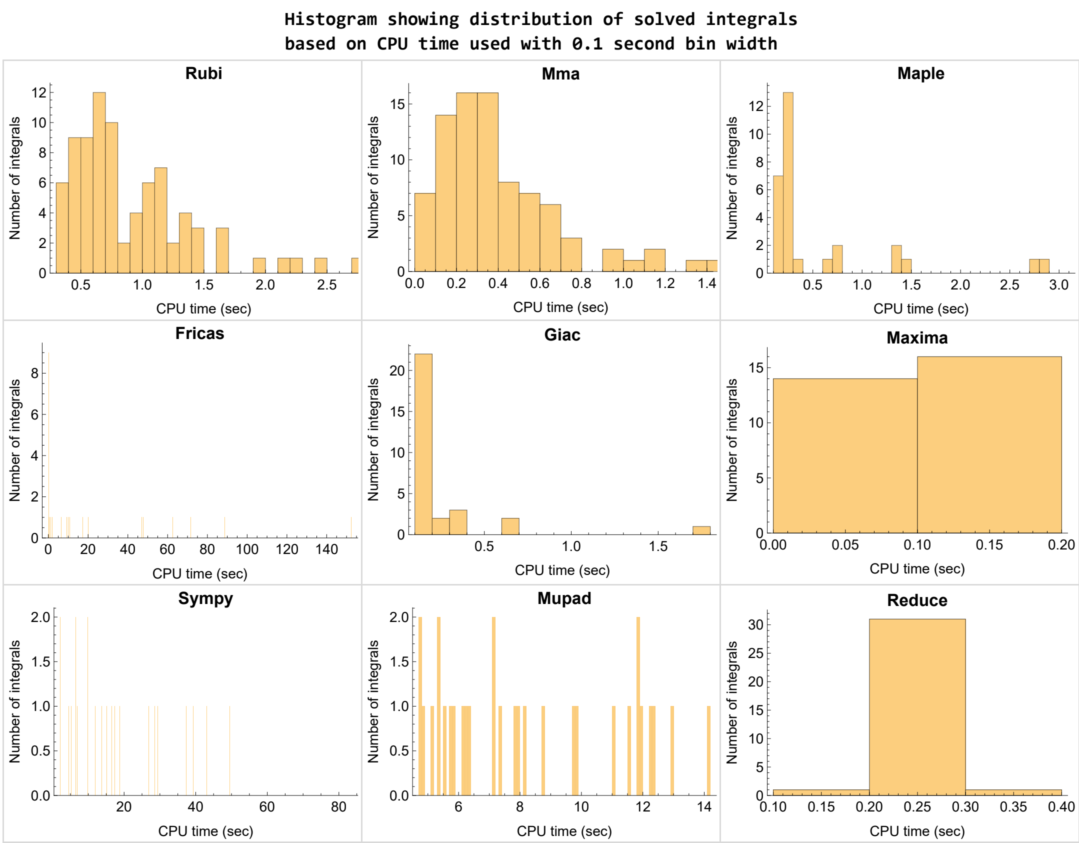


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

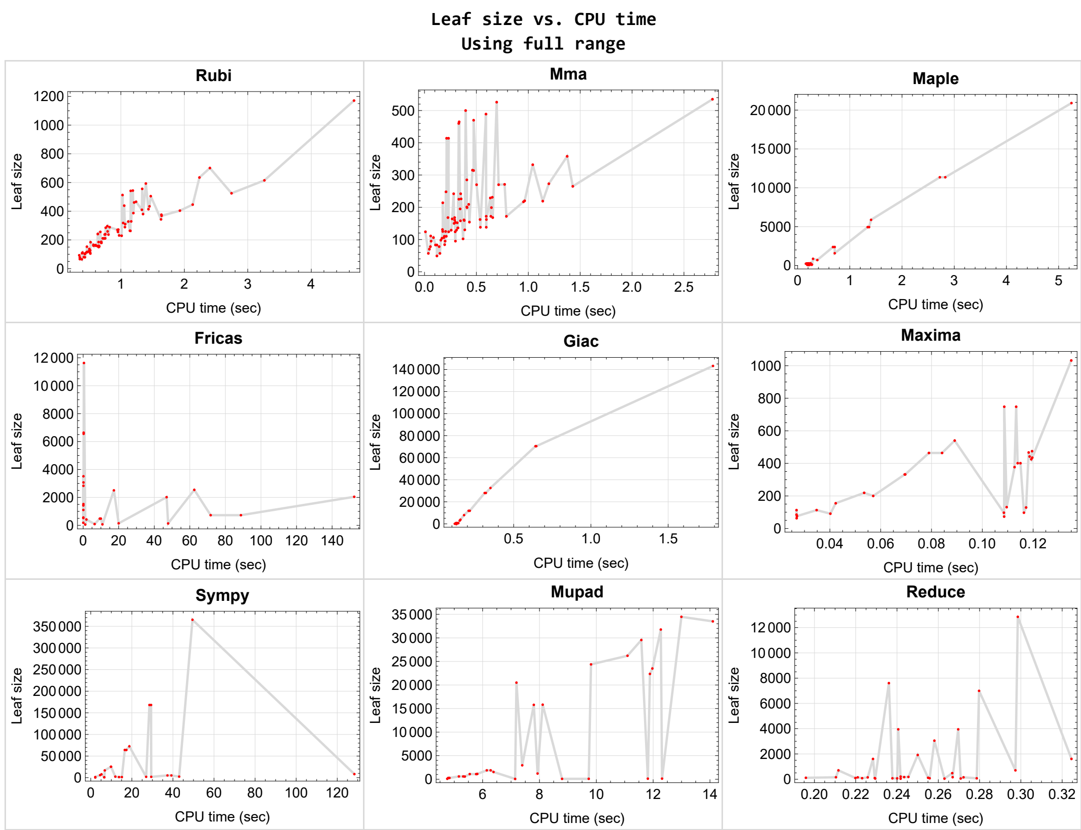


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{88, 89\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{76, 77, 80, 81, 83, 84\}$

Mathematica $\{73, 74, 76, 77, 82, 83, 84\}$

Maple $\{19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

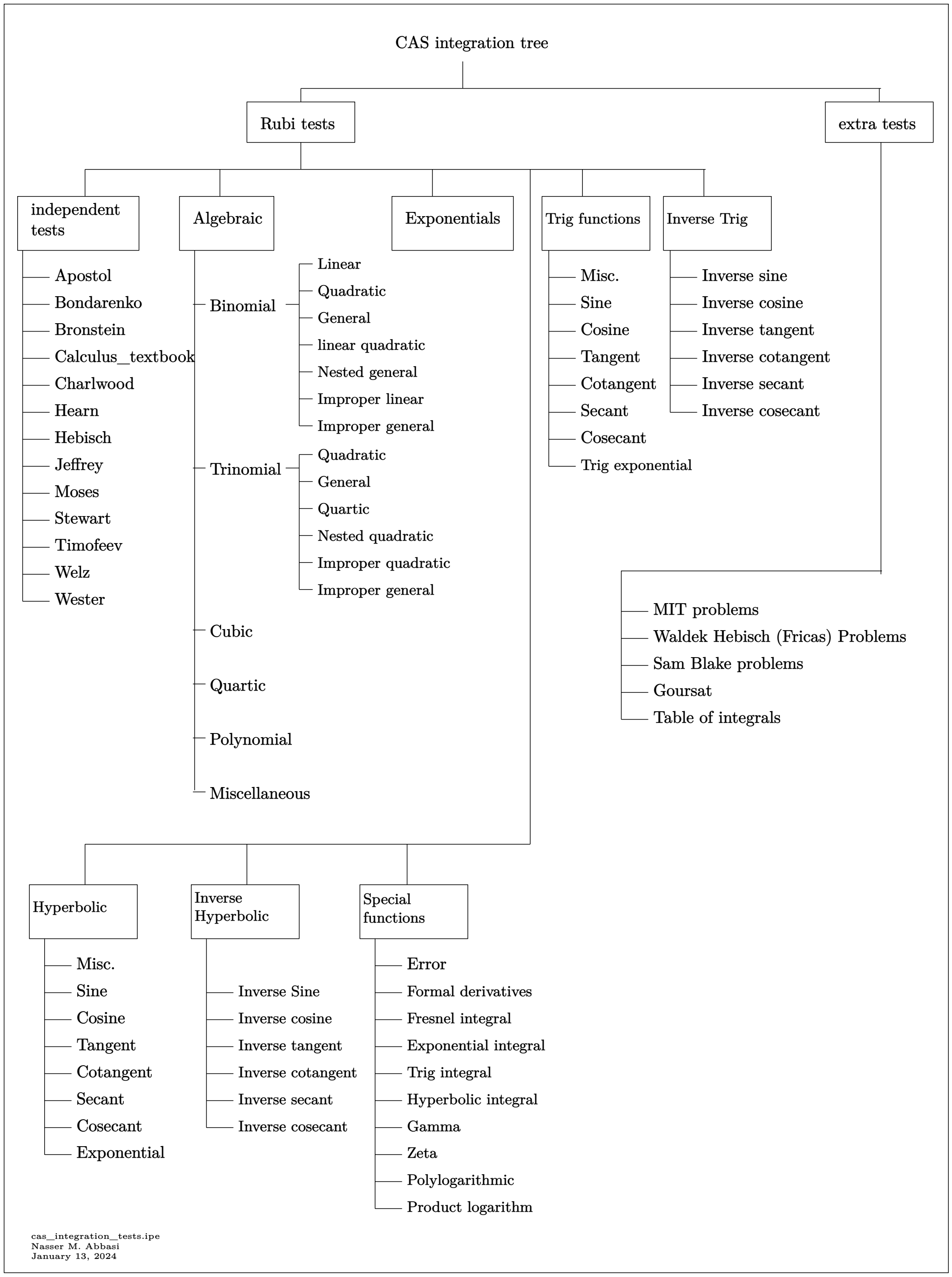
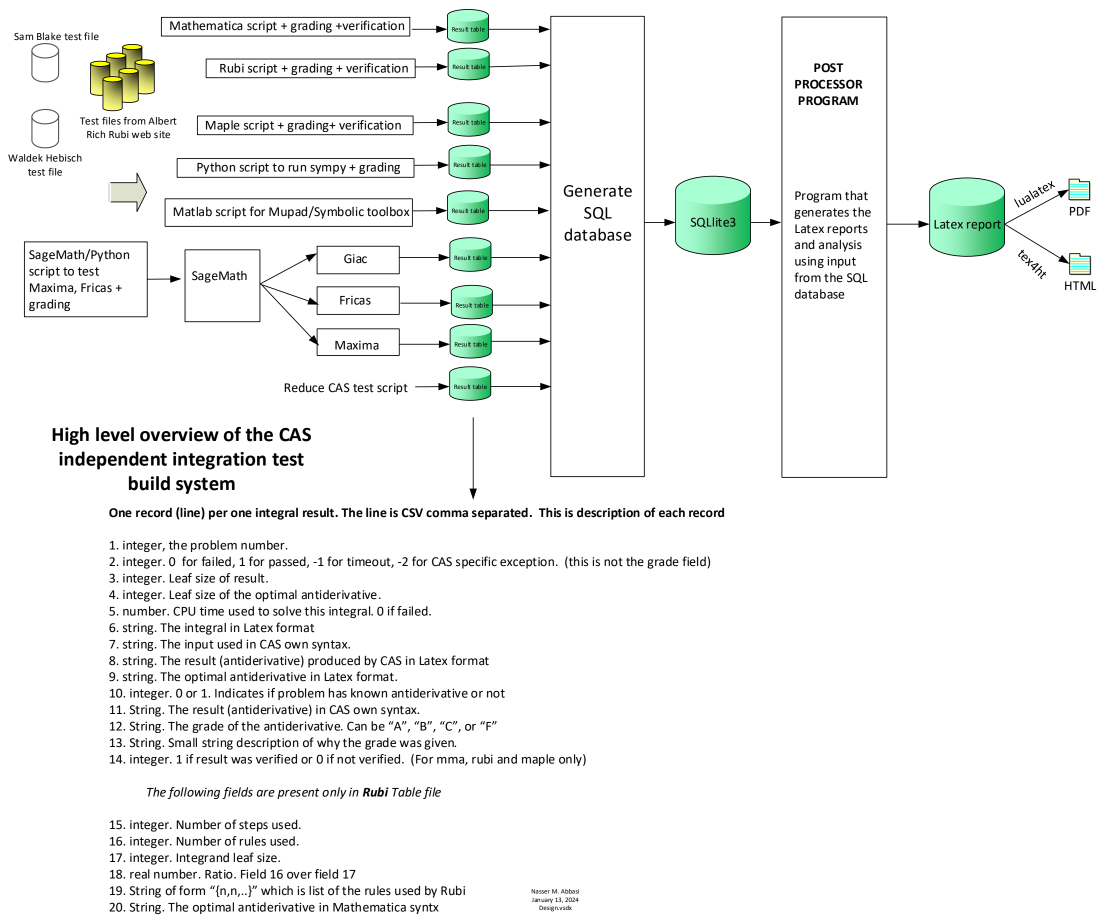


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 90, 91 }

B grade { }

C grade { }

F normal fail { 63, 64, 65, 66, 85 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

B grade { }

C grade { }

F normal fail { 63, 64, 65, 66 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 }

B grade { }

C grade { 19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36 }

F normal fail { 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

B grade { 19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36 }

C grade { 13, 14, 15, 16 }

F normal fail { 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

F(-1) timedout fail { 11, 12, 17, 18 }

F(-2) exception fail { 65, 66 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 29, 36 }

B grade { 19, 26, 27, 33, 34, 35 }

C grade { }

F normal fail { 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 17, 18 }

B grade { 11, 13, 14, 15, 16, 19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36 }

C grade { }

F normal fail { 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 86, 87, 90, 91 }

F(-1) timedout fail { 80 }

F(-2) exception fail { 59, 85 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36 }

C grade { }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 19, 20, 21, 22, 26, 27, 28, 29, 33, 34, 35, 36 }

C grade { 23, 24, 30, 37, 39, 40, 41, 42, 43, 49, 50, 56 }

F normal fail { 31, 48 }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 32, 53, 54, 55, 58, 59, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 90, 91 }

F(-2) exception fail { 38, 44, 45, 46, 47, 51, 52, 57, 60, 61, 62, 63, 64, 65, 66, 85, 86, 87, 88 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37 }

C grade { }

F normal fail { 24, 25, 31, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	110	106	105	113	146	0	119	71	121
N.S.	1	0.96	0.93	0.92	0.99	1.28	0.00	1.04	0.62	1.06
time (sec)	N/A	0.517	0.086	0.273	0.027	19.990	0.000	0.127	0.279	12.313

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	80	78	78	85	93	0	87	58	85
N.S.	1	0.96	0.94	0.94	1.02	1.12	0.00	1.05	0.70	1.02
time (sec)	N/A	0.416	0.057	0.185	0.027	6.470	0.000	0.131	0.271	9.734

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	65	57	64	63	57	0	65	47	63
N.S.	1	0.97	0.85	0.96	0.94	0.85	0.00	0.97	0.70	0.94
time (sec)	N/A	0.385	0.037	0.196	0.027	1.194	0.000	0.118	0.263	7.144

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	79	70	74	76	71	0	90	55	73
N.S.	1	1.03	0.91	0.96	0.99	0.92	0.00	1.17	0.71	0.95
time (sec)	N/A	0.434	0.047	0.222	0.027	10.848	0.000	0.127	0.229	8.795

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	105	111	102	113	137	0	151	71	111
N.S.	1	0.98	1.04	0.95	1.06	1.28	0.00	1.41	0.66	1.04
time (sec)	N/A	0.520	0.060	0.256	0.035	47.796	0.000	0.131	0.238	11.814

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	155	131	122	129	728	0	147	88	20507
N.S.	1	1.14	0.96	0.90	0.95	5.35	0.00	1.08	0.65	150.79
time (sec)	N/A	0.661	0.173	0.275	0.117	71.715	0.000	0.128	0.229	7.190

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	115	104	94	97	483	0	97	75	15787
N.S.	1	1.06	0.95	0.86	0.89	4.43	0.00	0.89	0.69	144.83
time (sec)	N/A	0.472	0.169	0.203	0.108	9.371	0.000	0.127	0.256	7.801

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	93	95	74	73	421	0	73	67	1201
N.S.	1	0.98	1.00	0.78	0.77	4.43	0.00	0.77	0.71	12.64
time (sec)	N/A	0.344	0.064	0.207	0.109	1.847	0.000	0.126	0.240	7.933

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	116	214	94	97	483	0	97	82	1537
N.S.	1	1.06	1.96	0.86	0.89	4.43	0.00	0.89	0.75	14.10
time (sec)	N/A	0.463	0.176	0.217	0.116	9.961	0.000	0.129	0.223	6.384

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	157	248	124	131	728	0	133	95	15821
N.S.	1	1.15	1.82	0.91	0.96	5.35	0.00	0.98	0.70	116.33
time (sec)	N/A	0.628	0.207	0.227	0.110	88.726	0.000	0.125	0.220	8.113

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	505	500	275	475	0	0	649	154	33507
N.S.	1	1.23	1.21	0.67	1.15	0.00	0.00	1.58	0.37	81.33
time (sec)	N/A	1.471	0.396	0.216	0.120	0.000	0.000	0.128	0.210	14.101

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	544	465	254	401	0	0	585	147	29549
N.S.	1	1.40	1.20	0.65	1.03	0.00	0.00	1.50	0.38	75.96
time (sec)	N/A	1.189	0.334	0.188	0.115	0.000	0.000	0.134	0.221	11.585

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	461	460	249	442	2045	0	582	144	31774
N.S.	1	1.20	1.20	0.65	1.15	5.33	0.00	1.52	0.38	82.74
time (sec)	N/A	1.209	0.330	0.176	0.119	152.429	0.000	0.134	0.226	12.270

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	513	414	240	377	2507	0	597	112	24377
N.S.	1	1.37	1.10	0.64	1.01	6.69	0.00	1.59	0.30	65.01
time (sec)	N/A	1.026	0.232	0.188	0.113	17.301	0.000	0.141	0.255	9.814

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	439	414	240	425	2019	0	581	112	22357
N.S.	1	1.17	1.10	0.64	1.13	5.38	0.00	1.55	0.30	59.62
time (sec)	N/A	1.053	0.213	0.171	0.119	46.944	0.000	0.124	0.196	11.887

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	542	470	252	401	2543	0	625	150	26209
N.S.	1	1.40	1.21	0.65	1.04	6.57	0.00	1.61	0.39	67.72
time (sec)	N/A	1.155	0.473	0.200	0.114	62.507	0.000	0.132	0.272	11.098

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	466	489	254	467	0	0	585	159	23502
N.S.	1	1.20	1.26	0.65	1.20	0.00	0.00	1.50	0.41	60.42
time (sec)	N/A	1.239	0.593	0.207	0.118	0.000	0.000	0.134	0.267	11.974

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	593	526	280	435	0	0	637	168	34477
N.S.	1	1.44	1.27	0.68	1.05	0.00	0.00	1.54	0.41	83.48
time (sec)	N/A	1.394	0.695	0.249	0.120	0.000	0.000	0.138	0.243	12.995

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	225	210	172	4939	464	3073	64068	27992	3053	1089
N.S.	1	0.93	0.76	21.95	2.06	13.66	284.75	124.41	13.57	4.84
time (sec)	N/A	0.744	0.789	1.369	0.084	0.200	17.321	0.320	0.258	5.547

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	160	129	2377	332	1524	25315	11834	1607	588
N.S.	1	0.93	0.75	13.82	1.93	8.86	147.18	68.80	9.34	3.42
time (sec)	N/A	0.596	0.308	0.678	0.069	0.172	9.840	0.208	0.325	5.164

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	117	108	84	858	200	562	7796	3764	700	271
N.S.	1	0.92	0.72	7.33	1.71	4.80	66.63	32.17	5.98	2.32
time (sec)	N/A	0.443	0.194	0.296	0.057	0.142	5.212	0.155	0.297	4.834

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	72	66	49	229	91	185	1498	763	208	91
N.S.	1	0.92	0.68	3.18	1.26	2.57	20.81	10.60	2.89	1.26
time (sec)	N/A	0.354	0.120	0.159	0.040	0.101	2.191	0.130	0.242	4.756

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	120	95	0	0	0	872	0	41	0
N.S.	1	0.98	0.77	0.00	0.00	0.00	7.09	0.00	0.33	0.00
time (sec)	N/A	0.491	0.199	0.000	0.000	0.000	6.597	0.000	0.242	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	179	110	0	0	0	5176	0	95	0
N.S.	1	1.02	0.63	0.00	0.00	0.00	29.58	0.00	0.54	0.00
time (sec)	N/A	0.693	0.191	0.000	0.000	0.000	39.242	0.000	0.224	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	238	136	0	0	0	0	0	0	0
N.S.	1	1.04	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.745	0.321	0.000	0.000	0.000	0.000	0.000	0.276	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	336	318	273	11356	748	6638	168099	70422	6996	1882
N.S.	1	0.95	0.81	33.80	2.23	19.76	500.29	209.59	20.82	5.60
time (sec)	N/A	1.036	1.198	2.724	0.113	0.291	29.351	0.646	0.280	6.282

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	252	237	199	5875	540	3515	72500	32523	3949	1119
N.S.	1	0.94	0.79	23.31	2.14	13.95	287.70	129.06	15.67	4.44
time (sec)	N/A	0.804	0.645	1.407	0.089	0.183	18.769	0.349	0.270	5.812

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	172	160	129	2377	332	1426	25315	11834	1916	588
N.S.	1	0.93	0.75	13.82	1.93	8.29	147.18	68.80	11.14	3.42
time (sec)	N/A	0.568	0.258	0.710	0.070	0.139	9.842	0.215	0.250	5.317

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	111	102	78	699	155	527	5882	2951	700	265
N.S.	1	0.92	0.70	6.30	1.40	4.75	52.99	26.59	6.31	2.39
time (sec)	N/A	0.430	0.141	0.378	0.042	0.114	4.536	0.150	0.212	4.789

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	185	153	0	0	0	1402	0	180	0
N.S.	1	0.97	0.80	0.00	0.00	0.00	7.34	0.00	0.94	0.00
time (sec)	N/A	0.649	0.299	0.000	0.000	0.000	15.134	0.000	0.246	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	264	159	0	0	0	0	0	454	0
N.S.	1	0.97	0.59	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	1.142	0.383	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	328	168	0	0	0	0	0	0	0
N.S.	1	1.02	0.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.117	0.657	0.000	0.000	0.000	0.000	0.000	0.253	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	431	410	358	20904	1032	11628	365145	143220	12856	2949
N.S.	1	0.95	0.83	48.50	2.39	26.98	847.20	332.30	29.83	6.84
time (sec)	N/A	1.331	1.374	5.245	0.135	0.467	49.518	1.790	0.299	7.393

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	328	310	265	11356	748	6557	168099	70422	7608	1882
N.S.	1	0.95	0.81	34.62	2.28	19.99	512.50	214.70	23.20	5.74
time (sec)	N/A	1.068	1.429	2.829	0.109	0.281	28.558	0.641	0.236	6.147

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	225	210	172	4939	464	2833	64068	27992	3949	1089
N.S.	1	0.93	0.76	21.95	2.06	12.59	284.75	124.41	17.55	4.84
time (sec)	N/A	0.728	0.639	1.342	0.079	0.172	16.551	0.312	0.241	5.772

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	149	137	106	1576	219	1104	16781	7893	1607	563
N.S.	1	0.92	0.71	10.58	1.47	7.41	112.62	52.97	10.79	3.78
time (sec)	N/A	0.503	0.187	0.711	0.053	0.129	6.806	0.179	0.228	5.372

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	270	229	0	0	0	1933	0	480	0
N.S.	1	0.97	0.82	0.00	0.00	0.00	6.93	0.00	1.72	0.00
time (sec)	N/A	0.946	0.634	0.000	0.000	0.000	29.431	0.000	0.267	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	395	376	217	0	0	0	0	0	1480	0
N.S.	1	0.95	0.55	0.00	0.00	0.00	0.00	0.00	3.75	0.00
time (sec)	N/A	1.639	0.954	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	392	380	332	0	0	0	2463	0	0	0
N.S.	1	0.97	0.85	0.00	0.00	0.00	6.28	0.00	0.00	0.00
time (sec)	N/A	1.349	1.043	0.000	0.000	0.000	43.028	0.000	0.261	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	281	272	231	0	0	0	1933	0	0	0
N.S.	1	0.97	0.82	0.00	0.00	0.00	6.88	0.00	0.00	0.00
time (sec)	N/A	0.960	0.653	0.000	0.000	0.000	26.956	0.000	0.255	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	187	154	0	0	0	1402	0	1480	0
N.S.	1	0.97	0.80	0.00	0.00	0.00	7.26	0.00	7.67	0.00
time (sec)	N/A	0.690	0.433	0.000	0.000	0.000	13.734	0.000	0.238	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	95	0	0	0	872	0	454	0
N.S.	1	0.98	0.76	0.00	0.00	0.00	6.98	0.00	3.63	0.00
time (sec)	N/A	0.504	0.298	0.000	0.000	0.000	6.523	0.000	0.257	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	0	0	0	377	0	95	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	4.83	0.00	1.22	0.00
time (sec)	N/A	0.354	0.149	0.000	0.000	0.000	2.189	0.000	0.292	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	102	0	0	0	0	0	19	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.516	0.372	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	229	152	0	0	0	0	0	36	0
N.S.	1	1.08	0.72	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	1.012	0.315	0.000	0.000	0.000	0.000	0.000	0.282	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	407	446	199	0	0	0	0	0	64	0
N.S.	1	1.10	0.49	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	2.132	0.410	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	387	368	220	0	0	0	0	0	0	0
N.S.	1	0.95	0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.642	0.965	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	270	263	161	0	0	0	0	0	0	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.151	0.380	0.000	0.000	0.000	0.000	0.000	0.268	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	176	180	110	0	0	0	5176	0	0	0
N.S.	1	1.02	0.62	0.00	0.00	0.00	29.41	0.00	0.00	0.00
time (sec)	N/A	0.693	0.210	0.000	0.000	0.000	37.325	0.000	0.271	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	83	0	0	0	2382	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	22.26	0.00	0.00	0.00
time (sec)	N/A	0.388	0.106	0.000	0.000	0.000	11.939	0.000	0.253	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	231	150	0	0	0	0	0	32	0
N.S.	1	1.09	0.71	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.973	0.285	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	315	343	209	0	0	0	0	0	64	0
N.S.	1	1.09	0.66	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	1.633	0.426	0.000	0.000	0.000	0.000	0.000	0.249	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	567	615	270	0	0	0	0	0	111	0
N.S.	1	1.08	0.48	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	3.262	0.714	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	322	329	172	0	0	0	0	0	0	0
N.S.	1	1.02	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.163	0.594	0.000	0.000	0.000	0.000	0.000	0.331	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	238	136	0	0	0	0	0	0	0
N.S.	1	1.04	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.754	0.322	0.000	0.000	0.000	0.000	0.000	0.270	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	83	0	0	0	8303	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	74.13	0.00	0.00	0.00
time (sec)	N/A	0.396	0.119	0.000	0.000	0.000	128.350	0.000	0.313	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	366	404	201	0	0	0	0	0	45	0
N.S.	1	1.10	0.55	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.930	0.408	0.000	0.000	0.000	0.000	0.000	0.286	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	525	271	0	0	0	0	0	92	0
N.S.	1	1.09	0.56	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	2.744	0.772	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	256	164	0	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.954	0.267	0.000	0.000	0.000	0.000	0.000	0.312	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	138	0	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	0.539	0.000	0.000	0.000	0.000	0.000	0.436	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	289	138	0	0	0	0	0	0	0
N.S.	1	0.95	0.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.064	0.595	0.000	0.000	0.000	0.000	0.000	0.949	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	211	162	0	0	0	0	0	33	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.754	0.597	0.000	0.000	0.000	0.000	0.000	200.036	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	0	0	0	0	0	0	0	78	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	0	0	0	0	0	0	0	35	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	0	0	0	0	0	0	0	80	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.539	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	0	0	0	0	0	0	0	35	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	200.030	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	241	195	0	0	0	0	0	0	0
N.S.	1	0.91	0.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.636	0.343	0.000	0.000	0.000	0.000	0.000	2.035	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	153	130	0	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	0.388	0.000	0.000	0.000	0.000	0.000	0.709	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	169	184	158	0	0	0	0	0	0	0
N.S.	1	1.09	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.346	0.000	0.000	0.000	0.000	0.000	0.408	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	242	160	0	0	0	0	0	0	0
N.S.	1	1.03	0.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	0.291	0.000	0.000	0.000	0.000	0.000	0.760	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	434	125	0	0	0	0	0	31	0
N.S.	1	0.94	0.27	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.451	0.204	0.000	0.000	0.000	0.000	0.000	200.024	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	259	125	0	0	0	0	0	0	0
N.S.	1	0.95	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.791	0.176	0.000	0.000	0.000	0.000	0.000	0.556	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	242	0	0	0	0	0	0	0
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.569	0.348	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	240	242	0	0	0	0	0	0	0
N.S.	1	0.98	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.756	0.281	0.000	0.000	0.000	0.000	0.000	2.876	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	387	124	0	0	0	0	0	31	0
N.S.	1	1.01	0.32	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.197	0.233	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	556	535	0	0	0	0	0	33	0
N.S.	1	0.97	0.93	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.336	2.775	0.000	0.000	0.000	0.000	0.000	200.027	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	289	270	0	0	0	0	0	0	0
N.S.	1	1.01	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	0.502	0.000	0.000	0.000	0.000	0.000	2.776	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	296	225	0	0	0	0	0	0	0
N.S.	1	1.14	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.331	0.000	0.000	0.000	0.000	0.000	1.280	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	285	226	0	0	0	0	0	0	0
N.S.	1	1.15	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	0.353	0.000	0.000	0.000	0.000	0.000	3.604	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1256	1171	168	0	0	0	0	0	33	0
N.S.	1	0.93	0.13	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	4.674	0.290	0.000	0.000	0.000	0.000	0.000	200.022	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	744	701	168	0	0	0	0	0	33	0
N.S.	1	0.94	0.23	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.403	0.226	0.000	0.000	0.000	0.000	0.000	200.035	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	285	0	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	0.408	0.000	0.000	0.000	0.000	0.000	0.862	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	258	414	314	0	0	0	0	0	0	0
N.S.	1	1.60	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.435	0.473	0.000	0.000	0.000	0.000	0.000	7.243	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	406	635	315	0	0	0	0	0	33	0
N.S.	1	1.56	0.78	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.238	0.462	0.000	0.000	0.000	0.000	0.000	200.031	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	F	F(-2)	F(-2)	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	0	219	0	0	0	0	0	35	0
N.S.	1	0.00	0.65	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	1.140	0.000	0.000	0.000	0.000	0.000	200.036	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	211	162	0	0	0	0	0	33	0
N.S.	1	0.99	0.76	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.714	0.535	0.000	0.000	0.000	0.000	0.000	200.029	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	98	0	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	0.158	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	35	0	35	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.06	0.00	1.06	1.06	1.06
time (sec)	N/A	0.347	2.205	0.208	0.105	0.113	0.000	0.168	200.027	5.033

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	48	0	35	35	35
N.S.	1	1.00	1.06	1.00	1.06	1.45	0.00	1.06	1.06	1.06
time (sec)	N/A	0.352	2.308	0.216	0.117	0.132	0.000	0.172	200.027	7.378

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	150	124	0	0	0	0	0	112	0
N.S.	1	1.17	0.97	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.650	0.296	0.000	0.000	0.000	0.000	0.000	0.212	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	159	124	0	0	0	0	0	112	0
N.S.	1	0.95	0.74	0.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.650	0.009	0.000	0.000	0.000	0.000	0.000	0.224	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [80] had the largest ratio of [.5806449999999999967]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	0.96	29	0.103
2	A	4	3	0.96	29	0.103
3	A	4	3	0.97	29	0.103
4	A	4	3	1.03	29	0.103
5	A	4	3	0.98	29	0.103
6	A	7	6	1.14	29	0.207
7	A	5	4	1.06	29	0.138
8	A	4	3	0.98	27	0.111
9	A	5	4	1.06	29	0.138
10	A	8	7	1.15	29	0.241
11	A	13	12	1.23	29	0.414
12	A	4	4	1.40	29	0.138
13	A	11	10	1.20	29	0.345
14	A	2	2	1.37	29	0.069
15	A	10	9	1.17	26	0.346
16	A	3	3	1.40	29	0.103
17	A	12	11	1.20	29	0.379
18	A	6	6	1.44	29	0.207
19	A	2	2	0.93	29	0.069
20	A	2	2	0.93	29	0.069
21	A	2	2	0.92	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	0.92	20	0.100
23	A	2	2	0.98	29	0.069
24	A	4	4	1.02	29	0.138
25	A	4	4	1.04	29	0.138
26	A	2	2	0.95	31	0.065
27	A	2	2	0.94	31	0.065
28	A	2	2	0.93	29	0.069
29	A	2	2	0.92	22	0.091
30	A	2	2	0.97	31	0.065
31	A	4	4	0.97	31	0.129
32	A	6	6	1.02	31	0.194
33	A	2	2	0.95	31	0.065
34	A	2	2	0.95	31	0.065
35	A	2	2	0.93	29	0.069
36	A	2	2	0.92	22	0.091
37	A	2	2	0.97	31	0.065
38	A	4	4	0.95	31	0.129
39	A	2	2	0.97	31	0.065
40	A	2	2	0.97	31	0.065
41	A	2	2	0.97	31	0.065
42	A	2	2	0.98	29	0.069
43	A	2	2	1.00	22	0.091
44	A	2	2	1.00	31	0.065
45	A	4	4	1.08	31	0.129
46	A	5	5	1.10	31	0.161
47	A	4	4	0.95	31	0.129
48	A	4	4	0.97	31	0.129
49	A	4	4	1.02	29	0.138
50	A	2	2	1.00	22	0.091
51	A	4	4	1.09	31	0.129
52	A	5	5	1.09	31	0.161
53	A	7	7	1.08	31	0.226

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	6	1.02	31	0.194
55	A	4	4	1.04	29	0.138
56	A	2	2	1.00	22	0.091
57	A	5	5	1.10	31	0.161
58	A	6	6	1.09	31	0.194
59	A	5	5	1.00	29	0.172
60	A	2	2	1.00	31	0.065
61	A	4	4	0.95	31	0.129
62	A	4	4	0.99	31	0.129
63	F	0	0	N/A	0.000	N/A
64	F	0	0	N/A	0.000	N/A
65	F	0	0	N/A	0.000	N/A
66	F	0	0	N/A	0.000	N/A
67	A	5	4	0.91	29	0.138
68	A	5	4	1.05	27	0.148
69	A	7	6	1.09	29	0.207
70	A	9	8	1.03	29	0.276
71	A	9	9	0.94	29	0.310
72	A	8	8	0.95	29	0.276
73	A	7	7	1.00	26	0.269
74	A	9	9	0.98	29	0.310
75	A	10	10	1.01	29	0.345
76	A	7	6	0.97	31	0.194
77	A	6	5	1.01	29	0.172
78	A	4	3	1.14	31	0.097
79	A	4	3	1.15	31	0.097
80	A	18	18	0.93	31	0.581
81	A	17	17	0.94	31	0.548
82	A	2	2	1.00	28	0.071
83	A	17	17	1.60	31	0.548
84	A	18	18	1.56	31	0.581
85	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	0.99	31	0.129
87	A	3	3	1.00	24	0.125
88	N/A	1	0	1.00	33	0.000
89	N/A	1	0	1.00	33	0.000
90	A	5	4	1.17	47	0.085
91	A	5	4	0.95	55	0.073

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 114

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(bBc-Abd+aBd)x^4}{4b^2d^2}+\frac{Bx^8}{8bd} + \frac{a^2(Ab-aB)\log(a+bx^4)}{4b^3(bc-ad)}+\frac{c^2(Bc-Ad)\log(c+dx^4)}{4d^3(bc-ad)}$$

output
```
-1/4*(-A*b*d+B*a*d+B*b*c)*x^4/b^2/d^2+1/8*B*x^8/b/d+1/4*a^2*(A*b-B*a)*ln(b
*x^4+a)/b^3/(-a*d+b*c)+1/4*c^2*(-A*d+B*c)*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{bd(bc-ad)x^4(-2bBc+2Abd-2aBd+bBdx^4)+2a^2(Ab-aB)d^3\log(a+bx^4)+2b^3c^2(Bc-Ad)\log}{8b^3d^3(bc-ad)}$$

input
```
Integrate[(x^11*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(b*d*(b*c - a*d)*x^4*(-2*b*B*c + 2*A*b*d - 2*a*B*d + b*B*d*x^4) + 2*a^2*(A
*b - a*B)*d^3*Log[a + b*x^4] + 2*b^3*c^2*(B*c - A*d)*Log[c + d*x^4])/(8*b^
3*d^3*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1043, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1043}$$

$$\frac{1}{4}\int \frac{x^8(Bx^4+A)}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{4}\int \left(\frac{Bx^4}{bd}+\frac{-bBc+Abd-aBd}{b^2d^2}-\frac{a^2(aB-Ab)}{b^2(bc-ad)(bx^4+a)}-\frac{c^2(Bc-Ad)}{d^2(ad-bc)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{a^2(Ab-aB)\log(a+bx^4)}{b^3(bc-ad)}-\frac{x^4(aBd-Abd+bBc)}{b^2d^2}+\frac{c^2(Bc-Ad)\log(c+dx^4)}{d^3(bc-ad)}+\frac{Bx^8}{2bd}\right)$$

input
```
Int[(x^11*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(((b*B*c - A*b*d + a*B*d)*x^4)/(b^2*d^2)) + (B*x^8)/(2*b*d) + (a^2*(A*b
- a*B)*Log[a + b*x^4])/(b^3*(b*c - a*d)) + (c^2*(B*c - A*d)*Log[c + d*x^4]
)/(d^3*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 1043
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simp
lify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /
; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/
n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

method	result
default	$\frac{\left(B\,x^{4} b d+A b d-B a d-B b c\right)^{2}}{8 b^{3} d^{3} B}-\frac{a^{2} \left(A b-B a\right) \ln \left(b\,x^{4}+a\right)}{4 b^{3} \left(a d-c b\right)}+\frac{c^{2} \left(A d-B c\right) \ln \left(d\,x^{4}+c\right)}{4 d^{3} \left(a d-c b\right)}$
norman	$\frac{B\,x^{8}}{8 b d}+\frac{\left(A b d-B a d-B b c\right) x^{4}}{4 b^{2} d^{2}}-\frac{a^{2} \left(A b-B a\right) \ln \left(b\,x^{4}+a\right)}{4 b^{3} \left(a d-c b\right)}+\frac{c^{2} \left(A d-B c\right) \ln \left(d\,x^{4}+c\right)}{4 d^{3} \left(a d-c b\right)}$
parallelrisch	$-\frac{-B\,x^{8} a\,b^{2} d^{3}+B\,x^{8} b^{3} c\,d^{2}-2 A\,x^{4} a\,b^{2} d^{3}+2 A\,x^{4} b^{3} c\,d^{2}+2 B\,x^{4} a^{2} b\,d^{3}-2 B\,x^{4} b^{3} c^{2} d+2 A \ln \left(b\,x^{4}+a\right) a^{2} b\,d^{3}-2 A \ln \left(d\,x^{4}+c\right) b^{3}}{8 b^{3} d^{3} \left(a d-c b\right)}$
risch	$\frac{B\,x^{8}}{8 b d}+\frac{A\,x^{4}}{4 b d}-\frac{B a\,x^{4}}{4 b^{2} d}-\frac{B c\,x^{4}}{4 b\,d^{2}}+\frac{A^{2}}{8 b d B}-\frac{A a}{4 b^{2} d}-\frac{A c}{4 b\,d^{2}}+\frac{B\,a^{2}}{8 b^{3} d}+\frac{B a c}{4 b^{2} d^{2}}+\frac{B\,c^{2}}{8 b\,d^{3}}+\frac{c^{2} \ln \left(d\,x^{4}+c\right) A}{4 d^{2} \left(a d-c b\right)}-\frac{c^{3} \ln (}{4 d^{3}}$

input
```
int(x^11*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*(B*b*d*x^4+A*b*d-B*a*d-B*b*c)^2/b^3/d^3/B-1/4*a^2*(A*b-B*a)/b^3/(a*d-b
*c)*ln(b*x^4+a)+1/4*c^2*(A*d-B*c)/d^3/(a*d-b*c)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.28

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{(Bb^3cd^2-Bab^2d^3)x^8-2\left(Bb^3c^2d-Ab^3cd^2-(Ba^2b-Aab^2)d^3\right)x^4-2\left(Ba^3-Aa^2b\right)d^3\log\left(bx^4+a\right)+}{8\left(b^4cd^3-ab^3d^4\right)}$$

input
```
integrate(x^11*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/8*((B*b^3*c*d^2 - B*a*b^2*d^3)*x^8 - 2*(B*b^3*c^2*d - A*b^3*c*d^2 - (B*a
^2*b - A*a*b^2)*d^3)*x^4 - 2*(B*a^3 - A*a^2*b)*d^3*log(b*x^4 + a) + 2*(B*b
^3*c^3 - A*b^3*c^2*d)*log(d*x^4 + c))/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**11*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba^3-Aa^2b)\log\left(bx^4+a\right)}{4\left(b^4c-ab^3d\right)} + \frac{(Bc^3-Ac^2d)\log\left(dx^4+c\right)}{4\left(bcd^3-ad^4\right)} + \frac{Bbdx^8-2\left(Bbc+(Ba-Ab)d\right)x^4}{8\,b^2d^2}$$

input
```
integrate(x^11*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*(B*a^3 - A*a^2*b)*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*(B*c^3 - A*c
^2*d)*log(d*x^4 + c)/(b*c*d^3 - a*d^4) + 1/8*(B*b*d*x^8 - 2*(B*b*c + (B*a
- A*b)*d)*x^4)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba^3-Aa^2b)\log(|bx^4+a|)}{4(b^4c-ab^3d)} + \frac{(Bc^3-Ac^2d)\log(|dx^4+c|)}{4(bcd^3-ad^4)} + \frac{Bbdx^8-2\,Bbcx^4-2\,Badx^4+2\,Abdx^4}{8\,b^2d^2}$$

input
```
integrate(x^11*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*(B*a^3 - A*a^2*b)*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*(B*c^3
- A*c^2*d)*log(abs(d*x^4 + c))/(b*c*d^3 - a*d^4) + 1/8*(B*b*d*x^8 - 2*B*b*
c*x^4 - 2*B*a*d*x^4 + 2*A*b*d*x^4)/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 12.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = x^4\left(\frac{A}{4\,b\,d}-\frac{B\,(a\,d+b\,c)}{4\,b^2\,d^2}\right) - \frac{\ln(b\,x^4+a)\,(B\,a^3-A\,a^2\,b)}{4\,b^4\,c-4\,a\,b^3\,d} - \frac{\ln(d\,x^4+c)\,(B\,c^3-A\,c^2\,d)}{4\,a\,d^4-4\,b\,c\,d^3} + \frac{B\,x^8}{8\,b\,d}$$

input
```
int((x^11*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
x^4*(A/(4*b*d) - (B*(a*d + b*c))/(4*b^2*d^2)) - (log(a + b*x^4)*(B*a^3 - A
*a^2*b))/(4*b^4*c - 4*a*b^3*d) - (log(c + d*x^4)*(B*c^3 - A*c^2*d))/(4*a*d
^4 - 4*b*c*d^3) + (B*x^8)/(8*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{2\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)c^2+2\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)c^2-2cd\,x^4+d^2x^8}{8d^3}$$

input
```
int(x^11*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(2*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*c**2 + 2*l
og(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*c**2 - 2*c*d*x**4
 + d**2*x**8)/(8*d**3)
```

3.2 $\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 83

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^4}{4bd} - \frac{a(Ab-aB)\log(a+bx^4)}{4b^2(bc-ad)} - \frac{c(Bc-Ad)\log(c+dx^4)}{4d^2(bc-ad)}$$

output
```
1/4*B*x^4/b/d-1/4*a*(A*b-B*a)*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c*(-A*d+B*c)*
ln(d*x^4+c)/d^2/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{a(-Ab+aB)d^2\log(a+bx^4)+b(Bd(bc-ad)x^4+bc(-Bc+Ad)\log(c+dx^4))}{4b^2d^2(bc-ad)}$$

input
```
Integrate[(x^7*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(a*(-(A*b) + a*B)*d^2*Log[a + b*x^4] + b*(B*d*(b*c - a*d)*x^4 + b*c*(-(B*c
) + A*d)*Log[c + d*x^4]))/(4*b^2*d^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1043, 159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^4)}{(a+bx^4)(c+dx^4)} dx$$

$$\Big\downarrow \text{1043}$$

$$\frac{1}{4}\int \frac{x^4 (Bx^4+A)}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{159}$$

$$\frac{1}{4}\int \left(\frac{B}{bd}+\frac{a(aB-Ab)}{b(bc-ad)(bx^4+a)}+\frac{c(Bc-Ad)}{d(ad-bc)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a(Ab-aB)\log(a+bx^4)}{b^2(bc-ad)}-\frac{c(Bc-Ad)\log(c+dx^4)}{d^2(bc-ad)}+\frac{Bx^4}{bd}\right)$$

input
```
Int[(x^7*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((B*x^4)/(b*d) - (a*(A*b - a*B)*Log[a + b*x^4])/(b^2*(b*c - a*d)) - (c*(B*
c - A*d)*Log[c + d*x^4])/(d^2*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 159
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n
*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ
[m, 0] || IntegersQ[m, n])
```

rule 1043
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simp
lify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /
; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/
n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{B\,x^{4}}{4bd}+\frac{a(Ab-Ba)\ln(b\,x^{4}+a)}{4(ad-cb)b^{2}}-\frac{c(Ad-Bc)\ln(d\,x^{4}+c)}{4d^{2}(ad-cb)}$	78
norman	$\frac{B\,x^{4}}{4bd}+\frac{a(Ab-Ba)\ln(b\,x^{4}+a)}{4(ad-cb)b^{2}}-\frac{c(Ad-Bc)\ln(d\,x^{4}+c)}{4d^{2}(ad-cb)}$	78
parallelrisch	$\frac{B\,x^{4}ab\,d^{2}-B\,x^{4}b^{2}cd+A\ln(b\,x^{4}+a)ab\,d^{2}-A\ln(d\,x^{4}+c)b^{2}cd-B\ln(b\,x^{4}+a)a^{2}d^{2}+B\ln(d\,x^{4}+c)b^{2}c^{2}}{4b^{2}d^{2}(ad-cb)}$	105
risch	$\frac{B\,x^{4}}{4bd}-\frac{c\ln(d\,x^{4}+c)A}{4d(ad-cb)}+\frac{c^{2}\ln(d\,x^{4}+c)B}{4d^{2}(ad-cb)}+\frac{a\ln(-b\,x^{4}-a)A}{4b(ad-cb)}-\frac{a^{2}\ln(-b\,x^{4}-a)B}{4b^{2}(ad-cb)}$	124

input
```
int(x^7*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*B*x^4/b/d+1/4*a*(A*b-B*a)/(a*d-b*c)/b^2*ln(b*x^4+a)-1/4*c*(A*d-B*c)/d^
2/(a*d-b*c)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{(Bb^2cd-Babd^2)x^4+(Ba^2-Aab)d^2\log\left(bx^4+a\right)-(Bb^2c^2-Ab^2cd)\log\left(dx^4+c\right)}{4\left(b^3cd^2-ab^2d^3\right)}$$

input
```
integrate(x^7*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*((B*b^2*c*d - B*a*b*d^2)*x^4 + (B*a^2 - A*a*b)*d^2*log(b*x^4 + a) - (B
*b^2*c^2 - A*b^2*c*d)*log(d*x^4 + c))/(b^3*c*d^2 - a*b^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^7(A + Bx^4)}{(a + bx^4)(c + dx^4)} dx = \text{Timed out}$$

input
```
integrate(x**7*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{x^7(A + Bx^4)}{(a + bx^4)(c + dx^4)} dx = \frac{Bx^4}{4\,bd} + \frac{(Ba^2 - Aab)\log(bx^4 + a)}{4\,(b^3c - ab^2d)} - \frac{(Bc^2 - Acd)\log(dx^4 + c)}{4\,(bcd^2 - ad^3)}$$

input
```
integrate(x^7*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*B*x^4/(b*d) + 1/4*(B*a^2 - A*a*b)*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1
/4*(B*c^2 - A*c*d)*log(d*x^4 + c)/(b*c*d^2 - a*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^4}{4\,bd} + \frac{(Ba^2-Aab)\log(|bx^4+a|)}{4\,(b^3c-ab^2d)} - \frac{(Bc^2-Acd)\log(|dx^4+c|)}{4\,(bcd^2-ad^3)}$$

input
```
integrate(x^7*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*B*x^4/(b*d) + 1/4*(B*a^2 - A*a*b)*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d
) - 1/4*(B*c^2 - A*c*d)*log(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{\ln(b\,x^4+a)\,\left(B\,a^2-A\,a\,b\right)}{4\,b^3\,c-4\,a\,b^2\,d} + \frac{\ln(d\,x^4+c)\,\left(B\,c^2-A\,c\,d\right)}{4\,a\,d^3-4\,b\,c\,d^2} + \frac{B\,x^4}{4\,b\,d}$$

input
```
int((x^7*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(a + b*x^4)*(B*a^2 - A*a*b))/(4*b^3*c - 4*a*b^2*d) + (log(c + d*x^4)*(
B*c^2 - A*c*d))/(4*a*d^3 - 4*b*c*d^2) + (B*x^4)/(4*b*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{x^7(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)c-\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)c+d\,x^4}{4d^2}$$

input
```
int(x^7*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*c - log(d
**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*c + d*x**4)/(4*d**2)
```

3.3 $\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 67

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{(Ab-aB)\log(a+bx^4)}{4b(bc-ad)} + \frac{(Bc-Ad)\log(c+dx^4)}{4d(bc-ad)}$$

output
```
1/4*(A*b-B*a)*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*(-A*d+B*c)*ln(d*x^4+c)/d/(-a*d+
b*c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{(Ab-aB)d\log(a+bx^4)+b(Bc-Ad)\log(c+dx^4)}{4bd(bc-ad)}$$

input
```
Integrate[(x^3*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((A*b - a*B)*d*Log[a + b*x^4] + b*(B*c - A*d)*Log[c + d*x^4])/(4*b*d*(b*c
- a*d))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1041, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 1041}$$

$$\frac{1}{4}\int \frac{Bx^4+A}{(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{4}\int \left(\frac{Ab-aB}{(bc-ad)(bx^4+a)}+\frac{Bc-Ad}{(bc-ad)(dx^4+c)}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{(Ab-aB)\log(a+bx^4)}{b(bc-ad)}+\frac{(Bc-Ad)\log(c+dx^4)}{d(bc-ad)}\right)$$

input
```
Int[(x^3*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(((A*b - a*B)*Log[a + b*x^4])/(b*(b*c - a*d)) + ((B*c - A*d)*Log[c + d*x^4
])/(d*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 1041
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*
x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m
, n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{(Ab-Ba)\ln(b\,x^4+a)}{4(ad-cb)b}+\frac{(Ad-Bc)\ln(d\,x^4+c)}{4(ad-cb)d}$	64
norman	$-\frac{(Ab-Ba)\ln(b\,x^4+a)}{4(ad-cb)b}+\frac{(Ad-Bc)\ln(d\,x^4+c)}{4(ad-cb)d}$	64
parallelrisch	$-\frac{A\ln(b\,x^4+a)bd-A\ln(d\,x^4+c)bd-B\ln(b\,x^4+a)ad+B\ln(d\,x^4+c)bc}{4(ad-cb)bd}$	70
risch	$-\frac{\ln(b\,x^4+a)A}{4(ad-cb)}+\frac{\ln(b\,x^4+a)Ba}{4(ad-cb)b}+\frac{\ln(-d\,x^4-c)A}{4ad-4cb}-\frac{\ln(-d\,x^4-c)Bc}{4(ad-cb)d}$	100

input
```
int(x^3*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(A*b-B*a)/(a*d-b*c)/b*ln(b*x^4+a)+1/4*(A*d-B*c)/(a*d-b*c)/d*ln(d*x^4+
c)
```

Fricas [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba-Ab)d\log(bx^4+a)-(Bbc-Abd)\log(dx^4+c)}{4(b^2cd-abd^2)}$$

input
```
integrate(x^3*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*((B*a - A*b)*d*log(b*x^4 + a) - (B*b*c - A*b*d)*log(d*x^4 + c))/(b^2*
c*d - a*b*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba-Ab)\log(bx^4+a)}{4(b^2c-abd)} + \frac{(Bc-Ad)\log(dx^4+c)}{4(bcd-ad^2)}$$

input
```
integrate(x^3*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*(B*a - A*b)*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*(B*c - A*d)*log(d*x^
4 + c)/(b*c*d - a*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba-Ab)\log(|bx^4+a|)}{4(b^2c-abd)} + \frac{(Bc-Ad)\log(|dx^4+c|)}{4(bcd-ad^2)}$$

input
```
integrate(x^3*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*(B*a - A*b)*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*(B*c - A*d)*log
(abs(d*x^4 + c))/(b*c*d - a*d^2)
```

Mupad [B] (verification not implemented)

Time = 7.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{\ln\left(b\,x^4+a\right)\,\left(A\,b-B\,a\right)}{4\,b^2\,c-4\,a\,b\,d}+\frac{\ln\left(d\,x^4+c\right)\,\left(A\,d-B\,c\right)}{4\,a\,d^2-4\,b\,c\,d}$$

input
```
int((x^3*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(a + b*x^4)*(A*b - B*a))/(4*b^2*c - 4*a*b*d) + (log(c + d*x^4)*(A*d -
B*c))/(4*a*d^2 - 4*b*c*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{x^3(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx = \frac{\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4d}$$

input
```
int(x^3*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2) + log(d**(1/
4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2))/(4*d)
```

3.4 $\int \frac{A+Bx^4}{x(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{A+Bx^4}{x\,(a+bx^4)\,(c+dx^4)}\,dx = \frac{A\log(x)}{ac}-\frac{(Ab-aB)\log\left(a+bx^4\right)}{4a(bc-ad)} - \frac{(Bc-Ad)\log\left(c+dx^4\right)}{4c(bc-ad)}$$

output
```
A*ln(x)/a/c-1/4*(A*b-B*a)*ln(b*x^4+a)/a/(-a*d+b*c)-1/4*(-A*d+B*c)*ln(d*x^4
+c)/c/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^4}{x\,(a+bx^4)\,(c+dx^4)}\,dx = \frac{A(-4bc+4ad)\log(x)+(Ab-aB)c\log\left(a+bx^4\right)+a(Bc-Ad)\log\left(c+dx^4\right)}{4ac(-bc+ad)}$$

input
```
Integrate[(A + B*x^4)/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(A*(-4*b*c + 4*a*d)*Log[x] + (A*b - a*B)*c*Log[a + b*x^4] + a*(B*c - A*d)*
Log[c + d*x^4])/(4*a*c*(-(b*c) + a*d))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1043, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x\left(a+bx^4\right)\left(c+dx^4\right)}dx$$

$$\Big\downarrow \text{1043}$$

$$\frac{1}{4}\int \frac{Bx^4+A}{x^4\left(bx^4+a\right)\left(dx^4+c\right)}dx^4$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{4}\int \left(\frac{A}{acx^4}-\frac{b(aB-Ab)}{a(ad-bc)\left(bx^4+a\right)}-\frac{d(Bc-Ad)}{c(bc-ad)\left(dx^4+c\right)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{(Ab-aB)\log\left(a+bx^4\right)}{a(bc-ad)}-\frac{(Bc-Ad)\log\left(c+dx^4\right)}{c(bc-ad)}+\frac{A\log\left(x^4\right)}{ac}\right)$$

input
```
Int[(A + B*x^4)/(x*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
((A*Log[x^4])/(a*c) - ((A*b - a*B)*Log[a + b*x^4])/(a*(b*c - a*d)) - ((B*c
 - A*d)*Log[c + d*x^4])/(c*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 1043
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simp
lify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /
; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/
n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{(Ab-Ba)\ln(b\,x^4+a)}{4a(ad-cb)}+\frac{A\ln(x)}{ac}-\frac{(Ad-Bc)\ln(d\,x^4+c)}{4c(ad-cb)}$	74
norman	$\frac{(Ab-Ba)\ln(b\,x^4+a)}{4a(ad-cb)}+\frac{A\ln(x)}{ac}-\frac{(Ad-Bc)\ln(d\,x^4+c)}{4c(ad-cb)}$	74
parallelrisch	$\frac{4A\ln(x)ad-4A\ln(x)bc+A\ln(b\,x^4+a)bc-A\ln(d\,x^4+c)ad-B\ln(b\,x^4+a)ac+B\ln(d\,x^4+c)ac}{4(ad-cb)ac}$	84
risch	$\frac{A\ln(x)}{ac}+\frac{\ln(b\,x^4+a)Ab}{4a(ad-cb)}-\frac{\ln(b\,x^4+a)B}{4(ad-cb)}-\frac{\ln(-d\,x^4-c)Ad}{4c(ad-cb)}+\frac{\ln(-d\,x^4-c)B}{4ad-4cb}$	110

input
```
int((B*x^4+A)/x/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(A*b-B*a)/a/(a*d-b*c)*ln(b*x^4+a)+A*ln(x)/a/c-1/4*(A*d-B*c)/c/(a*d-b*c
)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

Time = 10.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^4}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{(Ba-Ab)c\log\left(bx^4+a\right)-(Bac-Aad)\log\left(dx^4+c\right)+4\left(Abc-Aad\right)\log\left(x\right)}{4\left(abc^2-a^2cd\right)}$$

input
```
integrate((B*x^4+A)/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*((B*a - A*b)*c*log(b*x^4 + a) - (B*a*c - A*a*d)*log(d*x^4 + c) + 4*(A*
b*c - A*a*d)*log(x))/(a*b*c^2 - a^2*c*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**4+A)/x/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^4}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{(Ba-Ab)\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)} - \frac{(Bc-Ad)\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)} + \frac{A\log\left(x^4\right)}{4\,ac}$$

input
```
integrate((B*x^4+A)/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/4*(B*a - A*b)*log(b*x^4 + a)/(a*b*c - a^2*d) - 1/4*(B*c - A*d)*log(d*x^4
 + c)/(b*c^2 - a*c*d) + 1/4*A*log(x^4)/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^4}{x(a+bx^4)(c+dx^4)}\,dx = \frac{(Bab-Ab^2)\log(|bx^4+a|)}{4(ab^2c-a^2bd)} - \frac{(Bcd-Ad^2)\log(|dx^4+c|)}{4(bc^2d-acd^2)} + \frac{A\log(x^4)}{4ac}$$

input
```
integrate((B*x^4+A)/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/4*(B*a*b - A*b^2)*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) - 1/4*(B*c*d -
 A*d^2)*log(abs(d*x^4 + c))/(b*c^2*d - a*c*d^2) + 1/4*A*log(x^4)/(a*c)
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^4}{x(a+bx^4)(c+dx^4)}\,dx = \frac{\ln(b\,x^4+a)\,(A\,b-B\,a)}{4\,a^2\,d-4\,a\,b\,c} + \frac{\ln(d\,x^4+c)\,(A\,d-B\,c)}{4\,b\,c^2-4\,a\,c\,d} + \frac{A\,\ln(x)}{a\,c}$$

input
```
int((A + B*x^4)/(x*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(log(a + b*x^4)*(A*b - B*a))/(4*a^2*d - 4*a*b*c) + (log(c + d*x^4)*(A*d -
B*c))/(4*b*c^2 - 4*a*c*d) + (A*log(x))/(a*c)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^4}{x\,(a+bx^4)\,(c+dx^4)}\,dx$$
$$= \frac{-\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+4\log(x)}{4c}$$

input
```
int((B*x^4+A)/x/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2) - log(d**
(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2) + 4*log(x))/(4*c)
```

3.5 $\int \frac{A+Bx^4}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 107

$$\int \frac{A+Bx^4}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{A}{4acx^4}+\frac{(aBc-A(bc+ad))\log(x)}{a^2c^2} + \frac{b(Ab-aB)\log(a+bx^4)}{4a^2(bc-ad)}+\frac{d(Bc-Ad)\log(c+dx^4)}{4c^2(bc-ad)}$$

output
```
-1/4*A/a/c/x^4+(a*B*c-A*(a*d+b*c))*ln(x)/a^2/c^2+1/4*b*(A*b-B*a)*ln(b*x^4+
a)/a^2/(-a*d+b*c)+1/4*d*(-A*d+B*c)*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^4}{x^5(a+bx^4)(c+dx^4)}\,dx = -\frac{A}{4acx^4}+\frac{(-Abc+aBc-aAd)\log(x)}{a^2c^2} + \frac{(-Ab^2+abB)\log(a+bx^4)}{4a^2(-bc+ad)} - \frac{(-Bcd+Ad^2)\log(c+dx^4)}{4c^2(bc-ad)}$$

input
```
Integrate[(A + B*x^4)/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/4*A/(a*c*x^4) + ((-(A*b*c) + a*B*c - a*A*d)*Log[x])/(a^2*c^2) + ((-(A*b
^2) + a*b*B)*Log[a + b*x^4])/(4*a^2*(-(b*c) + a*d)) - ((-(B*c*d) + A*d^2)*
Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1043, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^5(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1043}$$

$$\frac{1}{4}\int \frac{Bx^4+A}{x^8(bx^4+a)(dx^4+c)}dx^4$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{4}\int \left(\frac{(aB-Ab)b^2}{a^2(ad-bc)(bx^4+a)}+\frac{d^2(Bc-Ad)}{c^2(bc-ad)(dx^4+c)}+\frac{aBc-A(bc+ad)}{a^2c^2x^4}+\frac{A}{acx^8}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\log(x^4)(aBc-A(ad+bc))}{a^2c^2}+\frac{b(Ab-aB)\log(a+bx^4)}{a^2(bc-ad)}+\frac{d(Bc-Ad)\log(c+dx^4)}{c^2(bc-ad)}-\frac{A}{acx^4}\right)$$

input
```
Int[(A + B*x^4)/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(A/(a*c*x^4)) + ((a*B*c - A*(b*c + a*d))*Log[x^4])/(a^2*c^2) + (b*(A*b -
 a*B)*Log[a + b*x^4])/(a^2*(b*c - a*d)) + (d*(B*c - A*d)*Log[c + d*x^4])/(
c^2*(b*c - a*d)))/4
```

Defintions of rubi rules used

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 1043
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simp
lify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /
; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/
n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

method	result
default	$-\frac{b(Ab-Ba)\ln(b\,x^4+a)}{4a^2(ad-cb)}-\frac{A}{4ac\,x^4}+\frac{(-Aad-Abc+aBc)\ln(x)}{a^2c^2}+\frac{d(Ad-Bc)\ln(d\,x^4+c)}{4c^2(ad-cb)}$
norman	$-\frac{A}{4ac\,x^4}-\frac{(Aad+Abc-aBc)\ln(x)}{a^2c^2}-\frac{b(Ab-Ba)\ln(b\,x^4+a)}{4a^2(ad-cb)}+\frac{d(Ad-Bc)\ln(d\,x^4+c)}{4c^2(ad-cb)}$
risch	$-\frac{A}{4ac\,x^4}-\frac{\ln(x)Ad}{a\,c^2}-\frac{\ln(x)Ab}{a^2c}+\frac{\ln(x)B}{ac}+\frac{d^2\ln(d\,x^4+c)A}{4c^2(ad-cb)}-\frac{d\ln(d\,x^4+c)B}{4c(ad-cb)}-\frac{b^2\ln(-b\,x^4-a)A}{4(ad-cb)a^2}+\frac{b\ln(-b\,x^4-}{4(ad-cb}$
parallelrisch	$-\frac{4A\ln(x)x^4a^2d^2-4A\ln(x)x^4b^2c^2+A\ln(b\,x^4+a)x^4b^2c^2-A\ln(d\,x^4+c)x^4a^2d^2-4B\ln(x)x^4a^2cd+4B\ln(x)x^4ab\,c^2-B\ln(b}{4a^2c^2x^4(ad-cb)}$

input
```
int((B*x^4+A)/x^5/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*(A*b-B*a)/a^2/(a*d-b*c)*ln(b*x^4+a)-1/4*A/a/c/x^4+1/a^2/c^2*(-A*a*d
-A*b*c+B*a*c)*ln(x)+1/4*d*(A*d-B*c)/c^2/(a*d-b*c)*ln(d*x^4+c)
```

Fricas [A] (verification not implemented)

Time = 47.80 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{(Bab-Ab^2)c^2x^4\log\left(bx^4+a\right)-(Ba^2cd-Aa^2d^2)x^4\log\left(dx^4+c\right)+4\left(Ba^2cd-Aa^2d^2-(Bab-Ab^2\right.}{4\left(a^2bc^3-a^3c^2d\right)x^4}$$

input
```
integrate((B*x^4+A)/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*((B*a*b - A*b^2)*c^2*x^4*log(b*x^4 + a) - (B*a^2*c*d - A*a^2*d^2)*x^4
*log(d*x^4 + c) + 4*(B*a^2*c*d - A*a^2*d^2 - (B*a*b - A*b^2)*c^2)*x^4*log(
x) + A*a*b*c^2 - A*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**4+A)/x**5/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{(Bab-Ab^2)\log\left(bx^4+a\right)}{4\left(a^2bc-a^3d\right)}+\frac{(Bcd-Ad^2)\log\left(dx^4+c\right)}{4\left(bc^3-ac^2d\right)} - \frac{(Aad-(Ba-Ab)c)\log\left(x^4\right)}{4\,a^2c^2}-\frac{A}{4\,acx^4}$$

input
```
integrate((B*x^4+A)/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/4*(B*a*b - A*b^2)*log(b*x^4 + a)/(a^2*b*c - a^3*d) + 1/4*(B*c*d - A*d^2
)*log(d*x^4 + c)/(b*c^3 - a*c^2*d) - 1/4*(A*a*d - (B*a - A*b)*c)*log(x^4)/
(a^2*c^2) - 1/4*A/(a*c*x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{\left(Bab^2-Ab^3\right)\log\left(\left|bx^4+a\right|\right)}{4\left(a^2b^2c-a^3bd\right)} + \frac{\left(Bcd^2-Ad^3\right)\log\left(\left|dx^4+c\right|\right)}{4\left(bc^3d-ac^2d^2\right)} + \frac{\left(Bac-Abc-Aad\right)\log\left(x^4\right)}{4\,a^2c^2} - \frac{Bacx^4-Abcx^4-Aadx^4+Aac}{4\,a^2c^2x^4}$$

input
```
integrate((B*x^4+A)/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/4*(B*a*b^2 - A*b^3)*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) + 1/4*(B*
c*d^2 - A*d^3)*log(abs(d*x^4 + c))/(b*c^3*d - a*c^2*d^2) + 1/4*(B*a*c - A*
b*c - A*a*d)*log(x^4)/(a^2*c^2) - 1/4*(B*a*c*x^4 - A*b*c*x^4 - A*a*d*x^4 +
 A*a*c)/(a^2*c^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{\ln\left(b\,x^4+a\right)\,\left(A\,b^2-B\,a\,b\right)}{4\,a^3\,d-4\,a^2\,b\,c} - \frac{\ln\left(d\,x^4+c\right)\,\left(A\,d^2-B\,c\,d\right)}{4\,b\,c^3-4\,a\,c^2\,d} - \frac{\ln\left(x\right)\,\left(A\,a\,d+A\,b\,c-B\,a\,c\right)}{a^2\,c^2} - \frac{A}{4\,a\,c\,x^4}$$

input
```
int((A + B*x^4)/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (log(a + b*x^4)*(A*b^2 - B*a*b))/(4*a^3*d - 4*a^2*b*c) - (log(c + d*x^4)
*(A*d^2 - B*c*d))/(4*b*c^3 - 4*a*c^2*d) - (log(x)*(A*a*d + A*b*c - B*a*c))
/(a^2*c^2) - A/(4*a*c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^4}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)d\,x^4+\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)d\,x^4-4\log(x)\,d\,x^4-c}{4c^2x^4}$$

input
```
int((B*x^4+A)/x^5/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*d*x**4 + log
(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*d*x**4 - 4*log(x)*d
*x**4 - c)/(4*c**2*x**4)
```

3.6 $\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 136

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(bBc-Abd+aBd)x^2}{2b^2d^2}+\frac{Bx^6}{6bd} + \frac{a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}$$

output
```
-1/2*(-A*b*d+B*a*d+B*b*c)*x^2/b^2/d^2+1/6*B*x^6/b/d+1/2*a^(3/2)*(A*b-B*a)*
arctan(b^(1/2)*x^2/a^(1/2))/b^(5/2)/(-a*d+b*c)+1/2*c^(3/2)*(-A*d+B*c)*arct
an(d^(1/2)*x^2/c^(1/2))/d^(5/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{6}\left(-\frac{3(bBc-Abd+aBd)x^2}{b^2d^2}+\frac{Bx^6}{bd} + \frac{3a^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{5/2}(-bc+ad)} + \frac{3c^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

input
```
Integrate[(x^9*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-3*(b*B*c - A*b*d + a*B*d)*x^2)/(b^2*d^2) + (B*x^6)/(b*d) + (3*a^(3/2)*(
-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3
*c^(3/2)*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(d^(5/2)*(b*c - a*d)))
/6
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1045, 444, 27, 444, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$

$$\Big\downarrow \text{ 1045}$$

$$\frac{1}{2}\int \frac{x^8(Bx^4+A)}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{ 444}$$

$$\frac{1}{2}\left(\frac{Bx^6}{3bd}-\frac{\int \frac{3x^4((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx^2}{3bd}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{Bx^6}{3bd}-\frac{\int \frac{x^4((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$

$$\Big\downarrow \; 444$$

$$\frac{1}{2}\left(\frac{Bx^6}{3bd}-\frac{\frac{x^2(aBd-Abd+bBc)}{bd}-\frac{\int \frac{\left(c(Bc-Ad)b^2+ad(Bc-Ad)b+a^2Bd^2\right)x^4+ac(bBc-Abd+aBd)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx^2}{bd}}{bd}\right)$$

$$\Big\downarrow \; 397$$

$$\frac{1}{2}\left(\frac{Bx^6}{3bd}-\frac{\frac{x^2(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\int \frac{1}{bx^4+a}dx^2}{bc-ad}+\frac{b^2c^2(Bc-Ad)\int \frac{1}{dx^4+c}dx^2}{bc-ad}}{bd}}{bd}\right)$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}\left(\frac{Bx^6}{3bd}-\frac{\frac{x^2(aBd-Abd+bBc)}{bd}-\frac{\frac{a^{3/2}d^2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}+\frac{b^2c^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}}{bd}}{bd}\right)$$

input
```
Int[(x^9*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((B*x^6)/(3*b*d) - (((b*B*c - A*b*d + a*B*d)*x^2)/(b*d) - ((a^(3/2)*(A*b -
 a*B)*d^2*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)) + (b^2*c^(3
/2)*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/(b*d
))/(b*d))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 1045
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Si
mp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
 + f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p,
q, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{B\,x^{6} b d}{6}+\frac{(Abd-Bad-Bbc)x^{2}}{2}}{b^{2} d^{2}}-\frac{a^{2}(Ab-Ba)\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2b^{2}(ad-cb)\sqrt{ab}}+\frac{c^{2}(Ad-Bc)\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2d^{2}(ad-cb)\sqrt{cd}}$	122
risch	Expression too large to display	2692

input
```
int(x^9*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d^2*(1/6*B*x^6*b*d+1/2*(A*b*d-B*a*d-B*b*c)*x^2)-1/2*a^2*(A*b-B*a)/b^
2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*c^2*(A*d-B*c)/d^2/(a
*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 71.71 (sec) , antiderivative size = 728, normalized size of antiderivative = 5.35

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \left[\frac{2\,(Bb^2cd-Babd^2)x^6+3\,(Ba^2-Aab)d^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)-6\,(Bb^2c^2-Ab^2cd-(Ba^2-Ac}{12\,(b^3cd^2-ab^2d^3)}\right.$$

input
```
integrate(x^9*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/12*(2*(B*b^2*c*d - B*a*b*d^2)*x^6 + 3*(B*a^2 - A*a*b)*d^2*sqrt(-a/b)*lo
g((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(B*b^2*c^2 - A*b^2*c*d
 - (B*a^2 - A*a*b)*d^2)*x^2 + 3*(B*b^2*c^2 - A*b^2*c*d)*sqrt(-c/d)*log((d*
x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b^3*c*d^2 - a*b^2*d^3), 1/12*
(2*(B*b^2*c*d - B*a*b*d^2)*x^6 - 6*(B*a^2 - A*a*b)*d^2*sqrt(a/b)*arctan(b*
x^2*sqrt(a/b)/a) - 6*(B*b^2*c^2 - A*b^2*c*d - (B*a^2 - A*a*b)*d^2)*x^2 + 3
*(B*b^2*c^2 - A*b^2*c*d)*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(
d*x^4 + c)))/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(B*b^2*c*d - B*a*b*d^2)*x^6
+ 3*(B*a^2 - A*a*b)*d^2*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b
*x^4 + a)) - 6*(B*b^2*c^2 - A*b^2*c*d - (B*a^2 - A*a*b)*d^2)*x^2 + 6*(B*b^
2*c^2 - A*b^2*c*d)*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c))/(b^3*c*d^2 - a*b^2
*d^3), 1/6*((B*b^2*c*d - B*a*b*d^2)*x^6 - 3*(B*a^2 - A*a*b)*d^2*sqrt(a/b)*
arctan(b*x^2*sqrt(a/b)/a) - 3*(B*b^2*c^2 - A*b^2*c*d - (B*a^2 - A*a*b)*d^2
)*x^2 + 3*(B*b^2*c^2 - A*b^2*c*d)*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c))/(b^
3*c*d^2 - a*b^2*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**9*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba^3-Aa^2b)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{(Bc^3-Ac^2d)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{Bbdx^6-3\,(Bbc+(Ba-Ab)d)x^2}{6\,b^2d^2}$$

input
```
integrate(x^9*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*(B*a^3 - A*a^2*b)*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b
)) + 1/2*(B*c^3 - A*c^2*d)*arctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt
(c*d)) + 1/6*(B*b*d*x^6 - 3*(B*b*c + (B*a - A*b)*d)*x^2)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba^3-Aa^2b)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^3c-ab^2d)\sqrt{ab}} + \frac{(Bc^3-Ac^2d)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd^2-ad^3)\sqrt{cd}} + \frac{Bb^2d^2x^6-3\,Bb^2cdx^2-3\,Babd^2x^2+3\,Ab^2d^2x^2}{6\,b^3d^3}$$

input
```
integrate(x^9*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(B*a^3 - A*a^2*b)*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b
)) + 1/2*(B*c^3 - A*c^2*d)*arctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt
(c*d)) + 1/6*(B*b^2*d^2*x^6 - 3*B*b^2*c*d*x^2 - 3*B*a*b*d^2*x^2 + 3*A*b^2*
d^2*x^2)/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 20507, normalized size of antiderivative = 150.79

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^9*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
x^2*(A/(2*b*d) - (B*(a*d + b*c))/(2*b^2*d^2)) - (atan((((A*d - B*c)*((4*x^
2*(B^5*a^3*b^9*c^12 + B^5*a^12*c^3*d^9 - A^5*a^2*b^10*c^8*d^4 - A^5*a^3*b^
9*c^7*d^5 - A^5*a^4*b^8*c^6*d^6 - A^5*a^6*b^6*c^4*d^8 - A^5*a^7*b^5*c^3*d^
9 - A^5*a^8*b^4*c^2*d^10 + B^5*a^5*b^7*c^10*d^2 + B^5*a^6*b^6*c^9*d^3 + B^
5*a^9*b^3*c^6*d^6 + B^5*a^10*b^2*c^5*d^7 - A*B^4*a^2*b^10*c^12 - A*B^4*a^1
2*c^2*d^10 + B^5*a^4*b^8*c^11*d + B^5*a^11*b*c^4*d^8 - 5*A*B^4*a^4*b^8*c^1
0*d^2 - 5*A*B^4*a^5*b^7*c^9*d^3 - 3*A*B^4*a^6*b^6*c^8*d^4 - 3*A*B^4*a^8*b^
4*c^6*d^6 - 5*A*B^4*a^9*b^3*c^5*d^7 - 5*A*B^4*a^10*b^2*c^4*d^8 + 4*A^2*B^3
*a^2*b^10*c^11*d + 4*A^2*B^3*a^11*b*c^2*d^10 + 4*A^4*B*a^2*b^10*c^9*d^3 +
5*A^4*B*a^3*b^9*c^8*d^4 + 5*A^4*B*a^4*b^8*c^7*d^5 + 2*A^4*B*a^5*b^7*c^6*d^
6 + 2*A^4*B*a^6*b^6*c^5*d^7 + 5*A^4*B*a^7*b^5*c^4*d^8 + 5*A^4*B*a^8*b^4*c^
3*d^9 + 4*A^4*B*a^9*b^3*c^2*d^10 + 10*A^2*B^3*a^3*b^9*c^10*d^2 + 10*A^2*B^
3*a^4*b^8*c^9*d^3 + 9*A^2*B^3*a^5*b^7*c^8*d^4 + 3*A^2*B^3*a^6*b^6*c^7*d^5
+ 3*A^2*B^3*a^7*b^5*c^6*d^6 + 9*A^2*B^3*a^8*b^4*c^5*d^7 + 10*A^2*B^3*a^9*b
^3*c^4*d^8 + 10*A^2*B^3*a^10*b^2*c^3*d^9 - 6*A^3*B^2*a^2*b^10*c^10*d^2 - 1
0*A^3*B^2*a^3*b^9*c^9*d^3 - 10*A^3*B^2*a^4*b^8*c^8*d^4 - 7*A^3*B^2*a^5*b^7
*c^7*d^5 - 2*A^3*B^2*a^6*b^6*c^6*d^6 - 7*A^3*B^2*a^7*b^5*c^5*d^7 - 10*A^3*
B^2*a^8*b^4*c^4*d^8 - 10*A^3*B^2*a^9*b^3*c^3*d^9 - 6*A^3*B^2*a^10*b^2*c^2*
d^10 - 5*A*B^4*a^3*b^9*c^11*d - 5*A*B^4*a^11*b*c^3*d^9))/(b^6*d^6) - (((16
*(B^4*a*b^12*c^13 + B^4*a^13*c*d^12 + 2*A^4*a^5*b^8*c^5*d^8 + 2*B^4*a^7...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

$$\int \frac{x^9(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)c-3\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)c-3cd\,x^2+d^2x^6}{6d^3}$$

input
```
int(x^9*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(
1/4)*c**(1/4)*sqrt(2)))*c - 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt
(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*c - 3*c*d*x**2 + d**2*x**6
)/(6*d**3)
```

3.7 $\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^2}{2bd} - \frac{\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{\sqrt{c}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

output
```
1/2*B*x^2/b/d-1/2*a^(1/2)*(A*b-B*a)*arctan(b^(1/2)*x^2/a^(1/2))/b^(3/2)/(-
a*d+b*c)-1/2*c^(1/2)*(-A*d+B*c)*arctan(d^(1/2)*x^2/c^(1/2))/d^(3/2)/(-a*d+
b*c)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{2}\left(\frac{Bx^2}{bd} + \frac{\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)} + \frac{\sqrt{c}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{d^{3/2}(-bc+ad)}\right)$$

input
```
Integrate[(x^5*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((B*x^2)/(b*d) + (Sqrt[a]*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(b
^(3/2)*(b*c - a*d)) + (Sqrt[c]*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/
(d^(3/2)*(-(b*c) + a*d)))/2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1045, 444, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$
$$\Big\downarrow \text{1045}$$
$$\frac{1}{2}\int \frac{x^4(Bx^4+A)}{(bx^4+a)(dx^4+c)}dx^2$$
$$\Big\downarrow \text{444}$$
$$\frac{1}{2}\left(\frac{Bx^2}{bd}-\frac{\int \frac{(bBc-Abd+aBd)x^4+aBc}{(bx^4+a)(dx^4+c)}dx^2}{bd}\right)$$
$$\Big\downarrow \text{397}$$
$$\frac{1}{2}\left(\frac{Bx^2}{bd}-\frac{\frac{ad(Ab-aB)\int \frac{1}{bx^4+a}dx^2}{bc-ad}+\frac{bc(Bc-Ad)\int \frac{1}{dx^4+c}dx^2}{bc-ad}}{bd}\right)$$
$$\Big\downarrow \text{218}$$
$$\frac{1}{2}\left(\frac{Bx^2}{bd}-\frac{\frac{\sqrt{a}d(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}+\frac{b\sqrt{c}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}}{bd}\right)$$

input
```
Int[(x^5*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((B*x^2)/(b*d) - ((Sqrt[a]*(A*b - a*B)*d*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(S
qrt[b]*(b*c - a*d)) + (b*Sqrt[c]*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]]
)/(Sqrt[d]*(b*c - a*d)))/(b*d))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 1045
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Si
mp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
 + f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p,
q, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{B\,x^{2}}{2bd}+\frac{a(Ab-Ba)\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2(ad-cb)b\sqrt{ab}}-\frac{c(Ad-Bc)\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2d(ad-cb)\sqrt{cd}}$	94
risch	Expression too large to display	2570

input
```
int(x^5*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b/d+1/2*a*(A*b-B*a)/(a*d-b*c)/b/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(
1/2))-1/2*c*(A*d-B*c)/d/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.43

$$\int\frac{x^5(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx$$
$$=\left[\frac{(Ba-Ab)d\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+2\,(Bbc-Bad)x^2+(Bbc-Abd)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right.}{4\,(b^2cd-abd^2)}\right.$$

input
```
integrate(x^5*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/4*((B*a - A*b)*d*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4
 + a)) + 2*(B*b*c - B*a*d)*x^2 + (B*b*c - A*b*d)*sqrt(-c/d)*log((d*x^4 - 2
*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b^2*c*d - a*b*d^2), 1/4*(2*(B*a - A*
b)*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 2*(B*b*c - B*a*d)*x^2 + (B*b*c
- A*b*d)*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b^
2*c*d - a*b*d^2), 1/4*((B*a - A*b)*d*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(
-a/b) - a)/(b*x^4 + a)) + 2*(B*b*c - B*a*d)*x^2 - 2*(B*b*c - A*b*d)*sqrt(c
/d)*arctan(d*x^2*sqrt(c/d)/c))/(b^2*c*d - a*b*d^2), 1/2*((B*a - A*b)*d*sqr
t(a/b)*arctan(b*x^2*sqrt(a/b)/a) + (B*b*c - B*a*d)*x^2 - (B*b*c - A*b*d)*s
qrt(c/d)*arctan(d*x^2*sqrt(c/d)/c))/(b^2*c*d - a*b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^2}{2\,bd} + \frac{(Ba^2-Aab)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^2c-abd)\sqrt{ab}} - \frac{(Bc^2-Acd)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd-ad^2)\sqrt{cd}}$$

input
```
integrate(x^5*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*B*x^2/(b*d) + 1/2*(B*a^2 - A*a*b)*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*
b*d)*sqrt(a*b)) - 1/2*(B*c^2 - A*c*d)*arctan(d*x^2/sqrt(c*d))/((b*c*d - a*
d^2)*sqrt(c*d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^2}{2\,bd} + \frac{(Ba^2-Aab)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(b^2c-abd)\sqrt{ab}} - \frac{(Bc^2-Acd)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bcd-ad^2)\sqrt{cd}}$$

input
```
integrate(x^5*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*B*x^2/(b*d) + 1/2*(B*a^2 - A*a*b)*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*
b*d)*sqrt(a*b)) - 1/2*(B*c^2 - A*c*d)*arctan(d*x^2/sqrt(c*d))/((b*c*d - a*
d^2)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 15787, normalized size of antiderivative = 144.83

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^5*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(atan((((A*d - B*c)*(-c*d^3)^(1/2)*(((A*d - B*c)*(-c*d^3)^(1/2)*((16*(B^4*
a*b^8*c^9 + B^4*a^9*c*d^8 + 2*A^4*a^3*b^6*c^3*d^6 + 2*B^4*a^5*b^4*c^5*d^4
+ A^4*a*b^8*c^5*d^4 + A^4*a^5*b^4*c*d^8 - 4*A*B^3*a^4*b^5*c^5*d^4 - 4*A*B^
3*a^5*b^4*c^4*d^5 + 6*A^2*B^2*a*b^8*c^7*d^2 + 6*A^2*B^2*a^7*b^2*c*d^8 - 4*
A^3*B*a^3*b^6*c^4*d^5 - 4*A^3*B*a^4*b^5*c^3*d^6 - 4*A*B^3*a*b^8*c^8*d - 4*
A*B^3*a^8*b*c*d^8 + 2*A^2*B^2*a^3*b^6*c^5*d^4 + 8*A^2*B^2*a^4*b^5*c^4*d^5
+ 2*A^2*B^2*a^5*b^4*c^3*d^6 - 4*A^3*B*a*b^8*c^6*d^3 - 4*A^3*B*a^6*b^3*c*d^
8))/(b^2*d^2) + ((A*d - B*c)*(((A*d - B*c)*((16*(32*B^2*a^4*b^7*c^5*d^6 -
32*A^2*a^5*b^6*c^2*d^9 - 32*B^2*a^2*b^9*c^7*d^4 - 32*B^2*a^3*b^8*c^6*d^5 -
 32*A^2*a^2*b^9*c^5*d^6 + 32*B^2*a^5*b^6*c^4*d^7 - 32*B^2*a^6*b^5*c^3*d^8
- 32*B^2*a^7*b^4*c^2*d^9 + 32*A^2*a*b^10*c^6*d^5 + 32*A^2*a^6*b^5*c*d^10 +
 32*B^2*a*b^10*c^8*d^3 + 32*B^2*a^8*b^3*c*d^10 - 64*A*B*a*b^10*c^7*d^4 - 6
4*A*B*a^7*b^4*c*d^10 + 64*A*B*a^2*b^9*c^6*d^5 + 64*A*B*a^3*b^8*c^5*d^6 - 1
28*A*B*a^4*b^7*c^4*d^7 + 64*A*B*a^5*b^6*c^3*d^8 + 64*A*B*a^6*b^5*c^2*d^9))
/(b^2*d^2) + ((A*d - B*c)*((4*x^2*(256*A*a*b^11*c^6*d^6 + 256*A*a^6*b^6*c*
d^11 - 768*A*a^2*b^10*c^5*d^7 + 512*A*a^3*b^9*c^4*d^8 + 512*A*a^4*b^8*c^3*
d^9 - 768*A*a^5*b^7*c^2*d^10))/(b^2*d^2) - (4*(A*d - B*c)*(-c*d^3)^(1/2)*(
512*a^2*b^11*c^6*d^7 - 256*a^7*b^6*c*d^12 - 256*a*b^12*c^7*d^6 + 256*a^3*b
^10*c^5*d^8 - 1024*a^4*b^9*c^4*d^9 + 256*a^5*b^8*c^3*d^10 + 512*a^6*b^7*c^
2*d^11))/(b^2*d^2*(a*d^4 - b*c*d^3)))*(-c*d^3)^(1/2))/(4*(a*d^4 - b*c*d...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x^5(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+d\,x^2}{2d^2}$$

input
```
int(x^5*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*
c**(1/4)*sqrt(2))) + sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*s
qrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) + d*x**2)/(2*d**2)
```

3.8 $\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)} + \frac{(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)}$$

output
```
1/2*(A*b-B*a)*arctan(b^(1/2)*x^2/a^(1/2))/a^(1/2)/b^(1/2)/(-a*d+b*c)+1/2*(
-A*d+B*c)*arctan(d^(1/2)*x^2/c^(1/2))/c^(1/2)/d^(1/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{(-Ab+aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(-bc+ad)} + \frac{(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)}$$

input
```
Integrate[(x*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*Sqrt[b]*(-(b*c)
+ a*d)) + ((B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*Sqrt[d]*(
b*c - a*d))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1045, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1045}$$

$$\frac{1}{2}\int \frac{Bx^4+A}{(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{(Ab-aB)\int \frac{1}{bx^4+a}dx^2}{bc-ad}+\frac{(Bc-Ad)\int \frac{1}{dx^4+c}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(bc-ad)}+\frac{(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(bc-ad)}\right)$$

input
```
Int[(x*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(((A*b - a*B)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(b*c - a*d))
 + ((B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]*(b*c - a*d
)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 1045
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Si
mp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
 + f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p,
q, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

method	result
default	$-\frac{(Ab-Ba)\arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2(ad-cb)\sqrt{ab}}+\frac{(Ad-Bc)\arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2(ad-cb)\sqrt{cd}}$
risch	$-\frac{\ln\left(-(-cd)^{\frac{3}{2}}x^{2}-c^{2}d\right)Ad}{4\sqrt{-cd}\,(ad-cb)}+\frac{\ln\left(-(-cd)^{\frac{3}{2}}x^{2}-c^{2}d\right)Bc}{4\sqrt{-cd}\,(ad-cb)}+\frac{\ln\left(-(-cd)^{\frac{3}{2}}x^{2}+c^{2}d\right)Ad}{4\sqrt{-cd}\,(ad-cb)}-\frac{\ln\left(-(-cd)^{\frac{3}{2}}x^{2}+c^{2}d\right)Bc}{4\sqrt{-cd}\,(ad-cb)}-\frac{\ln\left(-(-c\right.}{4\sqrt{-}}$

input
```
int(x*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(A*b-B*a)/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))+1/2*(A*d-B*
c)/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 421, normalized size of antiderivative = 4.43

$$\int \frac{x(A + Bx^4)}{(a + bx^4)(c + dx^4)} dx$$
$$= \left[-\frac{(Ba - Ab)\sqrt{-ab}cd \log\left(\frac{bx^4 + 2\sqrt{-ab}x^2 - a}{bx^4 + a}\right) + (Babc - Aabd)\sqrt{-cd}\log\left(\frac{dx^4 - 2\sqrt{-cd}x^2 - c}{dx^4 + c}\right)}{4(ab^2c^2d - a^2bcd^2)}, \right.$$
$$-\frac{(Ba - Ab)\sqrt{-ab}cd \log\left(\frac{bx^4 + 2\sqrt{-ab}x^2 - a}{bx^4 + a}\right) + 2(Babc - Aabd)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}}{dx^2}\right)}{4(ab^2c^2d - a^2bcd^2)}, \frac{2(Ba - Ab)\sqrt{abcd}\,\mathrm{arc}$$

input
```
integrate(x*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[-1/4*((B*a - A*b)*sqrt(-a*b)*c*d*log((b*x^4 + 2*sqrt(-a*b)*x^2 - a)/(b*x^
4 + a)) + (B*a*b*c - A*a*b*d)*sqrt(-c*d)*log((d*x^4 - 2*sqrt(-c*d)*x^2 - c
)/(d*x^4 + c)))/(a*b^2*c^2*d - a^2*b*c*d^2), -1/4*((B*a - A*b)*sqrt(-a*b)*
c*d*log((b*x^4 + 2*sqrt(-a*b)*x^2 - a)/(b*x^4 + a)) + 2*(B*a*b*c - A*a*b*d
)*sqrt(c*d)*arctan(sqrt(c*d)/(d*x^2)))/(a*b^2*c^2*d - a^2*b*c*d^2), 1/4*(2
*(B*a - A*b)*sqrt(a*b)*c*d*arctan(sqrt(a*b)/(b*x^2)) - (B*a*b*c - A*a*b*d)
*sqrt(-c*d)*log((d*x^4 - 2*sqrt(-c*d)*x^2 - c)/(d*x^4 + c)))/(a*b^2*c^2*d
- a^2*b*c*d^2), 1/2*((B*a - A*b)*sqrt(a*b)*c*d*arctan(sqrt(a*b)/(b*x^2)) -
 (B*a*b*c - A*a*b*d)*sqrt(c*d)*arctan(sqrt(c*d)/(d*x^2)))/(a*b^2*c^2*d - a
^2*b*c*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A + Bx^4)}{(a + bx^4)(c + dx^4)} dx = \text{Timed out}$$

input
```
integrate(x*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba-Ab)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{(Bc-Ad)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

input
```
integrate(x*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*(B*
c - A*d)*arctan(d*x^2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ba-Ab)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}(bc-ad)} + \frac{(Bc-Ad)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

input
```
integrate(x*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(B*a - A*b)*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*(B*
c - A*d)*arctan(d*x^2/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 1201, normalized size of antiderivative = 12.64

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(B*a*c*d*atan((A^2*a^2*b^4*d^2*x^2*1i + B^2*a^2*b^4*c^2*x^2*1i - B^2*a^3*b
^3*c*d*x^2*1i - A^2*a*b^5*c*d*x^2*1i)/(B^2*a^3*d^2*(-a*b)^(3/2) + A^2*b^3*
c*d*(-a*b)^(3/2) + A^2*a*b^2*d^2*(-a*b)^(3/2) + B^2*a^4*b*d^2*(-a*b)^(1/2)
 + 2*A^2*a^2*b^3*d^2*(-a*b)^(1/2) + B^2*a^2*b^3*c^2*(-a*b)^(1/2) - 2*A*B*a
^2*b*d^2*(-a*b)^(3/2) + B^2*a^2*b*c*d*(-a*b)^(3/2) - 2*A*B*a^3*b^2*d^2*(-a
*b)^(1/2) - 2*A*B*a*b^2*c*d*(-a*b)^(3/2) - 2*A*B*a^2*b^3*c*d*(-a*b)^(1/2))
)*(-a*b)^(1/2)*1i)/(2*a*b^2*c^2*d - 2*a^2*b*c*d^2) - (A*b*c*d*atan((A^2*a^
2*b^4*d^2*x^2*1i + B^2*a^2*b^4*c^2*x^2*1i - B^2*a^3*b^3*c*d*x^2*1i - A^2*a
*b^5*c*d*x^2*1i)/(B^2*a^3*d^2*(-a*b)^(3/2) + A^2*b^3*c*d*(-a*b)^(3/2) + A^
2*a*b^2*d^2*(-a*b)^(3/2) + B^2*a^4*b*d^2*(-a*b)^(1/2) + 2*A^2*a^2*b^3*d^2*
(-a*b)^(1/2) + B^2*a^2*b^3*c^2*(-a*b)^(1/2) - 2*A*B*a^2*b*d^2*(-a*b)^(3/2)
 + B^2*a^2*b*c*d*(-a*b)^(3/2) - 2*A*B*a^3*b^2*d^2*(-a*b)^(1/2) - 2*A*B*a*b
^2*c*d*(-a*b)^(3/2) - 2*A*B*a^2*b^3*c*d*(-a*b)^(1/2)))*(-a*b)^(1/2)*1i)/(2
*a*b^2*c^2*d - 2*a^2*b*c*d^2) + (A*a*b*d*atan((A^2*b^2*c^2*d^4*x^2*1i + B^
2*a^2*c^2*d^4*x^2*1i - B^2*a*b*c^3*d^3*x^2*1i - A^2*a*b*c*d^5*x^2*1i)/(B^2
*b^2*c^3*(-c*d)^(3/2) + A^2*a*b*d^3*(-c*d)^(3/2) + A^2*b^2*c*d^2*(-c*d)^(3
/2) + B^2*b^2*c^4*d*(-c*d)^(1/2) + 2*A^2*b^2*c^2*d^3*(-c*d)^(1/2) + B^2*a^
2*c^2*d^3*(-c*d)^(1/2) - 2*A*B*b^2*c^2*d*(-c*d)^(3/2) + B^2*a*b*c^2*d*(-c*
d)^(3/2) - 2*A*B*b^2*c^3*d^2*(-c*d)^(1/2) - 2*A*B*a*b*c*d^2*(-c*d)^(3/2) -
 2*A*B*a*b*c^2*d^3*(-c*d)^(1/2)))*(-c*d)^(1/2)*1i)/(2*a*b^2*c^2*d - 2*a...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int \frac{x(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\sqrt{d}\,\sqrt{c}\left(atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)\right)}{2cd}$$

input
```
int(x*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - sqrt(d)*sqrt(c)*(atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1
/4)*c**(1/4)*sqrt(2))) + atan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d
**(1/4)*c**(1/4)*sqrt(2)))))/(2*c*d)
```

3.9 $\int \frac{A+Bx^4}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 109

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{A}{2acx^2}-\frac{\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}-\frac{\sqrt{d}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

output
```
-1/2*A/a/c/x^2-1/2*b^(1/2)*(A*b-B*a)*arctan(b^(1/2)*x^2/a^(1/2))/a^(3/2)/(
-a*d+b*c)-1/2*d^(1/2)*(-A*d+B*c)*arctan(d^(1/2)*x^2/c^(1/2))/c^(3/2)/(-a*d
+b*c)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\frac{-\sqrt{b}(Ab-aB)c^{3/2}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-\sqrt{b}(Ab-aB)c^{3/2}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{a}\Big(A\sqrt{c}(bc}{2a^{3/2}c^{3/2}(-bc}$$

input
```
Integrate[(A + B*x^4)/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(Sqrt[b]*(A*b - a*B)*c^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
) - Sqrt[b]*(A*b - a*B)*c^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] + Sqrt[a]*(A*Sqrt[c]*(b*c - a*d) + a*Sqrt[d]*(-(B*c) + A*d)*x^2*ArcTan[1
 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + a*Sqrt[d]*(-(B*c) + A*d)*x^2*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)]))/(2*a^(3/2)*c^(3/2)*(-(b*c) + a*d)*x^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1045, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^3(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1045}$$

$$\frac{1}{2}\int \frac{Bx^4+A}{x^4(bx^4+a)(dx^4+c)}dx^2$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2}\left(-\frac{\int \frac{Abdx^4+Abc-aBc+aAd}{(bx^4+a)(dx^4+c)}dx^2}{ac}-\frac{A}{acx^2}\right)$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(-\frac{\frac{bc(Ab-aB)\int \frac{1}{bx^4+a}dx^2}{bc-ad}+\frac{ad(Bc-Ad)\int \frac{1}{dx^4+c}dx^2}{bc-ad}}{ac}-\frac{A}{acx^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{\frac{\sqrt{b}c(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}+\frac{a\sqrt{d}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{A}{acx^2}\right)$$

input
```
Int[(A + B*x^4)/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-(A/(a*c*x^2)) - ((Sqrt[b]*(A*b - a*B)*c*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(
Sqrt[a]*(b*c - a*d)) + (a*Sqrt[d]*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(Sqrt[c]*(b*c - a*d)))/(a*c))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 1045
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Si
mp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
 + f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p,
q, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{(Ab-Ba)b\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2a(ad-cb)\sqrt{ab}}-\frac{A}{2ac\,x^2}-\frac{(Ad-Bc)d\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2c(ad-cb)\sqrt{cd}}$	94
risch	Expression too large to display	1282

input
```
int((B*x^4+A)/x^3/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*b/a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/2*A/a/
c/x^2-1/2*(A*d-B*c)*d/c/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.43

$$\int \frac{A+Bx^4}{x^3\,(a+bx^4)\,(c+dx^4)}\,dx$$
$$=\left[\frac{(Ba-Ab)cx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+(Bac-Aad)x^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-2\,Abc+2\,A}{4\,(abc^2-a^2cd)x^2}\right.$$

input
```
integrate((B*x^4+A)/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/4*((B*a - A*b)*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b
*x^4 + a)) + (B*a*c - A*a*d)*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c
) - c)/(d*x^4 + c)) - 2*A*b*c + 2*A*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*((
B*a - A*b)*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
a)) - 2*(B*a*c - A*a*d)*x^2*sqrt(d/c)*arctan(x^2*sqrt(d/c)) - 2*A*b*c + 2*
A*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*(B*a - A*b)*c*x^2*sqrt(b/a)*arcta
n(x^2*sqrt(b/a)) + (B*a*c - A*a*d)*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqr
t(-d/c) - c)/(d*x^4 + c)) - 2*A*b*c + 2*A*a*d)/((a*b*c^2 - a^2*c*d)*x^2),
1/2*((B*a - A*b)*c*x^2*sqrt(b/a)*arctan(x^2*sqrt(b/a)) - (B*a*c - A*a*d)*x
^2*sqrt(d/c)*arctan(x^2*sqrt(d/c)) - A*b*c + A*a*d)/((a*b*c^2 - a^2*c*d)*x
^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**4+A)/x**3/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\left(Bab-Ab^2\right)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(abc-a^2d\right)\sqrt{ab}} - \frac{\left(Bcd-Ad^2\right)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^2-acd\right)\sqrt{cd}} - \frac{A}{2\,acx^2}$$

input
```
integrate((B*x^4+A)/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/2*(B*a*b - A*b^2)*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) -
1/2*(B*c*d - A*d^2)*arctan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) -
1/2*A/(a*c*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\left(Bab-Ab^2\right)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(abc-a^2d\right)\sqrt{ab}} - \frac{\left(Bcd-Ad^2\right)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^2-acd\right)\sqrt{cd}} - \frac{A}{2\,acx^2}$$

input
```
integrate((B*x^4+A)/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*(B*a*b - A*b^2)*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) -
1/2*(B*c*d - A*d^2)*arctan(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) -
1/2*A/(a*c*x^2)
```

Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 1537, normalized size of antiderivative = 14.10

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (atan((A^4*a^2*d^5*x^2*(-a^3*b)^(7/2)*1i + A^4*b^6*c^5*x^2*(-a^3*b)^(5/2
)*1i - B^4*a*b*c^5*x^2*(-a^3*b)^(7/2)*1i + A*B^3*b^2*c^5*x^2*(-a^3*b)^(7/2
)*4i + B^4*a^2*c^4*d*x^2*(-a^3*b)^(7/2)*1i + A^4*b^2*c^2*d^3*x^2*(-a^3*b)^
(7/2)*1i + A^4*a*b^5*c^4*d*x^2*(-a^3*b)^(5/2)*1i - A*B^3*a^2*c^3*d^2*x^2*(
-a^3*b)^(7/2)*4i - A^2*B^2*b^2*c^4*d*x^2*(-a^3*b)^(7/2)*4i + A^2*B^2*a^2*b
^4*c^5*x^2*(-a^3*b)^(5/2)*6i + A^2*B^2*a^2*c^2*d^3*x^2*(-a^3*b)^(7/2)*6i +
 A^4*a*b*c*d^4*x^2*(-a^3*b)^(7/2)*1i + A^4*a^2*b^4*c^3*d^2*x^2*(-a^3*b)^(5
/2)*1i - A^3*B*a*b^5*c^5*x^2*(-a^3*b)^(5/2)*4i - A^3*B*a^2*c*d^4*x^2*(-a^3
*b)^(7/2)*4i - A^3*B*a*b*c^2*d^3*x^2*(-a^3*b)^(7/2)*4i + A^2*B^2*a*b*c^3*d
^2*x^2*(-a^3*b)^(7/2)*4i - A^3*B*a^2*b^4*c^4*d*x^2*(-a^3*b)^(5/2)*4i)/(a^6
*b^2*(a^3*b*(A^4*a^4*d^5 - a^3*b*(B^4*c^5 - A^4*c*d^4 - 4*A^2*B^2*c^3*d^2
+ 4*A^3*B*c^2*d^3) + 4*A^3*B*b^4*c^5 - A^4*b^4*c^4*d + B^4*a^4*c^4*d + A^4
*a^2*b^2*c^2*d^3 - 4*A^3*B*a^4*c*d^4 + 4*A*B^3*a^2*b^2*c^5 - 6*A^2*B^2*a*b
^3*c^5 - 4*A*B^3*a^4*c^3*d^2 - A^4*a*b^3*c^3*d^2 + 6*A^2*B^2*a^4*c^2*d^3 -
 4*A^2*B^2*a^2*b^2*c^4*d + 4*A^3*B*a*b^3*c^4*d) - A^4*a^2*b^6*c^5)))*(A*b
- B*a)*(-a^3*b)^(1/2)*1i)/(2*(a^4*d - a^3*b*c)) - (atan((A^4*b^5*c^2*x^2*(
-c^3*d)^(7/2)*1i + A^4*a^5*d^6*x^2*(-c^3*d)^(5/2)*1i + A^4*a^2*b^3*d^2*x^2
*(-c^3*d)^(7/2)*1i - B^4*a^5*c*d*x^2*(-c^3*d)^(7/2)*1i + A*B^3*a^5*d^2*x^2
*(-c^3*d)^(7/2)*4i + B^4*a^4*b*c^2*x^2*(-c^3*d)^(7/2)*1i + A^4*a^4*b*c*d^5
*x^2*(-c^3*d)^(5/2)*1i - A*B^3*a^3*b^2*c^2*x^2*(-c^3*d)^(7/2)*4i - A^2*...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^4}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^2+\sqrt{d}\,\sqrt{c}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^2-c}{2c^2x^2}$$

input
```
int((B*x^4+A)/x^3/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(1/4)*
c**(1/4)*sqrt(2)))*x**2 + sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2)
+ 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*x**2 - c)/(2*c**2*x**2)
```

3.10 $\int \frac{A+Bx^4}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 136

$$\int \frac{A+Bx^4}{x^7(a+bx^4)(c+dx^4)}\,dx = -\frac{A}{6acx^6} + \frac{Abc-aBc+aAd}{2a^2c^2x^2} + \frac{b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)} + \frac{d^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}$$

output
```
-1/6*A/a/c/x^6+1/2*(A*a*d+A*b*c-B*a*c)/a^2/c^2/x^2+1/2*b^(3/2)*(A*b-B*a)*a
rctan(b^(1/2)*x^2/a^(1/2))/a^(5/2)/(-a*d+b*c)+1/2*d^(3/2)*(-A*d+B*c)*arcta
n(d^(1/2)*x^2/c^(1/2))/c^(5/2)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx^4}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$= \frac{3b^{3/2}(Ab-aB)c^{5/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+3b^{3/2}(Ab-aB)c^{5/2}x^6\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+\sqrt{a}\Big(-\sqrt{c}(}{}$$

input
```
Integrate[(A + B*x^4)/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(3*b^(3/2)*(A*b - a*B)*c^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 + 3*b^(3/2)*(A*b - a*B)*c^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4
)] + Sqrt[a]*(-(Sqrt[c]*(-(b*c) + a*d)*(-3*A*b*c*x^4 + 3*a*B*c*x^4 + a*A*(
c - 3*d*x^4))) + 3*a^2*d^(3/2)*(B*c - A*d)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)
*x)/c^(1/4)] + 3*a^2*d^(3/2)*(B*c - A*d)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)]))/(6*a^(5/2)*c^(5/2)*(-(b*c) + a*d)*x^6)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1045, 445, 27, 445, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

$$\Big\downarrow \text{1045}$$

$$\frac{1}{2}\int \frac{Bx^4+A}{x^8\left(bx^4+a\right)\left(dx^4+c\right)}dx^2$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{2}\left(-\frac{\int \frac{3\left(Abdx^4+Abc-aBc+aAd\right)}{x^4(bx^4+a)(dx^4+c)}dx^2}{3ac}-\frac{A}{3acx^6}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\int \frac{Abdx^4+Abc-aBc+aAd}{x^4(bx^4+a)(dx^4+c)}dx^2}{ac}-\frac{A}{3acx^6}\right)$$

$\Big\downarrow$ 445

$$\frac{1}{2}\left(-\frac{-\frac{\int -\frac{-bd(Abc-aBc+aAd)x^4+aBc(bc+ad)-A\left(b^2c^2+abdc+a^2d^2\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx^2}{ac}-\frac{aAd-aBc+Abc}{acx^2}}{ac}-\frac{A}{3acx^6}\right)$$

$\Big\downarrow$ 25

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{-bd(Abc-aBc+aAd)x^4+aBc(bc+ad)-A\left(b^2c^2+abdc+a^2d^2\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx^2}{ac}-\frac{aAd-aBc+Abc}{acx^2}}{ac}-\frac{A}{3acx^6}\right)$$

$\Big\downarrow$ 397

$$\frac{1}{2}\left(-\frac{\frac{-\frac{a^2d^2(Bc-Ad)\int \frac{1}{dx^4+c}dx^2}{bc-ad}-\frac{b^2c^2(Ab-aB)\int \frac{1}{bx^4+a}dx^2}{bc-ad}}{ac}-\frac{aAd-aBc+Abc}{acx^2}}{ac}-\frac{A}{3acx^6}\right)$$

$\Big\downarrow$ 218

$$\frac{1}{2}\left(-\frac{\frac{-\frac{a^2d^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}-\frac{b^{3/2}c^2(Ab-aB)\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}}{ac}-\frac{aAd-aBc+Abc}{acx^2}}{ac}-\frac{A}{3acx^6}\right)$$

input
```
Int[(A + B*x^4)/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-1/3*A/(a*c*x^6) - (-((A*b*c - a*B*c + a*A*d)/(a*c*x^2)) + (-((b^(3/2)*(A
*b - a*B)*c^2*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d))) - (a^2
*d^(3/2)*(B*c - A*d)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))
/(a*c))/(a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 1045
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Si
mp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q*(e
 + f*x^(n/k))^r, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, f, p,
q, r}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{b^2(Ab-Ba)\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2a^2(ad-cb)\sqrt{ab}}-\frac{A}{6ac\,x^6}-\frac{-Aad-Abc+aBc}{2a^2c^2x^2}+\frac{d^2(Ad-Bc)\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2c^2(ad-cb)\sqrt{cd}}$	124
risch	Expression too large to display	1453

input
```
int((B*x^4+A)/x^7/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*(A*b-B*a)/a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x^2/(a*b)^(1/2))-1/6
*A/a/c/x^6-1/2/a^2/c^2*(-A*a*d-A*b*c+B*a*c)/x^2+1/2*d^2*(A*d-B*c)/c^2/(a*d
-b*c)/(c*d)^(1/2)*arctan(d*x^2/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 88.73 (sec) , antiderivative size = 728, normalized size of antiderivative = 5.35

$$\int \frac{A+Bx^4}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \left[\frac{3\,(Bab-Ab^2)c^2x^6\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+3\,(Ba^2cd-Aa^2d^2)x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-2}{12\,(a^2bc^3-a^3c^2d)x^6}\right.$$

$$-\frac{6\,(Bab-Ab^2)c^2x^6\sqrt{\frac{b}{a}}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)-3\,(Ba^2cd-Aa^2d^2)x^6\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)+2\,Aabc^2}{12\,(a^2bc^3-a^3c^2d)x^6}$$

$$-\frac{3\,(Bab-Ab^2)c^2x^6\sqrt{\frac{b}{a}}\arctan\left(x^2\sqrt{\frac{b}{a}}\right)-3\,(Ba^2cd-Aa^2d^2)x^6\sqrt{\frac{d}{c}}\arctan\left(x^2\sqrt{\frac{d}{c}}\right)+Aabc^2-Aa^2cd}{6\,(a^2bc^3-a^3c^2d)x^6}$$

input
```
integrate((B*x^4+A)/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
[1/12*(3*(B*a*b - A*b^2)*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a
) - a)/(b*x^4 + a)) + 3*(B*a^2*c*d - A*a^2*d^2)*x^6*sqrt(-d/c)*log((d*x^4
+ 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*A*a*b*c^2 + 2*A*a^2*c*d + 6*(B*
a^2*c*d - A*a^2*d^2 - (B*a*b - A*b^2)*c^2)*x^4)/((a^2*b*c^3 - a^3*c^2*d)*x
^6), 1/12*(3*(B*a*b - A*b^2)*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(
-b/a) - a)/(b*x^4 + a)) + 6*(B*a^2*c*d - A*a^2*d^2)*x^6*sqrt(d/c)*arctan(x
^2*sqrt(d/c)) - 2*A*a*b*c^2 + 2*A*a^2*c*d + 6*(B*a^2*c*d - A*a^2*d^2 - (B*
a*b - A*b^2)*c^2)*x^4)/((a^2*b*c^3 - a^3*c^2*d)*x^6), -1/12*(6*(B*a*b - A*
b^2)*c^2*x^6*sqrt(b/a)*arctan(x^2*sqrt(b/a)) - 3*(B*a^2*c*d - A*a^2*d^2)*x
^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2*A*a*b*
c^2 - 2*A*a^2*c*d - 6*(B*a^2*c*d - A*a^2*d^2 - (B*a*b - A*b^2)*c^2)*x^4)/(
(a^2*b*c^3 - a^3*c^2*d)*x^6), -1/6*(3*(B*a*b - A*b^2)*c^2*x^6*sqrt(b/a)*ar
ctan(x^2*sqrt(b/a)) - 3*(B*a^2*c*d - A*a^2*d^2)*x^6*sqrt(d/c)*arctan(x^2*s
qrt(d/c)) + A*a*b*c^2 - A*a^2*c*d - 3*(B*a^2*c*d - A*a^2*d^2 - (B*a*b - A*
b^2)*c^2)*x^4)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**4+A)/x**7/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^4}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=-\frac{\left(Bab^2-Ab^3\right)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{ab}}+\frac{\left(Bcd^2-Ad^3\right)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3-ac^2d\right)\sqrt{cd}}+\frac{3\left(Aad-(Ba-Ab)c\right)x^4-Aac}{6\,a^2c^2x^6}$$

input
```
integrate((B*x^4+A)/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/2*(B*a*b^2 - A*b^3)*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b
)) + 1/2*(B*c*d^2 - A*d^3)*arctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt
(c*d)) + 1/6*(3*(A*a*d - (B*a - A*b)*c)*x^4 - A*a*c)/(a^2*c^2*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^4}{x^7(a+bx^4)(c+dx^4)}\,dx = -\frac{(Bab^2-Ab^3)\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2(a^2bc-a^3d)\sqrt{ab}} + \frac{(Bcd^2-Ad^3)\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2(bc^3-ac^2d)\sqrt{cd}} - \frac{3\,Bacx^4-3\,Abcx^4-3\,Aadx^4+Aac}{6\,a^2c^2x^6}$$

input
```
integrate((B*x^4+A)/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*(B*a*b^2 - A*b^3)*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b
)) + 1/2*(B*c*d^2 - A*d^3)*arctan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt
(c*d)) - 1/6*(3*B*a*c*x^4 - 3*A*b*c*x^4 - 3*A*a*d*x^4 + A*a*c)/(a^2*c^2*x^
6)
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 15821, normalized size of antiderivative = 116.33

$$\int \frac{A+Bx^4}{x^7(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
(atan((((x^2*(4*A^5*a^15*b^13*c^19*d^9 + 4*A^5*a^19*b^9*c^15*d^13 + 4*A*B^
4*a^18*b^10*c^20*d^8 + 4*A*B^4*a^20*b^8*c^18*d^10 - 4*A^4*B*a^15*b^13*c^20
*d^8 - 12*A^4*B*a^16*b^12*c^19*d^9 - 12*A^4*B*a^19*b^9*c^16*d^12 - 4*A^4*B
*a^20*b^8*c^15*d^13 - 12*A^2*B^3*a^17*b^11*c^20*d^8 - 4*A^2*B^3*a^18*b^10*
c^19*d^9 - 4*A^2*B^3*a^19*b^9*c^18*d^10 - 12*A^2*B^3*a^20*b^8*c^17*d^11 +
12*A^3*B^2*a^16*b^12*c^20*d^8 + 12*A^3*B^2*a^17*b^11*c^19*d^9 + 12*A^3*B^2
*a^19*b^9*c^17*d^11 + 12*A^3*B^2*a^20*b^8*c^16*d^12) + ((A*d - B*c)*(-c^5*
d^3)^(1/2)*(((x^2*(64*A^3*a^17*b^13*c^24*d^6 - 64*A^3*a^16*b^14*c^25*d^5 +
 64*A^3*a^19*b^11*c^22*d^8 - 64*A^3*a^20*b^10*c^21*d^9 - 64*A^3*a^21*b^9*c
^20*d^10 + 64*A^3*a^22*b^8*c^19*d^11 + 64*A^3*a^24*b^6*c^17*d^13 - 64*A^3*
a^25*b^5*c^16*d^14 + 64*B^3*a^19*b^11*c^25*d^5 - 64*B^3*a^20*b^10*c^24*d^6
 + 64*B^3*a^21*b^9*c^23*d^7 - 128*B^3*a^22*b^8*c^22*d^8 + 64*B^3*a^23*b^7*
c^21*d^9 - 64*B^3*a^24*b^6*c^20*d^10 + 64*B^3*a^25*b^5*c^19*d^11 - 192*A*B
^2*a^18*b^12*c^25*d^5 + 192*A*B^2*a^19*b^11*c^24*d^6 + 192*A*B^2*a^24*b^6*
c^19*d^11 - 192*A*B^2*a^25*b^5*c^18*d^12 + 192*A^2*B*a^17*b^13*c^25*d^5 -
192*A^2*B*a^18*b^12*c^24*d^6 - 64*A^2*B*a^19*b^11*c^23*d^7 + 64*A^2*B*a^20
*b^10*c^22*d^8 + 64*A^2*B*a^22*b^8*c^20*d^10 - 64*A^2*B*a^23*b^7*c^19*d^11
 - 192*A^2*B*a^24*b^6*c^18*d^12 + 192*A^2*B*a^25*b^5*c^17*d^13) + ((A*d -
B*c)*(-c^5*d^3)^(1/2)*(((A*d - B*c)*(-c^5*d^3)^(1/2)*(x^2*(2048*A*a^21*b^1
1*c^27*d^5 - 1024*A*a^20*b^12*c^28*d^4 - 2048*A*a^23*b^9*c^25*d^7 + 204...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^4}{x^7 (a+bx^4) (c+dx^4)} dx = \frac{-3\sqrt{d}\sqrt{c}\, atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\, x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right) d\, x^6 - 3\sqrt{d}\sqrt{c}\, atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\, x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right) d\, x^6 - c^2 + 3cd\, x^4}{6c^3x^6}$$

input
```
int((B*x^4+A)/x^7/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x)/(d**(
1/4)*c**(1/4)*sqrt(2)))*d*x**6 - 3*sqrt(d)*sqrt(c)*atan((d**(1/4)*c**(1/4)
*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*d*x**6 - c**2 + 3*c*d
*x**4)/(6*c**3*x**6)
```

3.11 $\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 412

$$\begin{aligned}\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx =& -\frac{(bBc-Abd+aBd)x}{b^2d^2}+\frac{Bx^5}{5bd}\\
&-\frac{a^{5/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{9/4}(bc-ad)}\\
&+\frac{a^{5/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{9/4}(bc-ad)}\\
&-\frac{c^{5/4}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{9/4}(bc-ad)}\\
&+\frac{c^{5/4}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{9/4}(bc-ad)}\\
&+\frac{a^{5/4}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{9/4}(bc-ad)}\\
&+\frac{c^{5/4}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}d^{9/4}(bc-ad)}\end{aligned}$$

output
```
-(-A*b*d+B*a*d+B*b*c)*x/b^2/d^2+1/5*B*x^5/b/d+1/4*a^(5/4)*(A*b-B*a)*arctan
(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(9/4)/(-a*d+b*c)+1/4*a^(5/4)*(A*b
-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/b^(9/4)/(-a*d+b*c)+1/4*c
^(5/4)*(-A*d+B*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(9/4)/(-a
*d+b*c)+1/4*c^(5/4)*(-A*d+B*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)
/d^(9/4)/(-a*d+b*c)+1/4*a^(5/4)*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*
x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/b^(9/4)/(-a*d+b*c)+1/4*c^(5/4)*(-A*d+B*c)
*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(9/4)/
(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.21

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-40\sqrt[4]{b}\sqrt[4]{d}(bc-ad)(bBc-Abd+aBd)x+8b^{5/4}Bd^{5/4}(bc-ad)x^5+10\sqrt{2}a^{5/4}(-Ab+aB)d^{9/4}\arctan\left(\right.}{}$$

input
```
Integrate[(x^8*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-40*b^(1/4)*d^(1/4)*(b*c - a*d)*(b*B*c - A*b*d + a*B*d)*x + 8*b^(5/4)*B*d
^(5/4)*(b*c - a*d)*x^5 + 10*Sqrt[2]*a^(5/4)*(-(A*b) + a*B)*d^(9/4)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*a^(5/4)*(A*b - a*B)*d^(9/4)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 10*Sqrt[2]*b^(9/4)*c^(5/4)*(B*c
- A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*b^(9/4)*c^(5/4
)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 5*Sqrt[2]*a^(5/4)*
(-(A*b) + a*B)*d^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2] + 5*Sqrt[2]*a^(5/4)*(A*b - a*B)*d^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2] - 5*Sqrt[2]*b^(9/4)*c^(5/4)*(B*c - A*d)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*b^(9/4)*c^(5/4)*
(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*b^
(9/4)*d^(9/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {1052, 27, 1052, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1052}$$

$$\frac{Bx^5}{5bd}-\frac{\int \frac{5x^4((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^5}{5bd}-\frac{\int \frac{x^4((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx}{bd}$$

$$\Big\downarrow \text{1052}$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\int \frac{\left(c(Bc-Ad)b^2+ad(Bc-Ad)b+a^2Bd^2\right)x^4+ac(bBc-Abd+aBd)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{bd}}{bd}$$

$$\Big\downarrow \text{1020}$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\int \frac{1}{bx^4+a}dx}{bc-ad}+\frac{b^2c^2(Bc-Ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{bd}}{bd}$$

$$\Big\downarrow \text{755}$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{b^2c^2(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{bd}}{bd}$$

$$\Big\downarrow \text{1476}$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{b^2c^2(Bc-Ad)\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}\cdots}{2\cdots}\cdots\right)}{\cdots}}{bd}}{bd}$$

$$\Big\downarrow \; 1082$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{b^2c^2(\cdots}{\cdots}}{bd}}{bd}$$

$$\Big\downarrow \; 217$$

$$\frac{Bx^5}{5bd}-\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{a^2d^2(Ab-aB)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{b^2c^2(Bc-Ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots}{\cdots}\right)}{bc-ad}}{bd}}{bd}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{Bx^5}{5bd}-\frac{a^2d^2(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{a}}\cdots\right)}{bc-ad}\cdots}{\cdots}}{b\cdots}$$

$\downarrow$ 25

$$\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{Bx^5}{5bd}-\frac{a^2d^2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\cdots\right)}{bc-ad}\cdots}{\cdots}}{bd}$$

$\downarrow$ 27

$$\frac{\frac{x(aBd-Abd+bBc)}{bd}-\frac{\frac{Bx^5}{5bd}-\frac{a^2d^2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{b^2c^2(Bc\cdots}{\cdots}}{bd}\cdots}{bd}$$

$\downarrow$ 1103

$$\frac{Bx^5}{5bd} - \frac{\frac{x(aBd-Abd+bBc)}{bd} - \frac{a^2d^2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}}{bd}$$

input
```
Int[(x^8*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(B*x^5)/(5*b*d) - (((b*B*c - A*b*d + a*B*d)*x)/(b*d) - ((a^2*(A*b - a*B)*d
^2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (b^2*c^2*(B*
c - A*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1
/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))
/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2
]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sq
rt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*d))/
(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.67

method	result
default	$\frac{\frac{1}{5}Bx^5bd+Abdx-Badx-Bbcx}{b^2d^2}-\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b^2(ad-cb)}$
risch	Expression too large to display

input
```
int(x^8*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d^2*(1/5*B*x^5*b*d+A*b*d*x-B*a*d*x-B*b*c*x)-1/8/b^2*a*(A*b-B*a)/(a*d
-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2
-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/d^2*c*(A*d-B*c)/(a*d-b*c)*(c/d)^(1/4)*
2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d
)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.15

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = -\frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,(b^3c-ab^2d)} + \frac{\left(\frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(Bc-Ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,(bcd^2-ad^3)} + \frac{Bbdx^5-5\,(Bbc+(Ba-Ab)d)x}{5\,b^2d^2}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1
/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sq
rt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a -
 A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1
/4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(1/4)))*a^2/(b^3*c - a*b^2*d) + 1/8*(2*sqrt(2)*(B*c - A
*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(B*c - A*d)*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(B*c - A*d)*log(sqrt(d)*x^2 +
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(B*c - A*
d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)
))*c^2/(b*c*d^2 - a*d^3) + 1/5*(B*b*d*x^5 - 5*(B*b*c + (B*a - A*b)*d)*x)/(
b^2*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(316) = 632$.

Time = 0.13 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.58

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2
*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) + 1/2*((c
*d^3)^(1/4)*B*c^2 - (c*d^3)^(1/4)*A*c*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)
*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*((c*d^3
)^(1/4)*B*c^2 - (c*d^3)^(1/4)*A*c*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/
d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) - 1/4*((a*b^3)^(1
/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) + 1/4*((a*b^3)^(1/4)*B*a^2 - (a*b^3)
^(1/4)*A*a*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c
- sqrt(2)*a*b^3*d) + 1/4*((c*d^3)^(1/4)*B*c^2 - (c*d^3)^(1/4)*A*c*d)*log(x
^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
- 1/4*((c*d^3)^(1/4)*B*c^2 - (c*d^3)^(1/4)*A*c*d)*log(x^2 - sqrt(2)*x*(c/d
)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/5*(B*b^4*d^4*x^
5 - 5*B*b^4*c*d^3*x - 5*B*a*b^3*d^4*x + 5*A*b^4*d^4*x)/(b^5*d^5)
```

Mupad [B] (verification not implemented)

Time = 14.10 (sec) , antiderivative size = 33507, normalized size of antiderivative = 81.33

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^8*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
x*(A/(b*d) - (B*(a*d + b*c))/(b^2*d^2)) + (atan(((((((-(B^4*a^9 + A^4*a^5
*b^4 + 6*A^2*B^2*a^7*b^2 - 4*A*B^3*a^8*b - 4*A^3*B*a^6*b^3)/(b^13*c^4 + a^
4*b^9*d^4 - 4*a^3*b^10*c*d^3 + 6*a^2*b^11*c^2*d^2 - 4*a*b^12*c^3*d))^(3/4)
*((4*x*(256*A^2*a^3*b^13*c^8*d^8 - 768*A^2*a^4*b^12*c^7*d^9 + 512*A^2*a^5*
b^11*c^6*d^10 + 512*A^2*a^6*b^10*c^5*d^11 - 768*A^2*a^7*b^9*c^4*d^12 + 256
*A^2*a^8*b^8*c^3*d^13 + 256*B^2*a^3*b^13*c^10*d^6 - 1024*B^2*a^4*b^12*c^9*
d^7 + 1536*B^2*a^5*b^11*c^8*d^8 - 768*B^2*a^6*b^10*c^7*d^9 - 768*B^2*a^7*b
^9*c^6*d^10 + 1536*B^2*a^8*b^8*c^5*d^11 - 1024*B^2*a^9*b^7*c^4*d^12 + 256*
B^2*a^10*b^6*c^3*d^13 - 512*A*B*a^3*b^13*c^9*d^7 + 2048*A*B*a^4*b^12*c^8*d
^8 - 3584*A*B*a^5*b^11*c^7*d^9 + 4096*A*B*a^6*b^10*c^6*d^10 - 3584*A*B*a^7
*b^9*c^5*d^11 + 2048*A*B*a^8*b^8*c^4*d^12 - 512*A*B*a^9*b^7*c^3*d^13))/(b^
5*d^5) - ((-(B^4*a^9 + A^4*a^5*b^4 + 6*A^2*B^2*a^7*b^2 - 4*A*B^3*a^8*b - 4
*A^3*B*a^6*b^3)/(b^13*c^4 + a^4*b^9*d^4 - 4*a^3*b^10*c*d^3 + 6*a^2*b^11*c^
2*d^2 - 4*a*b^12*c^3*d))^(1/4)*(256*B*a^3*b^14*c^9*d^8 - 1536*B*a^4*b^13*c
^8*d^9 + 3840*B*a^5*b^12*c^7*d^10 - 5120*B*a^6*b^11*c^6*d^11 + 3840*B*a^7*
b^10*c^5*d^12 - 1536*B*a^8*b^9*c^4*d^13 + 256*B*a^9*b^8*c^3*d^14)*4i)/(b^5
*d^5))*1i)/64 - (16*(B^5*a^4*b^9*c^13 + B^5*a^13*c^4*d^9 - A^5*a^3*b^10*c^
9*d^4 + A^5*a^4*b^9*c^8*d^5 + A^5*a^8*b^5*c^4*d^9 - A^5*a^9*b^4*c^3*d^10 -
 A*B^4*a^3*b^10*c^13 - A*B^4*a^13*c^3*d^10 - B^5*a^5*b^8*c^12*d - B^5*a^12
*b*c^5*d^8 + 4*A*B^4*a^5*b^8*c^11*d^2 + 4*A*B^4*a^11*b^2*c^5*d^8 + 4*A^...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.37

$$\int \frac{x^8(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{-10d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+10d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)-5d^{\frac{3}{4}}c^{\frac{5}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\right.}{40d^3}$$

input
```
int(x^8*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
( - 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*c + 10*d**(3/4)*c**(1/4)*sqrt(2)*atan((
d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*c -
5*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) +
 sqrt(d)*x**2)*c + 5*d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(
2)*x + sqrt(c) + sqrt(d)*x**2)*c - 40*c*d*x + 8*d**2*x**5)/(40*d**3)
```

3.12 $\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 389

$$\begin{aligned}\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx &= \frac{Bx^3}{3bd}+\frac{a^{3/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}(bc-ad)}\\
&\quad-\frac{a^{3/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{7/4}(bc-ad)}\\
&\quad+\frac{c^{3/4}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{7/4}(bc-ad)}\\
&\quad-\frac{c^{3/4}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{7/4}(bc-ad)}\\
&\quad+\frac{a^{3/4}(Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{7/4}(bc-ad)}\\
&\quad+\frac{c^{3/4}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}d^{7/4}(bc-ad)}\end{aligned}$$

output
```
1/3*B*x^3/b/d-1/4*a^(3/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2
^(1/2)/b^(7/4)/(-a*d+b*c)-1/4*a^(3/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x
/a^(1/4))*2^(1/2)/b^(7/4)/(-a*d+b*c)-1/4*c^(3/4)*(-A*d+B*c)*arctan(-1+2^(1
/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(7/4)/(-a*d+b*c)-1/4*c^(3/4)*(-A*d+B*c)*a
rctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(7/4)/(-a*d+b*c)+1/4*a^(3/4)*
(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)
/b^(7/4)/(-a*d+b*c)+1/4*c^(3/4)*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)
*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(7/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.20

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{8b^{3/4}Bd^{3/4}(bc-ad)x^3-6\sqrt{2}a^{3/4}(-Ab+aB)d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+6\sqrt{2}a^{3/4}(-Ab+aB)d^{7/4}\operatorname{arct}}{}$$

input
```
Integrate[(x^6*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(8*b^(3/4)*B*d^(3/4)*(b*c - a*d)*x^3 - 6*Sqrt[2]*a^(3/4)*(-(A*b) + a*B)*d^
(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*a^(3/4)*(-(A*b)
+ a*B)*d^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*b^(7/4)
*c^(3/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 6*Sqrt[2]*b
^(7/4)*c^(3/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 3*Sqr
t[2]*a^(3/4)*(-(A*b) + a*B)*d^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2] + 3*Sqrt[2]*a^(3/4)*(A*b - a*B)*d^(7/4)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - 3*Sqrt[2]*b^(7/4)*c^(3/4)*(B*c - A
*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 3*Sqrt[2]*b^(
7/4)*c^(3/4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(24*b^(7/4)*d^(7/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A + Bx^4)}{(a + bx^4)(c + dx^4)} dx$$

$$\Big\downarrow \text{ 1052}$$

$$\frac{Bx^3}{3bd} - \frac{\int \frac{3x^2((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx}{3bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bx^3}{3bd} - \frac{\int \frac{x^2((bBc-Abd+aBd)x^4+aBc)}{(bx^4+a)(dx^4+c)}dx}{bd}$$

$$\Big\downarrow \text{ 1054}$$

$$\frac{Bx^3}{3bd} - \frac{\int \left(\frac{a(Ab-aB)dx^2}{(bc-ad)(bx^4+a)} + \frac{bc(Bc-Ad)x^2}{(bc-ad)(dx^4+c)}\right)dx}{bd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bx^3}{3bd} - \frac{-\frac{a^{3/4}d(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)} + \frac{a^{3/4}d(Ab-aB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{3/4}(bc-ad)} + \frac{a^{3/4}d(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)} - \frac{a^{3/}}{}}{}$$

input
```
Int[(x^6*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(B*x^3)/(3*b*d) - (-1/2*(a^(3/4)*(A*b - a*B)*d*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*(A*b - a*B)*d*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) - (b*c
^(3/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*d^(
3/4)*(b*c - a*d)) + (b*c^(3/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/
c^(1/4)])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) + (a^(3/4)*(A*b - a*B)*d*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*(A*b - a*B)*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (b*c^(3/4)*(B*c - A*d)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b
*c - a*d)) - (b*c^(3/4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d)))/(b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65

method	result
default	$\frac{B\,x^{3}}{3bd}+\frac{a(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{c(Ad-Bc)\sqrt{2}\left(\ln\left(\frac{x^{2}-}{x^{2}+}\right.\right.}{}$
risch	Expression too large to display

input
```
int(x^6*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*B*x^3/b/d+1/8*a*(A*b-B*a)/(a*d-b*c)/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))
+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8*
c*(A*d-B*c)/d^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/
2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/
(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{6}(A+Bx^{4})}{(a+bx^{4})\,(c+dx^{4})}\,dx=\text{Timed out}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.03

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{Bx^3}{3\,bd} + \frac{(Ba^2-Aab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \right.}{8\,(b^2c-abd)} - \frac{(Bc^2-Acd)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}} + \right.}{8\,(bcd-ad^2)}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/3*B*x^3/(b*d) + 1/8*(B*a^2 - A*a*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)
) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4
)*b^(3/4)))/(b^2*c - a*b*d) - 1/8*(B*c^2 - A*c*d)*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - s
qrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(
1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt
(c))/(c^(1/4)*d^(3/4)))/(b*c*d - a*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.50

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/3*B*x^3/(b*d) + 1/2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*s
qrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^5*c - sqrt(2)*a
*b^4*d) + 1/2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d)
- 1/2*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/2*((
c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c
/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/4*((a*b^3)^(
3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + 1/4*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4
)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(
2)*a*b^4*d) + 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 + sqrt(2
)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/4*((c*d
^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5)
```

Mupad [B] (verification not implemented)

Time = 11.58 (sec) , antiderivative size = 29549, normalized size of antiderivative = 75.96

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^6*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(B*x^3)/(3*b*d) - 2*atan(((((256*A^3*a^3*b^10*c^7*d^6 - 1024*A^3*a^4*b^9*c
^6*d^7 + 1536*A^3*a^5*b^8*c^5*d^8 - 1024*A^3*a^6*b^7*c^4*d^9 + 256*A^3*a^7
*b^6*c^3*d^10 - 256*B^3*a^3*b^10*c^10*d^3 + 768*B^3*a^4*b^9*c^9*d^4 - 768*
B^3*a^5*b^8*c^8*d^5 + 256*B^3*a^6*b^7*c^7*d^6 + 256*B^3*a^7*b^6*c^6*d^7 -
768*B^3*a^8*b^5*c^5*d^8 + 768*B^3*a^9*b^4*c^4*d^9 - 256*B^3*a^10*b^3*c^3*d
^10 + 768*A*B^2*a^3*b^10*c^9*d^4 - 2304*A*B^2*a^4*b^9*c^8*d^5 + 2304*A*B^2
*a^5*b^8*c^7*d^6 - 1536*A*B^2*a^6*b^7*c^6*d^7 + 2304*A*B^2*a^7*b^6*c^5*d^8
 - 2304*A*B^2*a^8*b^5*c^4*d^9 + 768*A*B^2*a^9*b^4*c^3*d^10 - 768*A^2*B*a^3
*b^10*c^8*d^5 + 2304*A^2*B*a^4*b^9*c^7*d^6 - 1536*A^2*B*a^5*b^8*c^6*d^7 -
1536*A^2*B*a^6*b^7*c^5*d^8 + 2304*A^2*B*a^7*b^6*c^4*d^9 - 768*A^2*B*a^8*b^
5*c^3*d^10)/(b^3*d^3) - (x*(-(B^4*a^7 + A^4*a^3*b^4 + 6*A^2*B^2*a^5*b^2 -
4*A*B^3*a^6*b - 4*A^3*B*a^4*b^3)/(256*b^11*c^4 + 256*a^4*b^7*d^4 - 1024*a^
3*b^8*c*d^3 + 1536*a^2*b^9*c^2*d^2 - 1024*a*b^10*c^3*d))^(1/4)*(512*A^2*a^
3*b^11*c^7*d^7 - 2048*A^2*a^4*b^10*c^6*d^8 + 3072*A^2*a^5*b^9*c^5*d^9 - 20
48*A^2*a^6*b^8*c^4*d^10 + 512*A^2*a^7*b^7*c^3*d^11 + 256*B^2*a^3*b^11*c^9*
d^5 - 1024*B^2*a^4*b^10*c^8*d^6 + 1792*B^2*a^5*b^9*c^7*d^7 - 2048*B^2*a^6*
b^8*c^6*d^8 + 1792*B^2*a^7*b^7*c^5*d^9 - 1024*B^2*a^8*b^6*c^4*d^10 + 256*B
^2*a^9*b^5*c^3*d^11 - 512*A*B*a^3*b^11*c^8*d^6 + 1536*A*B*a^4*b^10*c^7*d^7
 - 1024*A*B*a^5*b^9*c^6*d^8 - 1024*A*B*a^6*b^8*c^5*d^9 + 1536*A*B*a^7*b^7*
c^4*d^10 - 512*A*B*a^8*b^6*c^3*d^11)*4i)/(b^3*d^3))*(-(B^4*a^7 + A^4*a^...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.38

$$\int \frac{x^6(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{6d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right) - 6d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right) - 3d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\right.}{24d^2}$$

input
```
int(x^6*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(6*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x
)/(d**(1/4)*c**(1/4)*sqrt(2))) - 6*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4
)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) - 3*d**(1/4
)*c**(3/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*
x**2) + 3*d**(1/4)*c**(3/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt
(c) + sqrt(d)*x**2) + 8*d*x**3)/(24*d**2)
```

3.13 $\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 384

$$
\begin{aligned}
\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = {} & \frac{Bx}{bd} + \frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} \\
& - \frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} \\
& + \frac{\sqrt[4]{c}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} \\
& - \frac{\sqrt[4]{c}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}d^{5/4}(bc-ad)} \\
& - \frac{\sqrt[4]{a}(Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}b^{5/4}(bc-ad)} \\
& - \frac{\sqrt[4]{c}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}d^{5/4}(bc-ad)}
\end{aligned}
$$

output
```
B*x/b/d-1/4*a^(1/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)
/b^(5/4)/(-a*d+b*c)-1/4*a^(1/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/
4))*2^(1/2)/b^(5/4)/(-a*d+b*c)-1/4*c^(1/4)*(-A*d+B*c)*arctan(-1+2^(1/2)*d^
(1/4)*x/c^(1/4))*2^(1/2)/d^(5/4)/(-a*d+b*c)-1/4*c^(1/4)*(-A*d+B*c)*arctan(
1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/d^(5/4)/(-a*d+b*c)-1/4*a^(1/4)*(A*b-B
*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/b^(5/
4)/(-a*d+b*c)-1/4*c^(1/4)*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^
(1/2)+d^(1/2)*x^2))*2^(1/2)/d^(5/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.20

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{8\sqrt[4]{b}B\sqrt[4]{d}(bc-ad)x-2\sqrt{2}\sqrt[4]{a}(-Ab+aB)d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}\sqrt[4]{a}(-Ab+aB)d^{5/4}\arctan}{}$$

input
```
Integrate[(x^4*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(8*b^(1/4)*B*d^(1/4)*(b*c - a*d)*x - 2*Sqrt[2]*a^(1/4)*(-(A*b) + a*B)*d^(5
/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*a^(1/4)*(-(A*b) +
a*B)*d^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(5/4)*c
^(1/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*Sqrt[2]*b^(
5/4)*c^(1/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + Sqrt[2]
*a^(1/4)*(A*b - a*B)*d^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + Sqrt[2]*a^(1/4)*(-(A*b) + a*B)*d^(5/4)*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(5/4)*c^(1/4)*(B*c - A*d)*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] - Sqrt[2]*b^(5/4)*c^(1/
4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(8*
b^(5/4)*d^(5/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1052, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \; 1052$$

$$\frac{Bx}{bd}-\frac{\int \frac{(bBc-Abd+aBd)x^4+aBc}{(bx^4+a)(dx^4+c)}dx}{bd}$$

$$\Big\downarrow \; 1020$$

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\int \frac{1}{bx^4+a}dx}{bc-ad}+\frac{bc(Bc-Ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{bd}$$

$$\Big\downarrow \; 755$$

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \; 1476$$

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}}{bd}$$

$$\Big\downarrow \; 1082$$

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\cdots\right.}{\cdots}}{bd}$$

$\big\downarrow$ 217

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\cdots}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{bd}$$

$\big\downarrow$ 1479

$$\frac{Bx}{bd}-\frac{\frac{ad(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-\cdots}{\cdots}}{bd}$$

$\big\downarrow$ 25

$$\frac{\frac{Bx}{bd}-\frac{ad(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)(\cdots}{\cdots}}{bd}$$

↓ 27

$$\frac{\frac{Bx}{bd}-\frac{ad(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{\cdots}}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}\cdots}}{2\sqrt{2}\sqrt[4]{c}\cdots}\cdots}{\cdots}\cdots\right)}{\cdots}}{bd}$$

↓ 1103

$$\frac{\frac{Bx}{bd}-\frac{ad(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc(Bc-Ad)(\cdots}{\cdots}}{bd}$$

input
```
Int[(x^4*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(B*x)/(b*d) - ((a*(A*b - a*B)*d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)
]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])
))/(b*c - a*d) + (b*c*(B*c - A*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/
4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(
Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c
])))/(b*c - a*d))/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.65

method	result
default	$\frac{Bx}{bd}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b(ad-cb)}-\frac{(Ad-Bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,(\mathrm{l}}{}$
risch	Expression too large to display

input
```
int(x^4*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
B*x/b/d+1/8/b*(A*b-B*a)/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)
*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(
2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/d*(A*d-B*c
)/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)
)/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 152.43 (sec) , antiderivative size = 2045, normalized size of antiderivative = 5.33

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(b*d*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3
 + A^4*a*b^4)/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d
^3 + a^4*b^5*d^4))^(1/4)*log(-(B*a - A*b)*x + (b^2*c - a*b*d)*(-(B^4*a^5 -
 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/(b^9*c^4
 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/
4)) - b*d*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3
 + A^4*a*b^4)/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d
^3 + a^4*b^5*d^4))^(1/4)*log(-(B*a - A*b)*x - (b^2*c - a*b*d)*(-(B^4*a^5 -
 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/(b^9*c^4
 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/
4)) - I*b*d*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b
^3 + A^4*a*b^4)/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c
*d^3 + a^4*b^5*d^4))^(1/4)*log(-(B*a - A*b)*x - (I*b^2*c - I*a*b*d)*(-(B^4
*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/(b
^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4
))^(1/4)) + I*b*d*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B
*a^2*b^3 + A^4*a*b^4)/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3
*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*log(-(B*a - A*b)*x - (-I*b^2*c + I*a*b*d)
*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*
b^4)/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A + Bx^4)}{(a + bx^4)(c + dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.15

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{8(b^2c-abd)}$$

$$-\frac{\left(\frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(Bc-Ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right.}{8(bcd-ad^2)}$$

$$+\frac{Bx}{bd}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/
4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqr
t(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a -
A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/
4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sq
rt(a))/(a^(3/4)*b^(1/4)))*a/(b^2*c - a*b*d) - 1/8*(2*sqrt(2)*(B*c - A*d)*a
rctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(B*c - A*d)*arctan(1/2*
sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(B*c - A*d)*log(sqrt(d)*x^2 + sqrt(
2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(B*c - A*d)*lo
g(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/
(b*c*d - a*d^2) + B*x/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(290) = 580$.

Time = 0.13 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.52

$$\int \frac{x^4(A + Bx^4)}{(a + bx^4)(c + dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*((a*
b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/2*((c*d^3)^(1/
4)*B*c - (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*((c*d^3)^(1/4)*B*c -
 (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/4*((a*b^3)^(1/4)*B*a - (a*b^3)
^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c -
sqrt(2)*a*b^2*d) - 1/4*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2 - s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*
((c*d^3)^(1/4)*B*c - (c*d^3)^(1/4)*A*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) +
sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/4*((c*d^3)^(1/4)*B*c - (c
*d^3)^(1/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
*d^2 - sqrt(2)*a*d^3) + B*x/(b*d)
```

Mupad [B] (verification not implemented)

Time = 12.27 (sec) , antiderivative size = 31774, normalized size of antiderivative = 82.74

$$\int \frac{x^4(A + Bx^4)}{(a + bx^4)(c + dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^4*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
(B*x)/(b*d) - 2*atan(((-(B^4*a^5 + A^4*a*b^4 + 6*A^2*B^2*a^3*b^2 - 4*A*B^3
*a^4*b - 4*A^3*B*a^2*b^3)/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*
d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*((-(B^4*a^5 + A^4*a*
b^4 + 6*A^2*B^2*a^3*b^2 - 4*A*B^3*a^4*b - 4*A^3*B*a^2*b^3)/(256*b^9*c^4 +
256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c
^3*d))^(1/4)*((-(B^4*a^5 + A^4*a*b^4 + 6*A^2*B^2*a^3*b^2 - 4*A*B^3*a^4*b -
 4*A^3*B*a^2*b^3)/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 15
36*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(((-(B^4*a^5 + A^4*a*b^4 + 6
*A^2*B^2*a^3*b^2 - 4*A*B^3*a^4*b - 4*A^3*B*a^2*b^3)/(256*b^9*c^4 + 256*a^4
*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^
(1/4)*(256*A*a^2*b^11*c^8*d^5 - 1536*A*a^3*b^10*c^7*d^6 + 3840*A*a^4*b^9*c
^6*d^7 - 5120*A*a^5*b^8*c^5*d^8 + 3840*A*a^6*b^7*c^4*d^9 - 1536*A*a^7*b^6*
c^3*d^10 + 256*A*a^8*b^5*c^2*d^11)*16i)/(b*d) - (4*x*(256*A^2*a^2*b^10*c^7
*d^5 - 768*A^2*a^3*b^9*c^6*d^6 + 512*A^2*a^4*b^8*c^5*d^7 + 512*A^2*a^5*b^7
*c^4*d^8 - 768*A^2*a^6*b^6*c^3*d^9 + 256*A^2*a^7*b^5*c^2*d^10 + 256*B^2*a^
3*b^9*c^8*d^4 - 768*B^2*a^4*b^8*c^7*d^5 + 512*B^2*a^5*b^7*c^6*d^6 + 512*B^
2*a^6*b^6*c^5*d^7 - 768*B^2*a^7*b^5*c^4*d^8 + 256*B^2*a^8*b^4*c^3*d^9 - 10
24*A*B*a^3*b^9*c^7*d^5 + 4096*A*B*a^4*b^8*c^6*d^6 - 6144*A*B*a^5*b^7*c^5*d
^7 + 4096*A*B*a^6*b^6*c^4*d^8 - 1024*A*B*a^7*b^5*c^3*d^9))/(b*d))*1i + (16
*(B^5*a^3*b^6*c^9 + B^5*a^9*c^3*d^6 - A^5*a^2*b^7*c^5*d^4 + A^5*a^3*b^6...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.38

$$\int \frac{x^4(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{2d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)-2d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x}{8d^2}$$

input
```
int(x^4*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x
)/(d**(1/4)*c**(1/4)*sqrt(2))) - 2*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4
)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) + d**(3/4)*
c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x*
*2) - d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c)
+ sqrt(d)*x**2) + 8*d*x)/(8*d**2)
```

3.14 $\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 375

$$
\begin{aligned}
\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = & -\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)} \\
& +\frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)} \\
& -\frac{(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)} \\
& +\frac{(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)} \\
& -\frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)} \\
& -\frac{(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)}
\end{aligned}
$$

output
```
1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)
/(-a*d+b*c)+1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(1
/4)/b^(3/4)/(-a*d+b*c)+1/4*(-A*d+B*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))
*2^(1/2)/c^(1/4)/d^(3/4)/(-a*d+b*c)+1/4*(-A*d+B*c)*arctan(1+2^(1/2)*d^(1/4
)*x/c^(1/4))*2^(1/2)/c^(1/4)/d^(3/4)/(-a*d+b*c)-1/4*(A*b-B*a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(1/4)/b^(3/4)/(-a*d
+b*c)-1/4*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^
2))*2^(1/2)/c^(1/4)/d^(3/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.10

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{-2(Ab-aB)\sqrt[4]{c}d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2(Ab-aB)\sqrt[4]{c}d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt[4]{a}b^{3/4}(Bc-}{}$$

input
```
Integrate[(x^2*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*(A*b - a*B)*c^(1/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
2*(A*b - a*B)*c^(1/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*
a^(1/4)*b^(3/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*a^
(1/4)*b^(3/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + (A*b -
 a*B)*c^(1/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] - (A*b - a*B)*c^(1/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] + a^(1/4)*b^(3/4)*(B*c - A*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] - a^(1/4)*b^(3/4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(3/4)*c^(1/4)*d^
(3/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^4)}{(a+bx^4)(c+dx^4)}dx$$
$$\Big\downarrow \ 1054$$
$$\int \left(\frac{x^2(Ab-aB)}{(a+bx^4)(bc-ad)}+\frac{x^2(Bc-Ad)}{(c+dx^4)(bc-ad)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)}-\frac{(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)}+\frac{(Bc-Ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)}+\frac{(Ab-aB)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)}-\frac{(Ab-aB)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}b^{3/4}(bc-ad)}+\frac{(Bc-Ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)}-\frac{(Bc-Ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)}$$

input
```
Int[(x^2*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/2*((A*b - a*B)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4
)*b^(3/4)*(b*c - a*d)) + ((A*b - a*B)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*(b*c - a*d)) - ((B*c - A*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)) + ((B*c
 - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*d^(3/4
)*(b*c - a*d)) + ((A*b - a*B)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(3/4)*(b*c - a*d)) - ((A*b - a*B)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*b^(3/4
)*(b*c - a*d)) + ((B*c - A*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sq
rt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)) - ((B*c - A*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*d^(3/4
)*(b*c - a*d))
```

Defintions of rubi rules used

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
 + b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{(Ad-Bc)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{}}{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{}$
risch	Expression too large to display

input
```
int(x^2*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(A*b-B*a)/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*(A*d-B*c)/(a*d-b*
c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+
(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.30 (sec) , antiderivative size = 2507, normalized size of antiderivative = 6.69

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a
^5*b^3*d^4))^(1/4)*log(-(B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3
)*x + (a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*(-(B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7
*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4
))^(3/4)) - 1/4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a
*b^3 + A^4*b^4)/(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b
^4*c*d^3 + a^5*b^3*d^4))^(1/4)*log(-(B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b
^2 - A^3*b^3)*x - (a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4
*b^4)/(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 +
 a^5*b^3*d^4))^(3/4)) + 1/4*I*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b
^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4))^(1/4)*log(-(B^3*a^3 - 3*A*B^2*a^2*b
 + 3*A^2*B*a*b^2 - A^3*b^3)*x - (I*a*b^5*c^3 - 3*I*a^2*b^4*c^2*d + 3*I*a^3
*b^3*c*d^2 - I*a^4*b^2*d^3)*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2
 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d
^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4))^(3/4)) - 1/4*I*(-(B^4*a^4 - 4*A*B^3*a
^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7*c^4 - 4*a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)\,(c+dx^4)}\,dx = -\frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\cdots\right.}{8\,(bc-ad)} + \frac{(Bc-Ad)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\cdots\right.}{8\,(bc-ad)}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1
/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(
b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*
d) + 1/8*(B*c - A*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*
c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) +
2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^
2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(s
qrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c
- a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(281) = 562$.

Time = 0.14 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.59

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c - sqrt(2)*a^2*b^3*d) - 1/2
*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c - sqrt(2)*a^2*b^3*d) + 1/2*((c
*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/
d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^3 - sqrt(2)*a*c*d^4) + 1/2*((c*d^3
)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^3 - sqrt(2)*a*c*d^4) + 1/4*((a*b^3)^(3
/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(sqrt(2)*a*b^4*c - sqrt(2)*a^2*b^3*d) - 1/4*((a*b^3)^(3/4)*B*a - (a*b^3)^(
3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^4*c -
sqrt(2)*a^2*b^3*d) - 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 +
 sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^3 - sqrt(2)*a*c*d^4)
+ 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1
/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^3 - sqrt(2)*a*c*d^4)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 24377, normalized size of antiderivative = 65.01

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((x^2*(A + B*x^4))/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
atan(((x*(4*A^6*a*b^6*c^2*d^5 + 4*A^6*a^2*b^5*c*d^6 + 4*B^6*a^3*b^4*c^6*d
+ 4*B^6*a^6*b*c^3*d^4 - 16*A*B^5*a^3*b^4*c^5*d^2 - 16*A*B^5*a^5*b^2*c^3*d^
4 - 16*A^3*B^3*a*b^6*c^5*d^2 - 16*A^3*B^3*a^5*b^2*c*d^6 + 24*A^4*B^2*a*b^6
*c^4*d^3 + 24*A^4*B^2*a^4*b^3*c*d^6 - 16*A^5*B*a^2*b^5*c^2*d^5 + 32*A^2*B^
4*a^2*b^5*c^5*d^2 + 24*A^2*B^4*a^3*b^4*c^4*d^3 + 24*A^2*B^4*a^4*b^3*c^3*d^
4 + 32*A^2*B^4*a^5*b^2*c^2*d^5 - 48*A^3*B^3*a^2*b^5*c^4*d^3 - 32*A^3*B^3*a
^3*b^4*c^3*d^4 - 48*A^3*B^3*a^4*b^3*c^2*d^5 + 36*A^4*B^2*a^2*b^5*c^3*d^4 +
 36*A^4*B^2*a^3*b^4*c^2*d^5 - 8*A*B^5*a^2*b^5*c^6*d - 8*A*B^5*a^6*b*c^2*d^
5 + 4*A^2*B^4*a*b^6*c^6*d + 4*A^2*B^4*a^6*b*c*d^6 - 16*A^5*B*a*b^6*c^3*d^4
 - 16*A^5*B*a^3*b^4*c*d^6) + (-(A^4*b^4 + B^4*a^4 + 6*A^2*B^2*a^2*b^2 - 4*
A*B^3*a^3*b - 4*A^3*B*a*b^3)/(256*a*b^7*c^4 + 256*a^5*b^3*d^4 - 1024*a^2*b
^6*c^3*d - 1024*a^4*b^4*c*d^3 + 1536*a^3*b^5*c^2*d^2))^(3/4)*(512*A^3*a^3*
b^7*c^4*d^6 - 768*A^3*a^2*b^8*c^5*d^5 - x*(-(A^4*b^4 + B^4*a^4 + 6*A^2*B^2
*a^2*b^2 - 4*A*B^3*a^3*b - 4*A^3*B*a*b^3)/(256*a*b^7*c^4 + 256*a^5*b^3*d^4
 - 1024*a^2*b^6*c^3*d - 1024*a^4*b^4*c*d^3 + 1536*a^3*b^5*c^2*d^2))^(1/4)*
(4096*A^2*a^2*b^9*c^6*d^5 - 7168*A^2*a^3*b^8*c^5*d^6 + 8192*A^2*a^4*b^7*c^
4*d^7 - 7168*A^2*a^5*b^6*c^3*d^8 + 4096*A^2*a^6*b^5*c^2*d^9 - 2048*B^2*a^3
*b^8*c^7*d^4 + 8192*B^2*a^4*b^7*c^6*d^5 - 12288*B^2*a^5*b^6*c^5*d^6 + 8192
*B^2*a^6*b^5*c^4*d^7 - 2048*B^2*a^7*b^4*c^3*d^8 - 1024*A^2*a*b^10*c^7*d^4
- 1024*A^2*a^7*b^4*c*d^10 + 2048*A*B*a^2*b^9*c^7*d^4 - 6144*A*B*a^3*b^8...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.30

$$\int \frac{x^2(A+Bx^4)}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\right.}{8d^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
int(x^2*(B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(d**(1/4)*c**(3/4)*sqrt(2)*( - 2*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) + 2*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2
*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) + log( - d**(1/4)*c**(1/4)*sqrt(2
)*x + sqrt(c) + sqrt(d)*x**2) - log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c)
+ sqrt(d)*x**2)))/(8*c*d)
```

3.15 $\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 375

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx = -\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}(bc-ad)} + \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}(bc-ad)} - \frac{(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)} + \frac{(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)} + \frac{(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}(bc-ad)} + \frac{(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)}$$

output
```
1/4*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)
/(-a*d+b*c)+1/4*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(3
/4)/b^(1/4)/(-a*d+b*c)+1/4*(-A*d+B*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))
*2^(1/2)/c^(3/4)/d^(1/4)/(-a*d+b*c)+1/4*(-A*d+B*c)*arctan(1+2^(1/2)*d^(1/4
)*x/c^(1/4))*2^(1/2)/c^(3/4)/d^(1/4)/(-a*d+b*c)+1/4*(A*b-B*a)*arctanh(2^(1
/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(3/4)/b^(1/4)/(-a*d
+b*c)+1/4*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^
2))*2^(1/2)/c^(3/4)/d^(1/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{-2(Ab-aB)c^{3/4}\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2(Ab-aB)c^{3/4}\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2a^{3/4}\sqrt[4]{b}(Bc-}{}$$

input
```
Integrate[(A + B*x^4)/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
(-2*(A*b - a*B)*c^(3/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] +
2*(A*b - a*B)*c^(3/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*
a^(3/4)*b^(1/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*a^
(3/4)*b^(1/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - (A*b -
 a*B)*c^(3/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] + (A*b - a*B)*c^(3/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x +
 Sqrt[b]*x^2] - a^(3/4)*b^(1/4)*(B*c - A*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + a^(3/4)*b^(1/4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(1/4)*c^(3/4)*d^
(1/4)*(b*c - a*d))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{1020}$$

$$\frac{(Ab-aB)\int \frac{1}{bx^4+a}dx}{bc-ad}+\frac{(Bc-Ad)\int \frac{1}{dx^4+c}dx}{bc-ad}$$

$$\Big\downarrow \text{755}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 217$$

$$\frac{(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1479$$

$$\frac{(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$\Big\downarrow$ 25

$$\frac{(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{\sqrt[4]{d}\left(x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$\Big\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}x}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}x+\sqrt[4]{c}}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

$$\Big\downarrow \; 1103$$

$$\frac{(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{(Bc-Ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}$$

input
```
Int[(A + B*x^4)/((a + b*x^4)*(c + d*x^4)),x]
```

output
```
((A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/
4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x
+ Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + ((
B*c - A*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^
(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)
))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x
^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8(ad-cb)a}+\frac{(Ad-Bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2}{x^2}\right.\right.}{}$
risch	Expression too large to display

input
```
int((B*x^4+A)/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-A*b+B*a)/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1
/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*(A*d-B*c)/(a*d-b*
c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-
(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.94 (sec) , antiderivative size = 2019, normalized size of antiderivative = 5.38

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
1/4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 +
 a^7*b*d^4))^(1/4)*log(-(B*a - A*b)*x + (a*b*c - a^2*d)*(-(B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5*c^4 - 4*a
^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4))^(1/4)) -
1/4*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 +
 a^7*b*d^4))^(1/4)*log(-(B*a - A*b)*x - (a*b*c - a^2*d)*(-(B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5*c^4 - 4*a
^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4))^(1/4)) -
1/4*I*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4
*b^4)/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3
 + a^7*b*d^4))^(1/4)*log(-(B*a - A*b)*x - (I*a*b*c - I*a^2*d)*(-(B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5*c^4
 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4))^(1/
4)) + 1/4*I*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3
 + A^4*b^4)/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2
*c*d^3 + a^7*b*d^4))^(1/4)*log(-(B*a - A*b)*x - (-I*a*b*c + I*a^2*d)*(-(B^
4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*
b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**4+A)/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,(bc-ad)}$$

$$+\frac{\frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(Bc-Ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\,(bc-ad)}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1
/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sq
rt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a -
 A*b)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1
/4)) - sqrt(2)*(B*a - A*b)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + s
qrt(a))/(a^(3/4)*b^(1/4)))/(b*c - a*d) + 1/8*(2*sqrt(2)*(B*c - A*d)*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(B*c - A*d)*arctan(1/2*sqrt(
2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)
*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(B*c - A*d)*log(sqrt(d)*x^2 + sqrt(2)*c^
(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(B*c - A*d)*log(sqr
t(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b*c -
a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(281) = 562$.

Time = 0.12 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.55

$$\int \frac{A + Bx^4}{(a + bx^4)(c + dx^4)} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*c - sqrt(2)*a^2*b*d) - 1/2*(
(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(
a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^2*c - sqrt(2)*a^2*b*d) + 1/2*((c*d^3
)^(1/4)*B*c - (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d - sqrt(2)*a*c*d^2) + 1/2*((c*d^3)^(1/4
)*B*c - (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(sqrt(2)*b*c^2*d - sqrt(2)*a*c*d^2) - 1/4*((a*b^3)^(1/4)*B*a
- (a*b^3)^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)
*a*b^2*c - sqrt(2)*a^2*b*d) + 1/4*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c - sqrt(2)*a^
2*b*d) + 1/4*((c*d^3)^(1/4)*B*c - (c*d^3)^(1/4)*A*d)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d - sqrt(2)*a*c*d^2) - 1/4*((c*d^3)
^(1/4)*B*c - (c*d^3)^(1/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d
))/(sqrt(2)*b*c^2*d - sqrt(2)*a*c*d^2)
```

Mupad [B] (verification not implemented)

Time = 11.89 (sec) , antiderivative size = 22357, normalized size of antiderivative = 59.62

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/((a + b*x^4)*(c + d*x^4)),x)
```

output
```
2*atan(-((-(A^4*d^4 + B^4*c^4 + 6*A^2*B^2*c^2*d^2 - 4*A*B^3*c^3*d - 4*A^3*
B*c*d^3)/(256*b^4*c^7*d + 256*a^4*c^3*d^5 - 1024*a*b^3*c^6*d^2 - 1024*a^3*
b*c^4*d^4 + 1536*a^2*b^2*c^5*d^3))^(1/4)*((-(A^4*d^4 + B^4*c^4 + 6*A^2*B^2
*c^2*d^2 - 4*A*B^3*c^3*d - 4*A^3*B*c*d^3)/(256*b^4*c^7*d + 256*a^4*c^3*d^5
 - 1024*a*b^3*c^6*d^2 - 1024*a^3*b*c^4*d^4 + 1536*a^2*b^2*c^5*d^3))^(1/4)*
(16*B^5*a^3*b^5*c^4*d^4 - 16*A^5*a^2*b^6*d^8 - 16*A^5*b^8*c^2*d^6 - 16*B^5
*a^2*b^6*c^5*d^3 - (x*(1024*A^2*a^7*b^4*d^11 + 1024*A^2*b^11*c^7*d^4 + 614
4*A^2*a^2*b^9*c^5*d^6 - 3072*A^2*a^3*b^8*c^4*d^7 - 3072*A^2*a^4*b^7*c^3*d^
8 + 6144*A^2*a^5*b^6*c^2*d^9 + 1024*B^2*a^2*b^9*c^7*d^4 - 3072*B^2*a^3*b^8
*c^6*d^5 + 2048*B^2*a^4*b^7*c^5*d^6 + 2048*B^2*a^5*b^6*c^4*d^7 - 3072*B^2*
a^6*b^5*c^3*d^8 + 1024*B^2*a^7*b^4*c^2*d^9 - 4096*A^2*a*b^10*c^6*d^5 - 409
6*A^2*a^6*b^5*c*d^10 - 2048*A*B*a*b^10*c^7*d^4 - 2048*A*B*a^7*b^4*c*d^10 +
 8192*A*B*a^2*b^9*c^6*d^5 - 14336*A*B*a^3*b^8*c^5*d^6 + 16384*A*B*a^4*b^7*
c^4*d^7 - 14336*A*B*a^5*b^6*c^3*d^8 + 8192*A*B*a^6*b^5*c^2*d^9) - (-(A^4*d
^4 + B^4*c^4 + 6*A^2*B^2*c^2*d^2 - 4*A*B^3*c^3*d - 4*A^3*B*c*d^3)/(256*b^4
*c^7*d + 256*a^4*c^3*d^5 - 1024*a*b^3*c^6*d^2 - 1024*a^3*b*c^4*d^4 + 1536*
a^2*b^2*c^5*d^3))^(1/4)*(20480*A*a^2*b^10*c^7*d^5 - 4096*A*a^8*b^4*c*d^11
- 4096*A*a*b^11*c^8*d^4 - 36864*A*a^3*b^9*c^6*d^6 + 20480*A*a^4*b^8*c^5*d^
7 + 20480*A*a^5*b^7*c^4*d^8 - 36864*A*a^6*b^6*c^3*d^9 + 20480*A*a^7*b^5*c^
2*d^10 + 4096*B*a^2*b^10*c^8*d^4 - 24576*B*a^3*b^9*c^7*d^5 + 61440*B*a^...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^4}{(a+bx^4)(c+dx^4)}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+\log\left(d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\right.}{8d^{\frac{1}{4}}c^{\frac{3}{4}}}$$

input
```
int((B*x^4+A)/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(d**(3/4)*c**(1/4)*sqrt(2)*( - 2*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) + 2*atan((d**(1/4)*c**(1/4)*sqrt(2) + 2
*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2))) - log( - d**(1/4)*c**(1/4)*sqrt(2
)*x + sqrt(c) + sqrt(d)*x**2) + log(d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c)
+ sqrt(d)*x**2)))/(8*c*d)
```

3.16 $\int \frac{A+Bx^4}{x^2(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 387

$$\begin{aligned}\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx =& -\frac{A}{acx}+\frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ &-\frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ &+\frac{\sqrt[4]{d}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} \\ &-\frac{\sqrt[4]{d}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{5/4}(bc-ad)} \\ &+\frac{\sqrt[4]{b}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{5/4}(bc-ad)} \\ &+\frac{\sqrt[4]{d}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{5/4}(bc-ad)}\end{aligned}$$

output
```
-A/a/c/x-1/4*b^(1/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2
)/a^(5/4)/(-a*d+b*c)-1/4*b^(1/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1
/4))*2^(1/2)/a^(5/4)/(-a*d+b*c)-1/4*d^(1/4)*(-A*d+B*c)*arctan(-1+2^(1/2)*d
^(1/4)*x/c^(1/4))*2^(1/2)/c^(5/4)/(-a*d+b*c)-1/4*d^(1/4)*(-A*d+B*c)*arctan
(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(5/4)/(-a*d+b*c)+1/4*b^(1/4)*(A*b-
B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(5
/4)/(-a*d+b*c)+1/4*d^(1/4)*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c
^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(5/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^4}{x^2(a+bx^4)(c+dx^4)}\,dx$$
$$= \frac{8\sqrt[4]{a}A\sqrt[4]{c}(-bc+ad)+2\sqrt{2}\sqrt[4]{b}(Ab-aB)c^{5/4}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{b}(Ab-aB)c^{5/4}x\arctan\Big(}{}$$

input
```
Integrate[(A + B*x^4)/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(8*a^(1/4)*A*c^(1/4)*(-(b*c) + a*d) + 2*Sqrt[2]*b^(1/4)*(A*b - a*B)*c^(5/4
)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*b^(1/4)*(A*b - a*B
)*c^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(5/4)*d^
(1/4)*(-(B*c) + A*d)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]
*a^(5/4)*d^(1/4)*(-(B*c) + A*d)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
- Sqrt[2]*b^(1/4)*(A*b - a*B)*c^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(1/4)*(A*b - a*B)*c^(5/4)*x*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(5/4)*d^(1/4)*(-(B*c
) + A*d)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2
]*a^(5/4)*d^(1/4)*(B*c - A*d)*x*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(8*a^(5/4)*c^(5/4)*(b*c - a*d)*x)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.40, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^2(a+bx^4)(c+dx^4)}dx$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{\int \frac{x^2(Abdx^4+Abc-aBc+aAd)}{(bx^4+a)(dx^4+c)}dx}{ac}-\frac{A}{acx}$$

$$\Big\downarrow \text{ 1054}$$

$$-\frac{\int \left(\frac{b(Ab-aB)cx^2}{(bc-ad)(bx^4+a)}+\frac{ad(Ad-Bc)x^2}{(ad-bc)(dx^4+c)}\right)dx}{ac}-\frac{A}{acx}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{\sqrt[4]{b}c(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}c(Ab-aB)\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{a\sqrt[4]{d}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a\sqrt[4]{d}(Bc-Ad)}{2\sqrt{}}}{}$$

$$\frac{A}{acx}$$

input
```
Int[(A + B*x^4)/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-(A/(a*c*x)) - (-1/2*(b^(1/4)*(A*b - a*B)*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(1/4)*(A*b - a*B)*c*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a*d^(1
/4)*(B*c - A*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4
)*(b*c - a*d)) + (a*d^(1/4)*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*(A*b - a*B)*c*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*
d)) - (b^(1/4)*(A*b - a*B)*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a*d^(1/4)*(B*c - A*d)*Log[Sq
rt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c
- a*d)) - (a*d^(1/4)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c)
```

Defintions of rubi rules used

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.65

method	result
default	$\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a(ad-cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{A}{acx}-\frac{(Ad-Bc)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{c}{d}\right.}{x^2+\left(\frac{c}{d}\right.}\right.\right.}{}$
risch	Expression too large to display

input
```
int((B*x^4+A)/x^2/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*(A*b-B*a)/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*2^(1/
2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-A/a/c/x-1/8*(A*d-B*c)/
c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)
)/(x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 62.51 (sec) , antiderivative size = 2543, normalized size of antiderivative = 6.57

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
-1/4*(a*c*x*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a
*b^4 + A^4*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8
*b*c*d^3 + a^9*d^4))^(1/4)*log(-(B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b
^3 - A^3*b^4)*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3
)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4
*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 +
 a^9*d^4))^(3/4)) - a*c*x*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b
^3 - 4*A^3*B*a*b^4 + A^4*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c
^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*log(-(B^3*a^3*b - 3*A*B^2*a^2*b^2
 + 3*A^2*B*a*b^3 - A^3*b^4)*x - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c
*d^2 - a^7*d^3)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3
*B*a*b^4 + A^4*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4
*a^8*b*c*d^3 + a^9*d^4))^(3/4)) + I*a*c*x*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 +
 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3
*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*log(-(B^3*a^3*b -
 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*x - (I*a^4*b^3*c^3 - 3*I*a^5*b
^2*c^2*d + 3*I*a^6*b*c*d^2 - I*a^7*d^3)*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6
*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
 + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - I*a*c*x*(-(B^4*a
^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**4+A)/x**2/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^4}{x^2\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{\left(Bab-Ab^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\right.}{8\left(abc-a^2d\right)} - \frac{\left(Bcd-Ad^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\right.}{8\left(bc^2-acd\right)} - \frac{A}{acx}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(B*a*b - A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2
*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2
 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sq
rt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c
 - a^2*d) - 1/8*(B*c*d - A*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
 + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*
sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log
(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sq
rt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3
/4)))/(b*c^2 - a*c*d) - A/(a*c*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(293) = 586$.

Time = 0.13 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.61

$$\int \frac{A + Bx^4}{x^2 (a + bx^4) (c + dx^4)} \, dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c - sqrt(2)*a^3*b^2*d) + 1/
2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c - sqrt(2)*a^3*b^2*d) - 1/2*
((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d^2 - sqrt(2)*a*c^2*d^3) - 1/2*((
c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c
/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d^2 - sqrt(2)*a*c^2*d^3) - 1/4*((a*
b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt
(a/b))/(sqrt(2)*a^2*b^3*c - sqrt(2)*a^3*b^2*d) + 1/4*((a*b^3)^(3/4)*B*a -
(a*b^3)^(3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a
^2*b^3*c - sqrt(2)*a^3*b^2*d) + 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d
)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d^2 - sqrt(2
)*a*c^2*d^3) - 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 - sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d^2 - sqrt(2)*a*c^2*d^3) - A/
(a*c*x)
```

Mupad [B] (verification not implemented)

Time = 11.10 (sec) , antiderivative size = 26209, normalized size of antiderivative = 67.72

$$\int \frac{A + Bx^4}{x^2 (a + bx^4) (c + dx^4)} \, dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
2*atan((1024*A^2*a*b^8*c^14*x*(-(A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d^3 -
 4*A^3*B*c*d^4 - 4*A*B^3*c^3*d^2)/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^
3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4) + 1024*B^2*a
^3*b^6*c^14*x*(-(A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d^3 - 4*A^3*B*c*d^4 -
 4*A*B^3*c^3*d^2)/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 15
36*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4) + 1024*A^2*a^9*c^6*d^8*x*(-(
A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d^3 - 4*A^3*B*c*d^4 - 4*A*B^3*c^3*d^2)
/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^
2 - 1024*a*b^3*c^8*d))^(5/4) + 4*A^6*b^5*c^6*d^4*x*(-(A^4*d^5 + B^4*c^4*d
+ 6*A^2*B^2*c^2*d^3 - 4*A^3*B*c*d^4 - 4*A*B^3*c^3*d^2)/(256*b^4*c^9 + 256*
a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d
))^(1/4) + 1024*B^2*a^9*c^8*d^6*x*(-(A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d
^3 - 4*A^3*B*c*d^4 - 4*A*B^3*c^3*d^2)/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 102
4*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4) + 4*A^4*
B^2*b^5*c^8*d^2*x*(-(A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d^3 - 4*A^3*B*c*d
^4 - 4*A*B^3*c^3*d^2)/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3
+ 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4) + 6144*A^2*a^3*b^6*c^12*
d^2*x*(-(A^4*d^5 + B^4*c^4*d + 6*A^2*B^2*c^2*d^3 - 4*A^3*B*c*d^4 - 4*A*B^3
*c^3*d^2)/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b
^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4) - 4096*A^2*a^4*b^5*c^11*d^3*x*(-(...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^4}{x^2(a+bx^4)(c+dx^4)}\,dx = \frac{2d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x-2d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x-d^{\frac{1}{4}}c^{\frac{3}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}+\right.}{8c^2x}$$

input
```
int((B*x^4+A)/x^2/(b*x^4+a)/(d*x^4+c),x)
```

output

```
(2*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x
)/(d**(1/4)*c**(1/4)*sqrt(2)))*x - 2*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1
/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*x - d**(1
/4)*c**(3/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d
)*x**2)*x + d**(1/4)*c**(3/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*sqrt(2)*x + sq
rt(c) + sqrt(d)*x**2)*x - 8*c)/(8*c**2*x)
```

3.17 $\int \frac{A+Bx^4}{x^4(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 389

$$
\begin{aligned}
\int \frac{A+Bx^4}{x^4(a+bx^4)(c+dx^4)}\,dx &= -\frac{A}{3acx^3}+\frac{b^{3/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad-\frac{b^{3/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad+\frac{d^{3/4}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} \\
&\quad-\frac{d^{3/4}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{7/4}(bc-ad)} \\
&\quad-\frac{b^{3/4}(Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{7/4}(bc-ad)} \\
&\quad-\frac{d^{3/4}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{7/4}(bc-ad)}
\end{aligned}
$$

output
```
-1/3*A/a/c/x^3-1/4*b^(3/4)*(A*b-B*a)*arctan(-1+2^(1/2)*b^(1/4)*x/a^(1/4))*
2^(1/2)/a^(7/4)/(-a*d+b*c)-1/4*b^(3/4)*(A*b-B*a)*arctan(1+2^(1/2)*b^(1/4)*
x/a^(1/4))*2^(1/2)/a^(7/4)/(-a*d+b*c)-1/4*d^(3/4)*(-A*d+B*c)*arctan(-1+2^(
1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/(-a*d+b*c)-1/4*d^(3/4)*(-A*d+B*c)*
arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(7/4)/(-a*d+b*c)-1/4*b^(3/4)
*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x/(a^(1/2)+b^(1/2)*x^2))*2^(1/2
)/a^(7/4)/(-a*d+b*c)-1/4*d^(3/4)*(-A*d+B*c)*arctanh(2^(1/2)*c^(1/4)*d^(1/4
)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(7/4)/(-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$= \frac{8a^{3/4}Ac^{3/4}(-bc+ad)+6\sqrt{2}b^{3/4}(Ab-aB)c^{7/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)-6\sqrt{2}b^{3/4}(Ab-aB)c^{7/4}x^3\operatorname{arct}}{}$$

input
```
Integrate[(A + B*x^4)/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(8*a^(3/4)*A*c^(3/4)*(-(b*c) + a*d) + 6*Sqrt[2]*b^(3/4)*(A*b - a*B)*c^(7/4
)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 6*Sqrt[2]*b^(3/4)*(A*b - a
*B)*c^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 6*Sqrt[2]*a^(7/4
)*d^(3/4)*(-(B*c) + A*d)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 6*S
qrt[2]*a^(7/4)*d^(3/4)*(-(B*c) + A*d)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] + 3*Sqrt[2]*b^(3/4)*(A*b - a*B)*c^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - 3*Sqrt[2]*b^(3/4)*(A*b - a*B)*c^(7/4)*x
^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 3*Sqrt[2]*a^(7
/4)*d^(3/4)*(B*c - A*d)*x^3*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + 3*Sqrt[2]*a^(7/4)*d^(3/4)*(-(B*c) + A*d)*x^3*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(24*a^(7/4)*c^(7/4)*(b*c - a*d)*x^3)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {1053, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}dx$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{\int \frac{3\left(Abdx^4+Abc-aBc+aAd\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{3ac}-\frac{A}{3acx^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{Abdx^4+Abc-aBc+aAd}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{A}{3acx^3}$$

$$\Big\downarrow \text{ 1020}$$

$$-\frac{\frac{bc(Ab-aB)\int \frac{1}{bx^4+a}dx}{bc-ad}+\frac{ad(Bc-Ad)\int \frac{1}{dx^4+c}dx}{bc-ad}}{ac}-\frac{A}{3acx^3}$$

$$\Big\downarrow \text{ 755}$$

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x^2+\sqrt{c}}{dx^4+c}dx}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{A}{3acx^3}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{b}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+\cdots}}{2\sqrt{d}}}{2\sqrt{c}}\cdots\right.}{bc-ad}}{ac}$$

$$\frac{A}{3acx^3}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}}{2\sqrt{c}}\right.}{}}{ac}$$

$$\frac{A}{3acx^3}$$

$\Big\downarrow$ 217

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{bx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x^2}{dx^4+c}dx}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(\right.}{\sqrt{2}}}{2\sqrt{c}}\right.}{bc-ad}}{ac}$$

$$\frac{A}{3acx^3}$$

$\Big\downarrow$ 1479

$$-\frac{\frac{bc(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-}{}}{ac}$$

$$\frac{A}{3acx^3}$$

$\Big\downarrow$ 25

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{\sqrt[4]{b}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\cdots}{\cdots}}{ac}-\frac{A}{3acx^3}$$

$\Big\downarrow$ 27

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}dx}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-\cdots}{x^2-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots}}{2\sqrt{2}\sqrt[4]{\cdots}}\cdots}{\cdots}\right)}{\cdots}}{ac}-\frac{A}{3acx^3}$$

$\Big\downarrow$ 1103

$$-\frac{\frac{bc(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad(Bc-Ad)\cdots}{\cdots}}{ac}-\frac{A}{3acx^3}$$

input
```
Int[(A + B*x^4)/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/3*A/(a*c*x^3) - ((b*(A*b - a*B)*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqr
t[a])))/(b*c - a*d) + (a*d*(B*c - A*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/
c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/
4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] +
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*S
qrt[c])))/(b*c - a*d))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.65

method	result
default	$\frac{(Ab-Ba)b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2(ad-cb)}-\frac{A}{3ac\,x^3}-\frac{(Ad-Bc)d\left(\frac{c}{d}\right)^{\frac{1}{4}}}{}$
risch	Expression too large to display

input
```
int((B*x^4+A)/x^4/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*(A*b-B*a)*b/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2
^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/3*A/a/c/x^3-1/8/
c^2*(A*d-B*c)*d/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x^2+(c/d)^(1/4)*x*2^(1/
2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/
(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx=\text{Timed out}$$

input
```
integrate((B*x**4+A)/x**4/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$
$$=\frac{\frac{2\sqrt{2}\left(Bab-Ab^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(Bab-Ab^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(Bab-Ab^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\left(abc-a^2d\right)}$$
$$-\frac{\frac{2\sqrt{2}\left(Bcd-Ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(Bcd-Ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(Bcd-Ad^2\right)\log\left(\sqrt{d}x^2+\sqrt{2}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{8\left(bc^2-acd\right)}$$
$$-\frac{A}{3\,acx^3}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
1/8*(2*sqrt(2)*(B*a*b - A*b^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2
*sqrt(2)*(B*a*b - A*b^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)
*(B*a*b - A*b^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(B*a*b - A*b^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b*c - a^2*d) - 1/8*(2*sqrt(2)
*(B*c*d - A*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(B*c*
d - A*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt
(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(B*c*d - A*d^
2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)
) - sqrt(2)*(B*c*d - A*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/(b*c^2 - a*c*d) - 1/3*A/(a*c*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
1/2*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/2*((a*
b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b
)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*((c*d^3)^(1/
4)*B*c - (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/2*((c*d^3)^(1/4)*B*c -
 (c*d^3)^(1/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/4*((a*b^3)^(1/4)*B*a - (a*b^3)
^(1/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c
- sqrt(2)*a^3*d) - 1/4*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(x^2 - s
qrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/4*
((c*d^3)^(1/4)*B*c - (c*d^3)^(1/4)*A*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) +
sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/4*((c*d^3)^(1/4)*B*c - (c
*d^3)^(1/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c
^3 - sqrt(2)*a*c^2*d) - 1/3*A/(a*c*x^3)
```

Mupad [B] (verification not implemented)

Time = 11.97 (sec) , antiderivative size = 23502, normalized size of antiderivative = 60.42

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^4*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
2*atan((A^6*a^2*b^5*d^7*x + A^6*b^7*c^2*d^5*x - 2*A^5*B*b^7*c^3*d^4*x + A^
4*B^2*a^4*b^3*d^7*x + A^4*B^2*b^7*c^4*d^3*x + 2*B^6*a^4*b^3*c^4*d^3*x - 2*
A^5*B*a^3*b^4*d^7*x - (256*A^2*a^2*b^9*c^11*x*(A^4*d^7 + B^4*c^4*d^3 + 6*A
^2*B^2*c^2*d^5 - 4*A^3*B*c*d^6 - 4*A*B^3*c^3*d^4))/(256*b^4*c^11 + 256*a^4
*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d)
 - (256*B^2*a^4*b^7*c^11*x*(A^4*d^7 + B^4*c^4*d^3 + 6*A^2*B^2*c^2*d^5 - 4*A
^3*B*c*d^6 - 4*A*B^3*c^3*d^4))/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*
b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (256*A^2*a^11*c^2*
d^9*x*(A^4*d^7 + B^4*c^4*d^3 + 6*A^2*B^2*c^2*d^5 - 4*A^3*B*c*d^6 - 4*A*B^3
*c^3*d^4))/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2
*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (256*B^2*a^11*c^4*d^7*x*(A^4*d^7 + B^4
*c^4*d^3 + 6*A^2*B^2*c^2*d^5 - 4*A^3*B*c*d^6 - 4*A*B^3*c^3*d^4))/(256*b^4*
c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*
a*b^3*c^10*d) + 7*A^2*B^4*a^2*b^5*c^4*d^3*x + 16*A^2*B^4*a^3*b^4*c^3*d^4*x
 + 7*A^2*B^4*a^4*b^3*c^2*d^5*x - 16*A^3*B^3*a^2*b^5*c^3*d^4*x - 16*A^3*B^3
*a^3*b^4*c^2*d^5*x + 12*A^4*B^2*a^2*b^5*c^2*d^5*x - 4*A^5*B*a*b^6*c^2*d^5*
x - 4*A^5*B*a^2*b^5*c*d^6*x - 6*A*B^5*a^3*b^4*c^4*d^3*x - 6*A*B^5*a^4*b^3*
c^3*d^4*x - 4*A^3*B^3*a*b^6*c^4*d^3*x - 4*A^3*B^3*a^4*b^3*c*d^6*x + 8*A^4*
B^2*a*b^6*c^3*d^4*x + 8*A^4*B^2*a^3*b^4*c*d^6*x + (512*A*B*a^11*c^3*d^8*x*
(A^4*d^7 + B^4*c^4*d^3 + 6*A^2*B^2*c^2*d^5 - 4*A^3*B*c*d^6 - 4*A*B^3*c^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^4}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{6d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^3-6d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^3+3d^{\frac{3}{4}}c^{\frac{1}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}\right.}{24c^2x^3}$$

input
```
int((B*x^4+A)/x^4/(b*x^4+a)/(d*x^4+c),x)
```

output
```
(6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(d)*x
)/(d**(1/4)*c**(1/4)*sqrt(2)))*x**3 - 6*d**(3/4)*c**(1/4)*sqrt(2)*atan((d*
*(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*x**3 +
 3*d**(3/4)*c**(1/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x + sqrt(c)
+ sqrt(d)*x**2)*x**3 - 3*d**(3/4)*c**(1/4)*sqrt(2)*log(d**(1/4)*c**(1/4)*s
qrt(2)*x + sqrt(c) + sqrt(d)*x**2)*x**3 - 8*c)/(24*c**2*x**3)
```

3.18 $\int \frac{A+Bx^4}{x^6(a+bx^4)(c+dx^4)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 413

$$\begin{aligned}\int \frac{A+Bx^4}{x^6(a+bx^4)(c+dx^4)}\,dx &= -\frac{A}{5acx^5}+\frac{Abc-aBc+aAd}{a^2c^2x} \\ &\quad -\frac{b^{5/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} \\ &\quad +\frac{b^{5/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} \\ &\quad -\frac{d^{5/4}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\ &\quad +\frac{d^{5/4}(Bc-Ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{9/4}(bc-ad)} \\ &\quad -\frac{b^{5/4}(Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} \\ &\quad -\frac{d^{5/4}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x}{\sqrt{c}+\sqrt{d}x^2}\right)}{2\sqrt{2}c^{9/4}(bc-ad)}\end{aligned}$$

output
```
-1/5*A/a/c/x^5+(A*a*d+A*b*c-B*a*c)/a^2/c^2/x+1/4*b^(5/4)*(A*b-B*a)*arctan(
-1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(9/4)/(-a*d+b*c)+1/4*b^(5/4)*(A*b-
B*a)*arctan(1+2^(1/2)*b^(1/4)*x/a^(1/4))*2^(1/2)/a^(9/4)/(-a*d+b*c)+1/4*d^
(5/4)*(-A*d+B*c)*arctan(-1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/c^(9/4)/(-a*
d+b*c)+1/4*d^(5/4)*(-A*d+B*c)*arctan(1+2^(1/2)*d^(1/4)*x/c^(1/4))*2^(1/2)/
c^(9/4)/(-a*d+b*c)-1/4*b^(5/4)*(A*b-B*a)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x
/(a^(1/2)+b^(1/2)*x^2))*2^(1/2)/a^(9/4)/(-a*d+b*c)-1/4*d^(5/4)*(-A*d+B*c)*
arctanh(2^(1/2)*c^(1/4)*d^(1/4)*x/(c^(1/2)+d^(1/2)*x^2))*2^(1/2)/c^(9/4)/(
-a*d+b*c)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx^4}{x^6\,(a+bx^4)\,(c+dx^4)}\,dx = \frac{8a^{5/4}Ac^{5/4}(-bc+ad)+40\sqrt[4]{a}\sqrt[4]{c}(bc-ad)(Abc-aBc+aAd)x^4-10\sqrt{2}b^{5/4}(Ab-aB)c^{9/4}x^5\arctan\left(1\right.}{}$$

input
```
Integrate[(A + B*x^4)/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
(8*a^(5/4)*A*c^(5/4)*(-(b*c) + a*d) + 40*a^(1/4)*c^(1/4)*(b*c - a*d)*(A*b*
c - a*B*c + a*A*d)*x^4 - 10*Sqrt[2]*b^(5/4)*(A*b - a*B)*c^(9/4)*x^5*ArcTan
[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*b^(5/4)*(A*b - a*B)*c^(9/4)
*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*a^(9/4)*d^(5/4)*
(-(B*c) + A*d)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*a^
(9/4)*d^(5/4)*(B*c - A*d)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 5*
Sqrt[2]*b^(5/4)*(A*b - a*B)*c^(9/4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2] - 5*Sqrt[2]*b^(5/4)*(A*b - a*B)*c^(9/4)*x^5*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*a^(9/4)*d^(5/4)*
(B*c - A*d)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5
*Sqrt[2]*a^(9/4)*d^(5/4)*(-(B*c) + A*d)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2])/(40*a^(9/4)*c^(9/4)*(b*c - a*d)*x^5)
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1053, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^4}{x^6 \left(a+bx^4\right) \left(c+dx^4\right)} \, dx$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{\int \frac{5\left(Abdx^4+Abc-aBc+aAd\right)}{x^2\left(bx^4+a\right)\left(dx^4+c\right)}dx}{5ac}-\frac{A}{5acx^5}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{Abdx^4+Abc-aBc+aAd}{x^2\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{A}{5acx^5}$$

$$\Big\downarrow \text{ 1053}$$

$$-\frac{-\frac{\int -\frac{x^2\left(-bd(Abc-aBc+aAd)x^4+aBc(bc+ad)-A\left(b^2c^2+abdc+a^2d^2\right)\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{aAd-aBc+Abc}{acx}}{ac}-\frac{A}{5acx^5}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{x^2\left(-bd(Abc-aBc+aAd)x^4+aBc(bc+ad)-A\left(b^2c^2+abdc+a^2d^2\right)\right)}{\left(bx^4+a\right)\left(dx^4+c\right)}dx}{ac}-\frac{aAd-aBc+Abc}{acx}}{ac}-\frac{A}{5acx^5}$$

$$\Big\downarrow \text{ 1054}$$

$$-\frac{\frac{\int \left(-\frac{b^2(Ab-aB)c^2x^2}{(bc-ad)\left(bx^4+a\right)}-\frac{a^2d^2(Ad-Bc)x^2}{(ad-bc)\left(dx^4+c\right)}\right)dx}{ac}-\frac{aAd-aBc+Abc}{acx}}{ac}-\frac{A}{5acx^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{a^2 d^{5/4}(Bc-Ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2 d^{5/4}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2 d^{5/4}(Bc-Ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2 d^{5/4}(Bc-Ad)\,\mathrm{l}}{4}}{}$$

$$\frac{A}{5acx^5}$$

input
```
Int[(A + B*x^4)/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

output
```
-1/5*A/(a*c*x^5) - (-((A*b*c - a*B*c + a*A*d)/(a*c*x)) + ((b^(5/4)*(A*b -
a*B)*c^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c
- a*d)) - (b^(5/4)*(A*b - a*B)*c^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
)/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a^2*d^(5/4)*(B*c - A*d)*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a^2*d^(5/4)
*(B*c - A*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(
b*c - a*d)) - (b^(5/4)*(A*b - a*B)*c^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(5/4)*(A*b - a*B
)*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a
^(1/4)*(b*c - a*d)) - (a^2*d^(5/4)*(B*c - A*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(5/4
)*(B*c - A*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*S
qrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.68

method	result
default	$-\frac{b(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^2(ad-cb)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{A}{5ac\,x^5}-\frac{-Aad-Abc+aBc}{a^2c^2x}+$
risch	Expression too large to display

input
```
int((B*x^4+A)/x^6/(b*x^4+a)/(d*x^4+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8*b*(A*b-B*a)/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^(1/4)*x*
2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/5*A/a/c/x^5-1/a
^2/c^2*(-A*a*d-A*b*c+B*a*c)/x+1/8*d*(A*d-B*c)/c^2/(a*d-b*c)/(c/d)^(1/4)*2^
(1/2)*(ln((x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2+(c/d)^(1/4)*x*2^(1/
2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+2*arctan(2^(1/2)/(c/d)^
(1/4)*x-1))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^4}{x^6\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \text{Timed out}$$

input
```
integrate((B*x**4+A)/x**6/(b*x**4+a)/(d*x**4+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^4}{x^6\,(a+bx^4)\,(c+dx^4)}\,dx =$$

$$-\frac{(Ba b^2 - Ab^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8\,(a^2bc-a^3d)}$$

$$+\frac{(Bcd^2 - Ad^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{8\,(bc^3-ac^2d)}$$

$$+\frac{5\,(Aad-(Ba-Ab)c)x^4-Aac}{5\,a^2c^2x^5}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

output
```
-1/8*(B*a*b^2 - A*b^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
+ 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*
x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^
2*b*c - a^3*d) + 1/8*(B*c*d^2 - A*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
rt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1
/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt
(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4
)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 1/5*(5*(A*a*d - (B*a - A*b)*c)*x^4 - A*a*
c)/(a^2*c^2*x^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^4}{x^6(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x^4+A)/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

output
```
-1/2*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c - sqrt(2)*a^4*b*d) - 1/2
*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2*c - sqrt(2)*a^4*b*d) + 1/2*((c
*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/
d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4*d - sqrt(2)*a*c^3*d^2) + 1/2*((c*d^3
)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4*d - sqrt(2)*a*c^3*d^2) + 1/4*((a*b^3)^(3
/4)*B*a - (a*b^3)^(3/4)*A*b)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(sqrt(2)*a^3*b^2*c - sqrt(2)*a^4*b*d) - 1/4*((a*b^3)^(3/4)*B*a - (a*b^3)^(
3/4)*A*b)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^3*b^2*c
- sqrt(2)*a^4*b*d) - 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 +
 sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4*d - sqrt(2)*a*c^3*d^2)
+ 1/4*((c*d^3)^(3/4)*B*c - (c*d^3)^(3/4)*A*d)*log(x^2 - sqrt(2)*x*(c/d)^(1
/4) + sqrt(c/d))/(sqrt(2)*b*c^4*d - sqrt(2)*a*c^3*d^2) - 1/5*(5*B*a*c*x^4
- 5*A*b*c*x^4 - 5*A*a*d*x^4 + A*a*c)/(a^2*c^2*x^5)
```

Mupad [B] (verification not implemented)

Time = 12.99 (sec) , antiderivative size = 34477, normalized size of antiderivative = 83.48

$$\int \frac{A+Bx^4}{x^6(a+bx^4)(c+dx^4)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^4)/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

output
```
- (A/(5*a*c) - (x^4*(A*a*d + A*b*c - B*a*c))/(a^2*c^2))/x^5 - 2*atan((1024
*A^2*a^11*b^10*c^13*x*(-(A^4*b^9 + B^4*a^4*b^5 + 6*A^2*B^2*a^2*b^7 - 4*A^3
*B*a*b^8 - 4*A*B^3*a^3*b^6)/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^
3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*A^6*a^11*b
^6*d^9*x*(-(A^4*b^9 + B^4*a^4*b^5 + 6*A^2*B^2*a^2*b^7 - 4*A^3*B*a*b^8 - 4*
A*B^3*a^3*b^6)/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 153
6*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*B^2*a^13*b^8*c^13*x*
(-(A^4*b^9 + B^4*a^4*b^5 + 6*A^2*B^2*a^2*b^7 - 4*A^3*B*a*b^8 - 4*A*B^3*a^3
*b^6)/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^
2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*A^2*a^21*c^3*d^10*x*(-(A^4*b^
9 + B^4*a^4*b^5 + 6*A^2*B^2*a^2*b^7 - 4*A^3*B*a*b^8 - 4*A*B^3*a^3*b^6)/(25
6*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2
 - 1024*a^12*b*c*d^3))^(5/4) + 1024*B^2*a^21*c^5*d^8*x*(-(A^4*b^9 + B^4*a^
4*b^5 + 6*A^2*B^2*a^2*b^7 - 4*A^3*B*a*b^8 - 4*A*B^3*a^3*b^6)/(256*a^13*d^4
 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^
12*b*c*d^3))^(5/4) - 8*A^5*B*a^12*b^5*d^9*x*(-(A^4*b^9 + B^4*a^4*b^5 + 6*A
^2*B^2*a^2*b^7 - 4*A^3*B*a*b^8 - 4*A*B^3*a^3*b^6)/(256*a^13*d^4 + 256*a^9*
b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3)
)^(1/4) - 4096*A^2*a^12*b^9*c^12*d*x*(-(A^4*b^9 + B^4*a^4*b^5 + 6*A^2*B^2*
a^2*b^7 - 4*A^3*B*a*b^8 - 4*A*B^3*a^3*b^6)/(256*a^13*d^4 + 256*a^9*b^4*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^4}{x^6(a+bx^4)(c+dx^4)}\,dx$$

$$= \frac{-10d^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}-2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^5+10d^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}+2\sqrt{d}\,x}{d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}}\right)x^5+5d^{\frac{5}{4}}c^{\frac{3}{4}}\sqrt{2}\log\left(-d^{\frac{1}{4}}c^{\frac{1}{4}}\sqrt{2}\,x+\right.}{40c^3x^5}$$

input
```
int((B*x^4+A)/x^6/(b*x^4+a)/(d*x^4+c),x)
```

output

```
( - 10*d**(1/4)*c**(3/4)*sqrt(2)*atan((d**(1/4)*c**(1/4)*sqrt(2) - 2*sqrt(
d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))*d*x**5 + 10*d**(1/4)*c**(3/4)*sqrt(2)*a
tan((d**(1/4)*c**(1/4)*sqrt(2) + 2*sqrt(d)*x)/(d**(1/4)*c**(1/4)*sqrt(2)))
*d*x**5 + 5*d**(1/4)*c**(3/4)*sqrt(2)*log( - d**(1/4)*c**(1/4)*sqrt(2)*x +
 sqrt(c) + sqrt(d)*x**2)*d*x**5 - 5*d**(1/4)*c**(3/4)*sqrt(2)*log(d**(1/4)
*c**(1/4)*sqrt(2)*x + sqrt(c) + sqrt(d)*x**2)*d*x**5 - 8*c**2 + 40*c*d*x**
4)/(40*c**3*x**5)
```

3.19 $\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 225

$$\begin{aligned}\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n) \, dx &= \frac{a^3Ac(ex)^{1+m}}{e(1+m)} + \frac{a^2(3Abc+aBc+aAd)x^n(ex)^{1+m}}{e(1+m+n)} \\ &+ \frac{a(3Ab(bc+ad)+aB(3bc+ad))x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\ &+ \frac{b(3aB(bc+ad)+Ab(bc+3ad))x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\ &+ \frac{b^2(bBc+Abd+3aBd)x^{4n}(ex)^{1+m}}{e(1+m+4n)} + \frac{b^3Bdx^{5n}(ex)^{1+m}}{e(1+m+5n)}\end{aligned}$$

output
```
a^3*A*c*(e*x)^(1+m)/e/(1+m)+a^2*(A*a*d+3*A*b*c+B*a*c)*x^n*(e*x)^(1+m)/e/(1
+m+n)+a*(3*A*b*(a*d+b*c)+a*B*(a*d+3*b*c))*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+
b*(3*a*B*(a*d+b*c)+A*b*(3*a*d+b*c))*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+b^2*(A
*b*d+3*B*a*d+B*b*c)*x^(4*n)*(e*x)^(1+m)/e/(1+m+4*n)+b^3*B*d*x^(5*n)*(e*x)^
(1+m)/e/(1+m+5*n)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.76

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n)(c+dx^n)\,dx$$
$$=x(ex)^m\left(\frac{a^3Ac}{1+m}+\frac{a^2(3Abc+aBc+aAd)x^n}{1+m+n}+\frac{a(3Ab(bc+ad)+aB(3bc+ad))x^{2n}}{1+m+2n}+\frac{b(3aB(bc+ad)+Ab(bc+3ad))x^{3n}}{1+m+3n}+\frac{b^2(bBc+Abd+3aBd)x^{4n}}{1+m+4n}+\frac{b^3Bdx^{5n}}{1+m+5n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n),x]
```

output
```
x*(e*x)^m*((a^3*A*c)/(1 + m) + (a^2*(3*A*b*c + a*B*c + a*A*d)*x^n)/(1 + m
+ n) + (a*(3*A*b*(b*c + a*d) + a*B*(3*b*c + a*d))*x^(2*n))/(1 + m + 2*n) +
 (b*(3*a*B*(b*c + a*d) + A*b*(b*c + 3*a*d))*x^(3*n))/(1 + m + 3*n) + (b^2*
(b*B*c + A*b*d + 3*a*B*d)*x^(4*n))/(1 + m + 4*n) + (b^3*B*d*x^(5*n))/(1 +
m + 5*n))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n)(c+dx^n)\,dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(a^3Ac(ex)^m+a^2x^n(ex)^m(aAd+aBc+3Abc)+b^2x^{4n}(ex)^m(3aBd+Abd+bBc)+ax^{2n}(ex)^m(3Ab(ad+bc)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac(ex)^{m+1}}{e(m+1)}+\frac{a^2x^{n+1}(ex)^m(aAd+aBc+3Abc)}{m+n+1}+\frac{b^2x^{4n+1}(ex)^m(3aBd+Abd+bBc)}{m+4n+1}+\frac{ax^{2n+1}(ex)^m(3Ab(ad+bc)+aB(ad+3bc))}{m+2n+1}+\frac{bx^{3n+1}(ex)^m(Ab(3ad+bc)+3aB(ad+bc))}{m+3n+1}+\frac{b^3Bdx^{5n+1}(ex)^m}{m+5n+1}$$

input

```
Int[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n),x]
```

output

```
(a^2*(3*A*b*c + a*B*c + a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (a*(3*A*b*
(b*c + a*d) + a*B*(3*b*c + a*d))*x^(1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (b*(
3*a*B*(b*c + a*d) + A*b*(b*c + 3*a*d))*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n)
+ (b^2*(b*B*c + A*b*d + 3*a*B*d)*x^(1 + 4*n)*(e*x)^m)/(1 + m + 4*n) + (b^3
*B*d*x^(1 + 5*n)*(e*x)^m)/(1 + m + 5*n) + (a^3*A*c*(e*x)^(1 + m))/(e*(1 +
m))
```

Defintions of rubi rules used

rule 1040

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.37 (sec) , antiderivative size = 4939, normalized size of antiderivative = 21.95

method	result	size
risch	Expression too large to display	4939
parallelrisch	Expression too large to display	6818
orering	Expression too large to display	10171

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(36*A*a*b^2*d*m^4*n*(x^n)^3+147*A*a*b^2*d*m^3*n^2*(x^n)^3+234*A*a*b^2*d*
m^2*n^3*(x^n)^3+120*A*a*b^2*d*m*n^4*(x^n)^3+120*B*a*b^2*c*m*n^4*(x^n)^3+13
2*B*a*b^2*d*m^3*n*(x^n)^4+144*B*a*b^2*c*m^3*n*(x^n)^3+441*B*a*b^2*c*m^2*n^
2*(x^n)^3+168*A*a^2*b*c*m*n*x^n+30*B*a*b^2*d*m^2*(x^n)^4+144*A*a*b^2*d*m*n
*(x^n)^3+234*B*a^2*b*c*m^2*n*(x^n)^2+308*B*a^3*c*m*n^3*x^n+78*B*a^3*d*m^2*
n*(x^n)^2+123*B*b^3*c*m^2*n^2*(x^n)^4+122*B*b^3*c*m*n^3*(x^n)^4+60*B*b^3*d
*m^2*n*(x^n)^5+105*B*b^3*d*m*n^2*(x^n)^5+3*A*a^2*b*d*m^5*(x^n)^2+3*A*a*b^2
*c*m^5*(x^n)^2+15*A*a*b^2*d*m^4*(x^n)^3+120*A*a*b^2*d*n^4*(x^n)^3+123*B*b^
3*c*m*n^2*(x^n)^4+183*B*a*b^2*d*m^2*n^3*(x^n)^4+90*B*a*b^2*d*m*n^4*(x^n)^4
+441*A*a*b^2*d*m^2*n^2*(x^n)^3+468*A*a*b^2*d*m*n^3*(x^n)^3+39*B*a^2*b*c*m^
4*n*(x^n)^2+321*A*a^2*b*d*m^2*n^3*(x^n)^2+30*A*a*b^2*d*m^2*(x^n)^3+147*A*a
*b^2*d*n^2*(x^n)^3+48*A*b^3*c*m*n*(x^n)^3+56*B*a^3*c*m^3*n*x^n+213*B*a^3*c
*m^2*n^2*x^n+24*B*b^3*d*n^4*(x^n)^5+A*b^3*c*m^5*(x^n)^3+5*A*b^3*d*m^4*(x^n
)^4+156*B*a^2*b*c*m^3*n*(x^n)^2+531*B*a^2*b*c*m*n^2*(x^n)^2+441*A*a*b^2*d*
m*n^2*(x^n)^3+180*B*a^2*b*c*n^4*(x^n)^2+30*B*a^2*b*d*m^3*(x^n)^3+234*B*a^2
*b*d*n^3*(x^n)^3+40*B*b^3*d*m^3*n*(x^n)^5+105*B*b^3*d*m^2*n^2*(x^n)^5+100*
B*b^3*d*m*n^3*(x^n)^5+3*A*a*b^2*d*m^5*(x^n)^3+12*A*b^3*c*m^4*n*(x^n)^3+49*
A*b^3*c*m^3*n^2*(x^n)^3+(x^n)^5*b^3*B*d+(x^n)^4*A*b^3*d+39*A*a*b^2*c*m^4*n
*(x^n)^2+147*B*a^2*b*d*m^3*n^2*(x^n)^3+234*B*a^2*b*d*m^2*n^3*(x^n)^3+10*B*
b^3*d*m^4*n*(x^n)^5+35*B*b^3*d*m^3*n^2*(x^n)^5+50*B*b^3*d*m^2*n^3*(x^n)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3073 vs. $2(225) = 450$.

Time = 0.20 (sec) , antiderivative size = 3073, normalized size of antiderivative = 13.66

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64068 vs. $2(214) = 428$.

Time = 17.32 (sec) , antiderivative size = 64068, normalized size of antiderivative = 284.75

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n)(c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**3*(A+B*x**n)*(c+d*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)**3*(c + d)*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A
*a**3*c*log(x) + A*a**3*d*x**n/n + 3*A*a**2*b*c*x**n/n + 3*A*a**2*b*d*x**(
2*n)/(2*n) + 3*A*a*b**2*c*x**(2*n)/(2*n) + A*a*b**2*d*x**(3*n)/n + A*b**3*
c*x**(3*n)/(3*n) + A*b**3*d*x**(4*n)/(4*n) + B*a**3*c*x**n/n + B*a**3*d*x*
*(2*n)/(2*n) + 3*B*a**2*b*c*x**(2*n)/(2*n) + B*a**2*b*d*x**(3*n)/n + B*a*b
**2*c*x**(3*n)/n + 3*B*a*b**2*d*x**(4*n)/(4*n) + B*b**3*c*x**(4*n)/(4*n) +
 B*b**3*d*x**(5*n)/(5*n))/e, Eq(m, -1)), (A*a**3*c*Piecewise((0**(-5*n - 1
)*x, Eq(e, 0)), (Piecewise((-1/(5*n*(e*x)**(5*n)), Ne(n, 0)), (log(e*x), T
rue))/e, True)) + A*a**3*d*Piecewise((-x*x**n*(e*x)**(-5*n - 1)/(4*n), Ne(
n, 0)), (x*x**n*(e*x)**(-5*n - 1)*log(x), True)) + 3*A*a**2*b*c*Piecewise(
(-x*x**n*(e*x)**(-5*n - 1)/(4*n), Ne(n, 0)), (x*x**n*(e*x)**(-5*n - 1)*log
(x), True)) + 3*A*a**2*b*d*Piecewise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n),
 Ne(n, 0)), (x*x**(2*n)*(e*x)**(-5*n - 1)*log(x), True)) + 3*A*a*b**2*c*Pi
ecewise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*x**(2*n)*(e*x)
**(-5*n - 1)*log(x), True)) + 3*A*a*b**2*d*Piecewise((-x*x**(3*n)*(e*x)**(
-5*n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(e*x)**(-5*n - 1)*log(x), True)) +
 A*b**3*c*Piecewise((-x*x**(3*n)*(e*x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x*
*(3*n)*(e*x)**(-5*n - 1)*log(x), True)) + A*b**3*d*Piecewise((-x*x**(4*n)*
(e*x)**(-5*n - 1)/n, Ne(n, 0)), (x*x**(4*n)*(e*x)**(-5*n - 1)*log(x), True
)) + B*a**3*c*Piecewise((-x*x**n*(e*x)**(-5*n - 1)/(4*n), Ne(n, 0)), (x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(225) = 450$.

Time = 0.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.06

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n)(c+dx^n)\,dx = \frac{Bb^3de^mxe^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{Bb^3ce^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,Bab^2de^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{Ab^3de^mxe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,Bab^2ce^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Ab^3ce^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Ba^2bde^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Aab^2de^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Ba^2bce^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Aab^2ce^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ba^3de^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Aa^2bde^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ba^3ce^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{3\,Aa^2bce^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Aa^3de^mxe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}\,Aa^3c}{e(m+1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
B*b^3*d*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + B*b^3*c*e^m*x*e^(m
*log(x) + 4*n*log(x))/(m + 4*n + 1) + 3*B*a*b^2*d*e^m*x*e^(m*log(x) + 4*n*
log(x))/(m + 4*n + 1) + A*b^3*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n +
 1) + 3*B*a*b^2*c*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + A*b^3*c*
e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*B*a^2*b*d*e^m*x*e^(m*log
(x) + 3*n*log(x))/(m + 3*n + 1) + 3*A*a*b^2*d*e^m*x*e^(m*log(x) + 3*n*log(
x))/(m + 3*n + 1) + 3*B*a^2*b*c*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n +
 1) + 3*A*a*b^2*c*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*a^3*d*
e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*A*a^2*b*d*e^m*x*e^(m*log
(x) + 2*n*log(x))/(m + 2*n + 1) + B*a^3*c*e^m*x*e^(m*log(x) + n*log(x))/(m
 + n + 1) + 3*A*a^2*b*c*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + A*a^3*
d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*a^3*c/(e*(m
+ 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27992 vs. $2(225) = 450$.

Time = 0.32 (sec) , antiderivative size = 27992, normalized size of antiderivative = 124.41

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(B*b^3*d*m^5*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 10*B*b^3*d*m^4*n*x*x^(5*n
)*e^(m*log(e) + m*log(x)) + 35*B*b^3*d*m^3*n^2*x*x^(5*n)*e^(m*log(e) + m*l
og(x)) + 50*B*b^3*d*m^2*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 24*B*b^3*d
*m*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + B*b^3*c*m^5*x*x^(4*n)*e^(m*log(
e) + m*log(x)) + 3*B*a*b^2*d*m^5*x*x^(4*n)*e^(m*log(e) + m*log(x)) + A*b^3
*d*m^5*x*x^(4*n)*e^(m*log(e) + m*log(x)) + B*b^3*d*m^5*x*x^(4*n)*e^(m*log(
e) + m*log(x)) + 11*B*b^3*c*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 33*B
*a*b^2*d*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 11*A*b^3*d*m^4*n*x*x^(4
*n)*e^(m*log(e) + m*log(x)) + 10*B*b^3*d*m^4*n*x*x^(4*n)*e^(m*log(e) + m*l
og(x)) + 41*B*b^3*c*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 123*B*a*b^
2*d*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 41*A*b^3*d*m^3*n^2*x*x^(4*
n)*e^(m*log(e) + m*log(x)) + 35*B*b^3*d*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*
log(x)) + 61*B*b^3*c*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 183*B*a*b
^2*d*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 61*A*b^3*d*m^2*n^3*x*x^(4
*n)*e^(m*log(e) + m*log(x)) + 50*B*b^3*d*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m
*log(x)) + 30*B*b^3*c*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 90*B*a*b^2
*d*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 30*A*b^3*d*m*n^4*x*x^(4*n)*e^
(m*log(e) + m*log(x)) + 24*B*b^3*d*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x))
 + 3*B*a*b^2*c*m^5*x*x^(3*n)*e^(m*log(e) + m*log(x)) + A*b^3*c*m^5*x*x^(3*
n)*e^(m*log(e) + m*log(x)) + B*b^3*c*m^5*x*x^(3*n)*e^(m*log(e) + m*log(...
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.84

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n) \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^3*(c + d*x^n),x)
```

output
```
(A*a^3*c*x*(e*x)^m)/(m + 1) + (b^2*x*x^(4*n)*(e*x)^m*(A*b*d + 3*B*a*d + B*
b*c)*(4*m + 11*n + 33*m*n + 82*m*n^2 + 33*m^2*n + 61*m*n^3 + 11*m^3*n + 6*
m^2 + 4*m^3 + m^4 + 41*n^2 + 61*n^3 + 30*n^4 + 41*m^2*n^2 + 1))/(5*m + 15*
n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*
m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n
^5 + 255*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (a*x*x^(2*n)*(e*x)^m*(3
*A*b^2*c + B*a^2*d + 3*A*a*b*d + 3*B*a*b*c)*(4*m + 13*n + 39*m*n + 118*m*n
^2 + 39*m^2*n + 107*m*n^3 + 13*m^3*n + 6*m^2 + 4*m^3 + m^4 + 59*n^2 + 107*
n^3 + 60*n^4 + 59*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*
n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5*m^4
+ m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 + 225*m^2*n^3 +
 85*m^3*n^2 + 1) + (b*x*x^(3*n)*(e*x)^m*(A*b^2*c + 3*B*a^2*d + 3*A*a*b*d +
 3*B*a*b*c)*(4*m + 12*n + 36*m*n + 98*m*n^2 + 36*m^2*n + 78*m*n^3 + 12*m^3
*n + 6*m^2 + 4*m^3 + m^4 + 49*n^2 + 78*n^3 + 40*n^4 + 49*m^2*n^2 + 1))/(5*
m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^
4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4
+ 120*n^5 + 255*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (a^2*x*x^n*(e*x)
^m*(A*a*d + 3*A*b*c + B*a*c)*(4*m + 14*n + 42*m*n + 142*m*n^2 + 42*m^2*n +
 154*m*n^3 + 14*m^3*n + 6*m^2 + 4*m^3 + m^4 + 71*n^2 + 154*n^3 + 120*n^4 +
 71*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3053, normalized size of antiderivative = 13.57

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n) \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n),x)
```

output
```
(x**m*e**m*x*(x**(5*n)*b**4*d*m**5 + 10*x**(5*n)*b**4*d*m**4*n + 5*x**(5*n
)*b**4*d*m**4 + 35*x**(5*n)*b**4*d*m**3*n**2 + 40*x**(5*n)*b**4*d*m**3*n +
 10*x**(5*n)*b**4*d*m**3 + 50*x**(5*n)*b**4*d*m**2*n**3 + 105*x**(5*n)*b**
4*d*m**2*n**2 + 60*x**(5*n)*b**4*d*m**2*n + 10*x**(5*n)*b**4*d*m**2 + 24*x
**(5*n)*b**4*d*m*n**4 + 100*x**(5*n)*b**4*d*m*n**3 + 105*x**(5*n)*b**4*d*m
*n**2 + 40*x**(5*n)*b**4*d*m*n + 5*x**(5*n)*b**4*d*m + 24*x**(5*n)*b**4*d*
n**4 + 50*x**(5*n)*b**4*d*n**3 + 35*x**(5*n)*b**4*d*n**2 + 10*x**(5*n)*b**
4*d*n + x**(5*n)*b**4*d + 4*x**(4*n)*a*b**3*d*m**5 + 44*x**(4*n)*a*b**3*d*
m**4*n + 20*x**(4*n)*a*b**3*d*m**4 + 164*x**(4*n)*a*b**3*d*m**3*n**2 + 176
*x**(4*n)*a*b**3*d*m**3*n + 40*x**(4*n)*a*b**3*d*m**3 + 244*x**(4*n)*a*b**
3*d*m**2*n**3 + 492*x**(4*n)*a*b**3*d*m**2*n**2 + 264*x**(4*n)*a*b**3*d*m*
*2*n + 40*x**(4*n)*a*b**3*d*m**2 + 120*x**(4*n)*a*b**3*d*m*n**4 + 488*x**(
4*n)*a*b**3*d*m*n**3 + 492*x**(4*n)*a*b**3*d*m*n**2 + 176*x**(4*n)*a*b**3*
d*m*n + 20*x**(4*n)*a*b**3*d*m + 120*x**(4*n)*a*b**3*d*n**4 + 244*x**(4*n)
*a*b**3*d*n**3 + 164*x**(4*n)*a*b**3*d*n**2 + 44*x**(4*n)*a*b**3*d*n + 4*x
**(4*n)*a*b**3*d + x**(4*n)*b**4*c*m**5 + 11*x**(4*n)*b**4*c*m**4*n + 5*x*
*(4*n)*b**4*c*m**4 + 41*x**(4*n)*b**4*c*m**3*n**2 + 44*x**(4*n)*b**4*c*m**
3*n + 10*x**(4*n)*b**4*c*m**3 + 61*x**(4*n)*b**4*c*m**2*n**3 + 123*x**(4*n
)*b**4*c*m**2*n**2 + 66*x**(4*n)*b**4*c*m**2*n + 10*x**(4*n)*b**4*c*m**2 +
 30*x**(4*n)*b**4*c*m*n**4 + 122*x**(4*n)*b**4*c*m*n**3 + 123*x**(4*n)*...
```

3.20 $\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 172

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n) \, dx = \frac{a^2Ac(ex)^{1+m}}{e(1+m)} + \frac{a(2Abc+aBc+aAd)x^n(ex)^{1+m}}{e(1+m+n)} + \frac{(aB(2bc+ad)+Ab(bc+2ad))x^{2n}(ex)^{1+m}}{e(1+m+2n)} + \frac{b(bBc+Abd+2aBd)x^{3n}(ex)^{1+m}}{e(1+m+3n)} + \frac{b^2Bdx^{4n}(ex)^{1+m}}{e(1+m+4n)}$$

output
```
a^2*A*c*(e*x)^(1+m)/e/(1+m)+a*(A*a*d+2*A*b*c+B*a*c)*x^n*(e*x)^(1+m)/e/(1+m
+n)+(a*B*(a*d+2*b*c)+A*b*(2*a*d+b*c))*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+b*(A
*b*d+2*B*a*d+B*b*c)*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+b^2*B*d*x^(4*n)*(e*x)^
(1+m)/e/(1+m+4*n)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n)(c+dx^n)\,dx = x(ex)^m \left(\frac{a^2Ac}{1+m}+\frac{a(2Abc+aBc+aAd)x^n}{1+m+n}+\frac{(aB(2bc+ad)+Ab(bc+2ad))x^{2n}}{1+m+2n}+\frac{b(bBc+Abd+2aBd)x^{3n}}{1+m+3n}+\frac{b^2Bdx^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n),x]
```

output
```
x*(e*x)^m*((a^2*A*c)/(1 + m) + (a*(2*A*b*c + a*B*c + a*A*d)*x^n)/(1 + m +
n) + ((a*B*(2*b*c + a*d) + A*b*(b*c + 2*a*d))*x^(2*n))/(1 + m + 2*n) + (b*
(b*B*c + A*b*d + 2*a*B*d)*x^(3*n))/(1 + m + 3*n) + (b^2*B*d*x^(4*n))/(1 +
m + 4*n))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n)(c+dx^n)\,dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(a^2Ac(ex)^m + x^{2n}(ex)^m(Ab(2ad+bc)+aB(ad+2bc)) + bx^{3n}(ex)^m(2aBd+Abd+bBc) + ax^n(ex)^m(aAd\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2Ac(ex)^{m+1}}{e(m+1)}+\frac{ax^{n+1}(ex)^m(aAd+aBc+2Abc)}{m+n+1}+\frac{x^{2n+1}(ex)^m(Ab(2ad+bc)+aB(ad+2bc))}{m+2n+1}+\frac{bx^{3n+1}(ex)^m(2aBd+Abd+bBc)}{m+3n+1}+\frac{b^2Bdx^{4n+1}(ex)^m}{m+4n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n),x]
```

output
```
(a*(2*A*b*c + a*B*c + a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + ((a*B*(2*b*c
 + a*d) + A*b*(b*c + 2*a*d))*x^(1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (b*(b*B*
c + A*b*d + 2*a*B*d)*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n) + (b^2*B*d*x^(1 +
4*n)*(e*x)^m)/(1 + m + 4*n) + (a^2*A*c*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 2377, normalized size of antiderivative = 13.82

method	result	size
risch	Expression too large to display	2377
parallelrisch	Expression too large to display	3344
orering	Expression too large to display	4757

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(48*A*a*b*d*m*n*(x^n)^2+8*B*a^2*d*m^3*n*(x^n)^2+19*B*a^2*d*m^2*n^2*(x^n)
^2+12*B*a^2*d*m*n^3*(x^n)^2+2*B*a*b*c*m^4*(x^n)^2+8*B*a*b*d*m^3*(x^n)^3+16
*B*a*b*d*n^3*(x^n)^3+8*B*b^2*c*m*n^3*(x^n)^3+18*B*b^2*d*m^2*n*(x^n)^4+22*B
*b^2*d*m*n^2*(x^n)^4+8*B*a^2*d*(x^n)^2*n+A*a^2*d*x^n+B*a^2*c*x^n+19*B*a^2*
d*n^2*(x^n)^2+4*B*b^2*c*(x^n)^3*m+7*B*b^2*c*(x^n)^3*n+12*A*b^2*c*m*n^3*(x^
n)^2+21*A*b^2*d*m^2*n*(x^n)^3+28*A*b^2*d*m*n^2*(x^n)^3+b^2*B*d*(x^n)^4+A*b
^2*d*(x^n)^3+B*b^2*c*(x^n)^3+A*b^2*c*(x^n)^2+B*a^2*d*(x^n)^2+24*A*a^2*c*n^
4+A*a^2*c*m^4+4*A*a^2*c*m^3+50*A*a^2*c*n^3+6*A*a^2*c*m^2+35*A*a^2*c*n^2+4*
a^2*A*c*m+10*a^2*A*c*n+8*A*a*b*d*m^3*(x^n)^2+24*A*a*b*d*n^3*(x^n)^2+24*A*b
^2*c*m^2*n*(x^n)^2+38*A*b^2*c*m*n^2*(x^n)^2+21*A*b^2*d*m*n*(x^n)^3+9*B*a^2
*c*m^3*n*x^n+a^2*A*c+26*B*a^2*c*m^2*n^2*x^n+24*B*a^2*c*m*n^3*x^n+24*B*a^2*
d*m^2*n*(x^n)^2+38*B*a^2*d*m*n^2*(x^n)^2+8*B*a*b*c*m^3*(x^n)^2+24*B*a*b*c*
n^3*(x^n)^2+12*B*a*b*d*m^2*(x^n)^3+11*B*b^2*d*m^2*n^2*(x^n)^4+6*B*b^2*d*m*
n^3*(x^n)^4+7*A*b^2*d*m^3*n*(x^n)^3+14*A*b^2*d*m^2*n^2*(x^n)^3+8*A*b^2*d*m
*n^3*(x^n)^3+28*B*a*b*d*n^2*(x^n)^3+14*B*a*b*d*m^3*n*(x^n)^3+28*B*a*b*d*m^
2*n^2*(x^n)^3+14*B*a*b*d*(x^n)^3*n+27*A*a^2*d*m*n*x^n+12*A*a*b*c*m^2*x^n+5
2*A*a*b*c*n^2*x^n+8*A*a*b*d*(x^n)^2*m+16*A*a*b*d*(x^n)^2*n+27*B*a^2*c*m*n*
x^n+16*B*a*b*d*m*n^3*(x^n)^3+16*A*a*b*d*m^3*n*(x^n)^2+38*A*a*b*d*m^2*n^2*(
x^n)^2+24*A*a*b*d*m*n^3*(x^n)^2+16*B*a*b*c*m^3*n*(x^n)^2+38*B*a*b*c*m^2*n^
2*(x^n)^2+8*A*a*b*c*m^3*x^n+48*A*a*b*c*n^3*x^n+12*A*a*b*d*m^2*(x^n)^2+3...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. $2(172) = 344$.

Time = 0.17 (sec) , antiderivative size = 1524, normalized size of antiderivative = 8.86

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
((B*b^2*d*m^4 + 4*B*b^2*d*m^3 + 6*B*b^2*d*m^2 + 4*B*b^2*d*m + B*b^2*d + 6*
(B*b^2*d*m + B*b^2*d)*n^3 + 11*(B*b^2*d*m^2 + 2*B*b^2*d*m + B*b^2*d)*n^2 +
 6*(B*b^2*d*m^3 + 3*B*b^2*d*m^2 + 3*B*b^2*d*m + B*b^2*d)*n)*x*x^(4*n)*e^(m
*log(e) + m*log(x)) + ((B*b^2*c + (2*B*a*b + A*b^2)*d)*m^4 + B*b^2*c + 4*(
B*b^2*c + (2*B*a*b + A*b^2)*d)*m^3 + 8*(B*b^2*c + (2*B*a*b + A*b^2)*d + (B
*b^2*c + (2*B*a*b + A*b^2)*d)*m)*n^3 + 6*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m
^2 + 14*(B*b^2*c + (B*b^2*c + (2*B*a*b + A*b^2)*d)*m^2 + (2*B*a*b + A*b^2)
*d + 2*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m)*n^2 + (2*B*a*b + A*b^2)*d + 4*(B
*b^2*c + (2*B*a*b + A*b^2)*d)*m + 7*(B*b^2*c + (B*b^2*c + (2*B*a*b + A*b^2
)*d)*m^3 + 3*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m^2 + (2*B*a*b + A*b^2)*d + 3
*(B*b^2*c + (2*B*a*b + A*b^2)*d)*m)*n)*x*x^(3*n)*e^(m*log(e) + m*log(x)) +
 (((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m^4 + 4*((2*B*a*b + A*b^2)*c
 + (B*a^2 + 2*A*a*b)*d)*m^3 + 12*((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*
d + ((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m)*n^3 + 6*((2*B*a*b + A*b
^2)*c + (B*a^2 + 2*A*a*b)*d)*m^2 + 19*(((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A
*a*b)*d)*m^2 + (2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d + 2*((2*B*a*b + A
*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m)*n^2 + (2*B*a*b + A*b^2)*c + (B*a^2 + 2*A
*a*b)*d + 4*((2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m + 8*(((2*B*a*b +
 A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*m^3 + 3*((2*B*a*b + A*b^2)*c + (B*a^2 + 2
*A*a*b)*d)*m^2 + (2*B*a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d + 3*((2*B*a*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25315 vs. $2(163) = 326$.

Time = 9.84 (sec) , antiderivative size = 25315, normalized size of antiderivative = 147.18

$$\int (ex)^m (a + bx^n)^2 (A + Bx^n)(c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)*(c+d*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)**2*(c + d)*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A
*a**2*c*log(x) + A*a**2*d*x**n/n + 2*A*a*b*c*x**n/n + A*a*b*d*x**(2*n)/n +
 A*b**2*c*x**(2*n)/(2*n) + A*b**2*d*x**(3*n)/(3*n) + B*a**2*c*x**n/n + B*a
**2*d*x**(2*n)/(2*n) + B*a*b*c*x**(2*n)/n + 2*B*a*b*d*x**(3*n)/(3*n) + B*b
**2*c*x**(3*n)/(3*n) + B*b**2*d*x**(4*n)/(4*n))/e, Eq(m, -1)), (A*a**2*c*P
iecewise((0**(-4*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(4*n*(e*x)**(4*n)), N
e(n, 0)), (log(e*x), True))/e, True)) + A*a**2*d*Piecewise((-x*x**n*(e*x)*
*(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(e*x)**(-4*n - 1)*log(x), True)) + 2
*A*a*b*c*Piecewise((-x*x**n*(e*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(e
*x)**(-4*n - 1)*log(x), True)) + 2*A*a*b*d*Piecewise((-x*x**(2*n)*(e*x)**(
-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-4*n - 1)*log(x), True)) +
 A*b**2*c*Piecewise((-x*x**(2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x*
*(2*n)*(e*x)**(-4*n - 1)*log(x), True)) + A*b**2*d*Piecewise((-x*x**(3*n)*
(e*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(e*x)**(-4*n - 1)*log(x), True
)) + B*a**2*c*Piecewise((-x*x**n*(e*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x*
*n*(e*x)**(-4*n - 1)*log(x), True)) + B*a**2*d*Piecewise((-x*x**(2*n)*(e*x
)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-4*n - 1)*log(x), True
)) + 2*B*a*b*c*Piecewise((-x*x**(2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)),
(x*x**(2*n)*(e*x)**(-4*n - 1)*log(x), True)) + 2*B*a*b*d*Piecewise((-x*x**
(3*n)*(e*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(e*x)**(-4*n - 1)*log...
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.93

$$\int (ex)^m (a + bx^n)^2 (A + Bx^n) (c + dx^n) \, dx = \frac{Bb^2de^m xe^{(m\log(x) + 4n\log(x))}}{m + 4n + 1} + \frac{Bb^2ce^m xe^{(m\log(x) + 3n\log(x))}}{m + 3n + 1} + \frac{2Babde^m xe^{(m\log(x) + 3n\log(x))}}{m + 3n + 1} + \frac{Ab^2de^m xe^{(m\log(x) + 3n\log(x))}}{m + 3n + 1} + \frac{2Babce^m xe^{(m\log(x) + 2n\log(x))}}{m + 2n + 1} + \frac{Ab^2ce^m xe^{(m\log(x) + 2n\log(x))}}{m + 2n + 1} + \frac{Ba^2de^m xe^{(m\log(x) + 2n\log(x))}}{m + 2n + 1} + \frac{2Aabde^m xe^{(m\log(x) + 2n\log(x))}}{m + 2n + 1} + \frac{Ba^2ce^m xe^{(m\log(x) + n\log(x))}}{m + n + 1} + \frac{2Aabce^m xe^{(m\log(x) + n\log(x))}}{m + n + 1} + \frac{Aa^2de^m xe^{(m\log(x) + n\log(x))}}{m + n + 1} + \frac{(ex)^{m+1} Aa^2c}{e(m + 1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
B*b^2*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + B*b^2*c*e^m*x*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 1) + 2*B*a*b*d*e^m*x*e^(m*log(x) + 3*n*lo
g(x))/(m + 3*n + 1) + A*b^2*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1
) + 2*B*a*b*c*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + A*b^2*c*e^m*
x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*a^2*d*e^m*x*e^(m*log(x) + 2*
n*log(x))/(m + 2*n + 1) + 2*A*a*b*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2
*n + 1) + B*a^2*c*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a*b*c*e^
m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + A*a^2*d*e^m*x*e^(m*log(x) + n*lo
g(x))/(m + n + 1) + (e*x)^(m + 1)*A*a^2*c/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11834 vs. $2(172) = 344$.

Time = 0.21 (sec) , antiderivative size = 11834, normalized size of antiderivative = 68.80

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(B*b^2*d*m^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 6*B*b^2*d*m^3*n*x*x^(4*n)
*e^(m*log(e) + m*log(x)) + 11*B*b^2*d*m^2*n^2*x*x^(4*n)*e^(m*log(e) + m*lo
g(x)) + 6*B*b^2*d*m*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + B*b^2*c*m^4*x*
x^(3*n)*e^(m*log(e) + m*log(x)) + 2*B*a*b*d*m^4*x*x^(3*n)*e^(m*log(e) + m*
log(x)) + A*b^2*d*m^4*x*x^(3*n)*e^(m*log(e) + m*log(x)) + B*b^2*d*m^4*x*x^
(3*n)*e^(m*log(e) + m*log(x)) + 7*B*b^2*c*m^3*n*x*x^(3*n)*e^(m*log(e) + m*
log(x)) + 14*B*a*b*d*m^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 7*A*b^2*d*m
^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 6*B*b^2*d*m^3*n*x*x^(3*n)*e^(m*lo
g(e) + m*log(x)) + 14*B*b^2*c*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) +
28*B*a*b*d*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 14*A*b^2*d*m^2*n^2*
x*x^(3*n)*e^(m*log(e) + m*log(x)) + 11*B*b^2*d*m^2*n^2*x*x^(3*n)*e^(m*log(
e) + m*log(x)) + 8*B*b^2*c*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 16*B*
a*b*d*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 8*A*b^2*d*m*n^3*x*x^(3*n)*
e^(m*log(e) + m*log(x)) + 6*B*b^2*d*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)
) + 2*B*a*b*c*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + A*b^2*c*m^4*x*x^(2*n
)*e^(m*log(e) + m*log(x)) + B*b^2*c*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x))
+ B*a^2*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 2*A*a*b*d*m^4*x*x^(2*n)*
e^(m*log(e) + m*log(x)) + 2*B*a*b*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x))
+ A*b^2*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*b^2*d*m^4*x*x^(2*n)*e^
(m*log(e) + m*log(x)) + 16*B*a*b*c*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(...
```

Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.42

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n)(c+dx^n)\,dx$$
$$= \frac{x\,x^{2\,n}\,(e\,x)^m\,(A\,b^2\,c+B\,a^2\,d+2\,A\,a\,b\,d+2\,B\,a\,b\,c)\,(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$
$$+\frac{A\,a^2\,c\,x\,(e\,x)^m}{m+1}$$
$$+\frac{a\,x\,x^n\,(e\,x)^m\,(A\,a\,d+2\,A\,b\,c+B\,a\,c)\,(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$
$$+\frac{b\,x\,x^{3\,n}\,(e\,x)^m\,(A\,b\,d+2\,B\,a\,d+B\,b\,c)\,(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$
$$+\frac{B\,b^2\,d\,x\,x^{4\,n}\,(e\,x)^m\,(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^2*(c + d*x^n),x)
```

output
```
(x*x^(2*n)*(e*x)^m*(A*b^2*c + B*a^2*d + 2*A*a*b*d + 2*B*a*b*c)*(3*m + 8*n
+ 16*m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m +
 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3
 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (A*a^2*c*x*(e*x)^m)/
(m + 1) + (a*x*x^n*(e*x)^m*(A*a*d + 2*A*b*c + B*a*c)*(3*m + 9*n + 18*m*n +
 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3 + 1))/(4*m + 10*n + 30
*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 3
5*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (b*x*x^(3*n)*(e*x)^m*(A*b*d +
2*B*a*d + B*b*c)*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 +
14*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3
 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2
+ 1) + (B*b^2*d*x*x^(4*n)*(e*x)^m*(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n
 + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30
*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*
n^4 + 35*m^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1607, normalized size of antiderivative = 9.34

$$\int (ex)^m (a + bx^n)^2 (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n),x)
```

output

```
(x**m*e**m*x*(x**(4*n)*b**3*d*m**4 + 6*x**(4*n)*b**3*d*m**3*n + 4*x**(4*n)
*b**3*d*m**3 + 11*x**(4*n)*b**3*d*m**2*n**2 + 18*x**(4*n)*b**3*d*m**2*n +
6*x**(4*n)*b**3*d*m**2 + 6*x**(4*n)*b**3*d*m*n**3 + 22*x**(4*n)*b**3*d*m*n
**2 + 18*x**(4*n)*b**3*d*m*n + 4*x**(4*n)*b**3*d*m + 6*x**(4*n)*b**3*d*n**
3 + 11*x**(4*n)*b**3*d*n**2 + 6*x**(4*n)*b**3*d*n + x**(4*n)*b**3*d + 3*x*
*(3*n)*a*b**2*d*m**4 + 21*x**(3*n)*a*b**2*d*m**3*n + 12*x**(3*n)*a*b**2*d*
m**3 + 42*x**(3*n)*a*b**2*d*m**2*n**2 + 63*x**(3*n)*a*b**2*d*m**2*n + 18*x
**(3*n)*a*b**2*d*m**2 + 24*x**(3*n)*a*b**2*d*m*n**3 + 84*x**(3*n)*a*b**2*d
*m*n**2 + 63*x**(3*n)*a*b**2*d*m*n + 12*x**(3*n)*a*b**2*d*m + 24*x**(3*n)*
a*b**2*d*n**3 + 42*x**(3*n)*a*b**2*d*n**2 + 21*x**(3*n)*a*b**2*d*n + 3*x**
(3*n)*a*b**2*d + x**(3*n)*b**3*c*m**4 + 7*x**(3*n)*b**3*c*m**3*n + 4*x**(3
*n)*b**3*c*m**3 + 14*x**(3*n)*b**3*c*m**2*n**2 + 21*x**(3*n)*b**3*c*m**2*n
 + 6*x**(3*n)*b**3*c*m**2 + 8*x**(3*n)*b**3*c*m*n**3 + 28*x**(3*n)*b**3*c*
m*n**2 + 21*x**(3*n)*b**3*c*m*n + 4*x**(3*n)*b**3*c*m + 8*x**(3*n)*b**3*c*
n**3 + 14*x**(3*n)*b**3*c*n**2 + 7*x**(3*n)*b**3*c*n + x**(3*n)*b**3*c + 3
*x**(2*n)*a**2*b*d*m**4 + 24*x**(2*n)*a**2*b*d*m**3*n + 12*x**(2*n)*a**2*b
*d*m**3 + 57*x**(2*n)*a**2*b*d*m**2*n**2 + 72*x**(2*n)*a**2*b*d*m**2*n + 1
8*x**(2*n)*a**2*b*d*m**2 + 36*x**(2*n)*a**2*b*d*m*n**3 + 114*x**(2*n)*a**2
*b*d*m*n**2 + 72*x**(2*n)*a**2*b*d*m*n + 12*x**(2*n)*a**2*b*d*m + 36*x**(2
*n)*a**2*b*d*n**3 + 57*x**(2*n)*a**2*b*d*n**2 + 24*x**(2*n)*a**2*b*d*n ...
```

3.21 $\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n) \, dx$

Optimal result

Integrand size = 27, antiderivative size = 117

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n) \, dx = \frac{aAc(ex)^{1+m}}{e(1+m)} + \frac{(Abc+aBc+aAd)x^n(ex)^{1+m}}{e(1+m+n)} + \frac{(bBc+Abd+aBd)x^{2n}(ex)^{1+m}}{e(1+m+2n)} + \frac{bBdx^{3n}(ex)^{1+m}}{e(1+m+3n)}$$

output
```
a*A*c*(e*x)^(1+m)/e/(1+m)+(A*a*d+A*b*c+B*a*c)*x^n*(e*x)^(1+m)/e/(1+m+n)+(A
*b*d+B*a*d+B*b*c)*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+b*B*d*x^(3*n)*(e*x)^(1+m
)/e/(1+m+3*n)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n) \, dx = x(ex)^m \left(\frac{aAc}{1+m} + \frac{(Abc+aBc+aAd)x^n}{1+m+n} + \frac{(bBc+Abd+aBd)x^{2n}}{1+m+2n} + \frac{bBdx^{3n}}{1+m+3n} \right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n),x]
```

output
```
x*(e*x)^m*((a*A*c)/(1 + m) + ((A*b*c + a*B*c + a*A*d)*x^n)/(1 + m + n) + (
(b*B*c + A*b*d + a*B*d)*x^(2*n))/(1 + m + 2*n) + (b*B*d*x^(3*n))/(1 + m +
3*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)\,dx$$

$$\Big\downarrow \ 1040$$

$$\int \left(x^{2n}(ex)^m(aBd+Abd+bBc)+x^n(ex)^m(aAd+aBc+Abc)+aAc(ex)^m+bBdx^{3n}(ex)^m\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^{n+1}(ex)^m(aAd+aBc+Abc)}{m+n+1}+\frac{x^{2n+1}(ex)^m(aBd+Abd+bBc)}{m+2n+1}+\frac{aAc(ex)^{m+1}}{e(m+1)}+\frac{bBdx^{3n+1}(ex)^m}{m+3n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n),x]
```

output
```
((A*b*c + a*B*c + a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + ((b*B*c + A*b*d
+ a*B*d)*x^(1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (b*B*d*x^(1 + 3*n)*(e*x)^m)/
(1 + m + 3*n) + (a*A*c*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.30 (sec) , antiderivative size = 858, normalized size of antiderivative = 7.33

method	result
risch	$\frac{x\left(10Aadmn\,x^{n}+10Abcmn\,x^{n}+10Bacmn\,x^{n}+3Bbd\,m^{2}n\,x^{3n}+8Badmn\,x^{2n}+8Bbcmn\,x^{2n}+4Bbc\,m^{2}n\,x^{2n}+3Bbcm\,n^{2}x^{2n}\cdots\right.}{}$
parallelrisch	Expression too large to display
orering	Expression too large to display

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(10*A*a*d*m*n*x^n+10*A*b*c*m*n*x^n+10*B*a*c*m*n*x^n+3*B*b*d*m^2*n*(x^n)^
3+B*b*c*m^3*(x^n)^2+3*B*b*d*m^2*(x^n)^3+2*B*b*d*n^2*(x^n)^3+A*a*d*m^3*x^n+
A*b*c*m^3*x^n+3*A*b*d*m^2*(x^n)^2+3*A*b*d*n^2*(x^n)^2+B*a*c*m^3*x^n+3*B*a*
d*m^2*(x^n)^2+3*B*a*d*n^2*(x^n)^2+A*a*c+8*B*a*d*m*n*(x^n)^2+8*B*b*c*m*n*(x
^n)^2+6*A*a*c*n+3*B*x^n*a*c*m+5*B*x^n*a*c*n+3*A*x^n*b*c*m+5*A*x^n*b*c*n+3*
A*x^n*a*d*m+5*A*x^n*a*d*n+4*B*(x^n)^2*b*c*n+3*B*(x^n)^3*b*d*n+3*A*(x^n)^2*
b*d*m+4*A*(x^n)^2*b*d*n+3*B*(x^n)^2*a*d*m+4*B*(x^n)^2*a*d*n+3*B*(x^n)^2*b*
c*m+3*B*(x^n)^3*b*d*m+3*A*a*c*m+3*B*a*c*m^2*x^n+6*B*a*c*n^2*x^n+B*a*d*(x^n
)^2+B*b*c*(x^n)^2+d*a*x^n*A+c*A*x^n*b+c*B*x^n*a+B*b*d*(x^n)^3+A*b*d*(x^n)^
2+A*a*c*m^3+3*A*a*c*m^2+11*A*a*c*n^2+6*A*a*c*n^3+3*B*b*c*m^2*(x^n)^2+3*B*b
*c*n^2*(x^n)^2+3*A*a*d*m^2*x^n+6*A*a*d*n^2*x^n+3*A*b*c*m^2*x^n+6*A*b*c*n^2
*x^n+12*A*a*c*m*n+4*B*b*c*m^2*n*(x^n)^2+3*B*b*c*m*n^2*(x^n)^2+6*B*b*d*m*n*
(x^n)^3+6*A*a*c*m^2*n+11*A*a*c*m*n^2+B*b*d*m^3*(x^n)^3+A*b*d*m^3*(x^n)^2+B
*a*d*m^3*(x^n)^2+3*B*a*d*m*n^2*(x^n)^2+5*A*a*d*m^2*n*x^n+6*A*a*d*m*n^2*x^n
+5*A*b*c*m^2*n*x^n+6*A*b*c*m*n^2*x^n+8*A*b*d*m*n*(x^n)^2+5*B*a*c*m^2*n*x^n
+6*B*a*c*m*n^2*x^n+2*B*b*d*m*n^2*(x^n)^3+4*A*b*d*m^2*n*(x^n)^2+3*A*b*d*m*n
^2*(x^n)^2+4*B*a*d*m^2*n*(x^n)^2)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*e^m*x^
m*exp(1/2*I*Pi*csgn(I*e*x)*m*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*
e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(117) = 234$.

Time = 0.14 (sec) , antiderivative size = 562, normalized size of antiderivative = 4.80

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n) \, dx = \frac{(Bbdm^3 + 3\,Bbdm^2 + 3\,Bbdm + Bbd + 2\,(Bbdm + Bbd)n^2 + 3\,(Bbdm^2 + 2\,Bbdm + Bbd)n)xx^{3\,n}e^{(m\log}}{}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
((B*b*d*m^3 + 3*B*b*d*m^2 + 3*B*b*d*m + B*b*d + 2*(B*b*d*m + B*b*d)*n^2 +
3*(B*b*d*m^2 + 2*B*b*d*m + B*b*d)*n)*x*x^(3*n)*e^(m*log(e) + m*log(x)) + (
(B*b*c + (B*a + A*b)*d)*m^3 + B*b*c + 3*(B*b*c + (B*a + A*b)*d)*m^2 + 3*(B
*b*c + (B*a + A*b)*d + (B*b*c + (B*a + A*b)*d)*m)*n^2 + (B*a + A*b)*d + 3*
(B*b*c + (B*a + A*b)*d)*m + 4*(B*b*c + (B*b*c + (B*a + A*b)*d)*m^2 + (B*a
+ A*b)*d + 2*(B*b*c + (B*a + A*b)*d)*m)*n)*x*x^(2*n)*e^(m*log(e) + m*log(x
)) + ((A*a*d + (B*a + A*b)*c)*m^3 + A*a*d + 3*(A*a*d + (B*a + A*b)*c)*m^2
+ 6*(A*a*d + (B*a + A*b)*c + (A*a*d + (B*a + A*b)*c)*m)*n^2 + (B*a + A*b)*
c + 3*(A*a*d + (B*a + A*b)*c)*m + 5*(A*a*d + (A*a*d + (B*a + A*b)*c)*m^2 +
 (B*a + A*b)*c + 2*(A*a*d + (B*a + A*b)*c)*m)*n)*x*x^n*e^(m*log(e) + m*log
(x)) + (A*a*c*m^3 + 6*A*a*c*n^3 + 3*A*a*c*m^2 + 3*A*a*c*m + A*a*c + 11*(A*
a*c*m + A*a*c)*n^2 + 6*(A*a*c*m^2 + 2*A*a*c*m + A*a*c)*n)*x*e^(m*log(e) +
m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 +
 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7796 vs. $2(109) = 218$.

Time = 5.21 (sec) , antiderivative size = 7796, normalized size of antiderivative = 66.63

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)*(c+d*x**n),x)
```

output
```
Piecewise(((A + B)*(a + b)*(c + d)*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A*a*
c*log(x) + A*a*d*x**n/n + A*b*c*x**n/n + A*b*d*x**(2*n)/(2*n) + B*a*c*x**n
/n + B*a*d*x**(2*n)/(2*n) + B*b*c*x**(2*n)/(2*n) + B*b*d*x**(3*n)/(3*n))/e
, Eq(m, -1)), (A*a*c*Piecewise((0**(-3*n - 1)*x, Eq(e, 0)), (Piecewise((-1
/(3*n*(e*x)**(3*n)), Ne(n, 0)), (log(e*x), True))/e, True)) + A*a*d*Piecew
ise((-x*x**n*(e*x)**(-3*n - 1)/(2*n), Ne(n, 0)), (x*x**n*(e*x)**(-3*n - 1)
*log(x), True)) + A*b*c*Piecewise((-x*x**n*(e*x)**(-3*n - 1)/(2*n), Ne(n,
0)), (x*x**n*(e*x)**(-3*n - 1)*log(x), True)) + A*b*d*Piecewise((-x*x**(2*
n)*(e*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(e*x)**(-3*n - 1)*log(x), T
rue)) + B*a*c*Piecewise((-x*x**n*(e*x)**(-3*n - 1)/(2*n), Ne(n, 0)), (x*x*
*n*(e*x)**(-3*n - 1)*log(x), True)) + B*a*d*Piecewise((-x*x**(2*n)*(e*x)**
(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(e*x)**(-3*n - 1)*log(x), True)) + B*
b*c*Piecewise((-x*x**(2*n)*(e*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(e*
x)**(-3*n - 1)*log(x), True)) + B*b*d*x*x**(3*n)*(e*x)**(-3*n - 1)*log(x),
 Eq(m, -3*n - 1)), (A*a*c*Piecewise((0**(-2*n - 1)*x, Eq(e, 0)), (Piecewis
e((-1/(2*n*(e*x)**(2*n)), Ne(n, 0)), (log(e*x), True))/e, True)) + A*a*d*P
iecewise((-x*x**n*(e*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(e*x)**(-2*n - 1
)*log(x), True)) + A*b*c*Piecewise((-x*x**n*(e*x)**(-2*n - 1)/n, Ne(n, 0))
, (x*x**n*(e*x)**(-2*n - 1)*log(x), True)) + A*b*d*x*x**(2*n)*(e*x)**(-2*n
 - 1)*log(x) + B*a*c*Piecewise((-x*x**n*(e*x)**(-2*n - 1)/n, Ne(n, 0)),...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.71

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)\,dx = \frac{Bbde^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Bbce^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Bade^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Abde^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Bace^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Abce^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Aade^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}Aac}{e(m+1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
B*b*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + B*b*c*e^m*x*e^(m*log
(x) + 2*n*log(x))/(m + 2*n + 1) + B*a*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m
 + 2*n + 1) + A*b*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*a*c*
e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + A*b*c*e^m*x*e^(m*log(x) + n*lo
g(x))/(m + n + 1) + A*a*d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x
)^(m + 1)*A*a*c/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3764 vs. $2(117) = 234$.

Time = 0.16 (sec) , antiderivative size = 3764, normalized size of antiderivative = 32.17

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(B*b*d*m^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*B*b*d*m^2*n*x*x^(3*n)*e^(
m*log(e) + m*log(x)) + 2*B*b*d*m*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) + B
*b*c*m^3*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*a*d*m^3*x*x^(2*n)*e^(m*log(
e) + m*log(x)) + A*b*d*m^3*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*b*d*m^3*x
*x^(2*n)*e^(m*log(e) + m*log(x)) + 4*B*b*c*m^2*n*x*x^(2*n)*e^(m*log(e) + m
*log(x)) + 4*B*a*d*m^2*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 4*A*b*d*m^2*n
*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*B*b*d*m^2*n*x*x^(2*n)*e^(m*log(e) +
 m*log(x)) + 3*B*b*c*m*n^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*B*a*d*m*n
^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*A*b*d*m*n^2*x*x^(2*n)*e^(m*log(e)
 + m*log(x)) + 2*B*b*d*m*n^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*a*c*m^3
*x*x^n*e^(m*log(e) + m*log(x)) + A*b*c*m^3*x*x^n*e^(m*log(e) + m*log(x)) +
 B*b*c*m^3*x*x^n*e^(m*log(e) + m*log(x)) + A*a*d*m^3*x*x^n*e^(m*log(e) + m
*log(x)) + B*a*d*m^3*x*x^n*e^(m*log(e) + m*log(x)) + A*b*d*m^3*x*x^n*e^(m*
log(e) + m*log(x)) + B*b*d*m^3*x*x^n*e^(m*log(e) + m*log(x)) + 5*B*a*c*m^2
*n*x*x^n*e^(m*log(e) + m*log(x)) + 5*A*b*c*m^2*n*x*x^n*e^(m*log(e) + m*log
(x)) + 4*B*b*c*m^2*n*x*x^n*e^(m*log(e) + m*log(x)) + 5*A*a*d*m^2*n*x*x^n*e
^(m*log(e) + m*log(x)) + 4*B*a*d*m^2*n*x*x^n*e^(m*log(e) + m*log(x)) + 4*A
*b*d*m^2*n*x*x^n*e^(m*log(e) + m*log(x)) + 3*B*b*d*m^2*n*x*x^n*e^(m*log(e)
 + m*log(x)) + 6*B*a*c*m*n^2*x*x^n*e^(m*log(e) + m*log(x)) + 6*A*b*c*m*n^2
*x*x^n*e^(m*log(e) + m*log(x)) + 3*B*b*c*m*n^2*x*x^n*e^(m*log(e) + m*lo...
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.32

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)\,dx = \frac{A\,a\,c\,x\,(e\,x)^m}{m+1} + \frac{x\,x^{2\,n}\,(e\,x)^m\,(A\,b\,d+B\,a\,d+B\,b\,c)\,(m^2+4\,m\,n+2\,m+3\,n^2+4\,n+1)}{m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+1} + \frac{x\,x^n\,(e\,x)^m\,(A\,a\,d+A\,b\,c+B\,a\,c)\,(m^2+5\,m\,n+2\,m+6\,n^2+5\,n+1)}{m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+1} + \frac{B\,b\,d\,x\,x^{3\,n}\,(e\,x)^m\,(m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1)}{m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+1}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)*(c + d*x^n),x)
```

output
```
(A*a*c*x*(e*x)^m)/(m + 1) + (x*x^(2*n)*(e*x)^m*(A*b*d + B*a*d + B*b*c)*(2*
m + 4*n + 4*m*n + m^2 + 3*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2
*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1) + (x*x^n*(e*x)^m*(A*a*d + A*b*c + B
*a*c)*(2*m + 5*n + 5*m*n + m^2 + 6*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^
2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1) + (B*b*d*x*x^(3*n)*(e*x)^m
*(2*m + 3*n + 3*m*n + m^2 + 2*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6
*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 700, normalized size of antiderivative = 5.98

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)\,dx = \frac{x^m e^m x(6x^{2n}abdm + 6x^{2n}abd\,n^2 + 8x^{2n}abdn + 4x^{2n}b^2c\,m^2n + 3x^{2n}b^2cm\,n^2 + 8x^{2n}b^2cmn + 5x^n a^2d\,m^2n}{}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n),x)
```

output

```
(x**m*e**m*x*(x**(3*n)*b**2*d*m**3 + 3*x**(3*n)*b**2*d*m**2*n + 3*x**(3*n)
*b**2*d*m**2 + 2*x**(3*n)*b**2*d*m*n**2 + 6*x**(3*n)*b**2*d*m*n + 3*x**(3*
n)*b**2*d*m + 2*x**(3*n)*b**2*d*n**2 + 3*x**(3*n)*b**2*d*n + x**(3*n)*b**2
*d + 2*x**(2*n)*a*b*d*m**3 + 8*x**(2*n)*a*b*d*m**2*n + 6*x**(2*n)*a*b*d*m*
*2 + 6*x**(2*n)*a*b*d*m*n**2 + 16*x**(2*n)*a*b*d*m*n + 6*x**(2*n)*a*b*d*m
+ 6*x**(2*n)*a*b*d*n**2 + 8*x**(2*n)*a*b*d*n + 2*x**(2*n)*a*b*d + x**(2*n)
*b**2*c*m**3 + 4*x**(2*n)*b**2*c*m**2*n + 3*x**(2*n)*b**2*c*m**2 + 3*x**(2
*n)*b**2*c*m*n**2 + 8*x**(2*n)*b**2*c*m*n + 3*x**(2*n)*b**2*c*m + 3*x**(2*
n)*b**2*c*n**2 + 4*x**(2*n)*b**2*c*n + x**(2*n)*b**2*c + x**n*a**2*d*m**3
+ 5*x**n*a**2*d*m**2*n + 3*x**n*a**2*d*m**2 + 6*x**n*a**2*d*m*n**2 + 10*x*
*n*a**2*d*m*n + 3*x**n*a**2*d*m + 6*x**n*a**2*d*n**2 + 5*x**n*a**2*d*n + x
**n*a**2*d + 2*x**n*a*b*c*m**3 + 10*x**n*a*b*c*m**2*n + 6*x**n*a*b*c*m**2
+ 12*x**n*a*b*c*m*n**2 + 20*x**n*a*b*c*m*n + 6*x**n*a*b*c*m + 12*x**n*a*b*
c*n**2 + 10*x**n*a*b*c*n + 2*x**n*a*b*c + a**2*c*m**3 + 6*a**2*c*m**2*n +
3*a**2*c*m**2 + 11*a**2*c*m*n**2 + 12*a**2*c*m*n + 3*a**2*c*m + 6*a**2*c*n
**3 + 11*a**2*c*n**2 + 6*a**2*c*n + a**2*c))/(m**4 + 6*m**3*n + 4*m**3 + 1
1*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6
*n**3 + 11*n**2 + 6*n + 1)
```

3.22 $\int (ex)^m (A+Bx^n) (c+dx^n) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (ex)^m (A+Bx^n) (c+dx^n) \, dx = \frac{Ac(ex)^{1+m}}{e(1+m)} + \frac{(Bc+Ad)x^n(ex)^{1+m}}{e(1+m+n)} + \frac{Bdx^{2n}(ex)^{1+m}}{e(1+m+2n)}$$

output
```
A*c*(e*x)^(1+m)/e/(1+m)+(A*d+B*c)*x^n*(e*x)^(1+m)/e/(1+m+n)+B*d*x^(2*n)*(e
*x)^(1+m)/e/(1+m+2*n)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int (ex)^m (A+Bx^n) (c+dx^n) \, dx = x(ex)^m \left(\frac{Ac}{1+m} + \frac{(Bc+Ad)x^n}{1+m+n} + \frac{Bdx^{2n}}{1+m+2n} \right)$$

input
```
Integrate[(e*x)^m*(A + B*x^n)*(c + d*x^n),x]
```

output
```
x*(e*x)^m*((A*c)/(1 + m) + ((B*c + A*d)*x^n)/(1 + m + n) + (B*d*x^(2*n))/(
1 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (A+Bx^n) (c+dx^n) \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(x^n(ex)^m(Ad+Bc)+Ac(ex)^m+Bdx^{2n}(ex)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{n+1}(ex)^m(Ad+Bc)}{m+n+1}+\frac{Ac(ex)^{m+1}}{e(m+1)}+\frac{Bdx^{2n+1}(ex)^m}{m+2n+1}$$

input
```
Int[(e*x)^m*(A + B*x^n)*(c + d*x^n),x]
```

output
```
((B*c + A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (B*d*x^(1 + 2*n)*(e*x)^m)/(1
 + m + 2*n) + (A*c*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.18

method	result
risch	$\frac{x\left(Bd\,m^{2}x^{2n}+Bdmn\,x^{2n}+Ad\,m^{2}x^{n}+2Admn\,x^{n}+Bc\,m^{2}x^{n}+2Bcmn\,x^{n}+2B\,x^{2n}dm+B\,x^{2n}dn+Ac\,m^{2}+3Acmn+2Ac\,n^{2}+\right.}{(1}$
parallelrisch	$\frac{Bx\,x^{2n}(ex)^{m}d\,m^{2}+Ax\,x^{n}(ex)^{m}d\,m^{2}+2Bx\,x^{2n}(ex)^{m}dm+Bx\,x^{2n}(ex)^{m}dn+Bx\,x^{n}(ex)^{m}c\,m^{2}+2Ax\,x^{n}(ex)^{m}dm+2Ax\,x^{n}(}{}$
orering	$\frac{x(3m^{2}+6mn+2n^{2}+3m+3n+1)(ex)^{m}(A+B\,x^{n})(c+d\,x^{n})}{(m^{2}+2mn+2m+2n+1)(1+m+n)}-\frac{3x^{2}(m+n)\left(\frac{(ex)^{m}m(A+B\,x^{n})(c+d\,x^{n})}{x}+\frac{(ex)^{m}B\,x^{n}n(c+d\,x^{n})}{x}\right.}{(m^{2}+2mn+2m+2n+1)(1+m+n)}$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
x*(B*d*m^2*(x^n)^2+B*d*m*n*(x^n)^2+A*d*m^2*x^n+2*A*d*m*n*x^n+B*c*m^2*x^n+2
*B*c*m*n*x^n+2*B*(x^n)^2*d*m+B*(x^n)^2*d*n+A*c*m^2+3*A*c*m*n+2*A*c*n^2+2*A
*x^n*d*m+2*A*x^n*d*n+2*B*x^n*c*m+2*B*x^n*c*n+d*(x^n)^2*B+2*A*c*m+3*A*c*n+d
*x^n*A+c*B*x^n+A*c)/(1+m)/(1+m+n)/(1+m+2*n)*e^m*x^m*exp(1/2*I*Pi*csgn(I*e*
x)*m*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(72) = 144.

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.57

$$\int (ex)^m (A+Bx^n)(c+dx^n)\,dx$$
$$=\frac{(Bdm^2+2\,Bdm+Bd+(Bdm+Bd)n)xx^{2\,n}e^{(m\log(e)+m\log(x))}+((Bc+Ad)m^2+Bc+Ad+2\,(Bc+A}{m^3+2\,(m+1)}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
((B*d*m^2 + 2*B*d*m + B*d + (B*d*m + B*d)*n)*x*x^(2*n)*e^(m*log(e) + m*log
(x)) + ((B*c + A*d)*m^2 + B*c + A*d + 2*(B*c + A*d)*m + 2*(B*c + A*d + (B*
c + A*d)*m)*n)*x*x^n*e^(m*log(e) + m*log(x)) + (A*c*m^2 + 2*A*c*n^2 + 2*A*
c*m + A*c + 3*(A*c*m + A*c)*n)*x*e^(m*log(e) + m*log(x)))/(m^3 + 2*(m + 1)
*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. $2(61) = 122$.

Time = 2.19 (sec) , antiderivative size = 1498, normalized size of antiderivative = 20.81

$$\int (ex)^m (A + Bx^n) (c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n),x)
```

output
```
Piecewise(((A + B)*(c + d)*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A*c*log(x) +
 A*d*x**n/n + B*c*x**n/n + B*d*x**(2*n)/(2*n))/e, Eq(m, -1)), (A*c*Piecewi
se((0**(-2*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(2*n*(e*x)**(2*n)), Ne(n, 0
)), (log(e*x), True))/e, True)) + A*d*Piecewise((-x*x**n*(e*x)**(-2*n - 1)
/n, Ne(n, 0)), (x*x**n*(e*x)**(-2*n - 1)*log(x), True)) + B*c*Piecewise((-
x*x**n*(e*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(e*x)**(-2*n - 1)*log(x), T
rue)) + B*d*x*x**(2*n)*(e*x)**(-2*n - 1)*log(x), Eq(m, -2*n - 1)), (A*c*Pi
ecewise((0**(-n - 1)*x, Eq(e, 0)), (Piecewise((-1/(n*(e*x)**n), Ne(n, 0)),
 (log(e*x), True))/e, True)) + A*d*x*x**n*(e*x)**(-n - 1)*log(x) + B*c*x*x
**n*(e*x)**(-n - 1)*log(x) + B*d*Piecewise((x*x**(2*n)*(e*x)**(-n - 1)/n,
Ne(n, 0)), (x*x**(2*n)*(e*x)**(-n - 1)*log(x), True)), Eq(m, -n - 1)), (A*
c*m**2*x*(e*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1) + 3*A*c*m*n*x*(e*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2
 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*c*m*x*(e*x)**m/(m**3 + 3*m**2*n +
 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*c*n**2*x*(e*x)*
*m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1)
+ 3*A*c*n*x*(e*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m +
2*n**2 + 3*n + 1) + A*c*x*(e*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 +
6*m*n + 3*m + 2*n**2 + 3*n + 1) + A*d*m**2*x*x**n*(e*x)**m/(m**3 + 3*m**2*
n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*A*d*m*n*x*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int (ex)^m (A + Bx^n)(c + dx^n) \, dx = \frac{Bde^m x e^{(m \log(x) + 2n \log(x))}}{m + 2n + 1} + \frac{Bce^m x e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{Ade^m x e^{(m \log(x) + n \log(x))}}{m + n + 1} + \frac{(ex)^{m+1} Ac}{e(m + 1)}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
B*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*c*e^m*x*e^(m*log(x)
+ n*log(x))/(m + n + 1) + A*d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) +
(e*x)^(m + 1)*A*c/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. 2(72) = 144.

Time = 0.13 (sec) , antiderivative size = 763, normalized size of antiderivative = 10.60

$$\int (ex)^m (A + Bx^n)(c + dx^n) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
(B*d*m^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*d*m*n*x*x^(2*n)*e^(m*log(e)
 + m*log(x)) + B*c*m^2*x*x^n*e^(m*log(e) + m*log(x)) + A*d*m^2*x*x^n*e^(m*
log(e) + m*log(x)) + B*d*m^2*x*x^n*e^(m*log(e) + m*log(x)) + 2*B*c*m*n*x*x
^n*e^(m*log(e) + m*log(x)) + 2*A*d*m*n*x*x^n*e^(m*log(e) + m*log(x)) + B*d
*m*n*x*x^n*e^(m*log(e) + m*log(x)) + A*c*m^2*x*e^(m*log(e) + m*log(x)) + B
*c*m^2*x*e^(m*log(e) + m*log(x)) + A*d*m^2*x*e^(m*log(e) + m*log(x)) + B*d
*m^2*x*e^(m*log(e) + m*log(x)) + 3*A*c*m*n*x*e^(m*log(e) + m*log(x)) + 2*B
*c*m*n*x*e^(m*log(e) + m*log(x)) + 2*A*d*m*n*x*e^(m*log(e) + m*log(x)) + B
*d*m*n*x*e^(m*log(e) + m*log(x)) + 2*A*c*n^2*x*e^(m*log(e) + m*log(x)) + 2
*B*d*m*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*d*n*x*x^(2*n)*e^(m*log(e) + m
*log(x)) + 2*B*c*m*x*x^n*e^(m*log(e) + m*log(x)) + 2*A*d*m*x*x^n*e^(m*log(
e) + m*log(x)) + 2*B*d*m*x*x^n*e^(m*log(e) + m*log(x)) + 2*B*c*n*x*x^n*e^(
m*log(e) + m*log(x)) + 2*A*d*n*x*x^n*e^(m*log(e) + m*log(x)) + B*d*n*x*x^n
*e^(m*log(e) + m*log(x)) + 2*A*c*m*x*e^(m*log(e) + m*log(x)) + 2*B*c*m*x*e
^(m*log(e) + m*log(x)) + 2*A*d*m*x*e^(m*log(e) + m*log(x)) + 2*B*d*m*x*e^(
m*log(e) + m*log(x)) + 3*A*c*n*x*e^(m*log(e) + m*log(x)) + 2*B*c*n*x*e^(m*
log(e) + m*log(x)) + 2*A*d*n*x*e^(m*log(e) + m*log(x)) + B*d*n*x*e^(m*log(
e) + m*log(x)) + B*d*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*c*x*x^n*e^(m*lo
g(e) + m*log(x)) + A*d*x*x^n*e^(m*log(e) + m*log(x)) + B*d*x*x^n*e^(m*log(
e) + m*log(x)) + A*c*x*e^(m*log(e) + m*log(x)) + B*c*x*e^(m*log(e) + m*...
```

Mupad [B] (verification not implemented)

Time = 4.76 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int (ex)^m (A+Bx^n)(c+dx^n)\,dx = (e\,x)^m\left(\frac{A\,c\,x}{m+1}+\frac{x\,x^n\,(A\,d+B\,c)\,(m+2\,n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}+\frac{B\,d\,x\,x^{2\,n}\,(m+n+1)}{m^2+3\,m\,n+2\,m+2\,n^2+3\,n+1}\right)$$

input
```
int((e*x)^m*(A + B*x^n)*(c + d*x^n),x)
```

output
```
(e*x)^m*((A*c*x)/(m + 1) + (x*x^n*(A*d + B*c)*(m + 2*n + 1))/(2*m + 3*n +
3*m*n + m^2 + 2*n^2 + 1) + (B*d*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n
+ m^2 + 2*n^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.89

$$\int (ex)^m (A+Bx^n)(c+dx^n)\,dx$$
$$= \frac{x^m e^m x(x^{2n}bd\,m^2 + x^{2n}bdmn + 2x^{2n}bdm + x^{2n}bdn + x^{2n}bd + x^n ad\,m^2 + 2x^n admn + 2x^n adm + 2x^n adr}{m^3 + 3m^2n + 2m\,n^2 + 3n}$$

input

```
int((e*x)^m*(A+B*x^n)*(c+d*x^n),x)
```

output

```
(x**m*e**m*x*(x**(2*n)*b*d*m**2 + x**(2*n)*b*d*m*n + 2*x**(2*n)*b*d*m + x*
*(2*n)*b*d*n + x**(2*n)*b*d + x**n*a*d*m**2 + 2*x**n*a*d*m*n + 2*x**n*a*d*
m + 2*x**n*a*d*n + x**n*a*d + x**n*b*c*m**2 + 2*x**n*b*c*m*n + 2*x**n*b*c*
m + 2*x**n*b*c*n + x**n*b*c + a*c*m**2 + 3*a*c*m*n + 2*a*c*m + 2*a*c*n**2
+ 3*a*c*n + a*c))/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n
**2 + 3*n + 1)
```

3.23 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)}{a+bx^n}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{a+bx^n}\,dx = \frac{(bBc+Abd-aBd)(ex)^{1+m}}{b^2e(1+m)}+\frac{Bdx^n(ex)^{1+m}}{be(1+m+n)} + \frac{(Ab-aB)(bc-ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ab^2e(1+m)}$$

output
```
(A*b*d-B*a*d+B*b*c)*(e*x)^(1+m)/b^2/e/(1+m)+B*d*x^n*(e*x)^(1+m)/b/e/(1+m+n
)+(A*b-B*a)*(-a*d+b*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x
^n/a)/a/b^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{a+bx^n}\,dx = \frac{x(ex)^m\left(\frac{bBc+Abd-aBd}{1+m}+\frac{bBdx^n}{1+m+n}+\frac{(-Ab+aB)(-bc+ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)}\right)}{b^2}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n),x]
```

output
```
(x*(e*x)^m*((b*B*c + A*b*d - a*B*d)/(1 + m) + (b*B*d*x^n)/(1 + m + n) + ((
-(A*b) + a*B)*(-(b*c) + a*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n
, -((b*x^n)/a)])/(a*(1 + m))))/b^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)}{a+bx^n} dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(\frac{(ex)^m(Ab-aB)(bc-ad)}{b^2 (a+bx^n)}+\frac{(ex)^m(-aBd+Abd+bBc)}{b^2}+\frac{Bdx^n(ex)^m}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}(Ab-aB)(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ab^2e(m+1)}+\frac{(ex)^{m+1}(-aBd+Abd+bBc)}{b^2e(m+1)}+\frac{Bdx^{n+1}(ex)^m}{b(m+n+1)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n),x]
```

output
```
(B*d*x^(1 + n)*(e*x)^m)/(b*(1 + m + n)) + ((b*B*c + A*b*d - a*B*d)*(e*x)^(
1 + m))/(b^2*e*(1 + m)) + ((A*b - a*B)*(b*c - a*d)*(e*x)^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^2*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B\,x^n)\,(c + d\,x^n)}{a + b\,x^n}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)\,(c + dx^n)}{a + bx^n}\,dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{bx^n + a}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*d*x^(2*n) + A*c + (B*c + A*d)*x^n)*(e*x)^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 872, normalized size of antiderivative = 7.09

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{a + bx^n} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)/(a+b*x**n),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c*e**m*m*x**(m + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n
)) + A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c*e**m*x**(m + 1)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*d*e**m*m*x**(m + n + 1)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(
n**2*gamma(m/n + 2 + 1/n)) + A*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*d*e*
*m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gam
ma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + A*a**(-m/n - 2 - 1/n)*a**(m/n
 + 1 + 1/n)*d*e**m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/
n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + B*a**(-m/n
 - 3 - 1/n)*a**(m/n + 2 + 1/n)*d*e**m*m*x**(m + 2*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n +
 3 + 1/n)) + 2*B*a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*d*e**m*x**(m + 2*n
 + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 +
 1/n)/(n*gamma(m/n + 3 + 1/n)) + B*a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*
d*e**m*x**(m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/
n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + B*a**(-m/n - 2 - 1/n
)*a**(m/n + 1 + 1/n)*c*e**m*m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)...
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{a + bx^n} \, dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{bx^n + a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

output
```
((b^2*c*e^m - a*b*d*e^m)*A - (a*b*c*e^m - a^2*d*e^m)*B)*integrate(x^m/(b^3
*x^n + a*b^2), x) + (B*b*d*e^m*(m + 1)*x*e^(m*log(x) + n*log(x)) + (A*b*d*
e^m*(m + n + 1) + (b*c*e^m*(m + n + 1) - a*d*e^m*(m + n + 1))*B)*x*x^m)/((
m^2 + m*(n + 2) + n + 1)*b^2)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{a + bx^n} \, dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{bx^n + a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)*(e*x)^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{a + bx^n} \, dx = \int \frac{(e\,x)^m\,(A + B\,x^n)\,(c + d\,x^n)}{a + b\,x^n} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{a + bx^n} dx = \frac{x^m e^m x(x^n dm + x^n d + cm + cn + c)}{m^2 + mn + 2m + n + 1}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n),x)
```

output
```
(x**m*e**m*x*(x**n*d*m + x**n*d + c*m + c*n + c))/(m**2 + m*n + 2*m + n +
1)
```

3.24 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)}{(a+bx^n)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 175

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{(a+bx^n)^2} dx = -\frac{d\left(A-\frac{aB(1+m+n)}{b(1+m)}\right)(ex)^{1+m}}{aben} + \frac{(Ab-aB)(ex)^{1+m}(c+dx^n)}{aben(a+bx^n)} + \frac{(bc(aB(1+m)-Ab(1+m-n))+ad(Ab(1+m)-aB(1+m+n)))(ex)^{1+m}\,\text{Hypergeometric2F1}\,(}{a^2b^2e(1+m)n}$$

output
```
-d*(A-a*B*(1+m+n)/b/(1+m))*(e*x)^(1+m)/a/b/e/n+(A*b-B*a)*(e*x)^(1+m)*(c+d*
x^n)/a/b/e/n/(a+b*x^n)+(b*c*(a*B*(1+m)-A*b*(1+m-n))+a*d*(A*b*(1+m)-a*B*(1+
m+n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/b^2/e/
(1+m)/n
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.63

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)}{(a + bx^n)^2} dx$$
$$= \frac{x(ex)^m \left(a^2Bd + a(bBc + Abd - 2aBd) \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right) + (Ab - aB)(bc - a\right.}{a^2b^2(1 + m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^2,x]
```

output
```
(x*(e*x)^m*(a^2*B*d + a*(b*B*c + A*b*d - 2*a*B*d)*Hypergeometric2F1[1, (1
+ m)/n, (1 + m + n)/n, -((b*x^n)/a)] + (A*b - a*B)*(b*c - a*d)*Hypergeomet
ric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)]))/(a^2*b^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1064, 25, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)}{(a + bx^n)^2} dx$$
$$\Big\downarrow \ 1064$$
$$\frac{(ex)^{m+1}(Ab - aB)(c + dx^n)}{aben(a + bx^n)} - \frac{\int -\frac{(ex)^m(c(aB(m+1) - Ab(m-n+1)) - d(Ab(m+1) - aB(m+n+1))x^n)}{bx^n + a}dx}{abn}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{(ex)^m(c(aB(m+1) - Ab(m-n+1)) - d(Ab(m+1) - aB(m+n+1))x^n)}{bx^n + a}dx}{abn} + \frac{(ex)^{m+1}(Ab - aB)(c + dx^n)}{aben(a + bx^n)}$$
$$\Big\downarrow \ 959$$

$$\frac{\frac{(Ab(ad(m+1)-bc(m-n+1))+aB(bc(m+1)-ad(m+n+1)))\int \frac{(ex)^m}{bx^n+a}dx}{b}-\frac{d(ex)^{m+1}(Ab(m+1)-aB(m+n+1))}{be(m+1)}}{abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{aben(a+bx^n)}$$

$$\Big\downarrow \; 888$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)(Ab(ad(m+1)-bc(m-n+1))+aB(bc(m+1)-ad(m+n+1)))}{abe(m+1)}-\frac{d(ex)^{m+1}(Ab(m+1)-aB}{be(m+1)}}{abn}$$
$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{aben(a+bx^n)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^n))/(a*b*e*n*(a + b*x^n)) + (-((d*(A*b
*(1 + m) - a*B*(1 + m + n))*(e*x)^(1 + m))/(b*e*(1 + m))) + ((A*b*(a*d*(1
+ m) - b*c*(1 + m - n)) + a*B*(b*c*(1 + m) - a*d*(1 + m + n)))*(e*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b*e*(1
 + m)))/(a*b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n)\,(c+d\,x^n)}{(a+b\,x^n)^2}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n)\,(c+dx^n)}{(a+bx^n)^2}\,dx = \int \frac{(Bx^n+A)(dx^n+c)(ex)^m}{(bx^n+a)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*d*x^(2*n) + A*c + (B*c + A*d)*x^n)*(e*x)^m/(b^2*x^(2*n) + 2*a*
b*x^n + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.24 (sec) , antiderivative size = 5176, normalized size of antiderivative = 29.58

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)/(a+b*x**n)**2,x)
```

output
```
A*c*(-a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*e**m*m**2*x**(m + 1)*lerchphi(b
*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n
+ 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n
 - 2 - 1/n)*e**m*m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n
+ 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m
/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*e**m*m*n*x**(m + 1)*
gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1
+ 1/n)) - 2*a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*e**m*m*x**(m + 1)*lerchph
i(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m
/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-
m/n - 2 - 1/n)*e**m*n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n
 + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(
m/n + 1 + 1/n)) + a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*e**m*n*x**(m + 1)*g
amma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 +
 1/n)) - a*a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*e**m*x**(m + 1)*lerchphi(b*x
**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n +
1 + 1/n) + b*n**3*x**n*gamma(m/n + 1 + 1/n)) - a**(m/n + 1/n)*a**(-m/n - 2
 - 1/n)*b*e**m*m**2*x**n*x**(m + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
m/n + 1/n)*gamma(m/n + 1/n)/(a*n**3*gamma(m/n + 1 + 1/n) + b*n**3*x**n*gam
ma(m/n + 1 + 1/n)) + a**(m/n + 1/n)*a**(-m/n - 2 - 1/n)*b*e**m*m*n*x**n...
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^2} dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{(bx^n + a)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-((b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m + 1))*A + (a^2*d*e^m*(m + n + 1) -
 a*b*c*e^m*(m + 1))*B)*integrate(x^m/(a*b^3*n*x^n + a^2*b^2*n), x) + (B*a*
b*d*e^m*n*x*e^(m*log(x) + n*log(x)) + ((b^2*c*e^m*(m + 1) - a*b*d*e^m*(m +
 1))*A + (a^2*d*e^m*(m + n + 1) - a*b*c*e^m*(m + 1))*B)*x*x^m)/((m*n + n)*
a*b^3*x^n + (m*n + n)*a^2*b^2)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^2} dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{(bx^n + a)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)*(e*x)^m/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^2} dx = \int \frac{(e\,x)^m\,(A + B\,x^n)\,(c + d\,x^n)}{(a + b\,x^n)^2} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{(a+bx^n)^2}\,dx$$
$$= \frac{e^m \left(x^m dx - \left(\int \frac{x^m}{x^n b+a}dx\right) adm - \left(\int \frac{x^m}{x^n b+a}dx\right) ad + \left(\int \frac{x^m}{x^n b+a}dx\right) bcm + \left(\int \frac{x^m}{x^n b+a}dx\right) bc\right)}{b(m+1)}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^2,x)
```

output
```
(e**m*(x**m*d*x - int(x**m/(x**n*b + a),x)*a*d*m - int(x**m/(x**n*b + a),x
)*a*d + int(x**m/(x**n*b + a),x)*b*c*m + int(x**m/(x**n*b + a),x)*b*c))/(b
*(m + 1))
```

3.25 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)}{(a+bx^n)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 228

$$\begin{aligned}\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{(a+bx^n)^3}\,dx &= -\frac{(Ab(bc(1+m-2n)-ad(1+m-n))-aB(bc(1+m)-ad(1+m+n)))(ex)^{1+m}}{2a^2b^2en^2(a+bx^n)} \\ &+ \frac{(Ab-aB)(ex)^{1+m}(c+dx^n)}{2aben(a+bx^n)^2} \\ &- \frac{(bc(aB(1+m)-Ab(1+m-2n))(1+m-n)+ad(1+m)(Ab(1+m-n)-aB(1+m+n)))(ex)}{2a^3b^2e(1+m)n^2}\end{aligned}$$

output
```
-1/2*(A*b*(b*c*(1+m-2*n)-a*d*(1+m-n))-a*B*(b*c*(1+m)-a*d*(1+m+n)))*(e*x)^(
1+m)/a^2/b^2/e/n^2/(a+b*x^n)+1/2*(A*b-B*a)*(e*x)^(1+m)*(c+d*x^n)/a/b/e/n/(
a+b*x^n)^2-1/2*(b*c*(a*B*(1+m)-A*b*(1+m-2*n))*(1+m-n)+a*d*(1+m)*(A*b*(1+m-
n)-a*B*(1+m+n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/
a^3/b^2/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{(a+bx^n)^3}\,dx = \frac{x(ex)^m \left(a^2 Bd \,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+a(bBc+Abd-2aBd)\,\text{Hypergeometric2F1}\right.}{a^3b^2(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^3,x]
```

output
```
(x*(e*x)^m*(a^2*B*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^
n)/a)] + a*(b*B*c + A*b*d - 2*a*B*d)*Hypergeometric2F1[2, (1 + m)/n, (1 +
m + n)/n, -((b*x^n)/a)] + (A*b - a*B)*(b*c - a*d)*Hypergeometric2F1[3, (1
+ m)/n, (1 + m + n)/n, -((b*x^n)/a)]))/(a^3*b^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1064, 25, 957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)}{(a+bx^n)^3}\,dx$$

$$\Big\downarrow \text{1064}$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{2aben(a+bx^n)^2}-\frac{\int -\frac{(ex)^m(c(aB(m+1)-Ab(m-2n+1))-d(Ab(m-n+1)-aB(m+n+1))x^n)}{(bx^n+a)^2}dx}{2abn}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(c(aB(m+1)-Ab(m-2n+1))-d(Ab(m-n+1)-aB(m+n+1))x^n)}{(bx^n+a)^2}dx}{2abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{2aben(a+bx^n)^2}$$

$$\downarrow 957$$

$$\frac{-\frac{(bc(m-n+1)(aB(m+1)-Ab(m-2n+1))+ad(m+1)(Ab(m-n+1)-aB(m+n+1)))\int\frac{(ex)^m}{bx^n+a}dx}{abn}-\frac{(ex)^{m+1}(Ab(bc(m-2n+1)-ad(m-n+1))}{aben(a+bx^n}}{2abn}$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{2aben(a+bx^n)^2}$$

$$\downarrow 888$$

$$\frac{-\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)(bc(m-n+1)(aB(m+1)-Ab(m-2n+1))+ad(m+1)(Ab(m-n+1)-aB(m+n+1)))}{a^2be(m+1)n}-}{2abn}$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)}{2aben(a+bx^n)^2}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^n))/(2*a*b*e*n*(a + b*x^n)^2) + (-(((A
*b*(b*c*(1 + m - 2*n) - a*d*(1 + m - n)) - a*B*(b*c*(1 + m) - a*d*(1 + m +
 n)))*(e*x)^(1 + m))/(a*b*e*n*(a + b*x^n))) - ((b*c*(a*B*(1 + m) - A*b*(1
+ m - 2*n))*(1 + m - n) + a*d*(1 + m)*(A*b*(1 + m - n) - a*B*(1 + m + n)))
*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
])/(a^2*b*e*(1 + m)*n))/(2*a*b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n)\,(c+d\,x^n)}{(a+b\,x^n)^3}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n)\,(c+dx^n)}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)(dx^n+c)(ex)^m}{(bx^n+a)^3}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*d*x^(2*n) + A*c + (B*c + A*d)*x^n)*(e*x)^m/(b^3*x^(3*n) + 3*a*
b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^3} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)}{(a + bx^n)^3} dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{(bx^n + a)^3} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^2*c*e^m - (m^2 - m*(n - 2) - n +
 1)*a*b*d*e^m)*A - ((m^2 - m*(n - 2) - n + 1)*a*b*c*e^m - (m^2 + m*(n + 2)
 + n + 1)*a^2*d*e^m)*B)*integrate(1/2*x^m/(a^2*b^3*n^2*x^n + a^3*b^2*n^2),
 x) + 1/2*(((a^2*b*d*e^m*(m - n + 1) - a*b^2*c*e^m*(m - 3*n + 1))*A - (a^3
*d*e^m*(m + n + 1) - a^2*b*c*e^m*(m - n + 1))*B)*x*x^m - ((b^3*c*e^m*(m -
2*n + 1) - a*b^2*d*e^m*(m + 1))*A + (a^2*b*d*e^m*(m + 2*n + 1) - a*b^2*c*e
^m*(m + 1))*B)*x*e^(m*log(x) + n*log(x)))/(a^2*b^4*n^2*x^(2*n) + 2*a^3*b^3
*n^2*x^n + a^4*b^2*n^2)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)}{(a + bx^n)^3} dx = \int \frac{(Bx^n + A)(dx^n + c)(ex)^m}{(bx^n + a)^3} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)*(e*x)^m/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)}{(a + bx^n)^3} dx = \int \frac{(e\,x)^m \left(A + B\,x^n\right)\left(c + d\,x^n\right)}{(a + b\,x^n)^3} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^3,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n))/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)}{(a + bx^n)^3} dx = \text{too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)/(a+b*x^n)^3,x)
```

output
```
(e**m*(x**(m + n)*a*d*m*x + x**(m + n)*a*d*x - x**(m + n)*b*c*m*x + x**(m
+ n)*b*c*n*x - x**(m + n)*b*c*x + x**m*a*c*m*x + x**m*a*c*n*x + x**m*a*c*x
 - x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**
2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*
a*b**2*d*m**3 - x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n +
 x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*
n + a**2),x)*a*b**2*d*m**2*n - 3*x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m +
x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b
 + a**2*m + a**2*n + a**2),x)*a*b**2*d*m**2 - 2*x**n*int(x**(m + 2*n)/(x**
(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b
*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a*b**2*d*m*n - 3*x**n*int(x**
(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*
m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a*b**2*d*m - x*
*n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2
*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a*b**
2*d*n - x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n
)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2
),x)*a*b**2*d + x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n +
 x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*
n + a**2),x)*b**3*c*m**3 + 3*x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x...
```

3.26 $\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 336

$$\begin{aligned}
&\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx \\
&= \frac{a^3Ac^2(ex)^{1+m}}{e(1+m)} + \frac{a^2c(3Abc+aBc+2aAd)x^n(ex)^{1+m}}{e(1+m+n)} \\
&\quad + \frac{a(aBc(3bc+2ad)+A(3b^2c^2+6abcd+a^2d^2))\,x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\
&\quad + \frac{(aB(3b^2c^2+6abcd+a^2d^2)+Ab(b^2c^2+6abcd+3a^2d^2))\,x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\
&\quad + \frac{b(3a^2Bd^2+3abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^{4n}(ex)^{1+m}}{e(1+m+4n)} \\
&\quad + \frac{b^2d(2bBc+Abd+3aBd)x^{5n}(ex)^{1+m}}{e(1+m+5n)} + \frac{b^3Bd^2x^{6n}(ex)^{1+m}}{e(1+m+6n)}
\end{aligned}$$

output
```
a^3*A*c^2*(e*x)^(1+m)/e/(1+m)+a^2*c*(2*A*a*d+3*A*b*c+B*a*c)*x^n*(e*x)^(1+m
)/e/(1+m+n)+a*(a*B*c*(2*a*d+3*b*c)+A*(a^2*d^2+6*a*b*c*d+3*b^2*c^2))*x^(2*n
)*(e*x)^(1+m)/e/(1+m+2*n)+(a*B*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)+A*b*(3*a^2*d^
2+6*a*b*c*d+b^2*c^2))*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+b*(3*a^2*B*d^2+3*a*b
*d*(A*d+2*B*c)+b^2*c*(2*A*d+B*c))*x^(4*n)*(e*x)^(1+m)/e/(1+m+4*n)+b^2*d*(A
*b*d+3*B*a*d+2*B*b*c)*x^(5*n)*(e*x)^(1+m)/e/(1+m+5*n)+b^3*B*d^2*x^(6*n)*(e
*x)^(1+m)/e/(1+m+6*n)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.81

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx = x(ex)^m \left(\frac{a^3Ac^2}{1+m}+\frac{a^2c(3Abc+aBc+2aAd)x^n}{1+m+n} + \frac{a(aBc(3bc+2ad)+A(3b^2c^2+6abcd+a^2d^2))\,x^{2n}}{1+m+2n} + \frac{(aB(3b^2c^2+6abcd+a^2d^2)+Ab(b^2c^2+6abcd+3a^2d^2))\,x^{3n}}{1+m+3n} + \frac{b(3a^2Bd^2+3abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^{4n}}{1+m+4n} + \frac{b^2d(2bBc+Abd+3aBd)x^{5n}}{1+m+5n}+\frac{b^3Bd^2x^{6n}}{1+m+6n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
x*(e*x)^m*((a^3*A*c^2)/(1 + m) + (a^2*c*(3*A*b*c + a*B*c + 2*a*A*d)*x^n)/(
1 + m + n) + (a*(a*B*c*(3*b*c + 2*a*d) + A*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^
2))*x^(2*n))/(1 + m + 2*n) + ((a*B*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2) + A*b
*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2))*x^(3*n))/(1 + m + 3*n) + (b*(3*a^2*B*d
^2 + 3*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))*x^(4*n))/(1 + m + 4*n) +
 (b^2*d*(2*b*B*c + A*b*d + 3*a*B*d)*x^(5*n))/(1 + m + 5*n) + (b^3*B*d^2*x^
(6*n))/(1 + m + 6*n))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(a^3Ac^2(ex)^m + ax^{2n}(ex)^m \left(A\left(a^2d^2+6abcd+3b^2c^2\right)+aBc(2ad+3bc)\right)+x^{3n}(ex)^m \left(Ab\left(3a^2d^2+6abcd+b^2\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac^2(ex)^{m+1}}{e(m+1)}+\frac{ax^{2n+1}(ex)^m \left(A\left(a^2d^2+6abcd+3b^2c^2\right)+aBc(2ad+3bc)\right)}{m+2n+1}+\frac{x^{3n+1}(ex)^m \left(Ab\left(3a^2d^2+6abcd+b^2c^2\right)+aB\left(a^2d^2+6abcd+3b^2c^2\right)\right)}{m+3n+1}+\frac{bx^{4n+1}(ex)^m \left(3a^2Bd^2+3abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{m+4n+1}+\frac{a^2cx^{n+1}(ex)^m(2aAd+aBc+3Abc)}{m+n+1}+\frac{b^2dx^{5n+1}(ex)^m(3aBd+Abd+2bBc)}{m+5n+1}+\frac{b^3Bd^2x^{6n+1}(ex)^m}{m+6n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
(a^2*c*(3*A*b*c + a*B*c + 2*a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (a*(a*
B*c*(3*b*c + 2*a*d) + A*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*x^(1 + 2*n)*(e*
x)^m)/(1 + m + 2*n) + ((a*B*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2) + A*b*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2))*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n) + (b*(3*a^
2*B*d^2 + 3*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))*x^(1 + 4*n)*(e*x)^m
)/(1 + m + 4*n) + (b^2*d*(2*b*B*c + A*b*d + 3*a*B*d)*x^(1 + 5*n)*(e*x)^m)/
(1 + m + 5*n) + (b^3*B*d^2*x^(1 + 6*n)*(e*x)^m)/(1 + m + 6*n) + (a^3*A*c^2
*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.72 (sec) , antiderivative size = 11356, normalized size of antiderivative = 33.80

method	result	size
risch	Expression too large to display	11356
parallelrisch	Expression too large to display	15203
orering	Expression too large to display	23014

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6638 vs. 2(336) = 672.

Time = 0.29 (sec) , antiderivative size = 6638, normalized size of antiderivative = 19.76

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168099 vs. $2(332) = 664$.

Time = 29.35 (sec) , antiderivative size = 168099, normalized size of antiderivative = 500.29

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**3*(A+B*x**n)*(c+d*x**n)**2,x)
```

output
```
Piecewise(((A + B)*(a + b)**3*(c + d)**2*log(x)/e, Eq(m, -1) & Eq(n, 0)),
((A*a**3*c**2*log(x) + 2*A*a**3*c*d*x**n/n + A*a**3*d**2*x**(2*n)/(2*n) +
3*A*a**2*b*c**2*x**n/n + 3*A*a**2*b*c*d*x**(2*n)/n + A*a**2*b*d**2*x**(3*n
)/n + 3*A*a*b**2*c**2*x**(2*n)/(2*n) + 2*A*a*b**2*c*d*x**(3*n)/n + 3*A*a*b
**2*d**2*x**(4*n)/(4*n) + A*b**3*c**2*x**(3*n)/(3*n) + A*b**3*c*d*x**(4*n)
/(2*n) + A*b**3*d**2*x**(5*n)/(5*n) + B*a**3*c**2*x**n/n + B*a**3*c*d*x**(
2*n)/n + B*a**3*d**2*x**(3*n)/(3*n) + 3*B*a**2*b*c**2*x**(2*n)/(2*n) + 2*B
*a**2*b*c*d*x**(3*n)/n + 3*B*a**2*b*d**2*x**(4*n)/(4*n) + B*a*b**2*c**2*x*
*(3*n)/n + 3*B*a*b**2*c*d*x**(4*n)/(2*n) + 3*B*a*b**2*d**2*x**(5*n)/(5*n)
+ B*b**3*c**2*x**(4*n)/(4*n) + 2*B*b**3*c*d*x**(5*n)/(5*n) + B*b**3*d**2*x
**(6*n)/(6*n))/e, Eq(m, -1)), (A*a**3*c**2*Piecewise((0**(-6*n - 1)*x, Eq(
e, 0)), (Piecewise((-1/(6*n*(e*x)**(6*n)), Ne(n, 0)), (log(e*x), True))/e,
 True)) + 2*A*a**3*c*d*Piecewise((-x*x**n*(e*x)**(-6*n - 1)/(5*n), Ne(n, 0
)), (x*x**n*(e*x)**(-6*n - 1)*log(x), True)) + A*a**3*d**2*Piecewise((-x*x
**(2*n)*(e*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-6*n - 1)*
log(x), True)) + 3*A*a**2*b*c**2*Piecewise((-x*x**n*(e*x)**(-6*n - 1)/(5*n
), Ne(n, 0)), (x*x**n*(e*x)**(-6*n - 1)*log(x), True)) + 6*A*a**2*b*c*d*Pi
ecewise((-x*x**(2*n)*(e*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(e*x)
**(-6*n - 1)*log(x), True)) + 3*A*a**2*b*d**2*Piecewise((-x*x**(3*n)*(e*x)
**(-6*n - 1)/(3*n), Ne(n, 0)), (x*x**(3*n)*(e*x)**(-6*n - 1)*log(x), Tr...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(336) = 672$.

Time = 0.11 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.23

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
B*b^3*d^2*e^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 2*B*b^3*c*d*e^m*
x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 3*B*a*b^2*d^2*e^m*x*e^(m*log(x
) + 5*n*log(x))/(m + 5*n + 1) + A*b^3*d^2*e^m*x*e^(m*log(x) + 5*n*log(x))/
(m + 5*n + 1) + B*b^3*c^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) +
6*B*a*b^2*c*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*A*b^3*c*d*
e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 3*B*a^2*b*d^2*e^m*x*e^(m*l
og(x) + 4*n*log(x))/(m + 4*n + 1) + 3*A*a*b^2*d^2*e^m*x*e^(m*log(x) + 4*n*
log(x))/(m + 4*n + 1) + 3*B*a*b^2*c^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m +
 3*n + 1) + A*b^3*c^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*B*
a^2*b*c*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*A*a*b^2*c*d*e^
m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + B*a^3*d^2*e^m*x*e^(m*log(x)
+ 3*n*log(x))/(m + 3*n + 1) + 3*A*a^2*b*d^2*e^m*x*e^(m*log(x) + 3*n*log(x)
)/(m + 3*n + 1) + 3*B*a^2*b*c^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n +
 1) + 3*A*a*b^2*c^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*B*a^
3*c*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 6*A*a^2*b*c*d*e^m*x*
e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + A*a^3*d^2*e^m*x*e^(m*log(x) + 2*
n*log(x))/(m + 2*n + 1) + B*a^3*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n +
 1) + 3*A*a^2*b*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a^3*c*
d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*a^3*c^2/(e*(
m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70422 vs. $2(336) = 672$.

Time = 0.65 (sec) , antiderivative size = 70422, normalized size of antiderivative = 209.59

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(B*b^3*d^2*m^6*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 15*B*b^3*d^2*m^5*n*x*x^
(6*n)*e^(m*log(e) + m*log(x)) + 85*B*b^3*d^2*m^4*n^2*x*x^(6*n)*e^(m*log(e)
 + m*log(x)) + 225*B*b^3*d^2*m^3*n^3*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 2
74*B*b^3*d^2*m^2*n^4*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 120*B*b^3*d^2*m*n
^5*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 2*B*b^3*c*d*m^6*x*x^(5*n)*e^(m*log(
e) + m*log(x)) + 3*B*a*b^2*d^2*m^6*x*x^(5*n)*e^(m*log(e) + m*log(x)) + A*b
^3*d^2*m^6*x*x^(5*n)*e^(m*log(e) + m*log(x)) + B*b^3*d^2*m^6*x*x^(5*n)*e^(
m*log(e) + m*log(x)) + 32*B*b^3*c*d*m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(x)
) + 48*B*a*b^2*d^2*m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 16*A*b^3*d^2*
m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 15*B*b^3*d^2*m^5*n*x*x^(5*n)*e^(
m*log(e) + m*log(x)) + 190*B*b^3*c*d*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*log
(x)) + 285*B*a*b^2*d^2*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 95*A*b^
3*d^2*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 85*B*b^3*d^2*m^4*n^2*x*x
^(5*n)*e^(m*log(e) + m*log(x)) + 520*B*b^3*c*d*m^3*n^3*x*x^(5*n)*e^(m*log(
e) + m*log(x)) + 780*B*a*b^2*d^2*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x))
 + 260*A*b^3*d^2*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 225*B*b^3*d^2
*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 648*B*b^3*c*d*m^2*n^4*x*x^(5*
n)*e^(m*log(e) + m*log(x)) + 972*B*a*b^2*d^2*m^2*n^4*x*x^(5*n)*e^(m*log(e)
 + m*log(x)) + 324*A*b^3*d^2*m^2*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 2
74*B*b^3*d^2*m^2*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 288*B*b^3*c*d*...
```

Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 1882, normalized size of antiderivative = 5.60

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^3*(c + d*x^n)^2,x)
```

output
```
(x*x^(3*n)*(e*x)^m*(A*b^3*c^2 + B*a^3*d^2 + 3*A*a^2*b*d^2 + 3*B*a*b^2*c^2
+ 6*A*a*b^2*c*d + 6*B*a^2*b*c*d)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^
2*n + 744*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^
4 + m^5 + 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^
3 + 121*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205
*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m
^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n
^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 +
735*m^3*n^3 + 175*m^4*n^2 + 1) + (A*a^3*c^2*x*(e*x)^m)/(m + 1) + (a*x*x^(2
*n)*(e*x)^m*(A*a^2*d^2 + 3*A*b^2*c^2 + 3*B*a*b*c^2 + 2*B*a^2*c*d + 6*A*a*b
*c*d)*(5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n
+ 702*m*n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3
 + 702*n^4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m
+ 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m
*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^
5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2
 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 +
 1) + (b*x*x^(4*n)*(e*x)^m*(3*B*a^2*d^2 + B*b^2*c^2 + 3*A*a*b*d^2 + 2*A*b^
2*c*d + 6*B*a*b*c*d)*(5*m + 17*n + 68*m*n + 321*m*n^2 + 102*m^2*n + 614*m*
n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 +...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6996, normalized size of antiderivative = 20.82

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^2,x)
```

output
```
(x**m*e**m*x*(x**(6*n)*b**4*d**2*m**6 + 15*x**(6*n)*b**4*d**2*m**5*n + 6*x
**(6*n)*b**4*d**2*m**5 + 85*x**(6*n)*b**4*d**2*m**4*n**2 + 75*x**(6*n)*b**
4*d**2*m**4*n + 15*x**(6*n)*b**4*d**2*m**4 + 225*x**(6*n)*b**4*d**2*m**3*n
**3 + 340*x**(6*n)*b**4*d**2*m**3*n**2 + 150*x**(6*n)*b**4*d**2*m**3*n + 2
0*x**(6*n)*b**4*d**2*m**3 + 274*x**(6*n)*b**4*d**2*m**2*n**4 + 675*x**(6*n
)*b**4*d**2*m**2*n**3 + 510*x**(6*n)*b**4*d**2*m**2*n**2 + 150*x**(6*n)*b*
*4*d**2*m**2*n + 15*x**(6*n)*b**4*d**2*m**2 + 120*x**(6*n)*b**4*d**2*m*n**
5 + 548*x**(6*n)*b**4*d**2*m*n**4 + 675*x**(6*n)*b**4*d**2*m*n**3 + 340*x*
*(6*n)*b**4*d**2*m*n**2 + 75*x**(6*n)*b**4*d**2*m*n + 6*x**(6*n)*b**4*d**2
*m + 120*x**(6*n)*b**4*d**2*n**5 + 274*x**(6*n)*b**4*d**2*n**4 + 225*x**(6
*n)*b**4*d**2*n**3 + 85*x**(6*n)*b**4*d**2*n**2 + 15*x**(6*n)*b**4*d**2*n
+ x**(6*n)*b**4*d**2 + 4*x**(5*n)*a*b**3*d**2*m**6 + 64*x**(5*n)*a*b**3*d*
*2*m**5*n + 24*x**(5*n)*a*b**3*d**2*m**5 + 380*x**(5*n)*a*b**3*d**2*m**4*n
**2 + 320*x**(5*n)*a*b**3*d**2*m**4*n + 60*x**(5*n)*a*b**3*d**2*m**4 + 104
0*x**(5*n)*a*b**3*d**2*m**3*n**3 + 1520*x**(5*n)*a*b**3*d**2*m**3*n**2 + 6
40*x**(5*n)*a*b**3*d**2*m**3*n + 80*x**(5*n)*a*b**3*d**2*m**3 + 1296*x**(5
*n)*a*b**3*d**2*m**2*n**4 + 3120*x**(5*n)*a*b**3*d**2*m**2*n**3 + 2280*x**
(5*n)*a*b**3*d**2*m**2*n**2 + 640*x**(5*n)*a*b**3*d**2*m**2*n + 60*x**(5*n
)*a*b**3*d**2*m**2 + 576*x**(5*n)*a*b**3*d**2*m*n**5 + 2592*x**(5*n)*a*b**
3*d**2*m*n**4 + 3120*x**(5*n)*a*b**3*d**2*m*n**3 + 1520*x**(5*n)*a*b**3...
```

3.27 $\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 252

$$\begin{aligned}\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx &= \frac{a^2Ac^2(ex)^{1+m}}{e(1+m)} + \frac{ac(aBc+2A(bc+ad))x^n(ex)^{1+m}}{e(1+m+n)} \\ &\quad + \frac{(2aBc(bc+ad)+A(b^2c^2+4abcd+a^2d^2))\,x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\ &\quad + \frac{(a^2Bd^2+2abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\ &\quad + \frac{bd(2bBc+Abd+2aBd)x^{4n}(ex)^{1+m}}{e(1+m+4n)} + \frac{b^2Bd^2x^{5n}(ex)^{1+m}}{e(1+m+5n)}\end{aligned}$$

output

```
a^2*A*c^2*(e*x)^(1+m)/e/(1+m)+a*c*(a*B*c+2*A*(a*d+b*c))*x^n*(e*x)^(1+m)/e/
(1+m+n)+(2*a*B*c*(a*d+b*c)+A*(a^2*d^2+4*a*b*c*d+b^2*c^2))*x^(2*n)*(e*x)^(1
+m)/e/(1+m+2*n)+(a^2*B*d^2+2*a*b*d*(A*d+2*B*c)+b^2*c*(2*A*d+B*c))*x^(3*n)*
(e*x)^(1+m)/e/(1+m+3*n)+b*d*(A*b*d+2*B*a*d+2*B*b*c)*x^(4*n)*(e*x)^(1+m)/e/
(1+m+4*n)+b^2*B*d^2*x^(5*n)*(e*x)^(1+m)/e/(1+m+5*n)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.79

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx = x(ex)^m \left(\frac{a^2Ac^2}{1+m} + \frac{ac(aBc+2A(bc+ad))x^n}{1+m+n} + \frac{(2aBc(bc+ad)+A(b^2c^2+4abcd+a^2d^2))\,x^{2n}}{1+m+2n} + \frac{(a^2Bd^2+2abd(2Bc+Ad)+b^2c(Bc+2Ad))\,x^{3n}}{1+m+3n} + \frac{bd(2bBc+Abd+2aBd)x^{4n}}{1+m+4n} + \frac{b^2Bd^2x^{5n}}{1+m+5n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
x*(e*x)^m*((a^2*A*c^2)/(1 + m) + (a*c*(a*B*c + 2*A*(b*c + a*d))*x^n)/(1 +
m + n) + ((2*a*B*c*(b*c + a*d) + A*(b^2*c^2 + 4*a*b*c*d + a^2*d^2))*x^(2*n
))/(1 + m + 2*n) + ((a^2*B*d^2 + 2*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*
d))*x^(3*n))/(1 + m + 3*n) + (b*d*(2*b*B*c + A*b*d + 2*a*B*d)*x^(4*n))/(1
+ m + 4*n) + (b^2*B*d^2*x^(5*n))/(1 + m + 5*n))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx$$

$$\Big\downarrow \; 1040$$

$$\int \left(x^{2n}(ex)^m \left(A\left(a^2d^2+4abcd+b^2c^2\right)+2aBc(ad+bc)\right)+x^{3n}(ex)^m \left(a^2Bd^2+2abd(Ad+2Bc)+b^2c(2Ad+B\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{2n+1}(ex)^m \left(A\left(a^2d^2+4abcd+b^2c^2\right)+2aBc(ad+bc)\right)}{m+2n+1}+ \frac{x^{3n+1}(ex)^m \left(a^2Bd^2+2abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{m+3n+1}+\frac{a^2Ac^2(ex)^{m+1}}{e(m+1)}+ \frac{acx^{n+1}(ex)^m(2A(ad+bc)+aBc)}{m+n+1}+\frac{bdx^{4n+1}(ex)^m(2aBd+Abd+2bBc)}{m+4n+1}+\frac{b^2Bd^2x^{5n+1}(ex)^m}{m+5n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
(a*c*(a*B*c + 2*A*(b*c + a*d))*x^(1 + n)*(e*x)^m)/(1 + m + n) + ((2*a*B*c*
(b*c + a*d) + A*(b^2*c^2 + 4*a*b*c*d + a^2*d^2))*x^(1 + 2*n)*(e*x)^m)/(1 +
 m + 2*n) + ((a^2*B*d^2 + 2*a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))*x^(
1 + 3*n)*(e*x)^m)/(1 + m + 3*n) + (b*d*(2*b*B*c + A*b*d + 2*a*B*d)*x^(1 +
4*n)*(e*x)^m)/(1 + m + 4*n) + (b^2*B*d^2*x^(1 + 5*n)*(e*x)^m)/(1 + m + 5*n
) + (a^2*A*c^2*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.41 (sec) , antiderivative size = 5875, normalized size of antiderivative = 23.31

method	result	size
risch	Expression too large to display	5875
parallelrisch	Expression too large to display	7994
orering	Expression too large to display	11761

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3515 vs. 2(252) = 504.

Time = 0.18 (sec) , antiderivative size = 3515, normalized size of antiderivative = 13.95

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72500 vs. 2(241) = 482.

Time = 18.77 (sec) , antiderivative size = 72500, normalized size of antiderivative = 287.70

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)*(c+d*x**n)**2,x)
```

output
```
Piecewise(((A + B)*(a + b)**2*(c + d)**2*log(x)/e, Eq(m, -1) & Eq(n, 0)),
((A*a**2*c**2*log(x) + 2*A*a**2*c*d*x**n/n + A*a**2*d**2*x**(2*n)/(2*n) +
2*A*a*b*c**2*x**n/n + 2*A*a*b*c*d*x**(2*n)/n + 2*A*a*b*d**2*x**(3*n)/(3*n)
 + A*b**2*c**2*x**(2*n)/(2*n) + 2*A*b**2*c*d*x**(3*n)/(3*n) + A*b**2*d**2*
x**(4*n)/(4*n) + B*a**2*c**2*x**n/n + B*a**2*c*d*x**(2*n)/n + B*a**2*d**2*
x**(3*n)/(3*n) + B*a*b*c**2*x**(2*n)/n + 4*B*a*b*c*d*x**(3*n)/(3*n) + B*a*
b*d**2*x**(4*n)/(2*n) + B*b**2*c**2*x**(3*n)/(3*n) + B*b**2*c*d*x**(4*n)/(
2*n) + B*b**2*d**2*x**(5*n)/(5*n))/e, Eq(m, -1)), (A*a**2*c**2*Piecewise((
0**(-5*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(5*n*(e*x)**(5*n)), Ne(n, 0)),
(log(e*x), True))/e, True)) + 2*A*a**2*c*d*Piecewise((-x*x**n*(e*x)**(-5*n
 - 1)/(4*n), Ne(n, 0)), (x*x**n*(e*x)**(-5*n - 1)*log(x), True)) + A*a**2*
d**2*Piecewise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*x**(2*n
)*(e*x)**(-5*n - 1)*log(x), True)) + 2*A*a*b*c**2*Piecewise((-x*x**n*(e*x)
**(-5*n - 1)/(4*n), Ne(n, 0)), (x*x**n*(e*x)**(-5*n - 1)*log(x), True)) +
4*A*a*b*c*d*Piecewise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*
x**(2*n)*(e*x)**(-5*n - 1)*log(x), True)) + 2*A*a*b*d**2*Piecewise((-x*x**
(3*n)*(e*x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(e*x)**(-5*n - 1)*lo
g(x), True)) + A*b**2*c**2*Piecewise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n),
 Ne(n, 0)), (x*x**(2*n)*(e*x)**(-5*n - 1)*log(x), True)) + 2*A*b**2*c*d*Pi
ecewise((-x*x**(3*n)*(e*x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x**(3*n)*(e...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(252) = 504$.

Time = 0.09 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.14

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx$$
$$= \frac{Bb^2d^2e^m xe^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{2\,Bb^2cde^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{2\,Babd^2e^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{Ab^2d^2e^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{Bb^2c^2e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{4\,Babcde^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{2\,Ab^2cde^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Ba^2d^2e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{2\,Aabd^2e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{2\,Babc^2e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ab^2c^2e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{2\,Ba^2cde^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{4\,Aabcde^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Aa^2d^2e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ba^2c^2e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{2\,Aabc^2e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{2\,Aa^2cde^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}\,Aa^2c^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
B*b^2*d^2*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 2*B*b^2*c*d*e^m*
x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*B*a*b*d^2*e^m*x*e^(m*log(x)
+ 4*n*log(x))/(m + 4*n + 1) + A*b^2*d^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m
 + 4*n + 1) + B*b^2*c^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 4*
B*a*b*c*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 2*A*b^2*c*d*e^m*
x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + B*a^2*d^2*e^m*x*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 1) + 2*A*a*b*d^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m
 + 3*n + 1) + 2*B*a*b*c^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) +
A*b^2*c^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*B*a^2*c*d*e^m*
x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 4*A*a*b*c*d*e^m*x*e^(m*log(x)
+ 2*n*log(x))/(m + 2*n + 1) + A*a^2*d^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m
 + 2*n + 1) + B*a^2*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a*
b*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a^2*c*d*e^m*x*e^(m*l
og(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*a^2*c^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32523 vs. $2(252) = 504$.

Time = 0.35 (sec) , antiderivative size = 32523, normalized size of antiderivative = 129.06

$$\int (ex)^m (a + bx^n)^2 (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(B*b^2*d^2*m^5*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 10*B*b^2*d^2*m^4*n*x*x^
(5*n)*e^(m*log(e) + m*log(x)) + 35*B*b^2*d^2*m^3*n^2*x*x^(5*n)*e^(m*log(e)
 + m*log(x)) + 50*B*b^2*d^2*m^2*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 24
*B*b^2*d^2*m*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 2*B*b^2*c*d*m^5*x*x^(
4*n)*e^(m*log(e) + m*log(x)) + 2*B*a*b*d^2*m^5*x*x^(4*n)*e^(m*log(e) + m*l
og(x)) + A*b^2*d^2*m^5*x*x^(4*n)*e^(m*log(e) + m*log(x)) + B*b^2*d^2*m^5*x
*x^(4*n)*e^(m*log(e) + m*log(x)) + 22*B*b^2*c*d*m^4*n*x*x^(4*n)*e^(m*log(e
) + m*log(x)) + 22*B*a*b*d^2*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 11*
A*b^2*d^2*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 10*B*b^2*d^2*m^4*n*x*x
^(4*n)*e^(m*log(e) + m*log(x)) + 82*B*b^2*c*d*m^3*n^2*x*x^(4*n)*e^(m*log(e
) + m*log(x)) + 82*B*a*b*d^2*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 4
1*A*b^2*d^2*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 35*B*b^2*d^2*m^3*n
^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 122*B*b^2*c*d*m^2*n^3*x*x^(4*n)*e^(
m*log(e) + m*log(x)) + 122*B*a*b*d^2*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log
(x)) + 61*A*b^2*d^2*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 50*B*b^2*d
^2*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 60*B*b^2*c*d*m*n^4*x*x^(4*n
)*e^(m*log(e) + m*log(x)) + 60*B*a*b*d^2*m*n^4*x*x^(4*n)*e^(m*log(e) + m*l
og(x)) + 30*A*b^2*d^2*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 24*B*b^2*d
^2*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + B*b^2*c^2*m^5*x*x^(3*n)*e^(m*
log(e) + m*log(x)) + 4*B*a*b*c*d*m^5*x*x^(3*n)*e^(m*log(e) + m*log(x)) ...
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 1119, normalized size of antiderivative = 4.44

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^2*(c + d*x^n)^2,x)
```

output
```
(x*x^(2*n)*(e*x)^m*(A*a^2*d^2 + A*b^2*c^2 + 2*B*a*b*c^2 + 2*B*a^2*c*d + 4*
A*a*b*c*d)*(4*m + 13*n + 39*m*n + 118*m*n^2 + 39*m^2*n + 107*m*n^3 + 13*m^
3*n + 6*m^2 + 4*m^3 + m^4 + 59*n^2 + 107*n^3 + 60*n^4 + 59*m^2*n^2 + 1))/(
5*m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*
n^4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^
4 + 120*n^5 + 255*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (x*x^(3*n)*(e*
x)^m*(B*a^2*d^2 + B*b^2*c^2 + 2*A*a*b*d^2 + 2*A*b^2*c*d + 4*B*a*b*c*d)*(4*
m + 12*n + 36*m*n + 98*m*n^2 + 36*m^2*n + 78*m*n^3 + 12*m^3*n + 6*m^2 + 4*
m^3 + m^4 + 49*n^2 + 78*n^3 + 40*n^4 + 49*m^2*n^2 + 1))/(5*m + 15*n + 60*m
*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n +
10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255
*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (A*a^2*c^2*x*(e*x)^m)/(m + 1) +
 (b*d*x*x^(4*n)*(e*x)^m*(A*b*d + 2*B*a*d + 2*B*b*c)*(4*m + 11*n + 33*m*n +
 82*m*n^2 + 33*m^2*n + 61*m*n^3 + 11*m^3*n + 6*m^2 + 4*m^3 + m^4 + 41*n^2
+ 61*n^3 + 30*n^4 + 41*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 + 90
*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5
*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 + 225*m^2*
n^3 + 85*m^3*n^2 + 1) + (B*b^2*d^2*x*x^(5*n)*(e*x)^m*(4*m + 10*n + 30*m*n
+ 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2
 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3949, normalized size of antiderivative = 15.67

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^2,x)
```

output

```
(x**m*e**m*x*(x**(5*n)*b**3*d**2*m**5 + 10*x**(5*n)*b**3*d**2*m**4*n + 5*x
**(5*n)*b**3*d**2*m**4 + 35*x**(5*n)*b**3*d**2*m**3*n**2 + 40*x**(5*n)*b**
3*d**2*m**3*n + 10*x**(5*n)*b**3*d**2*m**3 + 50*x**(5*n)*b**3*d**2*m**2*n*
*3 + 105*x**(5*n)*b**3*d**2*m**2*n**2 + 60*x**(5*n)*b**3*d**2*m**2*n + 10*
x**(5*n)*b**3*d**2*m**2 + 24*x**(5*n)*b**3*d**2*m*n**4 + 100*x**(5*n)*b**3
*d**2*m*n**3 + 105*x**(5*n)*b**3*d**2*m*n**2 + 40*x**(5*n)*b**3*d**2*m*n +
 5*x**(5*n)*b**3*d**2*m + 24*x**(5*n)*b**3*d**2*n**4 + 50*x**(5*n)*b**3*d*
*2*n**3 + 35*x**(5*n)*b**3*d**2*n**2 + 10*x**(5*n)*b**3*d**2*n + x**(5*n)*
b**3*d**2 + 3*x**(4*n)*a*b**2*d**2*m**5 + 33*x**(4*n)*a*b**2*d**2*m**4*n +
 15*x**(4*n)*a*b**2*d**2*m**4 + 123*x**(4*n)*a*b**2*d**2*m**3*n**2 + 132*x
**(4*n)*a*b**2*d**2*m**3*n + 30*x**(4*n)*a*b**2*d**2*m**3 + 183*x**(4*n)*a
*b**2*d**2*m**2*n**3 + 369*x**(4*n)*a*b**2*d**2*m**2*n**2 + 198*x**(4*n)*a
*b**2*d**2*m**2*n + 30*x**(4*n)*a*b**2*d**2*m**2 + 90*x**(4*n)*a*b**2*d**2
*m*n**4 + 366*x**(4*n)*a*b**2*d**2*m*n**3 + 369*x**(4*n)*a*b**2*d**2*m*n**
2 + 132*x**(4*n)*a*b**2*d**2*m*n + 15*x**(4*n)*a*b**2*d**2*m + 90*x**(4*n)
*a*b**2*d**2*n**4 + 183*x**(4*n)*a*b**2*d**2*n**3 + 123*x**(4*n)*a*b**2*d*
*2*n**2 + 33*x**(4*n)*a*b**2*d**2*n + 3*x**(4*n)*a*b**2*d**2 + 2*x**(4*n)*
b**3*c*d*m**5 + 22*x**(4*n)*b**3*c*d*m**4*n + 10*x**(4*n)*b**3*c*d*m**4 +
82*x**(4*n)*b**3*c*d*m**3*n**2 + 88*x**(4*n)*b**3*c*d*m**3*n + 20*x**(4*n)
*b**3*c*d*m**3 + 122*x**(4*n)*b**3*c*d*m**2*n**3 + 246*x**(4*n)*b**3*c*...
```

3.28 $\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 172

$$\begin{aligned}\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^2 \, dx &= \frac{aAc^2(ex)^{1+m}}{e(1+m)} + \frac{c(Abc+aBc+2aAd)x^n(ex)^{1+m}}{e(1+m+n)} \\ &+ \frac{(ad(2Bc+Ad)+bc(Bc+2Ad))x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\ &+ \frac{d(2bBc+Abd+aBd)x^{3n}(ex)^{1+m}}{e(1+m+3n)} + \frac{bBd^2x^{4n}(ex)^{1+m}}{e(1+m+4n)}\end{aligned}$$

output
```
a*A*c^2*(e*x)^(1+m)/e/(1+m)+c*(2*A*a*d+A*b*c+B*a*c)*x^n*(e*x)^(1+m)/e/(1+m
+n)+(a*d*(A*d+2*B*c)+b*c*(2*A*d+B*c))*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+d*(A
*b*d+B*a*d+2*B*b*c)*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+b*B*d^2*x^(4*n)*(e*x)^
(1+m)/e/(1+m+4*n)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^2 \, dx$$
$$= x(ex)^m \left(\frac{aAc^2}{1+m} + \frac{c(Abc+aBc+2aAd)x^n}{1+m+n} + \frac{(ad(2Bc+Ad)+bc(Bc+2Ad))x^{2n}}{1+m+2n} + \frac{d(2bBc+Abd+aBd)x^{3n}}{1+m+3n} + \frac{bBd^2x^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
x*(e*x)^m*((a*A*c^2)/(1 + m) + (c*(A*b*c + a*B*c + 2*a*A*d)*x^n)/(1 + m +
n) + ((a*d*(2*B*c + A*d) + b*c*(B*c + 2*A*d))*x^(2*n))/(1 + m + 2*n) + (d*
(2*b*B*c + A*b*d + a*B*d)*x^(3*n))/(1 + m + 3*n) + (b*B*d^2*x^(4*n))/(1 +
m + 4*n))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^2 \, dx$$

$$\Big\downarrow \text{ 1040}$$

$$\int \left(x^{2n}(ex)^m(ad(Ad+2Bc)+bc(2Ad+Bc)) + dx^{3n}(ex)^m(aBd+Abd+2bBc) + cx^n(ex)^m(2aAd+aBc+Ab\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{cx^{n+1}(ex)^m(2aAd+aBc+Abc)}{m+n+1}+\frac{x^{2n+1}(ex)^m(ad(Ad+2Bc)+bc(2Ad+Bc))}{m+2n+1}+\frac{dx^{3n+1}(ex)^m(aBd+Abd+2bBc)}{m+3n+1}+\frac{aAc^2(ex)^{m+1}}{e(m+1)}+\frac{bBd^2x^{4n+1}(ex)^m}{m+4n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
(c*(A*b*c + a*B*c + 2*a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + ((a*d*(2*B*c
 + A*d) + b*c*(B*c + 2*A*d))*x^(1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (d*(2*b*
B*c + A*b*d + a*B*d)*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n) + (b*B*d^2*x^(1 +
4*n)*(e*x)^m)/(1 + m + 4*n) + (a*A*c^2*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 2377, normalized size of antiderivative = 13.82

method	result	size
risch	Expression too large to display	2377
parallelrisch	Expression too large to display	3344
orering	Expression too large to display	4757

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(7*A*b*d^2*(x^n)^3*n+4*B*a*c^2*m^3*x^n+24*B*a*c^2*n^3*x^n+4*B*a*d^2*(x^n
)^3*m+7*B*a*d^2*(x^n)^3*n+38*B*a*c*d*n^2*(x^n)^2+24*B*b*c^2*m*n*(x^n)^2+8*
A*a*d^2*m^3*n*(x^n)^2+6*B*b*d^2*m*n^3*(x^n)^4+7*A*b*d^2*m^3*n*(x^n)^3+6*B*
a*c^2*m^2*x^n+26*B*a*c^2*n^2*x^n+4*B*b*c^2*(x^n)^2*m+8*B*b*c^2*(x^n)^2*n+4
*A*b*c^2*x^n*m+9*A*b*c^2*x^n*n+6*A*b*c^2*m^2*x^n+50*A*a*c^2*n^3+6*A*a*c^2*
m^2+35*A*a*c^2*n^2+10*A*a*c^2*m^3*n+38*B*a*c*d*m^2*n^2*(x^n)^2+24*B*a*c*d*
m*n^3*(x^n)^2+42*B*b*c*d*m^2*n*(x^n)^3+14*B*a*d^2*m^2*n^2*(x^n)^3+8*B*a*d^
2*m*n^3*(x^n)^3+2*B*b*c*d*m^4*(x^n)^3+24*A*a*c^2*n^4+A*a*c^2*m^4+4*A*a*c^2
*m^3+4*A*a*c^2*m+10*A*a*c^2*n+21*A*b*d^2*m*n*(x^n)^3+9*B*a*c^2*m^3*n*x^n+2
6*B*a*c^2*m^2*n^2*x^n+B*a*d^2*m^4*(x^n)^3+4*B*b*d^2*m^3*(x^n)^4+8*A*a*d^2*
(x^n)^2*n+8*A*a*c*d*m^3*x^n+48*A*a*c*d*n^3*x^n+24*A*a*d^2*m*n*(x^n)^2+27*A
*b*c^2*m^2*n*x^n+52*A*b*c^2*m*n^2*x^n+24*B*a*c^2*m*n^3*x^n+38*A*b*c*d*m^2*
n^2*(x^n)^2+24*A*b*c*d*m*n^3*(x^n)^2+16*B*a*c*d*m^3*n*(x^n)^2+24*A*a*d^2*m
^2*n*(x^n)^2+8*A*a*c*d*x^n*m+18*A*a*c*d*x^n*n+8*B*a*c*d*m^3*(x^n)^2+2*B*a*
c*d*m^4*(x^n)^2+21*B*a*d^2*m^2*n*(x^n)^3+28*B*a*d^2*m*n^2*(x^n)^3+8*B*b*c^
2*m^3*n*(x^n)^2+6*B*b*d^2*m^3*n*(x^n)^4+11*B*b*d^2*m^2*n^2*(x^n)^4+56*B*b*
c*d*m*n^2*(x^n)^3+18*A*a*c*d*m^3*n*x^n+52*A*a*c*d*m^2*n^2*x^n+48*A*a*c*d*m
*n^3*x^n+48*A*b*c*d*m^2*n*(x^n)^2+76*A*b*c*d*m*n^2*(x^n)^2+48*B*a*c*d*m^2*
n*(x^n)^2+24*B*a*c*d*n^3*(x^n)^2+14*B*a*d^2*n^2*(x^n)^3+4*B*b*c^2*m^3*(x^n
)^2+12*B*b*c^2*n^3*(x^n)^2+4*m*b*B*d^2*(x^n)^4+6*b*B*d^2*(x^n)^4*n+6*A*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 1426, normalized size of antiderivative = 8.29

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
((B*b*d^2*m^4 + 4*B*b*d^2*m^3 + 6*B*b*d^2*m^2 + 4*B*b*d^2*m + B*b*d^2 + 6*
(B*b*d^2*m + B*b*d^2)*n^3 + 11*(B*b*d^2*m^2 + 2*B*b*d^2*m + B*b*d^2)*n^2 +
 6*(B*b*d^2*m^3 + 3*B*b*d^2*m^2 + 3*B*b*d^2*m + B*b*d^2)*n)*x*x^(4*n)*e^(m
*log(e) + m*log(x)) + ((2*B*b*c*d + (B*a + A*b)*d^2)*m^4 + 2*B*b*c*d + 4*(
2*B*b*c*d + (B*a + A*b)*d^2)*m^3 + 8*(2*B*b*c*d + (B*a + A*b)*d^2 + (2*B*b
*c*d + (B*a + A*b)*d^2)*m)*n^3 + (B*a + A*b)*d^2 + 6*(2*B*b*c*d + (B*a + A
*b)*d^2)*m^2 + 14*(2*B*b*c*d + (B*a + A*b)*d^2 + (2*B*b*c*d + (B*a + A*b)*
d^2)*m^2 + 2*(2*B*b*c*d + (B*a + A*b)*d^2)*m)*n^2 + 4*(2*B*b*c*d + (B*a +
A*b)*d^2)*m + 7*(2*B*b*c*d + (2*B*b*c*d + (B*a + A*b)*d^2)*m^3 + (B*a + A*
b)*d^2 + 3*(2*B*b*c*d + (B*a + A*b)*d^2)*m^2 + 3*(2*B*b*c*d + (B*a + A*b)*
d^2)*m)*n)*x*x^(3*n)*e^(m*log(e) + m*log(x)) + ((B*b*c^2 + A*a*d^2 + 2*(B*
a + A*b)*c*d)*m^4 + B*b*c^2 + A*a*d^2 + 4*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*
b)*c*d)*m^3 + 12*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d + (B*b*c^2 + A*a*d
^2 + 2*(B*a + A*b)*c*d)*m)*n^3 + 2*(B*a + A*b)*c*d + 6*(B*b*c^2 + A*a*d^2
+ 2*(B*a + A*b)*c*d)*m^2 + 19*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d + (B*
b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d)*m^2 + 2*(B*b*c^2 + A*a*d^2 + 2*(B*a +
 A*b)*c*d)*m)*n^2 + 4*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d)*m + 8*(B*b*c
^2 + A*a*d^2 + (B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d)*m^3 + 2*(B*a + A*b)
*c*d + 3*(B*b*c^2 + A*a*d^2 + 2*(B*a + A*b)*c*d)*m^2 + 3*(B*b*c^2 + A*a*d^
2 + 2*(B*a + A*b)*c*d)*m)*n)*x*x^(2*n)*e^(m*log(e) + m*log(x)) + ((2*A*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25315 vs. $2(163) = 326$.

Time = 9.84 (sec) , antiderivative size = 25315, normalized size of antiderivative = 147.18

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)*(c+d*x**n)**2,x)
```

output
```
Piecewise(((A + B)*(a + b)*(c + d)**2*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A
*a*c**2*log(x) + 2*A*a*c*d*x**n/n + A*a*d**2*x**(2*n)/(2*n) + A*b*c**2*x**
n/n + A*b*c*d*x**(2*n)/n + A*b*d**2*x**(3*n)/(3*n) + B*a*c**2*x**n/n + B*a
*c*d*x**(2*n)/n + B*a*d**2*x**(3*n)/(3*n) + B*b*c**2*x**(2*n)/(2*n) + 2*B*
b*c*d*x**(3*n)/(3*n) + B*b*d**2*x**(4*n)/(4*n))/e, Eq(m, -1)), (A*a*c**2*P
iecewise((0**(-4*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(4*n*(e*x)**(4*n)), N
e(n, 0)), (log(e*x), True))/e, True)) + 2*A*a*c*d*Piecewise((-x*x**n*(e*x)
**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(e*x)**(-4*n - 1)*log(x), True)) +
A*a*d**2*Piecewise((-x*x**(2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**
(2*n)*(e*x)**(-4*n - 1)*log(x), True)) + A*b*c**2*Piecewise((-x*x**n*(e*x)
**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(e*x)**(-4*n - 1)*log(x), True)) +
2*A*b*c*d*Piecewise((-x*x**(2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x*
*(2*n)*(e*x)**(-4*n - 1)*log(x), True)) + A*b*d**2*Piecewise((-x*x**(3*n)*
(e*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(e*x)**(-4*n - 1)*log(x), True
)) + B*a*c**2*Piecewise((-x*x**n*(e*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x*
*n*(e*x)**(-4*n - 1)*log(x), True)) + 2*B*a*c*d*Piecewise((-x*x**(2*n)*(e*
x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-4*n - 1)*log(x), Tru
e)) + B*a*d**2*Piecewise((-x*x**(3*n)*(e*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x
**(3*n)*(e*x)**(-4*n - 1)*log(x), True)) + B*b*c**2*Piecewise((-x*x**(2*n)
*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-4*n - 1)*log(...
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.93

$$\begin{aligned}
&\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^2 \, dx \\
&= \frac{Bbd^2e^m xe^{(m\log(x)+4n\log(x))}}{m+4n+1} + \frac{2\,Bbcde^m xe^{(m\log(x)+3n\log(x))}}{m+3n+1} \\
&+ \frac{Bad^2e^m xe^{(m\log(x)+3n\log(x))}}{m+3n+1} + \frac{Abd^2e^m xe^{(m\log(x)+3n\log(x))}}{m+3n+1} \\
&+ \frac{Bbc^2e^m xe^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{2\,Bacde^m xe^{(m\log(x)+2n\log(x))}}{m+2n+1} \\
&+ \frac{2\,Abcde^m xe^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{Aad^2e^m xe^{(m\log(x)+2n\log(x))}}{m+2n+1} \\
&+ \frac{Bac^2e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Abc^2e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} \\
&+ \frac{2\,Aacde^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}\,Aac^2}{e(m+1)}
\end{aligned}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
B*b*d^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*B*b*c*d*e^m*x*e^
(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + B*a*d^2*e^m*x*e^(m*log(x) + 3*n*lo
g(x))/(m + 3*n + 1) + A*b*d^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1
) + B*b*c^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*B*a*c*d*e^m*
x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*A*b*c*d*e^m*x*e^(m*log(x) +
2*n*log(x))/(m + 2*n + 1) + A*a*d^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2
*n + 1) + B*a*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + A*b*c^2*e^m*
x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*a*c*d*e^m*x*e^(m*log(x) + n*lo
g(x))/(m + n + 1) + (e*x)^(m + 1)*A*a*c^2/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11834 vs. $2(172) = 344$.

Time = 0.22 (sec) , antiderivative size = 11834, normalized size of antiderivative = 68.80

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(B*b*d^2*m^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 6*B*b*d^2*m^3*n*x*x^(4*n)
*e^(m*log(e) + m*log(x)) + 11*B*b*d^2*m^2*n^2*x*x^(4*n)*e^(m*log(e) + m*lo
g(x)) + 6*B*b*d^2*m*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 2*B*b*c*d*m^4*
x*x^(3*n)*e^(m*log(e) + m*log(x)) + B*a*d^2*m^4*x*x^(3*n)*e^(m*log(e) + m*
log(x)) + A*b*d^2*m^4*x*x^(3*n)*e^(m*log(e) + m*log(x)) + B*b*d^2*m^4*x*x^
(3*n)*e^(m*log(e) + m*log(x)) + 14*B*b*c*d*m^3*n*x*x^(3*n)*e^(m*log(e) + m
*log(x)) + 7*B*a*d^2*m^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 7*A*b*d^2*m
^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 6*B*b*d^2*m^3*n*x*x^(3*n)*e^(m*lo
g(e) + m*log(x)) + 28*B*b*c*d*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) +
14*B*a*d^2*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 14*A*b*d^2*m^2*n^2*
x*x^(3*n)*e^(m*log(e) + m*log(x)) + 11*B*b*d^2*m^2*n^2*x*x^(3*n)*e^(m*log(
e) + m*log(x)) + 16*B*b*c*d*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 8*B*
a*d^2*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 8*A*b*d^2*m*n^3*x*x^(3*n)*
e^(m*log(e) + m*log(x)) + 6*B*b*d^2*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)
) + B*b*c^2*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 2*B*a*c*d*m^4*x*x^(2*n
)*e^(m*log(e) + m*log(x)) + 2*A*b*c*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)
) + 2*B*b*c*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + A*a*d^2*m^4*x*x^(2*n
)*e^(m*log(e) + m*log(x)) + B*a*d^2*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x))
+ A*b*d^2*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*b*d^2*m^4*x*x^(2*n)*e^
(m*log(e) + m*log(x)) + 8*B*b*c^2*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(x...
```

Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.42

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^2\,dx$$
$$= \frac{x\,x^{2n}\,(e\,x)^m\,(A\,a\,d^2+B\,b\,c^2+2\,A\,b\,c\,d+2\,B\,a\,c\,d)\,(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n}$$
$$+\frac{A\,a\,c^2\,x\,(e\,x)^m}{m+1}$$
$$+\frac{c\,x\,x^n\,(e\,x)^m\,(2\,A\,a\,d+A\,b\,c+B\,a\,c)\,(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$
$$+\frac{d\,x\,x^{3n}\,(e\,x)^m\,(A\,b\,d+B\,a\,d+2\,B\,b\,c)\,(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$
$$+\frac{B\,b\,d^2\,x\,x^{4n}\,(e\,x)^m\,(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)*(c + d*x^n)^2,x)
```

output
```
(x*x^(2*n)*(e*x)^m*(A*a*d^2 + B*b*c^2 + 2*A*b*c*d + 2*B*a*c*d)*(3*m + 8*n
+ 16*m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m +
 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3
 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (A*a*c^2*x*(e*x)^m)/
(m + 1) + (c*x*x^n*(e*x)^m*(2*A*a*d + A*b*c + B*a*c)*(3*m + 9*n + 18*m*n +
 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3 + 1))/(4*m + 10*n + 30
*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 3
5*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (d*x*x^(3*n)*(e*x)^m*(A*b*d +
B*a*d + 2*B*b*c)*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 +
14*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3
 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2
+ 1) + (B*b*d^2*x*x^(4*n)*(e*x)^m*(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n
 + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30
*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*
n^4 + 35*m^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1916, normalized size of antiderivative = 11.14

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^2 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^2,x)
```

output

```
(x**m*e**m*x*(x**(4*n)*b**2*d**2*m**4 + 6*x**(4*n)*b**2*d**2*m**3*n + 4*x*
*(4*n)*b**2*d**2*m**3 + 11*x**(4*n)*b**2*d**2*m**2*n**2 + 18*x**(4*n)*b**2
*d**2*m**2*n + 6*x**(4*n)*b**2*d**2*m**2 + 6*x**(4*n)*b**2*d**2*m*n**3 + 2
2*x**(4*n)*b**2*d**2*m*n**2 + 18*x**(4*n)*b**2*d**2*m*n + 4*x**(4*n)*b**2*
d**2*m + 6*x**(4*n)*b**2*d**2*n**3 + 11*x**(4*n)*b**2*d**2*n**2 + 6*x**(4*
n)*b**2*d**2*n + x**(4*n)*b**2*d**2 + 2*x**(3*n)*a*b*d**2*m**4 + 14*x**(3*
n)*a*b*d**2*m**3*n + 8*x**(3*n)*a*b*d**2*m**3 + 28*x**(3*n)*a*b*d**2*m**2*
n**2 + 42*x**(3*n)*a*b*d**2*m**2*n + 12*x**(3*n)*a*b*d**2*m**2 + 16*x**(3*
n)*a*b*d**2*m*n**3 + 56*x**(3*n)*a*b*d**2*m*n**2 + 42*x**(3*n)*a*b*d**2*m*
n + 8*x**(3*n)*a*b*d**2*m + 16*x**(3*n)*a*b*d**2*n**3 + 28*x**(3*n)*a*b*d*
*2*n**2 + 14*x**(3*n)*a*b*d**2*n + 2*x**(3*n)*a*b*d**2 + 2*x**(3*n)*b**2*c
*d*m**4 + 14*x**(3*n)*b**2*c*d*m**3*n + 8*x**(3*n)*b**2*c*d*m**3 + 28*x**(
3*n)*b**2*c*d*m**2*n**2 + 42*x**(3*n)*b**2*c*d*m**2*n + 12*x**(3*n)*b**2*c
*d*m**2 + 16*x**(3*n)*b**2*c*d*m*n**3 + 56*x**(3*n)*b**2*c*d*m*n**2 + 42*x
**(3*n)*b**2*c*d*m*n + 8*x**(3*n)*b**2*c*d*m + 16*x**(3*n)*b**2*c*d*n**3 +
 28*x**(3*n)*b**2*c*d*n**2 + 14*x**(3*n)*b**2*c*d*n + 2*x**(3*n)*b**2*c*d
+ x**(2*n)*a**2*d**2*m**4 + 8*x**(2*n)*a**2*d**2*m**3*n + 4*x**(2*n)*a**2*
d**2*m**3 + 19*x**(2*n)*a**2*d**2*m**2*n**2 + 24*x**(2*n)*a**2*d**2*m**2*n
 + 6*x**(2*n)*a**2*d**2*m**2 + 12*x**(2*n)*a**2*d**2*m*n**3 + 38*x**(2*n)*
a**2*d**2*m*n**2 + 24*x**(2*n)*a**2*d**2*m*n + 4*x**(2*n)*a**2*d**2*m +...
```

3.29 $\int (ex)^m (A+Bx^n) (c+dx^n)^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int (ex)^m (A+Bx^n) (c+dx^n)^2 \, dx = \frac{Ac^2(ex)^{1+m}}{e(1+m)} + \frac{c(Bc+2Ad)x^n(ex)^{1+m}}{e(1+m+n)} + \frac{d(2Bc+Ad)x^{2n}(ex)^{1+m}}{e(1+m+2n)} + \frac{Bd^2x^{3n}(ex)^{1+m}}{e(1+m+3n)}$$

output
```
A*c^2*(e*x)^(1+m)/e/(1+m)+c*(2*A*d+B*c)*x^n*(e*x)^(1+m)/e/(1+m+n)+d*(A*d+2
*B*c)*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+B*d^2*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n
)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.70

$$\int (ex)^m (A+Bx^n) (c+dx^n)^2 \, dx = x(ex)^m \left(\frac{Ac^2}{1+m} + \frac{c(Bc+2Ad)x^n}{1+m+n} + \frac{d(2Bc+Ad)x^{2n}}{1+m+2n} + \frac{Bd^2x^{3n}}{1+m+3n}\right)$$

input
```
Integrate[(e*x)^m*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
x*(e*x)^m*((A*c^2)/(1 + m) + (c*(B*c + 2*A*d)*x^n)/(1 + m + n) + (d*(2*B*c
 + A*d)*x^(2*n))/(1 + m + 2*n) + (B*d^2*x^(3*n))/(1 + m + 3*n))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (A + Bx^n) (c + dx^n)^2 \, dx$$

$$\Big\downarrow \; 950$$

$$\int \left(dx^{2n}(ex)^m(Ad + 2Bc) + cx^n(ex)^m(2Ad + Bc) + Ac^2(ex)^m + Bd^2x^{3n}(ex)^m\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{cx^{n+1}(ex)^m(2Ad + Bc)}{m + n + 1} + \frac{dx^{2n+1}(ex)^m(Ad + 2Bc)}{m + 2n + 1} + \frac{Ac^2(ex)^{m+1}}{e(m + 1)} + \frac{Bd^2x^{3n+1}(ex)^m}{m + 3n + 1}$$

input
```
Int[(e*x)^m*(A + B*x^n)*(c + d*x^n)^2,x]
```

output
```
(c*(B*c + 2*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (d*(2*B*c + A*d)*x^(1 +
2*n)*(e*x)^m)/(1 + m + 2*n) + (B*d^2*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n) +
(A*c^2*(e*x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 699, normalized size of antiderivative = 6.30

method	result
risch	$\frac{x(6A\,c^2n+10Acd\,m^2n\,x^n+6B\,c^2m\,n^2x^n+12Acdm\,n^2x^n+3B\,c^2x^nm+5B\,c^2x^nn+2Acd\,x^n+3B\,c^2m^2x^n+6B\,c^2n^2x^n+B\,c^2}{}$
parallelrisch	$\frac{6Ax(ex)^mc^2m^2n+11Ax(ex)^mc^2m\,n^2+3Bx\,x^n(ex)^mc^2m^2+6Bx\,x^n(ex)^mc^2n^2+4Ax\,x^{2n}(ex)^md^2m^2n+3Bx\,x^{3n}(ex)^md^2m}{}$
orering	Expression too large to display

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(6*A*c^2*n+10*A*c*d*m^2*n*x^n+6*B*c^2*m*n^2*x^n+12*A*c*d*m*n^2*x^n+3*B*c
^2*x^n*m+5*B*c^2*x^n*n+2*B*c*d*(x^n)^2+2*A*c*d*x^n+3*B*d^2*(x^n)^3*n+3*A*d
^2*(x^n)^2*m+4*A*d^2*(x^n)^2*n+3*B*c^2*m^2*x^n+6*B*c^2*n^2*x^n+3*A*d^2*m^2
*(x^n)^2+3*A*d^2*n^2*(x^n)^2+B*c^2*m^3*x^n+3*m*B*d^2*(x^n)^3+2*A*c*d*m^3*x
^n+8*A*d^2*m*n*(x^n)^2+5*B*c^2*m^2*n*x^n+6*A*c^2*m^2*n+16*B*c*d*m*n*(x^n)^
2+20*A*c*d*m*n*x^n+8*B*c*d*m^2*n*(x^n)^2+6*B*c*d*m*n^2*(x^n)^2+2*B*d^2*m*n
^2*(x^n)^3+4*A*d^2*m^2*n*(x^n)^2+3*A*d^2*m*n^2*(x^n)^2+2*B*c*d*m^3*(x^n)^2
+6*B*d^2*m*n*(x^n)^3+3*A*c^2*m+B*d^2*m^3*(x^n)^3+A*d^2*m^3*(x^n)^2+3*B*d^2
*m^2*(x^n)^3+2*B*d^2*n^2*(x^n)^3+11*A*c^2*m*n^2+12*A*c^2*m*n+6*A*c^2*n^3+3
*A*c^2*m^2+11*A*c^2*n^2+A*c^2*m^3+(x^n)^3*B*d^2+A*d^2*(x^n)^2+B*c^2*x^n+A*
c^2+12*A*c*d*n^2*x^n+10*B*c^2*m*n*x^n+6*B*c*d*(x^n)^2*m+8*B*c*d*(x^n)^2*n+
6*A*c*d*x^n*m+10*A*c*d*x^n*n+3*B*d^2*m^2*n*(x^n)^3+6*B*c*d*m^2*(x^n)^2+6*B
*c*d*n^2*(x^n)^2+6*A*c*d*m^2*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*e^m*x^
m*exp(1/2*I*Pi*csgn(I*e*x)*m*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*
e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(111) = 222$.

Time = 0.11 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.75

$$\int (ex)^m (A + Bx^n) (c + dx^n)^2 \, dx = \frac{(Bd^2m^3 + 3\,Bd^2m^2 + 3\,Bd^2m + Bd^2 + 2\,(Bd^2m + Bd^2)n^2 + 3\,(Bd^2m^2 + 2\,Bd^2m + Bd^2)n)xx^{3\,n}e^{(m\log(}}{}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
((B*d^2*m^3 + 3*B*d^2*m^2 + 3*B*d^2*m + B*d^2 + 2*(B*d^2*m + B*d^2)*n^2 +
3*(B*d^2*m^2 + 2*B*d^2*m + B*d^2)*n)*x*x^(3*n)*e^(m*log(e) + m*log(x)) + (
(2*B*c*d + A*d^2)*m^3 + 2*B*c*d + A*d^2 + 3*(2*B*c*d + A*d^2)*m^2 + 3*(2*B
*c*d + A*d^2 + (2*B*c*d + A*d^2)*m)*n^2 + 3*(2*B*c*d + A*d^2)*m + 4*(2*B*c
*d + A*d^2 + (2*B*c*d + A*d^2)*m^2 + 2*(2*B*c*d + A*d^2)*m)*n)*x*x^(2*n)*e
^(m*log(e) + m*log(x)) + ((B*c^2 + 2*A*c*d)*m^3 + B*c^2 + 2*A*c*d + 3*(B*c
^2 + 2*A*c*d)*m^2 + 6*(B*c^2 + 2*A*c*d + (B*c^2 + 2*A*c*d)*m)*n^2 + 3*(B*c
^2 + 2*A*c*d)*m + 5*(B*c^2 + 2*A*c*d + (B*c^2 + 2*A*c*d)*m^2 + 2*(B*c^2 +
2*A*c*d)*m)*n)*x*x^n*e^(m*log(e) + m*log(x)) + (A*c^2*m^3 + 6*A*c^2*n^3 +
3*A*c^2*m^2 + 3*A*c^2*m + A*c^2 + 11*(A*c^2*m + A*c^2)*n^2 + 6*(A*c^2*m^2
+ 2*A*c^2*m + A*c^2)*n)*x*e^(m*log(e) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 +
4*m^3 + 11*(m^2 + 2*m + 1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m
 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5882 vs. $2(99) = 198$.

Time = 4.54 (sec) , antiderivative size = 5882, normalized size of antiderivative = 52.99

$$\int (ex)^m (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**2,x)
```

output
```
Piecewise(((A + B)*(c + d)**2*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A*c**2*lo
g(x) + 2*A*c*d*x**n/n + A*d**2*x**(2*n)/(2*n) + B*c**2*x**n/n + B*c*d*x**(
2*n)/n + B*d**2*x**(3*n)/(3*n))/e, Eq(m, -1)), (A*c**2*Piecewise((0**(-3*n
 - 1)*x, Eq(e, 0)), (Piecewise((-1/(3*n*(e*x)**(3*n)), Ne(n, 0)), (log(e*x
), True))/e, True)) + 2*A*c*d*Piecewise((-x*x**n*(e*x)**(-3*n - 1)/(2*n),
Ne(n, 0)), (x*x**n*(e*x)**(-3*n - 1)*log(x), True)) + A*d**2*Piecewise((-x
*x**(2*n)*(e*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(e*x)**(-3*n - 1)*lo
g(x), True)) + B*c**2*Piecewise((-x*x**n*(e*x)**(-3*n - 1)/(2*n), Ne(n, 0)
), (x*x**n*(e*x)**(-3*n - 1)*log(x), True)) + 2*B*c*d*Piecewise((-x*x**(2*
n)*(e*x)**(-3*n - 1)/n, Ne(n, 0)), (x*x**(2*n)*(e*x)**(-3*n - 1)*log(x), T
rue)) + B*d**2*x*x**(3*n)*(e*x)**(-3*n - 1)*log(x), Eq(m, -3*n - 1)), (A*c
**2*Piecewise((0**(-2*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(2*n*(e*x)**(2*n
)), Ne(n, 0)), (log(e*x), True))/e, True)) + 2*A*c*d*Piecewise((-x*x**n*(e
*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(e*x)**(-2*n - 1)*log(x), True)) + A
*d**2*x*x**(2*n)*(e*x)**(-2*n - 1)*log(x) + B*c**2*Piecewise((-x*x**n*(e*x
)**(-2*n - 1)/n, Ne(n, 0)), (x*x**n*(e*x)**(-2*n - 1)*log(x), True)) + 2*B
*c*d*x*x**(2*n)*(e*x)**(-2*n - 1)*log(x) + B*d**2*Piecewise((x*x**(3*n)*(e
*x)**(-2*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(e*x)**(-2*n - 1)*log(x), True))
, Eq(m, -2*n - 1)), (A*c**2*Piecewise((0**(-n - 1)*x, Eq(e, 0)), (Piecewis
e((-1/(n*(e*x)**n), Ne(n, 0)), (log(e*x), True))/e, True)) + 2*A*c*d*x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.40

$$\int (ex)^m (A+Bx^n)(c+dx^n)^2\,dx = \frac{Bd^2e^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{2\,Bcde^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Ad^2e^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Bc^2e^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{2\,Acde^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}Ac^2}{e(m+1)}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
B*d^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 2*B*c*d*e^m*x*e^(m*l
og(x) + 2*n*log(x))/(m + 2*n + 1) + A*d^2*e^m*x*e^(m*log(x) + 2*n*log(x))/
(m + 2*n + 1) + B*c^2*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 2*A*c*d*
e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*c^2/(e*(m + 1)
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2951 vs. $2(111) = 222$.

Time = 0.15 (sec) , antiderivative size = 2951, normalized size of antiderivative = 26.59

$$\int (ex)^m (A + Bx^n) (c + dx^n)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

output
```
(B*d^2*m^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*B*d^2*m^2*n*x*x^(3*n)*e^(
m*log(e) + m*log(x)) + 2*B*d^2*m*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 2
*B*c*d*m^3*x*x^(2*n)*e^(m*log(e) + m*log(x)) + A*d^2*m^3*x*x^(2*n)*e^(m*lo
g(e) + m*log(x)) + B*d^2*m^3*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 8*B*c*d*m
^2*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 4*A*d^2*m^2*n*x*x^(2*n)*e^(m*log(
e) + m*log(x)) + 3*B*d^2*m^2*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 6*B*c*d
*m*n^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*A*d^2*m*n^2*x*x^(2*n)*e^(m*lo
g(e) + m*log(x)) + 2*B*d^2*m*n^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*c^2
*m^3*x*x^n*e^(m*log(e) + m*log(x)) + 2*A*c*d*m^3*x*x^n*e^(m*log(e) + m*log
(x)) + 2*B*c*d*m^3*x*x^n*e^(m*log(e) + m*log(x)) + A*d^2*m^3*x*x^n*e^(m*lo
g(e) + m*log(x)) + B*d^2*m^3*x*x^n*e^(m*log(e) + m*log(x)) + 5*B*c^2*m^2*n
*x*x^n*e^(m*log(e) + m*log(x)) + 10*A*c*d*m^2*n*x*x^n*e^(m*log(e) + m*log(
x)) + 8*B*c*d*m^2*n*x*x^n*e^(m*log(e) + m*log(x)) + 4*A*d^2*m^2*n*x*x^n*e^
(m*log(e) + m*log(x)) + 3*B*d^2*m^2*n*x*x^n*e^(m*log(e) + m*log(x)) + 6*B*
c^2*m*n^2*x*x^n*e^(m*log(e) + m*log(x)) + 12*A*c*d*m*n^2*x*x^n*e^(m*log(e)
 + m*log(x)) + 6*B*c*d*m*n^2*x*x^n*e^(m*log(e) + m*log(x)) + 3*A*d^2*m*n^2
*x*x^n*e^(m*log(e) + m*log(x)) + 2*B*d^2*m*n^2*x*x^n*e^(m*log(e) + m*log(x
)) + A*c^2*m^3*x*e^(m*log(e) + m*log(x)) + B*c^2*m^3*x*e^(m*log(e) + m*log
(x)) + 2*A*c*d*m^3*x*e^(m*log(e) + m*log(x)) + 2*B*c*d*m^3*x*e^(m*log(e) +
 m*log(x)) + A*d^2*m^3*x*e^(m*log(e) + m*log(x)) + B*d^2*m^3*x*e^(m*log...
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.39

$$\int (ex)^m (A+Bx^n) (c+dx^n)^2 \, dx$$
$$= \frac{A c^2 x (e x)^m}{m+1} + \frac{c x x^n (e x)^m (2 A d + B c) (m^2 + 5 m n + 2 m + 6 n^2 + 5 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1} + \frac{d x x^{2 n} (e x)^m (A d + 2 B c) (m^2 + 4 m n + 2 m + 3 n^2 + 4 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1} + \frac{B d^2 x x^{3 n} (e x)^m (m^2 + 3 m n + 2 m + 2 n^2 + 3 n + 1)}{m^3 + 6 m^2 n + 3 m^2 + 11 m n^2 + 12 m n + 3 m + 6 n^3 + 11 n^2 + 6 n + 1}$$

input
```
int((e*x)^m*(A + B*x^n)*(c + d*x^n)^2,x)
```

output
```
(A*c^2*x*(e*x)^m)/(m + 1) + (c*x*x^n*(e*x)^m*(2*A*d + B*c)*(2*m + 5*n + 5*
m*n + m^2 + 6*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 +
 m^3 + 11*n^2 + 6*n^3 + 1) + (d*x*x^(2*n)*(e*x)^m*(A*d + 2*B*c)*(2*m + 4*n
 + 4*m*n + m^2 + 3*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*
m^2 + m^3 + 11*n^2 + 6*n^3 + 1) + (B*d^2*x*x^(3*n)*(e*x)^m*(2*m + 3*n + 3*
m*n + m^2 + 2*n^2 + 1))/(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 +
 m^3 + 11*n^2 + 6*n^3 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 700, normalized size of antiderivative = 6.31

$$\int (ex)^m (A+Bx^n) (c+dx^n)^2 \, dx$$
$$= \frac{x^m e^m x(8x^{2n} bcd\, m^2 n + 6x^{2n} bcdm\, n^2 + 16x^{2n} bcdmn + 10x^n acd\, m^2 n + 12x^n acdm\, n^2 + 20x^n acdmn + 3x}{}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2,x)
```

output
```
(x**m*e**m*x*(x**(3*n)*b*d**2*m**3 + 3*x**(3*n)*b*d**2*m**2*n + 3*x**(3*n)
*b*d**2*m**2 + 2*x**(3*n)*b*d**2*m*n**2 + 6*x**(3*n)*b*d**2*m*n + 3*x**(3*
n)*b*d**2*m + 2*x**(3*n)*b*d**2*n**2 + 3*x**(3*n)*b*d**2*n + x**(3*n)*b*d*
*2 + x**(2*n)*a*d**2*m**3 + 4*x**(2*n)*a*d**2*m**2*n + 3*x**(2*n)*a*d**2*m
**2 + 3*x**(2*n)*a*d**2*m*n**2 + 8*x**(2*n)*a*d**2*m*n + 3*x**(2*n)*a*d**2
*m + 3*x**(2*n)*a*d**2*n**2 + 4*x**(2*n)*a*d**2*n + x**(2*n)*a*d**2 + 2*x*
*(2*n)*b*c*d*m**3 + 8*x**(2*n)*b*c*d*m**2*n + 6*x**(2*n)*b*c*d*m**2 + 6*x*
*(2*n)*b*c*d*m*n**2 + 16*x**(2*n)*b*c*d*m*n + 6*x**(2*n)*b*c*d*m + 6*x**(2
*n)*b*c*d*n**2 + 8*x**(2*n)*b*c*d*n + 2*x**(2*n)*b*c*d + 2*x**n*a*c*d*m**3
 + 10*x**n*a*c*d*m**2*n + 6*x**n*a*c*d*m**2 + 12*x**n*a*c*d*m*n**2 + 20*x*
*n*a*c*d*m*n + 6*x**n*a*c*d*m + 12*x**n*a*c*d*n**2 + 10*x**n*a*c*d*n + 2*x
**n*a*c*d + x**n*b*c**2*m**3 + 5*x**n*b*c**2*m**2*n + 3*x**n*b*c**2*m**2 +
 6*x**n*b*c**2*m*n**2 + 10*x**n*b*c**2*m*n + 3*x**n*b*c**2*m + 6*x**n*b*c*
*2*n**2 + 5*x**n*b*c**2*n + x**n*b*c**2 + a*c**2*m**3 + 6*a*c**2*m**2*n +
3*a*c**2*m**2 + 11*a*c**2*m*n**2 + 12*a*c**2*m*n + 3*a*c**2*m + 6*a*c**2*n
**3 + 11*a*c**2*n**2 + 6*a*c**2*n + a*c**2))/(m**4 + 6*m**3*n + 4*m**3 + 1
1*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*n**3 + 22*m*n**2 + 18*m*n + 4*m + 6
*n**3 + 11*n**2 + 6*n + 1)
```

3.30 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^2}{a+bx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 191

$$\begin{aligned}\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^2}{a+bx^n}\,dx &= \frac{(a^2Bd^2-abd(2Bc+Ad)+b^2c(Bc+2Ad))(ex)^{1+m}}{b^3e(1+m)} \\ &\quad+\frac{d(2bBc+Abd-aBd)x^n(ex)^{1+m}}{b^2e(1+m+n)}+\frac{Bd^2x^{2n}(ex)^{1+m}}{be(1+m+2n)} \\ &\quad+\frac{(Ab-aB)(bc-ad)^2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ab^3e(1+m)}\end{aligned}$$

output
```
(a^2*B*d^2-a*b*d*(A*d+2*B*c)+b^2*c*(2*A*d+B*c))*(e*x)^(1+m)/b^3/e/(1+m)+d*
(A*b*d-B*a*d+2*B*b*c)*x^n*(e*x)^(1+m)/b^2/e/(1+m+n)+B*d^2*x^(2*n)*(e*x)^(1
+m)/b/e/(1+m+2*n)+(A*b-B*a)*(-a*d+b*c)^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n
],[(1+m+n)/n],-b*x^n/a)/a/b^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} \, dx$$
$$= \frac{x(ex)^m \left(\frac{a^2Bd^2-abd(2Bc+Ad)+b^2c(Bc+2Ad)}{1+m} + \frac{bd(2bBc+Abd-aBd)x^n}{1+m+n} + \frac{b^2Bd^2x^{2n}}{1+m+2n} + \frac{(Ab-aB)(bc-ad)^2 \text{Hypergeometric2F1}\left(1, \right.}{a(1+m)}\right.}{b^3}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n),x]
```

output
```
(x*(e*x)^m*((a^2*B*d^2 - a*b*d*(2*B*c + A*d) + b^2*c*(B*c + 2*A*d))/(1 + m
) + (b*d*(2*b*B*c + A*b*d - a*B*d)*x^n)/(1 + m + n) + (b^2*B*d^2*x^(2*n))/
(1 + m + 2*n) + ((A*b - a*B)*(b*c - a*d)^2*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m))))/b^3
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} \, dx$$
$$\Big\downarrow \text{1040}$$
$$\int \left(\frac{(ex)^m \left(a^2Bd^2-abd(Ad+2Bc)+b^2c(2Ad+Bc)\right)}{b^3} + \frac{(ex)^m(Ab-aB)(bc-ad)^2}{b^3(a+bx^n)} + \frac{dx^n(ex)^m(-aBd+Ab\right.}{b^2}$$
$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}\left(a^2Bd^2 - abd(Ad + 2Bc) + b^2c(2Ad + Bc)\right)}{b^3e(m+1)} + \frac{(ex)^{m+1}(Ab - aB)(bc - ad)^2 \text{Hypergeometric2F1}\left(1, \frac{m+1}{n}, \frac{m+n+1}{n}, -\frac{bx^n}{a}\right)}{ab^3e(m+1)} + \frac{dx^{n+1}(ex)^m(-aBd + Abd + 2bBc)}{b^2(m+n+1)} + \frac{Bd^2x^{2n+1}(ex)^m}{b(m+2n+1)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n),x]
```

output
```
(d*(2*b*B*c + A*b*d - a*B*d)*x^(1 + n)*(e*x)^m)/(b^2*(1 + m + n)) + (B*d^2
*x^(1 + 2*n)*(e*x)^m)/(b*(1 + m + 2*n)) + ((a^2*B*d^2 - a*b*d*(2*B*c + A*d
) + b^2*c*(B*c + 2*A*d))*(e*x)^(1 + m))/(b^3*e*(1 + m)) + ((A*b - a*B)*(b*
c - a*d)^2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
(b*x^n)/a)])/(a*b^3*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m \left(A + B\,x^n\right)\left(c + d\,x^n\right)^2}{a + b\,x^n}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{a + bx^n} \, dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{bx^n + a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*d^2*x^(3*n) + A*c^2 + (2*B*c*d + A*d^2)*x^(2*n) + (B*c^2 + 2*A
*c*d)*x^n)*(e*x)^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.13 (sec) , antiderivative size = 1402, normalized size of antiderivative = 7.34

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{a + bx^n} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**2/(a+b*x**n),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**2*e**m*m*x**(m + 1)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**2*e**m*x**(m + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n
+ 1 + 1/n)) + A*a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*d**2*e**m*m*x**(m +
 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n +
 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 2*A*a**(-m/n - 3 - 1/n)*a**(m/n +
2 + 1/n)*d**2*e**m*x**(m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n + 3 + 1/n)) + A*a**(-m/n
- 3 - 1/n)*a**(m/n + 2 + 1/n)*d**2*e**m*x**(m + 2*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n +
 3 + 1/n)) + 2*A*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*c*d*e**m*m*x**(m +
 n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1
 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + 2*A*a**(-m/n - 2 - 1/n)*a**(m/n + 1
+ 1/n)*c*d*e**m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n +
 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + 2*A*a**(-m/n - 2
 - 1/n)*a**(m/n + 1 + 1/n)*c*d*e**m*x**(m + n + 1)*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1
/n)) + B*a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*d**2*e**m*m*x**(m + 3*n +
1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3 +...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} \, dx = \int \frac{(Bx^n+A)(dx^n+c)^2(ex)^m}{bx^n+a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
```

output
```
((b^3*c^2*e^m - 2*a*b^2*c*d*e^m + a^2*b*d^2*e^m)*A - (a*b^2*c^2*e^m - 2*a^
2*b*c*d*e^m + a^3*d^2*e^m)*B)*integrate(x^m/(b^4*x^n + a*b^3), x) + ((m^2
+ m*(n + 2) + n + 1)*B*b^2*d^2*e^m*x*e^(m*log(x) + 2*n*log(x)) + ((2*(m^2
+ m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c*d*e^m - (m^2 + m*(3*n + 2) + 2*n^2
+ 3*n + 1)*a*b*d^2*e^m)*A + ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^2
*e^m - 2*(m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a*b*c*d*e^m + (m^2 + m*(3*n
 + 2) + 2*n^2 + 3*n + 1)*a^2*d^2*e^m)*B)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n
 + 1)*A*b^2*d^2*e^m + (2*(m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c*d*e^m - (m^2
+ 2*m*(n + 1) + 2*n + 1)*a*b*d^2*e^m)*B)*x*e^(m*log(x) + n*log(x)))/((m^3
+ 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^3)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} dx = \int \frac{(Bx^n+A)(dx^n+c)^2(ex)^m}{bx^n+a} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)^2*(e*x)^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(c+d\,x^n)^2}{a+b\,x^n} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{a+bx^n} \, dx$$
$$= \frac{x^m e^m x(x^{2n}d^2m^2 + x^{2n}d^2mn + 2x^{2n}d^2m + x^{2n}d^2n + x^{2n}d^2 + 2x^n cd\, m^2 + 4x^n cdmn + 4x^n cdm + 4x^n cdn}{m^3 + 3m^2n + 2m\, n^2 + 3m^2 + 6mn + 2n^2 + 3m + 3n}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n),x)
```

output
```
(x**m*e**m*x*(x**(2*n)*d**2*m**2 + x**(2*n)*d**2*m*n + 2*x**(2*n)*d**2*m +
 x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m**2 + 4*x**n*c*d*m*n + 4*x*
*n*c*d*m + 4*x**n*c*d*n + 2*x**n*c*d + c**2*m**2 + 3*c**2*m*n + 2*c**2*m +
 2*c**2*n**2 + 3*c**2*n + c**2))/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*
m*n + 3*m + 2*n**2 + 3*n + 1)
```

3.31 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^2}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 271

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)^2}{(a+bx^n)^2}\,dx = -\frac{d(Ab(2bc(1+m)-ad(1+m+n))-aB(2bc(1+m+n)-ad(1+m+2n)))(ex)^{1+m}}{ab^3e(1+m)n} - \frac{d^2(Ab(1+m+n)-aB(1+m+2n))x^n(ex)^{1+m}}{ab^2en(1+m+n)} + \frac{(Ab-aB)(ex)^{1+m}(c+dx^n)^2}{aben(a+bx^n)} - \frac{(bc-ad)(Ab(bc(1+m-n)-ad(1+m+n))-aB(bc(1+m)-ad(1+m+2n)))(ex)^{1+m}\,\text{Hyperge}}{a^2b^3e(1+m)n}$$

output
```
-d*(A*b*(2*b*c*(1+m)-a*d*(1+m+n))-a*B*(2*b*c*(1+m+n)-a*d*(1+m+2*n)))*(e*x)
^(1+m)/a/b^3/e/(1+m)/n-d^2*(A*b*(1+m+n)-a*B*(1+m+2*n))*x^n*(e*x)^(1+m)/a/b
^2/e/n/(1+m+n)+(A*b-B*a)*(e*x)^(1+m)*(c+d*x^n)^2/a/b/e/n/(a+b*x^n)-(-a*d+b
*c)*(A*b*(b*c*(1+m-n)-a*d*(1+m+n))-a*B*(b*c*(1+m)-a*d*(1+m+2*n)))*(e*x)^(1
+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/b^3/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)^2}{(a+bx^n)^2} \, dx$$
$$= \frac{x(ex)^m \left(\frac{d(2bBc+Abd-2aBd)}{1+m} + \frac{bBd^2x^n}{1+m+n} + \frac{(bc-ad)(bBc+2Abd-3aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(1+m)} + \frac{(Ab-aB)(bc}{\cdots}\right.}{b^3}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^2,x]
```

output
```
(x*(e*x)^m*((d*(2*b*B*c + A*b*d - 2*a*B*d))/(1 + m) + (b*B*d^2*x^n)/(1 + m
 + n) + ((b*c - a*d)*(b*B*c + 2*A*b*d - 3*a*B*d)*Hypergeometric2F1[1, (1 +
 m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m)) + ((A*b - a*B)*(b*c - a*d
)^2*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a^2*(1
+ m))))/b^3
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1064, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)^2}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{1064}$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{aben(a+bx^n)} - \frac{\int -\frac{(ex)^m(dx^n+c)(c(aB(m+1)-Ab(m-n+1))-d(Ab(m+n+1)-aB(m+2n+1))x^n)}{bx^n+a}dx}{abn}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(dx^n+c)(c(aB(m+1)-Ab(m-n+1))-d(Ab(m+n+1)-aB(m+2n+1))x^n)}{bx^n+a}dx}{abn}+\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^2}{aben\left(a+bx^n\right)}$$

$\downarrow$ 1040

$$\frac{\int \left(\frac{d^2(aB(m+2n+1)-Ab(m+n+1))x^n(ex)^m}{b}+\frac{d(aB(2bc(m+n+1)-ad(m+2n+1))-Ab(2bc(m+1)-ad(m+n+1)))(ex)^m}{b^2}+\frac{(bc-ad)(aB(b}{}\right.}{abn}$$
$$\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^2}{aben\left(a+bx^n\right)}$$

$\downarrow$ 2009

$$\frac{-\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)(Ab(bc(m-n+1)-ad(m+n+1))-aB(bc(m+1)-ad(m+2n+1)))}{ab^2e(m+1)}-\frac{d(ex)^{m+1}}{}}{abn}$$
$$\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^2}{aben\left(a+bx^n\right)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^n)^2)/(a*b*e*n*(a + b*x^n)) + (-((d^2*
(A*b*(1 + m + n) - a*B*(1 + m + 2*n))*x^(1 + n)*(e*x)^m)/(b*(1 + m + n)))
- (d*(A*b*(2*b*c*(1 + m) - a*d*(1 + m + n)) - a*B*(2*b*c*(1 + m + n) - a*d
*(1 + m + 2*n)))*(e*x)^(1 + m))/(b^2*e*(1 + m)) - ((b*c - a*d)*(A*b*(b*c*(
1 + m - n) - a*d*(1 + m + n)) - a*B*(b*c*(1 + m) - a*d*(1 + m + 2*n)))*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(
a*b^2*e*(1 + m)))/(a*b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n) (c + d x^n)^2}{(a + b x^n)^2} dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{(bx^n + a)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*d^2*x^(3*n) + A*c^2 + (2*B*c*d + A*d^2)*x^(2*n) + (B*c^2 + 2*A
*c*d)*x^n)*(e*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} dx = \int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} dx$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**2/(a+b*x**n)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**n)*(c + d*x**n)**2/(a + b*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{(bx^n + a)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
-((a^2*b*d^2*e^m*(m + n + 1) + b^3*c^2*e^m*(m - n + 1) - 2*a*b^2*c*d*e^m*(
m + 1))*A - (a^3*d^2*e^m*(m + 2*n + 1) - 2*a^2*b*c*d*e^m*(m + n + 1) + a*b
^2*c^2*e^m*(m + 1))*B)*integrate(x^m/(a*b^4*n*x^n + a^2*b^3*n), x) + ((m*n
 + n)*B*a*b^2*d^2*e^m*x*e^(m*log(x) + 2*n*log(x)) + (((m^2 + m*(n + 2) + n
 + 1)*b^3*c^2*e^m - 2*(m^2 + m*(n + 2) + n + 1)*a*b^2*c*d*e^m + (m^2 + 2*m
*(n + 1) + n^2 + 2*n + 1)*a^2*b*d^2*e^m)*A - ((m^2 + m*(n + 2) + n + 1)*a*
b^2*c^2*e^m - 2*(m^2 + 2*m*(n + 1) + n^2 + 2*n + 1)*a^2*b*c*d*e^m + (m^2 +
 m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^3*d^2*e^m)*B)*x*x^m + ((m*n + n^2 + n)*A
*a*b^2*d^2*e^m + (2*(m*n + n^2 + n)*a*b^2*c*d*e^m - (m*n + 2*n^2 + n)*a^2*
b*d^2*e^m)*B)*x*e^(m*log(x) + n*log(x)))/((m^2*n + (n^2 + 2*n)*m + n^2 + n
)*a*b^4*x^n + (m^2*n + (n^2 + 2*n)*m + n^2 + n)*a^2*b^3)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} \, dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{(bx^n + a)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)^2*(e*x)^m/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^2} \, dx = \int \frac{(e\,x)^m\,\left(A + B\,x^n\right)\,\left(c + d\,x^n\right)^2}{\left(a + b\,x^n\right)^2} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)(c + dx^n)^2}{(a + bx^n)^2}\,dx$$
$$= \frac{e^m \left(x^{m+n} b\, d^2 m x + x^{m+n} b\, d^2 x - x^m a\, d^2 m x - x^m a\, d^2 n x - x^m a\, d^2 x + 2x^m b c d m x + 2x^m b c d n x + 2x^m b c d x\right.}{}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^2,x)
```

output
```
(e**m*(x**(m + n)*b*d**2*m*x + x**(m + n)*b*d**2*x - x**m*a*d**2*m*x - x**
m*a*d**2*n*x - x**m*a*d**2*x + 2*x**m*b*c*d*m*x + 2*x**m*b*c*d*n*x + 2*x**
m*b*c*d*x + int(x**m/(x**n*b + a),x)*a**2*d**2*m**2 + int(x**m/(x**n*b + a
),x)*a**2*d**2*m*n + 2*int(x**m/(x**n*b + a),x)*a**2*d**2*m + int(x**m/(x*
*n*b + a),x)*a**2*d**2*n + int(x**m/(x**n*b + a),x)*a**2*d**2 - 2*int(x**m
/(x**n*b + a),x)*a*b*c*d*m**2 - 2*int(x**m/(x**n*b + a),x)*a*b*c*d*m*n - 4
*int(x**m/(x**n*b + a),x)*a*b*c*d*m - 2*int(x**m/(x**n*b + a),x)*a*b*c*d*n
 - 2*int(x**m/(x**n*b + a),x)*a*b*c*d + int(x**m/(x**n*b + a),x)*b**2*c**2
*m**2 + int(x**m/(x**n*b + a),x)*b**2*c**2*m*n + 2*int(x**m/(x**n*b + a),x
)*b**2*c**2*m + int(x**m/(x**n*b + a),x)*b**2*c**2*n + int(x**m/(x**n*b +
a),x)*b**2*c**2))/(b**2*(m**2 + m*n + 2*m + n + 1))
```

3.32 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^2}{(a+bx^n)^3} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 322

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3} \, dx$$
$$= \frac{d(bc(1+m)-ad(1+m+n))(Ab(1+m)-aB(1+m+2n))(ex)^{1+m}}{2a^2b^3e(1+m)n^2}$$
$$+ \frac{(Ab-aB)(ex)^{1+m} (c+dx^n)^2}{2aben (a+bx^n)^2}$$
$$+ \frac{(bc-ad)(ex)^{1+m} (c(aB(1+m)-Ab(1+m-2n))-d(Ab(1+m)-aB(1+m+2n))x^n)}{2a^2b^2en^2 (a+bx^n)}$$
$$+ \frac{(bc(aB(1+m)-Ab(1+m-2n))(ad(1+m)-bc(1+m-n))-ad(bc(1+m)-ad(1+m+n))(\ldots}{\ldots 2a^3b^3e(1+m)n^2}$$

output
```
1/2*d*(b*c*(1+m)-a*d*(1+m+n))*(A*b*(1+m)-a*B*(1+m+2*n))*(e*x)^(1+m)/a^2/b^
3/e/(1+m)/n^2+1/2*(A*b-B*a)*(e*x)^(1+m)*(c+d*x^n)^2/a/b/e/n/(a+b*x^n)^2+1/
2*(-a*d+b*c)*(e*x)^(1+m)*(c*(a*B*(1+m)-A*b*(1+m-2*n))-d*(A*b*(1+m)-a*B*(1+
m+2*n))*x^n)/a^2/b^2/e/n^2/(a+b*x^n)+1/2*(b*c*(a*B*(1+m)-A*b*(1+m-2*n))*(a
*d*(1+m)-b*c*(1+m-n))-a*d*(b*c*(1+m)-a*d*(1+m+n))*(A*b*(1+m)-a*B*(1+m+2*n)
))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/b^3/e/(1+m
)/n^2
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.52

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3} \, dx = \frac{x(ex)^m \left(Bd^2 + \frac{d(2bBc+Abd-3aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a} + \frac{(bc-ad)(bBc+2Abd-3aBd)\,\text{Hypergeometric2F1}}{a^2}\right.}{b^3(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^3,x]
```

output
```
(x*(e*x)^m*(B*d^2 + (d*(2*b*B*c + A*b*d - 3*a*B*d)*Hypergeometric2F1[1, (1
 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/a + ((b*c - a*d)*(b*B*c + 2*A*b*d -
 3*a*B*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/a^
2 + ((A*b - a*B)*(b*c - a*d)^2*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/a^3))/(b^3*(1 + m))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1064, 25, 1064, 25, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3} \, dx$$

$$\Big\downarrow \; 1064$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2} - \frac{\int -\frac{(ex)^m(dx^n+c)(c(aB(m+1)-Ab(m-2n+1))-d(Ab(m+1)-aB(m+2n+1))x^n)}{(bx^n+a)^2}dx}{2abn}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m(dx^n+c)(c(aB(m+1)-Ab(m-2n+1))-d(Ab(m+1)-aB(m+2n+1))x^n)}{(bx^n+a)^2}dx}{2abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2}$$

$\Big\downarrow$ 1064

$$\frac{\frac{(ex)^{m+1}(bc-ad)(c(aB(m+1)-Ab(m-2n+1))-dx^n(Ab(m+1)-aB(m+2n+1)))}{aben(a+bx^n)}-\frac{\int -\frac{(ex)^m(d(bc(m+1)-ad(m+n+1))(Ab(m+1)-aB(m+2n+1))\cdots}{bx^n\cdots}}{abn\cdots}}{2abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{(ex)^m(d(bc(m+1)-ad(m+n+1))(Ab(m+1)-aB(m+2n+1))x^n+c(aB(m+1)-Ab(m-2n+1))(ad(m+1)-bc(m-n+1)))}{bx^n+a}dx}{abn}+\frac{(ex)^{m+1}(bc-ad)(c(aB(m+\cdots}{\cdots}}{2abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2}$$

$\Big\downarrow$ 959

$$\frac{\frac{\left(c(aB(m+1)-Ab(m-2n+1))(ad(m+1)-bc(m-n+1))-\frac{ad(Ab(m+1)-aB(m+2n+1))(bc(m+1)-ad(m+n+1))}{b}\right)\int \frac{(ex)^m}{bx^n+a}dx+\frac{d(ex)^{m+1}(Ab(m+1)-aB\cdots}{\cdots}}{abn}}{2abn}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2}$$

$\Big\downarrow$ 888

$$\frac{\frac{\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)\left(c(aB(m+1)-Ab(m-2n+1))(ad(m+1)-bc(m-n+1))-\frac{ad(Ab(m+1)-aB(m+2n+1))(bc(m+1)-ad(m+\cdots}{b}\cdots\right)}{ae(m+1)}\cdots}{abn}}{\cdots}+\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^2}{2aben(a+bx^n)^2}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^3,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^n)^2)/(2*a*b*e*n*(a + b*x^n)^2) + (((b
*c - a*d)*(e*x)^(1 + m)*(c*(a*B*(1 + m) - A*b*(1 + m - 2*n)) - d*(A*b*(1 +
 m) - a*B*(1 + m + 2*n))*x^n))/(a*b*e*n*(a + b*x^n)) + ((d*(b*c*(1 + m) -
a*d*(1 + m + n))*(A*b*(1 + m) - a*B*(1 + m + 2*n))*(e*x)^(1 + m))/(b*e*(1
+ m)) + ((c*(a*B*(1 + m) - A*b*(1 + m - 2*n))*(a*d*(1 + m) - b*c*(1 + m -
n)) - (a*d*(b*c*(1 + m) - a*d*(1 + m + n))*(A*b*(1 + m) - a*B*(1 + m + 2*n
)))/b)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x
^n)/a)])/(a*e*(1 + m)))/(a*b*n))/(2*a*b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n) (c+d\,x^n)^2}{(a+b\,x^n)^3}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3}\,dx = \int \frac{(Bx^n+A)(dx^n+c)^2(ex)^m}{(bx^n+a)^3}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*d^2*x^(3*n) + A*c^2 + (2*B*c*d + A*d^2)*x^(2*n) + (B*c^2 + 2*A
*c*d)*x^n)*(e*x)^m/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n + a^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**2/(a+b*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^3} dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{(bx^n + a)^3} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x, algorithm="maxima")
```

output
```
(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c^2*e^m - 2*(m^2 - m*(n - 2) -
 n + 1)*a*b^2*c*d*e^m + (m^2 + m*(n + 2) + n + 1)*a^2*b*d^2*e^m)*A - ((m^2
 - m*(n - 2) - n + 1)*a*b^2*c^2*e^m - 2*(m^2 + m*(n + 2) + n + 1)*a^2*b*c*
d*e^m + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a^3*d^2*e^m)*B)*integrate(1/
2*x^m/(a^2*b^4*n^2*x^n + a^3*b^3*n^2), x) + 1/2*(2*B*a^2*b^2*d^2*e^m*n^2*x
*e^(m*log(x) + 2*n*log(x)) - (((m^2 - m*(3*n - 2) - 3*n + 1)*a*b^3*c^2*e^m
 - 2*(m^2 - m*(n - 2) - n + 1)*a^2*b^2*c*d*e^m + (m^2 + m*(n + 2) + n + 1)
*a^3*b*d^2*e^m)*A - ((m^2 - m*(n - 2) - n + 1)*a^2*b^2*c^2*e^m - 2*(m^2 +
m*(n + 2) + n + 1)*a^3*b*c*d*e^m + (m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a
^4*d^2*e^m)*B)*x*x^m - (((m^2 - 2*m*(n - 1) - 2*n + 1)*b^4*c^2*e^m - 2*(m^
2 + 2*m + 1)*a*b^3*c*d*e^m + (m^2 + 2*m*(n + 1) + 2*n + 1)*a^2*b^2*d^2*e^m
)*A - ((m^2 + 2*m + 1)*a*b^3*c^2*e^m - 2*(m^2 + 2*m*(n + 1) + 2*n + 1)*a^2
*b^2*c*d*e^m + (m^2 + 2*m*(2*n + 1) + 4*n^2 + 4*n + 1)*a^3*b*d^2*e^m)*B)*x
*e^(m*log(x) + n*log(x)))/((m*n^2 + n^2)*a^2*b^5*x^(2*n) + 2*(m*n^2 + n^2)
*a^3*b^4*x^n + (m*n^2 + n^2)*a^4*b^3)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^2}{(a + bx^n)^3} dx = \int \frac{(Bx^n + A)(dx^n + c)^2 (ex)^m}{(bx^n + a)^3} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)^2*(e*x)^m/(b*x^n + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,\left(c+d\,x^n\right)^2}{(a+b\,x^n)^3}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^3,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^2)/(a + b*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^2}{(a+bx^n)^3}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^2/(a+b*x^n)^3,x)
```

output
```
(e**m*(2*x**(m + n)*a*c*d*m*x + 2*x**(m + n)*a*c*d*x - x**(m + n)*b*c**2*m
*x + x**(m + n)*b*c**2*n*x - x**(m + n)*b*c**2*x + x**m*a*c**2*m*x + x**m*
a*c**2*n*x + x**m*a*c**2*x + x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(
2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a
**2*m + a**2*n + a**2),x)*a**2*b*d**2*m**3 + 2*x**n*int(x**(m + 2*n)/(x**(
2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*
n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a**2*b*d**2*m**2*n + 3*x**n*in
t(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n
*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a**2*b*d**
2*m**2 + x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*
n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**
2),x)*a**2*b*d**2*m*n**2 + 4*x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(
2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a
**2*m + a**2*n + a**2),x)*a**2*b*d**2*m*n + 3*x**n*int(x**(m + 2*n)/(x**(2
*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m + 2*x**n*a*b*n
 + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a**2*b*d**2*m + x**n*int(x**(m
+ 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**2 + 2*x**n*a*b*m +
 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),x)*a**2*b*d**2*n**2 +
 2*x**n*int(x**(m + 2*n)/(x**(2*n)*b**2*m + x**(2*n)*b**2*n + x**(2*n)*b**
2 + 2*x**n*a*b*m + 2*x**n*a*b*n + 2*x**n*a*b + a**2*m + a**2*n + a**2),...
```

3.33 $\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 431

$$
\begin{aligned}
\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx
&= \frac{a^3Ac^3(ex)^{1+m}}{e(1+m)} + \frac{a^2c^2(aBc+3A(bc+ad))x^n(ex)^{1+m}}{e(1+m+n)} \\
&+ \frac{3ac(aBc(bc+ad)+A(b^2c^2+3abcd+a^2d^2))\,x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\
&+ \frac{(3aBc(b^2c^2+3abcd+a^2d^2)+A(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3))\,x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\
&+ \frac{(a^3Bd^3+9ab^2cd(Bc+Ad)+3a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad))\,x^{4n}(ex)^{1+m}}{e(1+m+4n)} \\
&+ \frac{3bd(a^2Bd^2+b^2c(Bc+Ad)+abd(3Bc+Ad))\,x^{5n}(ex)^{1+m}}{e(1+m+5n)} \\
&+ \frac{b^2d^2(3bBc+Abd+3aBd)x^{6n}(ex)^{1+m}}{e(1+m+6n)} + \frac{b^3Bd^3x^{7n}(ex)^{1+m}}{e(1+m+7n)}
\end{aligned}
$$

output
```
a^3*A*c^3*(e*x)^(1+m)/e/(1+m)+a^2*c^2*(a*B*c+3*A*(a*d+b*c))*x^n*(e*x)^(1+m
)/e/(1+m+n)+3*a*c*(a*B*c*(a*d+b*c)+A*(a^2*d^2+3*a*b*c*d+b^2*c^2))*x^(2*n)*
(e*x)^(1+m)/e/(1+m+2*n)+(3*a*B*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)+A*(a^3*d^3+9*
a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3))*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+(a^3*B
*d^3+9*a*b^2*c*d*(A*d+B*c)+3*a^2*b*d^2*(A*d+3*B*c)+b^3*c^2*(3*A*d+B*c))*x^
(4*n)*(e*x)^(1+m)/e/(1+m+4*n)+3*b*d*(a^2*B*d^2+b^2*c*(A*d+B*c)+a*b*d*(A*d+
3*B*c))*x^(5*n)*(e*x)^(1+m)/e/(1+m+5*n)+b^2*d^2*(A*b*d+3*B*a*d+3*B*b*c)*x^
(6*n)*(e*x)^(1+m)/e/(1+m+6*n)+b^3*B*d^3*x^(7*n)*(e*x)^(1+m)/e/(1+m+7*n)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.83

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx = x(ex)^m \left(\frac{a^3Ac^3}{1+m} + \frac{a^2c^2(aBc+3A(bc+ad))x^n}{1+m+n} + \frac{3ac(aBc(bc+ad)+A(b^2c^2+3abcd+a^2d^2))\,x^{2n}}{1+m+2n} + \frac{(3aBc(b^2c^2+3abcd+a^2d^2)+A(b^3c^3+9ab^2c^2d+9a^2bcd^2+a^3d^3))\,x^{3n}}{1+m+3n} + \frac{(a^3Bd^3+9ab^2cd(Bc+Ad)+3a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad))\,x^{4n}}{1+m+4n} + \frac{3bd(a^2Bd^2+b^2c(Bc+Ad)+abd(3Bc+Ad))\,x^{5n}}{1+m+5n} + \frac{b^2d^2(3bBc+Abd+3aBd)x^{6n}}{1+m+6n} + \frac{b^3Bd^3x^{7n}}{1+m+7n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
x*(e*x)^m*((a^3*A*c^3)/(1 + m) + (a^2*c^2*(a*B*c + 3*A*(b*c + a*d))*x^n)/(
1 + m + n) + (3*a*c*(a*B*c*(b*c + a*d) + A*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)
)*x^(2*n))/(1 + m + 2*n) + ((3*a*B*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2) + A*(
b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3))*x^(3*n))/(1 + m + 3*n)
 + ((a^3*B*d^3 + 9*a*b^2*c*d*(B*c + A*d) + 3*a^2*b*d^2*(3*B*c + A*d) + b^3
*c^2*(B*c + 3*A*d))*x^(4*n))/(1 + m + 4*n) + (3*b*d*(a^2*B*d^2 + b^2*c*(B*
c + A*d) + a*b*d*(3*B*c + A*d))*x^(5*n))/(1 + m + 5*n) + (b^2*d^2*(3*b*B*c
 + A*b*d + 3*a*B*d)*x^(6*n))/(1 + m + 6*n) + (b^3*B*d^3*x^(7*n))/(1 + m +
7*n))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(a^3Ac^3(ex)^m+3acx^{2n}(ex)^m \left(A\left(a^2d^2+3abcd+b^2c^2\right)+aBc(ad+bc)\right)+3bdx^{5n}(ex)^m \left(a^2Bd^2+abd(Ad+3\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3Ac^3(ex)^{m+1}}{e(m+1)}+\frac{3acx^{2n+1}(ex)^m \left(A\left(a^2d^2+3abcd+b^2c^2\right)+aBc(ad+bc)\right)}{m+2n+1}+\frac{3bdx^{5n+1}(ex)^m \left(a^2Bd^2+abd(Ad+3Bc)+b^2c(Ad+Bc)\right)}{m+5n+1}+\frac{a^2c^2x^{n+1}(ex)^m(3A(ad+bc)+aBc)}{m+n+1}+\frac{x^{4n+1}(ex)^m \left(a^3Bd^3+3a^2bd^2(Ad+3Bc)+9ab^2cd(Ad+Bc)+b^3c^2(3Ad+Bc)\right)}{m+4n+1}+\frac{x^{3n+1}(ex)^m \left(3aBc\left(a^2d^2+3abcd+b^2c^2\right)+A\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)\right)}{m+3n+1}+\frac{b^2d^2x^{6n+1}(ex)^m(3aBd+Abd+3bBc)}{m+6n+1}+\frac{b^3Bd^3x^{7n+1}(ex)^m}{m+7n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^3*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
(a^2*c^2*(a*B*c + 3*A*(b*c + a*d))*x^(1 + n)*(e*x)^m)/(1 + m + n) + (3*a*c
*(a*B*c*(b*c + a*d) + A*(b^2*c^2 + 3*a*b*c*d + a^2*d^2))*x^(1 + 2*n)*(e*x)
^m)/(1 + m + 2*n) + ((3*a*B*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2) + A*(b^3*c^3
 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3))*x^(1 + 3*n)*(e*x)^m)/(1 + m +
 3*n) + ((a^3*B*d^3 + 9*a*b^2*c*d*(B*c + A*d) + 3*a^2*b*d^2*(3*B*c + A*d)
+ b^3*c^2*(B*c + 3*A*d))*x^(1 + 4*n)*(e*x)^m)/(1 + m + 4*n) + (3*b*d*(a^2*
B*d^2 + b^2*c*(B*c + A*d) + a*b*d*(3*B*c + A*d))*x^(1 + 5*n)*(e*x)^m)/(1 +
 m + 5*n) + (b^2*d^2*(3*b*B*c + A*b*d + 3*a*B*d)*x^(1 + 6*n)*(e*x)^m)/(1 +
 m + 6*n) + (b^3*B*d^3*x^(1 + 7*n)*(e*x)^m)/(1 + m + 7*n) + (a^3*A*c^3*(e*
x)^(1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.24 (sec) , antiderivative size = 20904, normalized size of antiderivative = 48.50

method	result	size
risch	Expression too large to display	20904
parallelrisch	Expression too large to display	27583
orering	Expression too large to display	43732

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11628 vs. $2(431) = 862$.

Time $= 0.47$ (sec) , antiderivative size $= 11628$, normalized size of antiderivative $= 26.98$

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365145 vs. $2(422) = 844$.

Time $= 49.52$ (sec) , antiderivative size $= 365145$, normalized size of antiderivative $= 847.20$

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**3*(A+B*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise(((A + B)*(a + b)**3*(c + d)**3*log(x)/e, Eq(m, -1) & Eq(n, 0)),
((A*a**3*c**3*log(x) + 3*A*a**3*c**2*d*x**n/n + 3*A*a**3*c*d**2*x**(2*n)/(
2*n) + A*a**3*d**3*x**(3*n)/(3*n) + 3*A*a**2*b*c**3*x**n/n + 9*A*a**2*b*c*
*2*d*x**(2*n)/(2*n) + 3*A*a**2*b*c*d**2*x**(3*n)/n + 3*A*a**2*b*d**3*x**(4
*n)/(4*n) + 3*A*a*b**2*c**3*x**(2*n)/(2*n) + 3*A*a*b**2*c**2*d*x**(3*n)/n
+ 9*A*a*b**2*c*d**2*x**(4*n)/(4*n) + 3*A*a*b**2*d**3*x**(5*n)/(5*n) + A*b*
*3*c**3*x**(3*n)/(3*n) + 3*A*b**3*c**2*d*x**(4*n)/(4*n) + 3*A*b**3*c*d**2*
x**(5*n)/(5*n) + A*b**3*d**3*x**(6*n)/(6*n) + B*a**3*c**3*x**n/n + 3*B*a**
3*c**2*d*x**(2*n)/(2*n) + B*a**3*c*d**2*x**(3*n)/n + B*a**3*d**3*x**(4*n)/
(4*n) + 3*B*a**2*b*c**3*x**(2*n)/(2*n) + 3*B*a**2*b*c**2*d*x**(3*n)/n + 9*
B*a**2*b*c*d**2*x**(4*n)/(4*n) + 3*B*a**2*b*d**3*x**(5*n)/(5*n) + B*a*b**2
*c**3*x**(3*n)/n + 9*B*a*b**2*c**2*d*x**(4*n)/(4*n) + 9*B*a*b**2*c*d**2*x*
*(5*n)/(5*n) + B*a*b**2*d**3*x**(6*n)/(2*n) + B*b**3*c**3*x**(4*n)/(4*n) +
 3*B*b**3*c**2*d*x**(5*n)/(5*n) + B*b**3*c*d**2*x**(6*n)/(2*n) + B*b**3*d*
*3*x**(7*n)/(7*n))/e, Eq(m, -1)), (A*a**3*c**3*Piecewise((0**(-7*n - 1)*x,
 Eq(e, 0)), (Piecewise((-1/(7*n*(e*x)**(7*n)), Ne(n, 0)), (log(e*x), True)
)/e, True)) + 3*A*a**3*c**2*d*Piecewise((-x*x**n*(e*x)**(-7*n - 1)/(6*n),
Ne(n, 0)), (x*x**n*(e*x)**(-7*n - 1)*log(x), True)) + 3*A*a**3*c*d**2*Piec
ewise((-x*x**(2*n)*(e*x)**(-7*n - 1)/(5*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**
(-7*n - 1)*log(x), True)) + A*a**3*d**3*Piecewise((-x*x**(3*n)*(e*x)**(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1032 vs. $2(431) = 862$.

Time = 0.14 (sec) , antiderivative size = 1032, normalized size of antiderivative = 2.39

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
B*b^3*d^3*e^m*x*e^(m*log(x) + 7*n*log(x))/(m + 7*n + 1) + 3*B*b^3*c*d^2*e^
m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*B*a*b^2*d^3*e^m*x*e^(m*log
(x) + 6*n*log(x))/(m + 6*n + 1) + A*b^3*d^3*e^m*x*e^(m*log(x) + 6*n*log(x)
)/(m + 6*n + 1) + 3*B*b^3*c^2*d*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n +
 1) + 9*B*a*b^2*c*d^2*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 3*A*
b^3*c*d^2*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 3*B*a^2*b*d^3*e^
m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 3*A*a*b^2*d^3*e^m*x*e^(m*log
(x) + 5*n*log(x))/(m + 5*n + 1) + B*b^3*c^3*e^m*x*e^(m*log(x) + 4*n*log(x)
)/(m + 4*n + 1) + 9*B*a*b^2*c^2*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n
 + 1) + 3*A*b^3*c^2*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 9*B*
a^2*b*c*d^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 9*A*a*b^2*c*d^
2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + B*a^3*d^3*e^m*x*e^(m*log
(x) + 4*n*log(x))/(m + 4*n + 1) + 3*A*a^2*b*d^3*e^m*x*e^(m*log(x) + 4*n*lo
g(x))/(m + 4*n + 1) + 3*B*a*b^2*c^3*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3
*n + 1) + A*b^3*c^3*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 9*B*a^
2*b*c^2*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 9*A*a*b^2*c^2*d*
e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*B*a^3*c*d^2*e^m*x*e^(m*l
og(x) + 3*n*log(x))/(m + 3*n + 1) + 9*A*a^2*b*c*d^2*e^m*x*e^(m*log(x) + 3*
n*log(x))/(m + 3*n + 1) + A*a^3*d^3*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3
*n + 1) + 3*B*a^2*b*c^3*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143220 vs. 2(431) = 862.

Time = 1.79 (sec) , antiderivative size = 143220, normalized size of antiderivative = 332.30

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(B*b^3*d^3*m^7*x*x^(7*n)*e^(m*log(e) + m*log(x)) + 21*B*b^3*d^3*m^6*n*x*x^
(7*n)*e^(m*log(e) + m*log(x)) + 175*B*b^3*d^3*m^5*n^2*x*x^(7*n)*e^(m*log(e
) + m*log(x)) + 735*B*b^3*d^3*m^4*n^3*x*x^(7*n)*e^(m*log(e) + m*log(x)) +
1624*B*b^3*d^3*m^3*n^4*x*x^(7*n)*e^(m*log(e) + m*log(x)) + 1764*B*b^3*d^3*
m^2*n^5*x*x^(7*n)*e^(m*log(e) + m*log(x)) + 720*B*b^3*d^3*m*n^6*x*x^(7*n)*
e^(m*log(e) + m*log(x)) + 3*B*b^3*c*d^2*m^7*x*x^(6*n)*e^(m*log(e) + m*log(
x)) + 3*B*a*b^2*d^3*m^7*x*x^(6*n)*e^(m*log(e) + m*log(x)) + A*b^3*d^3*m^7*
x*x^(6*n)*e^(m*log(e) + m*log(x)) + B*b^3*d^3*m^7*x*x^(6*n)*e^(m*log(e) +
m*log(x)) + 66*B*b^3*c*d^2*m^6*n*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 66*B*
a*b^2*d^3*m^6*n*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 22*A*b^3*d^3*m^6*n*x*x
^(6*n)*e^(m*log(e) + m*log(x)) + 21*B*b^3*d^3*m^6*n*x*x^(6*n)*e^(m*log(e)
+ m*log(x)) + 570*B*b^3*c*d^2*m^5*n^2*x*x^(6*n)*e^(m*log(e) + m*log(x)) +
570*B*a*b^2*d^3*m^5*n^2*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 190*A*b^3*d^3*
m^5*n^2*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 175*B*b^3*d^3*m^5*n^2*x*x^(6*n
)*e^(m*log(e) + m*log(x)) + 2460*B*b^3*c*d^2*m^4*n^3*x*x^(6*n)*e^(m*log(e)
 + m*log(x)) + 2460*B*a*b^2*d^3*m^4*n^3*x*x^(6*n)*e^(m*log(e) + m*log(x))
+ 820*A*b^3*d^3*m^4*n^3*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 735*B*b^3*d^3*
m^4*n^3*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 5547*B*b^3*c*d^2*m^3*n^4*x*x^(
6*n)*e^(m*log(e) + m*log(x)) + 5547*B*a*b^2*d^3*m^3*n^4*x*x^(6*n)*e^(m*log
(e) + m*log(x)) + 1849*A*b^3*d^3*m^3*n^4*x*x^(6*n)*e^(m*log(e) + m*log(...
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 2949, normalized size of antiderivative = 6.84

$$\int (ex)^m (a+bx^n)^3 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^3*(c + d*x^n)^3,x)
```

output
```
(x*x^(3*n)*(e*x)^m*(A*a^3*d^3 + A*b^3*c^3 + 3*B*a*b^2*c^3 + 3*B*a^3*c*d^2
+ 9*A*a*b^2*c^2*d + 9*A*a^2*b*c*d^2 + 9*B*a^2*b*c^2*d)*(6*m + 25*n + 125*m
*n + 988*m*n^2 + 250*m^2*n + 3657*m*n^3 + 250*m^3*n + 6224*m*n^4 + 125*m^4
*n + 3796*m*n^5 + 25*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 247*
n^2 + 1219*n^3 + 3112*n^4 + 3796*n^5 + 1680*n^6 + 1482*m^2*n^2 + 3657*m^2*
n^3 + 988*m^3*n^2 + 3112*m^2*n^4 + 1219*m^3*n^3 + 247*m^4*n^2 + 1))/(7*m +
 28*n + 168*m*n + 1610*m*n^2 + 420*m^2*n + 7840*m*n^3 + 560*m^3*n + 20307*
m*n^4 + 420*m^4*n + 26264*m*n^5 + 168*m^5*n + 13068*m*n^6 + 28*m^6*n + 21*
m^2 + 35*m^3 + 35*m^4 + 21*m^5 + 7*m^6 + m^7 + 322*n^2 + 1960*n^3 + 6769*n
^4 + 13132*n^5 + 13068*n^6 + 5040*n^7 + 3220*m^2*n^2 + 11760*m^2*n^3 + 322
0*m^3*n^2 + 20307*m^2*n^4 + 7840*m^3*n^3 + 1610*m^4*n^2 + 13132*m^2*n^5 +
6769*m^3*n^4 + 1960*m^4*n^3 + 322*m^5*n^2 + 1) + (x*x^(4*n)*(e*x)^m*(B*a^3
*d^3 + B*b^3*c^3 + 3*A*a^2*b*d^3 + 3*A*b^3*c^2*d + 9*A*a*b^2*c*d^2 + 9*B*a
*b^2*c^2*d + 9*B*a^2*b*c*d^2)*(6*m + 24*n + 120*m*n + 904*m*n^2 + 240*m^2*
n + 3168*m*n^3 + 240*m^3*n + 5090*m*n^4 + 120*m^4*n + 2952*m*n^5 + 24*m^5*
n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 226*n^2 + 1056*n^3 + 2545*n^4
 + 2952*n^5 + 1260*n^6 + 1356*m^2*n^2 + 3168*m^2*n^3 + 904*m^3*n^2 + 2545*
m^2*n^4 + 1056*m^3*n^3 + 226*m^4*n^2 + 1))/(7*m + 28*n + 168*m*n + 1610*m*
n^2 + 420*m^2*n + 7840*m*n^3 + 560*m^3*n + 20307*m*n^4 + 420*m^4*n + 26264
*m*n^5 + 168*m^5*n + 13068*m*n^6 + 28*m^6*n + 21*m^2 + 35*m^3 + 35*m^4 ...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12856, normalized size of antiderivative = 29.83

$$\int (ex)^m (a + bx^n)^3 (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)*(c+d*x^n)^3,x)
```

output
```
(x**m*e**m*x*(x**(7*n)*b**4*d**3*m**7 + 21*x**(7*n)*b**4*d**3*m**6*n + 7*x
**(7*n)*b**4*d**3*m**6 + 175*x**(7*n)*b**4*d**3*m**5*n**2 + 126*x**(7*n)*b
**4*d**3*m**5*n + 21*x**(7*n)*b**4*d**3*m**5 + 735*x**(7*n)*b**4*d**3*m**4
*n**3 + 875*x**(7*n)*b**4*d**3*m**4*n**2 + 315*x**(7*n)*b**4*d**3*m**4*n +
 35*x**(7*n)*b**4*d**3*m**4 + 1624*x**(7*n)*b**4*d**3*m**3*n**4 + 2940*x**
(7*n)*b**4*d**3*m**3*n**3 + 1750*x**(7*n)*b**4*d**3*m**3*n**2 + 420*x**(7*
n)*b**4*d**3*m**3*n + 35*x**(7*n)*b**4*d**3*m**3 + 1764*x**(7*n)*b**4*d**3
*m**2*n**5 + 4872*x**(7*n)*b**4*d**3*m**2*n**4 + 4410*x**(7*n)*b**4*d**3*m
**2*n**3 + 1750*x**(7*n)*b**4*d**3*m**2*n**2 + 315*x**(7*n)*b**4*d**3*m**2
*n + 21*x**(7*n)*b**4*d**3*m**2 + 720*x**(7*n)*b**4*d**3*m*n**6 + 3528*x**
(7*n)*b**4*d**3*m*n**5 + 4872*x**(7*n)*b**4*d**3*m*n**4 + 2940*x**(7*n)*b*
*4*d**3*m*n**3 + 875*x**(7*n)*b**4*d**3*m*n**2 + 126*x**(7*n)*b**4*d**3*m*
n + 7*x**(7*n)*b**4*d**3*m + 720*x**(7*n)*b**4*d**3*n**6 + 1764*x**(7*n)*b
**4*d**3*n**5 + 1624*x**(7*n)*b**4*d**3*n**4 + 735*x**(7*n)*b**4*d**3*n**3
 + 175*x**(7*n)*b**4*d**3*n**2 + 21*x**(7*n)*b**4*d**3*n + x**(7*n)*b**4*d
**3 + 4*x**(6*n)*a*b**3*d**3*m**7 + 88*x**(6*n)*a*b**3*d**3*m**6*n + 28*x*
*(6*n)*a*b**3*d**3*m**6 + 760*x**(6*n)*a*b**3*d**3*m**5*n**2 + 528*x**(6*n
)*a*b**3*d**3*m**5*n + 84*x**(6*n)*a*b**3*d**3*m**5 + 3280*x**(6*n)*a*b**3
*d**3*m**4*n**3 + 3800*x**(6*n)*a*b**3*d**3*m**4*n**2 + 1320*x**(6*n)*a*b*
*3*d**3*m**4*n + 140*x**(6*n)*a*b**3*d**3*m**4 + 7396*x**(6*n)*a*b**3*d...
```

3.34 $\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 328

$$
\begin{aligned}
&\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx \\
&= \frac{a^2Ac^3(ex)^{1+m}}{e(1+m)} + \frac{ac^2(2Abc+aBc+3aAd)x^n(ex)^{1+m}}{e(1+m+n)} \\
&\quad + \frac{c(aBc(2bc+3ad)+A(b^2c^2+6abcd+3a^2d^2))\,x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\
&\quad + \frac{(6abcd(Bc+Ad)+a^2d^2(3Bc+Ad)+b^2c^2(Bc+3Ad))\,x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\
&\quad + \frac{d(a^2Bd^2+3b^2c(Bc+Ad)+2abd(3Bc+Ad))\,x^{4n}(ex)^{1+m}}{e(1+m+4n)} \\
&\quad + \frac{bd^2(3bBc+Abd+2aBd)x^{5n}(ex)^{1+m}}{e(1+m+5n)} + \frac{b^2Bd^3x^{6n}(ex)^{1+m}}{e(1+m+6n)}
\end{aligned}
$$

output
```
a^2*A*c^3*(e*x)^(1+m)/e/(1+m)+a*c^2*(3*A*a*d+2*A*b*c+B*a*c)*x^n*(e*x)^(1+m
)/e/(1+m+n)+c*(a*B*c*(3*a*d+2*b*c)+A*(3*a^2*d^2+6*a*b*c*d+b^2*c^2))*x^(2*n
)*(e*x)^(1+m)/e/(1+m+2*n)+(6*a*b*c*d*(A*d+B*c)+a^2*d^2*(A*d+3*B*c)+b^2*c^2
*(3*A*d+B*c))*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+d*(a^2*B*d^2+3*b^2*c*(A*d+B*
c)+2*a*b*d*(A*d+3*B*c))*x^(4*n)*(e*x)^(1+m)/e/(1+m+4*n)+b*d^2*(A*b*d+2*B*a
*d+3*B*b*c)*x^(5*n)*(e*x)^(1+m)/e/(1+m+5*n)+b^2*B*d^3*x^(6*n)*(e*x)^(1+m)/
e/(1+m+6*n)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.81

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = x(ex)^m \left(\frac{a^2Ac^3}{1+m}+\frac{ac^2(2Abc+aBc+3aAd)x^n}{1+m+n} + \frac{c(aBc(2bc+3ad)+A(b^2c^2+6abcd+3a^2d^2))\,x^{2n}}{1+m+2n} + \frac{(6abcd(Bc+Ad)+a^2d^2(3Bc+Ad)+b^2c^2(Bc+3Ad))\,x^{3n}}{1+m+3n} + \frac{d(a^2Bd^2+3b^2c(Bc+Ad)+2abd(3Bc+Ad))\,x^{4n}}{1+m+4n} + \frac{bd^2(3bBc+Abd+2aBd)x^{5n}}{1+m+5n}+\frac{b^2Bd^3x^{6n}}{1+m+6n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
x*(e*x)^m*((a^2*A*c^3)/(1 + m) + (a*c^2*(2*A*b*c + a*B*c + 3*a*A*d)*x^n)/(
1 + m + n) + (c*(a*B*c*(2*b*c + 3*a*d) + A*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^
2))*x^(2*n))/(1 + m + 2*n) + ((6*a*b*c*d*(B*c + A*d) + a^2*d^2*(3*B*c + A*
d) + b^2*c^2*(B*c + 3*A*d))*x^(3*n))/(1 + m + 3*n) + (d*(a^2*B*d^2 + 3*b^2
*c*(B*c + A*d) + 2*a*b*d*(3*B*c + A*d))*x^(4*n))/(1 + m + 4*n) + (b*d^2*(3
*b*B*c + A*b*d + 2*a*B*d)*x^(5*n))/(1 + m + 5*n) + (b^2*B*d^3*x^(6*n))/(1
+ m + 6*n))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(cx^{2n}(ex)^m \left(A\left(3a^2d^2+6abcd+b^2c^2\right)+aBc(3ad+2bc)\right)+x^{3n}(ex)^m \left(a^2d^2(Ad+3Bc)+6abcd(Ad+Bc)+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{cx^{2n+1}(ex)^m \left(A\left(3a^2d^2+6abcd+b^2c^2\right)+aBc(3ad+2bc)\right)}{m+2n+1}+\frac{x^{3n+1}(ex)^m \left(a^2d^2(Ad+3Bc)+6abcd(Ad+Bc)+b^2c^2(3Ad+Bc)\right)}{m+3n+1}+\frac{dx^{4n+1}(ex)^m \left(a^2Bd^2+2abd(Ad+3Bc)+3b^2c(Ad+Bc)\right)}{m+4n+1}+\frac{a^2Ac^3(ex)^{m+1}}{e(m+1)}+\frac{ac^2x^{n+1}(ex)^m(3aAd+aBc+2Abc)}{m+n+1}+\frac{bd^2x^{5n+1}(ex)^m(2aBd+Abd+3bBc)}{m+5n+1}+\frac{b^2Bd^3x^{6n+1}(ex)^m}{m+6n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^2*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
(a*c^2*(2*A*b*c + a*B*c + 3*a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (c*(a*
B*c*(2*b*c + 3*a*d) + A*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2))*x^(1 + 2*n)*(e*
x)^m)/(1 + m + 2*n) + ((6*a*b*c*d*(B*c + A*d) + a^2*d^2*(3*B*c + A*d) + b^
2*c^2*(B*c + 3*A*d))*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n) + (d*(a^2*B*d^2 +
3*b^2*c*(B*c + A*d) + 2*a*b*d*(3*B*c + A*d))*x^(1 + 4*n)*(e*x)^m)/(1 + m +
 4*n) + (b*d^2*(3*b*B*c + A*b*d + 2*a*B*d)*x^(1 + 5*n)*(e*x)^m)/(1 + m + 5
*n) + (b^2*B*d^3*x^(1 + 6*n)*(e*x)^m)/(1 + m + 6*n) + (a^2*A*c^3*(e*x)^(1
+ m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.83 (sec) , antiderivative size = 11356, normalized size of antiderivative = 34.62

method	result	size
risch	Expression too large to display	11356
parallelrisch	Expression too large to display	15203
orering	Expression too large to display	23014

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6557 vs. $2(328) = 656$.

Time = 0.28 (sec) , antiderivative size = 6557, normalized size of antiderivative = 19.99

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168099 vs. 2(321) = 642.

Time = 28.56 (sec) , antiderivative size = 168099, normalized size of antiderivative = 512.50

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise(((A + B)*(a + b)**2*(c + d)**3*log(x)/e, Eq(m, -1) & Eq(n, 0)),
((A*a**2*c**3*log(x) + 3*A*a**2*c**2*d*x**n/n + 3*A*a**2*c*d**2*x**(2*n)/(
2*n) + A*a**2*d**3*x**(3*n)/(3*n) + 2*A*a*b*c**3*x**n/n + 3*A*a*b*c**2*d*x
**(2*n)/n + 2*A*a*b*c*d**2*x**(3*n)/n + A*a*b*d**3*x**(4*n)/(2*n) + A*b**2
*c**3*x**(2*n)/(2*n) + A*b**2*c**2*d*x**(3*n)/n + 3*A*b**2*c*d**2*x**(4*n)
/(4*n) + A*b**2*d**3*x**(5*n)/(5*n) + B*a**2*c**3*x**n/n + 3*B*a**2*c**2*d
*x**(2*n)/(2*n) + B*a**2*c*d**2*x**(3*n)/n + B*a**2*d**3*x**(4*n)/(4*n) +
B*a*b*c**3*x**(2*n)/n + 2*B*a*b*c**2*d*x**(3*n)/n + 3*B*a*b*c*d**2*x**(4*n
)/(2*n) + 2*B*a*b*d**3*x**(5*n)/(5*n) + B*b**2*c**3*x**(3*n)/(3*n) + 3*B*b
**2*c**2*d*x**(4*n)/(4*n) + 3*B*b**2*c*d**2*x**(5*n)/(5*n) + B*b**2*d**3*x
**(6*n)/(6*n))/e, Eq(m, -1)), (A*a**2*c**3*Piecewise((0**(-6*n - 1)*x, Eq(
e, 0)), (Piecewise((-1/(6*n*(e*x)**(6*n)), Ne(n, 0)), (log(e*x), True))/e,
 True)) + 3*A*a**2*c**2*d*Piecewise((-x*x**n*(e*x)**(-6*n - 1)/(5*n), Ne(n
, 0)), (x*x**n*(e*x)**(-6*n - 1)*log(x), True)) + 3*A*a**2*c*d**2*Piecewis
e((-x*x**(2*n)*(e*x)**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-6*
n - 1)*log(x), True)) + A*a**2*d**3*Piecewise((-x*x**(3*n)*(e*x)**(-6*n -
1)/(3*n), Ne(n, 0)), (x*x**(3*n)*(e*x)**(-6*n - 1)*log(x), True)) + 2*A*a*
b*c**3*Piecewise((-x*x**n*(e*x)**(-6*n - 1)/(5*n), Ne(n, 0)), (x*x**n*(e*x
)**(-6*n - 1)*log(x), True)) + 6*A*a*b*c**2*d*Piecewise((-x*x**(2*n)*(e*x)
**(-6*n - 1)/(4*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-6*n - 1)*log(x), Tr...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(328) = 656$.

Time = 0.11 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.28

$$\int (ex)^m (a + bx^n)^2 (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
B*b^2*d^3*e^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*B*b^2*c*d^2*e^
m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 2*B*a*b*d^3*e^m*x*e^(m*log(x
) + 5*n*log(x))/(m + 5*n + 1) + A*b^2*d^3*e^m*x*e^(m*log(x) + 5*n*log(x))/
(m + 5*n + 1) + 3*B*b^2*c^2*d*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1
) + 6*B*a*b*c*d^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 3*A*b^2*
c*d^2*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + B*a^2*d^3*e^m*x*e^(m
*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*A*a*b*d^3*e^m*x*e^(m*log(x) + 4*n*
log(x))/(m + 4*n + 1) + B*b^2*c^3*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n
 + 1) + 6*B*a*b*c^2*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*A*
b^2*c^2*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*B*a^2*c*d^2*e^
m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*A*a*b*c*d^2*e^m*x*e^(m*log
(x) + 3*n*log(x))/(m + 3*n + 1) + A*a^2*d^3*e^m*x*e^(m*log(x) + 3*n*log(x)
)/(m + 3*n + 1) + 2*B*a*b*c^3*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1
) + A*b^2*c^3*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*B*a^2*c^2*
d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 6*A*a*b*c^2*d*e^m*x*e^(m
*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*A*a^2*c*d^2*e^m*x*e^(m*log(x) + 2*
n*log(x))/(m + 2*n + 1) + B*a^2*c^3*e^m*x*e^(m*log(x) + n*log(x))/(m + n +
 1) + 2*A*a*b*c^3*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 3*A*a^2*c^2*
d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*a^2*c^3/(e*(
m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70422 vs. 2(328) = 656.

Time = 0.64 (sec) , antiderivative size = 70422, normalized size of antiderivative = 214.70

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(B*b^2*d^3*m^6*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 15*B*b^2*d^3*m^5*n*x*x^
(6*n)*e^(m*log(e) + m*log(x)) + 85*B*b^2*d^3*m^4*n^2*x*x^(6*n)*e^(m*log(e)
 + m*log(x)) + 225*B*b^2*d^3*m^3*n^3*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 2
74*B*b^2*d^3*m^2*n^4*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 120*B*b^2*d^3*m*n
^5*x*x^(6*n)*e^(m*log(e) + m*log(x)) + 3*B*b^2*c*d^2*m^6*x*x^(5*n)*e^(m*lo
g(e) + m*log(x)) + 2*B*a*b*d^3*m^6*x*x^(5*n)*e^(m*log(e) + m*log(x)) + A*b
^2*d^3*m^6*x*x^(5*n)*e^(m*log(e) + m*log(x)) + B*b^2*d^3*m^6*x*x^(5*n)*e^(
m*log(e) + m*log(x)) + 48*B*b^2*c*d^2*m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(
x)) + 32*B*a*b*d^3*m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 16*A*b^2*d^3*
m^5*n*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 15*B*b^2*d^3*m^5*n*x*x^(5*n)*e^(
m*log(e) + m*log(x)) + 285*B*b^2*c*d^2*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*l
og(x)) + 190*B*a*b*d^3*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 95*A*b^
2*d^3*m^4*n^2*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 85*B*b^2*d^3*m^4*n^2*x*x
^(5*n)*e^(m*log(e) + m*log(x)) + 780*B*b^2*c*d^2*m^3*n^3*x*x^(5*n)*e^(m*lo
g(e) + m*log(x)) + 520*B*a*b*d^3*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x))
 + 260*A*b^2*d^3*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 225*B*b^2*d^3
*m^3*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 972*B*b^2*c*d^2*m^2*n^4*x*x^(
5*n)*e^(m*log(e) + m*log(x)) + 648*B*a*b*d^3*m^2*n^4*x*x^(5*n)*e^(m*log(e)
 + m*log(x)) + 324*A*b^2*d^3*m^2*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 2
74*B*b^2*d^3*m^2*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 432*B*b^2*c*d^...
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 1882, normalized size of antiderivative = 5.74

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^2*(c + d*x^n)^3,x)
```

output
```
(x*x^(3*n)*(e*x)^m*(A*a^2*d^3 + B*b^2*c^3 + 3*A*b^2*c^2*d + 3*B*a^2*c*d^2
+ 6*A*a*b*c*d^2 + 6*B*a*b*c^2*d)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^
2*n + 744*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^
4 + m^5 + 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^
3 + 121*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205
*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m
^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n
^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 +
735*m^3*n^3 + 175*m^4*n^2 + 1) + (A*a^2*c^3*x*(e*x)^m)/(m + 1) + (c*x*x^(2
*n)*(e*x)^m*(3*A*a^2*d^2 + A*b^2*c^2 + 2*B*a*b*c^2 + 3*B*a^2*c*d + 6*A*a*b
*c*d)*(5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n
+ 702*m*n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3
 + 702*n^4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m
+ 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m
*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^
5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2
 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 +
 1) + (d*x*x^(4*n)*(e*x)^m*(B*a^2*d^2 + 3*B*b^2*c^2 + 2*A*a*b*d^2 + 3*A*b^
2*c*d + 6*B*a*b*c*d)*(5*m + 17*n + 68*m*n + 321*m*n^2 + 102*m^2*n + 614*m*
n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 +...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7608, normalized size of antiderivative = 23.20

$$\int (ex)^m (a+bx^n)^2 (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)*(c+d*x^n)^3,x)
```

output

```
(x**m*e**m*x*(x**(6*n)*b**3*d**3*m**6 + 15*x**(6*n)*b**3*d**3*m**5*n + 6*x
**(6*n)*b**3*d**3*m**5 + 85*x**(6*n)*b**3*d**3*m**4*n**2 + 75*x**(6*n)*b**
3*d**3*m**4*n + 15*x**(6*n)*b**3*d**3*m**4 + 225*x**(6*n)*b**3*d**3*m**3*n
**3 + 340*x**(6*n)*b**3*d**3*m**3*n**2 + 150*x**(6*n)*b**3*d**3*m**3*n + 2
0*x**(6*n)*b**3*d**3*m**3 + 274*x**(6*n)*b**3*d**3*m**2*n**4 + 675*x**(6*n
)*b**3*d**3*m**2*n**3 + 510*x**(6*n)*b**3*d**3*m**2*n**2 + 150*x**(6*n)*b*
*3*d**3*m**2*n + 15*x**(6*n)*b**3*d**3*m**2 + 120*x**(6*n)*b**3*d**3*m*n**
5 + 548*x**(6*n)*b**3*d**3*m*n**4 + 675*x**(6*n)*b**3*d**3*m*n**3 + 340*x*
*(6*n)*b**3*d**3*m*n**2 + 75*x**(6*n)*b**3*d**3*m*n + 6*x**(6*n)*b**3*d**3
*m + 120*x**(6*n)*b**3*d**3*n**5 + 274*x**(6*n)*b**3*d**3*n**4 + 225*x**(6
*n)*b**3*d**3*n**3 + 85*x**(6*n)*b**3*d**3*n**2 + 15*x**(6*n)*b**3*d**3*n
+ x**(6*n)*b**3*d**3 + 3*x**(5*n)*a*b**2*d**3*m**6 + 48*x**(5*n)*a*b**2*d*
*3*m**5*n + 18*x**(5*n)*a*b**2*d**3*m**5 + 285*x**(5*n)*a*b**2*d**3*m**4*n
**2 + 240*x**(5*n)*a*b**2*d**3*m**4*n + 45*x**(5*n)*a*b**2*d**3*m**4 + 780
*x**(5*n)*a*b**2*d**3*m**3*n**3 + 1140*x**(5*n)*a*b**2*d**3*m**3*n**2 + 48
0*x**(5*n)*a*b**2*d**3*m**3*n + 60*x**(5*n)*a*b**2*d**3*m**3 + 972*x**(5*n
)*a*b**2*d**3*m**2*n**4 + 2340*x**(5*n)*a*b**2*d**3*m**2*n**3 + 1710*x**(5
*n)*a*b**2*d**3*m**2*n**2 + 480*x**(5*n)*a*b**2*d**3*m**2*n + 45*x**(5*n)*
a*b**2*d**3*m**2 + 432*x**(5*n)*a*b**2*d**3*m*n**5 + 1944*x**(5*n)*a*b**2*
d**3*m*n**4 + 2340*x**(5*n)*a*b**2*d**3*m*n**3 + 1140*x**(5*n)*a*b**2*d...
```

3.35 $\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 29, antiderivative size = 225

$$\begin{aligned}\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx &= \frac{aAc^3(ex)^{1+m}}{e(1+m)} + \frac{c^2(Abc+aBc+3aAd)x^n(ex)^{1+m}}{e(1+m+n)} \\ &+ \frac{c(3ad(Bc+Ad)+bc(Bc+3Ad))x^{2n}(ex)^{1+m}}{e(1+m+2n)} \\ &+ \frac{d(3bc(Bc+Ad)+ad(3Bc+Ad))x^{3n}(ex)^{1+m}}{e(1+m+3n)} \\ &+ \frac{d^2(3bBc+Abd+aBd)x^{4n}(ex)^{1+m}}{e(1+m+4n)} + \frac{bBd^3x^{5n}(ex)^{1+m}}{e(1+m+5n)}\end{aligned}$$

output
```
a*A*c^3*(e*x)^(1+m)/e/(1+m)+c^2*(3*A*a*d+A*b*c+B*a*c)*x^n*(e*x)^(1+m)/e/(1
+m+n)+c*(3*a*d*(A*d+B*c)+b*c*(3*A*d+B*c))*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+
d*(3*b*c*(A*d+B*c)+a*d*(A*d+3*B*c))*x^(3*n)*(e*x)^(1+m)/e/(1+m+3*n)+d^2*(A
*b*d+B*a*d+3*B*b*c)*x^(4*n)*(e*x)^(1+m)/e/(1+m+4*n)+b*B*d^3*x^(5*n)*(e*x)^
(1+m)/e/(1+m+5*n)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.76

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx = x(ex)^m \left(\frac{aAc^3}{1+m} + \frac{c^2(Abc+aBc+3aAd)x^n}{1+m+n} + \frac{c(3ad(Bc+Ad)+bc(Bc+3Ad))x^{2n}}{1+m+2n} + \frac{d(3bc(Bc+Ad)+ad(3Bc+Ad))x^{3n}}{1+m+3n} + \frac{d^2(3bBc+Abd+aBd)x^{4n}}{1+m+4n} + \frac{bBd^3x^{5n}}{1+m+5n}\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
x*(e*x)^m*((a*A*c^3)/(1 + m) + (c^2*(A*b*c + a*B*c + 3*a*A*d)*x^n)/(1 + m
+ n) + (c*(3*a*d*(B*c + A*d) + b*c*(B*c + 3*A*d))*x^(2*n))/(1 + m + 2*n) +
 (d*(3*b*c*(B*c + A*d) + a*d*(3*B*c + A*d))*x^(3*n))/(1 + m + 3*n) + (d^2*
(3*b*B*c + A*b*d + a*B*d)*x^(4*n))/(1 + m + 4*n) + (b*B*d^3*x^(5*n))/(1 +
m + 5*n))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx$$

$$\Big\downarrow \; 1040$$

$$\int \left(c^2x^n(ex)^m(3aAd+aBc+Abc) + d^2x^{4n}(ex)^m(aBd+Abd+3bBc) + cx^{2n}(ex)^m(3ad(Ad+Bc)+bc(3Ad+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{c^2x^{n+1}(ex)^m(3aAd+aBc+Abc)}{m+n+1}+\frac{d^2x^{4n+1}(ex)^m(aBd+Abd+3bBc)}{m+4n+1}+\frac{cx^{2n+1}(ex)^m(3ad(Ad+Bc)+bc(3Ad+Bc))}{m+2n+1}+\frac{dx^{3n+1}(ex)^m(ad(Ad+3Bc)+3bc(Ad+Bc))}{m+3n+1}+\frac{aAc^3(ex)^{m+1}}{e(m+1)}+\frac{bBd^3x^{5n+1}(ex)^m}{m+5n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
(c^2*(A*b*c + a*B*c + 3*a*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (c*(3*a*d*
(B*c + A*d) + b*c*(B*c + 3*A*d))*x^(1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (d*(
3*b*c*(B*c + A*d) + a*d*(3*B*c + A*d))*x^(1 + 3*n)*(e*x)^m)/(1 + m + 3*n)
+ (d^2*(3*b*B*c + A*b*d + a*B*d)*x^(1 + 4*n)*(e*x)^m)/(1 + m + 4*n) + (b*B
*d^3*x^(1 + 5*n)*(e*x)^m)/(1 + m + 5*n) + (a*A*c^3*(e*x)^(1 + m))/(e*(1 +
m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.34 (sec) , antiderivative size = 4939, normalized size of antiderivative = 21.95

method	result	size
risch	Expression too large to display	4939
parallelrisch	Expression too large to display	6818
orering	Expression too large to display	10171

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(15*A*a*c*d^2*(x^n)^2*m+39*A*a*c*d^2*(x^n)^2*n+56*A*b*c^3*m*n*x^n+15*A*b
*c^2*d*(x^n)^2*m+39*A*b*c^2*d*(x^n)^2*n+71*B*a*c^3*n^2*x^n+156*A*b*c^2*d*m
^3*n*(x^n)^2+531*A*b*c^2*d*m^2*n^2*(x^n)^2+642*A*b*c^2*d*m*n^3*(x^n)^2+216
*A*b*c*d^2*m^2*n*(x^n)^3+10*B*a*c^3*m^2*x^n+5*m*b*B*d^3*(x^n)^5+11*B*a*d^3
*m^4*n*(x^n)^4+41*B*a*d^3*m^3*n^2*(x^n)^4+61*B*a*d^3*m^2*n^3*(x^n)^4+30*B*
a*d^3*m*n^4*(x^n)^4+3*B*b*c*d^2*m^5*(x^n)^4+40*B*b*d^3*m^3*n*(x^n)^5+105*B
*b*d^3*m^2*n^2*(x^n)^5+100*B*b*d^3*m*n^3*(x^n)^5+321*A*a*c*d^2*m^2*n^3*(x^
n)^2+180*A*a*c*d^2*m*n^4*(x^n)^2+39*A*b*c^2*d*m^4*n*(x^n)^2+177*A*b*c^2*d*
m^3*n^2*(x^n)^2+321*A*b*c^2*d*m^2*n^3*(x^n)^2+441*A*b*c*d^2*m*n^2*(x^n)^3+
156*B*a*c^2*d*m^3*n*(x^n)^2+531*B*a*c^2*d*m^2*n^2*(x^n)^2+642*B*a*c^2*d*m*
n^3*(x^n)^2+216*B*a*c*d^2*m^2*n*(x^n)^3+123*B*a*d^3*m^2*n^2*(x^n)^4+122*B*
a*d^3*m*n^3*(x^n)^4+3*B*b*c^2*d*m^5*(x^n)^3+15*B*b*c*d^2*m^4*(x^n)^4+90*B*
b*c*d^2*n^4*(x^n)^4+60*B*b*d^3*m^2*n*(x^n)^5+105*B*b*d^3*m*n^2*(x^n)^5+3*A
*a*c*d^2*m^5*(x^n)^2+48*A*a*d^3*m^3*n*(x^n)^3+147*A*a*d^3*m^2*n^2*(x^n)^3+
156*A*a*d^3*m*n^3*(x^n)^3+213*A*a*c^2*d*m^3*n^2*x^n+462*A*a*c^2*d*m^2*n^3*
x^n+360*A*a*c^2*d*m*n^4*x^n+156*A*a*c*d^2*m^3*n*(x^n)^2+531*A*a*c*d^2*m^2*
n^2*(x^n)^2+642*A*a*c*d^2*m*n^3*(x^n)^2+308*A*b*c^3*m*n^3*x^n+30*A*b*c^2*d
*m^3*(x^n)^2+321*A*b*c^2*d*n^3*(x^n)^2+30*A*b*c*d^2*m^2*(x^n)^3+147*A*b*c*
d^2*n^2*(x^n)^3+56*B*a*c^3*m^3*n*x^n+213*B*a*c^3*m^2*n^2*x^n+44*B*a*d^3*m^
3*n*(x^n)^4+120*A*a*c^3*n^5+A*a*c^3*m^5+5*A*a*c^3*m^4+274*A*a*c^3*n^4+1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2833 vs. $2(225) = 450$.

Time = 0.17 (sec) , antiderivative size = 2833, normalized size of antiderivative = 12.59

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
((B*b*d^3*m^5 + 5*B*b*d^3*m^4 + 10*B*b*d^3*m^3 + 10*B*b*d^3*m^2 + 5*B*b*d^
3*m + B*b*d^3 + 24*(B*b*d^3*m + B*b*d^3)*n^4 + 50*(B*b*d^3*m^2 + 2*B*b*d^3
*m + B*b*d^3)*n^3 + 35*(B*b*d^3*m^3 + 3*B*b*d^3*m^2 + 3*B*b*d^3*m + B*b*d^
3)*n^2 + 10*(B*b*d^3*m^4 + 4*B*b*d^3*m^3 + 6*B*b*d^3*m^2 + 4*B*b*d^3*m + B
*b*d^3)*n)*x*x^(5*n)*e^(m*log(e) + m*log(x)) + ((3*B*b*c*d^2 + (B*a + A*b)
*d^3)*m^5 + 3*B*b*c*d^2 + 5*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^4 + 30*(3*B*
b*c*d^2 + (B*a + A*b)*d^3 + (3*B*b*c*d^2 + (B*a + A*b)*d^3)*m)*n^4 + (B*a
+ A*b)*d^3 + 10*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^3 + 61*(3*B*b*c*d^2 + (B
*a + A*b)*d^3 + (3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^2 + 2*(3*B*b*c*d^2 + (B*
a + A*b)*d^3)*m)*n^3 + 10*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^2 + 41*(3*B*b*
c*d^2 + (B*a + A*b)*d^3 + (3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^3 + 3*(3*B*b*c
*d^2 + (B*a + A*b)*d^3)*m^2 + 3*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m)*n^2 + 5
*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m + 11*(3*B*b*c*d^2 + (3*B*b*c*d^2 + (B*a
 + A*b)*d^3)*m^4 + (B*a + A*b)*d^3 + 4*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^3
 + 6*(3*B*b*c*d^2 + (B*a + A*b)*d^3)*m^2 + 4*(3*B*b*c*d^2 + (B*a + A*b)*d^
3)*m)*n)*x*x^(4*n)*e^(m*log(e) + m*log(x)) + ((3*B*b*c^2*d + A*a*d^3 + 3*(
B*a + A*b)*c*d^2)*m^5 + 3*B*b*c^2*d + A*a*d^3 + 5*(3*B*b*c^2*d + A*a*d^3 +
 3*(B*a + A*b)*c*d^2)*m^4 + 40*(3*B*b*c^2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^
2 + (3*B*b*c^2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m)*n^4 + 3*(B*a + A*b)*c
*d^2 + 10*(3*B*b*c^2*d + A*a*d^3 + 3*(B*a + A*b)*c*d^2)*m^3 + 78*(3*B*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64068 vs. $2(214) = 428$.

Time = 16.55 (sec) , antiderivative size = 64068, normalized size of antiderivative = 284.75

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise(((A + B)*(a + b)*(c + d)**3*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A
*a*c**3*log(x) + 3*A*a*c**2*d*x**n/n + 3*A*a*c*d**2*x**(2*n)/(2*n) + A*a*d
**3*x**(3*n)/(3*n) + A*b*c**3*x**n/n + 3*A*b*c**2*d*x**(2*n)/(2*n) + A*b*c
*d**2*x**(3*n)/n + A*b*d**3*x**(4*n)/(4*n) + B*a*c**3*x**n/n + 3*B*a*c**2*
d*x**(2*n)/(2*n) + B*a*c*d**2*x**(3*n)/n + B*a*d**3*x**(4*n)/(4*n) + B*b*c
**3*x**(2*n)/(2*n) + B*b*c**2*d*x**(3*n)/n + 3*B*b*c*d**2*x**(4*n)/(4*n) +
 B*b*d**3*x**(5*n)/(5*n))/e, Eq(m, -1)), (A*a*c**3*Piecewise((0**(-5*n - 1
)*x, Eq(e, 0)), (Piecewise((-1/(5*n*(e*x)**(5*n)), Ne(n, 0)), (log(e*x), T
rue))/e, True)) + 3*A*a*c**2*d*Piecewise((-x*x**n*(e*x)**(-5*n - 1)/(4*n),
 Ne(n, 0)), (x*x**n*(e*x)**(-5*n - 1)*log(x), True)) + 3*A*a*c*d**2*Piecew
ise((-x*x**(2*n)*(e*x)**(-5*n - 1)/(3*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-
5*n - 1)*log(x), True)) + A*a*d**3*Piecewise((-x*x**(3*n)*(e*x)**(-5*n - 1
)/(2*n), Ne(n, 0)), (x*x**(3*n)*(e*x)**(-5*n - 1)*log(x), True)) + A*b*c**
3*Piecewise((-x*x**n*(e*x)**(-5*n - 1)/(4*n), Ne(n, 0)), (x*x**n*(e*x)**(-
5*n - 1)*log(x), True)) + 3*A*b*c**2*d*Piecewise((-x*x**(2*n)*(e*x)**(-5*n
 - 1)/(3*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-5*n - 1)*log(x), True)) + 3*A
*b*c*d**2*Piecewise((-x*x**(3*n)*(e*x)**(-5*n - 1)/(2*n), Ne(n, 0)), (x*x*
*(3*n)*(e*x)**(-5*n - 1)*log(x), True)) + A*b*d**3*Piecewise((-x*x**(4*n)*
(e*x)**(-5*n - 1)/n, Ne(n, 0)), (x*x**(4*n)*(e*x)**(-5*n - 1)*log(x), True
)) + B*a*c**3*Piecewise((-x*x**n*(e*x)**(-5*n - 1)/(4*n), Ne(n, 0)), (x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(225) = 450$.

Time = 0.08 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.06

$$\int (ex)^m (a+bx^n)(A+Bx^n)(c+dx^n)^3 \, dx = \frac{Bbd^3e^m xe^{(m\log(x)+5\,n\log(x))}}{m+5\,n+1} + \frac{3\,Bbcd^2e^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{Bad^3e^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{Abd^3e^m xe^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1} + \frac{3\,Bbc^2de^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Bacd^2e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{3\,Abcd^2e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Aad^3e^m xe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1} + \frac{Bbc^3e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Bac^2de^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Abc^2de^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{3\,Aacd^2e^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1} + \frac{Bac^3e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{Abc^3e^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{3\,Aac^2de^m xe^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}Aac^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
B*b*d^3*e^m*x*e^(m*log(x) + 5*n*log(x))/(m + 5*n + 1) + 3*B*b*c*d^2*e^m*x*
e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + B*a*d^3*e^m*x*e^(m*log(x) + 4*n*
log(x))/(m + 4*n + 1) + A*b*d^3*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n +
 1) + 3*B*b*c^2*d*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*B*a*c*
d^2*e^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 3*A*b*c*d^2*e^m*x*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 1) + A*a*d^3*e^m*x*e^(m*log(x) + 3*n*log(
x))/(m + 3*n + 1) + B*b*c^3*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1)
+ 3*B*a*c^2*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*A*b*c^2*d*
e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*A*a*c*d^2*e^m*x*e^(m*log
(x) + 2*n*log(x))/(m + 2*n + 1) + B*a*c^3*e^m*x*e^(m*log(x) + n*log(x))/(m
 + n + 1) + A*b*c^3*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + 3*A*a*c^2*
d*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (e*x)^(m + 1)*A*a*c^3/(e*(m
+ 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27992 vs. $2(225) = 450$.

Time = 0.31 (sec) , antiderivative size = 27992, normalized size of antiderivative = 124.41

$$\int (ex)^m (a + bx^n) (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(B*b*d^3*m^5*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 10*B*b*d^3*m^4*n*x*x^(5*n
)*e^(m*log(e) + m*log(x)) + 35*B*b*d^3*m^3*n^2*x*x^(5*n)*e^(m*log(e) + m*l
og(x)) + 50*B*b*d^3*m^2*n^3*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 24*B*b*d^3
*m*n^4*x*x^(5*n)*e^(m*log(e) + m*log(x)) + 3*B*b*c*d^2*m^5*x*x^(4*n)*e^(m*
log(e) + m*log(x)) + B*a*d^3*m^5*x*x^(4*n)*e^(m*log(e) + m*log(x)) + A*b*d
^3*m^5*x*x^(4*n)*e^(m*log(e) + m*log(x)) + B*b*d^3*m^5*x*x^(4*n)*e^(m*log(
e) + m*log(x)) + 33*B*b*c*d^2*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 11
*B*a*d^3*m^4*n*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 11*A*b*d^3*m^4*n*x*x^(4
*n)*e^(m*log(e) + m*log(x)) + 10*B*b*d^3*m^4*n*x*x^(4*n)*e^(m*log(e) + m*l
og(x)) + 123*B*b*c*d^2*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 41*B*a*
d^3*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 41*A*b*d^3*m^3*n^2*x*x^(4*
n)*e^(m*log(e) + m*log(x)) + 35*B*b*d^3*m^3*n^2*x*x^(4*n)*e^(m*log(e) + m*
log(x)) + 183*B*b*c*d^2*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 61*B*a
*d^3*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 61*A*b*d^3*m^2*n^3*x*x^(4
*n)*e^(m*log(e) + m*log(x)) + 50*B*b*d^3*m^2*n^3*x*x^(4*n)*e^(m*log(e) + m
*log(x)) + 90*B*b*c*d^2*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 30*B*a*d
^3*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 30*A*b*d^3*m*n^4*x*x^(4*n)*e^
(m*log(e) + m*log(x)) + 24*B*b*d^3*m*n^4*x*x^(4*n)*e^(m*log(e) + m*log(x))
 + 3*B*b*c^2*d*m^5*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*B*a*c*d^2*m^5*x*x
^(3*n)*e^(m*log(e) + m*log(x)) + 3*A*b*c*d^2*m^5*x*x^(3*n)*e^(m*log(e) ...
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 1089, normalized size of antiderivative = 4.84

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)*(c + d*x^n)^3,x)
```

output
```
(A*a*c^3*x*(e*x)^m)/(m + 1) + (d^2*x*x^(4*n)*(e*x)^m*(A*b*d + B*a*d + 3*B*
b*c)*(4*m + 11*n + 33*m*n + 82*m*n^2 + 33*m^2*n + 61*m*n^3 + 11*m^3*n + 6*
m^2 + 4*m^3 + m^4 + 41*n^2 + 61*n^3 + 30*n^4 + 41*m^2*n^2 + 1))/(5*m + 15*
n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*
m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n
^5 + 255*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (c*x*x^(2*n)*(e*x)^m*(3
*A*a*d^2 + B*b*c^2 + 3*A*b*c*d + 3*B*a*c*d)*(4*m + 13*n + 39*m*n + 118*m*n
^2 + 39*m^2*n + 107*m*n^3 + 13*m^3*n + 6*m^2 + 4*m^3 + m^4 + 59*n^2 + 107*
n^3 + 60*n^4 + 59*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*
n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5*m^4
+ m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 + 225*m^2*n^3 +
 85*m^3*n^2 + 1) + (d*x*x^(3*n)*(e*x)^m*(A*a*d^2 + 3*B*b*c^2 + 3*A*b*c*d +
 3*B*a*c*d)*(4*m + 12*n + 36*m*n + 98*m*n^2 + 36*m^2*n + 78*m*n^3 + 12*m^3
*n + 6*m^2 + 4*m^3 + m^4 + 49*n^2 + 78*n^3 + 40*n^4 + 49*m^2*n^2 + 1))/(5*
m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^
4 + 15*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4
+ 120*n^5 + 255*m^2*n^2 + 225*m^2*n^3 + 85*m^3*n^2 + 1) + (c^2*x*x^n*(e*x)
^m*(3*A*a*d + A*b*c + B*a*c)*(4*m + 14*n + 42*m*n + 142*m*n^2 + 42*m^2*n +
 154*m*n^3 + 14*m^3*n + 6*m^2 + 4*m^3 + m^4 + 71*n^2 + 154*n^3 + 120*n^4 +
 71*m^2*n^2 + 1))/(5*m + 15*n + 60*m*n + 255*m*n^2 + 90*m^2*n + 450*m*n...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3949, normalized size of antiderivative = 17.55

$$\int (ex)^m (a+bx^n) (A+Bx^n) (c+dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)*(c+d*x^n)^3,x)
```

output
```
(x**m*e**m*x*(x**(5*n)*b**2*d**3*m**5 + 10*x**(5*n)*b**2*d**3*m**4*n + 5*x
**(5*n)*b**2*d**3*m**4 + 35*x**(5*n)*b**2*d**3*m**3*n**2 + 40*x**(5*n)*b**
2*d**3*m**3*n + 10*x**(5*n)*b**2*d**3*m**3 + 50*x**(5*n)*b**2*d**3*m**2*n*
*3 + 105*x**(5*n)*b**2*d**3*m**2*n**2 + 60*x**(5*n)*b**2*d**3*m**2*n + 10*
x**(5*n)*b**2*d**3*m**2 + 24*x**(5*n)*b**2*d**3*m*n**4 + 100*x**(5*n)*b**2
*d**3*m*n**3 + 105*x**(5*n)*b**2*d**3*m*n**2 + 40*x**(5*n)*b**2*d**3*m*n +
 5*x**(5*n)*b**2*d**3*m + 24*x**(5*n)*b**2*d**3*n**4 + 50*x**(5*n)*b**2*d*
*3*n**3 + 35*x**(5*n)*b**2*d**3*n**2 + 10*x**(5*n)*b**2*d**3*n + x**(5*n)*
b**2*d**3 + 2*x**(4*n)*a*b*d**3*m**5 + 22*x**(4*n)*a*b*d**3*m**4*n + 10*x*
*(4*n)*a*b*d**3*m**4 + 82*x**(4*n)*a*b*d**3*m**3*n**2 + 88*x**(4*n)*a*b*d*
*3*m**3*n + 20*x**(4*n)*a*b*d**3*m**3 + 122*x**(4*n)*a*b*d**3*m**2*n**3 +
246*x**(4*n)*a*b*d**3*m**2*n**2 + 132*x**(4*n)*a*b*d**3*m**2*n + 20*x**(4*
n)*a*b*d**3*m**2 + 60*x**(4*n)*a*b*d**3*m*n**4 + 244*x**(4*n)*a*b*d**3*m*n
**3 + 246*x**(4*n)*a*b*d**3*m*n**2 + 88*x**(4*n)*a*b*d**3*m*n + 10*x**(4*n
)*a*b*d**3*m + 60*x**(4*n)*a*b*d**3*n**4 + 122*x**(4*n)*a*b*d**3*n**3 + 82
*x**(4*n)*a*b*d**3*n**2 + 22*x**(4*n)*a*b*d**3*n + 2*x**(4*n)*a*b*d**3 + 3
*x**(4*n)*b**2*c*d**2*m**5 + 33*x**(4*n)*b**2*c*d**2*m**4*n + 15*x**(4*n)*
b**2*c*d**2*m**4 + 123*x**(4*n)*b**2*c*d**2*m**3*n**2 + 132*x**(4*n)*b**2*
c*d**2*m**3*n + 30*x**(4*n)*b**2*c*d**2*m**3 + 183*x**(4*n)*b**2*c*d**2*m*
*2*n**3 + 369*x**(4*n)*b**2*c*d**2*m**2*n**2 + 198*x**(4*n)*b**2*c*d**2...
```

3.36 $\int (ex)^m (A+Bx^n) (c+dx^n)^3 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int (ex)^m (A+Bx^n) (c+dx^n)^3 \, dx = \frac{Ac^3(ex)^{1+m}}{e(1+m)} + \frac{c^2(Bc+3Ad)x^n(ex)^{1+m}}{e(1+m+n)} + \frac{3cd(Bc+Ad)x^{2n}(ex)^{1+m}}{e(1+m+2n)} + \frac{d^2(3Bc+Ad)x^{3n}(ex)^{1+m}}{e(1+m+3n)} + \frac{Bd^3x^{4n}(ex)^{1+m}}{e(1+m+4n)}$$

output
```
A*c^3*(e*x)^(1+m)/e/(1+m)+c^2*(3*A*d+B*c)*x^n*(e*x)^(1+m)/e/(1+m+n)+3*c*d*
(A*d+B*c)*x^(2*n)*(e*x)^(1+m)/e/(1+m+2*n)+d^2*(A*d+3*B*c)*x^(3*n)*(e*x)^(1
+m)/e/(1+m+3*n)+B*d^3*x^(4*n)*(e*x)^(1+m)/e/(1+m+4*n)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

$$\int (ex)^m (A+Bx^n) (c+dx^n)^3 \, dx = x(ex)^m \left(\frac{Ac^3}{1+m} + \frac{c^2(Bc+3Ad)x^n}{1+m+n} + \frac{3cd(Bc+Ad)x^{2n}}{1+m+2n} + \frac{d^2(3Bc+Ad)x^{3n}}{1+m+3n} + \frac{Bd^3x^{4n}}{1+m+4n}\right)$$

input
```
Integrate[(e*x)^m*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
x*(e*x)^m*((A*c^3)/(1 + m) + (c^2*(B*c + 3*A*d)*x^n)/(1 + m + n) + (3*c*d*
(B*c + A*d)*x^(2*n))/(1 + m + 2*n) + (d^2*(3*B*c + A*d)*x^(3*n))/(1 + m +
3*n) + (B*d^3*x^(4*n))/(1 + m + 4*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {950, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (A+Bx^n)(c+dx^n)^3 \, dx$$

$$\Big\downarrow \text{950}$$

$$\int \left(c^2x^n(ex)^m(3Ad+Bc)+d^2x^{3n}(ex)^m(Ad+3Bc)+3cdx^{2n}(ex)^m(Ad+Bc)+Ac^3(ex)^m+Bd^3x^{4n}(ex)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2x^{n+1}(ex)^m(3Ad+Bc)}{m+n+1}+\frac{d^2x^{3n+1}(ex)^m(Ad+3Bc)}{m+3n+1}+\frac{3cdx^{2n+1}(ex)^m(Ad+Bc)}{m+2n+1}+\frac{Ac^3(ex)^{m+1}}{e(m+1)}+\frac{Bd^3x^{4n+1}(ex)^m}{m+4n+1}$$

input
```
Int[(e*x)^m*(A + B*x^n)*(c + d*x^n)^3,x]
```

output
```
(c^2*(B*c + 3*A*d)*x^(1 + n)*(e*x)^m)/(1 + m + n) + (3*c*d*(B*c + A*d)*x^(
1 + 2*n)*(e*x)^m)/(1 + m + 2*n) + (d^2*(3*B*c + A*d)*x^(1 + 3*n)*(e*x)^m)/
(1 + m + 3*n) + (B*d^3*x^(1 + 4*n)*(e*x)^m)/(1 + m + 4*n) + (A*c^3*(e*x)^(
1 + m))/(e*(1 + m))
```

Defintions of rubi rules used

rule 950
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.71 (sec) , antiderivative size = 1576, normalized size of antiderivative = 10.58

method	result	size
risch	Expression too large to display	1576
parallelrisch	Expression too large to display	2207
orering	Expression too large to display	2976

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(84*B*c*d^2*m*n^2*(x^n)^3+27*A*c^2*d*m^3*n*x^n+78*A*c^2*d*m^2*n^2*x^n+24
*B*c*d^2*m*n^3*(x^n)^3+24*A*c*d^2*m^3*n*(x^n)^2+57*A*c*d^2*m^2*n^2*(x^n)^2
+36*A*c*d^2*m*n^3*(x^n)^2+27*B*c^3*m*n*x^n+24*A*c^3*n^4+4*A*c^3*m^3+B*d^3*
(x^n)^4+A*d^3*(x^n)^3+24*B*c^2*d*(x^n)^2*n+21*B*c*d^2*(x^n)^3*n+3*A*c^2*d*
m^4*x^n+B*c^3*m^4*x^n+24*B*c^2*d*m^3*n*(x^n)^2+30*A*c^3*m^2*n+70*A*c^3*m*n
^2+30*A*c^3*m*n+10*A*c^3*m^3*n+35*A*c^3*m^2*n^2+50*A*c^3*m*n^3+21*B*c*d^2*
m^3*n*(x^n)^3+42*B*c*d^2*m^2*n^2*(x^n)^3+114*B*c^2*d*m*n^2*(x^n)^2+63*B*c*
d^2*m*n*(x^n)^3+81*A*c^2*d*m^2*n*x^n+A*c^3*m^4+4*A*c^3*m+B*c^3*x^n+12*A*c^
2*d*m^3*x^n+21*A*d^3*m*n*(x^n)^3+9*B*c^3*m^3*n*x^n+26*B*c^3*m^2*n^2*x^n+12
*B*c^2*d*m^3*(x^n)^2+12*A*c*d^2*m^3*(x^n)^2+36*A*c*d^2*n^3*(x^n)^2+21*A*d^
3*m^2*n*(x^n)^3+9*B*c^3*x^n*n+12*B*c*d^2*m^3*(x^n)^3+24*B*c*d^2*n^3*(x^n)^
3+18*B*d^3*m*n*(x^n)^4+4*B*c^3*m^3*x^n+27*B*c^3*m^2*n*x^n+52*B*c^3*m*n^2*x
^n+18*B*c^2*d*m^2*(x^n)^2+57*B*c^2*d*n^2*(x^n)^2+36*B*c^2*d*n^3*(x^n)^2+18
*B*c*d^2*m^2*(x^n)^3+42*B*c*d^2*n^2*(x^n)^3+3*x^n*A*c^2*d+12*A*c^2*d*x^n*m
+28*A*d^3*m*n^2*(x^n)^3+A*c^3+3*B*c^2*d*m^4*(x^n)^2+18*A*c^2*d*m^2*x^n+4*m
*B*d^3*(x^n)^4+27*A*c^2*d*x^n*n+156*A*c^2*d*m*n^2*x^n+72*A*c*d^2*m*n*(x^n)
^2+72*B*c^2*d*m*n*(x^n)^2+81*A*c^2*d*m*n*x^n+57*B*c^2*d*m^2*n^2*(x^n)^2+36
*B*c^2*d*m*n^3*(x^n)^2+14*A*d^3*n^2*(x^n)^3+6*A*d^3*m^2*(x^n)^3+22*B*d^3*m
*n^2*(x^n)^4+3*A*c*d^2*m^4*(x^n)^2+12*B*c*d^2*(x^n)^3*m+4*B*c^3*x^n*m+26*B
*c^3*n^2*x^n+3*B*c*d^2*(x^n)^3+50*A*c^3*n^3+6*A*c^3*m^2+35*A*c^3*n^2+24...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(149) = 298$.

Time = 0.13 (sec) , antiderivative size = 1104, normalized size of antiderivative = 7.41

$$\int (ex)^m (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
((B*d^3*m^4 + 4*B*d^3*m^3 + 6*B*d^3*m^2 + 4*B*d^3*m + B*d^3 + 6*(B*d^3*m +
 B*d^3)*n^3 + 11*(B*d^3*m^2 + 2*B*d^3*m + B*d^3)*n^2 + 6*(B*d^3*m^3 + 3*B*
d^3*m^2 + 3*B*d^3*m + B*d^3)*n)*x*x^(4*n)*e^(m*log(e) + m*log(x)) + ((3*B*
c*d^2 + A*d^3)*m^4 + 3*B*c*d^2 + A*d^3 + 4*(3*B*c*d^2 + A*d^3)*m^3 + 8*(3*
B*c*d^2 + A*d^3 + (3*B*c*d^2 + A*d^3)*m)*n^3 + 6*(3*B*c*d^2 + A*d^3)*m^2 +
 14*(3*B*c*d^2 + A*d^3 + (3*B*c*d^2 + A*d^3)*m^2 + 2*(3*B*c*d^2 + A*d^3)*m
)*n^2 + 4*(3*B*c*d^2 + A*d^3)*m + 7*(3*B*c*d^2 + A*d^3 + (3*B*c*d^2 + A*d^
3)*m^3 + 3*(3*B*c*d^2 + A*d^3)*m^2 + 3*(3*B*c*d^2 + A*d^3)*m)*n)*x*x^(3*n)
*e^(m*log(e) + m*log(x)) + 3*((B*c^2*d + A*c*d^2)*m^4 + B*c^2*d + A*c*d^2
+ 4*(B*c^2*d + A*c*d^2)*m^3 + 12*(B*c^2*d + A*c*d^2 + (B*c^2*d + A*c*d^2)*
m)*n^3 + 6*(B*c^2*d + A*c*d^2)*m^2 + 19*(B*c^2*d + A*c*d^2 + (B*c^2*d + A*
c*d^2)*m^2 + 2*(B*c^2*d + A*c*d^2)*m)*n^2 + 4*(B*c^2*d + A*c*d^2)*m + 8*(B
*c^2*d + A*c*d^2 + (B*c^2*d + A*c*d^2)*m^3 + 3*(B*c^2*d + A*c*d^2)*m^2 + 3
*(B*c^2*d + A*c*d^2)*m)*n)*x*x^(2*n)*e^(m*log(e) + m*log(x)) + ((B*c^3 + 3
*A*c^2*d)*m^4 + B*c^3 + 3*A*c^2*d + 4*(B*c^3 + 3*A*c^2*d)*m^3 + 24*(B*c^3
+ 3*A*c^2*d + (B*c^3 + 3*A*c^2*d)*m)*n^3 + 6*(B*c^3 + 3*A*c^2*d)*m^2 + 26*
(B*c^3 + 3*A*c^2*d + (B*c^3 + 3*A*c^2*d)*m^2 + 2*(B*c^3 + 3*A*c^2*d)*m)*n^
2 + 4*(B*c^3 + 3*A*c^2*d)*m + 9*(B*c^3 + 3*A*c^2*d + (B*c^3 + 3*A*c^2*d)*m
^3 + 3*(B*c^3 + 3*A*c^2*d)*m^2 + 3*(B*c^3 + 3*A*c^2*d)*m)*n)*x*x^n*e^(m*lo
g(e) + m*log(x)) + (A*c^3*m^4 + 24*A*c^3*n^4 + 4*A*c^3*m^3 + 6*A*c^3*m^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16781 vs. $2(134) = 268$.

Time = 6.81 (sec) , antiderivative size = 16781, normalized size of antiderivative = 112.62

$$\int (ex)^m (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**3,x)
```

output
```
Piecewise(((A + B)*(c + d)**3*log(x)/e, Eq(m, -1) & Eq(n, 0)), ((A*c**3*lo
g(x) + 3*A*c**2*d*x**n/n + 3*A*c*d**2*x**(2*n)/(2*n) + A*d**3*x**(3*n)/(3*
n) + B*c**3*x**n/n + 3*B*c**2*d*x**(2*n)/(2*n) + B*c*d**2*x**(3*n)/n + B*d
**3*x**(4*n)/(4*n))/e, Eq(m, -1)), (A*c**3*Piecewise((0**(-4*n - 1)*x, Eq(
e, 0)), (Piecewise((-1/(4*n*(e*x)**(4*n)), Ne(n, 0)), (log(e*x), True))/e,
 True)) + 3*A*c**2*d*Piecewise((-x*x**n*(e*x)**(-4*n - 1)/(3*n), Ne(n, 0))
, (x*x**n*(e*x)**(-4*n - 1)*log(x), True)) + 3*A*c*d**2*Piecewise((-x*x**(
2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0)), (x*x**(2*n)*(e*x)**(-4*n - 1)*log
(x), True)) + A*d**3*Piecewise((-x*x**(3*n)*(e*x)**(-4*n - 1)/n, Ne(n, 0))
, (x*x**(3*n)*(e*x)**(-4*n - 1)*log(x), True)) + B*c**3*Piecewise((-x*x**n
*(e*x)**(-4*n - 1)/(3*n), Ne(n, 0)), (x*x**n*(e*x)**(-4*n - 1)*log(x), Tru
e)) + 3*B*c**2*d*Piecewise((-x*x**(2*n)*(e*x)**(-4*n - 1)/(2*n), Ne(n, 0))
, (x*x**(2*n)*(e*x)**(-4*n - 1)*log(x), True)) + 3*B*c*d**2*Piecewise((-x*
x**(3*n)*(e*x)**(-4*n - 1)/n, Ne(n, 0)), (x*x**(3*n)*(e*x)**(-4*n - 1)*log
(x), True)) + B*d**3*x*x**(4*n)*(e*x)**(-4*n - 1)*log(x), Eq(m, -4*n - 1))
, (A*c**3*Piecewise((0**(-3*n - 1)*x, Eq(e, 0)), (Piecewise((-1/(3*n*(e*x)
**(3*n)), Ne(n, 0)), (log(e*x), True))/e, True)) + 3*A*c**2*d*Piecewise((-
x*x**n*(e*x)**(-3*n - 1)/(2*n), Ne(n, 0)), (x*x**n*(e*x)**(-3*n - 1)*log(x
), True)) + 3*A*c*d**2*Piecewise((-x*x**(2*n)*(e*x)**(-3*n - 1)/n, Ne(n, 0
)), (x*x**(2*n)*(e*x)**(-3*n - 1)*log(x), True)) + A*d**3*x*x**(3*n)*(e...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.47

$$\int (ex)^m (A+Bx^n)(c+dx^n)^3\,dx = \frac{Bd^3e^m x e^{(m\log(x)+4n\log(x))}}{m+4n+1} + \frac{3Bcd^2e^m x e^{(m\log(x)+3n\log(x))}}{m+3n+1} + \frac{Ad^3e^m x e^{(m\log(x)+3n\log(x))}}{m+3n+1} + \frac{3Bc^2de^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{3Acd^2e^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1} + \frac{Bc^3e^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{3Ac^2de^m x e^{(m\log(x)+n\log(x))}}{m+n+1} + \frac{(ex)^{m+1}Ac^3}{e(m+1)}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
B*d^3*e^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 3*B*c*d^2*e^m*x*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 1) + A*d^3*e^m*x*e^(m*log(x) + 3*n*log(x)
)/(m + 3*n + 1) + 3*B*c^2*d*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1)
+ 3*A*c*d^2*e^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + B*c^3*e^m*x*e^
(m*log(x) + n*log(x))/(m + n + 1) + 3*A*c^2*d*e^m*x*e^(m*log(x) + n*log(x)
)/(m + n + 1) + (e*x)^(m + 1)*A*c^3/(e*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7893 vs. $2(149) = 298$.

Time = 0.18 (sec) , antiderivative size = 7893, normalized size of antiderivative = 52.97

$$\int (ex)^m (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

output
```
(B*d^3*m^4*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 6*B*d^3*m^3*n*x*x^(4*n)*e^(
m*log(e) + m*log(x)) + 11*B*d^3*m^2*n^2*x*x^(4*n)*e^(m*log(e) + m*log(x))
+ 6*B*d^3*m*n^3*x*x^(4*n)*e^(m*log(e) + m*log(x)) + 3*B*c*d^2*m^4*x*x^(3*n
)*e^(m*log(e) + m*log(x)) + A*d^3*m^4*x*x^(3*n)*e^(m*log(e) + m*log(x)) +
B*d^3*m^4*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 21*B*c*d^2*m^3*n*x*x^(3*n)*e
^(m*log(e) + m*log(x)) + 7*A*d^3*m^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) +
 6*B*d^3*m^3*n*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 42*B*c*d^2*m^2*n^2*x*x^
(3*n)*e^(m*log(e) + m*log(x)) + 14*A*d^3*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m
*log(x)) + 11*B*d^3*m^2*n^2*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 24*B*c*d^2
*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 8*A*d^3*m*n^3*x*x^(3*n)*e^(m*lo
g(e) + m*log(x)) + 6*B*d^3*m*n^3*x*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*B*c
^2*d*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 3*A*c*d^2*m^4*x*x^(2*n)*e^(m*
log(e) + m*log(x)) + 3*B*c*d^2*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + A*d
^3*m^4*x*x^(2*n)*e^(m*log(e) + m*log(x)) + B*d^3*m^4*x*x^(2*n)*e^(m*log(e)
 + m*log(x)) + 24*B*c^2*d*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 24*A*c
*d^2*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 21*B*c*d^2*m^3*n*x*x^(2*n)*
e^(m*log(e) + m*log(x)) + 7*A*d^3*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(x))
+ 6*B*d^3*m^3*n*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 57*B*c^2*d*m^2*n^2*x*x
^(2*n)*e^(m*log(e) + m*log(x)) + 57*A*c*d^2*m^2*n^2*x*x^(2*n)*e^(m*log(e)
+ m*log(x)) + 42*B*c*d^2*m^2*n^2*x*x^(2*n)*e^(m*log(e) + m*log(x)) + 14...
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 563, normalized size of antiderivative = 3.78

$$\int (ex)^m (A+Bx^n) (c+dx^n)^3 \, dx = \frac{A\,c^3\,x\,{(e\,x)}^m}{m+1} + \frac{d^2\,x\,x^{3\,n}\,{(e\,x)}^m\,(A\,d+3\,B\,c)\,(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{c^2\,x\,x^n\,{(e\,x)}^m\,(3\,A\,d+B\,c)\,(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{B\,d^3\,x\,x^{4\,n}\,{(e\,x)}^m\,(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50} + \frac{3\,c\,d\,x\,x^{2\,n}\,{(e\,x)}^m\,(A\,d+B\,c)\,(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50}$$

input
```
int((e*x)^m*(A + B*x^n)*(c + d*x^n)^3,x)
```

output
```
(A*c^3*x*(e*x)^m)/(m + 1) + (d^2*x*x^(3*n)*(e*x)^m*(A*d + 3*B*c)*(3*m + 7*
n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14*n^2 + 8*n^3 + 1))/(4*m
+ 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^
3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^n*(e*x)^m*
(3*A*d + B*c)*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*
n^2 + 24*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 +
1) + (B*d^3*x*x^(4*n)*(e*x)^m*(3*m + 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3
*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2
*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4
+ 35*m^2*n^2 + 1) + (3*c*d*x*x^(2*n)*(e*x)^m*(A*d + B*c)*(3*m + 8*n + 16*m
*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1607, normalized size of antiderivative = 10.79

$$\int (ex)^m (A + Bx^n) (c + dx^n)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3,x)
```

output
```
(x**m*e**m*x*(x**(4*n)*b*d**3*m**4 + 6*x**(4*n)*b*d**3*m**3*n + 4*x**(4*n)
*b*d**3*m**3 + 11*x**(4*n)*b*d**3*m**2*n**2 + 18*x**(4*n)*b*d**3*m**2*n +
6*x**(4*n)*b*d**3*m**2 + 6*x**(4*n)*b*d**3*m*n**3 + 22*x**(4*n)*b*d**3*m*n
**2 + 18*x**(4*n)*b*d**3*m*n + 4*x**(4*n)*b*d**3*m + 6*x**(4*n)*b*d**3*n**
3 + 11*x**(4*n)*b*d**3*n**2 + 6*x**(4*n)*b*d**3*n + x**(4*n)*b*d**3 + x**(
3*n)*a*d**3*m**4 + 7*x**(3*n)*a*d**3*m**3*n + 4*x**(3*n)*a*d**3*m**3 + 14*
x**(3*n)*a*d**3*m**2*n**2 + 21*x**(3*n)*a*d**3*m**2*n + 6*x**(3*n)*a*d**3*
m**2 + 8*x**(3*n)*a*d**3*m*n**3 + 28*x**(3*n)*a*d**3*m*n**2 + 21*x**(3*n)*
a*d**3*m*n + 4*x**(3*n)*a*d**3*m + 8*x**(3*n)*a*d**3*n**3 + 14*x**(3*n)*a*
d**3*n**2 + 7*x**(3*n)*a*d**3*n + x**(3*n)*a*d**3 + 3*x**(3*n)*b*c*d**2*m*
*4 + 21*x**(3*n)*b*c*d**2*m**3*n + 12*x**(3*n)*b*c*d**2*m**3 + 42*x**(3*n)
*b*c*d**2*m**2*n**2 + 63*x**(3*n)*b*c*d**2*m**2*n + 18*x**(3*n)*b*c*d**2*m
**2 + 24*x**(3*n)*b*c*d**2*m*n**3 + 84*x**(3*n)*b*c*d**2*m*n**2 + 63*x**(3
*n)*b*c*d**2*m*n + 12*x**(3*n)*b*c*d**2*m + 24*x**(3*n)*b*c*d**2*n**3 + 42
*x**(3*n)*b*c*d**2*n**2 + 21*x**(3*n)*b*c*d**2*n + 3*x**(3*n)*b*c*d**2 + 3
*x**(2*n)*a*c*d**2*m**4 + 24*x**(2*n)*a*c*d**2*m**3*n + 12*x**(2*n)*a*c*d*
*2*m**3 + 57*x**(2*n)*a*c*d**2*m**2*n**2 + 72*x**(2*n)*a*c*d**2*m**2*n + 1
8*x**(2*n)*a*c*d**2*m**2 + 36*x**(2*n)*a*c*d**2*m*n**3 + 114*x**(2*n)*a*c*
d**2*m*n**2 + 72*x**(2*n)*a*c*d**2*m*n + 12*x**(2*n)*a*c*d**2*m + 36*x**(2
*n)*a*c*d**2*n**3 + 57*x**(2*n)*a*c*d**2*n**2 + 24*x**(2*n)*a*c*d**2*n ...
```

3.37 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^3}{a+bx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 279

$$
\begin{aligned}
\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^3}{a+bx^n}\,dx
&= -\frac{(a^3Bd^3+3ab^2cd(Bc+Ad)-a^2bd^2(3Bc+Ad)-b^3c^2(Bc+3Ad))(ex)^{1+m}}{b^4e(1+m)} \\
&\quad+\frac{d(a^2Bd^2+3b^2c(Bc+Ad)-abd(3Bc+Ad))x^n(ex)^{1+m}}{b^3e(1+m+n)} \\
&\quad+\frac{d^2(3bBc+Abd-aBd)x^{2n}(ex)^{1+m}}{b^2e(1+m+2n)}+\frac{Bd^3x^{3n}(ex)^{1+m}}{be(1+m+3n)} \\
&\quad+\frac{(Ab-aB)(bc-ad)^3(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{ab^4e(1+m)}
\end{aligned}
$$

output

```
-(a^3*B*d^3+3*a*b^2*c*d*(A*d+B*c)-a^2*b*d^2*(A*d+3*B*c)-b^3*c^2*(3*A*d+B*c
))*(e*x)^(1+m)/b^4/e/(1+m)+d*(a^2*B*d^2+3*b^2*c*(A*d+B*c)-a*b*d*(A*d+3*B*c
))*x^n*(e*x)^(1+m)/b^3/e/(1+m+n)+d^2*(A*b*d-B*a*d+3*B*b*c)*x^(2*n)*(e*x)^(
1+m)/b^2/e/(1+m+2*n)+B*d^3*x^(3*n)*(e*x)^(1+m)/b/e/(1+m+3*n)+(A*b-B*a)*(-a
*d+b*c)^3*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/b^4/e
/(1+m)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{a+bx^n}\,dx$$

$$= \frac{x(ex)^m \left(\frac{-a^3Bd^3-3ab^2cd(Bc+Ad)+a^2bd^2(3Bc+Ad)+b^3c^2(Bc+3Ad)}{1+m} + \frac{bd(a^2Bd^2+3b^2c(Bc+Ad)-abd(3Bc+Ad))x^n}{1+m+n} + \frac{b^2d^2(3bBc+\ldots}{1+\ldots}\right.}{b^4}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n),x]
```

output
```
(x*(e*x)^m*((-(a^3*B*d^3) - 3*a*b^2*c*d*(B*c + A*d) + a^2*b*d^2*(3*B*c + A
*d) + b^3*c^2*(B*c + 3*A*d))/(1 + m) + (b*d*(a^2*B*d^2 + 3*b^2*c*(B*c + A*
d) - a*b*d*(3*B*c + A*d))*x^n)/(1 + m + n) + (b^2*d^2*(3*b*B*c + A*b*d - a
*B*d)*x^(2*n))/(1 + m + 2*n) + (b^3*B*d^3*x^(3*n))/(1 + m + 3*n) + ((-(A*b
) + a*B)*(-(b*c) + a*d)^3*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*(1 + m))))/b^4
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{a+bx^n}\,dx$$

$$\Big\downarrow \; 1040$$

$$\int \left(\frac{dx^n(ex)^m \left(a^2Bd^2-abd(Ad+3Bc)+3b^2c(Ad+Bc)\right)}{b^3} + \frac{(ex)^m \left(-a^3Bd^3+a^2bd^2(Ad+3Bc)-3ab^2cd(Ad\ldots\right.}{b^4}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{dx^{n+1}(ex)^m\left(a^2Bd^2-abd(Ad+3Bc)+3b^2c(Ad+Bc)\right)}{b^3(m+n+1)}-\frac{(ex)^{m+1}\left(a^3Bd^3-a^2bd^2(Ad+3Bc)+3ab^2cd(Ad+Bc)+b^3\left(-c^2\right)(3Ad+Bc)\right)}{b^4e(m+1)}+\frac{(ex)^{m+1}(Ab-aB)(bc-ad)^3\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ab^4e(m+1)}+\frac{d^2x^{2n+1}(ex)^m(-aBd+Abd+3bBc)}{b^2(m+2n+1)}+\frac{Bd^3x^{3n+1}(ex)^m}{b(m+3n+1)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n),x]
```

output
```
(d*(a^2*B*d^2 + 3*b^2*c*(B*c + A*d) - a*b*d*(3*B*c + A*d))*x^(1 + n)*(e*x)
^m)/(b^3*(1 + m + n)) + (d^2*(3*b*B*c + A*b*d - a*B*d)*x^(1 + 2*n)*(e*x)^m
)/(b^2*(1 + m + 2*n)) + (B*d^3*x^(1 + 3*n)*(e*x)^m)/(b*(1 + m + 3*n)) - ((
a^3*B*d^3 + 3*a*b^2*c*d*(B*c + A*d) - a^2*b*d^2*(3*B*c + A*d) - b^3*c^2*(B
*c + 3*A*d))*(e*x)^(1 + m))/(b^4*e*(1 + m)) + ((A*b - a*B)*(b*c - a*d)^3*(
e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])
/(a*b^4*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n) (c+d\,x^n)^3}{a+b\,x^n}dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{a+bx^n}\,dx = \int \frac{(Bx^n+A)(dx^n+c)^3(ex)^m}{bx^n+a}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

output
```
integral((B*d^3*x^(4*n) + A*c^3 + (3*B*c*d^2 + A*d^3)*x^(3*n) + 3*(B*c^2*d
 + A*c*d^2)*x^(2*n) + (B*c^3 + 3*A*c^2*d)*x^n)*(e*x)^m/(b*x^n + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.43 (sec) , antiderivative size = 1933, normalized size of antiderivative = 6.93

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{a+bx^n}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**3/(a+b*x**n),x)
```

output
```
A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**3*e**m*m*x**(m + 1)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**(m/n + 1/n)*a**(-m/n - 1 - 1/n)*c**3*e**m*x**(m + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n
+ 1 + 1/n)) + A*a**(-m/n - 4 - 1/n)*a**(m/n + 3 + 1/n)*d**3*e**m*m*x**(m +
 3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n +
 3 + 1/n)/(n**2*gamma(m/n + 4 + 1/n)) + 3*A*a**(-m/n - 4 - 1/n)*a**(m/n +
3 + 1/n)*d**3*e**m*x**(m + 3*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(n*gamma(m/n + 4 + 1/n)) + A*a**(-m/n
- 4 - 1/n)*a**(m/n + 3 + 1/n)*d**3*e**m*x**(m + 3*n + 1)*lerchphi(b*x**n*e
xp_polar(I*pi)/a, 1, m/n + 3 + 1/n)*gamma(m/n + 3 + 1/n)/(n**2*gamma(m/n +
 4 + 1/n)) + 3*A*a**(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c*d**2*e**m*m*x**(
m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/
n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 6*A*a**(-m/n - 3 - 1/n)*a**(m/n
 + 2 + 1/n)*c*d**2*e**m*x**(m + 2*n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a
, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n + 3 + 1/n)) + 3*A*a*
*(-m/n - 3 - 1/n)*a**(m/n + 2 + 1/n)*c*d**2*e**m*x**(m + 2*n + 1)*lerchphi
(b*x**n*exp_polar(I*pi)/a, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*ga
mma(m/n + 3 + 1/n)) + 3*A*a**(-m/n - 2 - 1/n)*a**(m/n + 1 + 1/n)*c**2*d*e*
*m*m*x**(m + n + 1)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, m/n + 1 + 1/n...
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{a + bx^n} \, dx = \int \frac{(Bx^n + A)(dx^n + c)^3 (ex)^m}{bx^n + a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

output
```
((b^4*c^3*e^m - 3*a*b^3*c^2*d*e^m + 3*a^2*b^2*c*d^2*e^m - a^3*b*d^3*e^m)*A
 - (a*b^3*c^3*e^m - 3*a^2*b^2*c^2*d*e^m + 3*a^3*b*c*d^2*e^m - a^4*d^3*e^m)
*B)*integrate(x^m/(b^5*x^n + a*b^4), x) + ((m^3 + 3*m^2*(n + 1) + (2*n^2 +
 6*n + 3)*m + 2*n^2 + 3*n + 1)*B*b^3*d^3*e^m*x*e^(m*log(x) + 3*n*log(x)) +
 ((3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n
 + 1)*b^3*c^2*d*e^m - 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n +
3)*m + 11*n^2 + 6*n + 1)*a*b^2*c*d^2*e^m + (m^3 + 3*m^2*(2*n + 1) + 6*n^3
+ (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*a^2*b*d^3*e^m)*A + ((m^3 + 3*m
^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*b^3*c^3*e
^m - 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6
*n + 1)*a*b^2*c^2*d*e^m + 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*
n + 3)*m + 11*n^2 + 6*n + 1)*a^2*b*c*d^2*e^m - (m^3 + 3*m^2*(2*n + 1) + 6*
n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*a^3*d^3*e^m)*B)*x*x^m + ((
m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*A*b^3*d^3*e^m
 + (3*(m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*b^3*c*
d^2*e^m - (m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*
b^2*d^3*e^m)*B)*x*e^(m*log(x) + 2*n*log(x)) + ((3*(m^3 + m^2*(5*n + 3) + (
6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*b^3*c*d^2*e^m - (m^3 + m^2*(5*n + 3
) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a*b^2*d^3*e^m)*A + (3*(m^3 + m
^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*b^3*c^2*d*e^m - ...
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{a+bx^n} \, dx = \int \frac{(Bx^n+A)(dx^n+c)^3(ex)^m}{bx^n+a} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)^3*(e*x)^m/(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{a + bx^n} \, dx = \int \frac{(e\,x)^m \left(A + B\,x^n\right) \left(c + d\,x^n\right)^3}{a + b\,x^n} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.72

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{a + bx^n} \, dx = \frac{x^m e^m x (3x^{3n} d^3 m^2 + 3x^{3n} d^3 m + 2x^{3n} d^3 n^2 + 3x^{3n} d^3 n + 3x^{2n} c\, d^2 + 3x^n c^2 d + 6c^3 m^2 n + 11c^3 m\, n^2 + 12c^3 \cdots}{\cdots}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n),x)
```

output
```
(x**m*e**m*x*(x**(3*n)*d**3*m**3 + 3*x**(3*n)*d**3*m**2*n + 3*x**(3*n)*d**
3*m**2 + 2*x**(3*n)*d**3*m*n**2 + 6*x**(3*n)*d**3*m*n + 3*x**(3*n)*d**3*m
+ 2*x**(3*n)*d**3*n**2 + 3*x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*
d**2*m**3 + 12*x**(2*n)*c*d**2*m**2*n + 9*x**(2*n)*c*d**2*m**2 + 9*x**(2*n
)*c*d**2*m*n**2 + 24*x**(2*n)*c*d**2*m*n + 9*x**(2*n)*c*d**2*m + 9*x**(2*n
)*c*d**2*n**2 + 12*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m
**3 + 15*x**n*c**2*d*m**2*n + 9*x**n*c**2*d*m**2 + 18*x**n*c**2*d*m*n**2 +
 30*x**n*c**2*d*m*n + 9*x**n*c**2*d*m + 18*x**n*c**2*d*n**2 + 15*x**n*c**2
*d*n + 3*x**n*c**2*d + c**3*m**3 + 6*c**3*m**2*n + 3*c**3*m**2 + 11*c**3*m
*n**2 + 12*c**3*m*n + 3*c**3*m + 6*c**3*n**3 + 11*c**3*n**2 + 6*c**3*n + c
**3))/(m**4 + 6*m**3*n + 4*m**3 + 11*m**2*n**2 + 18*m**2*n + 6*m**2 + 6*m*
n**3 + 22*m*n**2 + 18*m*n + 4*m + 6*n**3 + 11*n**2 + 6*n + 1)
```

3.38 $\int \frac{(ex)^m(A+Bx^n)(c+dx^n)^3}{(a+bx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 395

$$\int \frac{(ex)^m (A+Bx^n)(c+dx^n)^3}{(a+bx^n)^2}\,dx = -\frac{d(Ab(3b^2c^2(1+m)-3abcd(1+m+n)+a^2d^2(1+m+2n))-aB(3b^2c^2(1+m+n)-3abcd(1+m}{ab^4e(1+m)n}$$
$$-\frac{d^2(Ab(3bc(1+m+n)-ad(1+m+2n))-aB(3bc(1+m+2n)-ad(1+m+3n)))x^n(ex)^{1+m}}{ab^3en(1+m+n)}$$
$$-\frac{d^3\left(A-\frac{aB(1+m+3n)}{b(1+m+2n)}\right)x^{2n}(ex)^{1+m}}{aben}+\frac{(Ab-aB)(ex)^{1+m}(c+dx^n)^3}{aben(a+bx^n)}$$
$$-\frac{(bc-ad)^2(Ab(bc(1+m-n)-ad(1+m+2n))-aB(bc(1+m)-ad(1+m+3n)))(ex)^{1+m}\,\text{Hyper}}{a^2b^4e(1+m)n}$$

output
```
-d*(A*b*(3*b^2*c^2*(1+m)-3*a*b*c*d*(1+m+n)+a^2*d^2*(1+m+2*n))-a*B*(3*b^2*c
^2*(1+m+n)-3*a*b*c*d*(1+m+2*n)+a^2*d^2*(1+m+3*n)))*(e*x)^(1+m)/a/b^4/e/(1+
m)/n-d^2*(A*b*(3*b*c*(1+m+n)-a*d*(1+m+2*n))-a*B*(3*b*c*(1+m+2*n)-a*d*(1+m+
3*n)))*x^n*(e*x)^(1+m)/a/b^3/e/n/(1+m+n)-d^3*(A-a*B*(1+m+3*n)/b/(1+m+2*n))
*x^(2*n)*(e*x)^(1+m)/a/b/e/n+(A*b-B*a)*(e*x)^(1+m)*(c+d*x^n)^3/a/b/e/n/(a+
b*x^n)-(-a*d+b*c)^2*(A*b*(b*c*(1+m-n)-a*d*(1+m+2*n))-a*B*(b*c*(1+m)-a*d*(1
+m+3*n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/b^4
/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.55

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{(a+bx^n)^2} \, dx = \frac{x(ex)^m \left(\frac{d(3a^2Bd^2+3b^2c(Bc+Ad)-2abd(3Bc+Ad))}{1+m} + \frac{bd^2(3bBc+Abd-2aBd)x^n}{1+m+n} + \frac{b^2Bd^3x^{2n}}{1+m+2n} + \frac{(bc-ad)^2(bBc+3Abd-4aBd)\,\text{Hype}}{a\ldots}\right.}{b^4}$$

input
```
Integrate[((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n)^2,x]
```

output
```
(x*(e*x)^m*((d*(3*a^2*B*d^2 + 3*b^2*c*(B*c + A*d) - 2*a*b*d*(3*B*c + A*d))
)/(1 + m) + (b*d^2*(3*b*B*c + A*b*d - 2*a*B*d)*x^n)/(1 + m + n) + (b^2*B*d
^3*x^(2*n))/(1 + m + 2*n) + ((b*c - a*d)^2*(b*B*c + 3*A*b*d - 4*a*B*d)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(1 + m)) + (
(-(A*b) + a*B)*(-(b*c) + a*d)^3*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/(a^2*(1 + m))))/b^4
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1064, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{(a+bx^n)^2} \, dx$$

$$\Big\downarrow \text{1064}$$

$$\frac{(ex)^{m+1}(Ab-aB)(c+dx^n)^3}{aben(a+bx^n)} - \frac{\int -\frac{(ex)^m(dx^n+c)^2(c(aB(m+1)-Ab(m-n+1))-d(Ab(m+2n+1)-aB(m+3n+1))x^n)}{bx^n+a}dx}{abn}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(dx^n+c)^2(c(aB(m+1)-Ab(m-n+1))-d(Ab(m+2n+1)-aB(m+3n+1))x^n)}{bx^n+a}dx}{abn}+$$
$$\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^3}{aben\left(a+bx^n\right)}$$

$\downarrow$ 1040

$$\frac{\int \left(\frac{d^2(aB(3bc(m+2n+1)-ad(m+3n+1))-Ab(3bc(m+n+1)-ad(m+2n+1)))x^n(ex)^m}{b^2}+\frac{d^3(aB(m+3n+1)-Ab(m+2n+1))x^{2n}(ex)^m}{b}+\frac{d(}{}\right.}{}$$
$$\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^3}{aben\left(a+bx^n\right)}$$

$\downarrow$ 2009

$$\frac{-\frac{d(ex)^{m+1}\left(Ab\left(a^2d^2(m+2n+1)-3abcd(m+n+1)+3b^2c^2(m+1)\right)-aB\left(a^2d^2(m+3n+1)-3abcd(m+2n+1)+3b^2c^2(m+n+1)\right)\right)}{b^3e(m+1)}-\frac{(ex)^{m+1}(bc-}{}}{}$$
$$\frac{(ex)^{m+1}(Ab-aB)\left(c+dx^n\right)^3}{aben\left(a+bx^n\right)}$$

input
```
Int[((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n)^2,x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*(c + d*x^n)^3)/(a*b*e*n*(a + b*x^n)) + (-((d^2*
(A*b*(3*b*c*(1 + m + n) - a*d*(1 + m + 2*n)) - a*B*(3*b*c*(1 + m + 2*n) -
a*d*(1 + m + 3*n)))*x^(1 + n)*(e*x)^m)/(b^2*(1 + m + n))) - d^3*(A - (a*B*
(1 + m + 3*n))/(b*(1 + m + 2*n)))*x^(1 + 2*n)*(e*x)^m - (d*(A*b*(3*b^2*c^2
*(1 + m) - 3*a*b*c*d*(1 + m + n) + a^2*d^2*(1 + m + 2*n)) - a*B*(3*b^2*c^2
*(1 + m + n) - 3*a*b*c*d*(1 + m + 2*n) + a^2*d^2*(1 + m + 3*n)))*(e*x)^(1
+ m))/(b^3*e*(1 + m)) - ((b*c - a*d)^2*(A*b*(b*c*(1 + m - n) - a*d*(1 + m
+ 2*n)) - a*B*(b*c*(1 + m) - a*d*(1 + m + 3*n)))*(e*x)^(1 + m)*Hypergeomet
ric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*b^3*e*(1 + m)))/(a*b
*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n) (c + d x^n)^3}{(a + b x^n)^2} dx$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{(a + bx^n)^2} \, dx = \int \frac{(Bx^n + A)(dx^n + c)^3 (ex)^m}{(bx^n + a)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*d^3*x^(4*n) + A*c^3 + (3*B*c*d^2 + A*d^3)*x^(3*n) + 3*(B*c^2*d
 + A*c*d^2)*x^(2*n) + (B*c^3 + 3*A*c^2*d)*x^n)*(e*x)^m/(b^2*x^(2*n) + 2*a*
b*x^n + a^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{(a + bx^n)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)*(c+d*x**n)**3/(a+b*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n) (c + dx^n)^3}{(a + bx^n)^2} \, dx = \int \frac{(Bx^n + A)(dx^n + c)^3 (ex)^m}{(bx^n + a)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

output
```
((a^3*b*d^3*e^m*(m + 2*n + 1) - 3*a^2*b^2*c*d^2*e^m*(m + n + 1) - b^4*c^3*
e^m*(m - n + 1) + 3*a*b^3*c^2*d*e^m*(m + 1))*A - (a^4*d^3*e^m*(m + 3*n + 1
) - 3*a^3*b*c*d^2*e^m*(m + 2*n + 1) + 3*a^2*b^2*c^2*d*e^m*(m + n + 1) - a*
b^3*c^3*e^m*(m + 1))*B)*integrate(x^m/(a*b^5*n*x^n + a^2*b^4*n), x) + ((m^
2*n + (n^2 + 2*n)*m + n^2 + n)*B*a*b^3*d^3*e^m*x*e^(m*log(x) + 3*n*log(x))
 + (((m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*b^4*c^3
*e^m - 3*(m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*a*b
^3*c^2*d*e^m + 3*(m^3 + m^2*(4*n + 3) + 2*n^3 + (5*n^2 + 8*n + 3)*m + 5*n^
2 + 4*n + 1)*a^2*b^2*c*d^2*e^m - (m^3 + m^2*(5*n + 3) + 4*n^3 + (8*n^2 + 1
0*n + 3)*m + 8*n^2 + 5*n + 1)*a^3*b*d^3*e^m)*A - ((m^3 + 3*m^2*(n + 1) + (
2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*a*b^3*c^3*e^m - 3*(m^3 + m^2*(4*n +
3) + 2*n^3 + (5*n^2 + 8*n + 3)*m + 5*n^2 + 4*n + 1)*a^2*b^2*c^2*d*e^m + 3*
(m^3 + m^2*(5*n + 3) + 4*n^3 + (8*n^2 + 10*n + 3)*m + 8*n^2 + 5*n + 1)*a^3
*b*c*d^2*e^m - (m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11
*n^2 + 6*n + 1)*a^4*d^3*e^m)*B)*x*x^m + ((m^2*n + 2*(n^2 + n)*m + 2*n^2 +
n)*A*a*b^3*d^3*e^m + (3*(m^2*n + 2*(n^2 + n)*m + 2*n^2 + n)*a*b^3*c*d^2*e^
m - (m^2*n + (3*n^2 + 2*n)*m + 3*n^2 + n)*a^2*b^2*d^3*e^m)*B)*x*e^(m*log(x
) + 2*n*log(x)) + ((3*(m^2*n + 2*n^3 + (3*n^2 + 2*n)*m + 3*n^2 + n)*a*b^3*
c*d^2*e^m - (m^2*n + 4*n^3 + 2*(2*n^2 + n)*m + 4*n^2 + n)*a^2*b^2*d^3*e^m)
*A + (3*(m^2*n + 2*n^3 + (3*n^2 + 2*n)*m + 3*n^2 + n)*a*b^3*c^2*d*e^m -...
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{(Bx^n+A)(dx^n+c)^3(ex)^m}{(bx^n+a)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)^3*(e*x)^m/(b*x^n + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{(a+bx^n)^2} \, dx = \int \frac{{\left (e\,x\right)}^m\,\left (A+B\,x^n\right)\,{\left (c+d\,x^n\right)}^3}{{\left (a+b\,x^n\right)}^2} \,d x$$

input
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(c + d*x^n)^3)/(a + b*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n) (c+dx^n)^3}{(a+bx^n)^2} \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)*(c+d*x^n)^3/(a+b*x^n)^2,x)
```

output
```
(e**m*(x**(m + 2*n)*b**2*d**3*m**2*x + x**(m + 2*n)*b**2*d**3*m*n*x + 2*x*
*(m + 2*n)*b**2*d**3*m*x + x**(m + 2*n)*b**2*d**3*n*x + x**(m + 2*n)*b**2*
d**3*x - x**(m + n)*a*b*d**3*m**2*x - 2*x**(m + n)*a*b*d**3*m*n*x - 2*x**(
m + n)*a*b*d**3*m*x - 2*x**(m + n)*a*b*d**3*n*x - x**(m + n)*a*b*d**3*x +
3*x**(m + n)*b**2*c*d**2*m**2*x + 6*x**(m + n)*b**2*c*d**2*m*n*x + 6*x**(m
 + n)*b**2*c*d**2*m*x + 6*x**(m + n)*b**2*c*d**2*n*x + 3*x**(m + n)*b**2*c
*d**2*x + x**m*a**2*d**3*m**2*x + 3*x**m*a**2*d**3*m*n*x + 2*x**m*a**2*d**
3*m*x + 2*x**m*a**2*d**3*n**2*x + 3*x**m*a**2*d**3*n*x + x**m*a**2*d**3*x
- 3*x**m*a*b*c*d**2*m**2*x - 9*x**m*a*b*c*d**2*m*n*x - 6*x**m*a*b*c*d**2*m
*x - 6*x**m*a*b*c*d**2*n**2*x - 9*x**m*a*b*c*d**2*n*x - 3*x**m*a*b*c*d**2*
x + 3*x**m*b**2*c**2*d*m**2*x + 9*x**m*b**2*c**2*d*m*n*x + 6*x**m*b**2*c**
2*d*m*x + 6*x**m*b**2*c**2*d*n**2*x + 9*x**m*b**2*c**2*d*n*x + 3*x**m*b**2
*c**2*d*x - int(x**m/(x**n*b + a),x)*a**3*d**3*m**3 - 3*int(x**m/(x**n*b +
 a),x)*a**3*d**3*m**2*n - 3*int(x**m/(x**n*b + a),x)*a**3*d**3*m**2 - 2*in
t(x**m/(x**n*b + a),x)*a**3*d**3*m*n**2 - 6*int(x**m/(x**n*b + a),x)*a**3*
d**3*m*n - 3*int(x**m/(x**n*b + a),x)*a**3*d**3*m - 2*int(x**m/(x**n*b + a
),x)*a**3*d**3*n**2 - 3*int(x**m/(x**n*b + a),x)*a**3*d**3*n - int(x**m/(x
**n*b + a),x)*a**3*d**3 + 3*int(x**m/(x**n*b + a),x)*a**2*b*c*d**2*m**3 +
9*int(x**m/(x**n*b + a),x)*a**2*b*c*d**2*m**2*n + 9*int(x**m/(x**n*b + a),
x)*a**2*b*c*d**2*m**2 + 6*int(x**m/(x**n*b + a),x)*a**2*b*c*d**2*m*n**2...
```

3.39 $\int \frac{(ex)^m(a+bx^n)^4(A+Bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 392

$$
\begin{aligned}
&\int \frac{(ex)^m\,(a+bx^n)^4\,(A+Bx^n)}{c+dx^n}\,dx \\
&= \frac{\left(a^4Bd^4+b^4c^3(Bc-Ad)-4ab^3c^2d(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)-4a^3bd^3(Bc-Ad)\right)(ex)^{1+m}}{d^5e(1+m)} \\
&\quad+\frac{b\left(4a^3Bd^3-b^3c^2(Bc-Ad)+4ab^2cd(Bc-Ad)-6a^2bd^2(Bc-Ad)\right)x^n(ex)^{1+m}}{d^4e(1+m+n)} \\
&\quad+\frac{b^2\left(6a^2Bd^2+b^2c(Bc-Ad)-4abd(Bc-Ad)\right)x^{2n}(ex)^{1+m}}{d^3e(1+m+2n)} \\
&\quad-\frac{b^3(bBc-Abd-4aBd)x^{3n}(ex)^{1+m}}{d^2e(1+m+3n)}+\frac{b^4Bx^{4n}(ex)^{1+m}}{de(1+m+4n)} \\
&\quad-\frac{(bc-ad)^4(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{cd^5e(1+m)}
\end{aligned}
$$

output
```
(a^4*B*d^4+b^4*c^3*(-A*d+B*c)-4*a*b^3*c^2*d*(-A*d+B*c)+6*a^2*b^2*c*d^2*(-A
*d+B*c)-4*a^3*b*d^3*(-A*d+B*c))*(e*x)^(1+m)/d^5/e/(1+m)+b*(4*a^3*B*d^3-b^3
*c^2*(-A*d+B*c)+4*a*b^2*c*d*(-A*d+B*c)-6*a^2*b*d^2*(-A*d+B*c))*x^n*(e*x)^(
1+m)/d^4/e/(1+m+n)+b^2*(6*a^2*B*d^2+b^2*c*(-A*d+B*c)-4*a*b*d*(-A*d+B*c))*x
^(2*n)*(e*x)^(1+m)/d^3/e/(1+m+2*n)-b^3*(-A*b*d-4*B*a*d+B*b*c)*x^(3*n)*(e*x
)^(1+m)/d^2/e/(1+m+3*n)+b^4*B*x^(4*n)*(e*x)^(1+m)/d/e/(1+m+4*n)-(-a*d+b*c)
^4*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/d
^5/e/(1+m)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n}\,dx = \frac{x(ex)^m\left(\frac{a^4Bd^4+b^4c^3(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)+4ab^3c^2d(-Bc+Ad)+4a^3bd^3(-Bc+Ad)}{1+m}+\frac{bd(4a^3Bd^3+4ab^2cd(Bc-Ad)+b^3c^2(-Bc}{1+m+n}\right.}{}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^4*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*((a^4*B*d^4 + b^4*c^3*(B*c - A*d) + 6*a^2*b^2*c*d^2*(B*c - A*d)
 + 4*a*b^3*c^2*d*(-(B*c) + A*d) + 4*a^3*b*d^3*(-(B*c) + A*d))/(1 + m) + (b
*d*(4*a^3*B*d^3 + 4*a*b^2*c*d*(B*c - A*d) + b^3*c^2*(-(B*c) + A*d) + 6*a^2
*b*d^2*(-(B*c) + A*d))*x^n)/(1 + m + n) + (b^2*d^2*(6*a^2*B*d^2 + b^2*c*(B
*c - A*d) + 4*a*b*d*(-(B*c) + A*d))*x^(2*n))/(1 + m + 2*n) + (b^3*d^3*(-(b
*B*c) + A*b*d + 4*a*B*d)*x^(3*n))/(1 + m + 3*n) + (b^4*B*d^4*x^(4*n))/(1 +
 m + 4*n) - ((b*c - a*d)^4*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/n, (1
+ m + n)/n, -((d*x^n)/c)])/(c*(1 + m))))/d^5
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n} \, dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(\frac{b^2x^{2n}(ex)^m \left(6a^2Bd^2-4abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3}+\frac{bx^n(ex)^m \left(4a^3Bd^3-6a^2bd^2(Bc-Ad)+4ab^2c\right.}{d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2x^{2n+1}(ex)^m \left(6a^2Bd^2-4abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3(m+2n+1)}+\frac{bx^{n+1}(ex)^m \left(4a^3Bd^3-6a^2bd^2(Bc-Ad)+4ab^2cd(Bc-Ad)+b^3\left(-c^2\right)(Bc-Ad)\right)}{d^4(m+n+1)}+\frac{(ex)^{m+1}\left(a^4Bd^4-4a^3bd^3(Bc-Ad)+6a^2b^2cd^2(Bc-Ad)-4ab^3c^2d(Bc-Ad)+b^4c^3(Bc-Ad)\right)}{d^5e(m+1)}-\frac{b^3x^{3n+1}(ex)^m(-4aBd-Abd+bBc)}{d^2(m+3n+1)}-\frac{(ex)^{m+1}(bc-ad)^4(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{cd^5e(m+1)}+\frac{b^4Bx^{4n+1}(ex)^m}{d(m+4n+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^4*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(b*(4*a^3*B*d^3 - b^3*c^2*(B*c - A*d) + 4*a*b^2*c*d*(B*c - A*d) - 6*a^2*b*
d^2*(B*c - A*d))*x^(1 + n)*(e*x)^m)/(d^4*(1 + m + n)) + (b^2*(6*a^2*B*d^2
+ b^2*c*(B*c - A*d) - 4*a*b*d*(B*c - A*d))*x^(1 + 2*n)*(e*x)^m)/(d^3*(1 +
m + 2*n)) - (b^3*(b*B*c - A*b*d - 4*a*B*d)*x^(1 + 3*n)*(e*x)^m)/(d^2*(1 +
m + 3*n)) + (b^4*B*x^(1 + 4*n)*(e*x)^m)/(d*(1 + m + 4*n)) + ((a^4*B*d^4 +
b^4*c^3*(B*c - A*d) - 4*a*b^3*c^2*d*(B*c - A*d) + 6*a^2*b^2*c*d^2*(B*c - A
*d) - 4*a^3*b*d^3*(B*c - A*d))*(e*x)^(1 + m))/(d^5*e*(1 + m)) - ((b*c - a*
d)^4*(B*c - A*d)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((d*x^n)/c)])/(c*d^5*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^4 (A+B\,x^n)}{c+d\,x^n}dx$$

input
```
int((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^4(ex)^m}{dx^n+c} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*b^4*x^(5*n) + A*a^4 + (4*B*a*b^3 + A*b^4)*x^(4*n) + 2*(3*B*a^2
*b^2 + 2*A*a*b^3)*x^(3*n) + 2*(2*B*a^3*b + 3*A*a^2*b^2)*x^(2*n) + (B*a^4 +
 4*A*a^3*b)*x^n)*(e*x)^m/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.03 (sec) , antiderivative size = 2463, normalized size of antiderivative = 6.28

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**4*(A+B*x**n)/(c+d*x**n),x)
```

output
```
A*a**4*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*m*x**(m + 1)*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**4*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*x**(m + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n
+ 1 + 1/n)) + 4*A*a**3*b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**
(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n
 + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + 4*A*a**3*b*c**(-m/n - 2 - 1/n)*c
**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1
, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + 4*A*a**3*
b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x*
*n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m
/n + 2 + 1/n)) + 6*A*a**2*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m
*m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 2 + 1/n)*g
amma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 12*A*a**2*b**2*c**(-m/n
- 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_po
lar(I*pi)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n + 3 + 1/n
)) + 6*A*a**2*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*x**(m + 2*n
 + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 +
 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 4*A*a*b**3*c**(-m/n - 4 - 1/n)*c**(m/n
 + 3 + 1/n)*e**m*m*x**(m + 3*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, ...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^4(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
((b^4*c^4*d*e^m - 4*a*b^3*c^3*d^2*e^m + 6*a^2*b^2*c^2*d^3*e^m - 4*a^3*b*c*
d^4*e^m + a^4*d^5*e^m)*A - (b^4*c^5*e^m - 4*a*b^3*c^4*d*e^m + 6*a^2*b^2*c^
3*d^2*e^m - 4*a^3*b*c^2*d^3*e^m + a^4*c*d^4*e^m)*B)*integrate(x^m/(d^6*x^n
 + c*d^5), x) + ((m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6*n^3
+ 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)*B*b^4*d^4*e^m*x*e^(m*
log(x) + 4*n*log(x)) - (((m^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n
+ 6)*m^2 + 50*n^3 + 2*(25*n^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*
b^4*c^3*d*e^m - 4*(m^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^
2 + 50*n^3 + 2*(25*n^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*a*b^3*c
^2*d^2*e^m + 6*(m^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 +
 50*n^3 + 2*(25*n^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*a^2*b^2*c*
d^3*e^m - 4*(m^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 + 50
*n^3 + 2*(25*n^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*a^3*b*d^4*e^m
)*A - ((m^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 + 50*n^3
+ 2*(25*n^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*b^4*c^4*e^m - 4*(m
^4 + 2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 + 50*n^3 + 2*(25*n
^3 + 35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*a*b^3*c^3*d*e^m + 6*(m^4 +
2*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 + 50*n^3 + 2*(25*n^3 +
35*n^2 + 15*n + 2)*m + 35*n^2 + 10*n + 1)*a^2*b^2*c^2*d^2*e^m - 4*(m^4 + 2
*m^3*(5*n + 2) + 24*n^4 + (35*n^2 + 30*n + 6)*m^2 + 50*n^3 + 2*(25*n^3 ...
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^4(ex)^m}{dx^n+c} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^4*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(e\,x)^m \,(A+B\,x^n)\,\left(a+b\,x^n\right)^4}{c+d\,x^n}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^4)/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^4)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^4 (A+Bx^n)}{c+dx^n}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^4*(A+B*x^n)/(c+d*x^n),x)
```

output
```
(e**m*(x**(m + 4*n)*b**5*d**4*m**4*x + 6*x**(m + 4*n)*b**5*d**4*m**3*n*x +
 4*x**(m + 4*n)*b**5*d**4*m**3*x + 11*x**(m + 4*n)*b**5*d**4*m**2*n**2*x +
 18*x**(m + 4*n)*b**5*d**4*m**2*n*x + 6*x**(m + 4*n)*b**5*d**4*m**2*x + 6*
x**(m + 4*n)*b**5*d**4*m*n**3*x + 22*x**(m + 4*n)*b**5*d**4*m*n**2*x + 18*
x**(m + 4*n)*b**5*d**4*m*n*x + 4*x**(m + 4*n)*b**5*d**4*m*x + 6*x**(m + 4*
n)*b**5*d**4*n**3*x + 11*x**(m + 4*n)*b**5*d**4*n**2*x + 6*x**(m + 4*n)*b*
*5*d**4*n*x + x**(m + 4*n)*b**5*d**4*x + 5*x**(m + 3*n)*a*b**4*d**4*m**4*x
 + 35*x**(m + 3*n)*a*b**4*d**4*m**3*n*x + 20*x**(m + 3*n)*a*b**4*d**4*m**3
*x + 70*x**(m + 3*n)*a*b**4*d**4*m**2*n**2*x + 105*x**(m + 3*n)*a*b**4*d**
4*m**2*n*x + 30*x**(m + 3*n)*a*b**4*d**4*m**2*x + 40*x**(m + 3*n)*a*b**4*d
**4*m*n**3*x + 140*x**(m + 3*n)*a*b**4*d**4*m*n**2*x + 105*x**(m + 3*n)*a*
b**4*d**4*m*n*x + 20*x**(m + 3*n)*a*b**4*d**4*m*x + 40*x**(m + 3*n)*a*b**4
*d**4*n**3*x + 70*x**(m + 3*n)*a*b**4*d**4*n**2*x + 35*x**(m + 3*n)*a*b**4
*d**4*n*x + 5*x**(m + 3*n)*a*b**4*d**4*x - x**(m + 3*n)*b**5*c*d**3*m**4*x
 - 7*x**(m + 3*n)*b**5*c*d**3*m**3*n*x - 4*x**(m + 3*n)*b**5*c*d**3*m**3*x
 - 14*x**(m + 3*n)*b**5*c*d**3*m**2*n**2*x - 21*x**(m + 3*n)*b**5*c*d**3*m
**2*n*x - 6*x**(m + 3*n)*b**5*c*d**3*m**2*x - 8*x**(m + 3*n)*b**5*c*d**3*m
*n**3*x - 28*x**(m + 3*n)*b**5*c*d**3*m*n**2*x - 21*x**(m + 3*n)*b**5*c*d*
*3*m*n*x - 4*x**(m + 3*n)*b**5*c*d**3*m*x - 8*x**(m + 3*n)*b**5*c*d**3*n**
3*x - 14*x**(m + 3*n)*b**5*c*d**3*n**2*x - 7*x**(m + 3*n)*b**5*c*d**3*n...
```

3.40 $\int \frac{(ex)^m(a+bx^n)^3(A+Bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 281

$$\begin{aligned}\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n}\,dx &= \frac{(a^3Bd^3-b^3c^2(Bc-Ad)+3ab^2cd(Bc-Ad)-3a^2bd^2(Bc-Ad))\,(ex)^{1+m}}{d^4e(1+m)} \\ &+ \frac{b(3a^2Bd^2+b^2c(Bc-Ad)-3abd(Bc-Ad))\,x^n(ex)^{1+m}}{d^3e(1+m+n)} \\ &- \frac{b^2(bBc-Abd-3aBd)x^{2n}(ex)^{1+m}}{d^2e(1+m+2n)} + \frac{b^3Bx^{3n}(ex)^{1+m}}{de(1+m+3n)} \\ &+ \frac{(bc-ad)^3(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{cd^4e(1+m)}\end{aligned}$$

output

```
(a^3*B*d^3-b^3*c^2*(-A*d+B*c)+3*a*b^2*c*d*(-A*d+B*c)-3*a^2*b*d^2*(-A*d+B*c
))*(e*x)^(1+m)/d^4/e/(1+m)+b*(3*a^2*B*d^2+b^2*c*(-A*d+B*c)-3*a*b*d*(-A*d+B
*c))*x^n*(e*x)^(1+m)/d^3/e/(1+m+n)-b^2*(-A*b*d-3*B*a*d+B*b*c)*x^(2*n)*(e*x
)^(1+m)/d^2/e/(1+m+2*n)+b^3*B*x^(3*n)*(e*x)^(1+m)/d/e/(1+m+3*n)+(-a*d+b*c)
^3*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/d
^4/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n} \, dx$$
$$= \frac{x(ex)^m \left(\frac{a^3Bd^3+3ab^2cd(Bc-Ad)+b^3c^2(-Bc+Ad)+3a^2bd^2(-Bc+Ad)}{1+m} + \frac{bd(3a^2Bd^2+b^2c(Bc-Ad)+3abd(-Bc+Ad))x^n}{1+m+n} + \frac{b^2d^2(-bB}{}\right.}{d^4}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^3*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*((a^3*B*d^3 + 3*a*b^2*c*d*(B*c - A*d) + b^3*c^2*(-(B*c) + A*d)
+ 3*a^2*b*d^2*(-(B*c) + A*d))/(1 + m) + (b*d*(3*a^2*B*d^2 + b^2*c*(B*c - A
*d) + 3*a*b*d*(-(B*c) + A*d))*x^n)/(1 + m + n) + (b^2*d^2*(-(b*B*c) + A*b*
d + 3*a*B*d)*x^(2*n))/(1 + m + 2*n) + (b^3*B*d^3*x^(3*n))/(1 + m + 3*n) +
((b*c - a*d)^3*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
-((d*x^n)/c)])/(c*(1 + m))))/d^4
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n} \, dx$$
$$\Big\downarrow \text{1040}$$
$$\int \left(\frac{bx^n(ex)^m (3a^2Bd^2-3abd(Bc-Ad)+b^2c(Bc-Ad))}{d^3} + \frac{(ex)^m (a^3Bd^3-3a^2bd^2(Bc-Ad)+3ab^2cd(Bc-}{d^4}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{bx^{n+1}(ex)^m\left(3a^2Bd^2-3abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3(m+n+1)}+\frac{(ex)^{m+1}\left(a^3Bd^3-3a^2bd^2(Bc-Ad)+3ab^2cd(Bc-Ad)+b^3\left(-c^2\right)(Bc-Ad)\right)}{d^4e(m+1)}-\frac{b^2x^{2n+1}(ex)^m(-3aBd-Abd+bBc)}{d^2(m+2n+1)}+\frac{(ex)^{m+1}(bc-ad)^3(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{cd^4e(m+1)}+\frac{b^3Bx^{3n+1}(ex)^m}{d(m+3n+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^3*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(b*(3*a^2*B*d^2 + b^2*c*(B*c - A*d) - 3*a*b*d*(B*c - A*d))*x^(1 + n)*(e*x)
^m)/(d^3*(1 + m + n)) - (b^2*(b*B*c - A*b*d - 3*a*B*d)*x^(1 + 2*n)*(e*x)^m
)/(d^2*(1 + m + 2*n)) + (b^3*B*x^(1 + 3*n)*(e*x)^m)/(d*(1 + m + 3*n)) + ((
a^3*B*d^3 - b^3*c^2*(B*c - A*d) + 3*a*b^2*c*d*(B*c - A*d) - 3*a^2*b*d^2*(B
*c - A*d))*(e*x)^(1 + m))/(d^4*e*(1 + m)) + ((b*c - a*d)^3*(B*c - A*d)*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(
c*d^4*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a + b\,x^n)^3 (A + B\,x^n)}{c + d\,x^n} dx$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a + bx^n)^3 (A + Bx^n)}{c + dx^n} dx = \int \frac{(Bx^n + A)(bx^n + a)^3 (ex)^m}{dx^n + c} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*b^3*x^(4*n) + A*a^3 + (3*B*a*b^2 + A*b^3)*x^(3*n) + 3*(B*a^2*b
 + A*a*b^2)*x^(2*n) + (B*a^3 + 3*A*a^2*b)*x^n)*(e*x)^m/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.96 (sec) , antiderivative size = 1933, normalized size of antiderivative = 6.88

$$\int \frac{(ex)^m (a + bx^n)^3 (A + Bx^n)}{c + dx^n} dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**3*(A+B*x**n)/(c+d*x**n),x)
```

output
```
A*a**3*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*m*x**(m + 1)*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**3*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*x**(m + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n
+ 1 + 1/n)) + 3*A*a**2*b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**
(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n
 + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + 3*A*a**2*b*c**(-m/n - 2 - 1/n)*c
**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1
, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + 3*A*a**2*
b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x*
*n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m
/n + 2 + 1/n)) + 3*A*a*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*m*
x**(m + 2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 2 + 1/n)*gamm
a(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 6*A*a*b**2*c**(-m/n - 3 - 1
/n)*c**(m/n + 2 + 1/n)*e**m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n + 3 + 1/n)) + 3*
A*a*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*x**(m + 2*n + 1)*lerc
hphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**
2*gamma(m/n + 3 + 1/n)) + A*b**3*c**(-m/n - 4 - 1/n)*c**(m/n + 3 + 1/n)*e*
*m*m*x**(m + 3*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 3 + 1...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^3(ex)^m}{dx^n+c} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-((b^3*c^3*d*e^m - 3*a*b^2*c^2*d^2*e^m + 3*a^2*b*c*d^3*e^m - a^3*d^4*e^m)*
A - (b^3*c^4*e^m - 3*a*b^2*c^3*d*e^m + 3*a^2*b*c^2*d^2*e^m - a^3*c*d^3*e^m
)*B)*integrate(x^m/(d^5*x^n + c*d^4), x) + ((m^3 + 3*m^2*(n + 1) + (2*n^2
+ 6*n + 3)*m + 2*n^2 + 3*n + 1)*B*b^3*d^3*e^m*x*e^(m*log(x) + 3*n*log(x))
+ (((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n
+ 1)*b^3*c^2*d*e^m - 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3
)*m + 11*n^2 + 6*n + 1)*a*b^2*c*d^2*e^m + 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3
 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*a^2*b*d^3*e^m)*A - ((m^3 + 3*
m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*b^3*c^3*
e^m - 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 +
6*n + 1)*a*b^2*c^2*d*e^m + 3*(m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12
*n + 3)*m + 11*n^2 + 6*n + 1)*a^2*b*c*d^2*e^m - (m^3 + 3*m^2*(2*n + 1) + 6
*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*a^3*d^3*e^m)*B)*x*x^m + (
(m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*A*b^3*d^3*e^
m - ((m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*b^3*c*d
^2*e^m - 3*(m^3 + m^2*(4*n + 3) + (3*n^2 + 8*n + 3)*m + 3*n^2 + 4*n + 1)*a
*b^2*d^3*e^m)*B)*x*e^(m*log(x) + 2*n*log(x)) - (((m^3 + m^2*(5*n + 3) + (6
*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*b^3*c*d^2*e^m - 3*(m^3 + m^2*(5*n +
3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a*b^2*d^3*e^m)*A - ((m^3 + m^
2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*b^3*c^2*d*e^m - 3...
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^3(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^3*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(e\,x)^m \, (A+B\,x^n) \, (a+b\,x^n)^3}{c+d\,x^n} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^3)/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^3)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{c+dx^n} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n),x)
```

output

```
(e**m*(x**(m + 3*n)*b**4*d**3*m**3*x + 3*x**(m + 3*n)*b**4*d**3*m**2*n*x +
 3*x**(m + 3*n)*b**4*d**3*m**2*x + 2*x**(m + 3*n)*b**4*d**3*m*n**2*x + 6*x
**(m + 3*n)*b**4*d**3*m*n*x + 3*x**(m + 3*n)*b**4*d**3*m*x + 2*x**(m + 3*n
)*b**4*d**3*n**2*x + 3*x**(m + 3*n)*b**4*d**3*n*x + x**(m + 3*n)*b**4*d**3
*x + 4*x**(m + 2*n)*a*b**3*d**3*m**3*x + 16*x**(m + 2*n)*a*b**3*d**3*m**2*
n*x + 12*x**(m + 2*n)*a*b**3*d**3*m**2*x + 12*x**(m + 2*n)*a*b**3*d**3*m*n
**2*x + 32*x**(m + 2*n)*a*b**3*d**3*m*n*x + 12*x**(m + 2*n)*a*b**3*d**3*m*
x + 12*x**(m + 2*n)*a*b**3*d**3*n**2*x + 16*x**(m + 2*n)*a*b**3*d**3*n*x +
 4*x**(m + 2*n)*a*b**3*d**3*x - x**(m + 2*n)*b**4*c*d**2*m**3*x - 4*x**(m
+ 2*n)*b**4*c*d**2*m**2*n*x - 3*x**(m + 2*n)*b**4*c*d**2*m**2*x - 3*x**(m
+ 2*n)*b**4*c*d**2*m*n**2*x - 8*x**(m + 2*n)*b**4*c*d**2*m*n*x - 3*x**(m +
 2*n)*b**4*c*d**2*m*x - 3*x**(m + 2*n)*b**4*c*d**2*n**2*x - 4*x**(m + 2*n)
*b**4*c*d**2*n*x - x**(m + 2*n)*b**4*c*d**2*x + 6*x**(m + n)*a**2*b**2*d**
3*m**3*x + 30*x**(m + n)*a**2*b**2*d**3*m**2*n*x + 18*x**(m + n)*a**2*b**2
*d**3*m**2*x + 36*x**(m + n)*a**2*b**2*d**3*m*n**2*x + 60*x**(m + n)*a**2*
b**2*d**3*m*n*x + 18*x**(m + n)*a**2*b**2*d**3*m*x + 36*x**(m + n)*a**2*b*
*2*d**3*n**2*x + 30*x**(m + n)*a**2*b**2*d**3*n*x + 6*x**(m + n)*a**2*b**2
*d**3*x - 4*x**(m + n)*a*b**3*c*d**2*m**3*x - 20*x**(m + n)*a*b**3*c*d**2*
m**2*n*x - 12*x**(m + n)*a*b**3*c*d**2*m**2*x - 24*x**(m + n)*a*b**3*c*d**
2*m*n**2*x - 40*x**(m + n)*a*b**3*c*d**2*m*n*x - 12*x**(m + n)*a*b**3*c...
```

3.41 $\int \frac{(ex)^m(a+bx^n)^2(A+Bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 193

$$
\begin{aligned}
\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx
&= \frac{(a^2Bd^2+b^2c(Bc-Ad)-2abd(Bc-Ad))\,(ex)^{1+m}}{d^3e(1+m)} \\
&\quad - \frac{b(bBc-Abd-2aBd)x^n(ex)^{1+m}}{d^2e(1+m+n)} + \frac{b^2Bx^{2n}(ex)^{1+m}}{de(1+m+2n)} \\
&\quad - \frac{(bc-ad)^2(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{cd^3e(1+m)}
\end{aligned}
$$

output

```
(a^2*B*d^2+b^2*c*(-A*d+B*c)-2*a*b*d*(-A*d+B*c))*(e*x)^(1+m)/d^3/e/(1+m)-b*
(-A*b*d-2*B*a*d+B*b*c)*x^n*(e*x)^(1+m)/d^2/e/(1+m+n)+b^2*B*x^(2*n)*(e*x)^(
1+m)/d/e/(1+m+2*n)-(-a*d+b*c)^2*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)
/n],[(1+m+n)/n],-d*x^n/c)/c/d^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n} \, dx = \frac{x(ex)^m \left(\frac{a^2Bd^2+b^2c(Bc-Ad)+2abd(-Bc+Ad)}{1+m} + \frac{bd(-bBc+Abd+2aBd)x^n}{1+m+n} + \frac{b^2Bd^2x^{2n}}{1+m+2n} - \frac{(bc-ad)^2(Bc-Ad)\,\text{Hypergeometric2F1}\left(\right.}{c(1+m)}\right.}{d^3}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*((a^2*B*d^2 + b^2*c*(B*c - A*d) + 2*a*b*d*(-(B*c) + A*d))/(1 +
m) + (b*d*(-(b*B*c) + A*b*d + 2*a*B*d)*x^n)/(1 + m + n) + (b^2*B*d^2*x^(2*
n))/(1 + m + 2*n) - ((b*c - a*d)^2*(B*c - A*d)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(1 + m))))/d^3
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n} \, dx$$

$$\Big\downarrow \; 1040$$

$$\int \left(\frac{(ex)^m \left(a^2Bd^2-2abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3} + \frac{(ex)^m(ad-bc)^2(Ad-Bc)}{d^3\left(c+dx^n\right)} + \frac{bx^n(ex)^m(2aBd+Abd\right.}{d^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{(ex)^{m+1}\left(a^2Bd^2-2abd(Bc-Ad)+b^2c(Bc-Ad)\right)}{d^3e(m+1)}-\frac{(ex)^{m+1}(bc-ad)^2(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{cd^3e(m+1)}-\frac{bx^{n+1}(ex)^m(-2aBd-Abd+bBc)}{d^2(m+n+1)}+\frac{b^2Bx^{2n+1}(ex)^m}{d(m+2n+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n),x]
```

output
```
-((b*(b*B*c - A*b*d - 2*a*B*d)*x^(1 + n)*(e*x)^m)/(d^2*(1 + m + n))) + (b^
2*B*x^(1 + 2*n)*(e*x)^m)/(d*(1 + m + 2*n)) + ((a^2*B*d^2 + b^2*c*(B*c - A*
d) - 2*a*b*d*(B*c - A*d))*(e*x)^(1 + m))/(d^3*e*(1 + m)) - ((b*c - a*d)^2*
(B*c - A*d)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((d*x^n)/c)])/(c*d^3*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m\,(a+b\,x^n)^2\,(A+B\,x^n)}{c+d\,x^n}dx$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*b^2*x^(3*n) + A*a^2 + (2*B*a*b + A*b^2)*x^(2*n) + (B*a^2 + 2*A
*a*b)*x^n)*(e*x)^m/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.73 (sec) , antiderivative size = 1402, normalized size of antiderivative = 7.26

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)/(c+d*x**n),x)
```

output
```
A*a**2*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*m*x**(m + 1)*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*a**2*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*x**(m + 1)*lerchphi
(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n
+ 1 + 1/n)) + 2*A*a*b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**(m
+ n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n +
1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + 2*A*a*b*c**(-m/n - 2 - 1/n)*c**(m/n
 + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n
+ 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + 2*A*a*b*c**(-m/
n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_po
lar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 +
1/n)) + A*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*m*x**(m + 2*n +
 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1
/n)/(n**2*gamma(m/n + 3 + 1/n)) + 2*A*b**2*c**(-m/n - 3 - 1/n)*c**(m/n + 2
 + 1/n)*e**m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n +
2 + 1/n)*gamma(m/n + 2 + 1/n)/(n*gamma(m/n + 3 + 1/n)) + A*b**2*c**(-m/n -
 3 - 1/n)*c**(m/n + 2 + 1/n)*e**m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_pol
ar(I*pi)/c, 1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1
/n)) + B*a**2*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**(m + n + 1)
*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
((b^2*c^2*d*e^m - 2*a*b*c*d^2*e^m + a^2*d^3*e^m)*A - (b^2*c^3*e^m - 2*a*b*
c^2*d*e^m + a^2*c*d^2*e^m)*B)*integrate(x^m/(d^4*x^n + c*d^3), x) + ((m^2
+ m*(n + 2) + n + 1)*B*b^2*d^2*e^m*x*e^(m*log(x) + 2*n*log(x)) - (((m^2 +
m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c*d*e^m - 2*(m^2 + m*(3*n + 2) + 2*n^2
+ 3*n + 1)*a*b*d^2*e^m)*A - ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^2
*e^m - 2*(m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*a*b*c*d*e^m + (m^2 + m*(3*n
 + 2) + 2*n^2 + 3*n + 1)*a^2*d^2*e^m)*B)*x*x^m + ((m^2 + 2*m*(n + 1) + 2*n
 + 1)*A*b^2*d^2*e^m - ((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c*d*e^m - 2*(m^2
+ 2*m*(n + 1) + 2*n + 1)*a*b*d^2*e^m)*B)*x*e^(m*log(x) + n*log(x)))/((m^3
+ 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n + 1)*d^3)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)^2}{c+d\,x^n}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{c+dx^n}\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n),x)
```

output
```
(e**m*(x**(m + 2*n)*b**3*d**2*m**2*x + x**(m + 2*n)*b**3*d**2*m*n*x + 2*x*
*(m + 2*n)*b**3*d**2*m*x + x**(m + 2*n)*b**3*d**2*n*x + x**(m + 2*n)*b**3*
d**2*x + 3*x**(m + n)*a*b**2*d**2*m**2*x + 6*x**(m + n)*a*b**2*d**2*m*n*x
+ 6*x**(m + n)*a*b**2*d**2*m*x + 6*x**(m + n)*a*b**2*d**2*n*x + 3*x**(m +
n)*a*b**2*d**2*x - x**(m + n)*b**3*c*d*m**2*x - 2*x**(m + n)*b**3*c*d*m*n*
x - 2*x**(m + n)*b**3*c*d*m*x - 2*x**(m + n)*b**3*c*d*n*x - x**(m + n)*b**
3*c*d*x + 3*x**m*a**2*b*d**2*m**2*x + 9*x**m*a**2*b*d**2*m*n*x + 6*x**m*a*
*2*b*d**2*m*x + 6*x**m*a**2*b*d**2*n**2*x + 9*x**m*a**2*b*d**2*n*x + 3*x**
m*a**2*b*d**2*x - 3*x**m*a*b**2*c*d*m**2*x - 9*x**m*a*b**2*c*d*m*n*x - 6*x
**m*a*b**2*c*d*m*x - 6*x**m*a*b**2*c*d*n**2*x - 9*x**m*a*b**2*c*d*n*x - 3*
x**m*a*b**2*c*d*x + x**m*b**3*c**2*m**2*x + 3*x**m*b**3*c**2*m*n*x + 2*x**
m*b**3*c**2*m*x + 2*x**m*b**3*c**2*n**2*x + 3*x**m*b**3*c**2*n*x + x**m*b*
*3*c**2*x + int(x**m/(x**n*d + c),x)*a**3*d**3*m**3 + 3*int(x**m/(x**n*d +
 c),x)*a**3*d**3*m**2*n + 3*int(x**m/(x**n*d + c),x)*a**3*d**3*m**2 + 2*in
t(x**m/(x**n*d + c),x)*a**3*d**3*m*n**2 + 6*int(x**m/(x**n*d + c),x)*a**3*
d**3*m*n + 3*int(x**m/(x**n*d + c),x)*a**3*d**3*m + 2*int(x**m/(x**n*d + c
),x)*a**3*d**3*n**2 + 3*int(x**m/(x**n*d + c),x)*a**3*d**3*n + int(x**m/(x
**n*d + c),x)*a**3*d**3 - 3*int(x**m/(x**n*d + c),x)*a**2*b*c*d**2*m**3 -
9*int(x**m/(x**n*d + c),x)*a**2*b*c*d**2*m**2*n - 9*int(x**m/(x**n*d + c),
x)*a**2*b*c*d**2*m**2 - 6*int(x**m/(x**n*d + c),x)*a**2*b*c*d**2*m*n**2...
```

3.42 $\int \frac{(ex)^m(a+bx^n)(A+Bx^n)}{c+dx^n} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 125

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} \, dx = -\frac{(bBc-Abd-aBd)(ex)^{1+m}}{d^2e(1+m)}+\frac{bBx^n(ex)^{1+m}}{de(1+m+n)} + \frac{(bc-ad)(Bc-Ad)(ex)^{1+m} \text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{cd^2e(1+m)}$$

output
```
-(-A*b*d-B*a*d+B*b*c)*(e*x)^(1+m)/d^2/e/(1+m)+b*B*x^n*(e*x)^(1+m)/d/e/(1+m
+n)+(-a*d+b*c)*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-
d*x^n/c)/c/d^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} \, dx = \frac{x(ex)^m \left(\frac{-bBc+Abd+aBd}{1+m}+\frac{bBdx^n}{1+m+n}+\frac{(bc-ad)(Bc-Ad) \text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(1+m)}\right)}{d^2}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*((-(b*B*c) + A*b*d + a*B*d)/(1 + m) + (b*B*d*x^n)/(1 + m + n) +
 ((b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((d*x^n)/c)])/(c*(1 + m))))/d^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{c+dx^n} dx$$

$$\Big\downarrow \text{1040}$$

$$\int \left(\frac{(ex)^m(ad-bc)(Ad-Bc)}{d^2 (c+dx^n)}+\frac{(ex)^m(aBd+Abd-bBc)}{d^2}+\frac{bBx^n(ex)^m}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ex)^{m+1}(bc-ad)(Bc-Ad)\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{cd^2e(m+1)}-\frac{(ex)^{m+1}(-aBd-Abd+bBc)}{d^2e(m+1)}+\frac{bBx^{n+1}(ex)^m}{d(m+n+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(b*B*x^(1 + n)*(e*x)^m)/(d*(1 + m + n)) - ((b*B*c - A*b*d - a*B*d)*(e*x)^(
1 + m))/(d^2*e*(1 + m)) + ((b*c - a*d)*(B*c - A*d)*(e*x)^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*d^2*e*(1 + m))
```

Defintions of rubi rules used

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n) (A+B\,x^n)}{c+d\,x^n}dx$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*b*x^(2*n) + A*a + (B*a + A*b)*x^n)*(e*x)^m/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.52 (sec) , antiderivative size = 872, normalized size of antiderivative = 6.98

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)/(c+d*x**n),x)
```

output
```
A*a*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*m*x**(m + 1)*lerchphi(d*x**n*e
xp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n
)) + A*a*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*x**(m + 1)*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 +
1/n)) + A*b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**(m + n + 1)*l
erchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(
n**2*gamma(m/n + 2 + 1/n)) + A*b*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e*
*m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gam
ma(m/n + 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + A*b*c**(-m/n - 2 - 1/n)*c**(m
/n + 1 + 1/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/
n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + B*a*c**(-m
/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**(m + n + 1)*lerchphi(d*x**n*exp
_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2
 + 1/n)) + B*a*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*x**(m + n + 1)*
lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/
(n*gamma(m/n + 2 + 1/n)) + B*a*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m
*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma
(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n)) + B*b*c**(-m/n - 3 - 1/n)*c**(
m/n + 2 + 1/n)*e**m*m*x**(m + 2*n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c,
1, m/n + 2 + 1/n)*gamma(m/n + 2 + 1/n)/(n**2*gamma(m/n + 3 + 1/n)) + 2*...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{dx^n+c} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
-((b*c*d*e^m - a*d^2*e^m)*A - (b*c^2*e^m - a*c*d*e^m)*B)*integrate(x^m/(d^
3*x^n + c*d^2), x) + (B*b*d*e^m*(m + 1)*x*e^(m*log(x) + n*log(x)) + (A*b*d
*e^m*(m + n + 1) - (b*c*e^m*(m + n + 1) - a*d*e^m*(m + n + 1))*B)*x*x^m)/(
(m^2 + m*(n + 2) + n + 1)*d^2)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{dx^n+c} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{c+dx^n} dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)}{c+d\,x^n} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a + bx^n) (A + Bx^n)}{c + dx^n} \, dx$$
$$= \frac{e^m \left(x^{m+n} b^2 dmx + x^{m+n} b^2 dx + 2x^m abdmx + 2x^m abdnx + 2x^m abdx - x^m b^2 cmx - x^m b^2 cnx - x^m b^2 cx + \right.}{}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n),x)
```

output
```
(e**m*(x**(m + n)*b**2*d*m*x + x**(m + n)*b**2*d*x + 2*x**m*a*b*d*m*x + 2*
x**m*a*b*d*n*x + 2*x**m*a*b*d*x - x**m*b**2*c*m*x - x**m*b**2*c*n*x - x**m
*b**2*c*x + int(x**m/(x**n*d + c),x)*a**2*d**2*m**2 + int(x**m/(x**n*d + c
),x)*a**2*d**2*m*n + 2*int(x**m/(x**n*d + c),x)*a**2*d**2*m + int(x**m/(x*
*n*d + c),x)*a**2*d**2*n + int(x**m/(x**n*d + c),x)*a**2*d**2 - 2*int(x**m
/(x**n*d + c),x)*a*b*c*d*m**2 - 2*int(x**m/(x**n*d + c),x)*a*b*c*d*m*n - 4
*int(x**m/(x**n*d + c),x)*a*b*c*d*m - 2*int(x**m/(x**n*d + c),x)*a*b*c*d*n
 - 2*int(x**m/(x**n*d + c),x)*a*b*c*d + int(x**m/(x**n*d + c),x)*b**2*c**2
*m**2 + int(x**m/(x**n*d + c),x)*b**2*c**2*m*n + 2*int(x**m/(x**n*d + c),x
)*b**2*c**2*m + int(x**m/(x**n*d + c),x)*b**2*c**2*n + int(x**m/(x**n*d +
c),x)*b**2*c**2))/(d**2*(m**2 + m*n + 2*m + n + 1))
```

3.43 $\int \frac{(ex)^m(A+Bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 78

$$\int \frac{(ex)^m (A+Bx^n)}{c+dx^n}\,dx = \frac{B(ex)^{1+m}}{de(1+m)} - \frac{(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{cde(1+m)}$$

output
```
B*(e*x)^(1+m)/d/e/(1+m)-(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+
m+n)/n],-d*x^n/c)/c/d/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m (A+Bx^n)}{c+dx^n}\,dx = \frac{x(ex)^m \left(Bc+(-Bc+Ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)\right)}{cd(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(B*c + (-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m +
n)/n, -((d*x^n)/c)]))/(c*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A + Bx^n)}{c + dx^n} dx$$

$$\Big\downarrow \text{959}$$

$$\frac{B(ex)^{m+1}}{de(m+1)} - \frac{(Bc - Ad) \int \frac{(ex)^m}{dx^n + c} dx}{d}$$

$$\Big\downarrow \text{888}$$

$$\frac{B(ex)^{m+1}}{de(m+1)} - \frac{(ex)^{m+1}(Bc - Ad)\,\text{Hypergeometric2F1}\left(1, \frac{m+1}{n}, \frac{m+n+1}{n}, -\frac{dx^n}{c}\right)}{cde(m+1)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(B*(e*x)^(1 + m))/(d*e*(1 + m)) - ((B*c - A*d)*(e*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*d*e*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n)}{c + d x^n} dx$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{c + dx^n} dx = \int \frac{(Bx^n + A)(ex)^m}{dx^n + c} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(d*x^n + c), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.19 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.83

$$\int \frac{(ex)^m (A + Bx^n)}{c + dx^n} dx$$
$$= \frac{Ac^{\frac{m}{n}+\frac{1}{n}}c^{-\frac{m}{n}-1-\frac{1}{n}}e^m m x^{m+1}\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{Ac^{\frac{m}{n}+\frac{1}{n}}c^{-\frac{m}{n}-1-\frac{1}{n}}e^m x^{m+1}\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{m}{n}+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}$$
$$+ \frac{Bc^{-\frac{m}{n}-2-\frac{1}{n}}c^{\frac{m}{n}+1+\frac{1}{n}}e^m m x^{m+n+1}\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$
$$+ \frac{Bc^{-\frac{m}{n}-2-\frac{1}{n}}c^{\frac{m}{n}+1+\frac{1}{n}}e^m x^{m+n+1}\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$
$$+ \frac{Bc^{-\frac{m}{n}-2-\frac{1}{n}}c^{\frac{m}{n}+1+\frac{1}{n}}e^m x^{m+n+1}\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{m}{n}+1+\frac{1}{n}\right)\Gamma\left(\frac{m}{n}+1+\frac{1}{n}\right)}{n^2\Gamma\left(\frac{m}{n}+2+\frac{1}{n}\right)}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(c+d*x**n),x)
```

output
```
A*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*m*x**(m + 1)*lerchphi(d*x**n*exp
_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n))
 + A*c**(m/n + 1/n)*c**(-m/n - 1 - 1/n)*e**m*x**(m + 1)*lerchphi(d*x**n*ex
p_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(n**2*gamma(m/n + 1 + 1/n)
) + B*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*m*x**(m + n + 1)*lerchph
i(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n + 1 + 1/n)/(n**2*g
amma(m/n + 2 + 1/n)) + B*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1/n)*e**m*x**(m
 + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/n)*gamma(m/n +
 1 + 1/n)/(n*gamma(m/n + 2 + 1/n)) + B*c**(-m/n - 2 - 1/n)*c**(m/n + 1 + 1
/n)*e**m*x**(m + n + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1 + 1/
n)*gamma(m/n + 1 + 1/n)/(n**2*gamma(m/n + 2 + 1/n))
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(Bx^n+A)(ex)^m}{dx^n+c} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
B*e^m*x*x^m/(d*(m + 1)) - (B*c*e^m - A*d*e^m)*integrate(x^m/(d^2*x^n + c*d
), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(Bx^n+A)(ex)^m}{dx^n+c} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{c+d\,x^n} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)}{c + dx^n} \, dx$$
$$= \frac{e^m \left(x^m bx + \left(\int \frac{x^m}{x^n d+c} dx\right) adm + \left(\int \frac{x^m}{x^n d+c} dx\right) ad - \left(\int \frac{x^m}{x^n d+c} dx\right) bcm - \left(\int \frac{x^m}{x^n d+c} dx\right) bc\right)}{d \left(m+1\right)}$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n),x)
```

output
```
(e**m*(x**m*b*x + int(x**m/(x**n*d + c),x)*a*d*m + int(x**m/(x**n*d + c),x
)*a*d - int(x**m/(x**n*d + c),x)*b*c*m - int(x**m/(x**n*d + c),x)*b*c))/(d
*(m + 1))
```

3.44 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)(c+dx^n)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 127

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)}\,dx = \frac{(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)} + \frac{(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}$$

output
```
(A*b-B*a)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d
+b*c)/e/(1+m)+(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d
*x^n/c)/c/(-a*d+b*c)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)}\,dx = \frac{x(ex)^m \left((-Abc+aBc)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right) + a(-Bc+Ad)\,\text{Hypergeometric2F1}\right.}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*((-(A*b*c) + a*B*c)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)] + a*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 +
m + n)/n, -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d)*(1 + m))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)} \, dx$$

$$\Big\downarrow \text{ 1067}$$

$$\int \left(\frac{(ex)^m(Ab-aB)}{(bc-ad)(a+bx^n)}+\frac{(ex)^m(Bc-Ad)}{(bc-ad)(c+dx^n)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex)^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
-((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m)) + ((B*c - A*d)*(e*x)^(1 + m)*Hype
rgeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e
*(1 + m))
```

Defintions of rubi rules used

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B\,x^n)}{(a + b\,x^n)\,(c + d\,x^n)}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)\,(c + dx^n)}\,dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)(c + dx^n)} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)(c + dx^n)} dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)(c + dx^n)} dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)} dx = \int \frac{(e\,x)^m \,(A+B\,x^n)}{(a+b\,x^n)\,(c+d\,x^n)} dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)} dx = e^m \left(\int \frac{x^m}{x^n d + c} dx \right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n),x)
```

output
```
e**m*int(x**m/(x**n*d + c),x)
```

3.45 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 212

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)}\,dx = \frac{(Ab-aB)(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{(Ab(ad(1+m-2n)-bc(1+m-n))+aB(bc(1+m)-ad(1+m-n)))(ex)^{1+m}\,\text{Hypergeometric2}}{a^2(bc-ad)^2e(1+m)n} - \frac{d(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^2e(1+m)}$$

output
```
(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+(A*b*(a*d*(1+m-2*n)-b*c*(
1+m-n))+a*B*(b*c*(1+m)-a*d*(1+m-n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(
1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(1+m)/n-d*(-A*d+B*c)*(e*x)^(1+m)*hy
pergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.72

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)} \, dx = -\frac{x(ex)^m \left(abc(-Bc+Ad) \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right) + a^2d(Bc-Ad) \text{Hypergeometric}\right.}{a^2c(bc-ad)^2}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
-((x*(e*x)^m*(a*b*c*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
+ n)/n, -((b*x^n)/a)] + a^2*d*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/n,
(1 + m + n)/n, -((d*x^n)/c)] - (A*b - a*B)*c*(b*c - a*d)*Hypergeometric2F1
[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)]))/(a^2*c*(b*c - a*d)^2*(1 + m)
))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1065, 25, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)} \, dx$$

$$\Big\downarrow \; 1065$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)} - \frac{\int -\frac{(ex)^m(-((Ab-aB)d(m-n+1)x^n)+aBc(m+1)-Abc(m-n+1)-aAdn)}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m(-((Ab-aB)d(m-n+1)x^n)+aBc(m+1)-A(bc(m-n+1)+adn))}{(bx^n+a)(dx^n+c)}dx}{an(bc-ad)} + \frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)}$$

$$\Big\downarrow \text{1067}$$

$$\frac{\int \left(\frac{(Ab(ad(m-2n+1)-bc(m-n+1))+aB(bc(m+1)-ad(m-n+1)))(ex)^m}{(bc-ad)(bx^n+a)}-\frac{ad(Ad-Bc)n(ex)^m}{(ad-bc)(dx^n+c)}\right)dx}{an(bc-ad)}+\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)(Ab(ad(m-2n+1)-bc(m-n+1))+aB(bc(m+1)-ad(m-n+1)))}{ae(m+1)(bc-ad)}-\frac{adn(ex)^{m+1}(Bc-Ac}{}}{an(bc-ad)}\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (((A*b*(a*d*
(1 + m - 2*n) - b*c*(1 + m - n)) + a*B*(b*c*(1 + m) - a*d*(1 + m - n)))*(e
*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (a*d*(B*c - A*d)*n*(e*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
)/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n)}{(a+b\,x^n)^2\,(c+d\,x^n)}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^
(2*n) + (2*a*b*c + a^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)**2/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

output
```
-(B*a*e^m - A*b*e^m)*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*
x^n) - ((b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*A + (a^2*d*e^m*(
m - n + 1) - a*b*c*e^m*(m + 1))*B)*integrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*
c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
 - (B*c*d*e^m - A*d^2*e^m)*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)} \, dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^2\,(c+d\,x^n)} \,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)} \, dx = e^m \left(\int \frac{x^m}{x^{2n}bd + x^n ad + x^n bc + ac} dx \right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n),x)
```

output
```
e**m*int(x**m/(x**(2*n)*b*d + x**n*a*d + x**n*b*c + a*c),x)
```

3.46 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 407

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)}\,dx = \frac{(Ab-aB)(ex)^{1+m}}{2a(bc-ad)en\,(a+bx^n)^2}$$
$$+\frac{(Ab(ad(1+m-4n)-bc(1+m-2n))+aB(bc(1+m)-ad(1+m-2n)))(ex)^{1+m}}{2a^2(bc-ad)^2en^2\,(a+bx^n)}$$
$$+\frac{(aB(2abcd(1+m)(1+m-2n)-b^2c^2(1+m)(1+m-n)-a^2d^2(1+m^2+m(2-3n)-3n+2n^2)}{}$$
$$+\frac{d^2(Bc-Ad)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(bc-ad)^3e(1+m)}$$

output
```
1/2*(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)^2+1/2*(A*b*(a*d*(1+m-
4*n)-b*c*(1+m-2*n))+a*B*(b*c*(1+m)-a*d*(1+m-2*n)))*(e*x)^(1+m)/a^2/(-a*d+b
*c)^2/e/n^2/(a+b*x^n)+1/2*(a*B*(2*a*b*c*d*(1+m)*(1+m-2*n)-b^2*c^2*(1+m)*(1
+m-n)-a^2*d^2*(1+m^2+m*(2-3*n)-3*n+2*n^2))+A*b*(b^2*c^2*(1+m^2+m*(2-3*n)-3
*n+2*n^2)-2*a*b*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)+a^2*d^2*(1+m^2+m*(2-5*n)-5
*n+6*n^2)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/(
-a*d+b*c)^3/e/(1+m)/n^2+d^2*(-A*d+B*c)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],
[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b*c)^3/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)}\,dx = \frac{x(ex)^m \left(\frac{bd(-Bc+Ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d^2(Bc-Ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c}+\frac{b(bc}{}\right.}{(bc-ad)^3(1}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
(x*(e*x)^m*((b*d*(-(B*c) + A*d)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/a + (d^2*(B*c - A*d)*Hypergeometric2F1[1, (1 + m)/n, (
1 + m + n)/n, -((d*x^n)/c)])/c + (b*(b*c - a*d)*(B*c - A*d)*Hypergeometric
2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/a^2 + ((A*b - a*B)*(b*c -
a*d)^2*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/a^3))
/((b*c - a*d)^3*(1 + m))
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1065, 25, 1065, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)}\,dx$$

$$\Big\downarrow \; 1065$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)(a+bx^n)^2}-\frac{\int -\frac{(ex)^m(-((Ab-aB)d(m-2n+1)x^n)+aBc(m+1)-Abc(m-2n+1)-2aAdn)}{(bx^n+a)^2(dx^n+c)}dx}{2an(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m(-((Ab-aB)d(m-2n+1)x^n)+aBc(m+1)-A(bc(m-2n+1)+2adn))}{(bx^n+a)^2(dx^n+c)}dx}{2an(bc-ad)} + \frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2}$$

$$\Big\downarrow \text{1065}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-4n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-2n+1)))}{aen(bc-ad)(a+bx^n)} - \frac{\int \frac{(ex)^m(d(Ab(ad(m-4n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-2n}{}}{}}{2an(bc-ad}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2}$$

$$\Big\downarrow \text{1067}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-4n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-2n+1)))}{aen(bc-ad)(a+bx^n)} - \frac{\int \Bigg(\frac{\left(-aB\left(-b^2(m+1)(m-n+1)c^2+2abd(m+1)(m-2n+1)c-a^2d^2\left(m^2\right.\right.\right.}{}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-4n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-2n+1)))}{aen(bc-ad)(a+bx^n)} - \frac{-\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)\left(Ab\left(a^2d^2\left(m\right.\right.\right.}{}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2}$$

input

```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*(b*c - a*d)*e*n*(a + b*x^n)^2) + (((A*b*(
a*d*(1 + m - 4*n) - b*c*(1 + m - 2*n)) + a*B*(b*c*(1 + m) - a*d*(1 + m - 2
*n)))*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) - (-(((a*B*(2*a*b*c*d
*(1 + m)*(1 + m - 2*n) - b^2*c^2*(1 + m)*(1 + m - n) - a^2*d^2*(1 + m^2 +
m*(2 - 3*n) - 3*n + 2*n^2)) + A*b*(b^2*c^2*(1 + m^2 + m*(2 - 3*n) - 3*n +
2*n^2) - 2*a*b*c*d*(1 + m^2 + m*(2 - 4*n) - 4*n + 3*n^2) + a^2*d^2*(1 + m^
2 + m*(2 - 5*n) - 5*n + 6*n^2)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m))) - (2*a^2*d^2
*(B*c - A*d)*n^2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(a*(b*c - a*d)*n))/(2*a*(b*c
 - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B\,x^n)}{(a + b\,x^n)^3 (c + d\,x^n)}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)}\,dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^3 (dx^n + c)}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b^3*d*x^(4*n) + a^3*c + (b^3*c + 3*a*b^2*d)*
x^(3*n) + 3*(a*b^2*c + a^2*b*d)*x^(2*n) + (3*a^2*b*c + a^3*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)**3/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^3 (dx^n + c)} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

output
```
-(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c^2*e^m - 2*(m^2 - 2*m*(2*n -
 1) + 3*n^2 - 4*n + 1)*a*b^2*c*d*e^m + (m^2 - m*(5*n - 2) + 6*n^2 - 5*n +
1)*a^2*b*d^2*e^m)*A - ((m^2 - m*(n - 2) - n + 1)*a*b^2*c^2*e^m - 2*(m^2 -
2*m*(n - 1) - 2*n + 1)*a^2*b*c*d*e^m + (m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
1)*a^3*d^2*e^m)*B)*integrate(-1/2*x^m/(a^3*b^3*c^3*n^2 - 3*a^4*b^2*c^2*d*n
^2 + 3*a^5*b*c*d^2*n^2 - a^6*d^3*n^2 + (a^2*b^4*c^3*n^2 - 3*a^3*b^3*c^2*d*
n^2 + 3*a^4*b^2*c*d^2*n^2 - a^5*b*d^3*n^2)*x^n), x) - (B*c*d^2*e^m - A*d^3
*e^m)*integrate(-x^m/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^
3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n), x) - 1/2
*(((a*b^2*c*e^m*(m - 3*n + 1) - a^2*b*d*e^m*(m - 5*n + 1))*A - (a^2*b*c*e^
m*(m - n + 1) - a^3*d*e^m*(m - 3*n + 1))*B)*x*x^m + ((b^3*c*e^m*(m - 2*n +
 1) - a*b^2*d*e^m*(m - 4*n + 1))*A + (a^2*b*d*e^m*(m - 2*n + 1) - a*b^2*c*
e^m*(m + 1))*B)*x*e^(m*log(x) + n*log(x)))/(a^4*b^2*c^2*n^2 - 2*a^5*b*c*d*
n^2 + a^6*d^2*n^2 + (a^2*b^4*c^2*n^2 - 2*a^3*b^3*c*d*n^2 + a^4*b^2*d^2*n^2
)*x^(2*n) + 2*(a^3*b^3*c^2*n^2 - 2*a^4*b^2*c*d*n^2 + a^5*b*d^2*n^2)*x^n)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^3 (dx^n + c)} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)^3*(d*x^n + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^3\,(c+d\,x^n)}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)}\,dx = e^m\left(\int \frac{x^m}{x^{3n}b^2d+2x^{2n}abd+x^{2n}b^2c+x^na^2d+2x^nabc+a^2c}dx\right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n),x)
```

output
```
e**m*int(x**m/(x**(3*n)*b**2*d + 2*x**(2*n)*a*b*d + x**(2*n)*b**2*c + x**n
*a**2*d + 2*x**n*a*b*c + a**2*c),x)
```

3.47 $\int \frac{(ex)^m(a+bx^n)^3(A+Bx^n)}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 387

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2}\,dx = -\frac{b(3a^2d^2(Ad(1+m)-Bc(1+m+n))-3abcd(Ad(1+m+n)-Bc(1+m+2n))+b^2c^2(Ad(1+m \cdots}{cd^4e(1+m)n} - \frac{b^2(3ad(Ad(1+m+n)-Bc(1+m+2n))-bc(Ad(1+m+2n)-Bc(1+m+3n)))x^n(ex)^{1+m}}{cd^3en(1+m+n)} - \frac{b^3\left(A-\frac{Bc(1+m+3n)}{d(1+m+2n)}\right)x^{2n}(ex)^{1+m}}{cden} - \frac{(Bc-Ad)(ex)^{1+m}(a+bx^n)^3}{cden(c+dx^n)} + \frac{(bc-ad)^2(ad(Bc(1+m)-Ad(1+m-n))+bc(Ad(1+m+2n)-Bc(1+m+3n)))(ex)^{1+m}\,\text{Hyp}\cdots}{c^2d^4e(1+m)n}$$

output
```
-b*(3*a^2*d^2*(A*d*(1+m)-B*c*(1+m+n))-3*a*b*c*d*(A*d*(1+m+n)-B*c*(1+m+2*n)
)+b^2*c^2*(A*d*(1+m+2*n)-B*c*(1+m+3*n)))*(e*x)^(1+m)/c/d^4/e/(1+m)/n-b^2*(
3*a*d*(A*d*(1+m+n)-B*c*(1+m+2*n))-b*c*(A*d*(1+m+2*n)-B*c*(1+m+3*n)))*x^n*(
e*x)^(1+m)/c/d^3/e/n/(1+m+n)-b^3*(A-B*c*(1+m+3*n)/d/(1+m+2*n))*x^(2*n)*(e*
x)^(1+m)/c/d/e/n-(-A*d+B*c)*(e*x)^(1+m)*(a+b*x^n)^3/c/d/e/n/(c+d*x^n)+(-a*
d+b*c)^2*(a*d*(B*c*(1+m)-A*d*(1+m-n))+b*c*(A*d*(1+m+2*n)-B*c*(1+m+3*n)))*(
e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/d^4/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} \, dx = \frac{x(ex)^m \left(\frac{b(3a^2Bd^2+b^2c(3Bc-2Ad)+3abd(-2Bc+Ad))}{1+m} + \frac{b^2d(-2bBc+Abd+3aBd)x^n}{1+m+n} + \frac{b^3Bd^2x^{2n}}{1+m+2n} - \frac{(bc-ad)^2(4bBc-3Abd-aBd)\,\mathrm{I}}{}\right.}{d^4}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^3*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*((b*(3*a^2*B*d^2 + b^2*c*(3*B*c - 2*A*d) + 3*a*b*d*(-2*B*c + A*
d)))/(1 + m) + (b^2*d*(-2*b*B*c + A*b*d + 3*a*B*d)*x^n)/(1 + m + n) + (b^3
*B*d^2*x^(2*n))/(1 + m + 2*n) - ((b*c - a*d)^2*(4*b*B*c - 3*A*b*d - a*B*d)
*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(1 + m))
 + ((b*c - a*d)^3*(B*c - A*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/
n, -((d*x^n)/c)])/(c^2*(1 + m))))/d^4
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1064, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} \, dx$$

$$\Big\downarrow \ 1064$$

$$-\frac{\int -\frac{(ex)^m(bx^n+a)^2(a(Bc(m+1)-Ad(m-n+1))-b(Ad(m+2n+1)-Bc(m+3n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1}(a+bx^n)^3(Bc-Ad)}{cden(c+dx^n)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m(bx^n+a)^2(a(Bc(m+1)-Ad(m-n+1))-b(Ad(m+2n+1)-Bc(m+3n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1}(a+bx^n)^3(Bc-Ad)}{cden(c+dx^n)}$$

$\Big\downarrow$ 1040

$$\frac{\int \left(\frac{b^2(bc(Ad(m+2n+1)-Bc(m+3n+1))-3ad(Ad(m+n+1)-Bc(m+2n+1)))x^n(ex)^m}{d^2} + \frac{b^3(Bc(m+3n+1)-Ad(m+2n+1))x^{2n}(ex)^m}{d} + \frac{b(\cdot}{}\right.}{} - \frac{(ex)^{m+1}(a+bx^n)^3(Bc-Ad)}{cden(c+dx^n)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{b(ex)^{m+1}(3a^2d^2(Ad(m+1)-Bc(m+n+1))-3abcd(Ad(m+n+1)-Bc(m+2n+1))+b^2c^2(Ad(m+2n+1)-Bc(m+3n+1)))}{d^3e(m+1)} - \frac{b^2x^{n+1}(ex)^m(3}{}}{} - \frac{(ex)^{m+1}(a+bx^n)^3(Bc-Ad)}{cden(c+dx^n)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^3*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
-(((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^n)^3)/(c*d*e*n*(c + d*x^n))) + (-((b
^2*(3*a*d*(A*d*(1 + m + n) - B*c*(1 + m + 2*n)) - b*c*(A*d*(1 + m + 2*n) -
 B*c*(1 + m + 3*n)))*x^(1 + n)*(e*x)^m)/(d^2*(1 + m + n))) - b^3*(A - (B*c
*(1 + m + 3*n))/(d*(1 + m + 2*n)))*x^(1 + 2*n)*(e*x)^m - (b*(3*a^2*d^2*(A*
d*(1 + m) - B*c*(1 + m + n)) - 3*a*b*c*d*(A*d*(1 + m + n) - B*c*(1 + m + 2
*n)) + b^2*c^2*(A*d*(1 + m + 2*n) - B*c*(1 + m + 3*n)))*(e*x)^(1 + m))/(d^
3*e*(1 + m)) + ((b*c - a*d)^2*(a*d*(B*c*(1 + m) - A*d*(1 + m - n)) + b*c*(
A*d*(1 + m + 2*n) - B*c*(1 + m + 3*n)))*(e*x)^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*d^3*e*(1 + m)))/(c*d*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a + b\,x^n)^3 (A + B\,x^n)}{(c + d\,x^n)^2} dx$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^3(ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*b^3*x^(4*n) + A*a^3 + (3*B*a*b^2 + A*b^3)*x^(3*n) + 3*(B*a^2*b
 + A*a*b^2)*x^(2*n) + (B*a^3 + 3*A*a^2*b)*x^n)*(e*x)^m/(d^2*x^(2*n) + 2*c*
d*x^n + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**3*(A+B*x**n)/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^3(ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
((b^3*c^3*d*e^m*(m + 2*n + 1) - 3*a*b^2*c^2*d^2*e^m*(m + n + 1) - a^3*d^4*
e^m*(m - n + 1) + 3*a^2*b*c*d^3*e^m*(m + 1))*A - (b^3*c^4*e^m*(m + 3*n + 1
) - 3*a*b^2*c^3*d*e^m*(m + 2*n + 1) + 3*a^2*b*c^2*d^2*e^m*(m + n + 1) - a^
3*c*d^3*e^m*(m + 1))*B)*integrate(x^m/(c*d^5*n*x^n + c^2*d^4*n), x) + ((m^
2*n + (n^2 + 2*n)*m + n^2 + n)*B*b^3*c*d^3*e^m*x*e^(m*log(x) + 3*n*log(x))
 - (((m^3 + m^2*(5*n + 3) + 4*n^3 + (8*n^2 + 10*n + 3)*m + 8*n^2 + 5*n + 1
)*b^3*c^3*d*e^m - 3*(m^3 + m^2*(4*n + 3) + 2*n^3 + (5*n^2 + 8*n + 3)*m + 5
*n^2 + 4*n + 1)*a*b^2*c^2*d^2*e^m + 3*(m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n
+ 3)*m + 2*n^2 + 3*n + 1)*a^2*b*c*d^3*e^m - (m^3 + 3*m^2*(n + 1) + (2*n^2
+ 6*n + 3)*m + 2*n^2 + 3*n + 1)*a^3*d^4*e^m)*A - ((m^3 + 3*m^2*(2*n + 1) +
 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)*b^3*c^4*e^m - 3*(m^3 +
m^2*(5*n + 3) + 4*n^3 + (8*n^2 + 10*n + 3)*m + 8*n^2 + 5*n + 1)*a*b^2*c^3*
d*e^m + 3*(m^3 + m^2*(4*n + 3) + 2*n^3 + (5*n^2 + 8*n + 3)*m + 5*n^2 + 4*n
 + 1)*a^2*b*c^2*d^2*e^m - (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n
^2 + 3*n + 1)*a^3*c*d^3*e^m)*B)*x*x^m + ((m^2*n + 2*(n^2 + n)*m + 2*n^2 +
n)*A*b^3*c*d^3*e^m - ((m^2*n + (3*n^2 + 2*n)*m + 3*n^2 + n)*b^3*c^2*d^2*e^
m - 3*(m^2*n + 2*(n^2 + n)*m + 2*n^2 + n)*a*b^2*c*d^3*e^m)*B)*x*e^(m*log(x
) + 2*n*log(x)) - (((m^2*n + 4*n^3 + 2*(2*n^2 + n)*m + 4*n^2 + n)*b^3*c^2*
d^2*e^m - 3*(m^2*n + 2*n^3 + (3*n^2 + 2*n)*m + 3*n^2 + n)*a*b^2*c*d^3*e^m)
*A - ((m^2*n + 6*n^3 + (5*n^2 + 2*n)*m + 5*n^2 + n)*b^3*c^3*d*e^m - 3*(...
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2}\, dx = \int \frac{(Bx^n+A)(bx^n+a)^3(ex)^m}{(dx^n+c)^2}\, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^3*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} \, dx = \int \frac{(e\,x)^m \, (A+B\,x^n) \, (a+b\,x^n)^3}{(c+d\,x^n)^2} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^3)/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^3)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^3 (A+Bx^n)}{(c+dx^n)^2} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^3*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
(e**m*(x**(m + 3*n)*b**4*c**2*d*m**2*x + x**(m + 3*n)*b**4*c**2*d*m*n*x +
2*x**(m + 3*n)*b**4*c**2*d*m*x + x**(m + 3*n)*b**4*c**2*d*n*x + x**(m + 3*
n)*b**4*c**2*d*x + 4*x**(m + 2*n)*a*b**3*c**2*d*m**2*x + 8*x**(m + 2*n)*a*
b**3*c**2*d*m*n*x + 8*x**(m + 2*n)*a*b**3*c**2*d*m*x + 8*x**(m + 2*n)*a*b*
*3*c**2*d*n*x + 4*x**(m + 2*n)*a*b**3*c**2*d*x - x**(m + 2*n)*b**4*c**3*m*
*2*x - 3*x**(m + 2*n)*b**4*c**3*m*n*x - 2*x**(m + 2*n)*b**4*c**3*m*x - 3*x
**(m + 2*n)*b**4*c**3*n*x - x**(m + 2*n)*b**4*c**3*x - x**(m + n)*a**4*d**
3*m**2*x - x**(m + n)*a**4*d**3*m*n*x - 2*x**(m + n)*a**4*d**3*m*x + 2*x**
(m + n)*a**4*d**3*n**2*x - x**(m + n)*a**4*d**3*n*x - x**(m + n)*a**4*d**3
*x + 4*x**(m + n)*a**3*b*c*d**2*m**2*x + 8*x**(m + n)*a**3*b*c*d**2*m*n*x
+ 8*x**(m + n)*a**3*b*c*d**2*m*x + 8*x**(m + n)*a**3*b*c*d**2*n*x + 4*x**(
m + n)*a**3*b*c*d**2*x + x**m*a**4*c*d**2*m**2*x + 3*x**m*a**4*c*d**2*m*n*
x + 2*x**m*a**4*c*d**2*m*x + 2*x**m*a**4*c*d**2*n**2*x + 3*x**m*a**4*c*d**
2*n*x + x**m*a**4*c*d**2*x + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(
2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c
**2*m + c**2*n + c**2),x)*a**4*d**5*m**4 + 2*x**n*int(x**(m + 2*n)/(x**(2*
n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n
+ 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**4*d**5*m**3*n + 4*x**n*int(x*
*(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d
*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**4*d**5*m...
```

3.48 $\int \frac{(ex)^m(a+bx^n)^2(A+Bx^n)}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 270

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2}\,dx = -\frac{b(2ad(Ad(1+m)-Bc(1+m+n))-bc(Ad(1+m+n)-Bc(1+m+2n)))(ex)^{1+m}}{cd^3e(1+m)n} - \frac{b^2(Ad(1+m+n)-Bc(1+m+2n))x^n(ex)^{1+m}}{cd^2en(1+m+n)} - \frac{(Bc-Ad)(ex)^{1+m}(a+bx^n)^2}{cden(c+dx^n)} - \frac{(bc-ad)(ad(Bc(1+m)-Ad(1+m-n))+bc(Ad(1+m+n)-Bc(1+m+2n)))(ex)^{1+m}\,\text{Hyperg}}{c^2d^3e(1+m)n}$$

output
```
-b*(2*a*d*(A*d*(1+m)-B*c*(1+m+n))-b*c*(A*d*(1+m+n)-B*c*(1+m+2*n)))*(e*x)^(
1+m)/c/d^3/e/(1+m)/n-b^2*(A*d*(1+m+n)-B*c*(1+m+2*n))*x^n*(e*x)^(1+m)/c/d^2
/e/n/(1+m+n)-(-A*d+B*c)*(e*x)^(1+m)*(a+b*x^n)^2/c/d/e/n/(c+d*x^n)-(-a*d+b*
c)*(a*d*(B*c*(1+m)-A*d*(1+m-n))+b*c*(A*d*(1+m+n)-B*c*(1+m+2*n)))*(e*x)^(1+
m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/d^3/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} \, dx$$
$$= \frac{x(ex)^m \left(\frac{b(-2bBc+Abd+2aBd)}{1+m} + \frac{b^2Bdx^n}{1+m+n} + \frac{(bc-ad)(3bBc-2Abd-aBd) \text{ Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c(1+m)} - \frac{(bc-ad)^2(}{}\right.}{d^3}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*((b*(-2*b*B*c + A*b*d + 2*a*B*d))/(1 + m) + (b^2*B*d*x^n)/(1 +
m + n) + ((b*c - a*d)*(3*b*B*c - 2*A*b*d - a*B*d)*Hypergeometric2F1[1, (1
+ m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(1 + m)) - ((b*c - a*d)^2*(B*c -
A*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c^2*(1
 + m))))/d^3
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1064, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} \, dx$$

$$\Big\downarrow \; 1064$$

$$-\frac{\int -\frac{(ex)^m (bx^n+a)(a(Bc(m+1)-Ad(m-n+1))-b(Ad(m+n+1)-Bc(m+2n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1} (a+bx^n)^2 (Bc-Ad)}{cden (c+dx^n)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m(bx^n+a)(a(Bc(m+1)-Ad(m-n+1))-b(Ad(m+n+1)-Bc(m+2n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{cden(c+dx^n)}$$

$\Big\downarrow$ 1040

$$\frac{\int \left(\frac{b^2(Bc(m+2n+1)-Ad(m+n+1))x^n(ex)^m}{d} + \frac{b(bc(Ad(m+n+1)-Bc(m+2n+1))-2ad(Ad(m+1)-Bc(m+n+1)))(ex)^m}{d^2} + \frac{(bc-ad)(-ad(}{}\right.}{cdn} \frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{cden(c+dx^n)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)(ad(Bc(m+1)-Ad(m-n+1))+bc(Ad(m+n+1)-Bc(m+2n+1)))}{cd^2e(m+1)} - \frac{b(ex)^{m+}}{}}{cdn} \frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{cden(c+dx^n)}$$

input

```
Int[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n)^2,x]
```

output

```
-(((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^n)^2)/(c*d*e*n*(c + d*x^n))) + (-((b
^2*(A*d*(1 + m + n) - B*c*(1 + m + 2*n))*x^(1 + n)*(e*x)^m)/(d*(1 + m + n)
)) - (b*(2*a*d*(A*d*(1 + m) - B*c*(1 + m + n)) - b*c*(A*d*(1 + m + n) - B*
c*(1 + m + 2*n)))*(e*x)^(1 + m))/(d^2*e*(1 + m)) - ((b*c - a*d)*(a*d*(B*c*
(1 + m) - A*d*(1 + m - n)) + b*c*(A*d*(1 + m + n) - B*c*(1 + m + 2*n)))*(e
*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/
(c*d^2*e*(1 + m)))/(c*d*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a + b\,x^n)^2 (A + B\,x^n)}{(c + d\,x^n)^2} dx$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*b^2*x^(3*n) + A*a^2 + (2*B*a*b + A*b^2)*x^(2*n) + (B*a^2 + 2*A
*a*b)*x^n)*(e*x)^m/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(ex)^m (A+Bx^n) (a+bx^n)^2}{(c+dx^n)^2} dx$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)/(c+d*x**n)**2,x)
```

output
```
Integral((e*x)**m*(A + B*x**n)*(a + b*x**n)**2/(c + d*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
-((b^2*c^2*d*e^m*(m + n + 1) + a^2*d^3*e^m*(m - n + 1) - 2*a*b*c*d^2*e^m*(
m + 1))*A - (b^2*c^3*e^m*(m + 2*n + 1) - 2*a*b*c^2*d*e^m*(m + n + 1) + a^2
*c*d^2*e^m*(m + 1))*B)*integrate(x^m/(c*d^4*n*x^n + c^2*d^3*n), x) + ((m*n
 + n)*B*b^2*c*d^2*e^m*x*e^(m*log(x) + 2*n*log(x)) + (((m^2 + 2*m*(n + 1) +
 n^2 + 2*n + 1)*b^2*c^2*d*e^m - 2*(m^2 + m*(n + 2) + n + 1)*a*b*c*d^2*e^m
+ (m^2 + m*(n + 2) + n + 1)*a^2*d^3*e^m)*A - ((m^2 + m*(3*n + 2) + 2*n^2 +
 3*n + 1)*b^2*c^3*e^m - 2*(m^2 + 2*m*(n + 1) + n^2 + 2*n + 1)*a*b*c^2*d*e^
m + (m^2 + m*(n + 2) + n + 1)*a^2*c*d^2*e^m)*B)*x*x^m + ((m*n + n^2 + n)*A
*b^2*c*d^2*e^m - ((m*n + 2*n^2 + n)*b^2*c^2*d*e^m - 2*(m*n + n^2 + n)*a*b*
c*d^2*e^m)*B)*x*e^(m*log(x) + n*log(x)))/((m^2*n + (n^2 + 2*n)*m + n^2 + n
)*c*d^4*x^n + (m^2*n + (n^2 + 2*n)*m + n^2 + n)*c^2*d^3)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)^2}{(c+d\,x^n)^2} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^2} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
(e**m*(x**(m + 2*n)*b**3*c**2*m*x + x**(m + 2*n)*b**3*c**2*x - x**(m + n)*
a**3*d**2*m*x + x**(m + n)*a**3*d**2*n*x - x**(m + n)*a**3*d**2*x + 3*x**(
m + n)*a**2*b*c*d*m*x + 3*x**(m + n)*a**2*b*c*d*x + x**m*a**3*c*d*m*x + x*
*m*a**3*c*d*n*x + x**m*a**3*c*d*x + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m
 + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*
c*d + c**2*m + c**2*n + c**2),x)*a**3*d**4*m**3 + 3*x**n*int(x**(m + 2*n)/
(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n
*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**3*d**4*m**2 - x**n*int
(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*
c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**3*d**4*m
*n**2 + 3*x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2
*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c*
*2),x)*a**3*d**4*m - x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**
2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m +
c**2*n + c**2),x)*a**3*d**4*n**2 + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m
+ x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c
*d + c**2*m + c**2*n + c**2),x)*a**3*d**4 - 3*x**n*int(x**(m + 2*n)/(x**(2
*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n
 + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**2*b*c*d**3*m**3 - 3*x**n*int
(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x*...
```

3.49 $\int \frac{(ex)^m(a+bx^n)(A+Bx^n)}{(c+dx^n)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 176

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^2} dx$$
$$= -\frac{B\left(a-\frac{bc(1+m+n)}{d(1+m)}\right)(ex)^{1+m}}{cden} - \frac{(bc-ad)(ex)^{1+m}(A+Bx^n)}{cden(c+dx^n)}$$
$$+ \frac{(Ad(bc(1+m)-ad(1+m-n))+Bc(ad(1+m)-bc(1+m+n)))(ex)^{1+m}\,\text{Hypergeometric2F1}\,(1}{c^2d^2e(1+m)n}$$

output
```
-B*(a-b*c*(1+m+n)/d/(1+m))*(e*x)^(1+m)/c/d/e/n-(-a*d+b*c)*(e*x)^(1+m)*(A+B
*x^n)/c/d/e/n/(c+d*x^n)+(A*d*(b*c*(1+m)-a*d*(1+m-n))+B*c*(a*d*(1+m)-b*c*(1
+m+n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/d^2/e
/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int \frac{(ex)^m (a + bx^n) (A + Bx^n)}{(c + dx^n)^2} dx$$
$$= \frac{x(ex)^m \left(bBc^2 + c(-2bBc + Abd + aBd) \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{dx^n}{c}\right) + (bc - ad)(Bc - A\right.}{c^2d^2(1 + m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*(b*B*c^2 + c*(-2*b*B*c + A*b*d + a*B*d)*Hypergeometric2F1[1, (1
 + m)/n, (1 + m + n)/n, -((d*x^n)/c)] + (b*c - a*d)*(B*c - A*d)*Hypergeome
tric2F1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(c^2*d^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1064, 25, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a + bx^n) (A + Bx^n)}{(c + dx^n)^2} dx$$

$$\Big\downarrow \text{1064}$$

$$-\frac{\int -\frac{(ex)^m (A(bc(m+1)-ad(m-n+1))-B(ad(m+1)-bc(m+n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1}(bc-ad)(A+Bx^n)}{cden(c+dx^n)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m (A(bc(m+1)-ad(m-n+1))-B(ad(m+1)-bc(m+n+1))x^n)}{dx^n+c}dx}{cdn} - \frac{(ex)^{m+1}(bc-ad)(A+Bx^n)}{cden(c+dx^n)}$$

$$\Big\downarrow \text{959}$$

$$\frac{\frac{(ad(Bc(m+1)-Ad(m-n+1))+bc(Ad(m+1)-Bc(m+n+1)))\int \frac{(ex)^m}{dx^n+c}dx}{d}-\frac{B(ex)^{m+1}(ad(m+1)-bc(m+n+1))}{de(m+1)}}{cdn}-\frac{(ex)^{m+1}(bc-ad)\left(A+Bx^n\right)}{cden\left(c+dx^n\right)}$$

$$\Big\downarrow \; 888$$

$$\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)(ad(Bc(m+1)-Ad(m-n+1))+bc(Ad(m+1)-Bc(m+n+1)))}{cde(m+1)}-\frac{B(ex)^{m+1}(ad(m+1)-b}{de(m+1)}}{cdn}-\frac{(ex)^{m+1}(bc-ad)\left(A+Bx^n\right)}{cden\left(c+dx^n\right)}$$

input

```
Int[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n)^2,x]
```

output

```
-(((b*c - a*d)*(e*x)^(1 + m)*(A + B*x^n))/(c*d*e*n*(c + d*x^n))) + (-((B*(
a*d*(1 + m) - b*c*(1 + m + n))*(e*x)^(1 + m))/(d*e*(1 + m))) + ((a*d*(B*c*
(1 + m) - A*d*(1 + m - n)) + b*c*(A*d*(1 + m) - B*c*(1 + m + n)))*(e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*d*e
*(1 + m)))/(c*d*n)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n) (A+B\,x^n)}{(c+d\,x^n)^2}dx$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*b*x^(2*n) + A*a + (B*a + A*b)*x^n)*(e*x)^m/(d^2*x^(2*n) + 2*c*
d*x^n + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.33 (sec) , antiderivative size = 5176, normalized size of antiderivative = 29.41

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)/(c+d*x**n)**2,x)
```

output
```
A*a*(-c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m**2*x**(m + 1)*lerchphi(d
*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n
+ 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n
 - 2 - 1/n)*e**m*m*n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n
+ 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m
/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m*n*x**(m + 1)*
gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1
+ 1/n)) - 2*c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m*x**(m + 1)*lerchph
i(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m
/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-
m/n - 2 - 1/n)*e**m*n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n
 + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(
m/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*n*x**(m + 1)*g
amma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 +
 1/n)) - c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*x**(m + 1)*lerchphi(d*x
**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n +
1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) - c**(m/n + 1/n)*c**(-m/n - 2
 - 1/n)*d*e**m*m**2*x**n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1,
m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gam
ma(m/n + 1 + 1/n)) + c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*d*e**m*m*n*x**n...
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
-((a*d^2*e^m*(m - n + 1) - b*c*d*e^m*(m + 1))*A + (b*c^2*e^m*(m + n + 1) -
 a*c*d*e^m*(m + 1))*B)*integrate(x^m/(c*d^3*n*x^n + c^2*d^2*n), x) + (B*b*
c*d*e^m*n*x*e^(m*log(x) + n*log(x)) - ((b*c*d*e^m*(m + 1) - a*d^2*e^m*(m +
 1))*A - (b*c^2*e^m*(m + n + 1) - a*c*d*e^m*(m + 1))*B)*x*x^m)/((m*n + n)*
c*d^3*x^n + (m*n + n)*c^2*d^2)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)}{(c+d\,x^n)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
(e**m*( - x**(m + n)*a**2*d*m*x + x**(m + n)*a**2*d*n*x - x**(m + n)*a**2*
d*x + 2*x**(m + n)*a*b*c*m*x + 2*x**(m + n)*a*b*c*x + x**m*a**2*c*m*x + x*
*m*a**2*c*n*x + x**m*a**2*c*x + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x
**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d
 + c**2*m + c**2*n + c**2),x)*a**2*d**3*m**3 + 3*x**n*int(x**(m + 2*n)/(x**
(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d
*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**2*d**3*m**2 - x**n*int(x**
(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*
m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a**2*d**3*m*n**
2 + 3*x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*
d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),
x)*a**2*d**3*m - x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n
 + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2
*n + c**2),x)*a**2*d**3*n**2 + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x*
*(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d +
 c**2*m + c**2*n + c**2),x)*a**2*d**3 - 2*x**n*int(x**(m + 2*n)/(x**(2*n)*
d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2
*x**n*c*d + c**2*m + c**2*n + c**2),x)*a*b*c*d**2*m**3 - 2*x**n*int(x**(m
 + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m +
 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a*b*c*d**2*m**2...
```

3.50 $\int \frac{(ex)^m(A+Bx^n)}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}}{cden(c+dx^n)} + \frac{(Bc(1+m)-Ad(1+m-n))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c^2de(1+m)n}$$

output
```
-(-A*d+B*c)*(e*x)^(1+m)/c/d/e/n/(c+d*x^n)+(B*c*(1+m)-A*d*(1+m-n))*(e*x)^(1
+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/d/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2}\,dx = \frac{x(ex)^m\left(Bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)+(-Bc+Ad)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+n}{n}\right.\right.}{c^2d(1+m)}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*(B*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c
)] + (-(B*c) + A*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^
n)/c)]))/(c^2*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2} dx$$

$$\Big\downarrow \; 957$$

$$\frac{(Bc(m+1)-Ad(m-n+1))\int \frac{(ex)^m}{dx^n+c}dx}{cdn}-\frac{(ex)^{m+1}(Bc-Ad)}{cden\,(c+dx^n)}$$

$$\Big\downarrow \; 888$$

$$\frac{(ex)^{m+1}(Bc(m+1)-Ad(m-n+1))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{c^2de(m+1)n}-\frac{(ex)^{m+1}(Bc-Ad)}{cden\,(c+dx^n)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
-(((B*c - A*d)*(e*x)^(1 + m))/(c*d*e*n*(c + d*x^n))) + ((B*c*(1 + m) - A*d
*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((d*x^n)/c)])/(c^2*d*e*(1 + m)*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m (A + B\,x^n)}{(c + d\,x^n)^2} dx$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^2}\, dx = \int \frac{(Bx^n + A)(ex)^m}{(dx^n + c)^2}\, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.94 (sec) , antiderivative size = 2382, normalized size of antiderivative = 22.26

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^2} dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(c+d*x**n)**2,x)
```

output
```
A*(-c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m**2*x**(m + 1)*lerchphi(d*x
**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n +
1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n -
 2 - 1/n)*e**m*m*n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n +
1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n
 + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m*n*x**(m + 1)*ga
mma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 +
1/n)) - 2*c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*m*x**(m + 1)*lerchphi(
d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n
 + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/
n - 2 - 1/n)*e**m*n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n +
 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/
n + 1 + 1/n)) + c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*n*x**(m + 1)*gam
ma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1
/n)) - c*c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*e**m*x**(m + 1)*lerchphi(d*x**
n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1
+ 1/n) + d*n**3*x**n*gamma(m/n + 1 + 1/n)) - c**(m/n + 1/n)*c**(-m/n - 2 -
 1/n)*d*e**m*m**2*x**n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/
n + 1/n)*gamma(m/n + 1/n)/(c*n**3*gamma(m/n + 1 + 1/n) + d*n**3*x**n*gamma
(m/n + 1 + 1/n)) + c**(m/n + 1/n)*c**(-m/n - 2 - 1/n)*d*e**m*m*n*x**n*x...
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
-(B*c*e^m - A*d*e^m)*x*x^m/(c*d^2*n*x^n + c^2*d*n) - (A*d*e^m*(m - n + 1)
- B*c*e^m*(m + 1))*integrate(x^m/(c*d^2*n*x^n + c^2*d*n), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(c+d\,x^n)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^2} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
(e**m*( - x**(m + n)*a*d*m*x + x**(m + n)*a*d*n*x - x**(m + n)*a*d*x + x**
(m + n)*b*c*m*x + x**(m + n)*b*c*x + x**m*a*c*m*x + x**m*a*c*n*x + x**m*a*
c*x + x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*
d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),
x)*a*d**3*m**3 + 3*x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*
n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c*
*2*n + c**2),x)*a*d**3*m**2 - x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**
(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d +
c**2*m + c**2*n + c**2),x)*a*d**3*m*n**2 + 3*x**n*int(x**(m + 2*n)/(x**(2*
n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n
+ 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a*d**3*m - x**n*int(x**(m + 2*n)
/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*d*m + 2*x**
n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a*d**3*n**2 + x**n*int(x
**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x**n*c*
d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*a*d**3 - x**n
*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*n)*d**2 + 2*x
**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**2),x)*b*c*d**
2*m**3 - x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(2*n)*d**2*n + x**(2*
n)*d**2 + 2*x**n*c*d*m + 2*x**n*c*d*n + 2*x**n*c*d + c**2*m + c**2*n + c**
2),x)*b*c*d**2*m**2*n - 3*x**n*int(x**(m + 2*n)/(x**(2*n)*d**2*m + x**(...
```

3.51 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 211

$$\int \frac{(ex)^m\left(A+Bx^n\right)}{\left(a+bx^n\right)\left(c+dx^n\right)^2}\,dx=\frac{(Bc-Ad)(ex)^{1+m}}{c(bc-ad)en\left(c+dx^n\right)}+\frac{b(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^2e(1+m)}+\frac{(bc(Ad(1+m-2n)-Bc(1+m-n))+ad(Bc(1+m)-Ad(1+m-n)))(ex)^{1+m}\,\text{Hypergeometric}}{c^2(bc-ad)^2e(1+m)n}$$

output
```
(-A*d+B*c)*(e*x)^(1+m)/c/(-a*d+b*c)/e/n/(c+d*x^n)+b*(A*b-B*a)*(e*x)^(1+m)*
hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)^2/e/(1+m)+(b*c*(
A*d*(1+m-2*n)-B*c*(1+m-n))+a*d*(B*c*(1+m)-A*d*(1+m-n)))*(e*x)^(1+m)*hyperg
eom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/(-a*d+b*c)^2/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx$$
$$= \frac{x(ex)^m \left(b(Ab-aB)c^2 \,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)+a(-Ab+aB)cd\,\text{Hypergeometric2F}\right.}{ac^2(bc-ad)^2(1}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
(x*(e*x)^m*(b*(A*b - a*B)*c^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, -((b*x^n)/a)] + a*(-(A*b) + a*B)*c*d*Hypergeometric2F1[1, (1 + m)/n, (1
 + m + n)/n, -((d*x^n)/c)] + a*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2
, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(a*c^2*(b*c - a*d)^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1065, 25, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx$$
$$\Big\downarrow \text{1065}$$
$$\frac{\int -\frac{(ex)^m (b(Bc-Ad)(m-n+1)x^n+a(Bc(m+1)-Ad(m-n+1))-Abcn)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)}{cen(bc-ad)(c+dx^n)}$$
$$\Big\downarrow \text{25}$$
$$\frac{(ex)^{m+1}(Bc-Ad)}{cen(bc-ad)(c+dx^n)}-\frac{\int \frac{(ex)^m (b(Bc-Ad)(m-n+1)x^n+aBc(m+1)-aAd(m-n+1)-Abcn)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}$$
$$\Big\downarrow \text{1067}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{cen(bc-ad)\left(c+dx^n\right)}-\frac{\int \left(\frac{(-bc(Ad(m-2n+1)-Bc(m-n+1))-ad(Bc(m+1)-Ad(m-n+1)))(ex)^m}{(bc-ad)(dx^n+c)}-\frac{b(Ab-aB)cn(ex)^m}{(bc-ad)(bx^n+a)}\right)dx}{cn(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{cen(bc-ad)\left(c+dx^n\right)}-\frac{-\frac{bcn(ex)^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)(ad(Bc(m+1)-}{ce(m+1)(bc-ad)}}{cn(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^2),x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m))/(c*(b*c - a*d)*e*n*(c + d*x^n)) - (-((b*(A*b -
 a*B)*c*n*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((
b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m))) - ((b*c*(A*d*(1 + m - 2*n) - B*c*(1
 + m - n)) + a*d*(B*c*(1 + m) - A*d*(1 + m - n)))*(e*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 +
m)))/(c*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B\,x^n)}{(a + b\,x^n)\,(c + d\,x^n)^2}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)\,(c + dx^n)^2}\,dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^
(2*n) + (b*c^2 + 2*a*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(B*c*e^m - A*d*e^m)*x*x^m/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x
^n) - ((a*d^2*e^m*(m - n + 1) - b*c*d*e^m*(m - 2*n + 1))*A + (b*c^2*e^m*(m
 - n + 1) - a*c*d*e^m*(m + 1))*B)*integrate(x^m/(b^2*c^4*n - 2*a*b*c^3*d*n
 + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n + a^2*c*d^3*n)*x^n), x)
- (B*a*b*e^m - A*b^2*e^m)*integrate(x^m/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2
 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2} \, dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)\,(c+d\,x^n)^2} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^2),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^2} \, dx = e^m \left(\int \frac{x^m}{x^{2n}d^2 + 2x^n cd + c^2} dx \right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^2,x)
```

output
```
e**m*int(x**m/(x**(2*n)*d**2 + 2*x**n*c*d + c**2),x)
```

3.52 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 315

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^2}\,dx = \frac{d(Abc-2aBc+aAd)(ex)^{1+m}}{ac(bc-ad)^2en\,(c+dx^n)} + \frac{(Ab-aB)(ex)^{1+m}}{a(bc-ad)en\,(a+bx^n)\,(c+dx^n)} + \frac{b(aB(bc(1+m)-ad(1+m-2n))+Ab(ad(1+m-3n)-bc(1+m-n)))(ex)^{1+m}\,\text{Hypergeometri}}{a^2(bc-ad)^3e(1+m)n} - \frac{d(bc(Ad(1+m-3n)-Bc(1+m-2n))+ad(Bc(1+m)-Ad(1+m-n)))(ex)^{1+m}\,\text{Hypergeomet}}{c^2(bc-ad)^3e(1+m)n}$$

output
```
d*(A*a*d+A*b*c-2*B*a*c)*(e*x)^(1+m)/a/c/(-a*d+b*c)^2/e/n/(c+d*x^n)+(A*b-B*
a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)/(c+d*x^n)+b*(a*B*(b*c*(1+m)-a*d*
(1+m-2*n))+A*b*(a*d*(1+m-3*n)-b*c*(1+m-n)))*(e*x)^(1+m)*hypergeom([1, (1+m
)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^3/e/(1+m)/n-d*(b*c*(A*d*(1+m-3*n
)-B*c*(1+m-2*n))+a*d*(B*c*(1+m)-A*d*(1+m-n)))*(e*x)^(1+m)*hypergeom([1, (1
+m)/n],[(1+m+n)/n],-d*x^n/c)/c^2/(-a*d+b*c)^3/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.66

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2} dx$$
$$= \frac{x(ex)^m \left(\frac{b(bBc-2Abd+aBd) \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{bx^n}{a}\right)}{a} - \frac{d(bBc-2Abd+aBd) \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\right.}{c}\right.}{(bc - }$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output
```
(x*(e*x)^m*((b*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[1, (1 + m)/n, (
1 + m + n)/n, -((b*x^n)/a)])/a - (d*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometr
ic2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c + (b*(-(A*b) + a*B)*(-
(b*c) + a*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])
/a^2 - (d*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m +
 n)/n, -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*(1 + m))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1065, 25, 1065, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2} dx$$
$$\Big\downarrow \; 1065$$
$$\frac{(ex)^{m+1}(Ab - aB)}{aen(bc - ad)(a + bx^n)(c + dx^n)} - \frac{\int -\frac{(ex)^m(-((Ab-aB)d(m-2n+1)x^n)+aBc(m+1)-Abc(m-n+1)-aAdn)}{(bx^n+a)(dx^n+c)^2}dx}{an(bc - ad)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(ex)^m(-((Ab-aB)d(m-2n+1)x^n)+aBc(m+1)-A(bc(m-n+1)+adn))}{(bx^n+a)(dx^n+c)^2}dx}{an(bc-ad)}+\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)(c+dx^n)}$$

$$\Big\downarrow \text{ 1065}$$

$$\frac{\frac{\int \frac{(ex)^m\left(n\left(aBc(bc+ad)(m+1)-A\left(b^2(m-n+1)c^2+2abdnc+a^2d^2(m-n+1)\right)\right)-bd(Abc-2aBc+aAd)(m-n+1)nx^n\right)}{(bx^n+a)(dx^n+c)}dx}{cn(bc-ad)}+\frac{d(ex)^{m+1}(aAd-2aBc+Abc)}{ce(bc-ad)(c+dx^n)}}{an(bc-ad)}+\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)(c+dx^n)}$$

$$\Big\downarrow \text{ 1067}$$

$$\frac{\frac{\int \left(\frac{bc(aB(bc(m+1)-ad(m-2n+1))+Ab(ad(m-3n+1)-bc(m-n+1)))n(ex)^m}{(bc-ad)(bx^n+a)}+\frac{ad(-bc(Ad(m-3n+1)-Bc(m-2n+1))-ad(Bc(m+1)-Ad(m-n+1)))n(ex)^m}{(bc-ad)(dx^n+c)}\right)c}{cn(bc-ad)}}{an(bc-ad)}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)(c+dx^n)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{bcn(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)(Ab(ad(m-3n+1)-bc(m-n+1))+aB(bc(m+1)-ad(m-2n+1)))}{ae(m+1)(bc-ad)}-\frac{adn(ex)^{m+1}\text{Hypergeometric2F1}\left(\right.}{}}{cn(bc-ad)}}{an(bc-ad)}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)(a+bx^n)(c+dx^n)}$$

input

```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

output

```
((A*b - a*B)*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)*(c + d*x^n)) +
((d*(A*b*c - 2*a*B*c + a*A*d)*(e*x)^(1 + m))/(c*(b*c - a*d)*e*(c + d*x^n))
 + ((b*c*(a*B*(b*c*(1 + m) - a*d*(1 + m - 2*n)) + A*b*(a*d*(1 + m - 3*n) -
 b*c*(1 + m - n)))*n*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
+ n)/n, -((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m)) - (a*d*(b*c*(A*d*(1 + m -
 3*n) - B*c*(1 + m - 2*n)) + a*d*(B*c*(1 + m) - A*d*(1 + m - n)))*n*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(
b*c - a*d)*e*(1 + m)))/(c*(b*c - a*d)*n))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n)}{(a + b x^n)^2 (c + d x^n)^2} dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b
*d^2)*x^(3*n) + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2
*c*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)^2} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
((b^3*c*e^m*(m - n + 1) - a*b^2*d*e^m*(m - 3*n + 1))*A + (a^2*b*d*e^m*(m -
 2*n + 1) - a*b^2*c*e^m*(m + 1))*B)*integrate(-x^m/(a^2*b^3*c^3*n - 3*a^3*
b^2*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d
*n + 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - ((a*d^3*e^m*(m - n + 1) -
 b*c*d^2*e^m*(m - 3*n + 1))*A + (b*c^2*d*e^m*(m - 2*n + 1) - a*c*d^2*e^m*(
m + 1))*B)*integrate(-x^m/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n
 - a^3*c^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n -
a^3*c*d^4*n)*x^n), x) + (((b^2*c^2*e^m + a^2*d^2*e^m)*A - (a*b*c^2*e^m + a
^2*c*d*e^m)*B)*x*x^m - (2*B*a*b*c*d*e^m - (b^2*c*d*e^m + a*b*d^2*e^m)*A)*x
*e^(m*log(x) + n*log(x)))/(a^2*b^2*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n
 + (a*b^3*c^3*d*n - 2*a^2*b^2*c^2*d^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*
c^4*n - a^2*b^2*c^3*d*n - a^3*b*c^2*d^2*n + a^4*c*d^3*n)*x^n)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^2\,(c+d\,x^n)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^2),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^2}\, dx$$
$$= e^m \left(\int \frac{x^m}{x^{3n} b\, d^2 + x^{2n} a\, d^2 + 2x^{2n} bcd + 2x^n acd + x^n b\, c^2 + a\, c^2} dx \right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

output
```
e**m*int(x**m/(x**(3*n)*b*d**2 + x**(2*n)*a*d**2 + 2*x**(2*n)*b*c*d + 2*x*
*n*a*c*d + x**n*b*c**2 + a*c**2),x)
```

3.53 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^3(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 567

$$\int \frac{(ex)^m\left(A+Bx^n\right)}{\left(a+bx^n\right)^3\left(c+dx^n\right)^2}\,dx$$
$$= \frac{d(aBc(bc(1+m)-ad(1+m-6n))+A(abcd(1+m-6n)-b^2c^2(1+m-2n)-2a^2d^2n))\,(ex)^{1+m}}{2a^2c(bc-ad)^3en^2\left(c+dx^n\right)}$$
$$+\frac{(Ab-aB)(ex)^{1+m}}{2a(bc-ad)en\left(a+bx^n\right)^2\left(c+dx^n\right)}$$
$$+\frac{(aB(bc(1+m)-ad(1+m-3n))+Ab(ad(1+m-5n)-bc(1+m-2n)))(ex)^{1+m}}{2a^2(bc-ad)^2en^2\left(a+bx^n\right)\left(c+dx^n\right)}$$
$$+\frac{b(aB(2abcd(1+m)(1+m-3n)-b^2c^2(1+m)(1+m-n)-a^2d^2(1+m^2+m(2-5n)-5n+6n^2}{}$$
$$+\frac{d^2(bc(Ad(1+m-4n)-Bc(1+m-3n))+ad(Bc(1+m)-Ad(1+m-n)))(ex)^{1+m}\,\text{Hypergeome}}{c^2(bc-ad)^4e(1+m)n}$$

output
```
1/2*d*(a*B*c*(b*c*(1+m)-a*d*(1+m-6*n))+A*(a*b*c*d*(1+m-6*n)-b^2*c^2*(1+m-2
*n)-2*a^2*d^2*n))*(e*x)^(1+m)/a^2/c/(-a*d+b*c)^3/e/n^2/(c+d*x^n)+1/2*(A*b-
B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)^2/(c+d*x^n)+1/2*(a*B*(b*c*(1+m
)-a*d*(1+m-3*n))+A*b*(a*d*(1+m-5*n)-b*c*(1+m-2*n)))*(e*x)^(1+m)/a^2/(-a*d+
b*c)^2/e/n^2/(a+b*x^n)/(c+d*x^n)+1/2*b*(a*B*(2*a*b*c*d*(1+m)*(1+m-3*n)-b^2
*c^2*(1+m)*(1+m-n)-a^2*d^2*(1+m^2+m*(2-5*n)-5*n+6*n^2))+A*b*(b^2*c^2*(1+m^
2+m*(2-3*n)-3*n+2*n^2)-2*a*b*c*d*(1+m^2+m*(2-5*n)-5*n+4*n^2)+a^2*d^2*(1+m^
2+m*(2-7*n)-7*n+12*n^2)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-
b*x^n/a)/a^3/(-a*d+b*c)^4/e/(1+m)/n^2+d^2*(b*c*(A*d*(1+m-4*n)-B*c*(1+m-3*n
))+a*d*(B*c*(1+m)-A*d*(1+m-n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n
)/n],-d*x^n/c)/c^2/(-a*d+b*c)^4/e/(1+m)/n
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.48

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2}\,dx = \frac{x(ex)^m \left(-\frac{bd(2bBc-3Abd+aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a}+\frac{d^2(2bBc-3Abd+aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+}{}\right.}{c}\right.}{}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)^2),x]
```

output
```
(x*(e*x)^m*(-((b*d*(2*b*B*c - 3*A*b*d + a*B*d)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, -((b*x^n)/a)])/a) + (d^2*(2*b*B*c - 3*A*b*d + a*B*d)*H
ypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c + (b*(b*c -
 a*d)*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n
)/n, -((b*x^n)/a)])/a^2 + (d^2*(b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[2
, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^2 + (b*(A*b - a*B)*(b*c - a*d
)^2*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)])/a^3))/((
b*c - a*d)^4*(1 + m))
```

Rubi [A] (verified)

Time = 3.26 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1065, 25, 1065, 1065, 25, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2} \, dx$$

$$\Big\downarrow \text{1065}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)(a+bx^n)^2(c+dx^n)}-\frac{\int -\frac{(ex)^m(-((Ab-aB)d(m-3n+1)x^n)+aBc(m+1)-Abc(m-2n+1)-2aAdn)}{(bx^n+a)^2(dx^n+c)^2}dx}{2an(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m(-((Ab-aB)d(m-3n+1)x^n)+aBc(m+1)-A(bc(m-2n+1)+2adn))}{(bx^n+a)^2(dx^n+c)^2}dx}{2an(bc-ad)}+\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)(a+bx^n)^2(c+dx^n)}$$

$$\Big\downarrow \text{1065}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-5n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-3n+1)))}{aen(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\int \frac{(ex)^m(d(aB(bc(m+1)-ad(m-3n+1))+Ab(ad(m-5n+1)-bc(m-2n\cdots}{\cdots}}{\cdots}}{2an(bc-a\cdots}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)(a+bx^n)^2(c+dx^n)}$$

$$\Big\downarrow \text{1065}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-5n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-3n+1)))}{aen(bc-ad)(a+bx^n)(c+dx^n)}-\frac{\int -\frac{(ex)^m\left(n\left(aBc(m+1)\left(-b^2(m-n+1)c^2+abd(m-5n+1)c-2a^2d^2n\right)\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)(a+bx^n)^2(c+dx^n)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-5n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-3n+1)))}{aen(bc-ad)(a+bx^n)(c+dx^n)} - \frac{-\frac{\int \frac{(ex)^m\left(n\left(aBc(m+1)\left(-b^2(m-n+1)c^2+abd(m-5n+1)c-2a^2d^2n\right)\right.\right.}{}}{}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2\left(c+dx^n\right)}$$

$\Big\downarrow$ 1067

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-5n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-3n+1)))}{aen(bc-ad)(a+bx^n)(c+dx^n)} - \frac{-\frac{\int \left(\frac{bcn\left(aB\left(-b^2(m+1)(m-n+1)c^2+2abd(m+1)(m-3n+1)c-a^2d^2\right.\right.}{}\right.}{}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2\left(c+dx^n\right)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{(ex)^{m+1}(Ab(ad(m-5n+1)-bc(m-2n+1))+aB(bc(m+1)-ad(m-3n+1)))}{aen(bc-ad)(a+bx^n)(c+dx^n)} - \frac{-\frac{bcn(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bx^n}{a}\right)\left(Ab\left(a^2d\right.\right.}{}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{2aen(bc-ad)\left(a+bx^n\right)^2\left(c+dx^n\right)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)^2),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(2*a*(b*c - a*d)*e*n*(a + b*x^n)^2*(c + d*x^n)
) + (((a*B*(b*c*(1 + m) - a*d*(1 + m - 3*n)) + A*b*(a*d*(1 + m - 5*n) - b*
c*(1 + m - 2*n)))*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)*(c + d*x^n
)) - (-((d*(a*B*c*(b*c*(1 + m) - a*d*(1 + m - 6*n)) + A*(a*b*c*d*(1 + m -
6*n) - b^2*c^2*(1 + m - 2*n) - 2*a^2*d^2*n))*(e*x)^(1 + m))/(c*(b*c - a*d)
*e*(c + d*x^n))) - ((b*c*n*(a*B*(2*a*b*c*d*(1 + m)*(1 + m - 3*n) - b^2*c^2
*(1 + m)*(1 + m - n) - a^2*d^2*(1 + m^2 + m*(2 - 5*n) - 5*n + 6*n^2)) + A*
b*(b^2*c^2*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 2*a*b*c*d*(1 + m^2 + m*
(2 - 5*n) - 5*n + 4*n^2) + a^2*d^2*(1 + m^2 + m*(2 - 7*n) - 7*n + 12*n^2))
)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) + (2*a^2*d^2*(b*c*(A*d*(1 + m - 4*n) - B*c*(
1 + m - 3*n)) + a*d*(B*c*(1 + m) - A*d*(1 + m - n)))*n^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*
e*(1 + m)))/(c*(b*c - a*d)*n))/(a*(b*c - a*d)*n))/(2*a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A+B\,x^n)}{(a+b\,x^n)^3 (c+d\,x^n)^2}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)^3(dx^n+c)^2}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b^3*d^2*x^(5*n) + a^3*c^2 + (2*b^3*c*d + 3*a
*b^2*d^2)*x^(4*n) + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^(3*n) + (3*a*b
^2*c^2 + 6*a^2*b*c*d + a^3*d^2)*x^(2*n) + (3*a^2*b*c^2 + 2*a^3*c*d)*x^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)**3/(c+d*x**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)^2} dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^3 (dx^n + c)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^4*c^2*e^m - 2*(m^2 - m*(5*n - 2)
 + 4*n^2 - 5*n + 1)*a*b^3*c*d*e^m + (m^2 - m*(7*n - 2) + 12*n^2 - 7*n + 1)
*a^2*b^2*d^2*e^m)*A - ((m^2 - m*(n - 2) - n + 1)*a*b^3*c^2*e^m - 2*(m^2 -
m*(3*n - 2) - 3*n + 1)*a^2*b^2*c*d*e^m + (m^2 - m*(5*n - 2) + 6*n^2 - 5*n
+ 1)*a^3*b*d^2*e^m)*B)*integrate(1/2*x^m/(a^3*b^4*c^4*n^2 - 4*a^4*b^3*c^3*
d*n^2 + 6*a^5*b^2*c^2*d^2*n^2 - 4*a^6*b*c*d^3*n^2 + a^7*d^4*n^2 + (a^2*b^5
*c^4*n^2 - 4*a^3*b^4*c^3*d*n^2 + 6*a^4*b^3*c^2*d^2*n^2 - 4*a^5*b^2*c*d^3*n
^2 + a^6*b*d^4*n^2)*x^n), x) - ((a*d^4*e^m*(m - n + 1) - b*c*d^3*e^m*(m -
4*n + 1))*A + (b*c^2*d^2*e^m*(m - 3*n + 1) - a*c*d^3*e^m*(m + 1))*B)*integ
rate(x^m/(b^4*c^6*n - 4*a*b^3*c^5*d*n + 6*a^2*b^2*c^4*d^2*n - 4*a^3*b*c^3*
d^3*n + a^4*c^2*d^4*n + (b^4*c^5*d*n - 4*a*b^3*c^4*d^2*n + 6*a^2*b^2*c^3*d
^3*n - 4*a^3*b*c^2*d^4*n + a^4*c*d^5*n)*x^n), x) - 1/2*(((a*b^3*c^3*e^m*(m
 - 3*n + 1) - a^2*b^2*c^2*d*e^m*(m - 7*n + 1) + 2*a^4*d^3*e^m*n)*A - (a^2*
b^2*c^3*e^m*(m - n + 1) - a^3*b*c^2*d*e^m*(m - 5*n + 1) + 2*a^4*c*d^2*e^m*
n)*B)*x*x^m + ((b^4*c^2*d*e^m*(m - 2*n + 1) - a*b^3*c*d^2*e^m*(m - 6*n + 1
) + 2*a^2*b^2*d^3*e^m*n)*A + (a^2*b^2*c*d^2*e^m*(m - 6*n + 1) - a*b^3*c^2*
d*e^m*(m + 1))*B)*x*e^(m*log(x) + 2*n*log(x)) + ((b^4*c^3*e^m*(m - 2*n + 1
) - a^2*b^2*c*d^2*e^m*(m - 7*n + 1) + 3*a*b^3*c^2*d*e^m*n + 4*a^3*b*d^3*e^
m*n)*A + (a^3*b*c*d^2*e^m*(m - 9*n + 1) - a*b^3*c^3*e^m*(m + 1) - 3*a^2*b^
2*c^2*d*e^m*n)*B)*x*e^(m*log(x) + n*log(x)))/(a^4*b^3*c^5*n^2 - 3*a^5*b...
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^3 (c + dx^n)^2} dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^3 (dx^n + c)^2} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)^3*(d*x^n + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^3\,(c+d\,x^n)^2}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)^2),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^3*(c + d*x^n)^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^3 (c+dx^n)^2}\,dx = e^m \left(\int \frac{x^m}{x^{4n}b^2d^2+2x^{3n}ab\,d^2+2x^{3n}b^2cd+x^{2n}a^2d^2+4x^{2n}abcd+x^{2n}b^2c^2+2x^na^2cd+2x^nab\,c^2+a^2c^2}dx\right.$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^3/(c+d*x^n)^2,x)
```

output
```
e**m*int(x**m/(x**(4*n)*b**2*d**2 + 2*x**(3*n)*a*b*d**2 + 2*x**(3*n)*b**2*
c*d + x**(2*n)*a**2*d**2 + 4*x**(2*n)*a*b*c*d + x**(2*n)*b**2*c**2 + 2*x**
n*a**2*c*d + 2*x**n*a*b*c**2 + a**2*c**2),x)
```

3.54 $\int \frac{(ex)^m(a+bx^n)^2(A+Bx^n)}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 322

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3}\,dx = \frac{b(ad(1+m)-bc(1+m+n))(Ad(1+m)-Bc(1+m+2n))(ex)^{1+m}}{2c^2d^3e(1+m)n^2} - \frac{(Bc-Ad)(ex)^{1+m}(a+bx^n)^2}{2cden(c+dx^n)^2} - \frac{(bc-ad)(ex)^{1+m}(a(Bc(1+m)-Ad(1+m-2n))-b(Ad(1+m)-Bc(1+m+2n))x^n)}{2c^2d^2en^2(c+dx^n)} + \frac{(ad(Bc(1+m)-Ad(1+m-2n))(bc(1+m)-ad(1+m-n))-bc(ad(1+m)-bc(1+m+n))(\ldots}{2c^3d^3e(1+m)n^2}$$

output
```
1/2*b*(a*d*(1+m)-b*c*(1+m+n))*(A*d*(1+m)-B*c*(1+m+2*n))*(e*x)^(1+m)/c^2/d^
3/e/(1+m)/n^2-1/2*(-A*d+B*c)*(e*x)^(1+m)*(a+b*x^n)^2/c/d/e/n/(c+d*x^n)^2-1
/2*(-a*d+b*c)*(e*x)^(1+m)*(a*(B*c*(1+m)-A*d*(1+m-2*n))-b*(A*d*(1+m)-B*c*(1
+m+2*n))*x^n)/c^2/d^2/e/n^2/(c+d*x^n)+1/2*(a*d*(B*c*(1+m)-A*d*(1+m-2*n))*(
b*c*(1+m)-a*d*(1+m-n))-b*c*(a*d*(1+m)-b*c*(1+m+n))*(A*d*(1+m)-B*c*(1+m+2*n
)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^3/d^3/e/(1+
m)/n^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.53

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3}\,dx$$
$$= \frac{x(ex)^m \left(b^2B - \frac{b(3bBc-Abd-2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c} + \frac{(bc-ad)(3bBc-2Abd-aBd)\,\text{Hypergeometric2F1}}{c^2}\right.}{d^3(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n)^3,x]
```

output
```
(x*(e*x)^m*(b^2*B - (b*(3*b*B*c - A*b*d - 2*a*B*d)*Hypergeometric2F1[1, (1
 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c + ((b*c - a*d)*(3*b*B*c - 2*A*b*d
 - a*B*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^
2 - ((b*c - a*d)^2*(B*c - A*d)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)
/n, -((d*x^n)/c)])/c^3))/(d^3*(1 + m))
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1064, 25, 1064, 25, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3}\,dx$$
$$\Big\downarrow \ 1064$$
$$-\frac{\int -\frac{(ex)^m (bx^n+a)(a(Bc(m+1)-Ad(m-2n+1))-b(Ad(m+1)-Bc(m+2n+1))x^n)}{(dx^n+c)^2}dx}{2cdn} - \frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$
$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m(bx^n+a)(a(Bc(m+1)-Ad(m-2n+1))-b(Ad(m+1)-Bc(m+2n+1))x^n)}{(dx^n+c)^2}dx}{2cdn}-\frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$

$\Big\downarrow$ 1064

$$\frac{-\frac{\int -\frac{(ex)^m(b(ad(m+1)-bc(m+n+1))(Ad(m+1)-Bc(m+2n+1))x^n+a(Bc(m+1)-Ad(m-2n+1))(bc(m+1)-ad(m-n+1)))}{dx^n+c}dx}{cdn}-\frac{(ex)^{m+1}(bc-ad)(a(Bc(}{}}{2cdn}-\frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{(ex)^m(b(ad(m+1)-bc(m+n+1))(Ad(m+1)-Bc(m+2n+1))x^n+a(Bc(m+1)-Ad(m-2n+1))(bc(m+1)-ad(m-n+1)))}{dx^n+c}dx}{cdn}-\frac{(ex)^{m+1}(bc-ad)(a(Bc(m+}{}}{2cdn}-\frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$

$\Big\downarrow$ 959

$$\frac{\frac{\left(a(bc(m+1)-ad(m-n+1))(Bc(m+1)-Ad(m-2n+1))-\frac{bc(ad(m+1)-bc(m+n+1))(Ad(m+1)-Bc(m+2n+1))}{d}\right)\int \frac{(ex)^m}{dx^n+c}dx+\frac{b(ex)^{m+1}(ad(m+1)-bc(}{}}{cdn}}{2cdn}-\frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$

$\Big\downarrow$ 888

$$\frac{\frac{\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)\left(a(bc(m+1)-ad(m-n+1))(Bc(m+1)-Ad(m-2n+1))-\frac{bc(ad(m+1)-bc(m+n+1))(Ad(m+1)-Bc(m+2}{d}\right.}{ce(m+1)}}{cdn}}{}-\frac{(ex)^{m+1}(a+bx^n)^2(Bc-Ad)}{2cden(c+dx^n)^2}$$

input

```
Int[((e*x)^m*(a + b*x^n)^2*(A + B*x^n))/(c + d*x^n)^3,x]
```

output
```
-1/2*((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^n)^2)/(c*d*e*n*(c + d*x^n)^2) + (
-(((b*c - a*d)*(e*x)^(1 + m)*(a*(B*c*(1 + m) - A*d*(1 + m - 2*n)) - b*(A*d
*(1 + m) - B*c*(1 + m + 2*n))*x^n))/(c*d*e*n*(c + d*x^n))) + ((b*(a*d*(1 +
 m) - b*c*(1 + m + n))*(A*d*(1 + m) - B*c*(1 + m + 2*n))*(e*x)^(1 + m))/(d
*e*(1 + m)) + ((a*(B*c*(1 + m) - A*d*(1 + m - 2*n))*(b*c*(1 + m) - a*d*(1
+ m - n)) - (b*c*(a*d*(1 + m) - b*c*(1 + m + n))*(A*d*(1 + m) - B*c*(1 + m
 + 2*n)))/d)*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
-((d*x^n)/c)])/(c*e*(1 + m)))/(c*d*n))/(2*c*d*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^2 (A+B\,x^n)}{(c+d\,x^n)^3}dx$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^2(ex)^m}{(dx^n+c)^3}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*b^2*x^(3*n) + A*a^2 + (2*B*a*b + A*b^2)*x^(2*n) + (B*a^2 + 2*A
*a*b)*x^n)*(e*x)^m/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*x**n)**2*(A+B*x**n)/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a + bx^n)^2 (A + Bx^n)}{(c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(bx^n + a)^2 (ex)^m}{(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
(((m^2 + m*(n + 2) + n + 1)*b^2*c^2*d*e^m - 2*(m^2 - m*(n - 2) - n + 1)*a*
b*c*d^2*e^m + (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*a^2*d^3*e^m)*A - ((m^2
 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^3*e^m - 2*(m^2 + m*(n + 2) + n + 1
)*a*b*c^2*d*e^m + (m^2 - m*(n - 2) - n + 1)*a^2*c*d^2*e^m)*B)*integrate(1/
2*x^m/(c^2*d^4*n^2*x^n + c^3*d^3*n^2), x) + 1/2*(2*B*b^2*c^2*d^2*e^m*n^2*x
*e^(m*log(x) + 2*n*log(x)) - (((m^2 + m*(n + 2) + n + 1)*b^2*c^3*d*e^m - 2
*(m^2 - m*(n - 2) - n + 1)*a*b*c^2*d^2*e^m + (m^2 - m*(3*n - 2) - 3*n + 1)
*a^2*c*d^3*e^m)*A - ((m^2 + m*(3*n + 2) + 2*n^2 + 3*n + 1)*b^2*c^4*e^m - 2
*(m^2 + m*(n + 2) + n + 1)*a*b*c^3*d*e^m + (m^2 - m*(n - 2) - n + 1)*a^2*c
^2*d^2*e^m)*B)*x*x^m - (((m^2 + 2*m*(n + 1) + 2*n + 1)*b^2*c^2*d^2*e^m - 2
*(m^2 + 2*m + 1)*a*b*c*d^3*e^m + (m^2 - 2*m*(n - 1) - 2*n + 1)*a^2*d^4*e^m
)*A - ((m^2 + 2*m*(2*n + 1) + 4*n^2 + 4*n + 1)*b^2*c^3*d*e^m - 2*(m^2 + 2*
m*(n + 1) + 2*n + 1)*a*b*c^2*d^2*e^m + (m^2 + 2*m + 1)*a^2*c*d^3*e^m)*B)*x
*e^(m*log(x) + n*log(x)))/((m*n^2 + n^2)*c^2*d^5*x^(2*n) + 2*(m*n^2 + n^2)
*c^3*d^4*x^n + (m*n^2 + n^2)*c^4*d^3)
```

Giac [F]

$$\int \frac{(ex)^m (a + bx^n)^2 (A + Bx^n)}{(c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(bx^n + a)^2 (ex)^m}{(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^2*(e*x)^m/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3} \, dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)^2}{(c+d\,x^n)^3} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n)^3,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^2)/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^2 (A+Bx^n)}{(c+dx^n)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^2*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
(e**m*(x**(m + 2*n)*b**3*d**2*m**2*x - 3*x**(m + 2*n)*b**3*d**2*m*n*x + 2*
x**(m + 2*n)*b**3*d**2*m*x + 2*x**(m + 2*n)*b**3*d**2*n**2*x - 3*x**(m + 2
*n)*b**3*d**2*n*x + x**(m + 2*n)*b**3*d**2*x + 3*x**(m + n)*a*b**2*d**2*m*
*2*x - 6*x**(m + n)*a*b**2*d**2*m*n*x + 6*x**(m + n)*a*b**2*d**2*m*x - 6*x
**(m + n)*a*b**2*d**2*n*x + 3*x**(m + n)*a*b**2*d**2*x - x**(m + n)*b**3*c
*d*m**2*x - 2*x**(m + n)*b**3*c*d*m*x + 4*x**(m + n)*b**3*c*d*n**2*x - x**
(m + n)*b**3*c*d*x + 3*x**m*a**2*b*d**2*m**2*x - 3*x**m*a**2*b*d**2*m*n*x
+ 6*x**m*a**2*b*d**2*m*x - 3*x**m*a**2*b*d**2*n*x + 3*x**m*a**2*b*d**2*x -
 3*x**m*a*b**2*c*d*m**2*x - 3*x**m*a*b**2*c*d*m*n*x - 6*x**m*a*b**2*c*d*m*
x - 3*x**m*a*b**2*c*d*n*x - 3*x**m*a*b**2*c*d*x + x**m*b**3*c**2*m**2*x +
3*x**m*b**3*c**2*m*n*x + 2*x**m*b**3*c**2*m*x + 2*x**m*b**3*c**2*n**2*x +
3*x**m*b**3*c**2*n*x + x**m*b**3*c**2*x + x**(2*n)*int(x**m/(x**(3*n)*d**3
*m**2 - 3*x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m + 2*x**(3*n)*d**3*n**2 - 3
*x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*m**2 - 9*x**(2*n)*c*d
**2*m*n + 6*x**(2*n)*c*d**2*m + 6*x**(2*n)*c*d**2*n**2 - 9*x**(2*n)*c*d**2
*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m**2 - 9*x**n*c**2*d*m*n + 6*x**n*c
**2*d*m + 6*x**n*c**2*d*n**2 - 9*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*m**2
 - 3*c**3*m*n + 2*c**3*m + 2*c**3*n**2 - 3*c**3*n + c**3),x)*a**3*d**5*m**
5 - 6*x**(2*n)*int(x**m/(x**(3*n)*d**3*m**2 - 3*x**(3*n)*d**3*m*n + 2*x**(
3*n)*d**3*m + 2*x**(3*n)*d**3*n**2 - 3*x**(3*n)*d**3*n + x**(3*n)*d**3 ...
```

3.55 $\int \frac{(ex)^m(a+bx^n)(A+Bx^n)}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 228

$$\int \frac{(ex)^m (a+bx^n)(A+Bx^n)}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)(ex)^{1+m}(A+Bx^n)}{2cden(c+dx^n)^2} - \frac{(ad(Ad(1+m-2n)-Bc(1+m-n))-bc(Ad(1+m)-Bc(1+m+n)))(ex)^{1+m}}{2c^2d^2en^2(c+dx^n)} - \frac{(Ad(bc(1+m)-ad(1+m-2n))(1+m-n)+Bc(1+m)(ad(1+m-n)-bc(1+m+n)))(ex)^{1}}{2c^3d^2e(1+m)n^2}$$

output
```
-1/2*(-a*d+b*c)*(e*x)^(1+m)*(A+B*x^n)/c/d/e/n/(c+d*x^n)^2-1/2*(a*d*(A*d*(1
+m-2*n)-B*c*(1+m-n))-b*c*(A*d*(1+m)-B*c*(1+m+n)))*(e*x)^(1+m)/c^2/d^2/e/n^
2/(c+d*x^n)-1/2*(A*d*(b*c*(1+m)-a*d*(1+m-2*n))*(1+m-n)+B*c*(1+m)*(a*d*(1+m
-n)-b*c*(1+m+n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)
/c^3/d^2/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx = \frac{x(ex)^m \left(bBc^2 \text{Hypergeometric2F1}\left(1, \frac{1+m}{n}, \frac{1+m+n}{n}, -\frac{dx^n}{c}\right) + c(-2bBc+Abd+aBd) \text{Hypergeometric2F}}{c^3d^2(1+m)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n)^3,x]
```

output
```
(x*(e*x)^m*(b*B*c^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^
n)/c)] + c*(-2*b*B*c + A*b*d + a*B*d)*Hypergeometric2F1[2, (1 + m)/n, (1 +
 m + n)/n, -((d*x^n)/c)] + (b*c - a*d)*(B*c - A*d)*Hypergeometric2F1[3, (1
 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(c^3*d^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1064, 25, 957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx$$

$$\Big\downarrow \text{1064}$$

$$-\frac{\int -\frac{(ex)^m (A(bc(m+1)-ad(m-2n+1))-B(ad(m-n+1)-bc(m+n+1))x^n)}{(dx^n+c)^2}dx}{2cdn} - \frac{(ex)^{m+1}(bc-ad)(A+Bx^n)}{2cden(c+dx^n)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(ex)^m (A(bc(m+1)-ad(m-2n+1))-B(ad(m-n+1)-bc(m+n+1))x^n)}{(dx^n+c)^2}dx}{2cdn} - \frac{(ex)^{m+1}(bc-ad)(A+Bx^n)}{2cden(c+dx^n)^2}$$

$\big\downarrow$ 957

$$\frac{-\frac{(Ad(m-n+1)(bc(m+1)-ad(m-2n+1))+Bc(m+1)(ad(m-n+1)-bc(m+n+1)))\int \frac{(ex)^m}{dx^n+c}dx}{cdn}-\frac{(ex)^{m+1}(ad(Ad(m-2n+1)-Bc(m-n+1))-}{cden(c+dx^n)}}{2cdn}$$

$$\frac{(ex)^{m+1}(bc-ad)\left(A+Bx^n\right)}{2cden\left(c+dx^n\right)^2}$$

$\big\downarrow$ 888

$$\frac{-\frac{(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)(Ad(m-n+1)(bc(m+1)-ad(m-2n+1))+Bc(m+1)(ad(m-n+1)-bc(m+n+1)))}{c^2de(m+1)n}-\frac{(}{}}{2cdn}$$

$$\frac{(ex)^{m+1}(bc-ad)\left(A+Bx^n\right)}{2cden\left(c+dx^n\right)^2}$$

input

```
Int[((e*x)^m*(a + b*x^n)*(A + B*x^n))/(c + d*x^n)^3,x]
```

output

```
-1/2*((b*c - a*d)*(e*x)^(1 + m)*(A + B*x^n))/(c*d*e*n*(c + d*x^n)^2) + (-(
((a*d*(A*d*(1 + m - 2*n) - B*c*(1 + m - n)) - b*c*(A*d*(1 + m) - B*c*(1 +
m + n)))*(e*x)^(1 + m))/(c*d*e*n*(c + d*x^n))) - ((A*d*(b*c*(1 + m) - a*d*
(1 + m - 2*n))*(1 + m - n) + B*c*(1 + m)*(a*d*(1 + m - n) - b*c*(1 + m + n
)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)
/c)])/(c^2*d*e*(1 + m)*n))/(2*c*d*n)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Maple [F]

$$\int \frac{(ex)^{m} \left(a+b\,x^{n}\right) \left(A+B\,x^{n}\right)}{\left(c+d\,x^{n}\right)^{3}}dx$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{(dx^n+c)^3} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*b*x^(2*n) + A*a + (B*a + A*b)*x^n)*(e*x)^m/(d^3*x^(3*n) + 3*c*
d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a + bx^n) (A + Bx^n)}{(c + dx^n)^3} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*x**n)*(A+B*x**n)/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (a + bx^n) (A + Bx^n)}{(c + dx^n)^3} dx = \int \frac{(Bx^n + A)(bx^n + a)(ex)^m}{(dx^n + c)^3} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
-(((m^2 - m*(n - 2) - n + 1)*b*c*d*e^m - (m^2 - m*(3*n - 2) + 2*n^2 - 3*n
+ 1)*a*d^2*e^m)*A - ((m^2 + m*(n + 2) + n + 1)*b*c^2*e^m - (m^2 - m*(n - 2
) - n + 1)*a*c*d*e^m)*B)*integrate(1/2*x^m/(c^2*d^3*n^2*x^n + c^3*d^2*n^2)
, x) + 1/2*(((b*c^2*d*e^m*(m - n + 1) - a*c*d^2*e^m*(m - 3*n + 1))*A - (b*
c^3*e^m*(m + n + 1) - a*c^2*d*e^m*(m - n + 1))*B)*x*x^m - ((a*d^3*e^m*(m -
 2*n + 1) - b*c*d^2*e^m*(m + 1))*A + (b*c^2*d*e^m*(m + 2*n + 1) - a*c*d^2*
e^m*(m + 1))*B)*x*e^(m*log(x) + n*log(x)))/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^
3*n^2*x^n + c^4*d^2*n^2)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx = \int \frac{(Bx^n+A)(bx^n+a)(ex)^m}{(dx^n+c)^3} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)*(e*x)^m/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)}{(c+d\,x^n)^3} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n)^3,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n))/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n) (A+Bx^n)}{(c+dx^n)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
(e**m*( - x**(m + n)*a**2*d*m*x + 2*x**(m + n)*a**2*d*n*x - x**(m + n)*a**
2*d*x + 2*x**(m + n)*a*b*c*m*x + 2*x**(m + n)*a*b*c*x + x**m*a**2*c*m*x +
x**m*a**2*c*n*x + x**m*a**2*c*x + x**(2*n)*int(x**(m + 2*n)/(x**(3*n)*d**3
*m**2 + x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m + x**(3*n)*d**3*n + x**(3*n)
*d**3 + 3*x**(2*n)*c*d**2*m**2 + 3*x**(2*n)*c*d**2*m*n + 6*x**(2*n)*c*d**2
*m + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m**2 + 3*x**n
*c**2*d*m*n + 6*x**n*c**2*d*m + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*m**
2 + c**3*m*n + 2*c**3*m + c**3*n + c**3),x)*a**2*d**4*m**4 - 2*x**(2*n)*in
t(x**(m + 2*n)/(x**(3*n)*d**3*m**2 + x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m
 + x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*m**2 + 3*x**(2*n)*c
*d**2*m*n + 6*x**(2*n)*c*d**2*m + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2
+ 3*x**n*c**2*d*m**2 + 3*x**n*c**2*d*m*n + 6*x**n*c**2*d*m + 3*x**n*c**2*d
*n + 3*x**n*c**2*d + c**3*m**2 + c**3*m*n + 2*c**3*m + c**3*n + c**3),x)*a
**2*d**4*m**3*n + 4*x**(2*n)*int(x**(m + 2*n)/(x**(3*n)*d**3*m**2 + x**(3*
n)*d**3*m*n + 2*x**(3*n)*d**3*m + x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(
2*n)*c*d**2*m**2 + 3*x**(2*n)*c*d**2*m*n + 6*x**(2*n)*c*d**2*m + 3*x**(2*n
)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m**2 + 3*x**n*c**2*d*m*n +
6*x**n*c**2*d*m + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*m**2 + c**3*m*n +
 2*c**3*m + c**3*n + c**3),x)*a**2*d**4*m**3 - x**(2*n)*int(x**(m + 2*n)/(
x**(3*n)*d**3*m**2 + x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m + x**(3*n)*d...
```

3.56 $\int \frac{(ex)^m(A+Bx^n)}{(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^3}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}}{2cden\,(c+dx^n)^2} + \frac{(Bc(1+m)-Ad(1+m-2n))(ex)^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{2c^3de(1+m)n}$$

output

```
-1/2*(-A*d+B*c)*(e*x)^(1+m)/c/d/e/n/(c+d*x^n)^2+1/2*(B*c*(1+m)-A*d*(1+m-2*
n))*(e*x)^(1+m)*hypergeom([2, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^3/d/e/(1+m)
/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^3}\,dx = \frac{x(ex)^m \left(Bc\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right) + (-Bc+Ad)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{n},\frac{1+n}{n}\right.\right.}{c^3d(1+m)}$$

input

```
Integrate[((e*x)^m*(A + B*x^n))/(c + d*x^n)^3,x]
```

output
```
(x*(e*x)^m*(B*c*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c
)] + (-(B*c) + A*d)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((d*x^
n)/c)]))/(c^3*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {957, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(c+dx^n)^3} \, dx$$

$$\Big\downarrow \text{957}$$

$$\frac{(Bc(m+1)-Ad(m-2n+1))\int \frac{(ex)^m}{(dx^n+c)^2}dx}{2cdn} - \frac{(ex)^{m+1}(Bc-Ad)}{2cden\,(c+dx^n)^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}(Bc(m+1)-Ad(m-2n+1))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{2c^3de(m+1)n} - \frac{(ex)^{m+1}(Bc-Ad)}{2cden\,(c+dx^n)^2}$$

input
```
Int[((e*x)^m*(A + B*x^n))/(c + d*x^n)^3,x]
```

output
```
-1/2*((B*c - A*d)*(e*x)^(1 + m))/(c*d*e*n*(c + d*x^n)^2) + ((B*c*(1 + m) -
 A*d*(1 + m - 2*n))*(e*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/n, (1 + m +
 n)/n, -((d*x^n)/c)])/(2*c^3*d*e*(1 + m)*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 957
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && N
eQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1
, m, (-n)*(p + 1)]))
```

Maple [F]

$$\int \frac{(ex)^m\,(A+B\,x^n)}{(c+d\,x^n)^3}dx$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m\,(A+Bx^n)}{(c+dx^n)^3}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(dx^n+c)^3}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n
+ c^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 128.35 (sec) , antiderivative size = 8303, normalized size of antiderivative = 74.13

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^3} \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(c+d*x**n)**3,x)
```

output
```
A*(c**2*c**(m/n + 1/n)*c**(-m/n - 3 - 1/n)*e**m*m**3*x**(m + 1)*lerchphi(d
*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*c**2*n**4*gamma
(m/n + 1 + 1/n) + 4*c*d*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*d**2*n**4*x**(2
*n)*gamma(m/n + 1 + 1/n)) - 3*c**2*c**(m/n + 1/n)*c**(-m/n - 3 - 1/n)*e**m
*m**2*n*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*gamma(
m/n + 1/n)/(2*c**2*n**4*gamma(m/n + 1 + 1/n) + 4*c*d*n**4*x**n*gamma(m/n +
 1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - c**2*c**(m/n + 1/
n)*c**(-m/n - 3 - 1/n)*e**m*m**2*n*x**(m + 1)*gamma(m/n + 1/n)/(2*c**2*n**
4*gamma(m/n + 1 + 1/n) + 4*c*d*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*d**2*n**
4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 3*c**2*c**(m/n + 1/n)*c**(-m/n - 3 - 1/
n)*e**m*m**2*x**(m + 1)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, m/n + 1/n)*g
amma(m/n + 1/n)/(2*c**2*n**4*gamma(m/n + 1 + 1/n) + 4*c*d*n**4*x**n*gamma(
m/n + 1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) + 2*c**2*c**(m
/n + 1/n)*c**(-m/n - 3 - 1/n)*e**m*m*n**2*x**(m + 1)*lerchphi(d*x**n*exp_p
olar(I*pi)/c, 1, m/n + 1/n)*gamma(m/n + 1/n)/(2*c**2*n**4*gamma(m/n + 1 +
1/n) + 4*c*d*n**4*x**n*gamma(m/n + 1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(m
/n + 1 + 1/n)) + 3*c**2*c**(m/n + 1/n)*c**(-m/n - 3 - 1/n)*e**m*m*n**2*x**
(m + 1)*gamma(m/n + 1/n)/(2*c**2*n**4*gamma(m/n + 1 + 1/n) + 4*c*d*n**4*x*
*n*gamma(m/n + 1 + 1/n) + 2*d**2*n**4*x**(2*n)*gamma(m/n + 1 + 1/n)) - 6*c
**2*c**(m/n + 1/n)*c**(-m/n - 3 - 1/n)*e**m*m*n*x**(m + 1)*lerchphi(d*x...
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
-((m^2 - m*(n - 2) - n + 1)*B*c*e^m - (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1
)*A*d*e^m)*integrate(1/2*x^m/(c^2*d^2*n^2*x^n + c^3*d*n^2), x) + 1/2*((B*c
^2*e^m*(m - n + 1) - A*c*d*e^m*(m - 3*n + 1))*x*x^m - (A*d^2*e^m*(m - 2*n
+ 1) - B*c*d*e^m*(m + 1))*x*e^(m*log(x) + n*log(x)))/(c^2*d^3*n^2*x^(2*n)
+ 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/(d*x^n + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^3} \, dx = \int \frac{(e\,x)^m\,(A + B\,x^n)}{(c + d\,x^n)^3} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n)^3,x)
```

output
```
int(((e*x)^m*(A + B*x^n))/(c + d*x^n)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(c + dx^n)^3} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(A+B*x^n)/(c+d*x^n)^3,x)
```

output
```
(e**m*( - x**(m + n)*a*d*m*x + 2*x**(m + n)*a*d*n*x - x**(m + n)*a*d*x + x
**(m + n)*b*c*m*x + x**(m + n)*b*c*x + x**m*a*c*m*x + x**m*a*c*n*x + x**m*
a*c*x + x**(2*n)*int(x**(m + 2*n)/(x**(3*n)*d**3*m**2 + x**(3*n)*d**3*m*n
+ 2*x**(3*n)*d**3*m + x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*
m**2 + 3*x**(2*n)*c*d**2*m*n + 6*x**(2*n)*c*d**2*m + 3*x**(2*n)*c*d**2*n +
 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m**2 + 3*x**n*c**2*d*m*n + 6*x**n*c**2*
d*m + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*m**2 + c**3*m*n + 2*c**3*m +
c**3*n + c**3),x)*a*d**4*m**4 - 2*x**(2*n)*int(x**(m + 2*n)/(x**(3*n)*d**3
*m**2 + x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m + x**(3*n)*d**3*n + x**(3*n)
*d**3 + 3*x**(2*n)*c*d**2*m**2 + 3*x**(2*n)*c*d**2*m*n + 6*x**(2*n)*c*d**2
*m + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d*m**2 + 3*x**n
*c**2*d*m*n + 6*x**n*c**2*d*m + 3*x**n*c**2*d*n + 3*x**n*c**2*d + c**3*m**
2 + c**3*m*n + 2*c**3*m + c**3*n + c**3),x)*a*d**4*m**3*n + 4*x**(2*n)*int
(x**(m + 2*n)/(x**(3*n)*d**3*m**2 + x**(3*n)*d**3*m*n + 2*x**(3*n)*d**3*m
+ x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*c*d**2*m**2 + 3*x**(2*n)*c*
d**2*m*n + 6*x**(2*n)*c*d**2*m + 3*x**(2*n)*c*d**2*n + 3*x**(2*n)*c*d**2 +
 3*x**n*c**2*d*m**2 + 3*x**n*c**2*d*m*n + 6*x**n*c**2*d*m + 3*x**n*c**2*d*
n + 3*x**n*c**2*d + c**3*m**2 + c**3*m*n + 2*c**3*m + c**3*n + c**3),x)*a*
d**4*m**3 - x**(2*n)*int(x**(m + 2*n)/(x**(3*n)*d**3*m**2 + x**(3*n)*d**3*
m*n + 2*x**(3*n)*d**3*m + x**(3*n)*d**3*n + x**(3*n)*d**3 + 3*x**(2*n)*...
```

3.57 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 366

$$\begin{aligned}\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3}\,dx &= \frac{(Bc-Ad)(ex)^{1+m}}{2c(bc-ad)en\,(c+dx^n)^2} \\ &+ \frac{(bc(Ad(1+m-4n)-Bc(1+m-2n))+ad(Bc(1+m)-Ad(1+m-2n)))(ex)^{1+m}}{2c^2(bc-ad)^2en^2\,(c+dx^n)} \\ &+ \frac{b^2(Ab-aB)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a(bc-ad)^3e(1+m)} \\ &- \frac{(b^2c^2(Ad(1+m-3n)-Bc(1+m-n))(1+m-2n)-a^2d^2(Bc(1+m)-Ad(1+m-2n))(1+m}{}\end{aligned}$$

output
```
1/2*(-A*d+B*c)*(e*x)^(1+m)/c/(-a*d+b*c)/e/n/(c+d*x^n)^2+1/2*(b*c*(A*d*(1+m
-4*n)-B*c*(1+m-2*n))+a*d*(B*c*(1+m)-A*d*(1+m-2*n)))*(e*x)^(1+m)/c^2/(-a*d+
b*c)^2/e/n^2/(c+d*x^n)+b^2*(A*b-B*a)*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(
1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)^3/e/(1+m)-1/2*(b^2*c^2*(A*d*(1+m-3*n)-B*c
*(1+m-n))*(1+m-2*n)-a^2*d^2*(B*c*(1+m)-A*d*(1+m-2*n))*(1+m-n)+2*a*b*c*d*(B
*c*(1+m)*(1+m-2*n)-A*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)))*(e*x)^(1+m)*hypergeom
([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^3/(-a*d+b*c)^3/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.55

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3} dx$$
$$= \frac{x(ex)^m \left(\frac{b^2(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a} - \frac{b(Ab-aB)d\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{c} - \frac{(Ab-\right.}{(bc-ad)^3(1-}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^3),x]
```

output
```
(x*(e*x)^m*((b^2*(A*b - a*B)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n
, -((b*x^n)/a)])/a - (b*(A*b - a*B)*d*Hypergeometric2F1[1, (1 + m)/n, (1 +
 m + n)/n, -((d*x^n)/c)])/c - ((A*b - a*B)*d*(b*c - a*d)*Hypergeometric2F1
[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^2 + ((b*c - a*d)^2*(B*c - A
*d)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^3))/((
b*c - a*d)^3*(1 + m))
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1065, 25, 1065, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3} dx$$

$$\Big\downarrow \text{1065}$$

$$\frac{\int -\frac{(ex)^m(b(Bc-Ad)(m-2n+1)x^n+aBc(m+1)-aAd(m-2n+1)-2Abcn)}{(bx^n+a)(dx^n+c)^2}dx}{2cn(bc-ad)} + \frac{(ex)^{m+1}(Bc-Ad)}{2cen(bc-ad)(c+dx^n)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Bc-Ad)}{2cen(bc-ad)\left(c+dx^n\right)^2}-\frac{\int \frac{(ex)^m\left(b(Bc-Ad)(m-2n+1)x^n+aBc(m+1)-aAd(m-2n+1)-2Abcn\right)}{\left(bx^n+a\right)\left(dx^n+c\right)^2}dx}{2cn(bc-ad)}$$

$\downarrow$ 1065

$$\frac{(ex)^{m+1}(Bc-Ad)}{2cen(bc-ad)\left(c+dx^n\right)^2}-$$

$$\frac{\frac{\int \frac{(ex)^m(b(bc(Ad(m-4n+1)-Bc(m-2n+1))+ad(Bc(m+1)-Ad(m-2n+1)))(m-n+1)x^n+a(m+1)(bc(Ad(m-4n+1)-Bc(m-2n+1))+ad(Bc(m+1)-Ad(m-2n}{(bx^n+a)(dx^n+c)}}{cn(bc-ad)}}{2cn(bc-a}$$

$\downarrow$ 1067

$$\frac{(ex)^{m+1}(Bc-Ad)}{2cen(bc-ad)\left(c+dx^n\right)^2}-$$

$$\frac{\frac{\int\left(\frac{\left(b^2(Ad(m-3n+1)-Bc(m-n+1))(m-2n+1)c^2+2abd\left(Bc(m+1)(m-2n+1)-Ad\left(m^2+(2-4n)m+3n^2-4n+1\right)\right)c-a^2d^2(Bc(m+1)-Ad(m-2n+1))(m-n+1)\right)}{(bc-ad)(dx^n+c)}\right.}{cn(bc-ad)}}{2cn(bc-ad)}$$

$\downarrow$ 2009

$$\frac{(ex)^{m+1}(Bc-Ad)}{2cen(bc-ad)\left(c+dx^n\right)^2}-$$

$$\frac{\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)\left(-a^2d^2(m-n+1)(Bc(m+1)-Ad(m-2n+1))+2abcd\left(Bc(m+1)(m-2n+1)-Ad\left(m^2+m(2-4n)+3n^2-4n\right.\right.\right.}{ce(m+1)(bc-ad)}}{cn(bc-ad)}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^3),x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m))/(2*c*(b*c - a*d)*e*n*(c + d*x^n)^2) - (-(((b*c
*(A*d*(1 + m - 4*n) - B*c*(1 + m - 2*n)) + a*d*(B*c*(1 + m) - A*d*(1 + m -
 2*n)))*(e*x)^(1 + m))/(c*(b*c - a*d)*e*n*(c + d*x^n))) + ((-2*b^2*(A*b -
a*B)*c^2*n^2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
-((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m)) + ((b^2*c^2*(A*d*(1 + m - 3*n) -
B*c*(1 + m - n))*(1 + m - 2*n) - a^2*d^2*(B*c*(1 + m) - A*d*(1 + m - 2*n))
*(1 + m - n) + 2*a*b*c*d*(B*c*(1 + m)*(1 + m - 2*n) - A*d*(1 + m^2 + m*(2
- 4*n) - 4*n + 3*n^2)))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 +
 m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m)))/(c*(b*c - a*d)*n))/(2
*c*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n)}{(a + b x^n)(c + d x^n)^3} dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n) (c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*
x^(3*n) + 3*(b*c^2*d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n) (c + dx^n)^3} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)/(c+d*x**n)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n) (c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)(dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
(((m^2 - m*(5*n - 2) + 6*n^2 - 5*n + 1)*b^2*c^2*d*e^m - 2*(m^2 - 2*m*(2*n
- 1) + 3*n^2 - 4*n + 1)*a*b*c*d^2*e^m + (m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
 1)*a^2*d^3*e^m)*A - ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^2*c^3*e^m -
2*(m^2 - 2*m*(n - 1) - 2*n + 1)*a*b*c^2*d*e^m + (m^2 - m*(n - 2) - n + 1)*
a^2*c*d^2*e^m)*B)*integrate(-1/2*x^m/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*
a^2*b*c^4*d^2*n^2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2
 + 3*a^2*b*c^3*d^3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) + (B*a*b^2*e^m - A*b^3*
e^m)*integrate(-x^m/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3
 + (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) - 1/2*
(((a*c*d^2*e^m*(m - 3*n + 1) - b*c^2*d*e^m*(m - 5*n + 1))*A - (a*c^2*d*e^m
*(m - n + 1) - b*c^3*e^m*(m - 3*n + 1))*B)*x*x^m + ((a*d^3*e^m*(m - 2*n +
1) - b*c*d^2*e^m*(m - 4*n + 1))*A + (b*c^2*d*e^m*(m - 2*n + 1) - a*c*d^2*e
^m*(m + 1))*B)*x*e^(m*log(x) + n*log(x)))/(b^2*c^6*n^2 - 2*a*b*c^5*d*n^2 +
 a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c^2*d^4*n^2)
*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^2)*x^n)
```

Giac [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{(Bx^n+A)(ex)^m}{(bx^n+a)(dx^n+c)^3}\,dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)*(d*x^n + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)\,(c+d\,x^n)^3}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^3),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)(c+dx^n)^3}\,dx = e^m\left(\int \frac{x^m}{x^{3n}d^3+3x^{2n}c\,d^2+3x^n c^2 d+c^3}dx\right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)/(c+d*x^n)^3,x)
```

output
```
e**m*int(x**m/(x**(3*n)*d**3 + 3*x**(2*n)*c*d**2 + 3*x**n*c**2*d + c**3),x
)
```

3.58 $\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^2(c+dx^n)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 482

$$\int \frac{(ex)^m(A+Bx^n)}{(a+bx^n)^2(c+dx^n)^3}\,dx = \frac{d(2Abc-3aBc+aAd)(ex)^{1+m}}{2ac(bc-ad)^2en(c+dx^n)^2}+\frac{(Ab-aB)(ex)^{1+m}}{a(bc-ad)en(a+bx^n)(c+dx^n)^2} - \frac{d(a^2d(Bc(1+m)-Ad(1+m-2n))-abc(Bc-Ad)(1+m-6n)-2Ab^2c^2n)(ex)^{1+m}}{2ac^2(bc-ad)^3en^2(c+dx^n)} + \frac{b^2(aB(bc(1+m)-ad(1+m-3n))+Ab(ad(1+m-4n)-bc(1+m-n)))(ex)^{1+m}\,\text{Hypergeometr}}{a^2(bc-ad)^4e(1+m)n} + \frac{d(b^2c^2(Ad(1+m-4n)-Bc(1+m-2n))(1+m-3n)-a^2d^2(Bc(1+m)-Ad(1+m-2n))(1+}{}$$

output
```
1/2*d*(A*a*d+2*A*b*c-3*B*a*c)*(e*x)^(1+m)/a/c/(-a*d+b*c)^2/e/n/(c+d*x^n)^2
+(A*b-B*a)*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)/(c+d*x^n)^2-1/2*d*(a^2*d
*(B*c*(1+m)-A*d*(1+m-2*n))-a*b*c*(-A*d+B*c)*(1+m-6*n)-2*A*b^2*c^2*n)*(e*x)
^(1+m)/a/c^2/(-a*d+b*c)^3/e/n^2/(c+d*x^n)+b^2*(a*B*(b*c*(1+m)-a*d*(1+m-3*n
))+A*b*(a*d*(1+m-4*n)-b*c*(1+m-n)))*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1
+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^4/e/(1+m)/n+1/2*d*(b^2*c^2*(A*d*(1+m-4*n
)-B*c*(1+m-2*n))*(1+m-3*n)-a^2*d^2*(B*c*(1+m)-A*d*(1+m-2*n))*(1+m-n)+2*a*b
*c*d*(B*c*(1+m)*(1+m-3*n)-A*d*(1+m^2+m*(2-5*n)-5*n+4*n^2)))*(e*x)^(1+m)*hy
pergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c^3/(-a*d+b*c)^4/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^3} \, dx = \frac{x(ex)^m \left(\frac{b^2(bBc-3Abd+2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{a} - \frac{bd(bBc-3Abd+2aBd)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m}{n}\right.}{c}\right.}{}$$

input
```
Integrate[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output
```
(x*(e*x)^m*((b^2*(b*B*c - 3*A*b*d + 2*a*B*d)*Hypergeometric2F1[1, (1 + m)/
n, (1 + m + n)/n, -((b*x^n)/a)])/a - (b*d*(b*B*c - 3*A*b*d + 2*a*B*d)*Hype
rgeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c + (b^2*(-(A*b)
 + a*B)*(-(b*c) + a*d)*Hypergeometric2F1[2, (1 + m)/n, (1 + m + n)/n, -((b
*x^n)/a)])/a^2 - (d*(b*c - a*d)*(b*B*c - 2*A*b*d + a*B*d)*Hypergeometric2F
1[2, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^2 + (d*(b*c - a*d)^2*(-(B*
c) + A*d)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/c^
3))/((b*c - a*d)^4*(1 + m))
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1065, 25, 1065, 1065, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^3} \, dx$$

$$\Big\downarrow \; 1065$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}-\frac{\int -\frac{(ex)^m\left(-\left((Ab-aB)d(m-3n+1)x^n\right)+aBc(m+1)-Abc(m-n+1)-aAdn\right)}{\left(bx^n+a\right)\left(dx^n+c\right)^3}dx}{an(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(ex)^m\left(-\left((Ab-aB)d(m-3n+1)x^n\right)+aBc(m+1)-A(bc(m-n+1)+adn)\right)}{\left(bx^n+a\right)\left(dx^n+c\right)^3}dx}{an(bc-ad)}+\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 1065$$

$$\frac{\frac{\int \frac{(ex)^m\left(n\left(aBc(2bc+ad)(m+1)-A\left(2b^2(m-n+1)c^2+4abdnc+a^2d^2(m-2n+1)\right)\right)-bd(2Abc-3aBc+aAd)(m-2n+1)nx^n\right)}{\left(bx^n+a\right)\left(dx^n+c\right)^2}dx}{2cn(bc-ad)}+\frac{d(ex)^{m+1}(aAd-3aBc+2A \cdots}{2ce(bc-ad)\left(c+dx^n\right)^2}}{an(bc-ad)}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 1065$$

$$\frac{\frac{\int \frac{(ex)^m\left(bd(m-n+1)n\left(d(Bc(m+1)-Ad(m-2n+1))a^2-bc(Bc-Ad)(m-6n+1)a-2Ab^2c^2n\right)x^n+n\left(ad(m+1)\left(d(Bc(m+1)-Ad(m-2n+1))a^2-bc(Bc-Ad)(m-6 \cdots\right.\right.\right.}{\left(bx^n+a\right)\left(dx^n+c\right)}}{cn(bc-ad)}}{\cdots}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 1067$$

$$\frac{\frac{\int \left(\frac{2b^2c^2(aB(bc(m+1)-ad(m-3n+1))+Ab(ad(m-4n+1)-bc(m-n+1)))n^2(ex)^m}{(bc-ad)\left(bx^n+a\right)}+\frac{adn\left(b^2(Ad(m-4n+1)-Bc(m-2n+1))(m-3n+1)c^2+2abd\left(Bc(m+1)(m-3 \cdots\right.\right.}{\cdots}\right.}{cn(bc-ad)}}{\cdots}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\left(a+bx^n\right)\left(c+dx^n\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{\frac{adn(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{dx^n}{c}\right)\left(-a^2d^2(m-n+1)(Bc(m+1)-Ad(m-2n+1))+2abcd\left(Bc(m+1)(m-3n+1)-Ad\left(m^2+m(2-5n)+4n^2\right.\right.\right.}{ce(m+1)(bc-ad)}}{}}{}$$

$$\frac{(ex)^{m+1}(Ab-aB)}{aen(bc-ad)\,(a+bx^n)\,(c+dx^n)^2}$$

input
```
Int[((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

output
```
((A*b - a*B)*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)*(c + d*x^n)^2)
+ ((d*(2*A*b*c - 3*a*B*c + a*A*d)*(e*x)^(1 + m))/(2*c*(b*c - a*d)*e*(c + d
*x^n)^2) + (-((d*(a^2*d*(B*c*(1 + m) - A*d*(1 + m - 2*n)) - a*b*c*(B*c - A
*d)*(1 + m - 6*n) - 2*A*b^2*c^2*n)*(e*x)^(1 + m))/(c*(b*c - a*d)*e*(c + d*
x^n))) + ((2*b^2*c^2*(a*B*(b*c*(1 + m) - a*d*(1 + m - 3*n)) + A*b*(a*d*(1
+ m - 4*n) - b*c*(1 + m - n)))*n^2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
 m)/n, (1 + m + n)/n, -((b*x^n)/a)])/(a*(b*c - a*d)*e*(1 + m)) + (a*d*n*(b
^2*c^2*(A*d*(1 + m - 4*n) - B*c*(1 + m - 2*n))*(1 + m - 3*n) - a^2*d^2*(B*
c*(1 + m) - A*d*(1 + m - 2*n))*(1 + m - n) + 2*a*b*c*d*(B*c*(1 + m)*(1 + m
 - 3*n) - A*d*(1 + m^2 + m*(2 - 5*n) - 5*n + 4*n^2)))*(e*x)^(1 + m)*Hyperg
eometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(
1 + m)))/(c*(b*c - a*d)*n))/(2*c*(b*c - a*d)*n))/(a*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (A + B x^n)}{(a + b x^n)^2 (c + d x^n)^3} dx$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^3} dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)^3} dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(e*x)^m/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2
*a*b*d^3)*x^(4*n) + (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c
^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^3} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(A+B*x**n)/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

output
```
(((m^2 - m*(7*n - 2) + 12*n^2 - 7*n + 1)*b^2*c^2*d^2*e^m - 2*(m^2 - m*(5*n
 - 2) + 4*n^2 - 5*n + 1)*a*b*c*d^3*e^m + (m^2 - m*(3*n - 2) + 2*n^2 - 3*n
+ 1)*a^2*d^4*e^m)*A - ((m^2 - m*(5*n - 2) + 6*n^2 - 5*n + 1)*b^2*c^3*d*e^m
 - 2*(m^2 - m*(3*n - 2) - 3*n + 1)*a*b*c^2*d^2*e^m + (m^2 - m*(n - 2) - n
+ 1)*a^2*c*d^3*e^m)*B)*integrate(1/2*x^m/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2
+ 6*a^2*b^2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6
*d*n^2 - 4*a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2
 + a^4*c^2*d^5*n^2)*x^n), x) - ((b^4*c*e^m*(m - n + 1) - a*b^3*d*e^m*(m -
4*n + 1))*A + (a^2*b^2*d*e^m*(m - 3*n + 1) - a*b^3*c*e^m*(m + 1))*B)*integ
rate(x^m/(a^2*b^4*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*
b*c*d^3*n + a^6*d^4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d
^2*n - 4*a^4*b^2*c*d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*(((a^3*c*d^3*e^m*(m
 - 3*n + 1) - a^2*b*c^2*d^2*e^m*(m - 7*n + 1) + 2*b^3*c^4*e^m*n)*A - (a^3*
c^2*d^2*e^m*(m - n + 1) - a^2*b*c^3*d*e^m*(m - 5*n + 1) + 2*a*b^2*c^4*e^m*
n)*B)*x*x^m + ((a^2*b*d^4*e^m*(m - 2*n + 1) - a*b^2*c*d^3*e^m*(m - 6*n + 1
) + 2*b^3*c^2*d^2*e^m*n)*A + (a*b^2*c^2*d^2*e^m*(m - 6*n + 1) - a^2*b*c*d^
3*e^m*(m + 1))*B)*x*e^(m*log(x) + 2*n*log(x)) + ((a^3*d^4*e^m*(m - 2*n + 1
) - a*b^2*c^2*d^2*e^m*(m - 7*n + 1) + 4*b^3*c^3*d*e^m*n + 3*a^2*b*c*d^3*e^
m*n)*A + (a*b^2*c^3*d*e^m*(m - 9*n + 1) - a^3*c*d^3*e^m*(m + 1) - 3*a^2*b*
c^2*d^2*e^m*n)*B)*x*e^(m*log(x) + n*log(x)))/(a^2*b^3*c^7*n^2 - 3*a^3*b...
```

Giac [F]

$$\int \frac{(ex)^m (A + Bx^n)}{(a + bx^n)^2 (c + dx^n)^3} \, dx = \int \frac{(Bx^n + A)(ex)^m}{(bx^n + a)^2 (dx^n + c)^3} \, dx$$

input
```
integrate((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(e*x)^m/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^3}\,dx = \int \frac{(e\,x)^m\,(A+B\,x^n)}{(a+b\,x^n)^2\,(c+d\,x^n)^3}\,dx$$

input
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^3),x)
```

output
```
int(((e*x)^m*(A + B*x^n))/((a + b*x^n)^2*(c + d*x^n)^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m (A+Bx^n)}{(a+bx^n)^2 (c+dx^n)^3}\,dx$$
$$= e^m\left(\int \frac{x^m}{x^{4n}b\,d^3+x^{3n}a\,d^3+3x^{3n}bc\,d^2+3x^{2n}ac\,d^2+3x^{2n}b\,c^2d+3x^n a\,c^2d+x^n b\,c^3+a\,c^3}dx\right)$$

input
```
int((e*x)^m*(A+B*x^n)/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

output
```
e**m*int(x**m/(x**(4*n)*b*d**3 + x**(3*n)*a*d**3 + 3*x**(3*n)*b*c*d**2 + 3
*x**(2*n)*a*c*d**2 + 3*x**(2*n)*b*c**2*d + 3*x**n*a*c**2*d + x**n*b*c**3 +
 a*c**3),x)
```

3.59 $\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n) \, dx$

Optimal result

Integrand size = 29, antiderivative size = 255

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n) \, dx$$
$$= \frac{(Abdn-aBd(1+m+n)+bBc(1+m+n(2+p)))(ex)^{1+m}(a+bx^n)^{1+p}}{b^2e(1+m+n+np)(1+m+n(2+p))}$$
$$+ \frac{d(ex)^{1+m}(a+bx^n)^{1+p}(A+Bx^n)}{be(1+m+n(2+p))}$$
$$- \frac{\left(aAd-\frac{Abc(1+m+n(2+p))}{1+m}+\frac{a(Abdn-aBd(1+m+n)+bBc(1+m+n(2+p)))}{b(1+m+n+np)}\right)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hyperg}}{be(1+m+n(2+p))}$$

output
```
(A*b*d*n-a*B*d*(1+m+n)+b*B*c*(1+m+n*(2+p)))*(e*x)^(1+m)*(a+b*x^n)^(p+1)/b^
2/e/(n*p+m+n+1)/(1+m+n*(2+p))+d*(e*x)^(1+m)*(a+b*x^n)^(p+1)*(A+B*x^n)/b/e/
(1+m+n*(2+p))-(A*a*d-A*b*c*(1+m+n*(2+p))/(1+m)+a*(A*b*d*n-a*B*d*(1+m+n)+b*
B*c*(1+m+n*(2+p)))/b/(n*p+m+n+1))*(e*x)^(1+m)*(a+b*x^n)^p*hypergeom([-p, (
1+m)/n],[(1+m+n)/n],-b*x^n/a)/b/e/(1+m+n*(2+p))/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.64

$$\int (ex)^m (a+bx^n)^p (A+Bx^n)(c+dx^n)\,dx$$
$$= x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(\frac{Ac\,\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{1+m} + x^n\left(\frac{(Bc+Ad)\,\text{Hypergeometric2F1}\left(\frac{1+m+n}{n},-p,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right)}{1+m+n} + \frac{Bdx^n\,\text{Hypergeometric2F1}\left(\frac{1+m+2n}{n},-p,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right)}{1+m+2n}\right)\right)$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(A + B*x^n)*(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*((A*c*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n
)/n, -((b*x^n)/a)])/(1 + m) + x^n*(((B*c + A*d)*Hypergeometric2F1[(1 + m +
 n)/n, -p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n) + (B*d*x^n*Hypergeo
metric2F1[(1 + m + 2*n)/n, -p, (1 + m + 3*n)/n, -((b*x^n)/a)])/(1 + m + 2*
n))))/(1 + (b*x^n)/a)^p
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1066, 25, 959, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (A+Bx^n)(c+dx^n)(a+bx^n)^p\,dx$$

$$\Big\downarrow \; 1066$$

$$\frac{\int -(ex)^m (bx^n+a)^p \left(A(ad(m+1)-bc(m+n(p+2)+1)) - (Abdn - aBd(m+n+1) + bBc(m+n(p+2)+\right.}{b(m+n(p+2)+1)}$$
$$\frac{d(ex)^{m+1}(A+Bx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)}$$

$\downarrow$ 25

$$\frac{d(ex)^{m+1}(A+Bx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\int (ex)^m (bx^n+a)^p \left((aBd(m+n+1)-b(Adn+Bc(m+n(p+2)+1)))x^n + A(ad(m+1)-bc(m+n(p+2)\right.}{b(m+n(p+2)+1)}$$

$\downarrow$ 959

$$\frac{d(ex)^{m+1}(A+Bx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\left(-\frac{a(m+1)(aBd(m+n+1)-b(Adn+Bc(m+n(p+2)+1)))}{b(m+np+n+1)} + aAd(m+1) - Abc(m+n(p+2)+1)\right)\int (ex)^m (bx^n+a)^p dx +}{b(m+n(p+2)+1)}$$

$\downarrow$ 889

$$\frac{d(ex)^{m+1}(A+Bx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-\frac{a(m+1)(aBd(m+n+1)-b(Adn+Bc(m+n(p+2)+1)))}{b(m+np+n+1)} + aAd(m+1) - Abc(m+n(p+2)+1)\right)}{b(m+n(p+2)+1)}$$

$\downarrow$ 888

$$\frac{d(ex)^{m+1}(A+Bx^n)(a+bx^n)^{p+1}}{be(m+n(p+2)+1)} - \frac{\frac{(ex)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)\left(-\frac{a(m+1)(aBd(m+n+1)-b(Adn+Bc(m+n(p+2)+1)))}{b(m+np+n+1)}+aAd(m+1)-\right.}{e(m+1)}}{b(m+n(p+2)+1)}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(A + B*x^n)*(c + d*x^n),x]
```

output
```
(d*(e*x)^(1 + m)*(a + b*x^n)^(1 + p)*(A + B*x^n))/(b*e*(1 + m + n*(2 + p))
) - (((a*B*d*(1 + m + n) - b*(A*d*n + B*c*(1 + m + n*(2 + p))))*(e*x)^(1 +
 m)*(a + b*x^n)^(1 + p))/(b*e*(1 + m + n + n*p)) + ((a*A*d*(1 + m) - A*b*c
*(1 + m + n*(2 + p)) - (a*(1 + m)*(a*B*d*(1 + m + n) - b*(A*d*n + B*c*(1 +
 m + n*(2 + p)))))/(b*(1 + m + n + n*p)))*(e*x)^(1 + m)*(a + b*x^n)^p*Hype
rgeometric2F1[(1 + m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/(e*(1 + m)*(1 +
 (b*x^n)/a)^p))/(b*(1 + m + n*(2 + p)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1066
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x
^n, c + d*x^n])
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (A+B\,x^n) (c+d\,x^n)\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)\;dx = \int (Bx^n+A)(dx^n+c)(bx^n+a)^p(ex)^m\;dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*d*x^(2*n) + A*c + (B*c + A*d)*x^n)*(b*x^n + a)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m (a + bx^n)^p (A + Bx^n) (c + dx^n) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(A+B*x**n)*(c+d*x**n),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m (a + bx^n)^p (A + Bx^n) (c + dx^n) \, dx = \int (Bx^n + A)(dx^n + c)(bx^n + a)^p (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(d*x^n + c)*(b*x^n + a)^p*(e*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (ex)^m (a + bx^n)^p (A + Bx^n) (c + dx^n) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,4,3,0,1,3,3,1,1,0,0]%%%}+%%%{-3,[1,0,4,3,0,1,2,3,1
,1,0,0]%%
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n) \, dx = \int (e\,x)^m \left(A+B\,x^n\right) \left(a+b\,x^n\right)^p \left(c+d\,x^n\right) \, dx$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^p*(c + d*x^n),x)
```

output
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^p*(c + d*x^n), x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n) \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n),x)
```

output

```
(e**m*(x**(m + 2*n)*(x**n*b + a)**p*b**2*d*m**2*x + 2*x**(m + 2*n)*(x**n*b
 + a)**p*b**2*d*m*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*m*n*x + 2*x*
*(m + 2*n)*(x**n*b + a)**p*b**2*d*m*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*
d*n**2*p**2*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*n**2*p*x + 2*x**(m + 2
*n)*(x**n*b + a)**p*b**2*d*n*p*x + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*n*x
 + x**(m + 2*n)*(x**n*b + a)**p*b**2*d*x + x**(m + n)*(x**n*b + a)**p*a*b*
d*m**2*x + 3*x**(m + n)*(x**n*b + a)**p*a*b*d*m*n*p*x + 2*x**(m + n)*(x**n
*b + a)**p*a*b*d*m*n*x + 2*x**(m + n)*(x**n*b + a)**p*a*b*d*m*x + 2*x**(m
+ n)*(x**n*b + a)**p*a*b*d*n**2*p**2*x + 2*x**(m + n)*(x**n*b + a)**p*a*b*
d*n**2*p*x + 3*x**(m + n)*(x**n*b + a)**p*a*b*d*n*p*x + 2*x**(m + n)*(x**n
*b + a)**p*a*b*d*n*x + x**(m + n)*(x**n*b + a)**p*a*b*d*x + x**(m + n)*(x*
*n*b + a)**p*b**2*c*m**2*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*m*n*p*x +
 2*x**(m + n)*(x**n*b + a)**p*b**2*c*m*n*x + 2*x**(m + n)*(x**n*b + a)**p*
b**2*c*m*x + x**(m + n)*(x**n*b + a)**p*b**2*c*n**2*p**2*x + 2*x**(m + n)*
(x**n*b + a)**p*b**2*c*n**2*p*x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*n*p*
x + 2*x**(m + n)*(x**n*b + a)**p*b**2*c*n*x + x**(m + n)*(x**n*b + a)**p*b
**2*c*x + x**m*(x**n*b + a)**p*a**2*d*n**2*p**2*x + x**m*(x**n*b + a)**p*a
**2*d*n**2*p*x + x**m*(x**n*b + a)**p*a*b*c*m**2*x + 3*x**m*(x**n*b + a)**
p*a*b*c*m*n*p*x + 3*x**m*(x**n*b + a)**p*a*b*c*m*n*x + 2*x**m*(x**n*b + a)
**p*a*b*c*m*x + 2*x**m*(x**n*b + a)**p*a*b*c*n**2*p**2*x + 5*x**m*(x**n...
```

3.60 $\int \frac{(ex)^m(a+bx^n)^p(A+Bx^n)}{c+dx^n}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 164

$$\int \frac{(ex)^m(a+bx^n)^p(A+Bx^n)}{c+dx^n}\,dx = -\frac{(Bc-Ad)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cde(1+m)} + \frac{B(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{de(1+m)}$$

output
```
-(-A*d+B*c)*(e*x)^(1+m)*(a+b*x^n)^p*AppellF1((1+m)/n,-p,1,(1+m+n)/n,-b*x^n
/a,-d*x^n/c)/c/d/e/(1+m)/((1+b*x^n/a)^p)+B*(e*x)^(1+m)*(a+b*x^n)^p*hyperge
om([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/d/e/(1+m)/((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\int \frac{(ex)^m(a+bx^n)^p(A+Bx^n)}{c+dx^n}\,dx = \frac{x(ex)^m(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\left(A(1+m+n)\operatorname{AppellF1}\left(\frac{1+m}{n},-p,1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+B(1+m)x^{n}\right.}{c(1+m)(1+m+n)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^p*(A + B*x^n))/(c + d*x^n),x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(A*(1 + m + n)*AppellF1[(1 + m)/n, -p, 1, (1 + m
+ n)/n, -((b*x^n)/a), -((d*x^n)/c)] + B*(1 + m)*x^n*AppellF1[(1 + m + n)/n
, -p, 1, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)]))/(c*(1 + m)*(1 + m
+ n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (a+bx^n)^p}{c+dx^n} \, dx$$

$$\Big\downarrow \text{1067}$$

$$\int \left(\frac{(ex)^m(Ad-Bc) (a+bx^n)^p}{d (c+dx^n)}+\frac{B(ex)^m (a+bx^n)^p}{d}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(ex)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-p,\frac{m+n+1}{n},-\frac{bx^n}{a}\right)}{de(m+1)}-\frac{(ex)^{m+1}(Bc-Ad) (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \text{AppellF1}\left(\frac{m+1}{n},-p,1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cde(m+1)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^p*(A + B*x^n))/(c + d*x^n),x]
```

output
```
-(((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 1, (1 +
 m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(c*d*e*(1 + m)*(1 + (b*x^n)/a)^p))
 + (B*(e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometric2F1[(1 + m)/n, -p, (1 + m
+ n)/n, -((b*x^n)/a)])/(d*e*(1 + m)*(1 + (b*x^n)/a)^p)
```

Defintions of rubi rules used

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^p (A+B\,x^n)}{c+d\,x^n}dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^p(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(A+B*x**n)/(c+d*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^p(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n}\,dx = \int \frac{(Bx^n+A)(bx^n+a)^p(ex)^m}{dx^n+c}\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n} \, dx = \int \frac{(e\,x)^m \,(A+B\,x^n)\,(a+b\,x^n)^p}{c+d\,x^n} \, dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^p)/(c + d*x^n),x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^p)/(c + d*x^n), x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{c+dx^n} \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n),x)
```

output

```
(e**m*(2*x**m*(x**n*b + a)**p*a*b*x - int((x**(m + 2*n)*(x**n*b + a)**p)/(
x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m + x**(2*n)*b
**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*d**2 + 2*x*
*n*a*b*c*d*m + x**n*a*b*c*d*n*p + 2*x**n*a*b*c*d + x**n*b**2*c**2*m + x**n
*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d + a*b*c**2*m + a*b
*c**2*n*p + a*b*c**2),x)*a**2*b**2*d**2*m**2 - 2*int((x**(m + 2*n)*(x**n*b
 + a)**p)/(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m +
 x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*
d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*n*p + 2*x**n*a*b*c*d + x**n*b**2*c*
*2*m + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d + a*b*c
**2*m + a*b*c**2*n*p + a*b*c**2),x)*a**2*b**2*d**2*m*n*p - 2*int((x**(m +
2*n)*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2 + x**(2*n)*
b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*m
+ x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*n*p + 2*x**n*a*b*c*d +
x**n*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2
*c*d + a*b*c**2*m + a*b*c**2*n*p + a*b*c**2),x)*a**2*b**2*d**2*m - 2*int((
x**(m + 2*n)*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2 + x
**(2*n)*b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2
*d**2*m + x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*n*p + 2*x**n*a*
b*c*d + x**n*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c...
```

3.61 $\int \frac{(ex)^m(a+bx^n)^p(A+Bx^n)}{(c+dx^n)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 304

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2}\,dx = \frac{(Bc-Ad)(ex)^{1+m}(a+bx^n)^{1+p}}{c(bc-ad)en(c+dx^n)}$$
$$-\frac{(ad(Bc(1+m)-Ad(1+m-n))+bc(Ad(1+m-n(1-p))-Bc(1+m+np)))(ex)^{1+m}(a+bx^n)}{c^2d(bc-ad)e(1+m)n}$$
$$-\frac{b(Bc-Ad)(1+m+np)(ex)^{1+m}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-p,\frac{1+m+n}{n},-\frac{bx^n}{a}\right)}{cd(bc-ad)e(1+m)n}$$

output
```
(-A*d+B*c)*(e*x)^(1+m)*(a+b*x^n)^(p+1)/c/(-a*d+b*c)/e/n/(c+d*x^n)-(a*d*(B*
c*(1+m)-A*d*(1+m-n))+b*c*(A*d*(1+m-n*(1-p))-B*c*(n*p+m+1)))*(e*x)^(1+m)*(a
+b*x^n)^p*AppellF1((1+m)/n,-p,1,(1+m+n)/n,-b*x^n/a,-d*x^n/c)/c^2/d/(-a*d+b
*c)/e/(1+m)/n/((1+b*x^n/a)^p)-b*(-A*d+B*c)*(n*p+m+1)*(e*x)^(1+m)*(a+b*x^n)
^p*hypergeom([-p, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/c/d/(-a*d+b*c)/e/(1+m)/n/
((1+b*x^n/a)^p)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} \, dx$$
$$= \frac{x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} \left(A(1+m+n) \text{AppellF1}\left(\frac{1+m}{n},-p,2,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+B(1+m)x^n\right.}{c^2(1+m)(1+m+n)}$$

input
```
Integrate[((e*x)^m*(a + b*x^n)^p*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(A*(1 + m + n)*AppellF1[(1 + m)/n, -p, 2, (1 + m
+ n)/n, -((b*x^n)/a), -((d*x^n)/c)] + B*(1 + m)*x^n*AppellF1[(1 + m + n)/n
, -p, 2, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)]))/(c^2*(1 + m)*(1 +
m + n)*(1 + (b*x^n)/a)^p)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1065, 25, 1067, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m (A+Bx^n) (a+bx^n)^p}{(c+dx^n)^2} \, dx$$

$$\Big\downarrow \text{1065}$$

$$\frac{\int -\frac{(ex)^m (bx^n+a)^p (b(Bc-Ad)(m+np+1)x^n+a(Bc(m+1)-Ad(m-n+1))-Abcn)}{dx^n+c}dx}{cn(bc-ad)}+\frac{(ex)^{m+1}(Bc-Ad)(a+bx^n)^{p+1}}{cen(bc-ad)(c+dx^n)}$$

$$\Big\downarrow \text{25}$$

$$\frac{(ex)^{m+1}(Bc-Ad)\,(a+bx^n)^{p+1}}{cen(bc-ad)\,(c+dx^n)}-\frac{\int \frac{(ex)^m(bx^n+a)^p(b(Bc-Ad)(m+np+1)x^n+aBc(m+1)-aAd(m-n+1)-Abcn)}{dx^n+c}dx}{cn(bc-ad)}$$

$$\Big\downarrow \text{ 1067}$$

$$\frac{(ex)^{m+1}(Bc-Ad)\,(a+bx^n)^{p+1}}{cen(bc-ad)\,(c+dx^n)}-\frac{\int \left(\frac{b(Bc-Ad)(m+np+1)(bx^n+a)^p(ex)^m}{d}+\frac{(d(aBc(m+1)-aAd(m-n+1)-Abcn)-bc(Bc-Ad)(m+np+1))(bx^n+a)^p(ex)^m}{d(dx^n+c)}\right)dx}{cn(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(ex)^{m+1}(Bc-Ad)\,(a+bx^n)^{p+1}}{cen(bc-ad)\,(c+dx^n)}-\frac{\frac{(ex)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(d(-aAd(m-n+1)+aBc(m+1)-Abcn)-bc(m+np+1)(Bc-Ad))\operatorname{AppellF1}\left(\frac{m+1}{n},-p,1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{cde(m+1)}}{cn(bc-ad)}$$

input
```
Int[((e*x)^m*(a + b*x^n)^p*(A + B*x^n))/(c + d*x^n)^2,x]
```

output
```
((B*c - A*d)*(e*x)^(1 + m)*(a + b*x^n)^(1 + p))/(c*(b*c - a*d)*e*n*(c + d*
x^n)) - (((d*(a*B*c*(1 + m) - a*A*d*(1 + m - n) - A*b*c*n) - b*c*(B*c - A*
d)*(1 + m + n*p))*(e*x)^(1 + m)*(a + b*x^n)^p*AppellF1[(1 + m)/n, -p, 1, (
1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(c*d*e*(1 + m)*(1 + (b*x^n)/a)^
p) + (b*(B*c - A*d)*(1 + m + n*p)*(e*x)^(1 + m)*(a + b*x^n)^p*Hypergeometr
ic2F1[(1 + m)/n, -p, (1 + m + n)/n, -((b*x^n)/a)])/(d*e*(1 + m)*(1 + (b*x^
n)/a)^p))/(c*(b*c - a*d)*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1065
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, q}, x] && LtQ[p, -1]
```

rule 1067
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex)^m (a+b\,x^n)^p (A+B\,x^n)}{(c+d\,x^n)^2}dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^p(ex)^m}{(dx^n+c)^2} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d^2*x^(2*n) + 2*c*d*x^n + c^2)
, x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(A+B*x**n)/(c+d*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} \, dx = \int \frac{(Bx^n+A)(bx^n+a)^p(ex)^m}{(dx^n+c)^2} \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d*x^n + c)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(Bx^n+A)(bx^n+a)^p (ex)^m}{(dx^n+c)^2} dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(e*x)^m/(d*x^n + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} dx = \int \frac{(e\,x)^m\,(A+B\,x^n)\,(a+b\,x^n)^p}{(c+d\,x^n)^2} dx$$

input
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^p)/(c + d*x^n)^2,x)
```

output
```
int(((e*x)^m*(A + B*x^n)*(a + b*x^n)^p)/(c + d*x^n)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m (a+bx^n)^p (A+Bx^n)}{(c+dx^n)^2} dx = \text{too large to display}$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)/(c+d*x^n)^2,x)
```

output

```
(e**m*(2*x**m*(x**n*b + a)**p*a*b*x - x**n*int((x**(m + 2*n)*(x**n*b + a)*
*p)/(x**(3*n)*a*b*d**3*m - x**(3*n)*a*b*d**3*n + x**(3*n)*a*b*d**3 + x**(3
*n)*b**2*c*d**2*m + x**(3*n)*b**2*c*d**2*n*p + x**(3*n)*b**2*c*d**2 + x**(
2*n)*a**2*d**3*m - x**(2*n)*a**2*d**3*n + x**(2*n)*a**2*d**3 + 3*x**(2*n)*
a*b*c*d**2*m + x**(2*n)*a*b*c*d**2*n*p - 2*x**(2*n)*a*b*c*d**2*n + 3*x**(2
*n)*a*b*c*d**2 + 2*x**(2*n)*b**2*c**2*d*m + 2*x**(2*n)*b**2*c**2*d*n*p + 2
*x**(2*n)*b**2*c**2*d + 2*x**n*a**2*c*d**2*m - 2*x**n*a**2*c*d**2*n + 2*x*
*n*a**2*c*d**2 + 3*x**n*a*b*c**2*d*m + 2*x**n*a*b*c**2*d*n*p - x**n*a*b*c*
*2*d*n + 3*x**n*a*b*c**2*d + x**n*b**2*c**3*m + x**n*b**2*c**3*n*p + x**n*
b**2*c**3 + a**2*c**2*d*m - a**2*c**2*d*n + a**2*c**2*d + a*b*c**3*m + a*b
*c**3*n*p + a*b*c**3),x)*a**2*b**2*d**3*m**2 - 2*x**n*int((x**(m + 2*n)*(x
**n*b + a)**p)/(x**(3*n)*a*b*d**3*m - x**(3*n)*a*b*d**3*n + x**(3*n)*a*b*d
**3 + x**(3*n)*b**2*c*d**2*m + x**(3*n)*b**2*c*d**2*n*p + x**(3*n)*b**2*c*
d**2 + x**(2*n)*a**2*d**3*m - x**(2*n)*a**2*d**3*n + x**(2*n)*a**2*d**3 +
3*x**(2*n)*a*b*c*d**2*m + x**(2*n)*a*b*c*d**2*n*p - 2*x**(2*n)*a*b*c*d**2*
n + 3*x**(2*n)*a*b*c*d**2 + 2*x**(2*n)*b**2*c**2*d*m + 2*x**(2*n)*b**2*c**
2*d*n*p + 2*x**(2*n)*b**2*c**2*d + 2*x**n*a**2*c*d**2*m - 2*x**n*a**2*c*d*
*2*n + 2*x**n*a**2*c*d**2 + 3*x**n*a*b*c**2*d*m + 2*x**n*a*b*c**2*d*n*p -
x**n*a*b*c**2*d*n + 3*x**n*a*b*c**2*d + x**n*b**2*c**3*m + x**n*b**2*c**3*
n*p + x**n*b**2*c**3 + a**2*c**2*d*m - a**2*c**2*d*n + a**2*c**2*d + a*...
```

3.62 $\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx$

Optimal result

Integrand size = 31, antiderivative size = 214

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx$$
$$= \frac{A(ex)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1+m}{n}, -p, -q, \frac{1+m+n}{n}, -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{e(1+m)}$$
$$+ \frac{Bx^n(ex)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1+m+n}{n}, -p, -q, \frac{1+m+2n}{n}, -\frac{bx^n}{a}, -\right.}{e(1+m+n)}$$

output
```
A*(e*x)^(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-b*
x^n/a,-d*x^n/c)/e/(1+m)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)+B*x^n*(e*x)^(1+m)*
(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m+n)/n,-p,-q,(1+m+2*n)/n,-b*x^n/a,-d*x
^n/c)/e/(1+m+n)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx$$
$$= \frac{x(ex)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(A(1+m+n)\,\text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\right.\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[(e*x)^m*(a + b*x^n)^p*(A + B*x^n)*(c + d*x^n)^q,x]
```

output
```
(x*(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(A*(1 + m + n)*AppellF1[(1 + m)/n,
-p, -q, (1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)] + B*(1 + m)*x^n*AppellF
1[(1 + m + n)/n, -p, -q, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)]))/((
1 + m)*(1 + m + n)*(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1068, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m (A+Bx^n) (a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{ 1068}$$

$$A\int (ex)^m (bx^n+a)^p (dx^n+c)^q dx + Bx^{-m}(ex)^m \int x^{m+n}(bx^n+a)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{ 1013}$$

$$A(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx + Bx^{-m}(ex)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^{m+n} \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$A(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\int(ex)^m\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^q dx+ Bx^{-m}(ex)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\int x^{m+n}\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{A(ex)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{n},-p,-q,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{e(m+1)}+\frac{Bx^{n+1}(ex)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{m+n+1}{n},-p,-q,\frac{m+2n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{m+n+1}$$

input
```
Int[(e*x)^m*(a + b*x^n)^p*(A + B*x^n)*(c + d*x^n)^q,x]
```

output
```
(A*(e*x)^(1 + m)*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (
1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(1 + m)*(1 + (b*x^n)/a)^p*(1
 + (d*x^n)/c)^q) + (B*x^(1 + n)*(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*Appell
F1[(1 + m + n)/n, -p, -q, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)])/((
1 + m + n)*(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1068
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*
x^n)^p*(c + d*x^n)^q, x], x] + Simp[f*((g*x)^m/x^m)   Int[x^(m + n)*(a + b*
x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x]
```

Maple [F]

$$\int (ex)^m (a+b\,x^n)^p (A+B\,x^n) (c+d\,x^n)^q\,dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q\,dx = \int (Bx^n+A)(bx^n+a)^p(dx^n+c)^q(ex)^m\,dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((B*x^n + A)*(b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x)**m*(a+b*x**n)**p*(A+B*x**n)*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx = \int (Bx^n + A)(bx^n + a)^p (dx^n + c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx = \int (Bx^n + A)(bx^n + a)^p (dx^n + c)^q (ex)^m \, dx$$

input
```
integrate((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((B*x^n + A)*(b*x^n + a)^p*(d*x^n + c)^q*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx = \int (e\,x)^m \,(A+B\,x^n)\,(a+b\,x^n)^p\,(c+d\,x^n)^q \, dx$$

input
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int((e*x)^m*(A + B*x^n)*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int (ex)^m (a+bx^n)^p (A+Bx^n) (c+dx^n)^q \, dx = \int (ex)^m (x^n b+a)^p (A+B\,x^n) (x^n d+c)^q \, dx$$

input
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x)
```

output
```
int((e*x)^m*(a+b*x^n)^p*(A+B*x^n)*(c+d*x^n)^q,x)
```

3.63 $\int \frac{x^n}{\sqrt{a+bx^n}(c+dx^n)(e+fx^n)}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 138

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = -\frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(de-cf)\sqrt{a+bx^n}} + \frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{fx^n}{e}\right)}{(de-cf)\sqrt{a+bx^n}}$$

output
```
-x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-d*x^n/c)/(-c*f+d*e
)/(a+b*x^n)^(1/2)+x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-f
*x^n/e)/(-c*f+d*e)/(a+b*x^n)^(1/2)
```

Mathematica [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$

input
```
Integrate[x^n/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
Integrate[x^n/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$\Big\downarrow \; 1073$$
$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}dx$$

input
```
Int[x^n/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x^n}{\sqrt{a+b\,x^n}\,(c+d\,x^n)\,(e+f\,x^n)}dx$$

input
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

Fricas [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^n + a)*x^n/(b*d*f*x^(3*n) + a*c*e + (b*d*e + (b*c + a*d)
*f)*x^(2*n) + (a*c*f + (b*c + a*d)*e)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**n/(a+b*x**n)**(1/2)/(c+d*x**n)/(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="maxima")
```

output
```
integrate(x^n/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Giac [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="giac")
```

output
```
integrate(x^n/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)\,\left(e+f\,x^n\right)}\,dx$$

input
```
int(x^n/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)),x)
```

output
```
int(x^n/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)), x)
```

Reduce [F]

$$\int \frac{x^n}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^n\sqrt{x^nb+a}}{x^{3n}bdf+x^{2n}adf+x^{2n}bcf+x^{2n}bde+x^nacf+x^nade+x^nbce+ace}dx$$

input
```
int(x^n/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int((x**n*sqrt(x**n*b + a))/(x**(3*n)*b*d*f + x**(2*n)*a*d*f + x**(2*n)*b*
c*f + x**(2*n)*b*d*e + x**n*a*c*f + x**n*a*d*e + x**n*b*c*e + a*c*e),x)
```

3.64 $\int \frac{x^{2n}}{\sqrt{a+bx^n}(c+dx^n)(e+fx^n)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 200

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \frac{cx\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{d(de-cf)\sqrt{a+bx^n}} - \frac{ex\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{fx^n}{e}\right)}{f(de-cf)\sqrt{a+bx^n}} + \frac{x\sqrt{1+\frac{bx^n}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{df\sqrt{a+bx^n}}$$

output
```
c*x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-d*x^n/c)/d/(-c*f+
d*e)/(a+b*x^n)^(1/2)-e*x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n
/a,-f*x^n/e)/f/(-c*f+d*e)/(a+b*x^n)^(1/2)+x*(1+b*x^n/a)^(1/2)*hypergeom([1
/2, 1/n],[1+1/n],-b*x^n/a)/d/f/(a+b*x^n)^(1/2)
```

Mathematica [F]

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$

input
```
Integrate[x^(2*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
Integrate[x^(2*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$\Big\downarrow \ 1073$$
$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}dx$$

input
```
Int[x^(2*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{2n}}{\sqrt{a+b\,x^{n}}\,\left(c+d\,x^{n}\right)\left(e+f\,x^{n}\right)}dx$$

input
```
int(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

Fricas [F]

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="fricas
")
```

output
```
integral(sqrt(b*x^n + a)*x^(2*n)/(b*d*f*x^(3*n) + a*c*e + (b*d*e + (b*c +
a*d)*f)*x^(2*n) + (a*c*f + (b*c + a*d)*e)*x^n), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)/(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="maxima
")
```

output
```
integrate(x^(2*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Giac [F]

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="giac")
```

output
```
integrate(x^(2*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2\,n}}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)\,\left(e+f\,x^n\right)}\,dx$$

input
```
int(x^(2*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)),x)
```

output
```
int(x^(2*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{2n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{2n}}{\sqrt{x^n b+a}\,(x^n d+c)\,(e+f\,x^n)}dx$$

input
```
int(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^(2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

3.65 $\int \frac{x^{3n}}{\sqrt{a+bx^n}(c+dx^n)(e+fx^n)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 259

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \frac{2x\sqrt{a+bx^n}}{bdf(2+n)} - \frac{c^2x\sqrt{1+\frac{bx^n}{a}}\,\mathrm{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{d^2(de-cf)\sqrt{a+bx^n}} + \frac{e^2x\sqrt{1+\frac{bx^n}{a}}\,\mathrm{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{fx^n}{e}\right)}{f^2(de-cf)\sqrt{a+bx^n}} - \frac{(2adf+b(de+cf)(2+n))x\sqrt{1+\frac{bx^n}{a}}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-\frac{bx^n}{a}\right)}{bd^2f^2(2+n)\sqrt{a+bx^n}}$$

output
```
2*x*(a+b*x^n)^(1/2)/b/d/f/(2+n)-c^2*x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1
,1+1/n,-b*x^n/a,-d*x^n/c)/d^2/(-c*f+d*e)/(a+b*x^n)^(1/2)+e^2*x*(1+b*x^n/a)
^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-f*x^n/e)/f^2/(-c*f+d*e)/(a+b*x^n
)^(1/2)-(2*a*d*f+b*(c*f+d*e)*(2+n))*x*(1+b*x^n/a)^(1/2)*hypergeom([1/2, 1/
n],[1+1/n],-b*x^n/a)/b/d^2/f^2/(2+n)/(a+b*x^n)^(1/2)
```

Mathematica [F]

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx=\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$

input
```
Integrate[x^(3*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
Integrate[x^(3*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$\Big\downarrow \ 1073$$
$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}dx$$

input
```
Int[x^(3*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{3n}}{\sqrt{a+b\,x^{n}}\,\left(c+d\,x^{n}\right)\left(e+f\,x^{n}\right)}dx$$

input
```
int(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{3n}}{\sqrt{a+bx^{n}}\,(c+dx^{n})\,(e+fx^{n})}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{3n}}{\sqrt{a+bx^{n}}\,(c+dx^{n})\,(e+fx^{n})}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)/(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx=\int \frac{x^{3\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="maxima
")
```

output
```
integrate(x^(3*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Giac [F]

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx=\int \frac{x^{3\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="giac")
```

output
```
integrate(x^(3*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx=\int \frac{x^{3\,n}}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)\,\left(e+f\,x^n\right)}\,dx$$

input
```
int(x^(3*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)),x)
```

output
```
int(x^(3*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{3n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$=\int \frac{x^{3n}\sqrt{x^nb+a}}{x^{3n}bdf+x^{2n}adf+x^{2n}bcf+x^{2n}bde+x^nacf+x^nade+x^nbce+ace}dx$$

input
```
int(x^(3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int((x**(3*n)*sqrt(x**n*b + a))/(x**(3*n)*b*d*f + x**(2*n)*a*d*f + x**(2*n
)*b*c*f + x**(2*n)*b*d*e + x**n*a*c*f + x**n*a*d*e + x**n*b*c*e + a*c*e),x
)
```

3.66 $\int \frac{x^{4n}}{\sqrt{a+bx^n}(c+dx^n)(e+fx^n)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 381

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$= \frac{c^3(4+8n+3n^2)\,x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{d^3(de-cf)(2+n)(2+3n)\sqrt{a+bx^n}}$$
$$-\frac{e^3(4+8n+3n^2)\,x\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{fx^n}{e}\right)}{f^3(de-cf)(2+n)(2+3n)\sqrt{a+bx^n}}$$
$$+\frac{x\Big(-2df(a+bx^n)\left(2adf(1+n)+b(de(2+3n)+cf(2+3n)-df(2+n)x^n)\right)+(4a^2d^2f^2(1+n)+2a \cdots}{b^2d^3f^3(2 \cdots}$$

output

```
c^3*(3*n^2+8*n+4)*x*(1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-d
*x^n/c)/d^3/(-c*f+d*e)/(2+n)/(2+3*n)/(a+b*x^n)^(1/2)-e^3*(3*n^2+8*n+4)*x*(
1+b*x^n/a)^(1/2)*AppellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-f*x^n/e)/f^3/(-c*f+d*e
)/(2+n)/(2+3*n)/(a+b*x^n)^(1/2)+x*(-2*d*f*(a+b*x^n)*(2*a*d*f*(1+n)+b*(d*e*
(2+3*n)+c*f*(2+3*n)-d*f*(2+n)*x^n))+(4*a^2*d^2*f^2*(1+n)+2*a*b*d*f*(c*f+d*
e)*(2+3*n)+b^2*(c^2*f^2+c*d*e*f+d^2*e^2)*(3*n^2+8*n+4))*(1+b*x^n/a)^(1/2)*
hypergeom([1/2, 1/n],[1+1/n],-b*x^n/a))/b^2/d^3/f^3/(2+n)/(2+3*n)/(a+b*x^n
)^(1/2)
```

Mathematica [F]

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$

input
```
Integrate[x^(4*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
Integrate[x^(4*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx$$
$$\Big\downarrow \text{1073}$$
$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}dx$$

input
```
Int[x^(4*n)/(Sqrt[a + b*x^n]*(c + d*x^n)*(e + f*x^n)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [F]

$$\int \frac{x^{4n}}{\sqrt{a+b\,x^{n}}\,(c+d\,x^{n})\,(e+f\,x^{n})}dx$$

input
```
int(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(x**(4*n)/(a+b*x**n)**(1/2)/(c+d*x**n)/(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{4\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="maxima
")
```

output
```
integrate(x^(4*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Giac [F]

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{4\,n}}{\sqrt{bx^n+a}(dx^n+c)(fx^n+e)}\,dx$$

input
```
integrate(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x, algorithm="giac")
```

output
```
integrate(x^(4*n)/(sqrt(b*x^n + a)*(d*x^n + c)*(f*x^n + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{4\,n}}{\sqrt{a+b\,x^n}\,\left(c+d\,x^n\right)\,\left(e+f\,x^n\right)}\,dx$$

input
```
int(x^(4*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)),x)
```

output
```
int(x^(4*n)/((a + b*x^n)^(1/2)*(c + d*x^n)*(e + f*x^n)), x)
```

Reduce [F]

$$\int \frac{x^{4n}}{\sqrt{a+bx^n}\,(c+dx^n)\,(e+fx^n)}\,dx = \int \frac{x^{4n}}{\sqrt{x^n b+a}\,\left(x^n d+c\right)\left(e+f\,x^n\right)}dx$$

input
```
int(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

output
```
int(x^(4*n)/(a+b*x^n)^(1/2)/(c+d*x^n)/(e+f*x^n),x)
```

3.67 $\int x^3(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 264

$$\int x^3\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx$$
$$=\frac{(bde(3+p+q)-f(bc(2+p)+ad(4+p+2q)))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)}$$
$$+\frac{f\left(a+bx^2\right)^{2+p}\left(c+dx^2\right)^{1+q}}{2b^2d(3+p+q)}$$
$$-\frac{(adf(2+p+q)(bc(2+p)+ad(1+q))+(bc(1+p)+ad(1+q))(bde(3+p+q)-f(bc(2+p)+ad(}{2b^2d^2(bc-ad)(1+p)(2+}$$

output
```
1/2*(b*d*e*(3+p+q)-f*(b*c*(2+p)+a*d*(4+p+2*q)))*(b*x^2+a)^(p+1)*(d*x^2+c)^
(1+q)/b^2/d^2/(2+p+q)/(3+p+q)+1/2*f*(b*x^2+a)^(2+p)*(d*x^2+c)^(1+q)/b^2/d/
(3+p+q)-1/2*(a*d*f*(2+p+q)*(b*c*(2+p)+a*d*(1+q))+(b*c*(p+1)+a*d*(1+q))*(b*
d*e*(3+p+q)-f*(b*c*(2+p)+a*d*(4+p+2*q))))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*
hypergeom([1, 2+p+q],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(p+
1)/(2+p+q)/(3+p+q)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.74

$$\int x^3(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx$$
$$=\frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\left((bc-ad)^2f\,\text{Hypergeometric2F1}\left(1+p,-2-q,2+p,\frac{d(a+bx^2)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*((b*c - a*d)^2*f*Hypergeometric2F1[1 +
p, -2 - q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(-(d*
e) + 2*c*f)*Hypergeometric2F1[1 + p, -1 - q, 2 + p, (d*(a + b*x^2))/(-(b*c
) + a*d)]) + b*c*(-(d*e) + c*f)*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(a
+ b*x^2))/(-(b*c) + a*d)])))/(2*b^3*d^2*(1 + p)*((b*(c + d*x^2))/(b*c - a*
d))^q)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {435, 164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(e+fx^2)(a+bx^2)^p(c+dx^2)^q\,dx$$
$$\Big\downarrow \text{435}$$
$$\frac{1}{2}\int x^2(bx^2+a)^p(dx^2+c)^q(fx^2+e)\,dx^2$$
$$\Big\downarrow \text{164}$$

$$\frac{1}{2}\left(\frac{\left(a^2d^2f(q+1)(q+2)+abd(q+1)(2cf(p+1)-de(p+q+3))+b^2c(p+1)(cf(p+2)-de(p+q+3))\right)\int (}{b^2d^2(p+q+2)(p+q+3)}\right.$$

$\big\downarrow$ 80

$$\frac{1}{2}\left(\frac{(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\left(a^2d^2f(q+1)(q+2)+abd(q+1)(2cf(p+1)-de(p+q+3))+b^2c(p+1)(cf(p+\right.}{b^2d^2(p+q+2)(p+q+3)}\right.$$

$\big\downarrow$ 79

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\left(a^2d^2f(q+1)(q+2)+abd(q+1)(2cf(p+1)-de(p+q+3))+b^2c(\right.}{b^3d^2(p+1)(p+q+2)(p+q}\right.$$

input
```
Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(-(((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q)*(b*c*f*(2 + p) + a*d*f*(2 + q)
 - b*d*e*(3 + p + q) - b*d*f*(2 + p + q)*x^2))/(b^2*d^2*(2 + p + q)*(3 + p
 + q))) + ((a^2*d^2*f*(1 + q)*(2 + q) + a*b*d*(1 + q)*(2*c*f*(1 + p) - d*e
*(3 + p + q)) + b^2*c*(1 + p)*(c*f*(2 + p) - d*e*(3 + p + q)))*(a + b*x^2)
^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2
))/(b*c - a*d))])/(b^3*d^2*(1 + p)*(2 + p + q)*(3 + p + q)*((b*(c + d*x^2)
)/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x^{3} \left(b\,x^{2}+a \right)^{p} \left(d\,x^{2}+c \right)^{q} \left(f\,x^{2}+e \right)dx$$

input
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int x^3 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^5 + e*x^3)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Giac [F]

$$\int x^3(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx = \int (fx^2+e)(bx^2+a)^p(dx^2+c)^q x^3\,dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx = \int x^3\,(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)\,dx$$

input
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int x^3(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx = \text{too large to display}$$

input
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
((c + d*x**2)**q*(a + b*x**2)**p*a**3*c*d**2*f*p*q + 2*(c + d*x**2)**q*(a
+ b*x**2)**p*a**3*c*d**2*f*p - (c + d*x**2)**q*(a + b*x**2)**p*a**3*d**3*f
*p*q**2*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**3*d**3*f*p*q*x**2 - 2*
(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c**2*d*f*p*q - (c + d*x**2)**q*(a +
 b*x**2)**p*a**2*b*c*d**2*e*p**2 - (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*
c*d**2*e*p*q - 3*(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*e*p - (c +
d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*f*p**2*q*x**2 - 2*(c + d*x**2)**q
*(a + b*x**2)**p*a**2*b*c*d**2*f*p**2*x**2 + 2*(c + d*x**2)**q*(a + b*x**2
)**p*a**2*b*c*d**2*f*p*q**2*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*
d**3*e*p**2*q*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*d**3*e*p*q**2*
x**2 + 3*(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*d**3*e*p*q*x**2 + (c + d*x
**2)**q*(a + b*x**2)**p*a**2*b*d**3*f*p**2*q*x**4 + (c + d*x**2)**q*(a + b
*x**2)**p*a**2*b*d**3*f*p*q**2*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a**2
*b*d**3*f*p*q*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**3*f*p*q + 2
*(c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**3*f*q - (c + d*x**2)**q*(a + b*
x**2)**p*a*b**2*c**2*d*e*p*q - (c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**2
*d*e*q**2 - 3*(c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**2*d*e*q + 2*(c + d
*x**2)**q*(a + b*x**2)**p*a*b**2*c**2*d*f*p**2*q*x**2 - (c + d*x**2)**q*(a
 + b*x**2)**p*a*b**2*c**2*d*f*p*q**2*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)
**p*a*b**2*c**2*d*f*q**2*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b**2*...
```

3.68 $\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx$

Optimal result

Integrand size = 27, antiderivative size = 146

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx = \frac{f(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(2+p+q)}$$
$$-\frac{(bcf(1+p)+adf(1+q)-bde(2+p+q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,2+p+\right.}{2bd(bc-ad)(1+p)(2+p+q)}$$

output
```
1/2*f*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(2+p+q)-1/2*(b*c*f*(p+1)+a*d*f*(
1+q)-b*d*e*(2+p+q))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[
2+p],-d*(b*x^2+a)/(-a*d+b*c))/b/d/(-a*d+b*c)/(p+1)/(2+p+q)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx$$
$$= \frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(bf(c+dx^2)+\frac{(-bcf(1+p)-adf(1+q)+bde(2+p+q))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2\right.}{1+p}\right.}{2b^2d(2+p+q)}$$

input
```
Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(b*f*(c + d*x^2) + ((-(b*c*f*(1 + p)) -
 a*d*f*(1 + q) + b*d*e*(2 + p + q))*Hypergeometric2F1[1 + p, -q, 2 + p, (d
*(a + b*x^2))/(-(b*c) + a*d)])/((1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)))
/(2*b^2*d*(2 + p + q))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {435, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(e+fx^2\right)\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx$$

$$\Big\downarrow \text{435}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q\left(fx^2+e\right)dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\left(e-\frac{f(ad(q+1)+bc(p+1))}{bd(p+q+2)}\right)\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q dx^2+\frac{f\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(p+q+2)}\right)$$

$$\Big\downarrow \text{80}$$

$$\frac{1}{2}\left(\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\left(e-\frac{f(ad(q+1)+bc(p+1))}{bd(p+q+2)}\right)\int \left(bx^2+a\right)^p\left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^2+\frac{f(\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{79}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\left(e-\frac{f(ad(q+1)+bc(p+1))}{bd(p+q+2)}\right)\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^2\ldots}{bc-a\ldots}\right.}{b(p+1)}\right.$$

input
```
Int[x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
((f*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(2 + p + q)) + ((e - (f*
(b*c*(1 + p) + a*d*(1 + q)))/(b*d*(2 + p + q)))*(a + b*x^2)^(1 + p)*(c + d
*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))
])/(b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)\,dx$$

input
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int x(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)\,dx=\int (fx^2+e)(bx^2+a)^p(dx^2+c)^q x\,dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^3 + e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)\,dx=\text{Timed out}$$

input
```
integrate(x*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx=\int \left(fx^2+e\right)\left(bx^2+a\right)^p\left(dx^2+c\right)^q x\,dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Giac [F]

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx=\int \left(fx^2+e\right)\left(bx^2+a\right)^p\left(dx^2+c\right)^q x\,dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx=\int x\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,\left(f\,x^2+e\right)\,dx$$

input
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \text{too large to display}$$

input
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
( - (c + d*x**2)**q*(a + b*x**2)**p*a**2*c*d*f*p + (c + d*x**2)**q*(a + b*
x**2)**p*a**2*d**2*f*p*q*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a*b*c**2*f
*q + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*p**2 + 2*(c + d*x**2)**q*(a
 + b*x**2)**p*a*b*c*d*e*p*q + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*
p + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*q**2 + 2*(c + d*x**2)**q*(a
+ b*x**2)**p*a*b*c*d*e*q + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f*p**2*
x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f*q**2*x**2 + (c + d*x**2)*
*q*(a + b*x**2)**p*a*b*d**2*e*p*q*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a
*b*d**2*e*q**2*x**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*q*x**2
+ (c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f*p*q*x**4 + (c + d*x**2)**q*(a
 + b*x**2)**p*a*b*d**2*f*q**2*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*d
**2*f*q*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f*p*q*x**2 + (c +
 d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e*p**2*x**2 + (c + d*x**2)**q*(a + b*
x**2)**p*b**2*c*d*e*p*q*x**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*
e*p*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*f*p**2*x**4 + (c + d*x
**2)**q*(a + b*x**2)**p*b**2*c*d*f*p*q*x**4 + (c + d*x**2)**q*(a + b*x**2)
**p*b**2*c*d*f*p*x**4 - 2*int(((c + d*x**2)**q*(a + b*x**2)**p*x**3)/(a**2
*c*d*p**2*q + 2*a**2*c*d*p*q**2 + 3*a**2*c*d*p*q + a**2*c*d*q**3 + 3*a**2*
c*d*q**2 + 2*a**2*c*d*q + a**2*d**2*p**2*q*x**2 + 2*a**2*d**2*p*q**2*x**2
+ 3*a**2*d**2*p*q*x**2 + a**2*d**2*q**3*x**2 + 3*a**2*d**2*q**2*x**2 + ...
```

3.69 $\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 169

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx = -\frac{e(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,1,-q,2+p,1+\frac{bx^2}{a},-\frac{d(a+bx^2)}{bc-ad}\right)}{2a(1+p)} + \frac{f(a+bx^2)^{1+p} (c+dx^2)^{1+q} \text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right)}{2(bc-ad)(1+p)}$$

output
```
-1/2*e*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,-q,1,2+p,-d*(b*x^2+a)/(-a*
d+b*c),1+b*x^2/a)/a/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)+1/2*f*(b*x^2+a)^(p+
1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/(-a
*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx$$
$$= \frac{1}{2}(a+bx^2)^p (c+dx^2)^q \left(fx^2 \left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(1,-p,-q,2,-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{e\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q} \text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{p+q} \right)$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*((f*x^2*AppellF1[1, -p, -q, 2, -((b*x^2)/a),
-((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (e*AppellF1[-p - q,
 -p, -q, 1 - p - q, -(a/(b*x^2)), -(c/(d*x^2))])/((p + q)*(1 + a/(b*x^2))^
p*(1 + c/(d*x^2))^q)))/2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {435, 175, 80, 79, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)(a+bx^2)^p(c+dx^2)^q}{x} \, dx$$

$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p(dx^2+c)^q(fx^2+e)}{x^2}dx^2$$

$$\Big\downarrow \text{ 175}$$

$$\frac{1}{2}\left(e\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}dx^2+f\int (bx^2+a)^p (dx^2+c)^q dx^2\right)$$

$$\Big\downarrow \text{80}$$

$$\frac{1}{2}\left(e\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}dx^2+f(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int (bx^2+a)^p\left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^2\right)$$

$$\Big\downarrow \text{79}$$

$$\frac{1}{2}\left(e\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}dx^2+\frac{f(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+\right.}{b(p+1)}\right.$$

$$\Big\downarrow \text{154}$$

$$\frac{1}{2}\left(e(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int \frac{(bx^2+a)^p\left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q}{x^2}dx^2+\frac{f(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{I}}{}\right.$$

$$\Big\downarrow \text{153}$$

$$\frac{1}{2}\left(\frac{f(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^2+a)}{bc-ad}\right)}{b(p+1)}-\frac{e(a+bx^2)^{p+1}(}{}\right.$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x,x]
```

output
```
(-((e*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d
*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c + d*x^2))/(b
*c - a*d))^q)) + (f*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1
+ p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d*x^2
))/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)}{x}\,dx = \int \frac{(fx^2+e)(bx^2+a)^p(dx^2+c)^q}{x}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)}{x}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x} \, dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x} \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x,x)
```

output
```
((c + d*x**2)**q*(a + b*x**2)**p*a*c*f*p + (c + d*x**2)**q*(a + b*x**2)**p
*a*c*f*q + (c + d*x**2)**q*(a + b*x**2)**p*a*d*e*p + (c + d*x**2)**q*(a +
b*x**2)**p*a*d*e*q + (c + d*x**2)**q*(a + b*x**2)**p*a*d*e + (c + d*x**2)*
*q*(a + b*x**2)**p*a*d*f*q*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*b*c*e*p
+ (c + d*x**2)**q*(a + b*x**2)**p*b*c*e*q + (c + d*x**2)**q*(a + b*x**2)**
p*b*c*e + (c + d*x**2)**q*(a + b*x**2)**p*b*c*f*p*x**2 + 2*int(((c + d*x**
2)**q*(a + b*x**2)**p*x**3)/(a**2*c*d*p*q + a**2*c*d*q**2 + a**2*c*d*q + a
**2*d**2*p*q*x**2 + a**2*d**2*q**2*x**2 + a**2*d**2*q*x**2 + a*b*c**2*p**2
 + a*b*c**2*p*q + a*b*c**2*p + a*b*c*d*p**2*x**2 + 2*a*b*c*d*p*q*x**2 + a*
b*c*d*p*x**2 + a*b*c*d*q**2*x**2 + a*b*c*d*q*x**2 + a*b*d**2*p*q*x**4 + a*
b*d**2*q**2*x**4 + a*b*d**2*q*x**4 + b**2*c**2*p**2*x**2 + b**2*c**2*p*q*x
**2 + b**2*c**2*p*x**2 + b**2*c*d*p**2*x**4 + b**2*c*d*p*q*x**4 + b**2*c*d
*p*x**4),x)*a**3*d**3*f*p**2*q**2 + 2*int(((c + d*x**2)**q*(a + b*x**2)**p
*x**3)/(a**2*c*d*p*q + a**2*c*d*q**2 + a**2*c*d*q + a**2*d**2*p*q*x**2 + a
**2*d**2*q**2*x**2 + a**2*d**2*q*x**2 + a*b*c**2*p**2 + a*b*c**2*p*q + a*b
*c**2*p + a*b*c*d*p**2*x**2 + 2*a*b*c*d*p*q*x**2 + a*b*c*d*p*x**2 + a*b*c*
d*q**2*x**2 + a*b*c*d*q*x**2 + a*b*d**2*p*q*x**4 + a*b*d**2*q**2*x**4 + a*
b*d**2*q*x**4 + b**2*c**2*p**2*x**2 + b**2*c**2*p*q*x**2 + b**2*c**2*p*x**
2 + b**2*c*d*p**2*x**4 + b**2*c*d*p*q*x**4 + b**2*c*d*p*x**4),x)*a**3*d**3
*f*p*q**3 + 2*int(((c + d*x**2)**q*(a + b*x**2)**p*x**3)/(a**2*c*d*p*q ...
```

3.70 $\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} \, dx$

Optimal result

Integrand size = 29, antiderivative size = 234

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} \, dx = -\frac{e(a+bx^2)^{1+p} (c+dx^2)^{1+q}}{2acx^2}$$
$$-\frac{(acf+bcep+adeq)(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,1,-q,2+p,1+\frac{bx^2}{a},-\frac{d(a+}{bc-}\right.}{2a^2c(1+p)}$$
$$+\frac{bde(1+p+q)(a+bx^2)^{1+p} (c+dx^2)^{1+q} \text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right)}{2ac(bc-ad)(1+p)}$$

output
```
-1/2*e*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x^2-1/2*(a*d*e*q+b*c*e*p+a*c*f)
*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,-q,1,2+p,-d*(b*x^2+a)/(-a*d+b*c)
,1+b*x^2/a)/a^2/c/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)+1/2*b*d*e*(1+p+q)*(b*
x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(b*x^2+a)/(-a*d
+b*c))/a/c/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3}\,dx$$
$$= \frac{\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}(a+bx^2)^p(c+dx^2)^q\left(f(-1+p+q)x^2\,\text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\right.\right.}{2(-1+p+q)(p+q)x^2}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^3,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*(f*(-1 + p + q)*x^2*AppellF1[-p - q, -p, -q,
1 - p - q, -(a/(b*x^2)), -(c/(d*x^2))] + e*(p + q)*AppellF1[1 - p - q, -p,
 -q, 2 - p - q, -(a/(b*x^2)), -(c/(d*x^2))]))/(2*(-1 + p + q)*(p + q)*(1 +
 a/(b*x^2))^p*(1 + c/(d*x^2))^q*x^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {435, 168, 25, 175, 80, 79, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)(a+bx^2)^p(c+dx^2)^q}{x^3}dx$$

$$\Big\downarrow \text{435}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p(dx^2+c)^q(fx^2+e)}{x^4}dx^2$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{(bx^2+a)^p(dx^2+c)^q(bde(p+q+1)x^2+bcep+a(cf+deq))}{x^2}dx^2}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{(bx^2+a)^p(dx^2+c)^q(bde(p+q+1)x^2+acf+bcep+adeq)}{x^2}dx^2}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx^2}\right)$$

$\downarrow$ 175

$$\frac{1}{2}\left(\frac{(acf+adeq+bcep)\int \frac{(bx^2+a)^p(dx^2+c)^q}{x^2}dx^2+bde(p+q+1)\int (bx^2+a)^p(dx^2+c)^q dx^2}{ac}-\frac{e(a+bx^2)^{p+1}(c}{acx^2}\right.$$

$\downarrow$ 80

$$\frac{1}{2}\left(\frac{(acf+adeq+bcep)\int \frac{(bx^2+a)^p(dx^2+c)^q}{x^2}dx^2+bde(p+q+1)(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int (bx^2+a)^p\left(\frac{bdx^2}{bc-ad}+\frac{b}{}\right.}{ac}\right.$$

$\downarrow$ 79

$$\frac{1}{2}\left(\frac{(acf+adeq+bcep)\int \frac{(bx^2+a)^p(dx^2+c)^q}{x^2}dx^2+\frac{de(p+q+1)(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,\right.}{p+1}}{ac}\right.$$

$\downarrow$ 154

$$\frac{1}{2}\left(\frac{(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}(acf+adeq+bcep)\int \frac{(bx^2+a)^p\left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q}{x^2}dx^2+\frac{de(p+q+1)(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+}{bc-}\right.}{}}{ac}\right.$$

$\downarrow$ 153

$$\frac{1}{2}\left(\frac{\frac{de(p+q+1)(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^2+a)}{bc-ad}\right)}{p+1}-\frac{(a+bx^2)^{p+1}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)}{}}{ac}\right.$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^3,x]
```

output
```
(-((e*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(a*c*x^2)) + (-(((a*c*f + b
*c*e*p + a*d*e*q)*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1,
 2 + p, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c
+ d*x^2))/(b*c - a*d))^q)) + (d*e*(1 + p + q)*(a + b*x^2)^(1 + p)*(c + d*x
^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])
/((1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q))/(a*c))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^3}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x^3} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x^3} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \int \frac{(fx^2+e)(bx^2+a)^p(dx^2+c)^q}{x^3} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^3} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^3,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^3} dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^3,x)
```

output
```
( - (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*e*q**2 + (c + d*x**2)**q*(a
+ b*x**2)**p*a**2*d**2*e*q + (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*f*q
*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*f*x**2 - 2*(c + d*x**2)*
*q*(a + b*x**2)**p*a*b*c*d*e*p*q + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d
*e*p + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*q - (c + d*x**2)**q*(a +
b*x**2)**p*a*b*c*d*f*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*p*x
**2 - (c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*e*p**2 + (c + d*x**2)**q*(
a + b*x**2)**p*b**2*c**2*e*p + (c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f
*p*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f*x**2 - (c + d*x**2)*
*q*(a + b*x**2)**p*b**2*c*d*e*q*x**2 - 2*int(((c + d*x**2)**q*(a + b*x**2)
**p*x**3)/(a**3*c*d**2*q**2 - a**3*c*d**2*q + a**3*d**3*q**2*x**2 - a**3*d
**3*q*x**2 + 2*a**2*b*c**2*d*p*q - a**2*b*c**2*d*p - a**2*b*c**2*d*q + 2*a
**2*b*c*d**2*p*q*x**2 - a**2*b*c*d**2*p*x**2 + a**2*b*c*d**2*q**2*x**2 - 2
*a**2*b*c*d**2*q*x**2 + a**2*b*d**3*q**2*x**4 - a**2*b*d**3*q*x**4 + a*b**
2*c**3*p**2 - a*b**2*c**3*p + a*b**2*c**2*d*p**2*x**2 + 2*a*b**2*c**2*d*p*
q*x**2 - 2*a*b**2*c**2*d*p*x**2 - a*b**2*c**2*d*q*x**2 + 2*a*b**2*c*d**2*p
*q*x**4 - a*b**2*c*d**2*p*x**4 - a*b**2*c*d**2*q*x**4 + b**3*c**3*p**2*x**
2 - b**3*c**3*p*x**2 + b**3*c**2*d*p**2*x**4 - b**3*c**2*d*p*x**4),x)*a**4
*b*d**5*f*p*q**3*x**2 + 4*int(((c + d*x**2)**q*(a + b*x**2)**p*x**3)/(a**3
*c*d**2*q**2 - a**3*c*d**2*q + a**3*d**3*q**2*x**2 - a**3*d**3*q*x**2 +...
```

3.71 $\int x^4(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 464

$$\int x^4(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)\,dx$$

$$= -\frac{(bcf(5+2p)+adf(5+2q)-bde(7+2p+2q))x(a+bx^2)^{1+p}\,(c+dx^2)^{1+q}}{b^2d^2(5+2p+2q)(7+2p+2q)}$$

$$+\frac{fx^3(a+bx^2)^{1+p}\,(c+dx^2)^{1+q}}{bd(7+2p+2q)}$$

$$+\frac{ac(bcf(5+2p)+adf(5+2q)-bde(7+2p+2q))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\text{Ap}}{b^2d^2(5+2p+2q)(7+2p+2q)}$$

$$+\frac{(a^2d^2f(15+16q+4q^2)+b^2c(3+2p)(cf(5+2p)-de(7+2p+2q))-abd(de(3+2q)(7+2p+2q)}{3b^2d^2(5-}$$

output
```
-(b*c*f*(5+2*p)+a*d*f*(5+2*q)-b*d*e*(7+2*p+2*q))*x*(b*x^2+a)^(p+1)*(d*x^2+
c)^(1+q)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)+f*x^3*(b*x^2+a)^(p+1)*(d*x^2+c)^(
1+q)/b/d/(7+2*p+2*q)+a*c*(b*c*f*(5+2*p)+a*d*f*(5+2*q)-b*d*e*(7+2*p+2*q))*x
*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/b^2/d^2
/(5+2*p+2*q)/(7+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*(a^2*d^2*f*(4
*q^2+16*q+15)+b^2*c*(3+2*p)*(c*f*(5+2*p)-d*e*(7+2*p+2*q))-a*b*d*(d*e*(3+2*
q)*(7+2*p+2*q)-c*f*(8*p*q+10*p+10*q+15)))*x^3*(b*x^2+a)^p*(d*x^2+c)^q*Appe
llF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)/((1+
b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.27

$$\int x^4(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx=\frac{1}{35}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\left(7e\,\mathrm{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+5fx^2\,\mathrm{AppellF1}\left(\frac{7}{2},-p,-q,\frac{9}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)$$

input
```
Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(x^5*(a + b*x^2)^p*(c + d*x^2)^q*(7*e*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)
/a), -((d*x^2)/c)] + 5*f*x^2*AppellF1[7/2, -p, -q, 9/2, -((b*x^2)/a), -((d
*x^2)/c)]))/(35*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {444, 444, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(e+fx^2\right) \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \ 444$$

$$\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf\right)dx}{bd(2p+2q+7)}$$

$$\Big\downarrow \ 444$$

$$\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q\left((ad(2q+3)(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7\right.\right.}{}}{bd(2p+2q+7)}$$

$$\Big\downarrow \ 406$$

$$\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q dx+(a}{}}{bd(2p+2q-}$$

$$\Big\downarrow \ 334$$

$$\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))\int \left(\frac{bx}{a}\right.}{}}{bd(}$$

$$\Big\downarrow \ 334$$

$$\frac{fx^3\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-$$

$$\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-ac\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}(adf(2q+5)+bcf(2p+5)-$$

$$\downarrow 333$$

$$\frac{fx^3\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-$$

$$\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-(ad(2q+3)(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))+b(4acdfp(q+1)+bc$$

$$\downarrow 395$$

$$\frac{fx^3\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-$$

$$\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ad(2q+3)(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7$$

$$\downarrow 395$$

$$\frac{fx^3\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-$$

$$\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}(ad(2q+3)(adf(2q+5)+bcf$$

$$\downarrow 394$$

$$\frac{fx^3\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)}-$$

$$\frac{x\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-acx\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},\right.$$

input
```
Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(f*x^3*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(7 + 2*p + 2*q)) - ((
(b*c*f*(5 + 2*p) + a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^2)^
(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - ((a*c*(b*c*f*(5 + 2*p
) + a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^2)^p*(c + d*x^2)^q
*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^
p*(1 + (d*x^2)/c)^q) + ((a*d*(3 + 2*q)*(b*c*f*(5 + 2*p) + a*d*f*(5 + 2*q)
- b*d*e*(7 + 2*p + 2*q)) + b*(4*a*c*d*f*p*(1 + q) + b*c*(3 + 2*p)*(c*f*(5
+ 2*p) - d*e*(7 + 2*p + 2*q))))*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3
/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d
*x^2)/c)^q))/(b*d*(5 + 2*p + 2*q)))/(b*d*(7 + 2*p + 2*q))
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{p} \left(d \,x^{2}+c \right)^{q} \left(f \,x^{2}+e \right)d x$$

input
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int \left(fx^{2} + e\right) \left(bx^{2} + a\right)^{p} \left(dx^{2} + c\right)^{q} x^{4} \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^6 + e*x^4)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) dx = \int \left(fx^2+e\right) \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) dx = \int \left(fx^2+e\right) \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx=\int x^4\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\left(f\,x^2+e\right)\,dx$$

input
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int x^4(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)\,dx=\int x^4\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\left(f\,x^2+e\right)dx$$

input
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

3.72 $\int x^2(a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx$

Optimal result

Integrand size = 29, antiderivative size = 273

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx = \frac{fx(a+bx^2)^{1+p} (c+dx^2)^{1+q}}{bd(5+2p+2q)}$$
$$-\frac{acfx(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{bd(5+2p+2q)}$$
$$-\frac{(bcf(3+2p)+adf(3+2q)-bde(5+2p+2q))x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \mathrm{App}}{3bd(5+2p+2q)}$$

output
```
f*x*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(5+2*p+2*q)-a*c*f*x*(b*x^2+a)^p*(d
*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/b/d/(5+2*p+2*q)/((1+b*
x^2/a)^p)/((1+d*x^2/c)^q)-1/3*(b*c*f*(3+2*p)+a*d*f*(3+2*q)-b*d*e*(5+2*p+2*
q))*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/
b/d/(5+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.46

$$\int x^2(a+bx^2)^p(c+dx^2)^q(e+fx^2)\,dx = \frac{1}{15}x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\left(5e\operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3fx^2\operatorname{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)$$

input
```
Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(5*e*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)
/a), -((d*x^2)/c)] + 3*f*x^2*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -((d
*x^2)/c)]))/(15*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {444, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(e+fx^2)(a+bx^2)^p(c+dx^2)^q\,dx$$

$$\Big\downarrow \text{444}$$

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\int (bx^2+a)^p(dx^2+c)^q((bcf(2p+3)+adf(2q+3)-bde(2p+2q+5))x^2+acf)\,dx}{bd(2p+2q+5)}$$

$$\Big\downarrow \text{406}$$

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q\,dx + acf\int (bx^2+a)^p(dx^2+c)^q\,dx}{bd(2p+2q+5)}$$

$\big\downarrow$ 334

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q\,dx + acf(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \Big(b}{bd(2p+2q+5)}$$

$\big\downarrow$ 334

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q\,dx + acf(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+}{bd(2p+2q+5)}$$

$\big\downarrow$ 333

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q\,dx + acfx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c}{bd(2p+2q+5)}$$

$\big\downarrow$ 395

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx + acfx(a}{bd(2p+2q+5)}$$

$\big\downarrow$ 395

$$\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)} - \frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2\left(\frac{bx^2}{a}+1\right)^p}{bd(2p+2q}$$

$\big\downarrow$ 394

$$\frac{fx(a+bx^2)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+5)} - \frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(adf(2q+3)+bcf(2p}{bd(2p}$$

input
```
Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(f*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - ((a*
c*f*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a),
 -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + ((b*c*f*(3 + 2*p)
+ a*d*f*(3 + 2*q) - b*d*e*(5 + 2*p + 2*q))*x^3*(a + b*x^2)^p*(c + d*x^2)^q
*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a
)^p*(1 + (d*x^2)/c)^q))/(b*d*(5 + 2*p + 2*q))
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

Maple [F]

$$\int x^2 (b x^2 + a)^p (d x^2 + c)^q (f x^2 + e) dx$$

input
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int \left(fx^2+e\right)\left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^4 + e*x^2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int \left(fx^2+e\right)\left(bx^2+a\right)^p \left(dx^2+c\right)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Giac [F]

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int (fx^2+e)(bx^2+a)^p (dx^2+c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \int x^2 (b x^2+a)^p (d x^2+c)^q (f x^2+e) \, dx$$

input
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2) \, dx = \text{too large to display}$$

input
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
( - 4*(c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*f*p*q*x - 6*(c + d*x**2)**
q*(a + b*x**2)**p*a**2*d**2*f*p*x + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*
c*d*f*p*q*x + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*p**2*x + 4*(c +
 d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*p*q*x + 10*(c + d*x**2)**q*(a + b*x
**2)**p*a*b*d**2*e*p*x + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f*p**2
*x**3 + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f*p*q*x**3 + 2*(c + d*x
**2)**q*(a + b*x**2)**p*a*b*d**2*f*p*x**3 - 4*(c + d*x**2)**q*(a + b*x**2)
**p*b**2*c**2*f*p*q*x - 6*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f*q*x
+ 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e*p*q*x + 4*(c + d*x**2)**q*(
a + b*x**2)**p*b**2*c*d*e*q**2*x + 10*(c + d*x**2)**q*(a + b*x**2)**p*b**2
*c*d*e*q*x + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*f*p*q*x**3 + 4*(c
+ d*x**2)**q*(a + b*x**2)**p*b**2*c*d*f*q**2*x**3 + 2*(c + d*x**2)**q*(a +
 b*x**2)**p*b**2*c*d*f*q*x**3 + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**
2*e*p**2*x**3 + 8*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e*p*q*x**3 + 1
2*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e*p*x**3 + 4*(c + d*x**2)**q*(
a + b*x**2)**p*b**2*d**2*e*q**2*x**3 + 12*(c + d*x**2)**q*(a + b*x**2)**p*
b**2*d**2*e*q*x**3 + 5*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e*x**3 +
4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*f*p**2*x**5 + 8*(c + d*x**2)**
q*(a + b*x**2)**p*b**2*d**2*f*p*q*x**5 + 8*(c + d*x**2)**q*(a + b*x**2)**p
*b**2*d**2*f*p*x**5 + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*f*q**...
```

3.73 $\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)\, dx = ex(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{1}{3}fx^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
e*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+
b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*f*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2
,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.46

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \frac{1}{3}x\left(a+bx^2\right)^p \left(c\right.$$

$$\left.+dx^2\right)^q \left(\frac{9ace \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcp \operatorname{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\cdots\right.}\right.$$

$$\left. +fx^2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(x*(a + b*x^2)^p*(c + d*x^2)^q*((9*a*c*e*AppellF1[1/2, -p, -q, 3/2, -((b*x
^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d
*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)
])) + (f*x^2*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 +
 (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)))/3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^2\right)\left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{406}$$

$$e\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx + f\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \; 334$$

$$e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx+f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx$$

$$\Big\downarrow \; 334$$

$$e(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+ f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx$$

$$\Big\downarrow \; 333$$

$$f\int x^2(bx^2+a)^p(dx^2+c)^q\,dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \; 395$$

$$f(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \; 395$$

$$f(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+ ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

$$\Big\downarrow \; 394$$

$$ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+ \frac{1}{3}fx^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x]
```

output
```
(e*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a),
-((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (f*x^3*(a + b*x^2)^
p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3
*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q} \left(f\,x^{2}+e\right)dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)\,dx = \int \left(fx^2+e\right)\left(bx^2+a\right)^p\left(dx^2+c\right)^q dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int (fx^2+e)\,(bx^2+a)^p\,(dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int (fx^2+e)\,(bx^2+a)^p\,(dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \int \left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,\left(f\,x^2+e\right) \,dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2),x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2), x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right) \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e),x)
```

output
```
(2*(c + d*x**2)**q*(a + b*x**2)**p*a*d*f*p*x + 2*(c + d*x**2)**q*(a + b*x*
*2)**p*b*c*f*q*x + 2*(c + d*x**2)**q*(a + b*x**2)**p*b*d*e*p*x + 2*(c + d*
x**2)**q*(a + b*x**2)**p*b*d*e*q*x + 3*(c + d*x**2)**q*(a + b*x**2)**p*b*d
*e*x + 2*(c + d*x**2)**q*(a + b*x**2)**p*b*d*f*p*x**3 + 2*(c + d*x**2)**q*
(a + b*x**2)**p*b*d*f*q*x**3 + (c + d*x**2)**q*(a + b*x**2)**p*b*d*f*x**3
- 16*int(((c + d*x**2)**q*(a + b*x**2)**p*x**2)/(4*a*c*p**2 + 8*a*c*p*q +
8*a*c*p + 4*a*c*q**2 + 8*a*c*q + 3*a*c + 4*a*d*p**2*x**2 + 8*a*d*p*q*x**2
+ 8*a*d*p*x**2 + 4*a*d*q**2*x**2 + 8*a*d*q*x**2 + 3*a*d*x**2 + 4*b*c*p**2*
x**2 + 8*b*c*p*q*x**2 + 8*b*c*p*x**2 + 4*b*c*q**2*x**2 + 8*b*c*q*x**2 + 3*
b*c*x**2 + 4*b*d*p**2*x**4 + 8*b*d*p*q*x**4 + 8*b*d*p*x**4 + 4*b*d*q**2*x*
*4 + 8*b*d*q*x**4 + 3*b*d*x**4),x)*a**2*d**2*f*p**3*q - 8*int(((c + d*x**2
)**q*(a + b*x**2)**p*x**2)/(4*a*c*p**2 + 8*a*c*p*q + 8*a*c*p + 4*a*c*q**2
+ 8*a*c*q + 3*a*c + 4*a*d*p**2*x**2 + 8*a*d*p*q*x**2 + 8*a*d*p*x**2 + 4*a*
d*q**2*x**2 + 8*a*d*q*x**2 + 3*a*d*x**2 + 4*b*c*p**2*x**2 + 8*b*c*p*q*x**2
 + 8*b*c*p*x**2 + 4*b*c*q**2*x**2 + 8*b*c*q*x**2 + 3*b*c*x**2 + 4*b*d*p**2
*x**4 + 8*b*d*p*q*x**4 + 8*b*d*p*x**4 + 4*b*d*q**2*x**4 + 8*b*d*q*x**4 + 3
*b*d*x**4),x)*a**2*d**2*f*p**3 - 32*int(((c + d*x**2)**q*(a + b*x**2)**p*x
**2)/(4*a*c*p**2 + 8*a*c*p*q + 8*a*c*p + 4*a*c*q**2 + 8*a*c*q + 3*a*c + 4*
a*d*p**2*x**2 + 8*a*d*p*q*x**2 + 8*a*d*p*x**2 + 4*a*d*q**2*x**2 + 8*a*d*q*
x**2 + 3*a*d*x**2 + 4*b*c*p**2*x**2 + 8*b*c*p*q*x**2 + 8*b*c*p*x**2 + 4...
```

3.74 $\int \frac{(a+bx^2)^p(c+dx^2)^q(e+fx^2)}{x^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 244

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} dx = -\frac{e(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{acx} + \frac{(acf+bce(1+2p)+ad(e+2eq))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-\right.}{ac} + \frac{bde(3+2p+2q)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right.}{3ac}$$

output
```
-e*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x+(a*c*f+b*c*e*(1+2*p)+a*d*(2*e*q+e
))*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/a/c
/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*b*d*e*(3+2*p+2*q)*x^3*(b*x^2+a)^p*(d*
x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/a/c/((1+b*x^2/a)^p)/((1
+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2}\,dx$$
$$= \frac{(a+bx^2)^p (c+dx^2)^q \left(-e\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{}{3ac\,\text{AppellF1}(}\right.}{x}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^2,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*(-((e*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a
), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)) + (3*a*c*f*x^2*Ap
pellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2,
 -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 -
 p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q,
5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/x
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {445, 25, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)(a+bx^2)^p(c+dx^2)^q}{x^2}\,dx$$

$$\Big\downarrow \; 445$$

$$-\frac{\int -(bx^2+a)^p(dx^2+c)^q\left(bde(2p+2q+3)x^2+bce(2p+1)+a(cf+d(2qe+e))\right)dx}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$$\Big\downarrow \; 25$$

$$\frac{\int (bx^2+a)^p (dx^2+c)^q \left(bde(2p+2q+3)x^2+acf+bce(2p+1)+ad(2qe+e)\right) dx}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 406

$$\frac{(acf+ad(2eq+e)+bce(2p+1))\int (bx^2+a)^p (dx^2+c)^q dx + bde(2p+2q+3)\int x^2(bx^2+a)^p (dx^2+c)^q dx}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 334

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (acf+ad(2eq+e)+bce(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q dx + bde(2p+2q+3)\int x^2(b}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 334

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} (acf+ad(2eq+e)+bce(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx +}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 333

$$\frac{bde(2p+2q+3)\int x^2(bx^2+a)^p (dx^2+c)^q dx + x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},\right.}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 395

$$\frac{bde(2p+2q+3)(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^2\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q dx + x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\right.}{ac} - \frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}$$

$\downarrow$ 395

$$\frac{\frac{bde(2p+2q+3)\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+x\left(a+bx^2\right)^p\left(\right.}{ac}}{\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{acx}}$$

$$\Big\downarrow \; 394$$

$$\frac{x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce}{\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{acx}}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^2,x]
```

output
```
-((e*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(a*c*x)) + (((a*c*f + b*c*e*
(1 + 2*p) + a*d*(e + 2*e*q))*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -
p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c
)^q) + (b*d*e*(3 + 2*p + 2*q)*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2
, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x
^2)/c)^q))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^2}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x^2} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x^2} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p (dx^2+c)^q}{x^2} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^2} \, dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^2} \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^2,x)
```

output
```
(2*(c + d*x**2)**q*(a + b*x**2)**p*a*c*f*p + 2*(c + d*x**2)**q*(a + b*x**2
)**p*a*c*f*q + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*d*e*p + 2*(c + d*x**2)*
*q*(a + b*x**2)**p*a*d*e*q + (c + d*x**2)**q*(a + b*x**2)**p*a*d*e + 2*(c
+ d*x**2)**q*(a + b*x**2)**p*a*d*f*q*x**2 - (c + d*x**2)**q*(a + b*x**2)**
p*a*d*f*x**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*b*c*e*p + 2*(c + d*x**2)*
*q*(a + b*x**2)**p*b*c*e*q + (c + d*x**2)**q*(a + b*x**2)**p*b*c*e + 2*(c
+ d*x**2)**q*(a + b*x**2)**p*b*c*f*p*x**2 - (c + d*x**2)**q*(a + b*x**2)**
p*b*c*f*x**2 + 16*int(((c + d*x**2)**q*(a + b*x**2)**p*x**2)/(4*a**2*c*d*p
*q - 2*a**2*c*d*p + 4*a**2*c*d*q**2 - a**2*c*d + 4*a**2*d**2*p*q*x**2 - 2*
a**2*d**2*p*x**2 + 4*a**2*d**2*q**2*x**2 - a**2*d**2*x**2 + 4*a*b*c**2*p**
2 + 4*a*b*c**2*p*q - 2*a*b*c**2*q - a*b*c**2 + 4*a*b*c*d*p**2*x**2 + 8*a*b
*c*d*p*q*x**2 - 2*a*b*c*d*p*x**2 + 4*a*b*c*d*q**2*x**2 - 2*a*b*c*d*q*x**2
- 2*a*b*c*d*x**2 + 4*a*b*d**2*p*q*x**4 - 2*a*b*d**2*p*x**4 + 4*a*b*d**2*q*
*2*x**4 - a*b*d**2*x**4 + 4*b**2*c**2*p**2*x**2 + 4*b**2*c**2*p*q*x**2 - 2
*b**2*c**2*q*x**2 - b**2*c**2*x**2 + 4*b**2*c*d*p**2*x**4 + 4*b**2*c*d*p*q
*x**4 - 2*b**2*c*d*q*x**4 - b**2*c*d*x**4),x)*a**3*d**3*f*p**2*q**2*x - 16
*int(((c + d*x**2)**q*(a + b*x**2)**p*x**2)/(4*a**2*c*d*p*q - 2*a**2*c*d*p
 + 4*a**2*c*d*q**2 - a**2*c*d + 4*a**2*d**2*p*q*x**2 - 2*a**2*d**2*p*x**2
+ 4*a**2*d**2*q**2*x**2 - a**2*d**2*x**2 + 4*a*b*c**2*p**2 + 4*a*b*c**2*p*
q - 2*a*b*c**2*q - a*b*c**2 + 4*a*b*c*d*p**2*x**2 + 8*a*b*c*d*p*q*x**2 ...
```

3.75 $\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 384

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4}\,dx = -\frac{e(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{3acx^3}$$
$$-\frac{(3acf-bce(1-2p)-ade(1-2q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{3a^2c^2x}$$
$$-\frac{(b^2c^2e(1-4p^2)-a^2d(3cf-de(1-2q))(1+2q)-abc(3cf(1+2p)-de(1-2p-2q-8pq)))\,x(a+}{3a^2c^2}$$
$$+\frac{bd(3acf-bce(1-2p)-ade(1-2q))(3+2p+2q)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q} A}{9a^2c^2}$$

output
```
-1/3*e*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x^3-1/3*(3*a*c*f-b*c*e*(1-2*p)-
a*d*e*(1-2*q))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a^2/c^2/x-1/3*(b^2*c^2*e*(-
4*p^2+1)-a^2*d*(3*c*f-d*e*(1-2*q))*(1+2*q)-a*b*c*(3*c*f*(1+2*p)-d*e*(-8*p*
q-2*p-2*q+1)))*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-
d*x^2/c)/a^2/c^2/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/9*b*d*(3*a*c*f-b*c*e*(1
-2*p)-a*d*e*(1-2*q))*(3+2*p+2*q)*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,
-p,-q,5/2,-b*x^2/a,-d*x^2/c)/a^2/c^2/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4}\,dx =$$
$$-\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \left(e\,\mathrm{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3fx^2\,\mathrm{Ap}\right.}{3x^3}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^4,x]
```

output
```
-1/3*((a + b*x^2)^p*(c + d*x^2)^q*(e*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2
)/a), -((d*x^2)/c)] + 3*f*x^2*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -(
(d*x^2)/c)]))/(x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {445, 25, 445, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)(a+bx^2)^p (c+dx^2)^q}{x^4}\,dx$$

$$\Big\downarrow \text{445}$$

$$-\frac{\int -\frac{(bx^2+a)^p (dx^2+c)^q (bde(2p+2q+1)x^2+3acf-bce(1-2p)-ad(e-2eq))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^p (dx^2+c)^q (bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{\frac{\int (bx^2+a)^p (dx^2+c)^q (-d(3cf-de(1-2q))(2q+1)a^2-bc(3cf(2p+1)-de(-8qp-2p-2q+1))a-bd(3acf-bce(1-2p)-ade(1-2q))(2p+2q+3)x^2 \cdots}{ac}}{3ac} \cdots$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 406

$$-\frac{\frac{(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2))\int (bx^2+a)^p (dx^2+c)^q dx-bd(2p+2q+3)(3acf-ade(\cdots}{ac}}{3ac} \cdots$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 334

$$-\frac{\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2))\int \left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q dx-bd \cdots}{ac}}{3ac} \cdots$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 334

$$-\frac{\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2))\int \left(\frac{bx^2}{a} \cdots\right.}{ac} \cdots}{\cdots}$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 333

$$-\frac{\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8p \cdots}{ac} \cdots}{\cdots}$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 395

$$-\frac{\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8p \cdots}{ac} \cdots}{\cdots}$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\big\downarrow$ 395

$$-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8p\right.}{}$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

$\Big\downarrow$ 394

$$-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8p\right.}{}$$

$$\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^4,x]
```

output
```
-1/3*(e*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(a*c*x^3) + (-(((3*a*c*f
- b*c*e*(1 - 2*p) - a*d*e*(1 - 2*q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 +
q))/(a*c*x)) - (((b^2*c^2*e*(1 - 4*p^2) - a^2*d*(3*c*f - d*e*(1 - 2*q))*(1
 + 2*q) - a*b*c*(3*c*f*(1 + 2*p) - d*e*(1 - 2*p - 2*q - 8*p*q)))*x*(a + b*
x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)
])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) - (b*d*(3*a*c*f - b*c*e*(1 - 2*p)
 - a*d*e*(1 - 2*q))*(3 + 2*p + 2*q)*x^3*(a + b*x^2)^p*(c + d*x^2)^q*Appell
F1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1
+ (d*x^2)/c)^q))/(a*c))/(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^4}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)}{x^4}\,dx = \int \frac{(fx^2+e)(bx^2+a)^p(dx^2+c)^q}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x, algorithm="fricas")
```

output
```
integral((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p(dx^2+c)^q}{x^4} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4} \, dx = \int \frac{(fx^2+e)(bx^2+a)^p(dx^2+c)^q}{x^4} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4} \, dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)}{x^4} \, dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)}{x^4}\,dx = \int \frac{(b\,x^2+a)^p (d\,x^2+c)^q (f\,x^2+e)}{x^4}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)/x^4,x)
```

3.76 $\int x^3(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx$

Optimal result

Integrand size = 31, antiderivative size = 576

$$\begin{aligned}
&\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx \\
&= \frac{af(bcf(2+p)-2bde(3+p+q)+adf(4+p+2q)) \left(a+bx^2\right)^{1+p} \left(c+dx^2\right)^{1+q}}{2b^3d^2(2+p+q)(3+p+q)} \\
&\quad - \frac{af^2\left(a+bx^2\right)^{2+p} \left(c+dx^2\right)^{1+q}}{2b^3d(3+p+q)} \\
&\quad - \frac{f(bcf(3+p)-2bde(4+p+q)+adf(5+p+2q)) \left(a+bx^2\right)^{2+p} \left(c+dx^2\right)^{1+q}}{2b^3d^2(3+p+q)(4+p+q)} \\
&\quad + \frac{f^2\left(a+bx^2\right)^{3+p} \left(c+dx^2\right)^{1+q}}{2b^3d(4+p+q)} \\
&\quad + \frac{a(d(2+p+q)\left(abcf^2(2+p)+a^2df^2(1+q)-b^2de^2(3+p+q)\right)-f(bc(1+p)+ad(1+q))(bcf(2+}{2b^3d^2(bc-ad} \\
&\quad - \frac{(d(3+p+q)\left(abcf^2(3+p)+a^2df^2(1+q)-b^2de^2(4+p+q)\right)-f(bc(2+p)+ad(1+q))(bcf(3+p}{2b^3d^2(bc-ad}
\end{aligned}$$

output
```
1/2*a*f*(b*c*f*(2+p)-2*b*d*e*(3+p+q)+a*d*f*(4+p+2*q))*(b*x^2+a)^(p+1)*(d*x
^2+c)^(1+q)/b^3/d^2/(2+p+q)/(3+p+q)-1/2*a*f^2*(b*x^2+a)^(2+p)*(d*x^2+c)^(1
+q)/b^3/d/(3+p+q)-1/2*f*(b*c*f*(3+p)-2*b*d*e*(4+p+q)+a*d*f*(5+p+2*q))*(b*x
^2+a)^(2+p)*(d*x^2+c)^(1+q)/b^3/d^2/(3+p+q)/(4+p+q)+1/2*f^2*(b*x^2+a)^(3+p
)*(d*x^2+c)^(1+q)/b^3/d/(4+p+q)+1/2*a*(d*(2+p+q)*(a*b*c*f^2*(2+p)+a^2*d*f^
2*(1+q)-b^2*d*e^2*(3+p+q))-f*(b*c*(p+1)+a*d*(1+q))*(b*c*f*(2+p)-2*b*d*e*(3
+p+q)+a*d*f*(4+p+2*q)))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+
q],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/b^3/d^2/(-a*d+b*c)/(p+1)/(2+p+q)/(3+p+q)
-1/2*(d*(3+p+q)*(a*b*c*f^2*(3+p)+a^2*d*f^2*(1+q)-b^2*d*e^2*(4+p+q))-f*(b*c
*(2+p)+a*d*(1+q))*(b*c*f*(3+p)-2*b*d*e*(4+p+q)+a*d*f*(5+p+2*q)))*(b*x^2+a)
^(2+p)*(d*x^2+c)^(1+q)*hypergeom([1, 3+p+q],[3+p],-d*(b*x^2+a)/(-a*d+b*c))
/b^3/d^2/(-a*d+b*c)/(2+p)/(3+p+q)/(4+p+q)
```

Mathematica [A] (warning: unable to verify)

Time = 2.77 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.93

$$\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx$$

$$= \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(\frac{b(c+dx^2)\left(d^2f(1+q)(bde(5+p+q)-f(bc(3+p)+ad(2+q)))\left(a+bx^2\right)^2+bd^3f(1+q)(3+p+q)\left(a+bx^2\right)^2\left(e+fx^2\right)+\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output

```
((a + b*x^2)^p*(c + d*x^2)^q*((b*(c + d*x^2)*(d^2*f*(1 + q)*(b*d*e*(5 + p
+ q) - f*(b*c*(3 + p) + a*d*(2 + q)))*(a + b*x^2)^2 + b*d^3*f*(1 + q)*(3 +
 p + q)*(a + b*x^2)^2*(e + f*x^2) + ((-(b*c) + a*d)*(a^2*d^2*f^2*(2 + 3*q
+ q^2) - 2*a*b*d*f*(1 + q)*(-(c*f*(2 + p)) + d*e*(4 + p + q)) + b^2*(c^2*f
^2*(6 + 5*p + p^2) - 2*c*d*e*f*(2 + p)*(4 + p + q) + d^2*e^2*(12 + p^2 + 7
*q + q^2 + p*(7 + 2*q))))*Hypergeometric2F1[-1 - p, 1 + q, 2 + q, (b*(c +
d*x^2))/(b*c - a*d)])/((d*(a + b*x^2))/(-(b*c) + a*d))^p))/((1 + q)*(4 + p
 + q)) - (a*d^2*(a + b*x^2)*(b*f*(1 + p)*(b*d*e*(4 + p + q) - f*(b*c*(2 +
p) + a*d*(2 + q)))*(c + d*x^2) + b^2*d*f*(1 + p)*(2 + p + q)*(c + d*x^2)*(
e + f*x^2) + ((a^2*d^2*f^2*(2 + 3*q + q^2) - 2*a*b*d*f*(1 + q)*(-(c*f*(1 +
 p)) + d*e*(3 + p + q)) + b^2*(c^2*f^2*(2 + 3*p + p^2) - 2*c*d*e*f*(1 + p)
*(3 + p + q) + d^2*e^2*(6 + p^2 + 5*q + q^2 + p*(5 + 2*q))))*Hypergeometri
c2F1[1 + p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/((b*(c + d*x^2))/(
b*c - a*d))^q))/((1 + p)*(2 + p + q))))/(2*b^4*d^4*(3 + p + q))
```

Rubi [A] (warning: unable to verify)

Time = 1.34 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {435, 170, 25, 164, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(e+fx^2\right)^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)^2 dx^2$$

$$\Big\downarrow \text{ 170}$$

$$\frac{1}{2}\left(\frac{\int -\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right) \left(-\left(\left(2bde-f(bc(p+3)+ad(q+3))\right)x^2\right)+2acf+e(bc(p+1)+ad(q+\right.}{bd(p+q+4)}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{(e+fx^2)^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+4)}-\frac{\int(bx^2+a)^p(dx^2+c)^q(fx^2+e)(-((2bde-bcf(p+3)-adf(q+\right.}{bd(p+q+4)}$$

$\downarrow$ 164

$$\frac{1}{2}\left(\frac{(e+fx^2)^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+4)}-\frac{(a+bx^2)^{p+1}(c+dx^2)^{q+1}(-bdfx^2(p+q+2)(-adf(q+3)-bcf(p+3)+2bde)+bcf(p+2)(-\right.}{}$$

$\downarrow$ 80

$$\frac{1}{2}\left(\frac{(e+fx^2)^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+4)}-\frac{(a+bx^2)^{p+1}(c+dx^2)^{q+1}(-bdfx^2(p+q+2)(-adf(q+3)-bcf(p+3)+2bde)+bcf(p+2)(-\right.}{}$$

$\downarrow$ 79

$$\frac{1}{2}\left(\frac{(e+fx^2)^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+4)}-\frac{(a+bx^2)^{p+1}(c+dx^2)^{q+1}(-bdfx^2(p+q+2)(-adf(q+3)-bcf(p+3)+2bde)+bcf(p+2)(-\right.}{}$$

input

```
Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
(((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q)*(e + f*x^2)^2)/(b*d*(4 + p + q))
 - (((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q)*(b*c*f*(2 + p)*(2*b*d*e - b*c
*f*(3 + p) - a*d*f*(3 + q)) + a*d*f*(2 + q)*(2*b*d*e - b*c*f*(3 + p) - a*d
*f*(3 + q)) - 2*b*d*(3 + p + q)*(b*e*(d*e - c*f*(2 + p)) - a*f*(c*f + d*e*
(2 + q))) - b*d*f*(2 + p + q)*(2*b*d*e - b*c*f*(3 + p) - a*d*f*(3 + q))*x^
2))/(b^2*d^2*(2 + p + q)*(3 + p + q)) - ((a^2*d^2*f*(1 + q)*(2 + q)*(2*b*d
*e - b*c*f*(3 + p) - a*d*f*(3 + q)) + a*b*d*(1 + q)*(2*c*f*(1 + p)*(2*b*d*
e - b*c*f*(3 + p) - a*d*f*(3 + q)) - 2*d*(3 + p + q)*(b*e*(d*e - c*f*(2 +
p)) - a*f*(c*f + d*e*(2 + q)))) - b^2*(d^2*e*(2 + p + q)*(3 + p + q)*(2*a*
c*f + b*c*e*(1 + p) + a*d*e*(1 + q)) - c^2*f*(1 + p)*(2 + p)*(2*b*d*e - b*
c*f*(3 + p) - a*d*f*(3 + q)) + 2*c*d*(1 + p)*(3 + p + q)*(b*e*(d*e - c*f*(
2 + p)) - a*f*(c*f + d*e*(2 + q)))))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hyp
ergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b^3*d^2
*(1 + p)*(2 + p + q)*(3 + p + q)*((b*(c + d*x^2))/(b*c - a*d))^q))/(b*d*(4
 + p + q)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x^{3} \left(b\,x^{2}+a \right)^{p} \left(d\,x^{2}+c \right)^{q} \left(f\,x^{2}+e \right)^{2}d x$$

input
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

Fricas [F]

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^7 + 2*e*f*x^5 + e^2*x^3)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Giac [F]

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^3 dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 dx = \int x^3 (b x^2 + a)^p (d x^2 + c)^q (f x^2 + e)^2 dx$$

input
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x)
```

output
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^3 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 dx = \int x^3 (b x^2 + a)^p (d x^2 + c)^q (f x^2 + e)^2 dx$$

input
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

3.77 $\int x(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 29, antiderivative size = 287

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)^2\,dx$$
$$= -\frac{f(bcf(2+p)-2bde(3+p+q)+adf(4+p+2q))\left(a+bx^2\right)^{1+p}\left(c+dx^2\right)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)}$$
$$+\frac{f^2\left(a+bx^2\right)^{2+p}\left(c+dx^2\right)^{1+q}}{2b^2d(3+p+q)}$$
$$-\frac{(d(2+p+q)\left(abcf^2(2+p)+a^2df^2(1+q)-b^2de^2(3+p+q)\right)-f(bc(1+p)+ad(1+q))(bcf(2+p}{2b^2d^2(bc-ad}$$

output
```
-1/2*f*(b*c*f*(2+p)-2*b*d*e*(3+p+q)+a*d*f*(4+p+2*q))*(b*x^2+a)^(p+1)*(d*x^
2+c)^(1+q)/b^2/d^2/(2+p+q)/(3+p+q)+1/2*f^2*(b*x^2+a)^(2+p)*(d*x^2+c)^(1+q)
/b^2/d/(3+p+q)-1/2*(d*(2+p+q)*(a*b*c*f^2*(2+p)+a^2*d*f^2*(1+q)-b^2*d*e^2*(
3+p+q))-f*(b*c*(p+1)+a*d*(1+q))*(b*c*f*(2+p)-2*b*d*e*(3+p+q)+a*d*f*(4+p+2*
q)))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(b*x^2+
a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(p+1)/(2+p+q)/(3+p+q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.94

$$\int x(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2\,dx$$

$$= \frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{f(bde(4+p+q)-f(bc(2+p)+ad(2+q)))(c+dx^2)}{bd(2+p+q)}+f(c+dx^2)(e+fx^2)+\frac{(a^2d^2f^2(2+3q+q^2)-2\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*((f*(b*d*e*(4 + p + q) - f*(b*c*(2 + p)
 + a*d*(2 + q)))*(c + d*x^2))/(b*d*(2 + p + q)) + f*(c + d*x^2)*(e + f*x^2
) + ((a^2*d^2*f^2*(2 + 3*q + q^2) - 2*a*b*d*f*(1 + q)*(-(c*f*(1 + p)) + d*
e*(3 + p + q)) + b^2*(c^2*f^2*(2 + 3*p + p^2) - 2*c*d*e*f*(1 + p)*(3 + p +
 q) + d^2*e^2*(6 + p^2 + 5*q + q^2 + p*(5 + 2*q))))*Hypergeometric2F1[1 +
p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/(b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q)))/(2*b*d*(3 + p + q))
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {435, 101, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(e+fx^2)^2(a+bx^2)^p(c+dx^2)^q\,dx$$

$$\Big\downarrow \text{435}$$

$$\frac{1}{2}\int (bx^2+a)^p(dx^2+c)^q(fx^2+e)^2\,dx^2$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{2}\left(\frac{\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(bd(p+q+3)e^2+f(bde(p+q+4)-f(bc(p+2)+ad(q+2)))x^2-f(acf+bce(p\right.}{bd(p+q+3)}\right.$$

$\Big\downarrow$ 90

$$\frac{1}{2}\left(\frac{\left(-f(acf+ade(q+1)+bce(p+1))+\frac{f(ad(q+1)+bc(p+1))(adf(q+2)+bcf(p+2)-bde(p+q+4))}{bd(p+q+2)}+bde^2(p+q+3)\right)\int \left(\right.}{bd(p+q+3)}\right.$$

$\Big\downarrow$ 80

$$\frac{1}{2}\left(\frac{\left(c+dx^2\right)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \left(-f(acf+ade(q+1)+bce(p+1))+\frac{f(ad(q+1)+bc(p+1))(adf(q+2)+bcf(p+2)-bde(p+q+4\right.}{bd(p+q+2)}\right.}{bd(p+}\right.$$

$\Big\downarrow$ 79

$$\frac{1}{2}\left(\frac{\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\left(-f(acf+ade(q+1)+bce(p+1))+\frac{f(ad(q+1)+bc(p+1))(adf(q+2)+bcf(p+2)-bde(p+q+4))}{bd(p+q+2)}+bde^2(p+q+3\right.}{b(p+1)}}{bd(p+q+3)}\right.$$

input
```
Int[x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
((f*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q)*(e + f*x^2))/(b*d*(3 + p + q))
 + ((f*(b*d*e*(4 + p + q) - f*(b*c*(2 + p) + a*d*(2 + q)))*(a + b*x^2)^(1
+ p)*(c + d*x^2)^(1 + q))/(b*d*(2 + p + q)) + ((b*d*e^2*(3 + p + q) - f*(a
*c*f + b*c*e*(1 + p) + a*d*e*(1 + q)) + (f*(b*c*(1 + p) + a*d*(1 + q))*(b*
c*f*(2 + p) + a*d*f*(2 + q) - b*d*e*(4 + p + q)))/(b*d*(2 + p + q)))*(a +
b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a +
 b*x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q))/(b*d*
(3 + p + q)))/2
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

Maple [F]

$$\int x(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2\,dx$$

input
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

Fricas [F]

$$\int x(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2\,dx=\int (fx^2+e)^2(bx^2+a)^p(dx^2+c)^q x\,dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^5 + 2*e*f*x^3 + e^2*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2\,dx=\text{Timed out}$$

input
```
integrate(x*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q x \, dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Giac [F]

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q x \, dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int x \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \left(f\,x^2+e\right)^2 \, dx$$

input
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x)
```

output
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \text{too large to display}$$

input
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
((c + d*x**2)**q*(a + b*x**2)**p*a**3*c*d**2*f**2*p*q + 2*(c + d*x**2)**q*
(a + b*x**2)**p*a**3*c*d**2*f**2*p - (c + d*x**2)**q*(a + b*x**2)**p*a**3*
d**3*f**2*p*q**2*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**3*d**3*f**2*p
*q*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c**2*d*f**2*p*q - 2*(c
+ d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*e*f*p**2 - 2*(c + d*x**2)**q*(a
 + b*x**2)**p*a**2*b*c*d**2*e*f*p*q - 6*(c + d*x**2)**q*(a + b*x**2)**p*a*
*2*b*c*d**2*e*f*p - (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*f**2*p**
2*q*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*f**2*p**2*x**2
+ 2*(c + d*x**2)**q*(a + b*x**2)**p*a**2*b*c*d**2*f**2*p*q**2*x**2 + 2*(c
+ d*x**2)**q*(a + b*x**2)**p*a**2*b*d**3*e*f*p**2*q*x**2 + 2*(c + d*x**2)*
*q*(a + b*x**2)**p*a**2*b*d**3*e*f*p*q**2*x**2 + 6*(c + d*x**2)**q*(a + b*
x**2)**p*a**2*b*d**3*e*f*p*q*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*b
*d**3*f**2*p**2*q*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*d**3*f**2*
p*q**2*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*b*d**3*f**2*p*q*x**4 +
(c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**3*f**2*p*q + 2*(c + d*x**2)**q*(
a + b*x**2)**p*a*b**2*c**3*f**2*q - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*
*2*c**2*d*e*f*p*q - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**2*d*e*f*q*
*2 - 6*(c + d*x**2)**q*(a + b*x**2)**p*a*b**2*c**2*d*e*f*q + 2*(c + d*x**2
)**q*(a + b*x**2)**p*a*b**2*c**2*d*f**2*p**2*q*x**2 - (c + d*x**2)**q*(a +
 b*x**2)**p*a*b**2*c**2*d*f**2*p*q**2*x**2 - 2*(c + d*x**2)**q*(a + b*x...
```

3.78 $\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} dx$

Optimal result

Integrand size = 31, antiderivative size = 259

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} dx = -\frac{e^2(a+bx^2)^{1+p}(c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,1,-q,2+p,1+\frac{bx^2}{a},-\frac{d(a+bx^2)}{bc-ad}\right)}{2a(1+p)} + \frac{f(2de-cf)(a+bx^2)^{1+p}(c+dx^2)^{1+q}\,\text{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right)}{2d(bc-ad)(1+p)} + \frac{f^2(a+bx^2)^{1+p}(c+dx^2)^{2+q}\,\text{Hypergeometric2F1}\left(1,3+p+q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right)}{2d(bc-ad)(1+p)}$$

output

```
-1/2*e^2*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,-q,1,2+p,-d*(b*x^2+a)/(-
a*d+b*c),1+b*x^2/a)/a/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)+1/2*f*(-c*f+2*d*e
)*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[2+p],-d*(b*x^2+a)/
(-a*d+b*c))/d/(-a*d+b*c)/(p+1)+1/2*f^2*(b*x^2+a)^(p+1)*(d*x^2+c)^(2+q)*hyp
ergeom([1, 3+p+q],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/d/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} dx$$
$$= \frac{1}{4}(a+bx^2)^p (c+dx^2)^q \left(4efx^2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(1,-p,-q,2,-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + f^2x^4\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(2,-p,-q,3,-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{2e^2\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q} \text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{p+q}\right)$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*((4*e*f*x^2*AppellF1[1, -p, -q, 2, -((b*x^2)/
a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (f^2*x^4*Appell
F1[2, -p, -q, 3, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x
^2)/c)^q) + (2*e^2*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^2)), -(c/(
d*x^2))])/((p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q)))/4
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {435, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2 (a+bx^2)^p (c+dx^2)^q}{x} dx$$
$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)^2}{x^2}dx^2$$

$$\Big\downarrow \text{198}$$

$$\frac{1}{2}\int \left(-\frac{f(cf-2de)\left(dx^2+c\right)^q\left(bx^2+a\right)^p}{d}+\frac{e^2\left(dx^2+c\right)^q\left(bx^2+a\right)^p}{x^2}+\frac{f^2\left(dx^2+c\right)^{q+1}\left(bx^2+a\right)^p}{d}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{e^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{a(p+1)}+\frac{f^2(bc-ad)\left(a\right.}{}\right.$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x,x]
```

output
```
(-((e^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -(
(d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c + d*x^2))/
(b*c - a*d))^q)) + ((b*c - a*d)*f^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hype
rgeometric2F1[1 + p, -1 - q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b^2*
d*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q) + (f*(2*d*e - c*f)*(a + b*x^2)^
(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2)
)/(b*c - a*d))])/(b*d*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x}\,dx = \int \frac{(fx^2+e)^2(bx^2+a)^p(dx^2+c)^q}{x}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x, algorithm="fricas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} \, dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} \, dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} \, dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x} \, dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x} \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x,x)
```

output
```
( - (c + d*x**2)**q*(a + b*x**2)**p*a**2*c*d*f**2*p + (c + d*x**2)**q*(a +
 b*x**2)**p*a**2*d**2*f**2*p*q*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a*b*
c**2*f**2*q + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*p**2 + 4*(c +
d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*p*q + 4*(c + d*x**2)**q*(a + b*x**2
)**p*a*b*c*d*e*f*p + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*q**2 +
4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*q + (c + d*x**2)**q*(a + b*x
**2)**p*a*b*c*d*f**2*p**2*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f
**2*q**2*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*p**2 + 2*(c
+ d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*p*q + 3*(c + d*x**2)**q*(a + b*
x**2)**p*a*b*d**2*e**2*p + (c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*q
**2 + 3*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*q + 2*(c + d*x**2)**
q*(a + b*x**2)**p*a*b*d**2*e**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d*
*2*e*f*p*q*x**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*q**2*x**2
 + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*q*x**2 + (c + d*x**2)**q
*(a + b*x**2)**p*a*b*d**2*f**2*p*q*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*
a*b*d**2*f**2*q**2*x**4 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f**2*q*
x**4 + (c + d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f**2*p*q*x**2 + (c + d*x*
*2)**q*(a + b*x**2)**p*b**2*c*d*e**2*p**2 + 2*(c + d*x**2)**q*(a + b*x**2)
**p*b**2*c*d*e**2*p*q + 3*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e**2*p
+ (c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e**2*q**2 + 3*(c + d*x**2)**...
```

3.79 $\int \frac{(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2}{x^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 248

$$\int \frac{(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2}{x^3}\,dx = -\frac{e^2(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2acx^2} - \frac{e(2acf+bcep+adeq)(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,1,-q,2+p,1+\frac{bx^2}{a},-\frac{d}{}\right.}{2a^2c(1+p)} + \frac{(acf^2+bde^2(1+p+q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}\operatorname{Hypergeometric2F1}\left(1,2+p+q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right.}{2ac(bc-ad)(1+p)}$$

output
```
-1/2*e^2*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x^2-1/2*e*(a*d*e*q+b*c*e*p+2*
a*c*f)*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,-q,1,2+p,-d*(b*x^2+a)/(-a*
d+b*c),1+b*x^2/a)/a^2/c/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)+1/2*(a*c*f^2+b*
d*e^2*(1+p+q))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)*hypergeom([1, 2+p+q],[2+p],
-d*(b*x^2+a)/(-a*d+b*c))/a/c/(-a*d+b*c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^3} \, dx = \frac{1}{2}(a+bx^2)^p (c$$
$$+ dx^2)^q \left(f^2x^2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(1,-p,-q,2,-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right.$$
$$+ \frac{e\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}\left(2f(-1+p+q)x^2 \text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)+e(p\right.}{(-1+p+q)(p+q)x^2}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^3,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*((f^2*x^2*AppellF1[1, -p, -q, 2, -((b*x^2)/a)
, -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (e*(2*f*(-1 + p +
 q)*x^2*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^2)), -(c/(d*x^2))] +
e*(p + q)*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/(b*x^2)), -(c/(d*x^2)
)]))/((-1 + p + q)*(p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q*x^2)))/2
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {435, 198, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2 (a+bx^2)^p (c+dx^2)^q}{x^3} \, dx$$

$$\Big\downarrow \; 435$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p (dx^2+c)^q (fx^2+e)^2}{x^4} dx^2$$

$$\Big\downarrow \; 198$$

$$\frac{1}{2}\int \left(f^2 \left(dx^2+c\right)^q \left(bx^2+a\right)^p + \frac{2ef\left(dx^2+c\right)^q \left(bx^2+a\right)^p}{x^2} + \frac{e^2\left(dx^2+c\right)^q \left(bx^2+a\right)^p}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{be^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{a^2(p+1)} - \frac{2ef\left(a+bx^2\right)^{p+}}{}\right.$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^3,x]
```

output
```
((-2*e*f*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -
((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c + d*x^2))
/(b*c - a*d))^q) + (b*e^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p
, -q, 2, 2 + p, -((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a^2*(1 +
p)*((b*(c + d*x^2))/(b*c - a*d))^q) + (f^2*(a + b*x^2)^(1 + p)*(c + d*x^2)
^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b
*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q))/2
```

Defintions of rubi rules used

rule 198
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c
 + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n}, x] && IntegersQ[p, q]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}\,(d\,x^{2}+c)^{q}\,(f\,x^{2}+e)^{2}}{x^{3}}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x^3}\,dx = \int \frac{(fx^2+e)^2(bx^2+a)^p(dx^2+c)^q}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x, algorithm="fricas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2/x**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^3} dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x^3} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^3} dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x^3} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^3} dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x^3} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^3,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^3} \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^3,x)
```

output
```
((c + d*x**2)**q*(a + b*x**2)**p*a**2*c*d*f**2*p*q*x**2 - (c + d*x**2)**q*
(a + b*x**2)**p*a**2*c*d*f**2*p*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a**
2*c*d*f**2*q**2*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a**2*c*d*f**2*q*x**
2 - (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*e**2*p*q**2 + (c + d*x**2)**
q*(a + b*x**2)**p*a**2*d**2*e**2*p*q - (c + d*x**2)**q*(a + b*x**2)**p*a**
2*d**2*e**2*q**3 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*e**2*q + 2*(c
 + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*e*f*p*q*x**2 - 2*(c + d*x**2)**q*(
a + b*x**2)**p*a**2*d**2*e*f*p*x**2 + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*
*2*d**2*e*f*q**2*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*e*f*x*
*2 + (c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*f**2*q**2*x**4 - (c + d*x**
2)**q*(a + b*x**2)**p*a**2*d**2*f**2*q*x**4 + (c + d*x**2)**q*(a + b*x**2)
**p*a*b*c**2*f**2*p**2*x**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c**2*f**
2*p*q*x**2 - (c + d*x**2)**q*(a + b*x**2)**p*a*b*c**2*f**2*p*x**2 - (c + d
*x**2)**q*(a + b*x**2)**p*a*b*c**2*f**2*q*x**2 - 2*(c + d*x**2)**q*(a + b*
x**2)**p*a*b*c*d*e**2*p**2*q + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e**
2*p**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e**2*p*q**2 + (c + d*x*
*2)**q*(a + b*x**2)**p*a*b*c*d*e**2*p + (c + d*x**2)**q*(a + b*x**2)**p*a*
b*c*d*e**2*q**2 + (c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e**2*q - 2*(c +
d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*p*x**2 - 2*(c + d*x**2)**q*(a + b*x
**2)**p*a*b*c*d*e*f*q*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*...
```

3.80 $\int x^4(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 1256

$$\int x^4\left(a+bx^2\right)^p\left(c+dx^2\right)^q\left(e+fx^2\right)^2\,dx=\text{Too large to display}$$

output
```
-e*(b*c*f*(5+2*p)+a*d*f*(5+2*q)-b*d*e*(7+2*p+2*q))*x*(b*x^2+a)^(p+1)*(d*x^
2+c)^(1+q)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)+f*(a*d*(5+2*q)*(b*c*f*(7+2*p)+a
*d*f*(7+2*q)-b*d*e*(9+2*p+2*q))+b*(4*a*c*d*f*p*(1+q)+b*c*(5+2*p)*(c*f*(7+2
*p)-d*e*(9+2*p+2*q))))*x*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b^3/d^3/(5+2*p+2*
q)/(7+2*p+2*q)/(9+2*p+2*q)+e*f*x^3*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(7+
2*p+2*q)-f*(b*c*f*(7+2*p)+a*d*f*(7+2*q)-b*d*e*(9+2*p+2*q))*x^3*(b*x^2+a)^(
p+1)*(d*x^2+c)^(1+q)/b^2/d^2/(7+2*p+2*q)/(9+2*p+2*q)+f^2*x^5*(b*x^2+a)^(p+
1)*(d*x^2+c)^(1+q)/b/d/(9+2*p+2*q)+a*c*e*(b*c*f*(5+2*p)+a*d*f*(5+2*q)-b*d*
e*(7+2*p+2*q))*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-
d*x^2/c)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)-a
*c*f*(a^2*d^2*f*(4*q^2+24*q+35)+b^2*c*(5+2*p)*(c*f*(7+2*p)-d*e*(9+2*p+2*q)
)-a*b*d*(d*e*(5+2*q)*(9+2*p+2*q)-c*f*(8*p*q+14*p+14*q+35)))*x*(b*x^2+a)^p*
(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/b^3/d^3/(5+2*p+2*q)/
(7+2*p+2*q)/(9+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*e*(a^2*d^2*f*(
4*q^2+16*q+15)+b^2*c*(3+2*p)*(c*f*(5+2*p)-d*e*(7+2*p+2*q))-a*b*d*(d*e*(3+2
*q)*(7+2*p+2*q)-c*f*(8*p*q+10*p+10*q+15)))*x^3*(b*x^2+a)^p*(d*x^2+c)^q*App
ellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)/((1
+b*x^2/a)^p)/((1+d*x^2/c)^q)-1/3*f*(a^3*d^3*f*(8*q^3+60*q^2+142*q+105)+b^3
*c^2*(4*p^2+16*p+15)*(c*f*(7+2*p)-d*e*(9+2*p+2*q))-a^2*b*d^2*(5+2*q)*(d*e*
(3+2*q)*(9+2*p+2*q)-c*f*(21+14*q+2*p*(7+6*q)))+a*b^2*c*d*(c*f*(5+2*p)*(...
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.13

$$\int x^4(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2\,dx = \frac{1}{315}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\left(63e^2\operatorname{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+5fx^2\left(18e\operatorname{AppellF1}\left(\frac{7}{2},-p,-q,\frac{9}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+7fx^2\operatorname{AppellF1}\left(\frac{9}{2},-p,-q,\frac{11}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)$$

input
```
Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
(x^5*(a + b*x^2)^p*(c + d*x^2)^q*(63*e^2*AppellF1[5/2, -p, -q, 7/2, -((b*x
^2)/a), -((d*x^2)/c)] + 5*f*x^2*(18*e*AppellF1[7/2, -p, -q, 9/2, -((b*x^2)
/a), -((d*x^2)/c)] + 7*f*x^2*AppellF1[9/2, -p, -q, 11/2, -((b*x^2)/a), -((
d*x^2)/c)])))/(315*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] (warning: unable to verify)

Time = 4.67 (sec) , antiderivative size = 1171, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {448, 444, 444, 406, 334, 334, 333, 395, 395, 394, 444, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(e+fx^2\right)^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f \int x^6 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right) dx}{e^2} + e \int x^4 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right) dx$$

$$\Big\downarrow \text{444}$$

$$\frac{f \left(\frac{fx^5 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+9)} - \frac{\int x^4 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^2+5acf\right) dx}{bd(2p+2q+9)}\right)}{e^2} +$$

$$e\left(\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)} - \frac{\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7)) \cdots\right.}{bd(2p+2q+7)} \cdots\right.$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{fx^5 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+9)} - \frac{\frac{x^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1} (adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)} - \frac{\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((ad(2q+5)(bcf(2p+ \cdots\right.}{\cdots}}{\cdots}\right.}{e^2} \cdots$$

$$e\left(\frac{fx^3 \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1}}{bd(2p+2q+7)} - \frac{\frac{x \left(a+bx^2\right)^{p+1} \left(c+dx^2\right)^{q+1} (adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)} - \frac{\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((a \cdots\right.}{\cdots}}{\cdots}\right.$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+}{}}{}\right.}{e^2}$$

$$e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(adf(2q+5)+bcf(2p+5}{}}{}\right.$$

$\downarrow$ 334

$$\frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+}{}}{}\right.}{e^2}$$

$$e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}}{}}{}\right.$$

$\downarrow$ 334

$$\frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+}{}}{}\right.}{e^2}$$

$$e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}}{}}{}\right.$$

$\downarrow$ 333

$$e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(ad(2q+3)(adf(2q+5)+b}{}}{}\right.$$

$$\frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+}{}}{}\right.}{e^2}$$

$\downarrow$ 395

$$\frac{e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(a\right.}{}\right.}{} \quad \frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+\right.}{}\right.}{e^2}$$

$\downarrow$ 395

$$\frac{e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c\right.}{}\right.}{} \quad \frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+\right.}{}\right.}{e^2}$$

$\downarrow$ 394

$$\frac{f\left(\frac{fx^5(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+9)}-\frac{x^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+7)+bcf(2p+7)-bde(2p+2q+9))}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((ad(2q+5)(bcf(2p+\right.}{}\right.}{e^2}$$

$$e\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{acx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-}\right.}{}$$

$\downarrow$ 444

$$e\left(\frac{fx^3(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5\right.}{}$$

$$f\left(\frac{fx^5(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+9)}-\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q\right.}{}$$

$\downarrow$ 406

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5}{}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q}{}\right.$$

$\downarrow$ 334

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5}{}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q}{}\right.$$

$\downarrow$ 334

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5}{}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q}{}\right.$$

$\downarrow$ 333

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5\ldots}{\ldots}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 395

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5\ldots}{\ldots}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 395

$$e\left(\frac{fx^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5\ldots}{\ldots}\right.$$

$$f\left(\frac{fx^5\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3\left(bx^2+a\right)^{p+1}\left(dx^2+c\right)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q\ldots}{\ldots}}{\ldots}\right.$$

$\downarrow$ 394

$$e\left(\frac{fx^3(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{(bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+5)}-\frac{ac(bcf(2p+5)+adf(2q+5}{}}{}\right.$$

$$f\left(\frac{fx^5(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+9)}-\frac{\frac{(bcf(2p+7)+adf(2q+7)-bde(2p+2q+9))x^3(bx^2+a)^{p+1}(dx^2+c)^{q+1}}{bd(2p+2q+7)}-\frac{(ad(2q+5)(bcf(2p+7)+adf(2q+7)-bde(2p+2q}{}}{}\right.$$

input
```
Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
e*((f*x^3*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(7 + 2*p + 2*q)) -
 (((b*c*f*(5 + 2*p) + a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^
2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - ((a*c*(b*c*f*(5 +
2*p) + a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^2)^p*(c + d*x^2
)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/
a)^p*(1 + (d*x^2)/c)^q) + ((a*d*(3 + 2*q)*(b*c*f*(5 + 2*p) + a*d*f*(5 + 2*
q) - b*d*e*(7 + 2*p + 2*q)) + b*(4*a*c*d*f*p*(1 + q) + b*c*(3 + 2*p)*(c*f*
(5 + 2*p) - d*e*(7 + 2*p + 2*q))))*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF
1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 +
 (d*x^2)/c)^q))/(b*d*(5 + 2*p + 2*q)))/(b*d*(7 + 2*p + 2*q))) + (f*((f*x^5
*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(9 + 2*p + 2*q)) - (((b*c*f
*(7 + 2*p) + a*d*f*(7 + 2*q) - b*d*e*(9 + 2*p + 2*q))*x^3*(a + b*x^2)^(1 +
 p)*(c + d*x^2)^(1 + q))/(b*d*(7 + 2*p + 2*q)) - (((a*d*(5 + 2*q)*(b*c*f*(
7 + 2*p) + a*d*f*(7 + 2*q) - b*d*e*(9 + 2*p + 2*q)) + b*(4*a*c*d*f*p*(1 +
q) + b*c*(5 + 2*p)*(c*f*(7 + 2*p) - d*e*(9 + 2*p + 2*q))))*x*(a + b*x^2)^(
1 + p)*(c + d*x^2)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - ((a*c*(a*d*(5 + 2*q)*(
b*c*f*(7 + 2*p) + a*d*f*(7 + 2*q) - b*d*e*(9 + 2*p + 2*q)) + b*(4*a*c*d*f*
p*(1 + q) + b*c*(5 + 2*p)*(c*f*(7 + 2*p) - d*e*(9 + 2*p + 2*q))))*x*(a + b
*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + ((b*c*(4*a^2*d^2*f*p*(7 + 9...
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{p} \left(d \,x^{2}+c \right)^{q} \left(f \,x^{2}+e \right)^{2}d x$$

input
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \int { {\left(f x^{2} + e\right)}^{2} {\left(b x^{2} + a\right)}^{p} {\left(d x^{2} + c\right)}^{q} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^8 + 2*e*f*x^6 + e^2*x^4)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \text{Timed out}$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int x^4 \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \left(f\,x^2+e\right)^2 dx$$

input
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x)
```

output
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^4 (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int x^4 \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \left(f\,x^2+e\right)^2 dx$$

input
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

3.81 $\int x^2(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 744

$$\int x^2(a+bx^2)^p\left(c+dx^2\right)^q\left(e+fx^2\right)^2\,dx = \frac{efx(a+bx^2)^{1+p}\left(c+dx^2\right)^{1+q}}{bd(5+2p+2q)}$$
$$-\frac{f(bcf(5+2p)+adf(5+2q)-bde(7+2p+2q))x(a+bx^2)^{1+p}\left(c+dx^2\right)^{1+q}}{b^2d^2(5+2p+2q)(7+2p+2q)}$$
$$+\frac{f^2x^3(a+bx^2)^{1+p}\left(c+dx^2\right)^{1+q}}{bd(7+2p+2q)}$$
$$-\frac{acefx(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{bd(5+2p+2q)}$$
$$+\frac{acf(bcf(5+2p)+adf(5+2q)-bde(7+2p+2q))x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\mathrm{A}}{b^2d^2(5+2p+2q)(7+2p+2q)}$$
$$-\frac{e(bcf(3+2p)+adf(3+2q)-bde(5+2p+2q))x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\mathrm{Ap}}{3bd(5+2p+2q)}$$
$$+\frac{f(a^2d^2f(15+16q+4q^2)+b^2c(3+2p)(cf(5+2p)-de(7+2p+2q))-abd(de(3+2q)(7+2p+2q)}{3b^2d^2(5}$$

output
```
e*f*x*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(5+2*p+2*q)-f*(b*c*f*(5+2*p)+a*d
*f*(5+2*q)-b*d*e*(7+2*p+2*q))*x*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b^2/d^2/(5
+2*p+2*q)/(7+2*p+2*q)+f^2*x^3*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(7+2*p+2
*q)-a*c*e*f*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x
^2/c)/b/d/(5+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+a*c*f*(b*c*f*(5+2*p)
+a*d*f*(5+2*q)-b*d*e*(7+2*p+2*q))*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-
p,-q,3/2,-b*x^2/a,-d*x^2/c)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)/((1+b*x^2/a)^p
)/((1+d*x^2/c)^q)-1/3*e*(b*c*f*(3+2*p)+a*d*f*(3+2*q)-b*d*e*(5+2*p+2*q))*x^
3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/b/d/(5
+2*p+2*q)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*f*(a^2*d^2*f*(4*q^2+16*q+15)
+b^2*c*(3+2*p)*(c*f*(5+2*p)-d*e*(7+2*p+2*q))-a*b*d*(d*e*(3+2*q)*(7+2*p+2*q
)-c*f*(8*p*q+10*p+10*q+15)))*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-
q,5/2,-b*x^2/a,-d*x^2/c)/b^2/d^2/(5+2*p+2*q)/(7+2*p+2*q)/((1+b*x^2/a)^p)/(
(1+d*x^2/c)^q)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.23

$$\int x^2(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2\,dx = \frac{1}{105}x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\left(35e^2\operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3fx^2\left(14e\operatorname{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+5fx^2\operatorname{AppellF1}\left(\frac{7}{2},-p,-q,\frac{9}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)$$

input
```
Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
(x^3*(a + b*x^2)^p*(c + d*x^2)^q*(35*e^2*AppellF1[3/2, -p, -q, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + 3*f*x^2*(14*e*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)
/a), -((d*x^2)/c)] + 5*f*x^2*AppellF1[7/2, -p, -q, 9/2, -((b*x^2)/a), -((d
*x^2)/c)])))/(105*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rubi [A] (warning: unable to verify)

Time = 2.40 (sec) , antiderivative size = 701, normalized size of antiderivative = 0.94, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.548, Rules used = {448, 444, 406, 334, 334, 333, 395, 395, 394, 444, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(e+fx^2\right)^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int x^4 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)dx}{e^2}+e\int x^2 \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)dx$$

$$\Big\downarrow \text{444}$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}+ e\left(\frac{fx\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+5)}-\frac{\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bcf(2p+3)+adf(2q+3)-bde(2p+2q+5))x^2\right.}{bd(2p+2q+5)}\right.$$

$$\Big\downarrow \text{406}$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}+ e\left(\frac{fx\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+5)}-\frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx}{bd(2p+2q+5)}\right.$$

$$\Big\downarrow \text{334}$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}+ e\left(\frac{fx\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(2p+2q+5)}-\frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx}{bd(2p+2q+5)}\right.$$

$$\Big\downarrow \text{334}$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}+$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q dx}{bd(2\right.$$

$$\Big\downarrow \text{333}$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2(bx^2+a)^p(dx^2+c)^q dx}{\right.$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}$$

$$\Big\downarrow \text{395}$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(adf(2q+3)+bcf(2p+3)-bde(2p+2q+5))\int x^2}{\right.$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}$$

$$\Big\downarrow \text{395}$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}(adf(2q+3)+bcf(2p+3)}{\right.$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}$$

$$\Big\downarrow \text{394}$$

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\int x^2(bx^2+a)^p(dx^2+c)^q((bcf(2p+5)+adf(2q+5)-bde(2p+2q+7))x^2+3acf)dx}{bd(2p+2q+7)}\right)}{e^2}+$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\right.}{\right.$$

$$\Big\downarrow \text{444}$$

$$f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{\int (bx^2+a)^p(dx^2+c)^q((ad(2q+3)(bcf(2p+5)+\cdots}{\cdots}}{\cdots}\right.$$

$$e^2$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.$$

$\downarrow$ 406

$$f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))\int(b\cdots}{\cdots}}{\cdots}\right.$$

$$e^2$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.$$

$\downarrow$ 334

$$f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(adf(2q+5)+bcf(2p+\cdots}{\cdots}}{\cdots}\right.$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.$$

$\downarrow$ 334

$$f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{ac(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)\cdots}{\cdots}}{\cdots}\right.$$

$$e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.$$

$\downarrow$ 333

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(ad(2q+3)(adf(2q+5)+bcf(2p+5)-bde(2p+2q+\cdots}{\cdots}}{\cdots}\right.}{e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.}$$

$\downarrow$ 395

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(ad(2q+3)(adf(2q+5)+\cdots}{\cdots}}{\cdots}\right.}{e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.}$$

$\downarrow$ 395

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q\cdots}}{\cdots}}{\cdots}\right.}{e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.}$$

$\downarrow$ 394

$$\frac{f\left(\frac{fx^3(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+7)}-\frac{\frac{x(a+bx^2)^{p+1}(c+dx^2)^{q+1}(adf(2q+5)+bcf(2p+5)-bde(2p+2q+7))}{bd(2p+2q+5)}-\frac{acx(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\cdots\right.}{\cdots}}{\cdots}\right.}{e\left(\frac{fx(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(2p+2q+5)}-\frac{\frac{1}{3}x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\cdots\right.}{\cdots}\right.}$$

input
```
Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
e*((f*x*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - (
(a*c*f*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + ((b*c*f*(3 + 2*
p) + a*d*f*(3 + 2*q) - b*d*e*(5 + 2*p + 2*q))*x^3*(a + b*x^2)^p*(c + d*x^2
)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2
)/a)^p*(1 + (d*x^2)/c)^q))/(b*d*(5 + 2*p + 2*q))) + (f*((f*x^3*(a + b*x^2)
^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(7 + 2*p + 2*q)) - (((b*c*f*(5 + 2*p) +
 a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^2)^(1 + p)*(c + d*x^2
)^(1 + q))/(b*d*(5 + 2*p + 2*q)) - ((a*c*(b*c*f*(5 + 2*p) + a*d*f*(5 + 2*q
) - b*d*e*(7 + 2*p + 2*q))*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p,
 -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^
q) + ((a*d*(3 + 2*q)*(b*c*f*(5 + 2*p) + a*d*f*(5 + 2*q) - b*d*e*(7 + 2*p +
 2*q)) + b*(4*a*c*d*f*p*(1 + q) + b*c*(3 + 2*p)*(c*f*(5 + 2*p) - d*e*(7 +
2*p + 2*q))))*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -
((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))/(b*d*
(5 + 2*p + 2*q)))/(b*d*(7 + 2*p + 2*q))))/e^2
```

Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [F]

$$\int x^2 (b x^2 + a)^p (d x^2 + c)^q (f x^2 + e)^2 dx$$

input
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

Fricas [F]

$$\int x^2 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^6 + 2*e*f*x^4 + e^2*x^2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 (a + bx^2)^p (c + dx^2)^q (e + fx^2)^2 \, dx = \int (fx^2 + e)^2 (bx^2 + a)^p (dx^2 + c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Giac [F]

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int x^2 (b\,x^2+a)^p \, (d\,x^2+c)^q \, (f\,x^2+e)^2 \, dx$$

input
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x)
```

output
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int x^2 (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int x^2 (b\,x^2+a)^p \, (d\,x^2+c)^q \, (f\,x^2+e)^2 \, dx$$

input
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

3.82 $\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx$

Optimal result

Integrand size = 28, antiderivative size = 256

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = e^2x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{2}{3}efx^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + \frac{1}{5}f^2x^5(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
e^2*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((
1+b*x^2/a)^p)/((1+d*x^2/c)^q)+2/3*e*f*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1
(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/5*f^2*
x^5*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(5/2,-p,-q,7/2,-b*x^2/a,-d*x^2/c)/((1+
b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \frac{1}{15} x (a+bx^2)^p (c+dx^2)^q \left(\frac{45ace^2 \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac \operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcp \operatorname{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+ \right.} + fx^2\left(1+\frac{bx^2}{a}\right)^{-p}\left(1+\frac{dx^2}{c}\right)^{-q}\left(10e \operatorname{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right) + 3fx^2 \operatorname{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)\right)$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
(x*(a + b*x^2)^p*(c + d*x^2)^q*((45*a*c*e^2*AppellF1[1/2, -p, -q, 3/2, -((
b*x^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -
((d*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)
/c)])) + (f*x^2*(10*e*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + 3*f*x^2*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -((d*x^2)/c)]))/((1
+ (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)))/15
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {433, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e+fx^2\right)^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \ 433$$

$$\int \left(e^2\left(a+bx^2\right)^p \left(c+dx^2\right)^q+2efx^2\left(a+bx^2\right)^p \left(c+dx^2\right)^q+f^2x^4\left(a+bx^2\right)^p \left(c+dx^2\right)^q\right) dx$$

$$\Big\downarrow \ 2009$$

$$e^2x\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{2}{3}efx^3\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{5}f^2x^5\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x]
```

output
```
(e^2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a)
, -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (2*e*f*x^3*(a + b
*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (f^2*x^5*(a + b*x^2)^p*(c + 
d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a), -((d*x^2)/c)])/(5*(1 + (
b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Defintions of rubi rules used

rule 433
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_
)^2)^(r_), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)
^q*(e + f*x^2)^r, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p,
 q, r}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q} \left(f\,x^{2}+e\right)^{2}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

Fricas [F]

$$\int (a+bx^2)^p (c+dx^2)^q (e+fx^2)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \int (fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \left(f\,x^2+e\right)^2 dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2,x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2, x)
```

Reduce [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \left(e+fx^2\right)^2 \, dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2,x)
```

output
```
( - 4*(c + d*x**2)**q*(a + b*x**2)**p*a**2*d**2*f**2*p*q*x - 6*(c + d*x**2
)**q*(a + b*x**2)**p*a**2*d**2*f**2*p*x + 8*(c + d*x**2)**q*(a + b*x**2)**
p*a*b*c*d*f**2*p*q*x + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*p**2
*x + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*p*q*x + 20*(c + d*x**2
)**q*(a + b*x**2)**p*a*b*d**2*e*f*p*x + 4*(c + d*x**2)**q*(a + b*x**2)**p*
a*b*d**2*f**2*p**2*x**3 + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f**2*
p*q*x**3 + 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*f**2*p*x**3 - 4*(c +
 d*x**2)**q*(a + b*x**2)**p*b**2*c**2*f**2*p*q*x - 6*(c + d*x**2)**q*(a +
b*x**2)**p*b**2*c**2*f**2*q*x + 8*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d
*e*f*p*q*x + 8*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e*f*q**2*x + 20*(c
 + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*e*f*q*x + 4*(c + d*x**2)**q*(a + b*
x**2)**p*b**2*c*d*f**2*p*q*x**3 + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c
*d*f**2*q**2*x**3 + 2*(c + d*x**2)**q*(a + b*x**2)**p*b**2*c*d*f**2*q*x**3
 + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e**2*p**2*x + 8*(c + d*x**2
)**q*(a + b*x**2)**p*b**2*d**2*e**2*p*q*x + 16*(c + d*x**2)**q*(a + b*x**2
)**p*b**2*d**2*e**2*p*x + 4*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e**2
*q**2*x + 16*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e**2*q*x + 15*(c +
d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e**2*x + 8*(c + d*x**2)**q*(a + b*x**
2)**p*b**2*d**2*e*f*p**2*x**3 + 16*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d*
*2*e*f*p*q*x**3 + 24*(c + d*x**2)**q*(a + b*x**2)**p*b**2*d**2*e*f*p*x*...
```

3.83 $\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} \, dx$

Optimal result

Integrand size = 31, antiderivative size = 258

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} \, dx = -\frac{e^2(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{acx}$$
$$+ \frac{e(2acf+bce(1+2p)+ad(e+2eq))x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p\right.}{ac}$$
$$+ \frac{(acf^2+bde^2(3+2p+2q))\,x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},\right.}{3ac}$$

output
```
-e^2*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x+e*(2*a*c*f+b*c*e*(1+2*p)+a*d*(2
*e*q+e))*x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/
c)/a/c/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)+1/3*(a*c*f^2+b*d*e^2*(3+2*p+2*q))*x
^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/a/c/(
(1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} dx$$
$$= \frac{(a+bx^2)^p (c+dx^2)^q \left(-3e^2\left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+fx^2\left(\frac{}{3ac}\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^2,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*((-3*e^2*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2
)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + f*x^2*((18*a*
c*e*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*a*c*AppellF
1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + 2*x^2*(b*c*p*AppellF1[3/
2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + a*d*q*AppellF1[3/2, -p, 1
 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])) + (f*x^2*AppellF1[3/2, -p, -q, 5/
2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))))/(
3*x)
```

Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.60, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {448, 406, 334, 334, 333, 395, 395, 394, 445, 25, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2 (a+bx^2)^p (c+dx^2)^q}{x^2} dx$$
$$\Big\downarrow \text{448}$$
$$\frac{f\int (bx^2+a)^p (dx^2+c)^q (fx^2+e)dx}{e^2}+e\int \frac{(bx^2+a)^p (dx^2+c)^q (fx^2+e)}{x^2}dx$$

$\big\downarrow$ 406

$$\frac{f\left(e\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx+f\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx\right)}{e^2}+e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\big\downarrow$ 334

$$\frac{f\left(e\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx+f\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx\right)}{e^2}+e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\big\downarrow$ 334

$$\frac{f\left(e\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx+f\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q \right.}{e^2}$$

$$e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\big\downarrow$ 333

$$\frac{f\left(f\int x^2\left(bx^2+a\right)^p \left(dx^2+c\right)^q dx+ex\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},\right.\right.}{e^2}$$

$$e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\big\downarrow$ 395

$$\frac{f\left(f\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx+ex\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\right.}{e^2}$$

$$e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\big\downarrow$ 395

$$\frac{f\left(f\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\left(c+dx^2\right)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx+ex\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p}\right.}{e^2}$$

$$e\int \frac{\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left(fx^2+e\right)}{x^2}dx$$

$\downarrow$ 394

$$e\int \frac{(bx^2+a)^p (dx^2+c)^q (fx^2+e)}{x^2}dx+$$
$$\frac{f\left(ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(\right.\right.}{e^2}$$

$\downarrow$ 445

$$e\left(-\frac{\int -(bx^2+a)^p (dx^2+c)^q (bde(2p+2q+3)x^2+bce(2p+1)+a(cf+d(2qe+e)))\,dx}{ac}-\frac{e(a+bx^2)^{p+1}(c}{acx}\right.$$
$$\frac{f\left(ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(\right.\right.}{e^2}$$

$\downarrow$ 25

$$e\left(\frac{\int (bx^2+a)^p (dx^2+c)^q (bde(2p+2q+3)x^2+acf+bce(2p+1)+ad(2qe+e))\,dx}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx}{acx}\right.$$
$$\frac{f\left(ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(\right.\right.}{e^2}$$

$\downarrow$ 406

$$e\left(\frac{(acf+ad(2eq+e)+bce(2p+1))\int (bx^2+a)^p (dx^2+c)^q\,dx+bde(2p+2q+3)\int x^2(bx^2+a)^p (dx^2+c)^q\,d}{ac}\right.$$
$$\frac{f\left(ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(\right.\right.}{e^2}$$

$\downarrow$ 334

$$e\left(\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (acf+ad(2eq+e)+bce(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q\,dx+bde(2p+2q+3)\int x}{ac}\right.$$
$$\frac{f\left(ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p \left(\right.\right.}{e^2}$$

$\downarrow$ 334

$$e\left(\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}(acf+ad(2eq+e)+bce(2p+1))\int\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q a}{ac}\right.$$

$$\frac{f\left(ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p\left(\right.\right.}{e^2}$$

$\downarrow$ 333

$$e\left(\frac{bde(2p+2q+3)\int x^2(bx^2+a)^p(dx^2+c)^q\,dx+x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}}{ac}\right.$$

$$\frac{f\left(ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p\left(\right.\right.}{e^2}$$

$\downarrow$ 395

$$e\left(\frac{bde(2p+2q+3)(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q\,dx+x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)}{ac}\right.$$

$$\frac{f\left(ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p\left(\right.\right.}{e^2}$$

$\downarrow$ 395

$$e\left(\frac{bde(2p+2q+3)(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+x(a+bx^2)}{}\right.$$

$$\frac{f\left(ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p\left(\right.\right.}{e^2}$$

$\downarrow$ 394

$$\frac{f\left(ex(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{1}{3}fx^3(a+bx^2)^p\left(\right.\right.}{e^2}$$

$$e\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+}{}\right.$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^2,x]
```

output
```
(f*((e*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (f*x^3*(a + b*x
^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
)/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)))/e^2 + e*(-((e*(a + b*x^2)^(1 +
 p)*(c + d*x^2)^(1 + q))/(a*c*x)) + (((a*c*f + b*c*e*(1 + 2*p) + a*d*(e +
2*e*q))*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)
/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) + (b*d*e*(3 + 2*
p + 2*q)*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x
^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))/(a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x^2}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} \, dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x^2} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x, algorithm="fricas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^2} \, dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x^2} \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x^2}\,dx = \int \frac{(fx^2+e)^2(bx^2+a)^p(dx^2+c)^q}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x^2}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^2,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q\,(e+fx^2)^2}{x^2}\,dx = \text{too large to display}$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^2,x)
```

output

```
( - 2*(c + d*x**2)**q*(a + b*x**2)**p*a**2*c*d*f**2*p + 4*(c + d*x**2)**q*
(a + b*x**2)**p*a**2*d**2*f**2*p*q*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**
p*a**2*d**2*f**2*p*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c**2*f**2*
q + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*p**2 + 16*(c + d*x**2)**
q*(a + b*x**2)**p*a*b*c*d*e*f*p*q + 12*(c + d*x**2)**q*(a + b*x**2)**p*a*b
*c*d*e*f*p + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*q**2 + 12*(c +
d*x**2)**q*(a + b*x**2)**p*a*b*c*d*e*f*q + 4*(c + d*x**2)**q*(a + b*x**2)*
*p*a*b*c*d*f**2*p**2*x**2 - 2*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f**2
*p*x**2 + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f**2*q**2*x**2 - 2*(c
+ d*x**2)**q*(a + b*x**2)**p*a*b*c*d*f**2*q*x**2 + 4*(c + d*x**2)**q*(a +
b*x**2)**p*a*b*d**2*e**2*p**2 + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2
*e**2*p*q + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*p + 4*(c + d*x
**2)**q*(a + b*x**2)**p*a*b*d**2*e**2*q**2 + 8*(c + d*x**2)**q*(a + b*x**2
)**p*a*b*d**2*e**2*q + 3*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e**2 + 8
*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*p*q*x**2 - 4*(c + d*x**2)**q
*(a + b*x**2)**p*a*b*d**2*e*f*p*x**2 + 8*(c + d*x**2)**q*(a + b*x**2)**p*a
*b*d**2*e*f*q**2*x**2 + 8*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*q*x
**2 - 6*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*e*f*x**2 + 4*(c + d*x**2)
**q*(a + b*x**2)**p*a*b*d**2*f**2*p*q*x**4 - 2*(c + d*x**2)**q*(a + b*x**2
)**p*a*b*d**2*f**2*p*x**4 + 4*(c + d*x**2)**q*(a + b*x**2)**p*a*b*d**2*...
```

3.84 $\int \frac{(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2}{x^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 406

$$\int \frac{(a+bx^2)^p(c+dx^2)^q(e+fx^2)^2}{x^4}\,dx = -\frac{e^2(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{3acx^3}$$
$$-\frac{e(6acf-bce(1-2p)-ade(1-2q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{3a^2c^2x}$$
$$-\frac{(b^2c^2e^2(1-4p^2)-abce(6cf(1+2p)-de(1-2p-2q-8pq))-a^2(3c^2f^2+6cdef(1+2q)-d^2e^2(1}{3a^2c^2}$$
$$+\frac{bde(6acf-bce(1-2p)-ade(1-2q))(3+2p+2q)x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}}{9a^2c^2}$$

output
```
-1/3*e^2*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a/c/x^3-1/3*e*(6*a*c*f-b*c*e*(1-2
*p)-a*d*e*(1-2*q))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/a^2/c^2/x-1/3*(b^2*c^2*
e^2*(-4*p^2+1)-a*b*c*e*(6*c*f*(1+2*p)-d*e*(-8*p*q-2*p-2*q+1))-a^2*(3*c^2*f
^2+6*c*d*e*f*(1+2*q)-d^2*e^2*(-4*q^2+1)))*x*(b*x^2+a)^p*(d*x^2+c)^q*Appell
F1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/a^2/c^2/((1+b*x^2/a)^p)/((1+d*x^2/c)^q
)+1/9*b*d*e*(6*a*c*f-b*c*e*(1-2*p)-a*d*e*(1-2*q))*(3+2*p+2*q)*x^3*(b*x^2+a
)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/a^2/c^2/((1+b*x^
2/a)^p)/((1+d*x^2/c)^q)
```

Mathematica [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4} dx$$
$$= \frac{(a+bx^2)^p (c+dx^2)^q \left(-e^2\left(1+\frac{bx^2}{a}\right)^{-p} \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-6efx^2\left(\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^4,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*(-((e^2*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2
)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)) - (6*e*f*x^2*A
ppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p
*(1 + (d*x^2)/c)^q) + (9*a*c*f^2*x^4*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^
2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
)/(3*x^3)
```

Rubi [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.56, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {448, 445, 25, 406, 334, 334, 333, 395, 395, 394, 445, 406, 334, 334, 333, 395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx^2)^2 (a+bx^2)^p (c+dx^2)^q}{x^4} dx$$

$$\Big\downarrow \text{448}$$

$$\frac{f\int \frac{(bx^2+a)^p (dx^2+c)^q (fx^2+e)}{x^2}dx}{e^2} + e\int \frac{(bx^2+a)^p (dx^2+c)^q (fx^2+e)}{x^4}dx$$

$$\Big\downarrow \text{ 445}$$

$$\frac{f\left(-\frac{\int -(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+3)x^2+bce(2p+1)+a(cf+d(2qe+e)))dx}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}\right)}{e^2}+e\left(-\frac{\int -\frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ad(e-2eq))}{x^2}dx}{3ac}-\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{3acx^3}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{f\left(\frac{\int (bx^2+a)^p(dx^2+c)^q(bde(2p+2q+3)x^2+acf+bce(2p+1)+ad(2qe+e))dx}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}\right)}{e^2}+e\left(\frac{\int \frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{3acx^3}\right)$$

$$\Big\downarrow \text{ 406}$$

$$\frac{f\left(\frac{(acf+ad(2eq+e)+bce(2p+1))\int (bx^2+a)^p(dx^2+c)^q dx+bde(2p+2q+3)\int x^2(bx^2+a)^p(dx^2+c)^q dx}{ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{acx}\right)}{e^2}+e\left(\frac{\int \frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{3acx^3}\right)$$

$$\Big\downarrow \text{ 334}$$

$$\frac{f\left(\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(acf+ad(2eq+e)+bce(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q dx+bde(2p+2q+3)\int x^2(bx^2+a)^p(dx^2+c)^q dx}{ac}-\frac{e(a+bx^2)^{p+1}\cdots}{ac\cdots}\right)}{e^2}$$
$$e\left(\frac{\int \frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{3acx^3}\right)$$

$$\Big\downarrow \text{ 334}$$

$$\frac{f\left(\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}(acf+ad(2eq+e)+bce(2p+1))\int \left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+bde(2p+2q+3)\int x^2(bx^2+a)^p(dx^2+\cdots}{ac}\right.}{e^2}$$
$$e\left(\frac{\int \frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{3acx^3}\right)$$

$$\Big\downarrow \text{ 333}$$

$$\frac{f\left(\frac{bde(2p+2q+3)\int x^2(bx^2+a)^p(dx^2+c)^q dx+x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq$$

$$e\left(\frac{\int\frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}\right)$$

$\downarrow$ 395

$$\frac{f\left(\frac{bde(2p+2q+3)(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^2\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q dx+x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{b}{}\right.$$

$$e\left(\frac{\int\frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}\right)$$

$\downarrow$ 395

$$\frac{f\left(\frac{bde(2p+2q+3)(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\int x^2\left(\frac{bx^2}{a}+1\right)^p\left(\frac{dx^2}{c}+1\right)^q dx+x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{Ap}\right.$$

$$e\left(\frac{\int\frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}\right)$$

$\downarrow$ 394

$$e\left(\frac{\int\frac{(bx^2+a)^p(dx^2+c)^q(bde(2p+2q+1)x^2+3acf-bce(1-2p)-ade(1-2q))}{x^2}dx}{3ac}-\frac{e(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{3acx^3}\right)+$$

$$\frac{f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b}{ac}\right.}{e^2}$$

$\downarrow$ 445

$$e\left(\frac{-\frac{\int(bx^2+a)^p(dx^2+c)^q(-d(3cf-de(1-2q))(2q+1)a^2-bc(3cf(2p+1)-de(-8qp-2p-2q+1))a-bd(3acf-bce(1-2p)-ade(1-2q))(2p+2q+3}{ac}}{3ac}\right.$$

$$\frac{f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b}{ac}\right.}{e^2}$$

$$\Big\downarrow \; 406$$

$$\frac{e\left(-\frac{\frac{\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2)\right)\int (bx^2+a)^p(dx^2+c)^q dx-bd(2p+2q+3)(3acf-\cdots}{ac}}{3ac}\right.}{e^2} \qquad f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b\cdots}{ac}\right.$$

$$\Big\downarrow \; 334$$

$$\frac{e\left(-\frac{\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2)\right)\int\left(\frac{bx^2}{a}+1\right)^p(dx^2+c)^q dx\cdots}{ac}}{3ac}\right.}{e^2} \qquad f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b\cdots}{ac}\right.$$

$$\Big\downarrow \; 334$$

$$\frac{e\left(-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(-8pq-2p-2q+1))+b^2c^2e(1-4p^2)\right)\int\left(\cdots\right.}{ac}\right.}{e^2} \qquad f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b\cdots}{ac}\right.$$

$$\Big\downarrow \; 333$$

$$\frac{e\left(-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(\cdots\right.}{ac}\right.}{e^2} \qquad f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+b\cdots}{ac}\right.$$

$$\Big\downarrow \; 395$$

$$\frac{e\left(-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(\right.}{ac}\right.}{}$$

$$f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+bx\right.}{ac}$$

$$e^2$$

$\downarrow$ 395

$$\frac{e\left(-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(\right.}{}\right.}{}$$

$$f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+bx\right.}{ac}$$

$$e^2$$

$\downarrow$ 394

$$\frac{e\left(-\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(a^2(-d)(2q+1)(3cf-de(1-2q))-abc(3cf(2p+1)-de(\right.}{}\right.}{}$$

$$f\left(\frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(c+dx^2)^q\left(\frac{dx^2}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)(acf+ad(2eq+e)+bce(2p+1))+\frac{1}{3}bdex^3(2p+2q+3)(a+bx\right.}{ac}$$

$$e^2$$

input

```
Int[((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^4,x]
```

output
```
(f*(-((e*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(a*c*x)) + (((a*c*f + b*
c*e*(1 + 2*p) + a*d*(e + 2*e*q))*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/
2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^
2)/c)^q) + (b*d*e*(3 + 2*p + 2*q)*x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1
[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 +
(d*x^2)/c)^q))/(a*c)))/e^2 + e*(-1/3*(e*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1
 + q))/(a*c*x^3) + (-(((3*a*c*f - b*c*e*(1 - 2*p) - a*d*e*(1 - 2*q))*(a +
b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(a*c*x)) - (((b^2*c^2*e*(1 - 4*p^2) -
a^2*d*(3*c*f - d*e*(1 - 2*q))*(1 + 2*q) - a*b*c*(3*c*f*(1 + 2*p) - d*e*(1
- 2*p - 2*q - 8*p*q)))*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q,
 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q) -
 (b*d*(3*a*c*f - b*c*e*(1 - 2*p) - a*d*e*(1 - 2*q))*(3 + 2*p + 2*q)*x^3*(a
 + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a), -((d*x^
2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))/(a*c))/(3*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 448
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*x
^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] + Simp[f/e^2   Int[(g*x)^(m
 + 2)*(a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^(r - 1), x], x] /; FreeQ[{a,
b, c, d, e, f, g, m, p, q}, x] && IGtQ[r, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q\,(f\,x^2+e)^2}{x^4}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4}\,dx = \int \frac{(fx^2+e)^2(bx^2+a)^p(dx^2+c)^q}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x, algorithm="fricas")
```

output
```
integral((f^2*x^4 + 2*e*f*x^2 + e^2)*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q*(f*x**2+e)**2/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4}\,dx = \int \frac{(fx^2+e)^2(bx^2+a)^p(dx^2+c)^q}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4} dx = \int \frac{(fx^2+e)^2 (bx^2+a)^p (dx^2+c)^q}{x^4} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x, algorithm="giac")
```

output
```
integrate((f*x^2 + e)^2*(b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4} dx = \int \frac{(b x^2+a)^p (d x^2+c)^q (f x^2+e)^2}{x^4} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^4,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q (e+fx^2)^2}{x^4} dx = \int \frac{(b x^2+a)^p (d x^2+c)^q (f x^2+e)^2}{x^4} dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q*(f*x^2+e)^2/x^4,x)
```

3.85 $\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx$

Optimal result

Integrand size = 33, antiderivative size = 336

$$
\begin{aligned}
&\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx \\
&= \frac{e^2 (gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{g(1+m)} \\
&\quad + \frac{2efx^n (gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m+n}{n},-p,-q,\frac{1+m+2n}{n},-\frac{bx^n}{a}\right.}{g(1+m+n)} \\
&\quad + \frac{f^2 x^{2n} (gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m+2n}{n},-p,-q,\frac{1+m+3n}{n},-\frac{bx^n}{a}\right.}{g(1+m+2n)}
\end{aligned}
$$

output

```
e^2*(g*x)^(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-
b*x^n/a,-d*x^n/c)/g/(1+m)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)+2*e*f*x^n*(g*x)^
(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m+n)/n,-p,-q,(1+m+2*n)/n,-b*x^n/
a,-d*x^n/c)/g/(1+m+n)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)+f^2*x^(2*n)*(g*x)^(1
+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m+2*n)/n,-p,-q,(1+m+3*n)/n,-b*x^n/
a,-d*x^n/c)/g/(1+m+2*n)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.65

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx$$
$$= x(gx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(\frac{e^2 \operatorname{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m} + fx^n\left(\frac{2e \operatorname{AppellF1}\left(\frac{1+m+n}{n},-p,-q,\frac{1+m+2n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m+n} + \frac{fx^n \operatorname{AppellF1}\left(\frac{1+m+2n}{n},-p,-q,\frac{1+m+3n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m+2n}\right)\right)$$

input
```
Integrate[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^2,x]
```

output
```
(x*(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*((e^2*AppellF1[(1 + m)/n, -p, -q, (
1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(1 + m) + f*x^n*((2*e*AppellF1[
(1 + m + n)/n, -p, -q, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(1 +
m + n) + (f*x^n*AppellF1[(1 + m + 2*n)/n, -p, -q, (1 + m + 3*n)/n, -((b*x^
n)/a), -((d*x^n)/c)])/(1 + m + 2*n))))/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^
q)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m (e+fx^n)^2 (a+bx^n)^p (c+dx^n)^q \, dx$$
$$\Big\downarrow \text{ 1073}$$
$$\int (gx)^m (e+fx^n)^2 (a+bx^n)^p (c+dx^n)^q dx$$

input
```
Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [F]

$$\int (gx)^m (a+b\,x^n)^p (c+d\,x^n)^q (e+f\,x^n)^2\,dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x)
```

Fricas [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx = \int (fx^n+e)^2 (bx^n+a)^p (dx^n+c)^q (gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x, algorithm="fricas
")
```

output
```
integral((f^2*x^(2*n) + 2*e*f*x^n + e^2)*(b*x^n + a)^p*(d*x^n + c)^q*(g*x)
^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m*(a+b*x**n)**p*(c+d*x**n)**q*(e+f*x**n)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx = \int (fx^n+e)^2 (bx^n+a)^p (dx^n+c)^q (gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x, algorithm="maxima
")
```

output
```
integrate((f*x^n + e)^2*(b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Giac [F(-2)]

Exception generated.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{-1,[1,0,5,3,0,10,3,4,4,10,3,0,2]%%%}+%%%{-4,[1,0,5,3,0,10,
3,4,3,10,
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m (a + bx^n)^p (c + dx^n)^q (e + fx^n)^2 \, dx$$
$$= \int (g\,x)^m (a + b\,x^n)^p (c + d\,x^n)^q (e + f\,x^n)^2 \,dx$$

input
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^2,x)
```

output
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^2, x)
```

Reduce [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)^2 \, dx = \int (gx)^m (x^n b+a)^p (x^n d+c)^q (e+f\,x^n)^2 \,dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n)^2,x)
```

3.86 $\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\, dx$

Optimal result

Integrand size = 31, antiderivative size = 214

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\, dx$$
$$= \frac{e(gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{g(1+m)}$$
$$+ \frac{fx^n(gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m+n}{n},-p,-q,\frac{1+m+2n}{n},-\frac{bx^n}{a},-\right.}{g(1+m+n)}$$

output
```
e*(g*x)^(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-b*
x^n/a,-d*x^n/c)/g/(1+m)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)+f*x^n*(g*x)^(1+m)*
(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m+n)/n,-p,-q,(1+m+2*n)/n,-b*x^n/a,-d*x
^n/c)/g/(1+m+n)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\,dx$$
$$= \frac{x(gx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \left(e(1+m+n)\,\text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\text{b}\right.\right.}{(1+m)(1+m+n)}$$

input
```
Integrate[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n),x]
```

output
```
(x*(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e*(1 + m + n)*AppellF1[(1 + m)/n,
-p, -q, (1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)] + f*(1 + m)*x^n*AppellF
1[(1 + m + n)/n, -p, -q, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)]))/((
1 + m)*(1 + m + n)*(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1068, 1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m (e+fx^n) (a+bx^n)^p (c+dx^n)^q\,dx$$

$$\Big\downarrow \text{1068}$$

$$e\int (gx)^m (bx^n+a)^p (dx^n+c)^q dx + fx^{-m}(gx)^m \int x^{m+n}(bx^n+a)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$e(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (gx)^m \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx +$$
$$fx^{-m}(gx)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int x^{m+n} \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$e(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\int (gx)^m\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^q dx+ fx^{-m}(gx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\int x^{m+n}\left(\frac{bx^n}{a}+1\right)^p\left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{e(gx)^{m+1}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{m+1}{n},-p,-q,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{g(m+1)}+\frac{fx^{n+1}(gx)^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(c+dx^n)^q\left(\frac{dx^n}{c}+1\right)^{-q}\text{AppellF1}\left(\frac{m+n+1}{n},-p,-q,\frac{m+2n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{m+n+1}$$

input
```
Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n),x]
```

output
```
(e*(g*x)^(1 + m)*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (
1 + m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(g*(1 + m)*(1 + (b*x^n)/a)^p*(1
 + (d*x^n)/c)^q) + (f*x^(1 + n)*(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*Appell
F1[(1 + m + n)/n, -p, -q, (1 + m + 2*n)/n, -((b*x^n)/a), -((d*x^n)/c)])/((
1 + m + n)*(1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1068
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e   Int[(g*x)^m*(a + b*
x^n)^p*(c + d*x^n)^q, x], x] + Simp[f*((g*x)^m/x^m)   Int[x^(m + n)*(a + b*
x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q}, x]
```

Maple [F]

$$\int (gx)^m (a+b\,x^n)^p (c+d\,x^n)^q (e+f\,x^n)\,dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x)
```

Fricas [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\ dx = \int (fx^n+e)(bx^n+a)^p(dx^n+c)^q(gx)^m\ dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x, algorithm="fricas")
```

output
```
integral((f*x^n + e)*(b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\ dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m*(a+b*x**n)**p*(c+d*x**n)**q*(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n) \, dx = \int (fx^n+e)(bx^n+a)^p(dx^n+c)^q(gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x, algorithm="maxima")
```

output
```
integrate((f*x^n + e)*(b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n) \, dx = \int (fx^n+e)(bx^n+a)^p(dx^n+c)^q(gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x, algorithm="giac")
```

output
```
integrate((f*x^n + e)*(b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n) \, dx = \int (g\,x)^m\,(a+b\,x^n)^p\,(c+d\,x^n)^q\,(e+f\,x^n) \, dx$$

input
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n),x)
```

output
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n), x)
```

Reduce [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q (e+fx^n)\ dx = \int (gx)^m (x^n b+a)^p (x^n d+c)^q (e+f\,x^n)\,dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q*(e+f*x^n),x)
```

3.87 $\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \frac{(gx)^{1+m} (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{g(1+m)}$$

output
```
(g*x)^(1+m)*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1((1+m)/n,-p,-q,(1+m+n)/n,-b*x^
n/a,-d*x^n/c)/g/(1+m)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \frac{x(gx)^m (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{n},-p,-q,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{1+m}$$

input
```
Integrate[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
(x*(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (1 + m
+ n)/n, -((b*x^n)/a), -((d*x^n)/c)])/((1 + m)*(1 + (b*x^n)/a)^p*(1 + (d*x^
n)/c)^q)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1013, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \int (gx)^m \left(\frac{bx^n}{a}+1\right)^p (dx^n+c)^q dx$$

$$\Big\downarrow \text{1013}$$

$$(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \int (gx)^m \left(\frac{bx^n}{a}+1\right)^p \left(\frac{dx^n}{c}+1\right)^q dx$$

$$\Big\downarrow \text{1012}$$

$$\frac{(gx)^{m+1} (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{n},-p,-q,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{g(m+1)}$$

input
```
Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x]
```

output
```
((g*x)^(1 + m)*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[(1 + m)/n, -p, -q, (1
+ m + n)/n, -((b*x^n)/a), -((d*x^n)/c)])/(g*(1 + m)*(1 + (b*x^n)/a)^p*(1 +
 (d*x^n)/c)^q)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int (gx)^m (a + b\,x^n)^p (c + d\,x^n)^q \,dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

Fricas [F]

$$\int (gx)^m (a + bx^n)^p (c + dx^n)^q \,dx = \int (bx^n + a)^p (dx^n + c)^q (gx)^m \,dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m*(a+b*x**n)**p*(c+d*x**n)**q,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q (gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Giac [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \int (bx^n+a)^p (dx^n+c)^q (gx)^m \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \int (g\,x)^m \, (a+b\,x^n)^p \, (c+d\,x^n)^q \, dx$$

input
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q,x)
```

output
```
int((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x)
```

Reduce [F]

$$\int (gx)^m (a+bx^n)^p (c+dx^n)^q \, dx = \text{too large to display}$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q,x)
```

output

```
(g**m*(x**m*(x**n*d + c)**q*(x**n*b + a)**p*a*d*x + x**m*(x**n*d + c)**q*(
x**n*b + a)**p*b*c*x - int((x**(m + 2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/
(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n
)*b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*
m + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*
n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*m + x**n*b**2*c**
2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d*n*q + a**2*c*d + a*b*c**2*m
 + a*b*c**2*n*p + a*b*c**2),x)*a**2*b*d**3*m*n*p - int((x**(m + 2*n)*(x**n
*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d**2*m + x**(2*n)*a*b*d**2*n*q +
 x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m + x**(2*n)*b**2*c*d*n*p + x**(2*n
)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*d**2*n*q + x**n*a**2*d**2 + 2*x*
*n*a*b*c*d*m + x**n*a*b*c*d*n*p + x**n*a*b*c*d*n*q + 2*x**n*a*b*c*d + x**n
*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b**2*c**2 + a**2*c*d*m + a**2*c*d
*n*q + a**2*c*d + a*b*c**2*m + a*b*c**2*n*p + a*b*c**2),x)*a**2*b*d**3*n**
2*p*q - int((x**(m + 2*n)*(x**n*d + c)**q*(x**n*b + a)**p)/(x**(2*n)*a*b*d
**2*m + x**(2*n)*a*b*d**2*n*q + x**(2*n)*a*b*d**2 + x**(2*n)*b**2*c*d*m +
x**(2*n)*b**2*c*d*n*p + x**(2*n)*b**2*c*d + x**n*a**2*d**2*m + x**n*a**2*d
**2*n*q + x**n*a**2*d**2 + 2*x**n*a*b*c*d*m + x**n*a*b*c*d*n*p + x**n*a*b*
c*d*n*q + 2*x**n*a*b*c*d + x**n*b**2*c**2*m + x**n*b**2*c**2*n*p + x**n*b*
*2*c**2 + a**2*c*d*m + a**2*c*d*n*q + a**2*c*d + a*b*c**2*m + a*b*c**2*...
```

3.88 $\int \frac{(gx)^m(a+bx^n)^p(c+dx^n)^q}{e+fx^n}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \frac{x^{-1-m}(gx)^{1+m}\text{Int}\left(\frac{x^m(a+bx^n)^p(c+dx^n)^q}{e+fx^n},x\right)}{g}$$

output
```
x^(-1-m)*(g*x)^(1+m)*Defer(Int)(x^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x)/g
```

Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx$$

input
```
Integrate[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n),x]
```

output
```
Integrate[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{1073\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n} \, dx$$

$$\Big\downarrow \text{ 1073}$$

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n} dx$$

input
```
Int[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(gx)^m (a+b\,x^n)^p (c+d\,x^n)^q}{e+f\,x^n}dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(bx^n+a)^p(dx^n+c)^q(gx)^m}{fx^n+e}\,dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f*x^n + e), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((g*x)**m*(a+b*x**n)**p*(c+d*x**n)**q/(e+f*x**n),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(bx^n+a)^p(dx^n+c)^q(gx)^m}{fx^n+e}\,dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f*x^n + e), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(bx^n+a)^p(dx^n+c)^q(gx)^m}{fx^n+e}\,dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f*x^n + e), x)
```

Mupad [N/A]

Not integrable

Time = 5.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(g\,x)^m\,(a+b\,x^n)^p\,(c+d\,x^n)^q}{e+f\,x^n}\,dx$$

input
```
int(((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n),x)
```

output
```
int(((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n), x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{e+fx^n}\,dx = \int \frac{(gx)^m (x^n b+a)^p (x^n d+c)^q}{e+f\,x^n}dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n),x)
```

3.89 $\int \frac{(gx)^m(a+bx^n)^p(c+dx^n)^q}{(e+fx^n)^2} dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} dx = \frac{x^{-1-m}(gx)^{1+m}\text{Int}\left(\frac{x^m(a+bx^n)^p(c+dx^n)^q}{(e+fx^n)^2},x\right)}{g}$$

output
```
x^(-1-m)*(g*x)^(1+m)*Defer(Int)(x^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x)
/g
```

Mathematica [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} dx = \int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} dx$$

input
```
Integrate[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n)^2,x]
```

output
```
Integrate[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {1073}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} \, dx$$
$$\Big\downarrow \; 1073$$
$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} dx$$

input
```
Int[((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1073
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Unintegrable[(g*x)^m
*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x] /; FreeQ[{a, b, c, d, e, f,
g, m, n, p, q, r}, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(gx)^m (a+b\,x^n)^p (c+d\,x^n)^q}{(e+f\,x^n)^2}dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2}\,dx = \int \frac{(bx^n+a)^p(dx^n+c)^q(gx)^m}{(fx^n+e)^2}\,dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x, algorithm="fricas
")
```

output
```
integral((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f^2*x^(2*n) + 2*e*f*x^n + e^
2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x)**m*(a+b*x**n)**p*(c+d*x**n)**q/(e+f*x**n)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} \, dx = \int \frac{(bx^n+a)^p (dx^n+c)^q (gx)^m}{(fx^n+e)^2} \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x, algorithm="maxima
")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f*x^n + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} \, dx = \int \frac{(bx^n+a)^p (dx^n+c)^q (gx)^m}{(fx^n+e)^2} \, dx$$

input
```
integrate((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*(d*x^n + c)^q*(g*x)^m/(f*x^n + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 7.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} \, dx = \int \frac{(g\,x)^m\,(a+b\,x^n)^p\,(c+d\,x^n)^q}{(e+f\,x^n)^2} \, dx$$

input
```
int(((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n)^2,x)
```

output
```
int(((g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q)/(e + f*x^n)^2, x)
```

Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(gx)^m (a+bx^n)^p (c+dx^n)^q}{(e+fx^n)^2} \, dx = \int \frac{(gx)^m (x^n b+a)^p (x^n d+c)^q}{(e+f\,x^n)^2} dx$$

input
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x)
```

output
```
int((g*x)^m*(a+b*x^n)^p*(c+d*x^n)^q/(e+f*x^n)^2,x)
```

3.90 $\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 128

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx=\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(-a+bx^{n/2}\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}}{x}-\frac{d\left(-a+bx^{n/2}\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{1}{n},-\frac{1-n}{n},-\frac{b^2x^n}{a^2-b^2x^n}\right)}{b^2x}$$

output
```
(c/a^2+d/b^2)*(-a+b*x^(1/2*n))^(1/n)*(a+b*x^(1/2*n))^(1/n)/x-d*(-a+b*x^(1/
2*n))^(1/n)*(a+b*x^(1/2*n))^(1/n)*hypergeom([1, -1/n],[-(1-n)/n],-b^2*x^n/
(a^2-b^2*x^n))/b^2/x
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx=\frac{\left(-a+bx^{n/2}\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(1-\frac{b^2x^n}{a^2}\right)^{-1/n}\left(c(-1\right.}{}$$

input
```
Integrate[((-a + b*x^(n/2))^(-1 + n^(-1))*(a + b*x^(n/2))^(-1 + n^(-1))*(c
 + d*x^n))/x^2,x]
```

output
```
((-a + b*x^(n/2))^n^(-1)*(a + b*x^(n/2))^n^(-1)*(c*(-1 + n)*(1 - (b^2*x^n)
/a^2)^n^(-1) - d*x^n*Hypergeometric2F1[(-1 + n)/n, (-1 + n)/n, 2 - n^(-1),
 (b^2*x^n)/a^2]))/(a^2*(-1 + n)*x*(1 - (b^2*x^n)/a^2)^n^(-1))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {2038, 954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^{n/2}-a\right)^{\frac{1}{n}-1}\left(a+bx^{n/2}\right)^{\frac{1}{n}-1}\left(c+dx^n\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2038}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{-1/n}\int \frac{\left(b^2x^n-a^2\right)^{\frac{1}{n}-1}\left(dx^n+c\right)}{x^2}dx$$

$$\Big\downarrow \text{ 954}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{-1/n}\left(\frac{d\int \frac{\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x^2}dx}{b^2}+\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x}\right)$$

$$\Big\downarrow \text{ 882}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{-1/n}\left(\frac{d\left(-\frac{x^n}{a^2-b^2x^n}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{\frac{1}{n}}\int \frac{\left(-\frac{x^n}{a^2-b^2x^n}\right)^{-1-\frac{1}{n}}}{\frac{b^2x^n}{a^2-b^2x^n}+1}d\left(-\frac{x^n}{a^2-b^2x^n}\right)}{b^2nx}+\right.$$

$$\Big\downarrow \text{ 74}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{-1/n}\left(\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x}-\frac{d\left(b^2x^n-a^2\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(\right.}{b^2x}\right.$$

input
```
Int[((-a + b*x^(n/2))^(-1 + n^(-1))*(a + b*x^(n/2))^(-1 + n^(-1))*(c + d*x
^n))/x^2,x]
```

output
```
((-a + b*x^(n/2))^n^(-1)*(a + b*x^(n/2))^n^(-1)*(((c/a^2 + d/b^2)*(-a^2 +
b^2*x^n)^n^(-1))/x - (d*(-a^2 + b^2*x^n)^n^(-1)*Hypergeometric2F1[1, -n^(-
1), -((1 - n)/n), -((b^2*x^n)/(a^2 - b^2*x^n))])/(b^2*x)))/(-a^2 + b^2*x^n
)^n^(-1)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

Maple [F]

$$\int \frac{\left(-a+b\,x^{\frac{n}{2}}\right)^{-1+\frac{1}{n}} \left(a+b\,x^{\frac{n}{2}}\right)^{-1+\frac{1}{n}} \left(c+d\,x^{n}\right)}{x^{2}}dx$$

input
```
int((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2,x)
```

output
```
int((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}} \left(a+bx^{n/2}\right)^{-1+\frac{1}{n}} \left(c+dx^n\right)}{x^2}\,dx = \int \frac{(dx^n+c)\left(bx^{\frac{1}{2}n}+a\right)^{\frac{1}{n}-1}\left(bx^{\frac{1}{2}n}-a\right)^{\frac{1}{n}-1}}{x^2}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2
,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/((b*x^(1/2*n) + a)^((n - 1)/n)*(b*x^(1/2*n) - a)^((n
- 1)/n)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a+b*x**(1/2*n))**(-1+1/n)*(a+b*x**(1/2*n))**(-1+1/n)*(c+d*x**n
)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx = \int \frac{(dx^n+c)\left(bx^{\frac{1}{2}n}+a\right)^{\frac{1}{n}-1}\left(bx^{\frac{1}{2}n}-a\right)^{\frac{1}{n}-1}}{x^2}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2
,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)*(b*x^(1/2*n) + a)^(1/n - 1)*(b*x^(1/2*n) - a)^(1/n -
 1)/x^2, x)
```

Giac [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx = \int \frac{(dx^n+c)\left(bx^{\frac{1}{2}n}+a\right)^{\frac{1}{n}-1}\left(bx^{\frac{1}{2}n}-a\right)^{\frac{1}{n}-1}}{x^2}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2
,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)*(b*x^(1/2*n) + a)^(1/n - 1)*(b*x^(1/2*n) - a)^(1/n -
 1)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx=\int \frac{\left(a+b\,x^{n/2}\right)^{\frac{1}{n}-1}\left(b\,x^{n/2}-a\right)^{\frac{1}{n}-1}(c+d\,x^n)}{x^2}\,dx$$

input
```
int(((a + b*x^(n/2))^(1/n - 1)*(b*x^(n/2) - a)^(1/n - 1)*(c + d*x^n))/x^2,
x)
```

output
```
int(((a + b*x^(n/2))^(1/n - 1)*(b*x^(n/2) - a)^(1/n - 1)*(c + d*x^n))/x^2,
 x)
```

Reduce [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}(c+dx^n)}{x^2}\,dx=\left(\int \frac{\left(x^{\frac{n}{2}}b+a\right)^{\frac{1}{n}}\left(x^{\frac{n}{2}}b-a\right)^{\frac{1}{n}}}{x^nb^2x^2-a^2x^2}dx\right)c$$
$$+\left(\int \frac{x^n\left(x^{\frac{n}{2}}b+a\right)^{\frac{1}{n}}\left(x^{\frac{n}{2}}b-a\right)^{\frac{1}{n}}}{x^nb^2x^2-a^2x^2}dx\right)d$$

input
```
int((-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(c+d*x^n)/x^2,x)
```

output
```
int(((x**(n/2)*b + a)**(1/n)*(x**(n/2)*b - a)**(1/n))/(x**n*b**2*x**2 - a*
*2*x**2),x)*c + int((x**n*(x**(n/2)*b + a)**(1/n)*(x**(n/2)*b - a)**(1/n))
/(x**n*b**2*x**2 - a**2*x**2),x)*d
```

3.91 $\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}\,dx$

Optimal result

Integrand size = 55, antiderivative size = 167

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}\,dx = -\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a^2-b^2x^n\right)}{x} + \frac{a^2d\left(-a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(a+bx^{n/2}\right)^{-1+\frac{1}{n}}\left(1-\frac{b^2x^n}{a^2}\right)^{-\frac{1-n}{n}}\text{Hypergeometric2F1}\left(-\frac{1}{n},-\frac{1}{n},-\frac{1-n}{n},\frac{b^2x^n}{a^2}\right)}{b^2x}$$

output
```
-(c/a^2+d/b^2)*(-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*(a^2-b^2
*x^n)/x+a^2*d*(-a+b*x^(1/2*n))^(-1+1/n)*(a+b*x^(1/2*n))^(-1+1/n)*hypergeom
([-1/n, -1/n],[-(1-n)/n],b^2*x^n/a^2)/b^2/x/((1-b^2*x^n/a^2)^((1-n)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}\,dx = \frac{\left(-a+bx^{n/2}\right)^{\frac{1}{n}}\left(a+bx^{n/2}\right)^{\frac{1}{n}}\left(1-\frac{b^2x^n}{a^2}\right)^{-1/n}\Big(c(-1+n$$

input

```
Integrate[((-a + b*x^(n/2))^((1 - n)/n)*(a + b*x^(n/2))^((1 - n)/n)*(c + d
*x^n))/x^2,x]
```

output

```
((-a + b*x^(n/2))^n^(-1)*(a + b*x^(n/2))^n^(-1)*(c*(-1 + n)*(1 - (b^2*x^n)
/a^2)^n^(-1) - d*x^n*Hypergeometric2F1[(-1 + n)/n, (-1 + n)/n, 2 - n^(-1),
 (b^2*x^n)/a^2]))/(a^2*(-1 + n)*x*(1 - (b^2*x^n)/a^2)^n^(-1))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {2038, 954, 882, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^{n/2}-a\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}dx$$

$$\Big\downarrow \text{2038}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}-1}\left(a+bx^{n/2}\right)^{\frac{1}{n}-1}\left(b^2x^n-a^2\right)^{-\frac{1-n}{n}}\int \frac{\left(b^2x^n-a^2\right)^{\frac{1}{n}-1}(dx^n+c)}{x^2}dx$$

$$\Big\downarrow \text{954}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}-1}\left(a+bx^{n/2}\right)^{\frac{1}{n}-1}\left(b^2x^n-a^2\right)^{-\frac{1-n}{n}}\left(\frac{d\int \frac{\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x^2}dx}{b^2}+\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x}\right)$$

$$\Big\downarrow \text{882}$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}-1}\left(a+bx^{n/2}\right)^{\frac{1}{n}-1}\left(b^2x^n-a^2\right)^{-\frac{1-n}{n}}\left(\frac{d\left(-\frac{x^n}{a^2-b^2x^n}\right)^{\frac{1}{n}}\left(b^2x^n-a^2\right)^{\frac{1}{n}}\int\frac{\left(-\frac{x^n}{a^2-b^2x^n}\right)^{-1-\frac{1}{n}}}{\frac{b^2x^n}{a^2-b^2x^n}+1}d\left(-\frac{x^n}{a^2-b^2x^n}\right.}{b^2nx}\right.$$

$$\Big\downarrow 74$$

$$\left(bx^{n/2}-a\right)^{\frac{1}{n}-1}\left(a+bx^{n/2}\right)^{\frac{1}{n}-1}\left(b^2x^n-a^2\right)^{-\frac{1-n}{n}}\left(\frac{\left(\frac{c}{a^2}+\frac{d}{b^2}\right)\left(b^2x^n-a^2\right)^{\frac{1}{n}}}{x}-\frac{d\left(b^2x^n-a^2\right)^{\frac{1}{n}}\text{Hypergeometric2}}{b^2x}\right.$$

input
```
Int[((-a + b*x^(n/2))^((1 - n)/n)*(a + b*x^(n/2))^((1 - n)/n)*(c + d*x^n))
/x^2,x]
```

output
```
((-a + b*x^(n/2))^(-1 + n^(-1))*(a + b*x^(n/2))^(-1 + n^(-1))*(((c/a^2 + d
/b^2)*(-a^2 + b^2*x^n)^n^(-1))/x - (d*(-a^2 + b^2*x^n)^n^(-1)*Hypergeometr
ic2F1[1, -n^(-1), -((1 - n)/n), -((b^2*x^n)/(a^2 - b^2*x^n))])/(b^2*x)))/(
-a^2 + b^2*x^n)^((1 - n)/n)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 882
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^Simplify[
(m + 1)/n + p]*x^m*(a + b*x^n)^p*((x^n/(a + b*x^n))^p/(n*x^Simplify[m + n*p
]))   Subst[Int[x^((m + 1)/n - 1)/(1 - b*x)^(Simplify[(m + 1)/n + p] + 1),
x], x, x^n/(a + b*x^n)], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simpli
fy[(m + 1)/n + p]]
```

rule 954
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
 1, 0] && NeQ[m, -1]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

Maple [F]

$$\int \frac{\left(-a+b\,x^{\frac{n}{2}}\right)^{\frac{1-n}{n}} \left(a+b\,x^{\frac{n}{2}}\right)^{\frac{1-n}{n}} \left(c+d\,x^{n}\right)}{x^{2}}dx$$

input
```
int((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x^2,x)
```

output
```
int((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x^2,x)
```

Fricas [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}} \left(a+bx^{n/2}\right)^{\frac{1-n}{n}} \left(c+dx^n\right)}{x^2}\,dx = \int \frac{dx^{n}+c}{\left(bx^{\frac{1}{2}\,n}+a\right)^{\frac{n-1}{n}} \left(bx^{\frac{1}{2}\,n}-a\right)^{\frac{n-1}{n}} x^{2}}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x
^2,x, algorithm="fricas")
```

output
```
integral((d*x^n + c)/((b*x^(1/2*n) + a)^((n - 1)/n)*(b*x^(1/2*n) - a)^((n
- 1)/n)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(c+dx^n\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-a+b*x**(1/2*n))**((1-n)/n)*(a+b*x**(1/2*n))**((1-n)/n)*(c+d*x*
*n)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(c+dx^n\right)}{x^2}\,dx = \int \frac{dx^n+c}{\left(bx^{\frac{1}{2}n}+a\right)^{\frac{n-1}{n}}\left(bx^{\frac{1}{2}n}-a\right)^{\frac{n-1}{n}}x^2}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x
^2,x, algorithm="maxima")
```

output
```
integrate((d*x^n + c)/((b*x^(1/2*n) + a)^((n - 1)/n)*(b*x^(1/2*n) - a)^((n
 - 1)/n)*x^2), x)
```

Giac [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(c+dx^n\right)}{x^2}\,dx = \int \frac{dx^n+c}{\left(bx^{\frac{1}{2}n}+a\right)^{\frac{n-1}{n}}\left(bx^{\frac{1}{2}n}-a\right)^{\frac{n-1}{n}}x^2}\,dx$$

input
```
integrate((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x
^2,x, algorithm="giac")
```

output
```
integrate((d*x^n + c)/((b*x^(1/2*n) + a)^((n - 1)/n)*(b*x^(1/2*n) - a)^((n
 - 1)/n)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}\,dx=\int \frac{c+d\,x^n}{x^2\left(a+b\,x^{n/2}\right)^{\frac{n-1}{n}}\left(b\,x^{n/2}-a\right)^{\frac{n-1}{n}}}\,dx$$

input
```
int((c + d*x^n)/(x^2*(a + b*x^(n/2))^((n - 1)/n)*(b*x^(n/2) - a)^((n - 1)/
n)),x)
```

output
```
int((c + d*x^n)/(x^2*(a + b*x^(n/2))^((n - 1)/n)*(b*x^(n/2) - a)^((n - 1)/
n)), x)
```

Reduce [F]

$$\int \frac{\left(-a+bx^{n/2}\right)^{\frac{1-n}{n}}\left(a+bx^{n/2}\right)^{\frac{1-n}{n}}(c+dx^n)}{x^2}\,dx=\left(\int \frac{\left(x^{\frac{n}{2}}b+a\right)^{\frac{1}{n}}\left(x^{\frac{n}{2}}b-a\right)^{\frac{1}{n}}}{x^n b^2x^2-a^2x^2}dx\right)c+\left(\int \frac{x^n\left(x^{\frac{n}{2}}b+a\right)^{\frac{1}{n}}\left(x^{\frac{n}{2}}b-a\right)^{\frac{1}{n}}}{x^n b^2x^2-a^2x^2}dx\right)d$$

input
```
int((-a+b*x^(1/2*n))^((1-n)/n)*(a+b*x^(1/2*n))^((1-n)/n)*(c+d*x^n)/x^2,x)
```

output
```
int((((x**(n/2)*b + a)**(1/n)*(x**(n/2)*b - a)**(1/n))/(x**n*b**2*x**2 - a*
*2*x**2),x)*c + int((x**n*(x**(n/2)*b + a)**(1/n)*(x**(n/2)*b - a)**(1/n))
/(x**n*b**2*x**2 - a**2*x**2),x)*d
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Exp
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file